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ABSTRACT

Feedback Effects and the Limits to Arbitrage*

This paper identifies a limit to arbitrage that arises because firm value is endogenous to the exploitation of arbitrage. Trading on private information reveals this information to managers and improves their real decisions, enhancing fundamental value. While this feedback effect increases the profitability of buying on good news, it reduces the profitability of selling on bad news. Thus, investors may refrain from trading on negative information, and so bad news is incorporated more slowly into prices than good news. This has potentially important real consequences -- if negative information is not incorporated into prices, inefficient projects are not canceled, leading to overinvestment.

JEL Classification: G14 and G34
Keywords: feedback effect, limits to arbitrage, overinvestment

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1 Introduction

Whether financial markets are informationally efficient is one of the most hotly-contested debates in finance. Proponents of market efficiency argue that profit opportunities in the financial market will lead speculators to trade in a way that eliminates any mispricing. For example, if speculators have negative information about a stock, and this information is not reflected in the price, they will find it profitable to sell the stock. This action will push down the price, reflecting the speculators' information. However, a sizable literature identifies various limits to arbitrage, which may deter speculators from trading on their information. (Here, we use "arbitrage" to refer to investors trading on their private information.[1]) Examples include holding costs, transactions costs, price impact, and short-sales constraints.

In this paper, we identify a quite distinct limit to arbitrage, which instead arises endogenously as part of the arbitrage process. It stems from the fact that the *value of the asset being arbitraged is endogenous to the act of exploiting the arbitrage.* By informed trading, speculators cause prices to move, which in turn reveals information to real decision makers, such as managers, board members, corporate raiders, and regulators. These decision makers then take actions based on the information revealed in the price, and these actions change the underlying asset value. This feedback effect might make the initial trading less profitable, deterring it from occurring in the first place.

To fix ideas, consider the following example. Suppose that a firm (acquirer) announces the acquisition of a target. Also assume that a large speculator has conducted analysis suggesting that this acquisition will be value-destructive. Traditional theory suggests that the speculator should sell the acquirer's stock. However, large-scale selling will convey to the acquirer that the speculator has discovered that the acquisition is a bad idea. As a result, the acquirer may end up cancelling the acquisition. In turn, cancellation of a bad acquisition will boost firm value, reducing the speculator's profit from her short position and in some cases even causing her to suffer a loss. Put differently, the acquirer's decision to cancel the acquisition means that the negative information possessed by the speculator is now less relevant, and hence she should not trade on it. Thus, her information ends up not being reflected in the price. Therefore, the market is not strong-form efficient in the Fama (1970) sense, in that private information is not reflected in the price. However, it is strong-form efficient in the Jensen (1978) sense, in that a privately-informed investor cannot earn profits by trading on her information.

A classic example of how information from the stock market can shape real decisions is Coca-Cola's attempted acquisition of Quaker Oats. On November 20, 2000, the *Wall Street Journal* reported that Coca-Cola was in talks to acquire Quaker Oats. Shortly thereafter, Coca-Cola confirmed such discussions. The market reacted negatively, sending Coca-Cola's shares down 8% on November 20th and 2% on November 21st. Coca-Cola's board rejected the acquisition later on November 21st, potentially due to the negative market reaction. The

[1] This notion of "arbitrage" is broader than the traditional textbook notion of risk-free arbitrage from trading two identical securities.

following day, Coca-Cola's shares rebounded 8%. Thus, speculators who had short-sold on the initial merger announcement, based on the belief that the acquisition would destroy value, lost money – precisely the effect modeled by this paper.[2] In Section 3.5, we discuss another similar example involving Hewlett Packard's (HP) acquisition of Compaq.

Our mechanism is based on the presence of a feedback effect from the financial market to real economic decisions – real decision makers learn from the market when deciding their actions. A common perception is that managers know more about their own firms than outsiders (e.g. Myers and Majluf (1984)). While this perception is plausible for internal information about the firm in isolation, optimal managerial decisions also depend on external information (such as market demand for a firm's products, the future prospects of the industry, or potential synergies with a target) which outsiders may possess more of. For example, a potential acquirer hires investment bank advisors at high fees because, while advisors have less internal information than the manager, they can add value on target selection, e.g. by evaluating which target will be the most synergistic. Note that we only require that outside investors possess some information that the manager does not have; they need not be more informed than the manager on an absolute basis. Luo (2005) provides large-sample evidence that an acquisition is more likely to be canceled if the market's reaction implies that it will be non-synergistic. The effect is stronger when the acquirer is more likely to have something to learn from the market, e.g., for non-high-tech deals and deals in which the bidder is small. Relatedly, Edmans, Goldstein, and Jiang (2012) demonstrate that a firm's market price affects the likelihood that it becomes a takeover target, which may arise because potential acquirers learn from the market price. More broadly, Chen, Goldstein, and Jiang (2007) show that the sensitivity of investment to price is higher when the price contains more private information not known to managers.

Moreover, our model can apply to corrective actions (i.e., actions that improve firm value upon learning negative information about firm prospects) undertaken by stakeholders other than the manager. Such stakeholders likely have less information than the manager and may be more reliant on information held by outsiders. Examples include managerial replacement (undertaken by the board, or by shareholders who lobby the board), a disciplinary takeover (undertaken by an acquirer), or the granting of a subsidy or a bail-out (undertaken by the government). We demonstrate a barrier to decision makers learning from investors – investors may choose not to impound their information into prices by trading. Furthermore, the model can apply in a non-corporate context. For example, in late 2011, investors sold Italian bonds due to concerns about Prime Minister Silvio Berlusconi's handling of the debt crisis. Commentators argue that his resignation on November 16 was due to pressure partly resulting from rising bond yields.[3]

[2] Our model predicts that speculators refrain from short-selling in expectation of deal cancelation, the direct evidence of which is not empirically detectable. However, the model prediction is consistent with the poor long-term performance of mergers and acquisitions (e.g., Andrade, Mitchell, and Stafford (2001)). In the above example, speculators who sold might have expected that the acquisition would go through due to managerial private benefits. Hence, the example should be used to demonstrate the losses incurred by speculators when a corrective action was unexpectedly adopted in response to their selling.

[3] For more of the story, see the news segment "Berlusconi Facing Intensified Pressure to Resign as Italian

After his resignation, bond yields fell from over 7% on November 16 to 6.6% on November 18 and below 6% in early December.

An important element of our theory is that it generates asymmetry between trading on positive and negative information. As we explained above, the feedback effect reduces the incentive of a speculator to trade on bad news, due to the possible correction of the problem after it is revealed in the price, but it actually increases the incentive of a speculator to trade on good news. The intuition is as follows. When a speculator trades on information, she improves the efficiency of the firm's decisions – regardless of the direction of her trade. If she has positive information on a firm's prospects, trading on it will reveal to the manager that investment is profitable. This revelation will cause the firm to invest more, thus increasing its value. If the speculator has negative information, trading on it will reveal to the manager that investment is unprofitable. This revelation will cause the firm to invest less, also increasing its value as contraction is the correct decision. When a speculator buys and takes a long position in a firm, she benefits not only from her positive information, but also from increasing the firm's value via the feedback effect. In contrast, when she sells and takes a short position, she loses from increasing the firm's value via the feedback effect. To convey this idea, our model features a firm that can either increase its investment (i.e., invest) or decrease it (i.e., disinvest). We show a clear asymmetry in equilibrium outcomes, whereby equilibria where the speculator trades on good news but refrains from trading on bad news are much more likely than equilibria where the speculator trades on bad news but refrains from trading on good news.

Even though the speculator's trading behavior is asymmetric, it is not automatic that the impact on prices will be asymmetric. The market maker is fully rational and takes into account the fact that the speculator trades more aggressively on positive information, and so he adjusts his pricing function accordingly. Therefore, it may seem that negative information will have the same price impact as positive information, because the market maker knows that a neutral order flow can stem from the speculator having negative information but choosing not to trade, and therefore should decrease the price accordingly. We show that asymmetry in trading behavior does translate into asymmetry in price impact. The crux is that the market maker cannot distinguish the case of a speculator who has negative information but chooses to withhold it, from the case in which the speculator is absent (i.e., there is no private information). Thus, a neutral order flow does not lead to a large stock price decrease, and so negative information has a smaller effect on prices. Indeed, Hong, Lim, and Stein (2000) show empirically that bad news is incorporated into prices more slowly than good news. They conjecture that this phenomenon arises because firm management possesses value-relevant information and publicizes it more enthusiastically for favorable than unfavorable information. Our paper presents a formal model that offers an alternative explanation. Here, key information is held by a firm's investors rather than its managers, who "publicize" it not through public news releases, but by trading on it.

Bond Rates Continue Climbing" on ForexTV on November 9, 2011, and the Yahoo Finance article "Berlusconi Urged To Quit As Bond Yields Climb" on October 31, 2011.

Investors also choose to disseminate good news more readily than bad news, but for a reason very different from that of management, i.e., because of the feedback effect and its implications for trading profits.

In addition to its effects on stock returns, the asymmetry of the speculator's trading strategy can also generate important real consequences. Since negative information is not incorporated into prices, it does not influence management decisions. Thus, while positive net present value ("NPV") projects will be encouraged, some negative-NPV projects will not be canceled, leading to overinvestment overall. In contrast to standard overinvestment theories based on the manager having private benefits (e.g., Jensen (1986), Stulz (1990), Zwiebel (1996)), here the manager is fully aligned with firm value and there are no agency problems. The manager wishes to maximize firm value by learning from prices, but is unable to do so since speculators refrain from revealing their information. Applied to M&A as well as organic investment, the theory may explain why M&A appears to be "excessive" and a large fraction of acquisitions destroy value (see, e.g., Andrade, Mitchell, and Stafford (2001)). While traditional finance theory would suggest that the market can prevent bad acquisitions by communicating negative information to the manager, our model shows that the market might fail to do so due to the adverse effect on speculators' trading incentives: revealing negative information to the firm is unprofitable for them if the information is then used to correct the underlying problem.

Our source of the limits to arbitrage – the feedback effect – is different from the mechanisms studied by prior research. Campbell and Kyle (1993) focus on fundamental risk, i.e., the risk that firm fundamentals will change while the arbitrage strategy is being pursued. In their model, such changes are unrelated to speculators' arbitrage activities. De Long, Shleifer, Summers, and Waldmann (1990) argue that noise-trading risk, i.e., the risk that noise trading will increase the degree of mispricing, may render arbitrage activities unprofitable. Noise trading only affects the asset's market price and not its fundamental value, which is again exogenous to the act of arbitrage. Shleifer and Vishny (1997) show that, even if an arbitrage strategy is sure to converge in the long-run, the possibility that mispricing may widen in the short-term may deter speculators from pursuing it, if they are concerned with short-term redemptions by their own investors. Similarly, Kondor (2009) demonstrates that financially-constrained arbitrageurs may stay out of a trade if they believe that it will become more profitable in the future. Many authors (e.g., Pontiff (1996), Mitchell and Pulvino (2001), and Mitchell, Pulvino, and Stafford (2002)) focus on the transaction costs and holding costs that arbitrageurs have to incur while pursuing an arbitrage strategy. Others (Geczy, Musto, and Reed (2002) and Lamont and Thaler (2003)) discuss the importance of short-sales constraints.

While many of these papers emphasize market frictions as the source of limits to arbitrage, our paper shows that limits to arbitrage arise when the market performs its utmost efficient role: guiding the allocation of real resources. Thus, while limits to arbitrage based on market frictions tend to attenuate with the development of financial markets, the effect identified by this paper will remain: informed investors might refrain from trading on negative information due to the feedback effect. This mechanism adds to "limit to arbitrage" studied in the vast literature following Kyle (1985), in which speculator are concerned about price impact (which moves prices closer to the fundamental value) and decrease their trading volumes to minimize it. Here, speculators are also concerned about the feedback effect on managerial decisions (which moves the fundamental value closer to the current price). Moreover, as investors become more sophisticated, managers will learn from them to a greater degree. As a result, the effect may even strengthen with the development of financial markets.

