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ABSTRACT

Buying First or Selling First in Housing Markets*

Housing transactions by existing homeowners take two steps, a purchase of a new property and sale of the old housing unit. These two decisions are not independent, and their sequence may depend on the state of the housing market. This paper shows how the sequence of buyer-seller decisions depends on, and in turn, affects housing market conditions in an equilibrium search-and-matching model of the housing market. Under a simple payoff condition, we show that the decisions to ``buy first'' or ``sell first'' among existing homeowners are strategic complements - homeowners prefer to ``buy first'' whenever there are more buyers than sellers in the market. This behavior leads to multiple steady state equilibria and to dynamic equilibria featuring low frequency self-fulfilling fluctuations in house prices and time on the market. The model is broadly consistent with stylized facts about the housing market.

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1 Introduction

Motivation. A large fraction of households move within the same local housing market in the U.S. every year. Many of these moves are by existing homeowners who buy a new property and sell their old housing unit.[1] However, it takes time to transact in the housing market, so a homeowner that moves may end up owning either two units or no unit for some period, depending on the sequence of transactions. Either of these two alternatives may be costly.[2]

Existing owners often engage in contracting arrangements that reflect the sequence of transactions they are making. Homeowners that buy a new property before selling their old one often apply for "bridging loans" from financial institutions. These are short-term mortgage loans to finance the new purchase before the sale of the old property is completed. Alternatively, homeowners that sell first may engage in a "rent back" agreement with the buyer of their property, allowing them to rent their old house after the official sale. These alternatives are also revealed in Internet searches for these terms as Figure 1 shows. The figure plots the relative monthly search frequencies for the terms "bridge loan" and "rent back" from 2006 to 2012 using data from the Google search engine. Both terms have a similar relative frequency overall and both follow a common seasonal pattern, which is a characteristic property of housing market transactions (Ngai and Tenreyro (2012)).

Importantly, however, relative searches for both terms appear to comove with the state of the housing market as proxied by the Case-Shiller house price index. Specifically, searches for "bridge loan" were substantially higher compared to "rent back" searches when the housing market was booming in 2006, and subsequently declined with the decline in house prices. Simultaneously "rent back" searches increased in frequency as house prices declined, overtaking "bridge loan" searches and remaining substantially higher in the post housing bust period. If one takes the two searches as proxies for the behavior of existing owners, Figure 1 reveals a dependence of their transaction sequence decisions on the state of the housing market.[3] However, given equilibrium feedbacks, these decisions must in turn affect the housing market. Therefore, the decisions of existing owners may be important for housing market dynamics.
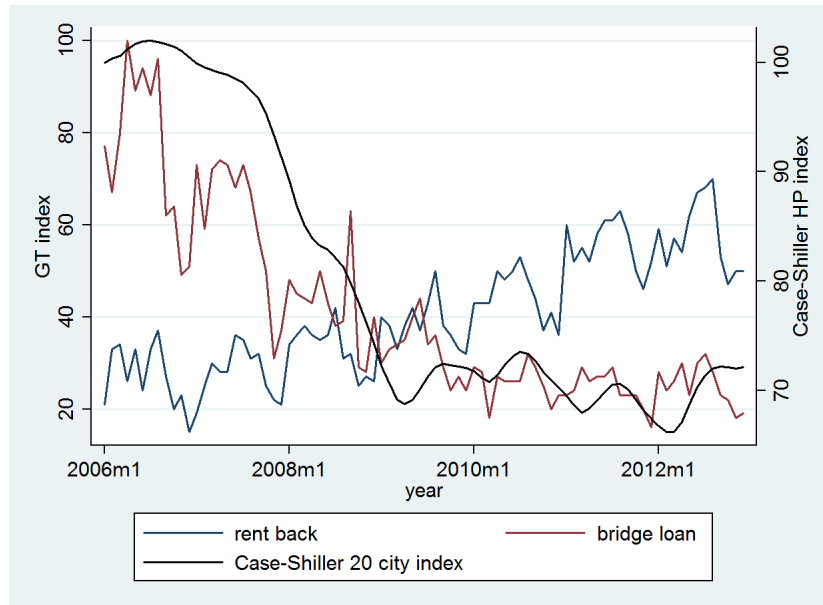
In this paper we examine theoretically this possibility in a tractable equilibrium model of a housing market, which explicitly features a transaction sequence decision for existing homeowners. In the model, agents continuously enter and exit a housing market with a fixed housing supply. Agents have a preference for owning housing over renting and consequently search for a housing unit to buy in a market characterized by a search-and-matching friction. The frictional trading process

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[1]For example, according to the CPS March supplement, on average, more than 7% of households moved within the same county in a year between 2000 and 2013. This constitutes around 60% of all moves in one year. Also, out of current homeowners, around 3% have moved within the same county in a year.

[2]The following quote from Realtor.com, an online real estate broker, highlights this issue: "If you sell first, you may find yourself under a tight deadline to find another house, or be forced in temporary quarters. If you buy first, you may be saddled with two mortgage payments for at least a couple months." (Dawson (2013))

[3]Røed Larsen (2014) provides more direct evidence on the response of homeowners' "buy first" or "sell first" decisions to housing market conditions using survey evidence for Norway. Additional anecdotal evidence from realtors in the U.S. points to a similar dependence. A common realtor advice to homeowners is to "buy first" in a "hot" market, when house prices are high or increasing and there are many buyers and few sellers, and "sell first" in a "cold" market, when house prices are falling or depressed and there are more sellers and few buyers.

Figure 1: Search trends for "bridge loan" and "rent back" and house prices

leads to a positive expected time on the market for both buyers and sellers, which is affected by the tightness in the market, the ratio of buyers to sellers. Once an agent becomes an owner, he may be hit by an idiosyncratic preference shock over his life cycle, which makes him dislike his current housing unit (the owner becomes mismatched). This induces existing owners to re-trade in the housing market. However, given a lack of double coincidence in housing preferences, a mismatched owner cannot simply exchange housing units with a counterparty. Instead, he must choose whether to buy the new unit first and then sell his old unit ("buy first"), or sell his old unit first and then buy ("sell first"). Given frictional search, this may lead to either owning two housing units or no housing for some time, respectively. The expected time of remaining in such a state depends on the market tightness.

In this standard setting, we show a simple condition, under which "buy first" is preferred to "sell first" whenever there are *more* buyers than sellers in the market, i.e. the market tightness is relatively high. The condition is a simple comparison of the flow disutility from remaining a mismatched owner for another instant and the flow disutility from having two units (or not owning a unit) for that instant. Whenever, the latter is more costly than the former, then mismatched owners prefer to "buy first" whenever there are more buyers than sellers, and consquently whenever other mismatched owners prefer to "buy first".

This behavior is intuitive when one considers how the expected time on the market for a buyer and a seller move with the ratio of buyers to sellers. Whenever there are more buyers than sellers, the expected time on the market for a seller is lower than that for a buyer. Consequently, if an

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owner chooses to "buy first" he expects to search longer for a housing unit to buy, and hence to remain mismatched longer. However, once he buys, he expects to wait less to find a buyer for his old property. Conversely, choosing to "sell first" in that case implies a short time of selling but a longer time of waiting to buy a new housing unit. If it is more costly to be left with two housing units (or to not own housing) than to be mismatched, then the decision to "buy first" naturally dominates the decision to "sell first".

As a result, under the simple condition of a higher disutility from owning two units (or no housing) compared to the disutility of being mismatched, there is a strategic complementarity in the decision of mismatched owners to "buy first" or "sell first". This in turn makes it possible for multiple steady state equilibria to exist. In one steady state equilibrium (a "buyers' market" equilibrium), mismatched owners prefer to "sell first", the market tightness is low and the expected time on the market for sellers is high. Therefore, the housing market is "illiquid" in the sense that it is harder to sell a housing unit.[4] In the other steady state equilibrium (a "sellers' market" equilibrium), mismatched owners prefer to "buy first", the market tightness is high and the expected time on the market for sellers is low.[5]

Next we show that this strategic complementarity, combined with a positive feedback from the market tightness to house prices, leads to dynamic equilibria with self-fulfilling fluctuations in prices and market liquidity.[6] We first show in a partial equilibrium setting that expectations about house price movements are important for the decision to "buy first" or "sell first". In particular, the decision to "buy first" or "sell first" exposes a mismatched owner to price risk, given the different exposure to housing that he would have at the intermediate stage when he owns two units or no units. For example, if an owner decides to "buy first" he essentially expects to be stuck with a long position in the housing market when he becomes an owner of two units. As a result an expected future house price depreciation (appreciation), biases a mismatched owner's decision towards choosing to "sell first" ("buy first"). This property of mismatched owners' decisions exerts a destabilizing force on house prices in the sense that mismatched owners prefer selling when house prices are expected to decline. If house prices respond negatively to decreases in market liquidity, this leads to further price declines.

This behavior is what makes self-fulfilling fluctuations possible. The fluctuations in such equilibria are driven purely by changes in agents' expectations about the future values of aggregate variables, which are in turn self-confirming. For example, the economy may currently be in a "sellers' market" regime with mismatched owners choosing to "buy first", a high market tightness and a higher price of housing. However, if agents begin to expect that a future reversal in the housing

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[4]The ease of selling is a natural measure of market liquidity in the housing market, since the seller side of the market is more easily observable compared to the buyer side.

[5]Note that we derive this multiplicity under an assumption of a constant returns to scale matching function. Therefore, the strategic complementarity does not arise from thick-market effects (Diamond (1982)).

[6]From a methodological point of view, such equilibrium fluctuations are very tractable to analyze as we show that they feature "simple" dynamics, in the sense that the payoff relevant state variable such as the market tightness, adjusts with a jump with dynamics only in non-payoff relevant stock variables. These "simple" dynamics are similar to the dynamics in the standard search-and-matching model of the labor market (Mortensen and Pissarides (1994), Menzio and Shi (2010)).

market is imminent, when the price of housing will be lower, they will start choosing to "sell first" instead. This change in behavior, however, drives down the market tightness and the house price, exactly confirming the agents' pessimism.

Importantly, this change in expectations (or regimes) occurs only with a low probability. Thus, the resulting dynamic equilibria feature a low frequency mean reversion in house prices and housing market liquidity. This low frequency mean reversion in housing market conditions (the fluctuations from "hot" to "cold" markets over low frequencies) is a key stylized fact about the behavior of housing markets (Krainer (2001), Gleaser, Gyourko, Morales, and Nathanson (2012)). Apart from this fact, and the motivation from Figure 1, our theoretical model is broadly consistent with other important facts about the housing market. In particular, equilibrium fluctuations in house prices are not driven by "fundamentals", such as rental rates or aggregate income (Shiller (2005), Campbell, Davis, Gallin, and Martin (2009)).[7] Also, house prices comove negatively with sellers' time on the market (Diaz and Jerez (2013)).[8]

Related Literature.   The paper is related to the growing literature on search-and-matching models of the housing market and fluctuations in housing market liquidity, initiated by the seminal work of Wheaton (1990).[9] This foundational paper is the first to consider a frictional model of the housing market to explain the existence of a "natural" vacancy rate in housing markets and the negative comovement between deviations from this natural rate and house prices. In that model, mismatched homeowners must also both buy and sell a housing unit. However, the model implicitly assumes that the cost of remaining with no housing is prohibitively large, so that mismatched owners always "buy first". As we show in this paper, allowing mismatched owners to endogenously chooses whether to "buy first" or "sell first" has important consequences for equilibrium behavior.

The paper is particularly related to the literature on search frictions and propagation and amplification of shocks in the housing market (Diaz and Jerez (2013), Head, Lloyd-Ellis, and Sun (forthcoming), Guren and McQuade (2013), and Anenberg and Bayer (2013)). This literature shows how search frictions naturally propagate aggregate shocks due to the slow adjustment in stock of buyers and sellers. Additionally, they can amplify price responses to aggregate shocks, which in Walrasian models would be fully absorbed by quantity responses.[10]

Diaz and Jerez (2013) calibrate a model of the housing market in the spirit of Wheaton (1990) where mismatched owners must "buy first" as well as a model where they must "sell first". They show that each model explains some aspects of the data on housing market dynamics pointing to the importance of a model that contains both. Other models of the housing market assume that the sequence of transactions are irrelevant, which implicitly assumes that the intermediate step of

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[7]Therefore, there is "excess volatility" in the model, in the sense of Shiller (1981).

[8]See Guren (2013) for a comprehensive list of key stylized facts about the housing market.

[9]It is hard to compile a fully exhaustive list of this large literature. Important recent contributions include Williams (1995), Krainer (2001), Novy-Marx (2009), Piazzesi and Schneider (2009), Ngai and Tenreyro (2012), Head and Lloyd-Ellis (2012), Diaz and Jerez (2013), Head, Lloyd-Ellis, and Sun (forthcoming), Anenberg and Bayer (2013), and Nenov, Larsen, and Sommervoll (2014) among others.

[10]The paper is also broadly related to the Walrasian literature on house price dynamics and volatility (Stein (1995), Ortalo-Magne and Rady (2006), Gleaser, Gyourko, Morales, and Nathanson (2012)).

a transaction for an existing owner is not costly (Ngai and Tenreyro (2012), Guren and McQuade (2013)).

Anenberg and Bayer (2013) and Maury and Tripier (2011) are two recent contributions that are closest to our paper, particularly, in terms of main motivation. The first paper studies a rich quantitative model of the housing market with two segments, in which some agents are sellers in the first segment, and simultaneously choose whether to also be buyers in the second segment. Shocks to the flow of new buyers in the first segment are transmitted and amplified onto the second segment through the decisions of these agents to participate as buyers in that second segment. Therefore, unlike our paper, there is no complementarity in the decisions of mismatched owners to transact given the market segmentation. As discussed above, the strategic complementarity in mismatched owners' actions is the main driver of multiplicity, self-fulfilling fluctuations, and volatility in our model. Also, in contrast to our model, buying-before-selling in that paper is a stochastic outcome rather than an endogenous choice.[11]

Maury and Tripier (2011) study a modification of the Wheaton (1990), in which the authors allow mismatched agents to buy and sell simultaneously. The authors use numerical examples to argue that whenever an equilibrium with simultaneous buying and selling exists in the Wheaton framework it Pareto dominates the "buy first" equilibrium in Wheaton (1990) and that it features equilibrium price dispersion. Similarly, to Anenberg and Bayer (2013) we complement this study by examining a tractable theoretical framework which clearly shows the incentives of angets to "buy first" or "sell first" and how these incentives interact with the state of housing markets and with agents' expectations regarding future market conditions.

The rest of the paper is organized as follows. Section 2 sets up the basic model of the housing market that we study. Section 3 contains the first main result of the paper, the condition under which mismatched owners' actions are strategic complements and shows that equilibrium multiplicity is possible in that situation. Section 4 contains the second main result, showing the existence of dynamic equilibria with self-fulfilling fluctuations in house prices. Section 5 includes extensions of the model, including allowing mismatched owners to simultaneously participate as buyers and sellers. Section 6 provides brief concluding remarks.

2 Basic Set-up

2.1 Agents, preferences and re-trading shocks

We start by setting up the basic model of a housing market characterized by trading frictions and re-trading shocks that will provide the main insights of our analysis. Time is continuous and runs forever, with $t \in [0, \infty)$. The housing market contains a unit measure of durable housing units that

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[11]The mechanism in Anenberg and Bayer (2013) is closer theoretically to the mechanism explored in Nenov (2014), in the context of liquidity provision by dealers in an over-the-counter market characterized by frictional trading, in the spirit of Duffie, Garleanu, and Pedersen (2005), Weill (2007), and Lagos, Rocheteau, and Weill (2011).

do not depreciate. In every instant there is a unit measure of agents in the economy.[12] Agents are risk neutral and discount the future at rate $r > 0$. They can borrow and lend without frictions at an interest rate of r.[13]

Agents in the economy derive a flow benefit from owning a housing unit. In particular, homeowners receive a flow utility of $u > 0$ in every instant that they are "matched" with the housing unit they reside in. However, a matched homeowner may become dissatisfied with the housing unit he owns, i.e. we say that he becomes "mismatched" with his current housing unit. This event occurs according to a Poisson process with rate γ. In that case the homeowner obtains a flow utility of $u - \chi$, for $0 < \chi < u$.

Note that taste shocks of this form are standard in search theoretic models of the housing market (Wheaton (1990)). They reflect a number of realistic events that take place over the life-cycle of a household, such as marriage or divorce, changes in household size that require moving to a housing unit of a different size, or job changes that require a move to reduce commuting distances. Such shocks create potential gains from trade for "mismatched" owners. Rather than introducing segmentation in the housing stock, we treat all housing units as homogenous, so that a "mismatched" owners participate in one integrated market with other agents.[14]

Upon becoming mismatched, the agent faces a set of choices, which we denote by $x \in \{0, b, s, bs\}$. First of all, he can choose not to enter the housing market and remain "passive" ($x = 0$). Alternatively, he can choose to enter the housing market as a "seller first" ($x = s$), selling his housing unit first and then buying a new one, or enter as a "buyer first" ($x = b$), buying a new housing unit first and then selling his old one. Importantly, we assume that the agent cannot simultaneously sell and buy a unit, whenever, for example, he meets another mismatched owner, that is, there is no double coincidence of housing needs among owners that want to switch houses.[15] Finally, a mismatched owner can choose to enter the housing market as a buyer and seller ($x = bs$). Note that this latter possibility does not imply that the agent can simultaneously sell and buy a house in the same instant in that case, only that he chooses to receive offers both from potential buyers and sellers.

