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ABSTRACT

Noisy Financial Signals and Persistent Effects of Nominal Shocks in Open Economies*

Floating exchange rates display substantial short-run volatility causing a non-trivial information problem in disentangling temporary from permanent changes. Although agents observe current market signals they are imperfectly informed about the future, but they accumulate information and learn over time. We analyse how this basic information problem in the presence of one-period nominal contracts affects the dynamic adjustment process to nominal shocks. Specifically we use a general equilibrium two-country model with specialized production and one-period nominal contracts and consider the propagation of nominal shocks. Informational problems are shown to have important qualitative and potentially strong quantitative importance for the propagation of nominal shocks.

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NON-TECHNICAL SUMMARY

Movements in exchange rates attract much attention and a great deal of resources are absorbed in interpreting the developments. Market participants and the media exert much effort in trying to interpret current market signals in an attempt to infer information of relevance for the future to answer the pertinent question of whether a given change is temporary or permanent. This information problem is complicated by the common perception that financial markets in general and exchange-rate markets in particular are significantly influenced by short-run noise of little relevance for market developments over the medium to long run.

This perception is confirmed by empirical evidence documenting that floating exchange rates display substantial short-run volatility but also that the adjustment of relative prices between countries (real exchange rates, terms of trade) to nominal exchange rate changes is very slow with half-lives as long as 3–4 years. Are the two observations related in the sense that inertia is induced by the information problem created by noise and the implied problem of distinguishing temporary from permanent changes?

Information problems arising due to confusion between temporary and permanent changes have direct implications for how markets operate but also wider macroeconomic implications since a number of decisions including wage and price decisions depend on current and future movements of the exchange rate. Hence, even though agents can observe all current market signals this does not imply that they are fully informed about future developments. However, information is accumulated over time and thereby agents may learn whether shocks were temporary or permanent. This learning process is critical for expectations formation and therefore the dynamic adjustment path in the economy.

The aim of this Paper is to analyse how imperfect information and learning affect the dynamic adjustment process, and in particular whether it can account for inertia and persistence in the effects of shocks. Specifically we consider the adjustment process generated by monetary shocks. This is motivated both by the fact that monetary shocks are often perceived to play an important role in observed exchange rate changes and partly by the fact that the theoretical literature has had difficulties in accounting for persistent effects of monetary shocks. To break classical neutrality such that nominal shocks have real effects we assume one-period nominal wage contracts. The contracts are thus so short as not to produce any interesting or plausible dynamic adjustment pattern by themselves.

To analyse this situation a general equilibrium framework is needed such that we can take account of the interdependency between information, exchange rates and crucial macrovariables including prices and wages. Specifically we develop a fully specified intertemporal two-country model with a flexible exchange rate. We assume rational expectations to demonstrate that informational problems can have crucial effects on the dynamics, even if all agents exploit the information contained in current information signals efficiently. A distinguishing feature of the present Paper is that we want to be precise about the process followed by endogenous variables including their persistence properties. To this end we solve explicitly for the analytical solution.

The main findings of the Paper are

(i)   Informational problems add interesting and plausible dynamics which would be absent under full information. Monetary shocks have effects which last well beyond the length of nominal contracts reflecting the inertia induced by learning. Numerical illustrations indicate that the quantitative importance is non-trivial.

(ii)   Noise (temporary shocks) leads to more volatility in nominal exchanges rates as well as longer adjustment lags for permanent shocks

(iii)   The adjustment pattern depends critically on the type of shock (temporary or permanent) and the information structure. Delayed overshooting of nominal exchange rates may arise.

(iv)   Interest rates may systematically differ and the spread displays strong persistency. A positive (negative) interest rate spread can arise alongside an appreciating (depreciating) currency.

(v)   Nominal shocks are not neutral in the long run. Due to short-run nominal rigidities it follows that nominal shocks induce wealth reallocation between countries via current account adjustment and due to consumption smoothing this translates into long-run real effects of nominal shocks. The size of the long-run effects depends critically on the information structure.

The present analysis has thus confirmed that the information content of the signals generated from the financial system are crucial for the adjustment process to shocks and therefore the business cycle. Even under rational expectations formation it follows that noise (temporary shocks) and imperfect information may be a source of volatility as well as persistence in the effects of shocks. We have considered nominal shocks in this Paper since it is most difficult to account for persistent effects of such shocks, but obviously the underlying information problems also apply to real shocks and the main findings can easily be generalized to such shocks.

1 Introduction

Movements in exchange rates attract much attention and a great deal of resources are absorbed in interpreting the developments. Interpretation of any change in the exchange rate inevitably raises the question whether it is temporary or permanent.[1] Information on this issue is clearly of value to traders in the foreign exchange markets, but also more widely as a number of decisions are dependent on exchange-rate movements in particular price and wage decisions. Time reveals whether given shocks were temporary or permanent, and this implies that information accumulation over time induces a learning process influencing the dynamic adjustment path.

Empirical evidence also supports that this is an important problem. Substantial short-run volatility is a well-known characteristic of foreign exchange markets as is large and lasting changes in exchange rates (Rogoff, 1996). It is also a well-established empirical fact that nominal exchange rates have persistent effects on relative prices between countries. It is an important puzzle to open-economy macroeconomics to explain the empirical evidence indicating that the half-life of shocks on relative prices can be as long as 3-4 years (see, for example, McDonald, 1999, for a survey). Since it moreover seems hard to explain the above-mentioned facts without leaving a role for nominal shocks, it is natural to question what role nominal shocks have when agents are unable to distinguish temporary from permanent changes.[2]

This paper studies how informational problems affect the transmission of nominal shocks into the real side of the economy within an explicit intertemporal general equilibrium model. The focus will in particular be on the adjustment process and the problem of persistence. To this end we need to model the sources of exchange-rate changes.

In a flexible exchange-rate regime changes in supply and demand translate immediately into changes in the exchange rate. It follows that changes in exchange rates may originate from various forms of shocks arising on either the demand or the supply side. These shocks could be real or monetary in nature and leave a non-trivial problem of separating temporary from permanent changes in the exchange rate. Building this problem into a fully specified

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[1] It is fairly agreed on that day-to-day movements in exchange rates are infected by noise and must be explained by market micro structure. But on medium and even long term traders do not think they can predict the exchange rate either (Cheung and Chinn, 1999), making the present analysis even more relevant.

[2] The importance of distinguishing between temporary and permanent shocks goes back to Muth's (1960) discussion of the possible optimality of adaptive expectations, see also Sargent (1982). In standard macromodels the idea has been explored by Andersen (1985), Brunner et al (1983) and Gertler (1982).

general equilibrium model is by no means trivial since it requires not only a specification of shocks which have temporary and permanent components but also an account of how these shocks affect the agents (preferences, endowments, technology, etc).

To simplify and to focus on the role of nominal shocks we analyze shocks arising in the financial sector. Thereby we also address the more difficult problem of explaining persistent effects of nominal shocks. An essential source of movements in exchange rates may be changes in the (relative) supply of liquidity due to changes in either the money base or the liquidity created by the financial system. A variety of causes can thus give rise to a change in liquidity and leave a problem of disentangling temporary from permanent changes. Since the information problem of interpreting changes is essential to our story we exploit the model simplification which can be achieved by assuming that changes in the available amount of liquidity are driven by changes in the money base which are either temporary or permanent. It is assumed that all current information is freely available, but agents face the problem of making inferences about its implications for the future. While the specific modeling approach taken here builds on the monetary (consumption) approach to exchange-rate determination, we think that the insights provided go beyond this specific way of modelling foreign exchange markets. The specific modeling approach adopted has the virtue of avoiding a specification of the financial system in great detail. The point that the determinants of exchange rates can have both temporary and permanent components is generic to any model of exchange-rate determination.

A substantial amount of effort has been put into analyzing the empirical role of monetary shocks for observed international business-cycle fluctuations including exchange-rate movements. Our reading of the available evidence is that nominal shocks play a role, while it is less clear to what extent they are the dominant type of shock.[3] In the following we analyze the interplay between information problems and monetary shocks, not because monetary shocks necessarily are the empirically dominant type of shock, but because the theoretical literature so far has been unable to give a convincing explanation why nominal shocks have persistent real effects. In the same vein we assume one-period nominal contracts since it is well known from the literature that this type of contract does not generate any interesting dynamics absent information problems. The reference point is thus a setting where nominal shocks may have an impact effect due to one-period nominal con-

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[3]Often cited papers are Clarida and Gali (1994) and Eichenbaum and Evans (1995). More recent contributions are Canova and de Nicoló (1998) and Rogers (1999). All find empirical support for the hypothesis that monetary shocks play a role (although to varying degrees).

tracts without generating any interesting dynamics, ie the impulse responses are trivial and implausible. We ask the simple question: to what extent is this changed when agents face the basic problem of disentangling temporary from permanent shocks?

The present analysis is related to the general discussion on propagation mechanisms in business-cycle models. It is a well-known fact that the inherent propagation mechanism in closed-economy models is weak (see eg Cogley and Nason, 1995). This implies that even though short-run nominal rigidities can induce nominal shocks to have real effects, the effects are not persistent since the propagation mechanism is too weak. For real shocks this problem can be circumvented by assuming persistence in the underlying shock. Obviously this procedure cannot readily be applied to nominal shocks since it is only the unanticipated part of the shock which has real effects. This has spurred an interest in strengthening the propagation mechanism via the introduction of staggered nominal contracts. While empirically relevant and strengthening propagation, it does not seem that this mechanism in itself can generate sufficient persistence in the adjustment process. The present paper takes an alternative route and shows that nominal shocks have persistent real effects when agents face the information problem of separating temporary from permanent shocks even if nominal contracts only have short duration.

The specific structure of the model builds on the so-called New Open-Economy Macroeconomics launched by Obstfeld and Rogoff (1995) and which is a rapidly expanding field.[4] The literature has explored the consequences of both nominal price and wage contracts. We assume nominal wage contracts (see section 2.3) and a brief list of related New Open-Economy Macroeconomics papers all with sticky wages and perfect information include; Andersen and Beier (1999) who investigate a version of this paper's model with staggering as the propagation strengthening mechanism; Hau (1999) focuses on the role of nontradables on exchange-rate dynamics in a deterministic model; Kollmann (1999) solves a stochastic model with both Calvo-style price and wage staggering numerically, and try to mimic cross country correlations of macro variables; and Obstfeld and Rogoff (1999) present a single-period stochastic model with exact closed form solutions, where they analyze the effects on welfare, expected output and expected terms of trade of the monetary regime. They rule out wealth reallocations by assumption.[5]

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[4]For an excellent survey see Lane (1999). Several papers can be found on the New Open-Economy Macroeconomics homepage http://www.princeton.edu/ ~bmdoyle/open.html.

[5]Apart form these sticky-wage papers there is a large literature on pricing-to-market and local-currency-pricing. From this literature Bergin and Feenstra (1999a,b) should be mentioned too. They consider pricing-to-market and find persistence in exchange rates

A basic lesson of open-economy models with one-period nominal wage or price contracts (see Lane, 1999) is that although nominal shocks have real effects, the dynamic implications are not very interesting since the steady-state effect is reached already after one period. Due to wealth reallocation induced by the impact effect and consumption smoothing, the shock has a permanent effect on relative prices (opposite in sign to the impact effect).

A distinguishing feature of the present paper is that we want to be precise about the processes followed by the endogenous variables including their persistence properties. To this end we solve explicitly for the analytical solution to an intertemporal general equilibrium open-economy model admitting wealth reallocations between countries and including nominal wage contracts as well as the information problem of separating temporary from permanent shocks. We show that this setting produces more rich and plausible dynamics.

