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NEW APPROACH TO THE BORDER
EFFECT**

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***INTERNATIONAL MACROECONOMICS,
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ABSTRACT

Regions are not countries: a new approach to the border effect*

We use a version of the Melitz (2003) model to calibrate the magnitude and impact of the border effect, the well-known empirical regularity that trade is much lower across a national border than would otherwise be expected. We calibrate total bilateral trade frictions as a parameter and show that frictions between nation states are systematically higher than those between sub-national states or regions. Using plausible counterfactual analysis, we assess the costs of independence for Scotland, Catalonia, & the Basque Country: the intellectual experiment that is performed is to suppose that the region on independence takes on the calibrated frictions of a counterfactual independent country. If the main change that comes with the independence of regions of larger countries is that their border with their former union partner comes to resemble a normal country border, then the trade costs of the break-up of countries into smaller states (even within the EU) are significant. The border effects associated with membership of the European Union or otherwise, are much lower than the border effect differences between countries and regions. As an illustration of this we produce a potential quantification of the trade costs of a British exit from the EU. Conversely, the potential gains from the European Union achieving the sort of integration seen within a nation state, a United States of Europe, are very large.

JEL Classification: F15 and R13

Keywords: border effect, independence and trade

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1. Introduction

Across Europe there is political momentum behind secessionist movements and some desire to achieve the creation of smaller states within the European Union.¹ Given this background, it is important to understand the differences in the level of economic integration that is entailed by sharing a country or nation state, compared with the integration that comes with membership of a supra-national organization like the European Union. What are the welfare consequences of these degrees of economic integration?

Using a structural model, this paper documents that there are large differences between national and regional borders (i.e. borders between entities within a country or nation state²) in terms of how frictional these borders seem to be for trade flows. We implement a framework that can quantify these frictions. We use counterfactual exercises to show that the differences between regional and country borders have important welfare effects. We ask what would be the consequences of regions taking on country level frictions, and countries taking region level frictions.

It is important to remark upon two things that we do not do in this paper. We do not provide an explanation for why frictions are different between countries and regions - we simply document these differences, and look at the effects of these differences disappearing (in either direction). Also, we do not look at the correlation (or lack thereof) of country size and income.

Whilst measurement of the difference between trade frictions of national and regional borders is a contribution to the economic calculus of a decision for or against independence, there are many other contributions to the costs and benefits of size, and there are many other considerations beyond economics.³ We abstract from all these issues and simply document the frictions across regional and national borders, and quantify the *ceteris paribus* welfare consequences of replacing regional borders with country borders and vice versa. This paper should not be read as implying a judgement upon the desirability or otherwise of independence of regions.

In order to quantify the value of the trade links that come from an integrated economy with a larger market, we consider the application of new trade theory, following [Anderson and van Wincoop \(2003\)](#), [Melitz \(2003\)](#) and [Arkolakis, Costinot, and Rodríguez-Clare \(2012\)](#) (ACR). Trade benefits can arise for many different underlying reasons, for example: a love of variety means that the available product range expands with the size of the market and leads to aggregate increasing returns to scale, as in [Krugman \(1980\)](#); a larger market can lead to better firm selection as

¹Scotland has a referendum on independence from the United Kingdom due to be held on the 18th of September 2014. The governing parties in the Spanish autonomous community of Catalonia have proposed that a referendum on Catalan independence be held on the 9th of November 2014. There is a long standing independence movement in the Basque Country of Spain. In Belgium, political disagreements between Flemish and Walloon parties prevented the formation of a national government for 18 months in 2010 and 2011.

²In this paper we label Scotland, Catalonia, and the Basque Country as *regions*, and the UK and Spain as *countries*. This is a point of nomenclature rather than any point that should be interpreted politically.

³E.g. the costs and failures that may arise from overly distant government under union (e.g. [Alesina, Spolaore, and Wacziarg \(2005\)](#)); the costs of “race to the bottom” fiscal competition (with independence); and other sources of economies and dis-economies of scale.

efficient firms expand to serve this larger market, putting upward pressure on wages, and lowering profitability of low productivity firms who exit, as in [Melitz \(2003\)](#); and traditional Ricardian trade explanations as in [Eaton and Kortum \(2002\)](#). [Arkolakis, Costinot, and Rodríguez-Clare \(2012\)](#) show that for all “gravity models” of trade, the microfoundations underlying gains from trade, conditional on the value of the elasticity of trade flows to trade frictions, do not affect the calibrated value of these gains. Therefore without loss of generality, we can select a specific model to work with.

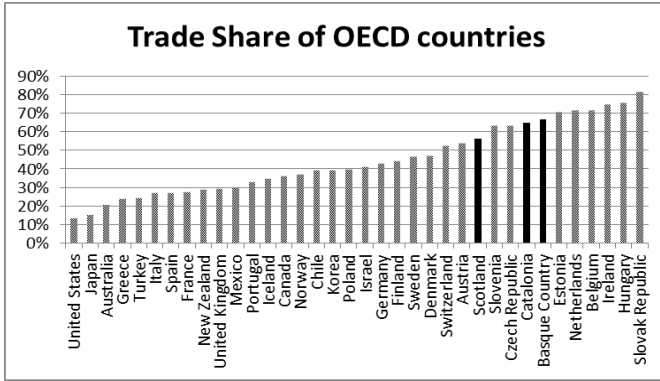
We develop a model that under some parameter restrictions falls within the class where the ACR formula for determining the gains from trade applies. In that case the numerical procedure is straightforward, and our contribution is simply to measure frictions across countries and regions, and to propose counterfactual scenarios and view its consequences within a wide class of well-understood models.

Nevertheless, under some parameter values of interest the ACR formula for gains for trade does not apply. In these cases the specificities of the model become more relevant and the numerical procedure is considerably more involved. Thus, our contribution extends to view the effects of trade frictions within a class of models that allows for capital transfers across economies.

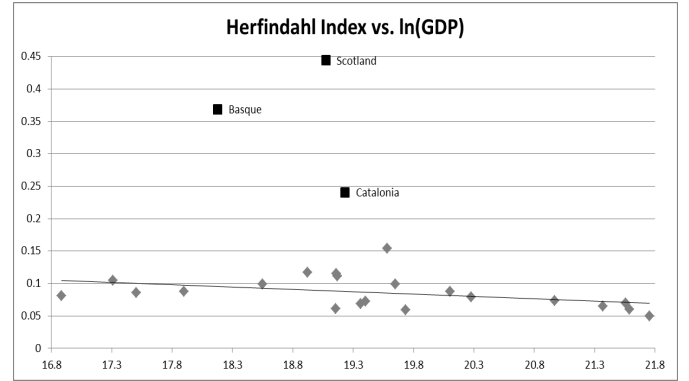
The paper is structured as follows. In section 2 we describe the empirical evidence that shows the extraordinarily high integration of regions relative to the independent countries of the EU. This evidence is the stylised fact that needs an explanation: why do the trading patterns of regions not look like the trading patterns of independent countries? Our answer is that regional borders are much “thinner” than country borders. Section 3 outlines the version of the [Melitz \(2003\)](#) model that we use. In section 4 we claim that regional borders and country borders do indeed look systematically different. In section 5 we measure the cost of regional borders becoming national borders by conducting a series of counterfactual experiments, substituting the borders of Scotland, Catalonia and the Basque Country have with the rest of the UK or Spain respectively, with the borders between the UK or Spain (respectively) and their closest independent trading partners. In section 6 we consider the impact of membership of the European Union, where our counterfactual exercises do not suggest the existence of a strong effect from membership upon trade with the EU. We evaluate a British exit from the EU by incorporating the small change in borders that this analysis suggests. In Section 7 we measure the benefit of country borders within the EU becoming regional borders by conducting the counterfactual experiment, substituting the national borders within the EU by reduced versions of themselves based on the observed regional borders. Section 8 concludes.

2. Openness and Trade Concentration

European regions, viewed as if they were independent countries, appear to highly integrated into the global economy. They have high imports and exports as a percentage of GDP. Comparing



(a) Trade:GDP ratio of OECD countries, plus Scotland, Catalonia & Basque Country



(b) Herfindahl Index of Trade Concentration of EU countries, plus Scotland, Catalonia & Basque Country

Figure 1: Trade and trade concentration in regions and countries

trade to GDP for Scotland, Catalonia and the Basque Country against the equivalent figures for the OECD countries (figure 1a) shows that these regions are open economies, but not outrageously so, and there are other small countries in the OECD which have similarly high trade shares⁴.

However, the trade of these regions is very concentrated with the rest of the country of which they are part: in all 3 cases, a high proportion of the total external trade is with the rest of UK/Spain rather than internationally. In order to quantify these trade concentration facts, we construct a Herfindahl Index of trade concentration⁵.

Figure 1b highlights how anomalous these regions' trade concentrations are compared with independent countries in the European Union. It shows the Herfindahl index against GDP on the x-axis, since we may expect small countries to trade more, and concentrate this trade with their large neighbours. We would expect regions to have relatively high index values, as they are relatively small, but not nearly as high as we observe. The regional Herfindahl Indices are much higher than that of the most trade concentrated independent EU member⁶: it's almost an order of magnitude type comparison.

The data is suggestive of high trade, concentrated with the rest of their country, being an artifact of current institutional arrangements contingent upon being part of the same country. The conclusion of the calibration exercise undertaken in this paper is that regions concentrate so much of their trade with the rest of the countries of which they are part because their calibrated bilateral frictions are much lower than the normal frictions between independent countries. The

⁴The data used in this section, and throughout the paper, is described in Appendix A

⁵If there are N countries, with the exports from country i to country j denoted X_j^i ($X_i^i \equiv 0$), then the Herfindahl Index for country i , $H_i = \sum_{j=1}^N [(X_j^i / \sum_{k=1}^N [X_k^i])^2]$. $H_i = 1$ indicates complete concentration of trade with a single trading partner. $H_i \rightarrow 0$ (equality only possible with infinitely many possible trading partners) indicates complete diversification of trade across all partners.

⁶The most trade concentrated independent EU member is Austria, a relatively small country which concentrates its trade with the EU's largest economy, Germany.

status of these regions as open economies is a function of the close integration with the rest of their countries and should not be automatically expected to survive in the long run after achieving independence.

3. Model

We develop a version of the standard Hopenhayn-Melitz model of firm heterogeneity and international trade. We model the world as consisting of N economies indexed by $i \in \{1, \dots, N\}$. These economies use a common currency (i.e. the nominal exchange rate is 1), but the purchasing power of this common currency can be different in the different economies. In each economy there are identical Dixit-Stiglitz consumers. Thus, the demand for any good from economy i , is given by the demand function: $q = \left(\frac{p}{P_i}\right)^{-\theta} \left(\frac{Y_i}{P_i}\right)$, where $\theta > 1$ is the elasticity of substitution across goods, p is the nominal price of the good, P_i is the price aggregator for economy i , q is the quantity of the good demanded in economy i , and Y_i is the nominal income in economy i .

3.1. Model Description

The supply side of the model consists of infinitesimal, monopolistically competitive, firms which take the demand for their goods from each economy in the world as given. There is a fixed cost for creating a firm, and existing firms pay a fixed cost per period to operate in each economy that is linear in the size of that economy. The operating profit of being active in an economy depends positively on that economy's demand, and depends negatively on the trade frictions between the economies of production and sale. A lower friction has positive effects at the macro level because: it increases the number of firms serving an economy; and it improves the quality of the firms that produce in that economy, as the more productive firms increase labour demand in order to export. This increases wages, and drives unproductive firms out of the market.

The life cycle of a firm consists of the following stages:

- A potential firm in i chooses whether to hire \tilde{c} units of labour locally, and incur the fixed cost, $\tilde{c}W_i$, to draw some productivity from a known distribution, $\phi \sim \text{Pareto}(k, b)$ (so that $F(\Phi) = 1 - \left(\frac{b}{\Phi}\right)^k$), which is common across all economies. W_i is the prevalent nominal wage in country i .
- If it pays that cost, it receives a productivity, ϕ .
- If this productivity is large enough it goes ahead with production in one or more markets. Otherwise, it disappears.
- Firms die exogenously with a fixed probability, $1 - \beta$, every period.

The only input is labour and production technology is constant returns to scale subject to fixed market access costs. The productivity of a firm from i selling in j is ϕ/δ_j^i , where ϕ is idiosyncratic

to the firm and δ_j^i is the trade friction experienced by firms in i selling to consumers in j . These trade frictions reflect how much easier it is to sell into a domestic market than to sell into a foreign market; and they have the following properties:

- Trade frictions are defined relative to domestic selling costs i.e. $\delta_i^i = 1, \forall i$
- We assume that trade frictions are positive i.e. $\delta_j^i > 1, j \neq i$
- This formulation is isomorphic to the standard *iceberg costs* concept, with the quantity of the good that “melts in transit” being, $\tau_j^i = 1 - 1/\delta_j^i$
- We do not impose symmetry in trade frictions i.e. $\delta_j^i \neq \delta_i^j$ in general, though we do for some of our empirical exercises.

δ_j^i measures the advantages than a local producer has versus a foreign producer if both have the same intrinsic quality ϕ . It will be increasing in all the contributions to trade barriers: geographic distance, language differences, regulatory differences, differences in consumer tastes and preferences, as well as, presumably, many other factors. We do not try to explain where it comes from - we simply use a structural model to measure it. This article is a measurement exercise in the absence of any definitive theory as to the full list of explanatory factors, and their relative contribution, which determine this *economic distance*. If we had a *theory of distance*, we would use it. Our point is to measure trade frictions in the context of the model, make comparisons and to undertake thought experiments.

