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SPACE AND UNEMPLOYMENT: THE LABOUR-MARKET EFFECTS OF SPATIAL MISMATCH

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OF SPATIAL MISMATCH

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ABSTRACT

Space And Unemployment: The Labour-Market Effects Of Spatial Mismatch*

The aim of this Paper is to analyse the effects of suburban housing discrimination on the wages and unemployment rates of black workers. In a duocentric city with efficiency wages, it is shown that, when blacks experience suburban housing discrimination, they face a higher unemployment rate in the central city than in the suburbs, also earning lower wages in the centre. An increase in commuting costs is shown to raise both these disparities, and a number of other results are established. The analysis thus generates a link between housing-market discrimination and a seemingly unrelated phenomenon: unemployment in the labour market. In doing so, the Paper provides new insight into the spatial mismatch hypothesis.

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NON-TECHNICAL SUMMARY

In most metropolitan areas, it is generally observed that the rich and the poor reside in very different areas of cities. In Europe and the US, the concentration of urban problems of ethnic minorities in the poor areas of many cities has intensified over the years and has attracted much attention over the issue of why urban poverty zones appear, how urban problems relate to one another and how they should efficiently be addressed. In the US, for example, there is clear distinction between the rich and generally white suburbs and the poor black downtown. In the UK, the spatially uneven distribution of ethnic minorities is of key significance, with particular concentration in Greater London and other metropolitan areas such as the West Midlands' metropolitan boroughs.

If we focus on the US pattern, then over the last 50 years, American metropolitan areas have undergone an important structural reorganization, with both residences and jobs moving from central cities to suburbs. Suburbanization of the population has been accompanied by job decentralization. Detroit is the most striking case. Over both periods, the number of central-city jobs in Detroit decreased at roughly 1% per year, while suburban jobs increased at a rate of about 3% per year. An important aspect of the suburbanization process is that household decentralization has been notably uneven across races. Black households have been largely left behind in central cities as whites have relocated to the suburbs. Several possible explanations may account for the failure of black households to follow whites in relocating to the suburbs. First, because of unfavourable black income trends, the desire of upwardly mobile households to consume larger dwellings at cheaper unit prices, which spurs suburbanization, may be largely missing for blacks. Another explanation is that suburban housing discrimination may generate a friction that impedes the relocation of those black households that might wish to move. Finally, the unfamiliarity of the predominantly white racial environment found in the suburbs may make blacks reluctant to leave the central city.

A natural question is then whether there is a link between these observed facts, i.e. between the suburbanization of jobs, residential concentration of blacks in central cities and poor labour-market outcomes for these black households. The spatial mismatch hypothesis, first advanced by Kain (1968), gives a positive answer to this question. The hypothesis argues that, because blacks reside in segregated areas distant from and poorly connected to major suburban centres of employment growth, they face strong geographic barriers to finding and keeping well-paid jobs.

Dozens of empirical studies have attempted to test the spatial mismatch hypothesis. The usual approach is to relate a measure of labour-market outcomes, based on either individual or aggregate data, to another measure of job access, typically some index that captures the distance from residences to centres of employment. The weight of the evidence suggests that poor job access indeed worsen labour-market outcomes, confirming the spatial mismatch hypothesis.

Despite the growth of a huge empirical literature, little effort has been spent in exploring the theoretical foundations of the spatial mismatch hypothesis (exceptions include Brueckner and Martin, 1997, Martin, 1997, Arnott, 1998, Coulson, Laing and Wang, 1997, and Anas, 1998)

The aim of this Paper is thus to provide a new theoretical analysis of the labour-market effects of spatial mismatch. The analysis is novel because these labour-market impacts arise from the spatial side of the model, where suburban housing discrimination concentrates black residences near the CBD. Because of the resulting remoteness of the suburban employment centre, the black labour force is skewed toward the CBD labour market and this in turn generates a host of labour-market effects. These include a higher black unemployment rate and a lower wage at the CBD. The analysis thus draws a connection between space and unemployment, providing the first well-rounded theoretical treatment of this important element of the spatial mismatch hypothesis.

The operation of the model is transparent. Because of suburban housing discrimination, the black labour force is skewed toward the central-city labour market. In order to induce central-city firms to absorb the resulting larger labour pool, a lower wage is required. But to prevent shirking, the reduction in work incentives caused by the lower wage must be offset by a higher unemployment rate, which raises the penalty from job termination. Because housing discrimination enlarges the central-city's black labour pool, black unemployment is therefore higher in the centre than in the suburbs, recapitulating one of the key stylized facts underlying the spatial mismatch hypothesis. In reaching this conclusion, the analysis forges a novel link between housing-market discrimination and an apparently unrelated phenomenon: unemployment in the labour market. The strength of this link in turn depends on the friction of space, as reflected in commuting costs. In particular, the analysis shows that the gap between the central and suburban unemployment rates grows as commuting costs rise. As noted above, the analysis also shows that the wages earned by blacks are lower in the central city than in the suburbs, capturing another tenet of the mismatch hypothesis. Overall, the discussion offers a compelling theoretical picture of the effects of spatial mismatch.

Space and Unemployment: The Labor-Market Effects of Spatial Mismatch

by

Jan K. Brueckner and Yves Zenou*

1. Introduction

Over the last fifty years, American metropolitan areas have undergone an important structural reorganization, with both residences and jobs moving from central cities to suburbs. While over 57 percent of urban residents lived in central cities in 1950, this proportion had declined to less than 40 percent by 1980 (Mills and Lubuele (1997)). Despite well-documented examples of central-city revitalization, suburbanization of households continued in the 1990s, with the suburbs growing twice as fast as central cities over the period 1990-1997. In 1996 alone, 2.7 million people moved from a central city to a suburban area, compared with only 800,000 people who moved in the other direction (U.S. Department of Housing and Urban Development, 1999).

Suburbanization of the population has been accompanied by job decentralization. Mills and Lubuele (1997) report that, while central cities contained 70 percent of MSA jobs in 1950, their job share in 1980 was only 50 percent. Table 1 gives more information on this decentralization phenomenon by tabulating the evolution of employment in the largest MSAs (Metropolitan Statistical Areas) over the 1969-1979 and 1979-1987 periods. Detroit is the most striking case. Over both periods, the number of central-city jobs in Detroit decreased at roughly 1 percent per year, while suburban jobs increased at rate of about 3 percent per year. While the contrast is less extreme in other MSAs, job growth in the suburbs outstripped that in the central city in each case. These patterns of change led Mieszkowski and Mills (1993) to state that the U.S. "is approaching the time where only about one-third of the residents within an MSA will live in central cities and only about 40 percent of the MSA jobs will be located there."

An important aspect of the suburbanization process is that household decentralization has

been notably uneven across races. Black households have been largely left behind in central cities as whites have relocated to the suburbs. Although the less than half of the MSA white population lived in the suburbs in 1950, 66 percent of white households resided in suburban communities by 1990. But in both years, more than 70 percent of black MSA households lived in central cities (Mills and Hamilton, 1994). Differences in suburbanization by race can also be seen by comparing the racial makeups of central city and suburban populations, as shown in Table 2 for the major MSAs in 1990. Even though the situation is quite different from city to city (for example in Detroit, 76 percent of central-city residents are black, compared to 5 percent in the suburbs), the black share of the suburban population is never larger than 20 percent.[1]

Several possible explanations may account for the failure of black households to follow whites in relocating to the suburbs. First, because of unfavorable black income trends (see below), the desire of upwardly-mobile households to consume larger dwellings at cheaper unit prices, which spurs suburbanization, may be largely missing for blacks.[2] Another explanation is that suburban housing discrimination may generate a friction that impedes the relocation of those black households that might wish to move. The existence of such segregation is well-documented (see, for example, Yinger (1995, 1997)).[3] Finally, the unfamiliarity of the predominantly-white racial environment found in the suburbs may make blacks reluctant to leave the central city.