Our paper is related to the literature exploring the theoretical implications of the feedback effects from market prices to real decision making. Several papers in this literature show that the feedback effect may have implications that are harmful for real efficiency: see Bond, Edmans, and Goldstein (2012) for a survey. Most closely related is Goldstein and Guembel (2008), who show that the feedback effect provides an incentive for uninformed speculators to short sell a stock, reducing its value by inducing a real decision (investment) based on false information. Their paper also highlights an asymmetry between buy-side and sell-side speculation, but only with respect to uninformed trading; here, we show that informed speculators are less likely to trade on bad news rather than good news, in turn generating implications for the speed of incorporation of news into prices. Bond, Goldstein, and Prescott (2010), Dow, Goldstein, and Guembel (2010), and Goldstein, Ozdenoren, and Yuan (2013) also model complexities arising from the feedback effect. Overall, the point in our paper – that negatively informed speculators will strategically withhold negative information from the market, because they know that the release of negative information will lead managers to fix the underlying problem – is new in this literature.[4]

This paper proceeds as follows. Section 2 presents the model. Section 3 contains the core analysis, demonstrating the asymmetric limit to arbitrage. Section 4 investigates the extent to which information affects beliefs and prices. Section 5 concludes. Appendix A contains all proofs not in the main text.

2 The Model

The model has three dates, $t \in \{0, 1, 2\}$. There is a firm whose stock is traded in the financial market. The firm's manager needs to take a decision as to whether to keep the current level of investment, increase it, or reduce it. The manager's goal is to maximize expected firm value; since there are no agency problems between the manager and the firm, we will use these two terms interchangeably. At $t = 0$, a risk-neutral speculator may be present in the financial market. If present, she is informed about the state of nature θ that determines both the value of

[4] Recently, Boleslavsky, Kelly, and Taylor (2013) followed our idea and developed a model with a related insight in the context of government intervention based on market prices. Their model only features "disinvestment" and not "investment" and so does not properly explore the asymmetry we analyze here. In addition, they only obtain equilibria in mixed strategies, in which the government is indifferent between intervention and no intervention, and so is actually never better off by using the information in the market.

the firm under the current investment level, and also the profitability of increasing or decreasing investment. She rationally anticipates the effect of her trading on the manager's investment level. Trading in the financial market occurs at $t = 1$. In addition to the speculator, two other types of agents participate in the financial market: a noise trader whose trades are unrelated to the realization of θ, and a risk-neutral market maker. The latter collects the orders from the speculator and noise trader, and sets a price at which he executes the orders out of his inventory. This price rationally anticipates the manager's investment decision. At $t = 2$, the manager takes the decision, which may be affected by the trading in the financial market at $t = 1$. Finally, all uncertainty is resolved and payoffs are realized. We now describe the firm's investment problem and the trading process in more detail.

2.1 The Firm's Decision

At $t = 2$, the manager takes an investment decision denoted by $d \in \{-1, 0, 1\}$, where $d = 0$ represents maintaining the current level of investment, $d = 1$ represents increasing investment (which we will often simply refer to as "investment"), and $d = -1$ represents reducing investment ("disinvestment"). Changing the level of investment in either direction (i.e., choosing $d \in \{-1, 1\}$) costs the firm $c \geq 0$.

The value of the firm, realized at $t = 2$, is denoted by $v(\theta, d)$. It depends on both the manager's action d and the state of nature $\theta \in \Theta \equiv \{H, L\}$ ("high" and "low"), and is summarized in Table 1. If the firm chooses $d = 0$, it is worth $v(H, 0) = R_H$ in state H and $v(L, 0) = R_L < R_H$ in state L. In state H, the correct action is to increase investment; doing so creates additional value of $x > 0$ (gross of the cost c) and so $v(H, 1) = R_H + x - c$. Reducing investment is the incorrect action and reduces firm value by x, and so $v(H, -1) = R_H - x - c$. Conversely, in state L, choosing $d = -1$ creates additional value of x, yielding a firm value of $v(L, -1) = R_L + x - c$; choosing $d = 1$ costs the firm x, yielding a firm value of $v(L, 1) = R_L - x - c$. We deliberately set the value created by correct investment in state H to equal the value created by correct disinvestment in state L, and to be the negative of the value destroyed by an incorrect investment decision, to avoid baking any asymmetries into the model. Instead, the asymmetric limit to arbitrage will stem entirely from the feedback effect.

		Investment d		
		1	0	-1
State θ	H	$R_H + x - c$	R_H	$R_H - x - c$
	L	$R_L - x - c$	R_L	$R_L + x - c$

Table 1: Firm value

Note that the above specification implies that:

$$v(H, 1) - v(L, 1) > v(H, 0) - v(L, 0) > v(H, -1) - v(L, -1). \quad (1)$$

Inequality (1) is the driving force behind our results. It means that increasing (reducing) investment increases (reduces) the dependence of firm value on the state. Thus, the speculator's private information on the state is less useful, the lower the investment level taken by the manager. In turn, inequality (1) incorporates two cases, depending on whether firm value is monotonic in the underlying state:

Case 1: $v(H,-1) > v(L,-1)$, i.e. $R_H - x > R_L + x$. In this case, state H entails higher firm value, no matter what action has been taken by the firm. Hence, disinvestment attenuates, but does not eliminate, the effect of the state on firm value. For example, state H can represent high demand for the firm's products, while state L represents low demand. Whether the firm increases or reduces its level of production, its value will be lower in state L, but the negative effect of state L (i.e., low demand) is attenuated if the firm operates at a lower scale. Note that $R_H - x > R_L + x$ implies $R_H - R_L > 2x$, i.e. the speculator's private information over assets in place is relatively more important than the manager's investment decision, and thus the feedback effect.

Case 2: $v(H,-1) < v(L,-1)$, i.e. $R_H - x < R_L + x$. In this case, if disinvestment occurs, firm value is higher in state L. The investment decision is sufficiently powerful to overturn the effect of the state on firm value. Firm value is now non-monotonic in the state: one state does not dominate the other. For example, consider the case where $d = 1$ implies proceeding with a takeover decision, $d = -1$ implies selling assets for cash, and $d = 0$ implies doing nothing. State H corresponds to a state in which current acquisition opportunities dominate future ones, and state L refers to the reverse. If the firm does nothing or makes an acquisition, its value is higher in state H. In contrast, if the firm chooses to sell assets to raise cash, its value is higher in state L since it can use the cash raised to exploit future acquisition opportunities. Another example is related to Aghion and Stein (2008): $d = 1$ corresponds to a growth strategy, and $d = -1$ corresponds to a strategy focused on current profit margins. Growth prospects are good if $\theta = H$ and bad if $\theta = L$. If the firm eschews the growth strategy ($d = -1$), its value is higher in the low state in which there are no growth opportunities. In contrast, in the high state its rivals could pursue the growth opportunities, in turn worsening its competitive position.

Case 1 is the common assumption in the literature (i.e., a "high" state dominates a "low" state), and will be the focus of our analyses. We have fully analyzed Case 2, but for brevity do not include it in the paper. Section 3.4 will briefly discuss the equilibria under Case 2 and explain how the fundamental intuition for our asymmetric limit to arbitrage becomes even stronger.

The prior probability that the state is $\theta = H$ is $y = \frac{1}{2}$, which is common knowledge. We use q to denote the posterior probability the manager assigns to the case $\theta = H$. The manager bases his decision on q, which is calculated using information arising from trades in the financial market. Let γ_1 denote the posterior belief that the state is H such that the manager is indifferent between investing and doing nothing, i.e.:

$$\gamma_1 R_H + (1-\gamma_1) R_L = \gamma_1 (R_H + x) + (1-\gamma_1)(R_L - x) - c, \tag{2}$$

which yields

$$\gamma_1 = \frac{1}{2} + \frac{c}{2x}.$$

Similarly, let γ_{-1} be the posterior belief on state H such that the manager is indifferent between disinvesting and doing nothing, i.e.:

$$\gamma_{-1} R_H + (1-\gamma_{-1}) R_L = \gamma_{-1}(R_H - x) + (1-\gamma_{-1})(R_L + x) - c,$$

which yields

$$\gamma_{-1} = \frac{1}{2} - \frac{c}{2x}.$$

For completeness and without loss of generality, if the manager is indifferent between doing nothing and changing the investment level, we will assume that he will maintain the status quo. The values of γ_1 and $\gamma_{-1} < \gamma_1$ represent "cutoffs" that determine the manager's action. If and only if $q > \gamma_1$, he will increase investment; if and only if $q < \gamma_{-1}$, he will reduce investment. For $\gamma_{-1} \leq q \leq \gamma_1$, he will maintain the current investment level.

Since $y = \frac{1}{2}$, the ex-ante net firm value created by changing investment in either direction is $\frac{1}{2}(x-c) + \frac{1}{2}(-x-c) = -c \leq 0$, and so the ex-ante optimal decision is for the firm not to change its investment level. As long as the information in the market does not cause the manager to update his prior much ($\gamma_{-1} \leq q \leq \gamma_1$), he will maintain the current investment level. As we can see from the definitions of γ_{-1} and γ_1, the range of posterior beliefs for which the firm remains with the status quo becomes larger when the cost of adjustment c is larger and the potential benefit from correctly changing investment x is smaller.

2.2 Trade in the Financial Market

At $t = 0$, a speculator arrives in the financial market with probability λ, where $0 < \lambda < 1$. Whether the speculator is present or not is unknown to anyone else.[5] If the speculator is present, she observes the state of nature θ with certainty. We will use the term "positively-informed speculator" to describe a speculator who observes $\theta = H$, and "negatively-informed speculator" to describe a speculator who observes $\theta = L$. The variable λ is a measure of market sophistication or the informedness of outside investors, and will generate a number of comparative statics. The speculator has no initial position in the firm. We have also fully analyzed the case where the speculator has an initial stake. Section 3.5 will discuss how the key intuition and results continue to hold under a positive initial stake; for brevity, we did not

[5]Since private information is not public knowledge, its existence is also unlikely to be public knowledge. Chakraborty and Yilmaz (2004) also feature uncertainty on whether the speculator is present, in an equilibrium in which informed insiders manipulate the market by trading in the wrong direction.

include this analysis in the paper.

Trading in the financial market happens at $t = 1$. Always present is a noise trader, who trades $z = -1$, 0, or 1 with equal probabilities. If the speculator is present, she makes an endogenous trading choice $s \in \{-1, 0, 1\}$. Trading either -1 or 1 is costly for the speculator and entails paying a cost of κ. The trading cost κ should be interpreted broadly. While direct transaction costs coming from commissions are typically small, other indirect costs can be large. These include borrowing costs (for short sales) and the opportunity costs of capital commitment (for purchases). These costs may differ between buying and selling, but the relative size is a priori unclear. Given our interest in exploring the endogenous asymmetry between buying and selling due to the feedback effect, we assume the same trading cost κ in both directions to avoid generating any asymmetry mechanically. Unless otherwise specified, we refer to trading profits and losses gross of the cost κ. If the speculator is indifferent between trading and not trading, we assume that she will not trade.

Following Kyle (1985), orders are submitted simultaneously to a market maker who sets the price and absorbs order flows out of his inventory. The orders are market orders and are not contingent on the price. The competitive market maker sets the price equal to expected asset value, given the information contained in the order flow. The market maker can only observe total order flow $X = s + z$, but not its individual components s and z. Possible order flows are $X \in \{-2, -1, 0, 1, 2\}$ and the pricing function is $p(X) = E(v|X)$. A critical departure from Kyle (1985) is that firm value here is endogenous, because it depends on the manager's action which is in turn based on information revealed during the trading process.

Specifically, the manager observes total order flow X, and uses the information in X to form his posterior q, which is then used in the investment decision. Allowing the manager to observe order flow X, rather than just the price p, simplifies the analysis without affecting its economic content. In the equilibria that we analyze, there is a one-to-one correspondence between the price and the order flow in most cases; in the few cases where two order flows correspond to the same price, the manager's decision is the same for both order flows. Under the alternative assumption that the manager observes p, other equilibria can arise, in which the market maker sets a price that is consistent with a different managerial decision (one that is suboptimal given the information in the order flow) and this becomes self-fulfilling due to the dependence of the manager's decision on the price. Since our interest is on the feedback effect, we focus on equilibria where the manager's decision responds optimally to the information in the order flow. Moreover, it seems reasonable to assume that managers have access to information about trading quantities in the financial market. First, market making is competitive and so there is little secrecy in the order flow; second, microstructure databases (such as TAQ) provide such information at a short lag – rapidly enough to guide investment decisions.

