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REAL EXCHANGE RATE POLICIES: A  
PECUNIARY EXTERNALITY  
PERSPECTIVE**

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# **OPTIMAL CAPITAL CONTROLS AND REAL EXCHANGE RATE POLICIES: A PECUNIARY EXTERNALITY PERSPECTIVE**

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## ABSTRACT

### Optimal Capital Controls and Real Exchange Rate Policies: A Pecuniary Externality Perspective\*

In response to the global financial crisis a new policy paradigm emerged in which capital controls and other quantitative restrictions on credit flows have become part of the standard crisis prevention policy toolkit. A new strand of theoretical literature studies the use of capital controls in a context in which pecuniary externality justifies policy interventions. Within the same theoretical framework adopted in this literature, we show that the optimal design of crisis prevention (ex-ante) policies depends on the effectiveness of crisis management (ex-post) policies. This interaction between ex-ante and ex-post policies gives rise to a new rationale for the use of capital controls. Specifically, we show that when ex-post policies are effective in containing crises, there is no need to intervene ex-ante with capital controls. On the other hand, if crises management policies entail efficiency costs and hence lose effectiveness, then the optimal policy mix consists of both ex-ante and ex-post interventions so that crises prevention policies become desirable. In our model, the optimal policy mix combines capital controls in tranquil times with real exchange rate support to limit its depreciation during crises times and yields welfare gains of more than 1% in consumption equivalence terms.

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# 1 Introduction

In response to the recent global financial crisis and its costly aftermath, a new policy paradigm has emerged in which old fashioned government distortions such as capital controls and other restrictions on credit flows are becoming part of the standard crisis prevention policy toolkit (the so called macro-prudential policies). Even the traditionally conservative IMF changed its orthodox views on capital controls and is now advocating the use of such measures when other tools are not available or have run their course of action—see Blanchard and Ostry (2012) and IMF (2012).

The key rationale underpinning the use of capital controls is financial stability.<sup>1</sup> The financial stability motive for capital controls is the focus of the influential papers of Korinek (2010) and Bianchi (2011).<sup>2</sup> Their analysis is based on variants of a common theoretical framework proposed by Mendoza (2002, 2010) in which the scope for policy intervention arises because of a pecuniary externality stemming from the presence of a key relative price in the collateral constraint faced by private agents. In this environment, prudential interventions (i.e., before a financial crisis occurs) may be desirable because they make agents internalize the aggregate consequences of their decisions, discourage financial excesses, reduce the amount that agents borrow and the probability of financial crises, thereby enhancing welfare.<sup>3</sup>

In this paper we consider the same framework in which government intervention is justified by the pecuniary externality, but examine the policy problem from a broader perspective. We study the role and the interaction of ex-ante and ex-post policy interventions and show that the optimal design of crisis prevention (ex-ante) policies depends on the effectiveness of crisis management (ex-post) policies. In our model, ex-ante interventions are identified with capital controls while ex-post interventions take the form of real exchange rate support policy (or price support policy).

Specifically, we first show that a price support policy in the event of a crisis (in our

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<sup>1</sup>As documented by Magud, Reinhart, and Rogoff (2011), historically, capital controls were adopted for fear of economic damage associated with reversal of capital flows, fear of excessive risk taking and to contain excessive capital inflows that fuel asset price bubbles.

<sup>2</sup>See also Lorenzoni (2008), Bianchi and Mendoza (2010), Jeanne and Korinek (2012, 2013) and Benigno et al. (2013).

<sup>3</sup>Blanchard and Ostry (2012) make explicit reference to the pecuniary externality perspective when motivating the IMF's view on the use of capital controls: "*If there are external effects from foreign borrowing (think of amplified crisis risks for the country, where the risks are not internalized by the borrower), then capital controls can act as Pigouvian taxes and constitute an optimal response at the country level, helping agents to internalize the external effects of their borrowing*". As Jeanne (2012) put it, this new literature "*transposes to international capital flows the closed-economy analysis of the macro-prudential policies that aim to curb the boom-bust cycle in credit and asset prices*".

model, a support to the real exchange rate in crises times to limit its depreciation) always welfare-dominates prudential capital controls. This is because by supporting the price of collateral these policies can achieve an allocation in which the collateral constraint is not binding. Contrary to what usually found in the literature on pecuniary externalities, the average welfare gains from price support policies are quantitatively large relative to the unregulated economy.

We then provide a new rationale for capital controls. Indeed, we show that when price support policies are costly, so that their effectiveness is reduced, capital controls become desirable. This rationale for capital controls depends on the effectiveness of ex-post policy intervention rather than the amount that agents borrow in the unregulated economy during tranquil times. This novel element of our analysis emphasizes the interaction between ex-ante (normal times) and ex-post (crises times) policy interventions: when price support policy is fully effective in crises times (i.e. is able to address the pecuniary externality distortion) there is no scope for ex-ante policy intervention. However, if the policy is relatively ineffective in crises times, it is optimal to adopt capital controls during normal times as a means to limit the occurrence of the crises, combined with price support policies in crises times to mitigate their severity. We find that the optimal combination of ex ante and ex post policy intervention achieves welfare gains of 1.10% of tradable consumption relative to the unregulated economy.

As the vehicle to convey our messages, we adopt the same model economy as in Bianchi (2011). This is a two-sector (tradables and nontradables) small open, endowment economy with an occasionally binding international borrowing constraint. Borrowing, denominated in units of tradable consumption is limited by the value of current income generated from both the tradable and nontradable sectors. When the borrowing constraint binds, the decline in the relative price of nontradables generates a balance sheet effect through the constraint leading to a Fisherian debt-deflation spiral. In this class of models, a financial crisis event (also labelled a Sudden Stop in capital or credit flows) occurs when the constraint binds. Quantitatively this model has been successful in reproducing the business cycle and the crisis dynamics properties of a typical emerging market economy.

While in this economy there is a well defined scope for government intervention because of the pecuniary externality associated with the borrowing constraint, there are multiple instruments or tools with which policy could be conducted. In fact, in our model, there are three types of taxes that can be used: a tax/subsidy on debt, a tax/subsidy on tradable consumption and a tax/subsidy on nontradable consumption. The tax on borrowing is usually interpreted as a capital control, while taxes on either tradable or nontradable consumption can be interpreted as a real exchange rate intervention because they affect the relative price

of nontradables directly.<sup>4</sup> In our policy analysis, we consider all of them, studying their relative effectiveness in welfare terms. To conduct the policy analysis we follow a Ramsey optimal taxation approach, assuming that the government budget is always balanced.

We first study the Ramsey problem when capital controls are the only policy tool available and the government budget constraint is balanced through lump-sum transfers/taxes. We find that it is optimal to limit the amount that agents borrow in normal times while no action is needed during crises times. (see also Korinek (2010) and Bianchi (2011)) The reason why capital controls are optimal under these assumptions is that, in this model environment, they cannot affect in a welfare-improving manner the allocation when a crises event occurs (i.e. when the constraint binds). Thus, in this setting, when capital controls are the only policy tool available, the best that the government can do is to reduce the probability that a crisis occurs. As a result, it becomes optimal to impose a tax on debt flows during tranquil times.

Next we show that a policy of supporting the real exchange rate during crisis times, and hence of relaxing the borrowing constraint when it binds, can achieve much higher welfare. In fact, we show that such a policy can undo the borrowing constraint completely and, as a result, support an equilibrium in which agents behave as if they were in the unconstrained allocation. Importantly, as we shall see, this policy is also time-consistent. This result hinges on the ability by the Ramsey planner to manipulate the value of collateral without creating other distortions, since intervention is financed through lump-sum transfers or taxes and does not entail further distortions.

Finally, we show that, when lump-sum transfers/taxes are no longer available, the effectiveness of price support policies is reduced and capital controls in normal times complement real exchange rate policies in crises times under the optimal policy. The interaction between ex-ante (pre-crisis) and ex-post (during crisis) policies gives rise to a new rationale for the use of capital controls along with price support policies when both are available. When ex-post policies are effective (they can relax the borrowing constraint), there is no need to engage in ex-ante policy interventions. But when the use of ex-post policies entails efficiency losses, then ex-ante policy intervention is required to limit the probability that a crisis might occur. As we shall see, our rationale for ex-ante policy intervention is not related to the amount that agents borrow in the unregulated economy.

The paper relates to a few other recent contributions in the literature on pecuniary externalities. Benigno et al. (2012) solve numerically for the Markov Perfect optimal

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<sup>4</sup>The interpretation of the relative price of nontradables as the real exchange rate is standard in the literature. See for instance Bianchi (2011), Caballero and Lorenzoni (2009), Mendoza (2002), Korinek (2010), Jeanne (2012). Alternatively, the consumption taxes (subsidies) could be interpreted literally as domestic fiscal policy tools.

policy problem in the context of a production version of our economy in which a time-consistency issue arises. Jeanne and Korinek (2013) study the time-consistent mix of ex-ante macroprudential regulation and ex-post bailout transfer in a three-period framework. Benigno et al. (2013) analyze the extent to which private agents overborrow or underborrow in a production version of our economy. Our paper also shares the emphasis on price support policies in terms of limiting the depreciation of the real exchange rate during crisis times with the work of Chang, Cespedes and Velasco (2012), who examine the role of other unconventional policy tools such as credit policies and direct interventions in the foreign exchange market. While they study more realistic forms of government intervention, they do not compute optimal policy but rather focus on the transmission mechanism of alternative policy tools.<sup>5</sup>

Other new theoretical approaches rationalized the adoption of capital controls. One approach motivates the use of capital controls with the possibility of manipulating the intertemporal terms of trade—conceptually analogous to the use of tariffs to manipulate the goods' terms of trade (Costinot, Lorenzoni and Werning (2014) and De Paoli and Lipinska (2013)). Other approaches focus on the role of capital controls when there are multiple distortions. For instance, Brunnermeir and Sannikov (2014) show that restrictions to capital flows can be welfare improving in an economy with multiple goods, incomplete financial markets and inefficient production. Schmitt-Grohe and Uribe (2012) examine the role of capital controls in an economy with downward nominal wage rigidity and a fixed exchange rate regime, while Fahri and Werning (2012) study capital controls as a way to address the impossibility to simultaneously have an open capital account, a fixed exchange regime, and an independent monetary policy (as known as the "impossible trilemma"). Finally, Devereux and Yetman (2013) analyze capital controls as a way to restore monetary policy effectiveness when the nominal interest rate reaches the zero lower bound in a global liquidity trap context.

More generally, the role of alternative policy tools is related to the work by Correia, Nicolini and Teles (2008) in which the role of price stickiness for the design of monetary policy depends on the existence of alternative fiscal policy tools. Finally, our paper relates to the important literature that analyze financial frictions in infinite horizon macroeconomic models from a positive perspective as in the seminal contributions of Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999) and, more recently, Brunnermeier and Sannikov (2013), Curdia and Woodford (2010), Gertler and

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<sup>5</sup>In an optimizing neoclassical framework without credit frictions, Calvo, Reinhart and Vegh (1995) also analyze the role of real exchange rate targeting as a temporary stabilization policy.



Karadi (2011), Gertler and Kiyotaki (2011) and Iacoviello (2005).

In terms of the solution techniques, in addition to Benigno et al. (2012), our paper is related to Klein, Krusell, and Rios-Rull (2009), Kim, Kim, and Kollman (2008) and Guerrieri and Iacoviello (2013): the main difference being that the algorithm that we use does not require that the policy functions are differentiable (which in general would not hold in our environment due to the occasionally-binding constraint) but only that they are continuous.

The rest of the paper is organized as follows. Section 2 describes the model environment, the scope for government intervention, and the alternative government instruments that we consider. Section 3 studies optimal capital control policy. Section 4 analyzes the optimal policy problem with real exchange rate management. Section 5 considers the case in which lump-sum transfers/taxes are not available and the joint use of capital controls and real exchange rate policies. Section 6 relates the main results of the paper to countries's experience with capital controls and price support policies. Section 7 concludes. The numerical solution methods we use as well as other technical material are reported in appendix.

## 2 The model environment

In this section we describe our model economy by presenting its structure and assumptions. Next we characterize the competitive equilibrium and the alternative equilibrium allocations that we examine. Then we identify the externality that gives rise to scope for government intervention. Finally, we present and discuss the alternative government policy instruments that we will analyze in the rest of the paper.

We consider a small open economy in which there is a continuum of households  $j \in [0, 1]$  that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_t^j)\}, \quad (1)$$

with  $C_t^j$  denoting the consumption basket for an individual  $j$  at time  $t$  and  $\beta$  the subjective discount factor.  $E_0$  denotes the conditional expectation at time 0. We assume that the period utility function is isoelastic:

$$u(C_t^j) \equiv \frac{1}{1-\rho} (C_t^j)^{1-\rho}.$$

The consumption basket,  $C_t$ , is a CES aggregate of tradable and nontradable goods:<sup>6</sup>

$$C_t \equiv \left[ \omega^{\frac{1}{\kappa}} (C_t^T)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}. \quad (2)$$

The parameter  $\kappa$  is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while  $\omega$  is the relative weight of the two goods in the utility function.

We normalize the price of tradable goods to 1 and denote the relative price of the nontradable goods with  $P^N$ . The aggregate price index is then given by

$$P_t = \left[ \omega + (1-\omega) (P_t^N)^{1-\kappa} \right]^{\frac{1}{1-\kappa}}.$$

Here we note that there is a one-to-one link between the aggregate price index  $P$  and the relative price  $P^N$ .

Households maximize utility subject to their budget constraint, which is expressed in units of tradable consumption, and a borrowing constraint. The asset menu includes only a one-period bond denominated in units of tradable consumption.

Each household has two stochastic endowment streams of tradable and non-tradable output,  $\{Y_t^T\}$  and  $\{Y_t^N\}$ . For simplicity, we assume that both  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support. Therefore the current state of the economy can be completely characterized by the triplet  $\{B_t, Y_t^T, Y_t^N\}$ . The budget constraint each household faces is

$$C_t^T + P_t^N C_t^N + B_{t+1} = Y_t^T + P_t^N Y_t^N + (1+r) B_t, \quad (3)$$

where  $B_{t+1}$  denotes the bond holding at the end of period  $t$ , and  $(1+r)$  is the given world gross interest rate.

Access to international financial markets is not only incomplete but also imperfect in the sense that we assume that the amount that each individual can borrow internationally is limited by a multiple of his current total income:

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N]. \quad (4)$$

One interpretation for the international borrowing constraint (4) relates it to the presence of liquidity constraints. By this interpretation, lenders require households to finance a

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<sup>6</sup>We omit the subscript  $j$  to simplify notation, but it is understood that all choices are made at the individual level.

fraction  $\phi$  of their current expenses out of current income, which includes consumption, debt repayments and taxes (see Mendoza (2002) for this interpretation)

$$\phi (Y_t^T + P_t^N Y_t^N) \geq C_t^T + P_t^N C_t^N - (1 + r)B_t. \quad (5)$$

By combining (5) with (3) we obtain (4). Another justification of this borrowing limit is provided by Bianchi (2011), who appeals to an environment in which the borrower engages in fraud activities in the period in which the debt is contracted and prevents creditors from seizing any future income.

At the empirical level, a specification in terms of current income is consistent with evidence on the determinants of access to credit markets (e.g., Jappelli 1990, Jappelli and Pagano, 1989) and lending criteria and guidelines used in mortgage and consumer financing as emphasized by Mendoza (2002). The assumption that nontradable goods are part of the collateral constraint is consistent with the evidence presented in Tornell and Westermann (2005) where external credit fuels credit booms in the nontradable sectors.

