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ABSTRACT

Environmental Policy and Directed Technical Change in a Global Economy: The Dynamic Impact of Unilateral Environmental Policies*

This paper builds a two-country (North, South), two-sector (polluting, nonpolluting) trade model with directed technical change, examining whether unilateral environmental policies can ensure sustainable growth. The polluting good is produced with a clean and a dirty input. I show that a temporary Northern policy combining clean research subsidies and a trade tax can ensure sustainable growth but Northern carbon taxes alone cannot. Trade and directed technical change accelerate environmental degradation either under laissez-faire or if the North implements carbon taxes, yet both help reduce environmental degradation under the appropriate unilateral policy. I characterize the optimal unilateral policy analytically and numerically using calibrated simulations.

JEL Classification: F18, F42, F43, O32, O33, O41, Q54 and Q55
Keywords: climate change, directed technical change, environment, innovation, trade and unilateral policy

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1 Introduction

Countries not subject to any binding constraints under the Kyoto protocol account for an increasing fraction of carbon dioxide (CO_2) emissions: their share in world emissions has risen from 31% in 1990 to 52% in 2010. Meanwhile, climate negotiations have stalled and no global agreement is in sight. In response, several countries either have undertaken unilateral actions or are considering doing so, and these policies increasingly harbor protectionist aspects. For instance, the American Clean Energy and Security Act—which was supposed to set up a cap-and-trade system in the United States—planned to implement trade barriers with countries that did not have a similar system, absent an international agreement, by 2018.[1] These responses raise two questions. First, can unilateral policies ensure sustainable growth? Second, are the calls for protectionism justified?

These questions are fundamentally about the economy's long-run behavior. Over the time period relevant to climate change, comparative advantages evolve with innovation, which itself responds to environmental policies. Therefore, a dynamic framework is necessary. This paper builds such a framework by integrating directed technical change with a trade model that features a global pollution externality. In doing so, it establishes two main points. First, intervening countries which only use unilateral (positive) carbon taxes typically fail to ensure sustainable growth; in fact such taxes are likely to accelerate environmental degradation because of the innovation response of nonintervening countries. Second, intervening countries can achieve sustainable growth without cooperation from the rest of the world by implementing a temporary industrial policy that combines clean research subsidies and a trade tax. Such a policy develops clean technologies in the polluting sector of the intervening countries, which leads to a long-run reduction of emissions not only in the intervening countries but also in nonintervening ones.

More formally, I consider a dynamic Ricardo–Heckscher–Ohlin model with two countries—North and South—and two sectors. The North represents countries willing to implement an environmental policy; the South represents countries that undertake no such policy. One sector never pollutes, whereas the other sector pollutes more or less depending on the country's balance between dirty and clean technologies. In practice, the polluting sector includes the manufacture of chemicals and chemical products, nonmetallic mineral products and basic metals. The distinction between clean and dirty technologies might refer to the use of renewable and nuclear instead of fossil fuel energy or to the use of bioplastics instead of traditional petroleum products. Innovation is undertaken in both countries by profit-maximizing firms that hire

[1] In this case, the trade barrier was an international reserve allowance. The bill passed the U.S. House in 2009 but was rejected by the U.S. Senate. Trade barriers have also been discussed for the European Union Emissions Trading System (EU ETS).

scientists, and it can be directed at the polluting or the nonpolluting sector. The allocation of scientists across these two sectors depends on the relative size of both sectors in the country as measured by their revenue share (Acemoglu, 1998). Under laissez-faire, the country exporting the polluting good has a relatively larger market size in the polluting than in the nonpolluting sector, which tends to amplify comparative advantage over time. Within the polluting sector, innovation can be directed at clean or dirty technologies, and the allocation of scientists across is tilted toward the most advanced of the two; this creates path dependence in innovation. For most of the analysis, innovation is completely local.

In laissez-faire, if clean technologies are initially less advanced than dirty ones in both countries, then most polluting sector innovations will be directed toward the dirty technologies. Emissions will continue to increase until the economy faces an "environmental disaster" as the quality of the environment falls below a critical threshold. In other words, economic growth is not sustainable. I first derive positive results by examining the impact of different policies unilaterally undertaken by the North. Suppose that the North implements a carbon tax, and that the South initially has a comparative advantage in the polluting sector. Such a policy leads to a reallocation of some of the polluting good's production from the North to the South—the "pollution haven effect"—thereby reinforcing the South's specialization in the polluting sector. Emissions still grow unboundedly in the South, which eventually causes an environmental disaster. Moreover, because reallocating production goes hand in hand with reallocating innovation, a Northern carbon tax actually increases dirty Southern innovations, and thereby may accelerate environmental degradation. The North could instead use a temporary combination of clean research subsidies and a trade tax, such a policy can help the North develop a comparative advantage in the polluting sector while making that sector cleaner at the same time. Once clean technologies in the North are sufficiently advanced and the initial comparative advantage is reversed, the market forces that previously drove the economy toward a disaster now work to averting it: emissions decrease both in the North (as innovation is directed to clean technologies) and in the South (as it specializes, over time, in the nonpolluting sector). If the initial environmental quality is high enough, then an environmental disaster can be averted.[2] Directed technical change is essential for this result; if technical change were exogenous, unilateral policies in the North would fail to prevent a disaster when the South initially has a sufficiently large comparative advantage in the polluting sector.

Then, I move to normative results, and study a social planner with two possible objectives. In the first case, the social planner's objective function depends only on world consumption

[2] Such a reversal of comparative advantage cannot always be ensured with clean research subsidies only. Indeed, under free trade, the South may fully specialize in the polluting sector, so that all its innovation is directed toward that sector. This makes it impossible for the North to reverse the pattern of comparative advantage without using a trade tax.

and environmental quality; in the second case, the planner maximizes a weighted sum of the utilities of infinitely lived representative agents in both countries, so that he is also concerned about the *distribution* of consumption across the two countries. In both cases, I characterize the first-best policy and the second-best policy under the constraint that no intervention can occur in the South. This second-best policy can be decentralized through a carbon tax and research subsidies in the North along with a trade tax on the polluting good.[3] Absent redistributive concerns, the trade tax typically takes the form of a tariff and then of an export subsidy, and its expression reflects two aims of the social planner: reducing emissions in the South and redirecting Southern innovation toward the nonpolluting sector. Yet when the social planner cares about the distribution of income, the optimal trade tax also reflects terms-of-trade considerations. A numerical exercise shows that, for reasonable parameter values, the welfare costs of not being able to intervene in the South are high. In addition, it highlights the double-edged nature of both trade and directed technical change: they accelerate environmental degradation under laissez-faire, but they also reduce environmental degradation when one country intervenes in an appropriate way.