Against the backdrop of this evolution in city structure, an additional phenomenon has attracted national attention: persistent poverty among central-city residents, largely concentrated among minority households. While central cities in 1996 contained 30 percent of America's MSA population, they were home to half of all low-income families in MSAs (Current Population Survey, March 1997). This concentration of the poor followed a 50 percent increase in central-city poverty rates between 1970 to 1993, which left 1 in 5 families living in poverty by 1996, compared with fewer than 1 in 10 suburban families. Of these poor families, minority households accounted for 72 percent (U.S. Department of Housing and Urban Development, 1999), a reflection mainly of the higher poverty rate among blacks. This rate was 25 percent among central-city black households in 1990, compared to 7.7 percent among white households

(see Glaeser, Kahn and Rappaport (1999)). Both rates were lower in the suburbs, where 18.8 percent of black households and 5.3 percent of white households were poor in 1990.

These outcomes are partly tied to differences in labor force participation and unemployment, as documented in Table 3. In absolute terms, black labor-force participation is low and unemployment high in the central city, partly explaining the high incidence of poverty among central-city blacks. Both numbers are more favorable for blacks in the suburbs, a pattern that is consistent with the lower incidence of black suburban poverty (18.8 percent vs. 25 percent). However, black unemployment in the suburbs is still double that of suburban whites. In contrast to the central-city/suburb disparity in black labor-force participation and unemployment, the numbers for white households show little difference between the two locations.

A natural question is whether there is a link between these observed facts, i.e., between the suburbanization of jobs, residential concentration of blacks in central cities, and poor labor-market outcomes for these black households. The spatial mismatch hypothesis, first advanced by Kain (1968), gives a positive answer to this question. The hypothesis argues that, because blacks reside in segregated areas distant from and poorly connected to major suburban centers of employment growth, they face strong geographic barriers to finding and keeping well-paid jobs.

Dozens of empirical studies, which are surveyed by Holzer (1991), Kain (1992) and Ihlanfeldt and Sjoquist (1998), have attempted to test the spatial mismatch hypothesis. The usual approach is to relate a measure of labor-market outcomes, based on either individual or aggregate data, to another measure of job access, typically some index that captures the distance from residences to centers of employment. The weight of the evidence suggests that poor job access indeed worsens labor-market outcomes, confirming the spatial mismatch hypothesis.[4]

Despite the growth of a huge empirical literature, little effort has been spent in exploring the theoretical foundations of the spatial mismatch hypothesis. The only existing studies are by Brueckner and Martin (1997) (with a follow-up paper by Martin (1997)), Arnott (1998), Coulson, Laing and Wang (1997), and Anas (1998). In its own way, each study attempts to provide a model that generates predictions consistent with the stylized facts outlined above.

Brueckner and Martin (1997) propose a spatial model where suburban housing discrimination prevents black households from following jobs to the suburbs. By itself, the resulting housing-market distortion leads to lower black welfare, and this outcome is compounded when labor-market effects are added to the analysis. However, use of a neoclassical market-clearing model rules out unemployment, a major focus of the empirical literature, making the model's depiction of spatial mismatch incomplete. The models of Arnott (1998) and Anas (1998) offer less spatial detail, and they also rely on simple labor markets that are incapable of generating unemployment effects as part of spatial mismatch.

The search model of Coulson, Laing and Wang (1997) comes closer to providing insight into some of the labor-market impacts that have concerned empirical researchers. These authors assume that the entry cost of firms is higher in the center than in the suburbs, and that some workers are willing to accept longer commutes than others. These assumptions affect the matching process between firms and workers, leading to a higher unemployment rate for central-city residents than for suburban residents, and the possibility of reverse commuting, with higher wages earned in the suburbs. The main drawback of the model is that there is no specific analysis of blacks versus whites, and the distortion is on the firms' side through a higher entry cost in the CBD.

The purpose of the present paper is to offer a new analysis that provides a simple and comprehensive picture of the effects of spatial mismatch in the labor market. The model, which borrows elements of the Brueckner-Martin (1997) framework, is explicitly spatial, and it realistically concentrates blacks in the central city via the assumption of suburban housing discrimination. To generate unemployment, the labor market is modeled using the efficiency wage framework of Shapiro and Stiglitz (1984), where unemployment serves as a "worker discipline device," keeping employees from shirking on the job.

The operation of the model is transparent. Because of suburban housing discrimination, the black labor force is skewed toward the central-city labor market. In order to induce central-city firms to absorb the resulting larger labor pool, a lower wage is required. But to prevent shirking, the reduction in work incentives caused by the lower wage must be offset by a higher unemployment rate, which raises the penalty from job termination. Because

housing discrimination enlarges the central-city's black labor pool, black unemployment is therefore higher in the center than in the suburbs, recapitulating one of the key stylized facts underlying the spatial mismatch hypothesis. In reaching this conclusion, the analysis forges a novel link between housing-market discrimination and an apparently unrelated phenomenon: unemployment in the labor market. The strength of this link in turn depends on the friction of space, as reflected in commuting costs. In particular, the analysis shows that the gap between the central and suburban unemployment rates grows as commuting costs rise. As noted above, the analysis also shows that the wages earned by blacks are lower in the central city than in the suburbs, capturing another tenet of the mismatch hypothesis. Overall, the discussion offers a compelling theoretical picture of the effects of spatial mismatch.

Section 2 of the paper develops the model and presents the equilibrium conditions. Section 3 derives the main results, including those above. While these results are derived analytically, section 4 provides numerical analysis of the one effect that cannot be derived in this fashion: the impact of spatial mismatch on black welfare. Section 5 offers conclusions.

2. The Model

As explained above, the analytical framework used in the paper comes from combining elements of the spatial model of Brueckner and Martin (1997) (hereafter BM) with the efficiency wage model of Shapiro and Stiglitz (1984). While the basic version of BM's model suppresses the labor market by assuming that incomes are fixed for blacks and whites, integration of the efficiency wage framework makes incomes endogenous and generates unemployment. To understand the resulting structure, it is helpful to first review aspects of the BM model, after which the efficiency wage model is discussed. For other applications of the efficiency wage model in a spatial context, see Zenou and Smith (1995), Smith and Zenou (1997), and Brueckner and Zenou (1999).

2.1. Spatial mismatch with fixed wages

For simplicity, the model focuses on a linear city with unit width. The city is occupied by K white residents, each of whom consumes one unit of land, and N black residents, whose individual land consumption equals θ, where $\theta < 1$. Since the city's employment areas take up

no space, its overall length is then $K + \theta N$. Initially, all jobs are located in an employment center at the left end of the city, denoted the CBD (central business district). Employment decentralization then occurs, as documented in Table 1, with some jobs moving to a suburban business district (SBD) located at the right end of the original city. Residences continue to occupy the same area after formation of the SBD, namely the interval from $x = 0$ to $x = K + \theta N \equiv \overline{x}$, where x denotes distance to the CBD. This simplifying assumption, which means that residences cannot relocate to the right of $\overline{x}$ after formation of the SBD, is justified by imagining that the city is located on an island, with a business district at each end.

As seen below, the assumption of lower black land consumption means that black bid-rent curves, which indicate willingness to pay for land at different locations, are steeper than those of whites. This in turn means that in the original city, the black residential area is closest to the CBD, with whites living farther out. The black area thus extends from $x = 0$ to $x = \theta N$, while the white area extends from $x = \theta N$ to $x = \overline{x}$.