As is standard in the feedback literature, we assume that the speculator cannot credibly communicate her information directly to the manager, since it is non-verifiable. Instead, she uses her information to maximize her trading profits (as in the theories of governance through trading/"exit" by Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011)). The trade-off between using private information to trade or intervene has been studied by Maug (1998) and Kahn and Winton (1998).

2.3 Equilibrium

The equilibrium concept we use is the Perfect Bayesian Nash Equilibrium. Here, it is defined as follows: (i) A trading strategy by the speculator: $S : \Theta \to \{-1, 0, 1\}$ that maximizes her expected final payoff $s(v - p) - |s|\kappa$, given the price setting rule, the strategy of the manager, and her information about the realization of θ. (ii) An investment strategy by the manager $D : \mathcal{Q} \to \{-1, 0, 1\}$ (where $\mathcal{Q} = \{-2, -1, 0, 1, 2\}$), that maximizes expected firm value v given the information in the order flow and all other strategies. (iii) A price setting strategy by the market maker $p : \mathcal{Q} \to \mathbb{R}$ that allows him to break even in expectation, given the information in the order flow and all other strategies. Moreover, (iv) the firm and the market maker use Bayes' rule in order to update their beliefs from the order they observe in the financial market, and (v) beliefs on outcomes not observed on the equilibrium path satisfy the Cho and Kreps (1987) intuitive criterion. Finally, (vi) all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium.

3 Feedback Effect and Asymmetric Limits to Arbitrage

In this section, we characterize the pure-strategy equilibria in our model. We demonstrate the emergence of asymmetric limits to arbitrage as a result of the feedback from market trading outcomes to the firm's investment decision. We focus on Case 1 ($R_H - x > R_L + x$) in our main analysis, that is, firm value is monotonic in states. We discuss Case 2 briefly in Section 3.4.

3.1 Overview of equilibria when firm value is monotonic in states

The equilibrium will depend on whether order flow is sufficiently informative to overturn the ex-ante optimal decision of $d = 0$. Hence, we distinguish between two cases. In the first ("feedback") case, $\frac{1}{2-\lambda} > \gamma_1$. As we will show, the quantity $\frac{1}{2-\lambda}$ represents the posterior probability of state H under an order flow of $X = 1$ in some equilibria. When $\frac{1}{2-\lambda} > \gamma_1$, the probability λ that the speculator is present is sufficiently high that $X = 1$ is sufficiently informative to induce the manager to increase investment. Thus, there is feedback from the market to real decisions. Since $\gamma_{-1} + \gamma_1 = 1$, $\frac{1}{2-\lambda} > \gamma_1$ is equivalent to $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$. In some equilibria, $\frac{1-\lambda}{2-\lambda}$ represents the posterior probability of state H under an order flow of $X = -1$. When $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$, the posterior is sufficiently low to induce the manager to disinvest. In the second ("no feedback") case, $\frac{1}{2-\lambda} \leq \gamma_1$ and $\frac{1-\lambda}{2-\lambda} \geq \gamma_{-1}$. Here, there is no feedback effect for $X \in \{-1, 1\}$: the order flow is not sufficiently informative to change the manager's decision from the status quo.

As we will show, depending on the values of κ, four equilibrium outcomes can arise:

1. No Trade Equilibrium NT: the speculator does not trade,

2. Trade Equilibrium T: the speculator buys when she knows that $\theta = H$ and sells when she knows that $\theta = L$,

3. Partial Trade Equilibrium BNS (Buy - Not Sell): the speculator buys when she knows that $\theta = H$ and does not trade when she knows that $\theta = L$,

4. Partial Trade Equilibrium SNB (Sell - Not Buy): the speculator does not trade when she knows that $\theta = H$ and sells when she knows that $\theta = L$.

3.2 No feedback equilibrium

Lemma 1 provides the characterization of equilibrium outcomes in the case of no feedback.

Lemma 1 *(Equilibrium, firm value is monotone in the state, no feedback). Suppose that* $R_H - x > R_L + x$ *and* $\frac{1}{2-\lambda} \leq \gamma_1$ $(\Leftrightarrow \frac{1-\lambda}{2-\lambda} \geq \gamma_{-1})$*. There exist cutoffs* $\kappa_{NF} < \kappa_{NT}$ *(defined in the proof) such that the trading game has the following pure-strategy equilibria:*

(a) When $\kappa < \kappa_{NF}$*, the only pure-strategy equilibrium is* T*.*

(b) When $\kappa \geq \kappa_{NT}$*, the only pure-strategy equilibrium is* NT*.*

(c) When $\kappa_{NF} \leq \kappa < \kappa_{NT}$*, the two pure strategy equilibria are* BNS *and* SNB*.*

There is no range of parameter values for which the BNS *equilibrium exists and the* SNB *equilibrium does not exist, or vice versa.*

Proof. This proof is incorporated in the proof of Proposition 1. ■

Two sources of limits to arbitrage are present in the no-feedback case, both of which are standard in the literature, and both of which are symmetric. The first source is the trading cost κ. As κ increases, we move to equilibria in which speculators trade less on their private information. κ_{NT} is the threshold for no trading, that is, when $\kappa \geq \kappa_{NT}$ there is no trading in either direction. Clearly, when speculators are subject to greater transaction costs, they have lower incentives to trade. At the other extreme, when trading cost is sufficiently low ($\kappa < \kappa_{NF}$, where the subscript indexes the "no feedback" regime), the speculator always trades because trading in the direction of private information is profitable.

The second source of limits to arbitrage is the price impact that speculators exert when they trade on their information. In the intermediate region $\kappa_{NF} \leq \kappa < \kappa_{NT}$, there are equilibria in which the speculator trades on one type of information but not the other. There is symmetry in that both types of asymmetric equilibria, BNS and SNB, are possible in exactly the same range of parameters. To understand the intuition behind these asymmetric equilibria, consider the BNS equilibrium (the case of the SNB equilibrium is analogous). Given that the market maker believes that the speculator buys on good news, a negative order flow is very revealing that the speculator is negatively informed and the price moves sharply to reflect this. Specifically, $X = -1$ is inconsistent with the speculator having positive information, and so she only receives $\frac{1-\lambda}{2-\lambda} R_H + \frac{1}{2-\lambda} R_L$. Thus, the speculator makes little profit from selling on bad news; knowing this, she chooses not to trade on bad news. Conversely, given that the market maker believes that the speculator does not sell on bad news, a positive order flow of $X = 1$ is consistent with the speculator being negatively informed and choosing not to trade. As a result, the market maker sets a relatively low price of $\frac{1}{2} R_H + \frac{1}{2} R_L$, which allows the speculator to make high profits by buying. Thus, the equilibrium is sustainable. In sum, in both partial trade equilibria, the order flow in the direction in which the speculator does not trade becomes particularly informative, leading to a larger price impact which reduces the potential trading profits. This force is symmetric in the absence of feedback: there is no value of κ in which there is one partial trade equilibrium but not the other.

3.3 Feedback equilibrium

3.3.1 Characterization of equilibrium outcomes

Proposition 1 provides the characterization of equilibrium outcomes in the case of feedback.

Proposition 1 *(Equilibrium, firm value is monotone in the state, feedback). Suppose that* $R_H - x > R_L + x$ *and* $\frac{1}{2-\lambda} > \gamma_1$ $(\Leftrightarrow \frac{1-\lambda}{2-\lambda} < \gamma_{-1})$*. There exist cutoffs* κ_{SNB}*,* κ_{NT}*, and* κ_T *(defined in the proof), where* $\kappa_T < \kappa_{SNB}$ *and* $\kappa_T < \kappa_{NT}$ *, such that the trading game has the following pure-strategy equilibria:*

(a) When $\kappa < \kappa_T$*, the only pure-strategy equilibrium is* T*.*

(b) When $\kappa \geq \kappa_{NT}$*, the only pure-strategy equilibrium is* NT*.*

(c) When $\kappa_T \leq \kappa < \kappa_{NT}$*,* BNS *is an equilibrium.*

(d) If $\kappa_{SNB} < \kappa_{NT}$*,* SNB *is also an equilibrium in the range* $\kappa_{SNB} \leq \kappa < \kappa_{NT}$*.*

There is a strictly positive range of parameter values $(\kappa_T \leq \kappa < \min(\kappa_{SNB}, \kappa_{NT}))$ *for which* BNS *is the only pure strategy equilibrium. There is no range of parameter values for which the* SNB *equilibrium exists but the* BNS *equilibrium does not exist. Equilibrium results are depicted in Figures 1 and 2.*

Proof. (This proof also incorporates the proof of Lemma 1 for ease of comparison. More details behind the calculations below are provided in Appendix A.) Since firm value is always higher when $\theta = H$ than when $\theta = L$, it is straightforward to show that the speculator will never buy when she knows that $\theta = L$ and will never sell when she knows that $\theta = H$. Then, the only possible pure-strategy equilibria are NT, T, BNS, and SNB. Below, we identify the conditions under which each one of these equilibria holds. If an order flow of $X = -2$ ($X = 2$) is observed off the equilibrium path, we assume that the beliefs of the market maker and the manager are that the speculator knows that the state is L (H). Since speculators always lose if they trade against their information, this is the only belief that is consistent with the intuitive criterion.

No Trade Equilibrium NT:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table (see Appendix A for the full calculations):

X	-2	-1	0	1	2
q	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
d	-1	0	0	0	1
p	$R_L + x - c$	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$R_H + x - c$

As shown in Appendix A, the gain to the negatively-informed speculator from deviating to selling is $\kappa_{NT} \equiv \frac{1}{3}(R_H - R_L)$, and this is also the gain to the positively-informed speculator from deviating to buying. Thus, this equilibrium holds if and only if $\kappa \geq \kappa_{NT}$.

Partial Trade Equilibrium BNS:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:

X	-2	-1	0
q	0	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$
d	-1	$\begin{cases} -1 & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma_{-1} \\ 0 & \text{if } \frac{1-\lambda}{2-\lambda} \geq \gamma_{-1} \end{cases}$	0
p	$R_L + x - c$	$\begin{cases} \frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma_{-1} \\ \frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L & \text{if } \frac{1-\lambda}{2-\lambda} \geq \gamma_{-1} \end{cases}$	$\frac{1}{2}R_H + \frac{1}{2}R_L$

X	1	2
q	$\frac{1}{2}$	1
d	0	1
p	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$R_H + x - c$

Calculating the gain to the negatively-informed speculator from deviating to selling and to the positively-informed speculator from deviating to not trading, we can see that this equilibrium holds if and only if $\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right] \equiv \kappa_T \leq \kappa < \kappa_{NT} \equiv \frac{1}{3}(R_H - R_L)$ for the case of feedback and if and only if $\frac{1}{3}\left[\left(\frac{1-\lambda}{2-\lambda} + \frac{1}{2}\right)(R_H - R_L)\right] \equiv \kappa_{NF} \leq \kappa < \kappa_{NT} \equiv \frac{1}{3}(R_H - R_L)$ for the case of no feedback.

Partial Trade Equilibrium SNB:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:

X	-2	-1	0
q	0	$\frac{1}{2}$	$\frac{1}{2}$
d	-1	0	0
p	$R_L + x - c$	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$\frac{1}{2}R_H + \frac{1}{2}R_L$

X	1	2
q	$\frac{1}{2-\lambda}$	1
d	$\begin{cases} 0 & \text{if } \frac{1}{2-\lambda} \leq \gamma_1 \\ 1 & \text{if } \frac{1}{2-\lambda} > \gamma_1 \end{cases}$	1
p	$\begin{cases} \frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L & \text{if } \frac{1}{2-\lambda} \leq \gamma_1 \\ \frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c & \text{if } \frac{1}{2-\lambda} > \gamma_1 \end{cases}$	$R_H + x - c$

Calculating the gain to the negatively-informed speculator from deviating to not trading and to the positively-informed speculator from deviating to buying, we can see that this equilibrium holds if and only if $\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) + \frac{1}{2}(R_H - R_L)\right] \equiv \kappa_{SNB} \leq \kappa < \kappa_{NT}$ for the case of feedback and if and only if $\kappa_{NF} \leq \kappa < \kappa_{NT}$ for the case of no feedback.

