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**TARGETING NOMINAL GDP OR
PRICES: GUIDANCE AND
EXPECTATION DYNAMICS**

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ABSTRACT

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JEL Classification: E52, E58 and E63

Keywords: adaptive learning, inflation targeting, monetary policy and zero interest rate lower bound

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Targeting Nominal GDP or Prices: Guidance and Expectation Dynamics*

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Abstract

We examine global dynamics under infinite-horizon learning in New Keynesian models where monetary policy practices either price-level or nominal GDP targeting and compare these regimes to inflation targeting. These interest-rate rules are subject to the zero lower bound. Robustness of the three rules in learning adjustment are compared using criteria for the domain of attraction of the targeted steady state, volatility of inflation and output and sensitivity to the speed of learning parameter. Performance of price-level and nominal GDP targeting significantly improves if the additional guidance in these regimes is incorporated in private agents' learning.

JEL Classification: E63, E52, E58.

Keywords: Adaptive Learning, Monetary Policy, Inflation Targeting, Zero Interest Rate Lower Bound

1 Introduction

The practical significance of the zero lower bound (ZLB) for policy interest rates has become evident in the US and Europe since the 2007-9 financial

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crisis and earlier in Japan since the mid 1990s. In monetary economics the Japanese experience initiated renewed interest in ways of getting out of or avoiding the ZLB constraint.¹ The long-standing Japanese experience of persistently very low inflation and occasional deflation took place nearly concurrently with the rise of inflation targeting as the most common, “best practice” monetary policy. Recently, the ongoing economic crisis in most advanced market economies and the already lengthy period of very low interest rates has led to suggestions that price-level or nominal GDP targeting can potentially be more appropriate frameworks for monetary policy.

History-dependence is a key feature of nominal income and price-level targeting as it can provide more guidance to the economy than inflation targeting.² This guidance can be helpful and is arguably good policy in a liquidity trap where the ZLB constrains on monetary policy. See Eggertsson and Woodford (2003) for a discussion of optimal monetary policy and price-level targeting. Carney (2012) and Evans (2012) provide broad discussions of the need for additional guidance for the price level and possibly other variables including the nominal GDP. Carney (2012) suggests that with policy rates at ZLB “there could be a more favorable case for nominal GDP targeting”.

If a move from inflation targeting to either nominal income or price-level targeting is contemplated, it is important to allow for the possibility that private agents face significant uncertainties when a new policy rule is adopted. Thus, it is assumed that private agents’ knowledge is imperfect. We analyze the properties of nominal GDP targeting and price-level targeting with specification of imperfect knowledge by means of the adaptive learning approach in which agents maximize anticipated utility or profit subject to expectations that are derived from an econometric model. The model is updated over time as new information becomes available.³

Our approach contrasts with the existing literature on nominal income

¹For prominent early analyzes, see for example Krugman (1998), Eggertsson and Woodford (2003), and Svensson (2003). Werning (2012) is a recent paper on optimal policies in a liquidity trap under rational expectations.

²Price-level targeting has received a fair amount of attention, see for example Svensson (1999), Vestin (2006). Nominal income targeting has been considered, for example Hall and Mankiw (1994), Jensen (2002) and Mitra (2003). A recent overview of nominal income targeting is given in Bean (2009).

³For discussion and analytical results concerning adaptive learning in a wide range of macroeconomic models, see for example Sargent (1993), Evans and Honkapohja (2001), Sargent (2008), and Evans and Honkapohja (2009).

and price-level targeting that is to a large extent based on the rational expectations (RE) hypothesis. RE is a very strong assumption about the agents' knowledge of the economy as the agents with RE are able to perfectly predict the future path of the economy, except for the consequences of unforecastable random shocks. We note that there has recently been increasing interest in relaxing the RE hypothesis in the context of macroeconomic policy analysis, see e.g. Taylor and Williams (2010) and Woodford (2013).⁴

We compare the properties of price-level and nominal income targeting to inflation targeting in a nonlinear New Keynesian (NK) model when private agents learn adaptively. The nonlinear framework is needed to assess the global properties of these two targeting regimes, including the possibility of encountering the ZLB. We argue that the good properties of nominal GDP or price-level targeting may not hold when imperfect knowledge prevails and agents behave in accordance with adaptive learning.⁵ The NK model is otherwise standard, so that there are no financial market imperfections. Our objective is to compare the three policy frameworks when there are no special financial-market problems.

A preliminary result is that, like inflation targeting, nominal income and price-level targeting regimes are subject to the problems caused by the ZLB. In a standard NK model a key equation for a nonstochastic steady state is the Fisher equation $R = \beta^{-1}\pi$, where R is the gross interest rate, β is the subjective discount factor and π is the gross inflation rate. Usually the interest rate rule has a specified target inflation rate $\pi^* \geq 1$ (and an associated output level) as a steady state. If policy sets $R = 1$, then $\pi = \beta < 1$ becomes a second steady state as the Fisher equation again holds. Eggertsson and Woodford (2003), pp.193-194 note that such a deflationary trap can exist but they do not analyze it in detail.⁶

As a first step for the analysis of learning we establish theoretically that the targeted steady state is locally stable under learning and the deflationary

⁴Some aspects of imperfect knowledge are included in the discussion of price-level targeting by Gaspar, Smets, and Vestin (2007). See also the literature they cite.

⁵Williams (2010) makes a similar argument about price-level targeting under imperfect knowledge and learning. His work relies on simulations of a linearized model.

⁶Analysis of the ZLB and multiple equilibria for an inflation targeting framework and a Taylor-type interest rate rule has been carried out by Reifschneider and Williams (2000), Benhabib, Schmitt-Grohe, and Uribe (2001) and Benhabib, Schmitt-Grohe, and Uribe (2002). These issues have been considered under learning in Evans and Honkapohja (2010), and Benhabib, Evans, and Honkapohja (2013).

steady state is unstable for both the price-level and nominal GDP targeting regimes. We then analyze the dynamics of learning under price-level and nominal GDP targeting regimes by using several robustness criteria, namely the size of the domain of attraction, volatility of aggregate variables during the adjustment, and maximal speed of private agent learning.

Very importantly, we show that the properties of learning depend on whether private agents include the further guidance provided by these targeting regimes into their forecasting and learning. If agents simply forecast expected inflation and aggregate output from available data in the same way as under inflation targeting, then the targeted steady state is locally stable under learning but the robustness characteristics of learning of price-level and nominal GDP targeting are worse or not much better than under inflation targeting. However, if agents incorporate either the target price level path or the target nominal GDP path, respectively, into their inflation forecasting when the central bank practices, respectively, price-level or nominal GDP targeting, then robustness properties of these regimes improve and are in important respects superior to those of inflation targeting.

2 A New Keynesian Model

We employ a standard New Keynesian model as the analytical framework.⁷ There is a continuum of household-firms, which produce a differentiated consumption good under monopolistic competition and price-adjustment costs. There is also a government which uses monetary policy, buys a fixed amount of output, finances spending by taxes, and issues of public debt, see below.

The objective for agent s is to maximize expected, discounted utility subject to a standard flow budget constraint (in real terms):

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t U_{t,s} \left(c_{t,s}, \frac{M_{t-1,s}}{P_t}, h_{t,s}, \frac{P_{t,s}}{P_{t-1,s}} - 1 \right) \quad (1)$$

$$\text{st. } c_{t,s} + m_{t,s} + b_{t,s} + \Upsilon_{t,s} = m_{t-1,s} \pi_t^{-1} + R_{t-1} \pi_t^{-1} b_{t-1,s} + \frac{P_{t,s}}{P_t} y_{t,s}, \quad (2)$$

where $c_{t,s}$ is the consumption aggregator, $M_{t,s}$ and $m_{t,s}$ denote nominal and real money balances, $h_{t,s}$ is the labor input into production, $b_{t,s}$ denotes the

⁷The same framework is developed in Evans, Guse, and Honkapohja (2008). It is also employed in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2013).

real quantity of risk-free one-period nominal bonds held by the agent at the end of period t , $\Upsilon_{t,s}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor between periods $t-1$ and t , $P_{t,s}$ is the price of consumption good s , $y_{t,s}$ is output of good s , P_t is the aggregate price level, and the inflation rate is $\pi_t = P_t/P_{t-1}$. The subjective discount factor is denoted by β . The utility function has the parametric form

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left(\frac{M_{t-1,s}}{P_t} \right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2} \left(\frac{P_{t,s}}{P_{t-1,s}} - 1 \right)^2,$$

where $\sigma_1, \sigma_2, \varepsilon, \gamma > 0$. The final term parameterizes the cost of adjusting prices in the spirit of Rotemberg (1982). We use the Rotemberg formulation rather than the Calvo model of price stickiness because it enables us to study global dynamics in the nonlinear system.⁸ The household decision problem is also subject to the usual “no Ponzi game” (NPG) condition.

Production function for good s is given by

$$y_{t,s} = h_{t,s}^\alpha,$$

where $0 < \alpha < 1$. Output is differentiated and firms operate under monopolistic competition. Each firm faces a downward-sloping demand curve

$$P_{t,s} = \left(\frac{y_{t,s}}{y_t} \right)^{-1/\nu} P_t. \quad (3)$$

Here $P_{t,s}$ is the profit maximizing price set by firm s consistent with its production $y_{t,s}$. The parameter ν is the elasticity of substitution between two goods and is assumed to be greater than one. y_t is aggregate output, which is exogenous to the firm.

The government’s flow budget constraint in real terms is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \quad (4)$$

where g_t denotes government consumption of the aggregate good, b_t is the real quantity of government debt, and Υ_t is the real lump-sum tax collected. We assume that fiscal policy follows a linear tax rule for lump-sum taxes as in Leeper (1991)

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1}, \quad (5)$$

⁸The linearizations at the targeted steady state are identical for the two approaches.

where we will assume that $\beta^{-1} - 1 < \kappa < 1$. Thus fiscal policy is “passive” in the terminology of Leeper (1991) and implies that an increase in real government debt leads to an increase in taxes sufficient to cover the increased interest and at least some fraction of the increased principal.

We assume that g is constant and given by

$$g_t = \bar{g}. \quad (6)$$

From market clearing we have

$$c_t + g_t = y_t. \quad (7)$$

2.1 Optimal decisions for private sector

As in Evans, Guse, and Honkapohja (2008), the first-order conditions for an optimum yield

$$0 = -h_{t,s}^\varepsilon + \frac{\alpha\gamma}{\nu}(\pi_{t,s} - 1)\pi_{t,s}\frac{1}{h_{t,s}} + \alpha \left(1 - \frac{1}{\nu}\right) y_t^{1/\nu} \frac{y_{t,s}^{(1-1/\nu)}}{h_{t,s}} c_{t,s}^{-\sigma_1} - \frac{\alpha\gamma\beta}{\nu} \frac{1}{h_{t,s}} E_{t,s}(\pi_{t+1,s} - 1)\pi_{t+1,s}. \quad (8)$$

$$c_{t,s}^{-\sigma_1} = \beta R_t E_{t,s} (\pi_{t+1}^{-1} c_{t+1,s}^{-\sigma_1}) \quad (9)$$

and

$$m_{t,s} = (\chi\beta)^{1/\sigma_2} \left(\frac{(1 - R_t^{-1}) c_{t,s}^{-\sigma_1}}{E_{t,s} \pi_{t+1}^{\sigma_2 - 1}} \right)^{-1/\sigma_2}, \quad (10)$$

where $\pi_{t+1,s} = P_{t+1,s}/P_{t,s}$.