The effect of job decentralization on the residential pattern depends on the extent of suburban housing discrimination against blacks. If blacks are free to live anywhere in the city, then the original residential pattern is replicated around the SBD as jobs decentralize. Blacks working at the SBD live closest to it, with white SBD commuters living outside them (i.e., closer to the CBD). The locations of CBD commuters mimic the original pattern, although over a smaller area. This "unrestricted" residential pattern, along with the associated bid-rent curves, is shown in Figure 1. Note that the bid-rent curves of SBD workers slope upward, while those of CBD workers are downward sloping.

Alternatively, blacks could face housing discrimination in the suburbs, as documented by Yinger (1995, 1997). To draw a sharp distinction from the unrestricted case, the analysis focuses on a situation where housing discrimination is so strong that landlords in the original white part of the city refuse to rent to blacks under any circumstances. In other words, blacks are prevented from living in the interval $[\theta N, \overline{x}]$ regardless of their willingness to pay for land in this area. The resulting "restricted" residential pattern, and the associated bid-rent curves, are shown in Figure 2. Note that in the restricted case, the racial makeups of the central and suburban parts of the city are skewed in favor of one group, reflecting the pattern seen in Table

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2 (the exact racial shares depend on the location of the suburban boundary).

To understand the bid-rent curves in Figure 2, observe that housing discrimination means that whites face no competition for suburban land. Blacks, however, must still outbid whites for land in the central part of the city. Therefore, the black bids in this area must be at least as large as the bids offered by white CBD commuters. This in turn implies that the minimum point of the black bid-rent curves must lie on the extension of the bid-rent curve of white CBD commuters, as shown in the Figure. Beyond this minimum point, which occurs at $x = \widetilde{x}$, Figure 2 shows that a black bid-rent curve slopes upward toward the SBD, indicating that black workers living between $\widetilde{x}$ and θN choose to commute to the SBD despite its remote location. The Figure also shows a dramatic bid-rent discontinuity at $x = \theta N$, with black SBD workers offering much more for land in the white area than the white residents themselves. This discrepancy, which would be unsustainable in a competitive market, is a consequence of discrimination by suburban landlords.

Fixed incomes are assumed in BM's analysis of Figures 1 and 2, with the incomes of whites and blacks at the CBD and SBD equal to constants that take the same value in the unrestricted and restricted cases. However, to generate the commuting patterns shown in the Figures, where each type of worker commutes to both employment centers, the CBD-SBD income differential for each type must not be too great in absolute value. Otherwise, all workers of a given type might favor one employment center over the other. As shown below, such restrictions are unneeded when incomes are endogenous. Under a mild assumption, the only admissible commute patterns in this case are those where whites and blacks work at both employment centers, as in Figures 1 and 2.

To generate the bid-rent curves shown in the Figures, it is assumed that city residents consume a composite good along with land, with the consumption level denoted Z for whites and z for blacks. With land consumption fixed, utility can be represented directly by the level of the composite good. Letting t denote the commuting cost per mile, the budget constraints of white and black CBD commuters are then given by $Z + P + tx = Y_c$ and $z + \theta p + tx = y_c$. In these constraints, Y_c and y_c are the fixed white and black incomes at the CBD, with P and p giving the rents per unit of land paid by whites and blacks (recall that land consumption levels

are fixed at 1 and θ respectively). Rearranging yields the bid-rent curves for white and black CBD commuters, which are written $P = Y_c - tx - Z$ and $p = (y_c - tx - z)/\theta$. These curves give the land rents in different locations consistent with given utility (composite consumption) levels for the groups. Note that since $\theta < 1$, the black bid-rent curve is steeper than the white curve, as noted above (the slopes are $-t/\theta$ and $-t$ respectively).

The bid-rent curves for the SBD workers, which are based on the fixed SBD income levels Y_s and y_s and the reverse distance measure $\overline{x} - x$, are derived similarly. Note that, because workers of a given type must receive the same utility regardless of where they are employed, the utility levels Z and z in these formulas do not have CBD and SBD indexes.

The bid-rent curves can be used to solve for the equilibrium shown in Figure 2. The equilibrium conditions require that (i) the black bid-rent curves intersect at $\widetilde{x}$ (the CBD-SBD commute boundary for black workers); (ii) the white bid-rent curves intersect each other as well as the horizontal axis at x^* (the latter requirement reflects a zero opportunity cost for land); (iii) the bid-rent curve of white CBD commuters intersects the black bid-rent curves at $\widetilde{x}$. These conditions determine $\widetilde{x}$, x^*, and the utility levels Z and z for the two types.[5] The $\widetilde{x}$ and z solutions are most relevant in the ensuing analysis, and they are given by

$$\widetilde{x} = (\Delta y + t\overline{x})/2t \tag{1}$$

$$z = [(1+\theta)y_c + (1-\theta)y_s - \theta\Delta Y - t\overline{x}]/2, \tag{2}$$

where $\Delta Y = Y_c - Y_s$ and $\Delta y = y_c - y_s$ denote the white and black CBD-SBD income differentials. Note that the black CBD-SBD commute boundary $\widetilde{x}$ diverges from the midpoint $\overline{x}/2$ of the city (where commuting costs to the two centers are equal) by a term that depends on the black income differential between the centers.[6]

A similar procedure can be used to solve for the unrestricted equilibrium shown in Figure 1. The black utility solution, which is relevant below, is given by

$$\widehat{z} = [y_c + y_s - t\theta(N + K)]/2. \tag{3}$$

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When the appropriate restrictions on incomes are imposed, (1) is less than (3), indicating that black utility is lower in the restricted case. This outcome shows the harmful effect of the housing-market distortion caused by suburban discrimination, abstracting from any labor-market effects of this discrimination.

To gauge the overall impact of spatial mismatch on black welfare, labor-market effects must be incorporated in the model, as is done below. However, incomes then become endogenous, and they differ between the restricted and unrestricted cases (i.e., y_c takes different values in (2) and (3), as does y_s). This precludes an analytical comparison of utilities in the two cases, requiring the use of simulation analysis instead (see Section 4).[7] However, the main results of the paper, which concern the effect of spatial mismatch on unemployment, are derived analytically, as seen in Section 3.

2.2. Incorporating efficiency wages

The next step is to integrate the spatial model with the efficiency wage framework in order to explore the effect of spatial mismatch on labor-market outcomes. A number of assumptions are imposed in order to carry out this integration in a tractable fashion. First, it is assumed that the urban economy has two types of jobs, one requiring high skills and the other low skills. Shirking is possible in the low-skill job, while high-skill workers do not have the opportunity to shirk. As a result, efficiency wages must be paid to prevent shirking by low-skill workers, while high-skill wages are determined by the usual marginal productivity conditions. Finally, because poor neighborhood schools and low family incomes impede accumulation of human capital, black workers are all assumed to have low skills, while all white workers have high skills. Together, the above assumptions yield the key implication that black workers are paid efficiency wages while white workers are not. Thus, black workers in model are doubly disadvantaged by housing-market discrimination and by the need to work at jobs where efficiency wages, and the associated unemployment, are required to maintain worker effort.

Although this structure is meant to reproduce the stylized facts of high black and low white unemployment, it could also be assumed that efficiency wages are paid to white workers, but that different job characteristics allow deterrence of shirking with relatively low white unemployment levels.[8] In any case, it should be recognized that the potential for shirking is

a characteristic of jobs and not a racial attribute.

The second assumption is that, for the purposes of determining efficiency wages, the CBD and SBD represent separate labor markets. Implicitly, it is assumed that when a black CBD worker is laid off, he searches for his next job at the CBD, not at the SBD. Analogous behavior applies to unemployed SBD workers. This behavior could reflect the accumulation of human capital that is specific to jobs at a given employment center, ruling out an easy switch to the other center's labor market. Alternatively, lack of familiarity with employers at the other center may reduce the effectiveness of job search there, causing the worker to remain attached to the center where he was laid off.[9] With the CBD and SBD constituting separate labor markets, each center has its own unemployment rate and labor pool.