Trade Equilibrium T:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:

X	-2	-1	0
q	0	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$
d	-1	$\begin{cases} -1 & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma_{-1} \\ 0 & \text{if } \frac{1-\lambda}{2-\lambda} \geq \gamma_{-1} \end{cases}$	0
p	$R_L + x - c$	$\begin{cases} \frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c & \text{if } \frac{1-\lambda}{2-\lambda} \leq \gamma_{-1} \\ \frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L & \text{if } \frac{1-\lambda}{2-\lambda} > \gamma_{-1} \end{cases}$	$\frac{1}{2}R_H + \frac{1}{2}R_L$

X	1	2
q	$\frac{1}{2-\lambda}$	1
d	$\begin{cases} 0 & \text{if } \frac{1}{2-\lambda} \leq \gamma_1 \\ 1 & \text{if } \frac{1}{2-\lambda} > \gamma_1 \end{cases}$	1
p	$\begin{cases} \frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L & \text{if } \frac{1}{2-\lambda} \leq \gamma_1 \\ \frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c & \text{if } \frac{1}{2-\lambda} > \gamma_1 \end{cases}$	$R_H + x - c$

Calculating the gain to the negatively-informed speculator from deviating to not trading and to the positively-informed speculator from deviating to not trading, we can see that this equilibrium holds if and only if $\kappa < \kappa_T \equiv \frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right]$ for the case of feedback and if and only if $\kappa < \kappa_{NF}$ for the case of no feedback. ■

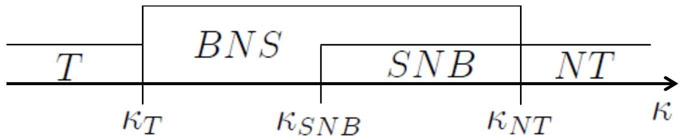

Figure 1: SNB Exists ($\kappa_{SNB} < \kappa_{NT}$)

Figure 2: SNB Does Not Exist ($\kappa_{SNB} \geq \kappa_{NT}$)

The relationship between the different threshold values is illustrated in Figure 1 (SNB exists) and Figure 2 (SNB does not exist). Proposition 1 introduces a new form of limit to arbitrage – the feedback effect – which is absent from the equilibria described in Lemma 1.

3.3.2 Discussion of the BNS and SNB equilibria

We start with the BNS equilibrium. Consider state L. If the negatively-informed speculator deviates to selling and the noise trader does not trade, we have $X = -1$, which induces the manager to disinvest in the case of feedback. Disinvestment is the optimal decision in state L and improves firm value. This decision reduces the speculator's profit in the node of $X = -1$ from $\frac{1-\lambda}{2-\lambda}(R_H - R_L)$ (in the case of no feedback) to only $\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$. While a transaction cost of $\kappa \geq \kappa_{NF}$ deterred the negatively-informed speculator from selling under no feedback, a transaction cost of only $\kappa \geq \kappa_T$ ($< \kappa_{NF}$) is sufficient to deter selling under feedback. The difference between κ_{NF} and κ_T is $\frac{1}{3}\frac{1-\lambda}{2-\lambda}2x$, the probability of $X = -1$ ($\frac{1}{3}$) multiplied by the decrease in trading profits in this node under feedback ($\frac{1-\lambda}{2-\lambda}2x$). Due to feedback, the range for the Partial Trade Equilibrium BNS is larger: it now becomes sustainable in the range $\kappa_T \leq \kappa < \kappa_{NF}$, whereas only the Trade Equilibrium T was sustainable in this range in the no-feedback case. The feedback effect thus provides an endogenous limit to arbitrage that is distinct from those identified in prior literature – *arbitrage is limited because the value of the asset being arbitraged is endogenous to the act of arbitrage.*

We now move to the SNB equilibrium. Consider state H. With $\frac{1}{2-\lambda} > \gamma_1$, an order flow of $X = 1$ provides enough positive information to induce the manager to invest, which is the optimal decision in state H. Improving the manager's decision *increases* the speculator's profit in the node of $X = 1$ for the one share she buys: from $\frac{1-\lambda}{2-\lambda}(R_H - R_L)$ to $\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x)$. While a transaction cost of $\kappa \geq \kappa_{NF}$ deters the positively-informed speculator from buying under no feedback, a higher transaction cost of $\kappa \geq \kappa_{SNB}$ ($> \kappa_{NF}$) is necessary to deter buying under feedback. The difference between κ_{NF} and κ_{SNB} is $\frac{1}{3}\frac{1-\lambda}{2-\lambda}2x$, the probability of $X = 1$ ($\frac{1}{3}$) multiplied by increase in trading profits in this node under feedback ($\frac{1-\lambda}{2-\lambda}2x$). This force contrasts the one in the BNS equilibrium, where improving the manager's decision decreases the speculator's profit on the one share she sells (if she deviates to selling).

In sum, due to the feedback effect, trading on information in either direction – buying on positive information or selling on negative information – puts information into prices, improving the manager's investment decision and thus firm value. More importantly, the feedback effect increases the profitability of informed buying relative to informed selling, and thus creates an asymmetric limit to arbitrage. In particular, if $\kappa_{SNB} \geq \kappa_{NT}$, the SNB equilibrium is never sustainable. This inequality is satisfied if x is large, so that the feedback effect creates significant value and thus significantly reduces (increases) the profitability of selling (buying). More specifically, if $x > \frac{\lambda}{4(1-\lambda)}(R_H - R_L)$, which implies $\kappa_{SNB} \geq \kappa_{NT}$, then SNB is never sustainable. Even if $\kappa_{SNB} < \kappa_{NT}$, there is still a nonempty region $\kappa_T \leq \kappa < \kappa_{SNB}$, where the BNS equilibrium is sustainable but the SNB equilibrium is not. The width of this range of parameter values is given by $\kappa_{SNB} - \kappa_T = \frac{4}{3}\frac{1-\lambda}{2-\lambda}x$. This wedge is increasing in x, the strength of the feedback effect.

The reason why the feedback effect reduces the trading profits is nuanced. Intuition may suggest that the market maker's pricing function will "undo" the feedback effect: since he is rational, the price he sets for a given order flow takes into account the order flow's effect on the manager's decision. Thus, the price received by the speculator will always reflect the manager's action d, and so it seems that the action should not affect her profits. Such intuition turns out to be incorrect. The source of the speculator's profits is not superior knowledge of the manager's action d, since the market maker can also perfectly predict this action from the order flow. The speculator's superior knowledge concerns the state – she directly observes θ, but the market maker can only imperfectly infer it from the order flow. In turn, the manager's action d (and thus the feedback effect on the manager's action) affects trading profits because it affects the dependence of the firm value on the state. From (1), firm value is more sensitive to the state – and thus the speculator makes greater profits from her information on the state – the greater the level of investment. Hence, buying and causing the manager to invest increases the profitability of buying, whereas selling and causing the manager to disinvest reduces the profitability of selling. Then, feedback effects make buying more profitable and selling less profitable, generating asymmetric limits to arbitrage in equilibrium.

3.3.3 Feedback effects and real efficiency

We finally discuss the implications of the limit to arbitrage on real efficiency. The feedback effect from stock prices to firm decisions increases real efficiency by providing the manager information to improve his investment choice. However, the limit to arbitrage deters the speculator from trading on her information, reducing the informativeness of stock prices and thus the feedback effect. Suppose the trading cost κ changes from $\kappa_T - \varepsilon$ to $\kappa_T + \varepsilon$ for an arbitrarily small positive ε. The equilibrium, in the case of feedback, will switch from T to BNS, which reduces the efficiency of the investment decision and thus firm value. The calculation of firm value in both equilibria is as follows. With probability $\frac{1}{2}$, $\theta = H$. In the T equilibrium, the manager invests unless $X = 0$, and so $v(H) = R_H + \frac{2}{3}(x - c)$; in the BNS equilibrium, the manager only invests when $X = 2$, so $v(H) = R_H + \frac{1}{3}(x - c)$. With probability $\frac{1}{2}$, $\theta = L$. In the T equilibrium, $X \in \{-2, -1, 0\}$ and so the manager correctly disinvests unless $X = 0$, so $v(L) = R_L + \frac{2}{3}(x - c)$. In the BNS equilibrium, $X \in \{-1, 0, 1\}$ and the manager correctly disinvests only if $X = -1$. Thus, $v(L) = R_L + \frac{1}{3}(x - c)$. Regardless of whether $\theta = \{H, L\}$, firm value is higher in the T equilibrium by $\frac{1}{3}(x - c)$, which reflects that correct decisions occur more frequently under T due to informed selling by the speculator.

Note that firm values in both equilibria remain higher than in the no-feedback case, in which $v(H) = R_H$ and $v(L) = R_L$. That is, the feedback effect adds value in general, since the stock market aids the manager to make better decisions. However, the feedback effect reduces the incentive of the speculator to trade on bad news, and this in itself has a negative effect on firm value that decreases its overall positive impact.

3.4 Equilibrium when firm value is non-monotonic in states

Though not the focus of our paper, for completeness we discuss the nature of the equilibria that arises when firm value is non-monotonic in the state, and outline the underlying intuition.[6] Under Case 2 ($R_H - x < R_L + x$), disinvestment not only mitigates the effect of the low state but is sufficiently powerful to overturn it, so that firm value is higher in the low state than in the high state when $d = -1$. As a result, the limit to arbitrage also becomes stronger. Now, if the speculator sells on negative information and we have $X = -1$ so that the manager disinvests, the speculator suffers a loss (rather than just a smaller profit) even before transaction costs. Even though both the speculator and market maker know that disinvestment will occur if $X = -1$, they have differing views on firm value conditional on disinvestment. The speculator knows that disinvestment will occur, *and* that disinvestment is desirable for firm value (since she knows that $\theta = L$), and $v = R_L + x - c$. In contrast, the market maker knows the disinvestment will occur but is not certain that it is optimal, because he is unsure of θ. Order flow $X = -1$ is consistent with a negatively-informed speculator, but because $\lambda < 1$ it is also consistent with an absent speculator and selling by the noise trader. Hence, it is possible that $\theta = H$, in which case disinvestment is undesirable and $v = R_H - x - c$. Therefore, the price set by the market maker is only $\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c$, which is less than $v = R_H - x - c$. The speculator's profit (before transaction costs) is $\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$, which is negative in Case 2. This result contrasts standard informed trading models where a speculator can never make a loss (before transactions costs) if she trades in the direction of her information. The key to this loss is the feedback effect. As a result, the minimum transaction cost required to

[6] The full analysis of this case is available upon request.

deter informed selling in the BNS equilibrium, $\kappa_T \equiv \frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right]$, is lower in Case 2 as the first term is now negative. Indeed, κ_T may be negative overall, in which case a negatively-informed speculator will not sell even if transactions costs are zero.

The non-monotonicity in Case 2 also introduces a new force: when the feedback effect is sufficiently strong, the positively-informed speculator may wish to manipulate the price by deviating (from her equilibrium action of buying in BNS or T, or no trade in SNB or NT) to selling.[7] If she sells when $\theta = H$, she potentially misleads the manager to believe that $\theta = L$ and disinvest. Since disinvestment is suboptimal when $\theta = H$, this decision reduces firm value and so the speculator may profit from her short position. Hence, for each of the four equilibria, an additional condition must be satisfied to rule out manipulation. A sufficient condition for all four equilibria is $R_H - R_L > \frac{4}{3}x$: the loss from trading against good news (which is proportional to $R_H - R_L$) is sufficiently high relative to the benefit from manipulation (which is proportional to x). In contrast, the same issue does not arise with the negatively-informed speculator, as she never has an incentive to deviate to buying. If she does so, she misleads the manager to believe that $\theta = H$ and incorrectly invest. This decision reduces firm value, causing the speculator to incur a loss on her long position.[8]

3.5 Discussion of Model Assumptions and Applicability

The above analysis has shown that the feedback effect discourages informed selling relative to informed buying. This section discusses which features of our setting are necessary for this result and which can be relaxed, thus highlighting the conditions under which the asymmetric limit to arbitrage likely exists in the real world.