All production occurs in the firm’s local labour market: the firm hires local labour at a nominal wage rate of W_i per unit of effective labour. The firm chooses its price, p , to maximise its nominal operating profits for selling into each distinct market. The fixed costs, per unit time, of running a firm in i that sells in market j , is the cost of hiring a labour force of fixed size to deal with the expenses associated with access to this market. These expenses depend on both the size of the market that it is going to be served, Y_j/P_j , and the trade frictions experienced by the firm in accessing this market, δ_j^i . The larger the market, the more complex it is to sell there (and the larger the reward). The more distant the market is, the more complicated it is to sell there. The quantity of effective labour necessary to deal with these access expenses is assumed to be:

$$f_j^i = c \times \delta_j^i \times \frac{Y_j}{P_j} \quad (\text{cost parameter, } c, \text{ is equal in all economies})$$

The per period net profit, labour demand and value of sales, for a firm in i , from selling to market j are:

$$\begin{aligned} \pi_j^i(\phi) &= \left[\Theta \left(\frac{\phi}{\delta_j^i} \right)^{\theta-1} \left(\frac{W_i}{P_j} \right)^{-\theta} - c\delta_j^i \right] W_i \frac{Y_j}{P_j} \\ L_j^i(\phi) &= (\theta - 1) \Theta \left(\frac{\phi}{\delta_j^i} \right)^{\theta-1} \left(\frac{W_i}{P_j} \right)^{-\theta} \frac{Y_j}{P_j} + c\delta_j^i \frac{Y_j}{P_j} \\ r_j^i(\phi) &= \theta \Theta \left(\frac{\phi}{\delta_j^i} \right)^{\theta-1} \left(\frac{W_i}{P_j} \right)^{1-\theta} Y_j, \end{aligned}$$

where $\Theta = \frac{(\theta-1)^{\theta-1}}{\theta^\theta}$.

Profits and labour demand are increasing in productivity, ϕ , and market size, Y_j . A firm from i will choose to operate in j only if its operating profit from that market exceeds its fixed cost for that market. That is, only if its productivity is high enough or the distance low enough:

$$\pi_j^i(\phi) \geq 0 \iff \phi > \Phi_j^i = \left(\frac{c}{\Theta}\right)^{\frac{1}{\theta-1}} \left(\delta_j^i \frac{P_i W_i}{P_j}\right)^{\frac{\theta}{\theta-1}}$$

Φ_j^i is the threshold of quality of a firm from i to operate in j . In matching the model to real world data we are claiming that all observed firms exporting from i to j have an intrinsic productivity larger than Φ_j^i . Φ_j^i will be larger (and hence we will observe a higher average quality for exporting firms) if: (i) the frictions experienced by firms in i exporting to j are larger; (ii) labour in i is more expensive; and/or (iii) the real exchange rate P_i/P_j is larger⁷.

Notice that the threshold Φ_j^i is independent of the size of both markets, in particular it is independent of the size of j . This because of our assumption that the fixed costs are linear in market size. This assumption simplifies the analysis enormously, but further, we believe that it is the correct assumption given the purpose of our exercise. The assumption of fixed costs linear in size ensures that the relative size of two economies is irrelevant if their trade frictions equals one. If this were not the case then there would be huge implications for the effect of size upon economic activity. A trade friction of 1 between two economies essentially means that the border is just a line on the map with no real meaning, and we want our model to be invariant to such lines on the map. This requires fixed costs be linear in size.

We assume that Φ_j^i acts as an existence threshold level of productivity⁸ i.e. those new firms who draw a productivity $\phi < \Phi_j^i$ choose not to produce anything. Notice further that firms that do choose existence (i.e. $\phi > \Phi_j^i$) may still make a realised loss, because their positive profits may not be sufficient to cover their sunk creation costs. The realised value of creating a firm in country i is given by:

$$V^i = \sum_{t=0}^{\infty} \beta^t \left(\sum_{j=1}^N \pi_j^i(\phi) \right) - W_i \tilde{c} = \frac{\sum_{j=1}^N \pi_j^i(\phi)}{1 - \beta} - W_i \tilde{c}$$

$\pi_j^i(\phi)$ is as previously defined for $\phi \geq \Phi_j^i$ and is equal to zero for $\phi < \Phi_j^i$. Notice that, since the lowest threshold of activity is to operate in the domestic market, all exporters also sell domestically, but not vice versa.

⁷ P_i/P_j is the rate of exchange rate for goods sold in j in terms of goods sold in i . Given that the marginal utilities of money in i and j are respectively $1/P_i$ and $1/P_j$, then P_i/P_j is the relative value of money in j with respect to i . A high value for P_i/P_j means that it is less attractive to sell to j instead of i : if P_j is low relative to P_i , then the price of your good in country j will necessarily be low - otherwise you do not sell much in j .

⁸This is the $\delta_j^i > 1, j \neq i$ assumption. Strictly this threshold productivity level is $\min\{\Phi_j^i\}$ over all j . This general formulation leads to complicated existence conditions. We short-circuit this complexity by assuming the simple case and checking the resulting calibrations. It turns out that the measured frictions under the simple model always satisfy the assumption used to derive it and so we never need to complicate the calibrations.

3.2. Equilibrium

The equilibrium conditions are that (i) the value of creating a firm equals zero, (ii) labor demand equals labor supply and (iii) that all income must be spent and there must be a balance of payments in all economies.

Since perfect financial markets drive the expected value of firm creation to zero, the nominal value of goods production in economy i must equal payments to labour in economy i , $S_i W_i$. Consumers in economy i spend some amount Y_i on goods, and the difference between labour income and goods expenditure in economy i , $T_i = S_i W_i - Y_i$, is some (exogenous) capital account or factor payment made by economy i into world capital markets. Balance of payments across the world implies that $\sum_{i=1}^N T_i = 0$.

For some of our exercises it is important to take into account the values of T_i . Due to the redistributive role of the state, a relatively rich region within a country (say, Catalonia within Spain) pays a fiscal transfer to the rest of the country. In order to treat the region as a country is important to cancel such a transfer, as it would not happen in case of independence. We do so by exogenous altering the value of T_i . As we see below the assumptions we make on T_i determine if our model is in the ACR class or not.

The following result characterizes the equilibrium:

Result 1. *The (unique) steady state equilibrium of the model is described by the following system of equations:*

$$D_i \equiv \sum_{j=1}^N \left(\frac{P_i}{P_j} \delta_j^i \right)^{1-\mu} Y_j \quad (1)$$

$$\left(\frac{W_i}{P_i} \right)^{-\mu} \frac{D_i}{P_i} = \frac{\tilde{c}(1-\beta)}{cb^k} \left(\frac{c}{\Theta} \right)^{\frac{\mu}{\theta}} \left(\frac{1}{\theta} - \frac{1}{\mu} \right) \mu \quad (2)$$

$$\left(\frac{1}{\theta} - \frac{1}{\mu} \right) S_i = cM_i \frac{D_i}{P_i} \quad (3)$$

$$\sum_{j=1}^N X_j^i = S_i W_i = Y_i - T_i \quad (4)$$

$$\sum_{j=1}^N X_i^j = Y_i = \sum_{j=1}^N cM_j \frac{W_j}{P_j} \left(\frac{P_j}{P_i} \delta_i^j \right)^{1-\mu} Y_i \left(\frac{1}{\theta} - \frac{1}{\mu} \right)^{-1} \quad (5)$$

where $\mu \equiv k \frac{\theta}{\theta-1}$. D_i , the effective demand or market potential for economy i , is defined by Equation (1). Equation (2) comes from financial market equilibrium, Equation (3) from labour market equilibrium, and Equations (4) and (5) from goods market equilibrium.

This defines a unique mapping from parameters $\{S_i, \delta_i^j, T_i, \tilde{c}, b, c, \theta, \beta, k\}_{\forall i,j}$ into endogenous variables $\{X_i^j, W_i, P_i, M_i, Y_i\}$

Proof: See Appendix B

Result 2. *The following gravity equation holds for any combination of parameters:*

$$\ln X_j^i = \ln (S_i W_i) + \ln Y_j - \ln D_i + (1 - \mu) \ln \left(\frac{P_i}{P_j} \delta_j^i \right) \quad (6)$$

Proof: See Appendix B

Model Solution

From Equation (6), we can immediately see that the elasticity of exports to variable trade costs is $1 - \mu$. This is the “trade elasticity” which [Arkolakis, Costinot, and Rodríguez-Clare \(2012\)](#) show is the crucial parameter for the calculation of welfare impacts of changes in trade flows. We fix the parameters that make up the trade elasticity with reference to the economic literature. We take the elasticity of substitution, θ , from the literature and follow the procedure that others have followed in determining the pareto distribution parameter, k . [Bernard, Eaton, Jensen, and Kortum \(2003\)](#) (BEJK) select a θ of 3.79 to match the size and productivity advantage of US firms that export⁹. Many papers use $\theta = 3.8$ following BEJK, see e.g. [Ghironi and Melitz \(2005\)](#), [Davis and Harrigan \(2011\)](#), and [Bernard, Redding, and Schott \(2007\)](#).

Some papers calibrate k to match the standard deviation of log domestic sales in the US (as found by BEJK). See e.g. [Davis and Harrigan \(2011\)](#) ($k = 3.4$), [Ghironi and Melitz \(2005\)](#) ($k = 3.4$), [Demidova \(2005\)](#) ($k = 3.3$), [Felbermayr and Jung \(2011\)](#) ($k = 3.3$). The Standard deviation of log firm sales in this model is given by $(\theta - 1)/k$.¹⁰ BEJK produce a simulated value of the stdev of log firm sales based on US data of 0.84, therefore conditional on $\theta = 3.8$, our model requires $k = 3.3$ in order to match this value.

Using $\theta = 3.8$ and $k = 3.3$ implies a trade elasticity, $1 - \mu = -3.47$. This is very close to the estimate of -3.41 made by [Simonovska and Waugh \(2013\)](#) for the Melitz model. All the results quoted in the main text are made using $\theta = 3.8$ and $k = 3.3$. In Appendix D we also provide bounds on the quantifications produced by also using the extreme values estimated by [Simonovska and Waugh \(2013\)](#) of -2.81 (estimated assuming a BEJK model), and -5.21 (estimated assuming an Armington or Krugman model).

It turns out that all the other parameters of the model can be ignored since they can be eliminated by a change of variables (see Appendix B for the proof of this claim). The model is used in two different ways. First we calibrate the trade frictions, δ_j^i , and effective labour supplies, S_i ; and then we perform counterfactual exercises varying these values.

If $T_i = 0, \forall i$, then the model is within the class of models characterised by [Arkolakis, Costinot, and Rodríguez-Clare \(2012\)](#). This means that calibration is easy and follows directly from data, bilateral trade flows and GDPs (see Appendix A), and from Equation (6) - see Appendix C.

⁹Though in BEJK markups (and not productivity) are drawn from a pareto distribution, so the shape parameter used in their paper is not applicable.

¹⁰See Appendix B for proof of this claim.

If $T_i \neq 0$ for some i 's, then we are not in the class of models characterised by [Arkolakis, Costinot, and Rodríguez-Clare \(2012\)](#) - models in this class satisfy several macro-level restrictions, the first of which is that trade in goods is balanced. For some of our analysis it is important to consider $T_i \neq 0$, in particular in relation to changes in fiscal transfers that would be expected in the event of constitutional change.

In the $T_i \neq 0$ case, we perform the calibrations by imposing data $\{X_j^i, (S_i W_i), Y_i, T_i, D_i\}, \forall i, j \in \{1, \dots, N\}$ (see Appendix A) and solving for the equilibrium of the model in which the unknowns are $\{S_i, W_i, P_i, M_i, \delta_j^i\}, \forall i, j \in \{1, \dots, N\}$. This is a total of $N(3 + N)$ unknowns - which is more than the number of conditions that we have so long as $N > 2$. However, it turns out that trade frictions have a transitive property (see Appendix B) that allows us to reduce the dimensionality of the problem to $5N$ unknowns, $\{S_i, W_i, P_i, M_i, \delta_i\}$. The $5N - 1$ equations (2) - (6) plus normalisation, $P_1 = 1$, can therefore be solved using numerical techniques (see Appendix C).

We perform the counterfactual policy experiments by proposing a set of bilateral trading frictions, effective labour supplies, and capital flows $\{\delta_j^i, S_i, T_i\}, \forall i, j \in \{1, \dots, N\}$ and solving for the equilibrium of the model with the $5N$ unknowns $\{Y_i, W_i, D_i, P_i, M_i\}$ using $5N$ equations (equations (1) to (4) give us $4N$ equations, and equation (5) gives us $N - 1$ equations¹¹, and normalisation, $P_1 = 1$, gives a final condition). The system can therefore be solved using numerical techniques (see Appendix C).

4. The bilateral trade frictions of countries and regions

We now use the model to measure the bilateral trade frictions between countries and between regions. We divide the world into a 'Rest of the World' aggregate, and the individual economies in the OECD (see Appendix A for details of the construction of this dataset). We want to look at the distribution of bilateral trade frictions between countries of the OECD (or between countries of the EU). It can be shown (see Appendix B) that the measured frictions between economy a and economy b do not depend upon how C , the set of other economies in the world, is aggregated i.e. C could be a single entity labelled 'the rest of the world' or it could be the entire set of other individual countries, it makes no difference to the measurement of δ_b^a or δ_a^b . We also use US, Canadian and Spanish inter-regional trade in goods (only) data (again see Appendix A) to disaggregate the US, Canadian and Spanish economies (for which we have data on international trade in goods and services) into the regions of these federations, and so also observe the distribution of bilateral trade frictions between the regions of the US, Canadian and Spanish federations. The results of this are shown in Figure 2. This shows that bilateral region-region frictions are generally lower than bilateral country-country frictions, and that there is a negative relationship between the GDPs of the entities and the measured trade frictions between them.

