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OPTIMAL MONETARY POLICY WITH STATE-DEPENDENT PRICING

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ABSTRACT

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JEL Classification: E31
Keywords: monetary policy, monopolistic competition and state-dependent pricing

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Abstract

This paper studies optimal monetary policy from the timeless perspective in a general model of state-dependent pricing. Firms are modeled as monopolistic competitors subject to idiosyncratic menu cost shocks. We find that, under certain conditions, a policy of zero inflation is optimal both in the long run and in response to aggregate shocks. Key to this finding is an "envelope" property: at zero inflation, a marginal increase in the rate of inflation has no effect on firms' profits and hence on their probability of repricing. We offer an analytic solution that does not require local approximation or efficiency of the steady state. Under more general conditions, we show numerically that the optimal commitment policy remains very close to strict inflation targeting.

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1 Introduction

A key normative question in monetary economics is the design of optimal monetary policy. An extensive amount of literature studies this question under the assumption that the timing of price changes is given exogenously, typically using the Calvo (1983) model with a constant adjustment rate.[1] Useful as it is as a first approximation, this literature nevertheless is subject to the Lucas (1976) critique: In principle, the frequency of price changes should not be treated as a parameter

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[1] For example, Clarida, Galï¿œ, and Gertler (1999); Woodford (2003); Yun (2005); Benigno and Woodford (2005).

which is independent of policy. Many economists, therefore, have argued against the use of the Calvo model, claiming that it provides a poor approximation to more elaborate models of price adjustment. For example, Golosov and Lucas (2007) show that the behavior of the price level in the Calvo model is very different from that in a "menu cost" model, when firms are subject to idiosyncratic productivity shocks as well as aggregate money growth shocks.

This paper studies optimal monetary policy in a model of state-dependent pricing (SDP) by monopolistically competitive firms. In models of this sort the frequency of adjustment is a statistic determined in equilibrium, not an exogenous parameter. In particular, we will work with a model in which individual prices are sticky because firms are subject to random idiosyncratic lump-sum costs of adjustment as in Dotsey, King, and Wolman (1999). Each firm would change its price only if the increase in the firm's value due to adjustment exceeds the "menu cost." As a result, the probability with which firms reprice depends on the gains from adjustment. This framework is very flexible because it nests a variety of pricing specifications, including the fixed menu cost model and the Calvo model as extreme limiting cases (Costain and Nakov, 2011).

Aside from pricing being state-dependent, our setup follows closely the standard New Keynesian model. In particular, the monetary authority is assumed to set the nominal interest rate, with money's role being only that of a unit of account. An important distinction with Clarida, Galï¿œ, and Gertler (1999) and Yun (2005) is that we assume no production subsidy to offset the markup distortion due to monopolistic competition. This implies that the steady state level of output is inefficiently low. Hence, the central bank has a constant temptation to inflate the economy so as to bring output closer to its efficient level.

We derive the optimal plan from the timeless perspective, as in Woodford (2003).[2] We demonstrate analytically that, if preferences are isoelastic, households consume all output, and there are no cost-push disturbances, then it is optimal to commit to zero inflation both in the long run and in reaction to shocks. Importantly, this result holds for a general specification of the menu cost distribution. In the optimal allocation, price markups are positive but constant, output is at its natural (flexible-price) level, and price dispersion is minimized. In earlier work, Benigno and Woodford (2005) found that, under the above conditions, zero inflation is optimal in the New Keynesian model with Calvo pricing.[3] Therefore, our analysis shows that the optimality of zero inflation carries over to the case of state-dependent pricing.

The reason why zero inflation is optimal in the SDP model is the following. Relative to models with exogenous timing of price changes, SDP implies two additional welfare effects of inflation.

[2]That is, the plan ignores policymakers' incentives to behave differently in the initial few periods, exploiting the private sector's expectations that had formed prior to the plan's starting date.

[3]Benigno and Woodford (2005) obtain their result in the context of a linear-quadratic approximation to the actual policy problem, whereas our finding is based on the exact non-linear welfare function and equilibrium conditions. However, Benigno and Woodford (2005) discuss conditions under which the solution to the linear-quadratic problem is identical to the first order approximation of the solution to the exact non-linear Ramsey problem.

First, firms must spend real resources (menu costs) on adjusting nominal prices. This distortion is minimized at zero inflation because under such a policy all firms end up at their optimal price, and hence they do not need to reprice. The second effect is somewhat more subtle. The main difference between exogenous-timing and SDP models is that price adjustment probabilities are endogenous in the latter. A priori, the monetary authority could have an incentive to use inflation so as to accelerate price adjustment and thus make prices more "flexible". However, the fact that adjusting firms set their prices in an optimal way implies that, in the timeless perspective regime with zero inflation, a marginal increase in the rate of inflation has no effect on firms' profits and hence on adjustment probabilities. This *envelope* property implies that the monetary authority has no incentive to deviate from zero inflation in order to affect the speed of price adjustment.

We also show that the same reasons for which zero inflation is optimal under Calvo pricing continue to hold under SDP. First, inefficient price dispersion is minimized at zero inflation. Second, in the timeless perspective regime with zero inflation, the marginal welfare gain from raising output toward its socially efficient level (i.e. a movement along the Phillips curve) exactly cancels out with the marginal welfare loss from generating expectations of future inflation (i.e., an upward shift of the Phillips curve). This finding echoes Kydland and Prescott's "rules rather than discretion" but it is independent of whether pricing is time- or state-dependent.[4]

We then study numerically the optimal monetary policy in the presence of government consumption and cost-push shocks, using a calibrated version of the model. We find that the zero inflation policy remains very close to the optimal commitment, both in terms of impulse-response dynamics and the associated welfare losses.[5] We also find that impulse-responses and welfare losses are very similar to those under Calvo pricing. This reflects the fact that the envelope property of the SDP framework continues to hold as long as one approximates equilibrium dynamics around the zero inflation steady-state, which is the one implied by the optimal policy. With trend inflation, however, the welfare cost of aggregate fluctuations under suboptimal policies is higher in the Calvo model; the welfare gap relative to SDP furthermore increases with trend inflation. Thus, the envelope property becomes less and less important in the dynamics of the SDP framework as trend inflation increases, with the resulting consequences for the welfare comparison with the Calvo framework.[6]

Relatedly, we show that the welfare cost of trend inflation itself is higher too under Calvo

[4]Independently, Lie (2009) studies numerically the optimal monetary policy in a New Keynesian model with stochastic menu costs and a monetary friction.

[5]Benigno and Woodford (2005) and Woodford (2003, ch. 6) show analytically that, even in the absence of cost-push shocks, a short-run trade-off exists between inflation and output stabilization when government spending is positive. However, they do not quantify the importance of this trade-off. Our numerical simulations suggest that the latter is negligible, both under SDP and Calvo pricing.

[6]For an analysis of log-linearized equilibrium dynamics under SDP in the presence of trend inflation, see Bakhshi, Khan and Rudolf (2007).

pricing, with the difference rising as trend inflation moves away from zero. Intuitively, trend inflation speeds up price adjustment in the SDP model, especially for those firms that are further away from their optimal price. This reduces the size of relative price distortions relative to the Calvo model. Finally, we provide some numerical analysis for the case with firm-level productivity shocks. We find again that welfare losses from trend inflation are larger under Calvo pricing vis-a-vis SDP, only now the latter is true even with zero trend inflation. This reflects the fact that price adjustment probabilities do react to firm-level shocks under SDP but not in the Calvo model.

The next section lays out the model and derives the conditions for equilibrium. Section 3 sets up the optimal monetary policy problem and obtains the main result regarding the optimality of zero inflation. It also formalizes the main intuition with a simplified version of the model (with the full proof included in the Appendix). Section 4 analyzes numerically the case with government expenditure and cost-push shocks, and also provides some analysis for the model extension to firm-level shocks. Section 5 concludes.

2 Model

There are three types of agents: households, firms, and a monetary authority. We begin by describing the behavior of households and firms.

2.1 Households

A representative household maximizes the expected flow of period utility $u(C_t) - x(N_t; \chi_t)$, discounted by β, subject to

$$C_t = \left(\int_0^1 C_{it}^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)},$$

where $\epsilon > 1$, and

$$\int_0^1 P_{it} C_{it} di + R_t^{-1} B_t = (1 - \tau_t) W_t N_t + B_{t-1} + \Pi_t,$$

where C_t is a basket of differentiated goods $i \in [0, 1]$, of quantity C_{it} and price P_{it}, N_t denotes hours worked, W_t is the nominal wage rate, τ_t is an exogenously-varying tax rate on wage income, χ_t is an exogenous shock to the disutility of labor,[7] B_t are nominally riskless bonds with price R_t^{-1}, and Π_t are the profits of firms owned by the household, net of lump-sum taxes.

The first order conditions are

$$u'(C_t) w_t = x'(N_t; \chi_t) u_t, \tag{1}$$

[7] Our results hold also in the case when the utility of consumption is affected by a preference shock; here we ommit such a shock for simplicity.

$$R_t^{-1} = \beta E_t \frac{u'(C_{t+1})}{\pi_{t+1} u'(C_t)}, \tag{2}$$

where $u_t \equiv (1-\tau_t)^{-1}$, $w_t \equiv W_t/P_t$ is the real wage, $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, and the aggregate price index is given by

$$P_t \equiv \left(\int_0^1 P_{it}^{1-\epsilon} di \right)^{1/(1-\epsilon)}.$$

2.2 Firms

There is a continuum of firms on the unit interval. Firm i's production function is

$$y_{it} = z_t n_{it},$$

where z_t is an exogenous aggregate productivity process.[8] The firm's labor demand thus equals $n_{it} = y_{it}/z_t$ and its real cost function is $w_t y_{it}/z_t$. The real marginal cost common to all firms is therefore w_t/z_t. Optimal allocation of expenditure across product varieties by households and the government implies that each individual firm faces a downward-sloping demand schedule for its good, given by $y_{it} = (P_{it}/P_t)^{-\epsilon} Y_t$, where Y_t is aggregate demand.

Following Dotsey et al. (1999), we assume that firms face random lump sum costs of adjusting prices ("menu costs"), distributed *i.i.d.* across firms and over time. Let $\Gamma(\kappa)$ and $g(\kappa)$ denote the cumulative distribution function and the probability density function, respectively, of the stochastic menu cost $\kappa \geq 0$. We assume that a positive random fraction of firms draw a zero menu cost, so that $\Gamma(0) > 0$.[9] Assuming that κ is measured in units of labor time, the total cost paid by a firm changing its price is $w_t \kappa$.[10]

Let v_{0t} denote the value of a firm that adjusts its price in period t before subtracting the menu cost. Let $v_{jt}(P)$ denote the value of a firm that has kept its nominal price unchanged at the level P in the last j periods. This firm will change its nominal price only if the value of adjustment, $v_{0t} - w_t \kappa$, exceeds the value of continuing with the current price, $v_{jt}(P)$. Therefore, from the set of firms that last reoptimized j periods ago (which we henceforth refer to as "vintage-j firms"), only those with a menu cost draw $\kappa \leq (v_{0t} - v_{jt}(P))/w_t$ will choose to change their price. The

[8] Our assumption of linear technology is only for ease of exposition. Our main analytical result, the optimality of strict inflation targeting from a timeless perspective, carries over to the more general case of a production function with decreasing marginal returns to labor. The proof is available upon request.

[9] We make this technical assumption to ensure a unique stationary distribution of firms over price vintages in the case of zero inflation. See Appendix B for details.

[10] Alternatively, we can assume that κ is measured in terms of the basket of final goods, in which case the total cost paid by a firm changing its price is simply κ. The results are not dependent on this assumption.

real value of an adjusting firm is given by

$$\begin{aligned} v_{0t} &= \max_{P}\left\{\Pi_t(P)+\beta E_t\frac{u'(C_{t+1})}{u'(C_t)}\left[\Gamma\left(\frac{v_{0,t+1}-v_{1,t+1}(P)}{w_{t+1}}\right)v_{0,t+1}-\Xi_{1,t+1}(P)\right]\right. \\ &\quad \left.+\beta E_t\frac{u'(C_{t+1})}{u'(C_t)}\left[1-\Gamma\left(\frac{v_{0,t+1}-v_{1,t+1}(P)}{w_{t+1}}\right)\right]v_{1,t+1}(P)\right\}, \end{aligned}$$

where $\beta u'(C_{t+s})/u'(C_t)$ is the stochastic discount factor between periods t and $t+s \geq t$,

$$\Pi_t(P) \equiv \left(\frac{P}{P_t}-\frac{w_t}{z_t}\right)\left(\frac{P}{P_t}\right)^{-\epsilon}Y_t$$

is the firm's real profit as a function of its nominal price P, and

$$\Xi_{j+1,t+1}(P) \equiv w_{t+1}\int_0^{(v_{0,t+1}-v_{j+1,t+1}(P))/w_{t+1}}\kappa g(\kappa)\,dk$$

is next period's expected adjustment cost for a firm currently in vintage j. The real value of a firm in vintage j, as a function of its current nominal price P, is given by

$$\begin{aligned} v_{jt}(P) &= \Pi_t(P)+\beta E_t\frac{u'(C_{t+1})}{u'(C_t)}\left[\Gamma\left(\frac{v_{0,t+1}-v_{j+1,t+1}(P)}{w_{t+1}}\right)v_{0,t+1}-\Xi_{j+1,t+1}(P)\right] \\ &\quad +\beta E_t\frac{u'(C_{t+1})}{u'(C_t)}\left[1-\Gamma\left(\frac{v_{0,t+1}-v_{j+1,t+1}(P)}{w_{t+1}}\right)\right]v_{j+1,t+1}(P). \end{aligned} \tag{3}$$

We assume that J periods after the last price adjustment, firms draw a zero menu cost.[11] This means that firms in vintage $J-1$ know that in the following period they will adjust their price with probability one at no cost. Therefore, expression (3) holds for vintages $j = 1, ..., J-2$, whereas for vintage-$(J-1)$ firms the corresponding value function is

$$v_{J-1,t}(P) = \Pi_t(P)+\beta E_t\frac{u'(C_{t+1})}{u'(C_t)}v_{0,t+1}. \tag{4}$$

The optimal price setting decision is given by

$$0 = \Pi'_t(P_t^*)+\beta E_t\frac{u'(C_{t+1})}{u'(C_t)}\left[1-\Gamma\left(\frac{v_{0,t+1}-v_{1,t+1}(P_t^*)}{w_{t+1}}\right)\right]v'_{1,t+1}(P_t^*), \tag{5}$$

where

$$\Pi'_t(P) = \left[\epsilon\frac{w_t}{z_t}-(\epsilon-1)\frac{P}{P_t}\right](P)^{-\epsilon-1}P_t^{\epsilon}Y_t.$$

[11] This is a tractability assumption which ensures a finite state space under zero inflation or when the support of the menu cost distribution is unbounded from above.