Since the above obstacles to switching between employment centers should become less important as the time horizon lengthens, the CBD and SBD labor markets ultimately should be equally attractive to black workers. Thus, despite being attached to a particular center's labor market in the short run, workers should be indifferent between centers in the long-run equilibrium. The utilities of black workers attached to the CBD and SBD should therefore be equal. Since this requirement is already built into the spatial model, no additional steps are needed to incorporate it.

The distinction between employed and unemployed black workers creates further heterogeneity in a model that already distinguishes individuals by their place of work. In order to keep the analysis manageable, this heterogeneity is collapsed by imposing a third assumption, namely that black workers engage in income smoothing as they cycle in and out of unemployment. Thus, black workers save while employed and draw down their savings when out of work, with their consumption expenses reflecting "permanent income." This means that all black workers attached to a given center have identical disposable incomes, equal to the average income over the job cycle.[10] The exact form of this average income is derived below.

The fourth assumption, which concerns the transportation costs of unemployed workers, is also designed to limit heterogeneity in the model. The assumption derives from the approach of Zenou and Smith (1995), who assume that unemployed workers incur transportation costs α times as large as the commuting costs of employed workers, where $\alpha \leq 1$. These costs

capture the cost of job search at the CBD as well as the cost of CBD shopping trips, which are combined with commute trips by employed workers. To make the present analysis manageable, the assumption $\alpha = 1$ is required, indicating that unemployed workers travel to the CBD just as frequently as those who are employed, carrying out job search, shopping, and other nonwork activities. Under this assumption, all workers residing at a given location incur the same transportation cost regardless of their employment status. Since these individuals also pay the same land rent, all location-related costs are invariant to employment status, as is income.

To develop the implications of these assumptions, further explanation of the efficiency wage model of Shapiro and Stiglitz (1984) is required. In the model, a worker can expend an effort level of $e > 0$ or shirk, which means setting effort at zero. Effort is viewed as generating an explicit dollar cost for the worker, so that a nonshirker's job income is equal to the wage minus e.[11] The firm monitors its work force, catching shirkers with a fixed probability m, and apprehended shirkers are fired, earning a zero unemployment benefit. Since shirkers contribute nothing to the employer's output, the firm offers incentives to make shirking unattractive. This involves setting a wage high enough so that, given the prevailing level of unemployment, the loss from being caught and fired offsets the worker's cost of exerting effort.

In addition to being fired for shirking, workers also face the threat of layoff because of exogenous job turnover, which occurs with probability v per period. Taking this additional factor into account, the wage that is sufficient to deter shirking is given by

$$w \ = \ e \ + \ \frac{e}{m}\left[\frac{v}{u} \ + \ r\right], \tag{4}$$

where r is the discount rate and u is the unemployment rate (see Shapiro and Stiglitz (1984)). Inspection of (4) shows that the efficiency wage is an increasing function of effort e, the job separation (turnover) rate v, and the discount rate r. The wage is also decreasing in monitoring efficiency m and the unemployment rate u. Note that an increase in either of the latter variables raises the potential loss from shirking, allowing the firm to pay a lower wage while still eliciting effort from the worker. Note also that, while transportation costs (which help determine net income) implicitly appear on both sides of (4), these costs cancel since they are the same for employed and unemployed workers.[12]

With shirking made unattractive, the unemployment occurring in the efficiency wage model is transitory, being the result of an exogenous flow of job separations combined with rehiring of laid-off workers.[13] The average income of a worker over the resulting employment cycle can be computed using the formulas of Shapiro and Stiglitz, and it equals $y = (1 - u)(w - e)$.[14] This value equals the constant level of income generated by the income-smoothing process discussed above, and as such, it equals the common income enjoyed by all workers, regardless of employment status. Note that while y gives the average intertemporal income, it also equals a worker's *expected income* at a given point in time (recall the unemployment benefit equals zero). Below, y is indexed by employment center.

The next step is to note that firms adjust employment until the marginal product of an additional worker equals the efficiency wage. Each firm views the unemployment rate upon which the efficiency wage depends as parametric and uninfluenced by its input choice. Letting $F(\cdot)$ denote the strictly concave aggregate production function and L denote total labor input, L then satisfies $F'(L) = w$, where w is given by (4). Since $L = (1 - u)n$, where n denotes the size of the labor pool, the previous equality can be written

$$F'[(1 - u)n] \;=\; e \;+\; \frac{e}{m}\left[\frac{v}{u} \;+\; r\right]. \tag{5}$$

Eq. (5) determines the equilibrium unemployment rate. Note that a high u decreases the efficiency wage, encouraging the firm to hire more workers, while at the same time decreasing the number of workers presumed to be employed. Equilibrium is achieved when these two effects are in balance.

Before applying (5) to the present model, a fifth assumption is needed. In particular, the aggregate production function, which is the same at the CBD and SBD, is assumed to be additively separable in black and white labor. Thus, in addition to the assumption that high and low-skill labor (provided by whites and blacks respectively) are distinct inputs, separability ensures that the marginal product of black labor is independent of the white labor input and vice versa. A further technical assumption is that the marginal product of each type of labor is infinite at a zero input level. This assumption ensures that, in any stable equilibrium, both blacks and whites must work at each employment center, as in Figures 1 and 2 above.[15]

Let $F(\cdot)$ from above represent the black portion of the production function. Then, recalling that the CBD and SBD represent separate labor markets, condition (5) applies separately to each market, yielding

$$F'[(1-u_c)N_c] \;=\; e \;+\; \frac{e}{m}\left[\frac{v}{u_c} \;+\; r\right] \tag{6}$$

$$F'[(1-u_s)N_s] \;=\; e \;+\; \frac{e}{m}\left[\frac{v}{u_s} \;+\; r\right], \tag{7}$$

where u_c, u_s, N_c, and N_s are the unemployment rates and labor pools for the CBD and SBD labor markets. Next, using the above income expression, incomes at the two centers are given by $y_c = (1-u_c)(w_c - e)$ and $y_s = (1-u_s)(w_s - e)$. Using (4) to eliminate w_c and w_s from these equations, they can be rewritten as

$$y_c \;=\; \frac{e}{m}\left[\frac{v(1-u_c)}{u_c} \;+\; r(1-u_c)\right] \tag{8}$$

$$y_s \;=\; \frac{e}{m}\left[\frac{v(1-u_s)}{u_s} \;+\; r(1-u_s)\right]. \tag{9}$$

The above approach allows efficiency wages to be embedded in the previous spatial model without changing its structure. The previous black incomes y_c and y_s, which were fixed exogenously, are now replaced by the endogenous expected incomes from (8) and (9), which depend on the unemployment rates. These equations are supplemented by (6) and (7), which determine the unemployment rates conditional on the labor-pool sizes. Then, the resulting system is augmented by previous equilibrium conditions from the spatial model, which determine the labor pools as functions of income levels. The first two of these equations relate N_c and N_s to the location of the CBD-SBD commute boundary $\widetilde{x}$ in Figure 2. Recalling that black land consumption equals θ, these conditions are

$$N_c \;=\; \widetilde{x}/\theta \tag{10}$$

$$N_c \;+\; N_s \;=\; N. \tag{11}$$

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The final equilibrium condition is the previous equation (1), which relates the commute boundary location to the black CBD-SBD income differential:

$$\widetilde{x} \;\; = \;\; (\Delta y \; + \; t\overline{x})/2t. \tag{12}$$

Eqs. (6)–(12) constitute seven conditions that determine equilibrium values for the variables u_c, u_s, y_c, y_s, N_c, N_s, and $\widetilde{x}$. Note that unlike in the standard efficiency wage model, the size of the labor pool at each employment center is endogenous, rather than exogenous. Once the equilibrium is determined, the black utility level can be found by substituting y_c and y_s into the utility expression (2) from above. However, since black utility also depends on ΔY, the CBD-SBD income differential for whites, this computation requires a prior determination of the equilibrium in the separate white labor markets.