Finally, I relax the assumption that knowledge is purely local by supposing that the less advanced country can partially catch up every period. The main results continue to hold: unilateral carbon taxes still fail to prevent an environmental disaster; whereas a combination of clean research subsidies and a carbon tariff can do so for sufficiently high initial environmental quality. In this scenario, however, the diffusion of knowledge can ensure a switch toward clean innovation in the South; hence an environmental disaster can be prevented even though the South still specializes in the polluting good.

This paper can be interpreted as a green version of the "infant industry argument," which claims that trade can be detrimental to growth if it leads countries to specialize in sectors with poor development prospects (Krugman, 1981; Young, 1991; Matsuyama, 1992; Galor and Mountford, 2008). Here as well, a country risks specializing in the "wrong" sector, not because that sector offers poor growth prospects, but because this country cannot prevent the environmental externality associated with production in that sector. The idea that free trade may amplify comparative advantages and that a temporary trade policy could permanently reverse the trade pattern was previously touched on by Krugman (1987), and Grossman and Helpman (1991, ch. 8).[4]

The literature on trade and the environment has long recognized that, in an open world,

[3] Production relies on monopolistically produced intermediate inputs (hereafter "intermediates"), so there is also a subsidy to correct for the monopoly distortion.

[4] Krugman's (1987) is based on learning-by-doing, and Grossman and Helpman's (1991) model features endogenous growth in one sector only. A few papers have built models with trade and directed technical change; examples include Acemoglu (2003), who studies the impact of trade on the skill bias of technological change, and Gancia and Bonfiglioli (2008), who show that trade amplifies international wage differences.

the effectiveness of unilateral policies for reducing world pollution can be hampered by the pollution haven effect; see, e.g., Pethig (1976). Empirical evidence is reported by Copeland and Taylor (2004) and more recently by Broner et al. (2012). Markusen (1975) and Hoel (1996) show that the optimal instrument for addressing the pollution haven effect is a tariff. In the specific context of global warming, where the pollutant (CO_2) enters differently at several stages of the production process, several papers use computable general equilibrium models to track carbon through the global economy; in this way they determine the pattern of trade and compute the carbon leakage rate, the rate at which emissions abroad increase after a domestic reduction. Developed countries are net carbon importers, which justifies the focus of the paper on the case where the South has a comparative advantage in the polluting sector: Atkinson et al. (2011) find that the net US imports of carbon from China in 2004 amounted to 244 million tons of CO_2 or 0.9 percent of total world emissions that year; the OECD STAN database estimates that for OECD countries net CO_2 imports represent 12.6% of CO_2 emissions from production. Elliott et al. (2010) compute a carbon leakage rate of 20 percent from a reduction in Annex I countries—i.e., the countries with binding constraints under the Kyoto protocol—and show that border tax adjustments eliminate half of it.[5] There are comparatively few empirical studies. Aichele and Felbermayr (2012) find that countries which committed to the Kyoto protocol reduced domestic CO_2 emissions by about 7 percent, but that their total CO_2 consumption did not change. The present paper is also related to the literature which addresses trade's impact on the environment (see Copeland and Taylor, 1995): in the absence of global cooperation, trade is necessary to avert an environmental disaster. However, trade needs to be managed in order to deliver the right outcome. This literature has focused on static models and has ignored the evolution of comparative advantage over time.

A growing literature has shown the importance of taking into account directed technical change when designing policies to combat climate change. On the empirical side, Popp (2002) shows that an increase in energy prices leads to more energy-saving innovation; similar results are found by Newell et al. (1999) in the air conditioner industry and by Hassler et al. (2012) using macroeconomic US data. Aghion et al. (2012) focus on the car industry and establish that *(a)* an increase in fuel prices leads to clean innovation at the expense of dirty innovation and *(b)* there is path dependence in clean versus dirty innovation—findings in line with the results reported here. Following this literature, several theoretical papers have integrated directed technical change in the study of climate change policies; here, I build on the model developed by Acemoglu et al. (2012a; henceforth AABH).[6] The final good in AABH and

[5] Among others, Babiker and Rutherford (2005), Böhringer et al. (2010), Böhringer et al. (2011) and Bucher and Schenker (2010) find similar results.

[6] Earlier work on the environment and directed technical change includes Bovenberg and Smulders (1995, 1996), Goulder and Schneider (1999), van der Zwaan et al. (2002) , Popp (2004), Grimaud and Rouge (2008),

the polluting sector in this paper are both produced with a clean and a dirty input, which are substitutes for each other. Because of knowledge externalities associated with "building on the shoulders of giants," there is path dependence in the direction of innovation (clean or dirty). Acemoglu et al. (2013) presents a two-country version of the model in which trade occurs between two substitutable goods, the polluting tradeable good cannot become less pollutive, and the South does not innovate, assumptions which are reversed here. Di Maria and Smulders (2004) and Di Maria and van der Werf (2008) also tackle the issue of modeling the interaction between directed technical change and international trade. These authors study the allocation of innovation between an energy-intensive sector and a non–energy-intensive sector, but overlook that innovations within the energy-intensive sector could either reduce or increase pollution.[7]

The is structured as follows. Section 2 presents the model. Section 3 studies the laissez-faire equilibrium, identifies which policies are able to ensure sustainable growth and discusses the model's main assumptions. Section 4 solves for the first- and second-best policies when the South is constrained to be in laissez-faire, and presents a numerical exercise which illustrates the workings of the model. Finally, Section 5 discusses how the main results generalize when knowledge flows across countries. Appendix A contains the main proofs. The online Appendix B features the more technical proofs, details on the calibration and some extensions.

2 Model

I consider a discrete-time, infinite-horizon version of a two-country (North, N, and South, S), two-sector (E and F), three-factor (capital, labor and scientists) Heckscher–Ohlin–Ricardo model in which sector E is similar to the economy of AABH. Each country is endowed with a fixed amount of labor and capital, L^N, K^N and L^S, K^S, and a mass 1 of scientists.