Equation (8) is the nonlinear New Keynesian Phillips curve describing the optimal price-setting by firms. The term $(\pi_{t,s} - 1)\pi_{t,s}$ arises from the quadratic form of the adjustment costs, and this expression is increasing in $\pi_{t,s}$ over the allowable range $\pi_{t,s} \geq 1/2$. To interpret this equation, note that the first term on the right-hand side is the marginal disutility of labor while the third term can be viewed as the product of the marginal revenue from an extra unit of labor with the marginal utility of consumption. The terms involving current and future inflation arise from the price-adjustment costs.

Equation (9) is the standard Euler equation giving the intertemporal first-order condition for the consumption path. Equation (10) is the money

demand function resulting from the presence of real balances in the utility function.

We now proceed to rewrite the decision rules for consumption and inflation so that they depend on forecasts of key variables over the infinite horizon (IH). The IH learning approach in New Keynesian models is emphasized by Preston (2005) and Preston (2006), and is used in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2013) to study the properties of a liquidity trap.

2.2 The infinite-horizon Phillips curve

Starting with (8), let

$$Q_{t,s} = (\pi_{t,s} - 1) \pi_{t,s}. \quad (11)$$

The appropriate root for given Q is $\pi \geq \frac{1}{2}$ and so we need to impose $Q \geq -\frac{1}{4}$ to have a meaningful model. Using the production function $h_{t,s} = y_{t,s}^{1/\alpha}$ we can rewrite (8) as

$$Q_{t,s} = \frac{\nu}{\alpha\gamma} y_{t,s}^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t^{1/\nu} y_{t,s}^{(\nu-1)/\nu} c_{t,s}^{-\sigma_1} + \beta E_{t,s} Q_{t+1,s}, \quad (12)$$

and using the demand curve $y_{t,s}/y_t = (P_{t,s}/P_t)^{-\nu}$ gives

$$Q_{t,s} = \frac{\nu}{\alpha\gamma} (P_{t,s}/P_t)^{-(1+\varepsilon)\nu/\alpha} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t (P_{t,s}/P_t)^{-(\nu-1)} c_{t,s}^{-\sigma_1} + \beta E_{t,s} Q_{t+1,s}.$$

Defining

$$x_{t,s} \equiv \frac{\nu}{\alpha\gamma} (P_{t,s}/P_t)^{-(1+\varepsilon)\nu/\alpha} Y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t (P_{t,s}/P_t)^{-(\nu-1)} c_{t,s}^{-\sigma_1}$$

and iterating the Euler equation yields

$$Q_{t,s} = x_{t,s} + \sum_{j=1}^{\infty} \beta^j E_{t,s} x_{t+j,s}, \quad (13)$$

provided that the transversality condition

$$\beta^j E_{t,s} x_{t+j,s} \rightarrow 0 \text{ as } j \rightarrow \infty \quad (14)$$

holds. It can be shown that the condition (14) is an implication of the necessary transversality condition for optimal price setting.⁹

The variable $x_{t+j,s}$ is a mixture of aggregate variables and the agent's own future decisions. Nonetheless this equation for $Q_{t,s}$ can be the basis for decision-making as follows. So far we have only used the agent's price-setting Euler equation and the above limiting condition (14). We now make some further adaptive learning assumptions.

First, agents are assumed to have point expectations, so that their decisions depend only on the mean of their subjective forecasts. Second, we assume that agents have learned from experience that in fact, in temporary equilibrium, it is always the case that $P_{t,s}/P_t = 1$. Therefore we assume that agents impose this in their forecasts in (13), i.e. they set $(P_{t+j,s}/P_{t+j})^e = 1$. Third, agents have learned from experience that in fact, in temporary equilibrium, it is always the case that $c_{t,s} = y_t - g_t$ in per capita terms. Therefore, agents impose in their forecasts that $c_{t+j,s}^e = y_{t,t+j}^e - g_{t,t+j}^e$. In the case of no fiscal policy change this becomes $c_{t+j,s}^e = y_{t+j}^e - \bar{g}$.

We now make use of the representative agent assumption, so that all agents s have the same utility functions, initial money and debt holdings, and prices. We assume also that they make the same forecasts $c_{t+1,s}^e$, $\pi_{t+1,s}^e$, as well as forecasts of other variables that will become relevant below. Under these assumptions all agents make the same decisions at each point in time, so that $h_{t,s} = h_t$, $y_{t,s} = y_t$, $c_{t,s} = c_t$ and $\pi_{t,s} = \pi_t$, and all agents make the same forecasts. For convenience, the utility of consumption and of money is also taken to be logarithmic ($\sigma_1 = \sigma_2 = 1$). Then (13) takes the form

$$\begin{aligned} Q_t &= \frac{\nu}{\alpha\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t (y_t - \bar{g})^{-1} + \\ &\quad \frac{\nu}{\gamma} \sum_{j=1}^{\infty} \alpha^{-1} \beta^j (y_{t+j}^e)^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} \sum_{j=1}^{\infty} \beta^j \left(\frac{y_{t+j}^e}{y_{t+j}^e - \bar{g}} \right) \\ &\equiv \tilde{K}(y_t, y_{t+1}^e, y_{t+2}^e \dots). \end{aligned} \tag{15}$$

The expectations are formed at time t and variables at time t are assumed to be in the information set of the agents when they make current decisions. We will treat (15), together with (11), as the temporary equilibrium equations that determine π_t , given expectations $\{y_{t+j}^e\}_{j=1}^{\infty}$.

⁹For further details see Benhabib, Evans, and Honkapohja (2013).

One might wonder why inflation does not also depend directly on the expected future aggregate inflation rate in the Phillip's curve relationship (15).¹⁰ Equation (8) is obtained from the first-order conditions using (3) to eliminate relative prices. Because of the representative agent assumption, each firm's output equals average output in every period. Since firms can be assumed to have learned this to be the case, we obtain (15).

2.3 The consumption function

To derive the consumption function from (9) we use the flow budget constraint and the NPG condition to obtain an intertemporal budget constraint. First, we define the asset wealth

$$a_t = b_t + m_t$$

as the sum of holdings of real bonds and real money balances and write the flow budget constraint as

$$a_t + c_t = y_t - \Upsilon_t + r_t a_{t-1} + \pi_t^{-1}(1 - R_{t-1})m_{t-1}, \quad (16)$$

where $r_t = R_{t-1}/\pi_t$. Note that we assume $(P_{jt}/P_t)y_{jt} = y_t$, i.e. the representative agent assumption is being invoked. Iterating (16) forward and imposing

$$\lim_{j \rightarrow \infty} (D_{t,t+j}^e)^{-1} a_{t+j}^e = 0, \quad (17)$$

where

$$D_{t,t+j}^e = \frac{R_t}{\pi_{t+1}^e} \prod_{i=2}^j \frac{R_{t+i-1}^e}{\pi_{t+i}^e}$$

with $r_{t+i}^e = R_{t+i-1}^e/\pi_{t+i}^e$, we obtain the life-time budget constraint of the household

$$0 = r_t a_{t-1} + \Phi_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} \Phi_{t+j}^e \quad (18)$$

$$= r_t a_{t-1} + \phi_t - c_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (\phi_{t+j}^e - c_{t+j}^e), \quad (19)$$

¹⁰There is an indirect effect of expected inflation on current inflation via current output.

where

$$\begin{aligned}\Phi_{t+j}^e &= y_{t+j}^e - \Upsilon_{t+j}^e - c_{t+j}^e + (\pi_{t+j}^e)^{-1}(1 - R_{t+j-1}^e)m_{t+j-1}^e, \\ \phi_{t+j}^e &= \Phi_{t+j}^e + c_{t+j}^e = y_{t+j}^e - \Upsilon_{t+j}^e + (\pi_{t+j}^e)^{-1}(1 - R_{t+j-1}^e)m_{t+j-1}^e.\end{aligned}\quad (20)$$

Here all expectations are formed in period t , which is indicated in the notation for $D_{t,t+j}^e$ but is omitted from the other expectational variables.

Invoking the relations

$$c_{t+j}^e = c_t \beta^j D_{t,t+j}^e, \quad (21)$$

which is an implication of the consumption Euler equation (9), we obtain

$$c_t(1 - \beta)^{-1} = r_t a_{t-1} + y_t - \Upsilon_t + \pi_t^{-1}(1 - R_{t-1})m_{t-1} + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} \phi_{t+j}^e. \quad (22)$$

As we have $\phi_{t+j}^e = y_{t+j}^e - \Upsilon_{t+j}^e + (\pi_{t+j}^e)^{-1}(1 - R_{t+j-1}^e)m_{t+j-1}^e$, the final term in (22) is

$$\sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (y_{t+j}^e - \Upsilon_{t+j}^e) + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (\pi_{t+j}^e)^{-1} (1 - R_{t+j-1}^e) m_{t+j-1}^e$$

and using (10) we have

$$\begin{aligned}& \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (\pi_{t+j}^e)^{-1} (1 - R_{t+j-1}^e) m_{t+j-1}^e \\ &= \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (\pi_{t+j}^e)^{-1} (-\chi \beta R_{t+j-1}^e c_{t+j-1}^e) = -\frac{\chi \beta}{1 - \beta} c_t.\end{aligned}$$

We obtain the consumption function

$$c_t \frac{1 + \chi \beta}{1 - \beta} = r_t b_{t-1} + \frac{m_{t-1}}{\pi_t} + y_t - \Upsilon_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (y_{t+j}^e - \Upsilon_{t+j}^e).$$

So far it is not assumed that households act in a Ricardian way, i.e. they have not imposed the intertemporal budget constraint (IBC) of the

government. To simplify the analysis, we assume that consumers are Ricardian, which allows us to modify the consumption function as in Evans and Honkapohja (2010).¹¹ From (4) one has

$$\begin{aligned} b_t + m_t + \Upsilon_t &= \bar{g} + m_{t-1}\pi_t^{-1} + r_t b_{t-1} \text{ or} \\ b_t &= \Delta_t + r_t b_{t-1} \text{ where} \\ \Delta_t &= \bar{g} - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}. \end{aligned}$$

By forward substitution, and assuming

$$\lim_{T \rightarrow \infty} D_{t,t+T} b_{t+T} = 0, \quad (23)$$

we get

$$0 = r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} D_{t,t+j}^{-1} \Delta_{t+j}. \quad (24)$$

Note that Δ_{t+j} is the primary government deficit in $t+j$, measured as government purchases less lump-sum taxes and less seigniorage. Under the Ricardian Equivalence assumption, agents at each time t expect this constraint to be satisfied, i.e.

$$\begin{aligned} 0 &= r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} \Delta_{t+j}^e, \text{ where} \\ \Delta_{t+j}^e &= \bar{g} - \Upsilon_{t+j}^e - m_{t+j}^e + m_{t+j-1}^e (\pi_{t+j}^e)^{-1} \text{ for } j = 1, 2, 3, \dots \end{aligned}$$

A Ricardian consumer assumes that (23) holds. His flow budget constraint (16) can be written as:

$$\begin{aligned} b_t &= r_t b_{t-1} + \psi_t, \text{ where} \\ \psi_t &= y_t - \Upsilon_t - m_t - c_t + \pi_t^{-1} m_{t-1}. \end{aligned}$$

The relevant transversality condition is now (23). Iterating forward and using (21) together with (23) yields the consumption function

$$c_t = (1 - \beta) \left(y_t - \bar{g} + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (y_{t+j}^e - \bar{g}) \right). \quad (25)$$

For further details see Evans and Honkapohja (2010).

¹¹Evans, Honkapohja, and Mitra (2012) state the assumptions under which Ricardian Equivalence holds along a path of temporary equilibria with learning if agents have an infinite decision horizon.

3 Temporary Equilibrium and Learning

We assume that agents form expectations using steady-state learning, which is formulated as follows. Steady-state learning with point expectations is formalized as

$$s_{t+j}^e = s_t^e \text{ for all } j \geq 1, \text{ and } s_t^e = s_{t-1}^e + \omega_t(s_{t-1} - s_{t-1}^e) \quad (26)$$

for $s = y, \pi, R$. It should be noted that expectations s_t^e refer to future periods (and not the current one). It is assumed that when forming s_t^e the newest available data point is s_{t-1} , i.e. expectations are formed in the beginning of the current period and current-period values of endogenous variables are not yet known.

