No. 2392

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FINANCIAL ECONOMICS

Centre for Economic Policy Research

ISSN 0265-8003

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Discussion Paper No. 2392
February 2000

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CEPR Discussion Paper No. 2392

February 2000

ABSTRACT

Risk Premia In The Term Structure Of Interest Rates: A Panel Data Approach*

This Paper proposes a panel data approach to modelling the risk premium in the term structure of interest rates. Specifically, we develop a fixed maturity/random time effects model that implies a time-invariant one-factor model. Our approach allows us to disentangle risk premia and unexpected excess returns, which is not possible in the standard time series approach. In addition, small sample bias is alleviated and statistical efficiency improved. Our results allow for interesting inferences about maturity-specific effects in the term structure. First, the expectations hypothesis is soundly rejected for our full data panel of US Treasury securities. Second, a considerable degree of mean reversion is present in the risk premia. Third, our findings shed new light on the magnitude of the slope coefficient in regressions of the yield onto the forward curve.

JEL Classification: E43, G12
Keywords: expectations hypothesis, risk premium

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* The authors wish to thank seminar participants at the 1997 Panel Data Conference in Paris, Maastricht University, and the National University of Singapore, and especially Peter C Schotman and Theo E Nijman for helpful comments. The usual caveat applies.

Submitted 20 January 2000

NON-TECHNICAL SUMMARY

The expectations hypothesis of the term structure of interest rates has been studied in the literature for a long time. The early literature deals with the pure expectations hypothesis, which is presented in different versions. The first version states that the excess holding period return on a bond is equal for all maturities. The second version of the pure expectations hypothesis states that the yield on a long-term bond is a weighted average of current and future expected short rates over the life of the yield of the long-term bond. In the third version, the forward rate is claimed to be an unbiased predictor of the future spot rate.

The very stringent hypothesis posed in either of the three variants appears not to hold and the inclusion of a constant risk premium has been adopted. Hicks (1939) postulated the liquidity premium theory, which states that borrowers of long-term capital are required to increase their promised return to compensate the preferred liquidity of lenders. Hence the longer the time to maturity of a bond, the greater the liquidity premium would need to be. Modigliani and Sutch (1966) argued that liquidity premia can be either positive or negative and there is no need for them to follow any systematic pattern. Their preferred habitat hypothesis recognizes that heterogeneous groups of borrowers and lenders prefer securities of different maturities.

By the early 1970s, a general consensus emerged that term premia exist and that they are usually positive. Most research activities have centred round the null hypothesis of rational expectations and time-invariant risk premia. In what follows we will refer to this as the Expectations hypothesis.

More evidence shows that time-varying risk premia exist and account for a substantial part of the variation in the term structure, see for example Fama (1984), Hamburger and Platt (1975) and Shiller, Campbell and Shoenholtz (1983). Although many researchers have found statistically significant evidence of time variation in term premia, the more important question is whether this time variation can be interpreted in an economically meaningful way. Much less work has been done on quantifying the importance of term premia as a source for variation in the term structure.

In this Paper we concentrate our efforts on the empirical properties of risk premia in a panel data framework. We focus on the variant of the expectations hypothesis in which the current forward rate is claimed to be the expected future interest rate plus some risk premium. Central to our analysis are the properties of the risk premia in the maturity dimension and how they vary over time. On the basis of this analysis we investigate whether the maturity-related properties that we uncover indeed cause rejection of the expectations hypothesis.

1 Introduction

The expectations hypothesis of the term structure of interest rates has been studied in the literature for a long time. The early literature[1] deals with the *pure* expectations hypothesis, which is presented in different versions. The first version states that the excess holding period return on a bond is equal for all maturities. The second version of the pure expectations hypothesis states that the yield on a long term bond is a weighted average of current and future expected short rates over the life of the yield of the long term bond. In the third version, the forward rate is claimed to be an unbiased predictor of the future spot rate.

Although Cox, Ingersoll and Ross (1981) show that these three variants are logically incompatible, Campbell (1986) demonstrates that they are not substantively dissimilar, as they are well approximated by a family of linear approximations which is internally consistent. In periods of high and volatile rates and for very long-term bonds care must be taken in using the linearization.

The very stringent hypothesis posed in either of the three variants appears not to hold and the inclusion of a constant risk premium has been adopted. Hicks (1939) postulated the liquidity premium theory, which states that borrowers of long term capital are required to increase their promised return to compensate the preferred liquidity of lenders. Hence the longer the time to maturity of a bond, the greater the liquidity premium would need to be. Modigliani and Sutch (1966) argued that liquidity premia can be either positive or negative and there is no need for them to follow any systematic pattern. Their preferred habitat hypothesis recognizes that heterogeneous groups of borrowers and lenders prefer securities of different maturities. By the early seventies, a general consensus emerged that term premia exist and that they are usually positive. Most research activities have centered around the null hypothesis of rational expectations and time invariant risk premia. In what follows we will refer to this as the expectations hypothesis.

More evidence shows that time-varying risk premia exist and account for a substantial part of the variation in the term structure, see for example Fama (1984), Hamburger and Platt (1975) and Shiller, Campbell and Shoenholtz (1983). Although many researchers have found statistically significant evidence of time variation in

[1] For a thorough overview of the literature on the expectations hypothesis we refer to Melino (1988).

term premia, the more important question is whether this time variation can be interexpreted in an economically meaningful way. Much less work has been done on quantifying the importance of term premia as a source for variation in the term structure.

Empirical tests have rejected the expectations hypothesis time and again. Mishkin (1988) provides some refinement of Fama's (1984) evidence on the information in the term structure. He proposes econometric techniques that properly correct standard errors for overlapping data and for conditional heteroskedasticity. Evans and Lewis (1994) test whether stationary risk premia alone can explain the predictable variation in excess returns. They reject this hypothesis and show that either permanent shocks in the risk premia and/or rationally anticipated shifts in the interest rate process could produce anomalous results. Furthermore, they show that time varying-risk premia which are correlated with forward rates contaminate regression results. Recently, rejection of the expectations hypothesis was attributed to small sample bias, see Bekaert, Hodrick and Marshall (1997). They documented that the asymptotic distributions of commonly used test of the expectations hypothesis are not to be relied upon.

In this paper we concentrate our efforts on the empirical properties of risk premia in a panel data framework. We focus on the variant of the expectations hypothesis in which the current forward rate is claimed to be the expected future interest rate plus some risk premium. Central to our analysis are the properties of the risk premia in the maturity dimension, *i.e.* how are risk premia of bonds with different time to maturity related to each other, and in the time series dimension, *i.e.* how do they vary over time. On the basis of this analysis we investigate whether the maturity-related properties that we uncover indeed cause rejection of the expectations hypothesis.

We employ panel data techniques, which seems natural for several reasons. First, in a panel data framework both the time series and cross section - i.e. maturity - dimensions are taken into account. Hence the model is more informative than models that only account for (univariate) time series behavior. Second, we are able to discern the risk premium from the unexpected excess return, which is not possible in pure time series approaches. This is achieved through the use of a random effects specification in order to model the unobservable time-varying risk premia. Third, since the expectations hypothesis essentially posits the same model for all maturities, pooling the data is appropriate and helps to avoid problems of small sample bias.

To avoid problems with overlapping data and to take the critique by Cox, Ingersoll and Ross (1981) seriously, we construct discount rates from raw bond data as proposed by McCulloch (1975). The use of discount rates avoids the use of linear approximations, which sometimes makes the different forms of the expectations hypothesis inconsistent with each other.

In the next section we provide some relevant background and explain our notation. In section 3 we formally set up the econometric framework, in which we will model and test the expectations hypothesis. In section 4 we show how the discount rates are constructed from coupon bearing bonds, in section 5 we present the estimation results and in section 6 we provide some concluding remarks.

