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ABSTRACT

Modelling Long Bonds - The Case of Optimal Fiscal Policy

We show how to model portfolio models in the presence of long bonds. Specifically we study optimal fiscal policy under incomplete markets where the government issues bonds of maturity N > 1. Assuming the existence of long bonds introduces an additional intertemporal mechanism that makes taxes more volatile in order to achieve lower debt management costs. In other words, fiscal policy is secondary to debt management. Modelling optimal policy with long term bonds is computationally demanding because of the promises made to cut future taxes. The longer the maturity of bonds the more promises need to be monitored and the larger the state space. We consider three means of overcoming this problem - a computational method using the “condensed PEA”, an approximation whereby long bonds are modelled as a sequence of geometrically declining coupons and a model of independent powers where the fiscal authority and interest rate setting authority are separate. We compare the accuracy and properties of solutions across these three approaches and examine how the properties of optimal fiscal policy differ in the case of long bonds compared to one period debt.

JEL Classification: E43, E62 and H63
Keywords: debt management, fiscal policy, government debt, maturity structure, tax smoothing and yield curve

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1 Introduction

Analysing government debt clearly requires an intertemporal approach and as a result the optimal fiscal policy literature has focused on the intertemporal properties of taxes and more recently on fluctuations in government debt. For instance, Barro (1979) and Lucas and Stokey (1983) show how the stochastic properties of tax rates vary under different assumptions about the structure of bond markets. Aiyagari, Marcet, Sargent and Seppälä (2002) show how government debt displays a martingale component depending on whether bond markets are characterised by a complete set of contingent claims or market incompleteness. Subsequently a number of papers have examined how debt management can support tax smoothing through issuing bonds of different maturities in order to exploit movements in the yield curve. Angeletos (2002), Barro (2003) and Buera and Nicolini (2004) do so in a complete market setting whilst Nosbusch (2008) and Lustig, Sleet and Yeltekin (2009) do so with models of incomplete markets.

This paper aims to complete this intertemporal analysis of taxation and debt management by focusing on an additional margin that arises when governments issue bonds of maturity N. Although a number of papers consider the case of long term bonds the mechanism we focus on has not been the primary focus of attention. We show that when governments issue bonds of maturity more than one period then concerns about debt management lead governments to introduce tax volatility in order to reduce funding costs. Specifically when the government is indebted and issues long bonds it announces its intention to lower future tax rates in order to lower future interest rates and so lower the cost of funding government debt. In other words, fiscal policy is subordinated to debt management concerns just as we see happening in the current European Sovereign Debt crisis.

This additional channel can only be observed when we consider the case of long term bonds. In the case of one period debt, when the government experiences an adverse expenditure shock there are two offsetting effects. The first is the need to raise taxes to finance the expenditure shock. The second is to lower taxes in order to lower interest rates and make it cheaper to fund the shock next period when debt is reissued. The former effect numerically dominates the latter and so we see tax rates increase as in the case of one period debt the effects are conflated and the second effect is not noticeable. If however long bonds are issued then these two effects are disentangled with an initial period increase in taxes and a later cut to lower future interest rates adding additional dynamics to tax rates.

Modelling bonds of long maturity creates serious computational issues and we investigate a variety of mechanisms to help overcome these challenges and compare outcomes and methods across them. These methods are of use beyond the case of optimal fiscal policy and can be used to solve general portfolio models in the presence of long term assets.

It is well known that solving for optimal policy under incomplete markets in the case of bonds of maturity N is difficult as the state space is of dimension $2N+1$. Given that the UK government issues bonds of 50 year maturity this is clearly a computationally demanding problem. To overcome this complexity we solve for optimal taxation in the case of long bonds using the "condensed PEA" of Faraglia, Marcet, Oikonomou and Scott (2013). This approach reduces the dimensionality of the state vector while allowing, in principle, for arbitrary precision. For instance, we will show how in the case of a twenty year bond the state space is effectively only four variables. We then compare the properties of optimal tax and debt behaviour with that when the government can only issue one period bonds and study the impact of maturity.

There are alternative approaches to reducing the complexity of studying bonds of varying maturity. We also consider the approach of Woodford (2001) and Arellano and Ramanararayanan (2008) who model bonds of different maturities by decaying coupon perpetuities where the decay rates are used to mimic maturity differences. We use our recursive contract approach to modelling long bonds and compare with the outcomes from this decaying coupon approach. We find important differences between our solutions and those produced by this approach - specifically the interest rate twisting effects that are based around specific maturity dates are absent and instead smoothed out across all periods in the case of decaying coupon perpetuities.

In the case of long bonds the high dimension of the state space arises from commitment issues. In the case of long term bonds the government makes a (time inconsistent) commitment to change future taxes and these promises have to be monitored. The longer the maturity of bonds the more past commitments have to be monitored and the larger the state space. Therefore another approach to reducing the complexity of the problem is to separate the ability to set taxes and the ability to influence interest rates. We therefore outline a model of "independent powers" where the fiscal authority is separate from the monetary authority setting interest rates. In this way the "twisting" of interest rates is not possible, since the fiscal authority takes interest rates as given and the state space reduces dramatically leading to a substantially easier model to solve. We compare the outcome from this independent powers model with that of Ramsey optimal taxation.

The structure of the paper is as follows. Section 2 outlines our main model - a Ramsey model with incomplete markets and long bonds. Under some simplifying assumptions we are able to provide some analytic results which offer insight into our key results. Section 3 describes how to solve this model using the "condensed PEA" and details the behavior of the model numerically. Section 4 studies the case where the model is solved by approximating long bonds by perpetuities with decaying coupons and Section 5 outlines and solves the model of independent powers whilst a final section concludes.

2 The Model - Analytic Results

Our benchmark model is of a Ramsey policy equilibrium with perfect commitment and coordination of policy authorities in which the government buys back all existing debt each period. The economy produces a single non-storable good with technology

$$c_t + g_t \leq A - x_t, \tag{1}$$

for all t, where x_t, c_t and g_t represent leisure, private consumption and government expenditure respectively. A is the total time available in every period. The exogenous stochastic process g_t is the only source of uncertainty. The representative consumer has utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u\left(c_t\right) + v\left(x_t\right) \right\} \tag{2}$$

and is endowed with A units of time that it allocates between leisure and labour and faces a proportional tax rate τ_t on labor income. The representative firm maximizes profits and both consumers and firms act competitively by taking prices and taxes as given. Consumers, firms and government all have full information, i.e they observe all shocks up to the current period, and all variables dated t are chosen contingent on histories $g^t = (g_t, ..., g_0)$. All agents have rational expectations.

Agents can only borrow and lend in the form of a zero-coupon, risk-free, N-period bond so that the government budget constraint is:

$$g_t + p_{N-1,t} b_{N,t-1} = \tau_t (A - x_t) + p_{N,t} b_{N,t} \tag{3}$$

where $b_{N,t}$ denotes the number of bonds the government issues at time t. Each bond pays one unit of consumption good in N periods time with complete certainty. The price of an i-period bond at time t is $p_{i,t}$. We assume that at the end of each period the government buys back the existing stock of debt and then reissues new debt of maturity N, these repurchases are reflected in the left side of the budget constraint (3). In addition government debt has to remain within upper and lower limits $\underline{M}$ and $\overline{M}$ so ruling out Ponzi schmes e.g

$$\underline{M} \leq \beta^N b_{N,t} \leq \overline{M} \tag{4}$$

The term β^N in this constraint reflects the value of the long bond at steady state so that the limits $\underline{M}$, $\overline{M}$ appropriately refer to the value of debt and are comparable across maturities.[1]

We assume after purchasing a long bond the household entertains only two possibilities: one is to resell the government bond in the secondary market in the period immediately after having purchased it, the other possibility is to hold the bond until maturity.[2] Letting $s_{N,t}$ be the sales in the secondary market the household's problem is to choose stochastic processes $\{c_t, x_t, s_{N,t}, b_{N,t}\}_{t=0}^{\infty}$ to maximize (2) subject to the sequence of budget constraints:

$$c_t + p_{N,t} b_{N,t} = (1 - \tau_t)(1 - x_t) + p_{N-1,t} s_{N,t} + b_{N,t-N} - s_{N,t-N+1}$$

with prices and taxes $\{p_{N,t}, p_{N-1,t}, \tau_t\}$ taken as given. The household also faces debt limits analogous to (4). We assume for simplicity that these limits are less stringent than those faced by the government, so that in equilibrium the household's problem always has an interior solution.

The consumer's first order conditions of optimality are given by

$$\frac{v_{x,t}}{u_{c,t}} = 1 - \tau_t \tag{5}$$

$$p_{N,t} = \frac{\beta^N E_t (u_{c,t+N})}{u_{c,t}} \tag{6}$$

$$p_{N-1,t} = \frac{\beta^{N-1} E_t (u_{c,t+N-1})}{u_{c,t}} \tag{7}$$

[1] Obviously the actual value of debt is $p_{N,t} b_{N,t}$, we substitute $p_{N,t}$ by its steady state value β^N for simplicity.

[2] We need to introduce secondary market sales $s_{N,t}$ in order to price the repurchase price of the bond.