ω_t is called the “gain sequence,” and measures the extent of adjustment of the estimates to the most recent forecast error. In stochastic systems one often sets $\omega_t = t^{-1}$ and this “decreasing gain” learning corresponds to least-squares updating. Also widely used is the case $\omega_t = \omega$, for $0 < \omega \leq 1$, called “constant gain” learning. In this case it is usually assumed that ω is small.

The temporary equilibrium equations with steady-state learning are:

1. The aggregate demand

$$\begin{aligned} y_t &= \bar{g} + (\beta^{-1} - 1)(y_t^e - \bar{g}) \left(\frac{\pi_t^e}{R_t} \right) \left(\frac{R_t^e}{R_t^e - \pi_t^e} \right) \\ &\equiv Y(y_t^e, \pi_t^e, R_t, R_t^e). \end{aligned} \quad (27)$$

Here it is assumed that consumers make forecasts of future nominal interest rates, which are equal for all future period, given that we are assuming steady-state learning.

2. The nonlinear Phillips curve

$$\pi_t = Q^{-1}[\tilde{K}(y_t, y_{t+1}^e, y_{t+2}^e \dots)] \equiv Q^{-1}[K(y_t, y_t^e)], \quad (28)$$

where $\tilde{K}(\cdot)$ is defined in (15) and

$$Q(\pi_t) \equiv (\pi_t - 1) \pi_t \quad (29)$$

$$\begin{aligned} K(y_t, y_t^e) &\equiv \frac{\nu}{\gamma} \left(\alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - (1 - \nu^{-1}) \frac{y_t}{(y_t - \bar{g})} \right) \\ &\quad + \frac{\nu}{\gamma} \left(\beta(1 - \beta)^{-1} \left(\alpha^{-1} (y_t^e)^{(1+\varepsilon)/\alpha} - (1 - \nu^{-1}) \frac{y_t^e}{y_t^e - \bar{g}} \right) \right). \end{aligned} \quad (30)$$

3. Bond dynamics

$$b_t + m_t = g - \Upsilon_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{m_{t-1}}{\pi_t}. \quad (31)$$

4. Money demand

$$m_t = \chi \beta \frac{R_t}{R_t - 1} c_t. \quad (32)$$

5. Different interest rate rules are considered below and are specified in the next section.

The state variables are b_{t-1} , m_{t-1} , and R_{t-1} . With Ricardian consumers the dynamics for bonds and money do not influence the dynamics of the endogenous variables, though clearly the evolution of b_t and m_t is influenced by the dynamics of inflation and output. The system in general has three expectational variables: output y_t^e , inflation π_t^e , and the interest rate R_t^e . We now assume that private agents formulate these expectations using available data on y_t , π_t and R_t . The evolution of expectations is then given by

$$y_t^e = y_{t-1}^e + \omega(y_{t-1} - y_{t-1}^e), \quad (33)$$

$$\pi_t^e = \pi_{t-1}^e + \omega(\pi_{t-1} - \pi_{t-1}^e), \quad (34)$$

$$R_t^e = R_{t-1}^e + \omega(R_{t-1} - R_{t-1}^e). \quad (35)$$

Analysis of E-stability is a convenient way to analyze conditions for convergence in these kinds of learning models.¹² Below we derive E-stability and instability results for the steady states. The E-stability conditions of RE equilibrium give conditions for local convergence of real-time learning rules such (33)-(35). The E-stability conditions apply directly for rules that employ a decreasing gain but the same conditions also apply to constant-gain rules in the limit, where the gain parameter is made arbitrarily small.

4 Monetary Policy Frameworks

Our aim is to compare the performance of price-level and nominal GDP targeting against each other and also against inflation targeting (IT). Initially,

¹²See e.g. Evans and Honkapohja (2001) and Evans and Honkapohja (2009) for details on E-stability analysis and its connections to real-time learning.

it is assumed that private agents learn according to (33)-(35). This case arises naturally when after a change in the IT interest rate rule to either price-level or nominal GDP targeting. For concreteness and simplicity of the comparisons we model IT in terms of the standard Taylor rule

$$R_t = 1 + \max[\bar{R} - 1 + \psi_\pi[(\pi_t - \pi^*)/\pi^*] + \psi_y[(y_t - y^*)/y^*], 0], \quad (36)$$

where we have introduced the ZLB, so that the gross interest rate cannot fall below one. For analytical ease, we adopt a piecewise linear formulation of the interest rate rule.

4.1 Price-level targeting

Starting with price-level targeting (PLT), we first note that a number of different formulations exist in the literature. We consider a simple formulation, where the policy maker sets the policy instrument with the intention to move the actual price level gradually toward a targeted price level path, which is specified exogenously. These kinds of instrument rules are called Wicksellian, see pp.260-61 of Woodford (2003) and Giannoni (2012) for discussions of Wicksellian rules. In particular, Giannoni (2012) analyses a number of different versions of the Wicksellian rules.

We assume that the target price-level path $\{\bar{p}_t\}$ involves constant inflation, so that

$$\bar{p}_t/\bar{p}_{t-1} = \pi^* \geq 1. \quad (37)$$

The interest rate, which is the policy instrument, is set above (below, respectively) the targeted steady-state value of the instrument when the actual price level is above (below, respectively) the targeted price-level path \bar{p}_t , as measured in percentage deviations. The interest rate also responds to the percentage gap between targeted and actual levels of output. The target level of output y^* is the steady state value associated with π^* . This leads to a **Wicksellian interest rate rule**

$$R_t = 1 + \max[\bar{R} - 1 + \psi_p[(p_t - \bar{p}_t)/\bar{p}_t] + \psi_y[(y_t - y^*)/y^*], 0], \quad (38)$$

where the max operation takes account of the ZLB on the interest rate and $\bar{R} = \beta^{-1}\pi^*$ is the gross interest rate at the targeted steady state. For analytical ease and to have comparability to IT, we adopt a piecewise linear formulation of the interest rate rule.¹³

¹³A fairly common formulation of PLT is advocated as a way to achieve optimal policy

4.2 Nominal GDP targeting

To have comparability to PLT, we model nominal GDP targeting in terms of an instrument rule similar to Wicksellian PLT rule, e.g., see Clark (1994) and Judd and Motley (1993). The monetary authority then sets the interest rate above (below) the targeted steady-state value \bar{R} if actual nominal GDP $p_t y_t$ is above (respectively, below) the targeted nominal GDP path $\{\bar{z}_t\}$. In line with a standard NK model, we assume that there is an associated inflation objective $\pi^* \geq 1$ calling for a non-negative (net) inflation rate and that the economy does not have any sources for trend real growth. Then the nominal GDP target is formally $\bar{z}_t = \bar{p}_t y^*$ with $\bar{p}_t/\bar{p}_{t-1} = \pi^*$. Taking the ratio, the path of nominal GDP growth satisfies

$$\bar{z}_t/\bar{z}_{t-1} = \Delta\bar{z} = \pi^*.$$

Taking into account the ZLB, such an interest rate rule takes the form

$$R_t = 1 + \max[\bar{R} - 1 + \psi[(p_t y_t - \bar{z}_t)/\bar{z}_t], 0], \quad (39)$$

where $\psi > 0$ is a policy parameter. Below we refer to (39) as the **NGDP interest rate rule**.

5 Steady States

A non-stochastic steady state (y, π, R) under PLT must satisfy the Fisher equation $R = \beta^{-1}\pi$, the interest rate rule (38), and steady-state form of the equations for output and inflation (27) and (28). One steady state clearly obtains when the actual inflation rate equals the inflation rate of the price-level target path, see equation (37). Then $R = R^*$, $\pi = \pi^*$ and $y = y^*$, where y^* is the unique solution to the equation

$$\pi^* = Q^{-1}[K(Y(y^*, \pi^*, R^*, R^*), y^*)].$$

Moreover, for this steady state $p_t = \bar{p}_t$ for all t .

with timeless perspective under RE locally near the targeted steady state. The learnability properties of this form of PLT depend on the implementation of the corresponding interest rate rule, see Evans and Honkapohja (2013), section 2.5.2 for an overview and further references. Global properties of this case have not been analyzed.

Then consider the possible steady states under NGDP targeting. One of the steady states obtains when the economy follows the targeted nominal GDP path, so that $R = R^*$, $\pi = \pi^*$ and $y = y^*$ and $\pi^* = \Delta\bar{z}$.

The targeted steady state under either PLT or NGDP rule is, however, not unique.¹⁴ It can be verified that there is a second steady state in which the ZLB condition is binding:

Lemma 1 (1) *Assume that $\beta^{-1}\pi^* - 1 < \psi_p$. Under the Wicksellian PLT rule (38), there exists a ZLB-constrained steady state in which $\hat{R} = 1$, $\hat{\pi} = \beta$, and \hat{y} solves the equation*

$$\hat{\pi} = Q^{-1}[K(Y(\hat{y}, \hat{\pi}, 1, 1), \hat{y})]. \quad (40)$$

(2) *Assume that $\beta^{-1}\pi^* - 1 < \psi$. The ZLB-constrained steady state \hat{R} , $\hat{\pi}$, and \hat{y} exists under the NGDP interest rate rule (39).*

In the ZLB-constrained steady state the price level p_t converges toward zero, so that the price-level target \bar{p}_t or NGDP target $\Delta\bar{z}$, respectively, is not met.

Proof. (1) Consider the interest rate rule (38). Imposing $\hat{\pi} = \beta < 1$ implies that $p_t \rightarrow 0$ while $\bar{p}_t \rightarrow \infty$ (or \bar{p} if $\pi^* = 1$) as $t \rightarrow \infty$. It follows that $\bar{R} - 1 + \psi_p[(p_t - \bar{p}_t)/\bar{p}_t] + \psi_y[(y_t - y^*)/y^*] < 0$ for t sufficiently large when $y_t \rightarrow \hat{y} < y^*$, so that $R_t = 1$ in the interest rate rule. A unique steady state satisfying (40) is obtained. Thus, \hat{y} , $\hat{\pi}$ and \hat{R} constitute a ZLB-constrained steady state.

(2) Now consider the economy under NGDP targeting and impose $\hat{R} = 1$, $\hat{\pi} = \beta$, and $y = \hat{y}$ where \hat{y} solves (40) with $\hat{\pi} = \beta$. Again $p_t \rightarrow 0$ while $\bar{p}_t \rightarrow \infty$ (or \bar{p} if $\pi^* = 1$) as $t \rightarrow \infty$. Inside the interest rate rule (39) we have

$$(p_t y_t - \bar{z}_t)/\bar{z}_t = (p_t/\bar{p}_t)(\hat{y}/y^*) - 1 \rightarrow -1,$$

so that for large enough t the interest rate from (39) must be $R_t = 1$. These requirements yield a steady state state for the economy. ■

We remark that the sufficient condition $\beta^{-1}\pi^* - 1 < \psi_p$ or $\beta^{-1}\pi^* - 1 < \psi$ is not restrictive e.g. when $\beta = 0.99$ and $\pi^* = 1.02$.¹⁵ The lemma states

¹⁴Existence of the two steady states under PLT was pointed out in Evans and Honkapohja (2013), section 2.5.3. This paper also considers E-stability of the steady states under short-horizon (so-called Euler equation) learning.

¹⁵For PLT a weaker sufficient condition is $\beta^{-1}\pi^* - 1 - \psi_p + \psi_y(\hat{y}/y^* - 1) < 0$, in which the term \hat{y}/y^* is complicated function of all model parameters.

that, like IT with a Taylor rule, commonly used formulations of price-level and NGDP targeting both suffer from global indeterminacy as the economy has two steady states under either monetary policy regime.