2 Background and notation

Let $P_t(\tau)$ denote the price of a discount bond at time t that matures at time $t+\tau$. The relation between the price of a discount bond and the yield-to-maturity is given by

$$P_t(\tau) = \exp[-\tau Y_t(\tau)] \tag{1}$$

where $Y_t(\tau)$ is the discount yield with time-to-maturity τ at time t. The discount yield is an instrument that is helpful in determining the current value of a future cashflow. Because of the one-to-one relation between prices and yields, both variables contain the same information. Another way to express information on the yield curve is through the holding period return of a bond with price $P_t(\tau)$ at time t

$$H_{t,t+1}(\tau) \equiv \ln\left[\frac{P_{t+1}(\tau-1)}{P_t(\tau)}\right] \tag{2}$$

The holding period return denotes the one-period return which follows from buying a bond at time t, and selling it one period later at $t+1$. The yield-to-maturity and the holding period return are different from each other. The yield-to-maturity is the average annual return on a discount bond when the bond is held until maturity, whereas the holding period return quantifies the change in price when holding the bond for one period. In the special case of a one year discount bond, yield-to-maturity and holding period return coincide.

The current term structure of interest rates also provides a forward rate curve. The forward rate at time t is the rate at which investors currently agree to enter into a contract at time $t+1$, that has maturity τ. In a risk neutral world, the current forward rate is equal to the expected future yield. The one-period forward rate, $F_{t,t+1}(\tau)$, is given by

$$(\tau - 1)F_{t,t+1}(\tau - 1) = \tau Y_t(\tau) - Y_t(1) \tag{3}$$

This definition of forward rates follows from two investment strategies that yield the same result. In the first strategy an investor decides to enter into a contract that holds for τ periods. In the second strategy, the investor decides to invest for one period and to invest the payoff of this investment for $\tau - 1$ periods, starting one period from now. Because both strategies start now and end at the same time, in the absence of arbitrage opportunities, their returns will be equal. Clearly, the forward rate on the left-hand-side of equation (3) is implied by current yields on the right-hand-side.

Consider an investor who faces at time t the possibility to invest in alternative discount bonds. Investing in a discount bond that has a remaining time-to-maturity of one period is riskless, since at maturity the principal is repaid. The one-period return on this discount bond is the one-period yield, $Y_t(1)$. The one-period return on a general discount bond is the holding period return. We define the value of the holding period return in excess of the return on a risk free investment as the excess holding period return

$$\zeta_{t+1}(\tau) = H_{t,t+1}(\tau) - Y_t(1) \tag{4}$$

The excess holding period return denotes the additional return on a risky investment over a certain investment for the same period. An interesting issue is whether this excess holding period return is zero on average (the pure expectations hypothesis), or positive (the expectations hypothesis). In the latter case the investor is rewarded with a higher expected return to compensate for the additional amount of risk. We refer to the expected excess holding period return as the risk premium, which quantifies the expected excess return over the riskfree rate. The risk premium, $\psi_t(\tau)$, is defined as

$$\psi_t(\tau) = E_t\left[\zeta_{t+1}(\tau)\right] = E_t\left[H_{t,t+1}(\tau)\right] - Y_t(1) \tag{5}$$

We focus on the properties of this risk premium. Using the relations in equations (1), (2) and (3) the risk premium as defined in equation (5) can be rewritten as

$$\psi_t(\tau) = (\tau - 1)\left[F_{t,t+1}(\tau - 1) - E_t\left(Y_{t+1}(\tau - 1)\right)\right] \tag{6}$$

This results in a second interpretation of the risk premium, which is the difference between the forward rate and the expected future yield, premultiplied with a maturity dependent factor. Under the pure expectations hypothesis the forward rate is assumed to be equal to the expected future yield and hence the risk premium is zero in that case. From equation (6) a testable expression of the expectations hypothesis follows, which links the future yield to the current forward rate

$$\tau Y_{t+1}(\tau) = \tau F_{t,t+1}(\tau) - \psi_t(\tau+1) + \tau \left[Y_{t+1}(\tau) - E_t \left(Y_{t+1}(\tau) \right) \right] \tag{7}$$

The future yield consists of three components: the current forward rate, a risk premium and an unexpected excess yield. This expression encompasses the pure expectation hypothesis, when $\psi_t(\tau+1) = 0$, and the expectations hypothesis, otherwise. The expected value of future interest rate is the current forward rate plus a risk premium. The relation in (7) is the fundamental relation that is our starting point for the empirical analysis. Note that we choose to run levels regressions because subtracting either yields or forward rates on both sides to render the regression variables "more stationary", as was suggested by Campbell and Shiller (1987), increases the potential for biases in small samples, see Bekaert, Hodrick and Marshall (1997).

3 The econometric framework

In the literature, testing the pure expectations hypothesis typically boils down to estimating the regression model

$$\tau Y_{t+1}(\tau) = \alpha + \beta \tau F_{t,t+1}(\tau) + \varepsilon_{t+1}(\tau) \tag{8}$$

where α and β are unknown parameters. Testing for the pure expectations hypothesis amounts to testing the hypothesis that $\alpha = 0$ and $\beta = 1$. The error term $\varepsilon_{t+1}(\tau)$ is given by

$$\varepsilon_{t+1}(\tau) = -\psi_t(\tau+1) + \tau \left[Y_{t+1}(\tau) - E_t \left(Y_{t+1}(\tau) \right) \right] \tag{9}$$

which implies that the error term consists of two components. One depends upon the risk premium, the other one is the unexpected excess yield. Leaving the risk premium in the error term does not allow us to gain insight into its properties, apart from its average value, that is measured through the estimate for α. In our view knowledge about the properties of the risk premium is important. It not only enables a better understanding of pricing of risk in the economy, but also it may be able to explain *why* the expectations hypothesis is rejected. Because both the risk premium and the unexpected excess yield are unobservable, it is in general not clear to which of the two we should attribute an effect in the error term as given in equation (9). Once we are able to discern the risk premium component from the unexpected excess yield component, the regression results could actually provide clear insight as to which component can account for the rejection of the expectations hypothesis, which is fairly common in the literature, see Melino (1988) for an overview. Panel data techniques are a convenient tool to disentangle risk premia and unexpected excess yields.

Next, we set up the panel data framework for the expectations hypothesis regression model of equations (8) and (9). For each time-to-maturity the pure expectations hypothesis states that $\alpha = 0$ and $\beta = 1$, hence pooling the data for different maturities gives the following model

$$\begin{bmatrix} \tau_1 Y_{t+1}(\tau_1) \\ \vdots \\ \tau_N Y_{t+1}(\tau_N) \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \alpha + \begin{bmatrix} \tau_1 F_{t,t+1}(\tau_1) \\ \vdots \\ \tau_N F_{t,t+1}(\tau_N) \end{bmatrix} \beta + \begin{bmatrix} \varepsilon_{t+1}(\tau_1) \\ \vdots \\ \varepsilon_{t+1}(\tau_N) \end{bmatrix} \tag{10}$$

or in matrix notation

$$Y_{t+1} = \iota\alpha + F_{t,t+1}\beta + \varepsilon_{t+1} \qquad t = 1, \ldots, T \tag{11}$$

where Y_{t+1} is the $(N \times 1)$ vector of maturities times yields at time $t+1$ for maturities $\tau_1, ..., \tau_N$, ι is the $(N \times 1)$ vector of ones, $F_{t,t+1}$ is the $(N \times 1)$ vector of maturities times forward rates at time t, and ε_{t+1} denotes the $(N \times 1)$ vector of error terms which consists of a risk premium and the unexpected excess yield. Stacking the vectors for all time observations $t = 1, ..., T$ results in the panel data model. We refer to this model as the pooled case. In this specification we do not explicitly account for the existence of the risk premium, but relegate it to the error term. Testing this specification results in more efficient parameter estimates than in the univariate time series models. The main motivation for considering the pooled case is because it may adress small sample biases in the parameter estimates, which were documented by Bekaert, Hodrick and Marshall (1997).

