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JEL Classification: C67, F12 and F63
Keywords: global supply chains, international trade and network effects

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A theory of trade in a global production network

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February 2014

Abstract

This paper develops a novel theory of trade in a global supply chain. We expand on a monopolistic competition trade model. Countries produce both intermediate and final goods that are sold domestically or, incurring country-pair specific trade costs, internationally. This links countries in a multi-stage production network. In the unique general equilibrium of the model, goods prices and wages in each country depend on the entire structure of trade connections. Drawing on methods from the social network literature, we then determine each country's importance in the global production network and analyse the welfare consequences of a further integration of the network. Our findings highlight the role of a few key countries that bring other nations closer together by intermediating their value added. Proximity to these key countries is crucial for other nations' income growth. An accompanying empirical analysis shows strong support in favor of the predicted network effects (JEL codes: C67, F12, F63).

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1 Introduction

Global supply chains are one of the defining characteristics of today's production processes. They carry many potential economic benefits. Most importantly, they allow countries to specialize in tasks in which they have comparative advantage. This increases the overall efficiency of production and the size of world welfare. However, it is not immediately

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clear that all countries benefit alike. In fact, increased production fragmentation might even hurt countries that do not manage to participate in one of the major global supply chains. Moreover, production fragmentation makes our economies more vulnerable to shocks hitting countries that play a key role in the world economy.[1]

This paper identifies the benefits and costs of production fragmentation within the confines of a general equilibrium model. Our point of departure is a model from the class of new "quantitative trade models" (Costinot and Rodríguez-Clare, 2013).[2] In particular, we expand on a monopolistic competition model of trade between an arbitrary number of countries that differ in terms of their productive capacity. Trade occurs in both intermediate and final products and involves country-pair-specific costs. The novel feature of our model is that it accommodates arbitrary degrees of production sharing between countries. In particular, the extent of fragmentation arises endogenously in our model. It depends on the size of a coordination cost that accrues, on top of the usual trade costs, when a firm uses a foreign intermediate good instead of a domestic factor for production.

We start by showing that the model admits a unique equilibrium in which all product and labor markets clear. Towards this end, we take advantage of the simple expressions for prices and trade volumes that our model yields for both the upstream and the downstream sectors. In fact, we are able to solve prices and quantities up to the still endogenous wage rates, which will remain the only implicitly defined variables in our model. To characterize the labor market equilibrium, we employ readily available results for Walrasian exchange economies to establish existence of a unique general equilibrium. Moreover, we show that this equilibrium admits the sort of comparative statics analysis that we are interested in: assessing the welfare effects of a country-specific, regional, or worldwide change in the ease of international production sharing. In fact, an important methodological contribution of our paper is that we develop the tools for comparative statics analysis for any initial trade cost configuration and any variation of the same. These tools are based on the natural link between our model and recent contributions in the social network literature (notably Ballester et al., 2006). This opens up an entire new set of possible counterfactual results beyond those provided in earlier trade models that typically only look at the move to global free trade or to autarky (e.g. Eaton and Kortum, 2002; Alvarez and Lucas, 2007) or assume all bilateral trade costs to be identical (e.g. Costinot et al., 2013). The main findings from

[1] Throughout the paper, we will use the terms global "supply chain", "production network", "fragmented production", and "shared production" interchangeably to denote a multi-stage, multi-country spanning production process leading to a final output.

[2] This is a class of trade models particularly suitable for empirical analysis. The seminal articles are Krugman (1980), Eaton and Kortum (2002), and Melitz (2003).

our analysis can be summarized as follows:

(i) Our model reveals a fundamental difference between trade in intermediate and final goods. Our expression for trade in final goods replicates the notion of prior trade theories that the "gravity" of a third country has a trade distorting effect (e.g. Anderson and van Wincoop, 2003). In contrast, the value of intermediate goods trade between any two nations increases in the size and productivity of a third country. Furthermore, a country's prices and trade volumes do not only depend on the trade costs along its direct routes to other nations, but also on the connections of those other nations to third countries. In fact, a country's access to foreign products in our model is related to a well-known power index from the social network literature, measuring the benefits an agent can draw from his or her entire network of peers (Katz, 1953; Bonacich, 1987). Combined, these findings already hint at the main message of this paper: in a global production network, a country's well-being does not primarily depend on its own state of technology or geographical location, but much more on the technology and geography of all countries that are part of the global supply chain.

(ii) Based on the link established to network theory, we introduce novel concepts into the realm of international trade. In particular, we extend the Ballester et al. (2006) concept of a "key player" and identify the importance of each country in the global supply chain by looking at how a removal of that country affects real incomes in all other nations. Our analysis shows that this does not only depend on the value added of the country's final and intermediate goods producers to the supply chain, but also on their roles as intermediaries for the valued added generated in other nations.

(iii) It has been argued that, by functioning as containers for foreign production technologies, the intermediate goods shipped between the members of a supply chain can mitigate country-specific productivity differentials. Consequently, production fragmentation might reduce world income inequality (Whittaker et al., 2010; Baldwin, 2011). We investigate this conjecture by considering a worldwide homogenous reduction of the coordination cost parameter and by identifying conditions on the matrix of pairwise trade costs under which one country catches up to another. Our findings suggest that a country unambiguously experiences a higher growth rate than a counterpart, if it has comparatively better access to important trade intermediaries (as defined under (ii)).

(iv) The finding of a trade-enhancing effect of a third nation's gravity on the flow of intermediate goods through the supply chain has interesting implications for trade policy. It suggests that negative third-country effects, such as concession erosion, are confined to countries trading only final products. We show that in an integrated supply chain a unilateral trade cost reduction typically results in a welfare increase in any other nation.

In sum, our model highlights the central factors that determine a country's welfare in a global production network. To get a feeling for whether these factors play a role in reality, we also take our model to the data and explore its predictions numerically for the real trade network of 2005. For this purpose, we develop an empirical strategy to estimate the model's main parameters and unobserved variables based on readily available data on bilateral trade cost components, trade flows, domestic output, technology proxies, and numbers of exporting firms. Our estimates show strong evidence in support of an integrated global supply chain and corroborate our theoretical predictions (ii)-(iv) numerically.

Of course, the significance of global supply chains has not gone unnoticed in academic circles. Already the early theories of Ethier (1979, 1982), and later Eaton and Kortum (2002), Yi (2003), Alvarez and Lucas (2007), and Baldwin and Venables (2013), have made clear that they have important implications for the sensitivity of national incomes to trade barriers and factor costs.[3] Also, it is well recognized since Krugman and Venables (1995) that cross-country production linkages shape the location of industries. Despite the valuable insights from these studies, they do not fully acknowledge the unique opportunities offered by global supply chains. Instead, the very same three factors are stressed that were already emphasized in earlier trade theories: a country's own state of technology, its own resources, and its own geographic location.[4]

More recent theoretical contributions have begun to explicitly analyse the novel welfare implications of production fragmentation. Triggered by the empirical study of Feenstra and Hanson (1996) much of this work has, however, focused on the economic fortunes of distinct groups of laborers within a nation (e.g. Antràs et al., 2006; Grossman and Rossi-Hansberg, 2008). Probably closest to our paper is a series of empirical studies following Hummels et al. (2001) that tries to attribute the value generated in a global supply chain to its constituent nations and sectors.[5] Our contribution to this literature is that we provide a sound theoretical foundation for the measures generated there (for more details, see Section 4.1).

[3]For recent empirical evidence on this literature, see Bems et al. (2011), Baldwin and Taglioni (2011), and Altomonte et al. (2012).

[4]One reason for this lies certainly in the methodological difficulties involved in solving an M-country general equilibrium model of trade. Being able to obtain closed-form solutions for all goods prices and trade volumes and for arbitrary trade cost configurations (up to the still endogenous wage rates), we can move a significant step ahead in this paper.

[5]Some recent studies along this line are Koopman et al. (2010), Antràs et al. (2012), Johnson and Noguera (2012), Baldwin and Lopez-Gonzales (2013), and Los et al. (2014).

On the theoretical side, Costinot et al. (2013) and Caliendo and Parro (2013) are recent exceptions that raise very similar questions as we do here. Costinot et al. (2013) study the endogenous sorting of countries into different stages of a global supply chain and the welfare effects of various technological shocks. The main difference to our paper is the following: while they investigate production fragmentation in a world without trade frictions, the focus of our paper is precisely on the implications of these frictions and on how changes in them can have effects that reverberate around the entire global production network. Caliendo and Parro (2013), on the other hand, use a setting similar to ours to investigate the welfare effects of NAFTA in the light of cross-border production linkages. The distinguishing methodological contribution of our paper is that we base our counterfactuals entirely on classic comparative statics analysis, rather than simulating some of the equilibrium equations. This enables us to (i) investigate the welfare consequences of various types of (hypothetical or real) changes in the trade cost matrix, (ii) to arbitrarily decompose the overall effects into e.g. supply and demand effects, or effects at different stages of the supply chain, and (iii) to derive several general propositions that highlight the consequences of a further integration of the world economy.

Finally, our paper is related in spirit and methodology to a growing literature emphasizing the consequences of interdependent decision making in social and economic networks.[6]

The remainder of the paper is organized in four sections. In Section 2, we present the theoretical model and derive its predictions concerning equilibrium trade volumes, prices, and income levels. Section 3 sets out our empirical strategy. Subsequently, in Section 4, we present our comparative static analysis. Besides deriving several counterfactual predictions analytically, we also calculate them numerically based on a combination of real world data and the estimates from Section 3. Section 5 concludes.

2 The Model

Consider a world of $i = 1, 2, ..., m$ countries, where each country i hosts a number of people, L_i, and where trade is subject to country-pair specific frictions. One category of products traded in our model are varieties of a final good that are used for consumption at home and abroad and that are each produced by a distinct monopolistic firm. These firms use domestic labor and varieties of an intermediate good in their production process.

[6]Goyal (2007) and Jackson (2008) summarize the state of the art of this literature. For two other recent articles on the relationship between the network structure of a national supply chain and macroeconomic outcomes, see Acemoglu et al. (2012) and Oberfield (2013).

The latter are produced by a distinct set of firms that sell both domestically and abroad. Moreover, as the intermediate goods producers themselves employ tradable intermediates, all countries are embedded in a deeply integrated global supply chain.

Final goods market: to set out the model, we begin with the demand for final manufactures. In line with much of the trade literature, we specify consumer preferences by the following Dixit and Stiglitz (1977) utility function:

$$U_i = \left[\sum_{z \in Z} \left(q_i^f(z) \right)^{(\sigma-1)/\sigma} \right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $q_i^f(z)$ depicts the quantity of the final goods variety $z \in Z$ consumed by an individual in country i and $\sigma > 1$ the elasticity of substitution between varieties. Both the set of varieties and σ are assumed to be common across individuals and countries. Given the assumed structure of preferences and costs (see the next subsection), the prices of all goods shipped from the same exporting nation to the same destination will be identical in equilibrium. Thus, we write more conveniently q_{ji}^f for the quantity of a typical consumption good imported from nation j and n_j^f for the number of final goods producers in j.

A consumer maximizes utility under the constraint that expenditures must not exceed w_i, the uniform wage rate of country i. Standard calculations show that indirect utility can be written as $U_i = w_i/P_i^c$, where P_i^c depicts the consumer price index:

$$P_i^c \;\; = \;\; \left[\sum_{j \in M} n_j^f (p_{ji}^f)^{1-\sigma} \right]^{1/(1-\sigma)} \tag{2}$$

Here, p_{ji}^f represents the profit-maximizing sales price of a typical producer from country j in country i. Throughout, we maintain the Dixit and Stiglitz (1977) assumption that the total number of producers is large so that the price index is inelastic with regard to changes in individual producer prices. Also, it is the firms that bear the trade costs when selling to a foreign destination. These costs accrue in Samuelson's iceberg form such that for q_{ji}^f units to be sold in country i, $\tau_{ji}^f q_{ji}^f$ units need to be shipped, where $\tau_{ji}^f \geq 1$. Thus, the profit-maximizing sales prices is given by $p_{ji}^f = (\sigma/(\sigma-1))(P_j^f/\kappa_j)\tau_{ji}^f$, where (P_j^f/κ_j) depicts the constant marginal production cost of a producer (specified in the next subsection) and $\sigma/(\sigma-1)$ the markup over costs. Moreover, the total export revenues of all firms from country j exporting to country i can be written as:

$$X_{ji}^f = n_j^f p_{ji}^f q_{ji}^f L_i = n_j^f \big(\frac{\sigma}{\sigma-1}\frac{P_j^f}{\kappa_j}\big)^{1-\sigma}(\tau_{ji}^f)^{1-\sigma} L_i w_i (P_i^c)^{\sigma-1} \tag{3}$$

We will call this equation henceforth the *final goods trade equation*. It states that a country's export revenue earned in country i increases in the size of the importer market, $L_i w_i$, and the productivity and the size of the exporter industry, $n_j^f(P_j^f/\kappa_j)^{1-\sigma}$. On the other hand, revenues deteriorate in trade cost, τ_{ji}^f, and in the importer's access to final goods from third nations, which is captured by the augmented price index $(P_i^c)^{1-\sigma}$.

Intermediate goods market: Next, we turn to the producer demand for domestic production factors and domestic and foreign intermediate inputs. Here, we assume that each country hosts separate intermediate and final goods industries. Moreover, unlike prior models in international economics, every (final and intermediate goods) producer operates with a CES production function subsuming all input factors under a single aggregator.[7] Specifically, to produce $Q_i^f > 0$ units of a final goods variety, a producer requires inputs according to:

$$l_i \geq 0, \big(q_{ji}^i \geq 0\big)_{j\in M} \quad \text{such that: } Q_i^f + \bar{Q}^f = \kappa_i \Big[l_i^{(\sigma-1)/\sigma} + \theta^f \sum_{j\in M} n_j^i (q_{ji}^i)^{(\sigma-1)/\sigma}\Big]^{\sigma/(\sigma-1)} \tag{4}$$

where $\bar{Q}^f > 0$ denotes a fixed amount of inputs required to get production started. A similar expression holds for an intermediate goods producer, where superscript f is substituted by i. Labor l_i is the sole domestic production factor in our model, whereas q_{ji}^i denotes the amount of intermediate goods purchased from one of the n_j^i upstream producers in country j. The parameter $\sigma > 1$ captures the elasticity of substitution among the production factors and $\kappa_i > 0$ denotes the total factor productivity in country i. For simplicity, we assume that σ is identical in both sectors and the same elasticity as in utility function (1).[8]

The central parameters of our model are θ^f and θ^i, which both satisfy $0 \leq \theta < 1$. They

[7] Earlier contributions typically assume nested production functions with labor and a CES aggregate of intermediates as the two inputs in a Cobb-Douglas technology (e.g. Krugman and Venables, 1995; Eaton and Kortum, 2002; Yi, 2003). As we see below, our different specification allows us to derive some closed-form solutions for equilibrium prices and outputs.

[8] We have also solved variants of our model where the elasticities in (1) and (4) are not the same and where the elasticities of (4) are sector-specific. The drawback of these models is that the empirical estimation and counterfactual analysis are significantly more complicated without adding any additional interesting insights.

measure the productivity of a (foreign) intermediate good relative to that of domestic labor. In fact, a meaningful interpretation of (4) is that producers can outsource parts of their production and that θ^f and θ^i reflect the additional coordination costs that accrue when a firm incorporates intermediate inputs from another producer into its own production process.[9] To be more precise, θ^f and θ^i reflect a worldwide homogeneous coordination cost component. The overall cost of using a foreign input is additionally determined by a country-pair specific component introduced below. To motivate our distinction between separate parameters per sector (θ^f and θ^i), prior research has shown that some production tasks lend themselves more easily to outsourcing than others (Leamer and Storper, 2001; Autor et al., 2003).[10] A second motivation stems from the meaningful interpretation of the two cases when θ^i is zero and when it is strictly positive, respectively. When $\theta^i = 0$ (and $\theta^f > 0$) only final goods producers use intermediate inputs and our supply chain consists of only two production stages. On the other hand, when $\theta^i > 0$ also the upstream firms use intermediate goods. And because their suppliers again use the inputs from other firms, our model captures in this way a supply chain of infinite length. As such, one could also interpret θ^i as a continuous measure for the *depth of the international supply chain*, where the value added at each production stage is inversely related to θ^i.

Taking their production technology as a given, the producers acquire a cost-minimizing input combination, whereby all intermediate and final goods producers from a certain country have access to the same input market. Standard calculations show that for a final goods manufacturer (and again similarly for an intermediate goods producer) the price index for the cost-minimizing input bundle is given by:

$$P_i^f = \left[w_i^{1-\sigma} + (\theta^f)^\sigma \sum_{j \in M} n_j^i (p_{ji}^i)^{1-\sigma} \right]^{1/(1-\sigma)} \tag{5}$$

[9] This interpretation is common in the literature on international supply chains and offshoring. According to Baldwin (2011), the reduction in the costs to coordinate distinct production processes was spurred by major breakthroughs in ICTs in the 1980s and key to the rise of international production fragmentation. Grossman and Rossi-Hansberg (2008) argue that for the very same reason also the costs of offshoring under the same ownership declined. Moreover, they argue that from the standpoint of a perfect competition model it does not make a difference whether a firm can more easily import inputs from a foreign firm or offshore its production to that country. However, this isomorphy between offshoring and foreign sourcing does not go through in our monopolistic-competition model, as offshoring avoids the price markup of a foreign firm at the expense of an additional setup cost.

