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No. 9943

**INTERNAL VERSUS EXTERNAL  
GROWTH IN INDUSTRIES WITH  
SCALE ECONOMIES: A  
COMPUTATIONAL MODEL OF  
OPTIMAL MERGER POLICY**

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# **INTERNAL VERSUS EXTERNAL GROWTH IN INDUSTRIES WITH SCALE ECONOMIES: A COMPUTATIONAL MODEL OF OPTIMAL MERGER POLICY**

**Ben Mermelstein, Northwestern University**  
**Volker Nocke, University of Mannheim and CEPR**  
**Mark Satterthwaite, Northwestern University**  
**Michael Whinston, Massachusetts Institute of Technology and CEPR**

Discussion Paper No. 9943  
April 2014

Centre for Economic Policy Research  
77 Bastwick Street, London EC1V 3PZ, UK  
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820  
Email: [cepr@cepr.org](mailto:cepr@cepr.org), Website: [www.cepr.org](http://www.cepr.org)

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CEPR Discussion Paper No. 9943

April 2014

## ABSTRACT

### Internal versus External Growth in Industries with Scale Economies: A Computational Model of Optimal Merger Policy\*

We study optimal merger policy in a dynamic model in which the presence of scale economies implies that firms can reduce costs through either internal investment in building capital or through mergers. The model, which we solve computationally, allows firms to invest or propose mergers according to the relative profitability of these strategies. An antitrust authority is able to block mergers at some cost. We examine the optimal policy when the antitrust authority can commit to a policy rule and when it cannot commit, and consider both consumer value and aggregate value as possible objectives of the antitrust authority. We find that optimal policy can differ substantially from what would be best considering only welfare in the period the merger is proposed. We also find that the ability to commit can lead to a significant welfare improvement. In general, antitrust policy can greatly affect firms' optimal investment behavior, and firms' investment behavior can in turn greatly affect the antitrust authority's optimal policy.

JEL Classification: L13 and L41

Keywords: antitrust, commitment, entry, investment and merger

Ben Mermelstein  
Northwestern University  
633 Clark St  
Evanston, IL 60208  
USA

Volker Nocke  
Chair in Microeconomics  
Department of Economics  
University of Mannheim  
L7, 3-5  
D-68131 Mannheim  
GERMANY

Email:  
benjaminmermelstein2014@u.northwestern.  
edu

Email: nocke@uni-mannheim.de

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Mark Satterthwaite  
Kellogg School of Management  
Northwestern University  
Evanston, IL 60208  
USA

Email: m-  
satterthwaite@kellogg.northwestern.edu

For further Discussion Papers by this author see:  
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Michael Whinston  
MIT Department of Economics  
Office E62-527  
50 Memorial Drive Cambridge  
Massachusetts 02142  
USA

Email: whinston@mit.edu

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\*We thank John Asker, Dennis Carlton, Alan Collard-Wexler, Uli Doraszelski, Gautam Gowrisankaran, Ariel Pakes and Patrick Rey for their comments, as well as seminar audiences at Bates White, Chicago, ECARES, Harvard, NYU, Penn, Stanford, Toulouse, UCLA, the 2012 Northwestern Searle Antitrust conference, the 2012 SciencesPo Workshop on Dynamic Models in IO, the 2013 CRESSE Conference, the 2013 NBER IO Summer Institute, the 2013 EARIE conference, the 2013 SFB-TR15 Meeting, and the 2013 Annual Meeting of the IO section of the German Economic Association. Mermelstein and Satterthwaite thank the General Motors Research Center for Strategy and Management at Northwestern University's Kellogg School of Management for financial support. Nocke gratefully acknowledges financial support from the European Research Council (ERC Starting Grant 313623). Whinston thanks the National Science Foundation and the Toulouse Network for Information Technology for financial support.

Submitted 08 April 2014

# 1 Introduction

Most analyses of optimal horizontal merger policy are static.<sup>1</sup> But many real-world mergers occur in markets in which dynamic issues are a central feature of competition among firms. In this paper, we analyze merger policy in the context of a model in which the presence of economies of scale presents firms with the opportunity to lower their average and marginal costs through capital accumulation. These scale economies are also the source of merger-related efficiencies, as a combination of capital lowers average and marginal costs. Thus, in such settings, an antitrust authority's merger approval decisions must weigh any extra cost reductions gained by allowing a merger (compared to insisting on internal growth) against the deadweight losses arising from increased market power.

As one example, consider the 2011 attempted merger between AT&T and T-Mobile USA.<sup>2</sup> The merger would have combined the network infrastructure of the two firms. Proponents of the merger argued that this combination would greatly improve both firms' service. Opponents countered that the merger would increase market power, and that absent the merger the two firms would each have incentives to independently increase their networks. Thus, the Federal Communications Commission and Department of Justice faced the question of whether any efficiency gains from increased infrastructure scale due to the merger (which in this case would be realized on the demand side through enhanced quality) were sufficient to justify the increase in market power.

We study these issues computationally in a dynamic industry model in which, in each period, active firms compete and also make investments to increase their capital stock. Economies of scale in production imply that mergers generate efficiencies. The model is similar to Pakes and McGuire (1994) and Ericson and Pakes (1995), but with some important differences. Most significantly, we modify the investment technology to make it *merger neutral*, so that mergers do not change the investment opportunities that are available in the market. Our investment technology also allows for significantly richer investment dynamics than do most computational dynamic models, as firms can increase their capital stocks by multiple units, and new entrants can choose endogeneously how many units of capital to build when entering.

In addition, we introduce the possibility for firms to merge as well as an antitrust authority who can block proposed mergers. The decision to propose a merger is endogenous and determined through a bargaining process. In general, bargaining over mergers involves externalities and the theory literature currently has few satisfying general solutions for such settings. For this reason, the present paper restricts attention to industries in which there are at most two active firms in any period. Doing so allows us to use the familiar Nash bargaining solution, and makes clear how the prospect of bargaining over mergers impacts investment incentives. Our approach to modeling the antitrust authority considers both the case in which the authority can commit to a policy rule and the case in which commitment is impossible. We refer to the policy that emerges in the latter case as a *Markov perfect policy*. While issues of commitment and Markov perfection in dynamic environments have been a focus of the macro and political economy literatures for some time [see, e.g., Kydland and Prescott (1977), Barro and Gordon (1983), and Krusell and Rios-Rull (1996)], they have received scant attention in the antitrust

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<sup>1</sup>For example, see the classic papers by Williamson (1968) and Farrell and Shapiro (1990).

<sup>2</sup>See Pittman and Li (2013) and DeGraba and Rossten (2014), and the references therein.

literature.<sup>3</sup> Within this framework, we consider both maximization of consumer surplus and of aggregate surplus as possible objectives of the authority.

We begin in Section 2 by describing our model. Section 3 formally defines a Markov perfect equilibrium and discusses how we construct these equilibria computationally.

Section 4 analyzes firm behavior under two extreme antitrust policies: one in which no mergers are allowed, and the other in which all are. We examine firms' investment behavior and merger decisions in three different markets: a large one, an intermediate one, and a small one. We first describe firm behavior and industry dynamics for these three markets when no mergers are allowed. In the large market steady state, the industry spends most of its time in states with relatively symmetric capital levels for the firms, although should the industry through chance depreciation end up at a highly asymmetric position, it stays there for a long time. In contrast, the small market steady state is highly asymmetric, with one firm often dominating the industry. The intermediate market no-mergers equilibrium lies between these two.

We then begin to explore the impact of merger policy by studying the Markov perfect equilibria when instead all mergers are allowed. We find a number of striking features. Not surprisingly, the steady state when all mergers are allowed involves a monopoly or near-monopoly market structure much more often than when mergers are prohibited. It also involves a lower average level of capital. This arises because total investment is lower in monopoly and near-monopoly states. Investment behavior also changes when mergers are allowed. Particularly striking is significantly greater investment by small firms in states in which one firm is very dominant, a form of "entry for buyout" [Rasmusen (1988)]. The steady state discounted expected value of consumer surplus, aggregate surplus, and incumbent firm profits all decline when all mergers are allowed, while the discounted expected value of total industry profit (including entrants) is essentially unchanged. The surprising reduction in incumbent profit is driven by the behavior, just noted, of new entrants after a merger. This investment is done at high investment costs and dissipates a great deal of industry profit.

In Section 5 we examine optimal merger policy, considering as objectives both discounted expected consumer surplus ("consumer value") and discounted expected aggregate surplus ("aggregate value"). We begin by looking at the static benchmark, examining which mergers would be approved from a myopic static perspective, considering only the effect on welfare in the period the merger is proposed. From that perspective, very few mergers are consumer surplus-enhancing, while many mergers increase aggregate surplus.

We then identify both the Markov perfect policy (the no-commitment policy outcome) and the optimal commitment policy under both objectives. With a consumer value objective, the Markov perfect policy and the commitment policy are almost identical and basically allow no mergers, just as with the static consumer surplus criterion.

With an aggregate value objective, however, the no-commitment and commitment policies each allow many fewer mergers than the optimal static policy. The reason is that, as seen in Section 4, allowing mergers leads to inefficient entry-for-buyout behavior, which makes mergers much less attractive socially, whether or not commitment is possible. Commitment can

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<sup>3</sup>We briefly discuss below the small literature on antitrust policy that has addressed such issues. Issues of commitment have been considered in the literature on optimal regulation; see, for example, Laffont and Tirole (1993).

lead to a significant gain in aggregate value. Moreover, we find that endowing the antitrust authority with a consumer value objective achieves a substantial gain in aggregate value when the antitrust authority cannot commit to its approval policy. This finding is consistent with the suggestion of Lyons (2002), but for a different reason: in Lyons (2002) this gets the firms to propose more attractive mergers, while here it induces much better investment behavior. Overall, we find that optimal merger policy — whether with commitment or without commitment — is significantly affected by firms’ investment behavior, and firms’ investment behavior is in turn significantly affected by merger policy.

Finally, in Section 6 we explore several extensions/robustness checks. First, we compare optimal merger policy to the optimal policy for a planner who can control investment and merger behavior but not output competition. Because this planner can control investment behavior, he is much more inclined to merge the firms to achieve short-run scale economies. Second, we study the effect of parameter changes that improve the investment efficiency of small vs. large firms. While entry-for-buyout behavior persists, this change reduces the social costs of entrants’ pre-merger capital investments. Third, we examine how the likelihood/speed of entry following a merger affects optimal policy. We find that while quicker entry raises consumer and aggregate surplus, under an aggregate value criterion it makes the Markov perfect policy less accepting of mergers as it exacerbates the entry-for-buyout problem. Fourth, we explore whether our model may have multiple equilibria. We find little evidence of multiplicity.

Section 7 concludes.

The paper is related to several strands of literature. The first is theoretical work on dynamic merger policy. Most of this work examines models in which two mergers between two non-overlapping pairs of firms can take place sequentially in static models of competition [e.g., Nilssen and Sorgard (1998), Motta and Vasconcelos (2005), and Matushima (2001)]. An exception is Nocke and Whinston (2010). They study a many-period dynamic model in which mergers become feasible stochastically through time and establish conditions under which the optimal dynamic (commitment) policy of an antitrust authority who maximizes consumer value is the fully myopic policy that approves a merger if and only if it would raise consumer surplus in the period it is proposed. The model in this paper departs from Nocke and Whinston (2010) in a number of ways, most notably in introducing investment by firms and in locating the efficiency gains from merger in the achievement of scale economies through capital acquisition.<sup>4</sup>

A second related strand of literature examines dynamic models of industry equilibrium with investment, most notably Pakes and McGuire (1994) and Ericson and Pakes (1995). Some of this literature has examined the effects of one-time mergers on industry evolution [e.g., Berry and Pakes (1993), Cheong and Judd (2000), Benkard, Bodoh-Creed, and Lazarek (2010)]. Closest to our work are Gowrisankaran (1997) and (1999). Gowrisankaran (1999) introduces an endogenous merger bargaining game into the Pakes-McGuire model and examines industry evolution when firms can choose whether, when, and with whom to merge. Our model differs in a number of respects: First, as mentioned above, we modify the Pakes-McGuire/Ericson-Pakes investment technology to make it merger neutral, and give entrants the same technology as incumbents with zero capital. Second, we locate the efficiency effects of mergers in scale

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<sup>4</sup>The model here also differs from Nocke and Whinston (2010) in that firms that do not merge in a given period may consummate a merger with different efficiencies (i.e., with different capital levels) in future periods.

economies achieved through capital acquisition, rather than in randomly drawn synergy gains. Third, we focus on settings with just two active firms and use the Nash bargaining solution over mergers. While restrictive, we do this because it allows us to examine a case in which the bargaining model is well accepted and easily understood. In unpublished work, Gowrisankaran (1997) introduces antitrust policy into the Gowrisankaran (1999) model. Specifically, he examines the effect of commitments to Herfindahl-based policies that block mergers if they result in a Herfindahl index above some maximum threshold and finds little effect of varying the threshold on welfare. We differ in considering a broader range of possible policy commitments and in examining the equilibrium policy when the antitrust authority cannot commit. We also find quite different results, with policy having significant effects. In both papers, optimal policy differs substantially from what would be myopically (i.e., statically) optimal.

## 2 The Model

We study a dynamic industry model in which firms may invest in capacity, or alternatively merge, to increase their capital stocks and harness scale economies. The model follows in broad outline Pakes and McGuire (1994) and Ericson and Pakes (1995), but with some important differences in its investment technology, as well as in the introduction of mergers and merger policy.

### 2.1 Static demand, costs, and competition

In each period, active firms produce a homogeneous good in a market in which demand is  $Q(p) = B(A - p)$ . The production technology, which requires capital and labor, is described by the production function  $F(K, L) = K^{\beta\theta}L^{(1-\beta)\theta}$ , where the capital share parameter is  $\beta \in (0, 1)$  and the scale economy parameter is  $\theta > 1$ . Normalizing the price of labor to be 1, for a fixed level of capital  $K$ , this production function gives rise to the short-run cost function

$$C(Q|K) = \frac{Q^{1/(1-\beta)\theta}}{K^{\beta/(1-\beta)}}$$

with marginal cost

$$C_Q(Q|K) = \left( \frac{1}{(1-\beta)\theta} \right) \frac{Q^{[1/(1-\beta)\theta]-1}}{K^{\beta/(1-\beta)}}.$$

With this technology, a merger that combines the capital of two identical firms reduces both average and marginal cost if their joint output remains unchanged. This effect will be the source of merger-related efficiencies in our model. Letting  $R$  measure the extent of this cost reduction, we have

$$R \equiv \frac{C_Q(2Q|2K)}{C_Q(Q|K)} = \frac{C(2Q|2K)/Q}{C(Q|K)/Q} = 2^{(\frac{1}{1-\beta})(\frac{1-\theta}{\theta})}.$$

Note in particular that the marginal cost reduction depends on the scale economy parameter  $\theta$  and capital share  $\beta$ , but is independent of the output level (and hence demand). In our computations we will focus on a case in which  $\beta = 1/3$  and  $\theta = 1.1$ . Given these values,  $R$  is 0.91.



In each period, active firms engage in Cournot competition given their capital stocks (a firm with no capital produces nothing), resulting in profit  $\pi(K_i, K_{-i})$  for a firm with capital stock  $K_i$  when its rival has capital stock  $K_{-i}$ .<sup>5</sup> The resulting consumer surplus is denoted  $CS(K_1, K_2)$ .

## 2.2 Investment and Depreciation

In Pakes and McGuire (1994) and Ericson and Pakes (1995) a firm chooses in each period how much money to invest, with the probability of successfully adding one unit of capital increasing in the investment level. We depart from this technology because in a model of mergers it would impose a significant inefficiency to mergers, as each merger between two firms would remove an investment possibility from the market.<sup>6</sup> Instead, we specify an investment technology that is *merger neutral* at a market level. By that we mean that a planner who controlled the firms and wanted to achieve at least cost any fixed increase in the market's aggregate capital stock would be indifferent about whether the firms merge. With this assumption we isolate the market-level efficiency effects of mergers fully in the scale economies of the production function.<sup>7</sup> Specifically, we imagine that there are two ways that a firm can invest.

The first is *capital augmentation*: each unit  $j$  of capital that a firm owns can be doubled at some cost  $c_j \in [\underline{c}, \bar{c}]$  drawn from a distribution  $F$ . The draws for different units of capital are independent and identically distributed. Thus, for a firm that has  $N_K$  units of capital, there are  $N_K$  cost draws. Given these draws, if the firm decides to augment  $m \leq N_K$  units of capital it will do so for the capital units with the cheapest cost draws. Note that capital augmentation is completely merger neutral: when two firms merge, their collective investment possibilities do not change.

The second is *greenfield investment*: a firm can build as many capital units as it wants at a cost  $c_g \in [\bar{c}, \bar{c}_g]$  drawn from a distribution  $G$ . Greenfield investment allows a firm whose capital stock is zero to invest, albeit at a cost that exceeds that of capital augmentation. We also choose the range of greenfield costs  $[\bar{c}, \bar{c}_g]$  to be small so that this investment technology is approximately merger neutral. (It would be fully merger neutral if  $\bar{c}_g = \bar{c}$ ; in our computations we introduce uncertain greenfield investment costs to ensure existence of equilibrium.)

Note that with our assumptions investment opportunities will be merger neutral at the market level, but not at the firm level – larger firms do have (stochastically) lower investment costs.

As we discuss shortly, our model allows for entry. In contrast to Pakes and McGuire (1994) and Ericson and Pakes (1995), we endow an entrant with the same investment technology as incumbents. The entrant, however, starts with no capital, so it must initially do greenfield investment.

Put together, the capital augmentation and greenfield investment processes allow for significantly richer investment dynamics than in the typical dynamic industry model. Firms can

<sup>5</sup> A firm's short-run cost is strictly convex if  $(1-\beta)\theta < 1$ , in which case there is a unique Cournot equilibrium if the demand function is weakly concave. In our analysis, these conditions are satisfied.

<sup>6</sup> Alternatively, if the merged firm kept both investment processes we would need to keep track, as a separate state variable, of how many investment processes a firm possesses, which has no natural bound.

<sup>7</sup> Of course, because of noncooperative investment behavior, there could be efficiency benefits of the merger in actually achieving a given amount of market-wide capital growth at least cost.

expand their capital by multiple units at a time through either investment method. And firms with no capital, including new entrants, can decide endogenously how far to jump up in their capital stock.

Capital also depreciates: in each period a unit of capital has a probability  $d > 0$  of becoming worthless (including for any future capital augmentation). Depreciation realizations are independent across units of capital. This depreciation process is also merger neutral. Finally, the firms discount the future according to discount factor  $\delta < 1$ .

A state is a capital stock pair  $(K_1, K_2)$ . In our computations firms will be restricted to an integer number of possible capital levels, with the maximal capital level  $S$  chosen to be non-binding. Since a firm may have zero capital, we define  $\mathcal{S} \equiv \{0, 1, 2, \dots, S\}$  to be the admissible values of  $K_i$  and  $\mathcal{S}^2 = S \times S$  to be the state space.

### 2.3 Mergers and Bargaining

In each period, firms can propose a merger that will combine their capital. Following a merger, a new entrant appears in the market with zero capital.<sup>8</sup>

Proposing a merger involves a cost  $\phi \in [\underline{\phi}, \bar{\phi}]$  drawn each period in an iid fashion from distribution  $\Phi$ . Firms engage in Nash bargaining to decide whether to merge. Thus, they propose a merger provided the expected gain in their joint continuation value, taking into account the likelihood the merger will be approved, exceeds  $\phi$ . If they merge, they make a side transfer to split evenly the joint value gain from the merger. In the event the antitrust authority rejects the proposed merger, they split the proposal cost evenly. The disagreement values in this bargaining are the two firms' continuation values in the event they do not merge this period. Let  $\bar{V}(K_1, K_2)$  denote the interim expected net present value (ENPV) of a firm with  $K_1$  units of capital when its rival has  $K_2$  units of capital. This interim value is measured just prior to the output competition stage. (In the timing given below, this is at the start of stage 5.) If the capital stocks prior to the merger stage are  $(K_1, K_2)$ , then the joint value gain from merging (gross of any proposal cost) is

$$\Delta_G(K_1, K_2) \equiv \{\bar{V}(K_1 + K_2, 0) - [\bar{V}(K_1, K_2) + \bar{V}(K_2, K_1)]\}, \quad (1)$$

where the first term is the joint (interim) value in case the merger takes place and the second term is the sum of the “disagreement payoffs” (i.e., the sum of the interim values if no merger occurs).

### 2.4 Merger Policy

The antitrust authority has the ability to block mergers. Blocking a proposed merger involves a cost  $b \in [\underline{b}, \bar{b}]$  drawn each period in an iid fashion from a distribution  $H$ . We consider two possible scenarios. In one, we suppose that the antitrust authority can commit to a deterministic policy  $a(\cdot) : \mathcal{S}^2 \rightarrow \{0, 1\}$  that specifies whether a proposed merger would be

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<sup>8</sup>Note that entry is allowed only once a merger has occurred, but not before. The reason is that we cannot evaluate the profitability of entry for a third firm without having a solution for the multi-firm bargaining with externalities problem that would arise after its entry. We have also analyzed the case in which only the two manager-owners possess the knowledge of how to operate a firm in this industry, and the new entrant is one of these owners. We get similar results (see the Appendix).

approved ( $a = 1$ ) or not ( $a = 0$ ) in each state  $(K_1, K_2)$ .<sup>9</sup> These commitment policies will be restricted further to two classes of policies described in Section 5. Under the commitment scenario the two firms never propose a merger that will be rejected because they will incur the proposal cost. Consequently, with commitment the antitrust authority never incurs a blocking cost.

We also consider cases in which the antitrust authority cannot commit to its policy. In that case, in any state  $(K_1, K_2)$  it will decide whether to block a merger by comparing the increase in its welfare criterion from blocking (we will consider both consumer value and aggregate value) to its blocking cost realization  $b$ . In that case, a Markovian strategy for the antitrust authority is a state contingent and history independent threshold  $\hat{b}(K_1, K_2)$  describing the highest blocking cost at which the authority will block a merger in a given state  $(K_1, K_2)$ . Equivalently, this can be translated into a merger acceptance probability  $a(K_1, K_2) \in [0, 1]$ . As we previously noted, we call the equilibrium policy that emerges a Markov perfect policy.

Identifying this policy is of interest for both positive and normative reasons. First, on a positive level, the antitrust authority may well lack an ability to commit to its future approval policy. For example, while both the Department of Justice and Federal Trade Commission in the U.S. periodically issue *Horizontal Merger Guidelines*, which may partially commit these agencies, over time their actual policy often comes to deviate substantially from the *Guidelines'* prescriptions. On a normative level, the possibility of gains from commitment may provide a justification for legislatively endowing the antitrust authority with an objective function different from the true social goal. In particular, specifying that the antitrust authority seek to maximize consumer value rather than aggregate value in deciding whether to approve a merger may result in greater expected aggregate value.

## 2.5 Timing

In each period, the timing of the model is as follows:

1. Firms observe each others' capital stocks.
2. Firms observe their proposal cost  $\phi$  and bargain over whether to propose a merger.
3. If a merger is proposed, the antitrust agency observes its blocking cost  $b$  and decides whether to block it. (This is when commitment is not possible; the antitrust authority simply follows its commitment strategy when commitment is possible.) If a merger is consummated in state  $(K_1, K_2)$ , the merged firm's capital stock is  $K_1 + K_2$ .
4. If a merger occurred, an entrant enters with no capital.
5. Firms choose their output levels simultaneously and the market price is determined. Firms with no capital, such as entrants, cannot produce.
6. Firms privately observe their capital augmentation and greenfield cost draws and decide on their investments.

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<sup>9</sup>Note that we restrict attention to history-independent policies. If this were not the case, firms would not generally employ Markovian policies.

7. Stochastic depreciation occurs, resulting in the capital levels at which firms begin the next period.

### 3 Equilibrium and Computation

In this section, we define equilibrium policies and values more formally and discuss our computational algorithm.