6 Expectations Dynamics: Theoretical Results

We now begin to consider global dynamics of the economy in these regimes under the hypothesis that agents operate under imperfect knowledge and form expectations in accordance with steady-state learning as described above. The formulation assumes that agents forecast output, inflation and the interest rate. This is a natural formulation when IT is the prevalent policy regime since the private sector knows that the central bank certainly focuses on inflation and possibly on aggregate activity, though the latter depends on its mandate. Agents also need to forecast the future interest rates if they do not know the policy rule. If the policy regime is changed to PLT or NGDP targeting, it is plausible to assume that private agents' forecasting will continue as before, given their imperfect knowledge. In this and the next section we analyze the properties of PLT and NGDP targeting when agents are assumed to learn in this way.

We start by developing theoretical results about the local stability of steady states under adaptive learning when either a PLT regime with rule (38) or NGDP regime with rule (39).¹⁶ The first step is to analyze local dynamics near the two possible steady states. Here some theoretical results can be obtained by exploiting theoretical stability methods that are commonly used in the study of adaptive least-squares learning.

By introducing the variable $X_t = p_t/\bar{p}_t$ it is possible to analyze even the case where the actual price level is explosive because of an explosive target $\pi^* > 1$. We then have a further equation

$$X_t = \pi_t X_{t-1} / \pi^* \tag{41}$$

and write the temporary equilibrium system (27), (28), (38), and (41) in the abstract form

$$F(x_t, x_t^e, x_{t-1}) = 0,$$

¹⁶The corresponding results for IT are known. See e.g. Evans and Honkapohja (2010) for the forward-looking version of the Taylor rule with IH learning.

where the vector x_t contains the dynamic variables. The vector of state variables is $x_t = (y_t, \pi_t, R_t, X_t)^T$. Linearizing around a steady state we obtain the system

$$x_t = (-DF_x)^{-1}(DF_{x^e}x_t^e + DF_{x_{-1}}x_{t-1}) \equiv Mx_t^e + Nx_{t-1}, \quad (42)$$

where for brevity we use the same notation for the deviations from the steady state. Recall that x_t^e refers to the expected future values of x_t and not the current one. This system is in a standard form for the analysis of learning stability of RE equilibrium.

The learning rules (33)-(35) can be written in vector form as

$$x_t^e = (1 - \omega)x_{t-1}^e + \omega x_{t-1}. \quad (43)$$

We remark that in the PLT case p_t is in fact a backward-looking variable and we will eliminate the redundant equation in (43) when doing the proof. Combining (42) and (43) we get the system

$$\begin{aligned} \begin{pmatrix} I & -M \\ 0 & I \end{pmatrix} \begin{pmatrix} x_t \\ x_t^e \end{pmatrix} &= \begin{pmatrix} N & 0 \\ \omega I & (1 - \omega)I \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-1}^e \end{pmatrix} \text{ or} \\ \begin{pmatrix} x_t \\ x_t^e \end{pmatrix} &= \begin{pmatrix} N + \omega M & (1 - \omega)M \\ \omega I & (1 - \omega)I \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-1}^e \end{pmatrix}. \end{aligned}$$

This system can be analyzed a system of difference equations and we are interested in "small gain" results. Alternatively, it would be possible to analyze convergence of learning dynamics using the so-called E-stability technique, see for example Evans and Honkapohja (2001).

6.1 Price-level targeting

We first analyze conditions for stability of learning of a steady state PLT formulated in Section (4.1) above. The system consists of equations (27), (28), (38) and (41), together with the adjustment of output, inflation and interest rate expectations given by (33), (34) and (35).¹⁷

In general, the analytical details for learning stability of PLT regime appear to be intractable, but results are available in the limiting case $\gamma \rightarrow 0$,

¹⁷Preston (2008) discusses local learnability of the targeted steady state with IH learning when the central bank employs PLT. In the earlier literature Evans and Honkapohja (2006) and Evans and Honkapohja (2013) consider E-stability of the targeted steady state under Euler equation learning for versions of PLT.

i.e. price adjustment costs are sufficiently small. It is possible to obtain the result:¹⁸

Proposition 2 *Assume $\pi^* \geq 1$ and $\gamma \rightarrow 0$. Then under PLT the targeted steady state $\pi = \pi^*$ and $R = \beta^{-1}$ is expectationally stable when $\psi_p > 0$.*

Proof. In the limit $\gamma \rightarrow 0$ for (42) we have the coefficient matrices

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 & 0 \\ \frac{\beta^2 \psi_y (\bar{g} - y^*) - y^*}{(\bar{g} - y^*)(\beta-1)\beta} & \frac{-1}{(\beta-1)\beta\psi_p} & \frac{\beta}{(\beta-1)\psi_p} & 0 \\ \frac{1}{(\bar{g} - y^*)(\beta-1)\beta} & \frac{-1}{(\beta-1)\beta} & \frac{\beta}{\beta-1} & 0 \\ \frac{y^* + \beta^2 \psi_y (y^* - \bar{g})}{(\bar{g} - y^*)(\beta-1)\beta\psi_p} & \frac{-1}{(\beta-1)\beta\psi_p} & \frac{\beta}{(\beta-1)\psi_p} & 0 \end{pmatrix}, N = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

It is seen that in the limit $\gamma \rightarrow 0$ the equation for y_t becomes

$$y_t = \frac{\beta}{\beta-1} y_t^e,$$

so that the movement of y_t under learning influences other variables but not *vice versa*. Moreover, it is seen that with learning rule (33) there is convergence to the steady state when ω is sufficiently small.

We can thus eliminate the sub-system for y_t from (42) and (43). We can also eliminate the equation for expectations of p_t since they do not appear in the system. This makes the system (42) and (43) five-dimensional. Computing the characteristic polynomial it can be seen that it two roots equal to 0. The roots of the remaining cubic equation are inside the unit circle provided the appropriate Schur-Cohn conditions are satisfied. Writing the cubic symbolically as $\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$, in general the S-C conditions are

$$\begin{aligned} SC1 &= 1 + a_1 - |a_0 + a_2| > 0 \\ SC2 &= 1 - a_0^2 - |a_1 - a_0 a_2| > 0. \end{aligned}$$

It can be computed that $SC1 = \omega^2/(1 - \beta)$. As regards the sign of $SC2$, in the limit $\omega \rightarrow 0$ we have $SC2 = 0$ and $d(SC2)/d\omega = (2 - \beta)/(1 - \beta)$ and so the conditions hold for all sufficiently small $\omega > 0$. ■

¹⁸Mathematica routines containing technical derivations in the proofs are available upon request from the authors.

We emphasize that by continuity of eigenvalues, learning stability of the targeted steady state also obtains when γ is sufficiently small. Below we carry out numerical simulations for other parameter configurations and with strictly increasing price-level target. The learning dynamics converge locally to the targeted steady state for π and y for cases with non-zero value of γ and a positive trend in the price level target.

As regards the other steady state in which ZLB binds, we have the result:

Proposition 3 *The steady state with binding ZLB $(\hat{y}, \beta, 1, \bar{p})$ is not expectationally stable under PLT.*

Proof. When the ZLB binds, the interest rate R_t is constant and the price level evolves exogenously in a neighborhood of the constrained steady state. The temporary equilibrium system and learning dynamics then reduce to two variables y_t and π_t together with their expectations. Moreover, no lags of these variables are present, so that the abstract system (42) is two dimensional with $x_t = (y_t, \pi_t)^T$ and $N = 0$. We analyze this by usual E-stability method, see e.g. Chapter 10 of Evans and Honkapohja (2001).

It can be shown that

$$\text{Det}(M - I) = \frac{\hat{y}^{(1+\varepsilon)/\alpha}(1 + \varepsilon)\nu(\bar{g} - \hat{y})^2 + \bar{g}\hat{y}\alpha^2(\nu - 1)}{(\bar{g} - \hat{y})\hat{y}\alpha^2(\beta - 1)^2\beta(2\beta - 1)\gamma}.$$

The numerator is positive whereas the denominator is negative. Thus, $\text{Det}(M - I) < 0$, which implies E-instability. ■

The preceding results show that the targeted steady state is locally but not globally stable under learning. In Section 7 we present a variety of numerical results for PLT. These results do not employ the restrictive assumption of a small γ that was invoked in Propositions 2 and 3.

6.2 Nominal GDP targeting

In this section, we consider the consequences of NGDP targeting under learning. We have the system (27), (28), (39) and (41), together with the adjustment of expectations given by (33), (34) and (35).

We now consider local dynamics near the two possible steady states by analyzing the system defined by (42) and (43). The state variable is again $x_t = (y_t, \pi_t, R_t, X_t)^T$. We assume that current values of the endogenous

variables are not in the information set at the moment of forecasting. The following result holds for the targeted steady state:¹⁹

Proposition 4 *Assume $\gamma \rightarrow 0$. Then the targeted steady state with $\pi^* \geq 1$ and $R = \beta^{-1}$ is expectationally stable under the NGDP rule (39) when $\psi > 0$.*

Proof. In the case $\gamma \rightarrow 0$ the coefficient matrices are given by

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 & 0 \\ \frac{\beta^2\psi\bar{g}y^* - \beta^2\psi(y^*)^2 - 1}{y^*(y^* - \bar{g})(\beta-1)\beta\psi} & \frac{1}{y^*\beta\psi(1-\beta)} & \frac{-\beta}{y^*\psi(1-\beta)} & 0 \\ \frac{1}{(\bar{g}-y^*)(\beta-1)\beta} & \frac{1}{(1-\beta)\beta} & \frac{\beta}{\beta-1} & 0 \\ \frac{\beta^2\psi\bar{g}y^* - \beta^2\psi(y^*)^2 - 1}{y^*(y^* - \bar{g})(\beta-1)\beta\psi} & \frac{1}{y^*\beta\psi(1-\beta)} & \frac{-\beta}{y^*\psi(1-\beta)} & 0 \end{pmatrix},$$

$$N = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Repeating the method using in the proof of Proposition 2, again the subsystem for y_t is independent from the other equations and expectations of p_t do not appear in the system. Eliminating the three equations one obtain a five-dimensional system. Its characteristic polynomial has two roots equal to 0, one root equal to $1 - \omega$, while the remaining roots satisfy a quadratic equation. The roots can be shown to be real and lie in the interval $(-1, 1)$ for all sufficiently small $\omega > 0$. ■

Recall that the ZLB-constrained steady state also exists under NGDP targeting. One then has $R_t = 1$ in the implied interest rate rule (39). The analysis of E-stability of this steady state is formally the same as in Proposition 3, and one has:

Proposition 5 *The steady state with binding ZLB $(\hat{y}, \beta, 1)$ is not expectationally stable under NGDP targeting.*

¹⁹Mitra (2003) discusses E-stability of the targeted steady state for nominal GDP targeting under Euler equation learning.

7 Numerical Analysis

7.1 Dynamics under PLT and Nominal GDP targeting

We adopt the following calibration: $\pi^* = 1.02$, $\beta = 0.99$, $\alpha = 0.7$, $\gamma = 350$, $\nu = 21$, $\varepsilon = 1$, and $g = 0.2$. The calibrations of π^* , β , α , and g are standard. We set the labor supply elasticity $\varepsilon = 1$. These values are taken from Benhabib, Evans, and Honkapohja (2013), who in turn use a 5% estimated markup of prices over marginal cost and the price adjustment costs estimated from the 14.5 months frequency of price changes. It is also assumed that interest rate expectations $r_{t+j}^e = R_{t+j-1}/\pi_{t+j}^e$ revert to the steady state value β^{-1} for $j \geq T$.²⁰ We use $T = 28$, which under a quarterly calibration corresponds to 7 years. To facilitate the numerical analysis the lower bound on the interest rate R is set slightly above 1 at value 1.001. The gain parameter is set at $\omega = 0.002$, which is a low value. Sensitivity of this choice is discussed below.