The key feature of the international borrowing constraint (4) is that it captures currency mismatches in the balance sheet of our small open economy model—see Krugman (1999). In fact borrowing is denominated in units of tradable consumption, while both the tradable and the nontradable endowment can be pledged as collateral. Indeed, currency mismatches have been one of the main vulnerability of emerging market economies in the numerous financial crises in the 1990s and the 2000s—See Cespedes, Chang and Velasco (2004, 2012) and Shin (2013) for a discussion.<sup>7</sup>

From a model perspective, a crisis occurs when the constraint binds; an event that is endogenous in the model. Yet the long-run business cycle features of the economy are only marginally affected by the crises events (Mendoza, 2010). A unique feature of the model environment, therefore, is to nest endogenous crises dynamics with “financial amplification” triggered by small exogenous disturbances within regular business cycles.

In our small open economy, the motive for borrowing arises from the assumption that  $\beta(1 + r) < 1$  so that agents are impatient compared to foreign lender. This assumption implies that their debt position will converge towards the natural debt limit, defined as level of debt  $B^n$  at which tradable consumption  $C_t^T$  equals zero, in the deterministic steady state of the model.<sup>8</sup> In our stochastic environment agents engage in precautionary saving behavior so that the probability of hitting the natural debt limit is zero.

We also assume that in our economy there is a lower bound on debt which is strictly

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<sup>7</sup>Shin (2013) emphasizes the role of dollar offshore borrowing by non financial firms in emerging market economies, as dollar funding costs were kept particularly low by the US monetary policy.

<sup>8</sup>In our model, this level equals (minus) the annuity value of the lowest tradable endowment value.

greater than the natural debt limit,  $\underline{B} > B^n$ , such that  $B_t \geq \underline{B}$ , for all  $t$ .<sup>9</sup> This lower bound guarantees that the competitive equilibrium allocation without government intervention and without the international borrowing constraint (4) (i.e. the unconstrained allocation) is well defined. In particular, it guarantees that this equilibrium has an ergodic distribution of debt with finite support, and both tradable and nontradable consumption have a strictly positive lower bound, while the nontradable price also has finite support with strictly positive lower bound. Finally, in order to focus on non-trivial policies, we also assume that, given  $Y_t^T$  and  $Y_t^N$ , when  $B_t = \underline{B}$ , the competitive equilibrium allocation always violates the borrowing constraint (4).<sup>10</sup>

Our calibration and in particular the assumption that tradable and nontradable goods are complement ( $\kappa < 1$ ) allows us to rule out the possibility of multiple equilibria (see also the discussion of Jeanne and Korinek (2012)).<sup>11</sup>

## 2.1 Competitive equilibrium

Consider first the competitive equilibrium of the model above without any government intervention. When there is no government intervention, households maximize (1) subject

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<sup>9</sup>If  $C^T$  and  $C^N$  are strong substitutes, this constraint may bind; since the evidence is that  $C^T$  and  $C^N$  are complements, we can ignore this possibility.

<sup>10</sup>This restriction amounts to a lower bound on  $\phi$ .

<sup>11</sup>The borrowing constraint can induce multiple equilibria due to the possibility of self-fulfilling decline in the relative price of nontradables that reduce the value of the collateral and the consumption of tradable goods in a manner compatible with the initial decline in the relative price of nontradables. More formally by combining the borrowing constraint (4), the budget constraint (3) and the pricing equation we obtain:

$$C_t^T = B_t(1+r) + \left(1 + \frac{1-\phi}{\phi}\right) \left[ Y_t^T + \frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} \right] \equiv f(C_t^T).$$

When the elasticity of intratemporal substitution is less than 1 (the goods are complement) then a sufficient condition for unicity is that the derivative of the RHS ( $f'(C^T)$ ) of the previous equation with respect to  $C^T$  evaluated at the intersection point with the LHS is greater than 1. Indeed when  $B_t(1+r) + \left(1 + \frac{1-\phi}{\phi}\right) Y_t^T < 0$  and  $\kappa < 1$ , we have that  $\lim_{C^T \rightarrow 0} f'(C^T) = 0$  and  $\lim_{C^T \rightarrow \infty} f'(C^T) = \infty$ . This assumption combined with the assumption that  $f'(C^T) > 1$  evaluated at the intersection point, guarantees that there is only one intersection between the RHS and the LHS.

Another issue that might arise given the specification of the borrowing constraint is the possibility that, when the amount that the planner borrows increases, then the relative price of nontradable rises the value of the collateral by more than the increase in  $B_{t+1}$  leading to a relaxation of the borrowing constraint. Our calibration rules out the possibility of such a perverse dynamic.

to (3) and (4) by choosing  $C_t^N$ ,  $C_t^T$  and  $B_{t+1}$ . The Lagrangian of this problem is

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \begin{array}{l} \frac{1}{1-\rho} C_t^{1-\rho} + \lambda_t \left( B_{t+1} + \frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N] \right) + \\ \mu_t (Y_t^T + P_t^N Y_t^N - B_{t+1} + (1+r) B_t - C_t^T - P_t^N C_t^N), \end{array} \right]$$

with  $\lambda_t$  and  $\mu_t$  denoting the multipliers on the borrowing constraint and the budget constraint, respectively. The first order conditions of this problem are:

$$C_T : u'(C_t) C_{C^T} = \mu_t, \quad (6)$$

$$C_N : u'(C_t) C_{C^N} = \mu_t P_t^N, \quad (7)$$

$$B_{t+1} : \mu_t = \lambda_t + \beta (1+r) E_t [\mu_{t+1}]. \quad (8)$$

$$\lambda_t \left\{ B_{t+1} + \frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N] \right\} = 0 \quad (9)$$

Combining (6) and (7) we have:

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N. \quad (10)$$

The competitive equilibrium allocation of the economy can now be characterized by the first order conditions (8), (9) and (10) and the goods market equilibrium conditions:

$$C_t^T = Y_t^T - B_{t+1} + (1+r) B_t, \quad (11)$$

and

$$C_t^N = Y_t^N. \quad (12)$$

The properties of the competitive equilibrium of this economy are well known (see Mendoza (2002), Korinek (2010) and Bianchi (2011)). But it is important to note here that, as Bianchi (2011) showed, this very same model can quantitatively account relatively well for some of the key business cycles statistics as well as the incidence and severity of financial crises in the data of a typical emerging market economy like Argentina.

### 2.1.1 Unconstrained Equilibrium

As we shall see below, two of the government policy instruments that we consider, when used optimally, can completely remove the effects of the constraint (4) and achieve an allocation that is identical to the competitive equilibrium of the model without the borrowing

constraint (4). We now characterize this allocation that we refer as the "unconstrained equilibrium" (UE).<sup>12</sup>

In terms of equilibrium conditions, the unconstrained allocation is characterized by the following equations:

$$C_T : u'(C_t^{UE}) C_{C^T}^{UE} = \mu_t^{UE}, \quad (13)$$

$$C_N : u'(C_t^{UE}) C_{C^N}^{UE} = \mu_t^{UE} (P_t^N)^{UE}, \quad (14)$$

$$B_{t+1} : \mu_t^{UE} = \beta (1 + r) E_t [\mu_{t+1}^{UE}], \quad (15)$$

along with the goods market equilibrium conditions (11) and (12).<sup>13</sup> We also note here that our unconstrained equilibrium characterizes an allocation in which financial markets are incomplete so that there are inefficient variations in consumption due to the lack of state contingent debt.

## 2.2 Pecuniary externality

In order to understand the rationale for policy intervention in our model, we follow the recent related literature—e.g., Lorenzoni (2008), Korinek (2010) and Bianchi (2011)—by focusing on a benevolent social planner problem with restricted planning abilities. In particular, we assume that the social planner can directly choose the level of debt subject to the credit constraint and allows goods markets to clear competitively. Unlike the representative agent in the competitive equilibrium of the model, the social planner internalizes the effects of his/her borrowing decisions on the equilibrium relative price of nontradables. This is relevant in our set up because, when the constraint binds, agents' borrowing capacity depends on the value of the collateral, which in turn is determined endogenously by the equilibrium relative price of nontradables. We now examine the social planner problem before focusing our analysis to the Ramsey problem.

### 2.2.1 Social planning problem

Specifically, the benevolent social planner maximizes (1) subject to the same borrowing constraint (4) that private agents face and the market clearing conditions for tradables and nontradables goods (11) and (12).

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<sup>12</sup>As we discussed above, the existence of a lower bound on debt which is strictly greater than the natural debt limit guarantees that the competitive allocation without borrowing constraint has an ergodic distribution of debt with finite support under the assumption that  $\beta(1+r) < 1$ .

<sup>13</sup>See Mendoza (2002) for a comparison between the constrained and the unconstrained competitive equilibrium of the model.

In specifying this problem, the equilibrium price of nontradables is determined competitively according to the pricing rule (10) that serves also as a constraint to the planning problem.<sup>14</sup> By substituting the relative price of nontradables,  $P_t^N$  in the borrowing constraint (4) with the competitive pricing rule (10) we can write the Lagrangian of the planning problem as

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \begin{aligned} & \frac{1}{1-\rho} (C_t)^{1-\rho} + \mu_{1,t}^{SP} (Y_t^T - B_{t+1} + (1+r)B_t - C_t^T) + \\ & + \mu_{2,t}^{SP} (Y_t^N - C_t^N) + \lambda_t^{SP} \left( B_{t+1} + \frac{1-\phi}{\phi} \left[ Y_t^T + \left( \frac{(1-\omega)(C_t^T)}{\omega Y_t^N} \right)^{\frac{1}{\kappa}} Y_t^N \right] \right) \end{aligned} \right],$$

where  $\mu_{1,t}^{SP}$ ,  $\mu_{2,t}^{SP}$  and  $\lambda_t^{SP}$  denote the multipliers with superscript  $SP$  to distinguish them from the multipliers in the competitive equilibrium allocation and the unconstrained equilibrium. The planner must choose the optimal path for  $C_t^T$ ,  $C_t^N$  and  $B_{t+1}$ , and the first order conditions for its problem are:

$$C_T : u'(C_t^{SP}) C_{C^T}^{SP} + \lambda_t^{SP} \Sigma_t^{SP} = \mu_{1,t}^{SP}, \quad (16)$$

$$C_N : u'(C_t^{SP}) C_{C^N}^{SP} = \mu_{2,t}^{SP}, \quad (17)$$

$$B_{t+1} : \mu_{1,t}^{SP} = \lambda_t^{SP} + \beta(1+r) E_t [\mu_{1,t+1}^{SP}]. \quad (18)$$

$$\lambda_t^{SP} \left\{ B_{t+1}^{SP} + \frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N] \right\} = 0 \quad (19)$$

where  $\Sigma_t^{SP} \equiv \frac{1-\phi}{\phi} \frac{\partial P_t^N}{\partial C_t^T} Y_t^N = \frac{1-\phi}{\phi} \frac{1}{\kappa} \frac{(1-\omega)}{\omega} \left( \frac{(1-\omega)(C_t^T)}{\omega} \right)^{\frac{1}{\kappa}-1} (Y_t^N)^{\frac{\kappa-1}{\kappa}}$ .

The key difference between the planning allocation and the competitive one follows from examining equations (16) and (6). From the perspective of the planner, who internalizes the consequences of her/his decisions on  $P_t^N$ , when the constraint binds ( $\lambda_t^{SP} > 0$ ), there is an additional marginal benefit in consuming an extra unit tradable consumption, represented by the term  $\lambda_t^{SP} \Sigma_t$ , which captures the increase in the price of non-tradable goods associated with the marginal increase in tradable consumption. As we shall see, this difference between the margin of the competitive equilibrium and the one of the social planner has intertemporal implications and affects agents behavior also when the constraint does not

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<sup>14</sup>This formulation is referred to as "constrained-efficient" planning problem. A second possibility, sometimes referred to as the "conditionally-efficient" problem, is to determine this relative market price by imposing as a constraint the competitive equilibrium *policy function* (in our case  $P_t^N = f^{CE}(B_t, Y_t^N, Y_t^T)$ ). In the case of the our endowment economy, these two definitions give exactly the same results and do not affect the normative analysis. See Kehoe and Levine (1993) and Lorenzoni (2008) for more details and a discussion.

bind.

We solve the competitive equilibrium of the model and the social planner equilibrium (and for comparability also the unconstrained equilibrium) with global solution methods that we describe in appendix. For illustrative purposes, the parameter values of the model are exactly as in Bianchi (2011), and a summary table is also reported in appendix.

To illustrate the consequences of the presence of the borrowing constraint and the resulting pecuniary externality, Figure 1 compares the policy functions of the endogenous variables  $(C_t^T, B_{t+1}, P_t^N)$  for one negative standard deviation shocks for three allocations: the competitive equilibrium (CE), the social planning problem (SP) and the unconstrained equilibrium (UE).<sup>15</sup> The figure shows the substantial difference between the policy functions of the unconstrained equilibrium (the UE) and the constrained ones (the CE and SP). In particular the unconstrained equilibrium features a much higher level of tradable consumption and debt, as well as a higher relative price of nontradable goods, compared to the CE and SP allocations. In the absence of the borrowing constraint, agents can borrow freely from international capital markets and sustain a much higher level of consumption for any given stock of existing debt. In contrast, the competitive and the social planner allocations are relatively close: they diverge slightly in the region in which the constraint is not binding but is expected to bind in the future, and otherwise coincide including in the region in which the constraint binds.

In the constrained allocations (in the CE and the SP), in the region in which the constraint binds (i.e., when there is a crisis), both consumption of tradables and the relative price of nontradables fall sharply.<sup>16</sup> This decline is the consequence of the so-called "Fisherian deflation" or fire sale mechanism emphasized in the financial crisis literature. When the amount that agents borrow is constrained, consumption is lower relative to the desired amount in the unconstrained equilibrium. Lower tradable consumption is accompanied by a decline in relative price of nontradables that reduces the value of the collateral, tightening agents' borrowing capacity and reducing further consumption of tradables. This feedback loop results in even lower relative price of nontradables and consumption. However, when the constraint binds for a given state  $\{B_t, Y_t^N, Y_t^T\}$ , the CE and the SP allocations coincide. For a given amount of debt, tradable consumption will be the same in the two allocations since it is constrained by the borrowing limit; also the resulting equilibrium price is equalized since the consumption on nontradables is determined by its endowment.<sup>17</sup>

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<sup>15</sup>A policy function is the non-linear equilibrium relation between the endogenous variables of the model and its exogenous and endogenous state variables (in our case, the triplet  $\{B_t, Y_t^N, Y_t^T\}$ ).

<sup>16</sup>In the figure, the binding region starts in correspondence of the kink in the policy functions.

<sup>17</sup>Recall that the relative price of nontradables is proportional to the ratio of tradable consumption over nontradable consumption.



As emphasized by Lorenzoni (2008), Korinek (2010) and Bianchi (2011), when the constraint does not bind (i.e. in normal times) but is expected to bind in the future with some positive probability, agents in the competitive equilibrium consume more than in the social planner allocation. This difference arises because the planner takes into account the additional marginal benefit of reducing consumption today, captured by the term  $E_t (\lambda_{t+1}^{SP} \Sigma_{t+1}^{SP})$  that represents the marginal benefit of consuming more when the constraint binds in the future.