It is useful to note how the above conditions would be affected if the production function were nonseparable, eliminating the separation of the black and white labor markets. In this case, the number of white CBD workers (K_c) would be an argument of the black marginal product expression on the LHS of (6), while the analogous variable K_s would appear in (7). In addition, white wages at the two centers would equal white marginal products, which would depend at each center on the inputs of both types of labor. Finally, K_c would equal $x^* - \theta N$ (see Figure 2), while K_s would equal $\overline{x} - x^*$, with x^* itself depending on the white income differential ΔY between the centers (see footnote 6). When these additional conditions are added to the above equilibrium conditions, the resulting complexity prevents derivation of many of the ensuing results. One key result, however, is unaffected, as noted below.

The above discussion has shown that integration of the spatial model with the efficiency wage framework leads to a model that incorporates (*i*) job decentralization, (*ii*) suburban housing discrimination, and (*iii*) endogenous unemployment at both centers. Analysis of the resulting equilibrium is carried out in the next section.

3. Analysis of the Restricted Equilibrium

As seen in Figure 2 and the above equations, the restricted equilibrium is fundamentally asymmetric, a consequence of the asymmetric locations of the residential areas of black and

white workers. This feature of the equilibrium can be expected to lead to asymmetric solutions for the endogenous variables at the CBD and SBD. These variables include the unemployment rate, wage, and expected income, along with the sizes of the labor pools at the two centers and their levels of total employment. The question is whether the resulting asymmetry confirms a widely held view in the literature on spatial mismatch, namely that mismatch leads to inferior labor-market outcomes for workers attached to the CBD. The following results show that this view is indeed confirmed by the model.

Proposition 1. *For black workers, the CBD has a higher unemployment rate, a lower wage and expected income, a larger labor pool, and a higher employment level than the SBD. In other words,*

$$u_c > u_s \tag{13}$$

$$w_c < w_s \tag{14}$$

$$y_c < y_s \tag{15}$$

$$N_c > N_s \tag{16}$$

$$(1 - u_c)N_c > (1 - u_s)N_s. \tag{17}$$

This and subsequent propositions are proved in the appendix.

Proposition 1 shows that workers attached to the CBD experience worse labor-market outcomes than SBD workers. These workers, who outnumber those attached to the SBD, experience a higher unemployment rate, a lower wage, and a lower expected income. As stressed in the introduction, the gap between CBD and SBD unemployment rates generated by the model mirrors the real-world pattern. As shown in Table 3, unemployment in 1997 among central-city blacks averaged 12.5 percent, while the unemployment rate for black suburban workers was a much-lower 7.6 percent.[16] The CBD-SBD wage differential implied by the model, where black workers earn less at the CBD, also conforms to existing evidence (see, for example, Price and Mills (1981)).

To better understand the sources of these disparities, it is helpful to state a second set of results related to Proposition 1. These results provide a comparison between the restricted

equilibrium and the unrestricted case in Figure 1, where suburban housing discrimination is absent. In effect, the results show that unrestricted labor-market outcomes represent an intermediate case that lies between the CBD and SBD outcomes in the restricted equilibrium.

The key to this comparison is the recognition that the CBD and SBD labor markets are symmetric in the unrestricted case, a consequence of identical production functions at the two centers and freedom of residential location for all workers. Since each center's labor pool thus contains half of the black population, the common unemployment rate, denoted $\widehat{u}$, is determined by the condition

$$F'[(1-\widehat{u})N/2] \;\;=\;\; e \;+\; \frac{e}{m}\left[\frac{v}{\widehat{u}} \;+\; r\right] \tag{18}$$

The following conclusions then apply:

Proposition 2. *The common unemployment rate $\widehat{u}$ at the two centers in the unrestricted equilibrium lies between the CBD and SBD unemployment rates in the restricted equilibrium, so that*

$$u_c \;\;>\;\; \widehat{u} \;\;>\;\; u_s. \tag{19}$$

The same conclusion applies to the common wage $\widehat{w}$, expected income $\widehat{y}$, labor pool $N/2$, and employment level $(1-\widehat{u})N/2$ at the two centers in the unrestricted equilibrium, each of which lies between the CBD and SBD values in the restricted equilibrium.

The results in Propositions 1 and 2 are noteworthy. They show that discrimination in the suburban housing market, which prevents black workers from living near the SBD, affects labor-market outcomes at the two centers. The wage falls and unemployment rises at the CBD relative to the unrestricted case, while the reverse effects occur at the SBD. Thus, the results provide theoretical confirmation of a central claim in the spatial mismatch literature, namely that the locational mismatch between jobs and minority residences leads to lower wages and higher unemployment for workers who remain attached to the CBD labor market.

It is especially striking that the results generate a link between housing-market discrimination and a seemingly unrelated phenomenon: unemployment in the labor market. Propositions 1 and 2 show that these phenomena are connected, a novel finding that is absent in previous

theoretical work on spatial mismatch. The intuitive explanation for this connection is that, by keeping black residences in close proximity to the CBD (and remote from the SBD), suburban housing discrimination enlarges the black CBD labor pool relative to the SBD pool. Because of the structure of the efficiency wage model, this enlargement in turn raises the CBD unemployment rate while lowering the rate at the SBD. This outcome can be seen by returning to the basic equation (5) of the general efficiency wage model. Differentiating (5) with respect to n shows the effect of a larger labor pool on the unemployment rate u. The result is

$$\frac{\partial u}{\partial n} = \frac{(1-u)F''}{nF'' - ev/mu^2} > 0, \tag{20}$$

which shows that a larger pool raises the unemployment rate. The intuition is that as n rises, the wage must fall to encourage firms to absorb more workers, and this requires a larger u from (4). Note that while the required reduction in the wage reduces the incentive to put forth effort on the job, the higher unemployment rate raises the penalty from job termination, maintaining the incentive against shirking. When combined with the mismatch-induced expansion of the CBD labor pool, and the offsetting contraction of the SBD pool, the effect in (20) accounts for the unemployment-rate differential between the two centers.

It should be noted that this unemployment effect persists under more general assumptions. In particular, if the production function, instead of being separable in white and black labor, is nonseparable but homothetic, then it can be shown that $u_c > \widehat{u} > u_s$ continues to hold. Interestingly, however, the results in (14)–(16) for the other endogenous variables do not necessarily carry over to this more general case.

Although spatial mismatch raises the unemployment rate among black workers attached to the CBD, what can be said about the effect on the total number of unemployed black workers in the city? The total number of unemployed, denoted U, is equal to

$$U = u_c N_c + u_s N_s. \tag{21}$$

In order to compare the value of U in the unrestricted equilibrium to the value in the restricted case, let N_c and N_s be replaced by $N/2 + \lambda$ and $N/2 - \lambda$, where $\lambda > 0$ given (16). Substituting

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these expressions in place of N_c and N_s, (6) and (7) then determine u_c and u_s as functions of λ, with the unrestricted case corresponding to $\lambda = 0$. Total unemployment in (21) can then be written

$$U(\lambda) \ = \ u_c(\lambda)(N/2 \ + \ \lambda) \ + \ u_s(\lambda)(N/2 \ - \ \lambda). \tag{22}$$

Using this approach, it can be shown that spatial mismatch raises total black unemployment in the city relative to the unrestricted case, in the following sense:

Proposition 3. *The relationships $U'(0) = 0$ and $U''(0) > 0$ hold, indicating that total unemployment reaches a local minimum at $\lambda = 0$, provided that the elasticity of F'' with respect to its argument exceeds or equals -2.*

The elasticity condition on F in Proposition 3 is weak, being satisfied by common functional forms such as the log function and power function. When the condition holds, the Proposition shows that the change from a zero to positive value of λ, which corresponds to a movement from the unrestricted to the restricted equilibrium, is likely to raise total unemployment in the city. However, because Proposition 3 says that $\lambda = 0$ represents a *local* minimum for total unemployment, the higher U is guaranteed only if the divergence between N_c and N_s is not too large. Otherwise, the implied λ may be far enough from zero that the local result ceases to hold. While it is not possible to prove that U is larger for all positive values of λ, the simulation analysis presented in Section 4 shows the local statement in Proposition 3 may be robust. The simulation shows the Proposition correctly predicts an increase in total unemployment in the cases considered.