2.1 The Ramsey problem

We assume the government has full commitment to implement the best sequence of (possibly time inconsistent) taxes and government debt knowing equilibrium relationships between prices and allocations. Using (5), (6) and (7) to substitute for taxes and consumption the Ramsey equilibrium can be found by solving

$$\max_{\{c_t, b_{N,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t) + v(x_t)\}$$

$$\text{s.t.} \qquad \beta^{N-1} E_t (u_{c,t+N-1}) b_{N,t-1} = S_t + \beta^N E_t (u_{c,t+N}) b_{N,t} \tag{8}$$

and (4) with x_t implicitly defined by (1).

To simplify the algebra we define $S_t = (u_{c,t} - v_{x,t})(c_t + g_t) - u_{c,t} g_t$ as the "discounted" surplus of the government and set up the Lagrangian

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + v(x_t) + \lambda_t \left[S_t + \beta^N u_{c,t+N} b_{N,t} - \beta^{N-1} u_{c,t+N-1} b_{N,t-1} \right] \right.$$
$$\left. + \nu_{1,t} \left(\overline{M} - \beta^N b_{N,t} \right) + \nu_{2,t} \left(\beta^N b_{N,t} - \underline{M} \right) \right\}$$

where λ_t is the Lagrange multiplier associated with the government budget constraint e.g the excess burden of taxation, and $\nu_{1,t}$ and $\nu_{2,t}$ are the multipliers associated with the debt limits.

The first-order conditions for the planner's problem with respect to c_t and $b_{N,t}$ are

$$u_{c,t} - v_{x,t} + \lambda_t (u_{cc,t} c_t + u_{c,t} + v_{xx,t} (c_t + g_t) - v_{x,t}) \tag{9}$$
$$+ u_{cc,t} (\lambda_{t-N} - \lambda_{t-N+1}) b_{N,t-N} = 0$$

$$E_t (u_{c,t+N} \lambda_{t+1}) = \lambda_t E_t (u_{c,t+N}) + \nu_{2,t} - \nu_{1,t} \tag{10}$$

with $\lambda_{-1} = ... = \lambda_{-N} = 0$.

These FOC help characterise some features of optimal fiscal policy with long bonds. Following the discussion in Aiyagari et al. (2002) we see that, in the case where debt limits are non binding, (10) implies λ_t is a risk-adjusted martingale, with risk-adjustment measure $\frac{u_{c,t+N}}{E_t(u_{c,t+N})}$, indicating that the presence of the state variable λ in the policy function imparts persistence in the variables of the model. The term

$$\mathcal{D}_t = (\lambda_{t-N} - \lambda_{t-N+1}) b_{N,t-N}$$

in (9) indicates that a feature of optimal fiscal policy will be that what happened in period $t - N$ has a specific impact on today's taxes. Since we have $u_{c,t} - v_{x,t} = 0$ and zero taxes in the first best, a high $\mathcal{D}_t$ pulls the model away from the first best and zero taxes. If $\mathcal{D}_t > 0$ it can be thought of as introducing a higher distortion in a given period. In periods when g_{t-N+1} is very high we have that the cost of the budget constraint is high so λ_{t-N+1} is high, and if the government is in debt $\mathcal{D}_t < 0$ so taxes should go down at t. Of course this is not a tight argument, as λ_t also responds to the shocks that have happened between t and $t - N$ and λ_t also plays a role in (9), but this argument is at the core of the interest rate twisting policy we identify below. In order to build up intuition for the role of commitment and to provide a tighter argument, we now show two examples that can be solved analytically.

2.2 A model under certainty

Assume for now that government spending is constant, $g_t = \overline{g}$ and the government is initally in debt such that $b^N_{-1} > 0$. In this case long bonds complete the market so that the only budget constraint of the government is :

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t \frac{u_{c,t}}{u_{c,0}} \widetilde{S}_t &= b_{N,-1}\ p_0^{N-1}, \text{ or} \\ \sum_{t=0}^{\infty} \beta^t\ S_t &= b_{N,-1}\ \beta^{N-1} u_{c,N-1} \qquad (11)\end{aligned}$$

where $\widetilde{S}_t = \frac{S_t}{u_{c,t}}$ is the "non-discounted" surplus of the government. This shows that for a given set of surpluses the funding costs of initial debt $b^N_{-1} > 0$ can be reduced by manipulating consumption such that $c_t < c_{N-1}$ for all $t \neq N$. As long as the elasticity of consumption with respect to wages is positive, as occurs with most utility functions, this will be achieved by promising a tax cut in period $N-1$ relative to other periods e.g

$$\begin{aligned}\tau_t &= \overline{\tau} \text{ for all } t \neq N-1 \qquad (12) \\ \overline{\tau} &> \tau_{N-1}\end{aligned}$$

This promise achieves a reduction of $u_{c,N-1}$, reducing the cost of outstanding debt. In other words, the long end of the yield curve needs to be twisted down.[3] Interestingly, even though there are no fluctuations in the economy, (12) shows that optimal policy implies that the government desires to introduce variability in taxes. In other words, optimal policy violates tax smoothing. This policy is clearly time inconsistent - if the government were able to reoptimize by surprise at some period $t' > 0$, $t' < N$ it will instead then promise a cut in taxes in period $t' + N - 1$.

2.3 A model with uncertainty at $t = 1$

The previous subsection abstracted from uncertainty. We now introduce uncertainty into our model. In the interest of obtaining analytic results we assume uncertainty occurs only in the first period, ie g is given by[4]:

$$\begin{cases} g_t = \overline{g} & \text{for } t = 0 \text{ and } t \geq 2 \\ g_1 \sim F_g \end{cases}$$

for some non-degenerate distribution F_g. Since future consumption and λ's are known the martingale condition implies $u_{c,t+N}\lambda_{t+1} = \lambda_t u_{c,t+N}$ and

$$\lambda_t = \lambda_1 \qquad t > 1$$

[3] This is, of course, a manifestation of the standard interest rate manipulation already noted by Lucas and Stokey (1983), except that in our case the twisting occurs in N periods.

[4] Formally this economy is very similar to that of Nosbusch (2008).

It is clear that in the case of short bonds ($N = 1$) equilibrium implies c_t and τ_t constant for $t \geq 2$, reflecting the fact that even though markets are incomplete the government smoothes taxes after the shock is realized.

For the case of long bonds when $N > 1$, the FOC with respect to consumption (9) is satisfied for $\mathcal{D}_t = (\lambda_{t-N} - \lambda_{t-N+1})\, b_{N,t-N}$

$$\mathcal{D}_t = 0 \quad \text{for } t \geq 0 \text{ and } t \neq N-1, N \tag{13}$$

$$\mathcal{D}_{N-1} = \lambda_0 b_{N,-1}\ , \quad \mathcal{D}_N = (\lambda_0 - \lambda_1)\, b_{N,0} \tag{14}$$

Hence equilibrium satisfies

$$c_t = \quad c^*(g_1) \quad \text{for } t \geq 2 \text{ and } t \neq N, N-1 \tag{15}$$

for a certain function c^* i.e consumption is the same in all periods $t \geq 2$ and $t \neq N, N-1$, although this level of constant consumption depends on the realization of the shock g_1. Clearly, c_{N-1}, c_N also depend on the realization of g_1.

In this model, when the shock g_1 is realised the government optimally spreads out the taxation cost of this shock over current and future periods. Typically the government gets in debt in period 1 if g_1 is high, so all future taxes for $t \geq 2$ are higher and future consumption lower. This would also happen with short bonds $N = 1$. What is new with long bonds is that optimal policy introduces tax volatility, since taxes vary in periods $N-1$ and N, even though by the time the economy arrives at these periods no more shocks have occured for a long time.

2.3.1 An Analytic Example

To make this argument precise consider the utility function and $A = 1$

$$\frac{c_t^{1-\gamma_c}}{1-\gamma_c} - B\frac{(1-x_t)^{1+\gamma_l}}{1+\gamma_l} \tag{16}$$

for $\gamma_c, \gamma_l, B > 0$.

Result 1 *Assume utility (16) and $b_{N,-1} > 0$.*
For a sufficiently high realization of g_1 we have

$$\begin{aligned} \tau_1 &= \tau_t \text{ for all } t \geq 1,\ t \neq N-1, N \\ \tau_1 &> \tau_{N-1}, \tau_N \end{aligned}$$

The inequalities are reversed if $b_{N,-1} < 0$ or if the realization of g_1 is sufficiently low.

Proof. Since $\lambda_t = \lambda_1$, for $t > 1$ the FOC of optimality yield

$$\frac{u_{c,t}}{v_{x,t}} - \frac{B + (\gamma_l + 1)\lambda_1}{(1 + (-\gamma_c + 1)\lambda_1)\, B} + (\lambda_{t-N} - \lambda_{t-N+1})\, \mathcal{F}_t = 0 \quad \text{for } t \geq 1$$

where $\mathcal{F}_t \equiv \frac{u_{cc,t} b_{N,t-N}}{(1+(-\gamma_c+1)\lambda_1)B}$.

Consider $t=1$. For any long maturity $N>1$ we have that $\lambda_{t-N} = \lambda_{t-N+1} = 0$ when $t=1$ so that

$$\frac{u_{c,1}}{v_{x,1}} = \frac{B + (\gamma_l + 1)\lambda_1}{(1 + (-\gamma_c + 1)\lambda_1)\, B} \tag{17}$$

Therefore we can write

$$\frac{u_{c,t}}{v_{x,t}} - \frac{u_{c,1}}{v_{x,1}} = (\lambda_{t-N+1} - \lambda_{t-N})\, \mathcal{F}_t = 0 \quad \text{for } t \geq 1 \tag{18}$$

That $\tau_t = \tau_1$ for all $t > 1$ and $t \neq N-1, N$ follows from (15).