Iterating (5) forward, and using the implications of (3) and (4) for the terms $v'_{j,t+j}(P_t^*)$, $j = 1, ..., J-1$, we can express the pricing decision as

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) u'(C_{t+j}) P_{t+j}^{\epsilon} Y_{t+j} (w_{t+j}/z_{t+j})}{\sum_{j=0}^{J-1} \beta^j E_t \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) u'(C_{t+j}) P_{t+j}^{\epsilon-1} Y_{t+j}},$$

where

$$\lambda_{jt} \equiv \Gamma\left(\frac{v_{0t} - v_{jt}}{w_t}\right) \tag{6}$$

denotes the period-t adjustment probability of firms in vintage $j = 1, ..., J-1$, and we define $v_{jt} \equiv v_{jt}(P_{t-j}^*)$ for short. As emphasized by Dotsey et al. (1999), this pricing decision is analogous to the one in the Calvo model. In particular, the term $\prod_{k=1}^{j}(1 - \lambda_{k,t+k})$ is the endogenous probability that the price chosen at t survives for the next j periods, thus replacing the exogenous probability $\left(1 - \lambda^C\right)^j$ where λ^C is the constant adjustment probability in the Calvo model. We can rewrite the price decision in terms of stationary variables as

$$p_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) \left(\prod_{k=1}^{j} \pi_{t+k}\right)^{\epsilon} u'(C_{t+j}) Y_{t+j} (w_{t+j}/z_{t+j})}{\sum_{j=0}^{J-1} \beta^j E_t \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) \left(\prod_{k=1}^{j} \pi_{t+k}\right)^{\epsilon-1} u'(C_{t+j}) Y_{t+j}}, \tag{7}$$

where $p_t^* \equiv P_t^*/P_t$ is the optimal relative price and $\prod_{k=1}^{j} \pi_{t+k} = P_{t+j}/P_t$ is accumulated inflation between periods t and $t+j$.

2.3 Market clearing

Labor input is required both for the production of goods and for changing prices. Labor demand for production by firm i is $n_{it} = y_{it}/z_t = (P_{it}/P_t)^{-\epsilon} Y_t/z_t$. Thus, total labor demand for production purposes equals $\Delta_t Y_t/z_t$, where $\Delta_t \equiv \int_0^1 (P_{it}/P_t)^{-\epsilon} di$ denotes relative price dispersion. At the same time, the total amount of labor used by vintage-j firms for pricing purposes equals $\psi_{jt} \int_0^{(v_{0t} - v_{jt})/w_t} \kappa g(\kappa) d\kappa$, where ψ_{jt} is the mass of firms in vintage j. Equilibrium in the labor market therefore implies

$$N_t = \frac{Y_t \Delta_t}{z_t} + \sum_{j=1}^{J-1} \psi_{jt} \int_0^{(v_{0t} - v_{jt})/w_t} \kappa g(\kappa) d\kappa. \tag{8}$$

Also, equilibrium in the goods market requires that

$$Y_t = C_t + G_t, \tag{9}$$

where G_t denotes government expenditure, which follows an exogenous process.

2.4 Inflation, price dispersion, and price distribution dynamics

All firms adjusting at time t choose the same nominal price, P_t^*. Given that no nominal price survives for longer than J periods by assumption, the finite set of beginning-of-period prices at any time t is $\left\{P_{t-1}^*, P_{t-2}^*, ..., P_{t-J}^*\right\}$. Let ψ_{jt} denote the time-t fraction of firms with beginning-of-period nominal price P_{t-j}^*, for $j = 1, 2, ..., J$, with $\sum_{j=1}^{J} \psi_{jt} = 1$. The price level evolves according to

$$P_t^{1-\epsilon} = \left(P_t^*\right)^{1-\epsilon} \sum_{j=1}^{J} \lambda_{jt}\psi_{jt} + \sum_{j=1}^{J-1} \left(P_{t-j}^*\right)^{1-\epsilon} \left(1-\lambda_{jt}\right)\psi_{jt},$$

where adjustment probabilities $\{\lambda_{jt}\}_{j=1}^{J-1}$ are given by (6), and where $\lambda_{J,t} = 1$. Rescaling both sides of the above equation by P_t, we obtain

$$1 = \left(p_t^*\right)^{1-\epsilon} \sum_{j=1}^{J} \lambda_{jt}\psi_{jt} + \sum_{j=1}^{J-1} \left(\frac{p_{t-j}^*}{\prod_{k=0}^{j-1}\pi_{t-k}}\right)^{1-\epsilon} \left(1-\lambda_{jt}\right)\psi_{jt}. \tag{10}$$

This equation determines the inflation rate π_t, given $\left\{p_{t-j}^*\right\}_{j=0}^{J-1}$ and $\{\pi_{t-j}\}_{j=1}^{J-2}$. Similarly, price dispersion follows

$$\Delta_t = \left(p_t^*\right)^{-\epsilon} \sum_{j=1}^{J} \lambda_{jt}\psi_{jt} + \sum_{j=1}^{J-1} \left(\frac{p_{t-j}^*}{\prod_{k=0}^{j-1}\pi_{t-k}}\right)^{-\epsilon} \left(1-\lambda_{jt}\right)\psi_{jt}, \tag{11}$$

where again $\lambda_{J,t} = 1$. The distribution of beginning-of-period prices evolves according to

$$\psi_{j,t} = \left(1-\lambda_{j-1,t-1}\right)\psi_{j-1,t-1} \tag{12}$$

for $j = 2, ..., J$, and

$$\psi_{1t} = 1 - \textstyle\sum_{j=2}^{J} \psi_{j,t} = \lambda_{1,t-1}\psi_{1,t-1} + \lambda_{2,t-1}\psi_{2,t-1} + ... + \psi_{J,t-1}. \tag{13}$$

2.5 Equilibrium

There are $8 + 2J + (J-1) = 7 + 3J$ stationary endogenous variables: C_t, N_t, Y_t, R_t, π_t, p_t^*, w_t, Δ_t, $\left\{\psi_{jt}\right\}_{j=1}^{J}$, $\{v_{jt}\}_{j=0}^{J-1}$, and $\{\lambda_{jt}\}_{j=1}^{J-1}$. The equilibrium conditions are (1), (2), the $J-1$ equations (6), equations (7) to (11), the J laws of motion (12) and (13), the value functions

$$\begin{aligned} v_{jt} \;=\; & \left(\frac{p_{t-j}^*}{\prod_{k=0}^{j-1}\pi_{t-k}} - \frac{w_t}{z_t}\right)\left(\frac{p_{t-j}^*}{\prod_{k=0}^{j-1}\pi_{t-k}}\right)^{-\epsilon} Y_t \\ & + \beta E_t \frac{u'\left(C_{t+1}\right)}{u'\left(C_t\right)} \left[\lambda_{j+1,t+1} v_{0,t+1} + \left(1-\lambda_{j+1,t+1}\right) v_{j+1,t+1} - w_{t+1} \int_0^{\left(v_{0,t+1}-v_{j+1,t+1}\right)/w_{t+1}} \kappa d\Gamma\left(\kappa\right)\right] \end{aligned}$$

for $j = 0, 1, ..., J-2$, and

$$v_{J-1,t} = \left(\frac{p^*_{t-(J-1)}}{\prod_{k=0}^{(J-1)-1} \pi_{t-k}} - \frac{w_t}{z_t} \right) \left(\frac{p^*_{t-(J-1)}}{\prod_{k=0}^{(J-1)-1} \pi_{t-k}} \right)^{-\epsilon} Y_t + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} v_{0,t+1};$$

plus a specification of monetary policy. If we were to close the model with a Taylor rule, this would give us a total of $2 + (J-1) + 5 + J + J + 1 = 7 + 3J$ equations. Instead, we will study the optimal state-contingent monetary policy plan, which will essentially double the number of equations and variables.

2.5.1 Equilibrium with flexible prices and no cost-push shocks

We now derive the equilibrium with flexible prices and no shocks to distortionary taxes ($u_t = 1$). The latter are an example of *cost-push* shocks. This equilibrium will be used later as a benchmark for measuring the welfare consequences of alternative monetary policy rules. In such an equilibrium, menu costs are zero and all firms choose the same nominal price $P^*_t = \frac{\epsilon}{\epsilon - 1} \frac{w_t}{z_t} P_t$ in each period t. All relative prices are one: $p^*_t = P^*_t / P_t = 1$. The equilibrium conditions simplify to

$$\begin{aligned} u'(C^p_t) z_t \frac{\epsilon - 1}{\epsilon} &= x'(N^p_t; \chi_t), \\ z_t N^p_t &= Y^p_t, \\ Y^p_t &= C^p_t + G_t, \end{aligned}$$

and so we obtain the classical decoupling of real and nominal variables. We refer to Y^p_t as the *potential* level of output, and use it to define the *output gap* as the ratio between actual output and its potential counterpart, Y_t / Y^p_t.[12]

3 Optimal monetary policy

3.1 The general problem

For the purpose of deriving the optimality conditions of the Ramsey plan, it is useful to define

$$\pi^{acc}_{jt} \equiv \prod_{k=0}^{j-1} \pi_{t-k} = \frac{P_t}{P_{t-j}}, \quad j = 1, ..., J-1,$$

[12] In the literature 'potential output' sometimes refers to the output level under perfect competition. Our definition of potential output thus differs from the latter in that it incorporates static monopolistic distortions. Our choice is motivated by our focus on the welfare losses due to trend inflation and aggregate fluctuations, as opposed to those caused by monopolistic distorsions in the steady state.

that is, the accumulated inflation between periods $t-j$ and t. This implies $\prod_{k=1}^{j}\pi_{t+k}=\pi_{j,t+j}^{acc}$. We also define

$$\theta_{jt}\equiv\prod_{k=0}^{j-1}\left(1-\lambda_{j-k,t-k}\right),\quad j=1,...,J-1,$$

that is, the probability that a price chosen at $t-j$ survives until t, which in turn implies $\prod_{k=1}^{j}\left(1-\lambda_{k,t+k}\right)=\theta_{j,t+j}$. These definitions allow us to express the optimal pricing decision in equation (7) in a more compact form,

$$p_t^*=\frac{\epsilon}{\epsilon-1}\frac{\sum_{j=0}^{J-1}\beta^j E_t\theta_{j,t+j}\left(\pi_{j,t+j}^{acc}\right)^{\epsilon}u'\left(C_{t+j}\right)Y_{t+j}\left(w_{t+j}/z_{t+j}\right)}{\sum_{j=0}^{J-1}\beta^j E_t\theta_{j,t+j}\left(\pi_{j,t+j}^{acc}\right)^{\epsilon-1}u'\left(C_{t+j}\right)Y_{t+j}}.$$

Similarly, we replace $\prod_{k=0}^{j-1}\pi_{t-k}$ by π_{jt}^{acc} in the laws of motion of inflation and price dispersion, and in the firms' value functions. It is useful to express the variables π_{jt}^{acc} and θ_{jt} recursively,

$$\pi_{jt}^{acc}=\pi_t\pi_{j-1,t-1}^{acc},\quad j=1,...,J-1,$$

$$\theta_{jt}^{acc}=\left(1-\lambda_{jt}\right)\theta_{j-1,t-1}^{acc},\quad j=1,...,J-1,$$

where the recursions start with $\pi_{0,t-1}^{acc}=1$ and $\theta_{0,t-1}^{acc}=1$, respectively. We use equation (1) to substitute for the real wage in the equilibrium conditions. In addition, we use (9) to substitute for C_t. Finally, we define $\tilde{v}_{jt}\equiv v_{jt}u'\left(C_t\right)$, $j=0,1,...,J-1$, such that $\left(v_{0t}-v_{jt}\right)/w_t=\left(\tilde{v}_{0t}-\tilde{v}_{jt}\right)/\left[x'\left(N_t;\chi_t\right)u_t\right]$. At time 0, the central bank chooses the state-contingent path of the

endogenous variables that maximizes the following Lagrangian:

$$
\begin{aligned}
&\mathcal{L}_0 = E_0 \textstyle\sum_{t=0}^{\infty} \beta^t \{ u\left(Y_t - G_t\right) - x\left(N_t; \chi_t\right) \\
&+\phi_t^{p^*} \left[p_t^* \sum_{j=0}^{J-1} \beta^j \theta_{j,t+j} (\pi_{j,t+j}^{acc})^{\epsilon-1} Y_{t+j} u'\left(Y_{t+j} - G_{t+j}\right) - \frac{\epsilon}{\epsilon - 1} \sum_{j=0}^{J-1} \beta^j \theta_{j,t+j} (\pi_{j,t+j}^{acc})^{\epsilon} Y_{t+j} \frac{x'\left(N_{t+j}; \chi_{t+j}\right)}{z_{t+j}/u_{t+j}} \right] \\
&+\phi_t^N \left[N_t - Y_t \Delta_t / z_t - \textstyle\sum_{j=1}^{J-1} \psi_{jt} \int_0^{(\tilde{v}_{0t} - \tilde{v}_{jt})/x'(N_t;\chi_t) u_t} \kappa g\left(\kappa\right) d\kappa \right] \\
&+\phi_t^{\pi} \left[\left(p_t^*\right)^{1-\epsilon} \textstyle\sum_{j=1}^{J} \lambda_{jt} \psi_{jt} + \sum_{j=1}^{J-1} \left(p_{t-j}^* / \pi_{jt}^{acc}\right)^{1-\epsilon} \left(1 - \lambda_{jt}\right) \psi_{jt} \right] \\
&+\phi_t^{\Delta} \left[\left(p_t^*\right)^{-\epsilon} \textstyle\sum_{j=1}^{J} \lambda_{jt} \psi_{jt} + \sum_{j=1}^{J-1} \left(p_{t-j}^* / \pi_{jt}^{acc}\right)^{-\epsilon} \left(1 - \lambda_{jt}\right) \psi_{jt} - \Delta_t \right] \\
&+ \sum_{j=1}^{J-1} \phi_t^{\lambda_j} \left[\lambda_{jt} - \Gamma\left(\frac{\tilde{v}_{0t} - \tilde{v}_{jt}}{x'\left(N_t; \chi_t\right) u_t} \right) \right] + \sum_{j=2}^{J} \phi_t^{\psi_j} \left[\psi_{j,t} - \left(1 - \lambda_{j-1,t-1}\right) \psi_{j-1,t-1} \right] + \phi_t^{\psi_1} \left[\psi_{1t} + \sum_{j=2}^{J} \psi_{j,t} \right] \\
&+ \sum_{j=0}^{J-2} \phi_t^{v_j} \left[\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}} u'\left(Y_t - G_t\right) - \frac{x'\left(N_t; \chi_t\right)}{z_t / u_t} \right) \left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}} \right)^{-\epsilon} Y_t - \tilde{v}_{jt} \right] \\
&+ \sum_{j=0}^{J-2} \phi_t^{v_j} \beta \left[\lambda_{j+1,t+1} \tilde{v}_{0,t+1} + \left(1\text{-}\lambda_{j+1,t+1}\right) \tilde{v}_{j+1,t+1} - \frac{x'\left(N_{t+1}; \chi_{t+1}\right)}{1/u_{t+1}} \int_0^{(\tilde{v}_{0t+1} - \tilde{v}_{j+1,t+1})/x'\left(N_{t+1};\chi_{t+1}\right) u_{t+1}} \kappa d\Gamma\left(\kappa\right) \right] \\
&+\phi_t^{v_{J-1}} \left[\left(\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}} u'\left(Y_t - G_t\right) - \frac{x'\left(N_t; \chi_t\right)}{z_t / u_t} \right) \left(\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}} \right)^{-\epsilon} Y_t - \tilde{v}_{J-1,t} + \beta \tilde{v}_{0,t+1} \right] \\
&+\phi_t^{\pi_1^{acc}} \left[\pi_{1t}^{acc} - \pi_t \right] + \textstyle\sum_{j=2}^{J-1} \phi_t^{\pi_j^{acc}} \left[\pi_{jt}^{acc} - \pi_t \pi_{j-1,t-1}^{acc} \right] \\
&+\phi_t^{\theta_1} \left[\theta_{1t} - \left(1 - \lambda_{1t}\right) \right] + \textstyle\sum_{j=2}^{J-1} \phi_t^{\theta_j} \left[\theta_{jt} - \left(1 - \lambda_{jt}\right) \theta_{j-1,t-1} \right] \}. \qquad (14)
\end{aligned}
$$

Since the nominal interest rate only appears in the consumption Euler equation, the latter is excluded from the set of constraints on the Ramsey problem and is used instead to back out the nominal interest rate path consistent with the optimal allocation. The first-order conditions of the above problem are derived in Appendix A.