The targeted steady state is $y^* = 0.944025$, $\pi^* = 1.02$ and the low steady state is $y_L = 0.942765$, $\pi_L = 0.99$. For most simulations we consider a grid of initial conditions and present mean paths.²¹ For policy parameters in the PLT regime we adopt the values $\psi_p = 0.25$ and $\psi_y = 1$, which are also used by Williams (2010). For NGDP targeting with the rule (39) the policy parameter is specified as $\psi = 0.5$, and for the IT rule (36) the parameter values are assumed to be the usual values $\psi_p = 1.5$ and $\psi_y = 0.5$.

Our focus is on the adjustment paths under learning that emerge when a shock has displaced the economy from the targeted steady state. We begin by considering the basic features of the adjustment paths under adaptive learning. It is assumed that the policy is not transparent, so that private agents must make direct forecasts of future interest rates. This assumption was also made above in Section 6. We note that since the underlying dynamics of the endogenous variables is simultaneous and nonlinear it is in general difficult to provide intuition for the dynamic paths. Under PLT the temporary equilibrium system is given by (27), (28), (38), and (41). If we substitute

²⁰The truncation is done to avoid the possibility of infinite consumption levels for some values of the expectations. See Evans and Honkapohja (2010) for more details.

²¹In some illustrations simulations from specific initial condition are shown: $y_0^e = 0.945$ and $y_0 = y_0^e + 0.001$, $\pi_0^e = 1.025$ and $\pi_0 = \pi_0^e + 0.001$, and $R_0^e = R_0 = R^*$. In PLT the initial deviation for the target path is set at $p_0/\bar{p}_0 = 1.003$ while $(y_0 - y^*)/y^*$ is determined by the value y_0 .

(38), and (41) into the aggregate demand curve and the Phillips curve (27) and (28) we obtain a system of two simultaneous nonlinear equations which is solved for π_t and y_t given agents' forecasts π_t^e , y_t^e and R_t^e formed at the beginning of t (based on data up to $t - 1$). Given π_t , then (41) determines the relative price X_t which along with y_t determines R_t .²²

Figures 1-3 illustrate the dynamics of inflation, output and the interest rate for IT, PLT and NGDP targeting. For generating these figures, we simulate the model for various values of initial inflation and output expectations, π_0^e and y_0^e , in the neighborhood of the desired steady state. π_0^e ranges in an interval of 1% around π^* i.e. from 1.01 to 1.03 at steps of 0.0005 while y_0^e varies in an interval around y^* ; specifically between 0.9439 and 0.945 at steps of 0.000025. The gain parameter is at the baseline value of 0.002. For initial output and inflation we set $y_0 = y_0^e + 0.001$ and $\pi_0 = \pi_0^e + 0.001$, and $R_0^e = R_0 = R^*$.²³ In PLT the initial deviation for the target path is set at $p_0/\bar{p}_0 = 1.003$ i.e. 0.3% off. The runs for all the grid points for π_0^e and y_0^e are done for a time interval of 1,500 periods and the mean values of the endogenous variables are reported on the basis of the first 500 periods. The figures show the mean paths of these variables.

It is seen that convergence for IT is monotonic after the initial jump, whereas for PLT and NGDP there is oscillatory convergence to the targeted steady state. The oscillations die away faster under NGDP than under PLT. We provide some brief intuition for the results. Broadly speaking, the dynamics of X_t (not shown in the figures) drive the dynamics of interest rates under PLT. The PLT interest rule (38) responds to both percentage deviations in X_t and y_t but the former effect dominates because it is much larger in quantitative terms. Initially by period 20, (mean) relative prices X_t increase gradually to almost 12% above its steady state value (of one) while y falls by just over 1%; R_t therefore rises gradually to almost 5% by period 20. Thereafter, till around period 70, y_t increases gradually above its (desired) steady state value. However, during this period relative prices X_t fall monotonically till it is more than 10% below its steady state value. This moves R_t on a downward path during this period. Then, X_t increases monotonically driving R_t upwards. These oscillations in X_t and R_t continue with the amplitude

²²The NGDP regime is also solved analogously; (27), (28) are solved for π and y given (39) and (41).

²³This means that mean paths for inflation and output below start from initial values that are above the corresponding steady state values. This delivers genuine adjustment dynamics.

diminishing over time leading to gradual convergence as shown in the figures. Over time the oscillations in y_t , π_t and R_t fall as the forecasts π_t^e , y_t^e and R_t^e converge to the steady state values.

Under NGDP, a similar phenomenon is present with deviations in GDP $p_t y_t / \bar{z}$ from the target value driving the dynamics of R_t by the rule (39).²⁴ Deviations in GDP from the target value are oscillatory initially increasing till around period 10 which puts R on an upward path. Thereafter, $p_t y_t / \bar{z}_t$ moves on a downward path which in turn puts R on a downward path. The oscillations gradually dampen over time leading to eventual convergence. Under IT, on the other hand, the monotonic downward path of π by virtue of the active Taylor rule (36) drives the monotonic dynamics of R .

FIGURES 1-3 ABOUT HERE

7.2 Robustness of the different rules

The differences in the adjustment dynamics just shown suggest that the PLT, NGDP and IT policy rules should be compared further in terms of the disequilibrium adjustment properties toward the targeted steady state. How robust is convergence to the targeted steady state? The robustness properties considered are as follows. (i) Domain of attraction: how far from the target can the initial conditions be and still deliver convergence to the target. (ii) Volatility: how big are the fluctuations during the adjustment path? (iii) Speed of learning: what is the maximum value of the gain parameter that still yields convergence to the target.

Looking first at the domain of attraction of the targeted steady state, we note that the system has six dynamic variables with initial conditions so it is impossible to visualize the whole domain of attraction. We focus on sensitivity with respect to initial inflation and output expectations π_0^e and y_0^e . Initial conditions on the interest rate R_0 and its expectations R_0^e are set at the target value, while initial conditions on actual inflation and output are set at $y_0 = y_0^e + 0.001$ and $\pi_0 = \pi_0^e + 0.001$. Also $p_0 / \bar{p}_0 = 1.003$ under PLT. For generating figures 4-6, we simulate the model for various values of initial inflation and output expectations, π_0^e and y_0^e . π_0^e ranges from 0.95 to

²⁴In fact, the dynamics of relative prices p_t / \bar{p}_t follows closely the dynamics of $p_t y_t / \bar{z}_t$ under NGDP.

1.08 at steps of 0.001 while y_0^e varies from 0.924025 and 0.964025 at steps of 0.0005. We say convergence has been attained when both π_t and y_t are within 0.5% of the targeted steady state; otherwise we say the dynamics does not converge.²⁵

Figures 4-6 present numerical computations of the domains of attraction for the three rules when policy not transparent. It is seen that PLT and NGDP regimes are clearly much less robustly convergent than IT. The NGDP rule is slightly more robust than PLT rule.

FIGURES 4-6 ABOUT HERE

The second robustness comparison is about volatility in inflation, output and interest rate during the learning adjustment. We also calculated a quadratic loss function in terms of the unconditional variances with the weights 0.5 for output, 0.1 for the interest rate and 1 for the inflation rate, following Williams (2010). The details for the grid searches of $\pi_0^e \in [1.01, 1.03]$ and $y_0^e \in [0.9439, 0.945]$ and the way the dynamics are generated are the same as those used to generate Figures 1-3 above ($R_0 = R^*$ as above). The reported results are the median volatilities based on a run of 1, 500 periods using our baseline gain of 0.002 for each monetary policy.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$
<i>IT</i>	23.9999	1.07756	52.8747	29.8261
<i>NGDP</i>	1.07151	5.2547	32.3791	6.93676
<i>PLT</i>	3.06211	6.19571	34.9104	9.651

Table 1: Volatility of inflation, output and interest rate for different policy rules without transparency.

Note: the numbers should be multiplied by 10^{-6} .

²⁵Convergence is relatively fast for NGDP targeting (even with small gains) and we simulate the runs for 600 periods with the baseline gain in this case. Convergence is slow with PLT; hence we do the runs for 1,000 periods with the baseline gain. With IT, convergence is slow for the baseline gain of 0.002 while it is fast for gains above 0.01; we use a gain of 0.01 and simulate the run for 300 periods.

It is seen from Table 1 that in terms of output fluctuations, IT does clearly best, but it does much worse in terms of inflation and interest rate fluctuations. NGDP targeting does best for the latter two variables. Given the weights in the loss function, the NGDP rule is the best rule overall followed by PLT in the volatility comparison.

The third robustness criterion we consider is with respect to magnitude of the speed of private agent learning, which is measured by the gain parameter ω . The initial conditions were set at values mentioned in the footnote in the beginning of Section 7. Table 2 presents the results.

<i>NGDP</i>	$0 < \omega \leq 0.022$
<i>IT</i>	$0 < \omega < 0.126$
<i>PLT</i>	$0 < \omega < 0.011$

Table 2: Robustness with respect to the gain parameter, non-transparent policy

The range of ω for which there is convergence to the targeted steady state under PLT and NGDP is clearly much smaller than the range under IT. The upper bound for ω under PLT is quite low and it is below the commonly used range but above the lowest values in the literature.²⁶ The NGDP rule is more robust as for it part of the range of ω is within the interval of values of the gain parameter that can be found in the empirical macro literature on learning. To check the sensitivity of the results presented in Table 2, we also did the following grid searches. We experimented over values of ω between 0.002 and 0.22 (increasing in intervals of 0.002 in this range) while at the same allowing both π_0^e and y_0^e to vary locally around the desired steady state.²⁷ This extensive grid search suggested that the results reported in Table 2 are robust to changes in initial conditions around the desired steady state.²⁸

²⁶Most commonly values $\omega \in [0.02, 0.05]$ are used, see Evans, Honkapohja, and Mitra (2009). Eusepi and Preston (2011) suggests a very low value $\omega = 0.002$ in the context of the RBC model.

²⁷In particular, we let π_0^e vary between 1.015 and 1.025 in intervals of 0.001 and y_0^e between 0.944 and 0.94405 in intervals of $5(10^{-6})$.

²⁸For PLT, there are some values of π_0^e and y_0^e which were stable for $\omega = 0.011$ and 0.012 but for all values of $\omega > 0.012$, all values of π_0^e and y_0^e were unstable. For NGDP, on the other hand, some values of π_0^e and y_0^e were stable for $\omega = 0.024$ and 0.026 but all values of π_0^e and y_0^e were unstable for $\omega \geq 0.028$.

8 Additional Guidance from Price-Level and Nominal GDP Targeting

The properties of learning dynamics under PLT or NGDP targeting have so far been analyzed under the assumption that private agents continue to learn in the same way as is natural under IT. Such assumption is plausible for describing the learning after a regime change from IT to either PLT or NGDP targeting.

This assumed way of learning has, however, the limitation that standard inflation and output forecasting on the basis of data does not take into consideration the additional guidance that the PLT or NGDP targeting regimes, respectively, can provide. If either PLT or NGDP targeting regime has been in operation for a long time, it is likely that private agents begin to take into account the guidance provided by such a regime so that their method of forecasting probably reflects the target path for the price level or nominal GDP, respectively.

We now formulate the inclusion of additional guidance in private agents' forecasting to see how it can affect the learning processes in the economy. The new result is that if agents incorporate the additional guidance of PLT or NGDP targeting into their learning, key robustness properties of these regimes are dramatically improved in comparison to the case of no guidance discussed above.