¹¹Since balance of payments in $N - 1$ economies combined with overall balance implies balance in the other economy.

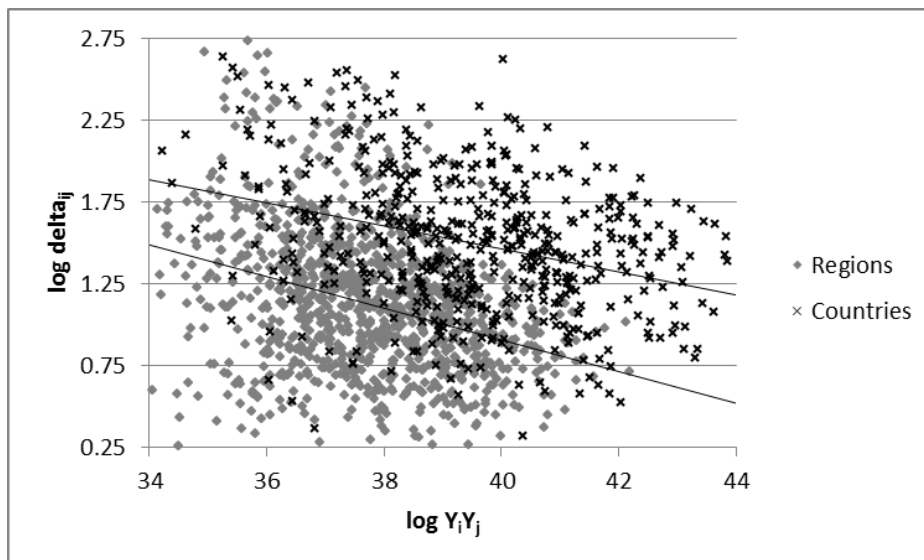


Figure 2: Scatter plot showing bilateral trade frictions against GDP split by countries and regions

Extra home bias in trade in services

The use of goods only inter-regional trade makes comparison between regional and country level frictions appear less stark than it actually is, and so is a conservative basis for conducting this comparison. We have used the ratio of, say, Ontarian goods trade with the rest of Canada to its goods trade with the rest of the world, combined with the Ontarian share of Canadian goods trade with the rest of the world, to generate a consistent measure of Ontario's internal trade, from Canada's external goods and services trade. This is a reasonable method to use to proxy internal trade, but it will understate internal trade if there is more home bias in trade in services than in trade in goods. We have this data for the Canadian Provinces and for the three regions which we consider in the following section: Scotland, Catalonia, and the Basque Country. Table 1 shows the ratio of internal trade to international trade if we consider only trade in goods, and if we consider trade in goods and services.

Every single region displays more home bias in its trade in services than it does in its trade in goods. Therefore, assuming that this is also true of the US states and the other Spanish Autonomous Communities, then differences between regional and country level frictions is actually higher than presented here.

A further case for the conservatism of the comparison that we do, is that sales across a border are more likely to be recorded and so we may expect any data quality issues to bias our results against finding a significant differences between regional and country level frictions.

Region	Goods Only	Goods & Services
Newfoundland	63%	89%
Prince Edward Island	104%	167%
Nova Scotia	71%	107%
New Brunswick	55%	80%
Quebec	44%	64%
Ontario	26%	41%
Manitoba	78%	118%
Saskatchewan	78%	106%
Alberta	49%	68%
British Columbia	48%	72%
Yukon	122%	192%
Northwest Territories	57%	108%
Nunavut	147%	296%
Scotland	136%	179%
Catalonia	126%	135%
Basque Country	74%	94%

Table 1: Ratio of internal to international trade for each region

Characterising the distribution of regional and country level bilateral trade frictions

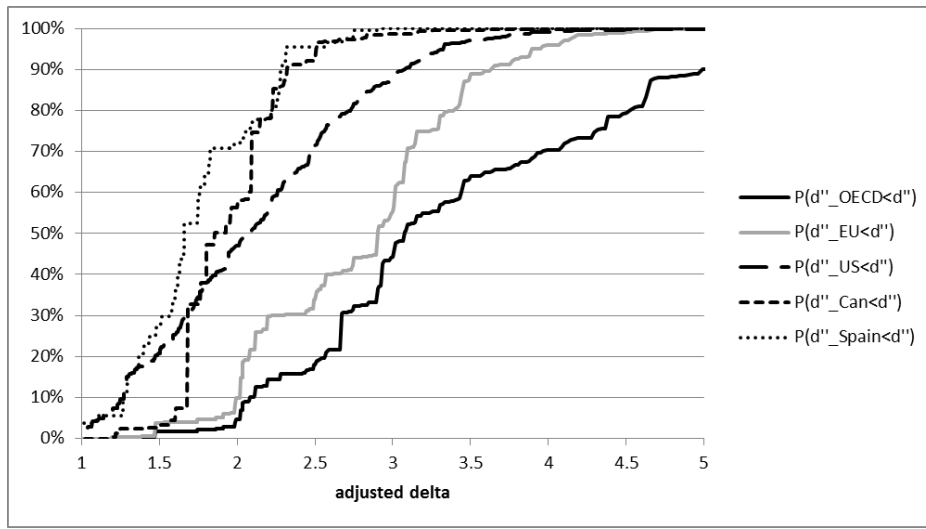
Figure 2 showed a negative dependence of measured trade frictions on GDPs of the trading partners. In an abstract, symmetrical version of the model, we can prove (see Appendix B), that for the same underlying trade frictions, larger entities are measured as having lower trade frictions. In this stylised model, we quantify a range that the negative relationship between size and frictions should fall in. The slopes observed in Figure 2 do indeed fall into this range. To ensure that we have a consistent measure of bilateral trade frictions that is uncontaminated by the size of the parties involved, we must control for the size of the parties when comparing frictions. We do this by showing *adjusted frictions* where the average slope of the log friction to log GDP relationship estimated over the OECD countries, has been used so that they all express the equivalent notional friction as at the same GDP (figures 3a and 3b).

$$\begin{aligned} \text{i.e. for } i, j \in \text{OECD, regress: } \ln \delta_{ij} &= \beta_0 + \beta_1 \ln Y_i Y_j + \epsilon_{ij} \\ \text{Then } \forall i, j \text{ define: } \ln \delta_{ij}^{adj} &= \ln \delta_{ij} - \hat{\beta}_1 (\ln Y_i Y_j - \ln \bar{Y} \bar{Y}) \end{aligned}$$

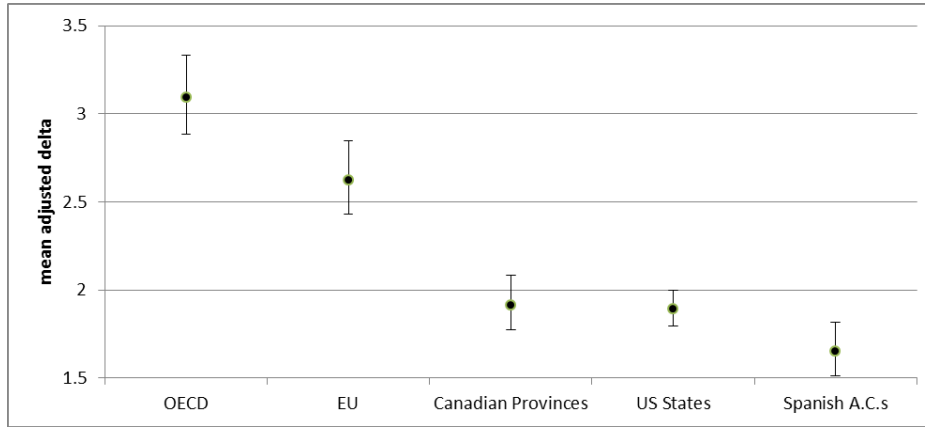
In order to be conservative, we use the observed slope, $\hat{\beta}_1$, for countries rather than the (steeper) observed slope for regions, when constructing the adjusted frictions.

The data are noisy and it does not seem reasonable to give equal weight to economically unimportant observations like the bilateral flow between Iceland & New Zealand, or between Yukon & Nunavut, as to economically important observations like the bilateral flows between USA & Germany, or between Ontario & Quebec. To deal with this, we weight the trade frictions by their share of overall trade within the appropriate grouping, giving each observation a weighting factor $X_{ij} / \sum_j \sum_{i \neq j} X_{ij}$ ¹².

¹²Using this weighting scheme seems appropriate since it is the weighting that would be used to calculate average



(a) Empirical CDF of adjusted bilateral trade frictions between economies of OECD, EU, US States, Canadian Provinces, and Spanish Autonomous Communities



(b) Weighted average adjusted bilateral trade friction between of OECD, EU, US States, Canadian Provinces, and Spanish Autonomous Communities

Figure 3: Frictions between countries and between regions

From Figure 2 it is clear that regions have lower frictions than countries conditional on size. In Figures 3a and 3b we can see that there is almost first order stochastic dominance for region-region adjusted trade frictions compared with country-country adjusted trade frictions¹³. The mean country-country adjusted trade frictions are well above the mean region-region ones. This is despite the fact that for both, regions and countries, we have compared trade frictions between entities that are spread across continental areas: the physical distances within the EU are less than those between US States or Canadian provinces.

losses actually incurred in trading through these frictions. The production in i that is sold to j but which *melts in transit* due to iceberg losses is $\tau_j^i X_j^i = (1 - 1/\delta_j^i) X_j^i$. Therefore the average iceberg cost, $\bar{\tau}$ is the mean value of τ_j^i when observation i, j is weighted by $X_{ij} / \sum_j \sum_{i \neq j} X_{ij}$.

¹³Not first order stochastic dominance for Canadian provinces only because of low trade frictions between Czech Republic & Slovakia, Hungary & Slovakia, Belgium & the Netherlands, and Estonia & Finland.

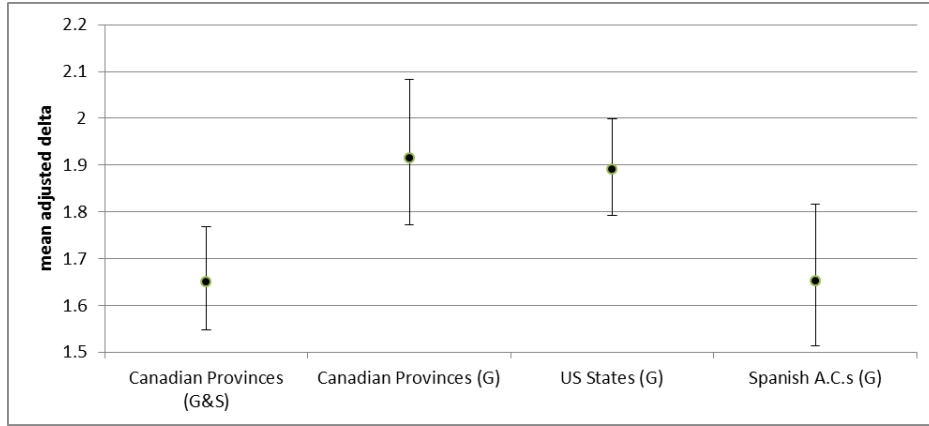


Figure 4: Weighted average adjusted bilateral trade friction between US States, Canadian Provinces, and Spanish Autonomous Communities, where we can compare estimate for the Canadian Provinces generated using the goods only data as per Figure 3b, and using goods and services data.

Based on this evidence, we claim that region-region borders are systematically less frictional than country-country borders. As described in the previous section, this is based on a conservative calculation given the excess home bias in trade in services. To provide some quantification of this degree of conservatism, Figure 4 shows the mean adjusted frictions for Canada calculated both using the goods only data of Figure 3b and now also using the interregional goods and services data. This substantially lowers average frictions and widens the countries versus regions gap.

5. From regional borders to country borders

The previous section gives us the empirical result that region-region borders are systematically less frictional than country-country borders. This motivates our exercise: we will change the frictions between an independence seeking region and its current union partner, to those that pertain between the union and its current least frictional independent trading partner. All other calibrated values will be kept constant in this counterfactual exercise. Implicitly, we are supposing that the independence seeking region’s border will come to resemble a normal country-country border, albeit one between close neighbours.

We do not try to capture transition dynamics because in the short and medium term it is difficult to guess the degree of interaction, as two forces operate in different directions. The process of separation may be expected to create tensions which would reduce the interactions between the former partners. On the other hand, history must have built strong links that may persist for some time. We do not know how long this transition period will last¹⁴. Therefore we

¹⁴A recent example that may shed light on the dynamics of the development of country level border frictions is the Czech Republic and Slovakia case. This is a recent case and did not involve a war, following the so-called “*Velvet Divorce*” of 1993. As referred to in footnote 13, the Czech Republic and Slovakia are still very close trading

focus solely on long run steady state. Further, focussing on this transition would require a view of all the aspects of the economic calculus of independence. We only measure the ceteris paribus cost of the border becoming like that of a normal country-country border and say very little about any other expected changes upon independence, other than controlling for large and fairly certain changes in the capital account that are explained by fiscal transfers.