#### 3.1 Firm Policies

Fix the symmetric merger policy  $a(\cdot)$ . We focus on Markov perfect equilibria in which the firms' merger and investment policies are symmetric with respect to the industry state  $(K_1, K_2)$ . We refer to the industry's state at the beginning of the period, just prior to stage 1, as its *ex ante state* and we refer to its state just after stage 4's entry activity concludes and before stage 5's production as its *interim state*. Firm  $i$ 's *ex ante value*  $V(K_i, K_{-i})$  is its ENPV at the beginning of the period and its *interim value*  $\bar{V}(K_i, K_{-i})$ —as defined in the previous section—is its ENPV before stage 5's production decisions. Thus, if the ex ante state is  $(K_1, K_2)$  and no merger occurs, then the firms' interim values are  $\bar{V}(K_1, K_2)$  and  $\bar{V}(K_2, K_1)$ . But if they do merge, then the industry transits to either interim state  $(K_1 + K_2, 0)$  or interim state  $(0, K_1 + K_2)$ , each with probability  $1/2$ . This transition rule ensures that the steady state distribution over states is indeed symmetric. The interim value of the firm that the merger created is  $\bar{V}(K_1 + K_2, 0)$  and the entrant's interim value is  $\bar{V}(0, K_1 + K_2)$ . Note that for notational simplicity we suppress the dependence of firms' values and behavior on the policy  $a(\cdot)$ .

Each period the firms make two decisions that affect the state to which the industry transits. After learning the proposal cost  $\phi$ , the firms jointly decide whether to propose a merger. Then each firm, after privately learning the  $K_i$  independent draws of its capital augmentation costs  $\{c_j\}_{j=1}^{K_i}$  and the single independent draw of its greenfield cost  $c_g$ , independently decides how many units of capital (if any) to add.<sup>10</sup>

**Merger proposals.** As outlined in the previous section, firms 1 and 2 propose a merger if and only if doing so increases their joint interim value net of the proposal cost. That is, if the antitrust authority's approval probability is  $a(K_1, K_2)$ , the firms propose a merger in ex ante state  $(K_1, K_2)$  if and only if the realized proposal cost  $\phi$  satisfies:

$$\phi < a(K_1, K_2)\Delta_G(K_1, K_2)$$

where  $\Delta_G(K_1, K_2)$  is the joint gain from merger (gross of the proposal cost) defined in equation (1). This implies that the ex ante probability of a merger proposal in state  $(K_1, K_2)$  is

$$\psi(K_1, K_2) \equiv \Phi(a(K_1, K_2)\Delta_G(K_1, K_2)) \quad (2)$$

and the ex ante probability of a merger occurring is  $a(K_1, K_2)\psi(K_1, K_2)$ . Nash bargaining over the gains from merging implies that firm  $i$ 's ex ante value is

$$V(K_i, K_{-i}) = \bar{V}(K_i, K_{-i}) + \psi(K_1, K_2)\frac{1}{2}\{a(K_1, K_2)\Delta_G(K_1, K_2) - \mathcal{E}[\phi|K_1, K_2]\} \quad (3)$$

<sup>10</sup>Each firm also decides on the quantity it produces. This decision is embedded in the firms' single-period profit function  $\pi(K_1, K_2)$ .

where  $\bar{V}(K_i, K_{-i})$  is the firm's disagreement value, the term in curly brackets is the merging firms' expected net gain from carrying out the merger (which they divide equally), and

$$\mathcal{E}[\phi|K_1, K_2] \equiv \frac{\int_{\phi}^{a(K_1, K_2)\Delta_G(K_1, K_2)} \phi d\Phi(\phi)}{\psi(K_1, K_2)}$$

is the expected proposal cost conditional on the merger being proposed.

**Investment.** We now turn to firms' investment decisions. Let firm  $i$ 's investment policy function at the interim stage be  $\xi_i(\cdot|K_i, K_{-i}) : \{0, 1, \dots, S - K_i\} \times \mathcal{S}^2 \rightarrow [0, 1]$ . Prior to the realization of its cost draws,  $\xi_i$  gives the probability  $\xi_i(k|K_i, K_{-i})$  of firm  $i$  adding  $k \in \{0, 1, \dots, S - K_i\}$  units of capital. Recall that at the end of each period each unit of capital depreciates with probability  $d$ , so if firm  $i$  enters the depreciation stage with  $K_i$  units of capital, then the probability it exits the stage with  $K'_i$  units of capital is

$$\kappa(K'_i|K_i) = \begin{cases} \binom{K_i}{K'_i}(1-d)^{K'_i}d^{K_i-K'_i} & \text{if } K'_i \in \{0, 1, \dots, K_i\}, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Given that firm  $i$  follows investment policy  $\xi_i$  in state  $(K_i, K_{-i})$ , the probability of firm  $i$  leaving the period with  $K'_i \in \mathcal{S}$  units of capital is given by the transition function

$$\tau_i(K'_i|K_i, K_{-i}, \xi_i) = \sum_{m=0}^{S-K_i} \xi_i(m|K_i, K_{-i})\kappa(K'_i|K_i + m). \quad (5)$$

Prior to making its investment decision, firm  $i$  privately observes  $K_i$  draws of the capital augmentation cost  $c_j \in [\underline{c}, \bar{c}]$  and one draw of its greenfield cost  $c_g \in [\bar{c}, \bar{c}_g]$ . For a given realization  $\tilde{c}$  of its  $(K_i + 1)$ -length vector of cost draws, let  $c_{K_i}(\cdot|\tilde{c})$  denote the resulting cost function where  $c_{K_i}(k|\tilde{c})$  is the minimum cost for firm  $i$  to add  $k$  units of capital. Let  $\mathcal{C}_{K_i}$  be the set of possible cost draws  $\tilde{c}$  and let  $h_{K_i}$  be the associated density that the distributions  $F$  and  $G$  of the cost draws determine. For a given draw  $\tilde{c}$ , cost function  $c_{K_i}(\cdot|\tilde{c})$ , rival's investment policy  $\xi_{-i}$ , and ex ante value function  $V(\cdot)$ , firm  $i$  chooses  $k_i$  so as to maximize its expected continuation value minus the investment cost:

$$\max_{k_i \in \{0, 1, \dots, S - K_i\}} -c_{K_i}(k_i|\tilde{c}) + \delta \sum_{K'_i \in \mathcal{S}} \sum_{K'_{-i} \in \mathcal{S}} \kappa(K'_i|K_i + k_i)\tau_{-i}(K'_{-i}|K_{-i}, K_i, \xi_{-i})V(K'_i, K'_{-i}).$$

Let  $k_i^*$  denote the solution to this optimization problem (which, generically, is unique), and define  $\omega(k_i, \tilde{c}, K_i, K_{-i}|\xi_{-i})$  to be the indicator function with value 1 if  $k_i = k_i^*$  and 0 otherwise. Firm  $i$ 's investment policy therefore is

$$\xi_i(k_i|K_i, K_{-i}) = \int_{\mathcal{C}_{K_i}} \omega(k_i, \tilde{c}, K_i, K_{-i}|\xi_{-i})h_{K_i}(\tilde{c})d\tilde{c}, \quad (6)$$

for  $k_i \in \{0, 1, \dots, S - K_i\}$ . This gives rise to the firm's expected investment cost in state  $(K_i, K_{-i})$ :

$$\mathcal{E}c(K_i, K_{-i}|\xi_{-i}) = \int_{\mathcal{C}_{K_i}} \sum_{k_i \in \{0, 1, \dots, S - K_i\}} \omega(k_i, \tilde{c}, K_i, K_{-i}|\xi_{-i})c_{K_i}(k_i|\tilde{c})h_{K_i}(\tilde{c})d\tilde{c}. \quad (7)$$

Given symmetric investment policies and transition functions  $\xi_i = \xi_{-i} \equiv \xi$  and  $\tau_i = \tau_{-i} \equiv \tau$ , we may write the firm's interim value in state  $(K_1, K_2)$  as its profits less its investment costs incurred plus its expected discounted ex ante value in the continuation game:

$$\begin{aligned} \bar{V}(K_i, K_{-i}) &= \pi(K_i, K_{-i}) - \mathcal{E}c(K_i, K_{-i}|\xi) \\ &+ \delta \sum_{K'_i \in \mathcal{S}} \sum_{K''_{-i} \in \mathcal{S}} \tau(K'_i|K_i, K_{-i}, \xi) \tau(K''_{-i}|K_{-i}, K_i, \xi) V(K'_i, K''_{-i}). \end{aligned} \quad (8)$$

Equation (3) gives a formula for  $V(K_i, K_{-i})$  in terms of  $\bar{V}(K_i, K_{-i})$  and  $\Delta_G(K_i, K_{-i})$ , where  $\Delta_G(K_i, K_{-i})$  itself is a function of interim values. Consequently equation (3) together with the formula (8) for  $\bar{V}(K_i, K_{-i})$  implicitly define the Bellman equation for the ex ante value  $V(K_i, K_{-i})$ .

In summary, given the fixed merger policy  $a(\cdot)$ , if for all states  $(K_1, K_2) \in \mathcal{S}^2$  the merger proposal probability  $\psi(K_i, K_{-i})$ , the firms' investment policy  $\xi(k_i|K_i, K_{-i})$ , each firm's ex ante value  $V(K_i, K_{-i})$ , and each firm's interim value  $\bar{V}(K_i, K_{-i})$  satisfy equations (2), (3), (6), and (8), then  $\{\psi, \xi, V, \bar{V}\}$  is a Markov perfect equilibrium that policy  $a(\cdot)$  induces.

**Transition array and steady state.** Given symmetric merger policy  $a(\cdot)$ , investment policy  $\xi$ , and transition function  $\tau$ , the transition array that the induced equilibrium generates is

$$\begin{aligned} T[(K_1, K_2), (K'_1, K'_2)] &= (1 - a(K_1, K_2)\psi(K_1, K_2)) [\tau(K'_1|K_1, K_2, \xi)\tau(K'_2|K_2, K_1, \xi)] \\ &+ \frac{1}{2}a(K_1, K_2)\psi(K_1, K_2)\{\tau(K'_1|0, K_1 + K_2, \xi)\tau(K'_2|K_1 + K_2, 0, \xi) \\ &+ \tau(K'_1|K_1 + K_2, 0, \xi)\tau(K'_2|0, K_1 + K_2, \xi)\}. \end{aligned}$$

where  $(K_1, K_2)$  is the beginning-of-period state and  $(K'_1, K'_2)$  is the state at the beginning of the next period.

To calculate welfare measures and statistics of the industry's dynamics we need the long run, steady state distribution that results from implementation of merger policy  $a(\cdot)$ . Let  $\Omega : \mathcal{S}^2 \rightarrow \{1, 2, \dots, (S+1)^2\}$  be an invertible mapping that maps the two-dimensional array of states  $(K_1, K_2)$  into a row vector. Then, for every  $\omega \in \Omega(K_1, K_2)$  and  $\omega' \in \Omega(K'_1, K'_2)$ , define the  $(S+1) \times (S+1)$  transition matrix  $\hat{T}$  to have element  $\hat{T}(\omega, \omega') = T[\Omega^{-1}(\omega), \Omega^{-1}(\omega')]$  at row  $\omega$  (the beginning-of-period state) and column  $\omega'$  (the state at the beginning of the next period). Let  $\hat{P}$  be a length  $(S+1)^2$  row vector. If  $\hat{P}\hat{T} = \hat{P}$ , then  $\hat{P}$  is a steady state distribution across states that the policy  $a(\cdot)$  induces. If  $\hat{P}$  is unique, then, for any probability vector  $P$ ,

$$\hat{P} = \lim_{t \rightarrow \infty} P \underbrace{\hat{T} \hat{T} \hat{T} \dots \hat{T}}_{t \text{ times}},$$

i.e., no matter what the initial probability distribution  $P$  over states is, the industry converges to the steady-state distribution  $\hat{P}$ .<sup>11</sup> A simpler representation of the steady state probabilities is the  $(S+1) \times (S+1)$  matrix  $\bar{P}$  whose entry in row  $K_1$  and column  $K_2$  is the steady state probability of state  $(K_1, K_2)$ :

$$\bar{P}(K_1, K_2) \equiv \hat{P}[\Omega(K_1, K_2)].$$

<sup>11</sup>In our model we cannot guarantee that, for some positive integer  $t$ , every element of  $\hat{T}^t$  is positive, i.e., we cannot guarantee that  $\hat{T}$  is a regular Markov transition matrix. If it were regular, then  $\hat{P}$  would be unique.

### 3.2 Welfare and Antitrust Policy

As discussed earlier, we consider two alternative objective functions for the antitrust authority: CV maximization and AV maximization. We also distinguish between settings in which the authority can commit to its future policy and those in which it cannot. In this subsection we first derive expressions for CV and AV given a fixed merger policy  $a(\cdot)$  and the firms' Markov perfect strategies. We then make the policy  $a(\cdot)$  variable and define the antitrust authority's optimal commitment policy and its optimal no-commitment (i.e., Markov perfect) policy. Throughout the discussion, whether  $a(\cdot)$  is fixed or variable,  $\{\psi, \xi, V, \bar{V}\}$  is the Markov perfect equilibrium that  $a(\cdot)$  induces the firms to follow.

**Consumer surplus, producer surplus, and aggregate surplus.** Recall that  $CS(K_1, K_2)$  is the consumer surplus that Cournot competition between the two firms generates given their capital levels  $K_1$  and  $K_2$  at the time of production in stage 5. If the ex ante state is  $(K_1, K_2)$  and no merger occurs, then the consumer surplus realized is  $CS(K_1, K_2)$ . If a merger occurs in the period, then the consumer surplus realized is  $CS(K_1 + K_2, 0)$ . The expected consumer surplus at the ex ante state  $(K_1, K_2)$  is therefore

$$\mathcal{E}CS(K_1, K_2) = [1 - a(K_1, K_2)\psi(K_1, K_2)]CS(K_1, K_2) + a(K_1, K_2)\psi(K_1, K_2)CS(K_1 + K_2, 0)$$

where  $a(K_1, K_2)\psi(K_1, K_2)$  is the probability a merger occurs. Similarly, expected producer surplus at ex ante state  $(K_1, K_2)$  is

$$\mathcal{E}PS(K_1, K_2) = [1 - a(K_1, K_2)\psi(K_1, K_2)]PS(K_1, K_2) + a(K_1, K_2)\psi(K_1, K_2)PS(K_1 + K_2, 0),$$

where  $PS(K_1, K_2) \equiv \pi(K_1, K_2) + \pi(K_2, K_1)$ . Aggregate surplus is the sum of consumer surplus and producer surplus:  $AS(K_1, K_2) = CS(K_1, K_2) + PS(K_1, K_2)$  where  $(K_1, K_2)$  is the interim state. Consequently, in ex ante state  $(K_1, K_2)$ , expected aggregate surplus is  $\mathcal{E}AS(K_1, K_2) = \mathcal{E}CS(K_1, K_2) + \mathcal{E}PS(K_1, K_2)$ .

**Consumer value and aggregate value.** We generalize these static criteria to their dynamic analogues,  $CV$  and  $AV$ , where the latter accounts not only for incumbents' profits at the output stage, but also investment costs, merger proposal costs, blocking costs, and future profits of new entrants. Ex ante consumer value,  $CV(K_1, K_2)$ , is the ENPV of current and future expected consumer surplus. Its Bellman equation is

$$CV(K_1, K_2) = \mathcal{E}CS(K_1, K_2) + \delta \sum_{K'_1 \in \mathcal{S}} \sum_{K'_2 \in \mathcal{S}} T[(K_1, K_2), (K'_1, K'_2)] CV(K'_1, K'_2) \quad (9)$$

for all  $(K_1, K_2) \in \mathcal{S}^2$ . Interim consumer value is, for all states  $(K_1, K_2)$ ,

$$\bar{CV}(K_1, K_2) = CS(K_1, K_2) + \delta \sum_{K'_1 \in \mathcal{S}} \sum_{K'_2 \in \mathcal{S}} \tau(K'_1 | K_1, K_2, \xi_1) \tau(K'_2 | K_1, K_2, \xi_2) CV(K'_1, K'_2). \quad (10)$$

Ex ante aggregate value  $AV(K_1, K_2)$  has four components: consumer value  $CV(K_1, K_2)$ , the sum of the incumbent firms' ex ante values  $V(K_1, K_2) + V(K_2, K_1)$ , the ENPV of all future entrants' cash flows  $\mathcal{E}EV(K_1, K_2)$ , and the ENPV of the antitrust authority's blocking costs  $\mathcal{E}BC(K_1, K_2)$ . Note that sum,  $V(K_1, K_2) + V(K_2, K_1)$ , fully accounts for the incumbents' expected merger proposal costs and expected capital investment costs. But neither  $\mathcal{E}CS$  nor

$V(K_1, K_2) + V(K_2, K_1)$  includes the latter two components,  $\mathcal{E}EV(K_1, K_2)$  and  $\mathcal{E}BC(K_1, K_2)$ , yet the authority should account for them.

Consider  $\mathcal{E}EV(K_1, K_2)$ . A new firm comes into existence in stage 4 of a period in which a merger occurs. This new firm's interim value is  $\bar{V}(0, K_1 + K_2)$  where  $K_1 + K_2$  is the merged firm's capital level. If the ex ante state is  $(K_1, K_2)$ , then the Bellman equation of the ex ante ENPV of all future entrants' cash flows is

$$\mathcal{E}EV(K_1, K_2) = a(K_1, K_2)\psi(K_1, K_2)\bar{V}(0, K_1 + K_2) + \delta \sum_{K'_1 \in \mathcal{S}} \sum_{K'_2 \in \mathcal{S}} T[(K_1, K_2), (K'_1, K'_2)] \mathcal{E}EV(K'_1, K'_2).$$

In interim state  $(K_1, K_2)$  the value is

$$\overline{\mathcal{E}EV}(K_1, K_2) = \delta \sum_{K'_1 \in \mathcal{S}} \sum_{K'_2 \in \mathcal{S}} T[(K_1, K_2), (K'_1, K'_2)] \mathcal{E}EV(K'_1, K'_2).$$

In Section 2.3 we sketched two scenarios for the antitrust authority. In the first, the authority commits to the policy  $a(\cdot)$  with the restriction that  $a(K_1, K_2) \in \{0, 1\}$  for all  $(K_1, K_2)$ . Given commitment, the firms know expending resources proposing a merger when  $a(K_1, K_2) = 0$  is hopeless because the authority will block the proposal with probability 1. Consequently the authority never has to block a proposal and incurs zero blocking costs. In the second scenario the authority does not commit and, in each ex ante state  $(K_1, K_2)$ , sets a threshold  $\hat{b}(K_1, K_2) \in [\underline{b}, \bar{b}]$  such that it blocks a proposed merger if and only if its private and random blocking cost  $b$  is less than  $\hat{b}$ . From the firms' viewpoint  $\hat{b}(K_1, K_2)$  generates the ex ante approval probability:  $a(K_1, K_2) = 1 - H[\hat{b}(K_1, K_2)]$  where  $H$  is the distribution function of  $b$ .<sup>12</sup> Conditional on a merger being proposed, the expected blocking cost in state  $(K_1, K_2)$  is

$$\mathcal{E}[b|K_1, K_2] = \int_{\underline{b}}^{H^{-1}(1-a(K_1, K_2))} b dH(b).$$

The Bellman equation for the ex ante ENPV of blocking costs in ex ante state  $(K_1, K_2)$  is therefore

$$\mathcal{E}BC(K_1, K_2) = \psi(K_1, K_2)\mathcal{E}[b|K_1, K_2] + \delta \sum_{K'_1 \in \mathcal{S}} \sum_{K'_2 \in \mathcal{S}} T[(K_1, K_2), (K'_1, K'_2)] \mathcal{E}BC(K'_1, K'_2).$$

In interim state  $(K_1, K_2)$  its value is

$$\overline{\mathcal{E}BC}(K_1, K_2) = \delta \sum_{K'_1 \in \mathcal{S}} \sum_{K'_2 \in \mathcal{S}} \tau(K'_1|K_1, K_2, \xi_1)\tau(K'_2|K_1, K_2, \xi_2)\mathcal{E}BC(K'_1, K'_2).$$

Given these definitions, ex ante aggregate value in state  $(K_1, K_2)$  is

$$AV(K_1, K_2) = CV(K_1, K_2) + V(K_1, K_2) + V(K_2, K_1) + \mathcal{E}EV(K_1, K_2) - \mathcal{E}BC(K_1, K_2) \quad (11)$$

and interim aggregate value is

$$\overline{AV}(K_1, K_2) = \overline{CV}(K_1, K_2) + \bar{V}(K_1, K_2) + \bar{V}(K_2, K_1) + \overline{\mathcal{E}EV}(K_1, K_2) - \overline{\mathcal{E}BC}(K_1, K_2). \quad (12)$$

**Optimal commitment policy.** The antitrust authority commits to a pure action  $a(K_1, K_2) \in \{0, 1\}$  for each state  $(K_1, K_2)$  so as to either (i) maximize the welfare criterion's steady state

<sup>12</sup>Hence  $a(\cdot) = 0$  corresponds to  $\underline{b}$  and  $a(\cdot) = 1$  corresponds to  $\bar{b}$ .



average value or (ii) maximize the welfare criterion's value at state  $(0, 0)$  where the latter goal represents a measure of a new industry's welfare contribution. Given a merger policy  $a(\cdot)$ , the two firms play their Markov perfect equilibrium  $\{\psi, \xi, V, \bar{V}\}$ . Let the authority's ex ante welfare criterion—whether consumer value or aggregate value—be denoted by  $W(K_1, K_2)$  and its interim welfare criterion by  $\bar{W}(K_1, K_2)$ . The industry's steady state distribution is the matrix  $\bar{P}$  whose elements are  $\bar{P}(K_1, K_2)$ . The steady state average value it achieves under policy  $a(\cdot)$  is

$$WSS = \sum_{K'_1 \in \mathcal{S}} \sum_{K'_2 \in \mathcal{S}} \bar{P}(K'_1, K'_2) W(K'_1, K'_2).$$

Recalling that the industry's behavior (including the value of  $WSS$ ) implicitly depends on  $a(\cdot)$ , if  $\mathcal{A}$  is the class of admissible commitment policies, then the optimal commitment policy  $a^*(\cdot)$  that maximizes steady state welfare is

$$a^*(\cdot) = \arg \max_{a(\cdot) \in \mathcal{A}} WSS.$$

Maximizing welfare of a new industry, the antitrust authority chooses  $a(\cdot)$  to maximize  $W(0, 0)$ .

**Markov perfect policy.** The antitrust authority acts as a third player who, unable to commit, makes its approval decision in every state  $(K_1, K_2)$  so as to maximize its welfare criterion, given the firms' Markov perfect equilibrium play in the continuation game. The resulting policy  $a(\cdot)$  and the firms' equilibrium actions together determine the welfare criterion's ex ante and interim values,  $W(K_1, K_2)$  and  $\bar{W}(K_1, K_2)$ .

A given merger policy  $a(\cdot)$  is a Markov perfect merger policy if it satisfies the one-step deviation principle when the two firms play the industry Markov perfect equilibrium  $\{\psi, \xi, V, \bar{V}\}$  that policy  $a(\cdot)$  induces. To check that  $a(\cdot)$  does satisfy the principle, calculate the welfare gain (gross of the blocking cost) from approving a proposed merger at beginning-of-period state  $(K_1, K_2)$ :

$$\Delta_W(K_1, K_2) \equiv \bar{W}(K_1 + K_2, 0) - \bar{W}(K_1, K_2).$$

Conditional on a merger being proposed at state  $(K_1, K_2)$ , the authority approves the proposal if and only if the realization  $b \in [\underline{b}, \bar{b}]$  of its blocking cost satisfies

$$b \geq \hat{b}(K_1, K_2) \equiv -\Delta_W(K_1, K_2),$$

i.e., it approves the merger if and only if blocking increases the welfare criterion net of the realized cost  $b$ . Therefore, before the blocking cost is observed, the probability the authority approves the merger is  $1 - H(\hat{b}(K_1, K_2)) = 1 - H(-\Delta_W(K_1, K_2))$ . This means that for the policy  $a(\cdot)$  not to violate the one-stage deviation principle and be optimal, it must solve the equation

$$a(K_1, K_2) = 1 - H(-\Delta_W(K_1, K_2)). \quad (13)$$

for all  $(K_1, K_2) \in \mathcal{S}^2$ .