A closely related paper considering how problems of imperfect information and learning affect exchange-rate dynamics is Gourinchas and Tornell (1996).[6] There are, however, some notable differences since; (i) they postulate an exogenous process for the interest rate and derive the exchange rate from the equilibrium condition for the bond market; (ii) intertemporal aspects or interactions between the real and nominal side (including nominal rigidities) are disregarded; (iii) they allow for agents misperceiving the parameters of the stochastic process. That said, the intuition underlying some of their results applies to our model as well.

The paper is organized as follows: The intertemporal two-country model with a flexible exchange rate is set up in section 2. The stochastic process for money and the information structure are defined in section 3. Section 4 describes the equilibrium. Section 5 considers the dynamics of nominal shocks. In section 6 we present numerical illustrations. Discussion and concluding remarks are offered in section 7.

2 A Stochastic Two-Country Model

Following Obstfeld and Rogoff (1995) we consider a symmetric two-country model with a flexible exchange rate and specialized production. There are two equally-sized countries and two goods, one produced by Home and one

in a model with price staggering, translog preferences and intermediate inputs. See Lane (1999) for other references and Obstfeld and Rogoff (1999) for a discussion on these pricing assumptions.

[6]For recent analyses of the relation between information and nonneutrality of nominal shocks, see Andersen (1997) and Kiley (1996).

produced by Foreign firms. There are two assets in the economy: money and a real bond, where the former is motivated through saved transactions costs (Feenstra, 1986) and the latter is traded in a perfect capital market. There is no real capital in the model and no internationally mobile labor.

2.1 Consumers

The countries are inhabited by consumers who consume goods, supply labor, and hold money and bonds. The consumers' behavior is determined through maximizing expected lifetime utility. Let E_t be the expectations operator conditional on period-t information (the information structure is defined below), N labor supplied, M nominal balances, C a real consumption index and P the consumer price index, then the consumer's objective function is

$$U_t = E_t \sum_{j=0}^{\infty} \delta^j \left[\frac{\sigma}{\sigma-1} C_{t+j}^{\frac{\sigma-1}{\sigma}} + \frac{\xi}{1-\beta} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-\beta} - \frac{\kappa}{1+\mu} N_{t+j}^{1+\mu} \right],$$

$$\sigma > 0, \quad \xi > 0, \quad \beta > 0, \quad \kappa < 0, \quad \mu > 0, \quad 0 < \delta \leq 1.$$

The real consumption index aggregates across consumption of the Home good (C_t^h) and the Foreign good (C_t^f)

$$C_t = \left[\left(\frac{1}{2} \right)^{\frac{1}{\rho}} \left(C_t^h \right)^{\frac{\rho-1}{\rho}} + \left(\frac{1}{2} \right)^{\frac{1}{\rho}} \left(C_t^f \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad \rho > 1,$$

where ρ is the elasticity of substitution between Home and Foreign goods.[7] The price index corresponding to composite consumption is

$$P_t = \left[\frac{1}{2} \left(P_t^h \right)^{1-\rho} + \frac{1}{2} \left(P_t^f \right)^{1-\rho} \right]^{\frac{1}{1-\rho}},$$

where $P_t^h(P_t^{*h})$ is the price of the Home good in Home (Foreign) currency and $P_t^f(P_t^{*f})$ is the price of the Foreign good in Home (Foreign) currency. As our focus will be on nominal wage rigidity we assume that the law of one price holds for both goods, ie

$$P_t^h = S_t P_t^{*h}, \quad P_t^f = S_t P_t^{*f}.$$

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[7]Assuming $\rho > 1$ ensures that the Marshall-Lerner condition is fulfilled, ie a nominal depreciation leads to an increase in the trade balance and, thus, a wealth reallocation in favor of Home. Andersen and Beier (1999) analyze the role of ρ for the adjustment process in more detail (see also Tille, 1999).

An asterisk denotes Foreign variables and h (f) refers to variables originating in Home (Foreign). S is the nominal exchange rate defined as the Home price of Foreign currency. A direct implication of law of one price is that purchasing power parity holds as well, that is, $P_t = S_t P_t^*$. As a consequence the subsequent analysis will focus on how nominal shocks affect the terms of trade. It can be shown in a setting including nontradables that the movements in the real exchange rate are qualitative equivalent to the movements in the terms of trade.

We assume that there is one internationally traded real bond denoted in the composite consumption good C. Let r_t be the consumption based real interest rate between dates t and $t + 1$. The consumer's budget constraint for any period t is given by

$$P_t B_t + M_t + P_t C_t = (1 + r_{t-1}) P_t B_{t-1} + M_{t-1} + W_t N_t + \Pi_t + P_t \tau_t.$$

The right-hand side gives available resources as the sum of the gross return on bond holdings $(1 + r_{t-1}) P_t B_{t-1}$, initial money holdings M_{t-1}, labor income $W_t N_t$, nominal profit income Π_t and transfers from the government $P_t \tau_t$. Resources are allocated to consumption $P_t C_t$, nominal money holdings M_t and bond holdings $P_t B_t$.

Given the constant elasticity consumption index Home consumers' demands for the Home good and the Foreign good are

$$D_t^h = \frac{1}{2} \left(\frac{P_t^h}{P_t} \right)^{-\rho} C_t, \quad D_t^f = \frac{1}{2} \left(\frac{P_t^f}{P_t} \right)^{-\rho} C_t,$$

respectively, and mutatis mutandis for the demands by Foreign consumers. Aggregating we find total demand for the Home good to be (and similarly for the Foreign good)

$$D_t \equiv D_t^h + D_t^{*h} = \frac{1}{2} \left(\frac{P_t^h}{P_t} \right)^{-\rho} C_t^W,$$

where world consumption $C_t^W \equiv \frac{1}{2} C_t + \frac{1}{2} C_t^*$.

The first-order conditions arising from consumer maximization, written in log-linear form[8], are the Euler equation

$$E_t c_{t+1} = c_t + \sigma \log (1 + r_t), \tag{1}$$

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[8]The model is specified so as to yield a log-linear model. However, log-linearizations are needed for money demand and the budget constraint. See appendix A for details and definitions of constants.

and money demand

$$m_t - p_t = \eta_{mc} c_t - \eta_{mc}^1 E_t c_{t+1} + \eta_{mp} \left(p_t - E_t p_{t+1} \right), \tag{2}$$

where lower-case letters denote the natural logarithms of the corresponding upper-case variables.[9] All constants are neglected, since our primary interest is the adjustment process to shocks.[10] The labor-supply decision and wage setting are dealt with in section 2.3.

For later reference it is noted that (1) implies that relative consumption follows a martingale ie

$$E_t \left(c_{t+1} - c_{t+1}^* \right) = c_t - c_t^*. \tag{3}$$

The log-linearized versions of the price indices are

$$p_t = \frac{1}{2} \left(p_t^h + s_t + p_t^{*f} \right), \quad p_t^* = \frac{1}{2} \left(p_t^h - s_t + p_t^{*f} \right),$$

and the terms of trade are defined as

$$q_t \equiv p_t^h - p_t^f = p_t^h - p_t^{*f} - s_t.$$

2.2 Firms

Firms demand labor and produce the Home good. There is perfect competition in the product markets implying that firms are price takers. The wage is taken as given as well. The good is produced subject to a decreasing returns technology linking output Y^h and labor input N[11]

$$Y_t^h = N_t^\gamma, \quad 0 < \gamma < 1.$$

Maximizing profits subject to technology yields the following labor demand and output supply (in logs)

$$n_t = \eta_{nw} \left(p_t^h - w_t \right), \quad \eta_{nw} = (1 - \gamma)^{-1}, \tag{4}$$

$$y_t^h = \eta_{yw} \left(p_t^h - w_t \right), \quad \eta_{yw} = \gamma \left(1 - \gamma \right)^{-1}.$$

Profits are distributed to households.

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[9]In the rest of the paper η_{xz} denotes the elasticity of the variable X with respect to the variable Z. Superscripts are included when the right hand side variable has more than one entry, eg lagged and leaded variables (cf η_{mc} and η_{mc}^1 in equation [2]).

[10]These constant terms include variance terms which are constant under the stochastic process considered.

[11]Y^h (Y^{*f}) is used as notation for Home (Foreign) output as we leave Y (Y^*) as notation for real incomes (see appendix A).

2.3   Wage Setting

To leave a role for nominal shocks we need to introduce nominal contracts. While nominal rigidities may prevail in both product and labor markets, we focus on one-period nominal wage contracts. Empirical evidence also supports that nominal wage rigidities play a role (see eg Estevão and Wilson, 1998; Obstfeld and Rogoff, 1999; Spencer, 1998). The problem of separating temporary and permanent influences on the exchange rate affects wage setting, and in this way informational problems get a non-trivial role for the interplay between the monetary and the real side of the economy.

We build on a rather extensive literature introducing imperfect competition into the labor market (see eg Moene and Wallerstein, 1993, for a survey and references). Workers are organized in (monopoly) unions, and each union represents a (small) subset of workers supplying labor to a given group of firms.[12] Each union is utilitarian and chooses a wage for period t given all available information in period $t-1$ to maximize the expected utility of workers which in turn depends on the wage income received and the disutility of work. The level of employment is determined by firms given the wage set by the union (right-to-manage structure). Since all unions are identical, we can write the wage decision problem of a representative union as maximizing (equivalent to maximizing the utility of a representative member)

$$E_{t-1} \left(\zeta_t \frac{W_t}{P_t} N_t - \frac{\kappa}{1+\mu} N_t^{1+\mu} \right),$$

where ζ_t measures the shadow value of wage income to the household ($\zeta_t = C_t^{-\frac{1}{\sigma}}$ cf the consumer's optimization problem). The union takes into account that employment is determined according to (4) and the optimal nominal wage to be quoted for period t can now be written

$$W_t = \kappa \frac{\eta_{nw}}{\eta_{nw} - 1} \frac{E_{t-1} \left(N_t^{1+\mu} \right)}{E_{t-1} \left(C_t^{-\frac{1}{\sigma}} \frac{N_t}{P_t} \right)},$$

It is noted that the wage demands of the union are increasing in its market power measured by the wage elasticity of labor demand $(-\eta_{nw})$. Using that all endogenous variables are log-normally distributed (cf below), we can by use of the labor demand (4) write the (log) nominal wage for period t as

$$w_t = E_{t-1} \left[\eta_{wp} p_t^h + \left(1 - \eta_{wp} \right) \left(s_t + p_t^{*f} \right) + \eta_{wc} c_t \right], \tag{5}$$

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[12]It is known from Hart (1982) that by assuming a sufficiently large number of unions, it is possible to maintain the property that they have market power in the labor market without introducing the possibility that they perceive that they can affect the whole economy (eg aggregate prices). By assuming symmetry we are able to attain a simple form for the optimal wage contract.

where

$$\eta_{wp} = (1 + \mu\eta_{nw})^{-1} (0.5 + \mu\eta_{nw}), \qquad \eta_{wc} = [\sigma (1 + \mu\eta_{nw})]^{-1}.$$

For a given wage, employment follows from the labor-demand equation. The wage equation implies that nominal wages and thus prices depend on expected exchange rates. This captures the channel through which exchange rates affect the real side of the economy. Note also that (5) captures the basic homogeneity property which will be generic to any micro-founded wage-setting model.[13]

2.4 Government

We assume that the government balances its budget each period, ie

$$M_t - M_{t-1} = P_t \tau_t.$$

In other words, the only role of the government is to issue money. Money is transferred to Home consumers in a lump-sum fashion. The stochastic process governing money supply along with the assumptions on the information structure are described in detail in the next section.