3.5.1 Condition for the feedback effect to exist

Our asymmetric limit to arbitrage requires feedback from the financial market to real decisions. This in turn arises if financial market trading conveys sufficient information to influence the manager's decision. The asymmetry between the BNS and SNB equilibria in Proposition 1 requires $\frac{1}{2-\lambda} > \gamma_1 = \frac{1}{2} + \frac{c}{2x} \iff \frac{1-\lambda}{2-\lambda} < \gamma_{-1} = \frac{1}{2} - \frac{c}{2x}$. These inequalities are more likely to be satisfied if x is large relative to c – the value created by improving the manager's investment decision is high relative to the cost of doing so – because then the feedback effect is more important. They are also more likely to be satisfied if λ, the probability that the speculator is present, is high, so that the order flow is sufficiently informative to change managerial decisions. The extent to which the manager will change his decision in response to trading will also depend on additional factors outside the model. If the investment is difficult to reverse (e.g., an M&A

[7] The positively-informed speculator will never sell *in equilibrium* because, if the market maker and manager believe that she is manipulating the price, she cannot profit from doing so, and so the set of pure-strategy equilibria remains unchanged at NT, T, SNB, and BNS. However, stronnger conditions are sometimes required to ensure that she is not tempted to *deviate* to selling in the above equilibria.

[8] This analysis is related to Goldstein and Guembel (2008), who analyze the possibility of manipulative trading in the presence of feedback effects.

deal in which there is a formal merger agreement or a termination fee, or an irreversible physical investment), or the manager is less likely to reverse it due to agency problems (e.g., weak governance allows him to pursue negative-NPV investment to maximize his private benefits), the feedback effect will be lower and so the limit to arbitrage will also be weaker.

Hewlett Packard's (HP) acquisition of Compaq illustrates the circumstances under which the feedback effect arises. HP's stock price fell 19% upon announcement on September 4, 2001. That HP's CEO conveyed the unanimous support of its high-profile board for the deal contributed to the magnitude of the decline, as traders did not fear that their selling would lead to the deal being canceled. To everyone's surprise, Walter Hewlett, who earlier voted in favor of the deal as a board member, announced opposition to the merger on behalf of the Hewlett Foundation in the wake of the stock price drop. As chairman of the second-largest shareholder and the son of the company's founder, he posed a credible threat to the deal. Shares of HP rose 17% in response, suggesting that the speculators would not have sold so aggressively had they known that the negative price impact could trigger a corrective action. The combination of rational investor expectation at the time of deal announcement and the expectation being ex post incorrect (due to the unexpected behavior of Walter Hewlett) offers a unique opportunity to observe the feedback effect.

3.5.2 Uncertainty regarding the presence of a speculator ($\lambda < 1$)

Another important assumption is $\lambda < 1$, so that there is uncertainty on whether there is an informed speculator in the market. To see this, note that the feedback effect only affects profits for the nodes of $X = \{-1, 1\}$. If $X = \{-2, 2\}$, the speculator is fully revealed and makes zero trading profit under both buying and selling. If $X = 0$, there is no feedback effect as the price is uninformative. Thus, the profits from informed buying equal the profits from informed selling, and again there is no asymmetry. In turn, $\lambda < 1$ is necessary for the speculator not to be fully revealed at $X = \{-1, 1\}$ and thus for trading profits to be non-zero. For example, consider the market maker's inference from seeing $X = -1$ in the BNS equilibrium. This order flow is consistent with either the speculator being absent (in which case the state may be H or L), or the speculator being present and negatively informed. If $\lambda = 1$, the first case is ruled out, and so the market maker knows for certain that $\theta = L$. Thus, $X = -1$ is fully revealing: the market maker knows *both* that disinvestment will occur, *and* that the state is L, and so sets the price exactly equal to the fundamental value of $R_L + x - c$. The speculator's profits are zero, and thus automatically unaffected by the manager's decision and the feedback effect. Indeed, if $\lambda = 1$, then $\kappa_T = \kappa_{SNB}$ and there is no range of parameter values in which there is a BNS equilibrium but no SNB equilibrium.

In contrast, if $\lambda < 1$, the market maker predicts the manager's action but does not know the state. Since $X = -1$ can be consistent with the speculator being absent and the state being H, the market maker allows for the possibility that $\theta = H$ and sets a price of $\frac{1-\lambda}{2-\lambda} v(H, d) + \frac{1}{2-\lambda} v(L, d)$. Because the speculator knows the state in addition to the action, she makes a profit of $\frac{1-\lambda}{2-\lambda}(v(H,d)-v(L,d))$. This profit is non-zero and depends on the decision d and thus the feedback effect, because the action affects the value of the speculator's information on θ.

The core interpretation of the parameter λ is the probability that an informed speculator is present in the market. Another interpretation is that the speculator is always present, but can only trade with probability λ. For example, with probability $1-\lambda$ she receives a liquidity shock that prevents her from trading: buying a share requires capital, and shorting a share requires posting margin. A third possibility is that the speculator is always present and can trade, but is informed only with probability λ. This alternative scenario, however, requires us to consider the possibility that the uninformed speculator will choose to sell to manipulate the price, as in Goldstein and Guembel (2008), because doing so may dupe the manager into disinvesting. Since $d=0$ is optimal in the absence of information, and such manipulation will enable the speculator to profit on a short position. To keep the paper focused on its primary contribution, we do not analyze this possibility here.

3.5.3 Zero initial position

The core model assumes that the speculator has a zero initial stake in the firm. We have fully analyzed the case in which the speculator owns an initial stake of $\alpha>0$ (i.e. is a blockholder) and show that the key results continue to hold.[9] The fundamental force of the model – the feedback effect increases the profitability of buying on positive information relative to selling on negative information – is independent of the speculator's initial stake. Indeed, the range of transactions costs in which the BNS equilibrium exists and the SNB equilibrium does not is independent of α; in addition, it remains the case that there is no range of κ for which the SNB equilibrium exists but the BNS equilibrium does not exist.

The intuition for the irrelevance of the initial stake is as follows. A positive initial stake increases a negatively-informed speculator's incentive to sell, because if selling leads to (correct) disinvestment, it increases the value of the speculator's initial stake. However, it also increases the positively-informed speculator's incentive to buy, because if buying leads to (correct) investment, it increases the value of the speculator's initial stake by the same margin. Specifically, if a negatively-informed speculator sells, she ends up with a final position of $\alpha-1$: her initial stake α plus her trade of -1. If a positively-informed speculator buys, she ends up with $\alpha+1$: her initial stake α plus her trade of $+1$. The incentive to trade on information to increase the value of her initial stake α (through the feedback effect) is symmetric across buying and selling, and so cancels out. We are thus left with the difference between trading -1 on negative information and trading $+1$ on positive information, which is the same as in the core model with $\alpha=0$. Hence, the asymmetry between buying on good news and selling on bad news remains despite the fact that both trading directions become more attractive when the speculator has an initial position. Overall, our results on the difference between BNS and SNB equilibria remain unchanged.

[9] The full analysis is available upon request.

3.5.4 Corrective action

In our model, the real decision is a corrective action in that it improves firm value in the low state. This case arises when the decision maker maximizes firm value. While we model a manager who attempts to maximize firm value via an investment decision, other potential applications include a board of directors firing an underperforming manager in the bad state or an outside blockholder engaging in activism to restore shareholder value. An alternative real decision is an amplifying action, where the decision maker's objective is something other than firm value, and maximizing this objective leads him to worsen firm value in the low state. For example, capital providers may withdraw their investment in the low state, reducing firm value further (Goldstein, Ozdenoren, and Yuan (2013)), or customers or employees could terminate their relationship with a troubled firm (Subrahmanyam and Titman (2001)). Our model provides distinctive insights on the feedback effect when real decisions are of the corrective nature. In a model with amplifying actions, the speculator will no longer be reluctant to sell on bad news if she has a zero initial stake, since the information will reduce firm value further, enabling her to profit more on her short position.

3.5.5 Other assumptions

Several other assumptions are made only for tractability and can be substantially weakened at the cost of complicating the model with little additional insight. The first such assumption is that the manager has no signal and the speculator has a perfect signal about the state of nature θ. All that is required for our results to go through is that the speculator has some important decision-relevant information that the manager does not have – it is not even necessary that the speculator be more informed than the manager.[10] Another non-critical assumption is discrete trading volumes (i.e., the speculator cannot trade an amount between 0 and 1). The results will likely continue to hold with continuous trading volumes. The speculator may be able to sell a small amount (rather than zero) on negative information without significantly increasing the probability of disinvestment, but she will buy a greater amount upon good information and so the asymmetry remains. Finally, while we assume that there is only one speculator, the results will likely continue to hold in a model with multiple speculators as long as each of them is large enough to have an effect on the total order flow (and hence on the firm's decision).

[10] For example, assume that the optimal decision d depends on both an internal state variable θ_i about the firm, and an external state variable θ_e about the industry's future prospects. Assume also that the manager has a perfect signal about θ_i and the speculator is completely uninformed about θ_i. In addition, the manager has a noisy signal about θ_e and the speculator has a less precise signal about θ_e which is conditionally uncorrelated with the manager's signal. Even though the manager is more informed than the speculator about both θ_i and θ_e, his decision will still be influenced by market prices as the speculator's information about θ_e is incremental and relevant for his decision.

4 Effect of Information on Beliefs and Prices

The previous section demonstrated that the feedback effect gives rise to an equilibrium in which a speculator buys on good news and does not trade on bad news. In this section, we study the implications of this equilibrium. The analysis that follows focuses on the BNS equilibrium in the case of feedback ($\frac{1}{2-\lambda} > \gamma_1 \Leftrightarrow \frac{1-\lambda}{2-\lambda} < \gamma_{-1}$), and considers both Case 1 and Case 2 together. Section 4.1 calculates the effect of good and bad news about the state on the posterior beliefs q, in order to study the extent to which information reaches the manager and affects real decisions. Section 4.2 analyzes the impact of news on prices to generate stock return predictions.

4.1 Beliefs

Since the manager uses the posterior belief q to guide his investment decision, we can interpret q as measuring the extent to which information reaches the manager and affects his actions. In a world in which no agent observes the state, or in which the manager does not learn from prices or order flows, the posterior q would equal the prior $y = \frac{1}{2}$. Conversely, in a world of perfect information transmission, $q = 1$ if $\theta = H$ and $q = 0$ if $\theta = L$. Our model, in which information is partially revealed through prices, lies in between these two polar cases. The absolute distance between q and $\frac{1}{2}$ measures the extent to which information reaches the manager.

Thus far, we have shown that good news received by the speculator has a different impact on her trades (and thus the total order flow) than bad news. However, it is not obvious that this difference will translate into a differential impact on the manager's beliefs. The manager is rational and takes into account the fact that the speculator does not sell on negative information: he updates his beliefs using the asymmetric equilibrium trading strategy. In the BNS equilibrium in the proof of Proposition 1, the manager recognizes that $X = 1$ could be consistent with a negatively-informed speculator who chooses not to trade, and so $q(1)$ is no higher than $q(0)$ (where $q(X)$ denotes the posterior at $t = 1$ upon observing order flow X). Thus, even though bad news can lead to a positive order flow of $X = 1$, the manager knows that such an order flow can stem from a negatively-informed and non-trading speculator, and will decrease his posterior accordingly. Put differently, although negative information does not cause a negative order flow (on average), it can still have a negative effect on beliefs and be fully conveyed to the manager. Thus, it may still seem possible for good and bad news to be conveyed symmetrically to the manager – by taking into account the speculator's asymmetric trading strategy, he can "undo" the asymmetry. Indeed, we start by showing that, if we do not condition on the presence of the speculator, the effects on beliefs of the high and low states being realized are symmetric. This is a direct consequence of the law of iterated expectations: the expected posterior must equal the prior.