2.1 Welfare

I consider two distinct problems. In the first problem, the economy admits, for each period t, a representative agent in the North who lives for one period and a like representative agent in the South.[8] The utility of time-t agent in country $X \in \{N, S\}$ is given by $\nu(S_t) C_t^X$, where S_t is the quality of the environment (identical in North and South) and C_t^X is the final good

and Aghion and Howitt (2009). A more recent work is that of Acemoglu et al. (2012b).

[7] In Di Maria and Smulders (2004), the North develops technologies that are imitated by the South, and so opening up to trade leads to a reallocation of innovation toward the sector that the North exports. Carbon leakage is reduced when the goods are substitutes and amplified otherwise. In Di Maria and van der Werf (2008), both countries innovate and carbon leakage is always reduced by the innovation response to a cut in emissions in a single country. Golombek and Hoel (2004) use a static model to study the interaction between environmental policy and innovation in an open world.

[8] As specified in what follows, only the social planner makes an intertemporal decision.

consumption in country X. The social welfare function aggregates these preferences according to:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{\left(v\left(S_t\right)\left(C_t^N + C_t^S\right)\right)^{1-\eta}}{1-\eta}; \tag{1}$$

where $\rho > 0$ is the discount rate and $\eta \geq 0$ is the inverse elasticity of intertemporal substitution ($\eta = 1$ corresponds to a logarithmic utility). Therefore, the social planner cares only about the time profile of world consumption and environmental quality.

In the second problem, the economy admits infinitely lived representative agents in each country, whose utilities are given by $\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{\left(v(S_t)C_t^X\right)^{1-\eta}}{1-\eta}$. The social planner maximizes a weighted sum of these utilities:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{v\left(S_t\right)^{1-\eta}}{1-\eta} \left(\Psi\left(C_t^N\right)^{1-\eta} + (1-\Psi)\left(C_t^S\right)^{1-\eta}\right), \tag{2}$$

where $\Psi \in [0,1]$ is the weight on the North's representative agent. In this case, the social planner also cares about the distribution of consumption across the two countries.

Consumption, C_t^X, and environmental quality, S_t, are weakly positive and v is increasing in S_t. There is an upper-bound on S_t, denoted $\overline{S}$, that corresponds to a pristine environment. I define an *environmental disaster* as an instance of environmental quality reaching zero in finite time. I assume that $v(0) = 0$ and $v'\left(\overline{S}\right) = 0$; hence a disaster is as detrimental to welfare as zero consumption and the marginal damage of the first unit of pollution is zero.[9]

2.2 Production

Final consumption is a CES (constant elasticity of substitution) aggregate of the consumption of two goods, E and F:

$$C^X = \left(\nu\left(C_E^X\right)^{\frac{\sigma-1}{\sigma}} + (1-\nu)\left(C_F^X\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}; \tag{3}$$

where C_Y^X represents the quantity of good $Y \in \{E, F\}$ consumed in country $X \in \{N, S\}$, and σ denotes the elasticity of substitution between goods E and F.[10] I restrict attention to the cases where the two goods are gross complements ($\sigma < 1$) or where final consumption is Cobb–Douglas ($\sigma = 1$), so that both goods are essential. Goods E and F are the only goods that are traded internationally. Good E represents the traded goods responsible for most of the emissions of greenhouse gases (in particular, energy-intensive goods), while good F represents traded goods that do not generate a lot of emissions. When the model is calibrated, good E

[9] A disaster puts the economy on an unsustainable path because the utility flow cannot be bounded away from the utility flow given by zero consumption.

[10] Whenever this does not lead to confusion, I drop the time subscript but it should be clear that allocations, technologies and policies are time-dependent (endowments are constant though).

is identified with the manufacture of chemicals and chemical products (ISIC code 24), other nonmetallic mineral products (26), and basic metals (27), good F is identified with the rest of manufacturing. The paper focuses on tradeable goods, since it is because of international trade that policymakers fear that unilateral policies may have adverse consequences. Emissions for the production of tradeable goods represent a large share of CO_2 emissions—once electricity and heat are allocated to consuming sectors, manufacturing and construction represented 36.9 % of world CO_2 emissions in 2010 according to the International Energy Agency.[11] The inclusion of nontradeable goods is discussed in Section 5.

Good F in country X is produced competitively according to

$$Y_F^X = \left(\int_0^1 A_{Fi}^X \left(x_i^X\right)^\gamma di \right) \left(\left(K_h^X\right)^\beta \left(L_h^X\right)^{1-\beta} \right)^{1-\gamma}. \tag{4}$$

Here K_h^X and L_h^X are the capital and labor employed in the assembly of good F in country X; x_{Fi}^X is the quantity of intermediates i employed in sector F; and A_{Fi}^X is the productivity of intermediate i, which is specific to the country and sector. The parameter γ is the factor share of intermediates. Intermediates are produced monopolistically according to

$$x_{Fi}^X = \psi^{-1} \left(K_{hi}^X\right)^\beta \left(L_{hi}^X\right)^{1-\beta}, \tag{5}$$

where K_{Fi}^X and L_{Fi}^X are the capital and labor employed in the production of intermediate i for good F in country X. Intermediates cannot be traded internationally. Since the same factor share is used in the production of intermediates and in the final assembly of the good, it follows that $\beta \in (0, 1)$ is the overall factor share of capital in sector F.[12] I use K_F^X to denote total employment of capital in sector F in country X:

$$K_F^X \equiv K_h^X + \int_0^1 K_{Fi}^X di; \tag{6}$$

similarly, L_F^X is total employment of labor in sector F in country X.

Good E is produced competitively with a clean input Y_c^X and a dirty input Y_d^X according to

$$Y_E^X = \left(\left(Y_c^X\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(Y_d^X\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{7}$$

where $\varepsilon > 1$ is the elasticity of substitution between the clean and the dirty input (the paper also discusses the perfect substitutes case, $\varepsilon = \infty$). The clean input models nonpolluting

[11] Construction is non-tradeable, but agriculture and forestry, which are tradeable activities, are not included in this figure. Using input-output tables Davis and Caldeira (2010) estimates that today, 23% of carbon emitted is attributable to the production of goods that will be exported.