Our object of interest is optimal monetary policy from a "timeless perspective." As explained by Woodford (2003), this type of policy does not exploit the private sector's expectations that formed prior to the particular date on which the plan was implemented. Instead, the central bank commits itself to behave, from date 0, in a way consistent with the way it would have chosen to behave had it committed to the optimal policy in the infinite past.

Appendix B proves the following result:

Proposition 1 *Let functional forms for preferences be of the constant elasticity type. Assume furthermore that households consume all output ($G_t = 0$) and that there are no cost-push shocks ($u_t = 1$). Then the zero inflation policy ($\pi_t = 1$) is optimal from the timeless perspective.*

There are two important aspects of the above proposition. The first is that optimal *trend*

inflation is zero. Therefore, the presence of monopolistic distortions does not justify a positive rate of trend inflation, and the optimal policy involves a commitment to eventually eliminating any inefficient price dispersion due to staggered price setting. Interestingly, this normative prescription coincides with the one implied by the standard New Keynesian model with Calvo pricing, as shown by Benigno and Woodford (2005).[13] The main insight of the Calvo framework, about the desirability of zero long-run inflation, thus continues to hold in a general model of state-dependent pricing. The key difference between exogenous-timing models of price adjustment such as Calvo's and SDP models is the endogeneity of the timing of price adjustment in the latter. A priori, the central bank could have an incentive to use trend inflation to influence the speed at which firms change prices, if such a policy were to have beneficial effects on society. The above result implies that the endogeneity of price adjustment frequencies does not affect the optimality of zero trend inflation.

To understand the intuition for this result, let us consider the different channels through which trend inflation affects welfare. Two of these channels are common to exogenous-timing models such as Calvo or Taylor. One is that, in the presence of staggered prices, inflation increases the extent of price dispersion, distorting the economy's pricing system. This leads to inefficient allocation of resources across product lines, and increases the total amount of (labor) resources needed to produce a given amount of the consumption basket; hence, it lowers welfare. Notice that inefficient price dispersion attains a global minimum at zero inflation because, under such a policy, all relative prices end up being equal.

The other common channel, through which trend inflation affects welfare, works through its two opposing effects on the inflation-output tradeoff: On the one hand, holding constant inflation expectations, a rise in current inflation allows the central bank to raise output toward its socially efficient level, thus reducing the monopolistic distortion and improving welfare; intuitively, the economy moves *along* the New Keynesian Phillips curve (NKPC).[14] On the other hand, choosing higher inflation raises the inflation expectations of price-setters; the latter produces an upward *shift* of the NKPC, thus worsening the short-run tradeoff between inflation and output. As it turns out, at zero inflation, the marginal welfare cost of raising inflation expectations exactly offsets the marginal welfare benefit of exploiting the short-run inflation-output tradeoff.

[13]The same result holds for another prominent exogenous-timing model of price adjustment, namely the Taylor model, where adjustment probabilities are zero for a number of periods after a price change and one afterwards. A proof of the latter result is available upon request from the authors.

[14]The "New Keynesian Phillips curve" is the structural relationship between inflation (current and expected) and output that arises in the standard New Keynesian model. Here, the optimal price decision (equation 7) and the relationship between inflation and the optimal relative price (equation 10) can be combined into a dynamic relationship between inflation and real marginal costs, where the latter can also be expressed in terms of aggregate output by using equations (1), (8), and (9). The resulting dynamic relationship between inflation and output may be interpreted as a "New Keynesian Phillips curve." Notice that the endogenous price adjustment frequencies, λ_{jt}, affect both the intercept and the slope of that curve.

While the former two effects of trend inflation are shared with exogenous-timing models, our framework with idiosyncratic menu cost shocks includes two additional channels through which trend inflation affects welfare. One is that inflation forces firms to spend real resources (menu costs) on adjusting their nominal prices; this distortion is minimized at zero inflation, because eventually all firms end up being at their optimal price.

The second additional channel is more subtle. In the stochastic menu costs model, adjustment frequencies are endogenous. In particular, trend inflation affects the relative prices of different cohorts of firms ($p_{t-j}/\prod_{k=0}^{j-1}\pi_{t-k}$, $j = 0, ..., J-1$), which has an effect on their profits, on their value functions, and ultimately on the gains from adjustment. A priori, the central bank may be tempted to use trend inflation to influence the speed of price adjustment, so as to shift the NKPC in a way that improves the inflation-output tradeoff. However, the fact that adjusting firms choose their prices in an *optimal* way implies that, in the steady state with zero inflation, all firms are maximizing profits. As a result, a marginal increase in the inflation rate has no effect on firms' profits, and therefore it has no effect on adjustment probabilities. This *envelope* property implies that the monetary authority has no incentive to create trend inflation so as to influence the speed with which firms change their prices.

The second important aspect of Proposition 1 is that the optimal *deviations* from zero inflation in response to technology or preference shocks are exactly zero as well. Therefore, the occurrence of these exogenous disturbances to preferences or technology does not justify temporary departures from strict inflation targeting. The intuition for this result, which coincides with that found by Benigno and Woodford (2005) for the Calvo model, is as follows. There are four potential inefficiencies in the present model, related to: (1) the level and volatility of price dispersion; (2) the volatility of the average markup; (3) the waste of resources due to menu costs; and (4) the level of the average markup due to monopolistic competition. Distortions (1) through (3) are directly related to the friction in price setting, and –absent idiosyncratic shocks to desired prices– a policy of zero inflation eliminates all three. It does so by replicating the flexible-price equilibrium and eliminating the incentives for price adjustment. Inefficiency (4) is a static markup distortion due to monopolistic competition. As we have just seen, the optimal plan does not involve a correction of this inefficiency because it is outweighed by the gains of committing to zero inflation and achieving the minimum possible price dispersion in the long run, independently of the price-setting policies followed by firms. The aforementioned envelope property, by which a marginal increase in inflation leaves price adjustment frequencies unaffected, continues to hold as the economy is hit by shocks to preferences or technology.

3.2 An illustration with two cohorts

While the appendix provides the proof of the optimality of zero inflation in the full-blown model, it is illustrative to formalize the above intuitions with a simplified version of the model. In particular, we consider the case of $J = 2$ cohorts, such that firms that adjust their nominal price today may or may not adjust in the following period, but adjust with certainty two periods after the last price change. To further simplify, we assume functional forms $u(C_t) = \log(C_t)$ and $x(N_t; \chi_t) = \chi_t N_t$. As in Proposition 1, we assume away government spending, $G_t = 0$, such that $C_t = Y_t$, and no cost push shocks ($u_t = 1$). The real wage is thus $w_t = x'(N_t; \chi_t)/u'(Y_t) = \chi_t Y_t$. To simplify the notation, let $\psi_t \equiv \psi_{1t}$ and $\lambda_t \equiv \lambda_{1t}$ denote the measure and adjustment probability of firms in vintage 1. The measure of firms in vintage 2 is then $\psi_{2t} = 1 - \psi_t$, and the law of motion of ψ_t is simply $\psi_t = 1 - (1 - \lambda_{t-1})\psi_{t-1}$. Let also $v_t \equiv v_{1t}$ denote the value of firms in vintage 1. Finally, we define $\tilde{v}_{0t} \equiv v_{0t}/Y_t$ and $\tilde{v}_t \equiv v_t/Y_t$, such that $(v_{0t} - v_t)/w_t = (\tilde{v}_{0t} - \tilde{v}_t)/\chi$. Taking all these elements, the central bank maximizes the following Lagrangian:

$$
\begin{aligned}
\mathcal{L}_0 = E_0 \textstyle\sum_{t=0}^{\infty} \beta^t \Bigg\{ & \log(Y_t) - \chi_t \frac{Y_t \Delta_t}{z_t} - \chi_t \psi_t \int_0^{(\tilde{v}_{0t} - \tilde{v}_t)/\chi} \kappa g(\kappa)\, d\kappa \\
& + \phi_t^{p^*} \left[p_t^* \left(1 + \beta (1 - \lambda_{t+1}) \pi_{t+1}^{\epsilon - 1} \right) - \frac{\epsilon}{\epsilon - 1} \left(\frac{\chi_t Y_t}{z_t} + \beta (1 - \lambda_{t+1}) \pi_{t+1}^{\epsilon} \frac{\chi_t Y_{t+1}}{z_{t+1}} \right) \right] \\
& + \phi_t^{\pi} \left[(p_t^*)^{1-\epsilon} (\lambda_t \psi_t + 1 - \psi_t) + \left(\frac{p_{t-1}^*}{\pi_t} \right)^{1-\epsilon} (1 - \lambda_t) \psi_t - 1 \right] \\
& + \phi_t^{\Delta} \left[(p_t^*)^{-\epsilon} (\lambda_t \psi_t + 1 - \psi_t) + \left(\frac{p_{t-1}^*}{\pi_t} \right)^{-\epsilon} (1 - \lambda_t) \psi_t - \Delta_t \right] \\
& + \phi_t^{\lambda} \left[\lambda_t - \Gamma \left(\frac{\tilde{v}_{0t} - \tilde{v}_t}{\chi_t} \right) \right] + \phi_t^{\psi} \left[\psi_t + (1 - \lambda_{t-1}) \psi_{t-1} \right] \\
& + \phi_t^{v_0} \left[\left(p_t^* - \frac{\chi_t Y_t}{z_t} \right) (p_t^*)^{-\epsilon} - \tilde{v}_{0t} + \beta \left(\lambda_{t+1} \tilde{v}_{0,t+1} + (1 - \lambda_{t+1}) \tilde{v}_{t+1} - \chi_t \int_0^{(\tilde{v}_{0,t+1} - \tilde{v}_{t+1})/\chi} \kappa g(\kappa)\, d\kappa \right) \right] \\
& + \phi_t^{v} \left[\left(\frac{p_{t-1}^*}{\pi_t} - \frac{\chi_t Y_t}{z_t} \right) \left(\frac{p_{t-1}^*}{\pi_t} \right)^{-\epsilon} - \tilde{v}_t + \beta \tilde{v}_{0,t+1} \right] \Bigg\}.
\end{aligned}
$$

For the present analysis, it suffices to differentiate the Lagrangian with respect to inflation and the optimal relative price for a particular state at time t. While the derivative of the Lagrangian with respect to π_t captures the direct marginal effect of inflation on welfare, the derivative with respect to p_t^* captures its indirect effect through its structural relationship with the optimal relative price. That relationship is given by the equation multiplied by ϕ_t^{π} in the Lagrangian. Indeed, if we use the latter equation to solve for the optimal relative price as a function of current and past inflation, and then use the resulting expression to substitute for p_t^* in the optimal price decision

(the equation multiplied by $\phi_t^{p^*}$), we obtain a dynamic relationship between inflation and aggregate activity. The latter may be interpreted as a "New Keynesian Phillips curve." The derivatives with respect to π_t and p_t^* are given by

$$\begin{aligned}\frac{\partial \mathcal{L}_0}{\partial \pi_t} &= \phi_{t-1}^{p^*}\left[\frac{p_{t-1}^*}{\pi_t}(\epsilon-1)-\frac{\epsilon}{\epsilon-1}\frac{\chi_t Y_t}{z_t}\epsilon\right]\pi_t^{\epsilon-1}(1-\lambda_t)\\ &+\left[\phi_t^{\pi}(\epsilon-1)\frac{p_{t-1}^*}{\pi_t}+\phi_t^{\Delta}\epsilon\right]\left(p_{t-1}^*\right)^{-\epsilon}\pi_t^{\epsilon-1}(1-\lambda_t)\psi_t\\ &+\phi_t^{v}\left[(\epsilon-1)\frac{p_{t-1}^*}{\pi_t}-\epsilon\frac{\chi_t Y_t}{z_t}\right]\left(p_{t-1}^*\right)^{-\epsilon}\pi_t^{\epsilon-1},\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_0}{\partial p_t^*} &= \phi_t^{p^*}\left[1+\beta(1-\lambda_{t+1})\pi_{t+1}^{\epsilon-1}\right]-\left[\phi_t^{\pi}(\epsilon-1)p_t^*+\phi_t^{\Delta}\epsilon\right](p_t^*)^{-\epsilon-1}(\lambda_t\psi_t+1-\psi_t)\\ &-\beta E_t\left[\phi_{t+1}^{\pi}(\epsilon-1)\frac{p_t^*}{\pi_{t+1}}+\phi_{t+1}^{\Delta}\epsilon\right](p_t^*)^{-\epsilon-1}\pi_{t+1}^{\epsilon}(1-\lambda_{t+1})\psi_{t+1}\\ &+\phi_t^{v_0}\left[\epsilon\frac{\chi_t Y_t}{z_t}-(\epsilon-1)p_t^*\right](p_t^*)^{-\epsilon-1}+\beta E_t\phi_{t+1}^{v}\left[\epsilon\frac{\chi_t Y_{t+1}}{z_{t+1}}-(\epsilon-1)\frac{p_t^*}{\pi_{t+1}}\right](p_t^*)^{-\epsilon-1}\pi_{t+1}^{\epsilon},\end{aligned} \tag{16}$$

respectively.[15] We now conjecture that the central bank commits to follow a policy of zero net inflation, or $\pi_t = 1$. It is straightforward to show that under such a policy the economy converges to an equilibrium in which $p_t^* = \Delta_t = 1$. That is, both firm vintages have the same relative price, and price dispersion is eliminated. Thus, both vintages end up having the same value, $v_{0t} = v_t$, which in turn implies $\lambda_t = \Gamma(0) \equiv \bar{\lambda} > 0$. The vintage distribution converges to $\psi_t = 1/\left(2-\bar{\lambda}\right) \equiv \bar{\psi}$. Finally, the real marginal cost equals the inverse of the monopolistic mark-up, $\chi_t Y_t/z_t = (\epsilon-1)/\epsilon$, implying that output equals its flexible-price level of section 2.5.1 at all times.