The severity of the distortion caused by suburban housing discrimination depends on the magnitude of the commuting cost parameter t. A large value for this parameter effectively increases the remoteness of the SBD from the black residential area. Intuition suggests that this should increase the extent to which the black labor force is skewed toward the CBD, amplifying the disparities between the two labor markets documented in Proposition 1. This conclusion is proved by comparative-static analysis of the restricted equilibrium, which establishes the following results:

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Proposition 4. *An increase in the commuting cost parameter t leads to a higher unemployment rate at the CBD and a lower rate at the SBD, with*

$$\frac{\partial u_c}{\partial t} \; > \; 0, \qquad \frac{\partial u_s}{\partial t} \; < \; 0. \tag{23}$$

Wages and expected incomes show corresponding changes, falling at the CBD and rising at the SBD, while the labor pools and total employment rise at the CBD and fall at the SBD.

As conjectured, Proposition 4 shows that the divergence in unemployment rates, wages, and expected incomes between the two centers, as documented in Proposition 1, is more pronounced when the cost of commuting is high. Thus, the effects of spatial mismatch are magnified when the friction of space, as captured in the t parameter, is more substantial.

Although Proposition 4 shows the effect of reducing the common level of transport costs for *all* workers in the city, it echoes a recommendation of a number of studies in the mismatch literature: cutting the transport costs of central-city residents, especially blacks, as a means of easing the effects of spatial mismatch (see Ihlanfeldt and Sjoquist (1998) and Pugh (1998)).[17] This recommendation grows out of the observation that public-transit access from downtown to many suburban locations is inconvenient or impossible, and that public investment to improve access may lead to better labor-market outcomes for central-city residents. The model supports this idea, showing that by decreasing t, an improvement in the transportation network reduces the gap between CBD and SBD unemployment rates.[18]

Finally, it is interesting to note that, in the present model, an increase in the required effort level e need not have usual effect on unemployment rates. Although an increase in e raises u in the standard efficiency wage framework (this follows from differentiation of (5)), the effect of such a change on u_c and u_s in the present setting is ambiguous.

4. Simulation Analysis

Despite the different labor-market outcomes at the CBD and SBD, black workers attached to the different centers enjoy the same (expected) utility, as discussed above. Their common utility level in the restricted equilibrium is given by (2), while their utility in the unrestricted case is given by (3). Because the values of y_c and y_s differ between the restricted and un-

restricted cases, and because the utility expressions for the two cases are not nested in the sense that one can be derived from the other by changing a parameter value, the only way to compare the utility magnitudes is via simulation analysis. Since such an analysis must be based on a variety of additional assumptions, the results it generates are at best suggestive.

In order to avoid consideration of the white labor markets, which generate the ΔY term in (2), it is assumed that the marginal product of white labor equals a constant. This assumption implies that CBD and SBD wages are equal for whites, yielding $\Delta Y = 0$. In addition, the function F is assumed to be quadratic, so that F' in (5) is given by the linear function $\delta - \beta(1 - u)n$. Note that both these functional-form assumptions are inconsistent with the previous requirement that white and black marginal products are infinite at a zero input level. However, the simulations are unworkable using functional forms that embody this assumption.

Parameter values in the base case are as follows. The production parameters δ and β are set at 15 and 1 respectively. The black population size N is set at 20 (which could represent units of 10 or 100 thousand to be realistic), and the black land consumption parameter θ equals 0.5. These values imply that the black residential area extends out to $x = 10$. The white population is equal to 11, implying that $\overline{x} = 21$. Since $\Delta Y = 0$ implies that x^* in Figure 2 equals $\overline{x}/2$ or 10.5, it follows that whites living in the x interval [10, 10.5] commute to the CBD, with the remainder commuting to the SBD. The commuting cost parameter t is set at 0.038, effort e equals 1.0, the discount rate r equals 0.05, the separation rate v equals 0.10, and monitoring efficiency m is 0.10.

The first two columns of Table 4 show the unrestricted and restricted solutions under the base-case parameter values. Inspection of the numbers shows that the results satisfy the inequalities (13)–(17) of Proposition 1 as well as conforming to Proposition 2. The results on total unemployment also affirm Proposition 3 in that the value of U is slightly higher in the restricted case, as predicted. It should be noted that the CBD-SBD differentials for most of the variables are not dramatic in size, an outcome that is probably due to the use of a linear form for F'. If a nonlinear F' with an infinite value at zero could have been used instead, the differentials would likely have been more dramatic.

The numbers in first row of Table 4 constitute the main reason for carrying out the sim-

ulation analysis. They show that black utility is lower in the restricted equilibrium than in the unrestricted case, indicating that the welfare impact of spatial mismatch is negative for blacks.[19] Brueckner and Martin (1997) establish this result analytically for the fixed-wage model and numerically for an endogenous-wage model without unemployment (see also Martin (1997)). However, there has been no previous welfare analysis of a model with unemployment.

The third column of Table 4 shows the effect of increasing t from 0.038 up to 0.057. A comparison of the second and third columns confirms the results of Proposition 4. The last column of the Table shows the effect of raising e from 1.0 to 3.0. Although the effect on u_c and u_s is ambiguous analytically, as noted above, the simulation results show large increases in the unemployment rates, as in the standard model. The other numbers show that a higher e has striking effects on the levels of the variables.

5. Conclusion

This paper has provided a new theoretical analysis of the labor-market effects of spatial mismatch. The analysis is novel because these labor-market impacts arise from the spatial side of the model, where suburban housing discrimination concentrates black residences near the CBD. Because of the resulting remoteness of the suburban employment center, the black labor force is skewed toward the CBD labor market, and this in turn generates a host of labor-market effects. These include a higher black unemployment rate and a lower wage at the CBD. The analysis thus draws a connection between "space" and unemployment, providing the first well-rounded theoretical treatment of this important element of the spatial mismatch hypothesis.

Further work could be devoted to development of other models that deepen our understanding of spatial mismatch. An important element missing from the current framework is job search, and further work could attempt to integrate the search process into a spatial model that also incorporates suburban housing discrimination, following the lead of Coulson et al. (1997). Another exercise could explore the effect of a gradual weakening of suburban discrimination within the current framework. This phenomenon could be modeled as a gradual shrinkage of the area where discrimination occurs, which would contract toward the SBD.

While such an exercise would be complex, it would offer a useful picture of the transition to an urban economy where spatial mismatch is absent.