Counterfactual exercises

The first stage of this is to identify the current least frictional trading partner of Spain (as a counterfactual for both Catalonia and the Basque Country) and the UK (as a counterfactual for Scotland). We do not claim that this represents the trade frictions that the independence seeking region will ultimately attain, merely that, given the data, it is plausible to propose that this is the case. We do not propose this counterfactual as a prediction, but as a benchmark of what other independent countries look like in trade terms. We do this by first running the simple regressions:

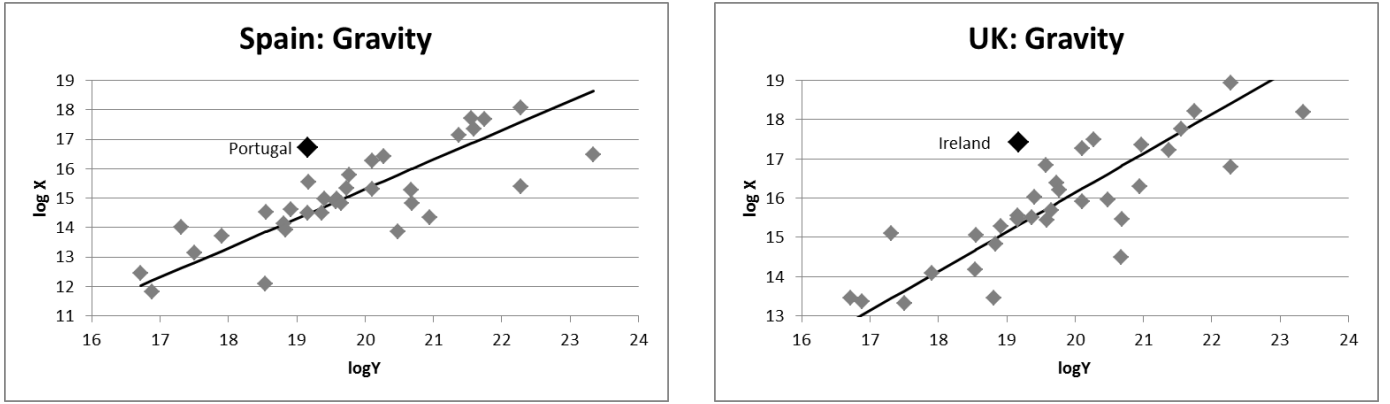
$$\ln X_i = \alpha + \ln Y_i + \epsilon_i$$

where X_i is the average of imports and exports to and from Spain or the UK with country i and Y_i is the national income measure (see Appendix A). The independent country that is chosen as the counterfactual for the independence seeking region is the country with the highest residual in these regressions. As shown in figures 5a and 5b, these countries are Portugal and Ireland respectively. The fact that Portugal and Catalonia, and Ireland and Scotland, are similar in size is helpful given the dependence of measured trade frictions upon sizes. When we come to the quantification exercises, these size considerations will mean that the quantification for Scotland is fairly unaffected by this issue given the similarity in size of Scotland and Ireland, and rUK¹⁵ and UK. However, the quantification for Catalonia's costs on independence could be underestimated due to this issue because although Catalonia and Portugal are similar in size, rSpain in this case is substantially smaller than Spain (smaller entities need a larger δ to model a given underlying trade friction). Likewise, the Basque quantification may underestimate the loss due to this issue because although in this case rSpain is similar in size to Spain, the Basque Country is substantially smaller than Portugal.

We do not choose Portugal or Ireland because they are geographically close to Spain or the UK, but because they trade a lot with Spain and the UK. We do not make any claim on why

partners with very low border frictions, however, data from the IMF Direction of Trade statistics shows that the share of bilateral trade in total trade fell dramatically between 1993 and 2003, with the share of total Czech exports going to Slovakia going from 22% to 8%, and the share of total Slovakian exports going to the Czech Republic going from 42% to 13%. This is not likely due to the opening of trade with the rest of the world following the fall of the Iron Curtain. The same data source suggests that the share of trade between other neighbours from the Eastern bloc, e.g. Poland and Hungary, held up much better, or actually increased, following the opening up to trade with the rest of the world.

¹⁵Denote the rest of the UK as rUK, and the rest of Spain as rSpain.



(a) Regression of log trade flows with Spain on log GDP

(b) log trade flows with the UK on log GDP

Figure 5: Gravity equations of UK and Spain

Portugal is so economically close to Spain, and Ireland to the UK. Total trade frictions may be determined by geography, history, culture, or simply by chance. We do not probe the reasons for these frictions in this paper: we simply measure them, and we note that frictions appear to be much greater between independent countries than between regions of the same country.

Despite choosing counterfactuals on the basis that they have the closest trade interaction with Spain and the UK, these trade relationships do not look anything like the level of interaction that Catalonia, the Basque Country and Scotland have with rSpain and rUK. Table 2a shows imports and exports as a percentage of GDP for Portugal & Spain, Catalonia & the rSpain, Basque Country & rSpain, Ireland & the UK, and Scotland & rUK. X_j^i (X_i^j) denotes exports (imports) from (to) i to (from) j , and in each case the smaller party is i . Therefore the first row in the table is the bilateral trade as a percentage of the smaller entity's GDP. The second row is bilateral trade as a percentage of the smaller entity's trade with the rest of the world. The third row shows λ , the smaller entity's *home share*. We see that these regions trade much much more with the rest of the country that they are part of than Portugal or Ireland do with their closest trading partners (though Ireland has strong trade interactions with the rest of the world, while Portugal is relatively closed).

The trade frictions, δ , are *measured* by requiring the model to reproduce the trade flows seen in the data. They are reported in Table 2b, where again i labels the small entity under consideration, j labels the larger partner, and R labels the rest of the world.

We also calibrate the model to modified data in which we force capital flows to equal zero and imports to equal exports (see Appendix A for details of this procedure). It can be shown (see Appendix B for proof) that with no capital flows and with bilaterally balanced trade flows, that $\delta_j^i = \delta_i^j \equiv \delta_{ij}$. The calibrated trade frictions in this case are shown in Table 2c, and can be seen to be approximately the average of δ_j^i and δ_i^j from Table 2b.

The counterfactual policy experiment exercises use the calibrated parameter values for Catalonia/rSpain, Basque/rSpain, and Scotland/rUK. These calibrations consist of the trade frictions

	Portugal/Spain	Catalonia/rSpain	Basque/rSpain	Ireland/UK	Scotland/rUK
$\frac{X_j^i + X_i^j}{GDP_i}$	18.5%	60.7%	82.5%	32.3%	78.8%
$\frac{X_j^i + X_i^j}{X_R^i + X_i^R}$	35.8%	91.3%	149.5%	30.0%	193.4%
$\lambda_i = \frac{2X_i^i}{GDP_i + GNT_i}$	66.7%	34.9%	33.1%	24.8%	43.8%

(a) Trade flows in data

	Portugal/Spain	Catalonia/rSpain	Basque/rSpain	Ireland/UK	Scotland/rUK
δ_j^i	2.88	1.42	1.63	1.93	1.61
δ_i^j	1.90	1.41	1.48	1.70	1.54
δ_R^i	3.28	2.68	3.09	2.03	3.03
δ_i^R	2.71	2.25	2.96	2.02	3.04

(b) Trade frictions in model required to match data

	Portugal/Spain	Catalonia/rSpain	Basque/rSpain	Ireland/UK	Scotland/rUK
δ_{ij}	2.32	1.41	1.55	1.81	1.57

(c) Trade frictions in model required to match modified data

Table 2: Trade flows and trade frictions

shown in Table 2b as well as effective labour supplies, and capital transfers expressed as a percentage of GDP. To model independence we replace δ_j^i and δ_i^j with the δ_{ij} from Table 2c (that of Portugal for Catalonia and the Basque Country, and that of Ireland for Scotland). For Catalonia, we also reduce the capital transfer percentage by 6.5% to account for the large fiscal transfer that Catalonia currently pays to the rest of Spain¹⁶ no longer being payable on independence. We leave the capital transfer percentage as calibrated for the Basque Country and for Scotland. The current arrangements for the Basque Country are highly decentralised with taxes raised in the Basque Country paying for spending in the Basque Country with only some payment to Madrid for “shared services”. In the case of Scotland, we are modelling Scotland’s “on-shore” GDP and trade, and the oil revenues that an independent Scottish Government will gain on independence are broadly similar to the fiscal transfer that a notional “on-shore” Scotland will lose given current government accounts¹⁷.

The results of the counterfactual policy experiment are shown in Table 3. The first three

¹⁶This is the official number for the transfer with the methodology of “flujo beneficio” for the year 2005. In 2009 this number would be 5.8% of the GDP of Catalonia.

¹⁷Currently this “on-shore” Scotland receives a fiscal transfer from the rest of the UK - since dividing the UK in this way means that rUK includes the “Ex-regio” region of the UK, from which North Sea oil is extracted. The rationale for modelling Scotland in this way is that we are supposing that the imposition of a national border between Scotland and the rest of the UK impedes cross border trade. We do this by imposing a model that has elastic demand for a continuum of differentiated goods. This may be a good model for manufactures, and it provides a rationale for intra-industry trade, but it is unlikely to be a good model for an essential homogenous good like oil. Trade flows related to oil are unlikely to be impeded by independence, and so including these in the calibration and then allowing them to be impeded in the counterfactual independence scenario would be to overstate the costs of the border.

	Catalonia/rSpain	Basque/rSpain	Scotland/rUK
$\frac{X_j^i + X_i^j}{GDP_i}$	13.9%	25.8%	53.7%
$\frac{X_j^i + X_i^j}{X_R^i + X_i^R}$	16.6%	36.2%	118.0%
$\lambda_i = \frac{2X_i^i}{GDP_i + GNI_i}$	51.4%	52.9%	53.4%
Δ real GDP i	-9.5%	-12.5%	-5.5%
Δ real GDP j	-1.9%	-0.6%	-0.4%
Δ real GNI i	-3.4%	-12.5%	-5.5%
Δ real GNI j	-3.1%	-0.6%	-0.4%
Φ_i^i	-12.7%	-16.6%	-7.3%
Φ_j^i	+76.6%	+54.9%	+16.9%
Φ_R^i	-7.7%	-4.5%	-3.3%

Table 3: Results of counterfactual independence scenarios

rows can be compared with the data in Table 2a. The next four rows show the impact upon production (GDP) and income (GNI) on both the now independent region, and on its former union partner. The final three rows show how the productivity thresholds for firm existence, selection into exporting to former union partner, and selection into exporting internationally, have changed.

The increase in distance with their largest trading partners has dramatic effects on trade, production and welfare. There is a large fall in productive capacity in these small economies, and a smaller loss in the larger economies of their former union partners. The degree of interaction between these former partners comes to resemble the degree of interaction between normal countries i.e. it is similar to the Portugal/Spain and Ireland/UK interactions seen in Table 2a. The explanation for these changes can be seen in the bottom three rows of the table: the productivity threshold for the existence of a firm has fallen, because wages have fallen, and so more low quality firms exist, reducing employment in high quality firms i.e. there has been a reallocation of resources away from quality which pushes TFP down. The productivity threshold for selecting into exports with former union partner has risen dramatically as the costs of doing this business have risen with δ . The productivity threshold for selecting into exports to the rest of the world has fallen slightly: the costs of doing this business are unchanged but local wages have fallen and so slightly lower quality firms than previously find it profitable to export.

The effect of eliminating the fiscal transfer from Catalonia to the rest of Spain is seen in the divergence of impact between GDP and GNI: the productivity impact of increased trade frictions for Catalonia is broadly similar to that for the Basque Country and for Scotland; but the welfare impact is muted by the gains from not paying the fiscal transfer. Obviously money that is not transferred to the rest of Spain increases income in Catalonia. What changes here is the distribution of the losses, which now fall more heavily on the rest of Spain. This distribution of losses does not greatly affect the impacts on the firm quality distribution. The loss in Catalonia is mitigated by not paying the fiscal transfer, whilst the loss in the rest of Spain is now larger.

	Catalonia/rSpain	Basque/rSpain	Scotland/rUK
$\frac{(X_j^i + X_i^j)'}{X_j^i + X_i^j}$	20.8%	27.3%	64.4%

Table 4: Trade flows with partner in counterfactual as a percentage of those flows in data

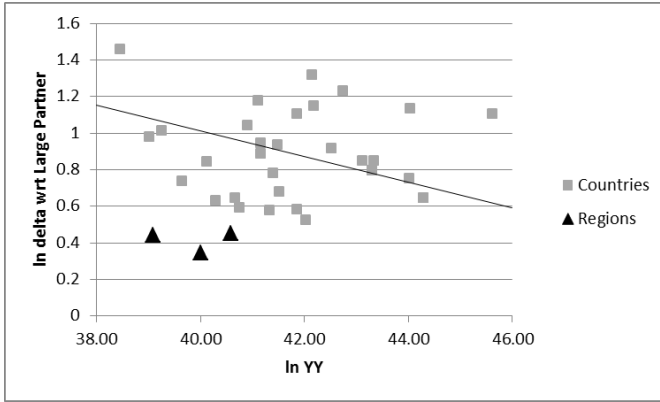
Comparison with existing literature

Our estimated impact upon regional trade is perfectly consistent with the literature on border effects taking into account endogeneity and trade with the rest of the world. [Anderson and van Wincoop \(2003\)](#) (AvW) estimate that we should expect to see a fall in trade of around 80% when comparing trade across a border to trade without the border. Our results, shown in Table 4, are entirely consistent with AvW for Catalonia and the Basque Country, and so they do not appear to be exceptional when viewed under the light of the border effect. The results for Scotland do not show such an extreme reduction in Scotland-rUK trade, which falls by only 35.6%. This is because Ireland is calibrated as having a relatively “thin” (but still of country-country level) border with the UK. The reason why these regions trade “so much” and why they concentrates their trade so much is due to the “thick” border with the rest of the world and the “thin” border with the rest of the country of which they are currently a part.