To summarize, a no-commitment Markov perfect merger policy involve policy and value functions  $\{\psi, \xi, V, \bar{V}, W, \bar{W}, a\}$  that, for all  $(K_1, K_2) \in \mathcal{S}^2$ , satisfy:

1. equations (2), (3), (6), (8), (9), (10), and (13) if the welfare criterion is consumer value, and
2. equations (2), (3), (6), (8), (11), (12), and (13) if the welfare criterion is aggregate value.

### 3.3 Computation

The algorithm that we use numerically to solve for equilibria is a version of the well-known Pakes-McGuire (1994) algorithm. It is a straightforward iterative process. For a given merger policy  $a(\cdot)$  the procedure works as follows. Pick an initial guess for the investment function  $\xi^{(0)}$  and the interim value function  $\bar{V}^{(0)}$  and then compute an initial value of the merger proposal function  $\psi^{(0)}$  using (2) and of the ex ante value function  $V^{(0)}$  using equation (3). Plugging  $V^{(0)}$  and  $\xi^{(0)}$  into the right-hand-side of (6), we then compute an updated estimate  $\xi^{(1)}$  of the investment policy function. As this is a difficult integral to evaluate, we use Monte Carlo integration at each interim state  $(K_1, K_2)$ . Specifically, for a given vector  $\tilde{c}$  of random cost draws, the ex ante value function  $V^{(0)}$ , and the rival's investment policy function  $\xi^{(0)}$  [which determines the rival's transition probabilities via equation (5)], we calculate firm  $i$ 's optimal investment decision  $k_i$  for that instance of  $\tilde{c}$ . Repeating this over and over with 5000 or more cost draws we use the proportion of cost draws for which  $k_i$  is optimal as our estimate of  $\xi^{(1)}(k_i|K_1, K_2, V^{(0)})$ . Inserting  $\xi^{(1)}$  into (8) yields an updated estimate  $\bar{V}^{(1)}$  of the interim value function. We continue with this iterative procedure until  $\|\bar{V}^{(\ell+1)} - \bar{V}^{(\ell)}\| \leq \varepsilon$  for some small  $\varepsilon > 0$ . The Appendix describes this algorithm in detail and includes a copy of the MatLab code.

## 4 Investment and Merger Incentives under Fixed Merger Policies

In this section we have three goals. First, we describe the specific parameterization of the model that we employ and discuss the properties of the static monopoly and Cournot equilibria that this parameterization implies. Second, we consider the Markov perfect equilibrium when mergers are prohibited—the “no-mergers” case. Third, we consider the Markov perfect equilibrium when firms are permitted to merge in any state in which it is profitable for them to do so—the “all-mergers-allowed” case. For each policy, we report its long-run steady state distribution over the state space  $\mathcal{S}^2$ , the producer and consumer values it generates, and the investment incentives it creates. The all-mergers-allowed equilibrium is very different than the no-mergers equilibrium in structure, incentives, and welfare measures. Merging causes the industry to be much more concentrated than in the no-merger case. Not surprisingly, expected consumer value is on average substantially reduced. But, surprisingly, expected incumbent value is also reduced, though not by nearly the same amount, and expected producer value (which includes the value of future entrants) is essentially unchanged. The key factor behind this result is the effect of merger policy on firms' investment behaviors.

### 4.1 Three Markets

In our main analysis, we examine three markets that are identical except for the level of market demand. The market demand takes the form  $Q(p) = B(3 - p)$  with  $B = 30$  for the “large” market,  $B = 26$  for the “intermediate” market, and  $B = 22$  for the “small” market. We will see that the small market is a natural monopoly, the large market is a workable duopoly, and the intermediate market is between those two. Firms' production function takes the Cobb-Douglas

form  $Q = (K^\beta L^{1-\beta})^\theta$  with capital share parameter  $\beta = 1/3$  and scale parameter  $\theta = 1.1$ . Thus, as noted earlier, a merger between two equal-sized firms who do not alter their output levels lowers marginal and average costs by 9 percent. The wage rate is normalized to 1.

Table 1 gives a sense of the intermediate market’s fundamental static properties with its strong economies of scale and linear demand. It shows the static Cournot equilibrium outcomes for three different states:  $(1, 0)$ ,  $(10, 0)$ , and  $(5, 5)$ . The first two states are monopoly states since the second firm has zero units of capital, while the third is a symmetric duopoly state. The comparison between the two monopoly states shows the substantial effects of the scale economies on marginal cost. It also shows for state  $(1, 0)$  the effect of linear demand when price is high and quantity small: demand is quite elastic causing a small price-cost markup.

**Table 1: Intermediate Market Static Equilibrium**

State	(1, 0)	(10, 0)	(5, 5)
Marginal Cost ( $MC$ )	2.56	1.32	1.54
Price ( $P$ )	2.78	2.16	2.02
$P \div MC$	1.09	1.63	1.32
Total Quantity	5.67	21.8	25.4
Total Profit	5.12	26.0	22.8
Consumer Surplus	0.619	9.14	12.4
Aggregate Static Surplus	5.74	35.12	35.16

The monopoly in state  $(10, 0)$  exerts its market power to restrict output and raise price to 2.16 compared to the duopoly’s 2.02. Per period consumer surplus as a consequence falls from 12.4 to 9.14, a change of 3.3. But the market’s strong scale economies gives the monopolist a marginal cost of 1.38 compared to the duopolists’ marginal cost of 1.54. This results in total profits in the  $(10, 0)$  monopoly being 26.0 instead of 22.8 in the  $(5, 5)$  duopoly, an increase of 3.2. Aggregate static surplus in the  $(10, 0)$  state is therefore almost identical to that in the  $(5, 5)$  state.

Turning to investment costs, we assume that the capital augmentation cost for a given unit of capital is independently drawn from a uniform distribution on the interval  $[3, 6]$ , while the greenfield investment cost  $c_g$  is drawn from a uniform distribution on the interval  $[6, 7]$ . Firms’ discount factor is  $\delta = 0.8$ , which corresponds to a period length of about 5 years. Each unit of capital depreciates independently with probability  $d = 0.2$  per period. We take the state space to be  $\{0, 1, \dots, 20\}^2$ , so each active firm can accumulate up to 20 units of capital. In these markets, firms almost never end up outside of the quadrant  $\{0, 1, \dots, 10\}^2$ ; we allow for capital levels up to 20 so that we can calculate values for mergers and avoid boundary effects.

Finally, we assume that proposal and blocking costs are uniformly distributed on  $[0, 1]$ .

## 4.2 Equilibrium with No Mergers Allowed

We begin by examining equilibria in these markets when no mergers are allowed.<sup>13</sup> Figure 1 shows the resulting beginning-of-period steady state distribution in the intermediate market.

<sup>13</sup>We have assembled the data that we have generated into large Excel workbooks that each contain for each equilibrium, first, a detailed description of the equilibrium strategies of the firms and, for Markov perfect merger policies, of the antitrust authority and, second, a full set of performance statistics. All of the tables and some of the figures that we include in the paper are lifted whole

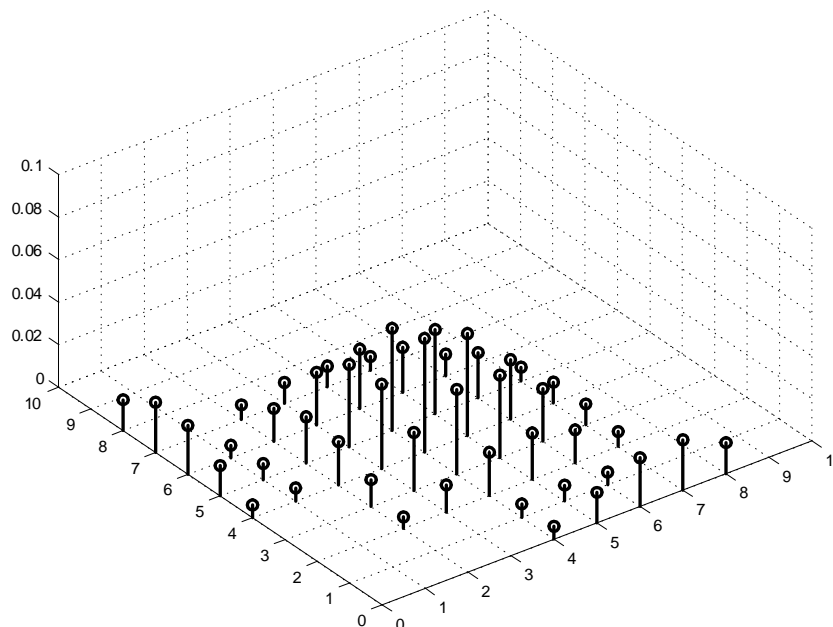


Figure 1: Beginning-of-period steady state distribution over states in the intermediate market with no mergers allowed. The height of each pin indicates the probability of being in a particular state.

The horizontal plane shows the quadrant  $\{0, 1, \dots, 10\}^2$  of the state space, while the height of each pin represents the probability that the industry is in a given state at the beginning of the period. As can be seen there, the industry spends most of its time in duopoly states in which both firms are active, but also spends roughly 18 percent of the time in monopoly states. In fact, if the industry finds itself in a monopoly state, it can stay there a long time; for example, starting in state  $(5, 0)$ , the probability that it is in a monopoly 5 periods later is 0.84. Figure 2 shows the one-period transition probabilities starting from state  $(5, 0)$ . Figure 3 illustrates the equilibrium transitions more generally. In that figure, each arrow represents the average movement over five periods starting in each state. The lack of movement toward duopoly from state  $(5, 0)$  is also evident there.

There are two cost-based reasons why it is so hard for an entrant starting in state  $(5, 0)$  to catch up. First, the entrant pays much more per unit of capital purchased: the large firm can add a unit of capital using the lowest of its five cost draws from the uniform distribution on  $[3, 6]$ , whereas the entrant (who chooses to add at most 1 unit) has to engage in greenfield investment using a cost draw from the uniform distribution on  $[6, 7]$ . Second, the large firm enjoys significant scale economies: with a capital level of 5 its marginal cost as a monopolist is 1.70 while setting a price of 2.35. If the potential entrant should enter with 2 units of capital, then at state  $(5, 2)$  the dominant firm sells quantity 14.6 at a price of 2.18 with a marginal cost of 1.62. The entering firm sells 6.7 units with a marginal cost of 1.92. Profits are 18.6 and 5.1

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from these workbooks. These notebooks are posted on the web as part of our Online Appendix at <http://www.kellogg.northwestern.edu/faculty/satterthwaite/research/researchpage.html>. They enable the reader to explore our results much as we have explored them.

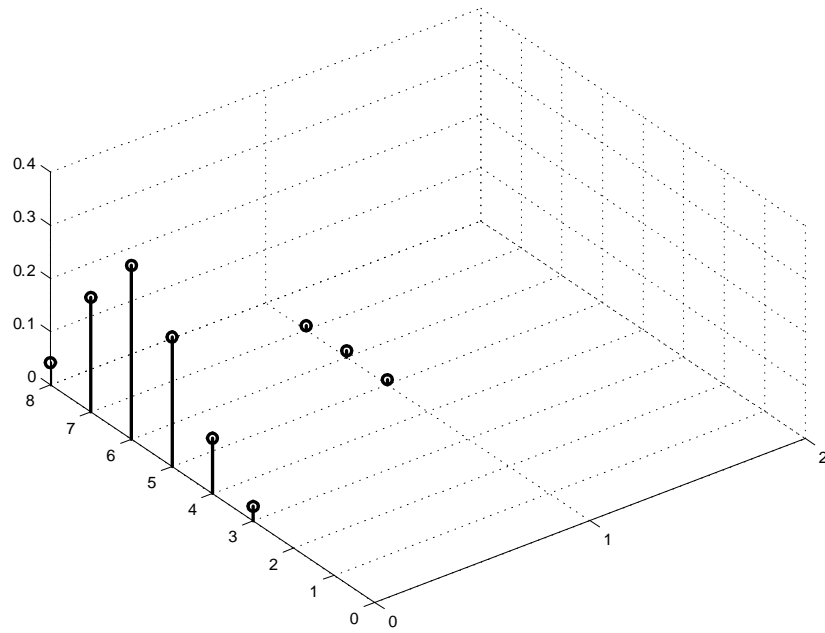


Figure 2: One-period transition from the state  $(5,0)$  in the intermediate market with no mergers allowed.

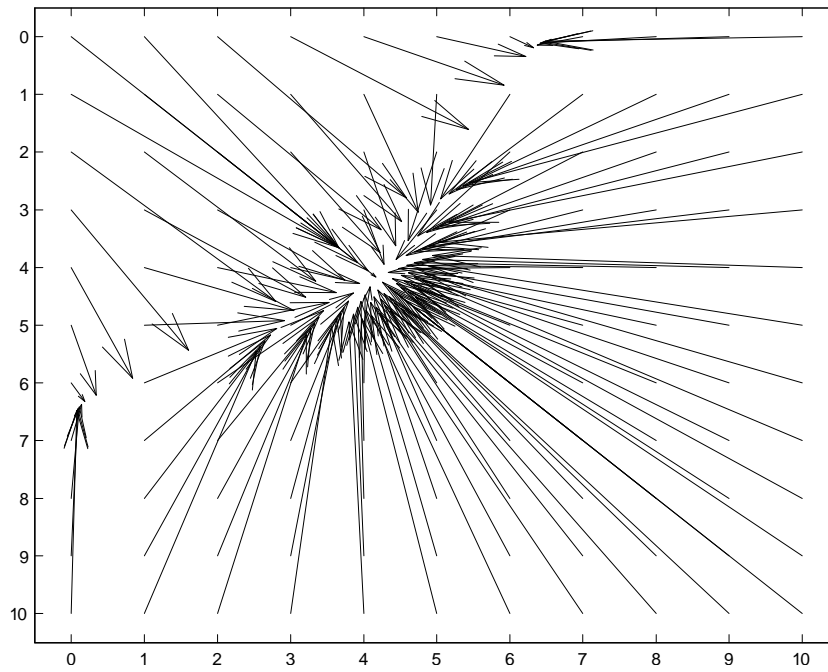


Figure 3: Arrows show the expected transitions over 5 periods in the intermediate market with no mergers allowed.

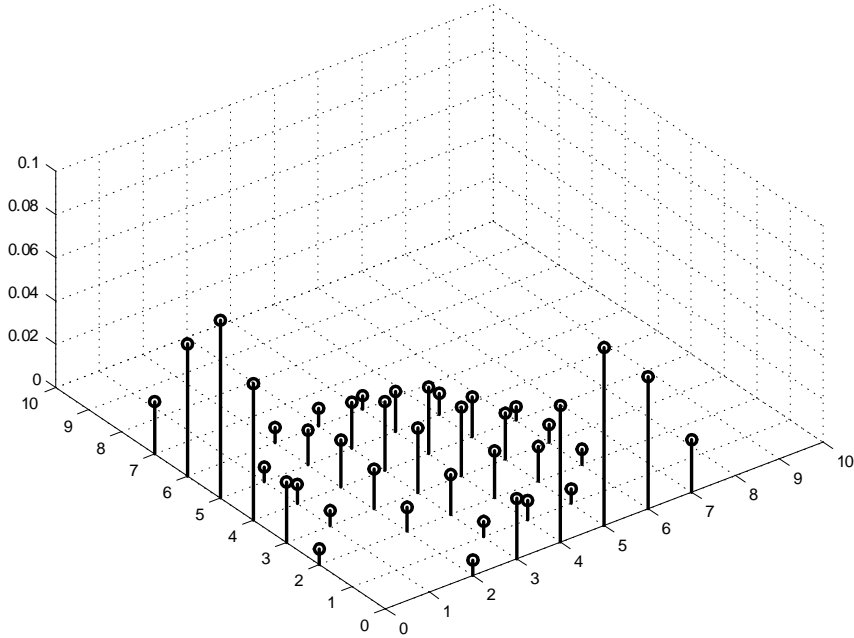


Figure 4: Beginning-of-period steady state distribution over states in the small market with no mergers allowed.

respectively.

Figures 4 through 7 show the steady state distributions and five-period transitions for the small and large markets. The small market is in a monopoly state almost 60 percent of the time, while the large market finds itself in such a state only a little over 2 percent of the time. The equilibria involve larger capital levels as the market size grows.

The left-hand side of Table 2 describes some features of the no-mergers equilibria in the three markets. The second and third rows from the bottom show the probability of being in a monopoly state at the output competition stage 5 (in the no-mergers case, this is the same as the probability of being in a monopoly state at the beginning of the period), as well as the probability of being in neither a monopoly nor a near-monopoly state (“%  $\min\{K_1, K_2\} \geq 2$ ”). Also shown are the beginning-of-period average total capital, average total output, aggregate value, and consumer value, each of which is not surprisingly increasing in market size. Finally, the average price is somewhat lower the larger the market.

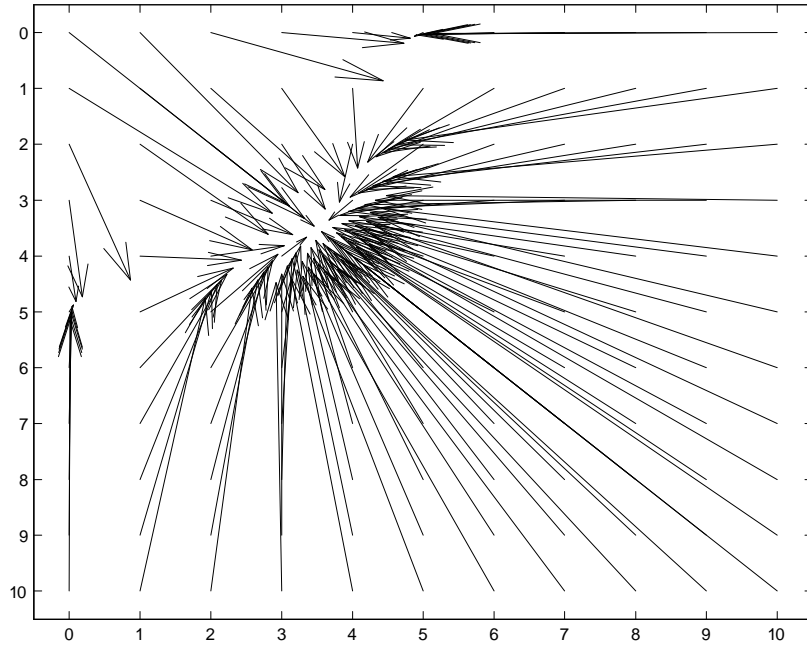


Figure 5: Arrows show the expected transitions over 5 periods in the small market with no mergers allowed.

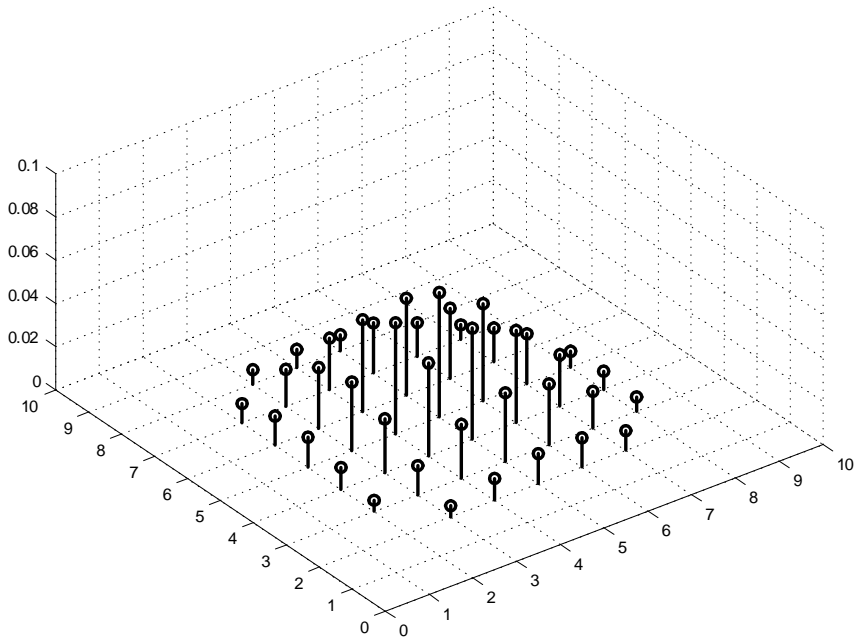


Figure 6: Beginning-of-period steady state distribution over states in the large market with no mergers allowed.

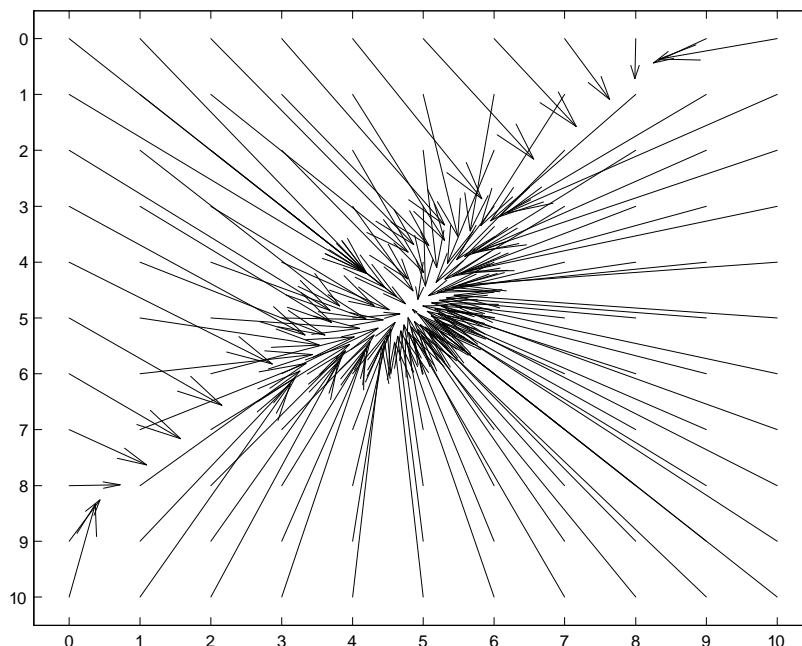


Figure 7: Arrows show the expected transitions over 5 periods in the large market with no mergers allowed.

**Table 2 : Performance Measures in Three Markets Under No Mergers Allowed and All Mergers Allowed**

<i>Performance measure</i> <sup>14</sup>	<i>No Mergers Allowed</i>			<i>All Mergers Allowed</i>		
	Small	Intermediate	Large	Small	Intermediate	Large
Avg. Consumer Value	31.8	48.1	61.3	28.8	35.8	44.1
Avg. Incumbent Value	57.8	69.4	81.0	56.3	68.1	80.8
Avg. Entrant Value	-	-	-	1.1	1.9	2.2
Avg. Aggregate Value	89.6	117.5	142.3	86.2	105.8	127.2
Avg. Price	2.25	2.15	2.10	2.28	2.26	2.24
Avg. Quantity	16.5	22.2	27.0	15.8	19.2	22.9
Avg. Total capital	5.79	7.98	9.58	5.88	7.01	8.29
Merger frequency	-	-	-	33.9%	37.7%	33.6%
% in monopoly	58.2%	18.6%	2.3%	95.2%	86.0%	68.4%
% $\min\{K_1, K_2\} \geq 2$	35.9%	75.7%	94.4%	0.1%	0.9%	3.8%

### 4.3 Equilibria with All Mergers Allowed

Equilibria with all mergers allowed are quite different. Figure 8 shows, for the intermediate market, the beginning-of-period steady state distribution over states that this equilibrium generates as well as the probability that a merger actually happens in each state. As before, the steady state distribution (at the start of the period, before mergers occur) is represented by the

<sup>14</sup>All values are ex ante (beginning-of-period) values, while the performance measures in the last two rows are at the output competition stage (stage 5).