We start with the PLT case where agents incorporate the target price level path in their learning. Agents are assumed to forecast the gap between the actual and targeted price levels, with the gap measured as the ratio $X_t \equiv p_t/\bar{p}_t$ or recursively

$$X_t \equiv X_{t-1} \times (\pi_t/\pi^*). \quad (44)$$

The variable X_t is a natural candidate for incorporating the guidance as it is also the relevant variable in the interest rate rule (38) when PLT is the central bank's monetary policy framework.

Using (44), agents can compute the inflation forecast as $(X_t^e \times \pi^*)/{}_tX_t^e$ assuming as before that information on current values of endogenous variables is not available at the time of forecasting. Here ${}_tX_t^e$ refers to the forecast of the current gap X_t in the beginning of period t . (Recall the notation convention that X_t^e is the forecast of X_{t+1} in the beginning of t , i.e., ${}_tX_{t+1}^e = X_t^e$ in more detailed notation.) We also assume that in specifying ${}_tX_t^e$ agents

simply use the latest actual observation and so ${}_tX_t^e = X_{t-1}$. This is plausible as using X_{t-1}^e would introduce a further information lag (X_{t-1}^e uses data only up to $t - 2$). Thus, the inflation forecast is assumed to be

$$\pi_t^e = (X_t^e \times \pi^*)/X_{t-1} \quad (45)$$

and these forecasts π_t^e are substituted into the aggregate demand function (27).

We remark that it could be assumed more generally that ${}_tX_t^e$ is a weighted average of the most recent observation X_{t-1} and the previous forecast X_{t-1}^e for period t . Simulations suggest that the results are mostly unchanged, but adjustment dynamics become more gradual as the weight on X_{t-1} is made smaller. Below we comment on the implications of this assumption on the detailed results.

Agents update the forecasts X_t^e by using steady-state learning, so that

$$X_t^e = X_{t-1}^e + \omega(X_{t-1} - X_{t-1}^e).$$

Output and interest rate expectations are assumed to be done as before, see equations (33) and (35). The temporary equilibrium is then given by equations (45), (27), (28), (38) and the actual relative price is given by (44).

We also analyze the case of NGDP targeting when agents are assumed to forecast future inflation by making use of the gap between actual and targeted level of nominal GDP. We measure the gap as the ratio $p_t y_t / \bar{z}_t \equiv Y_t$. Using

$$\pi_t \frac{y_t}{y_{t-1}} \equiv \frac{Y_t \bar{z}_t}{Y_{t-1} \bar{z}_{t-1}} \text{ or } \pi_t = \Delta \bar{z} \left(\frac{y_{t-1}}{y_t} \right) \left(\frac{Y_t}{Y_{t-1}} \right) \quad (46)$$

where $\bar{z}_{t+1}/\bar{z}_t = \Delta \bar{z}$, and a steady-state forecast Y_t^e , agents compute the forecast π_t^e from

$$\pi_t^e = \Delta \bar{z} \left(\frac{y_{t-1}}{y_t^e} \right) \left(\frac{Y_t^e}{Y_{t-1}} \right).$$

We are making the same assumption about available information at the moment of forecasting as in (45). Agents are assumed to use steady-state learning for the gap forecast

$$Y_t^e = Y_{t-1}^e + \omega(Y_{t-1} - Y_{t-1}^e)$$

and they forecast output and the interest rate using the earlier learning rules (33) and (35). The actual value of the nominal GDP gap in temporary equilibrium is recursively computed from (46) as

$$Y_t = (\Delta \bar{z})^{-1} \pi_t (y_t / y_{t-1}) Y_{t-1}.$$

8.1 Dynamics of learning

Our focus is the same as in Section 7.1, i.e., what kind of learning adjustment paths emerge when a shock has displaced the economy from the targeted steady state. We start by looking at the basic features of the adjustment paths under adaptive learning. The first comparison is between IT, PLT and NGDP targeting when agents incorporate the additional guidance provided in the latter two regimes.

FIGURES 7-9 ABOUT HERE

Figures 7-9 illustrate the dynamics of inflation, output and the interest rate π , y , and R for IT, PLT and NGDP targeting. The mean paths under IT are the same as in Figures 1-3 and they are included to facilitate comparisons. In PLT we simulate the model for various values of y_0^e and X_0^e ; the range for y_0^e is the same as under IT while that for X_0^e is between 0.99 and 1.01 (i.e. within 1% of its steady state value) at intervals of 0.0005 and $R_0^e = R_0 = R^*$. The initial relative price is set at $X_0 = X_0^e + 0.0001$. For NGDP we simulate the model for various values of y_0^e and Y_0^e ; the range for y_0^e is again the same as under IT while that for Y_0^e is between 0.99 and 1.01 (within 1% of its steady state value) at intervals of 0.0005 and $R_0^e = R_0 = R^*$.²⁹ The initial relative output is set at $Y_0 = Y_0^e + 0.0001$. The runs are done for a period of 1,500 periods and the mean values of the endogenous variables are reported on the basis of the first 100 periods. The figures show the mean paths of these variables.³⁰

Again since the underlying dynamics of the endogenous variables is simultaneous and nonlinear it is in general difficult to provide intuition for the dynamic paths. Both under PLT and NGDP equations (27) and (28) are solved simultaneously for π_t and y_t given agents' forecasts and the respective interest rate rules. It is, however, seen that the dynamics in PLT and NGDP regimes to the targeted steady state are significantly altered when additional

²⁹Initial relative price or output expectations under PLT or NGDP are allowed to vary 1% around its steady state values to make it consistent with the fluctuations of inflation expectations under IT which is also around 1%.

³⁰We remark that properties of the dynamics in Figures 7-9 and Table 3 below are affected if agents form expectations ${}_tX_t^e$ as a weighted average of X_{t-1} and X_{t-1}^e . The domain of attraction results in Figure 10 mostly go through.

guidance is included in learning. Comparing Figures 7-9 to 1-3, it is seen that the oscillations under PLT and NGDP die out much faster when additional guidance is used by private agents. This happens, for example, under PLT because inflation expectations and inflation are directly influenced by the lagged value X_{t-1} , see (44) and (45), which induces relatively fast adjustments also in output expectations and output and in turn leads to rapid convergence in the stable case. Without guidance inflation (respectively, output) expectations depend only on past inflation (respectively, output) and the movement is far more gradual than with guidance.³¹

In terms of the magnitude of oscillations the results are mixed: for inflation oscillations are smaller whereas they are slightly larger for output and the interest rate. Also, the paths of inflation and output under PLT and NGDP regimes are virtually indistinguishable for the initial periods and convergence is quite rapid within 30 periods or so (for an explanation, see section 8.2.1). Fluctuations of interest rate are slightly higher under PLT. In contrast convergence of all the variables is much slower under IT.

An interesting aspect is that under PLT the dynamics of R_t are now dominated by the dynamics of y_t rather than that of relative prices X_t (in contrast to the case without guidance shown in Figures 1-3 where the dynamics of X_t dominated the R_t dynamics). The dynamics of X_t oscillate much less and increase by less than 2% with guidance. The dynamics of X_t initially increase and then come down whereas R_t initially decreases and then increases; in fact the dynamics of R_t mirrors that of y_t under PLT. Under NGDP, on the other hand, the dynamics of R_t are driven by that of relative GDP $p_t y_t / \bar{z}_t$ which follows the same qualitative pattern as that of R_t shown in Figure 9 (in fact X_t follows a similar qualitative path as that of $p_t y_t / \bar{z}_t$ as well).

8.2 New robustness from guidance

We now take up the robustness properties of learning for PLT and NGDP targeting when private agents' have incorporated the guidance from specified price-level or nominal GDP paths into their learning. The robustness properties we look at are the same as before: (i) Domain of attraction, (ii) Volatility, and (iii) Speed of learning.

³¹This can be seen e.g. from constructing in the (π, y) - or (π^e, y^e) -space line plots for path for the same initial conditions in the two cases. Details are available on request.

8.2.1 Healthy stability under PLT and NGDP rules with guidance

We first look at the domain of attraction of the targeted steady state under PLT and NGDP rules when private agents incorporate in their learning the additional guidance from either price-level or nominal GDP target. We focus on sensitivity with respect to displacements of initial output and relative price level expectations y_0^e and X_0^e (or Y_0^e) in analogy with what was done in Figures 4-6 for the case without guidance.

Quite dramatically, we discovered that the domain of attraction is large under both the PLT and NGDP rules and seemed to contain values for y_0^e below the low steady state. To illustrate this the domain of initial conditions for X_0^e was made quite large and we also set the initial values of the variables at the deflationary steady state $\hat{R} = 1$, $\hat{\pi} = \beta$, and $y = \hat{y}$, except that the gap variables were set at values slightly above 0. Also $R_0 = R_0^e = 1$ and $X_0 = X_0^e$.

Figure 10 presents numerical illustration of the domain of attraction for the PLT policy rule with these initial conditions and grids for y_0^e and X_0^e . The horizontal axis gives the initial output expectations y_0^e and vertical axis gives the initial relative price expectations X_0^e . The grid search for y_0^e was over the range 0.935 to 1 at intervals of 0.0005 and that for X_0^e over the range 0.1 to 2 at intervals of 0.02.³²

FIGURE 10 ABOUT HERE

It is seen that the domain continues to cover the whole area above values $y_0^e = 0.935$, except for the unstable low steady state. In fact, the initial value for y_0^e can be much lower than this, for example $y_0^e \geq 0.90$, but with these initial conditions learning becomes extremely slow which makes the numerical computations extremely involved.³³

The corresponding result for NGDP targeting turns out to be the same, so we do not report the corresponding figure. In fact, the learning with

³²For PLT the domain of attraction was computed with the baseline gain of 0.002 and a run of 100 periods; for NGDP a gain of 0.002 and a run of 300 periods is used.

³³It is possible to speed up learning by raising the value of the gain, but this offers limited help. For example with gain $\omega = 0.01$ and initial conditions $y_0^e = 0.90$ convergence takes about 2500 periods or more. We plan to examine these boundaries further in the future.

guidance under our PLT and NGDP rules yield identical dynamics when the interest rate is at the ZLB provided that the initial conditions are identical. This can be seen by inspecting the formulas (45) and (46). Writing (45) using implied price level forecasts and actual values, we get

$$PLT : \quad \pi_t^e = \pi^* \frac{p_t^e / \bar{p}_{t+1}}{p_{t-1} / \bar{p}_{t-1}} = (\pi^*)^{-1} \frac{p_t^e}{p_{t-1}}.$$

For (46) we analogously get

$$NGDP : \quad \pi_t^e = \Delta \bar{z} \left(\frac{y_{t-1}}{y_t^e} \right) \left(\frac{p_t^e y_t^e / z_{t+1}}{p_{t-1} y_{t-1} / z_{t-1}} \right) = (\Delta \bar{z})^{-1} \frac{p_t^e}{p_{t-1}}.$$

The two formulas give the same result as $\Delta \bar{z} = \pi^*$, which shows that the dynamics must be identical if initial conditions are the same. Outside the ZLB the interest rate rules differ, so the dynamics also differ. However, the interest rate rules (38) and (39) are similar. Differences are because the policy parameter values differ and $p_t y_t / (\bar{p}_t y^*)$ is not equal to $p_t / \bar{p}_t + y_t / y^*$, though near the targeted steady state the deviation is small.

The results suggest a further analysis of the global aspects of the dynamics. Focusing on the PLT rule (38), recall that there exist two steady states under mild additional conditions. The deflationary steady state $\hat{R} = 1$, $\hat{\pi} = \beta$, and $y = \hat{y}$ under the PLT formally requires that the actual price level has converged to zero, so that agents do not use the additional guidance in this RE equilibrium. This possibility is a singularity for the learning dynamics with guidance and a consequent sequence of temporary equilibria since forecasting with (45) the gap variable X_{t-1} is not defined in the limit $p_t \rightarrow 0$.