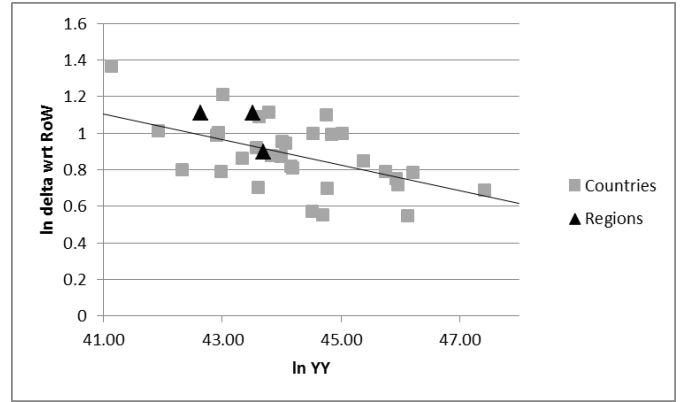
The gains from trade in the literature are usually expressed as a welfare cost on moving to an autarkic state i.e $\lambda' \rightarrow 100\%$. Using the ACR formula, we can simply calculate the welfare loss on moving from trade flows seen in the data, to a state of autarky. These results are shown in Table 5. The point of this exercise is to show the relative value of low region-region borders with a single party compared with country-country borders with every other country in the world. Therefore it is easier to simply use the ACR formula for Catalonia, since distortions caused by the change of capital flow will appear in every result and will to first order cancel out. The first row repeats data from Table 2a, the second row uses this in the ACR formula with $\lambda' = 100\%$. The third row calculates the cost of independence using the ACR formula on λ' from Table 3, and the forth row shows the required residual losses to go from independence to autarky. The final row shows that the losses on independence represent a significant percentage of the total losses that these regions stand to lose on complete autarky.

	Portugal	Catalonia	Basque	Ireland	Scotland
λ_i	66.7%	34.9%	33.1%	24.8%	43.8%
Δ real GNI: Autarky	-10.9%	-25.8%	-26.9%	-32.7%	-20.9%
Δ real GNI: Independence		-10.4%	-12.5%		-5.5%
Δ real GNI: Autarky once independent	-10.9%	-17.2%	-16.5%	-32.7%	-16.3%
Independence / Autarky		40.2%	46.3%		26.2%

Table 5: Losses on independence relative to losses on autarky



(a) Regions are better than expected at trading with their largest partner



(b) Regions do not seem to be any worse than expected at trade with the rest of the world

Figure 6: Distances with main trading partner and rest of the world

Regional intergration with the rest of the world

The cost of autarky for the regions is high, but this is because it is the composition of two losses. The first loss is the increase in frictions with their union partner, that were very low (much lower than the normal between countries), and are then those of a normal country. The second loss is the increase in frictions from having normal country borders with their union partner and with the rest of the world, to having infinite frictions. This composite effect means that we may expect the cost of autarky for sub-national regions to be higher than is estimated for independent countries of equivalent size. However, this is conditional on regions within larger nations being as economically integrated with the rest of the world as independent nations. Given the data we have collected in this exercise, we can provide some evidence as to whether this is true or not. Figure 6a shows the OECD countries alongside Scotland, Catalonia and the Basque Country, in terms of their measured trade frictions with their largest trading partner. As can be seen, and consistent with the above results, our regions have substantially lower frictions with their largest partners than the countries.

Figure 6b then shows the corresponding friction with the rest of the world, after stripping out the largest trading partner. Now we can see that the regions have roughly the expected level of trading frictions with respect to the rest of the world. There is no evidence, from this exercise, that newly independent regions can expect to substitute higher frictions with their former union partners for lower frictions with the rest of the world.

Alternative counterfactual scenarios

The gains from trade to a region from the economic integration it enjoys with its union partner is a significant proportion of its total gains from trade. These gains from trade are clearly a function of what we believe the border frictions will be on independence. We can show this graphically by generating a series of welfare results for Scotland, changing only frictions with the rest of the

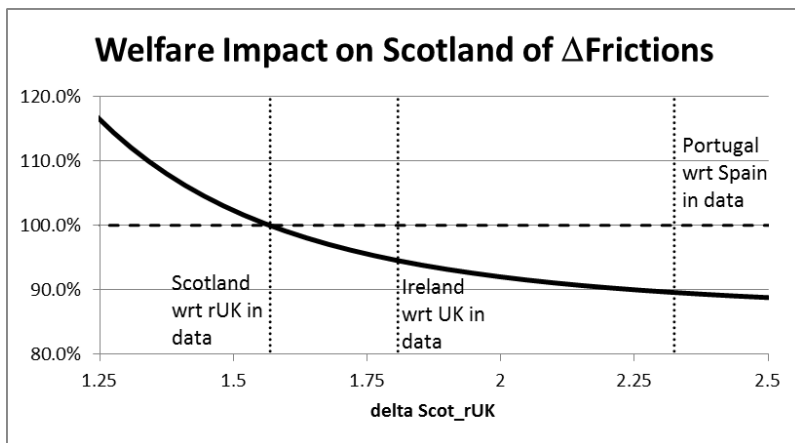


Figure 7: Scottish GNI as a function of the trade friction with the rest of the UK.

UK between each result and graphing on a single plot, see Figure 7¹⁸. Perhaps it was felt that, as trade in goods between Ireland and the UK is largely across a sea, a lower increase in frictions was appropriate for Scotland? Conversely, it could be argued that Ireland’s exceptionally high level of openness means that a higher increase in frictions is appropriate for an independent Scotland? The results of these alternative counterfactual scenarios can be read off from Figure 7. It could be argued that Scotland trade with the rest of the world will change in one way or the other - this would correspond to a vertical shift in the curve in Figure 7, the precise quantification of which would be generated by explicitly changing the friction with the rest of the world in the model.

In general, the relationship between independent countries is such that their trade exists along the ‘flat part’ of the ‘frictions vs income’ curve, whereas sub-national regions exist in the steeper region where gains from trade are larger. This suggests that there may exist large economic benefits, through closer trade links, of further integration at the EU level (since, as per Figure 3b, we observe that the borders between EU member states are of the “thick”, independent country type).

6. Trade Costs in the EU

The single market in the EU is an attempt to create the trade benefits of a single country across Europe. Are national borders within the EU “thinner” than borders across the EU/non-EU divide? Table 6 shows the model measured frictions between some potential candidates for EU membership¹⁹ and the EU, and the frictions between some selected EU members (selected on the basis of *matching* the non-EU members to some extent) and the rest of the EU.

¹⁸Note that the limit of infinite frictions on this plot does not correspond to the autarkic Scotland - the only friction that is being changed in this plot is the friction with the rest of the UK, the friction with the rest of the world is unaltered.

¹⁹This is a limited set given our OECD dataset.

Non EU	δ_{EU}	$Adj \delta_{EU}$	Matched EU	δ_{rEU}	$Adj \delta_{rEU}$
Norway	2.27	2.90	Sweden	2.12	2.74
Iceland	3.64	3.81	Finland	2.41	2.99
			Denmark	2.22	2.80
Switzerland	1.95	2.53	Austria	1.96	2.51
Turkey	2.78	3.69	Greece	2.90	3.64

Table 6: Frictions across and within EU borders

These results are suggestive of a small border effect from the EU, though not on the same scale as the internal border effect described in the previous section. Obviously there are many potential endogeneity issues here e.g. perhaps Switzerland did not join the EU since there were some costs in other spheres, and it already had all the trade benefits that it could get from the EU countries, whilst countries that could gain trade benefits on joining did so and have now reduced their frictions to something akin to Switzerland’s. Conversely there could be dynamic issues and a process may be underway which has reduced the frictions within the EU and may reduce them further.

The correlation between membership or otherwise of the EU and the log of adjusted trade frictions is such that membership of the EU is associated with perhaps a 3% reduction in size adjusted trade frictions. Clearly, we cannot claim that this is causative. However it can form the basis for starting to consider the trade implications of a British exit from the EU, so-called “Brexit”, a referendum on which is proposed by the Conservative Party for 2017. Further, because the UK makes a net contribution to EU finances, these trade costs can be compared with the benefits of not paying the UK’s net contribution in the same manner as we compared the trade costs to Catalonia with the benefits they receive from not paying the fiscal transfer. The UK’s net contribution²⁰ is estimated at 0.2% of UK GDP, and our counterfactual Brexit scenario is to increase frictions by 3%. The results of this exercise are shown in Table 7.

	UK in EU	UK out EU
δ_{rEU}	1.97	2.03
λ_{UK}	70%	71%
Δ real GDP		-0.5%
Δ real GNI		-0.3%

Table 7: The trade cost of Brexit

It is interesting that the net contribution required of the UK is of the same order as the trade benefits that the UK gains from EU membership under this estimate of the magnitude that trade frictions are reduced by EU membership. Clearly however, whatever the magnitude of the trade gains from EU membership, they do not reduce trade frictions to the level of internal borders within a country.

²⁰Reported on the BBC as €3.5b, see <http://news.bbc.co.uk/1/hi/world/europe/8036097.stm>

7. The United States of Europe

Suppose, a United States of Europe could be achieved. What are the potential gains? We can estimate these by reducing all measured frictions within the EU by the average considered in our counterfactual exercises of Section 5 in the following manner²¹:

$$\delta'_{ij} = \delta_{ij}^{\exp(-58\%)}$$

The Scottish friction represents a reduction of 27% compared to the Irish friction, whilst the Catalan friction represents a reduction of 89% compared to the Portuguese friction. Therefore we quantify the impact of achieving a *United States of Europe* by considering an across the board reduction in frictions of 58%, which is the average of these two figures. Performing this policy experiment (see Appendix C for the numerical procedure) produces a gain in GNI of approximately 17%. Analysing the resulting distribution of trade frictions in the manner of Section 4 suggests that this calculation is fair: Figure 8 shows that the mean (size adjusted) bilateral trade friction for the United States of Europe is approximately equal to the average Canadian provincial, or US State bilateral trade frictions²².

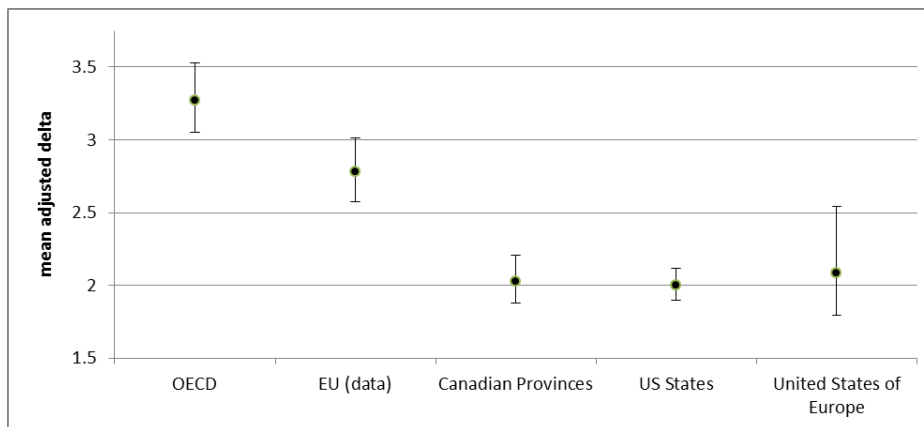


Figure 8: As Figure 3b, now also showing estimated United States of Europe

8. Conclusion

We have shown evidence that the independence seeking regions of Europe are typical open economies if we include trade with the rest of the country of which they are currently part. However, these regions have an exceptionally high level of economic integration with their union

²¹This method ensures that large frictions see proportionately larger reductions, and as the friction, $\delta \rightarrow 1$, no reduction is seen

²²Error bars are constructed using the Irish to Scottish reduction of 27% for the upper bound, and the Portuguese to Catalan reduction of 89% for the lower bound.

partners. The level of this integration is such their trade patterns are atypical of independent countries. To maintain these levels of trade concentration after a move to independence would make these countries highly anomalous. We implement a model of these regions as independent countries which are relatively close to their former union partners, but not exceptionally close compared with other independent countries. Natural counterfactual examples exist, which are economically close to these countries, but not exceptionally close like these regions.

We provide evidence for this being a reasonable exercise by using our structural model, based on Melitz (2003), to measure trade frictions across groups of countries and regions. We show that country-country borders are systematically substantially and significantly more frictional than region-region borders. Further, we demonstrate that this exercise is on the conservative side by presenting evidence that a large degree of home bias in services trade enhances the border effect. Given that country-country and region-region borders appear to be systematically different, we propose that we calibrate the model to a region of interest, the rest of the country of which it is part, and the rest of the world; and also to a plausible counterfactual small country, our large country of interest, and the rest of the world. These calibrations produce a set of parameters which, when plugged into the model, reproduce the incomes and trade flows seen in the data. In the calibrations, the counterfactual countries have much more frictional trade with the whole unions, than the independence seeking regions have with their union partners. The policy experiment undertaken is simply to replace the region's friction with its union partner with the small independent country's friction with the whole union, keeping all other parameters unchanged. Observing the impacts upon incomes and trade flows, we see that the economic integration of these regions with their union partners represents a significant fraction of their overall gains from trade.

On the other hand, the borders of the EU do not seem especially frictional compared with normal country-country borders. There is a weak correlation between EU membership and lower frictions in trading with the EU, but it is very weak. Using this observed correlation we estimate that the trade benefits that the UK receives from the EU are likely of the same order as its net contribution to the EU, but could very easily be larger. If however, the EU could evolve such that it comes to resemble a United States of Europe then the welfare gains could be large, we estimate a level effect of perhaps 17% of income.

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Appendices

A. Data

The data is all from years 2005 - 2007 (recent but not so recent as to be subject to further revision and unaffected by disruptions due to financial crisis from 2008 onwards).