### 2.3 Alternative policy instruments

While in this economy there is a well defined scope for government intervention, there is a variety of instruments or tools with which policy could be conducted. In fact, in the context of our model economy, there are three types of taxes that can be used: a tax/subsidy on debt, a tax/subsidy on tradable consumption and a tax/subsidy on nontradable consumption. In our policy analysis, in the rest of the paper, we are going to consider all of them, studying their relative effectiveness in welfare terms as well as their joint use.

To conduct the policy analysis we take a Ramsey optimal taxation approach, assuming that the government budget is always balanced. For given policy instrument(s), the Ramsey planner maximizes households' utility function subject to the resources constraints and the private agents' first order conditions.

**Tax on borrowing** The first policy tool that we examine is a tax  $\tau_t^B$  ( $< 0$ ) or a subsidy ( $> 0$ ) on one-period debt issued at time  $t$ ,  $B_{t+1}$ . This instrument is usually referred to as a capital control policy.<sup>18</sup> Since we allow for lump-sum transfers/taxation, the government budget constraint is:

$$T_t = \tau_t^B B_{t+1}, \quad (20)$$

where  $T_t$  denotes the lump sum transfer or tax. Because of this tax the household's budget constraint in the competitive equilibrium of the model becomes

$$C_t^T + P_t^N C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1}(1 + \tau_t^B) + (1 + r) B_t. \quad (21)$$

Nothing else changes compared to the environment that we described earlier: in particular we assume that the access to international financial market continues to be constrained by (4) as before.

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<sup>18</sup>A recent example of the active use of such a tax is Brazil. See Harris and Pereira (2012) for a detailed account of the Brazilian case.

**Taxes on consumption** The other policy tools that we study are a consumption tax on non-tradable and on tradable goods. Both policy tools influence directly equation (10) and affect the relative price of nontradable goods,  $P_t^N$ , which in the context of our economy is the proxy for the real exchange rate (see for example Mendoza (2002), Caballero and Lorenzoni (2009), Korinek (2010), Bianchi (2011), Jeanne (2012), and Schmitt-Grohe and Uribe (2012) for the same interpretation). In what follows we refer to these set of policy tools as “real exchange rate policy” or "exchange rate policy" for brevity.

With the tax on nontradable consumption,  $(1 + \tau_t^N)$ , the constraint that each household faces becomes

$$C_t^T + P_t^N(1 + \tau_t^N)C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1} + (1 + r)B_t, \quad (22)$$

where  $\tau_t^N > (<) 0$  is a tax (or a subsidy) on nontradable consumption and  $T_t > (<) 0$  is a government lump-sum transfer (or tax). As in the case of capital controls, we assume that the government runs a balanced budget period by period:

$$T_t = \tau_t^N P_t^N C_t^N. \quad (23)$$

Again, nothing else changes compared to the environment that we described above: in particular we continue to assume that international financial market access is constrained by (4). Indeed, our liquidity constraint now becomes

$$\phi(Y_t^T + P_t^N Y_t^N) \geq C_t^T + (1 + \tau_t^N)P_t^N C_t^N - (1 + r)B_t + T_t, \quad (24)$$

which combined with the individual and the government budget constraint will determine the same international borrowing constraint as before.

With a tax on tradable consumption as the government’s policy tool, each household now faces the following budget constraint:

$$(1 + \tau_t^T)C_t^T + P_t^T C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1} + (1 + r)B_t. \quad (25)$$

The government budget constraint continues to be balanced:

$$T_t = \tau_t^T C_t^T, \quad (26)$$

and the borrowing constraint remains as in (4).

### 3 Capital controls

We now study the optimal policy problem when the policy tool is given by  $\tau_t^B$ . The Ramsey problem for  $\tau_t^B$  is to choose a competitive equilibrium that maximizes (1). More formally:

**Definition 1.** *For a given  $\{B_0\}$  and assuming that  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support, the Ramsey problem for  $\tau_t^B$  is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_t)\},$$

subject to the resource constraints (11) and (12), the government budget constraint (20), the borrowing constraint

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N], \quad (27)$$

and the first order conditions of the household

$$u'(C_t) C_{CT} (1 + \tau^B) = \lambda_t + \beta (1 + r) E_t [u'(C_{t+1}) C_{CT}], \quad (28)$$

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N. \quad (29)$$

We can now state the following proposition, which qualifies the main result of Bianchi (2011).

**Proposition 1.** *The Ramsey optimal policy with  $\tau^B$  as the government policy instrument replicates the social planner allocation (SP). Moreover the optimal policy is time-consistent.*

PROOF: Let's consider first a less restricted version of the Ramsey problem in which the planner maximizes (1) subject to (11) and (12), (27) and (29). This problem corresponds to the social planner one (SP) defined above. The solution of the Ramsey problem for  $\tau^B$  cannot achieve a higher welfare than the social planner allocation because the Ramsey problem is more restricted than the social planner problem—by equation (28).

We conjecture that the two allocation coincide. To verify this, note that the Euler equation for the social planner problem is

$$u'(C_t^{SP}) C_{CT}^{SP} + \lambda_t^{SP} \Sigma_t^{SP} = \lambda_t^{SP} + \beta(1+r) E_t [u'(C_{t+1}^{SP}) C_{CT}^{SP} + \lambda_{t+1}^{SP} \Sigma_{t+1}^{SP}]. \quad (30)$$

It is easy to see that, if the Ramsey planner chooses  $(1 + \tau_t^B)$  in equation (28) so that

$$\tau_t^B = (u'(C_t^{SP}) C_{CT}^{SP})^{-1} (\lambda_t^{SP} \Sigma_t^{SP} - \beta(1+r) E_t [\lambda_{t+1}^{SP} \Sigma_{t+1}^{SP}]), \quad (31)$$

the Euler equations (28) and (30) become identical. It follows that the solution of the Ramsey problem for  $\tau^B$  and the social planner problem above coincide, and the expression (31) is the Ramsey optimal policy for  $\tau_t^B$ .

Moreover, since the optimal policy for  $\tau^B$  decentralizes the social planner problem, which is a recursive problem that can be represented by value function iteration and only depends on the current state ( $\{B_t, Y_t^N, Y_t^T\}$ ), the equilibrium is subgame perfect and the policy rule (31) is time-consistent.

QED.

A few remarks are in order here. From a policy perspective, as discussed by Bianchi (2011) and noted in the previous section, when the constraint binds (i.e.  $\lambda_t^{SP} > 0$ ), the social planner allocation coincides with the competitive equilibrium allocation, and therefore it is optimal to set  $\tau_t^B = 0$ . When the constraint does not bind, but it can bind with positive probability in the next period (i.e.  $\lambda_t^{SP} = 0$ , but  $E_t[\lambda_{t+1}^{SP} \Sigma_{t+1}^{SP}] > 0$ ), the optimal state contingent  $\tau_t^B$  is a tax on borrowing ( $\tau_t^B < 0$ ). Thus, it is optimal to engage in a policy intervention even when the constraint does not bind but might bind in the future. In this sense the optimal policy is “prudential” or “precautionary” in nature. Intuitively, since  $\tau_t^B$  is impotent during the crisis, the best thing that policy can do, conditional on having only the tax on debt as the government instrument, is to reduce the probability that a crisis occurs by limiting the amount that agents borrow in equilibrium (i.e. by taxing  $B_{t+1}$ ).

Figure 2 plots the policy function for  $\tau_t^B$  that solves the optimal policy problem above and replicates the SP allocation, as well as the welfare gains for  $\tau_t^B$  as a function of current bond holdings for a negative one standard deviation shocks. Figure 3 reports the ergodic distribution of debt. Table 1 reports the ergodic mean of debt as a share of (annual) income in units of tradable consumption, the unconditional probability of a financial crisis, as well as the average welfare gain associated with this policy instrument relative to the CE.<sup>19</sup>

Intuitively, when the economy approaches the binding constraint (which is where the tax rate peaks in Figure 2), the higher is the probability that the constraint binds the higher is the tax on borrowing. Looking at the welfare gains we can see that they also peak when the constraint binds, but reverts to zero slower than the tax rate. The welfare benefit of capital controls persists past the level of debt at which the constraint binds because entering a crisis with less debt burden makes the crisis relatively less costly. In fact, as we can see from Figure 3 and Table 1, the policy intervention decreases the likelihood of crises events and reduces the average debt/income ratio, implying that the economy, on average, will borrow less under the optimal capital control policy than in the competitive equilibrium and will experience fewer and less costly financial crises (see Figure 8 below

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<sup>19</sup>See appendix for a description of the solution method and the computation of the welfare gains.

and its discussion on the latter point).

## 4 Real exchange rate policy

We now consider the use of consumption taxes or equivalently real exchange rate intervention. We first examine the nontradable consumption tax. As we shall see, the tax on tradable goods is equivalent to the tax on non-tradable goods when used optimally.

### 4.1 Nontradable tax

As before we first define the Ramsey problem when  $\tau_t^N$  is the policy instrument.

**Definition 2.** *For a given  $\{B_0\}$  and assuming that  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support, the Ramsey problem for  $\tau_t^N$  is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_j)\},$$

*subject to the resource constraints (11) and (12), the government budget constraint (23), the borrowing constraint*

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N]. \quad (32)$$

*and the first order conditions of the household*

$$u'(C_t)C_{CT} = \lambda_t + \beta(1+r) E_t [u'(C_{t+1})C_{CT}], \quad (33)$$

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N (1 + \tau_t^N). \quad (34)$$

It is important to note here that the non-tradable consumption tax directly affects the relative price of nontradables (i.e. the real exchange rate). In normal times (when the constraint does not bind), or in the unconstrained competitive equilibrium,  $\tau_t^N$  is neutral in the sense that the determination of the consumption of tradable and non-tradable goods is independent from  $P_t^N$ . In fact, the Euler equation and the goods market equilibrium conditions are all that is needed to determine consumption of tradables and nontradables. In contrast, when the constraint binds,  $\tau_t^N$  is no longer neutral because changes in  $P_t^N$  affect the value of the collateral and hence the consumption of tradable goods.

In the next proposition we show that, when used optimally, the consumption tax can achieve the unconstrained allocation (i.e., it assures that the borrowing constraint is never

strictly binding in the equilibrium of our economy so that  $\lambda_t = 0$  for all  $t$ ). Again, as in the previous proposition, to characterize the solution of this Ramsey problem we follow two steps. First we characterize a policy rule for  $\tau_t^N$  that decentralizes the unconstrained competitive equilibrium. Then we will show that this competitive equilibrium is the one chosen by the Ramsey planner above.

**Proposition 2.** *In an economy defined by (1), (4), (22) and (23) with a tax on non-tradable consumption  $\tau_t^N$  as the government policy instrument, there exists a policy for  $\tau_t^N$  that decentralizes the unconstrained allocation. This policy is Ramsey optimal and time-consistent.*

PROOF: For a given state  $\{Y_t^N, Y_t^T, B_t\}$ , let  $B_{t+1}^{UE}$  be the next-period debt and  $P_t^{N,UE}$  the current period relative price of non tradable goods in the economy defined by (1) and (3) but without the credit constraint (4)—i.e., in the unconstrained economy satisfying (13)-(15).

Next, let  $\hat{P}_t^N$  be the minimum price such that the credit constraint would be met if it were present. Thus:

$$\hat{P}_t^N = \max \left\{ 0, -\frac{B_{t+1}^{UE} + \frac{1-\phi}{\phi} Y_t^T}{\frac{1-\phi}{\phi} Y_t^N} \right\}.$$

If we set  $\tau^N$  such that  $\hat{P}_t^N(1 + \tau_t^N) \leq P_t^{N,UE}$ , then the credit constraint does not bind. In other words, let  $\hat{\tau}_t^N = P_t^{N,UE}/\hat{P}_t^N - 1$ , then any  $\tau_t^N \in (-1, \hat{\tau}_t^N]$  would eliminate the credit constraint ( $\lambda_t = 0 \forall t$ ) if it were present, and the competitive equilibrium of the economy would coincide with the unconstrained allocation, which eventually converges to the finite support.

Now, in the economy with the credit constraint, the Ramsey planner maximizes (1) subject to (11) and (12), (34), (32), and (33). In this problem, any policy schedule such that  $\tau_t^N \in (-1, \hat{\tau}_t^N]$  can achieve an allocation that satisfies the first order conditions (13)-(15) of the unconstrained competitive equilibrium. Since  $\tau_t^N$  can affect the allocation only when the constraint binds, but it is neutral when the constraint does not bind, the Ramsey planner can achieve at best the unconstrained allocation and the tax policy  $\tau_t^N \in (-1, \hat{\tau}_t^N]$  is the optimal solution of the Ramsey problem. Moreover, any  $\tau_t^N \in (-1, \hat{\tau}_t^N]$  is completely determined by the current state  $\{B_t, Y_t^T, Y_t^N\}$  and therefore it is time-consistent.

QED.

The proposition above implies that real exchange rate policy always dominates capital control policy in welfare terms. Under this policy, it is possible to undo the constraint completely and replicate the unconstrained equilibrium. In contrast, capital controls can only remove the distortionary effects of the pecuniary externality associated with the constraint,

but not the constraint itself. As we shall see, these welfare differences are quantitatively very large.

But how does this policy work? The intuition for the result is that (34) directly links the relative price of nontradables to the tax on nontradables. When the borrowing constraint does not bind, this policy tool is neutral in the sense that it does not affect the consumption allocation, but only the relative price  $P_t^N$ . When the constraint binds, however, this tax is no longer neutral and can be used to affect the value of collateral in the borrowing constraint, and hence also the allocation of tradable consumption. By subsidizing the consumption of non-tradable goods, the policy increases its relative price. Crucially, when the constraint binds, this policy supports the relative price of nontradables to counteract the debt-deflation mechanism that would otherwise lead to a decline in tradable consumption and a fall in the relative price of nontradables. For this reason we also interpret this policy in broader terms as a collateral price support policy, or “price support” policy for brevity.

In equilibrium agents anticipate that policy will undo the constraint when this binds and will behave as if the constraint does not exist (i.e. like in the unconstrained allocation). For a given endowment of nontradable goods, the allocation with policy intervention entails relatively higher price of nontradables during tranquil times (i.e. a relatively less depreciated real exchange rate) and higher consumption of tradable goods. Eventually (i.e. in finite time) our economy will hit the borrowing constraint because agents are relatively impatient. When that happens, under the optimal policy,  $\tau^N$  will be set so that the multiplier on the constraint is zero (i.e. the constraint is just binding).

Note also that the policy function for  $\tau^N$  is time-consistent, and hence promising to eliminate the borrowing constraint by supporting the relative price of nontradable whenever the constraint binds is fully credible in equilibrium.

Figure 4 plots the implied  $\tau^N$  and the associated welfare gains as a function of current bond holdings for negative one-standard deviation shocks. The implied subsidy and the welfare gains are increasing in the amount that agents borrow. Optimal policy in this case subsidizes nontradable consumption, limiting the downward pressure on the relative price of non tradable goods. As a result, agents can borrow and consume much more in both good and bad times (Figure 1). In this case, the probability of a crisis goes to zero, while borrowing and consumption are much higher than in the CE or the SP. As a result, welfare gains from this policy intervention are two orders of magnitude higher compared to the gains from implementing the SP allocations (Table 1).