We will focus on the case where mismatched owners' choices are restricted to the first three options $x \in \{0, b, s\}$, that is we assume that choosing $x = bs$ is prohibitively costly. The reason for this is to convey the main mechanisms in the model more clearly. We extend the analysis to the full choice set in Section 5.

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[12]One can think of this population size as arising from a combination of labor market conditions and limited available housing, which we abstract from in the model. There are alternative set-ups of the model that will lead to the same results as the ones we present here. For example, one can consider a model that features constant population growth and exogenous housing construction, so that the economy is on a balanced growth path.

[13]Therefore, we are dealing with a small open economy with interest rate equal to the rate of time preferences of agents. This appears a reasonable assumption when considering a local housing market.

[14]Although, in reality agents move across housing market segments (whether geographic or unit size-based) in response to a taste shock of the type we have in mind, modeling explicitly several types of housing would substantially reduce the tractability of the model. Furthermore, defining empirically distinct market segments is not straightforward as in reality households often search in several segments simultaneously (Piazzesi, Schneider, and Stroebel (2013)).

[15]This is similar to the lack of double coincidence of needs used in money-search models (Kiyotaki and Wright (1993)).

We assume that participating in the housing market is costly, with agents that choose to participate incurring a flow cost of $k \geq 0$. This creates some opportunity cost of transacting so that choosing $x = 0$ need not be a dominated action. One can think of this as a transaction cost that sellers and buyers incur, for example, by paying real estate brokers to search for counter-parties on their behalf.

A mismatched owner who chooses to be a "buyer first" may end up holding two housing units simultaneously for some period. Similarly, choosing to be a "seller first" may result in owning no housing unit. We assume that in the former case, an owner of two housing units receives a flow utility of $0 \leq u_2 < u$ and in the latter case, the non-owner receives a flow utility of $0 \leq u_0 < u$. Both of these reflect non-pecuniary costs, such as maintenance costs in the former case, or restrictions on the use of the rental property imposed by a landlord in the latter case.[16]

We assume that in each instant a measure g of new agents are born and enter the housing market. They start out their life without owning housing and may choose to become homeowners and derive homeownership benefits. New non-owners receive the same flow utility as old non-owners. Therefore, since agents' utilities are time invariant, there is no heterogeneity between new and old non-owners. To keep population constant over time, we assume that all agents in the economy suffer a death/exit shock with Poisson rate g. Upon such a shock, an agent exits the economy immediately and obtains a reservation utility normalized to 0. If they own housing, their housing units are taken over by a real estate firm, which immediately places them on the market for sale.[17] Real estate firms are owned by the agents of the economy. Note that given the exit shock, agents will effectively discount future flow payoffs at a rate $\rho \equiv r + g$. For notational convenience, we will directly use ρ later on. Also, we assume that agents are free to exit the economy in every instant and obtain their reservation utility of 0.

Finally, we assume that there exists a frictionless rental market with a rental rate of R. Non-owners rent a housing unit in the rental market in any given instant they do not own housing. Similarly, owners with two units can rent out one of their units, as do real estate firms. For simplicity, we assume that there is no opportunity cost to renting out a vacant unit, and agents and real estate firms can simultaneously rent out a unit and have it up for sale. Free exit from the economy by non-owners and a zero opportunity cost for renting out a unit imply that the equilibrium rental rate can take multiple values. In particular, if $\mathcal{R}$ is the set of possible equilibrium rental rates, we have that $[0, u_0] \subset \mathcal{R}$.[18] We will consider equilibrium rental rates in the set $[0, u_0]$.

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[16]For simplicity we also assume that an owner of two housing units does not experience mismatching shocks. This ensures that the maximum holdings of housing by an agent will not exceed two units in equilibrium.

[17]For simplicity, we assume that agents are not compensated for their housing upon exiting the economy. We extend our results in Section 5.3 to a case where exiting agents are compensated for their housing by real estate firms. Also, as a technical assumption, we assume that real estate firms do not incur the flow cost k from participating in the market.

[18]The equilibrium rental rate R may be higher than u_0 because of the additional value from homeownership that a non-owner anticipates.

2.2 Trading Frictions and Aggregate Variables

The inherent heterogeneity in the housing stock and agents' preferences naturally lead to the assumptions that the housing market is subject to trading frictions, and that there is no immediacy in housing transactions. To capture these trading frictions in a reduced-form way, we follow the vast literature on search-and-matching models. In particular, the frictional process of matching buyers and sellers of housing units in the housing market is summarized by a standard constant returns to scale matching function $m(B(t), S(t))$, where $B(t)$ and $S(t)$ is the measure of buyers and sellers in a given instant t, respectively, and which gives the (rate of) successful meetings of buyer and sellers in the housing market in a given instant. Furthermore, there is no directed search (Moen (1997)), and meetings are random, so different types of agents meet with probabilities that are proportional to their mass in the population of sellers or buyers. We naturally define the market tightness in the housing market as the buyer-seller ratio, $\theta(t) \equiv \frac{B(t)}{S(t)}$. Additionally, $\mu(\theta(t)) \equiv m\left(\frac{B(t)}{S(t)}, 1\right) = \frac{m(B(t), S(t))}{S(t)}$ is defined as the Poisson rate with which a seller successfully transacts with a buyer. Similarly, $q(\theta(t)) \equiv \frac{m(B(t), S(t))}{B(t)} = \frac{\mu(\theta(t))}{\theta(t)}$ is the rate with which a buyer meets a seller and transacts.

Beside the market tightness $\theta(t)$, which will be relevant for agents' equilibrium payoffs, we keep track of the following aggregate stock variables.

- $B_0(t)$ - measure of non-owners;

- $B_1(t)$ - measure of mismatched owners who choose to be "buyers first";

- $S_1(t)$ - measure of mismatched owners who chooses to be "sellers first";

- $S_2(t)$ - measure of owners with two housing units;

- $O(t)$ - measure of matched owners;

- $O_m(t)$ - measure of mismatched owners who choose to be "passive";

- $A(t)$ - measure of housing units that are sold by real-estate firms;

Therefore, the total measure of buyers is $B(t) = B_0(t) + B_1(t)$ and the total measure of sellers is $S(t) = S_1(t) + S_2(t) + A(t)$. Also, since the total population is assumed to be constant and equal to 1 in every instant, it follows that

$$B_0 + B_1 + S_1 + S_2 + O + O_m = 1 \tag{1}$$

Finally, since the housing stock does not shrink or expand over time, the following housing ownership condition must hold in every instant,

$$B_1 + S_1 + O + O_m + A + 2S_2 = 1. \tag{2}$$
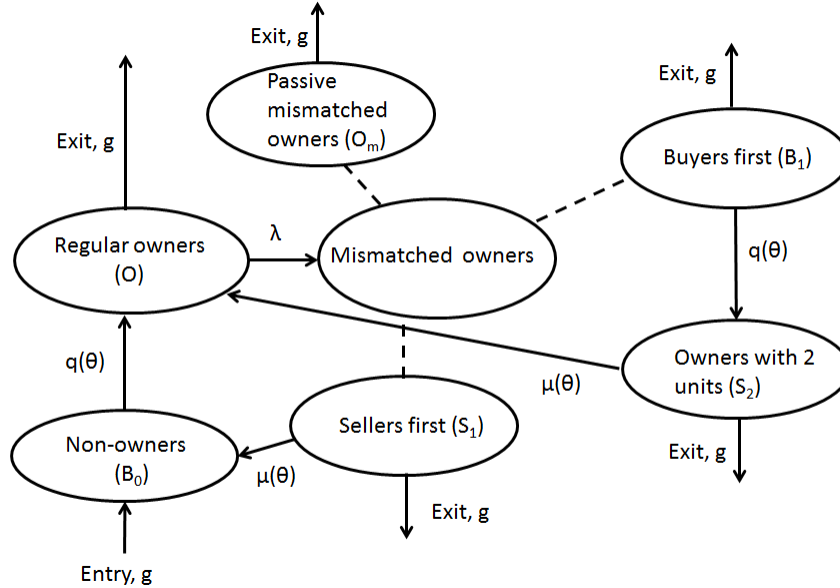
Figure 2: Housing market transitions

Figure 2 below summarizes the agent flows across different types. Agents begin their life as non-owners. With rate $q(\theta)$, they become regular owners. Regular owners become mismatched with rate λ. Once mismatched, they can choose to either remain "passive", become a "buyer first" or a "seller first". A "buyer first" becomes an owner of two units with rate $q(\theta)$, who in turn manages to sell one of the units and reverts to being a regular owner at rate $\mu(\theta)$. A "seller first" sells his unit at rate $\mu(\theta)$ and becomes a non-owner. In every stage of life an agent can exit the economy at rate g.

2.3   House price determination

We will conduct most of our analysis by assuming that the house price p is fixed and does not vary with the market tightness θ. However, similarly to the literature on rigid wages in search-and-matching models (Hall (2005), Gertler and Trigari (2009)), the price p does not violate the individual rationality of any agent in the economy that is a counterparty to a transaction.

Given that the price does not violate agent individual rationality it can be considered as the market clearing price in a competitive market with frictional entry of counterparties. In particular, similarly to Rocheteau and Wright (2005), the total measure of participants in that competitive market is determined by the matching function $M(B, S)$ with buyers and sellers of different types entering according to their fractions in the population of buyers and sellers, respectively. Once in the market, there is anonymity between buyers and sellers of different types, and the buyers and sellers take the transaction price as given. The transaction price leaves all counterparties (weakly) better off from transacting at that price compared to not transacting.[19] However, given the limited

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[19]This is the sense in which the price does not violate agent individual rationality.

heterogeneity of agents, there are generically many market clearing prices that leave agents (weakly) better off from transacting, so the equilibrium price is not pinned down in that case, which generates some indeterminacy in the price level. We resolve this indeterminacy by selecting a particular price p from the set of market clearing prices and examining equilibria in that case.

Therefore, in our framework search frictions will only affect the probability that agents transact in a given time interval but will not have an effect on the transaction prices. However, as we show below, even with this restriction of the effects of search frictions there are important implications for the equilibrium behavior of agents.

3 Steady State Equilibria

We first consider steady state equilibria of this economy. Informally, in a steady state equilibrium, agents (most importantly mismatched agents) make choices that maximize their discounted payoffs given the market tightness θ, the market tightness θ is constant over time, and so are the stocks of agents of different types, which are determined by a system of flow conditions that reflect agents' optimal actions, and finally, the house price, p, is such that it is individually rational for all agents to transact.[20] Similarly, agents' expected utility is constant over time. We will first discuss the value functions of different types of agents. A complete definition of a steady state equilibrium of this economy given these value functions and some parametric restrictions can be found in the Appendix.

3.1 Value functions

Given the heterogeneity over agent types, there is a number of value functions to consider. We start by introducing the notation for the steady state value functions of different agents in the economy. We have:

- V^{B0}- value function of a non-owner;

- V^{B1} - value function of an owner who is a "buyer first";

- V^{S1} - value function of an owner who is a "seller first";

- V^{S2} - value function of an owner of two housing units;

- V - value function of matched owner;

- V^m - value function of a mis-matched owner who is "passive";

- V^A - value function of a real-estate firm that holds one housing unit;

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[20]Also the equilibrium rental rate R is constant over time.

Given these notations, we have a standard set of Bellman equations for the agents' value functions in a steady state equilibrium.[21]

First of all, for a non-owner we have that:

$$\rho V^{B0} = u_0 - R - k + q(\theta)\left(-p + V - V^{B0}\right), \tag{3}$$

where the flow term $u_0 - R - k$ reflects the flow utility from being a non-owner net of the rental cost and housing market participation cost k. With rate $q(\theta)$, a non-owner is successfully matched with a seller in which case he transacts with the seller, paying a price p and switches to a matched owner, thus incurring a utility increase of $V - V^{B0}$.[22] Similarly, the value function of a "buyer first" satisfies the equation:

$$\rho V^{B1} = u - \chi - k + q(\theta)\left(-p + V^{S2} - V^{B1}\right) \tag{4}$$

where the flow term $u - \chi - k$ reflects the flow utility from being mismatched net of the housing market participation cost k. Similarly to the case of a non-owner, upon matching with a seller, a "buyer first" purchases a housing unit at price p, in which case he becomes an owner of two housing units, incurring a utility change of $V^{S2} - V^{B1}$.

An owner of two housing units incurs a flow utility of $u_2 + R - k$, while searching for a counterparty. Upon finding a buyer, he sells his second unit and becomes a matched owner. Therefore, his value function satisfies the equation:[23]

$$\rho V^{S2} = u_2 + R - k + \mu(\theta)\left(p + V - V^{S2}\right) \tag{5}$$

The value function of a "seller first" is analogous to that of a "buyer first" apart from the fact that a "seller first" enters on "the other side" of the housing market and upon transacting becomes a non-onwer. Hence, we have:

$$\rho V^{S1} = u - \chi - k + \mu(\theta)\left(p + V^{B0} - V^{S1}\right) \tag{6}$$

Finally, a mismatched owner who remains passive has a straightforward value function satisfying:

$$\rho V^m = u - \chi \tag{7}$$

A mismatched owner does not incur the market participation cost k unlike a "buyer first" or a "seller first". The remaining value functions are straightforward and are given in the Appendix.

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[21]Note that we will abstract from steady state equilibria, in which a mismatched owners that is indifferent between some action mixes over these actions over time. This restriction is without loss of generality.

[22]Note that we assume that in every steady state equilibrium non-owners strictly prefer to own a unit of housing, or $V - p \geq \frac{u_0 - R}{\rho}$, where the right-hand side is the utility from remaining a non-owner forever. The Appendix provides a sufficient condition for this to hold.

[23]Note that similarly to the case of a non-owner, we require that in every steady state equilibrium, $V + p \geq \frac{u_2 + R}{\rho}$.

It is important to note that in any steady state equilibrium

$$\rho p \geq R \tag{8}$$

The rason for this is that the house price cannot be lower than the present discounted value of rental income, since otherwise real estate agents would not find it individually rational to sell housing. However, the condition can hold with a strict inequality. The reason for this is that the search-and-matching frictions create a positive match surplus, so potential buyers of housing are willing to accept a price higher than the present discounted value of rental rates. In fact non-owners are willing to accept a price as high as $p = V - \frac{u_0}{\rho} + \frac{R}{\rho} > \frac{R}{\rho}$, since the value of homeownership, V, is higher than the value of remaining a non-owner in any steady state equilibrium that we consider.

3.2 Characterizing the Decision of a Mismatched Owner

In a steady state equilibrium, the optimal decision of mismatched owners depends on the comparison

$$V^m \gtreqless \max\left\{V^{B1}, V^{S1}\right\} \tag{9}$$

Condition (9) can be thought of as an entry condition where mismatched agents have an opportunity cost V^m to enter the housing market and transact. Note that if the condition holds with equality, then in equilibrium mismatched owners are indifferent between remaining "passive" and entering the market, so the equilibrium market tightness θ will reflect this indifference and will be pinned down by it. In the case that the condition does not hold with equality, then market tightness θ will be pinned down by a set of flow equations. We postpone the discussion about the various possible equilibrium configurations to Sections 3.3 and 3.4 and first consider the right-hand side of condition (9).

We can substitute for V^{B0} and V^{S2} from equations (3) and (5) into the value functions for a "buyer first" and "seller first", V^{B1} and V^{S1} to obtain:

$$V^{B1} = \frac{u - \chi - k}{\rho + q(\theta)} + \frac{q(\theta)(u_2 - k - (\rho p - R))}{(\rho + \mu(\theta))(\rho + q(\theta))} + \frac{q(\theta)\mu(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))}V \tag{10}$$

and

$$V^{S1} = \frac{u - \chi - k}{\rho + \mu(\theta)} + \frac{\mu(\theta)(u_0 - k + (\rho p - R))}{(\rho + \mu(\theta))(\rho + q(\theta))} + \frac{q(\theta)\mu(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))}V \tag{11}$$

There are several important observations to be made. First, even though the flow utility from ending with two housing units is u_2, the effective utility flow is $u_2 - (\rho p - R)$, and similarly the effective utility flow from ending as a non-owner is $u_0 + (\rho p - R)$. Therefore, even if the non-pecuniary utility flows, u_0 and u_2, are equal it is still (weakly) more costly to end with two housing units than as a non-owner. The reason is that an owner with two units faces a potentially lower rental income than the user cost of owning a housing unit, while a non-owner benefits from this possibility. Therefore, even with frictionless financing, and a frictionless rental market, an environment with search-and-matching frictions may make owning two units more costly than being a non-owner.

Hence, we define the effective utility flows from remaining a non-owner versus an owner with two units as $\tilde{u}_0 \equiv u_0 + \triangle$, and $\tilde{u}_2 \equiv u_2 - \triangle$, respectively, where $\triangle \equiv \rho p - R$ is the "ownership premium" that an agent who owns a housing unit must pay relative to renting. Whenever $\rho p = R$, then the ownership premium is zero. Also, if $u_0 = u_2 - 2\triangle$, then $\tilde{u}_0 = \tilde{u}_2$, so the effective utility flow from owning two units versus remaining a non-owner is the same. This particular case will serve as an important benchmark.