In the following, this model specification is extended to incorporate explicitly the risk premia as incidental variables. In a panel data framework the usual approach to model incidental variables is either as fixed effects, including a dummy for each observation, or as random effects by specifying a stochastic process for the variables in question. Direct application of either approach, however, is not possible in our model. In order to take account of the risk premium we extract it from the error term and rewrite the model as

$$\begin{bmatrix} \tau_1 Y_{t+1}(\tau_1) \\ \vdots \\ \tau_N Y_{t+1}(\tau_N) \end{bmatrix} = \begin{bmatrix} \tau_1 F_{t,t+1}(\tau_1) \\ \vdots \\ \tau_N F_{t,t+1}(\tau_N) \end{bmatrix} \beta - \begin{bmatrix} \psi_t(\tau_1) \\ \vdots \\ \psi_t(\tau_N) \end{bmatrix} + \begin{bmatrix} \mathrm{UEY}_{t+1}(\tau_1) \\ \vdots \\ \mathrm{UEY}_{t+1}(\tau_N) \end{bmatrix} \qquad (12)$$

where we introduce

$$\mathrm{UEY}_{t+1}(\tau) \equiv \tau \left(Y_{t+1}(\tau) - E_t[Y_{t+1}(\tau)] \right) \quad \tau = \tau_1, ..., \tau_N \qquad (13)$$

for the unexpected excess yield. In matrix notation it holds that

$$Y_{t+1} = F_{t,t+1}\beta - \psi_t + \mathrm{UEY}_{t+1} \qquad t = 1, \ldots, T \qquad (14)$$

Note that we have split the error term from equation (9) into two parts, the risk premium ψ_t and the unexpected excess yield UEY_{t+1}. The unexpected excess yield is treated as an error term that takes account of the cross sectional relations between the univariate series, through the adoption of a covariance matrix specification. We adopt the following specification for the error terms

$$\mathrm{UEY}_{t+1} \sim N(0, \Sigma)$$

where we specify the exact form of the covariance matrix below.

Our goal is to model the risk premium explicitly, where the fixed maturity effects estimator seems a natural candidate to learn about its maturity properties. The fixed maturity effects estimator treats the risk premium as a constant in time and assumes that it is only different whenever the maturity of the yield is different, hence

$$\psi_1(\tau) = \cdots = \psi_T(\tau) \qquad \tau = \tau_1, ..., \tau_N \qquad (15)$$

This estimator quantifies the relative levels of risk premia for yields with different time-to-maturity. The pure expectations hypothesis postulates that they are zero for all maturities. The liquidity preference theory suggests that the risk premium increases with time to maturity. In the appendix we derive the panel data estimator[2]

[2]See Baltagi (1995) and Hsiao (1986) for textbook treatments of panel data models.

for the fixed effects, the associated estimator for β, and expressions for standard errors. This results in the following estimator for the fixed maturity effects

$$\begin{bmatrix} \hat{\psi}(\tau_1) \\ \vdots \\ \hat{\psi}(\tau_N) \end{bmatrix} = \frac{1}{T}\sum_{t=1}^{T}(Y_t - F_{t,t+1}\beta) - \frac{1}{T}\sum_{t=1}^{T}\mathrm{UEY}_t \tag{16}$$

We split the estimator into an observed part and an unobserved part. The unobserved part represents the sample average of the unexpected excess return, which approaches zero by the law of large numbers. The fixed maturity effects estimator is equal to the estimator we find when the unexpected excess yield would not be incorporated in the model, and hence this estimator specifically describes the behavior of the risk premium in the maturity dimension.

The fixed maturity effects specification is interesting in itself, but at the same time somewhat restrictive: the risk premia are allowed to vary only with maturity and not with time. In what follows, we introduce time variation into the model by employing ingredients from a time effects panel model. The use of a random time effects model lies in *combining* it with the fixed maturity effects model in order to incorporate both the maturity and time dimensions appropriately.

Using equation (4) and the fact that $\psi_t(\tau) = E_t\zeta_{t+1}(\tau)$, we derive

$$Y_{t+1} = F_{t,t+1}\beta - \zeta_{t+1} + \mathrm{UEY}_{t+1} + \mathrm{UEH}_{t+1} \tag{17}$$

where the term given by

$$\mathrm{UEH}_{t+1}(\tau) = [\zeta_{t+1}(\tau) - E_t\zeta_{t+1}(\tau)] \qquad \tau = \tau_1, ..., \tau_N \tag{18}$$

denotes the unexpected excess holding period return. Obviously, the right-hand-side now consists of three terms which are indiscernible: the excess holding period return, ζ_{t+1}, the unexpected excess yield, UEY_{t+1}, and the unexpected excess holding period return, UEH_{t+1}. The last two terms are unexpected effects and are easily eliminated by taking conditional expectations. We impose an AR(1) process for the sum of excess holding period return, unexpected excess yield and unexpected excess return, denoted by

$$\lambda_{t+1} = -\zeta_{t+1} + \mathrm{UEY}_{t+1} + \mathrm{UEH}_{t+1} \tag{19}$$

This reduces the number of parameters to the number of unknown parameters in an AR(1) process which is only three. Note that taking conditional expectations in the estimated time series process results in a time series of risk premia

$$E_t\lambda_{t+1} = -E_t\zeta_{t+1} = -\psi_t \qquad t = 1, \ldots, T-1 \tag{20}$$

since the conditional expectations of the second and third components are zero. Because we are only interested in the risk premia, the modeling decision relies heavily on the assumptions we make for the excess holding period returns, whereas the second and third components are not relevant since they are eliminated from the equation by taking conditional expectations. The risk premium is allowed to be different from period to period with the arrival of news or with changes in the economic regime. The AR(1) process allows for such changes and provides us with a measure of persistence of the risk premium over time.

We assume that the excess holding period returns for yields with different time to maturity are driven by the same dynamics: we impose a time-invariant one-factor model in time dimension. We also incorporate the maturity dimension of risk premia. The excess holding period returns of the long term yield serves as a base case. The risk premia of all other yields are assumed to be related to this process through a scaling factor

$$\zeta_{t+1}(\tau) = Z(\tau)\zeta_{t+1} \qquad \tau = \tau_1, ..., \tau_N \tag{21}$$

with $Z(\tau_N) = 1$. Note that these scaling factors will be estimated from our term structure data and that they imply a time-invariant one-factor model. The model is expressed in state-space form, which makes it possible to apply Kalman Filter techniques. Furthermore, it allows for the simultaneous inclusion of time series effects and cross sectional effects of the risk premium. The excess holding period return is divided into a component ζ_{t+1}, which only depends on time, and a component $Z(\tau)$, which only depends on maturity. The process $\{E_t\zeta_{t+1}\}_{t=1}^{T}$ shows evolution of the risk premium over time, whereas $\{Z(\tau)\}_{\tau=\tau_1}^{\tau_N}$ shows the relation between risk premia of bonds with different time to maturity, where the long term yield serves as the reference point.

Altogether our fixed maturity/random time effects panel model is given by

$$\begin{aligned} Y_{t+1} &= F_{t,t+1}\beta - Z\lambda_{t+1} + \eta_{t+1} \\ \lambda_{t+1} &= (1-\rho)\mu + \rho\lambda_t + \nu_{t+1} \\ \eta_{t+1} &\sim N(0, \Sigma) \\ \nu_{t+1} &\sim N(0, \sigma^2) \end{aligned} \tag{22}$$

where ρ, μ and σ^2 are the parameters in the AR(1) process and Σ denotes the covariance matrix of the cross sectional error term. We assume that η_{t+1} and ν_{t+1} are independent. Once the process for the excess holding period has been estimated, the process for the risk premium at different maturities follows directly from the relations stated in equations (5) and in (21).

Of course, the error terms $\eta_{t+1}$are not cross-sectionally independent. We model the cross-sectional error term following the specification in Bams and Schotman (1997). In order to keep the number of parameters to be estimated tractable, and to gain further efficiency, we assume that the correlation between error terms $\eta_{t+1}(\tau_i)$ and $\eta_{t+1}(\tau_j)$ depends on the distance between the terms to maturity:

$$\mathsf{Corr}\left(\eta_{t+1}(\tau_i), \eta_{t+1}(\tau_j)\right) = \phi^{|\tau_i - \tau_j|} \tag{23}$$

with $0 < \varphi < 1$. Yields that are very close show high correlation, whereas yields that are far apart are less correlated. The specification resembles a cross-sectional AR(1) error term.

Besides cross sectional correlation we also account for possible cross sectional heteroskedasticity. Long term yields show less variance than short term yields. For that reason we specify the variance of the error term, $\eta_{t+1}(\tau)$, as a function of τ

$$\mathsf{Var}\left(\eta_{t+1}(\tau)\right) = \omega^2 \tau^{-2d} \ , \tag{24}$$

where ω^2 is a scale parameter, and d determines the sensitivity of the variance for the term to maturity. We estimate d along with the other parameters. If $d = 0$ then the error terms are homoskedastic in a model for the yields. In case $d = 1$, the model is homoskedastic in a regression model for (log-) bond prices. The heteroskedasticity implies a weighting scheme on the maturities. With $d > 0$ more emphasis is put on long term yields.