We note here that, for our calibration (which is the same as in Bianchi, 2011), agents are very impatient and the incentive to borrow dominates the precautionary motive that tends to reduce their borrowing. The relative strength of the “impatience” effect implies that

even when the initial net foreign assets position is positive, agents will borrow up to the borrowing limit so that a tax subsidy on nontradable consumption is needed to relax the credit constraint. As the debt position worsens, the state contingent tax subsidy becomes bigger, tending towards the lower bound of -1.

To quantify what a more realistic policy can achieve in welfare terms, we consider the case of a fix, 10 percent non tradable subsidy. Such a policy yields an average relative price of on nontradables that is approximately 10 percent less depreciated than in the competitive equilibrium with an average welfare gain of 0.4 percent of permanent consumption, similar to that attained with the optimal capital control policy, which however is state contingent tax schedule (Table 1).

## 4.2 Tradable tax

We now consider the last policy tool available. Define the Ramsey problem when  $\tau_t^T$  is the policy instrument as follows.

**Definition 3.** *For a given  $\{B_0\}$  and assuming that  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support, the Ramsey problem for  $\tau_t^T$  is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_t)\},$$

*subject to the resource constraints*

$$C_t^T = Y_t^T - B_{t+1} + (1+r)B_t, \quad (35)$$

$$C_t^N = Y_t^N, \quad (36)$$

*the borrowing constraint*

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N]. \quad (37)$$

*the government budget constraint (26) and the first order conditions of the household*

$$\frac{u'(C_t)C_{C_t^T}}{1+\tau_t^T} = \lambda_t + \beta(1+r)E_t \left[ \frac{u'(C_{t+1})C_{C_{t+1}^T}}{1+\tau_{t+1}^T} \right]. \quad (38)$$

*with*

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = \frac{P_t^N}{1+\tau_t^T}. \quad (39)$$



The tax on tradable consumption affects not only the intratemporal relative price in (39), but also the intertemporal allocation of resources in (38). Despite this difference, the next proposition shows that it is possible to find a policy for  $\tau_t^T$  that replicates the unconstrained allocation like in the case of the nontradable consumption tax  $\tau_t^N$ .

**Proposition 3.** *In an economy defined by (1), (3), (25) and (26) with a tax on tradable consumption  $\tau_t^T$  as the government instrument, there exists a policy for  $\tau_t^T$  that decentralizes the unconstrained allocation and is time-consistent.*

PROOF: Let the optimal non-tradable consumption tax be  $\hat{\tau}_t^N$ . In the Ramsey problem for  $\tau_t^T$ , if we set  $\frac{1}{1+\tau_t^T} = 1 + \tau_t^N$  we can achieve the unconstrained allocation, and  $\lambda_t \equiv 0 \forall t$ . However, since  $\tau_t^T$  affects also the intertemporal allocation of resources (38) we need to show that there is a constant  $\tau_t^T$  such that the intertemporal margin is not affected.

To do so, we first note that, by imposing  $\lambda_t \equiv 0$  and setting  $\tau_t^T$  so that

$$\frac{1}{1 + \tau_t^T} = \frac{\beta(1 + r)E_t \left[ \frac{u'(C_{t+1}^{UN})C_{t+1}^{UN}}{1 + \tau_{t+1}^T} \right]}{E_t[u'(C_{t+1}^{UN})C_{t+1}^{UN}]}, \quad (40)$$

the Euler equations of the Ramsey problem and the unconstrained equilibrium coincide. It follows that the tax rate  $\tau_t^T$  that satisfies (40) must be constant (otherwise the intertemporal margin would be distorted).

By inspection of the unconstrained allocation, the non-tradable price has a strictly positive lower limit. Therefore there exists  $\underline{\tau}^T$  (this is the lower level of the tax on tradables compatible with the strictly positive lower limit on the relative price of nontradables) such that the borrowing constraint (4) is always satisfied for any  $\tau^T \geq \underline{\tau}^T$ . Thus, any constant tax policy of the form  $\tau_t^T \equiv \tau^T \geq \underline{\tau}^T$  is an optimal policy such that the competitive equilibrium replicates the unconstrained equilibrium. As  $\underline{\tau}^T$  is completely determined by the current state  $\{B_t, Y_t^T, Y_t^N\}$  it is time-consistent.

QED.

While the two consumption taxes operate through the same mechanism (i.e. by increasing the relative price of nontradables), one is a state contingent subsidy while the other is a constant tax on tradable consumption. A second important difference is in terms of financing: the subsidy on nontradable consumption requires financing through lump sum taxes, while the revenues from the tax on tradables will be rebated as lump sum transfer to private agents.

## 5 Optimal capital controls and real exchange rate policy: the case of distortionary financing

Our analysis in the previous section showed that real exchange rate policy dominates capital control policy in welfare terms. Intuitively, in a debt-deflation environment, optimal policy aims at supporting prices that influence agents' borrowing decisions. The result hinges on the ability of the Ramsey planner to manipulate the price that enters the borrowing constraint without costs because the subsidy on the relative price of nontradables is financed with lump sum taxes.

We now depart from this key assumption by considering an environment in which lump-sum transfers/taxes are not available, so that it is costly to manipulate the relative price of nontradables. In this more general environment, there are efficiency losses due to the use of distortionary financing. By doing so, we characterize a situation in which managing the real exchange rate is costly during crisis times although we don't model these costs explicitly.

Given the structure of our endowment economy, we consider two possibilities for the government budget constraint. The first possibility is one in which the set of taxes is arbitrarily restricted to  $\tau_t^B$  and  $\tau_t^N$ . The second possibility is one in which we use all the taxes discussed thus far,  $\tau_t^B, \tau_t^N$ , and  $\tau_t^T$ .

### 5.1 Two distortionary instruments

In the first case, the government budget constraint becomes:

$$\tau_t^B B_{t+1} = \tau_t^N P_t^N C_t^N, \quad (41)$$

where the budget is balanced by combining the tax on borrowing with a subsidy on nontradables. The following definition states the corresponding Ramsey problem.

**Definition 4.** *For a given  $\{B_0\}$  and assuming that  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support, the Ramsey problem for  $\tau_t^N$  and  $\tau_t^B$  when (41) holds is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_j)\},$$

subject to (11) and (12) and (32), and the first order conditions of the households

$$u'(C_t)C_{CT}(1 + \tau_t^B) = \lambda_t + \beta(1 + r) E_t[u'(C_{t+1})C_{CT}], \quad (42)$$

$$\frac{(1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N(1 + \tau_t^N). \quad (43)$$

As we cannot characterize the solution of this problem analytically, we must rely on numerical simulations. To do so, we note first that, given our chosen instruments (i.e.  $\tau_t^N$  and  $\tau_t^B$ ), the problem is time consistent.<sup>20</sup> We then use a computational method that exploits the Markov-Perfect nature of the equilibrium, that we have developed in Benigno et al. (2012) and it is described in the appendix. Here we report and discuss the solution.

Figure 5 plots the policy function for  $\tau_t^N$  and  $\tau_t^B$  under the optimal policy and the welfare gains in terms of tradeable consumption as a function of current bond holdings for negative one standard deviation shocks. Figure 6 describes the policy function for  $B_{t+1}$ ,  $C_t^T$  and  $P_t^N$  under the optimal policy (OP, dashed line) and the competitive allocation (CE, solid line). Again for comparison purposes, the economy is calibrated exactly as in Bianchi (2011). Figure 7 reports the ergodic distribution of debt. In order to assess the severity of the crisis, Figure 8 reports the ergodic distribution of total consumption growth in unit of tradable consumption in crisis times (i.e., the change in consumption from  $t - 1$  to  $t$ , given that the economy is in a financial crisis in period  $t$ ). For this purpose, a crisis is identified, like in Bianchi (2011), by a constraint that binds strictly and a debt reduction larger than one standard deviation.

From Figure 5, we can see that there is scope for both ex-ante and ex-post policy intervention. During normal times (again, the constraint binds at a level of debt of about -.95, where the policy rules display a kink), the optimal policy requires capital controls whose revenues are rebated in the form of subsidies to nontradable consumption; during crises times, the optimal policy requires subsidies to non-tradable consumption to limit the depreciation of the real exchange rate, financed by a tax on the amount that agents borrow.

The optimal policy depends crucially on the interaction between ex-ante and ex-post interventions. In the context of our simple economy, this interaction is affected by the way the policy interventions are financed. When financing is not distortionary (i.e. there are lump-sum taxes) policies aimed at supporting the market price of collateral that af-

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<sup>20</sup>To see this, we can reduce the optimal control problem to a time-consistent static problem by considering the restricted problem in which the Ramsey planner maximizes agents' utility subject to (11), (12), (32) and (43). We can then solve for the allocations, the multiplier on the credit constraint and the relative price and then use (41) and (42) to retrieve the path of taxes. In the appendix we provide an alternative proof based on the equivalence between the commitment and the time-consistent problem.

fect borrowing decisions are fully effective and can achieve the unconstrained allocation. In contrast, when financing is distortionary, preventing excessive depreciation of the real exchange rate is costly, and the optimal policy weights the marginal benefit of relaxing the borrowing constraint with the distortion introduced by capital controls. Indeed, when the constraint binds, the tax on debt affects  $C_{t+1}$  through (42). Since ex-post policy is costly, it is not fully effective in addressing the distortion coming from the pecuniary externality and it becomes optimal to intervene during normal times to reduce the probability of meeting the international borrowing constraint. We can also see from a comparison of Figure 2 and Figure 5 that the optimal capital control tax rate, in the region where the constraint is not binding, is smaller than the case in which the planner can also intervene ex post.

There are two other features of the optimal policy that are noteworthy. First, under the optimal policy, agents borrow more than in the competitive equilibrium allocation during normal times even though optimal policy requires a tax on the amount agents borrow (Figure 6 and 7). Intuitively, on the one hand, agents want to borrow less because their borrowing is taxed; on the other hand, they are willing to borrow more since crises events are mitigated (only in part in this case) by policy intervention (see Figure 8 and Table 1, respectively). Indeed the real exchange rate depreciates less during crises times compared to the competitive equilibrium allocation and allows agents to consume more (Figure 6). As in the previous cases, the welfare gains from optimal policy are increasing the more indebted is the economy. Quantitatively, the welfare gains of the optimal policy are more than twice as large as those in which only capital controls are used (Figure 5 and Table 1).

The second important point to note is that in the economy with optimal policy there is more borrowing and consumption than in the competitive equilibrium despite fewer and less severe crises (see Table 1 and Figure 8). The Ramsey planner achieves this by choosing a different relative allocation of consumption between tradable and non tradable goods, with relatively more consumption of tradable compared to the competitive equilibrium allocation.

In sum, in this setting a new rationale for capital controls arises in which the effectiveness of ex-post policies determines the need for ex-ante policy intervention. As our analysis has shown, this rationale is not related to the amount agents borrow in the unregulated economy.

## 5.2 Three distortionary instruments

Consider now a second possibility in which all available distortionary taxes can be combined to balance the budget:

$$\tau_t^B B_{t+1} = \tau_t^N P_t^N C_t^N + \tau_t^T C_t^T. \quad (44)$$

In this situation, it is possible to show that there is a combination of policy tools that can achieve the unconstrained allocation even if there are no lump sum transfers/taxes. In the appendix, we prove that we can always combine the triplet of policy tools  $(\tau_t^N, \tau_t^T, \tau_t^B)$  to undo the international borrowing constraint.

The policy implication of this last exercise echoes what we emphasized earlier: the set of instruments and their effectiveness during crises times is crucial for determining the optimal policy design. The third instrument deals with the distortions introduced by the second in crisis times. Intuitively, it is possible to use the tax on tradable goods to undo the efficiency losses caused by the use of tax on borrowing when policy aims at supporting the real exchange rate.

## 6 Discussion

Our suggested policies are not inconsistent with the experience of many emerging market economies over the past 20 years or so.

A first implication of our analysis is on the role of price support policies when they can be implemented in a cost effective way. In the context of our model, these policies take the form of government interventions financed in a lump sum manner aimed at limiting the real exchange rate depreciation that occurs during a sudden stop in capital inflows. If such a policy is feasible, our analysis shows that it not only contains the crisis when one occurs, but also eliminates the scope for prudential capital controls. This is because it effectively removes the borrowing constraint, which is the only source of the inefficiency in our model economy.

Our model's prescriptions for price support policies is consistent with many countries' experience during the emerging market crises of the 1990s and the 2000s. In those episodes of financial crises, a key policy concern was the defense of the exchange rate from excessive depreciation. For instance, the defense of the exchange rate was a crucial component of the adjustment programs supported by the IMF in Indonesia, South Korea, and Brazil during the period 1997-1999 even after the initial exit from the respective currency pegs (IMF Independent Evaluation Office, 2003). In the specific case of Brazil, faced with the prospect of a new financial crisis ahead of the 2002 presidential election, "*the authorities ... responded ... proactively and ... maintained a firm monetary policy to limit the inflationary impact of the weakening Real*" (IMF, 2002). To support these policies, the IMF approved a US\$30.4 Billion Stand-By arrangement in September 2002, which was the largest loan ever to that date by the IMF to any member country.<sup>21</sup>

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<sup>21</sup>The 2002 loan turned out to be so successful that eventually was not drawn fully and was repaid well

More recently Brazil faced another sudden halt in private capital inflows following the Lehmann collapse in September 2008. The Real depreciated by more than 20 percent in a month and the Banco Central do Brasil intervened heavily in both the spot and repo market for the US dollar. Mesquita and Toros (2010) emphasize the vulnerability of the non-financial corporate sector to the depreciation of the Real because of their exposure to US dollar swaps (proxied in our model with borrowing in units of tradable consumption).

A similar experience was shared by Mexico when large corporate entities were also exposed to foreign currency derivatives at the time of Lehmann collapse in September 2008. In their account of the Mexican experience, Chang, Cespedes and Velasco (2012) emphasize how the response of the policy authorities consisted in intervention in the foreign exchange market to limit the depreciation of the Mexican Peso.

More broadly, in the context of the recent US and European financial crises, the prescription of our model can be interpreted as interventions that avoid the collapse of asset prices when a crisis occurs. In this sense, our results not only rationalizes the need to set a floor under the exchange rate as in the emerging market crises of the 1990s and the 2000s, but also non-conventional policies of purchases of risky assets to contain the "fire sales" and the asset deflation spirals that characterized the recent United States and European crises.<sup>22</sup>

A second main policy implication of our analysis is that, if financial crises cannot be contained without significant costs, a policy of prevention becomes desirable. Indeed, in our model, when these costs are taken into account by assuming that the financing of price support policies is costly, a new rationale for capital controls emerges in which capital controls and real exchange rate policy are used in a complementary way in both normal and crises times. When both ex ante and ex post interventions are used jointly, the optimal capital control becomes much smaller than when it used in solation from other tools, and yet the welfare gains achieved by their joint use with other tools are more than twice as large.

Indeed, emerging countries have adopted a variety of policy tools to prevent the occurrence of financial crises. One policy that countries have adopted over the past 20 years is the accumulation of a very large pools of official reserves to deploy in support of the exchange rate in the case of sudden halt in capital inflows.<sup>23</sup> While the spectacular accumulation of

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ahead of schedule by the Brazilian authorities.

<sup>22</sup>See Bianchi (2011) for a case in which there is an isomorphism between the small open economy environment we analyzed in this paper and a simple banking environment in which the tax on debt can be interpreted as a domestic macro-prudential policy tool.