Country Data

- Have GDP and total (goods and services) bilateral trade flows for 2006 from the OECD

$$\left(\tilde{S}_i \tilde{W}_i\right) \& X_j^i, \forall i \& \forall j \neq i$$

- Goods trade from the OECD's "STAN Bilateral Trade Database by Industry and End-use category"
- Services trade from the OECD's "EBOPS 2002 - Trade in Services by Partner Country"
- Consistent GDP measures inferred from
http://www.oecd-ilibrary.org/trade/exports-of-goods-and-services_exp-gds-serv-table-en
and
http://www.oecd-ilibrary.org/trade/imports-of-goods-and-services_imp-gds-serv-table-en
- "Rest of the world GDP" inferred from the OECD's "Economic Outlook No 93 - June 2013 - Long-term baseline projections"

US Data

- 2007 State GDP as percentage of US GDP, from Bureau of Economic Analysis <http://www.bea.gov/itable/>
- 2007 bilateral trade flows (goods only) between states and internationally from the Freight Analysis Framework <http://faf.ornl.gov/fafweb/Extraction0.aspx>
- State GDP for use in the model is taken to be the US GDP from the Country data above multiplied by the state percentage of total GDP from this dataset
- State exports to other states and internationally, for use in this model, is taken to be US international exports from the country data above multiplied by the individual trade flow from this dataset divided by total US international exports from this dataset.

Canadian Data

- 2006 Data from Statistics Canada, Table 386-0002 Interprovincial and international trade flows at producer prices
<http://www5.statcan.gc.ca/cansim/a05?lang=eng&id=3860002>
- This gives bilateral trade flows (either goods only, or goods and services) between provinces and internationally, as well as provincial GDPs
- Provincial GDP for use in the model is taken to be the Canadian GDP from the Country data above multiplied by the provincial share of total GDP from this dataset

- Provincial exports to other provinces and internationally, for use in this model, is taken to be Canadian international exports from the country data above multiplied by the individual trade flow from this dataset divided by total Canadian international exports from this dataset.

Spanish Data

- Goods only trade data, as at 2006, for all Spanish Autonomous Communities in terms of imports and exports to the rest of Spain and internationally, from C-Intereg 2008 (Table 6 on p28 of http://www.c-intereg.es/El_Comercio_Interregional_en_Espa%C3%B1a_1995-2006_29_10_08.pdf)
- A matrix of goods only bilateral trade flows between Autonomous Communities from C-Intereg statistics query: "Filas: CCAA origen: Todos; Columnas: Unidades; CCAA destino: Todos; Flujo: Inter + Intra; Dato acumulado de los años: 2005" on http://212.227.102.53/explotacion_multidimensional_comercio_interregional/estadisticas.aspx
- Autonomous Community GDPs from Eurostat ("Regional gross domestic product by NUTS 2 regions - million EUR, Code: tgs00003")
- Separately, we have goods & services data for Catalonia and the Basque Country.

Autonomous Community Goods only data

- Autonomous Community GDP for use in the model is taken to be the Spanish GDP from the Country Data above multiplied by the Autonomous Community share of total GDP in 2006 from Eurostat data
- The 2006 trade data is used to split Spanish trade with the rest of the world by A.C. and the trade with the rest of Spain is inferred from the comparison between rSpain and RoW trade by A.C. from this table
- The trade with the rest of Spain is split into bilateral trade across all A.C.s using matrix of bilateral trade flows

Catalonia

- Use Eurostat GDP split, now as at 2005, to calculate Catalan and rSpain GDP from Spanish GDP from Country Data
- Goods and services trade with rest of Spain and with rest of the world from Comptes econòmics simplificats de l'economia catalana 2005, expressed as % of Catalan GDP allows us to calculate Catalan trade with rSpain and RoW from Catalan GDP
- rSpain trade with rest of the world taken to be Spanish trade from Country Data minus Catalan trade from above
- Split Catalan trade with rest of the world, as calculated above, using data from C-Intereg (Table 4 of http://www20.gencat.cat/docs/economia/Documents/Arxius/doc_14603187_1.pdf) for use in Herfindahl calc

Basque Country

- Use Eurostat GDP split, now as at 2005, to calculate Basque and rSpain GDP from Spanish GDP from Country Data
- Goods and services trade with rest of Spain and with rest of the world from http://en.eustat.es/elementos/ele0010000/ti_Gross_Domestic_Product_of_the_Basque_Country_by_components_Supply_and_demand_Current_prices_thousands_of_euros_2005-2012a/tbl0010072_i.html#axzz2vHsbZz as % of Basque GDP allows us to calculate Basque trade with rSpain and RoW from Basque GDP
- rSpain trade with rest of the world taken to be Spanish trade from Country Data minus Basque trade from above
- Split Basque trade with rest of the world, as calculated above, by destination for imports and exports from Eustat (http://en.eustat.es/ci_ci/estadisticas/tema_374/opt_0/tipo_1/ti_Foreign_Trade/temas.html#axzz2vHsbZzjn) for use in Herfindahl calc

Scottish Data

- Use GERS (<http://www.scotland.gov.uk/Publications/2013/03/1859>) ratio of "Scotland - Excluding North Sea GDP" to UK GDP, for 2006-07, to split the UK GDP from Country Data
- Goods and services trade with rest of UK and with rest of the world from Scottish Government's Input-Output tables for 2007 (<http://www.scotland.gov.uk/Topics/Statistics/Browse/Economy/Input-Output/Downloads/I01998-2009A11>), expressed as % of Scottish GDP allows us to calculate Scottish trade with rUK and RoW from Scottish GDP
- rUK trade with rest of the world taken to be UK trade from Country Data minus Scottish trade from above
- Split Scottish trade with rest of the world, as calculated above, by destination from Global Connections Survey 2011 data for 2007 (<http://www.scotland.gov.uk/Topics/Statistics/Browse/Economy/Exports/GCSIntroduction/GCS2011pdf>) for use in Herfindahl calc

3 Country Model Data Procedure

- Suppose

$$X_i^i = \left(\tilde{S}_i \tilde{W}_i \right) - \sum_{j \neq i} X_j^i$$

$$Y_i = \sum_{j=1}^n X_i^j$$

$$T_i = Y_i - \left(\tilde{S}_i \tilde{W}_i \right)$$

$$D_i = \frac{\left(\tilde{S}_i \tilde{W}_i \right) Y_i}{X_i^i} \text{ (derived from gravity equation - See Appendix B)}$$

n Country Model Data Procedure

Use an alternative procedure that eliminates capital flows, equates GDP and GNI and equates imports and exports:

- As before, calculate from data

$$X_i^i = \left(\tilde{S}_i \tilde{W}_i \right) - \sum_{j \neq i} X_j^i$$

- Now however let

$$\begin{aligned} X_{ii} &= X_i^i \\ X_{ij} &= \frac{1}{2} \left(X_j^i + X_i^j \right) = X_{ji} \\ Y_i &= \left(\tilde{S}_i \tilde{W}_i \right)' = \sum_{j=1}^n X_{ij} \\ \text{so } T_i &= 0 \\ \& \text{ } D_i &= \frac{Y_i^2}{X_{ii}} \end{aligned}$$

B. Proofs

Result 1: Model defined by system of equations

We can evaluate the distribution function for productivity and hence the probability of a firm receiving a productivity draw that will lead it to choose to produce:

$$Pr(\phi \geq \Phi_i^i) = 1 - F(\Phi_i^i) = \left(\frac{b}{\Phi_i^i} \right)^k = b^k \left(\frac{\Theta}{c} \right)^{-\frac{\mu}{\theta}} \left(\frac{W_i}{P_i} \right)^{-\mu}; \quad \text{where } \mu = k \frac{\theta}{\theta - 1}$$

We assume $k > \theta - 1$ so that average profits, revenues and labour demand are defined²³.

The average realised per period net profit, labour demand²⁴, and value of sales, of the observed firms is the expected value of these quantities, conditional on existence:

$$\begin{aligned} \bar{\pi}^i &= \int_b^\infty \sum_{j=1}^N \pi_j^i(\phi) \frac{dF(\phi)}{1 - F(\Phi_i^i)} = cW_i \left[\frac{1}{\theta} - \frac{1}{\mu} \right]^{-1} \frac{1}{\mu} \frac{D_i}{P_i} \\ \bar{L}^i &= \int_b^\infty \sum_{j=1}^N L_j^i(\phi) \frac{dF(\phi)}{1 - F(\Phi_i^i)} = c \left[\frac{1}{\theta} - \frac{1}{\mu} \right]^{-1} \left(1 - \frac{1}{\mu} \right) \frac{D_i}{P_i} \\ \bar{r}^i &= \int_b^\infty \sum_{j=1}^N r_j^i(\phi) \frac{dF(\phi)}{1 - F(\Phi_i^i)} = cW_i \left[\frac{1}{\theta} - \frac{1}{\mu} \right]^{-1} \frac{D_i}{P_i} \end{aligned}$$

Where we define effective (nominal) demand in economy i as:

$$D_i = \sum_{j=1}^N \left(\frac{P_i}{P_j} \delta_j^i \right)^{1-\mu} Y_j$$

The world economy is in steady state general equilibrium if the following conditions hold:

(i) Within each country there are perfect financial markets which allow prospective firms to borrow to finance firm creation via contingent contracts which, in equilibrium, will be repaid using any realised

²³Profits, revenues and labour demand also have a Pareto distribution, but with exponent $k + 2 - \theta$. The means of these Pareto distributions are only defined if $k + 2 - \theta > 1$ i.e. $k > \theta - 1$.

²⁴Notice that labour demand from the average firm does not depend on the wage. The total labour demand it is going to depend on wages though because the number of firms will depend on wage.

profits. Free entry for entrepreneurs to create firms therefore means that the expected value of firm creation will be driven to zero:

$$E[V^i] = F(\Phi_i^i) \times 0 + (1 - F(\Phi_i^i)) \times \frac{\bar{\pi}^i}{1 - \beta} - W_i \tilde{c} = 0$$

$$\text{i.e.} \quad \left(\frac{W_i}{P_i}\right)^{-\mu} \frac{D_i}{P_i} = \frac{\tilde{c}(1 - \beta)}{cb^k} \left(\frac{c}{\Theta}\right)^{\frac{\mu}{\theta}} \left(\frac{1}{\theta} - \frac{1}{\mu}\right) \mu$$

(ii) Labour supply equals labor demand in all countries. Let the number of firms producing in economy i be M_i , then we know that in each period $(1 - \beta)M_i$ firms die. In steady state, M_i is constant and so the number of entrepreneurs who hire labour in an attempt to create a firm must be such that the resulting number of firms that choose to operate (i.e. who have $\phi > \Phi_i^i$) is equal to $(1 - \beta)M_i$ i.e. the labour employed in paying the fixed creation cost:

$$L_{creation} = \frac{1 - \beta}{1 - F(\Phi_i^i)} M_i \tilde{c} = cM_i \left(\frac{D_i}{P_i}\right) \left(\frac{1}{\theta} - \frac{1}{\mu}\right)^{-1} \frac{1}{\mu}$$

Total demand for effective labour is the sum of $L_{creation}$ and the labour employed by the firms that have decided to go ahead ($M_i \bar{L}^i$). Effective labour supply, S_i is an exogenous parameter, different in different economies, that we will calibrate to. Equilibrium means that effective labour supply equals demand for effective labour:

$$\left(\frac{1}{\theta} - \frac{1}{\mu}\right) S_i = cM_i \frac{D_i}{P_i}$$

(iii) All income must be spent and there must be a balance of payments in all economies. Since perfect financial markets drive the expected value of firm creation to zero, the nominal value of goods production in economy i must equal payments to labour in economy i , $S_i W_i$. Consumers in economy i spend some amount Y_i on goods, and the difference between labour income and goods expenditure in economy i , $T_i = S_i W_i - Y_i$, is some (exogenous) capital account or factor payment made by economy i into world capital markets. Balance of payments across the world implies that $\sum_{i=1}^N T_i = 0$. The nominal value of exports from economy i to economy j is:

$$X_j^i = M_i \frac{1 - F(\Phi_j^i)}{1 - F(\Phi_i^i)} \int_{\Phi_j^i} r_j^i(\phi) \frac{dF(\phi)}{1 - F(\Phi_j^i)} = cM_i \frac{W_i}{P_i} \left(\frac{P_i}{P_j} \delta_j^i\right)^{1-\mu} Y_j \left(\frac{1}{\theta} - \frac{1}{\mu}\right)^{-1}$$

A balance of payments in economy i implies that its imports (including imports from itself, X_i^i) are equal to its total expenditure, whilst its exports (including its exports to itself, X_i^i) are equal to its total labour income i.e.

$$\sum_{j=1}^N X_j^i = S_i W_i = Y_i - T_i$$

$$\sum_{j=1}^N X_i^j = Y_i = \sum_{j=1}^N cM_j \frac{W_j}{P_j} \left(\frac{P_j}{P_i} \delta_i^j\right)^{1-\mu} Y_i \left(\frac{1}{\theta} - \frac{1}{\mu}\right)^{-1}$$

Result 2: Gravity equation of the model

The equation for X_j^i in the proof of Result 1 above can be manipulated by substituting in from equation (3) to get:

$$\ln X_j^i = \ln(S_i W_i) + \ln Y_j - \ln D_i + (1 - \mu) \ln \left(\frac{P_i}{P_j} \delta_j^i\right)$$