Appendix

Proof of Proposition 1: The proof is by contradiction. Suppose contrary to (15) that $\Delta y = y_c - y_s \geq 0$. Then, since the expressions on the RHS of (8) and (9) are decreasing in the respective unemployment rates, it follows that $u_c \leq u_s$. But since the LHS expressions in (6) and (7) are also decreasing in the u's, this implies that $w_c = F'[(1-u_c)N_c] \geq F'[(1-u_s)N_s] = w_s$ must hold. From strict concavity of F, it then follows that $(1-u_c)N_c \leq (1-u_s)N_s$. However, given $u_c \leq u_s$, the only way the previous inequality can hold is if $N_c \leq N_s$. The next step is to note from (12) that $\widetilde{x} \geq \overline{x}/2$ must hold given $\Delta y \geq 0$. Using the two previous inequalities, it follows that the land area taken up by blacks, which equals $\widetilde{x} + \theta N_s$, satisfies $\widetilde{x} + \theta N_s \geq \widetilde{x} + \theta N_c = 2\widetilde{x} \geq 2(\overline{x}/2) = \overline{x}$, where the first equality uses (10). This conclusion, however, says that land area occupied by blacks equals or exceeds the area of the city. The resulting contradiction invalidates the initial assumption that $\Delta y \geq 0$. Thus, $\Delta y < 0$ must hold instead, which reverses all of the relevant inequalities above, establishing the Proposition. ∎

Proof of Proposition 2: To establish (19), suppose that $\widehat{u} \geq u_c$ holds. Then, noting that (16) along with (11) implies $N_c > N/2 > N_s$, it follows that $(1-\widehat{u})N/2 < (1-u_c)N_c$ holds. But, given $F'' < 0$, this inequality implies that the RHS of (6) exceeds the RHS of (18), contradicting the assumption that $\widehat{u} \geq u_c$. Since $u_s \geq \widehat{u}$ leads to a similar contradiction, (19) follows. The results on $\widehat{w}$, $\widehat{y}$, and $(1-\widehat{u})N/2$ then follow directly. ∎

Proof of Proposition 3: Note from (22) that

$$U'(\lambda) = u_c - u_s + \frac{N}{2}\left[\frac{\partial u_c}{\partial \lambda} + \frac{\partial u_s}{\partial \lambda}\right] + \lambda\left[\frac{\partial u_c}{\partial \lambda} - \frac{\partial u_s}{\partial \lambda}\right]. \tag{a1}$$

Since $u_c = u_s = \widehat{u}$ when $\lambda = 0$, and since $\partial u_c/\partial\lambda$ and $\partial u_s/\partial\lambda$ are equal and opposite in sign in this case, it follows that $U'(0) = 0$. Differentiating (a1) and evaluating the result at $\lambda = 0$ yields

$$U''(0) = 2\left[\frac{\partial u_c}{\partial \lambda} - \frac{\partial u_s}{\partial \lambda}\right] + \frac{N}{2}\left[\frac{\partial^2 u_c}{\partial \lambda^2} + \frac{\partial^2 u_s}{\partial \lambda^2}\right]. \tag{a2}$$

The derivative in (20) (with $n = N/2$) gives $\partial u_c/\partial\lambda$ evaluated at $\lambda = 0$, which in turn equals $-\partial u_s/\partial\lambda$. Eq. (20) is also used to compute the second derivatives in (a2), which are equal

when evaluated at $\lambda = 0$. After substitution of the resulting expressions, extensive and tedious manipulations show that $(a2)$ reduces to an expression with the sign of

$$\frac{ev}{m\widehat{u}}(1 \; + \; \epsilon/2) \; - \; (N/2)F'', \tag{a3}$$

where $\epsilon = F'''[(1 - \widehat{u})N/2]/F''$ is the elasticity of F'' evaluated at $\lambda = 0$. The Proposition follows from inspection of $(a3)$. ∎

Proof of Proposition 4: The first step is to use (10)–(12) to eliminate N_c and N_s in (6) and (7), which allows these equations to be written

$$\Gamma \;\; = \;\; F'\left[(1 - u_c)\frac{\Delta y + t\overline{x}}{2\theta t}\right] \; - \; e \; - \; \frac{e}{m}\left[\frac{v}{u_c} + r\right] \;\; = \;\; 0 \tag{a4}$$

$$\Phi \;\; = \;\; F'\left[(1 - u_s)\left(N - \frac{\Delta y + t\overline{x}}{2\theta t}\right)\right] \; - \; e \; - \; \frac{e}{m}\left[\frac{v}{u_s} + r\right] \;\; = \;\; 0, \tag{a5}$$

where

$$\Delta y \;\; = \;\; \frac{e}{m}\left[\frac{v(1 - u_c)}{u_c} - \frac{v(1 - u_s)}{u_s} + r(u_s - u_c)\right]. \tag{a6}$$

Differentiating $(a1)$ and $(a2)$ yields the following results (subscripts denote partial derivatives):

$$\Gamma_{u_c} \;\; = \;\; -\frac{F''}{2\theta t}\left[\Delta y \; + \; t\overline{x} \; + \; \frac{(1 - u_c)e}{m}\left(\frac{v}{u_c^2} + r\right)\right] \; + \; \frac{ev}{mu_c^2} \tag{a7}$$

$$\Gamma_{u_s} \;\; = \;\; \frac{F''}{2\theta t}\frac{(1 - u_c)e}{m}\left(\frac{v}{u_s^2} + r\right) \tag{a8}$$

$$\Gamma_t \;\; = \;\; -\frac{F''}{2\theta t^2}(1 - u_c)\Delta y \tag{a9}$$

$$\Phi_{u_c} \;\; = \;\; \frac{F''}{2\theta t}\frac{(1 - u_s)e}{m}\left(\frac{v}{u_c^2} + r\right) \tag{a10}$$

$$\Phi_{u_s} \;\; = \;\; -\frac{F''}{2\theta t}\left[2\theta tN \; - \; \Delta y \; - \; t\overline{x} \; + \; \frac{(1 - u_s)e}{m}\left(\frac{v}{u_s^2} + r\right)\right] \; + \; \frac{ev}{mu_s^2} \tag{a11}$$
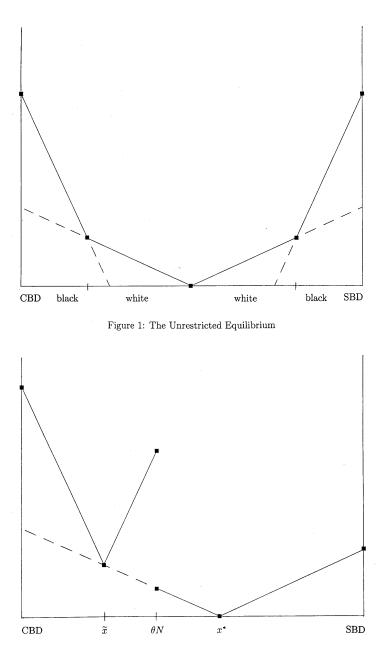
$$\Phi_t \;\; = \;\; \frac{F''}{2\theta t^2}(1 - u_s)\Delta y. \tag{a12}$$

Application of Cramer's rule then yields

$$\frac{\partial u_c}{\partial t} = \frac{\Gamma_{u_s}\Phi_t - \Gamma_t\Phi_{u_s}}{\Gamma_{u_c}\Phi_{u_s} - \Gamma_{u_s}\Phi_{u_c}} \qquad (a13)$$

$$\frac{\partial u_s}{\partial t} = \frac{\Gamma_t\Phi_{u_c} - \Gamma_{u_c}\Phi_t}{\Gamma_{u_c}\Phi_{u_s} - \Gamma_{u_s}\Phi_{u_c}}. \qquad (a14)$$

Using $(a7)$–$(a12)$, tedious but routine computations show that the denominator expression in $(a13)$ and $(a14)$ is positive, which establishes stability of the equilibrium. Similar calculations show that the numerator expression in $(a13)$ is positive, yielding $\partial u_c/\partial t > 0$, and that the numerator expression in $(a14)$ is negative, yielding $\partial u_s/\partial t < 0$. The remainder of Proposition 4 follows from (6)–(9). ∎
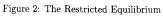
CBD   black   white   white   black   SBD

Figure 1: The Unrestricted Equilibrium

CBD $\widetilde{x}$ θN x^* SBD

Figure 2: The Restricted Equilibrium

Table 1.