<sup>23</sup>Emerging markets official reserves (excluding gold) increased from about one trillion US dollar in 2000 to over 6 trillions in 2012 according to IMF IFS data (or about a third of world GDP valued at current US dollars). While this spectacular accumulation of reserve assets cannot be explained entirely by prudential

official reserve after the emerging market crises of the 1990s and the 2000s is well known, the simultaneous use of prudential capital controls is lesser known and more controversial. Aizenman and Pasricha (2013), for instance, report some evidence of countercyclical use of prudential interventions (including capital controls) during the 2000s, because of concerns about net capital inflows. Similarly, Federico, Vegh, and Vuletin (2012), analyze the use and cyclical properties of reserve requirements as a macro-economic stabilization tool. They find that 74 percent of developing countries use reserve requirements counter-cyclically compared to just 38 percent that have engaged in countercyclical monetary policy. They interpret the latter finding as reflecting the reluctance of many emerging market economies to reduce interest rates in bad times for fear of letting their currency depreciate too rapidly. Fernandez, Rebucci, Uribe (2013), however, examine the behavior of capital controls in a large number of countries over the period 1995-2011. They find that boom-bust episodes in output, the current account, or the real exchange rate are associated with virtually no movements in capital controls. They also document a near complete acyclicity of capital controls in the run up to the recent global crisis of 2007-2009.<sup>24</sup>

We conclude from this brief review of the experience of emerging market countries over the past 20 years of so there has been widespread use of price support policies to contain financial crises in emerging markets, some use of macro-prudential policies, and limited use of prudential capital controls on transactions between non residents. Echoing Fernandez, Rebucci, Uribe (2013), the use capital controls for prudential purposes is at best a case of theory ahead of policy in the sense that it might emerge in the future but thus far there has been little or no use in practice.

## 7 Conclusion

In response to the recent global financial crisis, a new policy paradigm has emerged in which old fashioned forms of government interventions such as capital controls and other quantitative restrictions on credit flows—the so called macro-prudential policies—have become part of the standard policy toolkit. Arguably macro-prudential policies are desirable because they can help prevent financial crises that otherwise would be too costly to endure or contain with only ex post interventions.

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or precautionary motives, most empirical studies concur that precautionary saving was the most important determinant of this process.

<sup>24</sup>The key difference between these studies is that Aizenman and Pasricha (2013) includes also restrictions on transactions between residents in their measure of prudential policies such as currency based measures, while Federico, Vegh, and Vuletin (2013) focuses only on a domestic macro-prudential measures. In contrast, Fernandez, Rebucci, Uribe (2013), look only at restrictions on capital controls strictly defined.

In this paper we analyze formally the interaction between ex post, crisis management policies and ex ante, crisis prevention policies. We first show that when the Ramsey planner can choose among different policy tools, ex post price support policies dominate prudential policy measures in welfare terms by two orders of magnitude. This dominance is conditional on the extent to which price support policies do not entail efficiency losses. Indeed, when price support policies can be used effectively, there is no need for macro prudential policies. In contrast, when crisis management policies are not fully effective because they are costly to implement, ex-ante policies (capital controls) can be rationalized as a complement to price support policies to limit the occurrence of crises. The joint use of capital controls and price support policy achieves a welfare gain of 1 percent of permanent consumption; a gain that is twice as large as the welfare gain of using only capital controls.

Our analysis is conducted in the context of a simple model, but in reality the trade-offs that policymakers face are richer than the ones implied by our framework. For instance, there are benefits from a more depreciated exchange rate in terms of the classical expenditure switching effect that are not incorporated into our analysis. To an extent, we can interpret our model as one in which balance-sheet consideration dominates other policy motives, but we acknowledge that a richer model would be needed to address these issues. We regard this as an area of fruitful future research.



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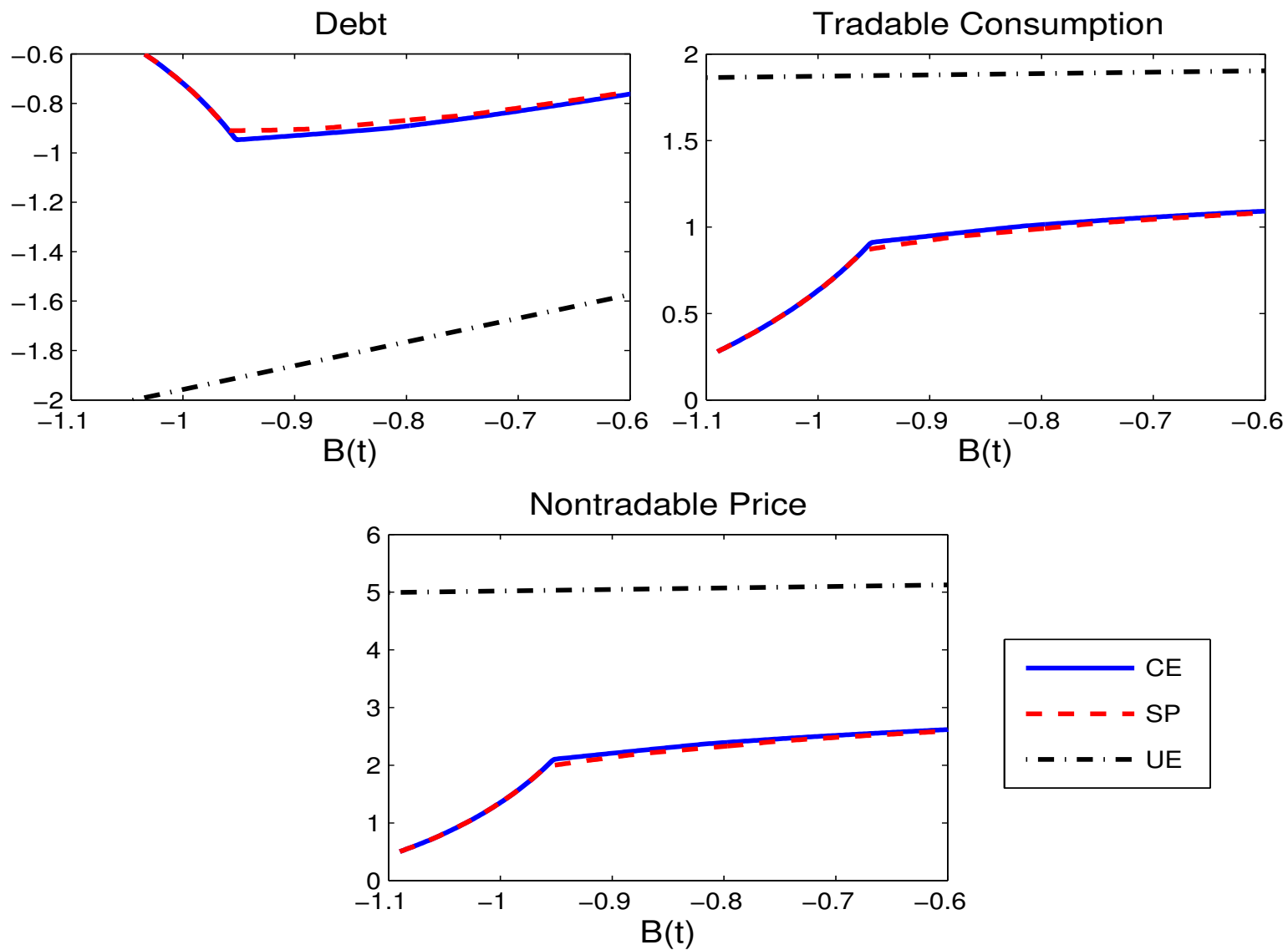
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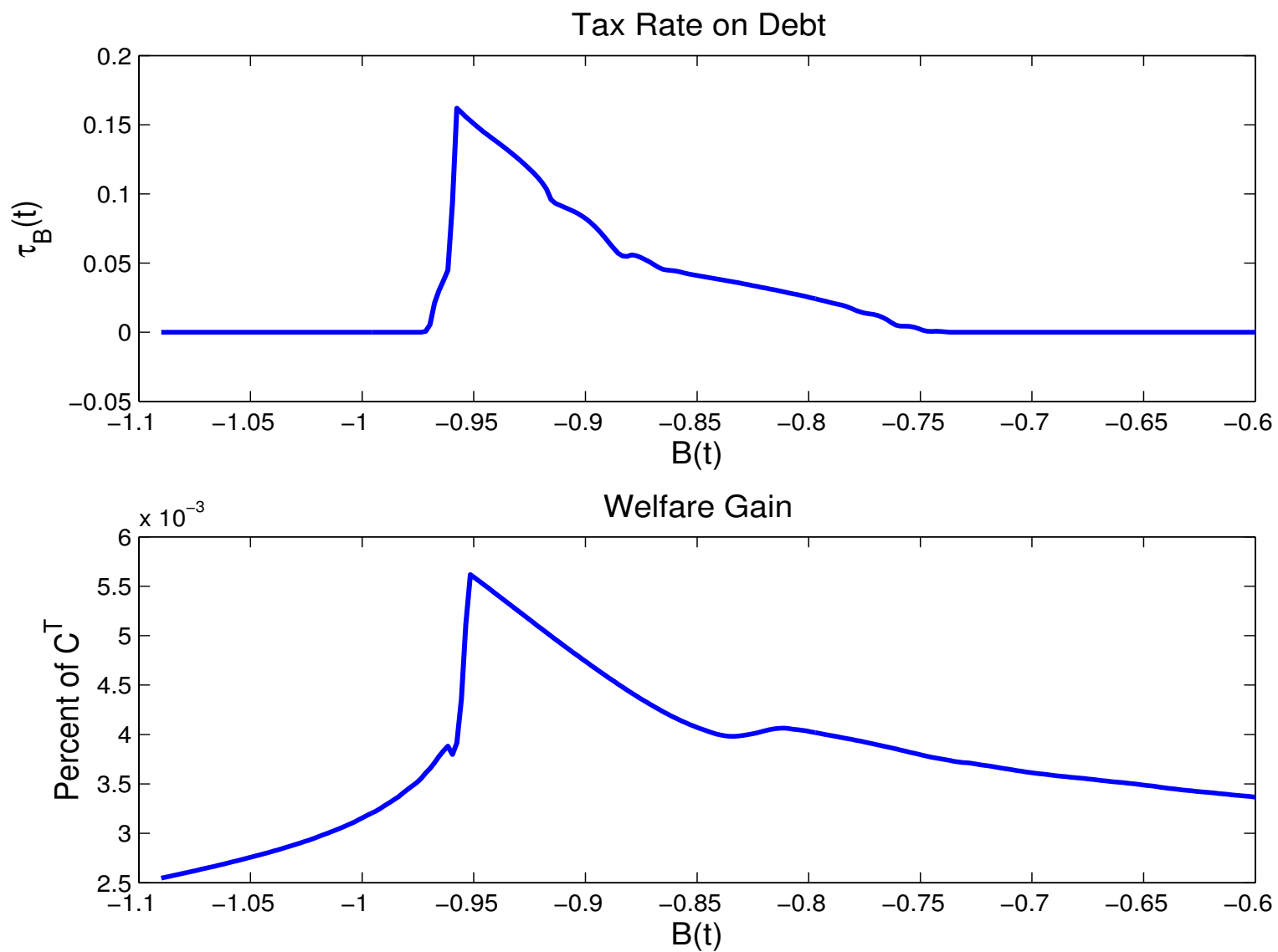
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Figure 1: Alternative Allocations: Decision Rules



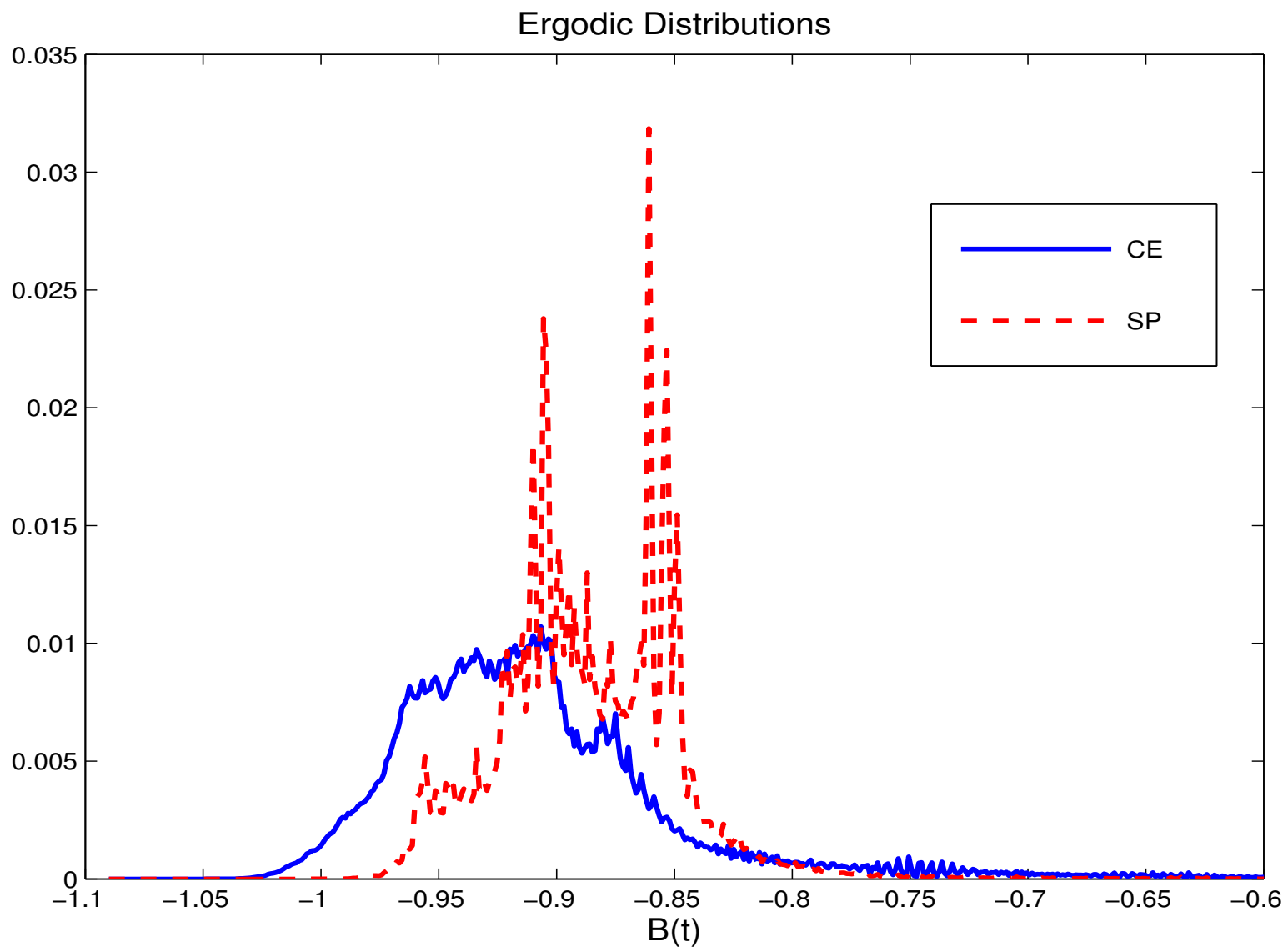
Notes: CE denotes the competitive equilibrium allocation; SP the social planner allocation; UE the unconstrained equilibrium. The figure plots the equilibrium decision rules or policy functions of the endogenous variables plotted conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 2: Optimal Capital Control Policy