Imposing the latter conjecture in expressions (15) and (16), we obtain

$$\frac{\partial \mathcal{L}_0}{\partial \pi_t} = -\phi_{t-1}^{p^*}\left(1-\bar{\lambda}\right)+\left[\phi_t^{\pi}(\epsilon-1)+\phi_t^{\Delta}\epsilon\right]\left(1-\bar{\lambda}\right)\bar{\psi}, \tag{17}$$

$$\frac{\partial \mathcal{L}_0}{\partial p_t^*} = \phi_t^{p^*}\left[1+\beta\left(1-\bar{\lambda}\right)\right]-\left[\phi_t^{\pi}(\epsilon-1)+\phi_t^{\Delta}\epsilon\right]\bar{\psi}-\beta\left(1-\bar{\lambda}\right)E_t\left[\phi_{t+1}^{\pi}(\epsilon-1)+\phi_{t+1}^{\Delta}\epsilon\right]\bar{\psi}, \tag{18}$$

where we have also used the fact that $\bar{\lambda}\bar{\psi}+1-\bar{\psi} = \bar{\psi}$. The first effect to notice is that, under our conjecture, all terms involving the Lagrange multipliers $\phi_t^{v_0}$ and ϕ_t^{v} in expressions (15) and (16) have disappeared. Such terms capture the marginal welfare effect of both variables through their effect on the value of both firm cohorts (v_{0t}, v_t). Therefore, once the economy has converged to the timeless perspective regime with zero inflation, a marginal deviation of inflation from zero has

[15] Both derivatives have been rescaled by β^t times the probability of reaching the particular state at time t conditional on the state at time 0.

no effect on the gains from adjustment, and hence it has no effect on the adjustment frequency either. This is the "envelope property" that we referred to before.

In equation (17), the term involving $\phi_{t-1}^{p^*}$ captures the marginal welfare effect from an increase in time $(t-1)$ *expectations* of inflation at time t, whereas the term involving $\phi_t^{p^*}$ in equation (18) reflects the marginal welfare effect from an increase in the optimal relative price (and thus in inflation) at time t. We show in the Appendix that, in the full-blown model, the multiplier $\phi_t^{p^*}$ converges to a constant value $\bar{\phi}^{p^*}$ in the timeless perspective regime, which is also true in this simplified version. Using this in (18), setting the resulting expression equal to zero (as required by the first-order optimality condition), and solving for ϕ_t^{π}, we obtain

$$\phi_t^{\pi} = \left(\bar{\phi}^{p^*}/\bar{\psi} - \phi_t^{\Delta}\epsilon\right)/(\epsilon - 1).$$

Using this to substitute for ϕ_t^{π} in (17), the latter becomes

$$\begin{aligned} \frac{\partial \mathcal{L}_0}{\partial \pi_t} &= \left[\bar{\phi}^{p^*}/\bar{\psi} - \phi_t^{\Delta}\epsilon + \phi_t^{\Delta}\epsilon\right]\left(1-\bar{\lambda}\right)\bar{\psi} - \bar{\phi}^{p^*}\left(1-\bar{\lambda}\right) \\ &= \phi_t^{\Delta}(\epsilon - \epsilon)\left(1-\bar{\lambda}\right)\bar{\psi} + \bar{\phi}^{p^*}(1-1)\left(1-\bar{\lambda}\right) \\ &= 0 + 0 = 0. \end{aligned} \tag{19}$$

Therefore, once the economy has converged to the timeless perspective regime with zero inflation, the central bank has no incentive to create positive or negative inflation at the margin, because the potential welfare costs cancel out the potential gains. The term involving ϕ_t^{Δ} in (19) captures the marginal welfare effect of inflation through its effect on price dispersion, which disappears under the timeless perspective regime with zero inflation. Finally, the term involving $\bar{\phi}^{p^*}$ is the difference between the positive marginal effect stemming from a movement along the NKPC, $\bar{\phi}^{p^*}\left(1-\bar{\lambda}\right)$, and the negative marginal effect due to the shift in the NKPC, $-\bar{\phi}^{p^*}\left(1-\bar{\lambda}\right)$. Under the zero inflation policy, both effects exactly cancel each other out.

The specific example above is intended to formalize the main intuition; more generally, the optimality of zero inflation from the timeless perspective holds for *any* number of cohorts and for standard (isoelastic) preferences, as shown in the Appendix.

4 Numerical analysis

The previous section derived the optimal policy under the assumption that the representative household consumes all output and there are no cost-push shocks. For the general case with both government consumption and cost-push shocks, we are no longer able to obtain analytical results, so we illustrate the nature of optimal monetary policy by numerical simulation. With this aim, we

first calibrate our model economy.

4.1 Calibration

We assume standard functional forms for preferences: $u(C_t) = \log(C_t)$, $x(N_t) = \chi N_t^{1+\varphi}/(1+\varphi)$.[16] Following Golosov and Lucas (2007), we set $\chi = 6$ and $\varphi = 1$. The discount factor is $\beta = 1.04^{-1/4}$. and the elasticity of substitution among product varieties is $\epsilon = 7$.

We assume that the cumulative distribution function of menu costs takes the form

$$\Gamma(\kappa) = \frac{\bar{\lambda}}{\bar{\lambda} + \left(1 - \bar{\lambda}\right) e^{-\kappa}},$$

which is a special case of the hazard function proposed by Woodford (2008). Unlike that considered by Costain and Nakov (2011), the function is bounded below not by 0 but by $\bar{\lambda} > 0$. We make this technical assumption to ensure a unique stationary distribution of firms over the (finite number of) price vintages in the case of zero inflation. We set $\bar{\lambda}$ so that, under a policy targeting 2% annual inflation (broadly consistent with the average observed rate in the United States since the mid-1980s), the model produces an average frequency of price changes in the steady state of once every 10 months, or 10/3 quarters, which is broadly consistent with the micro evidence found e.g. by Nakamura and Steinsson (2008). Figure 1 shows the adjustment hazard function λ_j and the distribution of firms by price vintage ψ_j in the steady state with 2% trend inflation. As shown in the right panel, our calibration implies that very few prices survive more than ten quarters.

Finally, our exogenous processes have law of motion $x_t/x = (x_{t-1}/x)^{\rho_x} \exp(\varepsilon_t^x)$, $\varepsilon_t^x \sim iid(0, \sigma_x)$, for $x = \{z, G, u\}$. We set the autocorrelation coefficients of productivity, government spending and cost-push shocks to $\rho_z = 0.95$, $\rho_g = 0.9$ and $\rho_u = 0.9$, respectively. For the purpose of illustration, we set the standard deviation of all three iid shocks to 1%. Steady-state government expenditure g is set to 0.1, so that it accounts for roughly 20% of GDP in the steady state, consistently with U.S. postwar experience.

4.2 Welfare losses from trend inflation

Proposition 1 implies that, in the absence of government consumption, the optimal rate of inflation in the steady state is zero. In fact, it is possible to show that zero steady-state inflation is optimal also in the presence of government consumption (Nakov and Thomas, 2010). Thus, we may first ask how much welfare is lost in the steady state from pursuing policy rules that imply nonzero trend inflation rates.

[16] For brevity, we omit the numerical analysis of preference shocks, and thus set $\chi_t = \chi$. Results are available upon request.

Figure 2 displays the welfare loss, relative to the flexible-price equilibrium, in the steady-state of the SDP model as a function of trend inflation. Welfare losses are expressed as a percent of steady-state consumption. For comparison, the figure also shows the welfare losses in the steady-state of the Calvo model. The equilibrium conditions in the latter model are given by equations (1), (2), (8, *without* menu costs), (9); as well as (7), (10) and (11), with $J = \infty$, a constant λ^C replacing $\lambda_{k,t+k}$ and λ_{jt}, and $\lambda^C(1-\lambda^C)^{j-1}$ replacing ψ_{jt}.[17] We calibrate λ^C to the same average adjustment frequency as the SDP model (10/3 quarters).

As the figure shows, welfare losses are minimized at zero inflation in both models, and increase in a convex manner as trend inflation departs from zero. With nonzero inflation, welfare losses are *higher* under Calvo pricing, with the gap widening as inflation moves away from zero. For instance, at 5% annual inflation, the SDP model generates welfare losses of almost 0.7% of steady-state consumption, whereas welfare losses are almost 1% of steady-state consumption in the Calvo model.

The difference in welfare loss between the SDP and Calvo model is due almost entirely to relative price dispersion. For instance, at 5% inflation, the excess price dispersion under Calvo pricing relative to SDP is equivalent to 0.9% of steady-state consumption. The intuition is simple. As inflation increases, firms adjust prices faster and faster under SDP, especially for those firms that are further away from the optimal price, but not under Calvo pricing, where price adjustment frequencies are constant. This increases the size of relative price distortions in the Calvo model vis–ï¿œ-vis the SDP model. *Ceteris paribus*, greater price dispersion reduces the amount of consumption that can be produced for given labor input, thus reducing welfare.[18]

4.3 Impulse-response analysis

We now study how the economy responds to aggregate shocks under different policy regimes. We consider three types of aggregate disturbances: (i) productivity shocks, (ii) government consumption shocks, and (iii) cost-push shocks. We also consider three policy rules: (i) the optimal monetary policy (i.e. the one that solves the Ramsey problem laid out in section 3), (ii) strict inflation targeting ($\pi_t = 1$), and (iii) a simple Taylor rule for the nominal interest rate,

$$\frac{R_t}{\bar{\pi}/\beta} = \left(\frac{R_{t-1}}{\bar{\pi}/\beta}\right)^{\phi_R} \pi_t^{(1-\phi_R)\phi_\pi}.$$

We set $\phi_\pi = 1.5$ and $\phi_R = 0.8$, which conforms well with estimated Taylor rules for the US economy for the Great Moderation period.

Both the optimal policy and strict inflation targeting imply zero inflation in the steady state.

[17] In fact, the counterparts of equations (7), (10) and (11) in the Calvo model can all be expressed recursively.

[18] Our calculations show that steady-state labor hours, N_{ss}, are in fact very similar under SDP and Calvo pricing.

Thus, in order to calculate impulse-responses we use a first-order Taylor expansion around the zero inflation steady state.[19] For comparability, we also approximate dynamics under the Taylor rule around the zero inflation steady state, which requires setting $\bar{\pi} = 1$ in the latter rule. We also set $J = 24$ in all our dynamic simulations, a number that is much greater than any observed price duration in recent U.S. evidence.

Figure 3 plots the responses of inflation, output and the output gap to a 1% positive productivity shock.[20] Each column displays the responses under each of the three policy rules. The upper-left and lower-left subplots reveal that the responses of output gap and inflation under the optimal policy are basically indistinguishable from zero. Thus, while strict inflation targeting is no longer exactly optimal in the presence of steady-state government consumption, it continues to provide a very good approximation of optimal policy in response to productivity shocks. By contrast, under the Taylor rule (third column) both output gap and inflation experience a sizable reduction, due to the failure of actual output to increase as much as its potential level.

Figure 4 shows the responses to a 1% increase in government consumption. Once again, we find that zero inflation provides a very good characterization of optimal policy. Under the Taylor rule, the actual response of output is now somewhat larger than that of potential output, which results in a slight increase in the output gap and hence in inflation.

While productivity and government spending shocks involve virtually no trade-off between stabilizing prices and stabilizing the output gap, cost-push shocks are more likely to generate a relevant trade-off. Figure 5 shows the impulse-responses to a 1% increase in the cost-push shifter u_t. As expected, this shocks does generate a sizable response in both inflation (about 16 basis points in annual terms) and the output gap under the optimal policy. Moreover, a policy of strict price stability now comes at the cost of a severe contraction in output. The Taylor rule produces a much larger inflation response than the optimal policy, and yet it does not stabilize the output gap much more than the latter.

In figures (3) to (5), we also display the impulse-responses in the Calvo framework. As the figures make clear, impulse-responses to all three shocks are virtually identical in both frameworks. The reason for this is again the 'envelope property' discussed in section 3. In particular, notice that both models are approximated around the zero inflation steady state. In the latter, all firms are at their optimal prices and are thus maximizing their value. As a result, in the SDP model the first-order approximate dynamics around the steady state feature very small (in fact, second-order) changes in adjustment gains and hence in adjustment frequencies.

[19] A second-order Taylor expansion yields virtually identical impulse responses, both under state-dependent and Calvo pricing.

[20] Output and output-gap responses are in percent, inflation responses are in percentage points and in annualized terms.

4.4 Welfare losses from aggregate fluctuations

We now analyze the welfare consequences of the alternative policy rules considered thus far. In this section, calculations are based on a second-order approximation of equilibrium dynamics. Table 1 displays mean welfare loses, relative to the equilibrium with flexible prices and no cost-push shocks, for each policy rule. Welfare losses are expressed again as a percent of steady-state consumption. Notice that, for the three policy rules, the implied steady state for the welfare-relevant variables is exactly the same as in the flexible-price equilibrium. Therefore, welfare losses are due exclusively to aggregate fluctuations around the zero-inflation steady state.

Table 1. Mean welfare losses for alternative policy rules

Policy rule \ Shock:	productivity	government	cost-push	unconditional
		SDP model		
Optimal policy	0.0000	0.0000	0.0121	0.0121
Zero inflation	0.0000	0.0000	0.0129	0.0129
Taylor rule ($\bar{\pi} = 1$)	0.0266	0.0005	0.0181	0.0452
		Calvo model		
Optimal policy	0.0000	0.0000	0.0122	0.0122
Zero inflation	0.0000	0.0000	0.0129	0.0129
Taylor rule ($\bar{\pi} = 1$)	0.0282	0.0005	0.0187	0.0473

Note: the table reports mean welfare losses relative to the flexible-price equilibrium without cost-push shocks, expressed as a percent of steady-state consumption

The table reveals that the zero inflation policy achieves the same welfare loss as the optimal monetary policy conditional on productivity or government spending shocks. In fact, such welfare losses are basically zero, as both policies manage to essentially replicate the flexible price allocation. Under this metric, strict inflation stabilization provides again a very good approximation of the optimal monetary policy. By contrast, the Taylor rule achieves somewhat higher welfare losses than either the optimal or the zero inflation policies, although the difference is of second order, as one would expect.

Welfare losses conditional on cost-push shocks are higher than for productivity or government shocks when the monetary authority follows the optimal or the zero inflation policy, reflecting the existence of a relevant output-inflation trade-off. The Taylor rule generates higher welfare losses than the other two policy rules, which reflects its poorer stabilizing performance.

The table also displays welfare losses in the Calvo model. Welfare losses are identical (i.e. zero) in both frameworks under the optimal or zero inflation policies conditional on productivity and

government shocks, and just slightly higher in the Calvo model conditional on cost-push shocks or under the Taylor rule. This similarity between both frameworks in terms of welfare losses stems again from the 'envelope property' of the SDP model. Since the latter is solved around a steady-state with zero trend inflation in which all firms are at their optimal prices, the approximate dynamics around that steady state involve very small fluctuations in adjustment gains and hence in endogenous adjustment frequencies. The resulting similarity in equilibrium dynamics carries over to the comparison of the associated welfare losses.