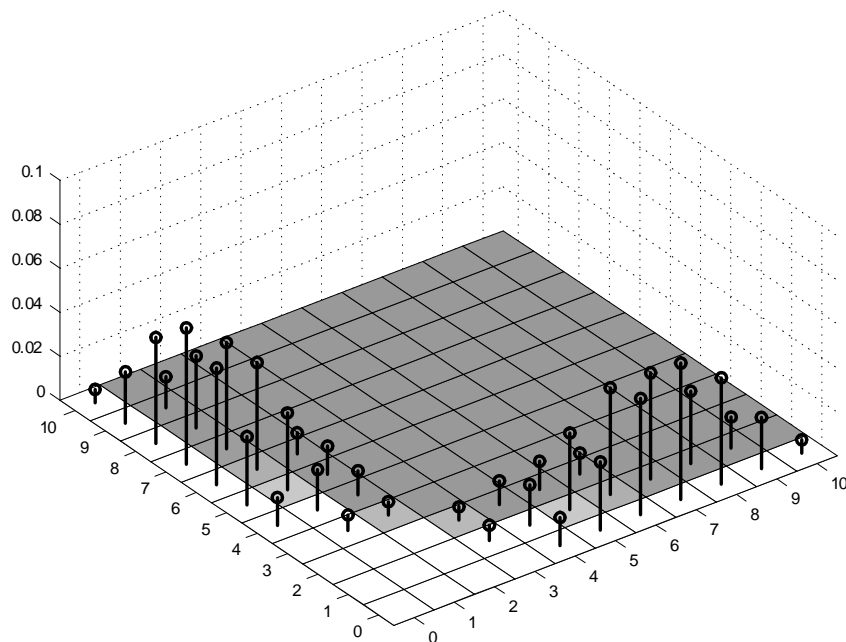


Figure 8: Beginning-of-period steady state distribution over states in the intermediate market with all mergers allowed. Darker shading indicates a merger is more likely to happen.

height of the pins. Now, in addition, each cell in which mergers occur with positive probability is shaded from light to dark grey, with a darker shade representing a higher probability of a merger happening in a state; cells in which mergers do not occur are unshaded. For example, a merger happens with probability 1 in state (3,3), with probability zero in state (2,2), and with probability 0.59 in state (2,3). As is apparent, the firms do not merge in all states, even though they would be allowed to. In particular, in states in which their capital stocks are both low, a merger would allow a new entrant to come into the industry, dissipating the gains from merger.

The next-to-last column of Table 2 shows the properties of the all-mergers-allowed equilibria in the intermediate market. The third row from the bottom shows that mergers happen 37.7% of the time. This results in the market being in a monopoly state (at the time of output competition) 86.0% of the time, and in a near monopoly 99.1% of the time. As a result of allowing mergers, average output falls from 22.2 to 19.2, while the average price rises from 2.15 to 2.26. Average total capital falls from 7.98 to 7.01. Also shown are beginning-of-period consumer, incumbent, entrant, and aggregate values. Not surprisingly, the change in policy leads to substantial negative changes in consumer value, which falls from 48.1 to 35.8. More surprisingly, average incumbent value falls despite the fact that the firms are now allowed to merge whenever they want. This is despite the success that unrestricted mergers have from the firms' point of view in raising expected price, reducing expected quantity, and limiting total capital. Even once one accounts for future entrants' value, producer value (the sum of incumbent and entrant values) barely rises. Combined with the dramatic reduction in consumer value, aggregate value falls substantially, from 117.5 to 105.8.

It is interesting to explore further the reasons behind these results. Consider, first, the

reduction in total capital. Allowing mergers does two things. First, it changes the states in which investments are taking place by moving the market to monopoly positions. The average capital addition in the no-mergers steady state is 1.99. If we keep firms' investment behavior fixed at their no-mergers equilibrium levels, but change the weighting over states to be that in the all-mergers-allowed steady state, the average capital addition drops to 1.46.<sup>15</sup> Second, firms' investment policies change; holding the distribution over states fixed at the all-mergers-allowed steady state and changing investment behavior to that in the all-mergers-allowed equilibrium increases average capital additions from 1.46 to 1.76. Thus, the reduction in total capital is due to the induced change in the steady state distribution rather than less aggressive investment behavior.

To understand the change in investment behavior, consider how the prospect of merger affects the incentive for a firm  $i$  to invest in state  $(K_i, K_{-i})$  if its rival does not. When mergers are not allowed, this incentive comes from the marginal effect on the firm's value,  $\partial\bar{V}(K_i, K_{-i})/\partial K_i$ . When, instead, a merger is certain to occur in the next period, the effect of investment on firm  $i$ 's gains from merger matters as well. Specifically, the marginal effect on a firm's value is  $\partial\bar{V}(K_i, K_{-i})/\partial K_i + (1/2)\partial\Delta_G(K_i, K_{-i})/\partial K_i$ , where  $\Delta_G(K_i, K_{-i})/\partial K_i$  is the effect of  $K_i$  on the gain from merger defined in (1). In the all-mergers-allowed steady state, the firms find themselves in states where  $\partial\Delta_G(K_i, K_{-i})/\partial K_i$  is positive 100% of the time.<sup>16,17</sup>

Why does average producer value not rise when all mergers are allowed? When all mergers are allowed, an entrant with zero capital frequently invests with the hope of being bought out, that is, we see a great deal of "entry for buyout" [Rasmusen (1988)]. Indeed, entrants invest much more than when no mergers are allowed. Figure 10 shows the one-period transition probabilities for an entrant in state  $(5, 0)$  when all mergers are allowed, which can be compared to Figure 2 that shows the same one-period transitions when no mergers are allowed. The probability that the entrant invests is 0.58 in the former case, versus 0.04 in the latter. Further, a merger happens 49% of the time after the entrant invests when all mergers are allowed. The two figures also show that the entrant's increased incentive lowers the incentive of the incumbent to invest.

Unfortunately for producer value, the entrant's investments are made, on average, at high cost. Figure 9 illustrates the destructiveness of this behavior. It shows for each state the change in the row firm's (firm 1) beginning-of-period value induced by a switch from a no-merger policy to an all-mergers-allowed policy. As can be seen there, in most states the row firm's value is enhanced by this change in policy, but in monopoly states in which the monopolist has at least 3 units of capital, the monopolist's value falls substantially due to the entry-for buyout behavior of the entrant who has no capital. This loss swamps the beneficial effect on entrant value of allowing mergers, resulting in large reductions in producer value in these monopoly states. Given the likelihood that the firms end up in such states, steady state producer value falls.

<sup>15</sup>The average capital addition in the all-mergers-allowed steady state is 1.76. Keeping investment behavior fixed at the all-mergers-allowed equilibrium behavior and reweighting by the steady state probabilities in the no-mergers equilibrium, the average capital addition increases from 1.76 to 2.24.

<sup>16</sup>In the no-mergers steady state, they are in states in which  $\partial\Delta_G(K_i, K_{-i})/\partial K_i$  is positive 97.5% of the time.

<sup>17</sup>This calculation looks at the effect of allowing mergers on investment incentives holding the interim value function fixed. Of course, in reality, allowing mergers will also alter this value function.

	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	1.1	1.1	1.6	1.7	1.4	1.3	1.1	1.0	1.0	0.9	0.8
k1=1	1.2	1.1	1.5	1.7	1.8	2.0	2.0	2.1	2.2	2.2	2.3
k1=2	(0.6)	0.8	1.5	1.9	2.1	2.3	2.4	2.5	2.6	2.7	2.8
k1=3	(5.4)	0.8	1.5	1.9	2.2	2.4	2.6	2.7	2.8	2.9	3.1
k1=4	(8.3)	0.4	1.5	2.0	2.3	2.5	2.7	2.8	2.9	3.1	3.3
k1=5	(10.1)	0.0	1.4	2.0	2.4	2.6	2.8	2.9	3.0	3.3	3.4
k1=6	(10.5)	(0.2)	1.4	2.1	2.4	2.7	2.9	3.0	3.1	3.4	3.5
k1=7	(10.4)	(0.4)	1.4	2.1	2.5	2.7	2.9	3.1	3.3	3.4	3.6
k1=8	(10.2)	(0.5)	1.4	2.1	2.5	2.8	3.0	3.1	3.4	3.5	3.7
k1=9	(10.1)	(0.6)	1.3	2.1	2.6	2.8	3.0	3.2	3.5	3.6	3.7
k1=10	(9.9)	(0.7)	1.3	2.1	2.6	2.9	3.1	3.2	3.5	3.7	3.8

Figure 9: Beginning-of-period value of the row firm (firm 1) in the all-mergers-allowed equilibrium minus its value in the no-mergers equilibrium.

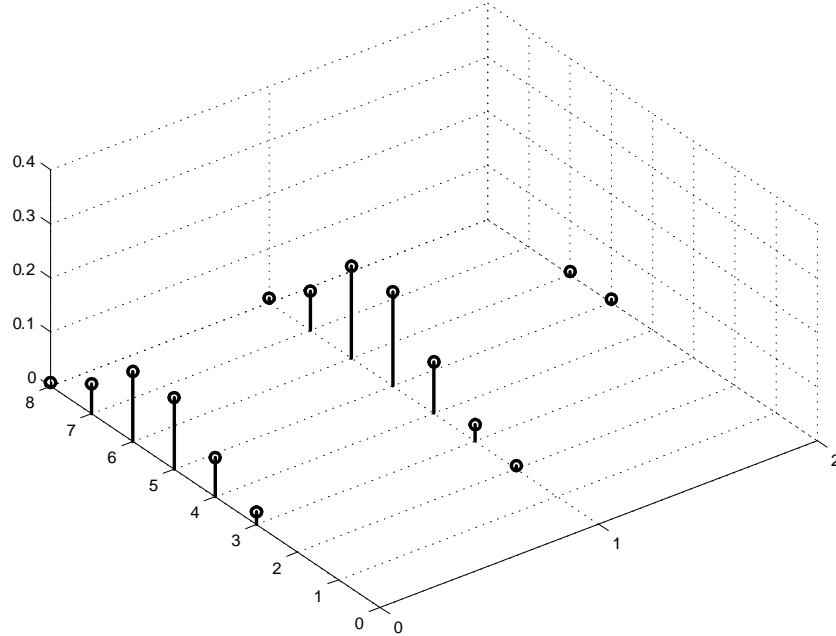


Figure 10: One-period transition from the state (5,0) in the intermediate market with all mergers allowed.

	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	(0.6)	(0.2)	0.5	1.2	2.2	3.0	3.5	3.9	4.2	4.4	4.6
k1=1	(1.2)	(1.3)	0.3	0.9	1.4	1.6	1.9	2.1	2.2	2.3	2.3
k1=2	(1.7)	(1.3)	(0.4)	0.1	0.4	0.7	0.9	1.0	1.2	1.3	1.3
k1=3	(1.4)	(1.2)	(0.6)	(0.2)	0.0	0.2	0.4	0.5	0.6	0.7	0.8
k1=4	(1.3)	(1.1)	(0.7)	(0.4)	(0.2)	0.0	0.1	0.3	0.3	0.4	0.4
k1=5	(1.3)	(1.0)	(0.7)	(0.4)	(0.3)	(0.1)	(0.0)	0.1	0.2	0.2	0.3
k1=6	(1.2)	(0.9)	(0.7)	(0.5)	(0.3)	(0.2)	(0.1)	(0.0)	0.1	0.1	0.1
k1=7	(1.1)	(0.9)	(0.6)	(0.5)	(0.4)	(0.3)	(0.2)	(0.1)	(0.0)	0.0	0.1
k1=8	(1.0)	(0.8)	(0.6)	(0.5)	(0.4)	(0.3)	(0.2)	(0.1)	(0.1)	(0.0)	0.0
k1=9	(0.9)	(0.8)	(0.7)	(0.6)	(0.5)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.0)
k1=10	(1.0)	(0.9)	(0.7)	(0.6)	(0.5)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)

Figure 11: Private incentive of the row firm (firm 1) to invest minus the social incentive for the row firm to invest in the intermediate market with all mergers allowed.

The entry-for-buyout incentive also reduces aggregate value. To illustrate why, Figure 11 shows the difference between the private and social incentives to invest when all mergers are allowed. Specifically, it shows, starting in each state  $(K_1, K_2)$ , the effects of the row firm adding one unit of capital on its value less its effect on aggregate value (so positive numbers indicate a socially excessive incentive to invest, while negative numbers indicate a socially insufficient incentive). As can be seen there, dominant firms generally have insufficient incentives, while entrants have excessive incentives. A very similar pattern emerges if instead no mergers are allowed. The entry-for-buyout phenomenon therefore causes a shift in investment away from the dominant firm, whose incentives are already insufficient, toward the entrant, whose incentives are excessive.

Similar welfare effects arise for identical reasons in the small and large markets, as is evident in Table 3.

## 5 Optimal Merger Policy

In this section we analyze the antitrust authority's optimal merger approval policy for both an AV and a CV welfare criterion. In our discussion we focus initially on the intermediate market analyzed in the previous section; we discuss the results for the small and large markets at the end of the section. For each market size we consider both no-commitment and commitment policies and contrast the resulting performance measures.

### 5.1 Feasible Policies

We consider two different types of settings, depending upon whether or not the authority can commit to its decision in a given state:

**No Commitment (Markov perfect policy).** In this setting the antitrust authority, like each of the firms, is a player in a dynamic stochastic game. Recall from Sections 2 and 3 that if at the beginning of the period the state is  $(K_1, K_2)$  and the firms submit a merger proposal, then the authority privately draws a random blocking cost  $b$  and blocks the merger only if the gain from the merger is less than the cost of blocking the merger:  $\Delta_W(K_1, K_2) < -b$ , where

$\Delta_W(K_1, K_2)$  is the welfare gain from the merger. This rule, from the firms' viewpoint, gives an approval policy of  $a(K_1, K_2) = 1 - H(-\Delta_W(K_1, K_2))$ . Appealing to the one-stage deviation principle,  $a(\cdot)$  is a Markov-perfect merger policy if and only if the authority has no incentive to deviate from  $a(\cdot)$  at any state  $(K_1, K_2)$ , assuming that in the continuation game it follows  $a(\cdot)$  and the firms follow their Markov perfect strategies that  $a(\cdot)$  induces.

**Commitment.** In this setting we assume that the antitrust authority commits to a pure action  $a(K_1, K_2) \in \{0, 1\}$  for each state  $(K_1, K_2)$  where  $a(K_1, K_2) = 1$  if the merger is approved and  $a(K_1, K_2) = 0$  if it is blocked. Observe that there are  $2^{100}$  possible deterministic symmetric merger policies. Thus, for computational reasons, we restrict the space of admissible commitment policies to two classes.<sup>18</sup>

1. *Herfindahl-Based Policy.* Under this type of policy, a proposed merger in state  $(K_1, K_2)$  is approved if and only if the induced change in the capital stock-based Herfindahl index is below a threshold  $\overline{\Delta H}$ :

$$\Delta H(K_1, K_2) \equiv H(K_1 + K_2, 0) - H(K_1, K_2) \leq \overline{\Delta H}$$

where  $H(K_1, K_2)$  is the capital stock-based Herfindahl index in state  $(K_1, K_2)$  and  $\overline{\Delta H}$  is the authority's policy variable.<sup>19,20</sup> For illustration, Figure 12(a) shows the policy  $\overline{\Delta H} = 0.35$  where states with  $a(K_1, K_2) = 1$  are shaded (only states with  $\max\{K_1, K_2\} \leq 10$  are shown), while Figure 12(b) shows the policy  $\overline{\Delta H} = 0.2$ .

2. *Capital-Stock-based Policy* Under this type of policy, a proposed merger in state  $(K_1, K_2)$  is approved if and only if  $K_1 + K_2 \notin [\underline{K}, \overline{K}]$  and  $\min\{K_1, K_2\} \geq \underline{K}_i$  where  $\underline{K}$ ,  $\overline{K}$ , and  $\underline{K}_i$  are the authority's policy variables.<sup>21</sup> Figure 12(c), for example depicts the policy  $(\underline{K}, \overline{K}, \underline{K}_i) = (4, 10, 1)$  where states with  $a(K_1, K_2) = 1$  are shaded (only states with  $\max\{K_1, K_2\} \leq 10$  are shown), while Figure 12(d) shows the policy  $(\underline{K}, \overline{K}, \underline{K}_i) = (4, 10, 3)$ .

As observed earlier, under a commitment policy the antitrust authority never incurs any blocking costs since if it commits to block a merger in state  $(K_1, K_2)$  the merger will not be proposed in the first place. We compute optimal commitment policies when the authority's goal is maximization of the steady-state level of its welfare criterion (AV or CV) and when, for the case of a new, infant industry, its goal is maximization of its welfare criterion in state  $(0, 0)$ .

<sup>18</sup>The particular form these simple commitment policies take is partly motivated by which mergers are AV-increasing as one-shot deviations.

<sup>19</sup>To retain computational tractability we discretize the policy space:  $\overline{\Delta H} \in \{0.075, 0.075 + \Delta, 0.075 + 2\Delta, \dots, 0.4 - \Delta, 0.4\}$ , where  $\Delta = 0.025$ .

<sup>20</sup>Because there are only two firms,  $H(K_1 + K_2, 0) = H(0, K_1 + K_2) = 1$ , so  $\Delta H(K_1, K_2) = 1 - H(K_1, K_2)$ . Therefore a merger is approved if and only if  $H(K_1, K_2) \geq 1 - \overline{\Delta H}$ . Thus, under the Herfindahl-based policy mergers are only approved if the beginning-of-period Herfindahl is sufficiently high. Consequently, the Herfindahl-based policy is, in effect, a policy that approves mergers between a dominant and a "failing" firm.

<sup>21</sup>To retain computational tractability we discretize the policy space:  $\underline{K} \in \{2, 4, \dots, 10, 12\}$ ,  $\overline{K} \in \{6, 8, \dots, 18, 20\}$  and  $\underline{K}_i \in \{1, 2, \dots, 6, 7\}$ .

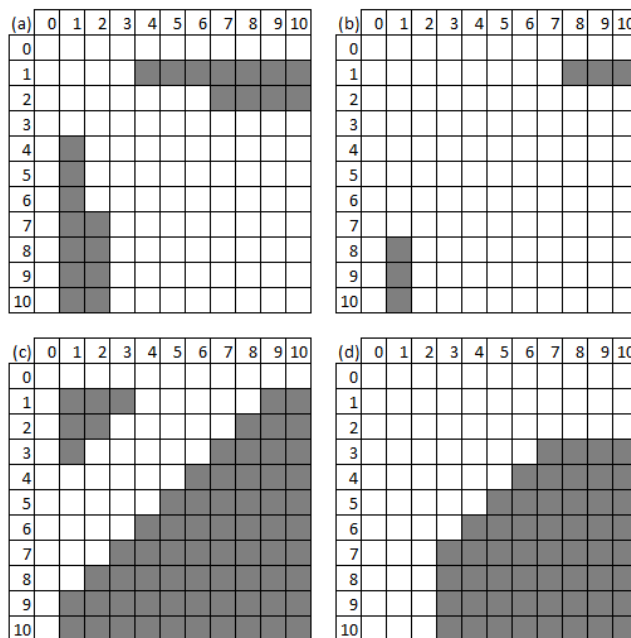


Figure 12: Panels (a) and (b) show Herfindahl-based commitment policies, whereas panels (c) and (d) show capital-stock-based commitment policies. (a) is  $\Delta\bar{H} = 0.35$ , (b) is  $\Delta\bar{H} = 0.2$ , (c) is  $(\underline{K}, \bar{K}, \underline{K}_i) = (4, 10, 1)$ , (d) is  $(\underline{K}, \bar{K}, \underline{K}_i) = (4, 10, 3)$ . The shaded states are those in which  $a(K_1, K_2) = 1$ .

## 5.2 Static Benchmarks

As a benchmark, and to understand some of the forces behind the optimal merger policy, Figure 13 shows for the intermediate market the static change in consumer surplus [panel (a)] and aggregate surplus [panel (b)] from allowing a merger (the figures show only states with  $\max\{K_1, K_2\} \leq 10$ ; states with positive surplus effects are shaded). This is the change in CS or AS due to production and consumption in the period the merger occurs.

In the intermediate market, only in state  $(K_1, K_2) = (1, 1)$  does a merger generate a static increase in consumer surplus, and there the gain is only 0.1.<sup>22</sup> In contrast, many mergers increase aggregate surplus. In general, these tend to be states in which the total capital in the industry is not too large: in the intermediate market, there is a static gain in aggregate surplus in any state in which total capital is not more than 10.<sup>23</sup> (There are also asymmetric states with total capital above that level in which a merger creates a static aggregate surplus gain.) The gains in aggregate surplus are generally smaller the larger is the total capital in the industry.<sup>24</sup>

<sup>22</sup>A merger among symmetrically-positioned firms increases consumer surplus if and only if the marginal cost reduction at the pre-merger output  $Q$  of the merging firms,  $C_Q(Q|K) - C_Q(2Q|2K)$ , exceeds the pre-merger price cost margin,  $P(2Q) - C_Q(Q|K)$ ; see Farrell and Shapiro (1990) and Nocke and Whinston (2010).

<sup>23</sup>The only exception is state (5, 5) where the static gain in aggregate surplus is approximately zero.

<sup>24</sup>To understand this result, observe that the change in aggregate surplus from a merger in a symmetric state is approximately

$$Q \left[ \left( \frac{\Delta Q}{Q} \right) (P - MC) - \left( 1 - \frac{\Delta Q}{Q} \right) \left( \frac{\Delta AC_M}{AC_M} \right) AC_M \right],$$

where  $(P - MC)$  is the premerger price-cost margin,  $AC_M$  is the average cost if no merger occurs but the

(a)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	0.1	(0.2)	(0.4)	(0.5)	(0.7)	(0.8)	(0.8)	(0.9)	(0.9)	(1.0)
k1=2	-	(0.2)	(0.6)	(1.0)	(1.3)	(1.5)	(1.7)	(1.9)	(2.0)	(2.0)	(2.1)
k1=3	-	(0.4)	(1.0)	(1.5)	(2.0)	(2.3)	(2.5)	(2.7)	(2.8)	(3.0)	(3.1)
k1=4	-	(0.5)	(1.3)	(2.0)	(2.4)	(2.8)	(3.1)	(3.3)	(3.5)	(3.7)	(3.8)
k1=5	-	(0.7)	(1.5)	(2.3)	(2.8)	(3.2)	(3.6)	(3.9)	(4.1)	(4.3)	(4.4)
k1=6	-	(0.8)	(1.7)	(2.5)	(3.1)	(3.6)	(4.0)	(4.3)	(4.5)	(4.7)	(4.9)
k1=7	-	(0.8)	(1.9)	(2.7)	(3.3)	(3.9)	(4.3)	(4.6)	(4.9)	(5.1)	(5.3)
k1=8	-	(0.9)	(2.0)	(2.8)	(3.5)	(4.1)	(4.5)	(4.9)	(5.2)	(5.4)	(5.6)
k1=9	-	(0.9)	(2.0)	(3.0)	(3.7)	(4.3)	(4.7)	(5.1)	(5.4)	(5.7)	(5.9)
k1=10	-	(1.0)	(2.1)	(3.1)	(3.8)	(4.4)	(4.9)	(5.3)	(5.6)	(5.9)	(6.2)

(b)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	1.7	1.6	1.4	1.2	1.0	0.9	0.8	0.7	0.6	0.5
k1=2	-	1.6	1.6	1.4	1.2	0.9	0.8	0.6	0.5	0.4	0.3
k1=3	-	1.4	1.4	1.1	0.9	0.6	0.4	0.2	0.1	0.0	(0.1)
k1=4	-	1.2	1.2	0.9	0.5	0.3	0.0	(0.1)	(0.3)	(0.4)	(0.5)
k1=5	-	1.0	0.9	0.6	0.3	(0.0)	(0.3)	(0.5)	(0.7)	(0.8)	(0.9)
k1=6	-	0.9	0.8	0.4	0.0	(0.3)	(0.6)	(0.8)	(1.0)	(1.2)	(1.3)
k1=7	-	0.8	0.6	0.2	(0.1)	(0.5)	(0.8)	(1.1)	(1.3)	(1.5)	(1.6)
k1=8	-	0.7	0.5	0.1	(0.3)	(0.7)	(1.0)	(1.3)	(1.5)	(1.7)	(1.9)
k1=9	-	0.6	0.4	0.0	(0.4)	(0.8)	(1.2)	(1.5)	(1.7)	(1.9)	(2.1)
k1=10	-	0.5	0.3	(0.1)	(0.5)	(0.9)	(1.3)	(1.6)	(1.9)	(2.1)	(2.3)

Figure 13: Static change in (a) consumer surplus and (b) aggregate surplus from a merger in the intermediate market.