[10] Assuming that final goods assembly consists of more routine tasks, whereas intermediate goods production is a more complicated, skill-intensive process, one would expect that $\theta^f > \theta^i$. Yet, we do not impose any direction of the inequality in our model.

where w_i depicts the domestic wage rate, and p^i_{ji} the price of a typical intermediate input from country j. P^f_i relates directly to the total production cost, C^f_i, of a final goods producer:

$$C^f_i = \frac{P^f_i}{\kappa_i}(Q^f_i + \bar{Q}^f) \tag{6}$$

The wage rate in (5) is endogenously determined in the domestic labor market and (as we show in the next subsection) our model admits a unique, implicitly defined equilibrium wage rate per country. The input prices, on the other hand, are determined in the international goods markets. In the following, we show that our production side specification allows for closed form solutions for the price index (5), individual input prices as well as quantities (up to the implicitly defined wage rates). Let us first remark, however, that our model admits three distinct price indices per country: the consumer price index, P^c_i, and according to our distinction between θ^f and θ^i, separate indices for intermediate and final goods producers, P^f_i and P^i_i.

To determine the equilibrium prices and quantities, note that by applying Shephard's Lemma to (6) the demand in country i for the intermediate goods produced by a typical producer from country j is given by:

$$(p^i_{ji})^{-\sigma}\left[n^f_i(\theta^f)^\sigma \frac{(P^f_i)^\sigma}{\kappa_i}\left(Q^f_i + \bar{Q}^f\right) + n^i_i(\theta^i)^\sigma \frac{(P^i_i)^\sigma}{\kappa_i}\left(Q^i_i + \bar{Q}^i\right)\right] \tag{7}$$

Let also the intermediate goods markets be monopolistically competitive, such as the final goods markets, the producer price indices be inelastic to individual prices, and transportation costs for intermediate goods of the iceberg form given by $\tau^i_{ji} \geq 1$. Then, the profit-maximizing sales price in country j becomes $p^i_{ji} = (\sigma/(\sigma-1))(P^i_j/\kappa_j)\tau^i_{ji}$. Moreover, based on (7) we can derive a *trade equation for intermediate goods* measuring the export revenues of country j's intermediate goods industry in country i:

$$\begin{aligned} X^i_{ji} = n^i_j\left(\frac{\sigma}{\sigma-1}\frac{P^i_j}{\kappa_j}\tau^i_{ji}\right)^{1-\sigma} \quad & \Bigg[\quad (\theta^f)^\sigma (P^f_i)^{\sigma-1}\left(\frac{\sigma-1}{\sigma}\sum_{k\in M} X^f_{ik} + n^f_i \frac{P^f_i}{\kappa_i}\bar{Q}^f\right) \\ & + \quad (\theta^i)^\sigma (P^i_i)^{\sigma-1}\left(\frac{\sigma-1}{\sigma}\sum_{k\in M} X^i_{ik} + n^i_i \frac{P^i_i}{\kappa_i}\bar{Q}^i\right)\Bigg] \end{aligned} \tag{8}$$

The equation nicely formalizes a distinctive feature of trade in a global supply chain. The demand for inputs from country j does not only depend on the size of the importer market

i, but also on the size of that country's own export markets and their distance to country i (reflected in $\sum_{k\in M} X_{ik}^{f}$ and $\sum_{k\in M} X_{ik}^{i}$). Ceteris paribus, the larger country i's export markets and the closer they are to i, the larger country i's demand for inputs from j. Hence, equation (8) suggests that the gravity of third nations has a trade-enhancing effect on the intermediate goods flows between i and j, rather than the distorting effect that is at the heart of some prior trade theories (e.g. Anderson and van Wincoop, 2003) and that can also be found in our final goods trade equation (3).[11] The order and magnitude of the trade-enhancing effect crucially hinges upon the size of the parameters θ^f and θ^i. In particular, if $\theta^i > 0$ the effect is of a higher order, because country j's inputs are incorporated in the intermediate goods shipped out of country i to be employed by country i's own trade partners. The larger $\theta^i > 0$, i.e. the deeper the global supply chain, the stronger the higher order effect is.

We continue the equilibrium characterization by exploiting the fact that the system of trade equations (8) and producer price indices (5) needs to clear in the global market. We present here the solution to the market-clearing price indices. The solution for the trade equations is given in equation (33) in the appendix. Substituting the profit-maximizing price, p_{ij}^{i}, into the price index of an intermediate goods producer, similar to (5), and taking both sides to the power of $(1-\sigma)$ we obtain the following:

$$(P_i^i)^{1-\sigma} = w_i^{1-\sigma} + \big(\frac{\sigma}{\sigma-1}\big)^{1-\sigma}(\theta^i)^{\sigma}\sum_{j\in M}(P_j^i)^{1-\sigma} n_j^i \kappa_j^{\sigma-1}(\tau_{ji}^i)^{1-\sigma} \tag{9}$$

The equation highlights the often encountered interdependence between the producer price indices in different nations, when production processes cross at least a single border (e.g., Krugman and Venables, 1995; Eaton and Kortum, 2002; Alvarez and Lucas, 2007). Given the specific functional form of the formerly studied price indices, an explicit solution was considered impossible, because of their non-linear interdependence. However, as becomes clear from (9), by virtue of our CES specification of production function (4) we obtain a linear equation system. This allows us to express the row vector of price indices in vector

[11] This exchange-enhancing effect is common in information diffusion theories on social networks, where the value of an information exchange tie between two individuals increases in the size of the network surrounding each one of the two (Jackson and Wolinsky, 1996; Bala and Goyal, 2000).
The corresponding equation (8) in the variant of our model with distinct substitution elasticities for final goods σ and intermediate goods γ highlights another trade-enhancing effect. As a price reduction by any single upstream producer from country j reduces the unit costs at all subsequent production stages, this increases the demand for all the individual inputs of the composite good, and hence even for other countries operating on the same stage of the value chain as country j.

nation as $(P^i)^{1-\sigma} = w^{1-\sigma} + (P^i)^{1-\sigma} A$, where matrix A is defined as:

$$A = \big(\frac{\sigma}{\sigma-1}\big)^{1-\sigma} (\theta^i)^\sigma N^i K^{\sigma-1} (T^i)^{1-\sigma} \tag{10}$$

and where $(\sigma/\sigma-1)^{1-\sigma}$ and $(\theta^i)^\sigma$ are scalars, N^i and $K^{\sigma-1}$ diagonal matrices with n_i^i-, respectively $\kappa_i^{\sigma-1}$-entries along their diagonals, and $(T^i)^{1-\sigma} = \big((\tau_{ij}^i)^{1-\sigma}\big)$ the full matrix of elasticity augmented trade costs for intermediate goods. Applying Neumann's series expansion for matrix inversion we get:[12]

$$(P^i)^{1-\sigma} = w^{1-\sigma} \big[I - A\big]^{-1} = w^{1-\sigma} \sum_{h=0}^{\infty} A^h \tag{11}$$

where I denotes the identity matrix. Let the ij'th entry in matrix A^h be denoted by $a_{ij}^{[h]}$ for any $h \geq 1$, where $a_{ij}^{[1]} = a_{ij}$ denotes a cell in matrix $A^1 = A$ and $A^0 = I$. Then, entry j of vector (11) can be written as:

$$(P_j^i)^{1-\sigma} = w_j^{1-\sigma} + \sum_{i \in M} w_i^{1-\sigma} \sum_{h=1}^{\infty} a_{ij}^{[h]} \tag{12}$$

and where we often refer to $SA_j^i \equiv \sum w_i^{1-\sigma} \sum_{h=1}^{\infty} a_{ij}^{[h]}$ as country j's *supplier access* to domestic and foreign intermediate inputs (Redding and Venables, 2004). Expanding on this, we also find closed-form solutions for the trade-elasticity augmented price indices of country j's final goods producers and consumers:

$$\begin{aligned} (P_j^f)^{1-\sigma} &= w_j^{1-\sigma} + SA_j^f = w_j^{1-\sigma} + \big(\frac{\theta^f}{\theta^i}\big)^\sigma SA_j^i \\ (P_j^c)^{1-\sigma} &= \sum_{i \in M} (P_i^f)^{1-\sigma} b_{ij} \end{aligned} \tag{13}$$

where b_{ij} denotes the ij'th entry in matrix:

$$B = \big(\frac{\sigma}{\sigma-1}\big)^{1-\sigma} N^f K^{\sigma-1} (T^f)^{1-\sigma}. \tag{14}$$

[12]Neumann's expansion requires that $\lim_{h\to\infty} A^h = 0$ for the inverse of matrix $I-A$ to have the functional form (11). By the spectral radius theorem, this is equivalent to requiring that the real components of all eigenvalues of A are strictly smaller than one and larger than minus one. A sufficient condition for this is that $(\frac{\sigma}{\sigma-1})^{1-\sigma}(\theta^i)^\sigma \sum_{j\in M} n_j^i \kappa_j^{\sigma-1} (\tau_{ji}^i)^{1-\sigma} < 1$ and $(\frac{\sigma}{\sigma-1})^{1-\sigma}(\theta^i)^\sigma n_i^i \kappa_i^{\sigma-1} \sum_{j\in M} (\tau_{ij}^i)^{1-\sigma} < 1$ for all $i \in M$. Thus, it is sufficient to have an upper bound on the parameter of interest θ^i.

To add meaning to the terms (12) and (13), the inverse matrix $[I - A]^{-1} = \sum_{h=0}^{\infty} A^h$ has long been of interest to regional and development economists who, dating back to the seminal work by Wassily Leontief (Leontief, 1936), studied the flow of factor content in a national supply chain. It has also received great attention in the sociological literature on power relations, where it is interpreted as a measure of the influence an actor can exert in his or her social network (Katz, 1953; Bonacich, 1987). Our interpretation combines these two views. In particular, note that cell ij, $j \neq i$, in this matrix can be written as:

$$\sum_{h=0}^{\infty} a_{ij}^{[h]} \; = \; a_{ij} \; + \; a_{ik} \sum_{k \in M} a_{kj} \; + \; a_{ik_1} \sum_{k_1 \in M} a_{k_1 k_2} \sum_{k_2 \in M} a_{k_2 j} \; + ...$$

Combined with (10), the interpretation is as follows: every input-producing nation contributes with its labor force to the productivity in other nations through the supply of intermediate inputs. The value added of a firm from country i is $w_i^{1-\sigma}$. Its output is used by all foreign manufacturers: some of them employ it directly, while others use it indirectly, embodied in the intermediate products of yet another firm. The matrix $[I - A]^{-1}$ keeps track of all the direct and indirect linkages across countries through which the goods flow, and its entry ij reflects the intensity with which the value added of country i is used in country j. The summand a_{ij} reflects the intensity of a direct link, which is inversely related to the pair-specific transportation cost, the producer price markup, and the level of coordination costs, but which increases in the number of producers and the productivity in country i. For $h > 1$, the term $a_{ij}^{[h]}$ displays the strength of an indirect path between two countries, where a path of length h is a connection via $h - 1$ other countries.

In other words, (12) and (13) reflect the idea that the output of a producer in our model is best labeled *"Made in the World"* (WTO, 2011). Moreover, the terms make clear that the well-being of a country is determined by factors that go beyond the size of its own export markets and the productivity of its immediate input suppliers.

Labor market and general equilibrium: Up to now we have characterized a partial equilibrium in the markets for tradable manufactures and determined the market clearing goods prices and quantities. We now turn to the labor market, pin down the equilibrium wage rate, and thereby close the entire economy.

We assume that workers are immobile between countries but free to move between the intermediate and final goods sectors, so that each country has a uniform wage rate w_i. As becomes clear from production function (4), labor is employed per unit of output and additionally to produce the required fixed amount of inputs $\bar{Q}$. Similar to (7), one can derive a firm's labor demand function and, based on this, express the labor cost share per unit of output as $w_i l_i/(w_i l_i + \sum_j n_j^i p_{ji}^i q_{ji}^i) = w_i^{1-\sigma}/(P_i)^{1-\sigma}$. From this, it follows that national labor income is made up of:

$$w_i L_i \;=\; \frac{w_i^{1-\sigma}}{(P_i^f)^{1-\sigma}} n_i^f C_i^f + \frac{w_i^{1-\sigma}}{(P_i^i)^{1-\sigma}} n_i^i C_i^i$$

where C_i^f and C_i^i are defined in (6). Another source of labor income are firm profits (or losses) given by:

$$n_i^f \pi_i^f + n_i^i \pi_i^i = \frac{1}{\sigma} \sum_{j \in M} X_{ij}^f - n_i^f \frac{P_i^f}{\kappa_i} \bar{Q}^f + \frac{1}{\sigma} \sum_{j \in M} X_{ij}^i - n_i^i \frac{P_i^i}{\kappa_i} \bar{Q}^i \tag{15}$$

To keep things simple, we assume that these profits accrue to a country's domestic work force according to the country's labor cost share, whereas the remaining profits go to the (domestic and foreign) suppliers of intermediate inputs (again, according to their respective cost shares). Thus, we add $(w_i^{1-\sigma}/(P_i^f)^{1-\sigma}) n_i^f \pi_i^f + (w_i^{1-\sigma}/(P_i^i)^{1-\sigma}) n_i^i \pi_i^i$ to (15).[13] Making use of trade equations (3) and (8), we thus obtain the following equation for total labor income:

$$w_i L_i \;=\; \frac{w_i^{1-\sigma}}{(P_i^f)^{1-\sigma}} \sum_{j \in M} X_{ij}^f + \frac{w_i^{1-\sigma}}{(P_i^i)^{1-\sigma}} \sum_{j \in M} X_{ij}^i \tag{16}$$

We are now in the position to define an equilibrium for our model as follows:

Definition 1. *For any transportation cost matrices (T^f, T^i), the tuple $(p^f, p^i, q^f, q^i, l, w)$ constitutes an equilibrium, if it satisfies (2), (3), (12), (13), and (33) as well as the implicit function defined by (16).*

[13] We have experimented with alternatives to this assumption, where either (a) profits are paid out to a separate group shareholders or (b) N^f and N^i are endogenous variables of the model and entry takes place until the marginal firm breaks even. Specification (a) bears the problem that shareholder profits depend on foreign incomes. Hence, regardless of whether they are saved or added to national expenditures on final manufactures in equation (3), this generates another higher-order interdependency in trade flows, next to the one that is at the heart of this paper. This makes things unnecessary complicated without adding any new insights. The problem with (b) is that, to the best of our knowledge, the international economics literature has not yet provided a general equilibrium characterization for an m country monopolistic-competition model with free entry and arbitrary pairwise trade frictions, even for a model with a single sector. All existing contributions (e.g. Negishi, 1972; Kehoe, 1985; Allen and Arkolakis, 2013; Arnold, 2013) have either just focussed on market-clearing wage rates, as we do, allowed for international labor mobility, or considered worldwide free trade.

Concerning the equilibrium characterization, we do have closed-form solutions for goods prices and quantities (see (13) and (33)), but are unable to provide analytical solutions for the equilibrium wage rates. However, following Alvarez and Lucas (2007) we can show that, under a mild condition on the trade intensity matrix A, a unique equilibrium exists:

Theorem 1. *Suppose that* $\lim_{h\to\infty} A^h = 0$. *There is a unique equilibrium satisfying Definition 1 and this equilibrium admits comparative statics analysis with regard to changes in* (T^f, T^i).

The proof can be found in the appendix. There, we show that the sufficient conditions for existence and uniqueness of a Walrasian equilibrium are met. The statement then follows from Propositions 17.C.1, 17.F.3, and 17.G.3 of Mas-Collel et al. (1995, p. 585, 613, 618).

In the remainder of the paper, we exploit this result and conduct several comparative statics analyses for our equilibrium. An important novelty of our paper with regard to the existing literature (Eaton and Kortum, 2002; Alvarez and Lucas, 2007; Costinot et al., 2013; Caliendo and Parro, 2013) is that we are able to derive comparative statics results for any initial transportation cost matrix and any marginal variation of the same. This will be appealing to the reader who is interested in less extreme counterfactual scenarios than the previously offered predictions for a world without any trade frictions, or in predictions that do not rely on simulating (some of) the equilibrium equations. Our analysis will shift the focus from the individual characteristics of a country to its embeddedness in the world production network as a central determinant of its well-being. To do this, we introduce several concepts and measures from the social network literature into the realm of trade theory.

Besides deriving several general propositions, we complement our analytic predictions by numerically exploring counterfactual situations for a real trade network. To do this, we first estimate our model to get numerical equivalents for important model parameters and variables that are not readily available in existing macroeconomic datasets. With our estimates in hand, we then turn to our comparative statics results in Section 4.