We end the description of the model by noting that Foreign is completely symmetric and that a (symmetric) equilibrium exists (see appendix A) in which money is neutral absent nominal rigidities.

3 Money Supply and Information Structure

The only source of uncertainty in the model is the (relative) money supply. A straightforward way by which to introduce the problem of separating between temporary and permanent influences is to assume that the relative money-supply process is[14]

$$m_t - m_t^* = z_t + u_t, \tag{6}$$

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[13]It follows that adopting the framework of eg Obstfeld and Rogoff (1999) does not change the results. In their framework there are differentiated labor inputs and CES production (linear in equilibrium). Workers have monopoly power in the labor market, set the wage a period in advance and firms are monopolistic in the product markets. In this setup we would get the same wage equation with $\frac{\eta_{nw}}{\eta_{nw}-1}$ replaced with $\frac{\phi}{\phi-1}$, where ϕ is the substitution elasticity between different kinds of labor inputs.

[14]Since information flows continuously in the foreign exchange market and the wage contracts are assumed to be fixed for a given period of time, it follows that some aggregation of information has already implicitly taken place in transforming financial data to match the length of wage contracts.

and

$$z_t = \theta z_{t-1} + \varepsilon_t, \quad 0 \le \theta \le 1,$$

where u and ε are independent and normally distributed mean-zero shocks with variances σ_u^2 and σ_ε^2.

The money-supply process captures that some changes are temporary (u) and some are permanent (z), and that agents cannot readily disentangle one type of shock from the other. They only observe the sum of the two components. The parameter θ determines the degree of persistence. Agents know current and past realizations of relative money supplies, but they do not know whether current changes are temporary or permanent. Accordingly, agents learn over time as they accumulate information. Notice that the present analysis does not build on the confusion between absolute and relative price changes underlying the New Classical macroeconomic models.

We distinguish between: (i) a *transitory* shock ($u > 0, 0 < \theta \le 1$); (ii) a *persistent* shock ($\varepsilon > 0, 0 < \theta < 1$); (iii) a *fully permanent* shock ($\varepsilon > 0, \theta = 1$). The need to introduce the latter distinction arises because the dynamic properties following a shock to z depend critically on whether θ is strictly less than one or exactly equal to one (see below). When we present our results it is implicit that we condition on the type of shock which has hit the economy. Note, that to avoid confusion we use the terminology *temporary* and *permanent* shocks when we in general discuss the information disentangling problem (cf, for example, the introduction) and when we in general discuss the money supply process (6).[15] Hence, u is labelled as the temporary part, and z the permanent part. When specific shocks are considered, we use the three labels: *transitory*, *persistent* and *fully permanent*.

The specification (6) can be interpreted in several ways. The immediate interpretation is that money-supply (monetary policy) changes may be either temporary or permanent. The temporary component may also represent short-run volatility or noise generated by the financial system (money multiplier), while the permanent part represents the underlying fundamental factors (money base)[16]. Finally, the temporary part may reflect measurement errors or errors included in preliminary data relative to the official data published with some lag.

[15]This means that both persistent and fully permanent shocks in the general discussion are labelled permanent shocks.

[16]The contemporaneous debate on transparency in monetary policy making can be interpreted as a way to minimize the noise component, and thereby provide more information on the fundamentals underlying monetary policy.

Predicting the future money supply is a question of predicting its permanent component, ie

$$E_t \left(m_{t+1} - m_{t+1}^* \right) = E_t \left(z_{t+1} \right).$$

Observations of the relative money supply are in the present set-up the only source of information on its future movements, and it can be shown that the conditional expectation can be written as[17]

$$
\begin{aligned}
& E_t \left(m_{t+1} - m_{t+1}^* \right) \hspace{4cm} (7) \\
& = \; \theta E_{t-1} \left(m_t - m_t^* \right) + \theta h \left[m_t - m_t^* - E_{t-1} \left(m_t - m_t^* \right) \right],
\end{aligned}
$$

where

$$0 \leq \theta h = \theta \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2} \leq \theta.$$

The h-coefficient is increasing in the variability of innovations to the permanent part of the (relative) money stock and decreasing in the variability of the temporary changes. That is, if all shocks are temporary we have $h = 0$, and if all shocks are permanent $h = 1$ reflecting that current signals contain all information of relevance for predicting future money supplies. Subsequently we label h as the signal-noise coefficient.

Expectations of tomorrow's relative money supply are given as a weighted sum of yesterday's expectations of today's relative money supply and the information obtained through today's relative money supply. The former is weighted by the degree of persistence in permanent shocks θ, and the latter is determined by the new information given as the difference between the actual period-t money supply and its expected value (conditional on period $t-1$ information) times θh, since a fraction h of this is perceived to be permanent of which a fraction θ carries forward to the next period.

If we interpret the temporary shock (u) as noise and the permanent part (z) as fundamentals, the updating formula has a very intuitive interpretation. The more noise (larger σ_u^2, smaller h), the less weight is put on the current observation of money supply since agents know that current movements tend to reflect noise, ie current signals have a low information content.

Since the updating of expectations to shocks is crucial to the results of this paper, it is useful to consider the process in some detail. Figure 1 describes the adjustment path for the actual and (un)expected money stock

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[17]All derivations and proofs can be found in a longer version at http://www.econ.au.dk/afn/default.htm (working paper 1999-28).

to transitory, persistent and fully permanent shocks, respectively. In all cases we consider a 1 percent positive shock to the relative money stock.[18]

Figure 1 about here

For a transitory shock (upper panel, drawn for $\theta = 1$, $h = 0.5$) we see that although the relative money supply is only affected in one period, it takes several periods for the agents to learn this. Accordingly a positive transitory shock will imply that the money stock is unanticipated low in subsequent periods until agents eventually learn that the shock was transitory. The learning period is prolonged and therefore persistence is increasing when θ is increasing and h decreasing. This follows since increasing θ leads to an increased weight on the first term in (7) and decreasing h leads to an decreased weight on the second term.

Next we turn to a persistent shock (middle panel, drawn for $\theta = 0.9$, $h = 0.5$) in which case it also takes several periods before expectations and de facto money converge, ie money is unexpectedly high. Furthermore, it is interesting to see that there is delayed overshooting. Initially expectations rise, and first several periods later they begin to fall. Delayed overshooting occurs when

$$E_t \left(m_{t+1} - m_{t+1}^* \right) < E_{t+1} \left(m_{t+2} - m_{t+2}^* \right),$$

and this is ensured if

$$1 - 2\theta + \theta h < 0.$$

This condition will turn up later when we reach exchange-rate dynamics. The effects of changes in θ and h are the same as for a transitory shock.

Finally, for a fully permanent shock (lower panel, drawn for $\theta = 1$, $h = 0.5$) we find that expectations slowly converge to the actual money stock from below. Furthermore, the learning period is prolonged the smaller is h. This is due to the fact that agents perceive current observations to be highly infected by noise. Despite being surprised the period after the shock, agents still perceive this 'new' surprise to be due to noise in the money market, until they eventually learn that it was a fully permanent shock.

Evidently, the dynamics of relative money and its expected value differ across the three types of shocks. These differences will turn out to be crucial when we turn to the dynamic adjustment of the other variables of the model.

[18]We look at dynamics of expectations under the assumption that they have been zero up to date 1, where a one-time 1 percent positive monetary shock hits the economy. No shock occurs after that.

4 Equilibrium

Characterizing the equilibrium analytically is not only complicated by the presence of nominal contracts and the information problem but also the intertemporal structure linking current and future decisions via changes in wealth and expectations. We demonstrate in appendix B how to find an analytical solution such that we can solve explicitly for the processes for the endogenous variables given the process for the exogenous variable (the relative money stock) and the information structure. Specifically it is demonstrated that the equilibrium process for the nominal exchange rate is given as[19]

$$\Phi_{ss}(L)s_t = \Phi_{sm}(L)\left(m_t - m_t^*\right),\tag{8}$$

where

$$\Phi_{ss}\left(L\right) = 1 - \phi_{ss}^1 L - \phi_{ss}^2 L^2, \quad \Phi_{sm}\left(L\right) = \phi_{sm} + \phi_{sm}^1 L + \phi_{sm}^2 L^2,$$

and L is the lag-operator.[20] Thus the dynamics of the nominal exchange rate is determined by second-order lag polynomials for both the autoregressive and moving average parts.

Similarly, the terms of trade can be written

$$\Phi_{qq}(L)q_t = \Phi_{qm}(L)\left(m_t - m_t^*\right),\tag{9}$$

where

$$\Phi_{qq}(L) = \Phi_{ss}\left(L\right), \quad \Phi_{qm}\left(L\right) = \phi_{qm} + \phi_{qm}^1 L + \phi_{qm}^2 L^2.$$

Finally, relative consumption can be written

$$\Phi_{cc}(L)\left(c_t - c_t^*\right) = \Phi_{cm}(L)\left(m_t - m_t^*\right),\tag{10}$$

where

$$\Phi_{cc}(L) = \Phi_{qq}(L) = \Phi_{ss}\left(L\right), \quad \Phi_{cm}\left(L\right) = \phi_{cm} + \phi_{cm}^1 L.$$

Informational problems generate a complicated dynamic adjustment path for both the nominal exchange rate, the terms of trade and relative consumption following nominal shocks. In particular we have non-trivial dynamics running beyond the length of nominal contracts (here one period).

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[19]We omit the exact derivations and expressions as they do not add enough information relative to the space they consume. The reader is referred to the appendix for more technical details and the longer version for in-depth proofs. See footnote 17.

[20]Along the same line as η_{xy} denotes the elasticity of a variable X with respect to another variable Y, $\Phi_{xx}\left(L\right)$ denotes the autoregressive lag polynomial determining the dynamics of the variable X. The entries are given by $\phi_{xx}^1 L$, $\phi_{xx}^2 L^2$, etc. Similarly, the moving-average part has entries ϕ_{xm}, $\phi_{xm}^1 L$, etc.

5   Dynamics Under Imperfect Information

We now turn to a detailed analysis of the dynamic adjustment path under imperfect information. In particular we lay out the economy's response following an increase in relative money supply and we focus on the impact effects, the dynamics and the long-run effects. Numerical illustrations are given in the next section.

For later comparison, note that the dynamics can easily be characterized under perfect information. Consider for instance the terms of trade which would respond to a one-time positive money shock as

$$q_t < 0, \text{ and } q_{t+j} = q_{t+i} > 0, \quad \forall j,i \quad j \geqslant 1, \quad i \geqslant 1,$$

that is, the impact effect of the shock - due to one-period nominal wage contracts - is a terms-of-trade deterioration while the long-run effect is an improvement attained already after one period.[21] In the absence of information problems the adjustment to unanticipated shocks is ended after a period of time equal to the contract length, that is, the dynamics is trivial and the impulse-response functions are implausible. This also applies for relative consumption and the nominal exchange rate. Next we explore how this is changed under imperfect information.

5.1   Impact Effects

The impact effect of an expansion in (relative) Home money supply (regardless of the type of shock) is a depreciation of the nominal exchange rate and a terms-of-trade deterioration

$$\frac{\partial s_t}{\partial (m_t - m_t^*)} = \phi_{sm} > 0, \quad \frac{\partial q_t}{\partial (m_t - m_t^*)} = \phi_{qm} < 0,$$

as a result of the one-period nominal wage contracts. Consequently, demand is switched towards the Home good, Home output increases and Home run a current-account surplus. This leads to a wealth increase which via consumption smoothing induces an increase in relative consumption

$$\frac{\partial (c_t - c_t^*)}{\partial (m_t - m_t^*)} = \phi_{cm} > 0.$$

Consumption only reacts to unanticipated changes in wealth and thus money supply. The impact effects differ across information regimes and the relative

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[21]The perfect information relative money supply is $m_t - m_t^* = \theta \left(m_{t-1} - m_{t-1}^* \right) + \varepsilon_t$, where $\theta \in [0,1]$.

magnitude of this difference is determined by two things; the persistence parameter (or type of shock) θ and the signal-noise coefficient h.