An increase in the asymmetry of capital positions, holding total capital fixed, has varying effects on the static gains in aggregate surplus from a merger. This gain gets smaller with increased asymmetry at low levels of total capital, but grows larger with increased asymmetry at greater levels of total capital.

Finally, firms always have a static profit gain from merging, as a merger creates a monopoly in the period in which it occurs.

### 5.3 Markov Perfect Policy

Turning to the optimal dynamic policy, we first examine the Markov perfect merger policy. To do so, we start with the policy of allowing no mergers and the associated equilibrium strategies for the firms (discussed in Section 4), and iteratively update the antitrust authority's policy and the firm's strategies until we converge to an equilibrium. In the first iteration, we identify for each state  $(K_1, K_2)$  the antitrust authority's optimal approval rule given its expectation that its own behavior in the future will be to approve no mergers and that the firms will conform to their equilibrium strategies given that no-mergers policy. We then update firms' equilibrium strategies given this new approval policy by the antitrust authority. We continue to iterate in

output level changes to its post-merger level, and  $\Delta AC_M$  is the change in average cost at the post-merger output level due to the combination of capital. At larger capital levels,  $(P - MC)$  and  $|\Delta Q/Q|$  are both greater,  $(\Delta AC_M/AC_M)$  is unchanged, and  $AC_M$  is smaller, making the sign of the effect on aggregate surplus more likely to be negative. For example,  $(P - MC)$  is 0.32 at state (2, 2) and 0.45 at state (4, 4),  $(\Delta Q/Q)$  is  $-0.062$  at (2, 2) and  $-0.125$  at (4, 4), and  $AC_M$  is 27% lower at (4, 4) than at (2, 2).

(a)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	1.1	(3.7)	(6.8)	(8.9)	(9.6)	(9.9)	(10.1)	(10.2)	(10.4)	(10.2)
k1=2	-	(3.7)	(8.1)	(11.0)	(12.3)	(13.1)	(13.6)	(13.9)	(14.3)	(14.2)	(14.2)
k1=3	-	(6.8)	(11.0)	(13.1)	(14.3)	(15.2)	(15.8)	(16.3)	(16.5)	(16.5)	(16.6)
k1=4	-	(8.9)	(12.3)	(14.3)	(15.6)	(16.6)	(17.4)	(17.7)	(17.9)	(18.0)	(18.2)
k1=5	-	(9.6)	(13.1)	(15.2)	(16.6)	(17.8)	(18.3)	(18.7)	(18.8)	(19.1)	(19.3)
k1=6	-	(9.9)	(13.6)	(15.8)	(17.4)	(18.3)	(18.9)	(19.3)	(19.5)	(19.8)	(20.1)
k1=7	-	(10.1)	(13.9)	(16.3)	(17.7)	(18.7)	(19.3)	(19.6)	(19.9)	(20.3)	(20.6)
k1=8	-	(10.2)	(14.3)	(16.5)	(17.9)	(18.8)	(19.5)	(19.9)	(20.4)	(20.9)	(21.3)
k1=9	-	(10.4)	(14.2)	(16.5)	(18.0)	(19.1)	(19.8)	(20.3)	(20.9)	(21.4)	(21.9)
k1=10	-	(10.2)	(14.2)	(16.6)	(18.2)	(19.3)	(20.1)	(20.6)	(21.3)	(21.9)	(22.5)

(b)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	1.8	2.3	3.0	3.4	3.2	3.0	2.8	2.6	2.4	2.5
k1=2	-	2.3	2.9	3.1	2.8	2.5	2.2	1.9	1.7	1.8	1.8
k1=3	-	3.0	3.1	2.6	2.0	1.5	1.1	0.8	0.8	0.8	0.8
k1=4	-	3.4	2.8	2.0	1.2	0.7	0.2	0.0	(0.0)	(0.1)	(0.2)
k1=5	-	3.2	2.5	1.5	0.7	(0.1)	(0.4)	(0.6)	(0.7)	(0.9)	(1.0)
k1=6	-	3.0	2.2	1.1	0.2	(0.4)	(0.7)	(1.0)	(1.3)	(1.5)	(1.8)
k1=7	-	2.8	1.9	0.8	0.0	(0.6)	(1.0)	(1.4)	(1.8)	(2.1)	(2.4)
k1=8	-	2.6	1.7	0.8	(0.0)	(0.7)	(1.3)	(1.8)	(2.2)	(2.7)	(3.0)
k1=9	-	2.4	1.8	0.8	(0.1)	(0.9)	(1.5)	(2.1)	(2.7)	(3.1)	(3.6)
k1=10	-	2.5	1.8	0.8	(0.2)	(1.0)	(1.8)	(2.4)	(3.0)	(3.6)	(4.1)

Figure 14: Effect of a one-time merger on (a) CV and (b) AV in the intermediate market. Each cell shows the change in CV or AV from a merger in a given state. Shaded cells are those in which the change is positive.

this fashion until the antitrust authority has no incentive to deviate from its current policy.<sup>25</sup>

Figures 14 and 15 show the first step in this iteration process. Figure 14 shows for each state the gain (before blocking costs) in CV or AV from a one-time merger approval given the expectation that no mergers will be approved in the future and that firms' strategies will be the ones that form an equilibrium given that no mergers would be allowed. For both the CV and AV welfare criteria, the set of states in which there is a gain (before blocking costs) from a one-time merger approval is very close to the set of states in which a merger is statically beneficial. For example, the merger increases CV in state (1, 1) where the gain is 1.1. So, with a CV criterion, a merger is approved with probability one in that state. In all other states the change in CV is less than -1, so with blocking costs drawn from the uniform distribution on  $[0, 1]$  a merger is blocked with probability 1 in all of these states. In contrast, every state with total capital no greater than 10 has an increase in AV from merger approval (except state (5, 5) where the gain is approximately zero). We will let  $a^{(1)}(\cdot)$  denote the policy that emerges from this first step in the iteration. Figure 15 shows the resulting probabilities of merger approval in each state with the AV criterion. Given this new policy  $a^{(1)}(\cdot)$ , we complete the iteration step by identifying firms' new equilibrium proposal and investment strategies, which we denote by  $[\psi^{(1)}(\cdot), \xi^{(1)}(\cdot)]$ .

In the next step of the iteration process we determine the gains from a one-time merger approval in each state given that the antitrust authority will follow policy  $a^{(1)}(\cdot)$  in the future, and firms' behavior is given by proposal and investment strategies  $(\psi^{(1)}(\cdot), \xi^{(1)}(\cdot))$ . When we

<sup>25</sup>For reasons of computational efficiency and to aid with convergence, this is not exactly what our code does. The discussion of these iterations serves to illustrate the economics.



(a)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=1	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=2	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=3	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=4	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=5	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=6	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=7	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=8	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=9	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=10	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

(b)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=1	0%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
k1=2	0%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
k1=3	0%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
k1=4	0%	100%	100%	100%	100%	100%	100%	100%	100%	92%	84%
k1=5	0%	100%	100%	100%	100%	96%	66%	46%	32%	14%	0%
k1=6	0%	100%	100%	100%	100%	66%	27%	0%	0%	0%	0%
k1=7	0%	100%	100%	100%	100%	46%	0%	0%	0%	0%	0%
k1=8	0%	100%	100%	100%	100%	32%	0%	0%	0%	0%	0%
k1=9	0%	100%	100%	100%	92%	14%	0%	0%	0%	0%	0%
k1=10	0%	100%	100%	100%	84%	0%	0%	0%	0%	0%	0%

Figure 15: First policy iteration according to (a) CV criterion and (b) AV criterion in the intermediate market. Each cell shows the probability  $a^{(1)}(K_1, K_2)$  (stated as a %) that a merger is approved.

do this for the CV criterion, there is no change in the approval probabilities, so the approval policy shown in Figure 15(a) is in fact a Markov perfect policy. This equilibrium is essentially identical to the no-mergers equilibrium of Section 4.<sup>26</sup>

For the AV criterion, however, the optimal policy changes dramatically in the second iteration. Figure 16 shows for each state the gain in AV (before blocking costs) from a one-time merger approval given the expectation that the antitrust authority will follow policy  $a^{(1)}(\cdot)$  in the future and that firms' strategies will be  $(\psi^{(1)}(\cdot), \xi^{(1)}(\cdot))$ . Only states in which  $\min\{K_1, K_2\} \leq 2$  (and not all of them) have gains in AV from merger approval. Due to blocking costs, in policy  $a^{(2)}(\cdot)$  the antitrust authority prevents with certainty only those mergers that cause AV to fall by at least 1.0. States with a positive probability of merger approval given the blocking costs are those with  $\min\{K_1, K_2\} \leq 3$ , plus state (4, 4).

While the analysis of Section 4 shows how merger policy can affect investment, we see here the reverse effect with the AV criterion: Firms' investment policies have a dramatic effect on the antitrust authority's best-response approval policy in the absence of commitment. To understand this effect, observe that once the merger policy changes from no mergers being allowed to policy  $a^{(1)}(\cdot)$ , the investment behavior of the firms (captured in  $\xi^{(1)}(\cdot)$ ) changes dramatically, especially for new entrants. As policy  $a^{(1)}(\cdot)$  allows many mergers, these changes in firms' behavior are similar to those we saw in Section 4 when all mergers were allowed.

<sup>26</sup>Under a Markov perfect policy, a merger is allowed in state (1,1), but it is not profitable for the firms.

	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	1.1	0.1	(0.0)	0.1	0.2	0.2	0.3	0.3	0.3	0.5
k1=2	-	0.1	0.1	0.1	(0.0)	(0.1)	(0.2)	(0.2)	(0.2)	(0.0)	0.2
k1=3	-	(0.0)	0.1	(0.2)	(0.4)	(0.6)	(0.7)	(0.8)	(0.7)	(0.5)	(0.4)
k1=4	-	0.1	(0.0)	(0.4)	(0.7)	(1.0)	(1.2)	(1.3)	(1.2)	(1.1)	(1.0)
k1=5	-	0.2	(0.1)	(0.6)	(1.0)	(1.4)	(1.6)	(1.6)	(1.6)	(1.6)	(1.6)
k1=6	-	0.2	(0.2)	(0.7)	(1.2)	(1.6)	(1.8)	(1.9)	(2.0)	(2.2)	(2.3)
k1=7	-	0.3	(0.2)	(0.8)	(1.3)	(1.6)	(1.9)	(2.2)	(2.5)	(2.9)	(3.1)
k1=8	-	0.3	(0.2)	(0.7)	(1.2)	(1.6)	(2.0)	(2.5)	(3.1)	(3.5)	(3.8)
k1=9	-	0.3	(0.0)	(0.5)	(1.1)	(1.6)	(2.2)	(2.9)	(3.5)	(4.0)	(4.4)
k1=10	-	0.5	0.2	(0.4)	(1.0)	(1.6)	(2.3)	(3.1)	(3.8)	(4.4)	(4.8)

Figure 16: Change in AV from a merger given firms' behavior  $(\psi^{(1)}(\cdot), \xi^{(1)}(\cdot))$  after the first policy iteration  $a^{(1)}(\cdot)$  in the intermediate market.

**Table 3: Investments Starting at  $(5, 0)$**

Capital addition	No-mergers equilibrium		Investment policy $\xi^{(1)}(\cdot)$	
	firm 0	firm 5	firm 0	firm 5
0	96%	0%	29%	3%
1	4%	8%	68%	39%
2	0%	66%	3%	48%
3	0%	25%	0%	9%
4	0%	1%	0%	0%

For example, starting at state  $(5, 0)$  the distributions of capital additions by the two firms when no mergers are allowed and under policy  $a^{(1)}(\cdot)$  are shown in Table 3. As can be seen in that table, the entrant invests only with a 4% probability at state  $(5, 0)$  when mergers are not allowed, but under policy  $a^{(1)}(\cdot)$  that probability changes to 71%. The incumbent, on the other hand, invests less under policy  $a^{(1)}(\cdot)$ . The entrant is doing this because of the prospect that he will get bought out. A merger happens with a high probability in the first period after this investment, provided the entrant's new unit of capital does not immediately depreciate. Even if it does not occur immediately, Table 4 shows that starting from state  $(5, 0)$  a merger is almost certain within a small number of periods.

**Table 4: Probability of a Merger if Industry is in State  $(5, 0)$  at Start of Period 1**

Period	1	2	3	4	5
Cumulative merger probability	29.1%	61.8%	80.5%	90.1%	95.0%

When the authority adopts policy  $a^{(1)}(\cdot)$ , monopolists generally have insufficient investment incentives, while entrants' incentives are too large leading them to invest at high cost. The policy-induced change in firms' investment behavior makes the movement to a monopoly state due to a merger much less attractive for the antitrust authority, causing the authority's second-step policy  $a^{(2)}(\cdot)$  to be much more restrictive in comparison to  $a^{(1)}(\cdot)$ .

(a)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=1	0%	100%	100%	93%	73%	67%	66%	69%	74%	80%	80%
k1=2	0%	100%	76%	22%	0%	0%	0%	0%	0%	0%	0%
k1=3	0%	93%	22%	0%	0%	0%	0%	0%	0%	0%	0%
k1=4	0%	73%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=5	0%	67%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=6	0%	66%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=7	0%	69%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=8	0%	74%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=9	0%	80%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=10	0%	80%	0%	0%	0%	0%	0%	0%	0%	0%	0%

(b)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=1	0%	0%	57%	100%	100%	100%	100%	100%	100%	100%	100%
k1=2	0%	57%	100%	100%	0%	0%	0%	0%	0%	0%	0%
k1=3	0%	100%	100%	0%	0%	0%	0%	0%	0%	0%	0%
k1=4	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=5	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=6	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=7	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=8	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=9	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=10	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%

(c)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=1	0%	0%	57%	93%	73%	67%	66%	69%	74%	80%	80%
k1=2	0%	57%	76%	22%	0%	0%	0%	0%	0%	0%	0%
k1=3	0%	93%	22%	0%	0%	0%	0%	0%	0%	0%	0%
k1=4	0%	73%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=5	0%	67%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=6	0%	66%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=7	0%	69%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=8	0%	74%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=9	0%	80%	0%	0%	0%	0%	0%	0%	0%	0%	0%
k1=10	0%	80%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Figure 17: Markov perfect policy (AV criterion, intermediate market): probabilities mergers are (a) allowed, (b) proposed, and (c) happen.

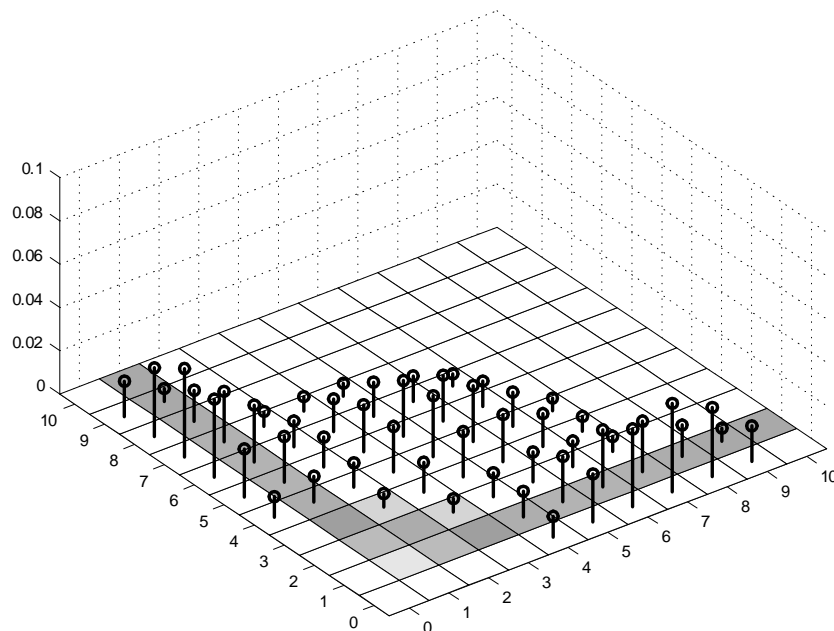


Figure 18: Beginning-of-period steady state distribution of the equilibrium generated by the Markov perfect policy (AV criterion) in the intermediate market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

The AV-maximizing Markov perfect policy is even more restrictive than policy  $a^{(2)}(\cdot)$ . Figure 17 shows the merger acceptance probabilities, merger proposal probabilities, and the probabilities a merger actually occurs in various states under this Markov perfect policy. For states in which each firm has no more than 10 units of capital, the antitrust authority approves a proposed merger with probability one only in states  $(1, 1)$ ,  $(2, 1)$ , and  $(1, 2)$ . The authority approves a proposed merger with positive probability in near-monopoly states in which  $\min\{K_1, K_2\} = 1$ , as well as in states  $(2, 2)$ ,  $(3, 2)$ , and  $(2, 3)$ . Overall, the policy resembles one in which mergers are approved if one of the firms is “failing.” Given this policy, mergers are proposed with probability one in all of these states, except in state  $(1, 1)$ , where a merger is never proposed, and in states  $(2, 1)$ , and  $(1, 2)$ , where a merger is proposed with less than full probability.

Figure 18 shows the steady state distribution for the Markov perfect policy. Table 5 shows some summary statistics for the Markov perfect policy equilibrium under the AV criterion, and for equilibria when either no mergers or all mergers are allowed. In the steady state induced by the Markov perfect policy, the industry is in a monopoly state at the time of static competition 49.4% of the time, and in near-monopoly states 55.8% of the time. Compared to the steady state induced when no mergers are allowed, the economy spends much more time in such states. In addition, the average aggregate capital level is lower (7.98 vs. 7.65). The reason is the shift in the steady state distribution toward more asymmetric states, in which investments are lower. For example, the average total capital addition (gross of depreciation) by the two firms in the no-mergers steady state is 1.99 units of capital. Keeping firms’ investment strategies fixed but

changing the steady state distribution to the one in the Markov perfect equilibrium lowers the average capital addition to 1.75. If we then change firms' investment strategies to that in the Markov perfect equilibrium, the average capital addition rises from 1.75 to 1.92.

**Table 5: Performance Measures for the Intermediate Market under Various Policies**

<i>Performance measure</i> <sup>27</sup>	<i>NoMergers/ MP-CV</i>	<i>All Mergers</i>	<i>MP-AV</i>	<i>Commitment (CV and AV)</i>	<i>Planner</i>
Avg. Consumer value	48.1	35.8	43.3	49.3	39.2
Avg. Incumbent value	69.4	68.1	69.9	68.8	82.1
Avg. Entrant value	0.0	1.9	0.5	0.0	0.0
Avg. Blocking cost	0.0	0.0	-0.1	0.0	0.0
Avg. Aggregate Value	117.5	105.8	113.6	118.1	121.3
Avg. Price	2.15	2.26	2.19	2.14	2.23
Avg. Quantity	22.2	19.2	21.0	22.5	20.1
Avg. Total capital	7.98	7.01	7.65	8.17	8.08
Merger frequency	0.0%	37.7%	16.1%	3.0%	0.0%
% in monopoly	18.6%	86.0%	49.4%	14.3%	100.0%
% $\min\{K_1, K_2\} \geq 2$	75.7%	0.9%	44.2%	78.8%	0.0%
State (0,0) Consumer Value	30.3	23.9	25.6	30.4	25.3
State (0,0) Aggregate Value	36.7	34.0	35.5	36.7	41.8

Most strikingly, the Markov perfect policy equilibrium with the AV criterion results in a level of steady state AV that is about 3% lower than with the no-mergers policy: AV is 113.6 compared to 117.5 when no mergers are allowed. Firms are slightly better off while consumers are much worse off: CV is 43.3 (vs. 48.1) and producer value is 70.4 (vs. 69.4). Consumers are harmed both from the monopoly pricing and the reduction in capital, both of which lead to higher prices.

The finding that the Markov perfect policy with the AV criterion performs worse than the no-mergers policy but better than the all-mergers-allowed policy holds not only for the steady state averages of AV and CV but also for a “new” industry: at state (0,0), the AV (resp. CV) value of the Markov perfect policy is 35.5 (25.6), that of the no-mergers policy 36.7 (30.3), while that of the all-mergers-allowed policy is only 34.0 (23.9).

## 5.4 Commitment Policy

We now turn to the optimal commitment policy in the intermediate market. By this we mean the policy that leads to the largest steady state level of expected welfare, either CV or AV depending on the welfare criterion.<sup>28</sup> We also consider the commitment policy that

<sup>27</sup>All values are ex ante (beginning-of-period) values, while the performance measures in the last two rows are at the output competition stage (stage 5).

<sup>28</sup>This policy will generally differ from the policy that would be optimal given that the industry is starting in a particular state  $(K_1, K_2)$ .

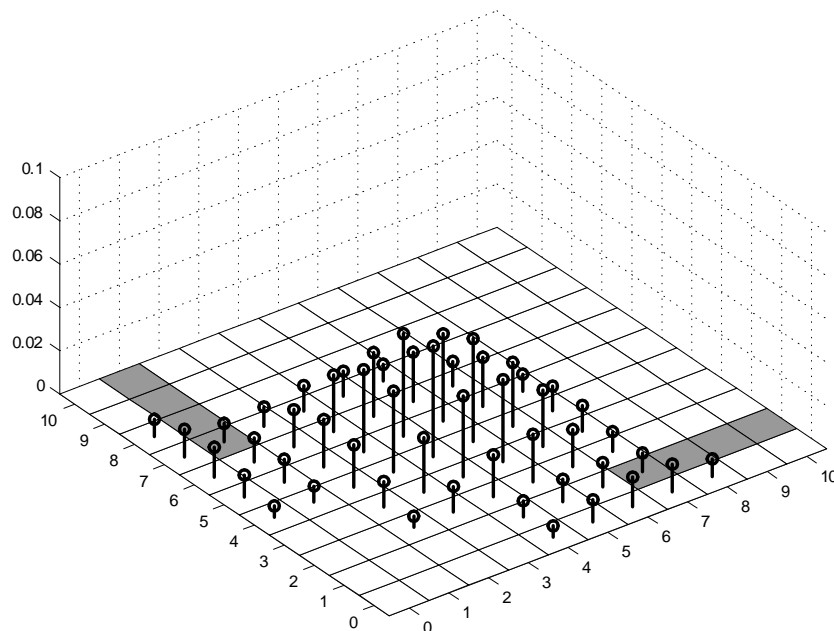


Figure 19: Beginning-of-period steady state distribution of the equilibrium generated by the optimal commitment policy (AV and CV criteria) in the intermediate market. The height of each pin indicates the steady state probability of that state. Cells in which mergers are proposed and approved are darkly shaded.

maximizes the expected welfare of a “new” industry at state (0,0). In contrast to the Markov perfect policy, the planner in the commitment case considers the impact his policy has on firms’ strategies.<sup>29</sup>

In the intermediate market, the optimal commitment policy — for either a CV or AV standard — is the Herfindahl-type policy  $\underline{\Delta H} = 0.225$ . For states in which each firm has no more than 10 units of capital, this policy involves approving a merger only when the smaller firm has one unit of capital and the larger firm has at least seven units. Wherever a merger is approved under this policy, it is also highly profitable to the merging firms and is proposed with probability one. With mergers occurring only 3 percent of the time, this policy is fairly close to the no-mergers policy.

Figure 19 shows the steady state distribution of the equilibrium induced by the optimal commitment policy. Table 5 shows steady state averages of various performance measures for this policy. The ability to commit leads to a 4% gain in AV compared to the Markov perfect policy with the AV criterion, and a 2.5% gain in CV compared to the Markov perfect policy with the CV criterion.

Strikingly, even though mergers move the industry to a monopoly state, the industry spends less time in a monopoly state (at the static competition stage) with the optimal commitment policy than under the no-mergers policy (14.3% vs. 18.6%), and capital levels are higher (8.17

<sup>29</sup>A less obvious difference is that under commitment the antitrust authority considers the impact its policy has on proposal costs, while without commitment those costs are considered to be sunk at the time a merger is reviewed. [A similar point arises in Besanko and Spulber (1992).]