However, the dynamics of learning with guidance are well-defined arbitrarily near the singularity and we next examine this case further. Except for the initial relative price variables X_0 and X_0^e , we set all other initial conditions at the deflationary steady state values $y_0 = \hat{y} = y_0^e$, $\pi_0 = \hat{\pi}$, $R_0 = R_0^e = \hat{R}$. Then set $X_0 = X_0^e$ at a very low positive value 0.001. We use a gain value of $\omega = 0.005$ and run the system for 70 periods. Figures 11-12 show how the learning system begins to move away from the vicinity of the deflationary steady state. In Figure 11 after an initial rise X_t fluctuates for some time after which it starts to rise gradually towards 1. Figure 12 show the dynamics of π_t and y_t in a phase plot. Inflation and output oscillate but these oscillations are damped and convergence of these variables to the targeted steady state is rapid. Given the very low starting point, X_t remains very low for a

prolonged period of time before gradually increasing to 1 asymptotically. R_t is correspondingly stuck at the ZLB for a prolonged period as well due to the interest rule (38). After a long interval of time as X_t converges towards close to one, R_t converges towards the targeted interest rate. We remark that IT is much less robust when the economy is initially near the deflationary steady state. The dynamics explode generally for inflation rates below 1.005.³⁴

FIGURES 11-12 ABOUT HERE

This analysis of initial dynamics indicates that with guidance the price gap variable X_t plays a key role also when the economy is in the liquidity trap. With guidance X_t continues to influence the economy through inflation forecasting even when ZLB is binding. The movements of X_t quickly induce significant movements in inflation, output and, as see from Figure 12, their expectations and the dynamics are in fact convergent to the targeted steady state in the long run. In contrast, without guidance the influence of X_t through the interest rate rule is not operative under ZLB which implies learning instability under PLT without guidance when initial conditions are close the deflationary steady state.

8.2.2 Other robustness criteria

The second comparison of robustness is about volatility in inflation, output and interest rate during the learning adjustment. As before we calculated a quadratic loss function in terms of these unconditional variances with the weights 0.5, 0.1 and 1 for output, interest rate and inflation. The details for the grid searches are largely the same as those in Table 1. However, with PLT the grid for relative price expectations X_0^e is $[0.99, 1.01]$ with steps of 0.0005 and the initial relative price is set at $X_0 = X_0^e + 0.0001$. For NGDP the grid for relative output expectations Y_0^e is the same as for X_0^e and the initial relative output is set at $Y_0 = Y_0^e + 0.0001$. The reported results are the median volatilities based on a run of 1,500 periods using our baseline gain of 0.002 for NGDP targeting and PLT. The final row reproduces the earlier results for IT from Table 1 to facilitate comparisons.

³⁴For economy of space we do not report the results for the longer runs here. The results are available on request.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$
<i>NGDP</i>	0.1027	3.8072	2.5518	2.2615
<i>PLT</i>	0.1215	3.7514	4.1768	2.4149
<i>IT</i>	23.9999	1.07756	52.8747	29.8261

Table 3: Volatility of inflation, output and interest rate for NGDP and PLT with guidance but without transparency.

Note: the numbers should be multiplied by 10^{-6} .

As can be seen, guidance has clear benefits for NGDP and PLT. Volatilities in π , y and R are all lower with guidance than without guidance (see Table 1). Thus, PLT and NGDP perform much better than IT and NGDP is the best regime because of the lower volatility of interest rate (as reflected in Figure 9). Note that inflation and interest rate volatilities under IT are much higher than both PLT and NGDP which is reflective of the initial wide fluctuations in these variables under IT.

The third robustness criterion is with respect to the magnitude of speed of private agent learning, which is measured by the gain parameter ω . The initial conditions were set at values mentioned in the footnote in the beginning of Section 7. Table 4 presents the results.

<i>NGDP</i>	$0 < \omega \leq 0.0115$
<i>PLT</i>	$0 < \omega \leq 0.0110$

Table 4: Robustness with respect to the gain parameter, guidance used

The range of ω for which there is convergence to the targeted steady state under PLT is approximately the same as the range without the further guidance. Surprisingly, the range for ω under the NGDP rule is smaller than the range without guidance. Table 4 is computed using the initial conditions as $y_0^e = 0.945$, $y_0 = y_0^e + 0.001$, $\pi_0 = 1.025$, and $R_0^e = R_0 = R^*$, which correspond to the values in the beginning of Section 7. These bounds are robust with respect to the way of forecasting ${}_tX_t^e$ in the sense that convergence obtains for all weights in the interval $(0, 1]$ when ${}_tX_t^e$ is a weighted average of X_{t-1}^e and X_{t-1} , see the comment after equation (45).

For specific values of the weight the maximal gain parameter for which stability obtains can be larger. Stability to the targeted steady state obtains under PLT for gain parameters ω up to 0.03 if the weight on X_{t-1} is 0.06. Correspondingly, we find that under NGDP stability can obtain for gain values up to 0.025.

9 Conclusion

Our study provides an assessment of price-level and nominal GDP targeting that have been recently suggested as improvements over inflation targeting policy. The results indicate that the performance of either price-level or nominal GDP targeting is on the whole better than inflation targeting, provided that private agents' learning has incorporated the guidance on the price level or nominal GDP that these regimes try to entail. If private agents' learning does not use the guidance, the results are not clear-cut; IT has a clearly bigger domain of attraction than PLT or NGDP targeting but is worse in terms of volatility. Thus, if a move to either price-level or nominal GDP targeting is contemplated, it is important to try to influence the way private agents' form inflation expectations, so that the additional guidance is incorporated into the learning.

It should be noted that though the learning viewpoint is natural for studying adjustment dynamics, these results about the key role of guidance come from comparing the properties of the different regimes. We have not formally modelled the dynamics that would follow from a shift from one regime to another. Analysis of how and why private agents might change their forecasting practice would be well worth studying.

Our analysis has two important starting points. It is assumed that agents have imperfect knowledge and therefore their expectations need not be rational. Agents make their forecasts using an econometric model that is updated over time. We have carefully introduced the nonlinear global aspects of a standard framework, so that the implications of the interest rate lower bound can be studied. As is well-known, inflation targeting with a Taylor rule suffers from global indeterminacy and it was shown here that the same problem exists for standard versions of price-level and NGDP targeting. Theoretical results for local (but not global) stability under learning of the targeted steady state were derived for the latter two policy rules, whereas the low inflation steady state is locally unstable under learning.

The current results are a first step in this kind of analysis. Several extensions can be considered. We have used standard policy rules and standard values for the policy parameters, but these do not represent optimal policies. Deriving globally optimal rules in a nonlinear setting like ours is clearly extremely demanding, but one could consider optimal simple rules, i.e. optimization of the parameter values of these instrument rules. One could also do away with the instrument rule formulations used in this paper and instead

postulate that the central bank employs a target rule whereby in each period the policy instrument is set to meet the target exactly unless the ZLB binds. Yet another extension would be the implications of transparency about the policy rule to the properties of learning dynamics.

There are naturally numerous more applied concerns that should be investigated before any final assessment. We just mention the issues connected with measurement and fluctuations of output and productivity. Orphanides (2003) and Orphanides and Williams (2007) discuss the measurement problems in output and output gap. Hall and Mankiw (1994) emphasize that the volatility in output and productivity measures can pose challenges in particular to nominal GDP targeting. Our non-stochastic model does not address these concerns.

We plan to address some of these extensions in the future.

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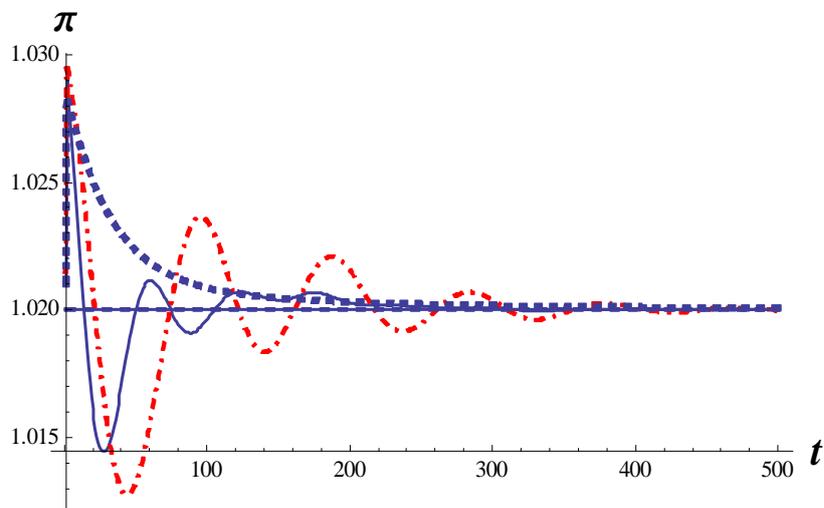


Figure 1: Inflation mean dynamics under IT, PLT, and NGDP. IT in dashed, PLT in mixed dashed and NGDP in solid line.

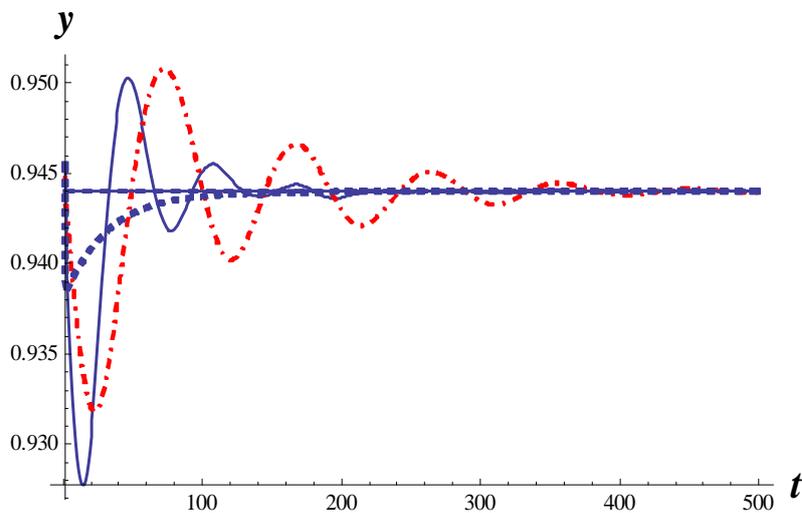


Figure 2: Output mean dynamics under IT, PLT, and NGDP. IT in dashed, PLT in mixed dashed and NGDP in solid line.

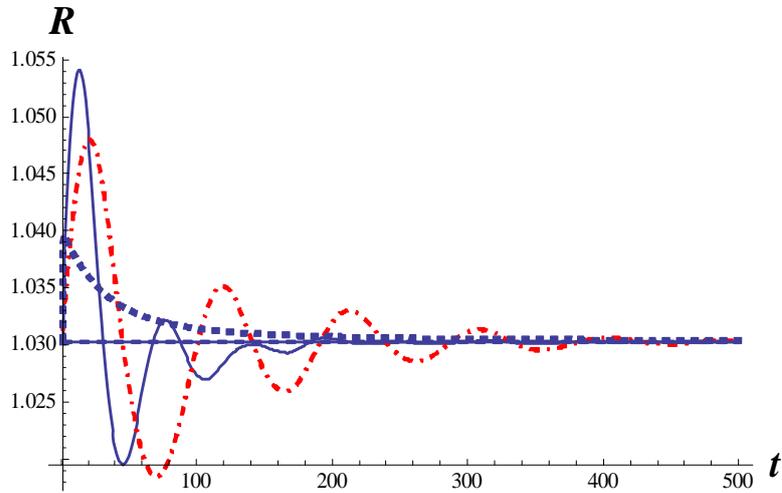


Figure 3: Interest rate mean dynamics under IT, PLT, and NGDP. IT in dashed, PLT in mixed dashed and NGDP in solid line.