A second important observation is that search-and-matching frictions may also affect the value of "selling first" versus "buying first" through the expected time on the market for a buyer and a seller, $\frac{1}{q(\theta)}$ and $\frac{1}{\mu(\theta)}$. To see this, consider the difference $D(\theta) \equiv V^{B1} - V^{S1}$, which gives the bias of a mismatched agent towards choosing to enter as "buyer first" versus "seller first" given θ. We have that

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))} \left[\left(1 - \frac{1}{\theta} \right) (u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 \right] \tag{12}$$

In the benchmark case, where $\tilde{u}_0 = \tilde{u}_2 = c$, equation (12) simplifies to

$$D(\theta) = \frac{(\mu(\theta) - q(\theta))(u - \chi - c)}{(\rho + q(\theta))(\rho + \mu(\theta))} \tag{13}$$

In the limiting case where the effective discount rate is small, $\rho \to 0$, we have that

$$D(\theta) = \left(\frac{1}{q(\theta)} - \frac{1}{\mu(\theta)} \right) (u - \chi - c) \tag{14}$$

Therefore, the value of being a "buyer first" versus a "seller first" depends on the difference in the expected time on the market for a buyer versus a seller, $\frac{1}{q(\theta)} - \frac{1}{\mu(\theta)}$. Furthermore, if the utility flow from being mismatched is higher than the utility flow from being an owner of two units or a non owner, so $u - \chi > c$, then the value of being a "buyer first" is higher than the value of being a "seller first" if the expected time on the market for *buyers* is higher than the expected time on the market for *sellers*.

The behavior of mismatched owners seems at first counter-intuitive. After all, if the expected time on the market for a buyer is longer than that for a seller, why would entering as a "buyer first" be preferred to entering as a "seller first". The reason for the counter-intuitive behavior is that a mismatched owner has to undergo two transactions on both sides of the market before he becomes a regular owner. If it is more costly to remain with two units or with no units than to remain mismatched, then a mismatched owner would care more about the expected time on the market for the second transaction.

In particular, consider the schematic representation of a mismatched owner's expected payoffs in Figure 3 in the cases when he chooses to be a "buyer first" and a "seller first" and $\theta < 1$. If the agent enters as a "buyer first", he has a short expected time on the market as a buyer. However, he anticipates a long expected time on the market in the next stage when he owns two units and has to dispose of his old housing unit. In contrast, entering as a "seller first" implies a long expected

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Figure 3: Buying first versus selling first when $\theta < 1$.
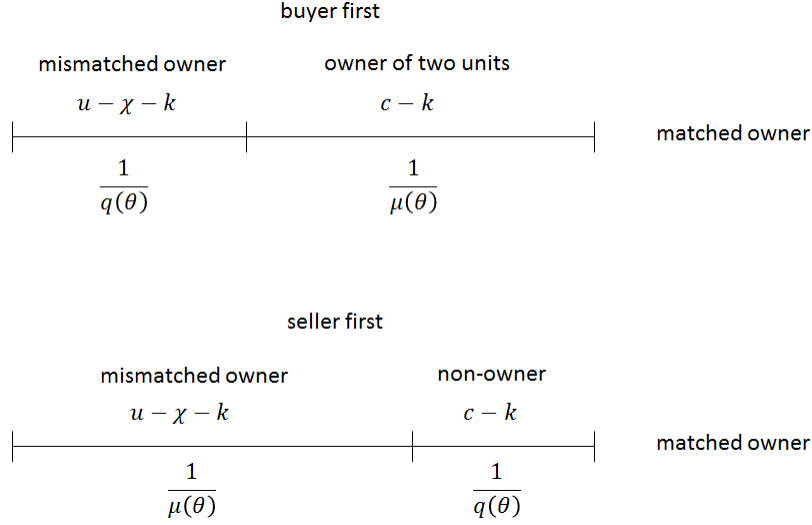
time on the market until the agent sells his property but a short time on the market when the agent is a non-owner and has to buy a new property. In the case where $u - \chi > c$, it is more costly to be stuck in the second stage for a long time (as an owner of two units or non-owner) rather than to remain mismatched and searching.

Therefore, being a "buyer first" is strictly preferred to being a "seller first", whenever $\theta > 1$. Note that θ is the buyer-seller ratio in the housing market, so it is increasing in the number of buyers that enter the market and decreasing in the number of sellers that enter the market. This behavior creates a form of strategic complementarity in mismatched owners' actions, which in turn leads to multiple steady state equilibria, as we show below.

The same insight applies away from the limit $\rho \to 0$. In particular, we have the following:

Lemma 1. *Suppose that $u - \chi > c$. Then, $V^{B1} > V^{S1} \iff \theta > 1$.*

Proof. Follows directly from a comparison of the sign of $D(\theta)$ in the limit as $\rho \to 0$. $\qquad\square$

Is the assumption that the utility flow from being a mismatched owner is higher than the utility flow from being a non-owner or the utility flow from owning two housing units reasonable? Anecdotal evidence points to being mismatched with ones home as not a particularly costly state for the majority of homeowners. In rare instances is the alternative of a household having to permanently reside in an owned property, which they are not fully satisfied with, worse than a situation, in which households are forced to permanently rent (despite preferring to own) or to permanently own two housing units. Therefore, our analysis focuses on this arguably more empirically relevant and realistic case, as we summarize in the following parametric restriction:[24]

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[24]This restriction is necessary for equilibrium multiplicity. One can show that if this restriction does not hold, then there is a unique steady state equilibrium only.

Assumption A1: $u - \chi \geq \max\{\tilde{u}_0, \tilde{u}_2\}$.

This assumption implies that the effective utility flow from owning two units $\tilde{u}_2$ is always lower than the utility flow from being a mismatched owner.

In the more general case when $\tilde{u}_0$ and $\tilde{u}_2$ are not equal, we can still use equation (12) to compare V^{B1} and V^{S1}. We define

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2}{u - \chi - \tilde{u}_0}. \tag{15}$$

Note that if $\tilde{u}_2 > \tilde{u}_0$, then $\tilde{\theta} < 1$ and vice versa if $\tilde{u}_2 < \tilde{u}_0$. Additionally, we observe that:

Lemma 2. $V^{B1} > V^{S1} \iff \theta > \tilde{\theta}$ and $V^{B1} = V^{S1} \iff \theta = \tilde{\theta}$.

Proof. See Appendix. □

Therefore, asymmetry in the flow values from being a non-owner versus an owner with two units, moves the value of the market tightness, θ, at which a mismatched agent is indifferent between buying first and selling first, away from $\theta = 1$. For example, if the effective flow utility from being a non-owner is lower relative to the effective flow utility from being an owner with two units, then at a market tightness $\theta = 1$, a mismatched owner is strictly better off buying first rather than selling first.[25]

In what follows we will characterize equilibria under the following condition on model primitives:

Assumption A2: $\frac{u-\chi}{\rho} < \frac{u-\chi-k}{\rho+\mu(1)} + \frac{\mu(1)}{(\rho+\mu(1))^2} \max\{\tilde{u}_0, \tilde{u}_2\} + \frac{\mu(1)^2}{(\rho+\mu(1))^2}\left(\frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}\right)$.

Assumption A2 is a necessary and sufficient condition for the non-existence of steady state equilibria, in which mismatched owners strictly prefer to remain passive. Although the existence of such equilibria is possible (for example, for a sufficiently high value of the market participation cost k), they are not particularly interesting either theoretically or empirically. Therefore, under A2, condition (9) has a clear sign for the inequality with $V^m \leq \max\{V^{B1}, V^{S1}\}$ in any equilibrium.[26]

3.3 Equilibria under symmetry $(\tilde{u}_0 = \tilde{u}_2)$

First of all, note that there always exists a steady state equilibrium with $\theta = 1$, in which mismatched owners are indifferent between "buying first" and "selling first". This is straightforward to see from Lemma 2 and from noting that the flow conditions for the aggregate stock variables (35) through (41) are satisfied given $\theta = 1$ and given the actions of mismatched owners. We summarize this implication in the following:

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[25] Apart from these results, Lemma 13 in Appendix contains a set of auxiliary results about agents' value functions that are necessary for equilibrium characterization for the case where $\tilde{\theta}$ is finite and positive (i.e. $u - \chi > \max\{\tilde{u}_0, \tilde{u}_2\}$).

[26] Also we will focus on a sufficiently small value of γ, so that both V^{B0} and V^{S2} are monotone in θ, and V^{B1} and V^{S1} will have a unique local maximizer, which is also a global maximizer. This particular restriction reduces the number of possible equilibria.

Proposition 3. *Consider the above economy and suppose that $\tilde{u}_0 = \tilde{u}_2 = c$. Then there exists a steady state equilibrium with $\theta = 1$. In that equilibrium mismatched owners are indifferent between entering as a "buyer first" and a "seller first".*

Proof. See Appendix. □

Besides the symmetric equilibrium with $\theta = 1$ there are several other possible equilibria, which involve steady state values of θ below or above $\theta = 1$. To characterize these equilibria, we define several important objects. First of all, we denote by $\bar{\theta}$ the solution to the equation:

$$\left(\frac{1}{q(\theta) + g} + \frac{1}{\gamma} \right) \theta + \left(\frac{1}{q(\theta) + g} - \frac{1}{\mu(\theta) + g} \right) = \frac{1}{g} + \frac{1}{\gamma} \tag{16}$$

and by $\underline{\theta}$, the solution to the equation:

$$\left(\frac{1}{\mu(\theta) + g} + \frac{1}{\gamma} \right) \frac{1}{\theta} = \frac{1}{g} + \frac{1}{\gamma} \tag{17}$$

These two equations arise from the flow conditions and population and housing conditions if all mismatched agents enter as "buyers first" and "sellers first", respectively. Importantly, as we show in Lemma 14 in the Appendix, the two equations have unique solutions with $\bar{\theta} > 1$ and $\underline{\theta} < 1$, with $\bar{\theta}$ increasing in γ and $\underline{\theta}$ decreasing in γ.

Secondly, we denote by θ^S the smallest solution to the equation:

$$\frac{u - \chi}{\rho} = \frac{u - \chi - k}{\rho + \mu(\theta)} + \frac{\mu(\theta)(\tilde{u}_0 - k)}{(\rho + \mu(\theta))(\rho + q(\theta))} +$$
$$+ \frac{\mu(\theta) q(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))} \left(\frac{u}{\rho} - \frac{\gamma}{\rho + \gamma} \frac{\chi}{\rho} \right) \tag{18}$$

and by θ^B the largest solution to the equation:

$$\frac{u - \chi}{\rho} = \frac{u - \chi - k}{\rho + q(\theta)} + \frac{q(\theta)(\tilde{u}_2 - k)}{(\rho + \mu(\theta))(\rho + q(\theta))} +$$
$$+ \frac{\mu(\theta) q(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))} \left(\frac{u}{\rho} - \frac{\gamma}{\rho + \gamma} \frac{\chi}{\rho} \right) \tag{19}$$

Note that θ^S is the smallest value of θ, which guarantees that mismatched owners are indifferent between remaining "passive" and entering as "sellers first" and similarly, θ^B is the largest value of θ, which guarantees that mismatched owners are indifferent between remaining "passive" and entering as "buyers first". Also, note that given condition A2 above, and given Lemma 13 in the Appendix, the two equations, (18) and (19), have a solution, so θ^S and θ^B exist, and also, $\theta^S < 1$ and $\theta^B > 1$.

Given these notations, we have the following important result.

Proposition 4. *Consider the above economy and suppose that $\tilde{u}_0 = \tilde{u}_2 = c$. Then there exists a steady state equilibrium with $\theta = \max\{\underline{\theta}, \theta^S\}$, in which mismatched owners prefer "selling first" to "buying first" whenever they choose to enter the housing market. There also exists a steady state*

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equilibrium with $\theta = \min\left\{\overline{\theta}, \theta^B\right\}$, in which mismatched owners prefer "buying first" to "selling first" whenever they choose to enter the housing market.

Proof. See Appendix. □

Therefore, Proposition 4 makes clear that there can be multiple steady state equilibria. In one steady state equilibrium mismatched owners are strictly better off entering as "sellers first" rather than "buyers first", even though the equilibrium market tightness $\theta < 1$, so that there are more sellers than buyers in the market. Conversely, in the other equilibrium mismatched owners are better off entering as "buyers first" rather than "sellers first", even though the equilibrium market tightness $\theta > 1$, so that there are more buyers than sellers in the market. This equilibrium behavior follows directly from the discussion in Section 3.2. To reiterate, since remaining without a housing unit or with two housing units is more costly than being mismatched and searching, mismatched agents want to minimize their expected time with no housing unit or with two housing units. This makes them prefer to enter as sellers (buyers) when the market tightness is low (high), reinforcing the low (high) ratio of buyers to sellers.

Given the steady state value of θ in the two steady state equilibria, we call the equilibrium with $\theta < 1$ a "Buyers' market" equilibrium, and the one with $\theta > 1$ a "Sellers' market" equilibrium. In the former, the expected time on the market is lower for buyers than for sellers and vice versa for the latter. Note again that in a "Buyers' market" equilibrium mismatched owners prefer to be "sellers first", while in a "Sellers' market" equilibrium mismatched owners prefer to be "buyers first". Also, note that depending on how $\underline{\theta}$ and θ^S compare, in the "Buyers' market" equilibrium mismatched agents are either strictly better off from participating in the market or indifferent between participating and remaining passive and similarly for the "Sellers' market" equilibrium.

Figure 4 illustrates this equilibrium multiplicity and the equilibrium value functions of mismatched owners for the case when $\underline{\theta} < \theta^S$ and $\overline{\theta} > \theta^B$ (Figure 4a) and $\underline{\theta} > \theta^S$ and $\overline{\theta} < \theta^B$ (Figure 4b). Since remaining passive dominates housing market participation if $\theta < \theta^S$ or $\theta > \theta^B$, it follows that in a steady state equilibrium, θ must lie in the set $\left[\theta^S, \theta^B\right]$. If $\underline{\theta} \in \left[\theta^S, \theta^B\right]$, then in a "Buyers' market" equilibrium $\theta = \underline{\theta}$, since at $\theta = \theta^S < \underline{\theta}$, the equilibrium flow conditions for aggregate stock variables fail to be satisfied. Similarly, if $\overline{\theta} \in \left[\theta^S, \theta^B\right]$, then in a "Sellers' market" equilibrium $\theta = \overline{\theta}$.

Figure 4 also shows the steady state equilibrium, in which $\theta = 1$ and mismatched agents are indifferent between "buying first" and "selling first" as shown in Proposition 3. However, this steady state equilibrium is unstable in the following sense: A small perturbation in θ around the equilibrium value of $\theta = 1$ will make mismatched agents either strictly better off from entering as "buyers first" or "sellers first", driving the value of θ away from $\theta = 1$ and towards $\min\left\{\underline{\theta}, \theta^S\right\}$ or $\max\left\{\overline{\theta}, \theta^B\right\}$, respectively. Therefore, if V^{B1} and V^{S1} have unique maxima, the "Buyers' market" and "Sellers' market" equilibria are the only stable steady state equilibria.

Figure 4: Equilibrium multiplicity with $\underline{\theta} < \theta^S$ and $\overline{\theta} > \theta^B$ (a) and $\underline{\theta} < \theta^S$ and $\overline{\theta} > \theta^B$ (b).
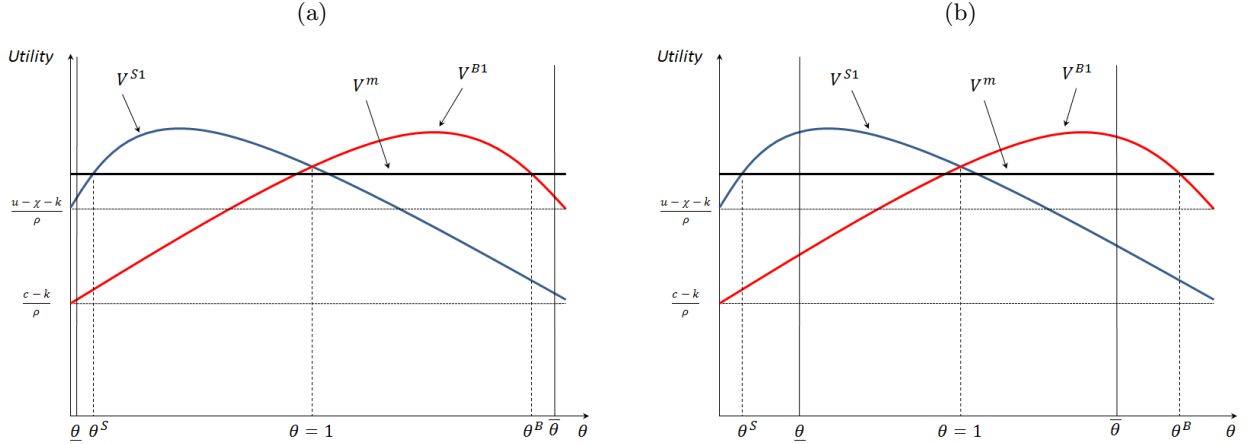

3.4 Asymmetric Equilibria ($\tilde{u}_0 \neq \tilde{u}_2$)

The results of Section 3.3 carry over for the case when the flow payoffs $\tilde{u}_0$ and $\tilde{u}_2$ are not equal to each other. In particular, there are still at most three equilibria, one in which mismatched owners enter as "buyer first" and "seller first", and two, in which they enter as either one or the other. However, if the payoff asymmetry is sufficiently strong, there will be a unique equilibrium. In particular, if $\tilde{u}_0$ is sufficiently low compared to $\tilde{u}_2$, there is a unique equilibrium in which mismatched owners enter as a "buyer first" and vice versa when $\tilde{u}_2$ is sufficiently low compared to $\tilde{u}_0$. Whether, there is equilibrium uniqueness or multiplicity depends on a comparison of the value of $\tilde{\theta}$, defined in condition (15) above, against the steady state equilibrium values of θ defined in conditions (16), (17), (18), and (19). We summarize the equilibrium characterization in this case in the following result.