[12] The Cobb–Douglas structure of production for intermediates is important because it ensures that monopolists get a constant share of the sector's revenues, which matters for the incentives to innovate. That being said, the analysis can be extended straightforwardly to production functions for which aggregation between capital and labor is not Cobb–Douglas.

inputs that could substitute for polluting inputs, for instance, renewable energies to replace fossil fuel energy or bioplastics to replace traditional petroleum products. Both inputs are produced competitively in a similar fashion to good F:

$$Y_{zt}^X = \left(\int_0^1 A_{zi}^X \left(x_{zi}^X\right)^\gamma di\right) \left(\left(K_z^X\right)^\alpha \left(L_z^X\right)^{1-\alpha}\right)^{1-\gamma} \text{ for } z \in \{c, d\}, \tag{8}$$

where K_z^X and L_z^X are the capital and labor employed in the assembly of input z in country X; x_{zi}^X is the quantity of intermediates i employed in sector z; and A_{zi}^X is the productivity of intermediate i. Both clean and dirty intermediates are produced by monopolists according to

$$x_{zi}^X = \psi^{-1} \left(K_{zi}^X\right)^\alpha \left(L_{zi}^X\right)^{1-\alpha}, \tag{9}$$

so that $\alpha \in (0,1)$ is the total factor share of capital in sector E. I assume throughout that $\alpha > \beta$, which is true empirically: sectors that pollute the most tend to be the most capital intensive. This assumption is without loss of generality, since all results hold when $\alpha < \beta$ and the analysis can be extended to a pure Ricardian model with $\alpha = \beta$ (as discussed below this creates some technical difficulties). I define K_E^X as the total employment of capital in sector E,

$$K_E^X \equiv K_c^X + K_d^X + \int_0^1 K_{ci}^X di + \int_0^1 K_{di}^X di, \tag{10}$$

and similarly define L_E^X as the total employment of labor in sector E in country X.

Market clearing for each factor in each country requires that

$$K_E^X + K_F^X \leq K^X \text{ and } L_E^X + L_F^X \leq L^X, \tag{11}$$

and market clearing for each good requires that

$$C_E^N + C_E^S \leq Y_E^N + Y_E^S \text{ and } C_F^N + C_F^S \leq Y_F^N + Y_F^S. \tag{12}$$

Furthermore, to simplify the exposition and focus the comparison between first-best and second-best on environmental issues, I assume throughout that the optimal subsidy to intermediates is implemented in *both countries*, that is the government implements a subsidy $1-\gamma$ to the purchase of all intermediates, so that they are priced at marginal cost. Since the share of intermediates γ is the same for all sectors and all countries, the monopoly distortion would only have a scale effect, so that this assumption is completely innocuous for my results.[13]

[13] Henceforth I abuse language by referring to the "laissez-faire" case as one where governments only implement the subsidy to the use of all intermediates.

2.3 Innovation

At the beginning of every period, one-period monopoly rights are allocated to entrepreneurs (such that each entrepreneur holds monopoly rights on only a finite number of intermediates). Entrepreneurs can hire scientists to increase the productivity of their variety. By hiring s_{zit}^X scientists, the entrepreneur holding the monopoly right on variety i in sector $z = F$ or subsectors $z \in \{c, d\}$ can increase the initial productivity $A_{zi(t-1)}^X$ of her intermediate to

$$A_{zit}^X = \left(1 + \kappa \left(s_{zit}^X\right)^\iota \left(\frac{A_{z(t-1)}^X}{A_{zi(t-1)}^X}\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma} A_{zi(t-1)}^X \text{ for } z \in \{c, d, F\}, \tag{13}$$

where $0 < \iota < 1$. A_{zt}^X is the average productivity of (sub)sector $z \in \{c, d, F\}$ at time t, and is defined as

$$A_{zt}^X \equiv \left(\int_0^1 \left(A_{zit}^X\right)^{\frac{1}{1-\gamma}} di\right)^{1-\gamma} \quad \text{for } z \in \{c, d, F\}. \tag{14}$$

The factor $\left(A_{zi(t-1)}^X\right)^{-\frac{1}{1-\gamma}}$ captures decreasing returns to scale in innovation (the more advanced is a technology, the more difficult it is to innovate further), and $\left(A_{z(t-1)}^X\right)^{\frac{1}{1-\gamma}}$ denotes knowledge spillovers from all the other intermediates in the same sector in the same country. The innovation technology exhibits decreasing returns to scale in the mass of scientists hired (e.g., because scientists hired for the same intermediate in the same period risk reproducing the same innovation). Since the mass of scientists is equal to 1 in both countries, the market clearing equation is given by

$$\int_0^1 \left(s_{Fit}^X + s_{cit}^X + s_{dit}^X\right) di \leq 1. \tag{15}$$

Because an entrepreneur has monopoly rights for one period only, she will hire scientists so as to maximize current profits instead of the entire flow of profits generated by the innovations of her scientists. The allocation of scientists across (sub)sectors is therefore myopic. One-period monopoly rights are the only inefficiency in innovation and they allow one to model as simply as possible the "building on the shoulder of giants" externality, whose existence has long been recognized by the endogenous growth literature. In the specific context of climate change, this externality plays a crucial role in explaining why clean technologies have so far failed to really take off, and why direct research incentives in addition to carbon taxes are welfare improving, a point made by AABH.[14]

There are no knowledge spillovers between sectors. Cross-country spillovers are absent for the moment but introduced in Section 5. A fixed mass of scientists in both countries allows to

[14] With permanent monopoly rights, infinitely lived agents, and no environmental externality, the efficient innovation allocation would be an equilibrium, although not usually a unique one.

focus on only the direction of technical change and ensures that one country does not become arbitrarily large relative to the other (this assumption is relaxed in Appendix B.4).

2.4 Environment

Within the two bounds 0 and $\overline{S}$, environmental quality evolves according to

$$S_t = (1+\Delta)\, S_{t-1} - \left(\xi^N Y_{dt}^N + \xi^S Y_{dt}^S\right). \tag{16}$$

The parameter $\xi^X > 0$ measures the rate of environmental degradation from the production of dirty inputs (which may be different in the two countries) and $\Delta > 0$ is the regeneration rate of the environment. Without loss of generality, I assume that $S_0 = \overline{S}$. Such a law of motion captures the idea that the environment's regeneration capacity decreases with greater environmental degradation — the type of negative feedback that climatologists worry about, e.g., the change in Earth's albedo and the release of captured greenhouse gases which may occur as the polar ice cap melts. It is adopted for simplicity's sake but, unless explicitly mentioned, the analytical results do not depend on it. The only important assumption is that if emissions become too large then S_t reaches the disaster level.