Interpretation is facilitated by noting that the impact effects can be written as a weighted average of the impact effects of a temporary and a permanent shock under perfect information. More specifically, assume that $\theta = \widetilde{\theta} \in (0, 1]$, then with obvious notation

$$\text{Impact Eff}\big|_{\theta=\widetilde{\theta}}^{\text{imp info}} = (1 - h)\, \text{Impact Eff}\big|_{\theta=0}^{\text{per info}} + h\, \text{Impact Eff}\big|_{\theta=\widetilde{\theta}}^{\text{per info}},$$

where[22]

$$h = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2}, \ \text{ and } \ \text{Impact Eff}\big|_{\theta=0}^{\text{per info}} < \text{Impact Eff}\big|_{\theta=\widetilde{\theta}}^{\text{per info}}.$$

It follows immediately that the impact effect of persistent or fully permanent shocks are smaller, and oppositely that the impact effect of a transitory shock is larger under imperfect information. The intuition is straightforward; part of a transitory shock is taken to be permanent and vice versa for a permanent shock. Moreover we have that more information confusion (low h) lowers the impact effects as more weight is put on a given shock being transitory. Similarly, the impact effects under imperfect information depend positively on θ since the impact effect under perfect information is increasing in it.

The larger sensitivity to transitory shocks due to imperfect information can be interpreted in the sense that the impact of noise on exchange rates and other macroeconomic variables is increased. This confirms the usual perception that noise and excess volatility are positively related.

5.2   Dynamics

The equilibrium process for the endogenous variables already showed that informational problems result in a much more complicated dynamic adjustment path displaying persistence compared to the simple and implausible dynamics arising in the benchmark case of full information. The reason

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[22]Even under perfect information the impact effect depends on the type of shock. Following a positive monetary shock the real affects depend on the nominal depreciation, which in turn depends on the change in the money stock, consumption and expected future exchange rates. The depreciation can not be independent of the type of shock for the simple reason that it would imply the same real effects and thus the same change in (expected) future consumption, but that is inconsistent with the condition for money market equilibrium in the future which depends on the extent to which the money change was permanent. The nominal depreciation therefore has to be smaller for a temporary than for a permanent change in the money supply to ensure equilibrium in the money market.

why the adjustment process to a shock is not ended after one period is that agents over time acquire more information of relevance in deciding whether changes were temporary or permanent. Thus, there is a non-trivial dynamic adjustment process driven by accumulation of information over time.

Considering the interim dynamic process for the variables in more detail, it is interesting to note that the autoregressive part of the nominal exchange rate, the terms of trade and relative consumption are equivalent. This indicates that persistence in the adjustment process spreads out to all variables. Furthermore, the autoregressive parameters depend only on the parameters characterizing the information structure (θ, h) since

$$\phi_{ii}^1 = 1 + \theta \left(1 - h\right) \in [1, 2], \quad i = s, q, c,$$

$$\phi_{ii}^2 = 1 - \phi_{ii}^1 = \theta \left(h - 1\right) \in [-1, 0], \quad i = s, q, c,$$

and there is a unit root in the autoregressive part, ie

$$\phi_{ii}^1 + \phi_{ii}^2 = 1, \quad i = s, q, c.$$

This brings out that the information structure has a potentially important role for the dynamic adjustment process, even if nominal contracts only have a duration of one period. Specifically we find that more persistence in the permanent part of the shocks (high θ) and more information confusion (low h) generate the strongest persistence in the response of the variables following nominal shocks. A high θ means that a large weight is put on changes being persistent and, therefore, on last periods expectations, and a low h means that little new information of relevance for predicting future money supplies is obtained from the most recent observations, cf (7).

In the New Open-Economy Macroeconomics literature there is a growing consensus that the determinants of persistence in models with staggering are (i) marginal costs' sensitivity with respect to output; (ii) prices' sensitivity with respect to marginal costs (see Lane, 1999), and researchers try to find conditions under which the sensitivity is low in both cases. In this model the persistence is generated without these considerations as the persistence parameters only depend on the information structure (θ, h).

5.3   Delayed Overshooting

Eichenbaum and Evans (1995) present impulse-response functions to monetary shocks which display delayed overshooting[23], that is, following an (unexpected) expansionary monetary policy the nominal exchange rate first follows

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[23]Faust and Rogers (1999) challenge whether this is a robust finding.

a depreciating path and then appreciates towards its long-run value. This is different from the overshooting phenomenon arising in the seminal work by Dornbusch (1976) in which the nominal exchange rate depreciates on impact and then appreciates. It turns out that a persistent shock may produce delayed overshooting, since we have that

$$\frac{\partial s_t}{\partial \varepsilon_t} < \frac{\partial s_{t+1}}{\partial \varepsilon_t},$$

and for some period in time $t + j$ (the shock fades out in the long run)

$$\frac{\partial s_{t+j+1}}{\partial \varepsilon_t} < \frac{\partial s_{t+j}}{\partial \varepsilon_t}, \quad j \geq 1,$$

is a possible dynamic adjustment pattern. A necessary condition for delayed overshooting is that

$$1 - 2\theta + \theta h < 0,$$

implying that a combination of a high degree of persistence in the shock (high θ) and much noise (low information content of signals, ie low h) tend to create this phenomenon. This condition on θ and h is the sufficient condition for generating delayed overshooting in money expectations (see section 3). Intuitively, this has to be fulfilled for delayed exchange-rate overshooting to occur at all, but it is not enough. The reason is that h can become too small, or equivalently, learning can become too slow.

To what extent does this capture the delayed overshooting found by Eichenbaum and Evans (1995)? There are two difficulties in interpreting their results relative to the present setting since their analysis does not allow for an explicit distinction between temporary and permanent shocks and since their results of course depend on the actual realization of temporary and permanent shocks over the sample period. With respect to the latter it might be that there are unusually many persistent shocks (compared to population moments) in the (small) sample they consider.

Finally, note that the result above relates to conditional delayed overshooting, that is, conditionally on a persistent shock delayed overshooting is possible. Unconditional delayed overshooting can be shown not to be possible in the present setting. Gourinchas and Tornell (1996) find that unconditional delayed overshooting can arise if agents misperceive the stochastic process underlying the shocks.

5.4 Long-Run Effects

Nominal shocks do in general have long-run real effects due to wealth reallocations between countries. Via consumption smoothing a change in wealth is transformed into a change in consumption. The unit root implied by consumption smoothing is thus recovered in the nominal exchange rate, the terms of trade and relative consumption. The long-run effects obviously depend on the type of shock as well as the information structure. The table given below summarizes the signs of the long-run effects

Table 5.1. Long-Run Effects.

	s	q	$c - c^*$	$\log(\frac{i}{i^*})$
Transitory	≤ 0	≥ 0	≥ 0	0
Persistent	< 0	> 0	> 0	0
Fully Permanent	$\gtreqless 0$	> 0	> 0	0

To see the intuition for these long-run effects note that the shock due to the nominal wage contracts creates a current account surplus which allows higher consumption in all future periods. This affects labor supply and therefore in turn all real variables. The unanticipated wealth changes induced by the nominal shock drive the long-run real effects. The interest rate spread $[\log\left(\frac{i}{i^*}\right)]$ is unaffected in the long run irrespective of the type of shock since the nominal exchange rate is constant in the long run and the interest rate spread is determined by uncovered interest rate parity (see next subsection).

Two special cases should be mentioned briefly. While even transitory shocks in general have long-run real effects this does not hold in the special case where the underlying permanent shocks are fully permanent ($\theta = 1$). The reason is that the initial positive impact effect on relative consumption is subsequently redressed as agents perceive part of the shock to be fully permanent ($\theta = 1$), when the shock is actually transitory. It turns out that the positive and negative surprises precisely balance in the long run for $\theta = 1$ while for $\theta < 1$ the positive surprises always dominate. Another puzzling finding is that a fully permanent shock may lead to an appreciation of the nominal exchange rate. While the likely long-run effect is a depreciation, overshooting and a long-run appreciation is possible for extreme parameter values, more specifically for a very low value of h.

Under imperfect information the long-run effects differ between the three types of shocks, reflecting that the learning processes are different in the three cases. The differences in the learning processes can be highlighted by writing

the long-run effect under imperfect information as (with obvious notation)[24]

$$\text{LR Effect}\big|_{\theta=\tilde{\theta}}^{\text{imp info}} = LE \cdot \left[(1-h)\,\text{LR Effect}\big|_{\theta=0}^{\text{per info}} + h\,\text{LR Effect}\big|_{\theta=\tilde{\theta}}^{\text{per info}} \right],$$

where

$$LE = \text{Learning Effect}.$$

That is, the long-run effect is the weighted average of the perfect information long-run effects multiplied by a term labelled "Learning Effect". This learning effect is dependent on the persistence of the shock and the informational setting, ie the degree of noise. More specifically it can be shown that for a;

$$\begin{aligned}
\text{transitory shock} \quad &: \quad LE = (1 - \theta + \theta h)^{-1}\,(1-\theta)\,; \\
\text{persistent shock} \quad &: \quad LE = (1 - \theta + \theta h)^{-1}\,; \\
\text{fully permanent shock} \quad &: \quad LE = (\theta h)^{-1} = h^{-1};
\end{aligned}$$

Comparing the relative sizes of the long-run effects, it turns out that they are smaller for transitory shock but larger for persistent and fully permanent shocks when compared to the long-run effects under perfect information.

The long-run effects following any shock are decreasing in h. There are two opposite effects. First, the impact effect increases. Secondly, learning is faster. The second effect outweighs the first. The dependence of the long-run effects on θ is a bit more mixed. For the transitory case the long-run effect is decreasing in θ. Again there are two effects; first, a positive effect since the impact effect increases, and, secondly, a negative effect since the period-after switch is increasing in θ. The second effect is strongest. For a persistent shock the long-run effects are increasing in θ.

5.5   Nominal Interest Rate Spread

The implications of nominal rigidities and imperfect information for the adjustment of nominal interest rates can easily be worked out by noting that the real asset available to households implies that it is possible to construct a nominal asset with a return determined as (in logs, disregarding constants)[25]

$$\log(1 + i_t) = \log(1 + r_t) + E_t p_{t+1} - p_t. \tag{11}$$

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[24]There is one slight exception: the nominal exchange rate following a fully permanent shock. In this case the long-run effect under perfect information consists of the flexible price long-run effect (1) and a negative component arising from labor-leisure substitution. Under imperfect information it is only the last component that can be written as the stipulated weighted average.

[25]For the nominal interest rate we deviate from letting lower-case letters denote natural logarithms of upper-case counterparts. It is denoted i_t.

It follows that the nominal interest rate spread can be written in accordance with uncovered interest rate parity as

$$\log\left(1+i_t\right) - \log\left(1+i_t^*\right) = E_t s_{t+1} - s_t, \tag{12}$$

where it has been used that $p_t - p_t^* = s_t$.

Using the equilibrium value for the exchange rate, it follows that the interest rate spread can be written in the following semi-reduced form

$$\begin{aligned}\log\left(1+i_t\right) - \log\left(1+i_t^*\right) &= (\theta - \theta h)\left[\log\left(1+i_{t-1}\right) - \log\left(1+i_{t-1}^*\right)\right] \\ &\quad + \Phi_{is}\left(L\right)s_t + \Phi_{im}\left(L\right)\left(m_t - m_t^*\right),\end{aligned} \tag{13}$$

where the lag polynomials are defined in appendix B.