Proposition 5. *Consider the above economy and suppose that $\tilde{u}_0 \neq \tilde{u}_2$. Let $\underline{\theta}$, $\overline{\theta}$, θ^S, and θ^B be defined by (16), (17), (18), and (19).*

1. *Suppose that $\tilde{\theta}$, defined as in condition (15) lies in the set $\left[\max\left\{ \underline{\theta}, \theta^S \right\}, \min\left\{ \overline{\theta}, \theta^B \right\} \right]$. Then there exist three steady state equilibria of this economy. In the first mismatched owners prefer "selling first" to "buying first" whenever they choose to enter the housing market. In the second mismatched owners prefer "buying first" to "selling first" whenever they choose to enter the housing market, and in the third mismatched owners are indifferent between entering as "buyers first" and "sellers first" so that the steady state value of $\theta = \tilde{\theta}$;*

2. *Suppose that $\tilde{\theta} < \max\left\{ \underline{\theta}, \theta^S \right\}$. Then there exists a unique steady state equilibrium, in which mismatched owners prefer "buying first" to "selling first" whenever they choose to enter the housing market;*

3. *Suppose that $\tilde{\theta} > \min\left\{ \overline{\theta}, \theta^B \right\}$. Then there exists a unique steady state equilibrium, in which mismatched owners "selling first" to "buying first" whenever they choose to enter the housing*

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Figure 5: Equilibria in the case of $\tilde{\theta} > \min\{\overline{\theta}, \theta^B\}$ (a) and $\tilde{\theta} < \max\{\underline{\theta}, \theta^S\}$ (b).
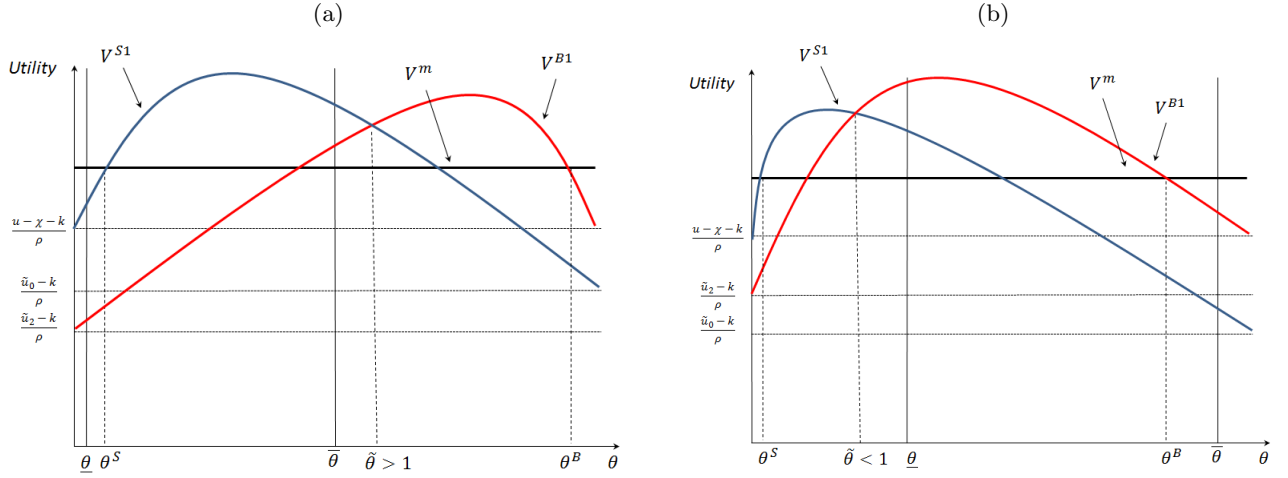
market.

Proof. See Appendix. □

Proposition 5 shows that the equilibrium multiplicity shown in the case where $\tilde{u}_0 = \tilde{u}_2$ holds under asymmetry with one important distinction. The difference between flow payoffs from owning no housing unit relative to owning two housing units can lead to equilibrium uniqueness. Figure 5 shows this particular possibility. Payoff asymmetry shifts the value of θ, $\tilde{\theta}$, for which an agent is indifferent between "buying first" and "selling first" away from $\theta = 1$. In particular, if $\tilde{u}_0 > \tilde{u}_2$, then $\tilde{\theta} > 1$ and vice versa for $\tilde{u}_0 < \tilde{u}_2$. Therefore, if the payoff asymmetry is sufficiently large, so that $\tilde{\theta} > \min\{\overline{\theta}, \theta^B\}$ or $\tilde{\theta} < \max\{\underline{\theta}, \theta^S\}$, then some of the equilibria that exist under $\tilde{u}_0 = \tilde{u}_2$ cease to exist in that case. For example, Figure 5a shows the case where $\tilde{\theta} > \min\{\overline{\theta}, \theta^B\} > 1$. In that case only the "buyers' market equilibrium" with $\theta = \max\{\underline{\theta}, \theta^S\}$ exists. Similarly, Figure 5b shows the case where $\tilde{\theta} < \max\{\underline{\theta}, \theta^S\} < 1$. In that case only the "sellers' market equilibrium" with $\theta = \min\{\overline{\theta}, \theta^B\}$ (or the "sellers' market equilibrium" in the case where $\overline{\theta} < \theta^B$) will exist.

Therefore, sufficiently strong payoff asymmetry between owning no housing units and owning two housing units can lead to equilibrium uniqueness.

3.5   Equilibrium transitions

Proposition 4 and 5 showed that multiple steady state equilibria are possible in the environment we consider. This possibility raises the question about equilibrium transitions between steady states and about the existance of dynamic equilibria with fluctuations in θ. In this section we address the first question by showing that there can exist a "simple" transition path between a "Buyers' market" steady state with $\theta = \theta^S$ and a "Sellers' market" steady state with $\theta = \theta^B$ (or vice versa). This simple path is characterized by a jump in the market tightness from θ^S to θ^B and a subsequent constant market tightness rate with dynamics only in non-payoff relevant aggregate stock variables.

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We will show this result for the case where the matching function $M(B, S)$ is symmetric, so that $M(B, S) = M(S, B)$. A symmetric matching function is an important theoretical benchmark. In particular, with a symmetric matching function, $\mu(\theta) = q\left(\frac{1}{\theta}\right)$, so the rate of matching for a seller, given a buyer-seller ratio of θ, equals the rate of matching for a buyer, provided that the buyers and sellers switch sides. In the context of a Cobb-Douglas matching function, this implies that the elasticities of matching with respect to buyers and sellers are equal. A symmetric matching function allows for a particularly clear comparison of θ^S and θ^B. As Lemma 16 in the Appendix shows, for $\tilde{u}_0 \geq \tilde{u}_2$ we have that $\theta^B \leq \frac{1}{\theta^S}$ with equality, iff $\tilde{u}_0 = \tilde{u}_2$.

We now show the following result:[27]

Proposition 6. *Suppose that the matching function $M(B, S)$ is symmetric, $\tilde{u}_0 \geq \tilde{u}_2$, and $\theta^S \geq \frac{1}{\theta}$. Consider the equilibrium transition from the "Sellers' market" steady state with $\theta = \theta^B$ to the "Buyers' market" steady state with $\theta = \theta^S$. There exists an equilibrium transition with $\theta(0) = \theta^B$, $\theta(t) = \theta^S$, for $t \in (0, \infty]$. There is a similar equilibrium transition from the "Buyers' market" steady state to the "Seller's market" steady state.*

Proof. See Appendix. $\square$

To understand this result it is best to first consider the case where $\tilde{u}_0 = \tilde{u}_2$.

Corollary 7. *Suppose that $\tilde{u}_0 = \tilde{u}_2$. Consider the equilibrium transition from the "Sellers' market" steady state with $\theta = \theta^B$ to the "Buyers' market" steady state with $\theta = \theta^S$. Then there exists an equilibrium transition with $\theta(0) = \theta^B$, $\theta(t) = \theta^S$, for $t \in (0, \infty]$, in which*

- $B_0(t) = B_0(0)$;

- $S_1(t) = B_1(0)$;

- $S_2(t) = S_2(0) \exp\left\{-\left(\mu\left(\theta^S\right) + g\right) t\right\}$;

- $A(t) = A(0) \exp\left\{-\left(\mu\left(\theta^S\right) + g\right) t\right\} + g \int_0^t \exp\left\{-\left(\mu\left(\theta^S\right) + g\right)(t - s)\right\} ds$.

There is a similar equilibrium transition from the "Buyers' market" steady state to the "Seller's market" steady state.

Proof. See Appendix. $\square$

To understand this result, notice first of all that the population and the housing ownership conditions, (1) and (2), imply that $B_0 = A + S_2$, that is the measure of non-owners equals the measure of real estate firms holding housing units for sale plus the measure of owners with 2 units. This means that the market tightness in a "Sellers' market" steady state equals $\theta^B = \frac{B_0 + B_1}{B_0}$, and the market tightness in the "Buyers' market" steady state equals $\theta^S = \frac{B_0}{B_0 + S_1}$.

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[27]Note that the values of θ^B and θ^S will be part of a steady state equilibrium whenever $\tilde{u}_0$ and $\tilde{u}_2$ are sufficiently close so that $\theta^S \leq \tilde{\theta}$ and $\theta^B \geq \tilde{\theta}$, with $\tilde{\theta}$ defined in equation (15). In particular, given that we will consider the case where $\tilde{u}_0 \geq \tilde{u}_2$, we will assume that $\tilde{u}_0 \leq \bar{u}_0$, for some $\bar{u}_0 > \tilde{u}_2$.

Suppose now that the economy starts in the "Sellers' market" steady state with $\theta = \theta^B$. At $t = 0$ all mismatched owners that enter the housing market move from entering as "buyers first" to entering as "sellers first". This leads to a market tightness of $\theta = \frac{B_0}{B_0 + B_1} = \frac{1}{\theta^B}$. However, a symmetric matching function implies that $\frac{1}{\theta^B} = \theta^S$. Therefore, this new market tightness is consistent with mismatched owners preferring to enter as "sellers first" rather than "buyers first". Furthermore, the constant market tightness is also consistent with the flow conditions and population and housing holding conditions (1) and (2) for aggregate stock variables with B_0 remaining constant over time.

More generally, when $\tilde{u}_0 > \tilde{u}_2$, it is no longer the case that moving mismatched owners from entering as "buyers first" to entering as "sellers first" will result in a market tightness equal to θ^S. However, as long as there are enough mismatched owners that remain "passive" in the "Sellers' market" steady state, there will exist a transitional path where some of these mismatched owners enter as "sellers first", keeping the market tightness at $\theta = \theta^S$. This is guaranteed under the condition $\theta^S \geq \frac{1}{\bar{\theta}}$.

4 House Price Fluctuations

Up to now we considered a constant house price p, which does not violate individual rationality of trading counterparties. In this section, we first examine the implications of expected changes in the house price for the behavior of mismatched owners. We then construct dynamic equilibria with self-confirming fluctuations in house prices and market tightness. Similarly to Section 3.5, for the results below, we assume that the matching function $M(B, S)$ is symmetric.

4.1 Exogenous house price movements

We first show that expected changes in the house price affect the incentives of mismatched owners to enter as "buyers first" versus "sellers first". In particular, even if there is symmetry in flow payoffs, an expected house price depreciation makes "selling first" dominate "buying first" even for values of the market tightness $\theta > 1$, and vice versa for an expected house price appreciation.

To show this, suppose that $u_0 = u_2$ and the house price $p = \frac{R}{\rho}$, so $\tilde{u}_0 = \tilde{u}_2 = c$. We consider a simple exogenous process for the house price p. We assume that with rate λ the house price p changes to a new level p_N and remains constant from them on.[28] We compare the utility from entering as a "buyer first" versus "seller first" for a mismatched owner before the price change.

For the value functions prior to the price change we have expressions similar to those in Section 3.1 but with an additional term reflecting the price uncertainty.[29] For example, the value function

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[28]In the case where $p = \frac{R}{\rho}$, one can think of a permanent change in the equilibrium rental rate to R_N, which leads to a house price change to $p_N = \frac{R_N}{\rho}$.

[29]We assume that θ remains constant over time, so the only change occurs in the house price p. Also, for this exercise, we implicitly assume that $\gamma \to 0$, so that V is indepdenent of the house price p.

of a mismatched owner who enters as a "buyer first" satisfies:

$$V^{B1} = \frac{u - \chi - k}{\rho + q(\theta) + \lambda} + q(\theta)\frac{c - k + \lambda(p_N - p) + \mu(\theta)V}{(\rho + q(\theta) + \lambda)(\rho + \mu(\theta) + \lambda)} + \\ + \frac{\lambda}{\rho + q(\theta) + \lambda}\left(\frac{q(\theta)v^{S2}}{\rho + \mu(\theta) + \lambda} + \overline{V}_N\right) \quad (20)$$

where $v^{S2} = \frac{c-k}{\rho+\mu(\theta)} + \frac{\mu(\theta)}{\rho+\mu(\theta)}V$, and $\bar{V}_N = \max\{V_N^{B1}, V_N^{S1}\}$, with V_N^{B1} and V_N^{S1} denoting the value functions from "buying first" and "selling first" after the price change.

Importantly, the value function of a "buyer first" depends on the expected price change $\lambda(p_N - p)$. Specifically, an expected price appreciation leads to a higher value for a "buyer first". The intuition for this dependence is that by choosing to enter as a "buyer first" a mismatched owner becomes potentially exposed to price risk. Once he buys a new housing unit at the current price p, he must sell a housing unit. However, he may end up selling his old housing unit at a price of p_N later on. If he expects house prices to depreciate, so $p_N < p$, this leads to a lower value from being a "buyer first" for any value of θ.

In contrast, the value of a "seller first" is decreasing in the expected price change.[30] The intuition for this is similar. A "seller first" becomes potentially exposed to price risk but with the opposite sign. If he sells his housing unit at the current price p, the agent must buy a housing unit but may end up buying at a price of p_N. A lower price $p_N < p$ leads to a higher expected value for the agent.

Therefore, the opposite loading on price risk by an owner of two units and a non-owner acts to create asymmetry in the payoff from being a "buyer first" and a "seller first". In particular, at $\theta = 1$, the difference between the two value functions $D(\theta) = V^{B1} - V^{S1}$ takes the form

$$D(1) = \frac{\mu(1)}{(\rho + q(1) + \lambda)(\rho + \mu(1) + \lambda)}2\lambda(p_N - p) \quad (22)$$

An expected price decrease, leads to a higher value of V^{S1} relative to V^{B1}, even if matching probabilities for a buyer and a seller are the same. Consequently, $V^{S1} > V^{B1}$ even for values of $\theta > 1$. If the expected price decrease is sufficiently large, so that even at $\theta = \bar{\theta}$, $D(\bar{\theta}) < 0$, then "selling first" will dominate "buying first" for any value of θ that is consistent with equilibrium. Similarly, a sufficiently large expected price increase, will imply that $D(\underline{\theta}) > 0$, so "buying first" will dominate "selling first" for any value of θ that is consistent with equilibrium. We summarize these observations in the following

Proposition 8. *Consider the modified economy with exogenous house price changes. Then for*

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[30]This value function is given by:

$$V^{S1} = \frac{u - \chi - k}{\rho + \mu(\theta) + \lambda} + \mu(\theta)\frac{c - k - \lambda(p_N - p) + q(\theta)V}{(\rho + \mu(\theta) + \lambda)(\rho + q(\theta) + \lambda)} + \\ + \frac{\lambda}{\rho + \mu(\theta) + \lambda}\left(\frac{\mu(\theta)v^{B0}}{\rho + q(\theta) + \lambda} + \overline{V}_N\right) \quad (21)$$

where $v^{B0} = \frac{c-k}{\rho+q(\theta)} + \frac{q(\theta)}{\rho+q(\theta)}V$ and $\bar{V}_N = \max\{V_N^{B1}, V_N^{S1}\}$.

every $\lambda > 0$, there exists a $\underline{p} < p$, such that for $p_N < \underline{p}$, a mismatched owner strictly prefers "selling first" to "buying first" for $1 < \theta \leq \overline{\theta}$. Furthermore, $\underline{p}$ is increasing in λ, with $\underline{p} \to p$ as $\lambda \to \infty$. Similarly, there exists a $\overline{p} > p$, such that for $p_N > \overline{p}$, a mismatched owner strictly prefers "buying first" to "selling first" for $\underline{\theta} \leq \theta < 1$. Furthermore, $\overline{p}$ is decreasing in λ, with $\overline{p} \to p$ as $\lambda \to \infty$.

Proof. See Appendix. □

Proposition 8 has two implications. First, variations in the expected future price of housing, p_N, influence mismatched owners' incentives to enter as "buyers first" versus "sellers first". If price increases are either expected to occur sooner (λ is high) or be large, then agents strictly prefer "buying first" to "selling first" even if the market tightness θ is unfavorably low and vice versa for price decreases. Secondly, the proposition implies that the actions of mismatched owners are destabilizing for house prices in the following sense. Suppose that the house price is an increasing function of market tightness θ. Then, if mismatched owners anticipate that the price will be decreasing for some exogenous reason, they will tend to prefer to "sell first" rather than "buy first". However, that behavior will tend to decrease the market tightness, which in turn would lower the house price even further. In the next section we show that this behavior of mismatched owners can lead to price fluctuations even without exogenous shocks to prices but due to self-fulfilling expectations about housing market conditions.