Now we show that $\mathcal{F}_t < 0$ for $t = N-1, N$. Since $\lambda_1, B, \gamma_l > 0$ we have that $B + (\gamma_l + 1)\lambda_1 > 0$. Since $u_{c,1}, v_{x,1} > 0$ clearly (17) implies that $(1 + (-\gamma_c + 1)\lambda_1)\, B > 0$. Since we consider the case of initial government debt $b_{N,-1} > 0$ this leads to $b_{N,0} > 0$ and since $u_{cc,1} < 0$ we have $\mathcal{F}_t < 0$ for $t = N-1, N$.

Since for $t = N-1$ we have $\lambda_{t-N} - \lambda_{t-N+1} = -\lambda_0 < 0$ it follows

$$\frac{u_{c,N-1}}{v_{x,N-1}} < \frac{u_{c,1}}{v_{x,1}} \Rightarrow \tau_{N-1} < \tau_t \text{ for all } t > 1,\ t \neq N-1, N.$$

Also, it is clear from (17) that high g_1 implies a high λ_1. Since the martingale condition implies $E_t\,(u_{c,N}\lambda_1) = \lambda_0 E_0\,(u_{c,N})$ for slightly high g_1 we have $\lambda_1 > \lambda_0$ Therefore, for $t = N$ and if g_1 was high enough we have $\lambda_{t-N} - \lambda_{t-N+1} = \lambda_0 - \lambda_1 < 0$ so that (18) implies

$$\frac{u_{c,N}}{v_{x,N}}, \frac{u_{c,N-1}}{v_{x,N-1}} < \frac{u_{c,1}}{v_{x,1}} \Rightarrow \tau_N, \tau_{N-1} < \tau_1$$

■

Intuitively, in period $t = N-1$ there is a tax cut for the same reasons as in Section 2.2. New in this section is the tax cut (for high g_1) at $t = N$. The intuition for this is clear: when an adverse shock to spending occurs at $t = 1$ the government uses debt as a buffer stock so $b_{N,1} > b_{N,0}$, as this allows tax smoothing by financing part of the adverse shock with higher future taxes. But since future surpluses are higher than expected as the higher interest has to be serviced, the government can lower the cost of existing debt by announcing a tax cut in period N, since this will reduce the price $p_{N-1,0}$ of period $t = 1$ outstanding bonds $b_{N,0}$. The tax cut at $t = N$ is a stochastic analog of the tax cut described in section 2.2.

2.3.2 Contradicting Tax Smoothing

The above result shows that in this model tax policy is subordinate to debt management. In models of optimal policy the government usually desires to smooth taxes. Taxes would be constant in the above model if the government had access to complete markets. But we find that the government increases tax volatility in period N, long after the economy has received any shock. Therefore, government forfeits tax smoothing in order to enhance a typical debt management concern, namely reducing the average cost of debt. Obviously this policy is time inconsistent: if the government could unexpectedly reoptimize in period $t = 2$ given its debt $b_{N,1}$ it would renege on the promise to cut taxes in period N, instead it would promise to lower taxes in period $N+1$. It is clear from this discussion that what will matter for the policy function is the term $\mathcal{D}_N = (\lambda_0 - \lambda_1) b_{N,0}$. Therefore it is the interaction between past λ's and past b's that determines the size and the sign of today's tax cut. A linear approximation to the policy function would fail to capture this feature of the model and it would be quite inaccurate.

To summarize, we have proved that in the presence of an adverse shock to spending the government has to take three actions: *i)* increase taxes permanently, *ii)* increase debt, *iii)* announce a tax cut when the outstanding debt matures. Effects *i)* and *ii)* are well known in the literature of optimal taxation under incomplete markets, effect *iii)* is clearly seen in this model with long bonds since the promise is made N periods ahead. Obviously in the case of short maturity $N = 1$ of Aiyagari et al. (2002) the effect of $\mathcal{D}_1$ would be felt in deciding optimally τ_1 but would be confounded with the fact g_1 is stochastic making *iii)* harder to see in a model with short bonds.

3 Optimal Policy - Simulation Results

We now turn to the case where g_t is stochastic in all periods. As is well known analytic solutions for this type of model are infeasible so we utilise numerical results. The objective is to compute a stochastic process $\{c_t, \lambda_t, b_{N,t}\}$ that solves the FOC of the Ramsey planner, namely (4), (8), (9) and (10). First we obtain a recursive formulation that makes computation possible, then we simulate the model using the method for reducing the dimensionality of the state space proposed by Faraglia et al (2013) and finally we discuss the behaviour of the economy.

3.1 Recursive Formulation

Using the recursive contract approach of Marcet and Marimon (2011) the Lagrangean can be rewritten as :

$$\begin{aligned} L = E_0 \sum_{t=0}^{\infty} \beta^t \{ & u(c_t) + v(x_t) + \lambda_t S_t + u_{c,t} (\lambda_{t-N} - \lambda_{t-N+1}) b_{N,t-N} \\ & + \nu_{1,t} \left(\overline{M} - \beta^N b_{N,t} \right) + \nu_{2,t} \left(\beta^N b_{N,t} - \underline{M} \right) \} \end{aligned} \tag{19}$$

for $\lambda_{-1} = ... = \lambda_{-N} = 0$.

Assuming g_t is a Markov process, as suggested by the form of this Lagrangean, Corollary 3.1 in Marcet and Marimon (2011) implies the solution satisfies the recursive structure:

$$\begin{aligned} \begin{bmatrix} b_{N,t} \\ \lambda_t \\ c_t \end{bmatrix} &= F(g_t, \lambda_{t-1}, ..., \lambda_{t-N}, b_{N,t-1}, ..., b_{N,t-N}) \\ \lambda_{-1} &= ... = \lambda_{-N} = 0, \quad \text{given } b_{N,-1} \end{aligned}$$

for a time-invariant policy function F. This allows for a simpler recursive formulation than the promised utility approach, as the co-state variables λ do not have to be restricted to belong to the set of feasible continuation variables. The state vector in this recursive formulation has dimension $2N + 1$.

3.2 Solving the Model with Condensed PEA

The utility function (16) was convenient for obtaining the analytic results of Section 2.3. In this section however we use a utility function more commonly used in DSGE models:

$$\frac{c_t^{1-\gamma_1}}{1-\gamma_1} + \eta \frac{x_t^{1-\gamma_2}}{1-\gamma_2}$$

We choose $\beta = 0.98$, $\gamma_1 = 1$, $\gamma_2 = 2$ and $A = 100$. We set η such that if the government's deficit equals zero in the non stochastic steady state agents work a fraction of leisure equal to 30% of their time endowment.

For the stochastic shock g we assume the following truncated AR(1) process:

$$g_t = \begin{cases} \overline{g} & \text{if } (1-\rho)\, g^* + \rho g_{t-1} + \varepsilon_t > \overline{g} \\ \underline{g} & \text{if } (1-\rho)\, g^* + \rho g_{t-1} + \varepsilon_t < \underline{g} \\ (1-\rho)\, g^* + \rho g_{t-1} + \varepsilon_t & \text{otherwise} \end{cases}$$

We assume $\varepsilon_t \sim N(0, 1.44)^2$, $g^* = 25$, with an upper bound $\overline{g}$ equal to 35% and a lower bound $\underline{g} = 15\%$ of average GDP and $\rho = 0.95$. $\overline{M}$ is set equal to 90% of average GDP and $\underline{M} = -\overline{M}$.

As highlighted in the previous section the dimension of the state vector of this model is $2N+1$. It is easy to realise that it is a high dimensional object - it includes the value of government spending a in period t, the history of debt obligations b_{t-j}^N, past values of the multipliers λ_{t-j} for $j = 1, ..., N$. To give an idea of its size note that if the maturity of government bonds is 10 periods then the state space consists of 21 variables. Faraglia, Marcet, Oikonomou and Scott (2013) show that in order to make the computation of models with large N manageable it is possible to reduce the number of states in the approximating polynomials. Using a refinement of the Parameterised Expectations Algorithm (PEA) called the "condensed PEA", their approach is to partition the state space into variables that are of primary importance and variables of secondary importance. The latter are introduced in the approximating functions as successive linear combinations.

The advantages of the condensed PEA are readily apparent. In nearly half the cases the core variables are sufficient to solve the model and at most only one linear combination of omitted variables required to improve accuracy. Clearly the condensed PEA can be used to solve models with large state spaces with relatively small computational cost, since the state vector is in principle of dimension 41 but utilising a dimension of 4 is sufficient.

We refer to Faraglia, Marcet, Oikonomou and Scott (2013) for a extensive discussion of the method and to the appendix to access the accuracy of the approximation of the current model.

3.3 Optimal Policy - The Impact of Maturity

3.3.1 Interest Rate Twisting

Figures 1 and 2 display the impulse response functions of key variables to an unexpected shock in g_t. The solution is computed using the condensed PEA[5]. The vertical axis is in units of each of the

[5] Since debt is very persistent, to ensure we visit all possible realizations in the long run simulations of PEA we initialize the model at 9 different initial conditions, simulate it for 200 periods for each initial condition, doing this 1000 times per initial condition and compute conditional expectations.

variables and expresses deviations from the value that would occur for the given initial condition if $g_t = g^{ss}$.