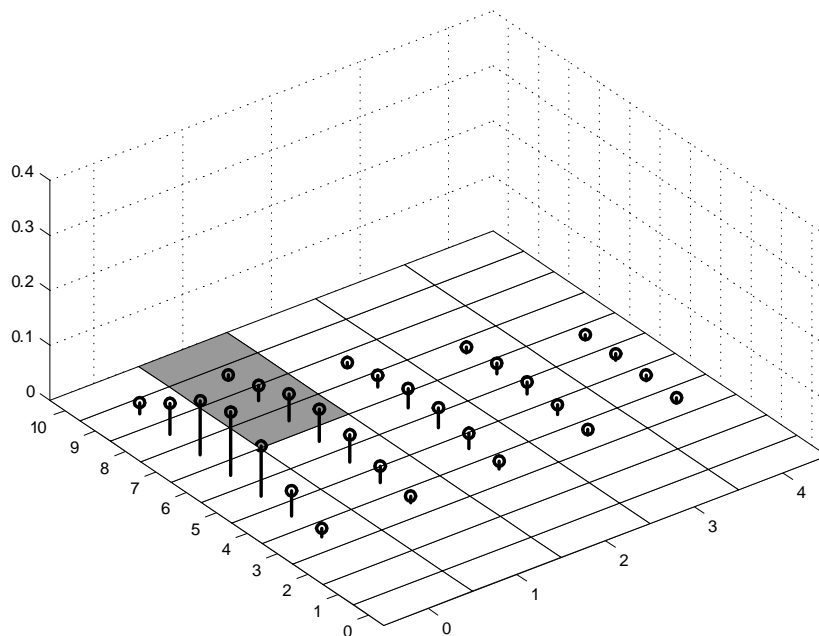


Figure 20: Five-period transitions from state (5,0) under the optimal commitment policy. The height of each pin indicates the probability of the industry being in that state. Cells in which mergers are proposed and approved are darkly shaded.

vs. 7.98). As can be seen in Figures 20 and 21, the reason there is less monopoly is that the prospect of merger induces entrants to invest, but the limited set of states in which mergers are allowed results in the industry often moving to symmetric duopoly positions following these investments. Indeed, the probability that the industry is in a monopoly state after five periods starting from state (5,0) is much lower than under the no-mergers policy: 0.45 vs. 0.84. The greater movement to symmetric, duopolistic states from monopoly ones can also be seen by comparing Figure 22 to Figure 3.

The greater permissiveness of the commitment policy compared to the no-mergers policy increases average AV because of this shift in the steady state distribution toward more symmetric duopoly states. As a general matter aggregate value falls in some states because of allowing these mergers and rises in others (aggregate value particularly falls in monopoly states because the commitment policy encourages entry for buyout). Were the distribution over states not to change, these changes in the value function would lead average aggregate value to fall from 117.5 to 116.9; the change in the steady state distribution, however, raises average aggregate value to 118.1.

While full commitment to a policy may be difficult to achieve, an alternative is to endow the antitrust authority with an objective that may not be the true social objective. In this regard, note that the steady-state level of AV under the Markov perfect merger policy when the antitrust authority has a CV objective is higher than that when it has an AV objective. Thus, when the antitrust authority cannot commit, a CV-maximizing antitrust authority is better for AV in this market than an AV-maximizing authority. This is consistent with a suggestion of Lyons (2002), but arises because of the policy's effect on investment, rather than by inducing

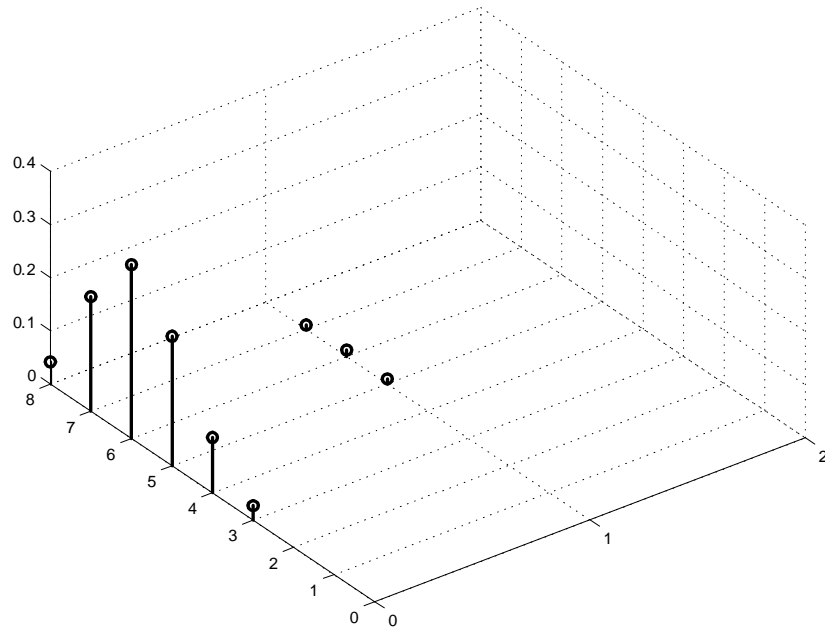


Figure 21: Five-period transitions from state (5,0) under the no-mergers policy. The height of each pin indicates the probability of the industry being in that state.

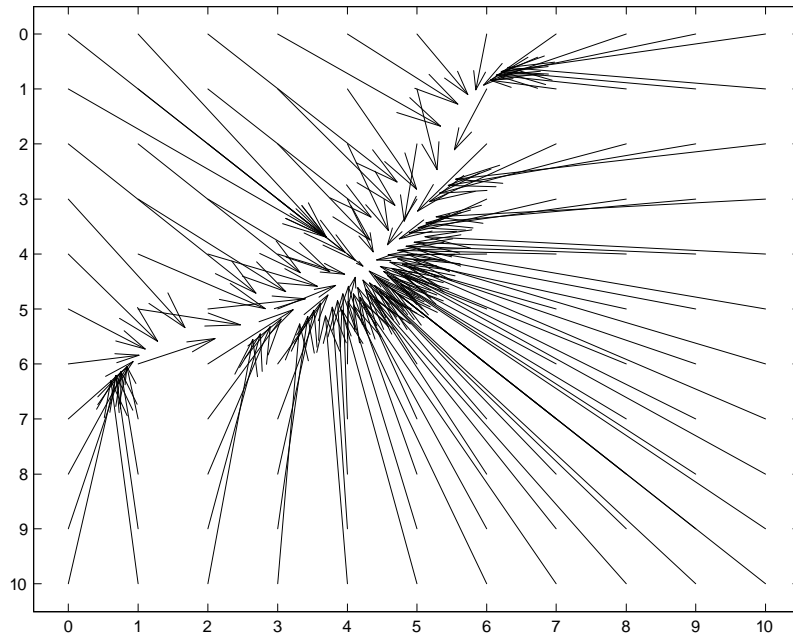


Figure 22: Arrows show the expected transitions over 5 periods under the optimal commitment policy.



more desirable merger proposals.

We also consider the optimal commitment policy for a new industry, which maximizes the welfare level (either AV or CV, depending on the authority’s objective function) at state (0,0). In searching for this policy, we proceed in two steps. First, we identify the state (0,0) welfare-maximizing policy in the class of Herfindahl-based or capital-stock-based commitment policies. Second, we identify the state-(0,0)-welfare-maximizing policy in the space of all feasible (history independent) commitment policies within the “box”  $\{(K_1, K_2) | 0 \leq K_i \leq 4, i = 1, 2\}$ , assuming that the authority commits to the policy identified in step 1 for states outside this box. The rationale for the second step is that the policy in states with small capital levels is likely to matter most when searching for the optimal commitment policy starting from state (0, 0).

It turns out that the optimal commitment policy from state (0,0) allows mergers in very few states. For the AV objective, the authority allows mergers only in states  $(K_1, K_2)$  such that  $K_i \in \{1, 2\}$ ,  $i = 1, 2$ . However, as a merger in state (1, 1) is never (and in states (1, 2) and (2, 1) only rarely) profitable, this is almost equivalent to allowing mergers only in state (2, 2). The resulting AV (resp. CV) level is 37.2 (26.6), whereas under the no-mergers policy it is 36.7 (30.3). For the CV objective, the state (0,0) optimal commitment policy is a no-mergers policy.

## 5.5 Merger Policy in the Small and Large Markets

In this subsection, we describe our results for the optimal merger policy in the small and large markets, and compare them to our results for the intermediate market.

The static welfare effects of mergers are very similar in the three markets: in all of them only a merger in state (1, 1) increases static consumer surplus, and in all of them, a merger in state  $(K_1, K_2)$  increases static aggregate surplus unless both  $K_1$  and  $K_2$  are “large,” with the set of statically aggregate surplus-increasing mergers being larger in larger markets. Figure 23 shows the set of aggregate surplus-increasing mergers in the small and large markets.

As in the intermediate market, if the antitrust authority pursues a CV goal and cannot commit, the Markov perfect merger policies in the small and large markets are essentially equivalent to the no-mergers policy.<sup>30</sup>

When the antitrust authority pursues instead an AV goal, the Markov perfect merger policy again results in mergers only in near-monopoly states in which the incumbent is sufficiently large. The larger the market, the more restrictive is the antitrust authority in equilibrium. Figures 24 and 25 show the steady state distribution and probabilities that a merger happens in the two markets, while Tables 6 and 7 provide some summary statistics of these equilibria. The average merger probability is 30.6% in the small market, but only 3.0% in the large market (versus 16.1% in the intermediate market). In the small market the industry is almost always (98.6% of the time) in a monopoly state at the (post-merger) output stage, compared to 49.4% in the intermediate market, and only 8.2% in the large market. Just as in the intermediate market, absent commitment, the optimal merger policy of a CV-oriented authority induces a higher value of AV than that of an AV-oriented authority: the respective AV values are 89.6 vs. 87.9 in the small market and 142.3 vs. 141.3 in the large market.

<sup>30</sup>In the large market, the authority would approve mergers in states (1, 1), (2, 1), and (1, 2) but such mergers are not value-enhancing for the firms and therefore never proposed.

(a)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	1.5	1.4	1.1	1.0	0.8	0.7	0.6	0.5	0.4	0.4
k1=2	-	1.4	1.3	1.1	0.8	0.6	0.5	0.4	0.3	0.2	0.1
k1=3	-	1.1	1.1	0.8	0.5	0.3	0.1	(0.0)	(0.1)	(0.2)	(0.3)
k1=4	-	1.0	0.8	0.5	0.2	(0.0)	(0.3)	(0.4)	(0.5)	(0.6)	(0.7)
k1=5	-	0.8	0.6	0.3	(0.0)	(0.3)	(0.6)	(0.7)	(0.9)	(1.0)	(1.1)
k1=6	-	0.7	0.5	0.1	(0.3)	(0.6)	(0.8)	(1.0)	(1.2)	(1.3)	(1.4)
k1=7	-	0.6	0.4	(0.0)	(0.4)	(0.7)	(1.0)	(1.3)	(1.4)	(1.6)	(1.7)
k1=8	-	0.5	0.3	(0.1)	(0.5)	(0.9)	(1.2)	(1.4)	(1.7)	(1.8)	(2.0)
k1=9	-	0.4	0.2	(0.2)	(0.6)	(1.0)	(1.3)	(1.6)	(1.8)	(2.0)	(2.2)
k1=10	-	0.4	0.1	(0.3)	(0.7)	(1.1)	(1.4)	(1.7)	(2.0)	(2.2)	(2.3)

(b)	k2=0	k2=1	k2=2	k2=3	k2=4	k2=5	k2=6	k2=7	k2=8	k2=9	k2=10
k1=0	-	-	-	-	-	-	-	-	-	-	-
k1=1	-	1.8	1.9	1.7	1.4	1.2	1.1	1.0	0.8	0.8	0.7
k1=2	-	1.9	2.0	1.8	1.5	1.3	1.1	0.9	0.8	0.6	0.6
k1=3	-	1.7	1.8	1.5	1.2	1.0	0.7	0.5	0.4	0.3	0.2
k1=4	-	1.4	1.5	1.2	0.9	0.6	0.4	0.1	(0.0)	(0.2)	(0.3)
k1=5	-	1.2	1.3	1.0	0.6	0.3	0.0	(0.2)	(0.4)	(0.6)	(0.7)
k1=6	-	1.1	1.1	0.7	0.4	0.0	(0.3)	(0.6)	(0.8)	(1.0)	(1.1)
k1=7	-	1.0	0.9	0.5	0.1	(0.2)	(0.6)	(0.8)	(1.1)	(1.3)	(1.5)
k1=8	-	0.8	0.8	0.4	(0.0)	(0.4)	(0.8)	(1.1)	(1.3)	(1.6)	(1.8)
k1=9	-	0.8	0.6	0.3	(0.2)	(0.6)	(1.0)	(1.3)	(1.6)	(1.8)	(2.0)
k1=10	-	0.7	0.6	0.2	(0.3)	(0.7)	(1.1)	(1.5)	(1.8)	(2.0)	(2.2)

Figure 23: Static change in aggregate surplus for (a) the small market and (b) the large market.

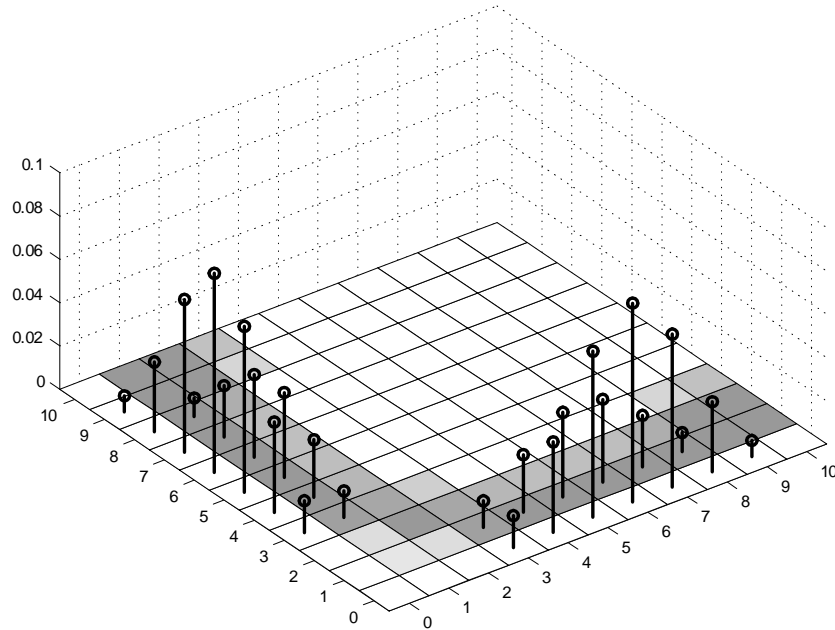


Figure 24: Beginning-of-period steady state distribution of the equilibrium generated by the Markov perfect policy (AV criterion) in the small market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

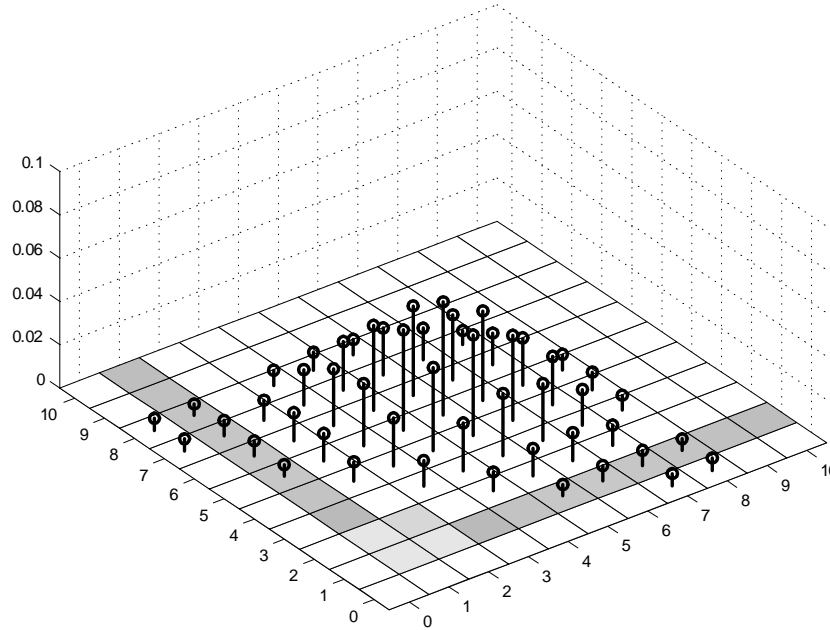


Figure 25: Beginning-of-period steady state distribution of the equilibrium generated by the Markov perfect policy (AV criterion) in the large market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

**Table 6: Performance Measures for the Small Market under Various Policies**

<i>Performance measure</i> <sup>31</sup>	<i>NoMergers/</i>	<i>MP-AV</i>	<i>Commitment</i>	<i>Commitment</i>
	<i>MP-CV</i>		<i>AV</i>	<i>CV</i>
Avg. Consumer value	31.8	29.1	32.9	33.2
Avg. Incumbent value	57.8	58.0	61.0	57.8
Avg. Entrant value	0.0	0.8	0.0	0.1
Avg. Blocking cost	0.0	0.0	0.0	0.0
Avg. Aggregate Value	89.6	87.9	94.0	91.1
Avg. Price	2.25	2.28	2.23	2.23
Avg. Quantity	16.5	15.9	16.9	16.9
Avg. Total capital	5.79	5.98	6.56	6.23
Merger frequency	0.0%	30.6%	6.8%	11.6%
% in monopoly	58.2%	98.6%	68.6%	60.8%
% $\min\{K_1, K_2\} \geq 2$	35.9%	0.3%	17.4%	32.3%
State (0,0) Consumer Value	24.0	19.4	5.9	24.1
State (0,0) Aggregate Value	28.4	26.8	27.7	28.3

<sup>31</sup>All values are ex ante (beginning-of-period) values, while the performance measures in the last two rows are at the output competition stage (stage 5).

**Table 7: Performance Measures for the Large Market under Various Policies**

<i>Performance measure</i> <sup>32</sup>	<i>NoMergers/</i>	<i>MP-AV</i>	<i>Commitment</i>	<i>Commitment</i>
	<i>MP-CV</i>		<i>AV</i>	<i>CV</i>
Avg. Consumer value	61.3	60.1	61.4	61.4
Avg. Incumbent value	81.0	81.1	81.1	80.8
Avg. Entrant value	0.0	0.1	0.0	0.0
Avg. Blocking cost	0.0	-0.1	0.0	0.0
Avg. Aggregate Value	142.3	141.3	142.5	142.3
Avg. Price	2.10	2.11	2.10	2.10
Avg. Quantity	27.0	26.7	27.0	27.0
Avg. Total capital	9.58	9.49	9.60	9.58
Merger frequency	0.0%	3.0%	0.0%	0.1%
% in monopoly	2.3%	8.2%	2.3%	1.1%
% $\min\{K_1, K_2\} \geq 2$	94.4%	87.9%	94.5%	95.5%
State (0,0) Consumer Value	36.4	35.5	36.5	36.4
State (0,0) Aggregate Value	45.6	45.2	45.6	45.6

If the antitrust authority can commit to its policy and pursues a CV goal, then in all three markets mergers are approved only in near-monopoly states in which the incumbent is sufficiently large. This policy is more restrictive the larger is the market, with the merger probabilities ranging from 0.1% in the large market to 11.6% in the small market. Figures 26 and 27 show the steady state distributions and optimal merger policy for the small and large markets.

If the antitrust authority can commit to its policy and pursues an AV goal instead, it essentially does not approve any mergers in the large market, whereas in the small market it does approve mergers in states in which both firms are sufficiently large (resulting in a merger probability of 6.8%), which boosts firms' investment incentives (resulting in an almost 10% higher capital level compared to the AV-maximizing Markov perfect policy). Figures 28 and 29 show the steady state distributions and optimal merger policies for the two markets. Observe that the optimal commitment policy is more restrictive in larger markets even though the set of states in which mergers increase static aggregate surplus is larger in larger markets.

Independently of whether the authority pursues a CV or AV objective, the advantage that commitment has over no commitment is decreasing (both in absolute as well as in relative terms) with the size of the market. For example, compared to the AV-maximizing Markov perfect policy, the AV-maximizing commitment policy induces an average AV that is 6.7% higher in the small market but only 0.8% higher in the large market.

## 6 Extensions and Robustness

In this section, we investigate several extensions and robustness issues. First, we consider the problem of a social planner who controls both merger and investment decisions. Second, we analyze the robustness of our results to reducing the difference in investment costs between

<sup>32</sup>All values are ex ante (beginning-of-period) values, while the performance measures in the last two rows are at the output competition stage (stage 5).