The preceding analysis shows that, as long as one approximates equilibrium dynamics around the zero inflation steady state, the envelope property holds. Things are different however if one approximates around a steady state with nonzero inflation. To see this, we now consider a scenario in which the monetary authority follows a Taylor rule with a nonzero trend inflation target, $\bar{\pi} > 1$. Notice that, in a steady state with trend inflation, firms are no longer at their optimal prices. In fact all firm vintages are below their optimal relative prices at the start of each period; they are thus situated on the upward-sloping part of their value functions, rather than at the maximum where the slope is zero. As a result, aggregate shocks may cause non-negligible changes in adjustment gains, and hence in adjustment probabilities.

Table 2 reports the difference in mean welfare losses between the Calvo and SDP models in such a scenario, for different levels of trend inflation. As discussed in section 4.2, steady-state welfare losses are higher in the Calvo model in the presence of trend inflation. Therefore, in order to isolate the welfare losses due only to aggregate fluctuations, we subtract the gap in steady-state welfare loss between both models from the overall difference in mean welfare losses.

Table 2. Mean welfare losses under Taylor rule with trend inflation: difference between Calvo and SDP model

Trend inflation	productivity	government	cost-push	unconditional
0	0.0016	0.0000	0.0005	0.0021
2%	0.0070	0.0030	0.0040	0.0082
5%	0.0376	0.0138	0.0184	0.0429

Note: the table reports the difference in mean welfare losses (expressed as a percent of steady-state consumption) between the Calvo model and the SDP model, adjusted for the difference in steady-state welfare loss

As the table shows, welfare differences between both models increase with trend inflation. Key to this result are the dynamics of relative price distorsions in each model. As an illustration, figure 6 displays the impulse-responses of relative price dispersion to a cost-push shock for different levels of trend inflation. With zero trend inflation, price dispersion barely responds in both models.

With 2% trend inflation, the response is non-negligible and it is somewhat larger in the Calvo model. With 5% trend inflation, responses are larger and so is the difference between both models. Therefore, the higher the trend inflation rate, the larger the relative price distortions in the Calvo model relative to the SDP model.

The difference in price dispersion dynamics between both models is in turn strongly related to the endogenous response of price adjustment probabilities in the SDP model. Figure 7 shows the responses of the adjustment probabilities of several cohorts for different trend inflation rates.[21] With zero trend inflation, the response of the adjustment probabilities is indistinguishable from zero, i.e. the envelope property holds almost exactly. With 2% trend inflation, their response is already non-negligible, and at 5% trend inflation the responses are even larger. Faster price adjustment under SDP implies smaller increases in price dispersion relative to the Calvo model.

The preceding analysis shows that the envelope property becomes less and less important in the dynamics of the SDP model as trend inflation increases. The endogeneity of price adjustment frequencies thus becomes more and more relevant, with the resulting consequences for the welfare comparison between the SDP and Calvo frameworks.

4.5 Firm-level shocks

For reasons of tractability the above analysis abstracted from firm-level shocks to desired prices despite the strong evidence in favor of their existence (e.g. Klenow and Kryvtsov, 2008, Golosov and Lucas, 2007). In this section we extend the model to include idiosyncratic productivity shocks, and analyze steady-state welfare as well as impulse-responses.[22] In particular, we assume that firm-level productivity follows an AR(1) process in logs,

$$\log A_{it} = \rho \log A_{it-1} + \varepsilon_{it}^{a}, \tag{20}$$

where $0 \leq \rho < 1$ and $\epsilon_{it}^{a} \sim i.i.d.N(0, \sigma_a^2)$.

In order to make the model consistent with the observed size distribution of price changes, we assume a more general hazard function, based on Woodford (2008). Specifically, we postulate

$$\Gamma(\kappa) = \frac{\bar{\lambda}}{\bar{\lambda} + \left(1 - \bar{\lambda}\right) e^{-\xi(\alpha - \kappa)}}.$$

[21] In figure 7, all adjustment probabilities increase on impact, as one would expect. After that, the adjustment probability of the j-th cohort experiences a sudden decline exactly j periods after the shock, because it is only then that cohort-j firms start the period with a price chosen *after* the arrival of the shock.

[22] We solve the model with idiosyncratic shocks using Reiter's (2009) method of projection and perturbation. The solution is non-linear in the idiosyncratic states (price and productivity) and linear with respect to the aggregate shocks. The linearity in the aggregate dynamics implies that we are not able to compute average welfare losses with a degree of accuracy of second order as we did in the baseline model without idiosyncratic shocks.

We take the estimates of $(\bar{\lambda},\, \alpha,\, \xi)$ and $(\rho,\, \sigma_a)$ from Costain and Nakov (2011), who estimate the model to match the average frequency and the size distribution of price changes in the AC Nielsen data reported by Midrigan (2011). The estimated values are $(\bar{\lambda},\, \alpha,\, \xi) = (0.0945, 0.0611, 1.3335)$ and $(\rho,\, \sigma_a) = (0.8575, 0.0924)$.[23]

Figure 8 shows the steady-state welfare losses induced by trend inflation in the Calvo and SDP models with idiosyncratic shocks. In both cases, welfare losses are a convex function of steady-state inflation, reaching a minimum at zero inflation, equivalent to around 0.9% of steady-state consumption. In the case of zero inflation the losses are exclusively due to the idiosyncratic shocks and the failure of firms to adjust continuously in response to these shocks. Notice that, contrary to the baseline case without firm-level shocks, welfare losses are higher in the Calvo model *even* at zero inflation, reflecting the fact that adjustment probabilities do not respond to idiosyncratic shocks in that model. Once again, the welfare gap between both models increases as trend inflation moves away from zero. For instance, for an annual inflation of 5%, welfare losses amount to about 1.5% of steady-state consumption under SDP and as much as 2.2% under Calvo pricing.

In turn, figure 9 shows the responses under Calvo pricing and SDP to a 1% positive shock to productivity. In both cases, the increase in actual output falls short of the increase in potential output, resulting in a falling output gap. As a result inflation declines. The fall in inflation is more pronounced in the SDP model due to the selection effect emphasized by Golosov and Lucas (2007): firms that adjust tend to be those for whom adjustment is most valuable and these are the firms whose prices are furthest out of line with the optimal price. As a result of the productivity shock there is a shift of adjustment opportunities from firms that were contemplating a relatively large price increase to firms that are contemplating a relatively large price decrease, resulting in more flexibility of the aggregate price level under SDP compared to Calvo.

5 Conclusion

We have analyzed optimal monetary policy in a general equilibrium model with state-dependent pricing by firms. In this model, firms choose the timing of price changes and therefore the overall frequency of adjustment is determined endogenously. Hence, unlike the Calvo model in which the frequency is an exogenous parameter, our model is not subject to the Lucas (1976) critique.

As it turns out, however, under certain conditions often assumed in the literature, and provided that monetary policy is set optimally, the probability of adjustment remains constant even if pricing is state-dependent. Hence, the optimal long-run rate of inflation is zero, and the optimal dynamic policy is strict price stability. Just like in the simpler Calvo model, the central bank should not

[23] We calibrate and simulate the model on a monthly frequency to match the monthly data on price changes used by Costain and Nakov (2011).

use inflation to offset the static distortion arising from monopolistic competition. These results lend support to more informal statements about the suitability of the Calvo model for studying optimal monetary policy despite its apparent conflict with the Lucas (1976) critique.

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Appendix

A. First-order conditions of Ramsey problem

The central bank maximizes the Lagrangian given by expression (14) in the main text. The first-order conditions are as follows (all expressions are equal to zero):

$$u'(C_t) + \sum_{j=0}^{J-1} \phi_{t-j}^{p^*} \left[\frac{p_{t-j}^*}{\pi_{jt}^{acc}} \left[u'(C_t) + Y_t u''(C_t) \right] - \frac{\epsilon}{\epsilon - 1} \frac{x'(N_t; \chi_t)}{z_t / u_t} \right] \theta_{jt} (\pi_{jt}^{acc})^{\epsilon} - \phi_t^N \frac{\Delta_t}{z_t}$$
$$+ \sum_{j=0}^{J-1} \phi_t^{v_j} \left[\frac{p_{t-j}^*}{\pi_{jt}^{acc}} \left[u'(C_t) + u''(C_t) Y_t \right] - \frac{x'(N_t; \chi_t)}{z_t / u_t} \right] \left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}} \right)^{-\epsilon}, \qquad (Y_t)$$

$$\phi_t^{p^*} E_t \sum_{j=0}^{J-1} \beta^j \theta_{j,t+j} (\pi_{j,t+j}^{acc})^{\epsilon-1} Y_{t+j} u'(C_{t+j}) - \left[\phi_t^{\pi} (\epsilon - 1) p_t^* + \phi_t^{\Delta} \epsilon \right] (p_t^*)^{-\epsilon-1} \sum_{j=1}^{J} \lambda_{jt} \psi_{jt}$$
$$- E_t \sum_{j=1}^{J-1} \beta^j \left[\phi_{t+j}^{\pi} (\epsilon - 1) \frac{p_t^*}{\pi_{j,t+j}^{acc}} + \phi_{t+j}^{\Delta} \epsilon \right] (p_t^*)^{-\epsilon-1} (\pi_{j,t+j}^{acc})^{\epsilon} (1 - \lambda_{j,t+j}) \psi_{j,t+j}$$
$$+ E_t \sum_{j=0}^{J-1} \beta^j \phi_{t+j}^{v_j} \left[\epsilon \frac{x'(N_{t+j}; \chi_{t+j})}{z_{t+j} u'(C_{t+j}) / u_{t+j}} - (\epsilon - 1) \frac{p_t^*}{\pi_{j,t+j}^{acc}} \right] (p_t^*)^{-\epsilon-1} (\pi_{j,t+j}^{acc})^{\epsilon} Y_{t+j} u'(C_{t+j}), \quad (p_t^*)$$

$$\phi_{t-j}^{p^*} \left[\frac{p_{t-j}^*}{\pi_{jt}^{acc}} (\epsilon - 1) - \frac{\epsilon}{\epsilon - 1} \epsilon \frac{x'(N_t; \chi_t)}{z_t u'(C_t) / u_t} \right] \theta_{jt} (\pi_{jt}^{acc})^{\epsilon-1} Y_t u'(C_t)$$
$$+ \left[\phi_t^{\pi} \frac{p_{t-j}^*}{\pi_{jt}^{acc}} (\epsilon - 1) + \phi_t^{\Delta} \epsilon \right] (p_{t-j}^*)^{-\epsilon} (\pi_{jt}^{acc})^{\epsilon-1} (1 - \lambda_{jt}) \psi_{jt}$$
$$+ \phi_t^{v_j} \left[\frac{p_{t-j}^*}{\pi_{jt}^{acc}} (\epsilon - 1) - \frac{x'(N_t; \chi_t)}{z_t u'(C_t) / u_t} \epsilon \right] (p_{t-j}^*)^{-\epsilon} (\pi_{jt}^{acc})^{\epsilon-1} Y_t u'(C_t)$$
$$+ \phi_t^{\pi_j^{acc}} - \beta E_t \phi_{t+1}^{\pi_{j+1}^{acc}} \pi_t, \qquad (\pi_{j=1,\ldots,J-2,\,t}^{acc})$$

$$\phi_{t-(J-1)}^{p^*} \left[\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}} (\epsilon - 1) - \frac{\epsilon}{\epsilon - 1} \epsilon \frac{x'(N_t; \chi_t)}{z_t u'(C_t) / u_t} \right] \theta_{J-1,t} (\pi_{J-1,t}^{acc})^{\epsilon-1} Y_t u'(C_t)$$
$$+ \left[\phi_t^{\pi} \frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}} (\epsilon - 1) + \phi_t^{\Delta} \epsilon \right] (p_{t-(J-1)}^*)^{-\epsilon} (\pi_{J-1,t}^{acc})^{\epsilon-1} (1 - \lambda_{J-1,t}) \psi_{J-1,t}$$
$$+ \phi_t^{v_j} \left[\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}} (\epsilon - 1) - \frac{x'(N_t; \chi_t)}{z_t u'(C_t) / u_t} \epsilon \right] (p_{t-(J-1)}^*)^{-\epsilon} \left(\pi_{J-1,t}^{acc} \right)^{\epsilon-1} Y_t u'(C_t) + \phi_t^{\pi_{J-1}^{acc}} (\pi_{J-1,\,t}^{acc})$$

$$\phi_{t-j}^{p^*} \left[\frac{p_{t-j}^*}{\pi_{jt}^{acc}} - \frac{\epsilon}{\epsilon - 1} \frac{x'(N_t; \chi_t)}{z_t u'(C_t) / u_t} \right] (\pi_{jt}^{acc})^{\epsilon} Y_t u'(C_t) + \phi_t^{\theta_j} - \beta E_t \phi_{t+1}^{\theta_{j+1}} (1 - \lambda_{j+1,t+1}), \quad (\theta_{j=1,\ldots,J-2,\,t})$$

$$\phi_{t-(J-1)}^{p^*}\left[\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}}-\frac{\epsilon}{\epsilon-1}\frac{x'\left(N_t;\chi_t\right)}{z_t u'\left(C_t\right)/u_t}\right](\pi_{J-1,t}^{acc})^{\epsilon}Y_t u'\left(C_t\right)+\phi_t^{\theta_{J-1}}, \qquad (\theta_{J-1,\,t})$$

$$\begin{aligned}&\phi_t^{\pi}\left[\left(p_t^*\right)^{1-\epsilon}-\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{1-\epsilon}\right]\psi_{jt}+\phi_t^{\Delta}\left[\left(p_t^*\right)^{-\epsilon}-\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{-\epsilon}\right]\psi_{jt}\\&+\phi_t^{\lambda_j}+\beta E_t\phi_{t+1}^{\psi_{j+1}}\psi_{jt}+\phi_{t-1}^{v_{j-1}}\left(\tilde{v}_{0,t}-\tilde{v}_{jt}\right)+\phi_t^{\theta_j}\theta_{j-1,t-1}, \qquad (\lambda_{j=1,\ldots,J-1,\,t})\end{aligned}$$

$$\begin{aligned}&-\phi_t^N\textstyle\int_0^{(\tilde{v}_{0t}-\tilde{v}_{1t})/x'(N_t;\chi_t)u_t}\kappa g\left(\kappa\right)d\kappa+\phi_t^{\pi}\left[\left(p_t^*\right)^{1-\epsilon}\lambda_{1t}+\left(\frac{p_{t-1}^*}{\pi_{1t}^{acc}}\right)^{1-\epsilon}\left(1-\lambda_{1t}\right)\right]\\&+\phi_t^{\Delta}\left[\left(p_t^*\right)^{-\epsilon}\lambda_{1t}+\left(\frac{p_{t-1}^*}{\pi_{1t}^{acc}}\right)^{-\epsilon}\left(1-\lambda_{1t}\right)\right]-\beta E_t\phi_{t+1}^{\psi_2}\left(1-\lambda_{1t}\right)+\phi_t^{\psi_1}, \qquad (\psi_{1,t})\end{aligned}$$