Altogether the cross sectional covariance matrix of the error terms for a model with maturities $\tau = \tau_1, \ldots, \tau_N$ is parametrized by the three parameters ϕ, ω and d. In matrix form the covariance structure for the cross sectional error terms is

$$\Sigma = \omega^2 S(\phi, d) \tag{25}$$

and

$$S(\phi,d)=\begin{bmatrix} \tau_1^{-2d} & \frac{\phi^{|\tau_2-\tau_1|}}{(\tau_2\tau_1)^d} & \cdots & \frac{\phi^{|\tau_N-\tau_1|}}{(\tau_N\tau_1)^d} \\ \frac{\phi^{|\tau_2-\tau_1|}}{(\tau_1\tau_2)^d} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{\phi^{|\tau_N-\tau_{N-1}|}}{(\tau_N\tau_{N-1})^d} \\ \frac{\phi^{|\tau_N-\tau_1|}}{(\tau_1\tau_N)^d} & \cdots & \frac{\phi^{|\tau_N-\tau_{N-1}|}}{(\tau_{N-1}\tau_N)^d} & \tau_N^{-2d} \end{bmatrix} \tag{26}$$

where Σ and S are matrices of order $(N \times N)$. We assume that the same covariance matrix is applicable in the regression equation for the pure expectations hypothesis of equation (8), where the error term is ε_{t+1}.

4 Data

Interest rates are only directly observable for short maturities, implied by money market instruments. The longer term yields are implicitly available in coupon bearing bond prices. Because the expectations hypothesis is stated in terms of discount yields and discount forward rates, the analysis is greatly simplified if we can work with discount rates. The alternative is to use approximate linear relations as proposed by Campbell (1986). Especially in periods of high and volatile interest rates care must be taken in applying the linear approximation.

In the literature there exist different methods to construct discount rates from coupon bearing bonds. Here we follow McCulloch (1975), who introduces cubic spline functions. A cubic spline is a functional form for the discount function like

$$\delta_t(\tau) = 1 + \alpha_1\tau + \alpha_2\tau^2 + \sum_{j=1}^{L} \alpha_{2+j}(\tau - c_j)_+^3 \tag{27}$$

where $\delta_t(\tau)$ denotes the discount function at time t for a cash flow of \$1 with maturity τ, c_j, $j = 1, \ldots, L$ are break-points, $(\cdot)_+ = \max[., 0]$ and α_j, $j = 1, \ldots, L+2$ denote the parameters to be estimated. The functional form for the discount function at time t is found by minimizing

$$\sum_{i=1}^{K} \left[P_{it} - \sum_{j=1}^{J_i} C_{ij}\delta_t(\tau_{ij}) \right]^2 \tag{28}$$

where K is the number of bonds, P_{it} denotes the price of bond i at time t, corrected for accrued interest, C_{ij} denotes the cash flow of bond i with term τ_{ij} and J_i is the number of cash flows for bond i. Finally, the yield curve at time t is obtained by

$$Y_t(\tau) = -\frac{\ln[\delta_t(\tau)]}{\tau} \tag{29}$$

The splines are an a-theoretical way to transform the information in bond prices to yield curve data. We have used US government bond data, available from the CRSP tapes for the period January 1970 until December 1994. These data are available on a monthly basis. For maturities less than a year we also include the observed Treasury bill rates. For each month the spline function contains 12 breakpoints, and thus 14 unrestricted parameters α_j. The parameters of the spline function are different for every month. As the fitted curve does not perfectly fit all available bond prices, the method already filters out some error terms implicit in the quadratic fit criterium (28). Using the estimated cubic spline functions, we have constructed a panel which includes time series of yields with maturities of 1 to 6 months and 1 to 10 years, a total of 16 time series. The number of maturities that we include in the sample corresponds with the number of parameters in the spline functions. In essence the data on a large cross section of bond prices are condensed to 16 observations. At every time t the yield curve is represented by a cross section of yields, denoted by $Y_t(\tau_1), \ldots, Y_t(\tau_N)$, where $N = 16$.

Figure 1 shows the full data panel. A number of issues are of interest. First, most of the yield curves are increasing. Second, in the period 1979-1984 the interest rates have been substantially larger than in the rest of the data sample. Third, for short maturities the yield curve is less smooth than for long maturities. Data at the short end of the yield curve are less reliable since they include monetary policy effects and the like.

In the top panel of table 1 summary statistics are presented for yields. The yield levels show an average term structure that is increasing. The term structure of volatilities is decreasing and from the last column we find that yields are highly autocorrelated. The bottom part of table 1 present the difference of yields and lagged forward rates. Under the pure expectations hypothesis the lagged forward rate is the expected value of the current yield. The second column denotes the average difference between the lagged forward rate and the current yield. It follows that next month's one month yield is on average 13 basis points lower than the current forward rate, next period's ten year yield is on average only 1 basis point lower than the current forward rate. A possible difference between the two is attributed to a risk premium. From equation (7) it follows that the average risk premium is given by

$$avg\left[\psi_t\left(\tau\right)\right] = \tau\left\{avg\left[F_{t,t+1}\left(\tau\right)\right] - avg\left[Y_{t+1}\left(\tau\right)\right]\right\} \tag{30}$$

In the last column the average risk premium is reported which follows from multiplication of the first and the second column. The statistics imply that the risk premium is on average increasing with maturity, the same holds for the associated standard errors. Equation (5) presents a definition of risk premium in a holding period return context. It follows that the average expected monthly excess return on a ten year bond over the risk free rate is 13 basis points.

In table 2 another feature of yield curve data is illustrated. The top panel shows that yields that have maturities that are close together, are highly correlated. The correlation decreases as the difference between the maturities of the bonds increases. For the yield minus the lagged forward rate, we observe the same pattern although the correlations are not as high as in the case of yield levels. This feature of the data is important since it shows that the information available in the data is limited. We take this explicitly into account when we model the cross sectional relation between the error terms.

5 Estimation results

This section presents empirical results for the various models that deal with tests of the expectations hypothesis. We start with simple univariate regressions. Next, we pool the data and test the expectations hypothesis for all yields simultaneously. Then models are considered in which the risk premium is explicitly modeled. First we consider a fixed effects estimator, then we deal with the case where the risk premium is modeled as a combined fixed maturity/random time effects model.

Table 3 shows the univariate regression results of yields on lagged forward rates for all maturities, as given by equation (8). Typically, the slope coefficient β is close to one, which implies that the lagged forward rate seems a reasonable predictor of the future yield. For most maturities at the short end of the yield curve, the pure expectations hypothesis (*i.e.* $H_0 : \alpha = 0,\ \beta = 1$) is rejected, which follows from the LR statistic, that is compared with a critical value of $\chi^2_{0.95}(2) = 5.99$. It seems that the results for very short maturities up to 4 months are imprecise, probably caused by transitory noise at the short end, or because of liquidity problems around maturity dates of bonds. Bekaert, Hodrick and Marshall (1997) point out that rejection of the expectations hypothesis may very well be caused by small sample biases.

The Durbin-Watson statistic indicates that for some maturities there is residual autocorrelation. This may point at dynamics in the error term that were not modeled adequately, perhaps a time-varying risk premium. The residuals for the separate maturities are clearly heteroskedastic, given the increment in volatility of the residual term when the maturity of the yield is higher. We have also calculated the cross-sectional correlation between error terms (not reported) and find high correlation for residuals which differ only a little in time-to-maturity. The correlation decreases as the residuals differ more in time-to-maturity.