The dirty input is directly responsible for environmental degradation. This specification is equivalent to one where a (cheap) fossil fuel resource can be combined with the dirty input in a Leontieff way. Given that most fossil fuel energy in manufacturing comes from coal and natural gas (which are in large supply relative to the time scale of critical environmental degradation), this is a plausible assumption; however, it is not a good approximation for oil, see Hassler and Krusell (2012).

2.5 Policy tools

Section 4 will solve the social planner's problem of maximizing (1) or (2), but Section 3 studies only whether or not an environmental disaster can be prevented with some specific policy instruments, the ones that will eventually be used to decentralize the optimal policy. More specifically, I introduce *ad valorem* taxes on the dirty input $\left(\tau_t^X\right)$, which are the equivalents of a carbon tax, as well as sector-specific *ad valorem* research subsidies or taxes on scientists' wages.[15] I also allow for an *ad valorem* trade tax on the polluting good E (by Lerner symmetry, doing so is without loss of generality; the trade tax could also be on the other good). Hence prices in the South are always equal to international prices: $p_{Et}^S = p_{Et}$ and $p_{Ft}^S = p_{Ft}$. In the North, the price of good F is also equal to the international price, $p_{Ft}^N = p_{Ft}$, but the

[15] In order to ensure uniqueness of the equilibrium allocation of scientists, I assume that it is possible to subsidize only a given mass of scientists; hence the social planner can use the subsidy to determine the exact allocation. If the subsidy is greater than 100 percent, then a monopolist may be willing to hire scientists even if she is not producing any good.

price of good E is given by $p^N_{Et} = p_{Et}(1 + b_t)$, where b_t is the trade tax. A positive trade tax corresponds to a tariff (resp., export subsidy) when the North imports (resp., exports) good E.[16] When the North is the only country intervening, I assume that trade balance must be maintained every period (there is no intertemporal trade):

$$p_{Et}\left(Y^S_{Et} - C^S_{Et}\right) + p_{Ft}\left(Y^S_{Ft} - C^S_{Ft}\right) = 0. \tag{17}$$

Note that the trade tax is *not* explicitly related to the carbon content of imports. If the South does not undertake any policy, then relating the tax to the *average* carbon content of imports from a given country and in a given sector would not alter the results; since each Southern firm is atomistic, its impact on average emission is infinitesimal and so its behavior will not affect the trade tax it pays. Changing the behavior of Southern firms would require either the North to know the exact carbon content of each individual import, which seems implausible, or the South to implement a policy in response to the North's tariff.

In short, a policy is characterized by a sequence of *ad valorem* taxes on the dirty input τ^X_t in each country, a sequence of subsidies for scientists in every subsector, and a sequence of trade taxes b_t on the polluting good. All subsidies and taxes are financed (or rebated) through lump-sum taxation at the country level.

3 Preventing an Environmental Disaster

This section presents positive results on whether certain type of policies can or cannot avert an environmental disaster. Section 3.1 details the behavior of the economy under laissez-faire. Section 3.2 explains why taxing the North's polluting sector likely fails to prevent a disaster. Section 3.3 describes how a disaster can be avoided using unilateral policies in the North, and Section 3.4 discusses some of the assumptions. For a given policy, the equilibrium is defined as follows.

Definition 1 *A* feasible allocation *is a sequence of demands for capital (K^X_{ht} K^X_{Fit}, K^X_{ct}, K^X_{cit}, K^X_{dt}, K^X_{dit}), demands for labor (L^X_{ht}, L^X_{Fit}, L^X_{ct}, L^X_{cit}, L^X_{dt}, L^X_{dit}), demands for intermediates (x^X_{zit} for $z \in \{c, d\}, F$), demands for inputs $\left(Y^X_{ct}, Y^X_{dt}\right)$, goods production $\left(Y^X_{Et}, Y^X_{Ft}\right)$, demands for goods $\left(C^X_{Et}, C^X_{Ft}\right)$, research allocations $\left(s^X_{zit} \text{ for } z \in \{c, d\}, F\right)$, and quality of the environment S_t such that, in each period t and in each country $X \in \{N, S\}$, factor and good markets clear (i.e.,* (11), (12)*, and* (15) *hold).*

Definition 2 *For a given policy, an* equilibrium *is given by a feasible allocation and sequences of wages of workers (w^X_t), returns to capital (r^X_t), wages of scientists (υ^X_t), consumer prices for*

[16] Starting from a situation where the North imports the polluting good under free trade, an increasingly higher trade tax corresponds to a positive tariff up to the point where it implements autarky. Beyond that point, the North begins to export the polluting good and the trade tax is a positive export subsidy.

intermediates $\left(\varphi_{zit}^X \text{ for } z \in \{c,d\}, F\right)$*, producer prices for clean and dirty inputs* (p_{ct}^X, p_{dt}^X)*, and international prices of goods* (p_{Et}, p_{Ft}) *for* $X \in \{N, S\}$ *such that: (i)* $\left(\varphi_{zit}^X, x_{zit}^X, s_{zit}^X, K_{zit}^X, L_{zit}^X\right)$ *maximizes profits by the producer of intermediate* i *in sector* $z \in \{c, d, F\}$ *in country* X*; (ii)* L_{zt}^X*, and* K_{zt}^X *maximize the profits of the producer of good* $z \in \{c, d, F\}$*; (iii)* Y_{ct}^X *and* Y_{dt}^X *maximize the profits of producer of good* E*; (iv)* C_{Et}^X *and* C_{Ft}^X *maximize consumers' utility under the trade balance constraint (17).*