Figure 1 is for the case when the government has zero inherited debt The only significant differences across maturity are on the face value of debt and interest rates. But these differences are immaterial: the face value of debt $b_{N,t}$ is obviously higher for long bonds, as long debt is discounted more heavily so its face value needs to be higher. What is relevant is the market value of debt, which is similar. As usual in endowment models the long interest rates respond less to shocks than the short interest rate. As usual in models of incomplete markets it is optimal to use debt as a buffer stock so that debt displays considerable persistence.

HERE FIGURE 1

Figure 2 shows the same impulse response functions but in this case we assume the government is indebted such that $b_{N,-1} = 0.5\ y^*/\beta^N$ where y^* is steady state output.

HERE FIGURE 2

With long bonds of maturity $N = 10$ there is a blip in taxes at the time of maturity of the outstanding bonds. This is a reflection of the promise to cut taxes with the aim to twist interest rates, as discussed in Section 2.3, only now the interest rate twisting occurs each period there is an adverse shock if the government is in debt. The size of the promised tax cut depends on how much larger is today's shock relative to yesterday's shock $(\lambda_{t-1} - \lambda_t)$ and the level of today's debt.

3.3.2 Optimal Policy with Short Bonds

This discussion helps to understand the role of commitment in the model of short term bonds as in Aiyagari et al (2002). Consider the case when the government is indebted when an adverse shock occurs, as in Figure 2. As we explained in Section 2.3 optimal policy is to increase current taxes but promise a tax cut in $N - 1$ periods. In the case of long bonds the promised tax cut is clearly distinct from the current increase in taxes. But in the case of short bonds $N = 1$ the two effects are confounded as they happen in the same period.

This can be seen in the response of taxes depicted in Figure 3 for maturities $N = 1, 5, 10, 20$. Given our previous discussion it is clear why the blip in taxes keeps moving to the left as we decrease the maturity until the blip simply reduces the reaction of taxes on impact at $N = 1$. Therefore optimal policy for short bonds is to increase taxes on impact but less than would be done if considerations of interest rate twisting were absent or if the debt were zero.

Figure 3 also shows that in the case when the government has assets the blip in taxes goes upwards, as the government desires to increase the value of assets. It is clear that for short bonds the increase in taxes on impact if the govenment initially has assets is much larger than if the government is indebted.

HERE FIGURES 3

3.3.3 Second Moments

Table 1 shows second moments for the economy at steady state distribution for different maturities[6]. With the exception of debt and deficit all the moments differ only to the second or third decimal place across maturities. This may be surprising, as we have seen that tax policy does change with maturity and since we know that under incomplete markets the way government finances its expenditure can affect the real economy. However with the government only issuing one type of bond in each case tax smoothing is mainly achieved by using debt as a buffer stock rather than through fiscal insurance (defined in Faraglia Marcet and Scott (2008) as achieving variations in the market value of debt which offset adverse expenditure or tax shocks). The fluctuations of all variables are driven mostly by the strong low frequency fluctuations of debt, so that the interest rate twisting plays relatively little role in these steady state second moments. We return to this issue later in the discussion of Figure 8.

The main exception are the levels of debt and deficit: government in the long run holds assets, but average asset holding are lower for higher maturities. The mean of assets at steady state for 20 year bonds is half the average assets compared for short bonds, due to the different opportunities for fiscal insurance that are offered by long bonds.

As is well known, in models of optimal policy with incomplete markets, there is a force pushing the government to *accumulate* long bonds in the long run. More precisely, extending the results in Aiyagari et al (2002) Section III one can easily prove that in the case of linear utility ($u(c) = c$) the government would purchase a very large amount of private long bonds in the long run, enough to abolish taxes. This accounts for the negative means for debt shown in Table 1. On the other hand, as argued in Angeletos (2002), Buera and Nicolini (2004) and Nosbusch (2008), if the term premium is negatively correlated with deficits (as it is in our model) it is optimal for the government to *issue* long bonds, as this provides fiscal insurance. Hence the government is aware that accumulating a very large amount of privately issued long bonds increases the volatility of taxes. This force accounts for the *lower* asset accumulation with *longer* maturities shown in Table 1.

HERE TABLE 1

Varying the average maturity of debt also has an influence on the persistence of debt. Marcet and Scott (2009) (MS from now on) show that measures of relative persistence are a good way of assessing the extent of market incompleteness and so Figure 4 shows for various variables the measure:

$$P_y^k = \frac{Var(y_t - y_{t-k})}{kVar(y_t - y_{t-1})}.$$

HERE FIGURE 4

The closer to 0 this measure the less persistence the variable shows, whereas the closer to 1 the measure the more the variable shows unit root persistence. MS show that observed k variances for

[6] These moments have been computed from very long simulations using the approximate policy function computed as described before.

US debt were even higher than 1, for example $P^{10}_{Debt} \approx 2.5$ (see Figure 2 in MS). Values of P^{k}_{Debt} higher than one are incompatible with complete market models and optimal policy, but they easily arise under incomplete markets. However MS also report a shortcoming of incomplete markets: debt display too much persistence under incomplete markets, as they report $P^{10}_{Debt} = 10$ (see Figure 6 in MS).

Figure 4 shows the small sample means of persistence measures for our model when the government is initially in debt[7]. Now $P^{10}_{Debt} = 4.1$ for 20 year bonds so the gap between the data and the model is now one fifth of the gap reported by MS. This improvement is in part due to our use of small sample moments, while MS reported k-variance ratios at steady state distribution. Note that even for a short maturity of N=2 (and also for N=1, not reported in Figure 4) we have $P^{10}_{Debt} \approx 5$, nearly cutting the persistence in half relative to MS.

4 Modelling Maturity With Decaying Coupon Perpetuities

In order to overcome the problem of dimensionality some authors model long bonds as perpetuities with decaying coupon payments where the rates of decay mimic differences in maturity (e.g Woodford (2001), Broner, Lorenzoni and Schmulker (2007), Arellano and Ramanarayanan (2008), Chen, Curdia and Ferrero (2012)).

In this formulation the government issues perpetuities, b, with coupon payments that decay geometrically i.e a bond with decay factor δ_L pays a coupon equal to $\delta_L^j b_j$ in period j. The decay rate determines the effective maturity of the bond as the bond's duration is defined by $1/(1-\delta_L)$ so that a bond of effective maturity 10 years has $\delta_L = 0.1$. In this case total payments from all previously and currently issued perpetuities are then given by $B_t = b_t + \delta_L b_{t-1} + \delta_L^2 b_{t-2} + ... + \delta_L^t b_0$ which follows the recursive structure $B_t = \delta_L B_{t-1} + b_t$. Treating this as the outstanding stock of the perpetuity we have a convenient way of dealing with long maturity bonds which dramatically reduces the state space as it is only necessary to keep track of the total number of bonds issued and not the number of bonds issued in each period. This reduction in the state space means that the "condensed PEA" is no longer required and the model can be solved using more conventional methods.

Whilst assuming decaying coupon payments has great computational merit it is not without modelling consequences. One justification for assuming decaying payoffs is that it mimics a bond portfolio with fixed shares that decay with maturity. However since our goal is to build a model of debt management where the object is precisely to study the appropriate portfolio weights, assuming fixed portfolio weights would be inappropriate. Further although modelling bond payoffs in this way would yield smaller state space vectors it is contrary to the structure of most government portfolios where most of the payoff occurs at the time of maturity, as in this model, whereas with decaying coupons the majority of cash flow is paid out in the early years.

The government budget constraint becomes:

$$B_{t-1} = \widetilde{S}_t + p_t \left(B_t - \delta_L B_{t-1}\right)$$

[7] The small sample means are found by fixing initial bonds at a level of debt equal to 0.5y*, obtain simulations of 50 periods, compute P_y^k for each realisation and average P_y^k over many realisations all starting at the same initial condition.

where $B_t - \delta_L B_{t-1} = b_t$ is the amount of bonds that the government issues in period t and B_{t-1} is the amount of coupons and maturing bonds that the government has to repay in the same period.

The Ramsey problem then becomes:

$$\max_{\{c_t,B_t,p_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t) + v(x_t)\}$$
$$B_{t-1} = \widetilde{S}_t + p_t (B_t - \delta_L B_{t-1})$$
$$p_t = \frac{\beta E_t (u_{c,t+1} (1 + \delta_L p_{t+1}))}{u_{c,t}}$$

The price of the bond can be rewritten as $p_t = \frac{\beta E_t\left(\sum_{j=0}^{\infty} (\beta\delta_L)^{j-1} u_{c,t+j}\right)}{u_{c,t}}$, that shows that it is a function of all the future marginal utilities since the bond will pay an income for the rest of the time.

We can then rewrite the Lagrangian of the problem as:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t) + v(x_t) + \lambda_t [S_t + u_{c,t} p_t (B_t - \delta_L B_{t-1}) - u_{c,t} B_{t-1}]$$
$$+\mu_t (u_{c,t} p_t - \beta u_{c,t+1} (1 + \delta_L p_{t+1}))\}$$

dropping for brevity the debt limits.