Remember that we employ panel data techniques in order to deal with two issues. First, pooling the data leads to more efficient estimates and will help to deal with the small sample bias problem. Furthermore, panel data techniques allow us to model the risk premium explicitly, whereas in the univariate regression tests it is included in the error term. Now, we pool the data and rerun the regression model of equation (8), estimating the slope parameter for all maturities simultaneously. Also, we take into account the heteroskedasticity and the cross-sectional correlation by specifying the error term as proposed by equations (25) and (26). The results are presented in the first column of table 4. The slope parameter β is very close to one. Pooling the data also results in an even lower associated standard error, which still leads to a rejection of the expectations hypothesis. The covariance matrix of the error terms properly accounts for the heteroskedasticity in the residuals, see figure 2 for the term structure of residual volatilities, both as observed in the residuals and implied by the cross-sectional covariance matrix specification. Also the covariance matrix appears to model the correlation between residuals adequately. Residuals which differ only one month in maturity have a correlation coefficient of $\phi^{\frac{1}{12}} = 0.997$. Clearly, since ϕ is close to one, the correlation between residuals decreases smoothly with difference in time-to-maturity, which is consistent with what we found in the univariate regression results.

In the second column of table 4 the parameter results for the fixed maturity effects model as specified by equation (16) are given. We find a drop in the estimate for β, which decreases to 0.945. The expectations hypothesis cannot be rejected in the case of fixed maturity effects. The standard errors are high enough, even after adjustment for low autocorrelation in the residuals, to favor the expectations hypothesis. The model explicitly includes the parameter $\psi_t(\tau)$ through inclusion of a dummy for each maturity. However, only if β is equal to 1, can we interpret the parameter $\psi_t(\tau)$ as a risk premium. Otherwise, we have to correct for the deviation of β from 1. Estimates for the parameter $\psi_t(\tau)$, along with the associated standard errors are given in table 5. In the last column we also quantify the risk premium, by taking account of the deviation. The levels of the risk premia are consistent with the historical average values we reported in the data section. In the case β is constrained to be 1, the fixed maturity estimator for the risk premium is nothing but the historical average value of yields minus lagged forwards, as given in the bottom part of table 1. The maturity effects show that risk premia increase with time-to-maturity τ. Testing whether the maturity effects are zero amounts to a likelihood ratio (LR) test between the pooled model and the fixed maturities effects model. The hypothesis is clearly rejected, since the regression results report an increase of two times the loglikelihood of about 158, while the critical value equals $\chi^2_{0.95}(16) = 26.30$. The LR test favors the presence of risk premia that vary with time-to-maturity. Note that estimation results for the covariance specification of the error term are almost identical to the *pooled* case.

In table 6 we report the estimation results for the full model that was presented in equations (22), (23) and (24). The model incorporates both fixed maturity effects and random time effects. The model is written in state space form and is estimated using the time-invariant Kalman filter. Model I in the first column of table 6 shows the regression results for the most general case. The slope parameter, β, shows a further decrease and is far away from the value where the expectations hypothesis would hold. The parameters ρ, μ and σ model the time series dynamics for ψ_t, which is the time series related to the longest maturity $\tau_N = 10$. We find moderate persistence in the process for ψ_t, with an AR(1) parameter of $\rho = 0.675$. Because β differs from 1, ψ_t cannot be interpreted as the risk premium for the long maturity yield. For the same reason, μ is not the average value of the long term risk premium. At time t, the bias is of the order $(1-\beta)\,\tau_N F_{t,t+1}(\tau_N)$, which is on average 17.96. The risk premium time series processes for the other maturities are linked to ψ_t through scaling with the maturity parameter $Z(\tau)$, which is found in table 7. As in the fixed maturity effects case, we find that Z increases with maturity. In figure 3 both the time series pattern and the cross sectional relation are depicted. The covariance matrix specification of the error term takes account of the high correlation between error terms that differ little in time to maturity. For error terms that differ 1 year in time to maturity, the implied correlation is $\hat{\phi} = 0.834$, for error terms that differ only one month in time to maturity the correlation is $\hat{\phi}^{\frac{1}{12}} = 0.985$, which corresponds to the data. The covariance specification is similar for restricted versions of the model that we consider next.

Model II in table 6 deals with a restricted version of model I. Since under the expectation hypothesis the theoretical value of the slope coefficient is $\beta = 1$, we restrict the parameter to this value. We find a lower AR(1) parameter estimate in the random effects specification of the risk premium, but it is significantly different from zero. This specification suggests that the associated risk premium still increases with maturity, and that there is also small but significant predictive power with respect to the time series dimension of the risk premium. This average value for the risk premium, $\mu = 0.206$, is similar to what we found in the fixed maturity case. The likelihood ratio test favors a model with time-varying risk premia and, thus, rejection of the expectations hypothesis, sinc the LR statistic is 232, whereas the critical value is $\chi^2_{0.95}(1) = 3.84$.

6 Concluding Remarks

In this paper we have developed a panel data model for the term structure of interest rates which combines fixed maturity effects with random time effects. Relative to standard regression approaches, an important advantage of the model is that it allows us to explicitly disentangle risk premia and unexpected excess returns. The panel setting also helps to mitigate small sample bias and to increase statistical efficiency in testing procedures.

Our empirical results demonstrate a resounding rejection of the expectations hypothesis in the multivariate panel setting, even though the rejection was not possible for the univariate models at the short end of the term structure. The point estimates indicate that a considerable degree of mean reversion is present in the risk premia and that the slope coefficient β,which is one under the expectations hypothesis, is estimated to be about 0.8 for the full datapanel.

Appendix

In this appendix we derive the first order conditions which are necessay to obtain parameter estimates. Furthermore we derive the Hessian matrix which is used to compute standard errors of the parameter estimates. We consider the case where the risk premia are treated as individual effects. The loglikelihood reads

$$\ln L = -\frac{1}{2}\ln|S| - \frac{1}{2}NT\ln(\omega^2) - \frac{1}{2\omega^2}\sum_{t=1}^{T-1}\left(Y_{t+1} - F_{t,t+1}\beta - \psi\right)' S^{-1}\left(Y_{t+1} - F_{t,t+1}\beta - \psi\right)$$

The first derivative with respect to ψ is

$$\frac{\partial \ln L}{\partial \psi} = \frac{1}{\omega^2}\sum_{t=1}^{T-1}\left(Y_{t+1} - F_{t,t+1}\beta - \psi\right)' S^{-1}$$

The first derivative with respect to β is

$$\frac{\partial \ln L}{\partial \beta} = \frac{1}{\omega^2}\sum_{t=1}^{T-1}\left(Y_{t+1} - F_{t,t+1}\beta - \psi\right)' S^{-1} F_{t,t+1}$$

For $\hat{\psi}$ it therefore holds that

$$\hat{\psi} = \frac{1}{T-1}\sum_{t=1}^{T-1}\left(Y_{t+1} - F_{t,t+1}\beta\right)$$

or

$$\hat{\psi} = \left(\frac{1}{T-1}\sum_{t=1}^{T-1} Y_{t+1}\right) - \left(\frac{1}{T-1}\sum_{t=1}^{T-1} F_{t,t+1}\right)\hat{\beta}$$

For $\hat{\beta}$ it holds that

$$\hat{\beta} = \left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}F_{t,t+1}\right)^{-1}\left(\sum_{t=1}^{T-1} Y'_{t+1}S^{-1}F_{t,t+1}\right) - \left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}F_{t,t+1}\right)^{-1}\left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}\right)\hat{\psi}$$

or

$$\begin{aligned}\hat{\beta} \;=\; & \left[I - \left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}F_{t,t+1}\right)^{-1}\left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}\right)\left(\frac{1}{T-1}\sum_{t=1}^{T-1} F_{t,t+1}\right)\right]^{-1}\left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}F_{t,t+1}\right)^{-1} \times \\ & \times\left[\left(\sum_{t=1}^{T-1} Y'_{t+1}S^{-1}F_{t,t+1}\right) - \left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}\right)\left(\frac{1}{T-1}\sum_{t=1}^{T-1} Y_{t+1}\right)\right]\end{aligned}$$

$$\begin{aligned}\hat{\beta} \;=\; & \left[\left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}F_{t,t+1}\right) - \left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}\right)\left(\frac{1}{T-1}\sum_{t=1}^{T-1} F_{t,t+1}\right)\right]^{-1} \times \\ & \times\left[\left(\sum_{t=1}^{T-1} Y'_{t+1}S^{-1}F_{t,t+1}\right) - \left(\sum_{t=1}^{T-1} F_{t,t+1}S^{-1}\right)\left(\frac{1}{T-1}\sum_{t=1}^{T-1} Y_{t+1}\right)\right]\end{aligned}$$