Annual Rates of Net Employment Change

	1969–1979		1979-1987	
	Central City	Suburbs	Central City	Suburbs
New York	−1.3	2.2	1.2	2.9
Chicago	0.4	3.5	0.3	3.5
Philadelphia	−2.0	−2.2	−0.2	2.8
Los Angeles	2.3	6.9	2.0	4.3
Atlanta	2.1	5.2	2.1	7.3
Boston	−0.3	2.1	1.6	3.0
Dallas	3.7	5.3	3.7	7.5
Detroit	−0.6	3.7	−1.6	2.5
Washington	0.4	3.9	0.9	4.8

[Source: Stanback (1991)]

Table 2.

Black Population Shares in Central Cities and Suburbs, 1990

	Percent Black in Central City	Percent Black in Suburbs
New York	29	12
Chicago	39	7
Philadelphia	40	9
Los Angeles	14	9
Atlanta	67	19
Boston	26	2
Dallas	30	7
Detroit	76	5
Washington	66	19

[Source: The State of the Nation's Cities, Version 2.2a (online database)]

Table 3.

Labor-Market Outcomes in the 25 Largest Cities and Their Suburbs, 1997

| | *Labor force participation rate* | | *Unemployment rate* | |
	Central City	Suburbs	Central City	Suburbs
Total	64.6	69.6	7.3	4.0
White	66.2	69.8	5.5	3.7
Black	60.2	73.3	12.5	7.6
Hispanic origin	64.3	71.3	8.1	6.1

[Source: Current Population Survey]

Table 4.
Simulation Results

Equilibrium:

Variable:	Unrestricted (base case)	Restricted (base case)	Restricted (higher t)	Restricted (higher e)
utility	4.476	4.273	4.029	2.958
u_c	0.186	0.193	0.197	0.539
u_s	0.186	0.180	0.176	0.507
U	3.720	3.740	3.740	10.480
w_c	6.864	6.668	6.577	10.068
w_s	6.864	7.068	7.167	10.414
y_c	4.771	4.571	4.478	3.260
y_s	4.771	4.978	5.079	3.653
N_c	10.000	10.331	10.489	10.692
N_s	10.000	9.669	9.511	9.308
$(1 - u_c)N_c$	8.136	8.332	8.423	4.932
$(1 - u_s)N_s$	8.136	7.932	7.833	4.586

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Footnotes

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[1]Another way of seeing the spatial concentration of blacks in central cities is through the "dissimilarity" index, which measures the share of the black population that would need to relocate in order to match the spatial distribution of the white population. According to Cutler, Glaeser and Vigdor (1999), this index increased from 72 percent in 1940 to 79 percent in 1970, reflecting the increasing spatial isolation of the black population.

[2]See Mieszkowski and Mills (1993) for an explanation of this force.

[3]Discrimination in mortgage markets may be a compounding factor. See Ladd (1998) for a survey of the evidence.

[4]One of the most striking empirical confirmations of the hypothesis is provided by Zax and Kain (1996), who show that when a Detroit firm relocated to the suburbs, its black employees were likely to quit rather than relocate their residences, in contrast to the behavior of the white employees. More generally, Cutler and Glaeser (1997) demonstrate that housing segregation lowers the welfare of blacks, and one of the reasons proposed is that segregation reduces job access for black workers.

[5]As in the standard urban model, absentee landownership is assumed, so that land rents do not appear as income for any of the urban residents.

[6]The solutions for the remaining variables are $x^* = (\Delta Y + t\overline{x})/2t$ and $Z = (Y_c + Y_s - t\overline{x})/2$.

[7]Brueckner and Martin (1997) and Martin (1997) also analyze a model with endogenous incomes via simulation, but they use a market-clearing framework without unemployment.

[8]The asymmetric treatment of the two types of workers is strictly necessary only in the simulation analysis of section 4, where black utilities are computed. The results of the main part of the analysis, which deals with black unemployment, do not actually depend on how white wages are determined given a separability assumption introduced below.

[9]The typically substantial distance between the CBD and suburban employment centers makes a lack of familiarity with the other center's labor market a plausible assumption for a given center's workers.

[10]A zero interest rate on savings is assumed for the purposes of income smoothing. In contrast to the above approach, other spatial models explicitly incorporate the heterogeneity of the employed and unemployed, whose different incomes and commuting costs lead them to occupy different areas of the city. See Zenou and Smith (1995) and Brueckner and Zenou (1999).

[11]The cost of effort is normalized to unity. Equivalently, effort can appear as a negative linear term in the utility function.

[12]Without this cancellation, it would be impossible to derive a common efficiency wage that applies to all locations.

[13]For this reason, the model may not offer an accurate portrayal of the chronically unemployed. However, despite this drawback, the model remains the most tractable framework available for the analysis of unemployment.

[14]To see this, let a denote the job acquisition rate, which equals the probability that an unemployed worker is rehired. Stiglitz and Shapiro show that, in order for the flows into and out of unemployment (which are governed by Poisson processes) to balance, a must satisfy the relationship $a/v = (1 - u)/u$. Given values of v and u then imply a particular value for a, which is thus endogenously determined once u is found. Since the expected durations of employment and unemployment equal $1/v$ and $1/a$ respectively, it follows that a worker spends a fraction $a/(a + v)$ of his time employed. Recalling that the unemployment benefit is zero, his average income over time thus equals $(w - e)a/(a + v)$, which reduces to $(w - e)(1 - u)$ using the above equilibrium condition.

[15]This assumption means that if one type of worker is not represented in the work force at one of the employment centers, the first added worker of that type would receive an enormous wage. It follows that any labor allocation where one type of worker is missing from a particular center is not stable, in the sense that a perturbation that adds a worker of the missing type will lead to further reallocation. Thus, the only stable allocations are those where both worker types are represented in both centers, as in Figures 1 and 2.

[16]A subtle point arises once this correspondence is noted. In particular, while the unemployment differential in the model is based on *place of work*, that in Table 3 is based on *place of residence*. If the central-city/suburban boundary were located at $\widetilde{x}$ in Figure 2, then the two criteria would be identical (all black CBD workers would live in the "central city," while

all suburban workers would live in the "suburbs"). This equivalence will be disrupted, however, with a different boundary location. Nevertheless, if the suburban boundary is located somewhere in the black area, then even though black suburban residents may include some central city workers (or vice versa), it will remain true that suburban residents have a lower (average) unemployment rate than central-city residents.

[17]In fact, the model can be modified so that this recommendation is exactly relevant. In particular, Propositions 1–4 are unaffected if the model is developed under the assumption that t differs between blacks and whites, presumably being higher for blacks. While the equilibrium conditions (6)–(12) are unchanged under this modification, the black utility solutions are altered, however. Thus, the utility results in the ensuing simulation analysis, which assume a common t, are not relevant in the modified model.

[18]It is interesting to observe that policy makers are beginning to pay more attention to the transportation challenges faced by low-income central-city residents. New programs to address these problems are targeted specifically at former welfare recipients, while others serve broader segments of the working poor. In addition, a number of states and counties have used welfare block grants and other federal funds to support urban transportation services for welfare recipients. Moreover, Congress has created a $750 million competitive grant program (called 'Access to Jobs') to fund transportation services for low-income workers (see Pugh (1998) for a complete description of these programs).

[19]Under the present assumptions, white welfare is unaffected by spatial mismatch. The reason is that the white bid-rent functions intersect the horizontal axis at the same point ($x = 10.5$) in both the restricted and unrestricted equilibria, implying that the rent level paid by whites is the same in both cases (along with their incomes).