$$\begin{aligned}&-\phi_t^N\textstyle\int_0^{(\tilde{v}_{0t}-\tilde{v}_{jt})/x'(N_t;\chi_t)u_t}\kappa g\left(\kappa\right)d\kappa+\phi_t^{\pi}\left[\left(p_t^*\right)^{1-\epsilon}\lambda_{jt}+\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{1-\epsilon}\left(1-\lambda_{jt}\right)\right]\\&+\phi_t^{\Delta}\left[\left(p_t^*\right)^{-\epsilon}\lambda_{jt}+\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{-\epsilon}\left(1-\lambda_{jt}\right)\right] \qquad (\psi_{j=2,\ldots,J-1,\,t})\\&+\phi_t^{\psi_j}-\beta E_t\phi_{t+1}^{\psi_{j+1}}\left(1-\lambda_{jt}\right)+\phi_t^{\psi_1},\end{aligned}$$

$$\phi_t^{\pi}\left(p_t^*\right)^{1-\epsilon}+\phi_t^{\Delta}\left(p_t^*\right)^{-\epsilon}+\phi_t^{\psi_J}+\phi_t^{\psi_1}, \qquad (\psi_{J,t})$$

$$-\phi_t^N\sum_{j=1}^{J-1}\frac{\psi_{jt}/u_t}{x'\left(N_t;\chi_t\right)}L_{jt}g\left(L_{jt}\right)-\sum_{j=1}^{J-1}\phi_t^{\lambda_j}g\left(L_{jt}\right)\frac{1/u_t}{x'\left(N_t;\chi_t\right)}-\phi_t^{v_0}+\sum_{j=0}^{J-2}\phi_{t-1}^{v_j}\left[\lambda_{j+1,t}-L_{j+1,t}g\left(L_{j+1,t}\right)\right]+\phi_{t-1}^{v_{J-1}}, \qquad (\tilde{v}_{0t})$$

$$\phi_t^N\psi_{jt}\frac{1/u_t}{x'\left(N_t;\chi_t\right)}L_{jt}g\left(L_{jt}\right)+\phi_t^{\lambda_j}g\left(L_{jt}\right)\frac{1/u_t}{x'\left(N_t;\chi_t\right)}-\phi_t^{v_j}+\phi_{t-1}^{v_{j-1}}\left[1-\lambda_{jt}+L_{jt}g\left(L_{jt}\right)\right], \quad (\tilde{v}_{j=1,\ldots,J-1,\,t})$$

$$\begin{aligned}&-x'\left(N_t;\chi_t\right)-\sum_{j=0}^{J-1}\phi_{t-j}^{p^*}\frac{\epsilon}{\epsilon-1}\theta_{jt}(\pi_{jt}^{acc})^{\epsilon}Y_t\frac{x''\left(N_t;\chi_t\right)}{z_t/u_t}+\phi_t^N\left[1+\sum_{j=1}^{J-1}\psi_{jt}\frac{x''\left(N_t;\chi_t\right)}{x'\left(N_t;\chi_t\right)}\left(L_{jt}\right)^2g\left(L_{jt}\right)\right]\\&+\sum_{j=1}^{J-1}\phi_t^{\lambda_j}g\left(L_{jt}\right)L_{jt}\frac{x''\left(N_t;\chi_t\right)}{x'\left(N_t;\chi_t\right)}-\sum_{j=0}^{J-1}\phi_t^{v_j}\frac{x''\left(N_t;\chi_t\right)}{z_t/u_t}\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{-\epsilon}Y_t\\&+\sum_{j=0}^{J-2}\phi_{t-1}^{v_j}x''\left(N_t;\chi_t\right)u_t\left[\left(L_{j+1,t}\right)^2g\left(L_{j+1,t}\right)-\textstyle\int_0^{L_{j+1,t}}\kappa g\left(\kappa\right)d\kappa\right], \qquad (N_t)\end{aligned}$$

$$-\phi_t^N\frac{Y_t}{z_t}-\phi_t^{\Delta}, \qquad (\Delta_t)$$

$$-\phi_t^{\pi_1^{acc}} - \sum_{j=2}^{J-1} \phi_t^{\pi_j^{acc}} \pi_{j-1,t-1}^{acc}, \qquad (\pi_t)$$

where we have defined the adjustment gain $L_{jt} \equiv (v_{0t} - v_{jt}) / w_t = (\tilde{v}_{0t} - \tilde{v}_{jt}) / [x'(N_t; \chi_t) u_t]$ for compactness.

B. Optimality of strict inflation targeting

Assume that there are no cost-push shocks, such that $u_t = 1$ at all times. We now conjecture that the timeless perspective optimal policy involves zero net inflation at all times, $\pi_t = 1$. Under such a policy, in the timeless perspective regime (that is, after all transitional dynamics have disappeared) the economy converges to the following equilibrium:

$$\pi_t = p_t^* = \Delta_t = \frac{\epsilon}{\epsilon - 1} \frac{x'(N_t; \chi_t)}{z_t u'(C_t)} = 1 = \pi_{jt}^{acc}, \quad j = 1, ..., J-1$$

$$v_{0t} = v_{jt} \Rightarrow L_{jt} = 0, \quad j = 1, ..., J-1$$

$$\lambda_{jt} = \Gamma(0) \equiv \bar{\lambda} > 0 \Rightarrow \theta_{jt} = \left(1 - \bar{\lambda}\right)^j, \quad j = 1, ..., J-1$$

$$\psi_{jt} = \frac{\left(1 - \bar{\lambda}\right)^{j-1}}{\sum_{k=0}^{J-1} \left(1 - \bar{\lambda}\right)^k} \equiv \bar{\psi}_j,$$

$$N_t = \frac{Y_t}{z_t} = \frac{C_t + G_t}{z_t}$$

for all t. Thus, all firms end up having the same relative prices. Price dispersion is eliminated; the average price markup is constant at the level $\epsilon/(\epsilon - 1)$, such that output, employment and consumption equal their flexible-price levels of section 2.5.1 at all times; adjustment gains are zero and the vintage distribution converges to a stationary distribution. Imposing our conjecture in the first-order conditions above, we obtain

$$0 = 1 + \frac{Y_t u''(C_t)}{u'(C_t)} \sum_{j=0}^{J-1} \phi_{t-j}^{p^*} \left(1 - \bar{\lambda}\right)^j - \frac{\phi_t^N}{z_t u'(C_t)} + \sum_{j=0}^{J-1} \phi_t^{v_j} \left[1 + \frac{u''(C_t) Y_t}{u'(C_t)} - \frac{\epsilon - 1}{\epsilon}\right], \qquad (21)$$

$$\begin{aligned} 0 \;=\; & \phi_t^{p^*} E_t \sum_{j=0}^{J-1} \beta^j \left(1 - \bar{\lambda}\right)^j Y_{t+j} u'(C_{t+j}) - \left[\phi_t^{\pi} (\epsilon - 1) + \phi_t^{\Delta} \epsilon\right] \left(\bar{\lambda} \sum_{j=1}^{J-1} \bar{\psi}_j + \bar{\psi}_J\right) \\ & - E_t \sum_{j=1}^{J-1} \beta^j \left[\phi_{t+j}^{\pi} (\epsilon - 1) + \phi_{t+j}^{\Delta} \epsilon\right] \left(1 - \bar{\lambda}\right) \bar{\psi}_j, \end{aligned} \qquad (22)$$

$$0 = -\phi_{t-j}^{p^*} \left(1 - \bar{\lambda}\right)^j Y_t u'(C_t) + \left[\phi_t^{\pi} (\epsilon - 1) + \phi_t^{\Delta} \epsilon\right] \left(1 - \bar{\lambda}\right) \bar{\psi}_j + \phi_t^{\pi_j^{acc}} - \beta E_t \phi_{t+1}^{\pi_{j+1}^{acc}}, \qquad (23)$$

$$0 = -\phi_{t-(J-1)}^{p^*} \left(1 - \bar{\lambda}\right)^{J-1} Y_t u'(C_t) + \left[\phi_t^{\pi} (\epsilon - 1) + \phi_t^{\Delta} \epsilon\right] \left(1 - \bar{\lambda}\right) \bar{\psi}_{J-1} + \phi_t^{\pi_{J-1}^{acc}}, \qquad (24)$$

$$0 = \phi_t^{\theta_j} - \beta E_t \phi_{t+1}^{\theta_{j+1}} \left(1 - \bar{\lambda}\right), \quad j = 1, ..., J-2, \tag{25}$$

$$0 = \phi_t^{\theta_{J-1}}, \tag{26}$$

$$0 = \phi_t^{\lambda_j} + \beta E_t \phi_{t+1}^{\psi_{j+1}} \bar{\psi}_j + \phi_t^{\theta_j} \left(1 - \bar{\lambda}\right)^{j-1}, \quad j = 1, ..., J-1, \tag{27}$$

$$0 = \phi_t^{\pi} + \phi_t^{\Delta} - \beta \left(1 - \bar{\lambda}\right) E_t \phi_{t+1}^{\psi_2} + \phi_t^{\psi_1}, \tag{28}$$

$$0 = \phi_t^{\pi} + \phi_t^{\Delta} + \phi_t^{\psi_j} - \beta \left(1 - \bar{\lambda}\right) E_t \phi_{t+1}^{\psi_{j+1}} + \phi_t^{\psi_1}, \quad j = 2, ..., J-1, \tag{29}$$

$$0 = \phi_t^{\pi} + \phi_t^{\Delta} + \phi_t^{\psi_J} + \phi_t^{\psi_1}, \tag{30}$$

$$0 = -\sum_{j=1}^{J-1} \phi_t^{\lambda_j} \frac{g(0)}{x'(N_t; \chi_t)} - \phi_t^{v_0} + \bar{\lambda} \sum_{j=0}^{J-2} \phi_{t-1}^{v_j} + \phi_{t-1}^{v_{J-1}}, \tag{31}$$

$$0 = \phi_t^{\lambda_j} \frac{g(0)}{x'(N_t; \chi_t)} - \phi_t^{v_j} + \left(1 - \bar{\lambda}\right) \phi_{t-1}^{v_{j-1}}, \tag{32}$$

$$0 = -1 - \sum_{j=0}^{J-1} \phi_{t-j}^{p^*} \frac{\epsilon}{\epsilon - 1} \left(1 - \bar{\lambda}\right)^j \frac{N_t x''(N_t; \chi_t)}{x'(N_t; \chi_t)} + \frac{\phi_t^N}{x'(N_t; \chi_t)} - \sum_{j=0}^{J-1} \phi_t^{v_j} \frac{x''(N_t; \chi_t) N_t}{x'(N_t; \chi_t)}, \tag{33}$$

$$0 = -\phi_t^N \frac{Y_t}{z_t} - \phi_t^{\Delta}, \tag{34}$$

$$0 = -\phi_t^{\pi_1^{acc}} - \textstyle\sum_{j=2}^{J-1} \phi_t^{\pi_j^{acc}}. \tag{35}$$

We now use equations (21) to (34) to solve for the Lagrange multipliers. From (26) and (25), it follows immediately that

$$\phi_t^{\theta_j} = 0, \quad j = 1, ..., J-1. \tag{36}$$

Equations (28) to (30) allow us to solve for the $\phi_t^{\psi_j}$ multipliers, obtaining

$$\phi_t^{\psi_1} = -\left(\phi_t^{\pi} + \phi_t^{\Delta}\right),$$

$$\phi_t^{\psi_j} = 0, \quad j = 2, ..., J. \tag{37}$$

Using (36) and (37) in equations (27), we obtain

$$\phi_t^{\lambda_j} = 0, \quad j = 1, ..., J-1. \tag{38}$$

Using the latter, equations (31) and (32) can be expressed compactly as $\boldsymbol{\phi}_t^v = \mathbf{A}\boldsymbol{\phi}_{t-1}^v$, where $\boldsymbol{\phi}_t^v = [\phi_t^{v_0}, \phi_t^{v_1}, ..., \phi_t^{v_{J-1}}]'$ and

$$\underset{J\times J}{\mathbf{A}} = \begin{bmatrix} \bar{\lambda} & \bar{\lambda} & \bar{\lambda} & ... & \bar{\lambda} & \bar{\lambda} & 1 \\ 1-\bar{\lambda} & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 1-\bar{\lambda} & 0 & ... & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1-\bar{\lambda} & 0 & 0 \\ 0 & 0 & 0 & .. & 0 & 1-\bar{\lambda} & 0 \end{bmatrix}.$$

The matrix $\mathbf{A}$ has $J-1$ eigenvalues with modulus equal to $1-\bar{\lambda} < 1$ and one unit eigenvalue.[24] The system is thus stable, and the elements in $\boldsymbol{\phi}_t^v$ converge to finite values that depend on initial conditions. Therefore, in the timeless perspective regime, in which all transitional dynamics have disappeared, the multipliers $\phi_t^{v_j}$ converge to constant values $\bar{\phi}^{v_j}$, $j = 0, ..., J-1$. We then use (33) to solve for ϕ_t^N, obtaining

$$\phi_t^N = x'(N_t;\chi_t)\left[1 + \frac{\epsilon}{\epsilon-1}\frac{N_t x''(N_t;\chi_t)}{x'(N_t;\chi_t)}\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j \phi_{t-j}^{p^*} + \frac{x''(N_t;\chi_t)N_t}{x'(N_t;\chi_t)}\sum_{j=0}^{J-1}\bar{\phi}^{v_j}\right].$$

Using the latter in (21), we obtain

$$\left[\frac{Y_t u''(C_t)}{(-)u'(C_t)} + \frac{N_t x''(N_t;\chi_t)}{x'(N_t;\chi_t)}\right]\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j \phi_{t-j}^{p^*} = \frac{1}{\epsilon} + \left[\frac{1}{\epsilon} - \frac{u''(C_t)Y_t}{(-)u'(C_t)} - \frac{\epsilon-1}{\epsilon}\frac{x''(N_t;\chi_t)N_t}{x'(N_t;\chi_t)}\right]\sum_{j=0}^{J-1}\bar{\phi}^{v_j},$$

where we have used the fact that, under our conjecture, $x'(N_t;\chi_t)/[z_t u'(C_t)] = (\epsilon-1)/\epsilon$. At this point, we assume away government spending, $G_t = 0$, such that $Y_t = C_t$. We also assume that functional forms for preferences are of the constant elasticity type. Let $\sigma \equiv (-)C_t u''(C_t)/u'(C_t) > 0$ and $\varphi \equiv N_t x''(N_t;\chi_t)/x'(N_t;\chi_t) > 0$ denote the constant elasticities of marginal consumption utility and marginal labor disutility, respectively. Then we have

$$\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j \phi_{t-j}^{p^*} = \frac{1/\epsilon}{\sigma+\varphi} + \frac{1/\epsilon - \sigma - \varphi(\epsilon-1)/\epsilon}{\sigma+\varphi}\sum_{j=0}^{J-1}\bar{\phi}^{v_j} \equiv \Xi.$$

It can be shown that all $J-1$ roots of the characteristic polynomial $\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j x^{J-1-j}$ have modulus equal to $1-\bar{\lambda} < 1$, hence they all lie inside the unit circle. Therefore, in the timeless perspective regime, the multiplier $\phi_t^{p^*}$ converges to the constant value $\bar{\phi}^{p^*} \equiv \Xi/\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j$.