The second derivatives are required to calculate standard errors for the parameter estimates.

$$\frac{\partial^2 \ln L}{\partial \psi \partial \psi'} = -\frac{1}{\omega^2}\left(T-1\right) S^{-1}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{1}{\omega^2}\sum_{t=1}^{T-1} F_{t,t+1} S^{-1} F_{t,t+1}$$

$$\frac{\partial^2 \ln L}{\partial \psi \partial \beta} = -\frac{1}{\omega^2}\sum_{t-1}^{T-1} F_{t,t+1} S^{-1}$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \psi'} = -\frac{1}{\omega^2}\sum_{t-1}^{T-1} S^{-1} F_{t,t+1}$$

and the covariance matrix of the parameter estimates reads

$$cov(\beta, \psi) = \omega^2 \begin{bmatrix} \sum_{t=1}^{T-1} F_{t,t+1} S^{-1} F_{t,t+1} & \sum_{t=1}^{T-1} F_{t,t+1} S^{-1} \\ \sum_{t=1}^{T-1} S^{-1} F_{t,t+1} & \left(T-1\right) S^{-1} \end{bmatrix}^{-1}$$

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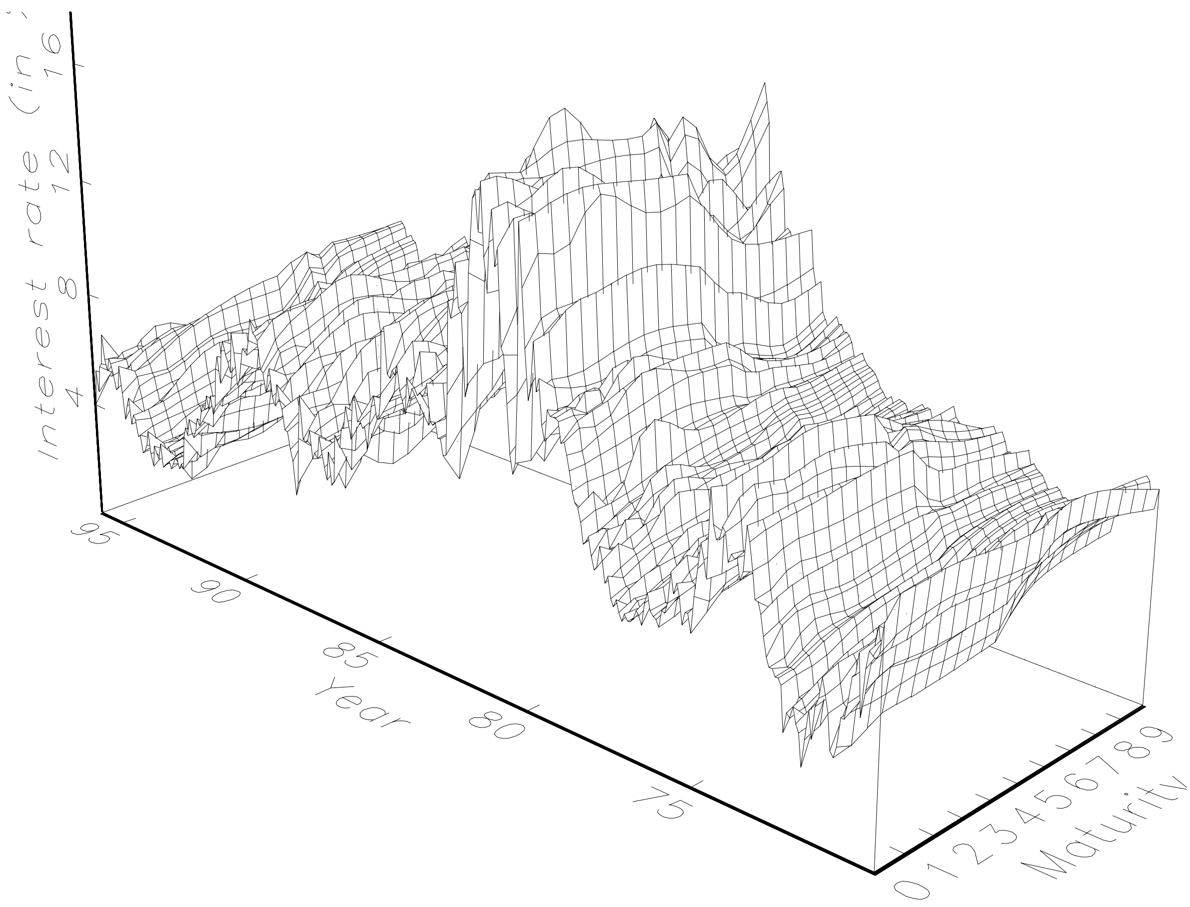


Figure 1: US Interest Rates

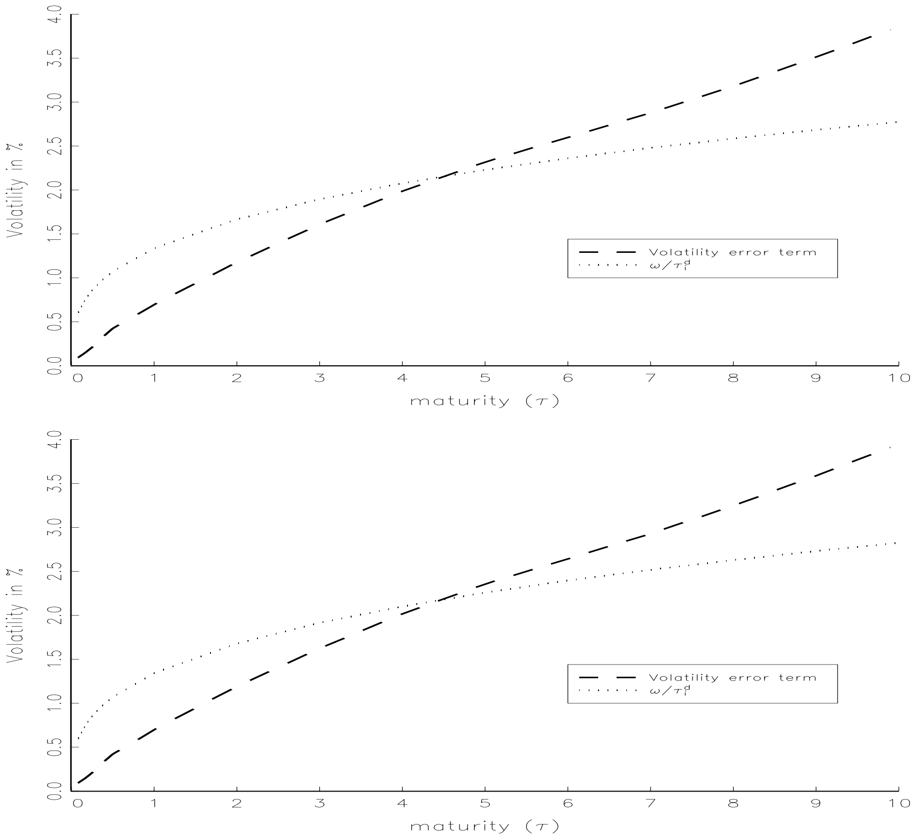
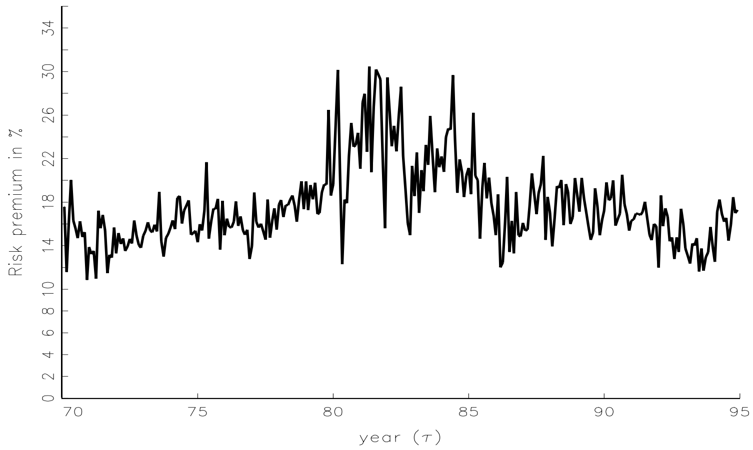


Figure 2: Volatilities error term

Each of the panels shows the implied volatility functions of the error terms for the panel data models. The top panel shows results for the pooled regression model and the bottom figure for the fixed maturity effects model.