Notes: The figure plots the optimal debt tax rate and the associated welfare gain relative to the competitive equilibrium conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

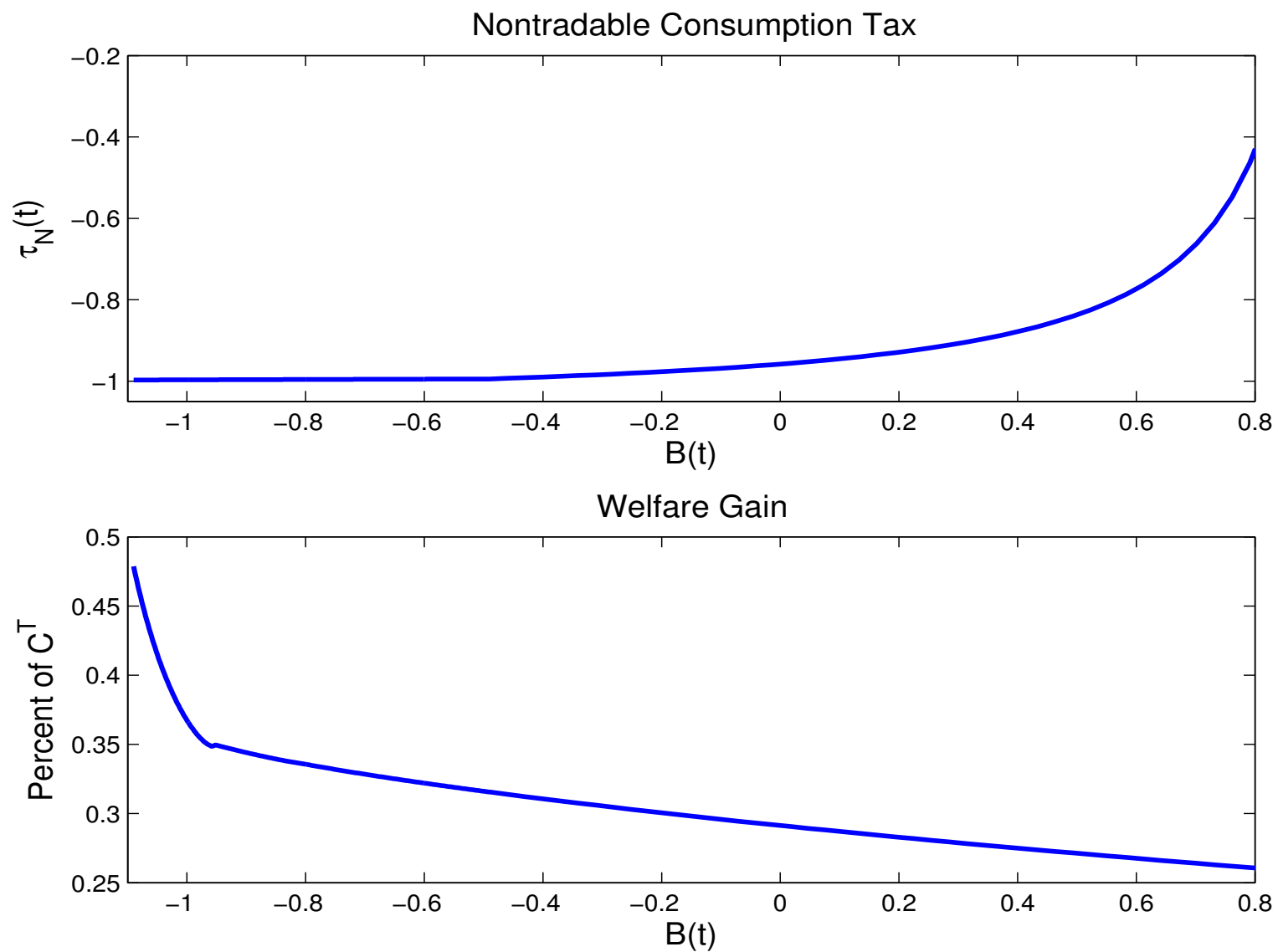
Figure 3: Optimal Capital Control Policy



Notes: The figure plots the ergodic distribution of debt in units of tradable consumption in the competitive equilibrium (CE) and the social planner allocations (SP). Borrowing decreases from left to right on the x-axis.

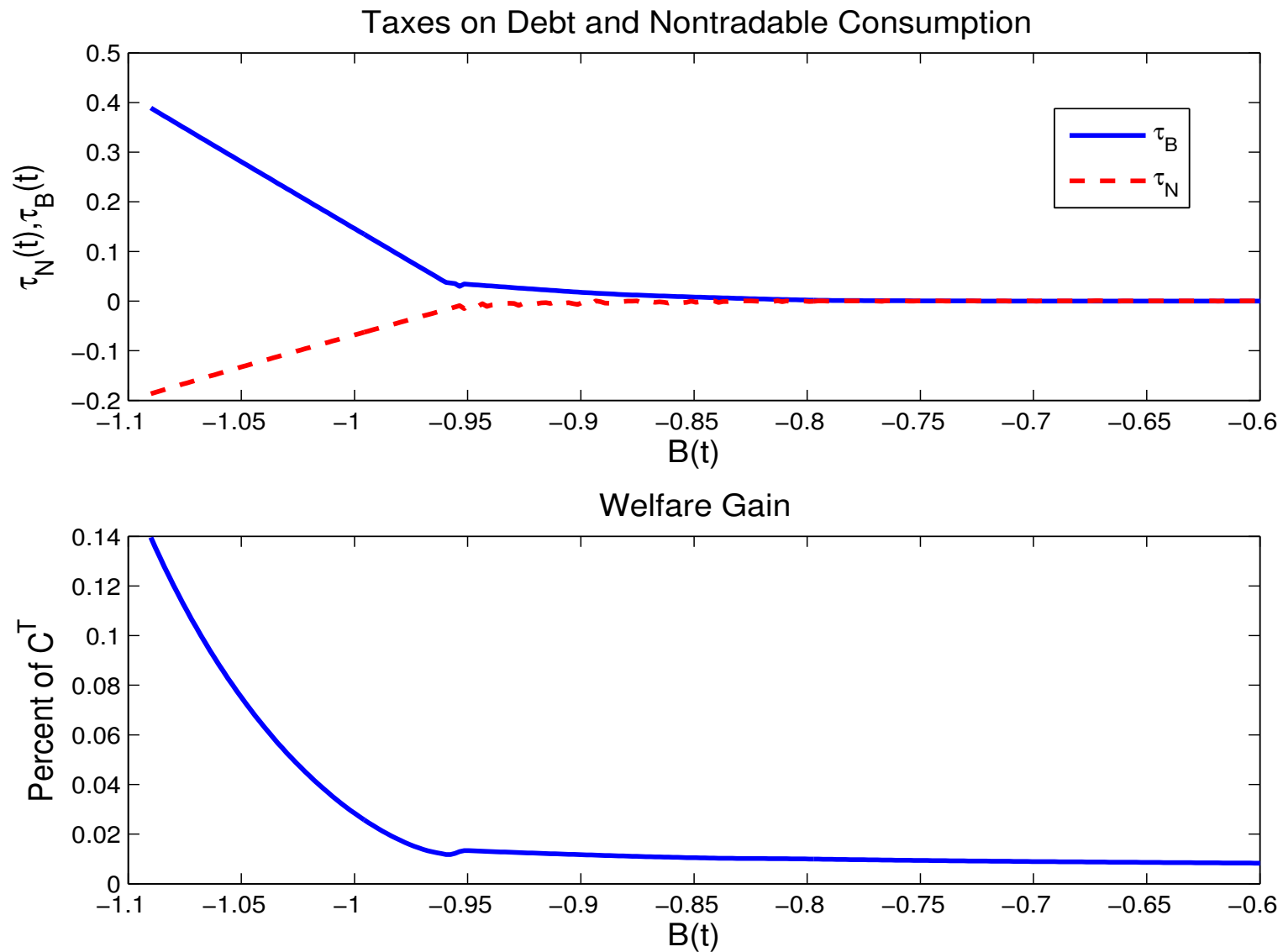


Figure 4: Optimal Exchange Rate Policy



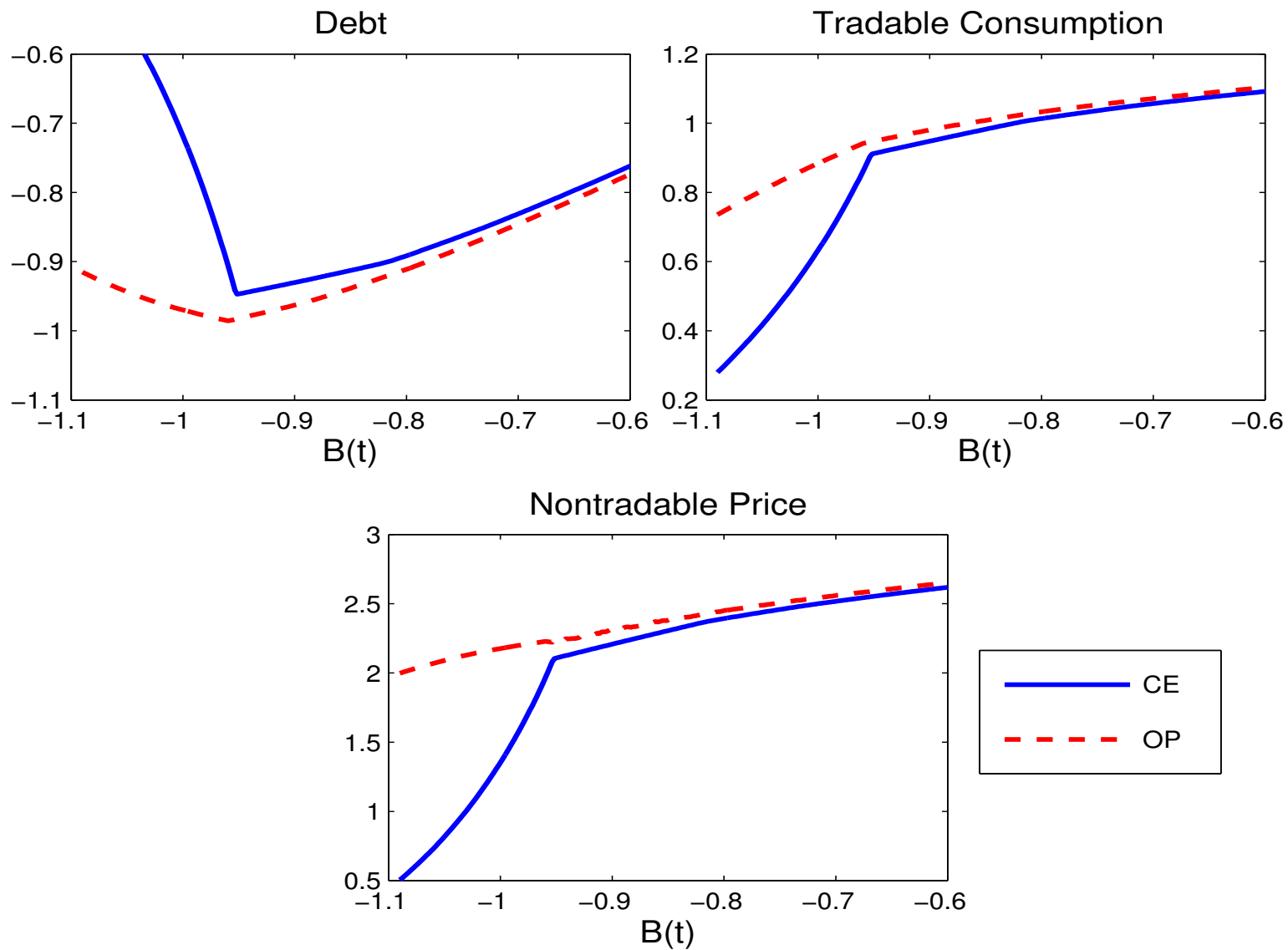
Notes: The figure plots the optimal nontradable consumption tax rate and the associated welfare gain relative to the competitive equilibrium conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 5: Optimal Capital Control and Exchange Rate Policy



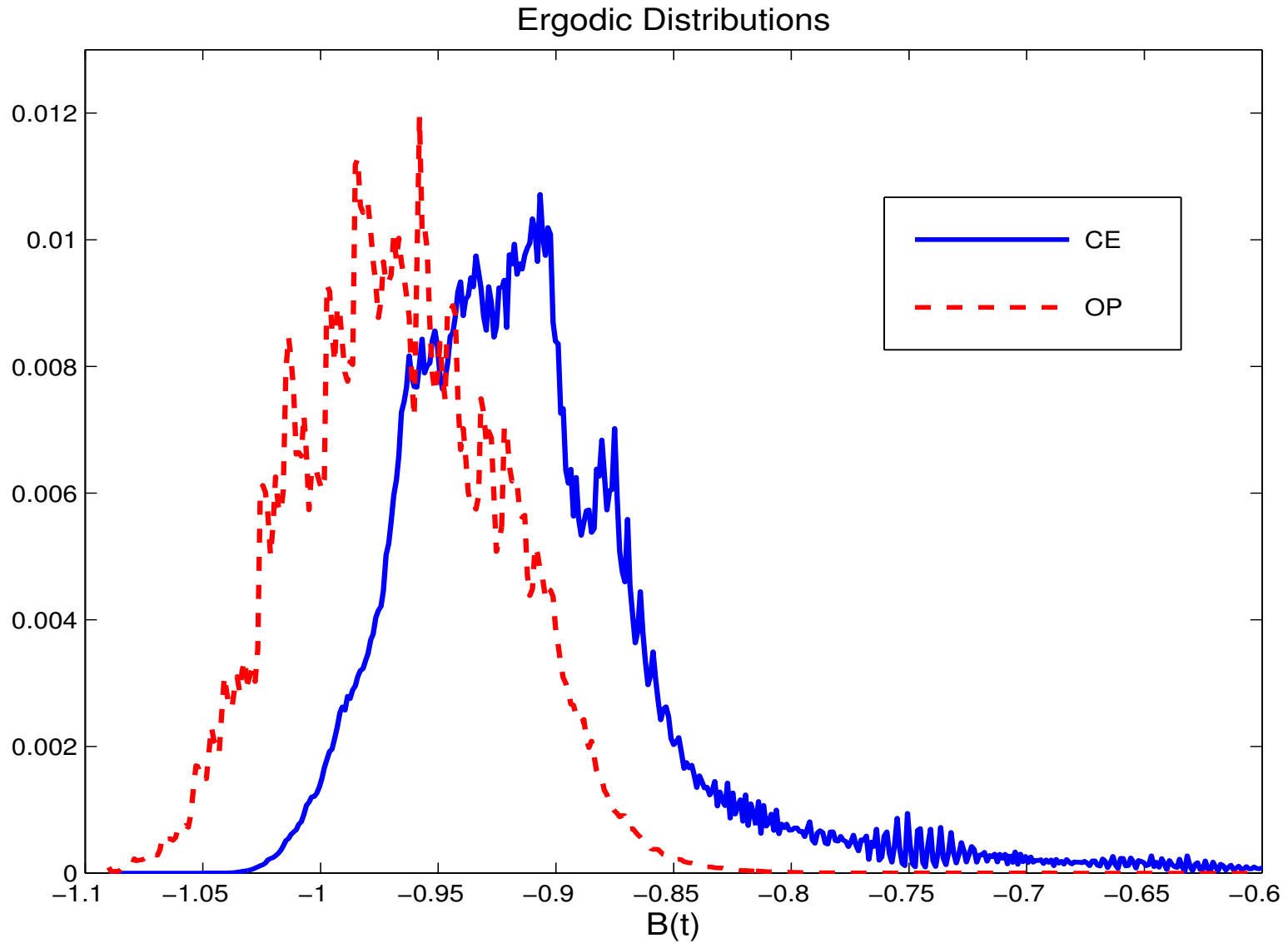
Notes: The figure plots the optimal debt and nontradable consumption tax rates and the associated welfare gain relative to the competitive equilibrium, conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 6: Optimal Capital Control and Exchange Rate Policy: Decision Rules