3.1 Laissez-Faire

Trade pattern. Here I analyze the laissez-faire equilibrium; the results are derived and generalized in Appendix A.1. In each country, aggregate production in each sector can be written as

$$Y_{Et}^X = \zeta A_{Et}^X \left(K_{Et}^X\right)^{\alpha} \left(L_{Et}^X\right)^{1-\alpha} \text{ and } Y_{Ft}^X = \zeta A_{Ft}^X \left(K_{Ft}^X\right)^{\beta} \left(L_{Ft}^X\right)^{1-\beta}, \tag{18}$$

where $\zeta \equiv \gamma^{\gamma} (1-\gamma)^{1-\gamma} \psi^{-\gamma}$ and $A_{Et}^X \equiv \left(\left(A_{ct}^X\right)^{\varepsilon-1} + \left(A_{dt}^X\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}$ is the average productivity of sector E. This formulation highlights that, in a given period, the model collapses to a Heckscher–Ohlin model with varying productivity across countries. The South has the comparative advantage in the polluting good E, and it exports E if and only if

$$\left(\frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}} \frac{K^S}{L^S} > \left(\frac{A_{Et}^N}{A_{Ft}^N}\right)^{\frac{1}{\alpha-\beta}} \frac{K^N}{L^N}. \tag{19}$$

Trade results from Ricardian forces (relative productivity) as well as Heckscher–Ohlin forces (relative factors endowment). Provided the difference in comparative advantage is not too large, both countries produce both goods. Yet once that difference becomes sufficiently large, one country fully specializes; if the difference in comparative advantage grows even more, then both countries fully specialize. Emissions are given by $E_t^X = \xi^X \left(\frac{A_{dt}^X}{A_{Et}^X}\right)^{\varepsilon} Y_{Et}^X$. Thus the emission rate in the polluting sector is increasing in the ratio of dirty to clean productivities A_{dt}^X / A_{ct}^X. Over time, innovation changes the comparative advantage and the emission rate.

Allocation of innovation. Entrepreneurs face a two-stage problem. In the second stage, they choose prices in order to maximize their profits given their productivity. Post-innovation profits in sector $z \in \{c, d, F\}$ are given by:

$$\pi_{zit}^X = (1-\gamma) \left(\frac{A_{zit}^X}{A_{zt}^X}\right)^{\frac{1}{1-\gamma}} p_{zt}^X Y_{zt}^X. \tag{20}$$

These profits are directly proportional to the revenues of the intermediate's (sub)sector (this follows from the Cobb-Douglas specification) and they are increasing in the productivity of the intermediate, A_{zit}^X. In the first stage, entrepreneurs hire scientists to increase the productivity of their intermediate. Thanks to the knowledge spillovers across varieties, all monopolists in a given (sub)sector hire the same number of scientists and so average productivity evolves according to

$$A_{zt}^X = \left(1 + \kappa \left(s_{zt}^X\right)^\iota\right)^{1-\gamma} A_{z(t-1)}^X \text{ for } z \in \{c, d, F\}\,.$$

Path dependence in clean versus dirty technologies. Assume that country X produces good E (otherwise, $s_{ct}^X = s_{dt}^X = 0$). Combining the first-order conditions with respect to the number of scientists in the clean and dirty subsector yields the following equation for the allocation of scientists within sector E:

$$\frac{\left(s_{ct}^X\right)^{1-\iota}\left(1+\kappa\left(s_{ct}^X\right)^\iota\right)}{\left(s_{dt}^X\right)^{1-\iota}\left(1+\kappa\left(s_{dt}^X\right)^\iota\right)} = \frac{p_{ct}^X Y_{ct}^X}{p_{dt}^X Y_{dt}^X} = \frac{\left(A_{ct}^X\right)^{\varepsilon-1}}{\left(A_{dt}^X\right)^{\varepsilon-1}}. \tag{21}$$

The second equality follows from the demand equation for both inputs in sector E (knowing that the production technologies differ only by their productivity level). The ratio of revenues in the clean sector to those in the dirty sector increases with the ratio of clean to dirty technologies. This association reflects two counteracting forces: a larger technology ratio leads to a larger market share ratio but also to a lower price ratio; the former effect dominates when the inputs are substitutes. Thus, for a sufficiently small innovation size κ, more scientists are allocated to the dirty than to the clean subsector if and only if $A_{d(t-1)}^X > A_{c(t-1)}^X$ (if κ is too large then there may be multiple equilibria when $A_{d(t-1)}^X$ and $A_{c(t-1)}^X$ are close to each other; see Appendix B.1). So, within the polluting sector, under laissez-faire, innovation tends to be allocated to the sector that is already the most advanced: there is path dependence.

Amplification of comparative advantage. Assume that production occurs in both sectors (otherwise, innovation occurs only in the active sector). By combining the first-order conditions with respect to the number of scientists in sector F and in subsectors c and d, I obtain

$$\frac{\left(s_{ct}^X\right)^{1-\iota}\left(1+\kappa\left(s_{ct}^X\right)^\iota\right) + \left(s_{dt}^X\right)^{1-\iota}\left(1+\kappa\left(s_{dt}^X\right)^\iota\right)}{\left(s_{Ft}^X\right)^{1-\iota}\left(1+\kappa\left(s_{Ft}^X\right)^\iota\right)} = \frac{p_{Et}^X Y_{Et}^X}{p_{Ft}^X Y_{Ft}^X}. \tag{22}$$

This equality implies that, for a given ratio $A_{d(t-1)}^X / A_{c(t-1)}^X$ of initial productivities within sector E, the number of scientists allocated to sector E is increasing in the ratio of sector E to sector F revenues. Under free trade, prices are equalized in North and South; hence each country tends to innovate relatively more in the sector it exports, and does so at an equal ratio of initial productivities within sector E. Hence, comparative advantages are typically amplified over time, so that one and eventually both countries fully specialize.[17] As mentioned in the introduction, developed countries are already net carbon importers, the model predicts that with no policy intervention, this will be more and more the case.

[17] This is the case for instance if initial comparative advantages are sufficiently marked; Appendix A.3 provides a formal lemma.

This contrasts with the autarky case, where consumer demand implies that

$$\frac{p^X_{Ft} Y^X_{Ft}}{p^X_{Et} Y^X_{Et}} = \frac{1-\nu}{\nu} \left(\frac{Y^X_E}{Y^X_F} \right)^{\frac{1-\sigma}{\sigma}} . \tag{23}$$

When $\sigma = 1$ the right-hand side is constant and the mass of scientists innovating in each sector (E and F) is bounded away from 0. When $\sigma < 1$, innovation tends to occur in the sector with the lowest productivity, therefore, in the long-run, the productivities of sectors E and F must grow at same rate. Since the two sectors are complements, innovation will not disappear in one sector over time (as it does in the case of clean versus dirty innovation).