3 Empirical framework

In this section, we set out an empirical strategy that, based on the structure dictated by our model, provides us with estimates of the main parameters and unobserved variables. Of particular interest are the two parameters θ^f and θ^i. They each capture the productivity discount incurred when using an intermediate product instead of a unit of domestic labor in the production of final and intermediate goods, respectively. θ^i is of particular interest as it can be interpreted as a measure for the depth of the global supply chain.

Our starting point is the intermediate goods trade equation (8). We rewrite the equation into logarithmic form and, in the absence of data on each country's use of domestically produced intermediates, substitute X^i_{ii} by its theoretical equivalent using (8).[14] Moreover, we make use of the additional assumption that a firm's profits are absorbed by its input suppliers in proportion to their cost shares (see below equation (15)). This leads to the following estimation equation:

$$\begin{aligned}\ln X^i_{ij} &= \ln\Phi - \ln\left[1-\Phi(\theta^i)^\sigma n^i_j \kappa_j^{\sigma-1}(\tau^i_{jj})^{1-\sigma}\right] + \ln\left[n^i_i (P^i_i)^{1-\sigma}\kappa_i^{\sigma-1}\right] \\ &+ \ln(\tau^i_{ij})^{1-\sigma} + \ln\left[(\theta^f)^\sigma (P^f_j)^{\sigma-1}\sum_{k\in M} X^f_{jk} + (\theta^i)^\sigma (P^i_j)^{\sigma-1}\sum_{k\neq j} X^i_{jk}\right]\end{aligned} \tag{17}$$

where $\Phi = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$. Still, direct identification of θ^f and θ^i from this equation is difficult. Even though we could capture the constant and the i-specific term in (17) by a full set of exporter dummies, we miss crucial pieces of information: the sector-specific price indices, P^i_j and P^f_j, trade costs, τ_{ij}, total factor productivity, κ_j, and the elasticity of substitution, σ. To overcome this problem, we propose a two-step procedure, which is very similar in spirit to the empirical implementations by Redding and Venables (2004) and Eaton and Kortum (2002). Our procedure relies on readily available data on intermediate and final goods trade flows (UN COMTRADE), observable trade cost components (CEPII), domestic output (WDI), productivity (PWT8.0), and the number of exporting firms by sector for a subset of nations (EUROSTAT).[15] Finally, we take different values for σ based

[14] $X^i_{ii} = \Phi\frac{(P^i_i)^{1-\sigma}n^i_i\kappa_i^{\sigma-1}(\tau^i_{ii})^{1-\sigma}}{1-(\theta^i)^\sigma\Phi n^i_i\kappa_i^{\sigma-1}(\tau^i_{ii})^{1-\sigma}}\left[(\theta^f)^\sigma(P^f_i)^{\sigma-1}\sum_{k\in M}X^f_{ik} + (\theta^i)^\sigma(P^i_i)^{\sigma-1}\sum_{k\neq i}X^i_{ik}\right]$. Contrary to X^i_{ii}, we can readily infer X^f_{ii} from the macroeconomic identity $X^f_{ii} = GDP_i - \sum_{k\neq i}\left[X^f_{ik} + X^i_{ik} - X^i_{ki}\right]$.

[15] We use the BEC classification of UN COMTRADE to distinguish between final and intermediate goods trade flows. Our definition of final goods corresponds to the BEC class consumption goods. For our intermediate goods flows, we add the BEC class capital goods to the UN's original definition of intermediates. The remaining non-classified goods are omitted.
To match the EUROSTAT data on exporting firms, which are classified according to the NACE industry classification, with UN COMTRADE's BEC classes, we make use of readily available concordance tables for NACE-SITC3 and SITC3-BEC. This creates many unique matchings for the complete set of NACE classes. In the few cases of multiple matches, we assume that each firm in such a NACE class produces all related BEC products.

on estimates from the existing empirical trade literature.

Step 1: We use trade equations (3) and (8) to obtain estimates for P_j^f, P_j^i, and for τ_{ij}^i. Towards this end, we follow common practice in the literature and capture the bilateral trade costs by a function of their observable components:

$$\tau_{ij}^i = (d_{ij})^{\delta^i} \exp(t^i \Lambda_{ij}), \qquad \tau_{ij}^f = (d_{ij})^{\delta^f} \exp(t^f \Lambda_{ij}) \tag{18}$$

where d_{ij} is the distance between countries i and j and Λ_{ij} is a vector of four other factors influencing trade costs: sharing a common border, language, or colonizer, and having (had) a colony-colonizer relationship. δ^i, t^i, δ^f and t^f are estimated. They determine the importance of each trade cost component in overall trade costs.

Next, we substitute (18) for τ_{ij}^i and rewrite equation (8) by expressing each country i's exports to country j relative to that of a reference exporter R:[16]

$$\ln \frac{X_{ij}^i}{X_{Rj}^i} = \underbrace{\ln \frac{n_i^i (P_i^i)^{1-\sigma} \kappa_i^{\sigma-1}}{n_R^i (P_R^i)^{1-\sigma} \kappa_R^{\sigma-1}}}_{s_i^i} + (1-\sigma)\left[\delta^i \ln \frac{d_{ij}}{d_{Rj}} + t^i(\Lambda_{ij} - \Lambda_{Rj})\right] + \ln \left[\frac{\epsilon_{ij}^i}{\epsilon_{Rj}^i}\right] \tag{19}$$

for all i, j with $i \neq R$ and $j \notin \{i, R\}$. The error terms, ϵ_{ij}^i, capture any i.i.d. measurement error in bilateral trade flows. We obtain a similar equation for final goods trade flows:

$$\ln \frac{X_{ij}^f}{X_{Rj}^f} = \underbrace{\ln \frac{n_i^f (P_i^f)^{1-\sigma} \kappa_i^{\sigma-1}}{n_R^f (P_R^f)^{1-\sigma} \kappa_R^{\sigma-1}}}_{s_i^f} + (1-\sigma)\left[\delta^f \ln \frac{d_{ij}}{d_{Rj}} + t^f(\Lambda_{ij} - \Lambda_{Rj})\right] + \ln \left[\frac{\epsilon_{ij}^f}{\epsilon_{Rj}^f}\right] \tag{20}$$

for all i, j with $i \neq R$ and $j \notin \{i, R\}$.

Estimating (19) and (20) with the help of UN COMTRADE and CEPII data, we obtain the empirical equivalents of our elasticity-augmented trade costs, $\widehat{(\tau_{ij}^f)^{1-\sigma}}$ and $\widehat{(\tau_{ij}^i)^{1-\sigma}}$. They are based on the observed cost components and their corresponding estimated coefficients. Moreover, by including a full set of exporter dummies in (19) and (20), we get estimates for each country's "competitive (dis-)advantage" in intermediate and final goods production over the reference country R: $\hat{s}_i^i$ and $\hat{s}_i^f$ for all $i \neq R$ and $\hat{s}_R^i = \hat{s}_R^f = 1$.

Step 2: Equipped with the results from step 1, which are summarized in Table 5 in the appendix, we return to equation (17). We substitute $\widehat{(\tau_{ij}^i)^{1-\sigma}}$ for $(\tau_{ij}^i)^{1-\sigma}$ and replace

[16] We take Germany as a reference country. This choice is based on the fact that, in our sample, German bilateral trade flows are best covered.

the unobserved augmented price indices of the importing country j by functions of their corresponding competitiveness expressions, $\hat{s}^i_j$ and $\hat{s}^f_j$, i.e. $(P^i_j)^{\sigma-1} = \frac{n^i_j \kappa_j^{\sigma-1}}{n^i_R (P^i_R)^{1-\sigma} \kappa_R^{\sigma-1} \hat{s}^i_j}$ and similarly for $(P^f_j)^{\sigma-1}$. This clearly shows that our estimated $\hat{s}^i_j$ and $\hat{s}^f_j$ from step 1 are not sufficient to fully capture the two price indices. We also need information on the number of trading firms in each importing country, n^i_j and n^f_j, and on the country's productivity, κ_j. Contrary to the two price indices, however, information on these variables is available.

We capture a country's productivity by a log linear function of its human capital index, $\kappa_j = \zeta \ln h_j$, where h_j is obtained for 121 countries from PWT8.0. Data on the number of trading firms is unfortunately not available for that many countries. We use the best available data on the number of exporting firms in 19 European countries (EUROSTAT). For this reason, our second step estimation is based on a restricted sample of all intermediate goods flows into one of these countries.[17]

Making all the substitutions outlined above, gives us the following estimation equation:

$$\begin{aligned}
\ln X^i_{ij} - \ln \hat{s}^j_j - \ln \widehat{(\tau^i_{ij})^{1-\sigma}} \;=\; & \ln\left[\Theta^f \left(\frac{n^f_j (\ln h_j)^{\sigma-1} n^i_R}{\hat{s}^f_j n^f_R} \sum_{k\in M} X^f_{jk}\right) + \Theta^i \left(\frac{n^i_j (\ln h_j)^{\sigma-1}}{\hat{s}^i_j} \sum_{k\neq j} X^i_{jk}\right)\right] \\
& - \ln\left[1 - \Phi\Theta^i n^i_j (\ln h_j)^{\sigma-1} \widehat{(\tau^i_{jj})^{1-\sigma}}\right] + \Phi + \ln \epsilon^i_{ij} \qquad (21)
\end{aligned}$$

for all i, j where j is one of the 19 European countries. We estimate (21) using nonlinear least squares, imposing the necessary parameter restriction on Θ^i. Two important notes remain before presenting our results. First, (21) clearly shows that we cannot separately identify our parameters of interest, Θ_i and Θ_f, as well as σ. We therefore preimpose a value for σ and, based on the estimates from the existing literature, take $\sigma = 5$ as our baseline, but also show results for $\sigma = 3$ and $\sigma = 8$. Second, even after fixing σ, our empirical equivalences of θ_i and θ_f are still unavoidably inflated by two factors:[18]

$$\Theta^f = (\theta^f)^\sigma \left(\zeta \frac{P^f_R}{P^i_R}\right)^{\sigma-1}, \qquad \Theta^i = (\theta^i)^\sigma \zeta^{\sigma-1} \qquad (22)$$

[17]Note however that under the maintained assumptions of our structural model (notably taking θ_i and θ_f to be homogenous across countries), the assumed functions for bilateral trade costs and countries' productivity, and of an i.i.d. measurement error, restricting our sample this way still gives us consistent estimates for θ_i and θ_f.

[18]One reason for this is that our estimates of s^i_i and s^f_i obtained are always *relative* to their corresponding value in the baseline country, R. The other is our assumption that each country's productivity can be captured by a log linear function of its human capital index.

Table 1: STEP 2 - ESTIMATING THE COORDINATION COST PARAMETERS

	Identified coefficients				$\sigma = 3$		$\sigma = 5$		$\sigma = 8$	
assumed:	$\sigma = 3$	$\sigma = 5$	$\sigma = 8$	P_R^f/P_R^i	$\hat{\theta}^f$	$\hat{\theta}^i$	$\hat{\theta}^f$	$\hat{\theta}^i$	$\hat{\theta}^f$	$\hat{\theta}^i$
$\hat{\Theta}^f$	0.0044***	0.0039***	0.0030***	0.75	0.20	0.17	0.42	0.34	0.62	0.49
	(0.00021)	(0.00019)	(0.00015)	1	0.16	0.17	0.33	0.34	0.48	0.49
$\hat{\Theta}^i$	0.0052***	0.0046***	0.0034***	1.25	0.14	0.17	0.28	0.34	0.40	0.49
	(0.00022)	(0.00019)	(0.00014)							
	observations	2127								

NOTES: The table shows the results from our estimation of equation (21). In all calculations of $\hat{\theta}^i$ and $\hat{\theta}^f$ we set $\zeta = 1$. Data stems from UN COMTRADE, CEPII, EUROSTAT, WDI, and PWT8.0. Our sample comprises only the export flows into 19 European countries for which we have information on the number of trading firms. Bootstrapped standard errors in parentheses, taking account of the fact that we use generated regressors in the second step of our empirical strategy. They are generated by randomly drawing (with replacement) 200 different samples of bilateral intermediate and final goods trade flows. For each of these samples we then estimate Θ^i and Θ^f using our 2-step procedure. The standard errors of the resulting 200 different estimates for Θ^i and Θ^f respectively, are reported in the table. *** p<0.01, ** p<0.05, * p<0.10.

As a result, we cannot identify θ^i and θ^f without imposing additional assumptions on ζ and P_R^f/P_R^i. However, a very important reason not to do so is that $\hat{\Theta}^i$ and $\hat{\Theta}^f$ already provide us with all the necessary information to conduct our counterfactual analyses in the next sections. These analyses do not depend in any way on the particular assumptions made on ζ and P_R^f/P_R^i.

Table 1 shows the estimation results. They lend strong support to our theory. First, irrespective of our assumption on σ, both $\hat{\Theta}^i$ and $\hat{\Theta}^f$ are significantly positive, which implies that θ_i and θ_f are positive as well (unless ζ is negative, which is highly unlikely). The significance of $\hat{\Theta}^i$ in particular suggests that a substantive share of intermediate goods passes multiple borders before being finally transformed into a final output: clear evidence of production fragmentation. Moreover, for the values assumed for P_R^f/P_R^i and ζ in Table 1, the implied θ^i and θ^f are both smaller than 1, suggesting that our model is well-specified. The estimated parameter values for θ^i and θ^f also suggest that there are still considerable coordination costs involved in the use of foreign intermediates. Moreover, they imply that the conditions for existence of an equilibrium are met (see Theorem 1), justifying the comparative statics analysis that we will do in the remainder of the paper.

4 Counterfactual analysis

We perform various comparative statics variations of the trade cost matrices (T^f, T^i) and investigate the implications around the unique equilibrium point in our model. The purpose of this analysis is to highlight the importance of the entire network structure of the global supply chain for the well-being of the embedded nations. We derive general results whenever possible. When this is not the case, we make use of the estimates from the previous section and study the counterfactual numerically for the trade network of 2005. In a nutshell, we start from the 2005 situation assuming that the world economy is in equilibrium. Then, we vary a model parameter, let prices and quantities adjust according to the equations of our model, and assess the welfare implications of the shock. Throughout, we distinguish between the effects on the supply of goods in every nation i, assuming flexible commodity prices but $dw_i = 0$ for all $i \in M$, and the effects on labor demand and thus wages.

We assess all comparative statics effects in terms of real labor income per capita, $U_i = w_i/P_i^c$. Since we hold population sizes fixed and assume full employment, there is no need to distinguish between income per worker or per capita and total labor income. Also, because workers absorb all profits, real income is equivalent to national welfare.

The effect of a trade cost shock on real labor income can be written as:

$$\begin{aligned} \ln \frac{U_i'}{U_i} &= \ln \frac{(w_i)'}{w_i} + \frac{1}{\sigma - 1} \ln \frac{\left((P_i^c)^{1-\sigma}\right)'}{(P_i^c)^{1-\sigma}} \\ &\approx \underbrace{\frac{dw_i}{w_i} + \frac{1}{\sigma - 1} \sum_{j \in M} \left[\frac{\partial (P_i^c)^{1-\sigma}}{\partial w_j} \frac{dw_j}{(P_i^c)^{1-\sigma}} \right.}_{\text{demand effect}} + \underbrace{\left. \frac{\partial (P_i^c)^{1-\sigma}}{\partial p_j} \frac{dp_j}{(P_i^c)^{1-\sigma}} \right]}_{\text{supply effect}} \end{aligned} \tag{23}$$

where x_i' denotes the counterfactual value of a variable x_i and $dx_i = x_i' - x_i$. In the second line, we decompose the total effect into a *supply effect* and a *labor demand effect*. The former reflects the immediate consequences of a trade cost variation for the prices of consumer goods in country i. The latter comprises the indirect effects on labor demand, and thus nominal income, dw_i/w_i, as well as any further price changes imposed by these wage adjustments.

Before we move to the results, it will be crucial to understand how a shock to (T^f, T^i) affects the inverse *trade intensity* matrix for intermediate goods, $[I - A]^{-1}$. The following lemma, which generalizes a central result of Ballester et al. (2006), shows that we can relate the effects of various types of shocks to the initial state of $[I - A]^{-1}$:

Lemma 1. *Consider square matrices A and A', such that $\lim_{h\to\infty} A^h = 0$ and $\lim_{h\to\infty}(A')^h = 0$. For scalars $x, y \in \Re$ and $A' = I_{xi} A I_{yi}$, where I_i denotes the matrix with a one in cell ii and zero everywhere else, $I_{xi} = (I + xI_i)$, and $I_{yi} = (I + yI_i)$, it holds:*

$$(i)\ \sum_{h=1}^{\infty} \big(I_{xi} A I_{yi}\big)^h - I_{xi}\big(\sum_{h=1}^{\infty} A^h\big) I_{yi} \quad = \quad \frac{(x+y+xy) I_{xi}\big(\sum_{h=1}^{\infty} A^h\big) I_i \big(\sum_{h=1}^{\infty} A^h\big) I_{yi}}{1 - (x+y+xy)\sum_{h=1}^{\infty} a_{ii}^{[h]}}$$

whereas for $z \in \Re$ and $A' = (1+z)A$:

$$(ii)\ \sum_{h=1}^{\infty}(1+z)^h A^h - \sum_{h=1}^{\infty} A^h \quad = \quad z\sum_{h=0}^{\infty}(1+z)^h A^h \sum_{h=1}^{\infty} A^h.$$

The proof is delegated to the appendix. We exploit property (i) in the following shock sensitivity analysis, and in our analysis of a unilateral trade cost reduction in Section 4.3. Property (ii) is of use in Section 4.2 where we investigate the welfare effects of a global coordination cost reduction.