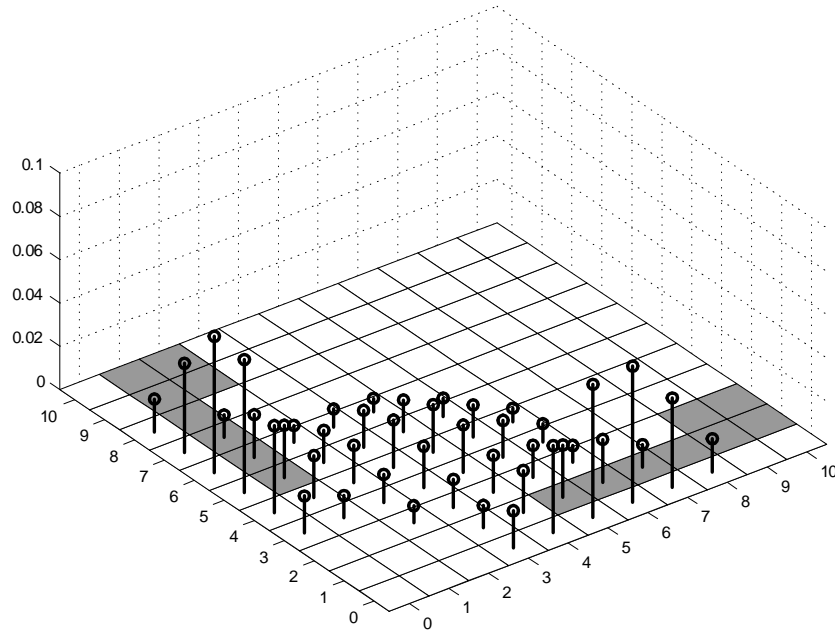


Figure 26: Beginning-of-period steady state distribution of the equilibrium generated by the best commitment policy (CV criterion) in the small market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

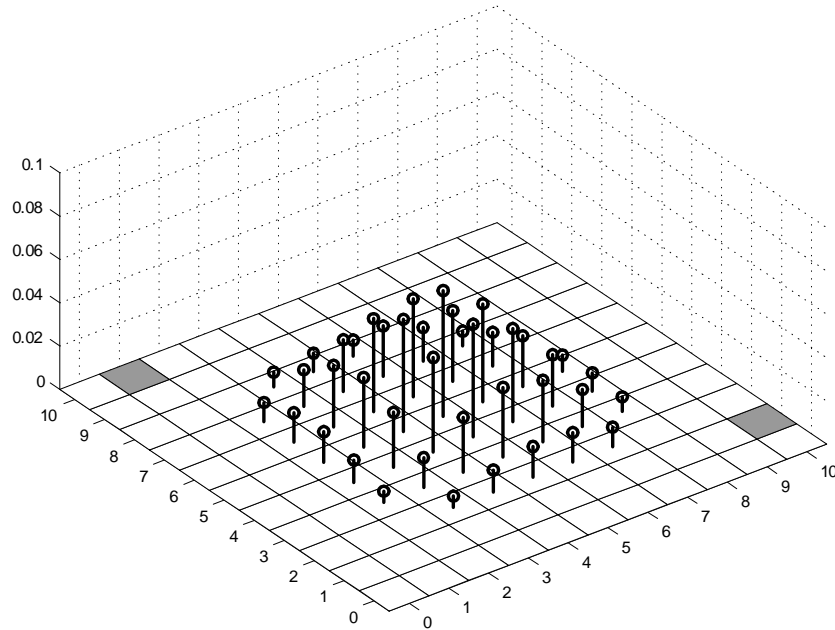


Figure 27: Beginning-of-period steady state distribution of the equilibrium generated by the best commitment policy (CV criterion) in the large market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

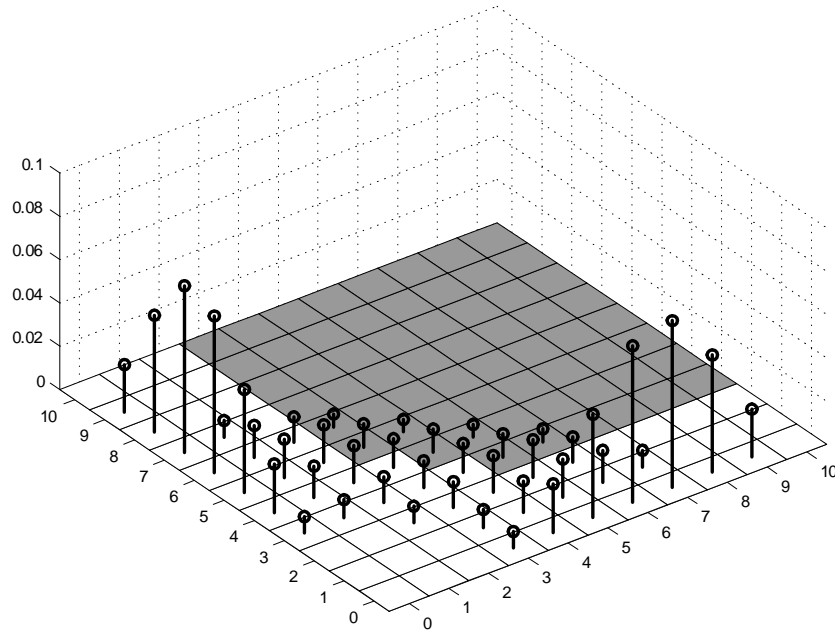


Figure 28: Beginning-of-period steady state distribution of the equilibrium generated by the best commitment policy (AV criterion) in the small market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

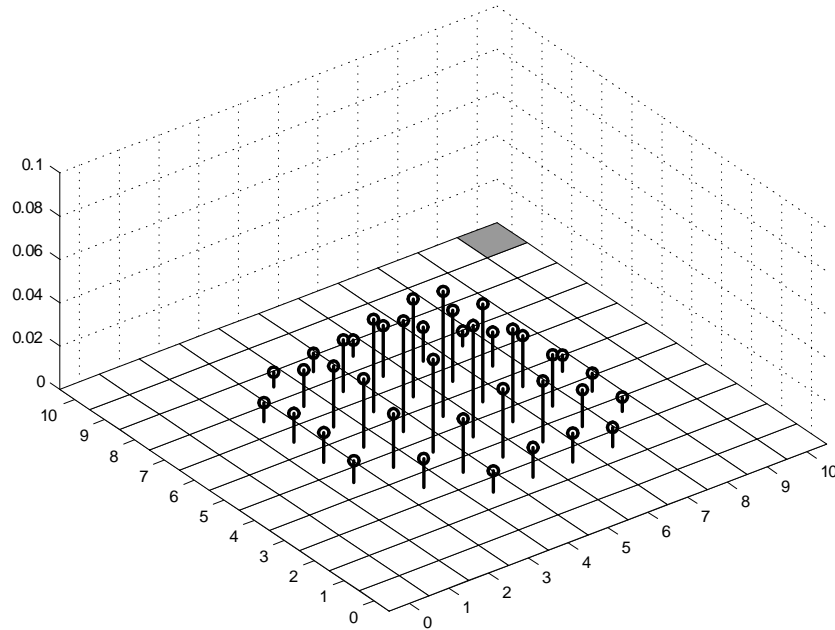


Figure 29: Beginning-of-period steady state distribution of the equilibrium generated by the best commitment policy (AV criterion) in the large market. The height of each pin indicates the steady state probability of that state. The shading of the cell reflects the probability of a merger happening (with a darker grey representing a higher probability).

incumbents and entrants. Third, we investigate changes in the arrival rate of a new entrant on the optimal merger policy (in the absence of commitment). Finally, we discuss the potential issue of multiplicity of equilibria. In the Appendix, we consider another variant of the entry process in which the know-how to produce in the industry is embodied in two owner-managers, thereby justifying the restriction to at most two active firms at any given point in time.

## 6.1 The Planner's Solution

In this subsection, we consider the solution to the second-best problem where the planner controls not only firms' merger decisions but also their investment decisions, taking as given firms' static competition. In our analysis, we confine attention to the intermediate market and the AV criterion.<sup>33</sup>

Our analysis above has revealed that the optimal merger policy in the intermediate market with commitment approves mergers only in near-monopoly states in which the incumbent is very large. This is even though in many more states a merger raises static aggregate surplus. As we have seen, the reason why the optimal commitment policy is so restrictive is that a more permissive policy would lead to adverse effects on investment incentives, and in particular inefficient entry for buyout. This raises the question of which mergers an AV-maximizing social planner would approve if he could control not only mergers (independently of their private profitability) but also firms' investment decisions (assuming the planner has perfect information about firms' private cost draws), taking as given only that, in every period, firms compete in a Cournot fashion at stage 5.

Figure 30 shows the steady state distribution for the solution of this second-best problem: the height of each pin gives the beginning-of-period probability of the corresponding state in the steady state generated by this policy; the cells in which mergers are approved are darkly shaded. Two comments are in order. First, as the planner controls not only merger decisions but also firms' investment decisions, the planner does not face a time inconsistency problem; i.e., the solution is independent of whether or not the planner can commit to his future decisions. Second, the existence of blocking costs is irrelevant for the solution to the second-best problem as it can never be optimal from the planner's point of view to propose a merger and subsequently block it in the event blocking costs are sufficiently low.

As Figure 30 shows, in the steady state generated by the planner's solution, the industry is always in a monopoly state. A merger is implemented in many states, unless these states involve high capital levels for both firms. In fact, the set of states in which mergers happen is almost identical to the set of states in which a merger is statically aggregate surplus-increasing (for reasons that will be discussed below). Table 5 summarizes various performance measures of the planner's solution. As can be seen from that table, the planner's solution does quite a bit better in terms of AV than the optimal merger policy with commitment (121.3 vs. 118.1). It does serve consumers very badly, however; worse in fact than the Markov perfect merger policy (39.2 vs. 43.3), despite a higher average capital level (8.08 vs. 7.65), and nearly as badly as allowing all mergers. The reason behind this is, of course, the monopolist's market power which leads to low output (20.1, compared to 21.0 under the AV-maximizing Markov perfect policy, and 22.5 under the optimal merger policy with commitment).

<sup>33</sup>Similar conclusions hold for the small and large markets.

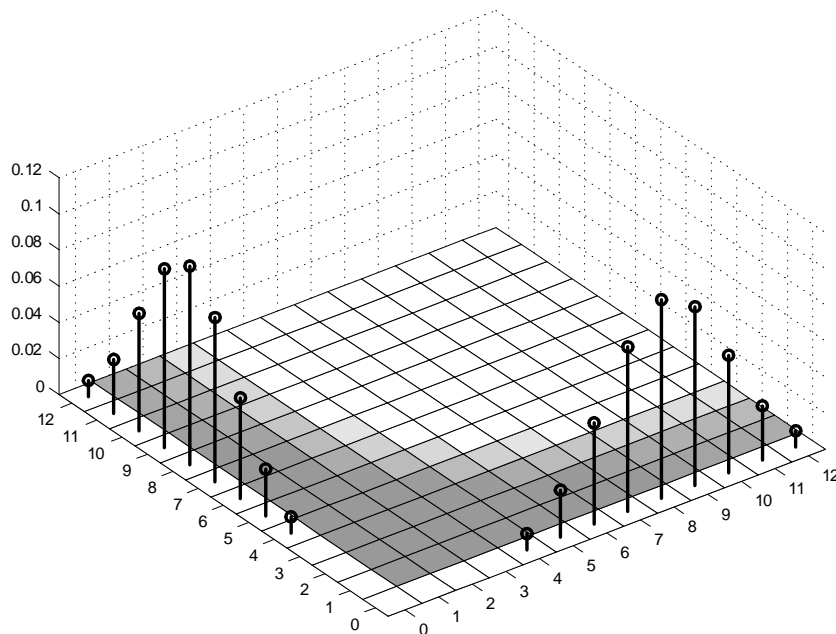


Figure 30: Solution to the planner’s second-best problem (AV criterion) in the intermediate market. The height of each pin gives the probability of the corresponding state in the steady state generated by the planner’s optimal policy. The shading of the cells indicates the merger probabilities, with a darker shading corresponding to a higher merger probability.

The fact that in the second-best solution the industry is always in a monopoly state may be surprising at first. After all, when mergers are not allowed the industry seems to be a workable duopoly, and in the equilibrium generated by the optimal merger policy with commitment, the industry spends only 14.3% of the time in a monopoly state. To understand this outcome, suppose first that the planner could not only control mergers but also costlessly undo previously approved mergers. Suppose also that there were no merger proposal costs. What would the planner’s optimal policy be in that case? In any state  $(K_1, K_2)$ , the planner would optimally implement a merger if and only if the merger increases static aggregate surplus as this is statically optimal and also does not impede dynamic optimality as the planner controls investment, the investment technology is merger neutral, and the planner can costlessly undo any previously approved merger. Now, previously we saw in Figure 13(b) that a merger increases static aggregate surplus in every state in which  $K_1 + K_2 \leq 10$  (except in state  $(5, 5)$  in which the gain is approximately zero) and, also, in several additional states in which  $K_1 + K_2 > 10$ . So, unless the planner wants to spend a large amount of time in states with more than 10 units of capital, the steady state generated by the planner’s policy will visit only monopoly states even if the planner cannot undo previously approved mergers and there are proposal costs — which is what is going on here.<sup>34</sup> Finally, note that this reasoning also explains why the set of states in which the planner implements mergers almost coincides with the set of statically

<sup>34</sup>In the steady state generated by the planner’s solution, the industry is sometimes (8.3% of the time) in a monopoly state with more than 10 units of capital, the joint frequencies of states  $(11, 0)$  and  $(0, 11)$  being 6.1%. But these are both states that are reachable by aggregate surplus increasing mergers.



aggregate surplus-increasing mergers. They do not coincide fully because of the presence of merger proposal costs, which the static criterion does not take into account.

## 6.2 Entrant Investment Efficiency

In our analysis of the welfare effects of various merger policies, “entry for buyout” plays a prominent role. When mergers are allowed a new entrant’s private benefit from investing significantly exceeds the incremental aggregate value that results from those investments, while the incremental aggregate value from an incumbent’s investment exceeds its private benefit to the incumbent. As a result, the entrant invests too much and the incumbent invests too little. The entrant’s high cost greenfield investment substitutes for the incumbent’s lower cost investment done through capital augmentation and directly causes waste.

In practice, however, entrants’ investments are not always less efficient than incumbents’ investments, and may even be more efficient.<sup>35</sup> In this section, we explore this point by changing the model’s parameters to close the gap between the investment costs entrants and incumbents face.

Focusing on the intermediate market, we examine whether this change largely eliminates the waste that entry for buyout causes by studying the effect of a change from the no-mergers-allowed policy to the all-mergers-allowed and Markov perfect policies when the antitrust authority’s criterion is AV maximization. Overall, we find that (i) entry-for-buyout behavior continues to be prevalent, (ii) its social costs are greatly reduced; (iii) the antitrust authority is much more willing to allow mergers in the Markov perfect policy; and (iv) with this change, consumer value falls somewhat more when moving from no mergers allowed to the Markov perfect policy.

Recall that capital augmentation each period enables a firm with  $K$  units of capital, if it wishes, to double each unit  $j$  at a cost  $c_j$  drawn independently and uniformly from the interval  $[\underline{c}, \bar{c}]$ . If it wants to more than double its current stock of capital, then it can purchase additional greenfield units at constant unit cost  $c_g$ , where  $c_g$  is uniformly drawn from  $[\bar{c}, \bar{c}_g]$ . Let  $s = \bar{c} - \underline{c}$  and  $s_g = \bar{c}_g - \bar{c}$  be the spread of capital augmentation costs and greenfield costs respectively. In the baseline industry analyzed in the previous sections the values are  $\underline{c} = 3$ ,  $\bar{c} = 6$ ,  $\bar{c}_g = 7$ ,  $s = 3$ , and  $s_g = 1$ . To close the gap between entrant and incumbent investment costs we reduce  $s$  to 1 and  $s_g$  to 0.25. Since this change, if  $\underline{c}$  were held fixed, would reduce firms’ investment costs, leading to less monopoly and very different merger behavior, we simultaneously raise  $\underline{c}$  to 5.645, which keeps the frequency of monopoly unchanged when no mergers are allowed. Thus, we have  $\underline{c} = 5.645$ ,  $\bar{c} = 6.645$ ,  $\bar{c}_g = 6.895$ ; we refer to these modified parameter values as the “efficient entry environment.”

Table 8 shows the results when we switch from our baseline environment (left three columns) to the efficient entry environment (right three columns). The table reports the same performance statistics as before, with the addition of one new measure: “Avg. Monop. to Merger Time.” This statistic measures the expected number of periods the industry takes to transition from a monopoly state to a state in which the incumbents merge.<sup>36</sup> Comparing the two

<sup>35</sup>Henderson (1993) provides dramatic evidence of this in the photolithographic alignment equipment industry where several generations of entrants supplanted incumbents by more efficiently using their knowledge capital.

<sup>36</sup>We use the steady state distribution over monopoly states as weights, and exclude state  $(0, 0)$ .

environments, we see that entry-for-buyout behavior actually increases when we move to the efficient entry environment; for example, when all mergers are allowed, the monopoly to merger time falls from 2.6 to 2.1. However, the costs of this behavior are greatly reduced: AV now falls only 0.6% when all mergers are allowed (from 87.9 with no mergers to 87.4 when all mergers are allowed), compared to 10.0% in our baseline case (from 117.5 with no mergers to 105.8 with all mergers allowed). Because of the reduction in the inefficiency of post-merger investment behavior, allowing mergers is much more attractive for the antitrust authority, and the Markov perfect policy results in far more mergers in the efficient entry environment: the probability of merger is now 42.6% in each period, compared to only 16.1% in our baseline case. Indeed, the equilibrium is essentially equivalent to the case in which all mergers are allowed. Finally, this increased merger activity results in a much greater likelihood of the industry being in a monopoly state (79.4% of the time in the efficient entry environment vs. 49.4% in our baseline case). As a consequence, there is a somewhat greater reduction in consumer value when moving from no mergers being allowed to the Markov perfect policy (a reduction of 13.6%, from 34.9 to 30.5 in the efficient entry environment, vs. a reduction of 10.0%, from 48.1 to 43.3).<sup>37</sup>

**Table 8 : Performance Measures for the Efficient Entry Environment  
in the Intermediate Market**

<i>Performance measure</i> <sup>38</sup>	Baseline Environment			Efficient Entry Environment		
	<i>No Mergers</i>	<i>All Mergers</i>	<i>MPP</i>	<i>No Mergers</i>	<i>All Mergers</i>	<i>MPP</i>
Avg. Consumer value	48.1	35.8	43.3	34.9	30.5	30.5
Avg. Incumbent value	69.4	68.1	69.9	53.1	54.9	54.9
Avg. Entrant value	-	1.9	0.5	-	2.0	2.0
Avg. Blocking Cost	-	-	-0.1	-	-	0.0
Avg. Aggregate Value	117.5	105.8	113.6	87.9	87.4	87.4
Avg. Price	2.25	2.36	2.19	2.14	2.20	2.20
Avg. Quantity	22.2	19.2	21.0	18.9	17.7	17.7
Avg. Total capital	7.98	7.01	7.65	5.58	5.66	5.66
Merger frequency	0.0%	37.7%	16.1%	6.8%	42.6%	42.6%
% in monopoly	18.6%	86.0%	49.4%	18.6%	79.4%	79.4%
% $\min\{K_1, K_2\} \geq 2$	75.7%	0.9%	44.2%	68.6%	4.2%	4.2%
Avg. Monop. to Merger Time	-	2.6	6.1	-	2.1	2.1

<sup>37</sup>The greater percentage reduction in consumer value in the Markov perfect policy compared to when no mergers are allowed depends on market size. In results not reported here, we find that it remains true in the large market, but in the small market there is no reduction in consumer value from allowing mergers in the efficient entry environment. The other effects we report here for the intermediate market (continued entry-for-buyout behavior, reduced cost of that behavior, and greater frequency of mergers) hold as well in the small and large markets.

<sup>38</sup>All values are ex ante (beginning-of-period) values, while the performance measures in second and third rows from the bottom are at the output competition stage (stage 5).

### 6.3 Entry Probability

It is generally perceived that the potential anticompetitive effects of horizontal mergers are mitigated by timely post-merger entry into the industry. For instance, the current (2010) U.S. Horizontal Merger Guidelines (which are largely based on a consumer welfare standard) state:

A merger is not likely to enhance market power if entry into the market is so easy that the merged firm and its remaining rivals in the market, either unilaterally or collectively, could not profitably raise price or otherwise reduce competition compared to the level that would prevail in the absence of the merger. Entry is that easy if entry would be timely, likely, and sufficient in its magnitude, character, and scope to deter or counteract the competitive effects of concern.

However, the appropriateness of this wisdom may depend on the welfare criterion and on whether dynamic investment incentives are taken into account.<sup>39</sup> To study how the timeliness of post-merger entry affects the optimal merger policy and the resulting performance of the industry, we extend the baseline model by introducing a probability  $e \geq 0$  with which a new entrant arrives at stage 4 whenever the current state of the industry has a single active firm.<sup>40</sup> We show that, contrary to what conventional wisdom may suggest, an increase in the timeliness of post-merger entry may result in fewer mergers occurring under the Markov perfect policy, for both an AV and a CV standard.

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<sup>39</sup>It has been remarked before that “the possibility of entry need not make a given merger more attractive” for an authority with an aggregate welfare rather than a consumer welfare standard [Whinston (2007); p. 2388]. While new entry is generally viewed as being price-reducing and thus beneficial to consumers, it may be excessive from an aggregate welfare point of view [Mankiw and Whinston (1986)].

<sup>40</sup>Formally, this requires extending the state space to  $\mathcal{S}' \equiv \{-1, 0, 1, \dots, 20\}^2$ , where  $K_i = -1$  means that firm  $i$  is an entrant who has not yet arrived. The firms’ expected gain from merging, equation (1), is therefore now given by

$$\Delta_G(K_1, K_2) = e\bar{V}(K_1 + K_2, 0) + (1 - e)\bar{V}(K_1 + K_2, -1) - [\bar{V}(K_1, K_2) + \bar{V}(K_2, K_1)],$$

where the first (second) term on the right-hand side is the probability of new entry (no new entry) occurring times the continuation value of the merged firm in that event.

**Table 9: Performance Measures for the Intermediate Market  
(Markov Perfect Policy, AV Criterion)**

<i>Performance measure</i> <sup>41</sup>	$e=1.0$	$e=0.8$	$e=0.6$	$e=0.4$	$e=0.2$	$e=0.0$
Avg. Consumer value	43.3	41.6	37.4	33.1	30.6	28.0
Avg. Incumbent value	69.9	70.3	70.7	69.6	69.7	70.5
Avg. Entrant value	0.5	0.5	0.9	1.8	1.4	0.0
Avg. Blocking cost	-0.1	0.0	0.0	0.0	0.0	0.0
Avg. Aggregate Value	113.6	112.4	109.0	104.6	101.7	98.5
Avg. Price	2.19	2.21	2.25	2.29	2.32	2.35
Avg. Quantity	21.0	20.6	19.6	18.5	17.7	16.9
Avg. Total capital	7.65	7.50	7.11	6.44	5.86	5.3
Merger frequency	16.1%	19.5%	30.3%	36.7%	20.0%	0.0%
% in monopoly	49.4%	58.9%	83.6%	99.4%	100%	100%
% $\min\{K_1, K_2\} \geq 2$	44.2%	35.3%	13.1%	0.1%	0.0%	0.0%

We begin our analysis with the Markov perfect policy of an authority that aims at maximizing AV in the intermediate market. Table 9 reports the performance measures of the intermediate market under this policy for different levels of the entry probability  $e$ . Despite the inefficiencies associated with entry for buyout, welfare declines as entry becomes less timely: the steady-state levels of CV and AV fall from 43.3 and 113.6, respectively, to 28.0 and 98.5 as  $e$  decreases from 1 to 0. The reason for this finding is that, as  $e$  decreases, the industry spends more and more time in a monopoly state; the steady-state probability of monopoly increases from 49.4% at  $e = 1$  to 100% at  $e = 0$ . This hurts consumers and society a lot in the short run (for a given level of capital) but even more so in the long run because a monopolist has little incentive to build capital in the absence of a threat of entry: the average total capital level decreases from 7.65 to 5.3 as  $e$  decreases from 1 to 0.

Table 9 also reveals that the frequency of mergers is non-monotonic in the timeliness of post-merger entry: as  $e$  decreases, the probability that a merger occurs in a randomly selected period first increases (from 16.1% at  $e = 1$  to 36.7% at  $e = 0.4$ ) and then decreases. As a merger is infeasible in states in which there is only one active firm, this steady-state weighted merger probability is equal to the probability that there are two active firms times the probability of a merger conditional on two firms being active, and is bounded from above by the entry probability  $e$ .<sup>42</sup> This explains why the merger frequency converges to zero as the entry probability  $e$  becomes small.

To understand why the merger frequency *increases* as  $e$  decreases from 1 to 0.4, consider the merger probability conditional on two firms being active, which is the product of two probabilities: the probability that the two active firms propose a merger and the probability that a proposed merger is approved.

<sup>41</sup>All values are ex ante (beginning-of-period) values, while the performance measures in the last two rows are at the output competition stage (stage 5).

<sup>42</sup>The steady-state weighted merger probability is maximized when the probability of a merger, conditional on there being two active firms, is equal to one. In that case, the probability that there are two active firms is equal to the entry probability  $e$ , implying that the steady-state weighted merger probability is equal to  $e$  as well.

Consider first states in which both firms have at least one unit of capital. As  $e$  decreases, mergers become more profitable in such states as the merged firm spends more time in a monopoly state before a new entrant appears. It follows that the probability of a merger proposal in any given state increases as entry becomes less timely. Moreover, the AV-maximizing Markov perfect policy tends to become *less* restrictive as  $e$  declines. When the entry probability  $e$  is high, the Markov perfect policy approves mergers only in states in which at least one of the firms is sufficiently small (as we have seen for  $e = 1$  in Section 5.3). As  $e$  decreases, this approval region increases. For example, a proposed merger in state (3,3) is never approved if  $e \geq 0.6$  but always approved if  $e \leq 0.35$ . In the limit as the entry probability  $e$  becomes small, the set of states in which the authority approves a merger with probability one is very similar to the set of states in which a merger raises static aggregate surplus: it includes the states in which total capacity satisfies  $K_1 + K_2 \leq 9$  and each firm has at least one unit of capacity,  $\min\{K_1, K_2\} \geq 1$ . In addition, the authority approves mergers with high probability in many other states. An important factor explaining why the Markov perfect policy becomes less restrictive as post-merger entry becomes less timely is that the entry-for-buyout phenomenon along with its investment distortions disappear in the limit as  $e$  becomes small.<sup>43</sup>

Consider now states in which an entrant has arrived but not yet built any capital. When the entry probability is one, the authority would always approve a proposed merger in such a state: approving the merger has no effect on AV, but blocking is costly. However, when the entry probability is one, such a merger would not be proposed as it is not profitable.<sup>44</sup> When the post-merger entry probability is sufficiently small, such a merger becomes profitable as the arrival of a new entrant following a merger takes time, allowing the merging firms to reap monopoly profits in the meantime. As  $e$  decreases, firms are therefore more likely to propose mergers between an entrant and an incumbent. At the same time, the antitrust authority starts to block mergers, but only in states with very low incumbent capital levels, in which entrant investment incentives are not excessive. Hence, the probability of a merger between an entrant and an incumbent becomes positive for  $e \leq 0.6$ .