All parameters other than μ and θ can be eliminated

The equations of the model are:

$$\begin{aligned}
 D_i &= \sum_{j=1}^N \left(\frac{P_i}{P_j} \delta_j^i \right)^{1-\mu} Y_j \\
 \left(\frac{W_i}{P_i} \right)^{-\mu} \frac{D_i}{P_i} &= \frac{\tilde{c}(1-\beta)}{cb^k} \left(\frac{c}{\Theta} \right)^{\frac{\mu}{\theta}} \left(\frac{1}{\theta} - \frac{1}{\mu} \right) \mu \\
 \left(\frac{1}{\theta} - \frac{1}{\mu} \right) S_i &= cM_i \frac{D_i}{P_i} \\
 \sum_{j=1}^N X_j^i &= S_i W_i = Y_i - T_i \\
 \sum_{j=1}^N X_i^j &= Y_i = \sum_{j=1}^N cM_j \frac{W_j}{P_j} \left(\frac{P_j}{P_i} \delta_i^j \right)^{1-\mu} Y_i \left(\frac{1}{\theta} - \frac{1}{\mu} \right)^{-1} \\
 \ln X_j^i &= \ln(S_i W_i) + \ln Y_j - \ln D_i + (1-\mu) \ln \left(\frac{P_i}{P_j} \delta_j^i \right)
 \end{aligned}$$

But the effective labour supplies, S_i , the wage per unit effective labour, W_i , and the measures of the continuum of firms, M_i , are in notional, model consistent units. We never equate these model consistent quantities with anything observable in real world data. Therefore we can define:

$$\begin{aligned}
 \tilde{W}_i &= \left[\frac{\tilde{c}(1-\beta)}{cb^k} \left(\frac{c}{\Theta} \right)^{\frac{\mu}{\theta}} \right]^{\frac{1}{\mu}} W_i \\
 \tilde{S}_i &= \left[\frac{\tilde{c}(1-\beta)}{cb^k} \left(\frac{c}{\Theta} \right)^{\frac{\mu}{\theta}} \right]^{-\frac{1}{\mu}} S_i \\
 \tilde{M}_i &= \left[\frac{\tilde{c}(1-\beta)}{cb^k} \left(\frac{c}{\Theta} \right)^{\frac{\mu}{\theta}} \right]^{-\frac{1}{\mu}} cM_i
 \end{aligned}$$

Making these substitutions produces the following equations for the model, which as required, have only the parameters μ and θ :

$$\begin{aligned}
 D_i &= \sum_{j=1}^N \left(\frac{P_i}{P_j} \delta_j^i \right)^{1-\mu} Y_j \\
 \left(\frac{\tilde{W}_i}{P_i} \right)^{-\mu} \frac{D_i}{P_i} &= \left(\frac{1}{\theta} - \frac{1}{\mu} \right) \mu \\
 \left(\frac{1}{\theta} - \frac{1}{\mu} \right) \tilde{S}_i &= \tilde{M}_i \frac{D_i}{P_i} \\
 \sum_{j=1}^N X_j^i &= \tilde{S}_i \tilde{W}_i = Y_i - T_i \\
 \sum_{j=1}^N X_i^j &= Y_i = \sum_{j=1}^N \tilde{M}_j \frac{\tilde{W}_j}{P_j} \left(\frac{P_j}{P_i} \delta_i^j \right)^{1-\mu} Y_i \left(\frac{1}{\theta} - \frac{1}{\mu} \right)^{-1} \\
 \ln X_j^i &= \ln(\tilde{S}_i \tilde{W}_i) + \ln Y_j - \ln D_i + (1-\mu) \ln \left(\frac{P_i}{P_j} \delta_j^i \right)
 \end{aligned}$$

D_i is data

The gravity equation of the model is:

$$\ln X_j^i = \ln(\tilde{S}_i \tilde{W}_i) + \ln Y_j - \ln D_i + (1 - \mu) \ln \left(\frac{P_i}{P_j} \delta_j^i \right)$$

Given a model consistent dataset, this equation holds for all bilateral trade flows. In particular it holds for X_i^i . Therefore, and using the fact that $\delta_i^i = 1$, we have:

$$\begin{aligned} \ln X_i^i &= \ln(\tilde{S}_i \tilde{W}_i) + \ln Y_i - \ln D_i + (1 - \mu) \ln \left(\frac{P_i}{P_i} \delta_i^i \right) \\ &= \ln(\tilde{S}_i \tilde{W}_i) + \ln Y_i - \ln D_i + (1 - \mu) \ln 1 \\ &= \ln(\tilde{S}_i \tilde{W}_i) + \ln Y_i - \ln D_i \\ \text{i.e. } D_i &= \frac{(\tilde{S}_i \tilde{W}_i) Y_i}{X_i^i} \end{aligned}$$

Since GDP, $(\tilde{S}_i \tilde{W}_i)$, GNI, $Y_i = \sum_{j=1}^n X_j^i$, and internal trade, $X_i^i = (\tilde{S}_i \tilde{W}_i) - \sum_{j \neq i} X_j^i$, are all data, so too is D_i .

Defining δ_j

Use normalisation, $P_1 = 1$ and the gravity equation to define δ_j as

$$\begin{aligned} \delta_j &\equiv \delta_j^1 = \left(\frac{X_j^1 D_1}{(\tilde{S}_1 \tilde{W}_1) Y_j} \right)^{\frac{1}{1-\mu}} \left(\frac{P_j}{P_1} \right) \\ &= \left(\frac{X_j^1 D_1}{(\tilde{S}_1 \tilde{W}_1) Y_j} \right)^{\frac{1}{1-\mu}} P_j \end{aligned}$$

Then we have:

$$\begin{aligned} \delta_j^i &= \left(\frac{X_j^i D_i}{(\tilde{S}_i \tilde{W}_i) Y_j} \right)^{\frac{1}{1-\mu}} \left(\frac{P_j}{P_i} \right) \\ &= \frac{\left(\frac{X_j^1 D_1}{(\tilde{S}_1 \tilde{W}_1) Y_j} \right)^{\frac{1}{1-\mu}} P_j}{\left(\frac{X_i^1 D_1}{(\tilde{S}_1 \tilde{W}_1) Y_i} \right)^{\frac{1}{1-\mu}} P_i} \left(\frac{X_i^1 X_j^i}{X_j^1 (\tilde{S}_i \tilde{W}_i) Y_j} \right)^{\frac{1}{1-\mu}} \\ &= \frac{\delta_j}{\delta_i} \left(\frac{X_i^1 X_j^i}{X_j^1 X_i^i} \right)^{\frac{1}{1-\mu}} \end{aligned}$$

Therefore, since in calibration mode X_j^i is data, we can express the $N(N-1)$ parameters, δ_j^i , in terms of the N unknowns, δ_j .

The Standard deviation of log firm sales in this model is given by $(\theta-1)/k$

Firm sales of a firm in i selling into j

$$r_j^i(\phi) = \theta \Theta \left(\frac{\phi}{\delta_j^i} \right)^{\theta-1} \left(\frac{W_i}{P_j} \right)^{1-\theta} Y_j$$

where

$$\phi \sim \text{Pareto}(b, k)$$

Therefore the total sales of any firm can be written as

$$\begin{aligned} r(\phi) &= \kappa \phi^{\theta-1} = \kappa b^{\theta-1} \left(\frac{\phi}{b} \right)^{\theta-1} \\ &= \kappa b^{\theta-1} Z \end{aligned}$$

for constant κ .

i.e.

$$\ln r = \ln \kappa b^{\theta-1} + \ln Z$$

so

$$\text{Std}[\ln r] = \text{Std}[\ln Z]$$

$$\begin{aligned} \Pr(\ln Z < z) &= \Pr\left(\ln\left(\frac{\phi}{b}\right)^{\theta-1} < z\right) = \Pr\left(\phi < b \exp\left(\frac{z}{\theta-1}\right)\right) \\ &= F\left(b \exp\left(\frac{z}{\theta-1}\right)\right) = 1 - \left(\frac{b}{b \exp\left(\frac{z}{\theta-1}\right)}\right)^k \\ \text{i.e. } G(z) &= 1 - \exp\left(-\frac{k}{\theta-1}z\right) \end{aligned}$$

i.e. $\ln Z \sim \text{Exp}\left(\frac{k}{\theta-1}\right)$

Therefore

$$\text{Std}[\ln r] = \text{Std}[\ln Z] = \frac{\theta-1}{k}$$

Symmetric trade frictions in model with no capital flows implies bilaterally balanced trade

This is the statement that:

$$(\tilde{S}_i \tilde{W}_i) = Y_i \text{ and } \delta_j^i = \delta_i^j \Rightarrow X_j^i = X_i^j$$

The previous equations for $\delta_j^1 = \delta_1^j$ give us

$$\delta_1^j = \frac{\delta_1}{\delta_j} \left(\frac{X_j^1 X_1^j}{X_1^1 X_j^1} \right)^{\frac{1}{1-\mu}} = \frac{\delta_j}{\delta_1} \left(\frac{X_1^1 X_j^1}{X_j^1 X_1^1} \right)^{\frac{1}{1-\mu}} = \delta_j^1$$

But $\delta_j^1 = \delta_j$ and $\delta_1 = 1$. Therefore

$$\delta_j = \left(\frac{X_j^1 X_1^j}{X_1^1 X_j^j} \right)^{\frac{1}{2(1-\mu)}}$$

$$\text{so } \delta_j^i = \left((X_j^i)^2 \frac{X_1^j X_i^1}{X_j^1 X_1^i X_i^j X_j^j} \right)^{\frac{1}{2(1-\mu)}}$$

Then $\delta_j^i = \delta_i^j$ implies

$$\frac{X_1^j X_i^1}{X_j^1 X_1^i} = \frac{X_i^j}{X_j^i}$$

Substituting this into the above expression for δ_j^i allows us to eliminate the trade flows with country 1 i.e.

$$\delta_j^i = \left(\frac{X_j^i X_i^j}{X_i^i X_j^j} \right)^{\frac{1}{2(1-\mu)}} = \delta_i^j$$

NB This is analogous and very similar to the equivalent formula derived in [Caselli, Koren, Lisicky, and Tenreyro \(2012\)](#).

The gravity equation with no capital flows is:

$$\ln X_j^i = \ln Y_i + \ln Y_j - \ln D_i + (1 - \mu) \ln \left(\frac{P_i}{P_j} \delta_j^i \right)$$

$$\ln X_i^j = \ln Y_i + \ln Y_j - \ln D_j + (1 - \mu) \ln \left(\frac{P_j}{P_i} \delta_j^i \right)$$

$$\text{so } \ln X_j^i - \ln X_i^j = \ln D_j - \ln D_i + 2(1 - \mu) \ln \left(\frac{P_i}{P_j} \right)$$

i.e. trade is bilaterally balanced if

$$\frac{D_i}{D_j} = \left(\frac{P_i}{P_j} \right)^{2(1-\mu)}$$

$$= \left(\frac{1}{\delta_j^i} \left(\frac{X_j^i D_i}{Y_i Y_j} \right)^{\frac{1}{1-\mu}} \right)^{2(1-\mu)}$$

$$= \left(\frac{X_i^i X_j^j}{X_j^i X_i^j} \right) \left(\frac{X_j^i D_i}{Y_i Y_j} \right)^2$$

Using the relation that $D_i = Y_i^2 / X_i^i$ in a model with no capital flows, this expression reduces to

$$1 = \frac{X_j^i}{X_i^j}$$

i.e. bilaterally balanced trade.

Independence from aggregation level of the rest of the world

It is immediately clear that, in the simple world of bilaterally balanced trade and no capital flows, the measured frictions between economy a and economy b do not depend upon how C , the set of other economies in the world, is aggregated. This is obvious from the formula, derived in Appendix C, which shows that C does not enter:

$$\delta_{ab} = \left(\frac{X_{ab}(D_a D_b)^{1/2}}{Y_a Y_b} \right)^{\frac{1}{1-\mu}} = \left(\frac{X_{ab}}{(X_{aa} X_{bb})^{1/2}} \right)^{\frac{1}{1-\mu}}$$

For the full model, with capital flows and without necessarily bilaterally balanced trade, things are more complicated. To simplify, note that the labelling of countries is arbitrary, and without loss of generality we can label country a as 1, and country b as 2. This has the advantage that we normalise with respect to country 1 and so $P_1 = 1$. We want to show that δ_2^1 and δ_1^2 are independent of how the set of countries $C = \{3, \dots, n\}$, are aggregated.