4.1 Shock sensitivity: key players in the global supply chain

Several recent events have made clear that our world economy is vulnerable to idiosyncratic shocks hitting any one nation. The consequences of the tsunami before the coast of Japan in March 2011 for example, or Thailand's flooding of September 2012, were not only borne by the afflicted nations themselves, but also by their trading partners who suffered significant disruptions in their production processes. Also, the worldwide recession and the excessive contraction of trade volumes in the aftermath of the US subprime mortgage crisis was, according to several experts, exacerbated by the ubiquity of international production linkages (Bems et al., 2011).

In this section, we aim to predict the welfare consequences of events like these with the help of our model.[19] The following question lies at the heart of our analysis: how sensitive is the world economy to an idiosyncratic demand and production shock in any single nation? Or, put differently, how dependent is the global production network on some key countries?

To answer these questions, we draw on concepts developed in a growing literature on

[19] Another example of such shocks is civil unrest. Political demonstrations, strikes, or in the worst case outright civil war can result in significant drops in a country's productive capacity. Finally, trade embargoes can effectively shut off nations from participating on world markets.

the robustness of social and economic networks.[20] In particular, in analogy to Ballester et al. (2006) we quantify the sensitivity of the world production network by identifying the *key player nation*, i.e. the country that when removed causes the largest welfare drop in all other countries.[21] To stress the impact of such a shock, we remove an entire nation – the demand from all its inhabitants and all its productive capacities – and calculate the real income losses incurred in the remaining nations.[22] Formally, denote by $((T^f)^{-i}, (T^i)^{-i})$ the trade network obtained from (T^f, T^i) after removing country i from it. The key player nation is the country satisfying:

$$i^* \;=\; \arg\min_{i \in M} \left[\sum_{j \neq i} \ln \frac{U_j((T^f)^{-i}, (T^i)^{-i})}{U_j(T^f, T^i)} \right] \tag{24}$$

Hence, we define the key player to be the country with the largest contribution to the real income of a representative inhabitant in every other nation. By summing up the welfare losses for a subset of nations, one could alternatively also identify the key player for a pre-defined world region.

Our ambition is to distinguish distinct channels through which world welfare is reduced. Let us begin by looking at the *supply side effects* on commodity prices, as defined in (23), and assume $dw_i = 0$ for all $i \in M$. Let us furthermore define matrix $d[I-A]^{-1} = [I - A^{-i}]^{-1} - [I-A]^{-1}$ to capture the changes in the trade intensities for intermediate goods along all paths of length $h \geq 1$ between any two countries, and let $-\sum_{h=1}^{\infty} a_{j(i)k}^{[h]}$

[20] See Albert et al. (2008); Goyal and Vigier (2010); Hoyer and De Jaegher (2010); Acemoglu et al. (2012).

[21] We should mention that the concept of a key player is not entirely new in the regional and international economics literature. The importance of key firms and sectors was already at the heart of early applications of Wassily Leontief's input-output analysis, for example for the French development plans of the 1950s (Paelinck et al., 1968). More recently, a series of papers has advanced the tools of I-O analysis to identify the contribution of a country/sector within an international supply chain (Hummels et al., 2001; Johnson and Noguera, 2012; Antràs et al., 2012). However, unlike those earlier approaches, our analysis is grounded in a general equilibrium framework. Moreover, based on the properties derived in Lemma 1, our Key Player formula allows for various types of analytic decompositions and experiments. We discuss the relationship to these earlier studies below in more detail.

[22] Alternatively, one could remove a fixed percentage of demand and supply from the afflicted nation. The techniques for both experiments are developed in Part (i) of Lemma 1: the shock to country i corresponds to a modification of row i and column i in the trade intensity matrix for final goods B and intermediate goods $[I-A]^{-1}$. In our experiment, where we remove an entire country, we set $x = y = -1$. Removing only part of a country corresponds to $-1 < x = y < 0$, but the direction of the effects and the relative impact on different nations are very similar.

denote an entry in this matrix. According to Property (i) of Lemma 1, it holds:

$$-\sum_{h=1}^{\infty} a_{j(i)k}^{[h]} = -\frac{\sum_{h=1}^{\infty} a_{ji}^{[h]} \sum_{h=1}^{\infty} a_{ik}^{[h]}}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \tag{25}$$

for any jk, with $j \neq i$ and $k \neq i$, whereas $-\sum_{h=1}^{\infty} a_{j(i)k}^{[h]} = -\sum_{h=1}^{\infty} a_{jk}^{[h]}$ for $j = i$ or $k = i$. Based on this, formula (24) can be written as:

$$\begin{aligned} i^* &\approx \arg\min_{i \in M} \left[\sum_{j \neq i} \left(\sum_{k \in M} \frac{\partial (P_j^c)^{1-\sigma}}{\partial p_k} \frac{dp_k}{(P_j^c)^{1-\sigma}} \right) \right] \\ &= \arg\min_{i \in M} \left[\sum_{j \neq i} \frac{-(P_i^f)^{1-\sigma} b_{ij} - \sum_{k \neq i} d_i S A_k^f b_{kj}}{(P_j^c)^{1-\sigma}} \right] \\ &= \arg\min_{i \in M} \left[\sum_{j \neq i} -\underbrace{\tilde{X}_{ij}^f}_{(i)} - \sum_{k \neq i} \left(\frac{\tilde{X}_{kj}^f}{(P_k^f)^{1-\sigma}} (\frac{\theta^f}{\theta^i})^{\sigma} \big(\underbrace{w_i^{1-\sigma} \sum_{h=1}^{\infty} a_{ik}^{[h]}}_{(ii)} \right.\right. \\ &\qquad \left.\left. + \underbrace{\sum_{l \neq i} w_l^{1-\sigma} \frac{\sum_{h=1}^{\infty} a_{li}^{[h]} \sum_{h=1}^{\infty} a_{ik}^{[h]}}{\sum_{h=0}^{\infty} a_{ii}^{[h]}}}_{(iii)} \big) \right) \right] \end{aligned} \tag{26}$$

where based on (3) and (13) we define:

$$\tilde{X}_{ij}^f \equiv \frac{X_{ij}^f}{\sum_{k \in M} X_{kj}^f} = \frac{(P_i^f)^{1-\sigma} b_{ij}}{(P_j^c)^{1-\sigma}} \tag{27}$$

Formula (26) shows that welfare in all other nations unambiguously declines after the removal of any nation i due to rising consumer prices. More importantly, however, it highlights three distinct channels underlying this price increase: (i) the lost access to the final manufactures from country i and (ii/iii) the lost access to the intermediate goods from country i affecting producer productivity in all other countries $k \neq i$. The latter channel can be further decomposed into (ii) the foregone access to the value added by country i's intermediate goods industry and (iii) the lost access to the value added of the producers from countries $l \neq i$ that was incorporated in the intermediate goods from country i and passed on by that country before the shock. Thus, the formula stresses the idea that countries can take in distinct roles in the world economy and contribute to world welfare in principally three different ways. In particular, the importance of a country not only derives from the productivity and abundance of its domestic production factors and/or its own centrality in the world production network. Countries can also be of mere systemic importance, i.e. they act as important intermediaries connecting other nations. This final point is highlighted in the following proposition, which is a variant of a key result in Ballester et al. (2006):

Proposition 1. *Suppose that $dw_i = 0$ for all $i \in M$. Suppose further that $\theta^f = \theta^i$. Then, the identity of the key player nation i^* is determined by its inter-centrality (Ballester et al., 2006, p. 1411) in the world trade network:*[23]

$$i^* \approx \arg\max_i \left[\frac{(P_i^i)^{1-\sigma} \big(\sum_{k \in M} \sum_{h=0}^{\infty} a_{ik}^{[h]} \sum_{j \neq i} b_{kj} (P_j^c)^{\sigma-1} \big)}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \right]. \tag{28}$$

The proof can be found in the appendix. When wages are flexible the identity of the key player nation additionally depends upon its influence on the demand for domestic labor in the remaining nations. The wage adjustments can be formally determined by the total differential of the system of wage equations (16), which we show in equation (44) of the appendix. Similar to (26), we can distinguish three distinct channels: (iv) the breakdown of the demand for final products from the removed nation i, (v) the foregone opportunity to ship intermediate products to nation i, which it passes on to the rest of the world, and (vi) the lost competition with the former value added by country i itself. While the first two effects clearly put the demand for labor in the remaining nations under pressure, effect (vi) actually reflects the fact that countries might benefit from 'jumping into' the gap left behind by their removed competitor.

Because the effects (i)-(vi) go in opposite directions, it is hard to make general statements about the identity of the overall key player nation. However, by making use of our estimates from Section 3 we can obtain numerical predictions for these effects for the real trade network of 2005. Before we move on to our findings, let us briefly discuss the relationship of our key player analysis to alternative approaches of the recent literature (Hummels et al., 2001; Johnson and Noguera, 2012; Antràs et al., 2012). Like the measures developed there, formula (26) can also be interpreted as a way to decompose the

[23] Formula (28) departs from the original inter-centrality measure, $\frac{\big(\sum_{k \in M} \sum_{h=0}^{\infty} a_{ki}^{[h]} \big) \big(\sum_{j \in M} \sum_{h=0}^{\infty} a_{ij}^{[h]} \big)}{\sum_{h=0}^{\infty} a_{ii}^{[h]}}$, in two minor respects: (i) it is based on a weighted Katz-Bonacich centrality index of the removed nation i, $(P_i^i)^{1-\sigma} = \sum_{j \in M} w_j^{1-\sigma} \sum_{h=0}^{\infty} a_{ji}^{[h]}$, as well as a weighted index for the recipient nations j, $\sum_{k \in M} \sum_{h=0}^{\infty} a_{ik}^{[h]} \sum_{j \neq i} b_{kj} (P_j^c)^{\sigma-1}$; and (ii) our measure disregards the impact of the shock to country i itself.

trade flows in a supply chain in order to identify the contribution of a certain country to it. Related to the "upstreamness" measure of Antràs et al. (2012), our distinction between effects (i) and (ii/iii) allows us to investigate whether a country is positioned more at the top or at the bottom of the global supply chain. Moreover, like in Hummels et al. (2001) and Johnson and Noguera (2012), our decomposition into effects (ii) and (iii) provides the means to track down the value added of a country to a complex supply chain with reciprocal input-output relationships. In comparison to these measures, a disadvantage of our key player formula is the coarseness of measurement as it is based on a simple model with a stylized supply chain, whereas the prior measures can be applied to realistic supply chains involving multiple sectors.[24] However, there are two major advantages to our approach. First, the theoretical foundation of formula (24), combined with the fact that we have not decomposed quantity flows but value flows according to their origin, gives our decomposition a straightforward interpretation: how important is a country for other nations' welfare based on one of the six presented channels. Second, unlike the earlier measures, our decomposition does not only apply to the observed input-output linkages in a supply chain. It also allows us to ask the counterfactual question: is a country really indispensable or can other countries fill its position? Based on our model, the key player formula (24) takes commodity and factor substitution into account.

Table 2 shows the 15 countries whose removal is predicted to cause the largest average welfare loss in all remaining countries. We also show two different decompositions of the overall welfare change. The first focuses on the importance of a country at different stages of the global supply chain (effects (i)-(vi) as distinguished above). The second decomposition instead distinguishes between the supply and demand effect set out in (23). Not surprisingly the USA, the large European economies, and China top the overall ranking, followed by the important emerging economies (BRICS, Thailand). Also, even the removal of the USA, the overall Key Player, results in a mere 2% welfare loss in other nations, which one could take as an indication that today's well-integrated global economy is not that dependent on any single nation.[25]

[24] We should note, however, that our model readily lends itself to extensions involving more than two sectors.

[25] Note that this calculation does not take the much larger welfare loss in the removed country itself into account. Also, we do not want to stress this interpretation too much. It strongly depends on the assumption of our model that each intermediate (final) is in principle an imperfect substitute for any other intermediate (final) good. These numbers are similar in magnitude to those found in other papers relying on a CES-production structure (see e.g. Arkolakis et al., 2012; Caliendo and Parro, 2013)

Table 2: TOP 15 KEY PLAYERS

		Supply chain stages				Supply vs. Demand	
Rank	Overall Δ Utility (%)	Final goods (i)+(iv)	Local VA (ii)	Intermediated VA (iii)+(v)	Competition (vi)	Supply (i)-(iii)	Demand (iv)-(vi)
1	USA (-2.04)	USA (-1.75)	DEU (-0.44)	BEL (-0.28)	DEU (0.23)	CHN (-0.9)	USA (-1.39)
2	DEU (-0.95)	CHN (-0.77)	USA (-0.36)	NLD (-0.23)	USA (0.2)	DEU (-0.86)	GBR (-0.32)
3	GBR (-0.86)	DEU (-0.54)	FRA (-0.27)	DEU (-0.19)	CHN (0.17)	USA (-0.65)	JPN (-0.18)
4	CHN (-0.83)	GBR (-0.54)	GBR (-0.26)	GBR (-0.18)	FRA (0.14)	FRA (-0.62)	RUS (-0.14)
5	FRA (-0.75)	FRA (-0.51)	CHN (-0.22)	USA (-0.12)	GBR (0.13)	GBR (-0.54)	FRA (-0.13)
6	ITA (-0.54)	ITA (-0.41)	ITA (-0.19)	FRA (-0.11)	ITA (0.09)	ITA (-0.45)	ESP (-0.11)
7	BEL (-0.4)	JPN (-0.24)	JPN (-0.07)	SGP (-0.06)	BEL (0.09)	NLD (-0.34)	ITA (-0.1)
8	NLD (-0.37)	ESP (-0.24)	SWE (-0.06)	CHE (-0.06)	NLD (0.09)	BEL (-0.34)	DEU (-0.09)
9	JPN (-0.3)	BEL (-0.22)	MYS (-0.05)	LUX (-0.05)	ESP (0.04)	THA (-0.18)	BEL (-0.07)
10	ESP (-0.26)	NLD (-0.22)	IND (-0.05)	IRL (-0.05)	SWE (0.03)	IND (-0.17)	ZAF (-0.05)
11	RUS (-0.24)	RUS (-0.19)	RUS (-0.05)	ITA (-0.04)	JPN (0.03)	ESP (-0.15)	CAN (-0.05)
12	IND (-0.19)	THA (-0.17)	KOR (-0.04)	AUT (-0.04)	CHE (0.03)	BRA (-0.13)	LUX (-0.05)
13	THA (-0.18)	IND (-0.16)	ESP (-0.04)	CAN (-0.03)	CAN (0.03)	JPN (-0.12)	SAU (-0.05)
14	BRA (-0.15)	BRA (-0.15)	TUR (-0.04)	CZE (-0.03)	IND (0.03)	SWE (-0.12)	AUS (-0.04)
15	ZAF (-0.12)	ZAF (-0.12)	FIN (-0.03)	SWE (-0.03)	DNK (0.02)	TUR (-0.11)	IRL (-0.03)
$\bar{\mu}$ - all	-0.07	-0.05	-0.02	-0.01	0.01	-0.07	-0.001

NOTES: The numbers in the cells represent average percentage losses in real income for the remaining 120 countries in our data, when removing country i from the trade network of 2005, $(\frac{1}{m-1}\sum_{j\neq i}\ln U'_j/U_j)\times$ 100%. To do our quantitative 'Key Player Analysis', we need to fix the numerical value for the elasticity of substitution parameter σ. The table shows the results for $\sigma = 5$. Also, not all countries report information on the number of final and intermediate goods producers. For those countries we proxy the number of final goods producers by multiplying a country's observed $\ln\left[\sum_{k\neq j} X^f_{jk}\right]$ by the estimated coefficients β_0 and β_1 obtained from running a simple linear regression $\ln n^f_j = \beta_0 + \beta_1 \ln\left[\sum_{k\neq j} X^f_{jk}\right] + \mu_i$ using all countries j reporting total exports and n^f_j. We proxy the number of intermediate goods producers similarly using $\ln\left[\sum_{k\neq j} X^i_{jk}\right]$. The results of this regression are available upon request.