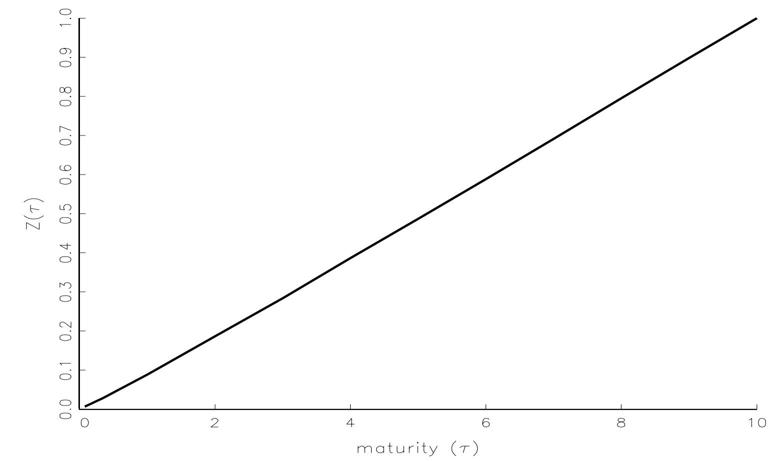


Figure 3: Risk premium specification

The top figure shows the time series process for the risk premium related to the 10 year yield. In the bottom figure the cross sectional relation to this risk premium is given.

Table 1: Summary statistics

(A) Yield Levels: $Y_t(\tau)$

Maturity	Average	Standard Deviation	Minimum	Maximum	Auto Correlation
1 month	6.99	2.98	2.49	16.97	0.904
2 months	7.05	2.87	2.52	16.37	0.962
3 months	7.13	2.83	2.64	15.93	0.971
4 months	7.21	2.82	2.90	15.96	0.968
5 months	7.30	2.82	2.89	16.21	0.962
6 months	7.38	2.83	2.92	16.55	0.960
1 year	7.54	2.65	2.94	15.89	0.968
2 years	7.89	2.52	3.77	16.31	0.973
3 years	8.07	2.39	4.20	15.88	0.975
4 years	8.23	2.33	4.52	16.00	0.978
5 years	8.33	2.23	4.81	15.50	0.979
6 years	8.42	2.16	4.99	14.93	0.980
7 years	8.49	2.14	5.16	14.71	0.982
8 years	8.54	2.14	5.32	14.80	0.983
9 years	8.56	2.13	5.47	15.04	0.983
10 years	8.58	2.12	5.57	15.28	0.983

(B) Lagged Forward Rates minus Yields : $F_{t-1,t}(\tau) - Y_t(\tau)$

Maturity	Average	Standard deviation	Minimum	Maximum	Auto Correlation	Average Risk Premium
1 month	0.13	1.11	-3.13	4.61	0.300	0.01
2 months	0.14	0.88	-2.77	4.75	0.287	0.02
3 months	0.16	0.84	-2.69	4.85	0.091	0.04
4 months	0.17	0.85	-2.55	4.87	-0.014	0.06
5 months	0.16	0.87	-2.47	4.82	-0.070	0.07
6 months	0.13	0.85	-2.72	4.72	-0.053	0.07
1 year	0.09	0.70	-2.92	4.21	0.113	0.09
2 years	0.05	0.59	-2.85	3.53	0.161	0.10
3 years	0.05	0.54	-2.42	2.78	0.115	0.15
4 years	0.04	0.50	-2.18	2.29	0.152	0.14
5 years	0.03	0.46	-2.08	1.93	0.160	0.16
6 years	0.03	0.43	-1.95	1.68	0.124	0.16
7 years	0.02	0.41	-1.78	1.59	0.083	0.16
8 years	0.02	0.40	-1.58	1.58	0.066	0.15
9 years	0.02	0.39	-1.38	1.57	0.079	0.14
10 years	0.01	0.39	-1.17	1.57	0.089	0.13

Table 2: Correlation matrix

(A) Yield Levels: $Y_t(\tau)$

τ	1/12	2/12	3/12	4/12	5/12	6/12	1	2	3	4	5	6	7	8	9	10
$\frac{1}{12}$	1.00	0.98	0.96	0.95	0.95	0.95	0.94	0.91	0.90	0.88	0.87	0.85	0.83	0.82	0.82	0.82
$\frac{2}{12}$	0.98	1.00	0.99	0.99	0.98	0.98	0.97	0.95	0.93	0.91	0.89	0.87	0.86	0.84	0.84	0.85
$\frac{3}{12}$	0.96	0.99	1.00	1.00	0.99	0.99	0.98	0.96	0.93	0.92	0.90	0.88	0.86	0.85	0.85	0.85
$\frac{4}{12}$	0.95	0.99	1.00	1.00	1.00	1.00	0.99	0.96	0.94	0.92	0.91	0.89	0.87	0.86	0.85	0.86
$\frac{5}{12}$	0.95	0.98	0.99	1.00	1.00	1.00	0.99	0.97	0.94	0.93	0.91	0.89	0.87	0.86	0.86	0.86
$\frac{6}{12}$	0.95	0.98	0.99	1.00	1.00	1.00	0.99	0.97	0.95	0.93	0.91	0.89	0.88	0.87	0.86	0.87
1	0.94	0.97	0.98	0.99	0.99	0.99	1.00	0.99	0.97	0.96	0.95	0.93	0.91	0.90	0.90	0.90
2	0.91	0.95	0.96	0.96	0.97	0.97	0.99	1.00	0.99	0.99	0.98	0.97	0.96	0.95	0.95	0.95
3	0.90	0.93	0.93	0.94	0.94	0.95	0.97	0.99	1.00	1.00	0.99	0.99	0.98	0.97	0.97	0.97
4	0.88	0.91	0.92	0.92	0.93	0.93	0.96	0.99	1.00	1.00	1.00	0.99	0.99	0.98	0.98	0.98
5	0.87	0.89	0.90	0.91	0.91	0.91	0.95	0.98	0.99	1.00	1.00	1.00	0.99	0.99	0.99	0.99
6	0.85	0.87	0.88	0.89	0.89	0.89	0.93	0.97	0.99	0.99	1.00	1.00	1.00	1.00	0.99	0.99
7	0.83	0.86	0.86	0.87	0.87	0.88	0.91	0.96	0.98	0.99	0.99	1.00	1.00	1.00	1.00	0.99
8	0.82	0.84	0.85	0.86	0.86	0.87	0.90	0.95	0.97	0.98	0.99	1.00	1.00	1.00	1.00	0.99
9	0.82	0.84	0.85	0.85	0.86	0.86	0.90	0.95	0.97	0.98	0.99	0.99	1.00	1.00	1.00	1.00
10	0.82	0.85	0.85	0.86	0.87	0.90	0.95	0.97	0.98	0.99	0.99	0.99	1.00	1.00	1.00	1.00