Lemma 2 *(Symmetric effect of high and low state on beliefs at $t = 1$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1$ (and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$). (i) If $\theta = H$, the manager's expected posterior* *probability of the high state is* $q^H = \frac{(1-\lambda)^2}{6-3\lambda} + \frac{1}{3} + \frac{\lambda}{3}$ *and is increasing in* λ*. (ii) If* $\theta = L$*, the manager's expected posterior probability of the high state is* $q^L = \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}$ *and is decreasing in* λ*. (iii) We have* $\frac{q^H+q^L}{2} = \frac{1}{2}$*: thus, the realization of state* H *has the same absolute impact on beliefs as the realization of state* L*.*

Proof. See Appendix A. ■

Of greater interest is to study the effect of the state realization conditional upon the speculator being present. We use the term "good news" to refer to $\theta = H$ being realized and the speculator being present, since in this case there is an agent in the economy who directly receives news on the state; "bad news" is defined analogously. While the above analysis studied the effect of the state being realized (regardless of whether the state is learned by any agent in the economy), this analysis studies the impact of the speculator receiving information about the state. The goal is to investigate the extent to which the speculator's good and bad news is conveyed to the manager at $t = 1$. The results are given in Proposition 2 below:

Proposition 2 *(Asymmetric effect of good and bad news on beliefs at* $t = 1$*). Consider the BNS equilibrium where* $\frac{1}{2-\lambda} > \gamma_1$ *(and* $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$*). (i) If* $\theta = H$ *and the speculator is present, the manager's expected posterior probability of the high state is* $q^{H,spec} = \frac{2}{3}$ *and is independent of* λ*. (ii) If* $\theta = L$ *and the speculator is present, the manager's expected posterior probability of the high state is* $q^{L,spec} = \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}$ *and is decreasing in* λ*. (iii) We have*

$$\frac{q^{H,spec} + q^{L,spec}}{2} = \frac{1 + \frac{1-\lambda}{6-3\lambda}}{2}, \tag{3}$$

which is decreasing in λ*. Since* $\frac{1+\frac{1-\lambda}{6-3\lambda}}{2} > \frac{1}{2}$*, (3) implies that* $\left|q^{H,spec} - y\right| - \left|q^{L,spec} - y\right| > 0$*, i.e. the absolute increase in the manager's posterior if the speculator receives good news exceeds the absolute decrease in his posterior if the speculator receives bad news. The difference is decreasing in* λ*.*

Proof. See Appendix A. ■

Proposition 2 shows that, conditional upon the speculator being present, the impact on beliefs of good news is greater in absolute terms than the impact of bad news, and the asymmetry is monotonically decreasing in the probability of the speculator's presence λ. Even though the manager takes the speculator's asymmetric trading strategy into account, he cannot distinguish the case of a negatively-informed (and non-trading) speculator from that of an absent speculator (i.e. no information) – both of these cases lead to the order flow being $\{-1, 0, 1\}$ with uniform probability. Thus, negative information has a smaller effect on his belief. In contrast, if the speculator is always present ($\lambda = 1$), the manager has no such inference problem and there is no asymmetry.

The above analysis considered the change in the manager's posterior at $t = 1$. At $t = 2$, the state is realized and the posterior becomes either 1 (if $\theta = H$) or 0 (if $\theta = L$). Since bad news is conveyed to the manager to a lesser extent at $t=1$, it seeps out to a greater extent ex post, between $t=1$ and $t=2$. Thus, bad news causes a greater change in the posterior between $t=1$ and $t=2$ than good news. This result is stated in Corollary 1 below:

Corollary 1 *(Asymmetric effect of high and low state on beliefs at $t=2$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1 \Leftrightarrow \frac{1-\lambda}{2-\lambda} < \gamma_{-1}$. The absolute impact on beliefs between $t=1$ and $t=2$ of the realization of the state is greater for the low state $\theta = L$ than for the high state $\theta = H$, i.e.*

$$\left|0 - q^{L,spec}\right| - \left|1 - q^{H,spec}\right| > 0.$$

The asymmetry is monotonically decreasing in the frequency of the speculator's presence λ.

Proof. Follows from simple calculations ■

The smaller effect of bad news on the posterior at $t=1$ is counterbalanced by its larger effect at $t=2$. As we will show in Section 4.2, surprisingly this result need not hold when we examine the effect of news on prices rather than posteriors.

4.2 Stock Returns

We now calculate the impact of the state realization and news on prices, in order to generate stock return implications. We study short-run stock returns between $t=0$ and $t=1$, and long-run drift between $t=1$ and $t=2$. While this analysis is similar to Section 4.1 but studying prices rather than beliefs, we will show that not all the results remain the same.

4.2.1 Short-Run Stock Returns

Lemma 3 is analogous to Lemma 2 and shows that, unconditionally, the good and bad states have the same absolute impact on prices, since the market maker takes the speculator's asymmetric trading strategy into account when devising his pricing function. Let p_0 denote the "ex ante" stock price at $t=0$, before the state has been realized.

Lemma 3 *(Symmetric effect of high and low state on returns between $t=0$ and $t=1$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1$ (and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$):*

(i) The stock price impact of the high state being realized is $p_1^H - p_0 = \frac{\lambda}{6}\left[p(2) - p(-1)\right] > 0$.

(ii) The stock price impact of the low state being realized is $p_1^L - p_0 = \frac{\lambda}{6}\left[p(-1) - p(2)\right] = -\left(p_1^H - p_0\right) < 0$.

Proof. See Appendix A. ■

We have $p_1^H - p_0 = -\left(p_1^L - p_0\right)$: the negative effect of the low state equals the positive effect of the high state. Thus, the unconditional expected return is zero. This is an inevitable consequence of market efficiency. The price at $t=0$ is an unbiased expectation of the $t=1$ expected price in the high state and the $t=1$ expected price in the low state. Since both states are equally likely, the absolute effect of the high state must equal the absolute effect of the low state. An uninformed investor cannot trade the stock at $t = 0$ and expect a non-zero average return at $t = 1$.

Proposition 3 is analogous to Proposition 2 and shows that, conditional on the speculator being present, good news has a greater effect than bad news:

Proposition 3 *(Asymmetric effect of good and bad news on returns between $t = 0$ and $t = 1$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1$ (and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$):*

(i) If $\theta = H$ and the speculator is present, the average return between $t = 0$ and $t = 1$ is $p_1^{H,spec} - p_0 = \frac{1}{3}\left(1 - \frac{\lambda}{2}\right)\left(p\left(2\right) - p\left(-1\right)\right) > 0$.

(ii) If $\theta = L$ and the speculator is present, the average return between $t = 0$ and $t = 1$ is $p_1^{L,spec} - p_0 = \frac{\lambda}{6}\left(p\left(-1\right) - p\left(2\right)\right) < 0$.

(iii) The difference in the absolute average returns between the speculator learning $\theta = H$ and $\theta = L$ is given by:

$$\left|p_1^{H,spec} - p_0\right| - \left|p_1^{L,spec} - p_0\right| = \frac{1}{3}\left(1 - \lambda\right)\left(p\left(2\right) - p\left(-1\right)\right) > 0, \tag{4}$$

i.e. the stock price increase upon good news exceeds the stock price decrease upon bad news. This difference is decreasing in λ.

(iv) The average return, conditional on the speculator being present, is positive:

$$p_1^{spec} - p_0 = \frac{1}{3}\frac{1-\lambda}{2}\left(p\left(2\right) - p\left(-1\right)\right) > 0. \tag{5}$$

This difference is decreasing in λ.

Proof. See Appendix A. ■

Proposition 3 states that the average return, *conditional* on the speculator being present, is positive – i.e., the stock price increase upon positive information exceeds the stock price decrease upon negative information (part (iii)). Put differently, positive news is impounded into prices to a greater degree than negative information, as found by Hong, Lim, and Stein (2000). Since good and bad news are equally likely, this means that the average return, conditional on the speculator being present, is positive (part (iv)). As with Proposition 2, the key to this result is that, even though the market maker is rational, he is unable to distinguish the case of a negatively-informed speculator from that of an absent speculator (i.e., no information). If $\lambda = 1$, equations (4) and (5) become zero and there is no asymmetry; the asymmetry is monotonically decreasing in λ. Note that the positive average return given in part (iv) is not inconsistent with market efficiency, because it is conditional upon the speculator being present, which is private information. An uninformed investor cannot buy the stock at $t = 0$ and expect to earn a positive return at $t = 1$ because she will not know whether the speculator is present.

4.2.2 Long-Run Drift

We now move from short-run returns to calculating the long-run drift of the stock price, to analyze the stock return analog of Corollary 1, i.e., the impact of the state realization on prices between $t = 1$ and $t = 2$.

Corollary 2 *(Asymmetric effect of good and bad news on returns between $t = 1$ and $t = 2$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1$ (and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$):*

(i) If $\theta = H$ and the speculator is present, the average return between $t = 1$ and $t = 2$ is $p_2^{H,spec} - p_1^{H,spec} = \frac{1}{3}(R_H - R_L) > 0$.

(ii) If $\theta = L$ and the speculator is present, the average return between $t = 1$ and $t = 2$ is

$$p_2^{L,spec} - p_1^{L,spec} = \frac{(3 - 2\lambda)(R_L - R_H) + 2(1 - \lambda)x}{3(2 - \lambda)}, \tag{6}$$

which is negative in Case 1, but can be positive or negative in Case 2.

(iii) If (6) < 0, the difference in the absolute average returns between the speculator learning $\theta = H$ and $\theta = L$ is given by:

$$\left|p_2^{H,spec} - p_1^{H,spec}\right| - \left|p_2^{L,spec} - p_1^{L,spec}\right| = \frac{(1 - \lambda)(R_L - R_H + 2x)}{3(2 - \lambda)},$$

which is positive in Case 2 and negative in Case 1. The magnitude of the difference is decreasing in λ.

(iv) Expected firm value at $t = 2$, conditional upon the speculator being present, is:

$$p_2^{spec} = \frac{1}{2}(R_H + R_L) + \frac{1}{3}(x - c),$$

and the average return between $t = 1$ and $t = 2$ if the speculator is present is:

$$p_2^{spec} - p_1^{spec} = \frac{1}{6}\frac{1 - \lambda}{2 - \lambda}(R_L - R_H + 2x),$$

which is positive in Case 2 and negative in Case 1. The magnitude of the difference is decreasing in λ.

Proof. See Appendix A. ■

Corollary 1 showed that the smaller effect of bad news on beliefs at $t = 1$ is counterbalanced by a larger effect on beliefs at $t = 2$, and so the average increase in beliefs in the short-run is reversed by an average decrease in beliefs in the long-run. Corollary 2 shows that this need not be the case for returns: it is possible for bad news to have a smaller effect than good news at both $t = 1$ and $t = 2$, and so the speculator's presence can lead to positive average returns in both the short-run and long-run.

The above result arises because the stock price depends not only on beliefs about the state, but also the manager's action. Thus, there is an additional effect of the speculator on prices that does not exist in the analysis of beliefs: not only does she convey information about the state, but also this information affects the manager's decision. In our default set up with $R_H - x > R_L + x$, the long-run drift to the low state is larger in magnitude, analogous to Corollary 1. Since state L is bad for firm value regardless of whether the manager disinvests, the realization of state L at $t = 2$ leads to a large decrease in the price. Thus, prices are too high at $t = 1$. Miller (1977) similarly shows that prices are too high if bad news is not traded upon. However, in his model, the lack of trading on bad news results from exogenous short-sales constraints; here, the reluctance to short-sell is generated endogenously.

Note that the long-term drift in returns does not violate market efficiency. The key to reconciling this result with market efficiency is that firm value is endogenous to trading. If the speculator sold aggressively upon observing $\theta = L$, the decline in the stock price will lead to disinvestment occurring. The market is not strong-form efficient in the Fama (1970) sense, since the speculator's private information is not incorporated into prices, but is strong-form efficient in the Jensen (1978) sense as the speculator cannot make profits on her information, due to the feedback effect. Since she does not trade on her information, the negative returns to $\theta = L$ must manifest predominantly at $t = 2$.

5 Conclusion

This paper has modeled a limit to arbitrage that stems from the fact that firm value is endogenous to the act of exploiting the arbitrage. Even if a speculator has negative information on the state, she may strategically refrain from trading on it, because doing so conveys her information to the manager. The manager may then disinvest, which improves firm value but reduces the profits from the speculator's sell order below the cost of trading, and may cause her to realize a loss. There are several important differences between the feedback-driven limit to arbitrage that we study, and the limits to arbitrage identified by prior literature. First, the effect is asymmetric. Trading in either direction impounds information into prices, which improves the manager's decision-making and increases fundamental value. This feedback effect increases the profitability of buying on good news but reduces the profitability of selling on bad news. Second, the effect does not rely on exogenous frictions or agency problems, but is instead generated endogenously as part of the arbitrage process. Thus, even if speculators have perfect private information and no wealth constraints or trading restrictions, they may choose not to trade on their information.