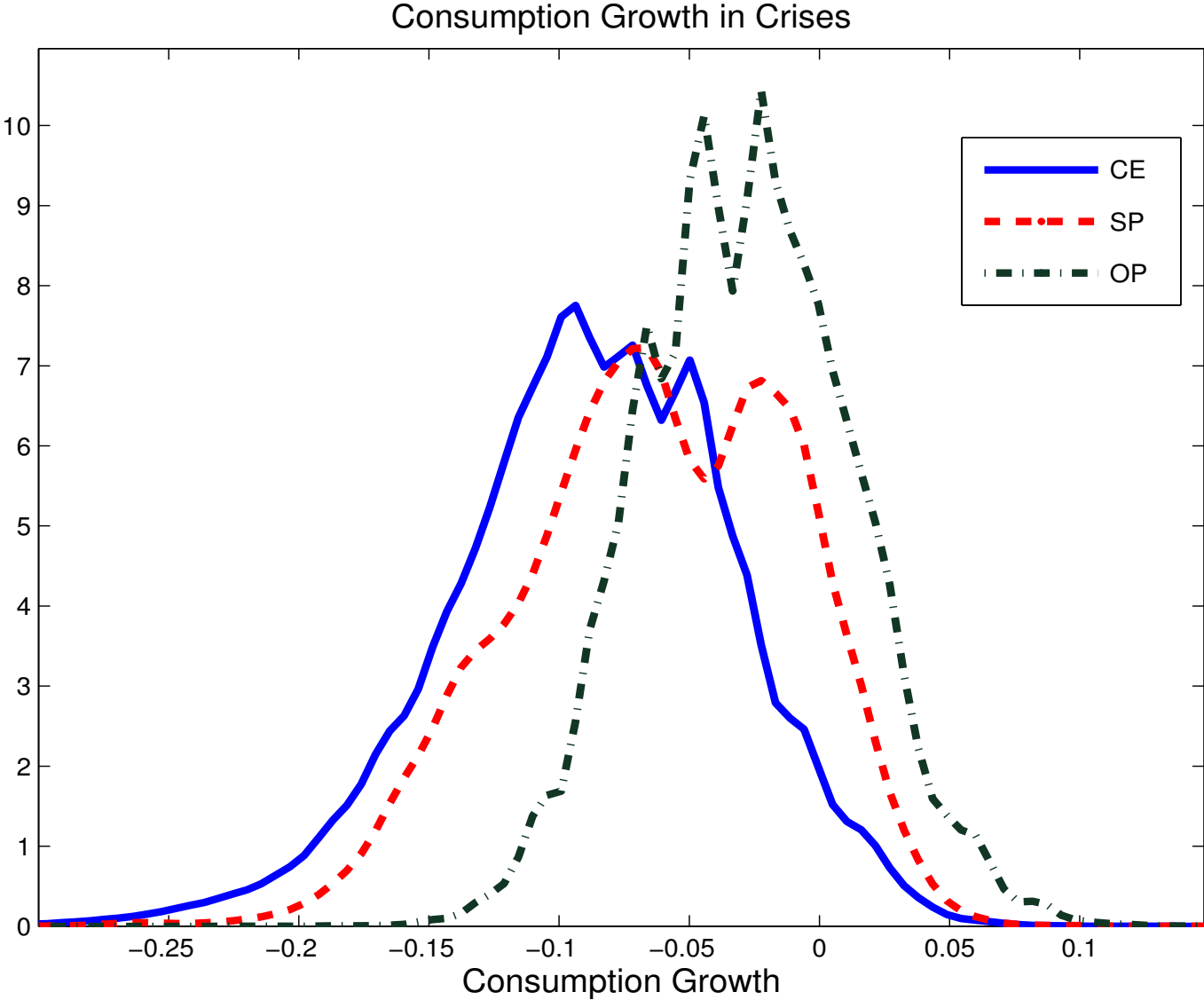
Notes: CE denotes the competitive equilibrium allocation; OP the optimal policy equilibrium with both debt tax and nontradable consumption tax. The figure plots the equilibrium decision rules of the endogenous variables plotted conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 7: Optimal Capital Control and Exchange Rate Policy



Notes: The figure plots the ergodic distribution of debt in units of tradable consumption in the competitive equilibrium (CE) and the optimal policy equilibrium with both debt tax and nontradable consumption tax (OP). Borrowing decreases from left to right on the x-axis.

Figure 8: Comparing Policy Regimes



Notes: CE denotes the competitive equilibrium allocation; SP the social planner allocation; OP the optimal policy equilibrium with both debt tax and nontradable consumption tax. The figure plots the ergodic distribution of consumption growth in the period after the constraint was binding. Borrowing decreases from left to right on the x-axis.

# A Appendix (for online publication)

## A.1 Parameter values and solution methods

The parameter values of the model are set as in Bianchi (2011):

### Structural parameters

|                                                                    |                     |
|--------------------------------------------------------------------|---------------------|
| Elasticity of substitution between tradable and non-tradable goods | $\kappa = .83$      |
| Intertemporal substitution and risk aversion                       | $\rho = 2$          |
| Credit constraint parameter                                        | $\phi = 0.75758$ 1/ |
| Relative weight of tradable and non-tradable goods                 | $\omega = 0.31$     |
| Discount factor                                                    | $\beta = 0.91$      |

### Values

### Exogenous variables

|                          |                 |
|--------------------------|-----------------|
| World real interest rate | $r = 0.04$      |
| Steady state endowments  | $Y^N = Y^T = 1$ |

### Values

### Endowment process

|                            |                                                                        |
|----------------------------|------------------------------------------------------------------------|
| Autocorrelation Matrix     | $\begin{bmatrix} 0.901 & 0.495 \\ -0.453 & 0.225 \end{bmatrix}$        |
| Variance-Covariance Matrix | $\begin{bmatrix} 0.00219 & 0.00162 \\ 0.00162 & 0.00167 \end{bmatrix}$ |

### Average values in the ergodic distribution

|                    |             |
|--------------------|-------------|
| Net foreign assets | $B = -0.91$ |
|--------------------|-------------|

### Values

1/ This value of  $\phi$  implies a value for  $\kappa = .32$  in Bianchi's (2011) notation.

We now describe how we compute the different equilibria. We start by rewriting the competitive equilibrium of the model with the borrowing constraint. The competitive equilibrium of the economy with the borrowing constraint can be summarized by the following set of nonlinear functional equations:

$$\begin{aligned}
\mu(B, Y^T, Y^N) &= \beta(1+r) E[\mu(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'})] + \max\{\lambda(B, Y^T, Y^N), 0\}^2 \\
\mu(B, Y^T, Y^N) &= \left(C^{\frac{\kappa-1}{\kappa}}\right)^{(1-\rho)\frac{\kappa}{1-\kappa}-1} \omega^{\frac{1}{\kappa}} C^T(B, Y^T, Y^N)^{-\frac{1}{\kappa}} \\
C^T(B, Y^T, Y^N) &= (1+r)B + Y^T - B'(B, Y^T, Y^N) \\
P^N(B, Y^T, Y^N) &= \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(\frac{Y^N}{C^T(B, Y^T, Y^N)}\right)^{-\frac{1}{\kappa}} \\
\max\{-\lambda(B, Y^T, Y^N), 0\}^2 &= B'(B, Y^T, Y^N) + \frac{1-\phi}{\phi} Y^T + P^N(B, Y^T, Y^N) Y^N,
\end{aligned}$$

where

$$C^{\frac{\kappa-1}{\kappa}} \equiv \left[\omega^{\frac{1}{\kappa}} (C^T(B, Y^T, Y^N))^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (Y^N)^{\frac{\kappa-1}{\kappa}}\right].$$

In these equations, following Garcia and Zangwill (1981), we converted the complementary slackness conditions for the borrowing constraint into a nonlinear equation.

### A.1.1 The constrained and unconstrained competitive equilibrium

Given an initial guess for the marginal utility of tradable consumption tomorrow  $\mu^0(B', Y^{T'}, Y^{N'})$ , the set of nonlinear functional equations above can be solved at each point in the state space  $(B, Y^T, Y^N)$  to obtain an updated function  $\mu^1(B, Y^T, Y^N)$ . This process is then iterated to convergence. We use a cubic spline to approximate the  $\mu^0(B', Y^{T'}, Y^{N'})$  function at values of  $B'$  that are not on the grid for  $B$ . We obtain the lifetime utility in the competitive equilibrium using the following Bellman equation:

$$V^{CE}(B, Y^T, Y^N) = \frac{1}{1-\rho} C^{1-\rho} + \beta E[V^{CE}(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'})].$$

The allocation corresponding to the unconstrained competitive equilibrium is computed in a similar fashion, except that the complementary slackness condition is omitted.

### A.1.2 The social planning problem

The solution of the social planning problem solves the following standard dynamic programming problem:

$$V^{SP}(B, Y^T, Y^N) = \max_{C^T, B', P^N} \left\{ \frac{1}{1-\rho} C^{1-\rho} + \beta E [V^{SP}(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'})] \right\}$$

subject to

$$\begin{aligned} C^T + B' &\leq (1+r)B + Y^T \\ B' &\geq -\frac{1-\phi}{\phi} (Y^T + P^N Y^N) \\ P^N &= \left( \frac{1-\omega}{\omega} \right)^{\frac{1}{\kappa}} \left( \frac{Y^N}{C^T} \right)^{-\frac{1}{\kappa}}. \end{aligned}$$

Again, we approximate the value function with a cubic spline and solve the constrained optimization problem using feasible sequential quadratic programming with analytical derivatives.

### A.1.3 Markov-Perfect optimal policy

To compute the Ramsey optimal control program with two instruments we exploit time-consistent nature of the problem and use the method proposed by Benigno et al. (2012). That method is related to Klein, Krusell, and Rios-Rull (2009): the main difference being that the algorithm that we use does not require that the policy functions are differentiable (which in general would not hold in our environment due to the occasionally-binding constraint) but only that they are continuous.<sup>25</sup>

The optimal policy problem for  $\tau_N$  and  $\tau_B$  is also solved iteratively. The current government solves the following problem

$$V^{OP}(B, Y^T, Y^N) = \max_{\tau_N, \tau_B, C^T, P^N, B', \mu, \lambda} \left\{ \frac{1}{1-\rho} C^{1-\rho} + \beta E [V^{OP}(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'})] \right\}$$

---

<sup>25</sup>Other methods for solving optimally policy problems with occasionally binding constraints relies on approximation techniques and as such they restrict the validity of the normative analysis to the neighborhood of the approximation point (see for instance Kim, Kim, and Kollman (2008) or Guerrieri and Iacoviello (2013)).



subject to

$$\begin{aligned}
(1 + \tau_B) \mu &= \beta (1 + r) E [\mu (B', Y^T, Y^N), Y^{T'}, Y^{N'}] + \max \{\lambda, 0\}^2 \\
\mu &= \left( C^{\frac{\kappa-1}{\kappa}} \right)^{(1-\rho)^{\frac{\kappa-1}{\kappa}-1}} \omega^{\frac{1}{\kappa}} (C^T)^{-\frac{1}{\kappa}} \\
C^T &= (1 + r) B + Y^T - B' \\
P^N &= \left( \frac{1 - \omega}{\omega} \right)^{\frac{1}{\kappa}} \left( \frac{Y^N}{C^T} \right)^{-\frac{1}{\kappa}} \frac{1}{1 + \tau_N} \\
\max \{-\lambda, 0\}^2 &= B' + \frac{1 - \phi}{\phi} (Y^T + P^N Y^N) \\
0 &= \tau_N P^N Y^N + \tau_B B' (B, Y^T, Y^N).
\end{aligned}$$

We then guess both the continuation value function and the future marginal utility function, solve the optimization problem using feasible sequential quadratic programming with analytical derivatives, and then update both functions to convergence.<sup>26</sup> Both functions are approximated with cubic splines. Note here that smooth equilibria of the sort considered by Klein, Krusell, and Ríos-Rull (2009) do not exist in this class of models because the policy functions are not differentiable at the point where the constraint binds exactly (that is, where  $\lambda^*(B, A^T) = 0$  in our model).

We set a large number of grid points in the  $B$  dimension (1550), with most of them concentrated at the lower end of the debt range where the constraint may bind. The joint process for  $(Y^T, Y^N)$  is approximated as a Markov chain with 49 states (7 in each dimension) using the method of Gospodinov and Lkhagvasuren (2013), which is an extension of the method from Rouwenhorst (1995) to vector autoregressive processes with correlated innovations. Invariant distributions were produced using the nonstochastic method from Young (2010), except for the frequency of crises which are estimated using a simulated sample of 10,000,000 observations.

#### A.1.4 Welfare calculations

To compute the welfare equivalents, we solve the following functional equation:

$$\tilde{V}^{CE}(B, Y^T, Y^N; \chi) = \frac{1}{1 - \rho} C^{1-\rho} + \beta E \left[ \tilde{V}^{CE}(B', Y^T, Y^N), Y^{T'}, Y^{N'}; \chi \right];$$

where  $\chi$  is a proportional increment to tradable consumption, and the decision rules are those from the competitive equilibrium. We use 200 grid points for  $\chi$ , evenly-spaced. We

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<sup>26</sup>We use analytical derivatives, particularly for the continuation value function, as numerical derivatives produce solutions that are "choppy" for the tax variables (but not the other endogenous variables).

then solve the following nonlinear equations for  $\chi(B, Y^T, Y^N)$ :

$$V^{SP}(B, Y^T, Y^N) = \tilde{V}^{CE}(B, Y^T, Y^N; \chi),$$

to obtain the welfare gain from moving to the SP allocation;

$$V^{OP}(B, Y^T, Y^N) = \tilde{V}^{CE}(B, Y^T, Y^N; \chi),$$

to obtain the welfare gain from moving to the OP allocation; and

$$V^{UA}(B, Y^T, Y^N) = \tilde{V}^{CE}(B, Y^T, Y^N; \chi)$$

to obtain the welfare gain from moving to the unconstrained allocation. These equations are solved using the Brent's method, with linear interpolation between grid points for  $\chi$ .

The Fortran code to replicate all the computations of the paper is available upon request from the authors.

## A.2 Distortionary taxation: three instruments case

Let's focus on the case in which all the distortionary policy tools are available to the policy maker (see (44)).