(B) Lagged Forward Rates minus Yield: $F_{t-1,t}(\tau) - Y_t(\tau)$

τ	1/12	2/12	3/12	4/12	5/12	6/12	1	2	3	4	5	6	7	8	9	10
$\frac{1}{12}$	1.00	0.86	0.58	0.43	0.42	0.44	0.46	0.42	0.40	0.39	0.40	0.38	0.35	0.33	0.33	0.33
$\frac{2}{12}$	0.86	1.00	0.90	0.75	0.68	0.67	0.69	0.63	0.58	0.56	0.54	0.52	0.50	0.47	0.45	0.44
$\frac{3}{12}$	0.58	0.90	1.00	0.95	0.87	0.84	0.81	0.75	0.69	0.65	0.60	0.58	0.57	0.55	0.52	0.50
$\frac{4}{12}$	0.43	0.75	0.95	1.00	0.97	0.95	0.84	0.79	0.73	0.68	0.61	0.59	0.59	0.57	0.55	0.52
$\frac{5}{12}$	0.42	0.68	0.87	0.97	1.00	0.99	0.82	0.79	0.73	0.68	0.61	0.59	0.59	0.58	0.56	0.54
$\frac{6}{12}$	0.44	0.67	0.84	0.95	0.99	1.00	0.84	0.81	0.76	0.71	0.64	0.63	0.63	0.61	0.59	0.57
1	0.46	0.69	0.81	0.84	0.82	0.84	1.00	0.94	0.89	0.85	0.81	0.79	0.78	0.77	0.74	0.71
2	0.42	0.63	0.75	0.79	0.79	0.81	0.94	1.00	0.97	0.94	0.89	0.88	0.87	0.85	0.82	0.80
3	0.40	0.58	0.69	0.73	0.73	0.76	0.89	0.97	1.00	0.98	0.93	0.92	0.91	0.90	0.88	0.86
4	0.39	0.56	0.65	0.68	0.68	0.71	0.85	0.94	0.98	1.00	0.98	0.96	0.95	0.93	0.91	0.89
5	0.40	0.54	0.60	0.61	0.61	0.64	0.81	0.89	0.93	0.98	1.00	0.99	0.96	0.93	0.91	0.88
6	0.38	0.52	0.58	0.59	0.59	0.63	0.79	0.88	0.92	0.96	0.99	1.00	0.99	0.96	0.92	0.89
7	0.35	0.50	0.57	0.59	0.59	0.63	0.78	0.87	0.91	0.95	0.96	0.99	1.00	0.99	0.96	0.92
8	0.33	0.47	0.55	0.57	0.58	0.61	0.77	0.85	0.90	0.93	0.93	0.96	0.99	1.00	0.99	0.95
9	0.33	0.45	0.52	0.55	0.56	0.59	0.74	0.82	0.88	0.91	0.91	0.92	0.96	0.99	1.00	0.99
10	0.33	0.44	0.50	0.52	0.54	0.57	0.71	0.80	0.86	0.89	0.92	0.95	0.99	1.00	1.00	1.00

Table 3: Results univariate regressions

τ	α	$s.e.(\alpha)$	β	$s.e.(\beta)$	$\sigma[\epsilon(\tau))]$	LR	R^2	DW
$\frac{1}{12}$	0.039	0.173	0.976	0.023	0.092	1.10	0.863	1.39
$\frac{2}{12}$	0.136	0.139	0.962	0.018	0.146	4.60	0.907	1.38
$\frac{3}{12}$	0.159	0.132	0.957	0.017	0.208	6.46	0.914	1.75
$\frac{4}{12}$	0.194	0.136	0.950	0.017	0.280	8.31	0.911	1.96
$\frac{5}{12}$	0.211	0.140	0.950	0.018	0.358	8.16	0.908	2.07
$\frac{6}{12}$	0.175	0.139	0.959	0.017	0.420	5.67	0.912	2.05
1	0.090	0.124	0.977	0.015	0.692	2.27	0.932	1.74
2	0.086	0.114	0.983	0.014	1.174	1.55	0.946	1.65
3	0.117	0.110	0.979	0.013	1.602	2.50	0.950	1.74
4	0.106	0.107	0.982	0.012	1.984	1.90	0.955	1.67
5	0.010	0.105	0.984	0.012	2.315	1.68	0.957	1.65
6	0.110	0.101	0.983	0.012	2.593	1.96	0.960	1.72
7	0.117	0.098	0.983	0.011	2.870	2.22	0.963	1.81
8	0.116	0.095	0.984	0.010	3.176	2.14	0.966	1.84
9	0.113	0.094	0.985	0.011	3.506	2.03	0.967	1.82
10	0.113	0.094	0.985	0.011	3.855	1.91	0.967	1.80

Notes: The table shows the estimation results for the univariate regression of a yield with maturity τ, $Y_{t+1}(\tau)$ on a constant plus the forward rate $F_{t,t+1}(\tau)$. The constant is denoted as α, the slope is β. With $\sigma[\epsilon(\tau))]$ we denote the standard deviation of the error term. We present the corresponding standard errors, R^2 and Durbin-Watson statistic (DW). LR denotes the values of the likelihood ratio statistic for the joint test of $\alpha = 0$ and $\beta = 1$, which has a critical value of $\chi^2_{0.95}(2) = 5.99$. The sample consists of monthly observations for the period January 1970-December 1994.

Table 4: Results pooled regression

	Pooled	Maturity effects
β	0.994	0.945
	(0.001)	(0.056)
ω	1.34	1.34
	(0.052)	(0.056)
ϕ	0.962	0.963
	(0.003)	(0.003)
d	-0.312	-0.323
	(0.003)	(0.003)
lnL	3306	3385

Notes: The table shows the estimation results for the pooled regression of $\tau Y_{t+1}(\tau)$ on $\tau F_{t,t+1}(\tau)$. With β we denote the regression coefficient. The covariance matrix is specified by the parameters ω, ϕ and d, lnL denotes the value of the loglikelihood function in the optimum. In the Pooled model, we pool the data for all yields with different maturities. The Maturity effects model incorporates a dummy parameter for each maturity. The sample consists of monthly observations for the period January 1970-December 1994. Standard errors are within parentheses.

Table 5: Results fixed maturity effects: $\psi(\tau)$

τ	$\psi(\tau)$	se($\psi(\tau)$)	risk premium
$\frac{1}{12}$	0.022	0.035	0.01
$\frac{2}{12}$	0.042	0.044	0.02
$\frac{3}{12}$	0.061	0.051	0.04
$\frac{4}{12}$	0.078	0.056	0.06
$\frac{5}{12}$	0.103	0.061	0.07
$\frac{6}{12}$	0.139	0.065	0.07
1	0.334	0.087	0.09
2	0.775	0.126	0.10
3	1.190	0.166	0.15
4	1.675	0.207	0.14
5	2.142	0.249	0.16
6	2.624	0.292	0.16
7	3.117	0.335	0.16
8	3.613	0.378	0.15
9	4.104	0.422	0.14
10	4.596	0.465	0.13

Notes: The table shows the estimation results for the fixed maturity effects $\psi(\tau)$ and the associated standard errors in the case of the pooled regressions with fixed individual effects. The third column quantifies the risk premium. The sample consists of monthly observations for the period January 1970 - December 1994.

Table 6: Results fixed maturity/random time effects panel model

	I	II
β	0.791	"1"
	(0.005)	-
ω	0.540	0.516
	(0.023)	(0.026)
ϕ	0.834	0.806
	(0.01)	(0.02)
d	-0.252	-0.246
	(0.004)	(0.005)
ρ	0.675	0.115
	(0.045)	(0.055)
μ	17.57	0.206
	(0.99)	(0.31)
σ	4.18	3.68
	(0.165)	(0.173)
lnL	4076	3960

Notes: The tabel reports estimation results for the pooled regression model with a covariance structure that takes account of the correlation between yields. An AR(1) process is incorporated for the 10 year yield risk premium. The remaining risk premia are related to this proces by multiplying with $Z(\tau)$. Model I is the general case, in model II we restrict β to 1. Standard errors are within parentheses.

Table 7: Results risk premium specification: $Z(\tau)$

τ	I (pars)	I (s.e.)	II (pars)	II (s.e.)
$\frac{1}{12}$	0.007	(0.008)	0.008	(0.044)
$\frac{2}{12}$	0.013	(0.008)	0.019	(0.043)
$\frac{3}{12}$	0.020	(0.011)	0.033	(0.062)
$\frac{4}{12}$	0.027	(0.009)	0.047	(0.051)
$\frac{5}{12}$	0.034	(0.011)	0.061	(0.060)
$\frac{6}{12}$	0.042	(0.011)	0.074	(0.059)
1	0.088	(0.012)	0.148	(0.061)
2	0.186	(0.011)	0.275	(0.057)
3	0.284	(0.010)	0.395	(0.048)
4	0.387	(0.009)	0.499	(0.044)
5	0.487	(0.007)	0.575	(0.035)
6	0.589	(0.010)	0.649	(0.049)
7	0.692	(0.011)	0.732	(0.064)
8	0.795	(0.009)	0.823	(0.053)
9	0.898	(0.005)	0.917	(0.032)
10	1	-	1	-

Notes: The tabel reports the estimation results for $Z(\tau)$, which relates the risk premium of the yield to the risk premium process for the 10 year yield. Model I is the most general case and in model II we restrict β to 1. Standard errors are within parentheses.