We have the plausible result that a (relative) monetary expansion leads to a fall in the interest rate spread, ie

$$\frac{\partial\left[\log\left(1+i_t\right) - \log\left(1+i_t^*\right)\right]}{\partial\left(m_t - m_t^*\right)} < 0,$$

Persistence in the adjustment of the interest rate spread is captured by the presence of both the lagged interest rate spread and nominal exchange-rate changes on top of money-supply changes. Notice that even under perfect information there is persistence in the interest rate spread following a persistent shock. This is simply the mirror image of (correct) appreciation expectations. In the transitory case (and perfect information still) there is one-period dynamics and in the fully permanent case the interest rate differential does not move at all since the exchange rate follows a random walk. The information problem of disentangling temporary from permanent shocks strengthens inertia across types of shocks. That is, systematic differences in nominal interest rates arise as a result of the failure of agents to separate temporary from permanent nominal shocks. This does not, however, leave any risk free arbitrage possibilities, since the real return is the same in both countries and the difference in nominal interest rates reflects the changes in the nominal exchange rate expected by all market participants.

It is a well established empirical fact that an appreciating (depreciating) exchange rate tends to be accompanied by a positive (negative) interest rate spread (see eg Frankel and Rose, 1995). The present setting explains that excess returns are persistent due to the interplay between nominal rigidities in the real side of the economy and the information problem of disentangling temporary from permanent shocks. To put it differently, we find that expectational errors - in a rational expectations setting - can account for systematic interest rate spreads. It is also worth pointing out that it is an

implication that the spread is time-varying reflecting the type of shock hitting the economy and the learning problem.

It is interesting to note that under imperfect information the impact effect on the nominal interest rate spread is decreasing (absolute value) in θ. The reason for this is simply that the nominal depreciation is increasing and, hence, the demand switching effect is increasing in θ. Given a small demand switching effect, the interest rate has to fall more to clear the money market. This effect is found too in the imperfect information case. The same applies for h. Comparing across regimes the impact effects are larger in the persistent and fully permanent cases and smaller in the transitory case. The reasoning is the same as above, the larger the effect on demand, the smaller the interest-rate movement warranted to clear the money market. This also means that the interest rate spread moves oppositely the three other variables in sensitivity analysis.

6 Numerical Illustrations

To make the preceding theoretical analysis more transparent it is useful to consider numerical illustrations of the effects of different types of shocks - transitory, persistent or fully permanent - to the relative money supply. The impulse-response functions for the nominal exchange rate, the terms of trade, relative consumption and the nominal interest rate spread are given in figures 2, 3 and 4, respectively. In all cases we consider a 1 percent increase in Home (relative) money in period 1. The parameter values used for the numerical illustrations are given in table 6.1

Table 6.1. Benchmark Values.[26]

ρ	γ	μ	σ	β	δ	h
2	0.67	10	0.75	9	1/1.05	0.5

In all illustrations it is assumed that $\theta = 1$ in the case of transitory and fully permanent shocks, while we generate a persistent shock by assuming that $\theta = 0.9$. The numerical illustrations are made for a given value of the signal-noise coefficient ($h = 0.5$). We then feed the model with shocks which by construction are either transitory, persistent or fully permanent. The dynamic responses thus reflect the learning process when agents over time acquire more information. The figures also briefly illustrate how the

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[26]The elasticity parameter ρ is chosen arbitrarily to be 2, γ is chosen to match the wage share of about 2/3 while μ is chosen so as to imply a labor-supply elasticity of 0.1. The next three coefficients correspond to those adopted in eg Hairault and Portier (1993) and Sutherland (1996). The last coefficient value is arbitrarily chosen to be 0.5.

impulse-response functions are affected by variations in the parameters underlying the information structure (θ, h), and make a comparison to the case of full information. In reading the subsequent figures it is useful to keep the adjustment of expectations (cf figure 1) in mind.

6.1 Transitory Shock

The impulse-response functions for a transitory shock show that the initial exchange-rate depreciation is gradually worked out of the system. Relative consumption follows the same pattern, that is, it increases on impact and slowly decays towards its long-run value. The terms of trade deteriorate on impact, but then improve. This reflects that the monetary change disappears but agents still expect the money shock to have increased. As a consequence demand switches towards the Foreign good and this explains why the long-run effects are smaller when information is imperfect. The subsequent terms-of-trade improvement is gradually worked out of the system. The interest rate spread dynamics is the same as for the terms of trade; it falls on impact and then rises (relative to the initial value). From then on the adjustment is gradual towards the long-run value. Note, that we have a positive interest rate spread and an appreciating currency.

Figure 2 about here

Comparing across information regimes we see that trivial dynamics have been replaced by more persistence and non-trivial dynamics for all variables, and the impact effects are larger for all variables except for the nominal interest rate spread, cf the theoretical analysis.

The sensitivity results with respect to the signal-noise coefficient h (only shown for the terms of trade) show two things. First, the impact effects are increasing in h, since agents tend to take the shock as being fully permanent. Secondly, persistence is decreasing in h. The reason for this is that although a higher h, lowers a priori belief that the shock is transitory noise, it implies that it takes less time for agents to learn that the shock in fact was transitory. Along the same lines we find that the impact effects are increasing in θ (only shown for the terms of trade), which reflects that a larger weight is attributed to the part of the shock perceived to be permanent. Persistence of the variables is increasing in θ reflecting that there is more persistence in the permanent part of the shock.

6.2   Persistent Shock

For a persistent shock we get on impact a nominal depreciation, a terms-of-trade deterioration, an increase in consumption and an interest-rate fall. The striking feature is the dynamics of the exchange rate, which is seen to display delayed overshooting. Following the initial depreciation the exchange rate depreciates even further in the next period before settling on an appreciating path towards the long-run value.

Figure 3 about here

Imperfect information generates delayed overshooting as well as much richer dynamics for relative consumption displaying a kind of undershooting path and the terms of trade gradually rising after the impact fall. The impact effects are, as analytically predicted, smaller under imperfect information for the first three variables and, hence, larger for the nominal interest rate spread.

The sensitivity results (only shown for the nominal exchange rate) show that the delayed overshooting behavior disappears for lower values of the θ parameter. It is also seen that the impact effects (except for the interest rate spread [not shown]) are increasing in θ.

The coefficient h affects the dynamics, and for low values of h delayed overshooting disappears as learning becomes too slow. The sensitivity of impact effects and persistence of the variables are as shown analytically; a lower h implies smaller impact effects (vice versa for the interest rate spread [not shown]) and more persistence as agents take longer to learn the exact nature of the shock.

6.3   Fully Permanent Shock

For a fully permanent shock the initial effect is slowly worked out of the system for all four variables. For the specific parameter constellation used we see that the long-run exchange-rate effect is a depreciation displaying undershooting. Again the dynamics are much richer under imperfect information. With perfect information the exchange rate and relative consumption jump immediately to their new long-run values, the nominal interest rate spread is unchanged and the terms of trade have one-period dynamics. All four variables display much more plausible dynamics under imperfect information.

Figure 4 about here

Again we see that increasing the degree of noise (lowering h) increases persistence, reflecting that the learning process is prolonged. Moreover, we have a depreciating currency and a negative interest rate spread.

The numerical illustrations displayed above show that informational problems imply that nominal shocks have effects running well beyond the length of contracts. This supports that informational problems can play an important role as a propagation mechanism, though, it is clearly not the whole story since the model does not in general produce terms-of-trade half-lives of 3-4 years to shocks.

7   Discussion and Conclusions

We have analyzed a situation where markets are affected by noise which makes the problem of distinguishing between temporary and permanent shocks essential for exchange-rate dynamics and wage formation and therefore in turn the dynamic adjustment path to nominal shocks. By adopting a general equilibrium framework, we endogenously related the adjustment of nominal exchange rates, wages, prices, and the nominal interest rate spread as well as real variables to monetary shocks. Informational problems add interesting and plausible dynamics which would be absent under full information. Monetary shocks have effects which last well beyond the length of nominal contracts reflecting inertia induced by learning. Numerical illustrations indicate that the quantitative importance is non-trivial. Other interesting results were that the model was able to generate: (i) more volatility in the nominal exchange rate via noise (temporary shocks); (ii) delayed nominal exchange-rate overshooting and persistence in the nominal interest rate differential as well as a positive (negative) nominal interest rate differential alongside an appreciating (depreciating) currency; (iii) non-neutrality of nominal shocks, where the sizes of the long-run effects depend critically on the information structure.

Our model should be seen as a complement rather than a substitute for other analyses (eg Bergin and Feenstra, 1999a,b; Kollmann, 1999) of persistence. We do not introduce persistence via staggered nominal contracts and the results of the paper can be interpreted as showing how information problems can circumvent the problem of creating conditions under which prices are insensitive to marginal cost and marginal cost insensitive to output changes. Persistence in our model is generated only by learning without direct assumptions on the above-mentioned connections and should be seen as adding to the other mechanisms in the literature.[27]

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[27]This is basically a question of the type: Is the glass half empty or half full? Learning in our model can be seen as driving a wedge in between marginal costs and output, or in other words, making marginal cost less dependent on output. When learning is slow (much noise) there are large and persistent effects on output without (relative) wage adjustment.

We model the information problem in a very stylized way, that is, the information structure (θ,h) is invariant over time. In reality the relative variances of temporary and permanent shocks might vary and agents might not learn the new structure right away. In this paper we therefore have explored a minimal change from perfect information and shown that this can have drastic effects for the dynamic adjustment path. An obvious next step would be to add misperception (Gourinchas and Tornell, 1996) and see how this affects the properties of the model.

We have chosen to consider the analytics in great detail. Much of the current business-cycle literature adopt the approach of proceeding directly to numerical simulations (calibration) after having set up a model (often featuring many new aspects). The precise working of the mechanisms and the critical parameters tend to become blurred by this approach. Since it is possible to find an analytical solution under assumptions no more restrictive than those employed in empirical simulations, we find that this yields more insights. Obviously, it does not make sense to take a - after all - simple model as the present directly to the data, except if one believes in a monocausal explanation of observed business-cycle facts (only nominal shocks, only informational problems as propagation mechanism). Still, the qualitative results and the quantitative illustrations in our numerical examples make us conclude that the realistic problem of interpreting whether changes are temporary or permanent adds important dynamics to the adjustment process.

A Steady State and Log-Linearization

Our analysis builds on a version of the model set up in section 2 in log-deviations from steady state. The consumers maximize expected utility subject to the budget constraint and, the first-order conditions determining the optimal choice of B_t and M_t are

$$C_t^{-\frac{1}{\sigma}} = \delta \left(1 + r_t\right) E_t \left(C_{t+1}^{-\frac{1}{\sigma}}\right), \tag{14}$$

$$C_t^{-\frac{1}{\sigma}} = \xi \left(\frac{M_t}{P_t}\right)^{-\beta} + E_t \left(\delta C_{t+1}^{-\frac{1}{\sigma}} \frac{P_t}{P_{t+1}}\right), \tag{15}$$

where it is assumed that the usual transversality condition is satisfied. Since not all expressions are log-linear, we have to approximate around a steady state. The steady-state version of the model is similar to that analyzed

Imperfect information implicitly makes marginal cost less sensitive to output changes.

in eg Obstfeld and Rogoff (1995) and Tille (1999). We focus on a symmetric steady state where $B = B^* = 0$ and

$$C = C^* = Y^h = Y^{*f} = Y = Y^* = \alpha_y \left[\alpha_n \left(\kappa \alpha_y^\sigma \right)^{\frac{1}{\mu}} \right]^{\frac{-\mu \eta_{yw}}{\mu \eta_{nw} + \sigma \eta_{yw} + 1}}, \qquad (16)$$

$$r = \delta^{-1} - 1, \qquad (17)$$

$$\frac{P^h}{P} = \frac{P^f}{P} = \frac{P^{*h}}{P^*} = \frac{P^{*f}}{P^*} = 1, \qquad (18)$$

$$\frac{W}{P} = \frac{W^*}{P^*} = \left[\alpha_n \left(\kappa \alpha_y^\sigma \right)^{\frac{1}{\mu}} \right]^{\frac{\mu}{\mu \eta_{nw} + \sigma \eta_{yw} + 1}}, \qquad (19)$$

where $\alpha_n = \gamma^{\frac{1}{1-\gamma}}$ and $\alpha_y = \gamma^{\frac{\gamma}{1-\gamma}}$.