It is much more interesting to look at the two decompositions of the overall welfare effects. Our first split shows that the losses to the intermediate goods producing sector, (ii),(iii), and (v), account, on average, for a nontrivial 40% of the overall welfare loss (i)-(v). Moreover, two-thirds of this loss in the upstream sector results from the foregone access to the local value added of the removed country; the other third stems from losing the country as an intermediary of other countries' value added. Finally, the competition effect (vi) shows that about 15% of the overall welfare loss is mitigated by the remaining countries' ability to fill the gap left by the removed country.

These averages do hide substantial heterogeneity in terms of the roles that individual countries take in the global supply chain. China's, and most other emerging markets', importance primarily stems from their roles as final goods exporters. These countries play a much smaller role in the upstream market. Here the developed economies top the ranking. Some of these countries even derive most of their importance for the global supply chain by their value added to the upstream sector (e.g. Germany, Malaysia, South Korea, Sweden, and Finland), while others primarily intermediate other nations' value (The Netherlands, Belgium, Singapore, Luxemburg, and Ireland).

Our second split complements these findings by showing that, on average, the lost supply of goods from the removed country almost completely determines the overall welfare loss in other nations. This average however hides an important difference between emerging and developed economies. The former's importance indeed predominantly stems from their role in world supply. For the developed economies this is much less the case. Their overall importance stems for a nontrivial part from their demand for foreign products ('Key Player' USA and Japan stand out here).

4.2 Income inequality

Our next counterfactual analysis investigates the interesting conjecture that the emergence of a globally integrated supply chain might lead to a convergence of national income levels (Whittaker et al., 2010; Baldwin, 2011). As the argument goes, it is easier to join a supply chain than to build one altogether, which was the only way for a developing country to compete with the industrialized nations about thirty years ago. In the twenty first century, a country only needs to contribute incremental value to an existing supply chain in order to make its products an export success. Low wage countries, for example, can specialize in the assembly of parts. A second related argument is that the proliferation of intermediate goods trade enables every nation to take advantage of the advanced technologies developed in other parts of the world. Put differently, intermediate goods are 'containers for foreign technologies' that help to equalize productivity differentials around the globe.

Does the emergence of a global supply chain inevitably lead to income convergence? And if not, how does this depend on the position of a country in the world production network? Our model allows us to look at this question from a general equilibrium perspective and to compare the relative welfare gains (or losses) across nations.

Motivated by the idea that the benefits from production fragmentation crucially depend on the cost of coordinating a geographically dispersed production processes (Grossman and Rossi-Hansberg, 2008; Baldwin, 2011), we approach these questions by considering an exogenous increase of θ^f and θ^i. Yet, to contrast our findings, we first begin by exploring the relative gains from a worldwide cost reduction for final goods shipments.[26] This exercise is directly comparable to earlier analyses on the gains from a global trade cost reduction (Krugman, 1980; Eaton and Kortum, 2002; Arkolakis et al., 2012). The crucial difference here is that we single out the effects of a cost reduction for final goods shipments, but leave the trade costs for intermediate goods unchanged:

Proposition 2. *Consider a homogenous transportation cost reduction for final goods shipments, such that $(\tau^f_{ij})' = \delta\tau^f_{ij}$ for all $ij \in T^f$ and $0 < \delta < 1$. For any two $i, j \in M$ it holds:*

$$\ln \frac{U_i(\delta T^f)}{U_i(T^f)} - \ln \frac{U_j(\delta T^f)}{U_j(T^f)} = \frac{1}{\sigma - 1}\left[\ln \frac{\left((P^c_i)^{1-\sigma}\right)'}{(P^c_i)^{1-\sigma}} - \ln \frac{\left((P^c_j)^{1-\sigma}\right)'}{(P^c_j)^{1-\sigma}}\right] = 0.$$

The result is proven in the appendix. A first surprising insight is that the welfare gains from this cost reduction are solely determined by the immediate effect on consumer prices. Labor demand, on the other hand, and hence wages are entirely unaffected. Moreover, the result states that all countries gain to an exactly equal extent. The intuition for the first part is that the final goods producers from all nations gain from an improved access to their overseas (and domestic) markets. As the cost reduction is proportional to their original level of transportation costs, each firm gains in proportion to its original market share so that no one attains any competitive advantage.[27] However, consumers from all

[26] To see why this is useful, note that our coordination cost parameters can also be interpreted as a homogenous component in the intermediate goods transport cost matrix T^i.

[27] Hence, the proposition corroborates one of the major insights of Krugman and Venables (1995) and Puga (1999) that the agglomeration forces, which are at the heart of their analysis, are due to geographically distinct effects of a global cost reduction on the profitability of the intermediate goods producing sector. In their models, this in turn leads to a concentration of the intermediate goods sector in a single location.

nation attain access to cheaper products, whereby according to the proposition the price effect is proportional to the initial level of a country's price index.

We move on to investigate the effects of changes in θ^f and θ^i. A coordination cost reduction directly improves the producer access to intermediate inputs (13). The more interesting question is which country benefits most, when considering the implications for consumer prices and for labor demand. Like in Section 4.1, we begin with the supply effect outlined in (23) and assume that wages stay put.

Based on Property (ii) of Lemma 1, a marginal increase in θ^f and θ^i has the following effect on consumer prices in any country $i \in M$:

$$\sum_{k\in M} \frac{\partial (P_i^c)^{1-\sigma}}{\partial p_k} \frac{dp_k}{(P_i^c)^{1-\sigma}} = \sum_{j\in M} \Bigg[\underbrace{\frac{SA_j^f}{(\theta^f)^\sigma}}_{(i)} + \underbrace{\frac{(\theta^f)^\sigma}{(\theta^i)^{2\sigma}} \sum_{k\in M} SA_k^i \Big(\sum_{h=1}^{\infty} a_{kj}^{[h]} \Big)}_{(ii)} \Bigg] (P_j^f)^{\sigma-1} \tilde{X}_{ji}^f \tag{29}$$

Cearly, all consumers benefit. Yet, they do so only in an indirect way through the improved access of their final goods suppliers, located in countries $j \in M$, to the intermediate inputs of their own suppliers. Two channels are at work: (i) the associated increase in θ^f affects the direct trade connections between country j's producers and their input suppliers. This effect is stronger the more favorable country j's supplier access from the outset. (ii) The increase in θ^i, on the other hand, triggers a higher-order effect, as it improves the flow of intermediate inputs along the entire supply chain. As shown in (29), not only the direct trade connections of country j, but much more its indirect connections become relevant for prices in country i. Or, to put it differently, an increase in θ^i is in the advantage of a country i which is closely linked to suppliers that take in the role of important intermediaries in the global supply chain.

The following result shows that the higher-order effect is of such importance for the development of prices that a country's comparative advantage in the access to intermediaries is the sole determinant of whether it will keep up with the rest of the world, or not:

Proposition 3. *Suppose that $dw_i = 0$ for all $i \in M$. Suppose further that $\theta^f = \theta^i$. Then, for a marginal $d\theta > 0$:*

$$\begin{aligned} & \ln \frac{U_i(\theta + d\theta)}{U_i(\theta)} > \ln \frac{U_j(\theta + d\theta)}{U_j(\theta)} \\ \Leftrightarrow \quad & \frac{\sum_{k\in M} \big(\sum_{l\in M} SA_l^i \sum_{h=0}^{\infty} a_{lk}^{[h]} \big) b_{ki}}{(P_i^c)^{1-\sigma}} > \frac{\sum_{k\in M} \big(\sum_{l\in M} SA_l^i \sum_{h=0}^{\infty} a_{lk}^{[h]} \big) b_{kj}}{(P_j^c)^{1-\sigma}} \end{aligned}$$

The proof is delegated to the appendix. When wages are flexible the relative income change additionally depends upon the effect of a worldwide cost reduction on countries' competitive positions in the intermediate goods market, and hence on the room for wage increases or the need for necessary cuts. The direction and the magnitude of the wage adjustments are determined by the total differential of labor income equation (16), which we present in (44) in the appendix. Just like with the supply side effects, the increase in θ^f will favor different nations than a similar increase in θ^i. The former triggers a first-order effect, which is in the advantage of a nation with an initially superior access to sales markets for its intermediate products (a superior *market access* in the terminology of Redding and Venables, 2004). The effect of θ^i is again of a higher order and improves in a country's initial access to important trade intermediaries. However, unlike for consumer prices where a country gains from the access to intermediaries on the supply side, what matters for wages is the access to countries that help boost a country's exports. A final difference between the wage and price effects is that some countries might actually need to accept a wage cut, because of the intensified competition in their sales markets. As a consequence, a coordination cost reduction might actually result in absolute welfare losses for some nations.

A general characterization of the direction and the size of wage adjustments proves to be difficult, because of the opposing effects mentioned above. Nevertheless, equipped with our empirical estimates from Section 3, we can predict the wage changes for the trade network of 2005. They are summarized in Table 3. It shows the countries that benefit most/least from a 10% increase in θ^f and θ^i. Besides showing the overall welfare effects, we also decompose them into the supply and the demand effects according to (23).

Overall, the 10% increase results in an average income growth of 6.6% per country. Much more interesting for our purposes, however, are the large differences observed between countries. They tend to support the notion that a further integrating global supply chain can indeed result in an income catch-up for the currently poorest nations. In fact, almost all countries benefit more from such a development in comparison to the USA.

Our split into supply and demand effects reveals that for many countries this catch-up is primarily explained by falling consumer goods prices. The extent of this benefit is, on the one hand, determined by the supply effect (see Proposition 3 and column 2). On the other hand, it depends on how much worldwide changes in labor demand and thus wages ripple through on consumer prices (column 5). As shown in column (2), developing countries with good access to final goods suppliers that are themselves well-connected benefit most from

Table 3: Welfare effects of a further deepening supply chain

		Supply	Demand		
Rank	(1) Overall		Δ wages		(5) Δ prices
	Δ Utility (%)	(2) θ_f & θ_i	(3) θ_f	(4) θ_i	θ_f & θ_i
1	MYS (61.6)	KHM (41.2)	IRQ (12.5)	MYS (39.2)	MDV (15.3)
2	KHM (44.7)	BWA (33.3)	QAT (11.2)	THA (3.1)	BEN (9.4)
3	BWA (24)	MDV (27.5)	TGO (10.7)	DEU (2.1)	NOR (3.4)
4	THA (18.7)	BEN (20.6)	BEN (10.2)	IND (1.6)	TTO (2.4)
5	KAZ (16.6)	MNG (17)	MYS (9.6)	CHN (0.9)	CYP (1.7)
6	ALB (15.8)	ALB (15.8)	NER (9.2)	FRA (0.6)	BRB (1.6)
7	BEN (15.7)	HRV (14.6)	UKR (9.1)	KOR (0.6)	ARM (1.6)
...					
117	ARG (1.3)	YEM (0.4)	BWA (-0.6)	PAN (-5.8)	DEU (-6.1)
118	SDN (1)	CHL (0.4)	JAM (-0.8)	NOR (-6.1)	SAU (-6.1)
119	USA (1)	ARG (0.3)	ARM (-0.9)	BWA (-10)	BRA (-7.0)
120	SGP (0.8)	BRA (0.2)	TTO (-1.1)	BEN (-24.4)	CHN (-7.1)
121	BRA (-0.1)	IRQ (0)	NOR (-1.2)	MDV (-30.2)	FRA (-7.6)
$\bar{\mu}$ - all	6.59	5.01	4.53	-1.47	-1.48
# (%) losers	1 (0.01)	0 (0)	8 (0.07)	105 (0.87)	94 (0.78)

Notes: The numbers in the cells represent the percentage gains/losses in real income for each of the 121 countries in our data, when increasing both θ_i and θ_f by 10% from the initial value in 2005, $\ln U_i'/U_i \times 100\%$. For this counterfactual analysis, we fix the numerical value for the elasticity of substitution parameter to $\sigma = 5$. Moreover, we proxy the number of firms for those countries, where we do not have information on, in the same way as detailed below Table 2.

the supply effect (Cambodia [Thailand, Vietnam], Maldives [India] and Mongolia [China] in Asia, Botswana [South Africa] and Benin [Nigeria] in Africa, and Albania and Croatia [Western Europe] in Europe). By contrast, countries in South America and the Middle East gain the least. The wage-induced price changes in column (5) are typically negative, but smaller in absolute terms. In 78% of the countries, they reduce the positive supply effect and mostly so in countries that experience the highest wage increase.

Columns (3) and (4) depict these wage changes. They clearly show that nominal wages might actually fall as a result of being exposed to fiercer competition from more efficient producers in other nations. As already theorized above, it does however matter quite substantially whether coordination costs only fall in the final production stage (column 3) or also in all other intermediate production stages (column 4).

Increasing θ^f only boosts the use of intermediate goods in final production and thus increases labor demand in the final stage of the upstream sector. Although this means that every country is also exposed to more heavy competition, only in 8 countries we see an actual loss in nominal wages. In 93% of the countries it leads to a wage increase. Countries gaining most are natural resource suppliers well-shielded from competition (Iraq, Qatar, and Niger). But also countries benefit a lot that are themselves the fiercest competitors in the intermediate goods markets. They add substantive value to the intermediate products passing through the supply chain (Malaysia, Germany).

Things are very different when increasing θ^i. This increases demand for intermediates in the upstream stages of the supply chain. Hence, just like with an increase of θ^f, a country actually needs to produce intermediate products to benefit. However, as already theorized above and confirmed in the table, the biggest gainers are those countries with the best access to the world's leading intermediaries for their own sales (Malaysia and Thailand [Singapore], Germany and France [the Netherlands and Belgium]). Since now competition is aggravated at any single stage of the supply chain, labor demand is much more negatively affected than when increasing θ^f. As a result, we find that wages go up in only a few countries.

4.3 Building blocks for free trade

As our final counterfactual, we explore the welfare effects of a unilateral trade cost reduction. Technically, we perform the somewhat stylized exercise to reduce the trade costs for the outgoing shipments of a focal nation to all other countries. Examples for this kind of intervention are improved export procedures or import tariff reductions negotiated with the country's trading partners. Naturally, this stimulates exports and increases welfare in the focal country. The more interesting question is whether its trading partners benefit as well. We show that this depends nontrivially on the extent of international production fragmentation. As in our previous counterfactuals we always distinguish between the supply and demand effect outlined in (23).

We start by considering a world without cross-border production linkages:

Proposition 4. *Suppose that $\theta^f = \theta^i = 0$ and consider a marginal transport cost reduction for all final goods exports originating from country i: $d_i = (T^f)' - T^f$, where $(T^f)'$ is such that $(\tau^f_{ik})' = \delta\tau^f_{ik}$, with $0 < \delta < 1$, and $(\tau^f_{lk})' = \tau^f_{lk}$ for all $k \in M$ and $l \in M\backslash\{i\}$.*
It follows for any $j \in M\backslash\{i\}$:

$$(i)\ \sum_{k\in M} \frac{\partial (P^c_j)^{1-\sigma}}{\partial p_k} \frac{d_i p_k}{(P^c_j)^{1-\sigma}} > 0 \quad and \quad (ii)\ \ d_i w_j < 0.$$

Our result, which is proven in the appendix, suggests an unambiguously positive suppy effect on prices: foreign consumers gain from an improved access to the final manufactures from the focal country (i). However, the downside of this is an intensified competition for their own domestic final goods industry. According to part (ii), we find an unambiguous negative externality on foreign wages. Hence, whether or not a foreign country gains or loses depends on the importance of the focal nation as a supplier of consumption goods versus its role as a competitor. In other words, our findings for a world without production linkages reproduce the well-known effect that a unilateral effort to boost exports might be vetoed by a country's trading partners. The reason is the trade diverting effects that put pressure on their domestic labor markets (Panagariya, 2000).

In an integrated production network, the effects of a comparable unilateral export cost reduction on both final and intermediate goods turn out to be quite different. Here, a general characterization is only possible for the supply side effect, which can be shown to lead to lower consumer prices all across the world.[28] This is reminiscent of our results from the Key Player analysis: opening (closing) the world to the products of a focal country improves (reduces) the world's direct access to consumption goods and facilitates (hampers) the flow of intermediate goods through the supply chain.[29]

[28] The proof of this is very similar to Proposition 4 and available upon request.

[29] In fact, the impact of a unilateral cost reduction for intermediate goods shipments on matrix $[I - A]^{-1}$ is mathematically similar to the removal of a country: in the unreported proof of a positive supply effect, we make use of Lemma 1 Property (i), fix $y = 0$, and investigate the effects of $x > 0$.

What makes the presence of an integrated supply chain very different are the externalities of a unilateral export cost reduction on other countries' labor markets (demand effect). When $\theta^f > 0$ and $\theta^i \geq 0$, these are no longer unambiguously negative. On the one hand, foreign workers from all countries benefit (even from a cost reduction on final goods exports), because this boosts the focal country's demand for their intermediate products. On the other hand, for the very same reason, they face increased competition in all their export markets. We can formally determine each country's wage adjustments by the total differential of the system of wage equations (16): see equation (44) in the Appendix. The equations show that the sign (and size) of the externality imposed on wages can be very different in different countries depending on a country's precise network position vis-à-vis the country actually lowering its export barriers.