The first order conditions then become:

$$u_{c,t} - v_{x,t} + \lambda_t (u_{cc,t} c_t + u_{c,t} + v_{xx,t} (c_t + g_t) - v_{x,t})$$
$$+u_{cc,t} \left[p_t (B_t - \delta_L B_{t-1}) - u_{c,t} B_{t-1} + \mu_t p_t - \mu_{t-1} \beta (1 + \delta_L p_{t+1})\right] = 0$$

$$\lambda_t u_{c,t} p_t = \beta E_t (\lambda_{t+1} u_{c,t+1} (1 + \delta_L p_{t+1})) \tag{20}$$

$$\mu_t = \mu_{t-1} - \lambda_t (B_t - \delta_L B_{t-1})$$

A new state emerges, μ_t, and the state space becomes $\{g_t, \mu_{t-1}, B_{t-1}\}$

Following the same steps taken in Section 2 we can write the implementability constraint:

$$E_t \sum_{j=0}^{\infty} \beta^j S_{t+j} = B_{t-1} E_t \sum_{j=0}^{\infty} (\delta_L \beta)^j u_{c,t+j} = \left(\sum_{i=1}^{t} \delta_L^{i-1} b_{t-i}\right) E_t \sum_{j=0}^{\infty} (\delta_L \beta)^j u_{c,t+j}$$

if we assume no uncertainty this becomes:

$$\sum_{j=0}^{\infty} \beta^j S_{t+j} = \left(\sum_{i=1}^{t} \delta_L^{i-1} b_{t-i}\right) \sum_{j=0}^{\infty} (\delta_L \beta)^j u_{c,t+j}.$$

It becomes clear that the government has an incentive to affect the interest rates and consequently the taxes on an infinite horizon with decaying weights.[8]

INSERT FIGURE 5

Figure 5 shows the impulse response functions of tax rates in response to a government expenditure shock for the case of a one year bond, a ten year bond (as above and solved for using the condensed PEA approximating $\Omega^1 = E_t\left(\lambda_{t+1}u_{c,t+1}\left(1+\delta_L p_{t+1}\right)\right)$ and $\Omega^2 = \beta E_t\left(u_{c,t+1}\left(1+\delta_L p_{t+1}\right)\right)$) and a bond with decaying coupons with duration set equal to 10 under the case of no debt, positive initial debt and a government that inherits a positive asset position. Focusing on the case with zero initial debt, in which case the government has no incentive to engage in interest rate twisting, we see taxes follow the same smooth path across all three models however whilst with the one and ten year bonds taxes follow a risk adjusted martingale that sees them slowly declining over time, for the case of decaying coupons we have taxes smoothly trending upwards. The more revealing differences are shown for the case where government debt is not initially zero. In the case where the government inherits positive debt it uses taxes to twist interest rates to reduce funding costs in the manner described above. However with decaying coupons taxes now decline smoothly across all periods. The logic is simple as now with decaying coupons the government has incentive to "twist" every period and so taxes fall smoothly each period as the interest rate twisting incentive occurs every period.

Even if the results of Figure 5 reveal that the behavior of the taxes and bonds are different in the two models, we have tested how substitutable the two modelling assumptions are. First we checked if the approximation of the decaying coupon model is accurate[9]. Satisfied by the accuracy results, we have approximated the expectations $\Phi^1 = \beta^N E_t(u_{c,t+N})$ $\Phi^2 = \beta^{N-1}E_t(u_{c,t+N-1})$ $\Phi^3 = \beta^N E_t(\lambda_{c,t+1}u_{c,t+N})$ with a polynomial function of the variables $\{g_t, \lambda_{t-1}, ..., \lambda_{t-N}, B_{t-1}, ..., B_{t-N}\}$ retrieved from the decaying coupon bond model. These approximations have allowed us to simulate the optimal model in Section 2. We have then checked the accuracy of the solution. The model rejects this approximation. The Euler equation errors are on average above 4%. We then compare a long simulation of our optimal model generated by the policy function retrieved in section 2 and the one generated by the decaying coupon model. We plot the path of debt, taxes and deficit assuming no inherited debt. Figure 6 shows the results. The dotted lines refer to the optimal model and the solid lines refer to the one generated by the policy function of the decaying coupon bond model. The paths are really different. We can conclude then that the two models cannot be substitutes.

INSERT FIGURE 6

[8] This is exactly the same as the case of a model with no buyback. The budget constraint there becomes:

$$\sum_{j=0}^{\infty}\beta^j S_{t+j} = \sum_{i=0}^{N} p_{N-i,t}b_{N,t-i}$$

[9] We used Euler Equation Errors to check the accuracy of our results. The appendix reports the results of the test.

5 Independent Powers

In Sections 2 and 3 we found that full commitment implied a tight connection between interest rate policy, debt management and tax policy. Specifically when the government is in debt and spending is high the government promises a tax cut in $N-1$ periods, knowing that this will increase future consumption and this decreases long interest rates in the current period. The reader may think that this optimal policy is not relevant for the "real world" for at least two reasons. First, different authorities influence interest rates and fiscal policy, it is unlikely that they will coordinate in the way described before and, secondly, it is unlikely that governments can commit to a tax cut in the distant future and actually carry through with the promise. Some papers in the literature react to this type of criticisms by writing down models where government policy is discretionary. But assuming that the government has no possibility of committing is also problematic, as governments frequently do things for the very reason they have previously committed to do so.

For these reasons in this section we change the way policy is decided in this model. We relax the assumption of perfect coordination and assume the presence of a third agent, a monetary authority that fixes interest rates in every period. The fiscal authority now takes interest rates as a given and implements optimal policy given these interest rates. We examine an equilibrium where the two policy makers play a dynamic Markov Nash equilibrium with respect to the strategy of the other policy power and they both play Stackelberg leaders with respect to the consumer. More precisely, the fiscal authority chooses taxes and debt given a sequence for interest rates, the monetary authority simply chooses interest rates that clear the market and the fiscal authority maximizes the utility of agents. This assumption sidesteps the issues of commitment, now there is no room for interest rate twisting on the part of the fiscal authority.

It is easy to think of models where even if the monetary authority is independent it can not deviate too much from equilibrium interest rates of the flexible price model. Therefore we take a limit case and assume that the monetary authority simply sets in equilibrium interest rates as:

$$\begin{aligned} p_{N,t} &= \frac{\beta^N E_t\left(u_{c,t+N}\right)}{u_{c,t}} && (21)\\ p_{N-1,t} &= \frac{\beta^{N-1} E_t\left(u_{c,t+N-1}\right)}{u_{c,t}}. \end{aligned}$$

given agents' consumption. To solve this model we are looking for an interest rate policy function $\mathcal{R} : R^2 \to R^2$ such that if long interest rates at t are given by

$$(p_{N,t}^{-1}, p_{N,t-1}^{-1}) = \mathcal{R}(g_t, b_{N,t-1}) \tag{22}$$

then (21) holds with the fiscal authority maximizing consumer utility in the knowledge of all market equilibrium conditions but taking the stochastic process for interest rates as given it chooses a bond policy such that (22) holds.

From the point of view of the fiscal authority the problem now is a standard dynamic programming problem such that the vector of state variables is now $(b_{N,t-1}, g_t)$. An advantage of this model is there is no longer any reason for longer lags to enter the state vector, as past Lagrange multipliers do not play a role. Therefore this separation of powers approach is an alternative way to reducing the state space and simplifying the solution of the model.

In this case of independent powers the Lagrangian of the Ramsey planner becomes

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + v(x_t) + \lambda_t \left[S_t + p_{N,t} b_{N,t} - p_{N,t-1} b_{N,t-1} \right] \right. \\ \left. + \nu_{1,t} \left(\overline{M} - \beta^N b_{N,t} \right) + \nu_{2,t} \left(\beta^N b_{N,t} - \underline{M} \right) \right\} \tag{23}$$

The first order condition with respect to consumption is

$$u_{c,t} - v_{x,t} + \lambda_t \left(u_{cc,t} c_t + u_{c,t} + v_{xx,t} (c_t + g_t) - v_{x,t} \right) + u_{cc,t} \lambda_t \left(p_{N,t} b_{N,t} - p_{N-1,t} b_{N,t-1} \right) = 0$$

and using the government's budget constraint gives

$$u_{c,t} - v_{x,t} + \lambda_t \left(u_{cc,t} c_t + u_{c,t} + v_{xx,t} (c_t + g_t) - v_{x,t} \right) + u_{cc,t} \lambda_t \left(g_t - \left(1 - \frac{v_{x,t}}{u_{c,t}} \right) (1 - x_t) \right) = 0 \tag{24}$$

To see the impact of Independent Powers we calibrate the model as in Section 3 and consider the case $N = 10$. Figure 7 compares the impulse responses to a one standard deviation shock to the innovation in the level of government spending (in the presence of government debt) between independent powers and the benchmark model of Section 3. As can be seen the model of independent powers does not show the blip in taxes at maturity. In this case debt management is subservient to tax smoothing and is aimed at lowering the variance of deficits.

HERE FIGURE 7

To better understand the magnitude of the interest twisting channel we can compare our independent powers model with our earlier benchmark model. We simulated the models at different time horizons $T = 40$, $T = 200$ and $T = 5000$ discarding the first 500 periods. We calculated the standard deviation of taxes for each realizations and averaged across simulations. We repeat the same exercise for $N = 2, 5, 10, 15, 20$. Figure 8 shows the results.