Money is neutral and the price level follows from (15). Real incomes are

$$Y = \frac{P^h Y^h}{P}, \qquad Y^* = \frac{P^{*f} Y^{*f}}{P^*}.$$

Steady-state values are indicated by omission of time subscripts. To rewrite the model in log-linear form we use the convenient formula for log-normally distributed variables

$$\log E \left(X^f \right) = f E \left[\log (X) \right] + \frac{f^2}{2} Var \left[\log (X) \right],$$

where f is a scalar and X is log-normally distributed. Since the money demand equation (15) is not log-linear we use the following approximation around a steady state (disregarding constants)

$$\log(X_t + Z_t) = \frac{X}{X + Z} \log(X_t) + \frac{Z}{X + Z} \log(Z_t).$$

Using this we get that

$$\log \left[\xi \left(\frac{M_t}{P_t} \right)^{-\beta} + E_t \left(\delta C_{t+1}^{-\frac{1}{\sigma}} \frac{P_t}{P_{t+1}} \right) \right]$$

$$= (1 - \delta) \log \left[\xi \left(\frac{M_t}{P_t} \right)^{-\beta} \right] + \delta \log \left[E_t \left(\delta C_{t+1}^{-\frac{1}{\sigma}} \frac{P_t}{P_{t+1}} \right) \right].$$

Equation (2) follows immediately with

$$\eta_{mc} = \frac{1}{\sigma (1 - \delta) \beta}, \qquad \eta_{mc}^1 = \frac{\delta}{\sigma (1 - \delta) \beta}, \qquad \eta_{mp} = \frac{\delta}{(1 - \delta) \beta}.$$

Note, that following a shock the economy moves away from the initial steady state and does not return. The log-linearized first-order condition for money demand (2) still holds, though. Log-linearizing (15) around any steady state with no inflation will yield (2) (disregarding constants).

While the model is specified so as to yield a log-linear structure, we have that the budget constraint is linear in levels, ie

$$B_t = (1 + r_{t-1}) B_{t-1} + Y_t - C_t. \tag{20}$$

Subtracting the steady-state version of the budget constraint from (20) and dividing by Y(=C) we get

$$
\begin{aligned}
\frac{B_t - B}{Y} &= (1 + r) \frac{B_{t-1} - B}{Y} + \frac{Y_t - Y}{Y} - \frac{C_t - C}{C} \\
&\quad + [(1 + r_{t-1}) - (1 + r)] \frac{B_{t-1} - B}{Y}.
\end{aligned}
$$

The last term on the right-hand side is negligible as we look at small deviations around steady state. We end up with

$$b_t = \delta^{-1} b_{t-1} + y_t - c_t, \tag{21}$$

as $1 + r = \delta^{-1}$ and

$$
\begin{aligned}
y_t &= \log \left(\frac{Y_t}{Y} \right) \approx \frac{Y_t - Y}{Y}, \quad b_t = \frac{B_t}{Y}, \\
c_t &= \log \left(\frac{C_t}{C} \right) \approx \frac{C_t - C}{C}.
\end{aligned}
$$

B Equilibrium with One-Period Nominal Wage Contracts and Imperfect Information

We solve for four variables: the nominal exchange rate, the terms of trade, relative consumption and the nominal interest rate spread. We use the method of undetermined coefficients and take each variable in turn. Our guesses are

$$s_t = \pi_{sc} (c_t - c_t^*) + \pi_{sm} (m_t - m_t^*) + \pi_{sm}^1 E_{t-1} (m_t - m_t^*), \tag{22}$$

$$q_t = \pi_{qc} (c_{t-1} - c_{t-1}^*) + \pi_{qm} (m_t - m_t^*) + \pi_{qm}^1 E_{t-1} (m_t - m_t^*), \tag{23}$$

and

$$
\begin{aligned}
c_t - c_t^* &= \pi_{cb} (b_{t-1} - b_{t-1}^*) + \pi_{cc} (c_{t-1} - c_{t-1}^*) \\
&\quad + \pi_{cm} (m_t - m_t^*) + \pi_{cm}^1 E_{t-1} (m_t - m_t^*).
\end{aligned} \tag{24}
$$

Using the Euler equation we can write relative consumption as

$$c_t - c_t^* = c_{t-1} - c_{t-1}^* + \pi_{cm} [m_t - m_t^* - E_{t-1} (m_t - m_t^*)]. \tag{25}$$

Next we consider the endogenous variables in turn.

B.1 Nominal Exchange Rate

The nominal exchange rate follows from the money market equilibrium condition yielding

$$s_t = \frac{1}{1 + \eta_{mp}} \left[\left(\eta_{mc}^1 - \eta_{mc} \right) \left(c_t - c_t^* \right) + \eta_{mp} E_t s_{t+1} + m_t - m_t^* \right].$$

Consistency with (22) requires

$$\pi_{sc} = \frac{\eta_{mp} \pi_{sc} + \eta_{mc}^1 - \eta_{mc}}{1 + \eta_{mp}} \quad \pi_{sm} = \frac{\eta_{mp} \left(\pi_{sm} + \pi_{sm}^1 \right) h + 1}{1 + \eta_{mp}} \quad \pi_{sm}^1 = \frac{\eta_{mp} \left(\pi_{sm} + \pi_{sm}^1 \right) (\theta - h)}{1 + \eta_{mp}}$$

Equation (22) can by use of (25) and

$$E_{t-1} \left(m_t - m_t^* \right) = \theta \left(1 - h \right) E_{t-2} \left(m_{t-1} - m_{t-1}^* \right) + \theta h \left(m_{t-1} - m_{t-1}^* \right), \quad (26)$$

be rewritten as (8), where the coefficients are

$\phi_{ss}^1 = 1 + \theta - \theta h$	$\phi_{ss}^2 = \theta h - \theta$
$\phi_{sm} = \pi_{sm} + \pi_{sc} \pi_{cm}$	$\phi_{sm}^1 = \left(-1 - \theta + h \right) \pi_{sm} + \theta h \pi_{sm}^1 - \theta \pi_{sc} \pi_{cm}$
	$\phi_{sm}^2 = - \left(\pi_{sm}^1 \frac{h}{1-h} - \pi_{sm} \right) \left(\theta - \theta h \right)$

B.1.1 Impact Effects, Dynamics and Long-Run Effects

For all three shocks the impact effect is given by

$$\frac{\partial s_t}{\partial \left(m_t - m_t^* \right)} = \pi_{sm} + \pi_{sc} \pi_{cm} > 0. \tag{27}$$

For a transitory shock the dynamics are described by an ARIMA(1,1,2)

$$\left(1 - L \right) \left[1 - \theta \left(1 - h \right) L \right] s_t = \phi_{sm} u_t + \phi_{sm}^1 u_{t-1} + \phi_{sm}^2 u_{t-2}, \tag{28}$$

for a persistent shock an ARIMA(2,1,2)

$$\left(1 - L \right) \left[1 - \theta \left(2 - h \right) L - \theta^2 \left(h - 1 \right) L^2 \right] s_t \tag{29}$$
$$= \phi_{sm} \varepsilon_t + \phi_{sm}^1 \varepsilon_{t-1} + \phi_{sm}^2 \varepsilon_{t-2},$$

and finally for a fully permanent shock an ARIMA(1,1,1)

$$\left(1 - L \right) \left[1 - \left(1 - \theta h \right) L \right] s_t = \phi_{sm} \varepsilon_t + \left(\phi_{sm} + \phi_{sm}^1 \right) \varepsilon_{t-1}. \tag{30}$$

The long-run effects in the three cases are

$$\left. \frac{\partial s_{t+j}}{\partial u_t} \right|_{\theta \in (0,1]} = \frac{\left(1 - \theta \right) \pi_{sc} \pi_{cm}}{1 - \theta + \theta h} \leq 0 \quad for \ j \to \infty, \tag{31}$$

$$\left. \frac{\partial s_{t+j}}{\partial \varepsilon_t} \right|_{\theta \in (0,1)} = \frac{\pi_{sc} \pi_{cm}}{1 - \theta + \theta h} < 0 \quad for \ j \to \infty, \tag{32}$$

$$\left. \frac{\partial s_{t+j}}{\partial \varepsilon_t} \right|_{\theta = 1} = 1 + \frac{\pi_{sc} \pi_{cm}}{h} \lesseqgtr 0 \quad for \ j \to \infty. \tag{33}$$

Conditional Delayed Overshooting In the case of a persistent shock $[\varepsilon_t > 0,\ \theta \in (0,1)]$ we cannot reject the possibility of delayed overshooting. We need

$$\frac{\partial s_t}{\partial\left(m_t - m_t^*\right)} = \phi_{sm} < \phi_{ss}^1 \phi_{sm} + \theta\phi_{sm} + \phi_{sm}^1 = \frac{\partial s_{t+1}}{\partial\left(m_t - m_t^*\right)},$$

or

$$\frac{1}{1+\eta_{mp}}\left[\left(\pi_{sm} + \pi_{sm}^1\right)\eta_{mp}\theta h\left(1 - 2\theta + \theta h\right) + 1 - \theta\right] < \left(\theta - \theta h\right)\pi_{sc}\pi_{cm} < 0.$$

Hence, a necessary condition for delayed overshooting is $1 - 2\theta + \theta h < 0$.