What happens if the authority adopts a CV standard instead? The parameterization of the intermediate-sized market may not seem ideal to study this question: After all, when post-merger entry is immediate ( $e = 1$ ), the CV-maximizing Markov perfect policy is outcome-equivalent to the no-mergers policy (it approves a merger only in state (1,1) but such a merger is not profitable), as we have seen in Section 5.3. Nevertheless, the relationship between the timeliness of entry and the steady-state probability of a merger is also non-monotonic under a CV-maximizing policy: as  $e$  decreases, the merger frequency first increases and then decreases. This is the result of opposing effects: as post-merger entry becomes less timely, mergers tend to become more profitable on the one hand, but on the other the authority's approval policy becomes more restrictive, and the industry spends more time in states in which a merger is

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<sup>43</sup>If  $e = 0$ , the authority approves a proposed merger with probability one in all states in which each firm has at least one unit of capital, and aggregate capital is less than 10, and even in most states in which one of the firms has zero capital. The approval region is thus similar to the set of states in which a merger does not decrease static aggregate surplus, even though the authority does take into account the effect of a merger to monopoly on future investment.

<sup>44</sup>When  $e = 1$ , a merger in state  $(K_1, 0)$  or  $(0, K_2)$  does not affect producer value because the old entrant gets immediately replaced by a new entrant. As the value of the new entrant is strictly positive, this implies that the merger must decrease the joint continuation values of the merging firms.

infeasible because a new entrant has not yet arrived. The only mergers that are sometimes both proposed and approved are those involving a newly arrived entrant with zero units of capital and a sufficiently large incumbent. For example, at  $e = 0.4$ , mergers sometimes happen in states  $(0, K_2)$  or  $(K_1, 0)$  with  $14 \geq K_i \geq 6$ , with the steady state probability of a merger being 1.2 percent. At  $e = 0.2$ , mergers happen only in states  $(0, K_2)$  or  $(K_1, 0)$  with  $20 \geq K_i \geq 7$ , with the steady state probability of a merger being 0.4 percent. Given that the merger probability is very low for all values of  $e$ , changes in the entry probability have only very small effects on the performance measures.

## 6.4 Multiplicity of Equilibria

Dynamic stochastic games with infinite horizons generally have multiple equilibria when players are patient. Within the context of the Ericson and Pakes model of computable Markov perfect equilibria, Besanko et al (2010, section 3.1) developed a homotopy based method for tracing out paths on the equilibrium manifold and systematically finding points in the parameter space for which multiple equilibria exist.<sup>45</sup> It does not, however, provide a guarantee that it will find all equilibria.

The homotopy technique depends on differentiating the equations that implicitly define the model's equilibria. This requirement makes it, as a practical matter, infeasible to apply to our merger model because a key step in numerically solving for equilibria is a Monte Carlo integration. Numerically differentiating this integral with reasonable accuracy is not possible with the computing power to which we have access. Consequently we implemented a cruder search for multiple equilibria that may fail to find cases of multiplicity that the homotopy technique would find if it were feasible.<sup>46</sup>

The idea is straightforward. Define a cube,

$$D = \{(B, \underline{c}, s) | B \in [22, 30] \ \& \ \underline{c} \in [2.0, 5.5] \ \& \ s \in [1.0, 3.0]\},$$

in the parameter space where, as in Section 6.2,  $s = \bar{c} - \underline{c}$ . Set all other parameters equal to their baseline values. Along lines within this cube calculate sequences of equilibria using the equilibrium values of one equilibrium as the starting points for the next equilibrium computation. For example, for each  $\lambda \in \{0, 0.025, 0.050, \dots, 0.975, 1\}$ , calculate equilibria along the line  $(B, \underline{c}, s) \in \langle 22(1 - \lambda) + 30\lambda, \underline{c}, s \rangle$  where  $\underline{c} \in [2.0, 5.5]$  and  $s \in [1.0, 3.0]$ . Start the equilibrium calculations from both the  $\lambda = 0$  and the  $\lambda = 1$  ends of the line and use the equilibrium values calculated for a particular  $\lambda$  as the initial values for calculating the equilibrium at the next  $\lambda$ . If equilibrium multiplicity exists along the line, then the equilibrium values for a particular  $\lambda$  reached from the line's left end *may* not equal the equilibrium values for that same  $\lambda$  reached from the line's right end.

Within  $D$ , in order to construct lines of search, we fix two of the three parameters  $(B, \underline{c}, s)$  at values within the three-dimensional array  $\{B \in [22.0, 23.6, 25.2, 26.8, 28.4, 30.0], \underline{c} \in [2.0, 2.7, 3.4, 4.1, 4.8, 5.5], s \in [1.0, 1.5, 2.0, 2.5, 3.0]\}$  and then varied the third parameter from its minimum to its maximum in 40 steps. This created 36 lines parallel to each axis. For the no-mergers policy we

<sup>45</sup>See Borkovsky, Doraszelski, and Kryukov (2010, 2012) for further discussion and illustration of how to use this homotopy technique.

<sup>46</sup>We thank Ulrich Doraszelski for suggesting this technique to us.

found multiplicity for some parameter values, but for the all-mergers-allowed policy we found no multiplicity. Additionally, because calculating Markov perfect policies is much more computer intensive than calculating fixed policy equilibria, we did a less dense search of the parameter space for Markov perfect policies. We found no multiplicity, but did find several regions in which our algorithm failed to calculate an equilibrium.

Each instance of multiplicity that we found for the no-mergers policy has a common structure. Whenever we found two equilibria for a parameter vector, then the distinguishing strategic difference was the investment behavior at state  $(1,0)$ . Total investment is always approximately the same, but in one equilibrium, the firm with 1 unit of capital invests more, and in the other equilibrium, the firm with 0 units of capital invests more. Each firm wants to have an aggressive investment policy if the other firm has a passive investment policy, and a passive policy if the other firm has an aggressive policy. Almost certainly a third equilibrium exists that is unstable and not computable with our algorithm.<sup>47</sup>

These instances of multiplicity are all found on lines parallel to the  $B$  axis. We found four regions in the parameter space with multiplicity. Lines where  $(\underline{c}, s)$  are  $(4.1, 2.2)$ ,  $(4.1, 2.6)$ ,  $(4.8, 1.0)$ , and  $(4.8, 1.4)$  all contain regions of multiplicity. To be specific, consider the line for which  $(\underline{c}, s) = (4.8, 1.4)$ . Multiplicity occurs for  $B \in [24.6, 28.0]$ . Fix  $B$  at 26.4 and let the state be  $(1, 0)$ . In “equilibrium 1” firm 1 with one unit of capital builds, in expectation, 2.00 units of capital while firm 2 with zero units of capital builds, in expectation, 0.61 units of capital. In “equilibrium 2” the behavior reverses: firm 1 builds, in expectation, 1.01 units of capital while firm 2 builds, in expectation, 2.01 units of capital. Table 10 shows performance measures for these two equilibria.

Finally, we point out that we found no multiplicity at or close to our baseline parameters.

**Table 10: Performance Measures for Two Pure Equilibria  
under No Mergers at  $(B, \underline{c}, s) = (26, 4.8, 1.4)$**

<i>Performance measure</i>	Equil. 1	Equil. 2
Avg. Consumer Value	22.9	24.9
Avg. Incumbent Value	51.3	51.8
Avg. Aggregate Value	74.2	76.8
Avg. Price	2.42	2.39
Avg. Quantity	15.1	15.8
Avg. Total capital	4.03	4.31
% in monopoly	80.6%	71.2%
% $\min\{K_1, K_2\} \geq 2$	14.8%	22.3%

## 7 Conclusion

We have studied the optimal merger policy in a dynamic industry model in which mergers offer the potential for cost reduction through the achievement of scale economies, but also increase market power. An antitrust authority must then weigh any potential gain in efficiency

<sup>47</sup>See Besanko et al. (2012, section 3.2) for a discussion of the inability of Pakes-McGuire-like algorithms to compute unstable equilibria.

generated by the merger, over that which would be achieved by internal growth, against the losses from increased market power.

In terms of the trade-off between internal and external growth we have seen several things. First, the very nature of this trade-off depends on whether we are taking the perspective of an antitrust authority that cannot commit and must decide what to do about a given proposed merger, or the perspective of an authority identifying an optimal commitment policy. From the former perspective, we have seen that the desirability of approving a merger can depend importantly on the investment behavior that will follow if it is or is not approved. However, this involves more than just the behavior of the merging firms, as the investment behavior of outsiders to the merger (here, new entrants) can have significant welfare effects. In the other direction, these investment behaviors can be importantly influenced by firms' beliefs about future merger policy. From the perspective of identifying an optimal commitment policy, the potential effects on investment behavior can make the optimal commitment policy differ substantially from the policy that emerges when the antitrust authority instead considers mergers on a case-by-case basis without commitment. Moreover, in cases in which commitment is not possible and aggregate value is the true social objective, it is often better in our model to endow the antitrust authority with a consumer value objective (which roughly corresponds to the objective of most antitrust authorities).

Whether with or without commitment, however, we have found that in our model the optimal antitrust policy for maximizing aggregate value is significantly more restrictive than the optimal static policy that considers a merger's effects only at the time it would be approved. In fact, as we have seen, even the comparative statics of the optimal dynamic policy can be very different from those of the optimal static policy: for example, an increase in market size leads to a larger set of states in which mergers increase static aggregate surplus but a smaller set of states in which an AV-maximizing authority approves a proposed merger. Finally, we have shown that, contrary to what conventional wisdom may suggest, less timely entry may induce more mergers to be proposed and approved under the Markov perfect policy (for both AV and CV criteria).

Our model leaves a number of important research directions open related to this issue. Most significant is the need to expand the analysis beyond the case of two active firms. This will require a model of bargaining with externalities among many parties which is tractable and offers sensible predictions.

Another direction for future research involves calibrating the model to match key features of an industry.

At a more general level, the existing literature on antitrust policy has largely neglected issues relating to investment or firm entry and exit that are inherently dynamic. In a world where the antitrust authority may not have the ability to commit fully to its future actions, analyzing such issues requires modeling the authority as a player who acts dynamically. The present paper is a first step in doing so in a truly dynamic setting. By proposing a merger-neutral investment technology that allows for complex multi-unit investment choices, and yet is tractable, it also contributes to the computational industrial organization literature more generally.



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## 8 Appendix: Entrant Identity

A key restriction in our model is that no more than two firms can be active at any one time. Throughout this restriction has been posed exogenously. Our baseline assumption is that the entering firm after a merger is owned by an entrepreneur who has never before been active within the industry. This assumption begs the question as to why he did not enter previously before the merger took place. A more satisfactory model would allow free entry with entry stopping only when the value of the potential entrant becomes negative. Implementing this creates two difficulties. With three or more active firms a merger between two of them may have positive externalities on one or more of the non-merging firms. A satisfactory model of bargaining with positive externalities and three or more principals has not yet been developed to our knowledge. Moreover, if one were developed, equilibrium behavior would almost certainly involve delay. That would create the additional difficulty of a second time scale within our model. Currently with a discount factor of  $\delta = 0.8$  periods are on the order of five years. This is reasonable for a capital intensive industry that for both physical and regulatory reasons has a very long capital planning and construction cycle. This, however, is a completely unreasonable period length for a merger negotiation between two ambitious CEOs and their boards. Incorporating this second time scale into our model will necessitate some modeling and computational innovations.

An alternative to the exogenous restriction we have used is to assume that only two entrepreneurs have the necessary skill and knowledge set to compete in the industry. If that is the case and both entrepreneurs are active in the industry, then the owner/manager of the acquired firm would become the new entrant following a merger. (We assume there is not a “no-compete” clause in the acquisition agreement.) Equation (1) giving the joint value gain from merging then becomes

$$\Delta_G(K_1, K_2) \equiv \{ [\bar{V}(K_1 + K_2, 0) + \bar{V}(0, K_1 + K_2)] - [\bar{V}(K_1, K_2) + \bar{V}(K_2, K_1)] \}.$$

New to the definition is the entrant’s ex ante value  $\bar{V}(0, K_1 + K_2)$ . It must be included because the entrepreneur who is bought out intends to re-enter. In other words, the two entrepreneurs will agree to merge—one buying out the other—if it pays them jointly to create temporarily a monopoly situation in the industry until that time the bought-out entrepreneur successfully returns to the industry. Since  $\bar{V}(0, K_1 + K_2) \geq 0$  this weakly increases the merger frequency (holding the policy and value function constant). Figure 31 shows a side-by-side comparison for the intermediate market of the equilibria for these two different assumptions concerning entry. When all mergers are allowed, this change increases the frequency of mergers. (Although note that in the AV-maximizing Markov perfect policy the merger frequency ends up lower than before.) Inspection shows that, overall, our results are not qualitatively different from our earlier results.

## 9 Appendix: Computational Algorithm

In this section, we describe the algorithm used to compute equilibrium in our model. The algorithm uses value function iteration and is similar to the Pakes and McGuire (1994) algorithm.

Intermediate Market: $B = 26$						
	New Entrant			Bought is Entrant		
	No M	All M	MPP	No M	All M	MPP
Consumer Value	48.1	35.8	43.3	48.2	35.6	45.4
Incumbent Value	69.4	68.1	69.9	69.4	68.7	69.6
Aggregate Value	117.5	105.8	113.6	117.6	104.3	115.0
Price	2.26	2.36	2.19	2.26	2.36	2.17
Quantity	22.2	19.2	21.0	22.2	19.2	21.5
Merger Freq	0.0%	37.7%	16.1%	0.0%	49.2%	11.8%
Inv Inc of Merger	97.5%	100.0%	99.8%	97.5%	100.0%	99.8%
Monopoly Freq	18.6%	86.0%	49.4%	18.3%	94.0%	35.9%
$K \geq 2$ Duopoly Freq	75.7%	0.9%	44.2%	76.0%	0.1%	57.0%
Ave Capital	8.0	7.0	7.7	8.0	7.0	7.8
Mon $\rightarrow$ Merg Time	10000.0	2.6	6.1	10000.0	2.0	8.5
Inv Distort All	0.1	1.2	0.1	0.1	1.2	-0.1
Inv Distort Small	1.1	3.5	1.1	1.1	3.5	0.7

Figure 31: Equilibrium in the intermediate market under two entry assumptions. The left column shows equilibria in which entry is by an entrepreneur who is new to the industry. The right column shows equilibria in which entry is by the entrepreneur who until earlier in the period was active in the industry and agreed to be bought out.

This section proceeds as follows. First, we list the algorithm's input parameters. Then, we explain the initial computations prior to value function iteration. Next, we explain the value function iteration itself and, finally, we define convergence of the algorithm.

## 9.1 Parameters

The algorithm makes use of the following parameters:

### 1. Demand and Production:

- Parameters  $(B, A)$  from the demand function  $Q = B \cdot (A - p)$ .
- Parameters  $(\beta, \theta, w)$  from the production function  $Q = K^{\beta\theta} L^{(1-\beta)\theta}$  with wage parameter  $w$ .
- Throughout the computations, we set  $(A, \beta, \theta, w) = (3, \frac{2}{3}, 1.1, 1)$  and  $B \in [22, 26, 30]$ .

### 2. Investment:

- Parameters  $(\underline{c}, \bar{c})$ , corresponding to the minimum and maximum values of capital augmentation cost draws. Augmentation costs are uniformly distributed in this interval.
- Parameters  $(\underline{c}_g, \bar{c}_g)$ , correspond to the minimum and maximum values of the greenfield cost draws. Greenfield costs are uniformly distributed in this interval.
- Throughout the computations, we set  $\underline{c}_g = \bar{c}$ , implying that the minimum greenfield cost draw is the same as the maximum augmentation cost draw.

### 3. Merger:

- Parameters  $(\underline{\phi}, \overline{\phi})$  and  $(\underline{b}, \overline{b})$  form the intervals of merger proposal and blocking costs, respectively. Both types of costs are uniformly distributed.
- $a$  is a matrix defining in which states mergers are allowed.
- $fixedMP$  is a binary variable indicating whether the authority's policy is a commitment policy ( $fixedMP = 1$ ) or not.
- $boughtIsEntrant$  is a binary variable indicating whether the acquired firm becomes the entrant ( $boughtIsEntrant = 1$ ) or if there is a new entrant.
- $cvCriterion$  is a binary variable indicating whether the merger authority uses a consumer value (CV) ( $cvCriterion = 1$ ) or aggregate value (AV) criterion.

### 4. Other:

- $S = \{0, 1, \dots, S\}$  is the set of possible capital levels  $K_i$  for each firm  $i$ .
- $\delta$  is the discount factor.
- $d$  is the depreciation factor.
- Throughout the computations, we set  $(S, \delta, d) = (20, 0.8, 0.2)$ .

## 9.2 Initial Calculations

Using the demand and production parameters, we start by calculating Cournot equilibrium quantities and profits for each interim state  $(K_1, K_2)$ . We define  $\pi$  as an  $(S + 1) \times (S + 1)$  matrix where an element of the matrix gives the profit of the row firm when the rival is the column firm. We then calculate the depreciation function  $\kappa$ , given by equation (4) in Section 3 and defining the transition probability from one capital level to another due to depreciation.

Next, we initialize variables used in the value function iteration.  $V$  is an  $(S + 1) \times (S + 1)$  matrix where an element of the matrix gives the beginning-of-period value of the row firm.  $\overline{V}$  is defined analogously for interim values.  $\xi$  is a firm's investment policy function.  $\mathcal{E}c$  is the expected investment cost function given interim capital levels and an investment policy  $\xi$ .  $a$  is an  $(S + 1) \times (S + 1)$  matrix where each element gives the probability that a merger in that state will be approved by the antitrust authority, and  $\psi$  is an  $(S + 1) \times (S + 1)$  matrix where each element gives the probability that a merger is proposed.

To start the iteration process we require initial values for all of these functions and matrices.  $V$  and  $\overline{V}$  are set equal to  $\pi/(1 - \delta)$ , the value that would result in each state if firms were not allowed to invest and capital did not depreciate.  $\xi$  is set to the identity transition, i.e.,  $\xi(k_i|\cdot, \cdot) = 1$  if  $k_i = 0$ , and  $\xi(k_i|\cdot, \cdot) = 0$  otherwise.  $\mathcal{E}c$  is set equal to 0 and  $a$  is set to the inputted  $a$  matrix.<sup>48</sup>

We also keep old matrices and functions from the previous iteration while we update in the current iteration.  $V'$  is the previous iteration's value for the matrix  $V$ .  $\overline{V}'$ ,  $\xi'$ ,  $\mathcal{E}c'$ ,  $a'$ , and  $\psi'$  are defined analogously.

<sup>48</sup>Experimenting with different starting values, the algorithm yielded the same equilibrium values.

### 9.3 Value Function Iteration

The value function iteration contains seven main steps. First, all of the old matrices and functions are set to the values calculated in the previous iteration. For example,  $V'$  is first set equal to  $V$ , and then a new  $V$  is calculated in the current iteration. Next, we calculate a new  $\xi$  and  $\mathcal{E}c$  based on the input parameters  $V'$  and  $\xi'$ . This is the most computationally intensive part of the iteration.

The investment transition probabilities and costs contained in  $\xi$  and  $\mathcal{E}c$  are computed using Monte Carlo integration to integrate equation (6) in Section 3, which gives  $\xi$ , and equation (7), which gives  $\mathcal{E}c$ . The Monte Carlo integration involves simulating thousands of vectors of investment cost draws using the investment parameters and, for each vector, identifying the optimal investment level based on the cost draws and on  $V'$ ,  $\xi'$ , and  $\kappa$ . By averaging over the large number of vectors of cost draws, we arrive at new values of  $\xi$  and  $\mathcal{E}c$ . The calculations are done for each pair  $(K_i, K_{-i})$  of capital levels.

The row firm gets  $K_i$  capital augmentation cost draws and one greenfield cost draw. (Recall that each capital augmentation draw can be used at most once, whereas a firm may build as many units of capital as it wants at the greenfield cost draw). When building new capital firms use the lowest cost draws sufficient to reach the desired capital level. Given the vector  $\tilde{c}$  of capital augmentation and greenfield cost draws, the investment cost function is given by  $c_{K_i}(\cdot|\tilde{c})$ . The benefits of investing are determined by the induced change in the firm's expected discounted ex ante value next period. Trading off the costs and benefits, firm  $i$  solves the following optimization problem:

$$\max_{k_i \in \{0,1,\dots,S-K_i\}} -c_{K_i}(k_i|\tilde{c}) + \delta \sum_{K'_i \in \mathcal{S}} \sum_{K'_{-i} \in \mathcal{S}} \kappa(K'_i|K_i + k_i) \tau'(K'_{-i}|K_{-i}, K_i, \xi_{-i}) V'(K'_i, K'_{-i}),$$

where  $\tau'$  is the rival firm's combined investment and depreciation transition probability defined in Section 3. The solution to this problem is denoted  $k_i^*$ . Using this solution, we derive an updated investment policy  $\xi$  and expected average investment cost  $\mathcal{E}c$ , where the average is taken over all vectors of cost draws.

To facilitate convergence of the algorithm, we dampen  $\xi$  and  $\mathcal{E}c$  with  $\xi'$  and  $\mathcal{E}c'$ . The amount of weight put on the old values of  $\xi'$  and  $\mathcal{E}c'$  is gradually increased from 0 to a maximum of .95 as the number of iterations increases.  $\xi$  and  $\mathcal{E}c$  are the only variables which are dampened, but the dampening propagates to the other variables which are calculated using  $\xi$  and  $\mathcal{E}c$ .

The third step in the value function iteration involves updating the interim value  $\bar{V}$  using equation (8) in Section 3, based on the updated functions  $\xi$  and  $\mathcal{E}c$  along with  $V'$ .  $\bar{V}$  is used in the next two steps to update the matrices  $\psi$  and  $a$ .

$\psi$  is calculated using  $\bar{V}$ ,  $a'$ , and the merger proposal cost interval  $[\underline{\phi}, \bar{\phi}]$ , according to equation (2) in Section 3:

$$\psi(K_1, K_2) \equiv \Phi(a'(K_1, K_2) \Delta_G(K_1, K_2))$$

where  $\Delta_G$  is the gain from merger calculated using  $\bar{V}$ .

In the case of a no-commitment merger policy,  $a$  is recomputed according to equation (13) in Section 3.

The sixth step of the value function iteration involves computing a new beginning-of-period value  $V$ , according to equation (3) in Section 3.

The final step involves comparing  $V$  and  $V'$  to check convergence. If  $V$  and  $V'$  are sufficiently close, the algorithm stops; otherwise, the algorithm continues with the next iteration.

## 9.4 Convergence

Our distance measure between  $V$  and  $V'$  is a modified Maximum metric, defined as the minimum of the standard Maximum metric and a maximum percentage change metric:

$$D(V, V') = \min \left\{ \max_{K_1, K_2} |V(K_1, K_2) - V'(K_1, K_2)|, \max_{K_1, K_2} \frac{|V(K_1, K_2) - V'(K_1, K_2)|}{|V'(K_1, K_2)|} \right\}$$

We use this minimum to avoid two problems with each of the two individual metrics. While the Maximum metric treats a value of 1000 as being just as close to 1000.1 as a value of 1 is to 1.1, when defining convergence we would like to consider 1000 and 1000.1 to be the same but 1 and 1.1 to be different. The maximum percentage change metric does not suffer from this issue. But there is a problem with the maximum percentage change metric when considering convergence to 0. Suppose that, after each iteration, the value of a variable is moving halfway to 0, in which case the variable gets arbitrarily close to 0 as the number of iterations becomes large but the percentage change will stay at 50%. In light of these issues, we use the minimum of the two metrics. For most of our computations, the algorithm stops when the distance measure is less than 0.0005.

As the iteration process proceeds we also increase the dampening ratio and increase the number of draws that are used. The dampening ratio gradually increases from 0 to 0.95 and the number of draws from 3,000 to 192,000. Convergence is frequently achieved before we reach the maximum number of draws and the maximum dampening ratio.