Suppose that the triplet of policy tools  $(\tau_t^N, \tau_t^T, \tau_t^B)$  can completely remove the borrowing constraint (4). The Euler equation for this economy would be:

$$\frac{1 + \tau_t^B}{1 + \tau_t^T} \mu_t = \beta(1 + r) E_t \frac{\mu_{t+1}}{1 + \tau_{t+1}^T}. \quad (45)$$

Remember now that the Euler equation for the unconstrained economy is:

$$\mu_t^{UE} = \beta(1 + r) E_t [\mu_{t+1}^{UE}]. \quad (46)$$

By comparing (45) and (46), we can see that in order to replicate the unconstrained equilibrium the triplet of policy tools  $(\tau_t^N, \tau_t^T, \tau_t^B)$  must satisfy:

$$\frac{1 + \tau_t^B}{1 + \tau_t^T} = \frac{E_t \frac{\mu_{t+1}}{1 + \tau_{t+1}^T}}{E_t [\mu_{t+1}^{UE}]}. \quad (47)$$

In addition, from the government budget constraint, we need to have

$$\tau_t^T (C_t^T)^{UE} + \tau_t^N (C_t^N)^{UE} + \tau_t^B B_{t+1}^{UE} = 0. \quad (48)$$

And from the borrowing constraint, we must have that

$$B_{t+1}^{UE} \geq -\frac{1-\phi}{\phi} \left[ Y_t^T + (P_t^N)^{UE} Y_t^N \frac{1+\tau_t^T}{1+\tau_t^N} \right]. \quad (49)$$

To find the tax policy  $\{\tau_t^B, \tau_t^T, \tau_t^N\}$  that solves (47) to (49) we proceed recursively as follows. Denote the stochastic steady state level of debt by  $\underline{B}$ , and by  $B_0$  the level of debt in the unconstrained equilibrium at which the constraint would become binding exactly in the constrained economy. Now define  $B_t = B^{UE}(B_{t-1})$ , where  $B^{UE}(\cdot)$  is the policy function in the unconstrained equilibrium. From this policy function, we can obtain  $B_0 > B_1 > \dots > B_t > B_{t+1} > \dots > \underline{B}$ , so that  $\{B_k\}$  is a debt trajectory in the unconstrained solution starting from  $B_0$ .

Starting from  $k = 0$ , for any  $B \in (B_1, B_0]$ , we can compute

$$\frac{1 + \tau^T(B)}{1 + \tau^N(B)} = -\frac{B^{UE}(B) + \frac{1-\phi}{\phi} Y_t^T}{\frac{1-\phi}{\phi} (P_t^N)^{UE}(B) Y_t^N}$$

from (49). Let's set  $\tau_0^T(B) \equiv 0$  in that interval and use the expression above to obtain  $\tau_0^N(B)$ . The value of  $\tau_0^B(B)$  in the  $(B_1, B_0]$  interval can then be determined by the government budget constraint (48).

Next, consider  $k = 1$  and the associated interval  $(B_2, B_1]$ . Since we have already determined the value of  $\tau_0^T(B)$  and  $\tau_0^B(B)$  for  $B \in (B_1, B_0]$ , by using (47), we can obtain the value of  $\tau_1^T(B)$ . Again by assuming the borrowing constraint (47) is binding exactly, we can determine the value of  $\tau_1^N(B)$ . Last, by using the government budget constraint (48) we can determine the value of  $\tau_1^B(B)$  and update to  $k = 2$ .

By iterating recursively, we can always find the tax policy that replicates the unconstrained solution in an economy with the borrowing constraint.

QED

### A.3 Time consistency of optimal policy

In this appendix we prove formally that, the Ramsey problem defined in (4) is time-consistent.

The Ramsey optimal policy solves the following problem:

$$(\{B_{t+1}^R\}, \{(\tau_t^N)^R\}, \{(\tau_t^B)^R\}) \doteq \arg \max_{(\{B_{t+1}\}, \{\tau_t^N\}, \{\tau_t^B\})} \sum_{t=0}^{\infty} \beta^t U(C_t^T, C_t^N), \quad (50)$$

subject to conditions (11), (12), (32), (42) and (43) for all  $t = 0, 1, \dots$ .

The time consistent optimal policy solves the following recursive problem

$$(B_{t+1}^C, (\tau_t^N)^C, (\tau_t^B)^C) \doteq \arg \max_{(B_{t+1}, \tau_t^N, \tau_t^B)} U(C_t^T, C_t^N) + \beta V^C(B_{t+1}) \quad (51)$$

subject to conditions (11), (12), (32), (42) and (43) at time  $t$ . Here  $V^C(\cdot)$  is the household value function under the time consistent optimal policy, i.e.

$$V^C = \sum_{t=0}^{\infty} \beta^t U(C_t^{TC}, C_t^{NC})$$

where  $\{C_t^{TC}\}$  and  $\{C_t^{NC}\}$  are sequences of tradeable and nontradable consumptions based on the time consistent optimal policy. It is important to note that the state of economy at time  $t$  is  $B_t$ , the current level of debt. Hence, the value function depends solely on  $B_t$ . We want to establish that, for the economy under consideration, the Ramsey optimal policy is time consistent, i.e.  $B_{t+1}^R = B_{t+1}^C, \tau_t^{NR} = \tau_t^{NC}, \tau_t^{BR} = \tau_t^{BC}$ .

To prove this, we shall take the following steps. First, we show that this is the case in a three-period version of these two problems. Second, we look at a four-period case and show that this can be reduced to the 3-period case. Next we show that we can always reduce an  $n$ -period case to a  $n - 1$ -period one for any  $n > 4$ . This establishes, by induction, that in any finite-period version of our model economy the two policy regimes coincide. Finally, under the auxiliary assumption that the period utility function and the marginal utility of consumption are bounded in the feasible set, we prove that Ramsey optimal policy in the finite-period model converges to Ramsey optimal policy in a infinite-horizon version of our economy.

### A.3.1 Three-period model

We start by examining the 3-period version of the original Ramsey optimal policy problem:

$$\begin{aligned} & (\{B_1^{R'}, B_2^{R'}\}, \{\tau_0^{NR'}, \tau_1^{NR'}\}, \{\tau_0^{BR'}, \tau_1^{BR'}\}) \doteq \\ & \arg \max_{(\{B_1, B_2\}, \{\tau_0^N, \tau_1^N\}, \{\tau_0^B, \tau_1^B\})} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 V^C(B_2), \end{aligned} \quad (52)$$

subject to (11), (12), (32), (42) and (43) for all  $t = 0, 1, \dots$ .

It is easy to see that the only potential source of difference between the two policy regimes comes from the Euler equation (42). In fact, when we optimize at time  $t = 1$  in the time-consistent regime, we do not take into account that the choice of  $B_2$  affects the Euler equation at time  $t = 0$ ,

$$U_{C_0^T}(1 + \tau_0^B) = \lambda_0 + \beta(1 + r)U_{C_1^T},$$

since from (11)  $B_2$  affects  $U_{C_1^T}$ .

However this can result in differences between the two policy regimes only if  $B_2$  affects  $U(C_0^T)$  and  $U(C_1^T, C_1^N) + \beta V^C(B_2)$  in opposite ways. Specifically, in order for the following two problems

$$\begin{aligned} & \max_{(\{B_1, B_2\}, \{\tau_0^N, \tau_1^N\}, \{\tau_0^B, \tau_1^B\})} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 V^C(B_2) \\ &= \max_{(B_1, \tau_0^N, \tau_0^B)} U(C_0^T, C_0^N) + \beta \left( \max_{(B_2, \tau_1^N, \tau_1^B)} U(C_1^T, C_1^N) + \beta^2 V^C(B_2) \right), \end{aligned} \quad (53)$$

to coincide, it is sufficient that the following derivatives have the same sign:

$$\frac{\partial U_0(B_2)}{\partial B_2} \quad \text{and} \quad \frac{\partial U_1(B_1, B_2)}{\partial B_2}, \quad (54)$$

where

$$U_0(B_2) \doteq \max_{(B_1, \tau_0^N, \tau_0^B)} U(C_0^T, C_0^N),$$

subject to (11), (12), (32), (42) and (43) at time  $t = 0$ , and

$$U_1(B_1, B_2) \doteq \max_{(\tau_1^N, \tau_1^B)} U(C_1^T, C_1^N) + \beta V^C(B_2)$$

subject to (11), (12), (32), (42) and (43) at time  $t = 1$ .

If this restriction holds, the maximization of  $U_1(B_1, B_2)$  with respect to  $B_2$ , yields the same optimal value of  $B_2$  that maximizes  $U_0(B_2)$ . Therefore the maximization can be done in a step-wise way (which gives the time consistent optimal policy) for the Ramsey program on the left hand side of the equality (53).

Thus, in order to show that in our economy Ramsey optimal policy is time consistent we need to establish (54). To do this, we are going to show that both  $U_0(B_2)$  and  $U_1(B_1, B_2)$  are decreasing functions of  $B_2$ , given  $B_1$ . In fact, it is straightforward to see that the function  $U_0(B_2)$  is a decreasing function of  $B_2$ , since if the household knows that in period

2 she can borrow more, she is able to consume more in period 1, and through the Euler equation (42), she can also consume more in period 0.

Next we want to show that  $U_1(B_1, B_2)$  is also a decreasing function of  $B_2$ , for given  $B_1$ . Let  $B_2^*$  be the borrowing level in the competitive equilibrium without the borrowing constraint or any tax intervention. So  $U_1(B_1, B_2)$  must achieve its maximum at  $B_2^*$ . Therefore  $U_1(B_1, B_2)$  decreases for any  $B_2 \geq B_2^*$ . We shall show that  $B_2^C \geq B_2^*$  in the optimal plan that maximizes  $U_1(B_1, B_2)$  subject to (11), (12), (32), (42) and (43) for  $t = 1$ .

We know from our optimal policy analysis on the individual tax instruments, that the optimal policy is such that  $\tau_1^{NC} \leq 0$  and  $\tau_1^{BC} \leq 0$ . If the borrowing constraint is not binding, we have from the Euler equation (42) that

$$U_{C_1^T}(1 + \tau_1^B) = \beta(1 + r)U_{C_2^T}.$$

And if  $\tau_1^B < 0$  and the  $B_2 < B_2^*$ , we would have

$$U_{C_1^T}(1 + \tau_1^B) < U_{C_1^{T*}} = \beta(1 + r)U_{C_2^{T*}} < \beta(1 + r)U_{C_2^T},$$

which is a contradiction.<sup>27</sup> Therefore we conclude that if the borrowing constraint is not binding,  $B_2^C \geq B_2^*$ .

If the constraint is binding, from the Euler equation (42) we have that  $\lambda > 0$ . Suppose that  $B_2' \geq B_2^*$  is optimal in the economy without the borrowing constraint. We want to show that the optimal policy in the economy with the borrowing constraint has  $B_2 \geq B_2'$ . Suppose this is not the case. Then we would have

$$U_{C_1^T}(1 + \tau_1^B) - \lambda < U_{C_1^{T'}}(1 + \tau_1^B) = \beta(1 + r)U_{C_2^{T'}} < U_{C_2^T},$$

which again contradicts the Euler equation (42).<sup>28</sup> Therefore we must have that  $B_2^C \geq B_2' \geq B_2^*$ .

Combining the previous two arguments, it follows that  $U_1(B_1, B_2)$  is also a decreasing function of  $B$  and hence has the same sign of  $U_0(B_2)$ , which proves that 54 holds.

### A.3.2 Finite-period model

Let us now look first at the case of a four-period model. We will show that this case can be reduced to the 3-period model above. In a four-period version of our model, the Ramsey

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<sup>27</sup>The first inequality comes from  $C_1^T > C_1^{T*}$  and the fact that  $U_{C_1^T}$  is a decreasing function of  $C_1^T$ . The second inequality comes from  $C_2^T < C_2^{T*}$  and the same logic.

<sup>28</sup>The inequalities follow from the same reasonings in the case of a nonbinding borrowing constraint.

program solves the following problem

$$\max_{(\{B_i\}_{i=1}^3, \{\tau_i^N\}_{i=0}^2, \{\tau_i^B\}_{i=0}^2)} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 U(C_2^T, C_2^N) + \beta^3 V^C(B_3),$$

subject to (11), (12), (32), (42) and (43) for  $t = 1, \dots, 3$ .

Now note first that

$$U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N)$$

is decreasing in  $B_3$  by the same reasoning as in the 3-period model, and that

$$U(C_2^T, C_2^N) + \beta V^C(B_3)$$

is also decreasing in  $B_3$ , since  $B_3 \geq B_3^*$  where  $B_3^*$  is the competitive equilibrium borrowing level without the borrowing constraint or tax interventions. Therefore we have that

$$\begin{aligned} & \max_{(\{B_i\}_{i=1}^3, \{\tau_i^N\}_{i=0}^2, \{\tau_i^B\}_{i=0}^2)} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 U(C_2^T, C_2^N) + \beta^3 V^C(B_3) \\ &= \max_{(\{B_i\}_{i=1}^2, \{\tau_i^N\}_{i=0}^1, \{\tau_i^B\}_{i=0}^1)} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 V^{C'}(B_2), \end{aligned}$$

where

$$V^{C'}(B_2) = \max_{B_3, \tau_2^N, \tau_2^B} U(C_2^T, C_2^N) + \beta V^C(B_3)$$

subject to (11), (12), (32), (42) and (43) for  $t = 3$ . Thus, we reduced a four-period model into a three-period model. It follows that the Ramsey optimal policy is time consistent in a four-period version of our model.

By using the same method, we can always reduce an  $n$ -period model into  $n - 1$ -period model for any  $n > 4$ . By induction, therefore, we showed that the Ramsey optimal policy for any finite-period version of our economy is time consistent.

### A.3.3 Infinite-horizon model

If we can establish the convergence of the Ramsey optimal policy problem for a finite-period version of our model to an infinite-period version, we will have established that Ramsey optimal policy is time consistent for (50). To do so, we need an additional assumption, i.e. that both  $U(\cdot)$  and  $U_{CT}(\cdot)$  are bounded in the feasible set.

Define now the following mapping  $T : \mathcal{F}_b \rightarrow \mathcal{F}_b$ , where  $\mathcal{F}_b$  is the set of bounded contin-

uous function defined on  $[\underline{B}, 0] \times \mathbb{R}^+$ ,

$$T(V)(B, \mu) \doteq \min_{\gamma \geq 0} \max_{B', \tau^B} U(C^T, C^N) - \mu(1+r)U_{C^T} + \gamma(U_{C^T} - \lambda) + \beta V(B', \gamma),$$

subject to (11), (12), (32), (42) and (43).

From the assumption that both  $U(\cdot)$  and  $U_{C^T}(\cdot)$  are bounded, it follows that  $\lambda$  is bounded. Following Marcet and Marimon (2011), we also conclude that  $T$  is a contraction mapping.

In addition, we note that  $T^n(V)(B, 0)$  is the welfare function of a Ramsey optimal plan for a  $n$ -period economy with  $V(\cdot)$  as the final period utility. Therefore from a standard contraction mapping argument we have that

$$V^*(\cdot) \doteq \lim_{n \rightarrow \infty} T^n(V)(\cdot)$$

is well defined and is uniformly converging.  $V^*(\cdot)$  will be the fixed point of the contraction mapping and is the welfare function of the infinite-period economy under the Ramsey optimal policy.

By the uniform convergence of the welfare function, the finite-period Ramsey optimal policy converges to the infinite-period Ramsey optimal policy. Therefore we established that the Ramsey optimal policy for (50) is time consistent.

QED