B.2   Terms of Trade

Product market equilibrium implies that

$$\begin{aligned}
q_t &= \frac{\eta_{yw}\eta_{wc}}{\rho + \eta_{yw}}\left(c_{t-1} - c_{t-1}^*\right) + \frac{\eta_{yw}\left(2\eta_{wp} - 1\right)}{\rho + \eta_{yw}}E_{t-1}q_t \\
&\quad - \frac{\eta_{yw}}{\rho + \eta_{yw}}\left(s_t - E_{t-1}s_t\right),
\end{aligned}$$

where we have invoked the Euler equation. Invoking the expression for the nominal exchange rate and (25) we obtain

$$\begin{aligned}
q_t &= \frac{\eta_{yw}\eta_{wc}}{\rho + \eta_{yw}}\left(c_{t-1} - c_{t-1}^*\right) + \frac{\eta_{yw}\left(2\eta_{wp} - 1\right)}{\rho + \eta_{yw}}E_{t-1}q_t \\
&\quad - \frac{\eta_{yw}}{\rho + \eta_{yw}}\left(\pi_{sm} + \pi_{sc}\pi_{cm}\right)\left[m_t - m_t^* - E_{t-1}\left(m_t - m_t^*\right)\right].
\end{aligned}$$

Using our guess (23) to find $E_{t-1}q_t$ and inserting, the restrictions are

$$\boxed{\pi_{qc} = \frac{\eta_{yw}\eta_{wc}}{\rho + \eta_{yw}}\pi_{qc} + \frac{\eta_{yw}\left(2\eta_{wp} - 1\right)}{\rho + \eta_{yw}} \quad \bigg| \quad \pi_{qm} = -\frac{\eta_{yw}\left(\pi_{sm} + \pi_{sc}\pi_{cm}\right)}{\rho + \eta_{yw}} \quad \bigg| \quad \pi_{qm}^1 = -\pi_{qm}}$$

Using (23), (25) and (26) we can rewrite the terms of trade as (9), where

$$\boxed{\begin{aligned}
\phi_{qm} &= \pi_{qm} \\
\phi_{qm}^1 &= \left(-1 - \theta + \theta h\right)\pi_{qm} + \frac{2\theta - \theta h(1-h) - h}{\theta(1-h)}\pi_{qm}^1 + \frac{\theta - h}{\theta - \theta h}\pi_{qc}\pi_{cm} \\
\phi_{qm}^2 &= \theta(1 - h)\pi_{qm} - \theta h\pi_{qm}^1 - \theta\pi_{qc}\pi_{cm}
\end{aligned}}$$

B.2.1 Impact Effects, Dynamics and Long-Run Effects

The impact effects in all three cases are

$$\frac{\partial q_t}{\partial (m_t - m_t^*)} = \pi_{qm} < 0. \tag{34}$$

The dynamics in the three cases are

$$(1 - L)\left[1 - \theta (1 - h) L\right] q_t = \phi_{qm} u_t + \phi_{qm}^1 u_{t-1} + \phi_{qm}^2 u_{t-2}, \tag{35}$$

$$(1 - L)\left[1 - \theta (2 - h) L - \theta^2 (h - 1) L^2\right] q_t \tag{36}$$
$$= \phi_{qm}\varepsilon_t + \phi_{qm}^1 \varepsilon_{t-1} + \phi_{qm}^2 \varepsilon_{t-2},$$

and

$$(1 - L)\left[1 - (1 - \theta h) L\right] q_t = \phi_{qm}\varepsilon_t + \left(\phi_{qm} + \phi_{qm}^1\right) \varepsilon_{t-1}, \tag{37}$$

respectively.

The long-run effect of a transitory shock is

$$\left.\frac{\partial q_{t+j}}{\partial u_t}\right|_{\theta \in (0,1]} = (1 - \theta) \frac{\pi_{qc}\pi_{cm}}{(1 - \theta + \theta h)} \geq 0 \quad for \ j \to \infty. \tag{38}$$

The long-run effect of a persistent shock is

$$\left.\frac{\partial q_{t+j}}{\partial \varepsilon_t}\right|_{\theta \in (0,1)} = \frac{\pi_{qc}\pi_{cm}}{(1 - \theta + \theta h)} > 0 \quad for \ j \to \infty. \tag{39}$$

The long-run effect of a fully permanent shock is

$$\left.\frac{\partial q_{t+j}}{\partial \varepsilon_t}\right|_{\theta = 1} = \frac{\pi_{qc}\pi_{cm}}{h} > 0 \quad for \ j \to \infty. \tag{40}$$

B.3 Relative Consumption

From equation (21) we have

$$b_t - b_t^* = \delta^{-1} \left(b_{t-1} - b_{t-1}^*\right) + (y_t - y_t^*) - (c_t - c_t^*),$$

and since $y_t - y_t^* = (1 - \rho) q_t$ we have that

$$b_t - b_t^* = \delta^{-1} \left(b_{t-1} - b_{t-1}^*\right)$$
$$+ (1 - \rho)\left[\pi_{qc}\left(c_{t-1} - c_{t-1}^*\right) + \pi_{qm}\left(m_t - m_t^*\right)\right.$$
$$\left. + \pi_{qm}^1 E_{t-1}\left(m_t - m_t^*\right)\right] - (c_t - c_t^*).$$

The next step is to find an expression for relative consumption consistent with the Euler equation. Leading our guess we find

$$
\begin{aligned}
E_t \left(c_{t+1} - c_{t+1}^* \right) &= \pi_{cb} \left(b_t - b_t^* \right) + \pi_{cc} \left(c_t - c_t^* \right) \\
&= \pi_{cb} \delta^{-1} \left(b_{t-1} - b_{t-1}^* \right) + \pi_{cb} \left(1 - \rho \right) \pi_{qc} \left(c_{t-1} - c_{t-1}^* \right) \\
&\quad + \left[\pi_{cb} \left(1 - \rho \right) \pi_{qm} + \theta h \left(\pi_{cm} + \pi_{cm}^1 \right) \right] \left(m_t - m_t^* \right) \\
&\quad + [\pi_{cb} \left(1 - \rho \right) \pi_{qm}^1 \\
&\quad + \left(\theta - \theta h \right) \left(\pi_{cm} + \pi_{cm}^1 \right)] E_{t-1} \left(m_t - m_t^* \right) \\
&\quad + \left(\pi_{cc} - \pi_{cb} \right) \left(c_t - c_t^* \right) .
\end{aligned}
$$

Consistency with the Euler equations (3) requires that

$\pi_{cb} = \dfrac{\delta^{-1} \pi_{cb}}{1 + \pi_{cb} - \pi_{cc}}$	$\pi_{cc} = \dfrac{\pi_{cb}(1-\rho)\pi_{qc}}{1 + \pi_{cb} - \pi_{cc}}$
$\pi_{cm} = \dfrac{\pi_{cb}(1-\rho)\pi_{qm} + \theta h \left(\pi_{cm} + \pi_{cm}^1 \right)}{1 + \pi_{cb} - \pi_{cc}}$	$\pi_{cm}^1 = -\pi_{cm}$

It is worth noticing that relative consumption does not follow a random walk (as it does under perfect information) but evolves according to richer dynamics. This is seen by subtracting the lagged version of (25) from itself. After some simple algebra we end up with (10) where

$\phi_{cc}^1 = 1 - \theta + \theta h$	$\phi_{cc}^2 = \theta h - \theta$	$\phi_{cm} = \pi_{cm}$	$\phi_{cm}^1 = -\theta \pi_{cm}$

.

B.3.1 Impact Effects, Dynamics and Long Run Effects

The impact effect is

$$
\frac{\partial \left(c_t - c_t^* \right)}{\partial \left(m_t - m_t^* \right)} = \pi_{cm} > 0. \tag{41}
$$

The dynamics for a transitory shock, persistent shock and fully permanent shock, respectively, are

$$
\left(1 - L \right) \left[1 - \theta \left(1 - h \right) L \right] \left(c_t - c_t^* \right) = \phi_{cm} u_t + \phi_{qm}^1 u_{t-1}, \tag{42}
$$

$$
\begin{aligned}
& \left(1 - L \right) \left[1 - \theta \left(2 - h \right) L - \theta^2 \left(h - 1 \right) L^2 \right] \left(c_t - c_t^* \right) \\
&= \phi_{qm} \varepsilon_t + \phi_{qm}^1 \varepsilon_{t-1},
\end{aligned} \tag{43}
$$

and

$$
\left(1 - L \right) \left[1 - \left(1 - h \right) L \right] \left(c_t - c_t^* \right) = \phi_{cm} \varepsilon_t. \tag{44}
$$

The long-run effects are given by

$$
\left. \frac{\partial \left(c_{t+j} - c_{t+j}^* \right)}{\partial u_t} \right|_{\theta \in (0,1]} = \frac{\pi_{cm} \left(1 - \theta \right)}{\left(1 - \theta + \theta h \right)} \geq 0 \quad for \ j \to \infty, \tag{45}
$$

in the transitory case and

$$\frac{\partial \left(c_{t+j} - c_{t+j}^*\right)}{\partial \varepsilon_t}\Bigg|_{\theta \in (0,1)} = \frac{\pi_{cm}}{(1 - \theta + \theta h)} > 0 \quad for \ j \to \infty, \tag{46}$$

in the persistent case. The long-run effect in the fully permanent case is

$$\frac{\partial \left(c_{t+j} - c_{t+j}^*\right)}{\partial \varepsilon_t}\Bigg|_{\theta = 1} = \frac{\pi_{cm}}{h} > 0 \quad for \ j \to \infty. \tag{47}$$

B.4 Nominal Interest Rate Spread

If we instead of a real bond assume a nominal bond with gross return $1 + i_t$ the Euler equation reads

$$C_t^{-\frac{1}{\sigma}} = \delta E_t \left[\frac{(1 + i_t) P_t}{P_{t+1}} C_{t+1}^{-\frac{1}{\sigma}}\right].$$

Combining with the Euler equation when using a real bond we get

$$1 + i_t = \left[E_t \left(\frac{C_{t+1}^{-\frac{1}{\sigma}}}{P_{t+1}}\right) P_t\right]^{-1} (1 + r_t) E_t \left(C_{t+1}^{-\frac{1}{\sigma}}\right).$$

Using the joint log-normality of C and P we get (11) disregarding constants. Subtracting the Foreign version we get (12). Finally, inserting for the nominal exchange rate we obtain (13), where

$$\Phi_{ij}(L) = \phi_{ij} + \phi_{ij}^1 L + \phi_{ij}^2 L^2 + \phi_{ij}^3 L^3, \quad j = s, m,$$

with entries

$\phi_{is} = \phi_{ss}^1$	$\phi_{im} = \phi_{sm}(\theta h - 1) + \phi_{sm}^1$
$\phi_{is}^1 = \phi_{ss}^2 - \left(\phi_{ss}^1\right)^2$	$\phi_{im}^1 = \theta(1 - h)\phi_{sm} - \phi_{ss}^1\phi_{sm}^1 + \phi_{sm}^2$
$\phi_{is}^2 = -2\phi_{ss}^1\phi_{ss}^2$	$\phi_{im}^2 = \theta(1 - h)\phi_{sm}^1 - \phi_{ss}^1\phi_{sm}^2$
$\phi_{is}^3 = -\left(\phi_{ss}^2\right)^2$	$\phi_{im}^3 = \theta(1 - h)\phi_{sm}^2$

Compared to the perfect information case we have non-trivial dynamics for all three kinds of shocks. The impact effect is

$$\frac{\partial \left[\log(1 + i_t) - \log(1 + i_t^*)\right]}{\partial (m_t - m_t^*)} = \phi_{im} + \phi_{is}\frac{\partial s_t}{\partial (m_t - m_t^*)} < 0, \tag{48}$$

and the long-run effect is zero

$$\frac{\partial \left[\log(1 + i_{t+j}) - \log(1 + i_{t+j}^*)\right]}{\partial (m_t - m_t^*)} = 0 \quad for \ j \to \infty. \tag{49}$$

B.5 Perfect Information

In the text we compare the outcome under imperfect information to that of perfect information. The latter situation is defined for a relative money-supply process where there is no problems in disentangling permanent and temporary shocks, ie

$$m_t - m_t^* = \theta \left(m_{t-1} - m_{t-1}^* \right) + \varepsilon_t, \quad \varepsilon_t \sim nid \left(0, \sigma_\varepsilon^2 \right), \quad \theta \in [0, 1],$$

The solution is this case can found as the solution under imperfect information by setting $\sigma_u^2 = 0$ and therefore $h = 1$. For details see Andersen and Beier (1999).

B.6 Information Structure

Finally we state some results (without proofs) on how changes in the information structure affect the impact and long-run effects. Effects on the dynamics are reported in the text.