HERE FIGURE 8

In shorter sample periods the effect of twisting interest rates in connection with initial period debt is significant and provides a higher level of tax volatility in the benchmark model. Naturally ss we increase the sample size the initial period effect diminishes.

The second moments of the model in this section are shown in Table 2. They are extremely similar to those of the benchmark model in Table 1. We have essentially a very similar amount of bond issuance, debt persistence, tax smoothing etc, the only difference being that the interest rate twisting adds some tax volatility, but this volatility only shows up in second moments with short samples as shown in Figure 8. We conclude that the model of independent powers may be a good model to have in the toolkit as it retains many of the interesting features of the Ramsey models, it has the same steady state moments, it avoids the technicalities arising from the very large state vector and it avoids discussion on the role to commitment at very long horizons. There are, however, issues of tax volatility showing up in small samples where the two models differ.

HERE TABLE 2

6 Conclusions

This paper has the aim to study optimal fiscal policy when governments issue bonds of long maturity.

In the analysis it becomes evident that number of different considerations arise when governments issue long term bonds. If the government inherits debt it has an incentive to twist interest rates to minimize the costs of funding debt. This is achieved by violating tax smoothing and promising a tax cut in $N-1$ periods, when existing bonds mature. A typical debt management concern, namely lowering the cost of debt, therefore shapes the path of fiscal policy. This suggests that it is important to consider debt management and fiscal policy jointly.

The model with long bonds helps to clarify the role of commitment in models of fiscal policy and incomplete markets. In the case of short bonds the change in taxes needed to adjust to a shock and the promise to cut taxes at time of maturity are conjoined, what is observed is that taxes increase on impact much less if the government is in debt.

In the case of long bonds these two effects are separated. The commitment to cut future taxes is time inconsistent and also leads to a potentially very large state space of dimension $2N+1$. Using the "condensed PEA" enables us to solve this model accurately with a much reduced state space allowing for the computation of non-linear numerical solutions.

We also propose an alternative model of government policy, where a central bank determines interest rates and a fiscal authority separately decides on debt and taxes. This model of independent powers is of interest per se, as policy authorities may not be able to coordinate as much as is required to implement the full commitment solution. Also, it does not display policies where promises that will be implemented very far in the future matter for today's solution. As such it serves to highlight the role of commitment and to look at a solution in which the state space is not enormous.

There is little quantitative difference in fiscal policy or economic allocations at steady state second moments as the maturity of debt is varied, justifying the observation in Table 1 that similar countries may have very different average maturity of debt. The main difference is in the steady state level of debt: longer maturities imply lower asset accumulation because long bonds provide a volatile deficit if the government holds assets. However, for second moments computed with short run moments we do find more tax volatility with long bonds.

A number of further issues remain. We have throughout this paper assumed the government can issue only one bond and have varied its maturity. In order to fully understand debt management we need to consider the case when the government can issue several bonds of different maturity and choose the optimal portfoliio. Another important issue is to consider why in practice governments do not buyback debt each period but hold bonds through to maturity. In Faraglia et al (2013) we combine both of these features and the methodologies and insights of this paper to develop a model of debt management able to characterise observed debt management.

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.

Table 1 Second Moments, Steady State

Model: **Benchmark Model**

	N	c	y	τ	**deficit**	R_N	**MV** $=p_N b_N$	$\boldsymbol{\lambda}$
mean								
	1	52.60	70.11	0.243	0.42	2.02	-24.68	0.057
	5	52.58	70.08	0.245	0.32	2.02	-19.21	0.058
	10	52.56	70.06	0.246	0.25	2.03	-16.28	0.058
	20	52.54	70.05	0.247	0.17	2.03	-12.46	0.059
std								
	1	3.49	0.35	0.044	1.46	0.5	27.26	0.013
	5	3.48	0.37	0.043	1.57	0.4	30.96	0.013
	10	3.48	0.38	0.044	1.59	0.3	31.97	0.013
	20	3.48	0.39	0.044	1.66	0.2	32.84	0.014

Note: to provide a more interpretable quantity we report annualized interest rates instead of bond prices, namely $R_N = \left((p_N)^{-\frac{1}{N}} - 1\right) 100$.

Table 2: Second Moments, Steady State

Model: **Independent Powers**

	N	c	y	τ	**deficit**	R_N	**MV** $=p_N b_N$	$\boldsymbol{\lambda}$
mean								
	1	52.60	70.10	0.244	0.41	2.02	-23.54	0.057
	5	52.58	70.08	0.245	0.32	2.02	-19.49	0.058
	10	52.56	70.07	0.246	0.26	2.03	-16.40	0.058
	20	52.54	70.05	0.247	0.17	2.03	-12.31	0.059
std								
	1	3.49	0.34	0.044	1.43	0.5	27.88	0.013
	5	3.48	0.36	0.044	1.51	0.4	31.11	0.013
	10	3.48	0.37	0.044	1.54	0.3	32.20	0.013
	20	3.49	0.37	0.044	1.56	0.2	33.20	0.014

Figure 1: Responses to a positive shock in g_t, Benchmark model
Maturities 1 and 10: $b_{N,-1} = 0$

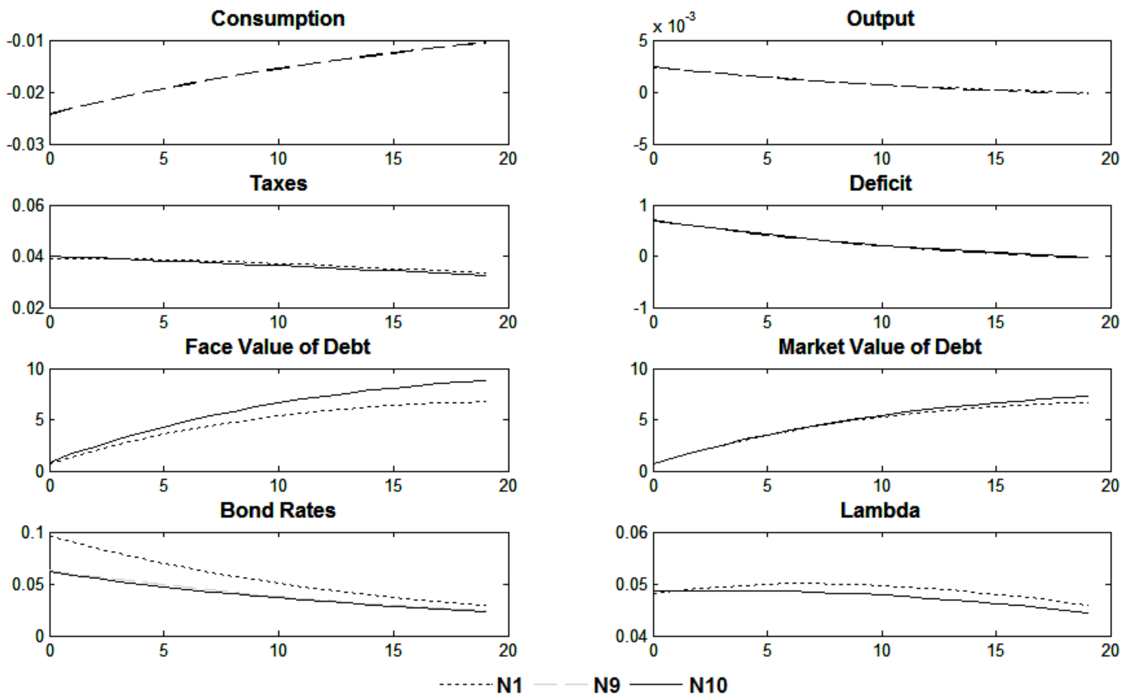


Figure 2: Responses to a positive shock in g_t, Benchmark model Maturities 1 and 10: $b_{N,-1} = \frac{0.5y^*}{\beta^N}$