[24] Every column of A sums to unity, which implies that unity is an eigenvalue of A (Hamilton, 1994, p. 681), but A is also a Leslie matrix, hence it has only one positive and dominant eigenvalue (Poole, 2006, p. 328). Hence, all other eigenvalues must lie inside the unit circle.

Using this in equation (22), together with $\bar{\lambda}\sum_{j=1}^{J-1}\bar{\psi}_j+\bar{\psi}_J=\bar{\psi}_1$ and $\bar{\psi}_j=\left(1-\bar{\lambda}\right)^{j-1}\bar{\psi}_1$, the latter equation can be expressed as

$$\begin{aligned} 0 &= E_t\sum_{j=0}^{J-1}\beta^j\left(1-\bar{\lambda}\right)^j\left\{\bar{\phi}^{p^*}Y_{t+j}u'\left(C_{t+j}\right)-\left[\phi_{t+j}^{\pi}\left(\epsilon-1\right)+\phi_{t+j}^{\Delta}\epsilon\right]\bar{\psi}_1\right\} \\ &= E_t\sum_{j=0}^{J-1}\beta^j\left(1-\bar{\lambda}\right)^j\Sigma_{t+j}, \end{aligned} \tag{39}$$

where we have defined $\Sigma_t\equiv\bar{\phi}^{p^*}Y_tu'\left(C_t\right)-\left[\phi_t^{\pi}\left(\epsilon-1\right)+\phi_t^{\Delta}\epsilon\right]\bar{\psi}_1$. All $J-1$ roots of the polynomial $\sum_{j=0}^{J-1}\beta^j\left(1-\bar{\lambda}\right)^j x^{J-1-j}$ have modulus equal to $\beta\left(1-\bar{\lambda}\right)<1$ and are thus inside the unit circle. Therefore, equation (39) has a unique solution given by $\Sigma_t=0$, or equivalently

$$\bar{\phi}^{p^*}Y_tu'\left(C_t\right)-\left[\phi_t^{\pi}\left(\epsilon-1\right)+\phi_t^{\Delta}\epsilon\right]\bar{\psi}_1=0, \tag{40}$$

which pins down the multiplier ϕ_t^{π} as a function of the variables $Y_tu'\left(C_t\right)$ and ϕ_t^{Δ}. The latter multiplier is in turn determined by equation (34).

Equation (24) can be solved for $\phi_t^{\pi_{J-1}^{acc}}$, obtaining

$$\phi_t^{\pi_{J-1}^{acc}}=\left(1-\bar{\lambda}\right)^{J-1}\left\{\bar{\phi}^{p^*}Y_tu'\left(C_t\right)-\left[\phi_t^{\pi}\left(\epsilon-1\right)+\phi_t^{\Delta}\epsilon\right]\bar{\psi}_1\right\}=0,$$

where we have used $\bar{\psi}_{J-1}=\left(1-\bar{\lambda}\right)^{J-2}\bar{\psi}_1$ and where the second equality follows from (40). Using $E_t\phi_{t+1}^{\pi_{J-1}^{acc}}=0$ and $\bar{\psi}_{J-2}=\left(1-\bar{\lambda}\right)^{J-3}\bar{\psi}_1$ in equation (23) for $j=J-2$, the latter implies $\phi_t^{\pi_{J-2}^{acc}}=0$. Operating in the same fashion, equations (23) for $j=1,...,J-3$ imply that $\phi_t^{\pi_j^{acc}}=0$ for $j=1,...,J-3$.

It only remains to verify that equation (35) holds given the solution of the Lagrange multipliers. This is obvious, as we have already shown that $\phi_t^{\pi_j^{acc}}=0$ for $j=1,...,J-1$.

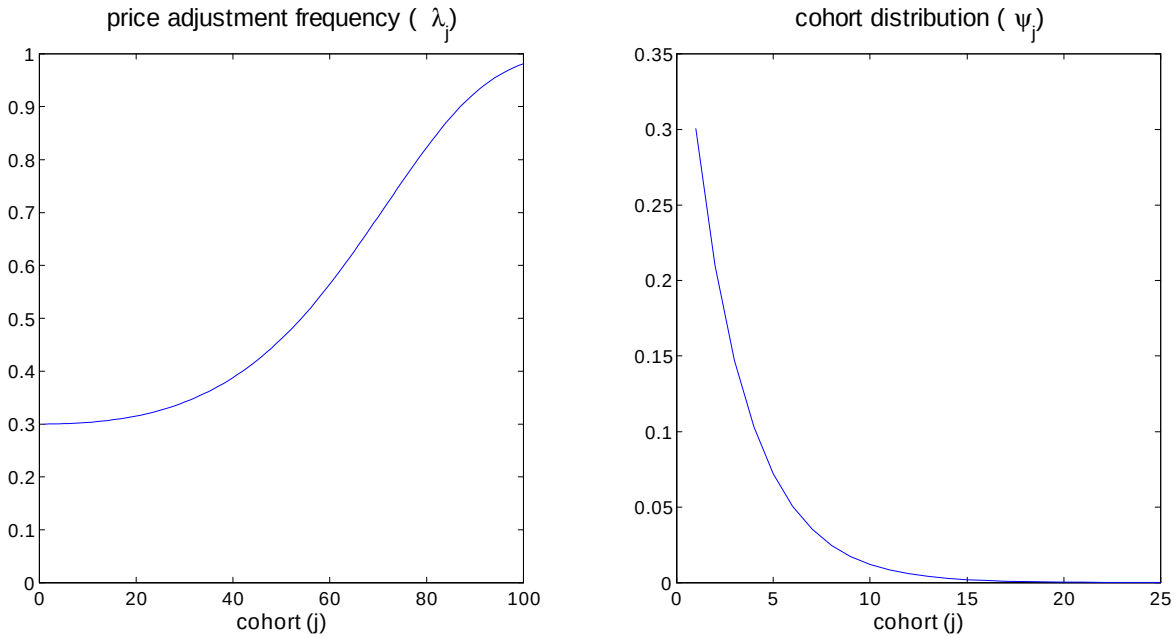

Figure 1: Price adjustment hazard and cohort density at 2% inflation

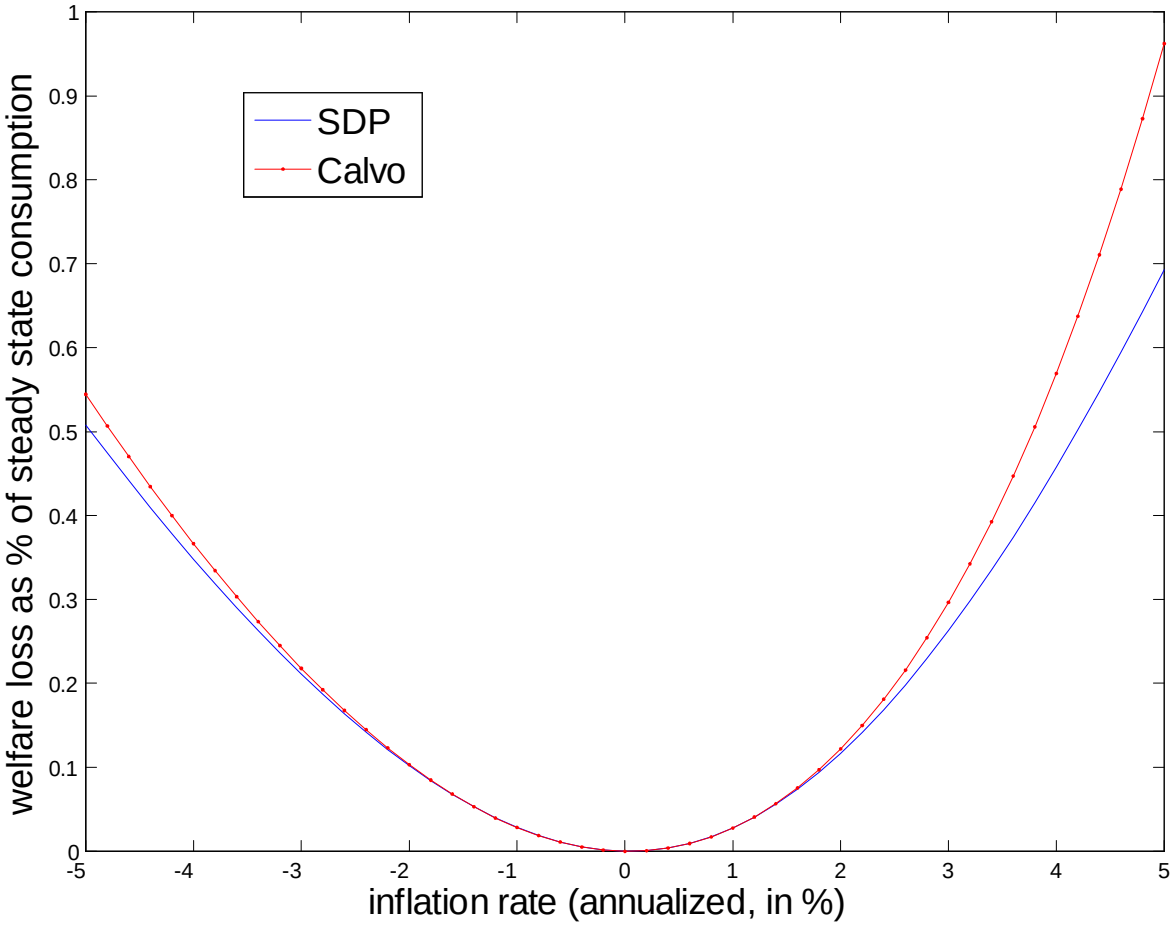

Figure 2: Welfare losses from trend inflation in Calvo and SDP models

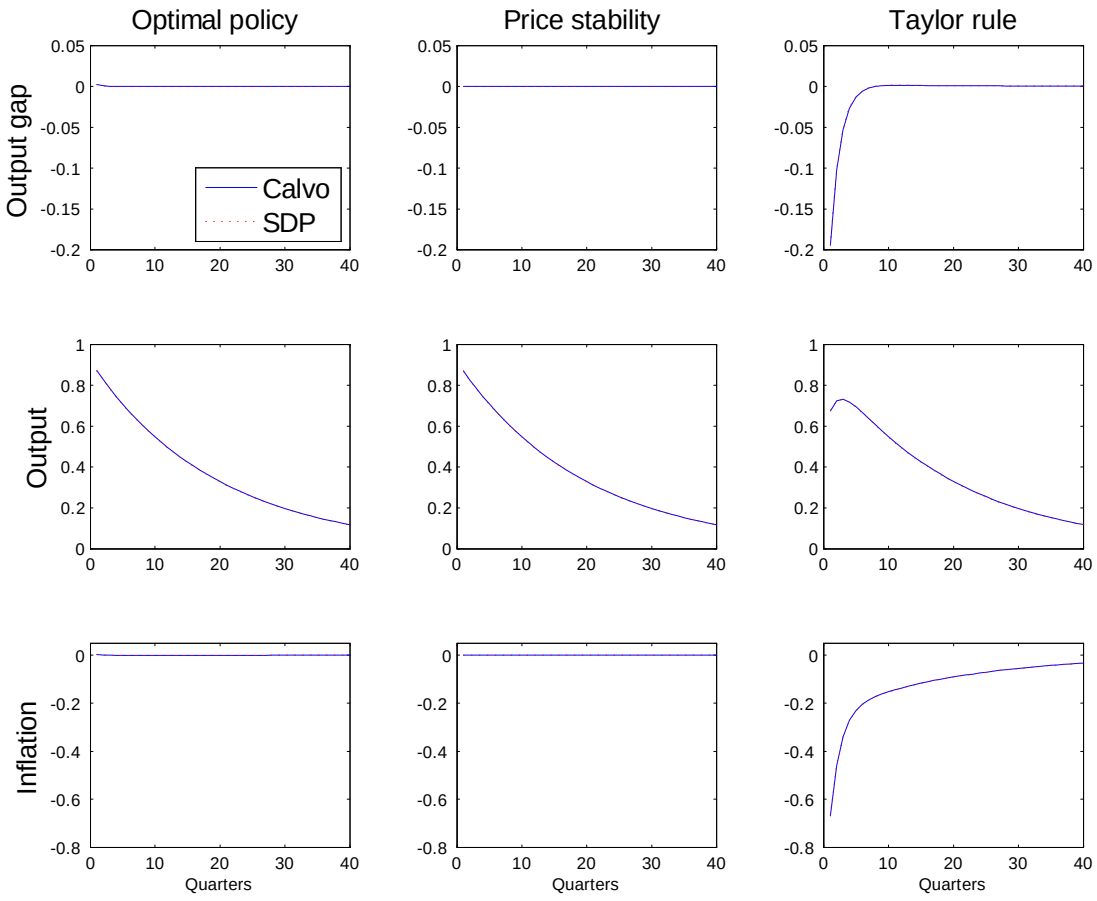

Figure 3: Responses to productivity shock under three monetary policy regimes

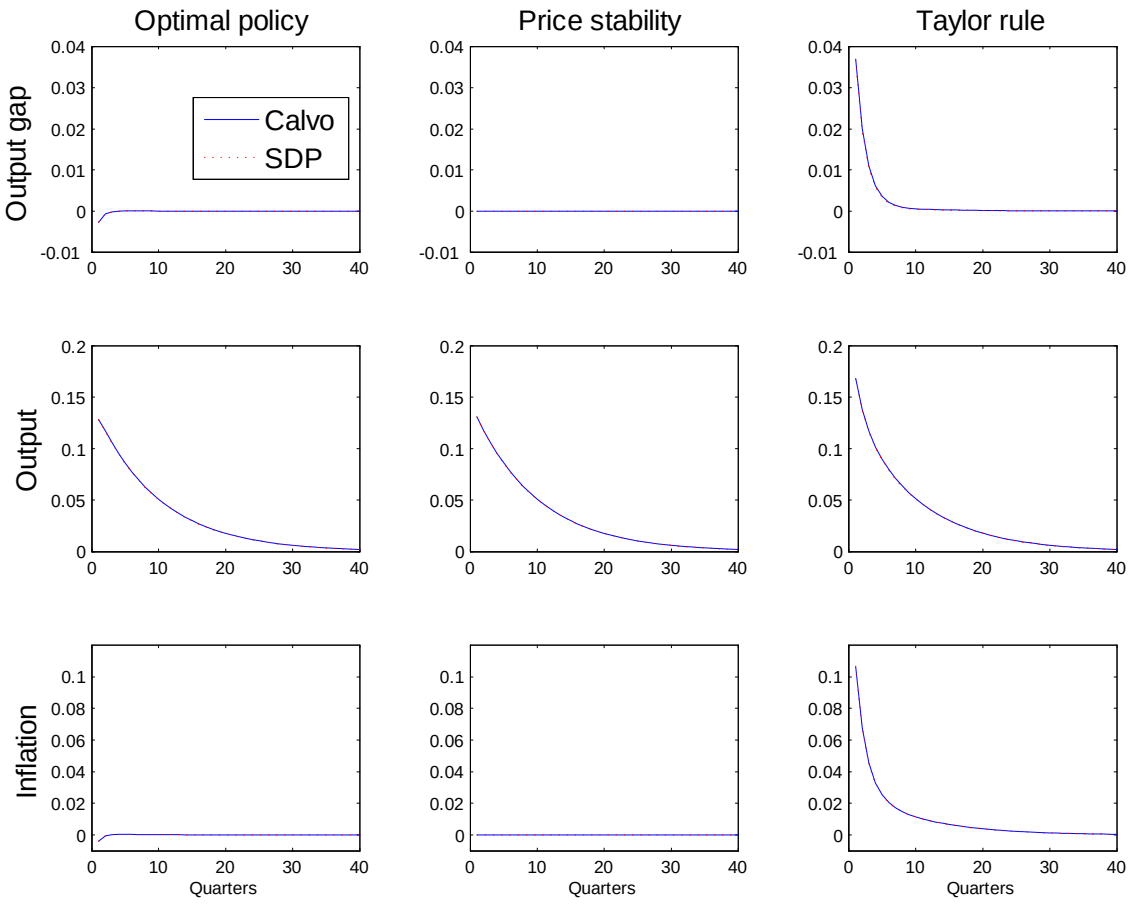

Figure 4: Responses to government spending shock under three monetary policy regimes