Equilibrium uniqueness. As innovating more in a sector increases a country's comparative advantage in that sector, which, in turn, prompts more innovation in the same sector, multiple equilibria could arise. The results of this section can easily be extended to a case with multiple equilibria, but focusing on a unique equilibrium simplifies the exposition. Henceforth, I assume that the conditions of the following lemma are satisfied, so that the equilibrium is unique.[18]

Lemma 1 *If κ is small enough and $\iota \geq 1/2$, the laissez-faire equilibrium is unique.*

Proof. See Appendix B.1. ■

Environmental disaster. Under laissez-faire, as long as dirty technologies are more advanced than clean ones in both countries, innovation in the polluting sector remain directed primarily toward dirty technologies. Since innovation in the polluting sector does not asymptotically vanish (the exporting country innovates more in the polluting good than it would under autarky), the production of good E grows unboundedly and so do emissions. At some point, the regenerative capacity of the environment becomes overwhelmed and the economy reaches an environmental disaster.

As in AABH, a global government could use clean research subsidies, taxes on dirty research and/or carbon taxes to redirect innovation from the dirty toward the clean subsector in countries that produce the polluting good. Once clean technologies acquire a sufficient lead over dirty intermediates, market forces will ensure that most research is directed toward the clean subsector, which is now the most advanced. Eventually, the emission rate of the polluting good approaches zero—sufficiently fast to offset the growth in the polluting good's production—and a disaster can be avoided for sufficiently high initial environmental quality (see Appendix B.2).

[18] A sufficiently small size of innovation κ ensures that changes in productivities during one period remain sufficiently small. The technical assumption $\iota \geq 1/2$ is further necessary to ensure that the equilibrium is unique when one country is close to a corner of specialization (i.e., to a point at which a producer of the imported good would break even only if he produces an infinitesimal amount of the good). The lemma does not extend to the Ricardian case where $\alpha = \beta$: in that case, no matter how small κ is, there are multiple equilibria when the initial comparative advantage is small.

3.2 Taxes on the Polluting Good in the North only

Assume now that only the North is able to implement some policy, which rules out the case where the North pays the South to implement some policy. Is this alone enough to avoid environmental disaster? Observe that, in autarky and without knowledge spillovers, no policy restricted to the North can prevent a disaster because Southern emissions grow unboundedly regardless of what the North does. Absent international cooperation, trade is necessary to avoid an environmental disaster.

The key to avoid environmental disaster with Northern policies only is ensuring that the South asymptotically fully specializes in the nonpolluting sector. Otherwise, innovation in the polluting sector always occurs in the South, so that the production of the polluting good and therefore emissions grow unboundedly. This is formalized in the following lemma.

Lemma 2 *If clean technologies are less developed than dirty ones in the South ($A_{c0}^S/A_{d0}^S \leq 1$), then a disaster can be averted only if (i) all factors in the South are asymptotically allocated to the nonpolluting sector F and (ii) in the long run, the North exports the polluting good.*

Proof. See Appendix A.2. ■

I first focus on the case where the North can implement a positive carbon tax and/or a positive tax on dirty research. Both instruments have no protectionist aspects, can reduce emissions in the North, and prompt clean innovation there; and both could prevent an environmental disaster if the North were the only country or if the South undertook the same policy. However, such policies may be incompatible with a South specializing in the nonpolluting sector and thus may fail to prevent an environmental disaster.

Proposition 1 *If innovation size κ is small enough then, no matter how high $\overline{S}$ is, no combination of a positive carbon tax and a positive tax on dirty research can prevent an environmental disaster if: (i) clean technologies are less developed than dirty ones in the North ($A_{c0}^N/A_{d0}^N \leq 1$); (ii) clean technologies are sufficiently less developed than dirty ones in the South (A_{c0}^S/A_{d0}^S is sufficiently small); and (iii) the South has a weak initial comparative advantage in the polluting sector (i.e., $\left(A_{E0}^S/A_{F0}^S\right)^{\frac{1}{\alpha-\beta}} K^S/L^S \geq \left(A_{E0}^N/A_{F0}^N\right)^{\frac{1}{\alpha-\beta}} K^N/L^N$).*

Proof. See Appendix A.3. ■

Under laissez-faire and with the assumptions of the proposition, the South would keep its comparative advantage in the polluting sector, eventually specializing in that sector. The North government cannot reverse this pattern simply by using a positive tax on dirty research or a positive carbon tax. On the contrary, a positive tax on dirty innovation drives scientists away from the polluting sector E toward the nonpolluting sector F; moreover, within the polluting sector it allocates innovation toward the initially backward clean subsector, which further reduces the growth rate of average productivity A^N_{Et}. A positive carbon tax has the same effect on innovation and also directly reduces the productivity of the polluting sector in the North. Since both instruments increase the costs of producing the polluting good in the North, they lead to an increase in its world relative price. This induces an increase in production of the polluting good E in the South and hence more emissions there, which is the classic pollution haven effect. As the relative revenues of the polluting sector increase in the South, and following equation (22), Southern innovation is further tilted toward the polluting sector, where it is mostly directed at the dirty technologies. Accordingly, positive Northern taxes on the polluting good can only accelerate the Southern specialization in the polluting sector.

In fact, such policies are likely to also accelerate environmental degradation because of the reallocation of innovation in the South. Indeed, the economy tends to grow faster when countries are more specialized since there is less overlap in the type of innovations being undertaken by both countries. In addition, the gap between clean and dirty technologies in the South grows faster, which increases the South's emissions rate. Both effects work towards an increase in emissions. Furthermore, although carbon taxes and taxes on dirty research can tilt innovation within the polluting sector toward clean technologies, they typically fail to ensure that such technologies get significantly developed. As production of the polluting sector moves to the South, the market size for clean technologies in the North becomes too small to attract much innovation.

In the proposition, the condition that A^S_{c0}/A^S_{d0} be sufficiently small (and not simply less than 1) is necessary because when the ratio of clean to dirty revenues is farther from unity in the North than in the South, more innovation in the polluting sector might take place in the former even if the latter exports the polluting good.[19] The condition could be dispensed with if the initial comparative advantage were sufficiently large.