4.2   Self-fulfilling house price fluctuations

We now show our second main result, the existence of dynamic equilibria with self-fulfilling fluctuations in house prices and housing market liquidity. For illustration, we show a result for a simpler environment with $u_0 = u_2 = c$ and a zero ownership premium, so the house price $p = \frac{R}{\rho}$ and $\tilde{u}_0 = \tilde{u}_2 = c$. In Section 5.2 we extend this result for the case of a positive ownership premium (i.e. $p > \frac{R}{\rho}$) and a constant rental rate R.

We assume that the house price p is increasing in the market tightness θ, that is $p = f(\theta)$, with $f(\theta)$ a strictly increasing function of θ. We take this relationship as exogenous and reduced-form to illustrate the equilibrium consequences of the interaction of this feedback between housing prices and market liquidity conditions with the transaction decisions of mismatched owners.[31]

We consider equilibria, in which a mismatched owner chooses to enter as a "buyer first" or a "seller first" depending on the realization of a two-state Markov chain $X(t) \in \{0, 1\}$. $X(t)$ starts in $X(t) = 0$ and with Poisson rate λ transitions permanently to $X(t) = 1$. The realization of $X(t)$ plays the role of a sunspot variable that helps coordinate mismatched agents actions.

We assume that if $X(t) = 0$, mismatched owners anticipate that other mismatched owners will "buy first", and if $X(t) = 1$, they anticipate that other mismatched owners will "sell first". Therefore, we will index equilibrium variables in both of these cases by the realization of the state $X(t)$, for example, the market tightness if $X(t) = 0$ is $\theta(t) = \theta_0$ and the price is $p(t) = p_0$.

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[31]Since $p = \frac{R}{\rho}$, this assumption also imposes a positive relation between R and θ. See Section 5.2 for equilibria with self-fulfilling fluctuations and a constant rental rate R.

We construct equilibria, in which θ (and p) take two different values, depending on the realization of $X(t)$. Specifically, θ_0 is the equilibrium market tightness in a "Sellers' market" regime that the economy starts in. In that regime: 1) mismatched owners strictly prefer entering as a "buyer first" to entering as a "seller first" and are indifferent between transacting and remaining "passive", and 2) agents expect that with rate λ, the economy permanently switches to a "Buyers' market" regime with market tightness θ_1. In that second regime, 1) a mismatched owners strictly prefers entering as a "seller first" to entering as a "buyer first" and is indifferent between transacting and remaining passive, and 2) agents expect that the economy will remain in the "Seller's market" regime forever. We describe these equilibria in Proposition 9 below.[32]

Proposition 9. *Consider the model economy with $u_0 = u_2 = c$ and with the sunspot process described above. Suppose that the matching function is symmetric and the house price $p = f(\theta)$, with $f'(\theta) > 0$. Then there is a $\overline{\lambda}$, such that for $\lambda < \overline{\lambda}$, there exists a dynamic equilibrium characterized by two regimes $x \in \{0, 1\}$. In the first regime, $\theta_0 > 1$, $p_0 = f(\theta_0)$, and mismatched owners either enter as "buyers first" or remain "passive". In the second regime, $\theta_1 < 1$, $p_1 < p_0$, and mismatched owners either enter as "sellers first" or remain "passive".*

Proof. See Appendix. $\square$

Proposition 9 shows that when prices are allowed to respond to changes in the market tightness, the actions of mismatched owners lead to self-fulfilling fluctuations in both market liquidity and house prices. Furthermore, given Proposition 6 above, moving from one regime to the other does not feature transitional dynamics in θ. Instead it occurs with an instantaneous jump in θ.[33]

The transition between the two regimes is broadly consistent with our motivating Figure 1. When the house price is high, owners prefer to enter as "buyers first". A decline in the house price is associated with a reversal of the incentives of owners and they prefer to enter as "sellers first". Additionally, there is a negative relation between expected seller time on the market and prices. This latter prediction is consistent with the observed behavior of average time on the market and house prices (Diaz and Jerez (2013)).

Since movements from the first regime to the second regime entail price depreciation, Proposition 8 above shows that if agents expected the change in regimes to occur sufficiently frequently, then it can be optimal for mismatched owners to enter as "sellers first" in the "Buyers' market" regime despite the high market tightness. This, however, is inconsistent with equilibrium. Therefore, an equilibrium with a transition between the two regimes exists only for a sufficiently low regime

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[32]Note that for both Propositions 9 and 11 we will be assuming that $c < \overline{c}$, where $\overline{c}$ is the solution to

$$\frac{u - \chi}{\rho} = \frac{u - \chi - k}{\rho + q(\overline{\theta})} + \frac{q(\overline{\theta})(\overline{c} - k)}{(\rho + \mu(\overline{\theta}))(\rho + q(\overline{\theta}))}$$

$$+ \frac{\mu(\overline{\theta}) q(\overline{\theta})}{(\rho + \mu(\overline{\theta}))(\rho + q(\overline{\theta}))} \left(\frac{u}{\rho} - \frac{\gamma}{\rho + \gamma} \frac{\chi}{\rho} \right) \tag{23}$$

and where $\overline{\theta}$ is the solution to equation (16). This restriction of the value of c ensures that in either of the two regimes mismatched owners are indifferent between entering the market and remaining passive.

[33]Note that one can construct other dynamic equilibria, for example with alternations in regimes.

switching rate λ. Therefore, a price decline must be expected to occur rarely when the house price is high and mismatched owners enter as "buyers first". As a result, the dynamic equilibria described in Proposition 8 features medium-to-low frequency mean reversion in house prices and market liquidity. The existence of such boom-bust transitions is an important feature of housing markets.[34]

The fluctuations in prices and liquidity are purely driven by changes in expectations. As we show in Section 5.2 they can occur even with a constant rental rate R. Therefore, the expectations and actions of mismatched owners can lead to volatility in house prices that is unrelated to changes in rental rates or other fundamentals (Shiller (2005), Campbell, Davis, Gallin, and Martin (2009)).

5 Extensions

5.1 Alllowing for Entry as both Buyer and Seller

Up to now, we assumed that there is a trade-off in the decision of a mismatched owner to enter the housing market as a buyer or as a seller. In this section, we allow for the possibility that households can choose to be both a buyer and a seller at the same time, and extend our main result about equilibrium multiplicity. Importantly, the main mechanisms investigated above carry through, since the decision to enter as both a buyer and a seller depends ultimately on the value from entering as a buyer only and the value from entering as a seller only.

We denote by SB the measure of agents who enter as both a seller and a buyer in the housing market.[35]

The value function V^{SB} satisfies the following equation in a steady state equilibrium

$$\rho V^{SB} = u - \chi - k + \mu(\theta)\left(p + V^{B0} - V^{SB}\right) + q(\theta)\left(-p + V^{S2} - V^{SB}\right) \tag{24}$$

where for simplicity we assume that entering as both a buyer and a seller results in paying the flow cost k only once. We solve for the value function to obtain the expression:

$$V^{SB} = \frac{u - \chi - k}{\rho + \mu(\theta) + q(\theta)} + \frac{q(\theta)}{\rho + \mu(\theta) + q(\theta)}v^{S2} + \frac{\mu(\theta)}{\rho + \mu(\theta) + q(\theta)}v^{B0} \tag{25}$$

where

$$v^{B0} \equiv \frac{\tilde{u}_0 - k}{\rho + q(\theta)} + \frac{q(\theta)}{\rho + q(\theta)}V \tag{26}$$

and

$$v^{S2} \equiv \frac{\tilde{u}_2 - k}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)}V \tag{27}$$

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[34]For example, price changes in housing markets are negatively correlated at a horizon higher than 3 years (Gleaser, Gyourko, Morales, and Nathanson (2012), Guren (2013)).

[35]Note that the definition of equilibrium requires a straightforward extension to accommodate this particular type of mismatched agents in the economy.

Figure 6: Equilibrium multiplicity when entry as both a buyers and seller is allowed and (a) $\tilde{u}_0 = \tilde{u}_2$ or (b) $\tilde{u}_0 > \tilde{u}_2$ with $\tilde{\theta} > \bar{\theta}$.
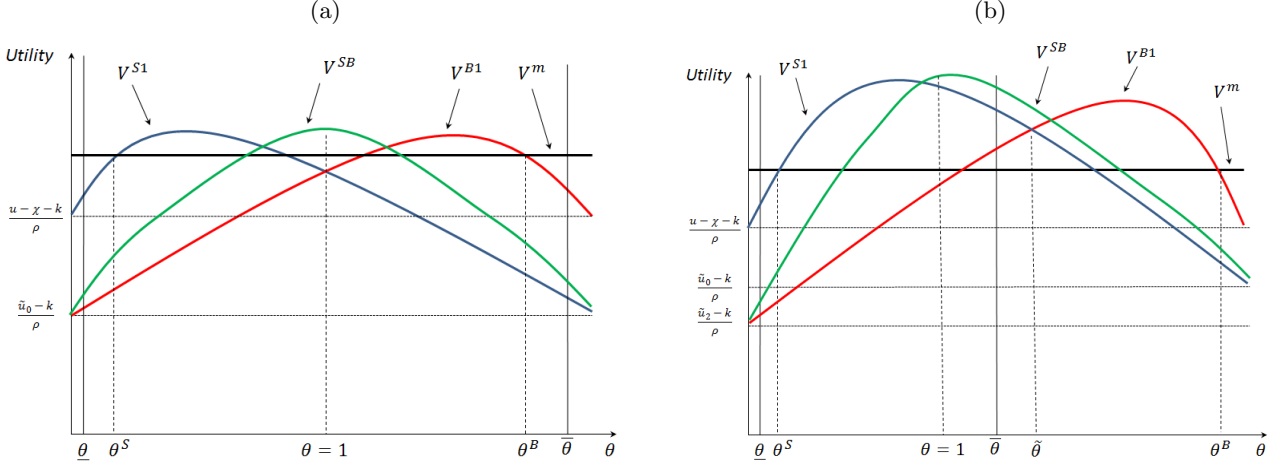
Note that

$$
\begin{aligned}
V^{SB} &= \frac{\rho + \mu(\theta)}{\rho + \mu(\theta) + q(\theta)} V^{S1} + \frac{q(\theta)}{\rho + \mu(\theta) + q(\theta)} v^{S2} \\
&= \frac{\rho + q(\theta)}{\rho + \mu(\theta) + q(\theta)} V^{B1} + \frac{\mu(\theta)}{\rho + \mu(\theta) + q(\theta)} v^{B0}
\end{aligned}
\tag{28}
$$

that is the value of simultaneous selling and buying can be written as a weighted average of the value of "selling first" and v^{S2} or the value of "buying first" and v^{B0}. Therefore, $V^{SB} \leq V^{S1} \iff v^{S2} \leq V^{S1}$ and $V^{SB} \leq V^{B1} \iff v^{B0} \leq V^{B1}$. We denote by θ_1^{SB} be the value of θ for which $v^{S2} = V^{S1}$ and by θ_2^{SB} the value of θ for which $v^{B0} = V^{B1}$. Note that $V^{SB} < V^{S1}$ for $\theta < \theta_1^{SB}$, and $V^{SB} < V^{B1}$ for $\theta > \theta_2^{SB}$. We now show the main result of this Section:

Proposition 10. *Consider the above economy. Let θ^{S2} be defined as the value of θ, at which $v^{S2} = \frac{u - \chi - k}{\rho} = V^{B1}$ and θ^{B0} be defined as the value of θ, at which $v^{B0} = \frac{u - \chi - k}{\rho} = V^{S1}$. Suppose that $\theta^{B0} < \theta^{S2}$. Then it is never optimal for a mismatched owner to enter as both a buyer and a seller. Suppose that $\theta^{B0} \geq \theta^{S2}$. If $\theta_1^{SB} \leq 1$, then there exists a steady state equilibrium with market tightness $\theta = 1$, in which mismatched owners enter as both a buyer and a seller. There can also exist "Buyers' market" and "Sellers' market" equilibria as described in Proposition 5.*

Proof. See Appendix. □

The existence of an equilibrium with $\theta = 1$, in which mismatched owners enter as both buyers and sellers changes the possible equilibria discussed above slightly. Figure 6 below shows some of these possible value function configurations. Most importantly, as Figure 6b shows, it is possible that this equilibrium coexists with the "Buyers' market" equilibrium even if the "Sellers' market" equilibrium does not exist. More specifically, note that if $\underline{\theta} > \theta_1^{SB}$, then a "Buyers' market" equilibrium does not exist, since entering as a "seller first" only is dominated by entering as both a buyer and a seller.

Similarly, if $\bar{\theta} < \theta_2^{SB}$, then a "Sellers' market" equilibrium does not exist, since entering as a "buyer first" is dominated by entering as both a buyer and a seller.

Also, note that whenever this equilibrium exists, the aggregate volume of transactions tends to be higher than in either the "Buyers' market" or "Sellers' market" equilibria. The reason for this is that since mismatched agents enter on both sides of the market, that increases the measure of both buyers and sellers, which mechanically increases the matching rate in the economy, and from there the total number of transactions.

Finally, self-fulfilling fluctuations in liquidity and house prices as in Section 4 can still be possible given the additional choice of entering as both buyers and sellers. However, in that case there will be non-trivial dynamics in the market tightness θ.

5.2 Self-Fulfilling Fluctuations with a Positive Ownership Premium

In this Section we extend the result from Section 4.2 to the case where the house price $p > \frac{R}{\rho}$, so there is a positive ownership premium. We assume that the house price p is a strictly increasing function of the market tightness θ, i.e. $p = \epsilon f(\theta) + \frac{R}{\rho}$, for some $\epsilon > 0$, where $f(\theta)$ is continuous and increasing in θ.

We proceed as in Section 4.2 and construct equilibria in which θ (and p) jump between two different values, θ_0 and θ_1, depending on the realization of the Markov chain $X(t)$. $X(t)$ starts in $X(t) = 0$ and with Poisson rate λ transitions permanently to $X(t) = 1$. We describe them in the following

Proposition 11. *Consider the model economy with $u_0 = u_2 = c$, a house price $p = \epsilon f(\theta) + \frac{R}{\rho}$ and the sunspot process described above. There is an $\bar{\epsilon}$ and $\bar{\lambda}$ such that for $\epsilon < \bar{\epsilon}$ and $\lambda < \bar{\lambda}$, there exists a dynamic equilibrium characterized by two regimes $x \in \{0, 1\}$. In the first regime, $\theta = \theta_0 > 1$, $p_0 = \epsilon f(\theta_0) + \frac{R}{\rho}$, and mismatched owners enter as "buyers first" or remain "passive". In the second regime, $\theta = \theta_1 < 1$, $p_1 < p_0$, and mismatched owners enter as "sellers first" or remain "passive".*

Proof. See Appendix. $\square$

Proposition 11 has a similar flavor to Proposition 9 and relies on a similar set of arguments. One important technical difference is that, since a price $p > \frac{R}{\rho}$ creates asymmetry in the flow payoffs of mismatched owners that enter as "buyers first" versus "sellers first", the homeownership premium $p - \frac{R}{\rho}$ must be sufficiently small for any value of θ, that is ϵ must be sufficiently small.

5.3 Homeowners compensated for their housing unit upon exit

The results presented up to now were derived under the assumption that homeowners who exit the economy are not compensated for the value of their housing units. In this section we show that this assumption is not important for the main mechanism we explored, namely that the incentives for a mismatched owner to "buy first" are increasing in the market tightness.

Suppose that upon exit homeowners receive bids for their housing unit(s) from a set of competitive real estate firms. Therefore, given that the to a real estate firm is $V^A(\theta)$, homeowners receive

$V^A(\theta)$ for each housing unit that they own. Again, we consider a steady state equilibrium with a fixed market tightness θ. We define $\tilde{u}_0(\theta, g) \equiv u_0 + \triangle - gV^A(\theta)$ and $\tilde{u}_2(\theta, g) = u_2 - \triangle + gV^A(\theta)$. Note that $V^A(\theta)$ is (weakly) increasing in θ, so $\tilde{u}_2$ is weakly increasing in θ and $\tilde{u}_0$ is weakly decreasing in θ;

Given this definition, the difference between the values from "buying first" and "selling first", $D(\theta) \equiv V^{B1} - V^{S1}$, can be written as

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))} \left[\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2(\theta, g)) - \tilde{u}_0(\theta, g) + \tilde{u}_2(\theta, g) \right]$$

Let $\tilde{\theta}$ be defined implicitly by

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2\left(\tilde{\theta}, g\right)}{u - \chi - \tilde{u}_0\left(\tilde{\theta}, g\right)}$$

whenever that equation has a solution. Note that the above equation for $\tilde{\theta}$, whenever it has a solution, has a unique solution for any $g \geq 0$, since given the properties of $\tilde{u}_0$ and $\tilde{u}_2$, it follows that the right hand side of this expression is (weakly) decreasing in θ. Furthermore, the right hand side is strictly decreasing in g for any $\theta > 0$, so by the implicit function theorem, $\tilde{\theta}$ is decreasing in g.

Finally, note that in the limit as $g \to 0$, assumption A1 will hold. Therefore, for g sufficiently close to zero, we will have that $u - \chi > \max\{\tilde{u}_0(\theta, g), \tilde{u}_2(\theta, g)\}$, for all $\theta \in [\underline{\theta}, \overline{\theta}]$, and so a version of Lemma 2 will hold in this case as well, with $V^{B1} > V^{S1} \iff \theta > \tilde{\theta}$ and $V^{B1} = V^{S1} \iff \theta = \tilde{\theta}$. Given this result one can then easily construct multiple steady state equilibria as in Proposition 5 or dynamic equilibria as in Proposition 9.