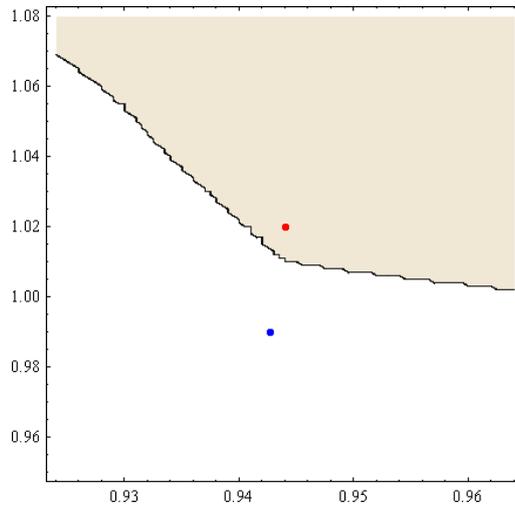


Figure 4: Domain of attraction for IT.

Horizontal axes gives y_0^e and vertical axis π_0^e . Shaded area indicates convergence. The red circle denotes the intended steady state and the blue circle the unintended one in this and subsequent figures.

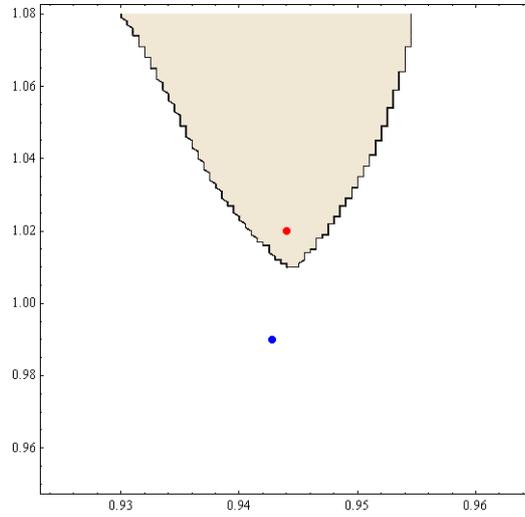


Figure 5: Domain of attraction for NGDP. Horizontal axes gives y_0^e and vertical axis π_0^e . Shaded area indicates convergence.

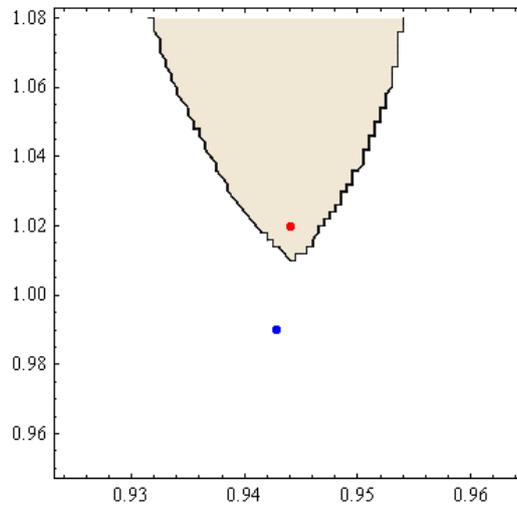


Figure 6: Domain of attraction for PLT. Horizontal axes gives y_0^e and vertical axis π_0^e . Shaded area indicates convergence.

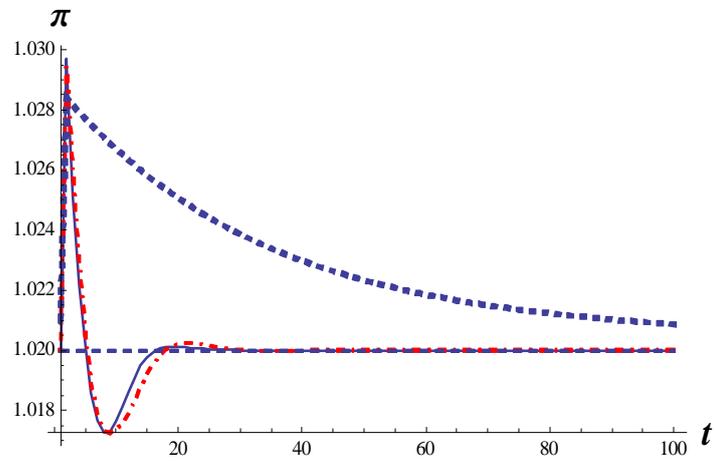


Figure 7: Inflation mean dynamics under IT, PLT, and NGDP (the latter two under gap forecasting). IT in dashed, NGDP in solid and PLT in mixed dashed line. The horizontal dashed line is the steady state. Note that the paths under PLT and NGDP are virtually indistinguishable in the first 20 periods or so and that convergence is quite fast.

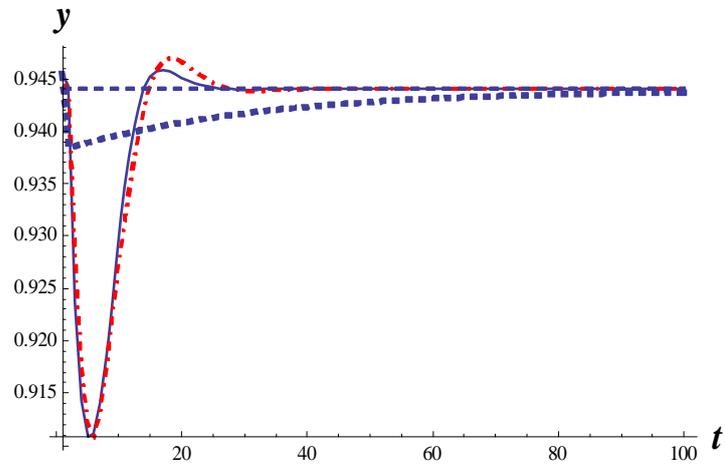


Figure 8: Output mean dynamics under IT, PLT, and NGDP (the latter two under gap forecasting). IT in dashed, NGDP in solid and PLT in mixed dashed line. The horizontal dashed line is the steady state. Note that the paths under PLT and NGDP are virtually indistinguishable in the first 10 periods or so and that convergence is quite fast.

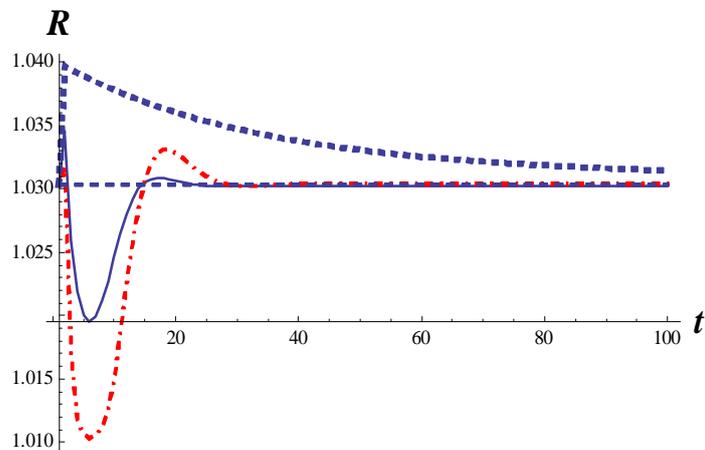


Figure 9: Interest rate mean dynamics under IT, PLT, and NGDP (the latter two under gap forecasting). IT in dashed, NGDP in solid and PLT in mixed dashed line. The horizontal dashed line is the steady state. Note that the paths under PLT and NGDP are quite different unlike that of inflation and output.

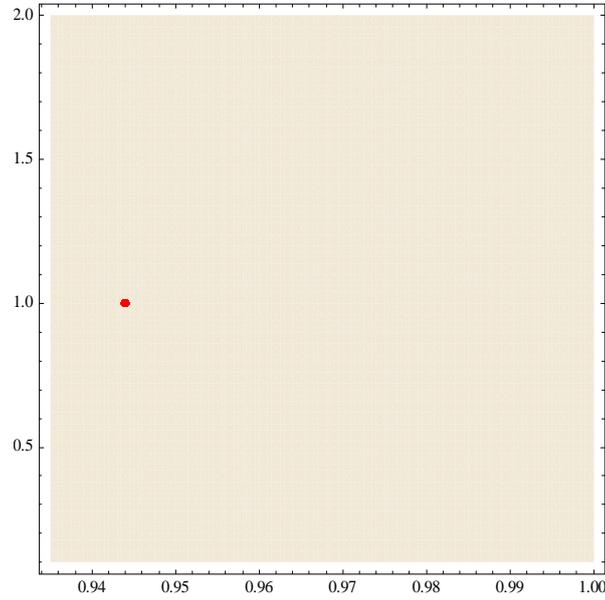


Figure 10: Domain of attraction for PLT with forecasting of gaps with initial conditions close to deflationary steady state. Horizontal axis gives the initial output expectations y_0^e and vertical axis the initial relative price expectations X_0^e . The red dot is the point where output expectation is at the targeted steady state and relative price expectation is one. Shaded area indicates convergence.

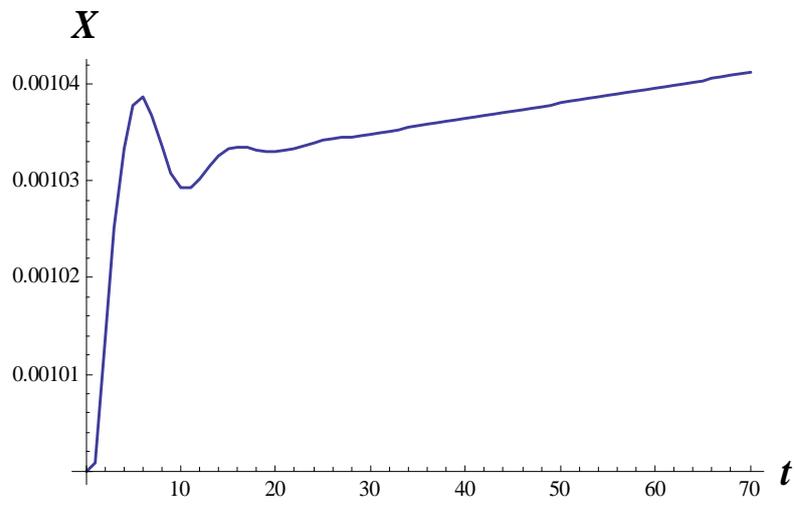


Figure 11: Initial X_t dynamics from near low steady state under PLT.

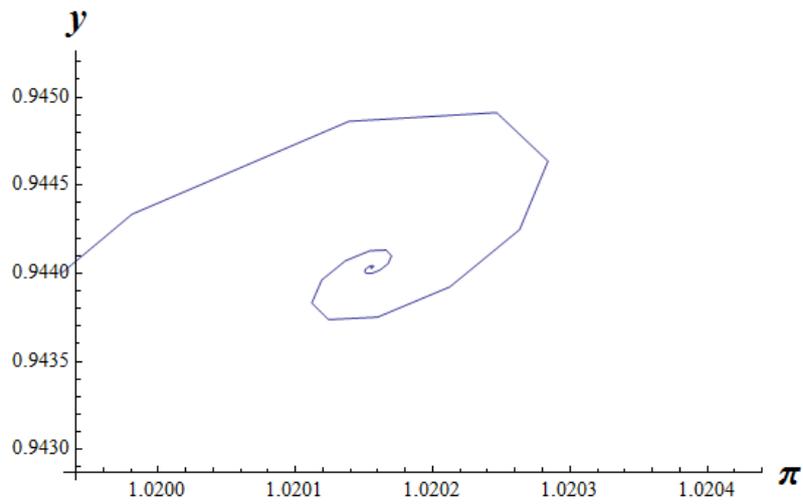


Figure 12: Initial π_t and y_t dynamics from near low steady state under PLT.