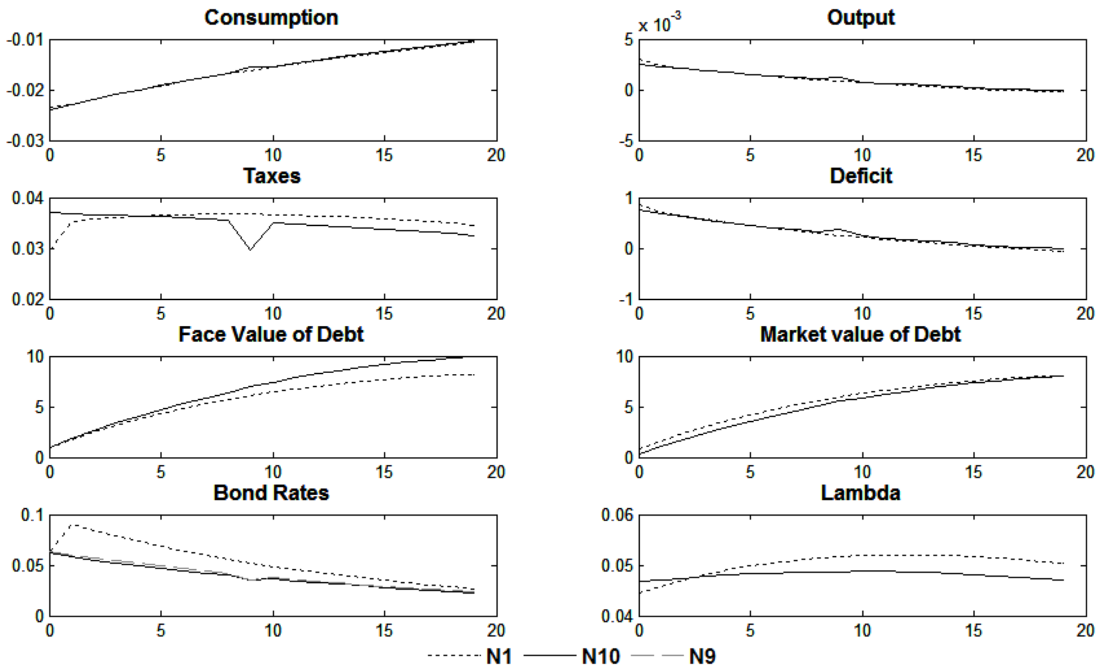
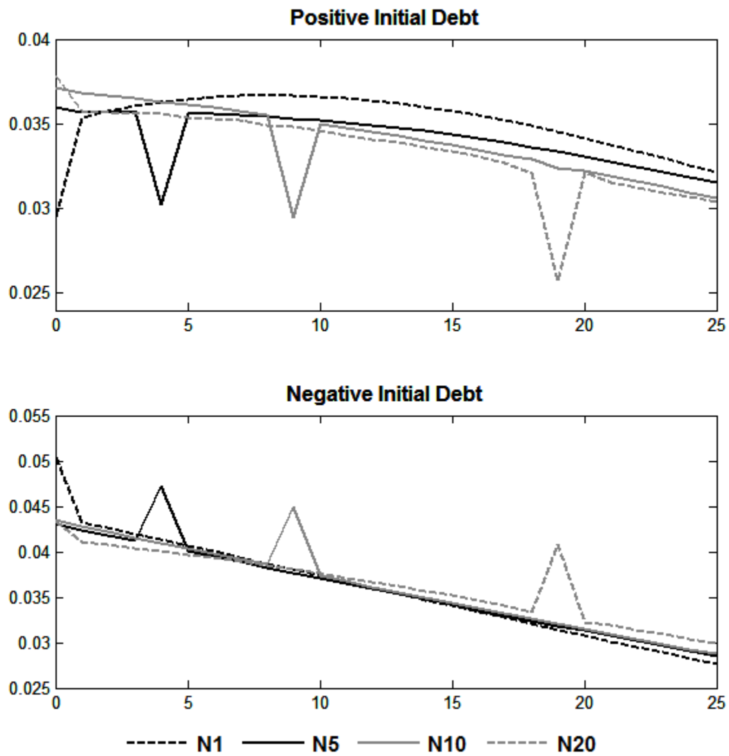


Figure 3: Responses to a positive shock in g_t, Benchmark model Maturities 1, 5, 10 and 20:



First panel $b_{N,-1} = \frac{0.5y^*}{\beta^N}$ Bottom panel $b_{N,-1} = -\frac{0.5y^*}{\beta^N}$

Figure 4: k-Variances, Benchmark model
Maturities 2, 5, 10 and 20

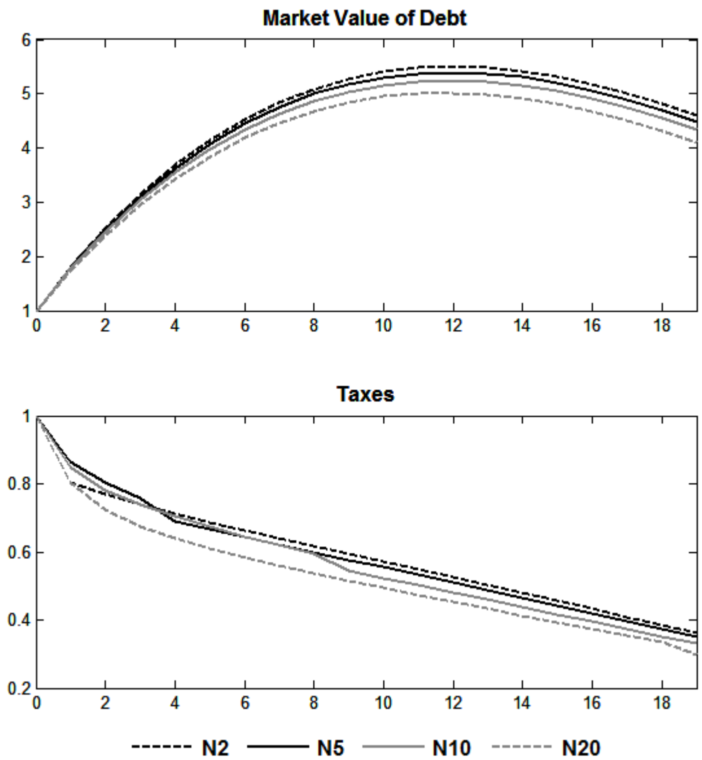


Figure 5: Responses to a positive shock in g_t, Benchmark and Deacaying Coupon Model

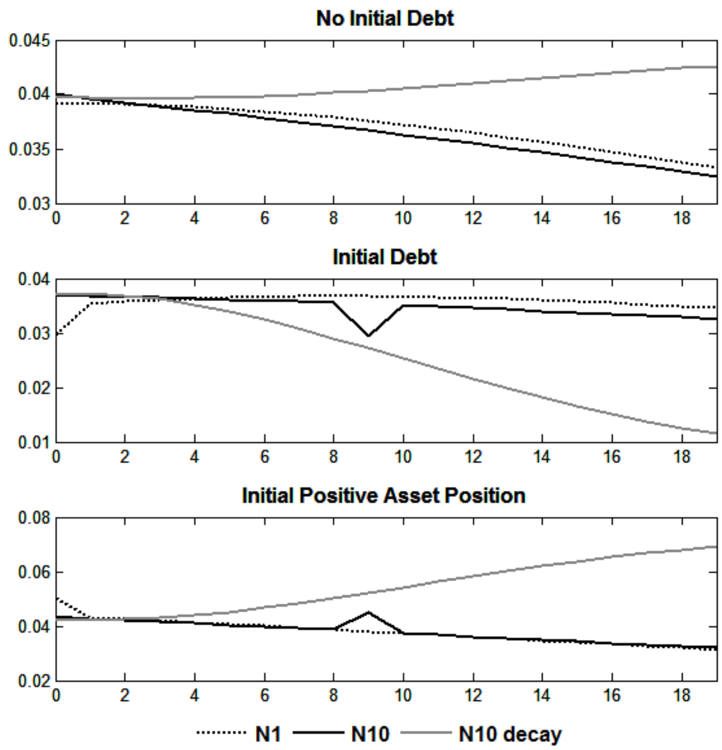


Figure 6: Paths generated with the policy of the optimal model and the policy of the Decaying Coupon Model

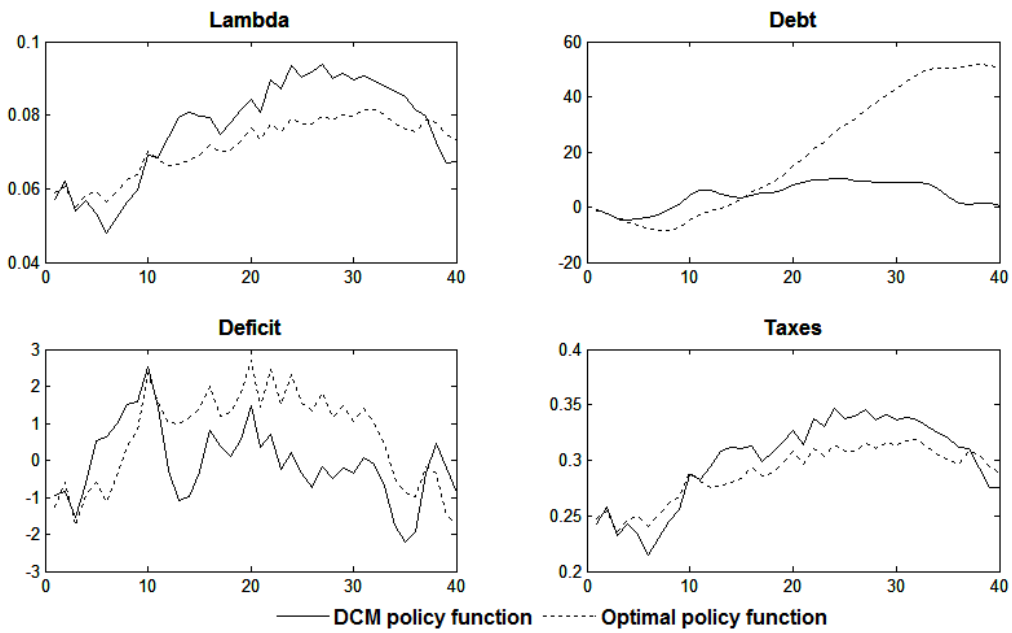


Figure 7: Responses to a positive shock in g_t, Benchmark and Independent Power Model