6   Concluding Comments

In this paper we study a tractable model of the housing market that explicitly features a "buy first"-"sell first" trade off for existing owners who have to re-trade in the housing market. We show that the decision to "buy first" or "sell first" is a strategic complement among such homeowners, whenever it is more costly to end up with two housing units or with no housing, compared to being imperfectly matched to one's current residence. This leads to both multiple steady state equilibria but also to dynamic equilibria with self-confirming fluctuations in house prices and market liquidity. The model is broadly consistent with key stylized facts about the housing market.

Whether the key condition, under which we study our model of the housing market, is valid is a ultimately a matter of empirical investigation. Nevertheless, one can conclude *a priori* that it should be fairly easily satisfied for a broad set of households. Very often households can fairly easily accommodate having an increase in household size or a job change that requires a longer commuting distance. In contrast, keeping two houses for a significant period or having to move into rental housing appear to be substantially more costly outcomes.

The model was deliberately simplified and so lacked household heterogeneity in these relative costs. Since for the most part, we considered equilibria, in which mismatched homeowners are

indifferent between participating in the housing market and not participating, including limited heterogeneity along that dimension should not affect the results greatly. If the heterogeneity is substantial, then it may be the case that some agents have dominant strategies, "selling first" or "buying first" regardless of the value of the market tightness. Enriching the model along this dimension is important for a thorough quantitative evaluation of the model, which is an important next step for future research.

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Appendix

Equilibrium concept and parameter restrictions

First of all, the steady state value functions of a matched owner and a real estate firm satisfy equations:

$$\rho V = u + \gamma \left(\overline{V} - V \right) \tag{29}$$

where $\overline{V} = \max \left\{ V^{B1}, V^{S1}, V^m \right\}$ and

$$\rho V^A = R + \mu \left(\theta \right) \left(p - V^A \right) \tag{30}$$

Importantly, in every steady state equilibrium, V satisfies $V \geq \tilde{V}$, where $\tilde{V} = \frac{u}{\rho+\gamma} + \frac{\gamma}{\rho+\gamma} V^m$, with $V^m = \frac{u-\chi}{\rho}$. Hence, $\tilde{V}$ is the value of a matched owner who will always choose to remain "passive" when mismatched. Therefore, $V \geq \tilde{V} = \frac{u}{\rho} - \frac{\gamma}{\rho+\gamma} \frac{\chi}{\rho}$ in any steady state equilibrium.

Parameter restrictions

Sufficient conditions for non-owners and owners of two housing units to prefer becoming regular owners are given by:

$$\frac{u_0 - R}{\rho} \leq \tilde{V} - p \tag{31}$$

and

$$\frac{u_2 + R}{\rho} \leq \tilde{V} + p \tag{32}$$

Equivalently, conditions (31) and (32) imply restrictions for the values of the house price, p, that are sufficient for p to satisfy agent individual rationality for non-owners and owners with two housing units, namely $p \in \left[\frac{u_2}{\rho} - \tilde{V} + \frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho} \right]$.

From (30) an individual rationality restriction for the price, p, for a real estate firm is given by $p \geq \frac{R}{\rho}$. Finally, note that the value functions of a "buyer first" and a "seller first" generally also depend on the house price, p. However, for the definition of equilibrium, we do not impose a specific

restriction that the price must satisfy for those agents, since they may endogenously choose to not participate in the housing market (remain "passive") in equilibrium. We discuss the effect of the price on mismatched owners value functions extensively in the paper.

Therefore, equilibrium is defined for a house price p, that satisfies:

$$p \in \left[\max \left\{ \frac{u_2}{\rho} - \tilde{V}, 0 \right\} + \frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho} \right] \tag{33}$$

For $u - \chi \geq \max \{u_0, u_2\}$, which is the condition we will use to characterize equilibria under, it follows that $\frac{u_2}{\rho} - \tilde{V} < 0$ and so the set for prices is given by

$$p \in \left[\frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho} \right] \tag{34}$$

Steady state flow conditions

Before moving to our formal definition, it is necessary to describe the flow conditions that the aggregate stock variables defined in Section 2.2 must satisfy. We have that in a steady state equilibrium, given a market tightness θ, the steady state values of B_0, B_1, S_1, S_2, O, O_m, and A must satisfy the following system of flow conditions:

$$g + \mu(\theta) S_1 = (q(\theta) + g) B_0 \tag{35}$$

$$\gamma x_b O = (q(\theta) + g) B_1 \tag{36}$$

$$\gamma x_s O = (\mu(\theta) + g) S_1 \tag{37}$$

$$\gamma x_0 O = g O_m \tag{38}$$

$$q(\theta) B_1 = (\mu(\theta) + g) S_2 \tag{39}$$

$$g(O + O_m + B_1 + S_1 + 2S_2) = \mu(\theta) A \tag{40}$$

$$x_0 + x_b + x_s = 1 \tag{41}$$

where x_0, x_b, and x_s are the equilibrium fractions of mis-matched owners that choose to be "passive", to be "buyers first", and "sellers first", respectively. Apart from these conditions, the aggregate variables must satisfy the population constancy and housing ownership conditions (1) and (2), respectively. Finally, the equilibrium market tightness θ, satisfies

$$\theta = \frac{B}{S} = \frac{B_0 + B_1}{S_1 + S_2 + A} \tag{42}$$

Equilibrium definition

We are now in a position to define a steady state equilibrium for this economy.

Definition 12. A steady state equilibrium given a house price p consists of equilibrium rental rate R, value functions V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^m, V^A, market tightness θ, fractions of mismatched owners that choose to be "passive", to be a "buyer first" or to be a "seller first", x_0, x_b, and x_s and aggregate stock variables, B_0, B_1, S_1, S_2, O, O_m, and A such that:

1. The equilibrium rental rate $R \in [0, u_0]$;

2. The value functions satisfy equations (3)-(7) and (29)-(30) given θ, and R;

3. Mismatched owners choose $x \in \{0, b, s\}$, to maximize $\overline{V} = \max\{V^{B1}, V^{S1}, V^m\}$ and the fractions x_0, x_b, and x_s reflect that, i.e.

$$x_0 = \int_i I\{x_i = 0\}\, di$$

where $i \in [0, 1]$ indexes the i-th mismatched owner and similarly for x_b and x_s;

4. The market tightness θ solves (42) given the aggregate stock variables, B_0, B_1, S_1, S_2, O, O_m, and A;

5. The aggregate stock variables B_0, B_1, S_1, S_2, O, O_m, and A, solve (35)-(40) given θ and mismatched owners' optimal decisions reflected in x_0, x_b, and x_s;

6. The house price p lies in the set given by (33).

Omitted Results and Proofs

Lemma 13. *Suppose that $u - \chi > \max\{\tilde{u}_0, \tilde{u}_2\}$. Then:*

1. *V^{B1} and V^{S1} cross only once at $\theta = \tilde{\theta} = \frac{u - \chi - \tilde{u}_2}{u - \chi - \tilde{u}_0}$;*

2. *$V^m > \lim_{\theta \to 0} V^{S1} = \lim_{\theta \to \infty} V^{B1}$;*

3. *If v^{S2} is monotone increasing in θ, thenV^{B1} has a unique maximum at a value of $\theta > \theta^{S2}$, where θ^{S2} is defined as the value of θ, at which $v^{S2} = \frac{u - \chi - k}{\rho} = V^{B1}$;*

4. *If v^{B0} is monotone decreasing in θ, then V^{S1} has a unique maximum at a value of $\theta < \theta^{B0}$, where θ^{B0} is defined as the value of θ, at which $v^{B0} = \frac{u - \chi - k}{\rho} = V^{S1}$;*

5. *If $\theta^{B0} < \theta^{S2}$ then at $\theta = \tilde{\theta}$, $V^{S1} = V^{B1} < V^m$.*

Proof. To show the the first claim, note that V^{B1} and V^{S1} clearly cross at $\theta = \tilde{\theta}$ by the definition of $\tilde{\theta}$. To show that they do not cross anywhere else. Note that for $\theta \to 0$,

$$\lim_{\theta \to 0} V^{B1} = \frac{\tilde{u}_2 - k}{\rho}$$

and

$$\lim_{\theta \to 0} V^{S1} = \frac{u - \chi - k}{\rho}$$

and so $\lim_{\theta \to 0} V^{B1} < \lim_{\theta \to 0} V^{S1}$. Similarly, for $\theta \to \infty$, $\lim_{\theta \to \infty} V^{B1} = \frac{u - \chi - k}{\rho} > \frac{\tilde{u}_0 - k}{\rho} = \lim_{\theta \to \infty} V^{S1}$. Therefore, $V^{B1} < V^{S1}$ for any $\theta < \tilde{\theta}$ and $V^{B1} > V^{S1}$ for any $\theta > \tilde{\theta}$. To show the second claim, let us express V^m as:

$$V^m = \frac{u - \chi}{\rho} \tag{43}$$

and so $V^m > \lim_{\theta \to 0} V^{S1} = \lim_{\theta \to \infty} V^{B1}$.

To show the third claim, note that since V^{B1} is a weighted average of $\frac{u - \chi - k}{\rho}$ and v^{S2} with $\lim_{\theta \to 0} V^{B1} = v^{S2}$ and $\lim_{\theta \to \infty} V^{B1} = \frac{u - \chi - k}{\rho}$, and $\lim_{\theta \to 0} v^{S2} < \frac{u - \chi - k}{\rho}$, and $\lim_{\theta \to \infty} v^{S2} = V > \frac{u - \chi - k}{\rho}$, then, if v^{S2} is monotone increasing in θ, V^{B1} has a unique maximum. Since V^{B1} is strictly increasing for $\theta < \theta^{S2}$, where θ^{S2} is defined as the value of θ, at which $v^{S2} = \frac{u - \chi - k}{\rho} = V^{B1}$ it follows that the value of θ that maximizes V^{B1} is $> \theta$. Showing the fourth claim is analogous. Finally, note that if $\theta^{B0} < \theta^{S2}$ then $\tilde{\theta} \in \left(\theta^{B0}, \theta^{S2}\right)$. However for $\theta \in \left(\theta^{B0}, \theta^{S2}\right)$, $V^{S1} < \frac{u - \chi - k}{\rho} < V^m$ and $V^{B1} < \frac{u - \chi - k}{\rho} < V^m$. Therefore, at $\theta = \tilde{\theta}$, $V^{B1} = V^{S1} < V^m$. $\qquad \square$

Lemma 14. *Consider equations (16) and (17). Each has a unique solution, denoted by $\bar{\theta}$ and $\underline{\theta}$, respectively. Furthermore, $\bar{\theta} > 1$, $\underline{\theta} < 1$, and $\bar{\theta}$ is increasing in γ and $\underline{\theta}$ is decreasing in γ.*

Proof. Consider first equation (16). At $\theta = 1$, the left-hand side equals

$$\frac{1}{q(1) + g} + \frac{1}{\gamma} < \frac{1}{g} + \frac{1}{\gamma}$$

Furthermore, note that $\left(\frac{1}{q(\theta) + g} + \frac{1}{\gamma}\right)\theta$ is strictly increasing in θ and also unbounded. Similarly, $\left(\frac{1}{q(\theta) + g} - \frac{1}{\mu(\theta) + g}\right)$ is strictly increasing in θ as well. Therefore, the left-hand side of (16) is strictly increasing in θ, unbounded, and lower than the right-hand side for $\theta = 1$. Therefore, it has a unique solution for $\theta > 1$. We call this solution $\bar{\theta}$. Furthermore, by the Implicit Function Theorem, it immediately follows that $\bar{\theta}$ is increasing in γ. Secondly, consider the equation (17). At $\theta = 1$, the left-hand side equals

$$\frac{1}{\mu(1) + g} + \frac{1}{\gamma} < \frac{1}{g} + \frac{1}{\gamma}$$

Note also that $\left(\frac{1}{\mu(\theta) + g} + \frac{1}{\gamma}\right)\frac{1}{\theta}$ is strictly decreasing in θ and goes to 0 as $\theta \to \infty$. Also it asymptotes to ∞ as $\theta \to 0$. Therefore, the equation has a unique solution for $\theta < 1$. We call this solution $\underline{\theta}$. By the Implicit Function Theorem, it immediately follows that $\underline{\theta}$ is decreasing in γ.

$\qquad \square$

Lemma 15. *Define*

$$\tilde{F}(\theta, \tilde{u}) \equiv \frac{u - \chi - k}{\rho + \mu(\theta)} + \frac{\mu(\theta)(\tilde{u} - k)}{(\rho + \mu(\theta))(\rho + q(\theta))}$$
$$+ \frac{\mu(\theta) q(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))} \left(\frac{u}{\rho} - \frac{\gamma}{\rho + \gamma} \frac{\chi}{\rho} \right)$$

(44)

and let $\hat{\theta}(\tilde{u})$ be the smallest solution to $\frac{u - \chi}{\rho} = \tilde{F}(\hat{\theta}, \tilde{u})$ whenever it exists. Then $\hat{\theta}$ is decreasing in $\tilde{u}$.

Proof. First, notice that by Lemma 13, $\tilde{F}(\theta, \tilde{u}) = V^{S1}$ is increasing in θ around $\hat{\theta}$, since $\hat{\theta}$ is to the left of the unique maximum of V^{S1}. Secondly, note that $\tilde{F}(\theta, \tilde{u})$ is everywhere increasing in $\tilde{u}$. Therefore, by the implicit function theorem, $\hat{\theta}$ is decreasing in $\tilde{u}$. □

Lemma 16. *Suppose that the matching function $M(B, S)$ is symmetric and that $\tilde{u}_0 \geq \tilde{u}_2$. Then, $\theta^B \leq \frac{1}{\theta^S}$ with equality, iff $\tilde{u}_0 = \tilde{u}_2$.*

Proof. Whenever the matching function is symmetric, the values of θ^S and θ^B can be determined by a single condition. In particular, defining, $\tilde{F}(\theta, \tilde{u})$ as in Lemma 15 and letting $\hat{\theta}(\tilde{u})$ be the smallest solution to

$$\frac{u - \chi}{\rho} = \tilde{F}(\hat{\theta}, \tilde{u})$$

(45)

we have that $\theta^S = \hat{\theta}(\tilde{u}_0)$ and $\theta^B = \frac{1}{\hat{\theta}(\tilde{u}_2)}$. Therefore, with a symmetric matching function, whenever $\tilde{u}_0 = \tilde{u}_2$, the value from being a "buyer first" given a market tightness of θ is equal to the value from being a "seller first" given the reciprocal market tightness. Furthermore, Lemma 15 shows that $\hat{\theta}$ is decreasing in $\tilde{u}$. This implies that θ^S is decreasing in $\tilde{u}_0$ and θ^B is increasing in $\tilde{u}_2$. Therefore, it follows that for $\tilde{u}_0 \geq \tilde{u}_2$, $\theta^S \leq \frac{1}{\theta^B}$, or $\theta^B \leq \frac{1}{\theta^S}$. □

Proof of Lemma 2

Proof. Using the expression for $D(\theta)$, equation (12), we have that $D(\theta) > 0$, whenever

$$\left(1 - \frac{1}{\theta} \right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 > 0$$

which is equivalent to $\theta > \tilde{\theta}$. Also $D(\theta) = 0$, whenever

$$\left(1 - \frac{1}{\theta} \right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 = 0$$

which is equivalent to $\theta = \tilde{\theta}$. □

Proof of Proposition 3

Proof. First of all, note that a value of $\theta = 1$, implies that

$$V^{B1} = V^{S1} = \frac{u - \chi - k}{\rho + \mu(1)} + \frac{\mu(1)}{(\rho + \mu(1))^2} \tilde{u}_0(p) + \frac{\mu(1)^2}{(\rho + \mu(1))^2} V$$

(46)

We define two values for V. First of all, we let $\rho V_1 = u + \gamma (V^m - V_1)$, where V^m is given in (43). Hence, V_1 so defined is the value of a matched owner that remains passive when mismatched. Therefore, solving for V_1, we have that $V_1 = \frac{u}{\rho} - \frac{\gamma}{\rho+\gamma} \frac{\chi}{\rho}$. Secondly, we let $\rho V_2 = u + \gamma (V^{B1} - V_2)$, V^{B1} is given in (46) but with V_2 substituted for V. Hence, V_2 is the value of a matched owner that enters as a "buyer first" (or equivalently "seller first") when mismatched. First of all, note that $V_1 < V_2$ iff condition A2 holds. Therefore, if condition A2 holds, $V^{B1}(V_1) < V^{B1}(V_2)$, where $V^{B1}(V_1)$ denotes (46) but with V_1 substituted for V and similarly for $V^{B1}(V_2)$. Note however, that condition A2 implies also that $V^m < V^{B1}(V_1)$.