The asymmetry of our effect has implications for both stock returns and real investment. In terms of stock returns, bad news has a smaller effect on short-run prices than good news, even though the market maker is rational and takes the speculator's trading strategy into account when devising his pricing function. Interestingly, in contrast to underreaction models, the smaller short-run reaction to bad news may also coincide with smaller long-run drift, since the manager can disinvest to attenuate the negative effect of the state on firm value. In terms of real investment, the manager may overinvest in negative-NPV projects, even though there are no agency problems and he is attempting to learn from the market to take the efficient decision. Even though there is an agent in the economy who knows with certainty that the investment is undesirable, and the manager is aware of the speculator's asymmetric trading strategy, this information is not conveyed to the manager and so the project is not abandoned.

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A Appendix: Proofs

Proof of Proposition 1

This proof only provides material supplementary to what is in the main text.

No Trade Equilibrium NT. The order flows of $X = -2$ and $X = 2$ are off the equilibrium path and the posteriors are given by 0 and 1, respectively, as these are the only posteriors that satisfy the Intuitive Criterion (as stated in the main proof). The order flows of $X \in \{-1, 0, 1\}$ are observed on the equilibrium path and so the posteriors can be calculated by Bayes' rule:

$$
\begin{aligned}
q(X) &= \Pr(H|X) \\
&= \frac{\Pr(X|H)}{\Pr(X|H) + \Pr(X|L)}.
\end{aligned}
$$

We thus have:

$$
\begin{aligned}
q(-1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} \\
&= \frac{1}{2},
\end{aligned}
$$

and $q(0)$ and $q(1)$ are calculated in exactly the same way. Sequential rationality leads to the decisions d and prices p as given by the Table in the proof in the main text.

We now turn to calculating the speculator's payoff under different trading strategies, which comprises of the value of her final stake (of -1, 0, or 1 share), plus (minus) the price received (paid) for any share sold (bought). Under the positively-informed speculator's equilibrium strategy of not trading, we have $X \in \{-1, 0, 1\}$ and so her payoff is 0. If she deviates to buying:

- With probability (w.p.) $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is $-\kappa$.
- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) - \kappa = \frac{1}{2}(R_H - R_L) - \kappa$.

Thus, her overall gain from deviation to buying is given by:

$$
\frac{1}{3}(R_H - R_L) \equiv \kappa_{NT}. \tag{7}
$$

A similar calculation shows that, if the negatively-informed speculator sells, her gross gain is also given by (7). Thus, if and only if $\kappa \geq \kappa_{NT}$, the no-trade equilibrium is sustainable. The above calculations apply both in the case of feedback ($\frac{1}{2-\lambda} > \gamma_1$ and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$) and no feedback ($\frac{1}{2-\lambda} \leq \gamma_1$ and $\frac{1-\lambda}{2-\lambda} \geq \gamma_{-1}$).

Partial Trade Equilibrium BNS. The order flow of $X = -2$ is off the equilibrium path and the posterior is given by 0. The posteriors of the other order flows are given as follows:

$$q(-1) = \frac{(1-\lambda)(1/3)}{(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1-\lambda}{2-\lambda},$$
$$q(0) = \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1}{2},$$
$$q(1) = \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1}{2},$$
$$q(2) = \frac{\lambda(1/3)}{\lambda(1/3)} = 1.$$

Under this equilibrium, the positively-informed speculator buys.

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is $-\kappa$.
- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) - \kappa = \frac{1}{2}(R_H - R_L) - \kappa$.

If the positively-informed speculator deviates from buying to not trading:

- W.p. 1, $X \in \{-1, 0, 1\}$, and she doesn't trade in the market. Her payoff is 0.

Thus, her overall gain from deviating to not trading is $-\kappa_{NT}$ (as given by (7)) in the cases of both feedback and no feedback.

The negatively-informed speculator's equilibrium action is not to trade.

- W.p. 1, $X \in \{-1, 0, 1\}$, and she doesn't trade in the market. Her payoff is 0.

If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and she is fully revealed. Her payoff is $-\kappa$.
- W.p. $\frac{1}{3}$, $X = -1$. In the case of feedback, she receives $\frac{1-\lambda}{2-\lambda}(R_H - x - c) + \frac{1}{2-\lambda}(R_L + x - c)$ per share, and so her payoff is $-(R_L + x - c) + (\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c) - \kappa = \frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) - \kappa$. In the case of no feedback, she receives $\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L$ per share, and so her payoff is $-R_L + (\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L) - \kappa = \frac{1-\lambda}{2-\lambda}(R_H - R_L) - \kappa$.
- W.p. $\frac{1}{3}$, $X = 0$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_L + (\frac{1}{2}R_H + \frac{1}{2}R_L) - \kappa = \frac{1}{2}(R_H - R_L) - \kappa$.

Thus, her overall gain from deviation to selling is:

$$\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right] \equiv \kappa_T \tag{8}$$

in the case of feedback, and

$$\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}+\frac{1}{2}\right](R_H-R_L)\equiv\kappa_{NF} \tag{9}$$

in the case of no feedback.

Thus, the BNS equilibrium is sustainable if and only if $\kappa_T \leq \kappa < \kappa_{NT}$ in the case of feedback, and $\kappa_{NF} \leq \kappa < \kappa_{NT}$ in the case of no feedback.

Partial Trade Equilibrium SNB. The order flow of $X = 2$ is off the equilibrium path and the posterior is given by 1. The posteriors of the other order flows are given as follows:

$$\begin{aligned}
q(-2) &= \frac{0}{\lambda(1/3)} = 0, \\
q(-1) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1}{2}, \\
q(0) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1}{2}, \\
q(1) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+(1-\lambda)(1/3)} = \frac{1}{2-\lambda}.
\end{aligned}$$

Under this equilibrium, the negatively-informed speculator sells.

- W.p. $\frac{1}{3}$, $X = -2$, and she is fully revealed. Her payoff is $-\kappa$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_L + \left(\frac{1}{2}R_H + \frac{1}{2}R_L\right) - \kappa = \frac{1}{2}(R_H - R_L) - \kappa$.

If the negatively-informed speculator deviates from selling to not trading:

- W.p. 1, $X \in \{-1, 0, 1\}$, and she doesn't trade in the market. Her payoff is 0.

Thus, her overall gain from deviating to not trading is $-\kappa_{NT}$ (as given by (7)) in the cases of both feedback and no feedback.

The positively-informed speculator's equilibrium action is not to trade.

- W.p. 1, $X \in \{-1, 0, 1\}$, and she doesn't trade in the market. Her payoff is 0.

If she deviates to buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is $-\kappa$.
- W.p. $\frac{1}{3}$, $X = 1$. In the case of feedback, she pays $\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c$ per share, and so her payoff is $(R_H + x - c) - (\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c) - \kappa = \frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) - \kappa$. In the case of no feedback, she pays $\frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L$ per share, and so her payoff is $R_H - (\frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L) - \kappa = \frac{1-\lambda}{2-\lambda}(R_H - R_L) - \kappa$.

- W.p. $\frac{1}{3}$, $X=0$, and she pays $\frac{1}{2}R_H+\frac{1}{2}R_L$ per share. Her payoff is $R_H-(\frac{1}{2}R_H+\frac{1}{2}R_L)-\kappa = \frac{1}{2}(R_H-R_L)-\kappa$.

Thus, her overall gain from deviation to buying is given by:

$$\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H-R_L+2x)+\frac{1}{2}(R_H-R_L)\right]\equiv\kappa_{SNB}$$

in the case of feedback and κ_{NF} (as given by (9)) in the case of no feedback.

Thus, the SNB equilibrium is sustainable if and only if $\kappa_{SNB}\leq\kappa<\kappa_{NT}$ in the case of feedback and $\kappa_{NF}\leq\kappa<\kappa_{NT}$ in the case of no feedback.

Trade Equilibrium T. All order flows are on the equilibrium path and so the posteriors are given as follows:

$$\begin{aligned}
q(-2) &= \frac{0}{\lambda(1/3)}=0,\\
q(-1) &= \frac{(1-\lambda)(1/3)}{(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)}=\frac{1-\lambda}{2-\lambda},\\
q(0) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)}=\frac{1}{2},\\
q(1) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+(1-\lambda)(1/3)}=\frac{1}{2-\lambda},\\
q(2) &= \frac{\lambda(1/3)}{\lambda(1/3)}=1.
\end{aligned}$$

Under this equilibrium, the negatively-informed speculator sells.

- W.p. $\frac{1}{3}$, $X=-2$, and she is fully revealed. Her payoff is $-\kappa$.
- W.p. $\frac{1}{3}$, $X=-1$. In the case of feedback, she receives $\frac{1-\lambda}{2-\lambda}(R_H-x)+\frac{1}{2-\lambda}(R_L+x)-c$ per share, and so her payoff is $-(R_L+x-c)+(\frac{1-\lambda}{2-\lambda}(R_H-x)+\frac{1}{2-\lambda}(R_L+x)-c)-\kappa = \frac{1-\lambda}{2-\lambda}(R_H-R_L-2x)-\kappa$. In the case of no feedback, she receives $\frac{1-\lambda}{2-\lambda}R_H+\frac{1}{2-\lambda}R_L$ per share, and so her payoff is $-R_L+(\frac{1-\lambda}{2-\lambda}R_H+\frac{1}{2-\lambda}R_L)-\kappa=\frac{1-\lambda}{2-\lambda}(R_H-R_L)-\kappa$.
- W.p. $\frac{1}{3}$, $X=0$, and she receives $\frac{1}{2}R_H+\frac{1}{2}R_L$ per share. Her payoff is $-R_L+(\frac{1}{2}R_H+\frac{1}{2}R_L)-\kappa=\frac{1}{2}(R_H-R_L)-\kappa$.

If she deviates to not trading:

- W.p. 1, $X\in\{-1,0,1\}$, and she doesn't trade in the market. Her payoff is 0.

Thus, her overall gain from deviation to not trading is $-\kappa_T$ (as given by (8)) in the case of feedback, and $-\kappa_{NF}$ (as given by (9)) in the case of no feedback.

A similar calculation shows that, if the positively-informed speculator deviates to not trading, her gross gain is $-\kappa_{SNB}$ ($\kappa_{SNB}>\kappa_T$) in the case of feedback and $-\kappa_{NF}$ in the case of no feedback. Thus, the trade equilibrium is sustainable if and only if $\kappa < \kappa_T$ in the case of feedback, and if and only if $\kappa < \kappa_{NF}$ in the case of no feedback.

We now turn to the range of parameter values in which BNS is the only pure strategy equilibrium in the case of feedback. If $\kappa_T \leq \kappa < \kappa_{NT}$, then the conditions for both the NT and T equilibrium to exist are violated. In addition, this is also the range where BNS equilibrium exists. We thus must derive conditions under which the SNB equilibrium does not hold, so that BNS is the unique equilibrium. There are two cases to consider. (i) If $\kappa_{SNB} \geq \kappa_{NT}$, the SNB equilibrium never exists, and so $\kappa_T \leq \kappa < \kappa_{NT}$ is sufficient for BNS to be the unique equilibrium. (ii) If $\kappa_T < \kappa_{SNB} \leq \kappa_{NT}$, the SNB equilibrium exists unless $\kappa < \kappa_{SNB}$. Thus, BNS is the unique equilibrium if $\kappa_T \leq \kappa < \kappa_{SNB}$. Combining the two cases gives the range, $\kappa_T \leq \kappa < \min(\kappa_{SNB}, \kappa_{NT})$, in the Proposition.