Maturity 10: $b_{N,-1} = \frac{0.5y^*}{\beta^N}$

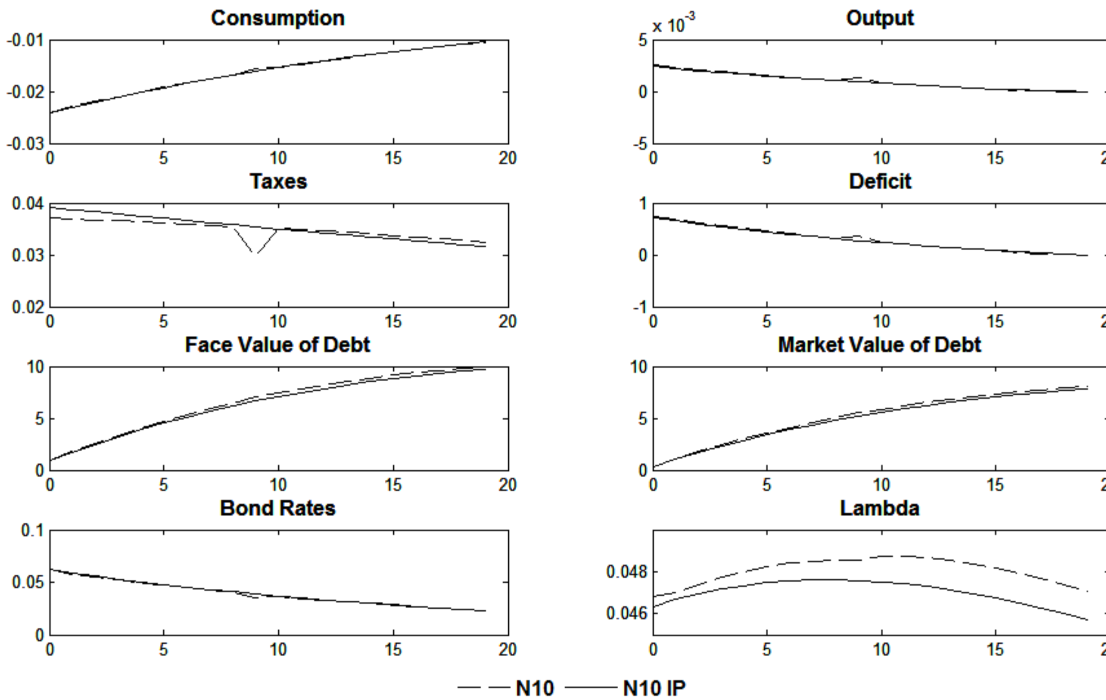
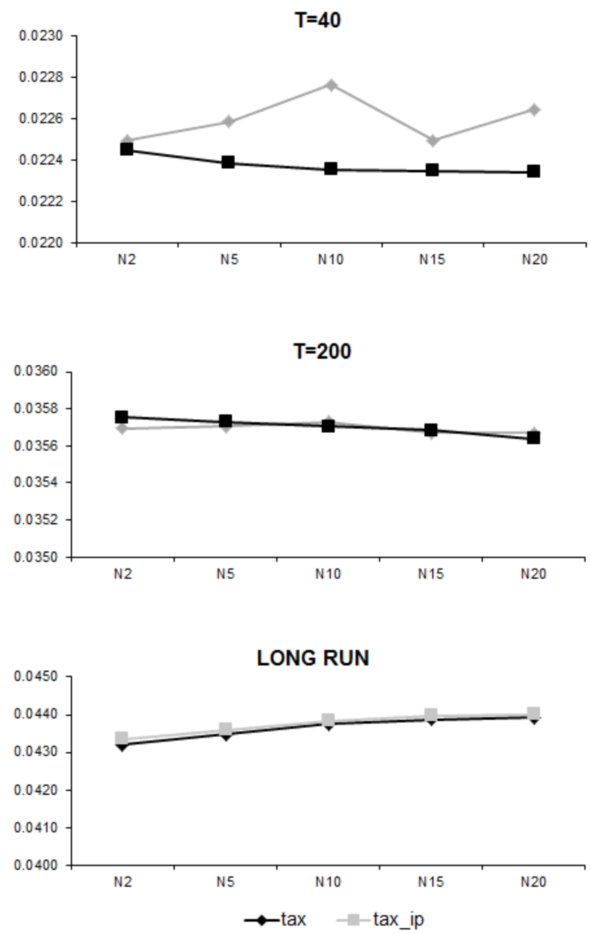


Figure 8: Tax Volatility at Different Horizons
Benchmark and Independent Powers Model



7 Appendix 1 - Accuracy of the solutions

In order to access the accuracy of our solutions we calculate the Euler Equation Errors generated by our models (Judd (1998)).

Given our approximated policy function, $\phi(s_t)$, where s_t is the vector of the states variables, and a chosen maturity N, we simulate the model for $T = 5000$ periods and we evaluate the Euler Equation Errors in $\widehat{T} = 50$ points equally spaced in the sample. We repeate the evaluations using 200 samples for each of the 9 different initial conditions. In total we check the errors in 90000 points. For every $\widehat{T}$ we recalculate the conditional expectations using $\phi(s_t)$

$$
\begin{aligned}
R_1 &= E_{\hat{t}}\left(\lambda_{\hat{t}+1} u_c\left(c_{\hat{t}+N}\right)\right) \\
R_2 &= E_{\hat{t}}\left(u_c\left(c_{\hat{t}+N}\right)\right) \\
R_3 &= E_{\hat{t}}\left(u_c\left(c_{\hat{t}+N-1}\right)\right)
\end{aligned}
$$

drawing 5000 realizations of the shocks for every $j = \hat{t}+1, .., \hat{t}+N$ using $\phi(s_{t+j})$ to pin down the allocation in every period and performing a Montecarlo integration. Given the new expectations we recalculate the implied solution in $\hat{t}$ evalutate the percentage difference between the allocation given by the approximated policy rule $\phi_{\hat{t}}$ and the one implied by R_1, R_2, R_3, $\phi_{\hat{t}}^{impl}$. The results are reported in tables 1A. Table 1A reports the average of the Euler Equation Errors $\left(EEE_{\hat{t}} = \left|\frac{\omega_{i,\hat{t}} - \omega_{i,\hat{t}}^{impl}}{\omega_{i,\hat{t}}}\right|\right)$ implied by the three Euler equations of the model in terms of the multiplier of the government budget constraint, $\omega_1 = \lambda$, and the price of an N and $N-1$ period bond, $\omega_2 = p^N$ and $\omega_3 = p^{N-1}$. We perform this test for all the models presented in the paper. The results show that the errors are low and the approximations are accurate. Moreover in all the models roughly 50% of the errors have a positive sign highlighing that the errors are equally distributed around the average. We have also checked that more than 90% of the times the errors are below one percent and no bigger than 2%. To further evaluate the accuracy of the solution he have calculated also the average and the standard deviation of the allocation of our approximation and the one implied by R_1, R_2, R_3.Table 1B reports the results. The averages are equal to each other up to the forth decimal as well as the standard deviations. This evidence confirms that our approximation is accurate enough.

Table 1A: Euler Equation Errors

	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$	$N=20$
Optimal Model						
λ	0.006	0.007	0.008	0.008	0.008	0.008
p^N	0.002	0.002	0.003	0.002	0.002	0.008
p^{N-1}		0.002	0.003	0.002	0.002	0.002
Independent Power Model						
λ		0.006	0.007	0.007	0.008	0.008
p^N		0.002	0.003	0.002	0.002	0.008
p^{N-1}		0.002	0.003	0.002	0.002	0.002
Decaying Bond Model						
λ				0.007		
p^N				0.001		

Table 1B: Implied Moments

	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$	$N=20$
Optimal Model						
λ	0.057 (0.013)	0.058 (0.013)	0.058 (0.013)	0.058 (0.013)	0.059 (0.014)	0.059 (0.014)
λ_{impl}	0.057 (0.0131)	0.058 (0.013)	0.058 (0.013)	0.058 (0.013)	0.059 (0.014)	0.059 (0.014)
p^N	0.980 (0.005)	0.961 (0.008)	0.905 (0.017)	0.819 (0.026)	0.740 (0.030)	0.670 (0.031)
p^N_{impl}	0.980 (0.005)	0.961 (0.008)	0.905 (0.017)	0.819 (0.026)	0.740 (0.030)	0.670 (0.031)
p^{N-1}		0.980 0.005	0.923 (0.014)	0.835 (0.024)	0.755 (0.029)	0.683 (0.031)
p^{N-1}_{impl}		0.980 0.004	0.923 (0.014)	0.835 (0.024)	0.755 (0.029)	0.683 (0.031)
Independent Power Model						
λ		0.057 (0.013)	0.058 (0.013)	0.058 (0.013)	0.058 (0.013)	0.058 (0.014)
λ_{impl}		0.057 (0.013)	0.058 (0.013)	0.058 (0.013)	0.058 (0.014)	0.058 (0.014)
p^N		0.961 (0.008)	0.905 (0.017)	0.819 (0.026)	0.740 (0.030)	0.670 (0.031)
p^N_{impl}		0.961 (0.008)	0.905 (0.017)	0.819 (0.026)	0.740 (0.030)	0.670 (0.031)
p^{N-1}		0.980 (0.005)	0.923 (0.014)	0.835 (0.025)	0.755 (0.030)	0.683 (0.031)
p^{N-1}_{impl}		0.980 (0.004)	0.923 (0.014)	0.835 (0.025)	0.755 (0.030)	0.683 (0.031)
Decaying Bond Model						
λ				0.058 (0.013)		
λ_{impl}				0.058 (0.013)		
p^N				8.0315 (0.198)		
p^N_{impl}				8.0315 (0.198)		