If condition A2 holds, then in every instant entering as a "buyer first" is preferred to remaining passive regardless of whether the agent will remain passive or enter the market in later instances when he becomes mismatched. The fact that mismatched agents are equally likely to enter as a "buyer first" and a "seller first", implies that

$$\gamma \frac{1}{2} O = (q(\theta) + g) B_1 \tag{47}$$

$$\gamma \frac{1}{2} O = (\mu(\theta) + g) S_1 \tag{48}$$

which for $\theta = 1$ gives $B_1 = S_1$. Furthermore, the housing holding and population conditions in this case are:

$$O + B_1 + S_1 + 2S_2 = 1 - A$$

and

$$B_0 + O + B_1 + S_1 + S_2 = 1$$

which implies that $B_0 = A + S_2$. This in turn means that $\theta = \frac{B_0 + B_1}{A + S_2 + S_1} = 1$. Therefore, the steady state value of θ is consistent with the flow conditions for the aggregate stock variables given the behavior of mismatched agents. $\qquad \square$

Proof of Proposition 4

Proof. First of all, suppose that $\max \{\theta^B, \bar{\theta}\} = \theta^B$, i.e. $\theta = \bar{\theta}$. Therefore, by Lemma 1, we have that $V^{B1} > V^{S1}$. Furthermore, by condition A2, $V^{B1} > V^m$. Therefore, mismatched agents strictly prefer entering as "buyers first". To see that this action and the market tightness $\bar{\theta}$ are consistent with population constancy, the housing condition and the flow conditions for aggregate stock variables, note that we have the following set of equations in this case:

$$O + B_1 + 2S_2 = 1 - A$$

$$B_0 + O + B_1 + S_2 = 1$$

as well as,

$$g = (q(\theta) + g) B_0$$

$$\gamma O = (q(\theta) + g) B_1$$

$$q(\theta) B_1 = (\mu(\theta) + g) S_2$$

$$g(O + B_1 + 2S_2) = \mu(\theta) A$$

From the first two conditions, we have that $B_0 = A + S_2$. Combining this with the flow conditions, we have that $B_0 = \frac{g}{q(\theta)+g}$ and $A = \frac{g}{\mu(\theta)+g}$, or $A = \frac{q(\theta)+g}{\mu(\theta)+g} B_0$, so $S_2 = \frac{g}{q(\theta)+g} - \frac{g}{\mu(\theta)+g}$. Therefore, from the equation for θ, we have that $B_1 = (\theta - 1) B_0$ and so $O = \frac{1}{\gamma} (q(\theta) + g) (\theta - 1) B_0$. Substituting into the population constancy condition, we have that

$$\theta B_0 + B_0 - \frac{q(\theta) + g}{\mu(\theta) + g} B_0 + \frac{1}{\gamma} (q(\theta) + g) (\theta - 1) B_0 = 1$$

which, after substituting for B_0 and re-arranging we can write as:

$$\left(\frac{1}{q(\theta) + g} + \frac{1}{\gamma} \right) \theta + \left(\frac{1}{q(\theta) + g} - \frac{1}{\mu(\theta) + g} \right) = \frac{1}{g} + \frac{1}{\gamma}$$

Note, however, that this is exactly equation (16), which has $\bar{\theta}$ as a solution. Therefore, the value of $\bar{\theta}$ and the actions of mismatched agents are consistent with the conditions for the aggregate stock variables.

Next, suppose that $\min\{\theta^S, \underline{\theta}\} = \theta^S$, i.e. $\theta = \underline{\theta}$. Therefore, by Lemma 1, we have that $V^{S1} > V^{B1}$. Furthermore, by condition A2, $V^{S1} > V^m$. Therefore, mismatched agents strictly prefer entering as "buyers first". To see that this action and the market tightness $\underline{\theta}$ are consistent with population constancy, the housing condition and the flow conditions for aggregate stock variables, note that we have the following set of equations in this case:

$$O + S_1 = 1 - A$$

$$B_0 + O + S_1 = 1$$

as well as,

$$g + \mu(\theta) S_1 = (q(\theta) + g) B_0$$

$$\gamma O = (\mu(\theta) + g) S_1$$

$$g(O + S_1) = \mu(\theta) A$$

By the population constancy and housing conditions, $B_0 = A$. Furthermore, from the flow equations, $A = \frac{g}{\mu(\theta)+g} = B_0$, $S_1 = \frac{1-\theta}{\theta} A$ and $O = \frac{1}{\gamma} (\mu(\theta) + g) \frac{1-\theta}{\theta} A$. Therefore, substituting for

these in the population constancy condition, we have that

$$\frac{1}{\theta}A + \frac{1}{\gamma}\left(\mu\left(\theta\right) + g\right)\frac{1-\theta}{\theta}A = 1$$

Substituting for A and re-arranging, we have the equation

$$\left(\frac{1}{\mu\left(\theta\right) + g} + \frac{1}{\gamma}\right)\frac{1}{\theta} = \frac{1}{g} + \frac{1}{\gamma}$$

However, $\underline{\theta}$ is defined exactly as the solution to this equation from (17). Therefore, the value of $\underline{\theta}$ and the actions of mismatched agents are consistent with the conditions for the aggregate stock variables.

Next, suppose that $\min\left\{\theta^S, \underline{\theta}\right\} = \underline{\theta}$, i.e. $\theta = \theta^S$. Clearly, given the definition of θ^S, we have that $\theta^S < 1$. Therefore, by Lemma 1, we have that $V^{S1} > V^{B1}$. Furthermore, by the definition of θ^S, $V^m = V^{S1}$. Therefore, mismatched agents' actions are optimal given the market tightness θ^S. To see that θ is consistent with the flow conditions for aggregate stock variables, note that the aggregate stock variables must satisfy the following conditions:

$$O + O_m + S_1 = 1 - A$$

$$B_0 + O + O_m + S_1 = 1$$

as well as

$$g + \mu\left(\theta^S\right)S_1 = \left(q\left(\theta^S\right) + g\right)B_0$$

$$q\left(\theta^s\right)B_0 = \left(\gamma + g\right)O$$

$$\gamma x_s O = \left(\mu\left(\theta^S\right) + g\right)S_1$$

$$\gamma\left(1 - x_s\right)O = gO_m$$

$$g = \left(\mu\left(\theta^S\right) + g\right)A$$

where $x_s \in [0, 1]$. Finally, $\theta^S = \frac{B_0}{A+S_1}$. Note that at $x_s = 0$, $\theta^S = 1$ and at $x_s = 1$, $\theta^S = \underline{\theta}$. Also, it is straightforward to show that x_s is a decreasing function of θ^S. To see this, note that $B_0 = \frac{g}{\mu(\theta^S)+g}$, which is decreasing in θ^S. Similarly, $S_1 = \frac{1-\theta^S}{\theta^S}\frac{g}{\mu(\theta^S)+g}$, which is also decreasing in θ^S. Finally, $O = \frac{g}{\gamma+g}\frac{q(\theta^S)}{\mu(\theta^S)+g}$, which is also decreasing in θ^s. Therefore, given population constancy it follows that O_m is increasing in θ^S, which using the flow equation $\gamma\left(1 - x_s\right)O = gO_m$, implies that x_s is

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decreasing in θ^S. One can also express x_s directly as a decreasing function of θ^S, namely:

$$x_s = \left(1 + \frac{g}{\gamma}\right)\left(\frac{g}{\mu\left(\theta^S\right)} + 1\right)\left(1 - \theta^S\right)$$

Therefore, the value of θ^S determined via equation (18) pins down $x_s \in [0, 1]$, which in turn ensures consistency with the above flow conditions. Showing that if $\max\left\{\theta^B, \overline{\theta}\right\} = \overline{\theta}$, i.e. $\theta = \theta^B$ in a steady state equilibrium, in which mismatched owners prefer to "buy first", is analogous.

$\square$

Proof of Proposition 5

Proof. The proof of the first claim is equivalent to the proof of Proposition 4 in the part about existence of a "Buyers' market" and "Sellers' market" equilibrium combined with the observation that if $\tilde{\theta} \geq \min\left\{\underline{\theta}, \theta^S\right\}$, then at $\theta = \min\left\{\underline{\theta}, \theta^S\right\}$ by Lemma 2, $V^{S1} \geq V^{B1}$ and if $\tilde{\theta} \leq \max\left\{\overline{\theta}, \theta^B\right\}$, $V^{B1} \geq V^{S1}$. Similarly, the proof of the second claim follows from the same observation, since if $\tilde{\theta} < \min\left\{\underline{\theta}, \theta^S\right\}$, then at $\theta = \min\left\{\underline{\theta}, \theta^S\right\}$, $V^{B1} > V^{S1}$, so entering as a "seller first" for a mismatched owner is dominated by entering as a "buyer first" for that value of θ, so a "Buyers' market" equilibrium fails to exist. The proof of the third claim is analogous.

It remains to show the last part of the first claim, that there exists an equilibrium with $\theta = \tilde{\theta}$, in which mismatched owners are indifferent between entering as "sellers first" and "buyers first". To see this, first of all note that at $\theta = \tilde{\theta}$, by Lemma 2 $V^{B1} = V^{S1}$ and by condition A2, $V^{B1} > V^m$. if $\tilde{\theta} > 1$ or $V^{S1} > V^m$, if $\tilde{\theta} < 1$. Note that given $\tilde{\theta}$ and the equilibrium conditions for the aggregate stock variables, one can find a $x_b \in [0, 1]$ such that:

$$\gamma x_b O = \left(q\left(\tilde{\theta}\right) + g\right) B_1 \tag{49}$$

$$\gamma\left(1 - x_b\right) O = \left(\mu\left(\tilde{\theta}\right) + g\right) S_1 \tag{50}$$

Therefore, having $\theta = \tilde{\theta}$ can be consistent with the equilibrium flow conditions. $\square$

Proof of Proposition 6

Proof. First of all, note that the instantaneous transition to $\theta\left(t\right) = \theta^S$ for $t \in (0, \infty]$ is consistent with mismatched agents' behavior, i.e. under $\theta = \theta^S$, $V^{S1} > V^{B1}$ and mismatched agents prefer "selling first" to "buying first" whenever they enter the housing market. Furthermore, they are indifferent between participating in the market and remaining "passive". It remains to show that $\theta\left(t\right) = \theta^S$ for $t \in (0, \infty]$ is consistent with the equilibrium flow conditions. Let us examine the behavior of stock variables given a jump from $\theta = \theta^B$ to $\theta = \theta^S$ at $t = 0$. We have that in every instant t the following equations hold:

$$O\left(t\right) + S_1\left(t\right) + 2S_2\left(t\right) = 1 - A\left(t\right)$$

Additionally, we have the population condition

$$B_0(t) + O(t) + S_1(t) + S_2(t) = 1$$

Therefore, $B_0(t) = A(t) + S_2(t)$. Also, note that $\theta^S = \frac{B_0(t)}{A(t)+S_1(t)+S_2(t)}$, so $S_1(t) = \left(\frac{1}{\theta^S} - 1\right) B_0(t)$ and $\dot{S}_1(t) = \left(\frac{1}{\theta^S} - 1\right) \dot{B}_0(t)$. Furthermore, we have the flow equations:

$$\dot{A}(t) = g - \left(\mu\left(\theta^S\right) + g\right) A(t)$$

$$\dot{S}_2(t) = - \left(\mu\left(\theta^S\right) + g\right) S_2(t)$$

so

$$\dot{B}_0(t) = g - \left(\mu\left(\theta^S\right) + g\right) B_0(t)$$

and also

$$\dot{S}_1(t) = \gamma x_s(t) O(t) - (\mu(\theta) + g) S_1(t)$$

where $x_s(t) \in [0,1]$ is the fraction of mismatched owners that participate in the market at time t. Multiplying the equation for $\dot{B}_0$ by $\left(\frac{1}{\theta^S} - 1\right)$ and using the relations, $S_1(t) = \frac{1-\theta^S}{\theta^S} B_0(t)$ and $\dot{S}_1(t) = \frac{1-\theta^S}{\theta^S} \dot{B}_0(t)$ we get that

$$\left(\frac{1}{\theta^S} - 1\right) g - \left(\mu\left(\theta^S\right) + g\right) S_1(t) = \gamma x_s(t) O(t) - (\mu(\theta) + g) S_1(t)$$

or

$$\left(\frac{1}{\theta^S} - 1\right) g = \gamma x_s(t) O(t) \tag{51}$$

Next, note that in a "Sellers' market" steady state the measure of regular owners, $O^B\left(\theta^B\right)$ equals $O^B\left(\theta^B\right) = \frac{g}{\gamma+g} \frac{\mu(\theta^B)}{q(\theta^B)+g} + \frac{g^2}{\gamma+g} \left(\frac{q(\theta^B)-\mu(\theta^B)}{(q(\theta^B)+g)(\mu(\theta^B)+g)}\right)$, while in a "Buyers' market" steady state that measure equals $O^S\left(\theta^S\right) = \frac{g}{\gamma+g} \frac{q(\theta^S)}{\mu(\theta^S)+g}$. Note that $\frac{q(\theta)}{\mu(\theta)+g}$ is decreasing in θ. Furthermore, Lemma 16 shows that for $\tilde{u}_0 \geq \tilde{u}_2$, $\theta^S \leq \frac{1}{\theta^B}$ so $\frac{\mu(\theta^B)}{q(\theta^B)+g} = \frac{q\left(\frac{1}{\theta^B}\right)}{\mu\left(\frac{1}{\theta^B}\right)+g} \leq \frac{q(\theta^S)}{\mu(\theta^S)+g}$ and so $\frac{q(\theta^B)-\mu(\theta^B)}{(q(\theta^B)+g)(\mu(\theta^B)+g)} \leq 0$. Thus, $O^B\left(\theta^B\right) \leq O^S\left(\theta^S\right)$. Furthermore, at $\theta = \bar{\theta}$, $O^B\left(\bar{\theta}\right) = \frac{g}{\gamma}\left(\bar{\theta}-1\right) < O^B\left(\theta^B\right)$ given that $O^B\left(\theta^B\right)$ is decreasing in θ^B. Similarly, we have that $B_0^S = \frac{g}{\mu(\theta^S)+g} > \frac{g}{\mu\left(\frac{1}{\theta^B}\right)+g} = B_0^B$. On the transition path the stock of regular owners $O(t)$ evolves according to

$$\dot{O} = - (\gamma + g) O(t) + \mu\left(\theta^S\right) S_2(t) + q\left(\theta^S\right) B_0(t)$$

We know that it starts from $O(0) = O^B$ and converges to $O^S > O^B$. Given that $S_2(t)$ and $B_0(t)$ are continuous and monotone, it follows that $O(t)$ will be monotone increasing over time. Therefore, $O(t) \geq O^B \geq O^B\left(\bar{\theta}\right)$. Therefore, as long as $\left(\frac{1}{\theta^S} - 1\right) g \leq \gamma O^B\left(\bar{\theta}\right)$, there is an $x_s(t) \in [0,1]$, $\forall t$, such that, condition (51) holds with equality given $O(t)$. Noting that $\left(\frac{1}{\theta^S} - 1\right) g = \gamma O^B\left(\bar{\theta}\right)$ is

equivalent to $\theta^S \geq \frac{1}{\theta}$, the result follows. To show that there exists a "simple" transition path from a $\theta = \theta^S$ to a $\theta = \theta^B$ steady state, it suffices to show that for this path to exist it must be the case that

$$\left(\theta^B - 1\right) g = \gamma x_b\left(t\right) O\left(t\right)$$

holds, $\forall t$, where $x_b\left(t\right) \in [0,1]$ is the fraction of mismatched owners that participate in the market at time t. Noting that $O\left(0\right) = O^S > O^B$, $\left(\theta^B - 1\right) g = \gamma x_b O^B$ for some $x_b \in [0,1]$ in a "Sellers' market" equilibrium, and that $O\left(t\right)$ is monotone decreasing in t, it follows that the condition is indeed satisfied, for some $x_b\left(t\right) \in [0,1]$, $\forall t$. $\qquad \square$

Proof of Corollary 7

Proof. The result follows from Proposition 6 and from noting that the solution to the equation for $B_0\left(t\right)$ as the economy transitions from $\theta = \theta^B$ to $\theta = \theta^S$ is:

$$B_0\left(t\right) = B_0\left(0\right) \exp\left\{-\left(\mu\left(\theta^S\right) + g\right) t\right\} + g \int_0^t \exp\left\{-\left(\mu\left(\theta^S\right) + g\right)\left(t - s\right)\right\} ds$$

where $B_0\left(0\right) = \frac{g}{(q(\theta^B)+g)}$. Simplifying further, we get that

$$B_0\left(t\right) = g \exp\left\{-\left(\mu\left(\theta^S\right) + g\right) t\right\} \left(\frac{1}{\left(q\left(\theta^B\right) + g\right)} - \frac{1}{\left(\mu\left(\theta^S\right) + g\right)}\right) + \frac{g}{\left(\mu\left(\theta^S\right) + g\right)}$$

Note, however, that with a symmetric matching function $\mu\left(\theta^S\right) = q\left(\theta^B\right)$ since $\theta^B = \frac{1}{\theta^S}$. Therefore, it follows that $B_0\left(t\right) = B_0\left(0\right) = \frac{g}{q(\theta^B)+g}$ and so $\dot{B}_0\left(t\right) = 0$, which implies that $\dot{S}_1\left(t\right) = 0$. Therefore, $S_1\left(t\right) = \frac{1-\theta^S}{\theta^S} B_0\left(0\right) = \left(\theta^B - 1\right) B_0\left(0\right) = B_1\left(0\right)$, and the only variables that adjust are $S_2\left(t\right)$ and $A\left(t\right)$ with $\dot{S}_2\left(t\right) = -\dot{A}\left(t\right)$. $\qquad \square$

Proof of Proposition 8

Proof. Consider the difference between the two value functions, $D\left(\theta\right) = V^{B1} - V^{S1}$.

$$
D\left(\theta\right) = \frac{\mu\left(\theta\right)\left[\left(1 - \frac{1}{\theta}\right)\left(u - \chi - c + \lambda\left(\overline{V}_N - v^{B0}\right)\right) +\right.}{\left(\rho + q\left(\theta\right) + \lambda\right)\left(\rho + \mu\left(\theta\right) + \lambda\right)}
$$

$$
\frac{\left.+\frac{\lambda\left(1 - \frac{1}{\theta}\right)q(\theta)}{(r+\mu(\theta))(r+q(\theta))}\left[\rho V - \left(c - k\right)\right] + \left(1 + \frac{1}{\theta}\right)\lambda\left(p_N - p\right)\right]}{\left(\rho + q\left(\theta\right) + \lambda\right)\left(\rho + \mu\left(\theta\right) + \lambda\right)} \tag{52}
$$

Consider the case of $1 < \theta \leq \overline{\theta}$, so $\overline{V}_N = V_N^{B1}$. If $\overline{V}_N = V_N^{B1}$, this difference simplifies further to

$$
D\left(\theta\right) = \frac{\mu\left(\theta\right)\left[\left(1 - \frac{1}{\theta}\right)\left(1 + \frac{\lambda}{\rho+q(\theta)}\right)\left(u - \chi - c\right) + \left(1 + \frac{1}{\theta}\right)\lambda\left(p_N - p\right)\right]}{\left(\rho + q\left(\theta\right) + \lambda\right)\left(\rho + \mu\left(\theta\right) + \lambda\right)} \tag{53}
$$

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Suppose that $p_N < p$ and define define θ_{B1}^{PR} as the solution to

$$\frac{\theta_{B1}^{PR} - 1}{\theta_{B1}^{PR} + 1}\left(1 + \frac{\lambda}{\rho + q\left(\theta_{B1}^{PR}\right)}\right) = \frac{\lambda\left(p - p_N\right)}{(u - \chi - c)} \tag{54}$$

Therefore, θ_{B1}^{PR} is the value of θ that leaves a mismatched owner indifferent between entering as a "buyer first" and a "seller first" if he anticipates a price change of $p_N - p$ and a market tightness of $\theta > 1$ after the price change. Note that θ_{B1}^{PR} is increasing in $p - p_N$ if $\theta_{B1}^{PR} \geq 1$. Therefore, a sufficient condition for mismatched owners to prefer "selling first" to "buying first", given $1 < \theta \leq \bar{\theta}$, is that $\theta_{B1}^{PR} > \bar{\theta}$. In that case, at $\theta = \bar{\theta}$ mismatched agents still prefer to enter as "sellers first" prior to the price change. Note that for any $\lambda > 0$, one can find a sufficiently low value of p_N relative to p, $\underline{p} < p$, so that $\theta_{B1}^{PR} > \bar{\theta}$ for $p_N < \underline{p}$. Since θ_{B1}^{PR} is increasing in λ, it follows that $\underline{p}$ is increasing in p with $\underline{p} \to p$ as $\lambda \to \infty$.