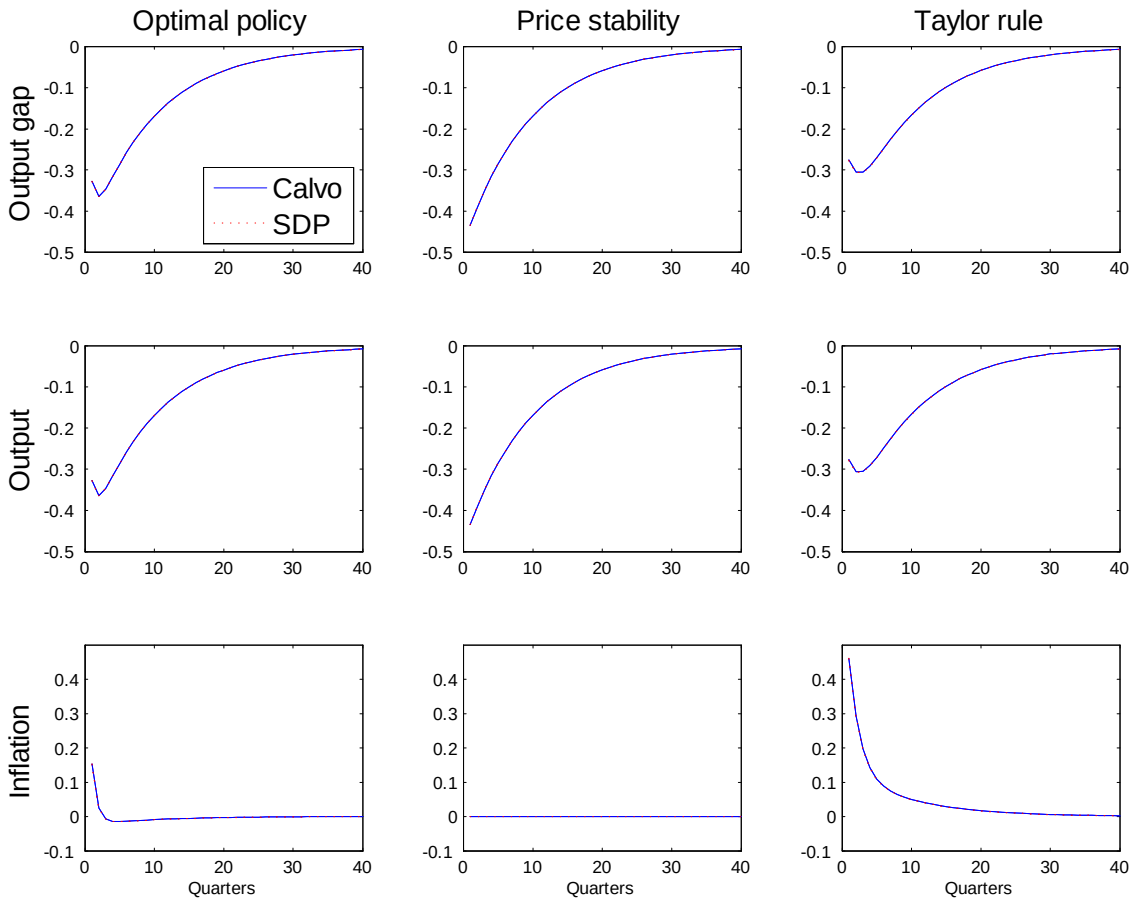

Figure 5: Responses to cost-push shock under three monetary policy regimes

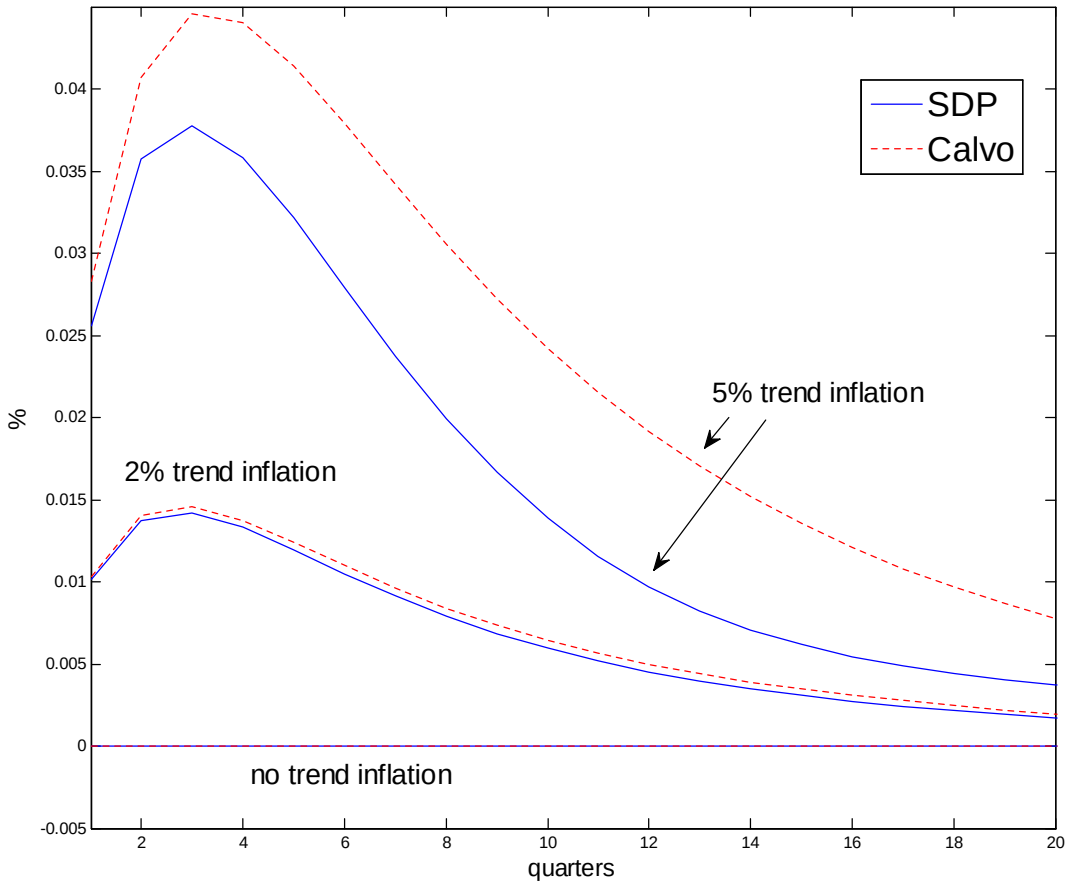

Figure 6: Response of relative price dispersion (Δ_t) to a cost-push shock under Taylor rule with trend inflation

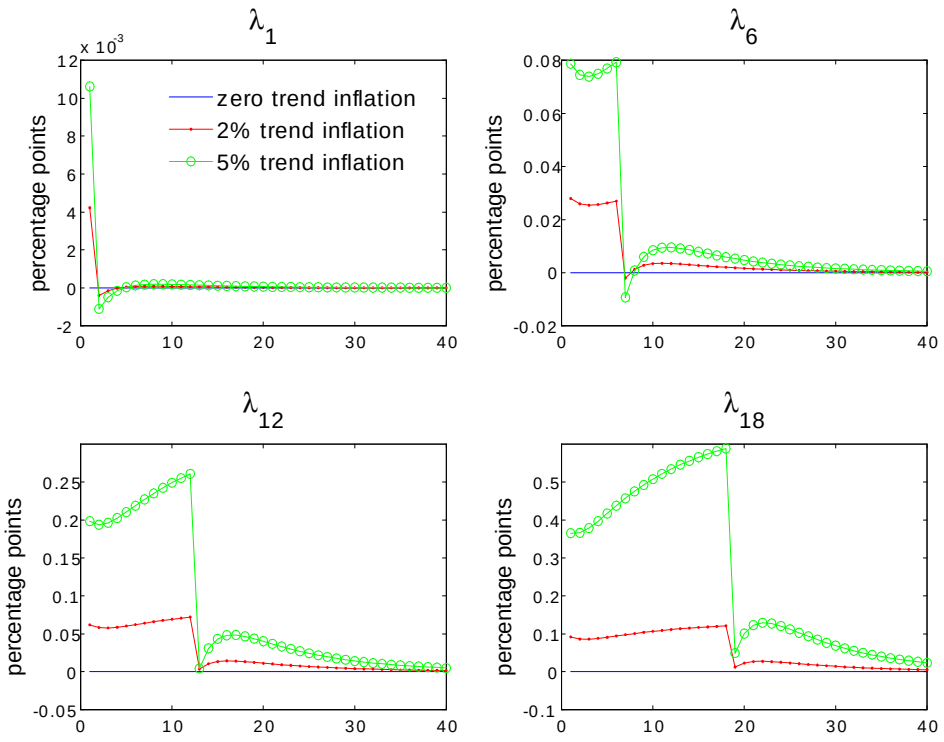

Figure 7: Responses of adjustment probabilities to a cost-push shock under Taylor rules with trend inflation

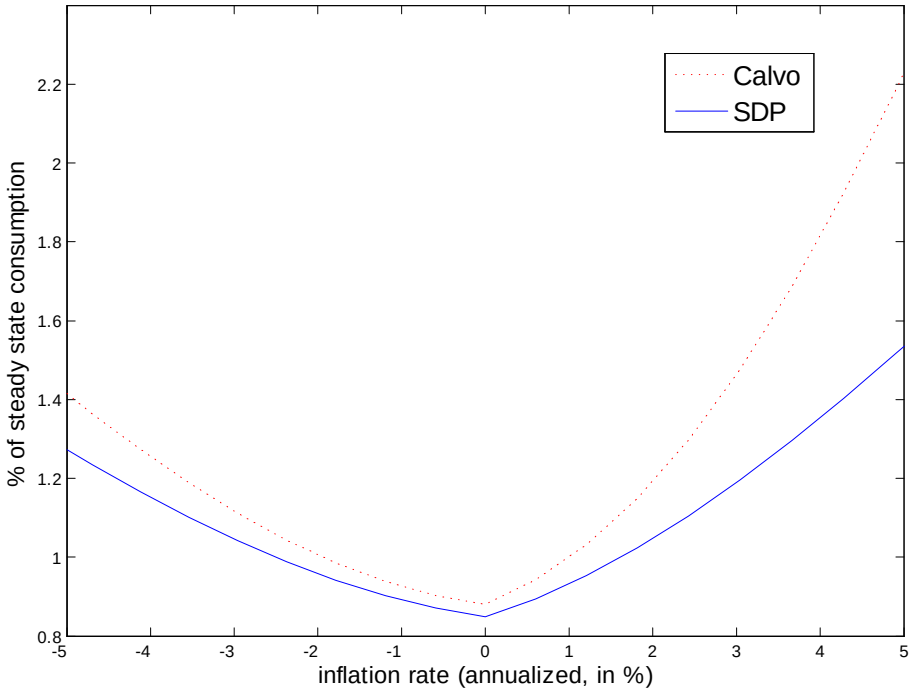

Figure 8: Welfare losses from trend inflation in models with idiosyncratic technology shocks

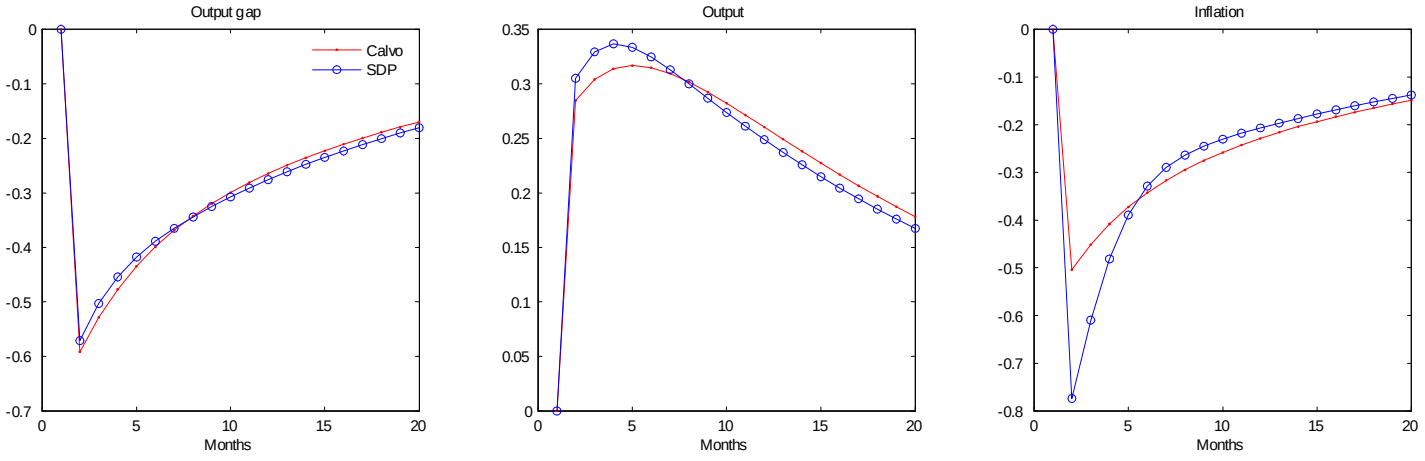

Figure 9: Responses to an aggregate TFP shock in Calvo and SDP models with idiosyncratic shocks