Table 4 illustrates the wage and overall utility externalities for the 2005 world trade network. We focus on the effects of a unilateral trade cost reduction in four different countries: China, the USA, Germany, and Singapore. They are chosen based on their different roles in the global production network (see Table 2). Our findings for Germany and Singapore clearly show the expected wage increases in countries other than the one reducing its export costs. This in itself suggests that the 2005 trade network shares indeed the features of an integrated global supply chain (at least in some parts of the world). Many more countries experience a wage increase when Singapore lowers its export costs. This is easily explained, since Germany is a much fiercer competitor on especially intermediate goods markets. However, do note that the countries incurring a wage loss from a Singaporian export cost reduction suffer, on average, much more than from a similar cost reduction in Germany. This reflects Singapore's role as an important trade intermediary: those countries whose local value added it intermediates are the ones whose wages go up most (Malaysia, Thailand, Indonesia, etc). All others are exposed to fiercer competition, not so much from Singaporian products but from *all* the products it intermediates. A Singaporian export cost reduction has for the same reason a much larger positive externality on consumer prices: this improves consumer access to *all* the goods the country intermediates. Moreover, as Singapore hardly adds any value to the global supply chain itself, it helps other nations more by reducing export costs than it benefits itself.

The case is different for China and the USA. Although their export cost reduction does increase the demand for foreign workers in the intermediate goods industry, this is more than outweighed by the increased competitiveness of the countries themselves. However, as China is the main supplier for final manufacturers in 2005, a Chinese export cost reduction

Table 4: WELFARE EFFECTS OF A UNILATERAL EXPORT COSTS REDUCTION

country:	China	USA	Germany	Singapore
		Δ Wages (%)		
own country	4.63	5.45	6.31	+0.0
$\bar{\mu}$ - other top 15	-0.01	-0.01	0.03	4.19
$\bar{\mu}$ - all other	-0.18	-0.2	-0.17	-0.47
# (%) losers	120 (all)	120 (all)	114 (0.95)	100 (0.83)
		Overall Δ Utility (%)		
own country	10.28	10.52	9.31	0.48
$\bar{\mu}$ -other top 15	1.65	1.01	1.68	13.29
$\bar{\mu}$ -all other	0.33	0.19	0.43	2.48
# (%) losers	2 (0.02)	14 (0.12)	5 (0.04)	1 (0.01)

NOTES: The numbers in the cells represent the percentage gains/losses in real income for the each countries in our data, when increasing τ_{ij}^{f} and τ_{ij}^{f} by 10% for all $j \in M$ compared to the situation in 2005: $\ln U_i'/U_i \times 100\%$. For this counterfactual analysis, we fix the numerical value for the elasticity of substitution parameter to $\sigma = 5$. Moreover, we proxy the number of firms for those countries, where we do not have information on, in the same way as below Table 2.

causes a larger positive externality on consumer prices, which explains why more countries suffer from a US export cost reduction.

5 Conclusion

In this paper, we present a novel theory about how the emergence of a global supply chain affects the welfare in different parts of the world. The main difference to prior theories on the topic is that ours stresses a central feature of trade in a global supply chain: the well-being of any one nation depends on the technologies and geographical locations of all other nations. We highlight these network characteristics of the supply chain by means of methods adopted from the social network literature. This allows us to perform a series of novel comparative statics analyses: we identify the key player nations in the global production network, show that proximity to these nations is crucial for a country's income development, and illustrate that in a deeply integrated supply chain a unilateral trade cost reduction can even have a positive effect on other nations' labor markets.

Even though our theory is based on many assumptions and mathematical specifications, the produced insights could also be found in more general setttings. For example, we could allow for (i) distinct elasticities of substitution for consumers and producers ($\sigma \neq \gamma$), (ii) a third sector producing non-tradable consumption goods, or (iii) country-pair specific access to traded manufactures (i.e. replacing n_i^f and n_i^i by n_{ij}^f and n_{ij}^i). None of these extensions would have an impact on our main findings. A further interesting extension of our model would be to allow for (iv) endogenous numbers of firms and international plant mobility, and (v) a supply chain of more than just two production stages such as in Caliendo and Parro (2013). In particular the last modification will bring our model much closer to reality and improve its suitability for quantitative exercises based on one of the world input-output datasets that are currently under development.

6 Appendix

6.1 The intermediate goods trade equation

Here, we solve for the market-clearing trade values X_{ij}^i in the *general equilibrium*, where the intermediate goods producers from country i collect a share of the profits from country j's producers according to their input cost share (given by $(P_i^i)^{1-\sigma} a_{ij}/(P_j^f)^{1-\sigma}$ for country i's final goods producers and by $(P_i^i)^{1-\sigma} a_{ij}/(P_j^i)^{1-\sigma}$ for the country's intermediate goods producers).

Substituting the expressions for the collected profit shares into equation (8) and summing over all importing countries $j \in M$, we get:

$$(P_i^i)^{\sigma-1} \sum_{j \in M} X_{ij}^i \;=\; \sum_{j \in M} \left[\Big(\frac{\theta^f}{\theta^i}\Big)^{\sigma} a_{ij} (P_j^f)^{\sigma-1} \sum_{k \in M} X_{jk}^f + a_{ij} (P_j^i)^{\sigma-1} \sum_{k \in M} X_{jk}^i \right]$$

for any exporter i. Similarly, after substitution and summation over the exporting countries $i \in M$, we get:

$$\sum_{i \in M} (P_i^i)^{\sigma-1} (a_{ij})^{-1} X_{ij}^i \;=\; m \left[\Big(\frac{\theta^f}{\theta^i}\Big)^{\sigma} (P_j^f)^{\sigma-1} \sum_{k \in M} X_{jk}^f + (P_j^i)^{\sigma-1} \sum_{k \in M} X_{jk}^i \right]$$

for any importer j. In vector notation, denote by X^i and Y^i the full matrices (X_{ij}^i) and $((a_{ij})^{-1} X_{ij}^i)$, respectively, by X^f the full matrix (X_{ij}^f), by $(P)^{\sigma-1}$ the diagonal matrix with $(P_i)^{\sigma-1}$ along its diagonal, and by 1 the row vector of ones. The previous expressions are equivalent to:

$$\begin{aligned} (P^i)^{\sigma-1} X^i 1^T &= \Big(\frac{\theta^f}{\theta^i}\Big)^{\sigma} A \left[(P^f)^{\sigma-1} X^f 1^T \right] \;+\; A \left[(P^i)^{\sigma-1} X^i 1^T \right] \\ &= \Big(\frac{\theta^f}{\theta^i}\Big)^{\sigma} \left[I - A \right]^{-1} A \left[(P^f)^{\sigma-1} X^f 1^T \right] \end{aligned} \tag{30}$$

and

$$\left[1(P^i)^{\sigma-1}Y^i\right]^T \;=\; m\Big(\frac{\theta^f}{\theta^i}\Big)^{\sigma}\left[(P^f)^{\sigma-1}X^f 1^T\right] \;+\; m\left[(P^i)^{\sigma-1}X^i 1^T\right] \tag{31}$$

where $[I-A]^{-1}$ is a variant of Wassily Leontief's inverse matrix (Leontief, 1936), where the difference lies in the fact that we look at a geographically dispersed two-sector economy with imperfect substitutes in the intermediate goods sector. Substituting (30) into (31) gives

$$\begin{aligned}\left[1(P^i)^{\sigma-1}Y^i\right]^T &= m\Big(\frac{\theta^f}{\theta^i}\Big)^{\sigma}\Big[I+\left[I-A\right]^{-1}A\Big]\;\left[(P^f)^{\sigma-1}X^f 1^T\right]\\ &= m\Big(\frac{\theta^f}{\theta^i}\Big)^{\sigma}\left[I-A\right]^{-1}\;\left[(P^f)^{\sigma-1}X^f 1^T\right] \end{aligned} \tag{32}$$

Equation (8) states that entry i in vector (32) consists of m equal summands. Hence, rearranging some of the left-hand terms to the right-hand side, the trade equation for any two countries i and j can be written as:

$$X^i_{ij} = \Big(\frac{\theta^f}{\theta^i}\Big)^{\sigma}(P^i_i)^{1-\sigma}a_{ij}\Big[\sum_{k\in M}\sum_{h=0}^{\infty}a^{[h]}_{jk}(P^f_k)^{\sigma-1}\sum_{l\in M}X^f_{kl}\Big]. \tag{33}$$

6.2 Proof of Theorem 1

Proof. To prove existence of at least one equilibrium as defined in Definition 1, we verify that there is a $w\in\Re^m_{++}$ such that the transformed equation (16):

$$Z_i(w) = \frac{w_i^{-\sigma}}{(P^f_i)^{1-\sigma}}\sum_{j\in M}X^f_{ij} + \frac{w_i^{-\sigma}}{(P^i_i)^{1-\sigma}}\sum_{j\in M}X^i_{ij} - L_i \tag{34}$$

satisfies the following properties. For all $i\in M$ and vectors $w=(w_1,w_2,...,w_m)$:

i) $Z_i(w)$ is continuous,

ii) $Z_i(w)$ is homogeneous of degree zero,

iii) $\sum_{i\in M}w_iZ_i(w)=0$ for all $w\in\Re^m_{++}$ (Walras' Law),

iv) for $k=\max_j L_j>0$, $Z_i(w)>-k$ for all $w\in\Re^m_{++}$ and

v) if $w^m\to w^0$, where $w^0_{-i}\neq 0$ and $w^0_i=0$ for some i, then $\max_j Z_j(w^m)\to\infty$.

Existence then follows from Proposition 17.C.1 of Mas-Collel et al. (1995, p. 585).

(i) The continuity of $Z_i(w)$ follows immediately from the convergence requirement $\lim_{h\to\infty}A^h=0$, which ensures that some continuous, vector-valued functions for X^i_{ij}, $(P^f_i)^{1-\sigma}$, and $(P^i_i)^{1-\sigma}$ exist, given in (12), (13), and (33), respectively. (ii) Since trade equations (3) and (33) are both homogeneous of degree one, such as P_i^f and P_i^i are, it follows immediately that $Z_i(w)$ is homogeneous of degree zero. (iii) To verify Walras' Law, we restate (34) by adding and deducting the intermediate goods imports in country i:

$$\sum_{j\in M} X_{ji}^i = \frac{SA_i^f}{(P_i^f)^{1-\sigma}} \sum_{k\in M} X_{ik}^f + \frac{SA_i^i}{(P_i^i)^{1-\sigma}} \sum_{k\in M} X_{ik}^i$$

which are derived from (8) in combination with the observations that $\sum_{j\in M}(P_j^i)^{1-\sigma} a_{ji}^{[1]} = SA_i^i$ and $w_i^{1-\sigma} + SA_i^i = (P_i^i)^{1-\sigma}$. Hence, we get:

$$w_i Z_i(w) = \sum_{j\in M} X_{ij}^f + \sum_{j\in M} X_{ij}^i - \sum_{j\in M} X_{ji}^i - L_i w_i$$

The property $\sum_i w_i Z_i(w) = 0$ follows from the fact that $\sum_j X_{ji}^f = L_i w_i$ and $\sum_i \sum_j X_{ji}^i = \sum_i \sum_j X_{ij}^i$. (iv) A lower bound on $Z_i(w)$ is implied by $Z_i(w) > -L_i$ for all $w \in \Re_{++}^m$. Thus, let $k = \max_j L_j$. It is $Z_i(w) > -k$ for all $i \in M$.

To prove part (v) suppose that $w^m \to w^0$, where $w_{-i}^0 \neq 0$ and $w_i^0 = 0$. For any $w \in \Re_{++}^m$ and $j, k \in M$ it holds:

$$\begin{aligned}
Z_i(w) &> \max_{j\in M} \frac{w_i^{-\sigma}}{(P_i^f)^{1-\sigma}} X_{ij}^f \;-\; \max_{k\in M} L_k \\
&= \max_{j\in M} \frac{b_{ij} L_j w_j}{w_i^\sigma \sum_{k\in M}(P_k^f)^{1-\sigma} b_{kj}} - \max_{k\in M} L_k \\
&= \max_{j\in M} \frac{b_{ij} L_j w_j}{w_i^\sigma \sum_{k\neq i}(w_k^{1-\sigma} + SA_k^f) b_{kj} + w_i^\sigma (w_i^{1-\sigma} + SA_i^f) b_{kj}} - \max_{k\in M} L_k
\end{aligned}$$

By looking at (13), it immediately becomes clear that the denominator approaches zero in the limit as w_i goes to zero. This implies that $\lim_{w^m \to w^0} Z_i(w^m) \to \infty$ and therefore establishes existence of an equilibrium.

To prove that there is exactly one equilibrium, we verify that $Z_i(w)$ has the gross substitution property:

$$\frac{\partial Z_i(w)}{\partial w_j} > 0 \quad \text{for all } i, j, \;\; i \neq j \quad \text{for all } w \in \Re_{++}^m.$$

Uniqueness follows then from Proposition 17.F.3 of Mas-Collel et al. (1995, p. 613).

For any $i \neq j$, the partial derivatives of the system of functions (34) are given by:

$$\frac{\partial Z_i}{\partial w_j} = \frac{1}{w_j}\left[\frac{w_i^{-\sigma}}{(P_i^f)^{1-\sigma}}\left(X_{ij}^f - \sum_{k\in M} X_{ik}^f \phi_{jk}\right) + \frac{w_i^{-\sigma}}{(P_i^i)^{1-\sigma}}\left(\varphi_{ij} - \sum_{k\in M} \varphi_{ik}\phi_{jk}\right)\right] \tag{35}$$

where we define:

$$\begin{aligned}\phi_{jk} &= (1-\sigma)\bigg(\frac{w_j^{1-\sigma}}{(P_j^f)^{1-\sigma}}\tilde{X}_{jk}^f + \sum_{l\in M}\frac{SA_{jl}^f}{(P_l^f)^{1-\sigma}}\tilde{X}_{lk}^f\bigg) \\ \varphi_{ik} &= (P_i^i)^{1-\sigma}\big(\frac{\theta^f}{\theta^i}\big)^{\sigma}\sum_{j\in M}\sum_{h=1}^{\infty} a_{ij}^{[h]}\frac{X_{jk}^f}{(P_j^f)^{1-\sigma}}\end{aligned} \tag{36}$$

and where $\tilde{X}_{ij}^f$ is defined in (27) and SA_{ji}^f denotes the j'th summand of SA_i^f. Since $\sigma > 1$, it immediately follows $\partial Z_i/\partial w_j > 0$ and $Z(w)$ therefore has the gross substitute property.

To prove that comparative statics analysis for this equilibrium is possible, it suffices to notice that the off-diagonal elements of the Jacobian matrix of $Z(w,\cdot)$ with regard to w, $D_wZ(w,\cdot)$, has positive off-diagonal entries, which follows from $\partial Z_i/\partial w_j > 0$. Moreover, since $Z(w,\cdot)$ is homogenous of degree zero, it is $\partial Z_i/\partial w_i < 0$ for any $i \in M$ and hence the Jacobian has negative diagonal entries. From Proposition 17.G.3 of Mas-Collel et al. (1995, p. 618) it then follows that $[D_wZ(w,\cdot)]^{-1}$ exists and has all its entries negative. Hence, if $D_TZ(\cdot,T)$ measures the shock to the exogenous matrices (T^f, T^i) we can determine dw by $dw = -[D_wZ(w,\cdot)]^{-1}\, D_TZ(\cdot,T)$. $\square$

6.3 Empirical Framework

Table 5: STEPS 1 AND 2 - ESTIMATING THE INTERMEDIATE AND FINAL GOODS TRADE EQUATIONS

VARIABLES	STEP 1 final goods	STEP 2 intermediates
ln Distance	-1.516***	-1.382***
	(0.000)	(0.000)
Border	-0.622*	-0.774*
	(0.066)	(0.072)
Common Language	1.040***	0.739***
	(0.000)	(0.001)
Colonial Relationship	0.878***	1.033***
	(0.000)	(0.000)
Common Colonizer	1.147***	0.875***
	(0.000)	(0.000)
FE	exporter	exporter
Observations	16511	17478
Nr. importers	212	213
Nr. exporters	167	167

NOTES: The dependent variable in steps 1 (2) are final (intermediate) goods exports from country i to country j relative to the exports from Germany to country j, X_{ij}^f/X_{GERj}^f (X_{ij}^i/X_{GERj}^i). Data stem from UN COMTRADE and CEPII and we include all countries available. The only country not represented is Palau. t statistics based on standard errors clustered at the importer level which are shown in the parentheses. *** p<0.01, ** p<0.05, * p<0.10.