- Firstly, from the gravity equation, it is clear that the quantity $\frac{P_i}{P_j} \delta_j^i = \left(\frac{X_j^i D_i}{(\tilde{S}_i \tilde{W}_i) Y_j} \right)^{1/(1-\mu)}$ is independent of the aggregation of countries $h \neq i, j$

- Then

$$\begin{aligned} D_1 &= \sum_{j=1}^n \left(\frac{P_1}{P_j} \delta_j^1 \right)^{1-\mu} Y_j \\ &= Y_1 + \left(\frac{P_1}{P_2} \delta_2^1 \right)^{1-\mu} Y_2 + \sum_{j \in C} \left(\frac{P_1}{P_j} \delta_j^1 \right)^{1-\mu} Y_j \\ &= Y_1 + \left(\frac{1}{P_2} \delta_2^1 \right)^{1-\mu} Y_2 + \frac{D_1}{(\tilde{S}_1 \tilde{W}_1)} \sum_{j \in C} X_j^1 \\ &= Y_1 + \left(\frac{1}{P_2} \delta_2^1 \right)^{1-\mu} Y_2 + A_{DATA} \end{aligned}$$

where clearly A_{DATA} does not depend on the aggregation within the set C

- Likewise

$$\begin{aligned} D_2 &= \sum_{j=1}^n \left(\frac{P_2}{P_j} \delta_j^2 \right)^{1-\mu} Y_j \\ &= \left(\frac{P_2}{P_1} \delta_1^2 \right)^{1-\mu} Y_1 + Y_2 + \sum_{j \in C} \left(\frac{P_2}{P_j} \delta_j^2 \right)^{1-\mu} Y_j \\ &= (P_2 \delta_1^2)^{1-\mu} Y_1 + Y_2 + \frac{D_2}{(\tilde{S}_2 \tilde{W}_2)} \sum_{j \in C} X_j^2 \\ &= (P_2 \delta_1^2)^{1-\mu} Y_1 + Y_2 + B_{DATA} \end{aligned}$$

where clearly B_{DATA} does not depend on the aggregation within the set C

- Combine two gravity equations to get a third condition

$$\begin{aligned}\ln X_1^2 &= \ln(\tilde{S}_2 \tilde{W}_2) + \ln Y_1 - \ln D_2 + (1 - \mu) \ln\left(\frac{P_2}{P_1} \delta_1^2\right) \\ \ln X_2^1 &= \ln(\tilde{S}_1 \tilde{W}_1) + \ln Y_2 - \ln D_1 + (1 - \mu) \ln\left(\frac{P_1}{P_2} \delta_2^1\right) \\ \delta_2^1 \delta_1^2 &= \left(\frac{D_1 D_2 X_1^2 X_2^1}{(\tilde{S}_1 \tilde{W}_1)(\tilde{S}_2 \tilde{W}_2) Y_1 Y_2}\right)^{\frac{1}{1-\mu}}\end{aligned}$$

- i.e. we have 3 equations in 3 unknowns, $\{\delta_2^1, \delta_1^2, P_2\}$, which can be solved without any consideration of how the set C is aggregated

$$\begin{aligned}D_1 &= Y_1 + \left(\frac{1}{P_2} \delta_2^1\right)^{1-\mu} Y_2 + A_{DATA} \\ D_2 &= (P_2 \delta_1^2)^{1-\mu} Y_1 + Y_2 + B_{DATA} \\ \delta_2^1 \delta_1^2 &= \left(\frac{D_1 D_2 X_1^2 X_2^1}{(\tilde{S}_1 \tilde{W}_1)(\tilde{S}_2 \tilde{W}_2) Y_1 Y_2}\right)^{\frac{1}{1-\mu}}\end{aligned}$$

Negative relationship between size and measured frictions

Suppose that the world consists of Ω identical small countries, each with $\text{GDP} = \text{GNI} = Y$, so that world GDP, $Y_W = \Omega Y$. Since all countries are identical, price level P and home share $\lambda = (Y - (\Omega - 1)X)/Y$, must be identical in each country. There is a constant bilateral trade friction, δ between any two countries which perfectly determines bilateral trade flows i.e.

$$\begin{aligned}\ln X &= 2 \ln Y - \ln D + (1 - \mu) \ln \delta \\ &= 2 \ln Y - \ln\left(\frac{Y}{\lambda}\right) + (1 - \mu) \ln \delta \\ &= \ln(\lambda Y) + (1 - \mu) \ln \delta \\ \text{i.e. } X &= \lambda Y \delta^{1-\mu}\end{aligned}$$

Now consider the case where we cannot observe these small identical countries, but instead the observed actual countries are aggregations $M = \{1, \dots, m\}$ and $N = \{m+1, \dots, m+n\}$ of non-overlapping underlying “countries”. In this case:

$$\begin{aligned}Y_M &= mY \\ Y_N &= nY \\ X_{MM} &= m\lambda Y + m(m-1)X \\ X_{NN} &= n\lambda Y + n(n-1)X \\ X_{MN} &= mnX = X_{NM}\end{aligned}$$

If we now use the model to measure the frictions associated with observed trade flows, using the procedure for calibrating the simplified model with bilaterally balanced trade and no capital flows from Appendix

C, we obtain

$$\begin{aligned}\delta_{MN} &= \left(\frac{X_{MN}}{X_{MM}^{1/2} X_{NN}^{1/2}} \right)^{\frac{1}{1-\mu}} \\ \ln \delta_{MN} &= \frac{1}{1-\mu} \ln \left(\frac{mn\lambda Y \delta^{1-\mu}}{(m\lambda Y + m(m-1)\lambda Y \delta^{1-\mu})^{1/2} (n\lambda Y + n(n-1)\lambda Y \delta^{1-\mu})^{1/2}} \right) \\ &= \frac{1}{1-\mu} \ln \left(\frac{m^{1/2} n^{1/2} \delta^{1-\mu}}{(1 + (m-1)\delta^{1-\mu})^{1/2} (1 + (n-1)\delta^{1-\mu})^{1/2}} \right)\end{aligned}$$

Differentiating $\ln \delta_{MN}$ by $\ln Y_M$ and evaluating this at $m = 1$ gives

$$\begin{aligned}\frac{\partial \ln \delta_{MN}}{\partial \ln Y_M} &= \frac{\partial \ln \delta_{MN}}{\partial \ln m} \\ &= \frac{1}{2(1-\mu)} \left(1 - \frac{\delta^{1-\mu} m}{1 + (m-1)\delta^{1-\mu}} \right) \\ \text{i.e. } \frac{\partial \ln \delta_{MN}}{\partial \ln Y_M} (m=1) &= \frac{1 - \delta^{1-\mu}}{2(1-\mu)}\end{aligned}$$

Therefore, purely from aggregation effects rather than any real frictions, we would expect to observe a negative relationship between log frictions and log incomes with a slope in the range $\frac{1}{2(1-\mu)} < 0$ (for high values of δ) to 0 (for a value of $\delta \approx 1$). This range, given the value of μ used in the main analysis, is $(-0.1419, 0)$. The empirically observed slopes shown in Figure 2 are -0.0700 for country-country frictions, and -0.0967 for region-region frictions.

C. Numerical procedures

Calibrating simplified model with bilaterally balanced trade and no capital flows

From data we have $\{Y_i, X_{ij}, D_i\}, \forall i, j$. We know that balanced trade and no capital flows is associated with symmetric trade frictions. Therefore, the gravity equation implies

$$\begin{aligned}\delta_j^i &= \left(\frac{X_{ij} D_i}{Y_i Y_j} \right)^{\frac{1}{1-\mu}} \frac{P_j}{P_i} = \left(\frac{X_{ij} D_j}{Y_i Y_j} \right)^{\frac{1}{1-\mu}} \frac{P_i}{P_j} = \delta_j^j \\ \text{therefore } \frac{P_i}{P_j} &= \left(\frac{D_i}{D_j} \right)^{\frac{1}{2(1-\mu)}} \\ \text{so } \delta_{ij} &= \left(\frac{X_{ij} (D_i D_j)^{1/2}}{Y_i Y_j} \right)^{\frac{1}{1-\mu}} = \left(\frac{X_{ij}}{(X_{ii} X_{jj})^{1/2}} \right)^{\frac{1}{1-\mu}}\end{aligned}$$

So we have the price indices and bilateral trade frictions directly from data. We can solve for the wage rate per unit effective labour, the quantity of effective labour and the measure of the continuum of firms

using:

$$\begin{aligned}\left(\frac{\tilde{W}_i}{P_i}\right)^{-\mu} \frac{D_i}{P_i} &= \left(\frac{1}{\theta} - \frac{1}{\mu}\right) \mu \\ Y_i &= \tilde{S}_i \tilde{W}_i \\ \left(\frac{1}{\theta} - \frac{1}{\mu}\right) \tilde{S}_i &= \tilde{M}_i \frac{D_i}{P_i}\end{aligned}$$

Calibrating full model to data that divides the world into 3 countries

From data we have $\{(\tilde{S}_i \tilde{W}_i, Y_i, X_{ij}, T_i, D_i), \forall i, j\}$. We then find the endogenous variables of the model $\{\tilde{S}_i, \tilde{W}_i, P_i, \tilde{M}_i, \delta_i\}$, by solving the following system of 15 equations simultaneously, using the results of the calibration under bilaterally balanced trade and no capital flows as an initial condition, using Excel's solver tool.

$$\begin{aligned}\tilde{S}_i &= \frac{(\tilde{S}_i \tilde{W}_i)}{\tilde{W}_i} \quad , \quad i = \{1, 2, 3\} \\ \tilde{M}_i &= \left(\frac{1}{\theta} - \frac{1}{\mu}\right) \frac{P_i}{D_i} \tilde{S}_i \quad , \quad i = \{1, 2, 3\} \\ \frac{\delta_i}{P_i} &= \left(\frac{X_i^1 D_1}{(\tilde{S}_1 \tilde{W}_1) Y_i}\right)^{\frac{1}{1-\mu}} \quad , \quad i = \{1, 2, 3\} \\ \left(\frac{\tilde{W}_i}{P_i}\right)^{-\mu} \frac{D_i}{P_i} &= \left(\frac{1}{\theta} - \frac{1}{\mu}\right) \mu \quad , \quad i = \{1, 2, 3\} \\ 1 &= \sum_j \frac{(\tilde{S}_j \tilde{W}_j)}{D_j} \left(\frac{X_j^1 X_i^j}{X_i^1 X_j^j}\right) \left(\frac{P_j \delta_i}{P_i \delta_j}\right)^{1-\mu} \quad , \quad i = \{2, 3\} \\ P_1 &= 1\end{aligned}$$

3 country model policy experiments

By assumption we have $\{\tilde{S}_i, \delta_j^i, T_i\}, \forall i, j$. We then find the endogenous variables of the model $\{\tilde{W}_i, \tilde{M}_i, P_i, Y_i, D_i\}$, by solving the following system of 15 equations simultaneously, using the results of the calibration before changing the parameters as an initial condition, using Excel's solver tool.

$$\begin{aligned}\left(\frac{\tilde{W}_i}{P_i}\right)^{-\mu} \frac{D_i}{P_i} &= \left(\frac{1}{\theta} - \frac{1}{\mu}\right) \mu \quad , \quad i = \{1, 2, 3\} \\ \tilde{M}_i &= \left(\frac{1}{\theta} - \frac{1}{\mu}\right) \frac{P_i}{D_i} \tilde{S}_i \quad , \quad i = \{1, 2, 3\} \\ D_i &= \sum_j \left(\frac{P_i \delta_j^i}{P_j}\right)^{1-\mu} Y_j \quad , \quad i = \{1, 2, 3\} \\ Y_i &= \tilde{S}_i \tilde{W}_i + T_i \quad , \quad i = \{1, 2, 3\} \\ 1 &= \sum_j \frac{\tilde{S}_j \tilde{W}_j}{D_j} \left(\frac{P_j \delta_i^j}{P_i}\right)^{1-\mu} \quad , \quad i = \{2, 3\} \\ P_1 &= 1\end{aligned}$$

n country model policy experiments (assuming bilaterally balanced trade and no capital flows)

By assumption we have $\{\tilde{S}_i, \delta_{ij}, T_i = 0\}, \forall i, j$. We then find the endogenous variables of the model $\{X_{ii}, D_i\}$, by solving the following system of $2n$ equations simultaneously, using the results of the calibration before changing the parameters as an initial condition, using Excel's solver tool.

$$\begin{aligned} \left(\frac{\mu}{\theta} - 1\right)^{-\frac{1}{\mu}} &= \left(D_1^{-\frac{1}{2\mu}} D_i^{-\frac{1}{2\mu}} \frac{X_{ii}}{S_i}\right) \sum_j \delta_{ij}^{1-\mu} \left(\frac{X_{jj}}{X_{ii}}\right)^{\frac{1}{2}}, \quad i = \{1, \dots, n\} \\ D_i &= \left(\sum_j \delta_{ij}^{1-\mu} X_{jj}^{\frac{1}{2}}\right)^2, \quad i = \{1, \dots, n\} \end{aligned}$$

Bilateral trade flows, nominal incomes, price levels, and real incomes can then be recovered using

$$\begin{aligned} X_{ij} &= \delta_{ij}^{1-\mu} (X_{ii} X_{jj})^{\frac{1}{2}} \\ Y_i &= (D_i X_{ii})^{\frac{1}{2}} \\ P_i &= \left(\frac{D_i}{D_1}\right)^{\frac{1}{2(1-\mu)}} \\ y_i &= \frac{Y_i}{P_i} \end{aligned}$$

D. Sensitivity Analysis

	Portugal /Spain	Catalonia /rSpain	Basque /rSpain	Ireland /UK	Scotland /rUK
Calibrated δ_{ij}	2.88	1.54	1.73	2.10	1.76
λ_i (data)	66.7%	34.9%	33.1%	24.8%	43.8%
λ_i (pol. exp.)		51.0%	52.7%		53.4%
Δ real GNI: Independence		-12.6%	-15.3%		-6.8%
Δ real GNI: Autarky	-13.4%	-31.2%	-32.6%	-39.1%	-25.5%
Independence / Autarky		40.4%	46.9%		26.7%

Table 8: Results using $\theta = 3.8$ and $k = 2.81$ so that trade elasticity = -2.81

	Portugal /Spain	Catalonia /rSpain	Basque /rSpain	Ireland /UK	Scotland /rUK
Calibrated δ_{ij}	1.77	1.26	1.35	1.49	1.36
λ_i (data)	66.7%	34.9%	33.1%	24.8%	43.8%
λ_i (pol. exp.)		51.6%	53.0%		53.3%
Δ real GNI: Independence		-7.2%	-8.7%		-3.7%
Δ real GNI: Autarky	-7.5%	-18.3%	-19.1%	-23.5%	-14.7%
Independence / Autarky		39.6%	45.3%		25.2%

Table 9: Results using $\theta = 3.8$ and $k = 4.58$ so that trade elasticity = -5.21

Note that the Independence / Autarky figures are relatively insensitive to changes in the value of the trade elasticity.