Proof of Lemma 2

For part (i), if $\theta = H$, the expected posterior is given by:

$$\begin{aligned} q^H &= (1-\lambda)\left[\frac{1}{3}q(-1) + \frac{1}{3}q(0) + \frac{1}{3}q(1)\right] + \lambda\left(\frac{1}{3}q(0) + \frac{1}{3}q(1) + \frac{1}{3}q(2)\right) \\ &= \frac{1-\lambda}{3}q(-1) + \frac{1}{3}q(0) + \frac{1}{3}q(1) + \frac{\lambda}{3}q(2) \\ &= \frac{(1-\lambda)^2}{6-3\lambda} + \frac{1}{3} + \frac{\lambda}{3}. \end{aligned} \tag{10}$$

We have:

$$\begin{aligned} \frac{\partial q^H}{\partial \lambda} &= \frac{1}{3} + \frac{1}{3}\left[\frac{-2(1-\lambda)(2-\lambda) + (1-\lambda)^2}{(2-\lambda)^2}\right] \\ &= \frac{1}{3}\left[1 + \left(\frac{1-\lambda}{2-\lambda}\right)^2 - 2\frac{1-\lambda}{2-\lambda}\right] \\ &= \frac{1}{3}\left[1 - \left(\frac{1-\lambda}{2-\lambda}\right)\right]^2 \\ &> 0. \end{aligned}$$

The expected posterior is increasing in λ: if the speculator is more likely to be present, she is more likely to impound her information into prices by trading.

Moving to part (ii), if $\theta = L$, we have:

$$\begin{aligned} q^L &= \frac{1}{3}\left(q(-1) + q(0) + q(1)\right) \\ &= \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}. \end{aligned} \tag{11}$$

This quantity is decreasing in λ. Even though the speculator does not trade upon $\theta = L$ if she is present, her information is still partially incorporated into prices. With $\theta = L$, there is a $\frac{1}{3}$

probability that the order flow is $X = -1$. This is consistent with the speculator being absent (in which case the state may be either H or L) or her being present and observing $\theta = L$; it is not consistent with the speculator observing $\theta = H$. The greater the likelihood that the speculator is present, the greater the likelihood that $X = -1$ stems from $\theta = L$, and thus the greater the decrease in the market maker's posterior. Part (iii) follows from simple calculations.

Proof of Proposition 2

For parts (i) and (ii), we have:

$$\begin{aligned} q^{H,spec} &= \frac{1}{3}\left(q\left(0\right)+q\left(1\right)+q\left(2\right)\right) \\ &= \frac{2}{3}, \end{aligned} \tag{12}$$

$$\begin{aligned} q^{L,spec} &= \frac{1}{3}\left(q\left(-1\right)+q\left(0\right)+q\left(1\right)\right) \\ &= \frac{1-\lambda}{6-3\lambda}+\frac{1}{3}. \end{aligned} \tag{13}$$

Note that $q^{H,spec}$ is independent of λ, but $q^{L,spec}$ is decreasing in λ. The variable λ can affect the expected posterior in two ways: first, it can change the relative likelihood of the different order flows, and second, it can change the actual posterior given a certain order flow. Since we are conditioning on the speculator being present, the first channel is ruled out: conditional on the speculator being present and $\theta = H$, $X \in \{0, 1, 2\}$ with uniform probability regardless of λ; conditional on the speculator being present and $\theta = L$, $X \in \{-1, 0, 1\}$ with uniform probability regardless of λ. Turning to the second channel, the only posterior that depends on λ is $q(-1)$: since $X = -1$ is inconsistent with the speculator being present and seeing $\theta = H$, it has a particularly negative impact on the likelihood of $\theta = H$ if the speculator is more likely to be present. In contrast, $X \in \{-2, 2\}$ is fully revealing and so the posterior is independent of λ; $X \in \{0, 1\}$ is completely uninformative and so the posterior is again independent of λ. Since $X = -1$ can only occur in the presence of a speculator if she has received bad news, only $q^{L,spec}$ depends on λ but $q^{H,spec}$ does not. Part (iii) follows from simple calculations.

Proof of Lemma 3

We start by calculating p_0. With probability $\frac{1}{2}$, the state will be $\theta = L$ and there is no trade, regardless of whether the speculator is present. Thus, $X \in \{-1, 0, 1\}$ with equal probability. With probability $\frac{1}{2}$, the state will be $\theta = H$. If the speculator is absent (w.p. $(1-\lambda)$), there is no trade and we again have $X \in \{-1, 0, 1\}$. If the speculator is present, $X \in \{0, 1, 2\}$. Letting $p(X)$ denote the stock price set by the market maker after observing order flow X at $t = 1$, the price at $t = 0$ will be the expectation over all possible future prices at $t = 1$, and is given as follows:

$$p_0 = \frac{\lambda}{2}\left(\frac{1}{3}p\left(0\right)+\frac{1}{3}p\left(1\right)+\frac{1}{3}p\left(2\right)\right)+\left(1-\frac{\lambda}{2}\right)\left(\frac{1}{3}p\left(-1\right)+\frac{1}{3}p\left(0\right)+\frac{1}{3}p\left(1\right)\right)$$
$$= \frac{1}{3}\left(\left(1-\frac{\lambda}{2}\right)p\left(-1\right)+p\left(0\right)+p\left(1\right)+\frac{\lambda}{2}p\left(2\right)\right)$$
$$= \frac{1}{6}\left[3R_H+3R_L-2c+2\lambda x\right]. \tag{14}$$

Even though the initial belief y is independent of λ, the initial stock price p_0 is increasing in λ, because the speculator provides information to improve the manager's decision.

For part (i), if $\theta = H$ is realized, the expected price at $t = 1$ is given by:

$$p_1^H = (1-\lambda)\left[\frac{1}{3}p\left(-1\right)+\frac{1}{3}p\left(0\right)+\frac{1}{3}p\left(1\right)\right]+\lambda\left(\frac{1}{3}p\left(0\right)+\frac{1}{3}p\left(1\right)+\frac{1}{3}p\left(2\right)\right)$$
$$= \frac{1-\lambda}{3}p\left(-1\right)+\frac{1}{3}p\left(0\right)+\frac{1}{3}p\left(1\right)+\frac{\lambda}{3}p\left(2\right)$$
$$= \frac{(3-\lambda)R_H+(3-2\lambda)(R_L+\lambda x)}{3(2-\lambda)}-\frac{c}{3}. \tag{15}$$

Note that:

$$\frac{\partial p_1^H}{\partial\lambda} = \frac{1}{3}p(2)-\frac{1}{3}p(-1)+\frac{1-\lambda}{3}\frac{\partial p(-1)}{\partial\lambda}$$
$$= \frac{R_H-R_L+2(3-4\lambda+\lambda^2)x}{3(2-\lambda)^2} > 0,$$

i.e., p_1^H is increasing in λ, since the speculator impounds information about the high state into prices.

Turning to part (ii), if $\theta = L$ is realized, the expected price at $t = 1$ is given by:

$$p_1^L = \frac{1}{3}\left(p\left(-1\right)+p\left(0\right)+p\left(1\right)\right). \tag{16}$$

We have $\frac{\partial p_1^L}{\partial\lambda} = \frac{R_L-R_H+2x}{3(2-\lambda)^2}$. If the speculator is more likely to be present, then $X = -1$ is more likely to result from $\theta = L$. Thus, the price is higher if and only if firm value is higher in this state, i.e., $R_L + x > R_H - x$ (Case 2).

The calculations of $p_1^H - p_0$ and $p_1^L - p_0$ follow automatically.

Proof of Proposition 3

For part (i), if the speculator receives positive information, she will buy one share and so the expected price becomes:

$$p_1^{H,spec} = \frac{1}{3}\left(p\left(0\right)+p\left(1\right)+p\left(2\right)\right). \tag{17}$$

Unlike p_1^H (equation (30)), this quantity is independent of λ, for the same reasons that $q^{H,spec}$ (equation (27)) is independent of λ. The stock return realized when the speculator receives good information is thus given by:

$$\begin{aligned}
p_1^{H,spec} - p_0 &= \frac{1}{3}\left(p\left(0\right)+p\left(1\right)+p\left(2\right)\right) - \frac{1}{3}\left(\left(1-\frac{\lambda}{2}\right)p\left(-1\right)+p\left(0\right)+p\left(1\right)+\frac{\lambda}{2}p\left(2\right)\right) \\
&= \frac{1}{3}\left(1-\frac{\lambda}{2}\right)\left(p\left(2\right)-p\left(-1\right)\right) \\
&= \frac{1}{6}\left(R_H - R_L + 2(1-\lambda)x\right) > 0, \qquad (18)
\end{aligned}$$

and we have

$$\frac{\partial\left(p_1^{H,spec} - p_0\right)}{\partial\lambda} = -\frac{1}{3}x < 0.$$

Equation (33) is decreasing in λ, whereas the stock return not conditioning on the speculator's presence, $p_1^H - p_0$, was increasing in λ. This reversal is because p_0 is increasing in λ, but $p_1^{H,spec}$ is independent of λ.

For part (ii), if the speculator is present and receives negative information, we have:

$$p_1^{L,spec} = \frac{1}{3}\left(p\left(-1\right)+p\left(0\right)+p\left(1\right)\right) = p_1^L, \qquad (19)$$

and

$$\begin{aligned}
p_1^{L,spec} - p_0 &= \frac{1}{3}\left(p\left(-1\right)+p\left(0\right)+p\left(1\right)\right) - \frac{1}{3}\left(\left(1-\frac{\lambda}{2}\right)p\left(-1\right)+p\left(0\right)+p\left(1\right)+\frac{\lambda}{2}p\left(2\right)\right) \\
&= \frac{\lambda}{6}\left(p\left(-1\right)-p\left(2\right)\right) = p_1^L - p_0 < 0.
\end{aligned}$$

Parts (iii) and (iv) follow from simple calculations.

Dropping constants, both equation (4) (the asymmetry between the price impact of good and bad news) and equation (5) (the average return, conditional on the speculator being present) become:

$$(1-\lambda)\left(\frac{R_H - R_L + 2(1-\lambda)x}{2-\lambda}\right).$$

Differentiating with respect to λ gives:

$$\frac{R_L - R_H - 2(3-4\lambda+\lambda^2)x}{(2-\lambda)^2} < 0.$$

Thus, both equations (4) and (5) are decreasing in λ.

Proof of Corollary 2

We start with part (i). If the speculator receives good news, she will buy and and the investment will be undertaken only if the noise trader buys. We thus have $p_2^{H,spec} = \frac{1}{3}(R_H +$

$x - c) + \frac{2}{3}R_H$. This observation yields:

$$p_2^{H,spec} - p_1^{H,spec} = R_H + \frac{1}{3}(x - c) - \frac{1}{3}\left(p\left(0\right) + p\left(1\right) + p\left(2\right)\right)$$
$$= \frac{1}{3}\left(R_H - R_L\right).$$

Moving to part (ii), if the speculator receives bad news, she will not trade. The firm reduces investment only if the noise trader sells. We thus have $p_2^{L,spec} = \frac{1}{3}(R_L + x - c) + \frac{2}{3}R_L$. This yields:

$$p_2^{L,spec} - p_1^{L,spec} = R_L + \frac{1}{3}(x - c) - \frac{1}{3}\left(p\left(-1\right) + p\left(0\right) + p\left(1\right)\right)$$
$$= \frac{(3 - 2\lambda)(R_L - R_H) + 2(1 - \lambda)x}{3(2 - \lambda)},$$

which is negative in Case 1, but can be positive or negative in Case 2. Part (iii) follows from simple calculations. For part (iv), we first calculate the expected firm value at $t = 2$ if the speculator is present, not conditioning on the state. If $\theta = H$, investment depends on the order flow: if $X = 2$, we have $d = 1$ and so firm value is $v = R_H + x - c$; if $X \in \{0, 1\}$, we have $d = 0$ and so $v = R_H$. If $\theta = L$, disinvestment depends on the order flow: if $X = -1$, we have $d = -1$ and so $v = R_L + x - c$; if $X \in \{0, 1\}$, we have $d = 0$ and so $v = R_L$. Expected firm value at $t = 2$ is thus given by:

$$p_2^{spec} = \frac{1}{2}(R_H + R_L) + \frac{1}{3}(x - c),$$

and so we have

$$p_2^{spec} - p_1^{spec} = \frac{1}{6}\frac{1 - \lambda}{2 - \lambda}(R_L - R_H + 2x),$$

which is positive if we are in Case 2 and negative if we are in Case 1.