Proposition 1 *The (absolute values of the) impact effects on the nominal exchange rate, the terms of trade and relative consumption are increasing in θ and h.*

Proposition 2 *The (absolute value of the) impact effect on the nominal interest rate spread is decreasing in θ and h.*

Proposition 3 *The (absolute values of the) long-run effects on the nominal exchange rate, the terms of trade and relative consumption following a transitory shock are decreasing in θ.*

Proposition 4 *The (absolute values of the) long-run effects on the nominal exchange rate, the terms of trade and relative consumption following a persistent shock are increasing in θ.*

Proposition 5 *The (absolute values of the) long-run effects on the nominal exchange rate, the terms of trade and relative consumption following any shock are decreasing in h.*

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Figure 1.
Dynamics of Expected and Unexpected Relative Money.
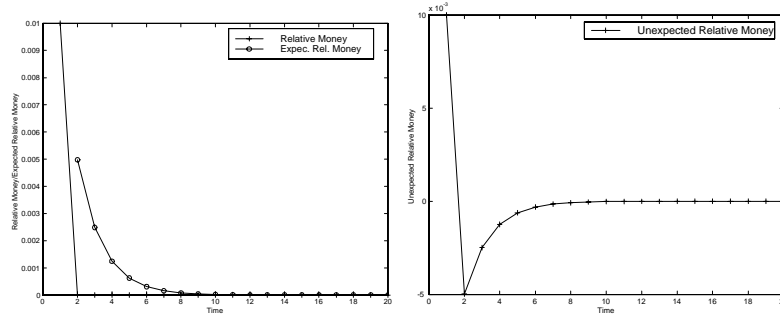
i) Transitory Shock

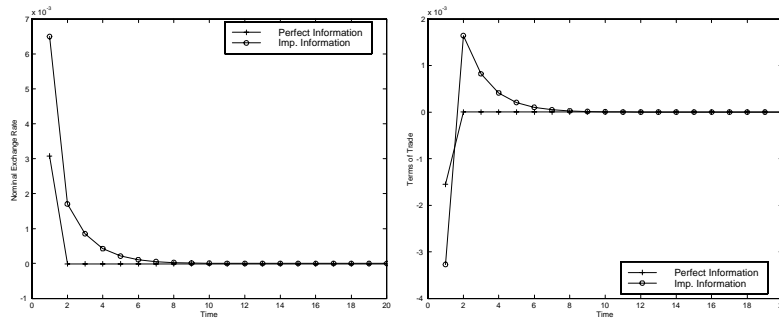ii) Transitory Shock
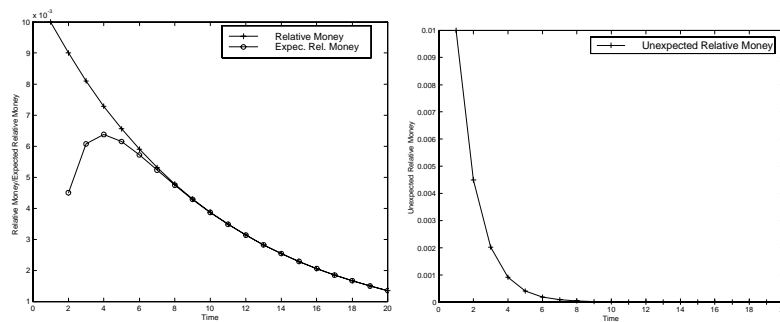
iii) Persistent Shock
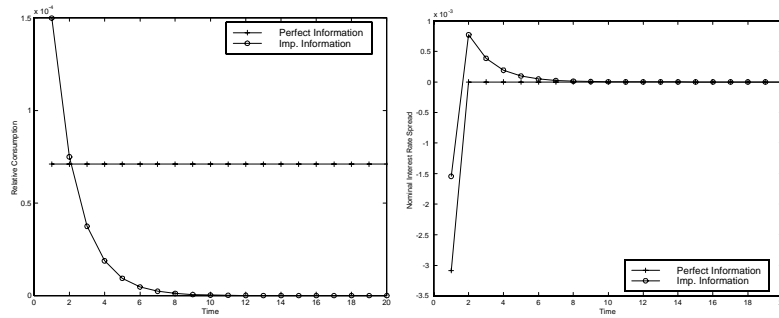
iv) Persistent Shock
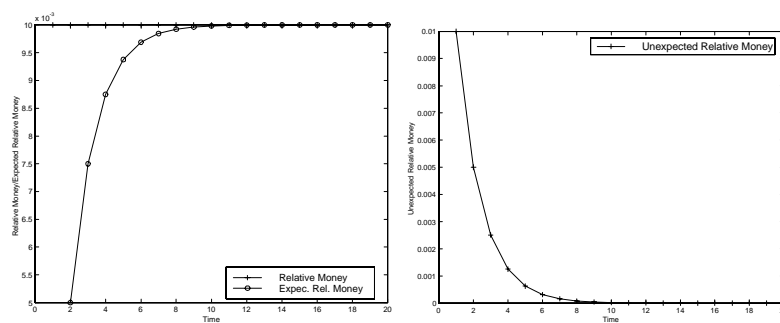
v) Fully Perm. Shock

vi) Fully Perm. Shock

Figure 2.
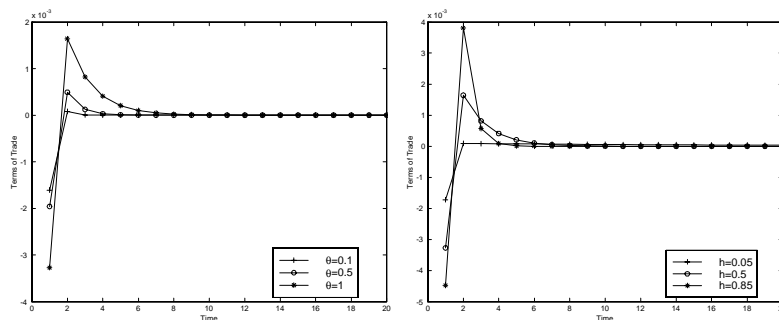**Dynamics - Transitory Shock.**[28]

i) Nominal Exchange Rate

ii) Terms of Trade

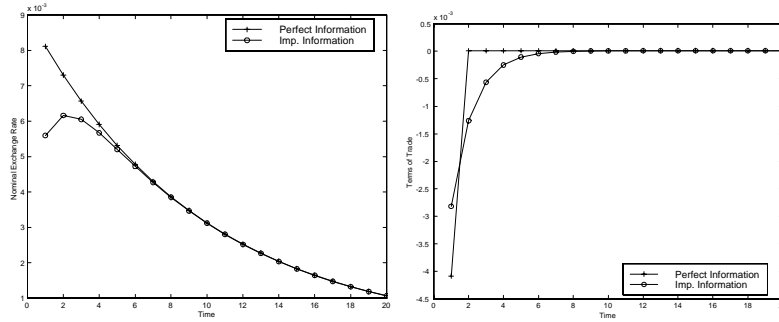iii) Relative Consumption

iv) Interest Rate Spread

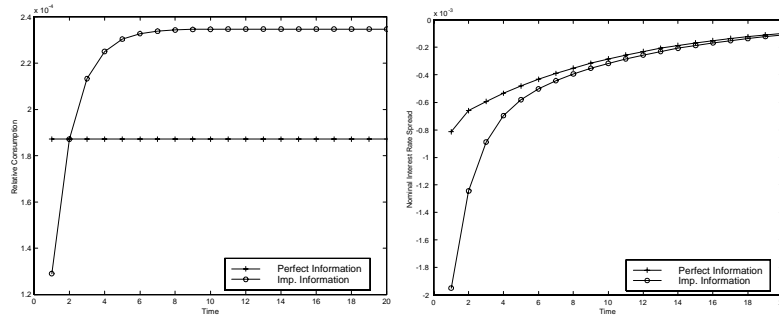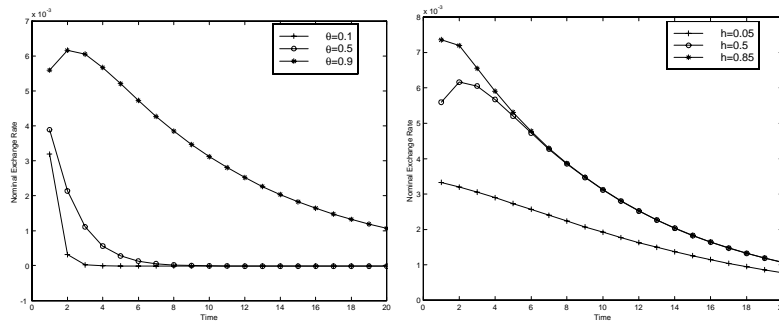v) Sensitivity of q wrt θ

vi) Sensitivity of q wrt h

[28]$\theta = 1$.

Figure 3.
Dynamics - Persistent Shock.[29]

i) Nominal Exchange Rate

ii) Terms of Trade

iii) Relative Consumption
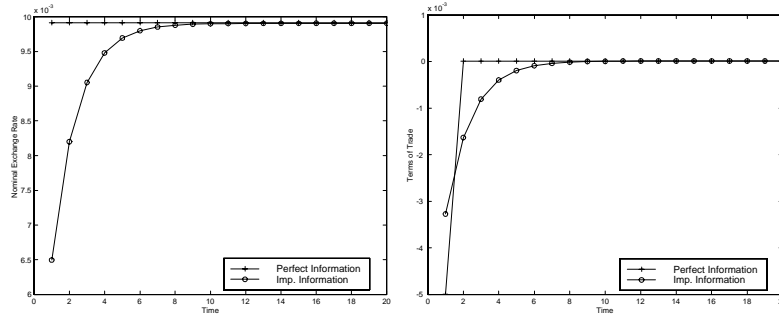

iv) Interest Rate Spread

v) Sensitivity of s wrt θ
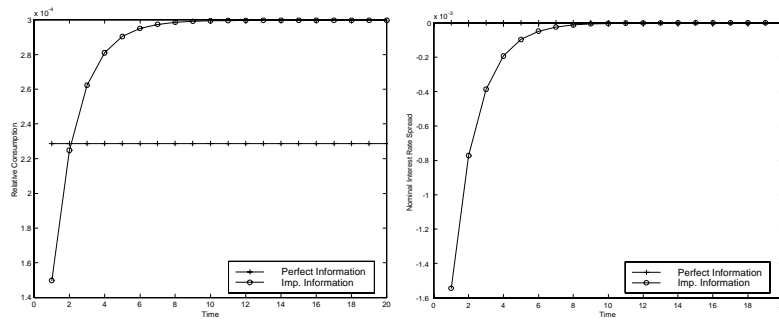

vi) Sensitivity of s wrt h

[29]$\theta = 0.9$.
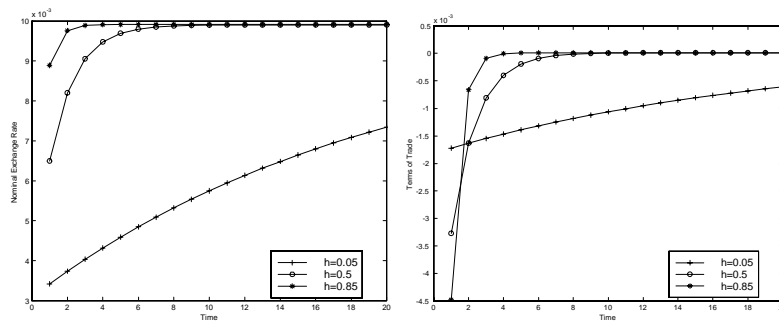
Figure 4.
Dynamics - Fully Permanent Shock.

i) Nominal Exchange Rate   ii) Terms of Trade

iii) Relative Consumption   iv) Interest Rate Spread

v) Sensitivity of s wrt h vi) Sensitivity of q wrt h