6.4 Proof of Lemma 1

Proof. of part (i). We prove the matrix identity cell by cell and in five steps. In so doing, let us denote cell ij in matrix $I_{xi}AI_{yi}$ by a'_{ij}, in matrix $\sum_{h=0}^{\infty}\left(I_{xi}AI_{yi}\right)^h$ by c'_{ij}, and correspondingly in $\sum_{h=0}^{\infty}A^h$ by c_{ij}. It follows for $j,k \neq i$:

$$a'_{jk} = a_{jk} \ , \ \ a'_{ij} = (1+x)a_{ij} \ , \ \ a'_{ji} = (1+y)a_{ji} \ , \ \ a'_{ii} = (1+y)(1+x)a_{ii} \tag{37}$$

Moreover, we make use of the following properties that immediately derive from the rules of matrix multipliation. For any ij with $i \neq j$:

$$\begin{aligned} c_{ij} &= \sum_{k\in M} c_{ik}a_{kj} = \sum_{k\in M} a_{ik}c_{kj} \\ c_{ii} &= 1 + \sum_{k\in M} c_{ik}a_{ki} = 1 + \sum_{k\in M} a_{ik}c_{ki} \end{aligned} \tag{38}$$

and similar for c'_{ij}.

Step 1: By (37) and (38) it holds for any $l, p \neq i$:

$$c'_{lp} - c_{lp} \quad = \quad a'_{li}c'_{ip} - a_{li}c_{ip} + \sum_{k \neq i} a_{lk}\big[c'_{kp} - c_{kp}\big]$$

Applying the same steps recursively to $[c'_{kp} - c_{kp}]$, we find:

$$\begin{aligned}
c'_{lp} - c_{lp} \quad &= \quad a'_{li}c'_{ip} - a_{li}c_{ip} + \sum_{k \neq i} a_{lk}\big[a'_{ki}c'_{ip} - a_{ki}c_{ip}\big] + \sum_{k_1 \neq i} a_{lk_1} \sum_{k \neq i} a_{k_1 k}\big[c'_{kp} - c_{kp}\big] \\
&= \quad a'_{li}c'_{ip} - a_{li}c_{ip} + \big[\sum_{k \neq i} a_{lk} + \sum_{k_1 \neq i} a_{lk_1} \sum_{k \neq i} a_{k_1 k}\big]\big[a'_{ki}c'_{ip} - a_{ki}c_{ip}\big] \\
&+ \quad \sum_{k_1 \neq i} a_{lk_1} \sum_{k_2 \neq i} a_{k_1 k_2} \sum_{k \neq i} a_{k_2 k}\big[c'_{kp} - c_{kp}\big] \\
&= \quad a'_{li}c'_{ip} - a_{li}c_{ip} + \big[\sum_{k \neq i} a_{lk} + \sum_{k_1 \neq i} a_{lk_1} \sum_{k \neq i} a_{k_1 k} + \sum_{k_1 \neq i} a_{lk_1} \sum_{k_2 \neq i} a_{k_1 k_2} \sum_{k \neq i} a_{k_2 k}\big] \\
&\times \quad \big[a'_{ki}c'_{ip} - a_{ki}c_{ip}\big] + \sum_{k_1 \neq i} a_{lk_1} \sum_{k_2 \neq i} a_{k_1 k_2} \sum_{k_3 \neq i} a_{k_2 k_3} \sum_{k \neq i} a_{k_3 k}\big[c'_{kp} - c_{kp}\big] \\
&\dots
\end{aligned}$$

By (38) and since $\lim_{h \to \infty} A^h = 0$, this simplifies to:

$$c'_{lp} - c_{lp} \quad = \quad \sum_{k \neq i} c_{l(i)k}\big[a'_{ki}c'_{ip} - a_{ki}c_{ip}\big] \tag{39}$$

where $c_{l(i)k}$ denotes the special case of c'_{lk} with player i being completely isolated ($x = y = -1$).

Step 2: From (39) it follows for the case $x = y = -1$ and any $l, p \neq i$:

$$c_{l(i)p} - c_{lp} = -\sum_{k \neq i} c_{l(i)k} a_{ki} c_{ip}$$

since $a'_{ki} = c'_{ip} = 0$. Post-multication by a_{pi} and summation over equations $p \in M \backslash \{i\}$ leads to:

$$\begin{aligned}
\sum_{p \neq i} c_{l(i)p} a_{pi} \quad &= \quad \sum_{p \neq i} c_{lp} a_{pi} - \sum_{k \neq i} c_{l(i)k} a_{ki} \sum_{p \neq i} c_{ip} a_{pi} \\
&= \quad \sum_{p \neq i} c_{lp} a_{pi} \left[1 + \sum_{p \neq i} c_{ip} a_{pi}\right]^{-1} \\
&= \quad \frac{c_{li}(1 - a_{ii})}{c_{ii}(1 - a_{ii})}
\end{aligned}$$

Hence,

$$c_{l(i)p} - c_{lp} = -\frac{c_{li}c_{ip}}{c_{ii}} \tag{40}$$

Step 3: Similar to *Step 1*, we can write $c'_{ip} - c_{ip}$ for $p \neq i$ as:

$$c'_{ip} - c_{ip} \;\; = \;\; a'_{ii}c'_{ip} - a_{ii}c_{ip} + \sum_{k \neq i} a_{ik}\left[c'_{kp} - c_{kp}\right] + x\sum_{k \neq i} a_{ik}c'_{kp}$$

and after substitution of (39) and (40):

$$\begin{aligned}
c'_{ip} \;\; &= \;\; a'_{ii}c'_{ip} + (1-a_{ii})c_{ip} + (1+x)\sum_{k\neq i} a_{ik}\sum_{l \neq i}\left[c_{kl} - \frac{c_{ki}c_{il}}{c_{ii}}\right]\left[a'_{li}c'_{ip} - a_{li}c_{ip}\right] + x\sum_{k \neq i} a_{ik}c_{kp} \\
&= \;\; \frac{(1-a_{ii})(1+x)c_{ip} - (1+x)\sum_{k\neq i} a_{ik}\sum_{l\neq i}\left[c_{kl} - \frac{c_{ki}c_{il}}{c_{ii}}\right]a_{li}c_{ip}}{1-(1+x)(1+y)\left[a_{ii} + \sum_{k\neq i} a_{ik}\sum_{l\neq i}\left[c_{kl} - \frac{c_{ki}c_{il}}{c_{ii}}\right]a_{li}\right]} \\
&= \;\; (1+x)c_{ip}\frac{1 - a_{ii} - \sum_{k\neq i} a_{ik}\left[c_{ki}(1-a_{ii}) - \frac{c_{ki}[c_{ii}(1-a_{ii})-1]}{c_{ii}}\right]}{1-(1+x)(1+y)\left[a_{ii} + \sum_{k\neq i} a_{ik}\left[c_{ki}(1-a_{ii}) - \frac{c_{ki}[c_{ii}(1-a_{ii})-1]}{c_{ii}}\right]\right]} \\
&= \;\; (1+x)c_{ip}\frac{1 - a_{ii} - \sum_{k\neq i} a_{ik}\frac{c_{ki}}{c_{ii}}}{1-(1+x)(1+y)\left[a_{ii} + \sum_{k\neq i} a_{ik}\frac{c_{ki}}{c_{ii}}\right]} \\
&= \;\; (1+x)c_{ip}\frac{1 - a_{ii} - \frac{c_{ii}(1-a_{ii})-1}{c_{ii}}}{1-(1+x)(1+y)\left[a_{ii} + \frac{c_{ii}(1-a_{ii})-1}{c_{ii}}\right]} \\
&= \;\; \frac{(1+x)c_{ip}}{c_{ii} - (1+x)(1+y)(c_{ii}-1)} \\
&= \;\; \frac{(1+x)c_{ip}}{1-(x+y+xy)(c_{ii}-1)}
\end{aligned}$$

where we have made repeated use of properties (37) and (38) in lines two to five. Thus:

$$c'_{ip} - (1+x)c_{ip} \;\; = \;\; \frac{(x+y+xy)(1+x)}{1-(x+y+xy)(c_{ii}-1)}(c_{ii}-1)c_{ip} \tag{41}$$

Step 4: From (39), (40), and (41) it follows for $l, p \neq i$:

$$\begin{aligned}
c'_{lp} - c_{lp} &= \sum_{k \neq i} \big[c_{lk} - \frac{c_{li}c_{ik}}{c_{ii}}\big] \big[a'_{ki} \frac{(1+x)}{1-(x+y+xy)(c_{ii}-1)} - a_{ki}\big] c_{ip} \\
&= \sum_{k \neq i} \big[c_{lk} - \frac{c_{li}c_{ik}}{c_{ii}}\big] a_{ki} \frac{(x+y+xy)c_{ii}}{1-(x+y+xy)(c_{ii}-1)} c_{ip} \\
&= \frac{(x+y+xy)}{1-(x+y+xy)(c_{ii}-1)} c_{li} c_{ip} \qquad (42)
\end{aligned}$$

Step 5: Finally, from (38), in combination with (37) and (41), it follows:

$$\begin{aligned}
c'_{ii} &= \frac{1 + \sum_{k \neq i} c'_{ik} a'_{ki}}{1 - a'_{ii}} \\
&= \frac{1-(x+y+xy)(c_{ii}-1) + (1+x)(1+y)\sum_{k \neq i} c_{ik} a_{ki}}{[1-(x+y+xy)(c_{ii}-1)][1-(1+x)(1+y)a_{ii}]} \\
&= \frac{1-(x+y+xy)(c_{ii}-1) + (1+x)(1+y)(c_{ii}(1-a_{ii})-1)}{[1-(x+y+xy)(c_{ii}-1)][1-(1+x)(1+y)a_{ii}]} \\
&= \frac{c_{ii}}{1-(x+y+xy)(c_{ii}-1)}
\end{aligned}$$

and hence

$$\begin{aligned}
c'_{ii} - 1 - (1+x)(1+y)(c_{ii}-1) &= \frac{c_{ii} - [1+(1+x)(1+y)(c_{ii}-1)][1-(x+y+xy)(c_{ii}-1)]}{1-(x+y+xy)(c_{ii}-1)} \\
&= \frac{(x+y+xy)(1+x)(1+y)}{1-(x+y+xy)(c_{ii}-1)} (c_{ii}-1)^2
\end{aligned}$$

Moreover, from (38), in combination with (37) and (42), it is for $l \neq i$:

$$\begin{aligned}
c'_{li} &= \frac{\sum_{k \neq i} c'_{lk} a'_{ki}}{1 - a'_{ii}} \\
&= (1+y) \frac{\sum_{k \neq i} \big[(x+y+xy)c_{li}c_{ik} + [1-(x+y+xy)(c_{ii}-1)]c_{lk}\big] a_{ki}}{[1-(x+y+xy)(c_{ii}-1)][1-(1+x)(1+y)a_{ii}]} \\
&= (1+y) c_{li} \frac{\big[(x+y+xy)[c_{ii}(1-a_{ii})-1] + [1-(x+y+xy)(c_{ii}-1)](1-a_{ii})\big]}{[1-(x+y+xy)(c_{ii}-1)][1-(1+x)(1+y)a_{ii}]} \\
&= \frac{(1+y)c_{li}}{1-(x+y+xy)(c_{ii}-1)}
\end{aligned}$$

and therefore

$$c'_{li} - (1+y)c_{li} = \frac{(1+y)(x+y+xy)c_{li}(c_{ii}-1)}{1-(x+y+xy)(c_{ii}-1)}.$$

Proof of part (ii). We move on to show that:

$$\sum_{h=1}^{\infty}(1+z)^h A^h - \sum_{h=1}^{\infty} A^h = z\sum_{h=0}^{\infty}(1+z)^h A^h \sum_{h=1}^{\infty} A^h$$

It is for any $s \geq 3$:

$$\begin{aligned}\sum_{h=1}^{s}(1+z)^h A^h &= (1+z)\sum_{h=1}^{s} A^h + (1+z)z\sum_{h=2}^{s} A^h + (1+z)^2 z\sum_{h=3}^{s} A^h + ...\\ &= (1+z)\sum_{h=1}^{s} A^h + (1+z)zA\big(\sum_{h=1}^{s-1} A^h\big) + (1+z)^2 zA^2\big(\sum_{h=1}^{s-2} A^h\big) + ...\end{aligned}$$

For $s \to \infty$, this becomes:

$$\begin{aligned}\sum_{h=1}^{\infty}(1+z)^h A^h &= \Big[1+z+z(1+z)A+z(1+z)^2A^2+...\Big]\sum_{h=1}^{\infty} A^h\\ &= \sum_{h=1}^{\infty} A^h + z\sum_{h=0}^{\infty}(1+z)^h A^h \sum_{h=1}^{\infty} A^h\end{aligned}$$

which was to be shown. $\square$

6.5 Proof of Proposition 1

Proof. Suppose that $dw_i = 0$ for all $i \in M$. Suppose further that $\theta^f = \theta^i$ such that $(P_i^i)^{1-\sigma} = (P_i^f)^{1-\sigma}$. Based on equation (26), the Key Player problem can be equivalently written as:

$$\begin{aligned}i^* &\approx \arg\min_i \Bigg[\sum_{j\neq i}\Bigg(\sum_{k\in M}\frac{\partial (P_j^c)^{1-\sigma}}{\partial p_k}\frac{dp_k}{(P_j^c)^{1-\sigma}}\Bigg)\Bigg]\\ &= \arg\max_i \Bigg[\sum_{j\neq i}\Bigg(\tilde{X}_{ij}^f + \sum_{k\neq i}\tilde{X}_{kj}^f (P_k)^{\sigma-1}\sum_{l\in M} w_l^{1-\sigma}\sum_{h=1}^{\infty} a_{l(i)k}^{[h]}\Bigg)\Bigg]\end{aligned}$$

where according to Property (i) of Lemma 1:

$$\sum_{h=1}^{\infty} a_{l(i)k}^{[h]} = \frac{\sum_{h=1}^{\infty} a_{li}^{[h]}\sum_{h=1}^{\infty} a_{ik}^{[h]}}{\sum_{h=0}^{\infty} a_{ii}^{[h]}}$$

for any cell $lk \in d[I-A]^{-1}$ with $l, k \neq i$, and $\sum_{h=1}^{\infty} a_{l(i)k}^{[h]} = \sum_{h=1}^{\infty} a_{lk}^{[h]}$, if $l = i$ or $k = i$. Therefore:

$$\begin{aligned}
i^* &\approx \arg\max_i \Bigg[\sum_{j\neq i} \Bigg(\tilde{X}_{ij}^f + \sum_{k\neq i} \tilde{X}_{kj}^f (P_k)^{\sigma-1} \Big(\sum_{l\neq i} w_l^{1-\sigma} \frac{\sum_{h=1}^{\infty} a_{li}^{[h]}}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} + w_i^{1-\sigma} \Big) \sum_{h=1}^{\infty} a_{ik}^{[h]} \Bigg) \Bigg] \\
&= \arg\max_i \Bigg[\sum_{j\neq i} \Bigg(\tilde{X}_{ij}^f + \sum_{k\neq i} \tilde{X}_{kj}^f (P_k)^{\sigma-1} \frac{(P_i)^{1-\sigma} \sum_{h=1}^{\infty} a_{ik}^{[h]}}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \Bigg) \Bigg] \\
&= \arg\max_i \Bigg[\sum_{j\neq i} \Bigg(\frac{(P_i)^{1-\sigma} b_{ij}}{(P_j^c)^{1-\sigma}} + \sum_{k\neq i} \frac{b_{kj}}{(P_j^c)^{1-\sigma}} \frac{(P_i)^{1-\sigma} \sum_{h=1}^{\infty} a_{ik}^{[h]}}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \Bigg) \Bigg] \\
&= \arg\max_i \Bigg[\sum_{j\neq i} \frac{(P_i)^{1-\sigma} \sum_{k\in M} \sum_{h=0}^{\infty} a_{ik}^{[h]} b_{kj}}{(P_j^c)^{1-\sigma} \sum_{h=0}^{\infty} a_{ii}^{[h]}} \Bigg]
\end{aligned}$$

since $\sum_{l\neq i} w_l^{1-\sigma} \frac{\sum_{h=1}^{\infty} a_{li}^{[h]}}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} + w_i^{1-\sigma} = \frac{(P_i)^{1-\sigma}}{\sum_{h=0}^{\infty} a_{ii}^{[h]}}$ and $\sum_{h=0}^{\infty} a_{ik}^{[h]} = \sum_{h=1}^{\infty} a_{ik}^{[h]}$ for $i \neq k$. $\square$

6.6 Proof of Proposition 2

Proof. Consider a worldwide homogenous transportation cost reduction by a factor δ such that $(\tau_{ij}^f)' = \delta \tau_{ij}^f$ for all $ij \in T^f$ with $0 < \delta < 1$. We verify that consumer prices reduce at the same rate for any $i \in M$, but that nominal wages stay constant.