Similarly, consider the case of $\underline{\theta} \leq \theta < 1$, so $\overline{V}_N = V_N^{S1}$. In that case the difference in value functions can be written as

$$D\left(\theta\right) = \frac{\mu\left(\theta\right)\left[\left(1 - \frac{1}{\theta}\right)\left(1 + \frac{\lambda}{\rho + \mu(\theta)}\right)(u - \chi - c) + \left(1 + \frac{1}{\theta}\right)\lambda\left(p_N - p\right)\right]}{(\rho + q\left(\theta\right) + \lambda)(\rho + \mu\left(\theta\right) + \lambda)} \tag{55}$$

Suppose that $p_N > p$ and define θ_{S1}^{PR} as the solution to

$$\frac{\theta_{S1}^{PR} - 1}{\theta_{S1}^{PR} + 1}\left(1 + \frac{\lambda}{\rho + \mu\left(\theta_{S1}^{PR}\right)}\right) = \frac{\lambda\left(p - p_N\right)}{(u - \chi - c)} \tag{56}$$

Similarly, to the case of θ_{B1}^{PR}, θ_{S1}^{PR} is increasing in $p - p_N$ if $\theta_{S1}^{PR} \leq 1$. Then, a sufficient condition for mismatched owners to prefer "buying first" to "selling first", given $\underline{\theta} \leq \theta < 1$ is that $\theta_{S1}^{PR} < \underline{\theta}$. For any $\lambda > 0$, one can find a sufficiently high value of p_N relative to p, $\bar{p} > p$, so that $\theta_{S1}^{PR} < \underline{\theta}$ for $p_N > \underline{p}$. Since θ_{S1}^{PR} is decreasing in λ, it follows that $\bar{p}$ is decreasing in p with $\bar{p} \to p$ as $\lambda \to \infty$. $\quad\square$

Proof of Proposition 9

Proof. To construct such equilibria, we proceed in three steps.

First, we consider the second regime $X\left(t\right) = 1$. In that regime the equilibrium market tightness, θ_1, is the smallest solution to

$$\frac{u - \chi}{\rho} = \frac{u - \chi - k}{\rho + \mu\left(\theta\right)} + \frac{\mu\left(\theta\right)\left(c - k + q\left(\theta\right)V\right)}{(\rho + \mu\left(\theta\right))(\rho + q\left(\theta\right))} \tag{57}$$

where $V = \frac{u}{\rho} - \frac{\gamma}{\rho + \gamma}\frac{\chi}{\rho}$. Also, given Lemma 15, $\theta_1 > \underline{\theta}$ for a sufficiently small value of c, i.e. for $c < \bar{c} < u - \chi$. Given the results in Proposition 4, for a market tightness of θ_1 mismatched owners are indifferent between remaining passive and entering the market, and conditional on entering will prefer to enter as "sellers first".

Second, consider the value function of a mismatched owner who enters as a "buyer first" in the

first regime. We have that:

$$V_0^{B1} = \frac{u - \chi - k}{\rho + q(\theta_0) + \lambda} + \frac{q(\theta_0)}{\rho + q(\theta_0) + \lambda} \left(V_0^{S2} - p_0\right) + \frac{\lambda}{\rho + q(\theta_0) + \lambda} V^m$$

where

$$V_0^{S2} = v^{S2}(\theta_0) + \frac{\lambda}{\rho + \mu(\theta_0) + \lambda} \left(v^{S2}(\theta_1) - v^{S2}(\theta_0) + p_1 - p_0\right) + p_0$$

with

$$v^{S2}(\theta_i) = \frac{c - k}{\rho + \mu(\theta_i)} + \frac{\mu(\theta_i)}{\rho + \mu(\theta_i)} V$$

The third term arises since in the second regime an agent is indifferent between entering as a seller first and remaining mismatched. If a mismatched owner is indifferent between remaining passive and entering as "buyers first" in the first regime, then $V^m = V_0^{B1}$, or $\theta_0 = \theta_0(\lambda)$, where $\theta_0(\lambda)$ is the largest solution to

$$\frac{u - \chi}{\rho} = \frac{u - \chi - k}{\rho + q(\theta)} + \frac{q(\theta)\left(\tilde{c}^2(\theta, \lambda) - k + \mu(\theta)V\right)}{(\rho + \mu(\theta))(\rho + q(\theta))} \tag{58}$$

where $\tilde{c}^2(\theta, \lambda) = c + \lambda \frac{\rho + \mu(\theta)}{\rho + \mu(\theta) + \lambda} \left(v^{S2}(\theta_1) - v^{S2}(\theta) + p_1 - f(\theta)\right) \leq c$ for $\theta > 1$. Note that for $\lambda = 0$, and given a symmetric matching function, the solution to this equation is $\theta_0(0) = \frac{1}{\theta_1}$ Furthermore, by Lemma 15, θ_1 is decreasing in c and so $\theta_0(0)$ is increasing in c, so for c sufficiently small, $1 < \theta_0(0) < \bar{\theta}$. Away from the limit $\lambda \to 0$, with $\tilde{c}(\theta, \lambda) < c$, we therefore, have that $\theta_0(\lambda) < \theta_0(0)$. By the implicit function theorem $\theta_0(\lambda)$ is continuous in λ.

Now, consider the difference $D_0(\theta_0) = V_0^{B1}(\theta_0) - V_0^{S1}(\theta_0)$, where

$$V_0^{S1}(\theta_0) = \frac{u - \chi - k}{\rho + \mu(\theta_0) + \lambda} + \frac{\mu(\theta_0)}{\rho + \mu(\theta_0) + \lambda} \left(V_0^{B0} + p_0\right) + \frac{\lambda}{\rho + \mu(\theta_0) + \lambda} V^m$$

where

$$V_0^{B0} = v^{B0}(\theta_0) + \frac{\lambda}{\rho + q(\theta_0) + \lambda} \left(v^{B0}(\theta_1) - v^{B0}(\theta_0) + p_0 - p_1\right) + p_0$$

with

$$v^{B0}(\theta_i) = \frac{c - k}{\rho + q(\theta_i)} + \frac{q(\theta_i)}{\rho + q(\theta_i)} V$$

and define $\tilde{c}^0(\theta, \lambda) = c + \lambda \frac{\rho + q(\theta)}{\rho + q(\theta) + \lambda} \left(v^{B0}(\theta_1) - v^{B0}(\theta) + f(\theta) - p_1\right) > c$ for $\theta > 1$. Then we have that

$$D_0(\theta_0) = \frac{\mu(\theta_0)\left(\left(1 - \frac{1}{\theta_0}\right)(u - \chi - \tilde{c}^2(\theta, \lambda) + \lambda V^m) - \tilde{c}^0(\theta, \lambda) + \tilde{c}^2(\theta, \lambda)\right)}{(\rho + q(\theta_0) + \lambda)(\rho + \mu(\theta_0) + \lambda)}$$

Note that $\lim_{\lambda \to 0} D_0(\theta_0(\lambda)) > 0$, so that will also be the case for λ sufficiently close to 0. Therefore, there exists a $\bar{\lambda}$ such that for $\lambda < \bar{\lambda}$, $V_0^{B1} > V_0^{S1}$, and $V_0^{B1} = V^m$ for θ_0 given as the largest solution to (58).

Third, we show that given the values of θ_0 and θ_1 in the two regimes, jumps from θ_0 to θ_1 are

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consistent with the evolution of stock variables. This follows directly from Proposition 6 above. $\quad\square$

Proof of Proposition 10

Proof. To show the first part, suppose that $\theta^{B0} < \theta^{S2}$. It follows that $v^{S2} < \frac{u-\chi-k}{\rho} < V^{S1}$ for $\theta < \theta^{B0}$, and so $\theta_1^{SB} > \theta^{B0}$. Also, since $V^{S1} > v^{B0}$ for $\theta > \theta^{B0}$, it follows that θ_1^{SB} lies to the right of the value of θ, at which v^{S2} and v^{B0} cross. Similarly, $\theta_2^{SB} < \theta^{S2}$ and θ_2^{SB} lies to the left of the point where v^{S2} and v^{B0} cross. Therefore, $\theta_1^{SB} > \theta_2^{SB}$ and so $V^{SB} < \max\{V^{B1}, V^{S1}\}$ for any θ and it is never optimal for a mismatched owner to enter as both a buyer and a seller.

To show the second part, suppose that $\theta^{S2} < \theta^{B0}$. It follows that $v^{S2} < \frac{u-\chi-k}{\rho} < V^{S1}$ for $\theta > \theta^{S2}$, and so $\theta_1^{SB} > \theta^{S2}$. Also, since $V^{S1} < v^{B0}$ for $\theta < \theta^{B0}$, it follows that θ_1^{SB} lies to the left of the value of θ, at which v^{S2} and v^{B0} cross. Similarly, $\theta_2^{SB} < \theta^{B0}$ and θ_2^{SB} lies to the right of the point where v^{S2} and v^{B0} cross. Therefore, $\theta_1^{SB} < \theta_2^{BS}$ and $V^{SB} \geq \max\{V^{B1}, V^{S1}\} > \frac{u-\chi-k}{\rho}$ for $\theta \in [\theta_1^{BS}, \theta_2^{BS}]$. In that case, depending on the value of θ_1^{SB}, it is possible for a steady state equilibrium to exist, in which agents enter as both buyers and sellers.

Note that in an equilibrium where agents enter as both buyers and sellers, we have the following flow conditions and housing and population conditions:

$$O + SB + 2S_2 = 1 - A$$

$$B_0 + O + SB + S_2 = 1$$

From these equations, it follows that $B_0 = A + S_2$. Therefore, $\theta = \frac{B_0 + SB}{B_0 + SB} = 1$. Given $\theta = 1$, one can solve for the aggregate stock variables given the flow conditions. $\quad\square$

Proof of Proposition 11

Proof. We define $\tilde{u}_0(\theta) = c + (\rho p(\theta) - R) = c + \epsilon \rho f(\theta)$ and $\tilde{u}_2(\theta) = u_2 - (\rho p(\theta) - R) = c - \epsilon \rho f(\theta)$. We proceed as in the proof of Proposition 11 above.

First, we consider the second regime $X(t) = 1$. In that regime the equilibrium market tightness, θ_1, is the smallest solution to

$$\frac{u-\chi}{\rho} = \frac{u-\chi-k}{\rho + \mu(\theta)} + \frac{\mu(\theta)\left(\tilde{u}_0(\theta) - k + q(\theta)V\right)}{(\rho + \mu(\theta))(\rho + q(\theta))} \tag{59}$$

where $V = \frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}$. Also, since $f(\theta)$, applying the implicit function theorem gives that θ_1 is decreasing in c, so $\theta_1 > \underline{\theta}$ for a sufficiently small value of c, i.e. for $c < \bar{c} < u - \chi$. Given the results in Proposition 4, for a market tightness of θ_1 mismatched owners are indifferent between remaining passive and entering the market, and conditional on entering will prefer to enter as "sellers first".

Second, we consider the value function of a mismatched owner who enters as a "buyer first" in the first regime. We have that:

$$V_0^{B1} = \frac{u-\chi-k}{\rho + q(\theta_0) + \lambda} + \frac{q(\theta_0)}{\rho + q(\theta_0) + \lambda}\left(V_0^{S2} - p_0\right) + \frac{\lambda}{\rho + q(\theta_0) + \lambda}V^m$$

where
$$V_0^{S2} = v^{S2}(\theta_0) + \frac{\lambda}{\rho + \mu(\theta_0) + \lambda}\left(v^{S2}(\theta_1) - v^{S2}(\theta_0) + p_1 - p_0\right) + p_0$$

with
$$v^{S2}(\theta_i) = \frac{\tilde{u}_2(\theta) - k}{\rho + \mu(\theta_i)} + \frac{\mu(\theta_i)}{\rho + \mu(\theta_i)}V$$

We define
$$\tilde{U}_0^0 \equiv c + \epsilon\rho f(\theta) + \lambda\left[\epsilon\left(f(\theta) - f(\theta_1)\right) + v_1^{B0}\right]$$

and
$$\tilde{U}_2^0 \equiv c - \epsilon\rho f(\theta) + \lambda\left[-\left(f(\theta) - f(\theta_1)\right) + v_1^{S2}\right]$$

where $v_1^{S2} = \frac{c - \epsilon\rho f(\theta_1) - k}{\rho + \mu(\theta_1)} + \frac{\mu(\theta_1)}{\rho + \mu(\theta_1)}V$ and $v_1^{B0} = \frac{c + \epsilon\rho f(\theta_1) - k}{\rho + q(\theta_1)} + \frac{q(\theta_1)}{\rho + q(\theta_1)}V$. Note that $\tilde{U}_0^0 \geq c$ and $\tilde{U}_2^0 \leq c$ for $\theta > 1$. Then $\theta_0 = \theta_0(\epsilon, \lambda)$, where $\theta_0(\epsilon, \lambda)$ is the largest solution to

$$\frac{u - \chi}{\rho} = \frac{u - \chi - k}{\rho + q(\theta_0)} + \frac{q(\theta_0)}{\rho + q(\theta_0)(\rho + \mu(\theta_0) + \lambda)}\left(\tilde{U}_2^0 - k + \mu(\theta_0)V\right) \quad (60)$$

For $\epsilon \to 0$ and $\lambda \to 0$, this condition becomes

$$\frac{u - \chi}{\rho} = \frac{u - \chi - k}{\rho + q(\theta_0)} + \frac{q(\theta_0)}{\rho + q(\theta_0)(\rho + \mu(\theta_0))}\left(c - k + \mu(\theta_0)V\right)$$

which has a solution $\theta_0(0,0) < \bar{\theta}$, for c sufficiently small. Away from this limit, given $\tilde{U}_2^0 \leq c$, we have that $\theta_0(\epsilon, \lambda) < \theta_0(0,0) < \bar{\theta}$. Also $\theta_0(\epsilon, \lambda)$ is continuous in ϵ and λ.

We can express the difference $D_0(\theta_0) = V_0^{B1}(\theta_0) - V_0^{S1}(\theta_0)$ as

$$D_0(\theta_0) = \frac{\mu(\theta_0)\left(\left(1 - \frac{1}{\theta_0}\right)\left(u - \chi - \tilde{U}_2^0 + \lambda V^m\right) - \tilde{U}_0^0 + \tilde{U}_2^0\right)}{(\rho + q(\theta_0) + \lambda)(\rho + \mu(\theta_0) + \lambda)}$$

Note that $\lim_{\epsilon \to 0, \lambda \to 0} D_0(\theta_0(\epsilon, \lambda)) > 0$, so by continuity of $D_0(\theta)$ and of $\theta_0(\epsilon, \lambda)$, that will also be the case for ϵ and λ sufficiently close to 0. Therefore, there will be an $\bar{\epsilon}$ and $\bar{\lambda}$ such that for $\epsilon < \bar{\epsilon}$ and $\lambda < \bar{\lambda}$, $V_0^{S1} > V_0^{B1}$, and $V_0^{B1} = V^m$ for θ_0 given as the solution to (60).

Third, we show that given the values of θ_0 and θ_1 in the two regimes, jumps from θ_0 to θ_1 are consistent with the evolution of stock variables. This follows directly from Proposition 6 above. $\qquad \square$