Concerning the price effect, note that $(P_i^c)^{1-\sigma} = \sum_{k\in M} (P_k^f)^{1-\sigma} b_{ki}$ is homogenous of degree one with regard to $(\tau_{ki}^f)^{1-\sigma}$ for all $k \in M$. Thus, for any $i \in M$ it is:

$$\ln \frac{\left((P_i^c)^{1-\sigma}\right)'}{(P_i^c)^{1-\sigma}} = \ln \delta^{1-\sigma} > 0$$

The wage effect is determined by the direct effect of the cost reduction, $D_{T^f} Z(\cdot, T^f, T^i)$, on the labor income equation (34):

$$Z_i(w, \tau^f) = \frac{w_i^{-\sigma}}{(P_i^f)^{1-\sigma}} \sum_{j\in M} X_{ij}^f + \frac{w_i^{-\sigma}}{(P_i^i)^{1-\sigma}} \sum_{j\in M} X_{ij}^i - L_i$$

However, since X_{ij}^f and X_{ij}^i are both homogenous of degree zero with regard to $(\tau_{kj}^f)^{1-\sigma}$ for any $k \in M$, and $(P_i^f)^{1-\sigma}$ and $(P_i^i)^{1-\sigma}$ are unaffected, it immediately follows $D_{T^f} Z(\cdot, T^f, T^i) = 0$. Thus, since dw is determined by $dw = -[D_w Z(w, \cdot)]^{-1} D_{T^f} Z(\cdot, T^f, T^i)$, also $dw = 0$. $\square$

6.7 Proof of Proposition 3

Proof. Suppose that $\theta^f = \theta^i$. Expression (29) then simplifies to:

$$\begin{aligned}\sum_{k\in M}\frac{\partial(P_i^c)^{1-\sigma}}{\partial p_k}\frac{dp_k}{(P_i^c)^{1-\sigma}} &= \frac{1}{\theta^\sigma}\sum_{k\in M}\left[\sum_{l\in M}SA_l^i\Big(\sum_{h=0}^{\infty}a_{lk}^{[h]}\Big)\right](P_k^f)^{\sigma-1}\tilde{X}_{ki}^f \\ &= \frac{1}{\theta^\sigma}\sum_{k\in M}\left[\sum_{l\in M}SA_l^i\Big(\sum_{h=0}^{\infty}a_{lk}^{[h]}\Big)\right]\frac{b_{ki}}{(P_i^c)^{1-\sigma}}\end{aligned}$$

Thus, for any two countries i and j we immediately find that:

$$\begin{aligned}\sum_{k\in M}\frac{\partial(P_i^c)^{1-\sigma}}{\partial p_k}\frac{dp_k}{(P_i^c)^{1-\sigma}} &> \sum_{k\in M}\frac{\partial(P_j^c)^{1-\sigma}}{\partial p_k}\frac{dp_k}{(P_j^c)^{1-\sigma}} \\ &\Leftrightarrow \\ \frac{\sum_{k\in M}\big(\sum_{l\in M}SA_l^i\sum_{h=0}^{\infty}a_{lk}^{[h]}\big)b_{ki}}{(P_i^c)^{1-\sigma}} &> \frac{\sum_{k\in M}\big(\sum_{l\in M}SA_l^i\sum_{h=0}^{\infty}a_{lk}^{[h]}\big)b_{kj}}{(P_j^c)^{1-\sigma}}.\end{aligned}$$

□

6.8 Proof of Proposition 4

Proof. of part (i). Suppose that $\theta^f = \theta^i = 0$ and consider a transport cost reduction $d_i = (T^f)' - T^f$, where $(T^f)'$ is such that $(\tau_{ik}^f)' = \delta\tau_{ik}^f$ for $0 < \delta < 1$, and $(\tau_{lk}^f)' = \tau_{lk}^f$ for all $k \in M$ and $l \in M\backslash\{i\}$.

The price effect in any $j \in M$ (when holding wages constant) can be written as:

$$\begin{aligned}\sum_{k\in M}\frac{\partial(P_j^c)^{1-\sigma}}{\partial p_k}\frac{d_i p_k}{(P_j^c)^{1-\sigma}} &= \frac{((P_j^c)^{1-\sigma})'}{(P_j^c)^{1-\sigma}} - 1 = \frac{\delta^{1-\sigma}(P_i^f)^{1-\sigma}b_{ij} + \sum_{k\neq i}(P_k^f)^{1-\sigma}b_{kj}}{(P_j^c)^{1-\sigma}} - 1 \\ &= (\delta^{1-\sigma}-1)\tilde{X}_{ij}^f > 0\end{aligned}$$

Proof of part (ii). To investigate the wage adjustments in countries $j \in M\backslash\{i\}$, we take advantage of Walras' Law, i.e. we normalize the wage rates to w_j/w_i and investigate the system of $m-1$ equations $Z_j(w/w_i, \tau) = 0$ for $j \in M\backslash\{i\}$. The wage adjustments are then given by the system:

$$dw = -[D_w Z(w/w_i,\cdot)]^{-1} D_{T^f} Z(\cdot, T^f, T^i) \tag{43}$$

where $D_w Z(w/w_i,\cdot)$ denotes the $(m-1)\times(m-1)$ Jacobian matrix of $Z(w/w_i, T^f, T^i)$. From Proposition 17.G.3 of Mas-Collel et al. (1995, p. 618), $[D_w Z(w/w_i,\cdot)]^{-1}$ exists and has all its entries negative. Moreover, for $\theta^f = \theta^i = 0$ the direct effect of the transport cost reduction on the modified (34) is given by:

$$D_{T^f} Z(\cdot, T^f, T^i) = -(\delta^{1-\sigma} - 1) \frac{(w_j/w_i)^{-\sigma}}{(P_j^f)^{1-\sigma}} \sum_{k \in M} \tilde{X}_{ik}^f X_{jk}^f$$

for any $j \neq i$ and $0 < \delta < 1$, which is strictly smaller zero. In combination with (43) this verifies that $d_i w_j < 0$ for all $j \neq i$. $\square$

6.9 Wage adjustments

Here, we present in detail the equations that pin down the wage adjustments $dw = (dw_1, dw_2, ..., dw_m)$ after one of the investigated shocks to the trade cost matrices (T^f, T^i). To calculate the direction and the magnitude of the adjustments, we make use of the total differential of the labor income equation (34) with respect to dw, dT^f, and dT^i, where $dT = (dT_1, dT_2, ..., dT_m)$ is our short-hand notation for the direct effect of a change in the trade cost matrix on (34), i.e. $dT = D_T Z(\cdot, T^f, T^i)$.

The wage adjustments can be determined as follows: let us restate (34) as $w_i Z_i(w, T^f, T^i) = f_i(w, T^f, T^i) - w_i L_i = 0$, where:

$$f_i(w, T^f, T^i) = \frac{w_i^{1-\sigma}}{(P_i^f)^{1-\sigma}} \sum_{j \in M} X_{ij}^f + \frac{w_i^{1-\sigma}}{(P_i^i)^{1-\sigma}} \sum_{j \in M} X_{ij}^i$$

We first wish to determine all the partial derivatives of the system of functions $WZ(w, \cdot)$ (where W denotes the diagonal matrix with w_i as its elements) with respect to the individual w_i's. Let Ψ therefore be the $m \times m$ diagonal matrix with elements $\psi_{ii} = (1-\sigma) f_i / w_i - L_i < 0$. Moreover, Λ is the $m \times m$ full matrix with elements:

$$\lambda_{ij} = \frac{1}{w_j} \left[\frac{w_i^{1-\sigma}}{(P_i^f)^{1-\sigma}} \left(X_{ij}^f - \sum_{k \in M} X_{ik}^f \phi_{kj} \right) + \frac{w_i^{1-\sigma}}{(P_i^i)^{1-\sigma}} \left(\varphi_{ij} - \sum_{k \in M} \varphi_{ik} \phi_{kj} \right) \right] > 0$$

where ϕ_{jk} and φ_{ik} are defined in (36). The Jacobian of the system $WZ(w, \cdot)$ is then $[\Psi + \Lambda]$. Hence, dw is given by $dw = -\big[\Psi + \Lambda\big]^{-1} [dT^f + dT^i]$. For our empirical implementation it will prove useful however to insert the diagonal matrices WW^{-1} such that we determine $dw/w = W^{-1} dw = (dw_1/w_1, dw_2/w_2, ..., dw_m/w_m)$ as:

$$\begin{aligned} \big[[\Psi + \Lambda] W\big] [W^{-1} dw] &= -[dT^f + dT^i] \\ \frac{dw}{w} &= -\big[[\Psi + \Lambda] W\big]^{-1} [dT^f + dT^i] \end{aligned} \tag{44}$$

The direct effects, dT^f and dT^i, are dependent on the type of shock:

(i) Removal of a nation: removing country i from the network affects $Z_j(\cdot)$, for any $j \in M$, in the following way:

$$d_i T_j^f = \frac{w_j^{1-\sigma}}{(P_j^f)^{1-\sigma}} \Big[- X_{ji}^f + \sum_{k \neq i} X_{jk}^f \tilde{X}_{ik}^f \Big] + \frac{w_j^{1-\sigma}}{(P_j^i)^{1-\sigma}} \Big[- \varphi_{ji} + \sum_{k \neq i} \varphi_{jk} \tilde{X}_{ik}^f \Big] \tag{45}$$

and

$$\begin{aligned} d_i T_j^i = \frac{w_j^{1-\sigma}}{(P_j^i)^{1-\sigma}} \Big[& - (P_j^i)^{1-\sigma} \big(\frac{\theta^f}{\theta^i}\big)^{\sigma} \sum_{k \neq i} \Big(\sum_{h=1}^{\infty} a_{ji}^{[h]} \frac{X_{ik}^f}{(P_i^f)^{1-\sigma}} + \sum_{l \neq i} \sum_{h=1}^{\infty} a_{j(i)l}^{[h]} \frac{X_{lk}^f}{(P_l^f)^{1-\sigma}} \Big) \\ & + \sum_{k \neq i} \varphi_{jk} \frac{\sum_{l \neq i} d_i S A_l^f \tilde{X}_{lk}^f}{(P_l^f)^{1-\sigma}} \Big] + \frac{w_j^{1-\sigma}}{(P_j^f)^{1-\sigma}} \Big[\sum_{k \neq i} X_{jk}^f \frac{\sum_{l \neq i} d_i S A_l^f \tilde{X}_{lk}^f}{(P_l^f)^{1-\sigma}} \Big] \end{aligned} \tag{46}$$

where all the terms are evaluated at the initial matrices (T^f, T^i) and where $\tilde{X}_{ik}^f$, $\sum_{h=0}^{\infty} a_{j(i)k}^{[h]}$, and $d_i S A_l^f$ are defined in (27), (25), and (26), respectively.

To further decompose $d_i T_j^f$, the two negative summands reflect the *lost demand* for final goods from country i, whereas the two positive summands capture the fact that country j has *lost a competitor* in all its other sales markets $k \neq i$.

Decomposing $d_i T_j^i$, the two negative summands in the first line of (46) are due to the fact that country i has *intermediated value added* from country j into the rest of the world. The two positive summands in the second line again reflect that competition for j becomes weaker, as all competing producers lose access to the intermediate goods produced or chanelled by the removed country.

(ii) Coordination cost reduction: a worldwide small increase in $(\theta^f)^\sigma$ has the following effect on $Z_i(\cdot)$, for any $i \in M$:

$$\begin{aligned} dT_i^i = & - \frac{w_i^{1-\sigma}}{(P_i^f)^{1-\sigma}} \sum_{j \in M} \Big[X_{ij}^f \frac{\sum_{k \in M} dSA_k^f \tilde{X}_{kj}}{(P_k^f)^{1-\sigma}} \Big] \\ & + \frac{w_i^{1-\sigma}}{(P_i^i)^{1-\sigma}} \sum_{j \in M} \Big[\frac{X_{ij}^i}{(\theta^f)^\sigma} - \varphi_{ij} \frac{\sum_{k \in M} dSA_k^f \tilde{X}_{kj}}{(P_k^f)^{1-\sigma}} \Big] \end{aligned}$$

where $dSA_k^f = SA_k^f / (\theta^f)^\sigma$. A comparable increase in $(\theta^i)^\sigma$ does the following:

$$\begin{aligned} dT_i^i = & \frac{w_i^{1-\sigma}}{(P_i^i)^{1-\sigma}} \sum_{j \in M} \Big[\frac{(P_i^i)^{1-\sigma}}{(\theta^i)^\sigma} \sum_{k \in M} \sum_{h=1}^{\infty} a_{ik}^{[h]} \frac{X_{kj}^i}{(P_k^i)^{1-\sigma}} - \varphi_{ij} \frac{\sum_{k \in M} dSA_k^f \tilde{X}_{kj}}{(P_k^f)^{1-\sigma}} \Big] \\ & - \frac{w_i^{1-\sigma}}{(P_i^f)^{1-\sigma}} \sum_{j \in M} \Big[X_{ij}^f \frac{\sum_{k \in M} dSA_k^f \tilde{X}_{kj}}{(P_k^f)^{1-\sigma}} \Big] \end{aligned}$$

where

$$dSA_k^f \quad = \quad \frac{(\theta^f)^\sigma}{(\theta^i)^{2\sigma}} \sum_{l \in M} SA_l^i \sum_{h=1}^{\infty} a_{lk}^{[h]}$$

(iii) Unilateral transport cost reduction: A marginally small reduction of the trade costs for country i's *final goods* exports, $d_i = (T^f)' - T^f$, where $(T^f)'$ is such that $(\tau_{ik}^f)' = \delta\tau_{ik}^f$ for $\delta^{1-\sigma} \to 1$, and $(\tau_{lk}^f)' = \tau_{lk}^f$ for all $k \in M$ and $l \in M\backslash\{i\}$, imposes the following effect on $Z_i(\cdot)$:

$$\begin{aligned} d_i T_i^f = & \quad \frac{w_i^{1-\sigma}}{(P_i^f)^{1-\sigma}} \sum_{k \in M} X_{ik}^f \big[1 - \tilde{X}_{ik}^f\big] \\ & + \frac{w_i^{1-\sigma}}{(P_i^i)^{1-\sigma}} \sum_{k \in M} \Bigg[(P_i^i)^{1-\sigma} \big(\frac{\theta^f}{\theta^i}\big)^\sigma \sum_{h=1}^{\infty} a_{ii}^{[h]} \frac{X_{ik}^f}{(P_i^f)^{1-\sigma}} - \varphi_{ik} \tilde{X}_{ik}^f\Bigg] \end{aligned}$$

The effect on $Z_j(\cdot)$, for $j \neq i$, is:

$$\begin{aligned} d_i T_j^f = & - \frac{w_j^{1-\sigma}}{(P_j^f)^{1-\sigma}} \sum_{k \in M} X_{jk}^f \tilde{X}_{ik}^f \\ & + \frac{w_j^{1-\sigma}}{(P_j^i)^{1-\sigma}} \sum_{k \in M} \Bigg[(P_j^i)^{1-\sigma} \big(\frac{\theta^f}{\theta^i}\big)^\sigma \sum_{h=1}^{\infty} a_{ji}^{[h]} \frac{X_{ik}^f}{(P_i^f)^{1-\sigma}} - \varphi_{jk} \tilde{X}_{ik}^f\Bigg] \end{aligned}$$

A corresponding cost reduction for *intermediate goods* shipments, $d_i = (T^i)' - T^i$, has the following effect on any $Z_j(\cdot)$, $j \in M$:

$$\begin{aligned} d_i T_j^i = & - \frac{w_j^{1-\sigma}}{(P_j^f)^{1-\sigma}} \sum_{k \in M} \Bigg[X_{jk}^f \frac{\sum_{l \in M} d_i SA_l^f \ \tilde{X}_{lk}^f}{(P_l^f)^{1-\sigma}}\Bigg] \\ & + \frac{w_j^{1-\sigma}}{(P_j^i)^{1-\sigma}} \sum_{k \in M} \Bigg[(P_j^i)^{1-\sigma} \sum_{h=0}^{\infty} a_{ji}^{[h]} \frac{X_{ik}^i}{(P_i^i)^{1-\sigma}} \ - \ \varphi_{jk} \frac{\sum_{l \in M} d_i SA_l^f \ \tilde{X}_{lk}^f}{(P_l^f)^{1-\sigma}}\Bigg] \end{aligned}$$

where

$$d_i SA_l^f \quad = \quad \big(\frac{\theta^f}{\theta^i}\big)^\sigma \sum_{m \in M} \big(w_m^{1-\sigma} \sum_{h=1}^{\infty} a_{mi}^{[h]} \sum_{h=1}^{\infty} a_{il}^{[h]}\big) + w_i^{1-\sigma} \sum_{h=1}^{\infty} a_{il}^{[h]}$$

The direction and magnitude of the wage adjustments are hard to predict analytically in most of our experiments. However, we can take advantage of the fact that the matrices and vectors in (44) have some real world correspondences and an thus be constructed numerically from our empirical estimates of Section 3. Hence, we are able to predict the wage changes for a realistic trade network, based on the state of the world economy before the shock. The implied real income changes can be inferred from (23), where $\partial (P_i^c)^{1-\sigma}/\partial w_j \times dw_j/(P_i^c)^{1-\sigma} = \phi_{ij} dw_j/w_j$ with ϕ_{ij} defined in (36).

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