3.3 Introducing Clean Research Subsidies and Trade Taxes

The previous policies could not prevent an environmental disaster—when the South had the initial comparative advantage in the polluting sector—because they could not reverse the pattern of trade. I now allow the North to use clean research subsidies and a trade tax.[20] Both

[19] More specifically: the incentive to innovate in sector G is, *ceteris paribus*, lower when the revenues in the clean and dirty subsectors are close to each other — that is when $\left(A^X_{c(t-1)}\right)^{\varepsilon-1}$ and $(1+\tau_{Xt})^{-\varepsilon}\left(A^X_{d(t-1)}\right)^{\varepsilon-1}$ are comparable. Given carbon taxes that are high enough or taxes on dirty research that are of sufficient duration, the ratio of clean to dirty revenues may become farther from unity in the North than in the South. In that event, the assumption on A^S_{c0}/A^S_{d0} ensures that, when this occurs, the difference in comparative advantages is large enough to ensure that there is more polluting innovation in the South.

[20] With a trade tax, the equilibrium may not be unique when $\sigma < 1/2$, but this does not affect the following analysis.

policies have protectionist aspects, in that the clean research subsidy is a conditional subsidy granted to the polluting sector, which is the sector facing import competition.

Proposition 2 *A combination of a temporary trade tax and a temporary clean research subsidy in the North can prevent an environmental disaster provided that the initial environmental quality $\overline{S}$ is sufficiently high.*

The key difference between clean research subsidies and the carbon tax or the tax on dirty research is that the former can also reallocate scientists who were working in the nonpolluting sector F toward the clean subsector. This boosts innovation in clean technologies in the North, even when the North does not have the comparative advantage in the polluting sector. Increasing innovation in clean technologies makes the polluting sector less polluting and helps build a comparative advantage in the polluting sector. In the meantime, a positive trade tax reduces production and therefore innovation in the polluting sector in the South, as a result, it also helps reverse the pattern of comparative advantage. For sufficiently high initial environmental quality, a policy combining these two instruments can prevent a disaster. To see this and prove Proposition 2, consider the following two-phase approach. In the first phase, a social planner implements a tariff large enough to shut down trade, so that innovation in the South must be balanced between the polluting and nonpolluting sectors. Simultaneously, she implements large clean research subsidies so that nearly all Northern scientists innovate in the clean subsector, and the North innovates more in the polluting sector E than the South. Once the North has acquired the comparative advantage in the polluting sector and $A^N_{c(t-1)}/A^N_{d(t-1)}$ is sufficiently large, the social planner can discontinue all policies and re-open up to trade. Market forces then ensure that the production of the polluting good eventually moves entirely to the North where it relies essentially on clean technologies,[21] emissions go down to zero in both countries, and a disaster can be avoided.

From this discussion one might think that clean research subsidies alone should be enough to prevent an environmental disaster. This is true if the initial comparative advantage of the South is not too large, but as the following remark stipulates, it does not always hold.

Remark 1 Suppose final consumption is Cobb–Douglas in both the polluting and nonpolluting goods ($\sigma = 1$). There exist initial factor endowments and technologies, such that no matter how high $\overline{S}$ is, no combination of a carbon tax, a tax on dirty research, and a subsidy for clean research can prevent a disaster.

Proof. See Appendix B.3. ■

[21] This follows lemma A.2, applied to the case where the North now has the comparative advantage in the polluting sector at some date τ, with $A^N_{d\tau}/A^N_{c\tau} < A^S_{c\tau}/A^S_{d\tau} < 1$.

Clean research subsidies alone cannot prevent a disaster when the South fully specializes in the polluting sector and clean technologies in the South are sufficiently less advanced than dirty ones. In that case, all Southern scientists are allocated to the polluting sector and, asymptotically, to dirty technologies. So even if the North were to allocate all its scientists to clean technologies, A_{Et}^{S} would grow as fast as A_{Et}^{N}. That situation is irreversible in the Cobb–Douglas case and an environmental disaster cannot be avoided. Full specialization in the South occurs in the first place when its initial comparative advantage in the polluting sector is sufficiently large or when clean technologies are sufficiently backward in the North, as the average productivity of the polluting sector in the North, A_{Et}^{N}, grows slowly during the period when clean technologies are catching up with dirty ones. This is not the case in the strict complement case, $\sigma < 1$, as the South cannot continue to fully specialize in the polluting sector if both countries innovate only in that sector, because the demand for the nonpolluting good becomes too large. Hence the mass of scientists innovating in dirty technologies in the South is bounded away from one, and the North can reverse the pattern of trade and prevent an environmental disaster for a sufficiently high level of initial environmental quality using clean research subsidies only.[22]

3.4 Discussion

This subsection discusses some of the assumptions of the model and presents additional results. Appendix B.4 relaxes the assumption of an equal measure of scientists in both countries.

Other instruments. It is clear that the reasoning behind Proposition 2 extends to the case where the North uses a combination of clean research subsidies and subsidies to the polluting good (which is relevant if trade taxes are impossible), a combination of carbon taxes and trade taxes (relevant if targeted research subsidies are impossible to implement),[23] or subsidies to the production of the clean input alone (relevant if both research subsidies and trade taxes are impossible). Paradoxically, a negative carbon tax combined with a positive tax on dirty research might also avert a disaster when a positive carbon tax could not: the negative carbon tax can be used to reverse the pattern of trade while the tax on dirty research can ensure that innovation occurs in clean technologies.[24] So far I have assumed that the North cannot find the carbon content of imports at the firm level. Under the opposite assumption, trade taxes

[22] See Appendix B.3. This result would not generalize to a case where the North could not incentivize firms to innovate in clean technology if there is no production of the clean input (maybe because subsidies greater than 100% cannot be implemented).

[23] For this combination to work, it is important that trade taxes large enough to reverse the pattern of trade immediately are allowed. A trade tax that implements autarky is generally insufficient to prevent a disaster when combined with carbon taxes (while it is sufficient when combined with clean research subsidies).

[24] This scenario is not very realistic: achieving the right combination of a large negative carbon tax and a large positive tax on dirty research seems difficult, moreover, since emissions are likely to increase considerably in the short-run, the initial level of environmental quality necessary to avert a disaster must be very high.

related to the emission content of imports could directly influence the behavior of Southern firms. In some cases this instrument (combined, for instance, with a carbon tax in the North) can prevent a disaster by inducing a switch to clean innovation in the South; but, this requires that clean technologies are not too backward in the South and that the export market is large.

Clean and dirty input. The results of the paper crucially depend on the assumption that innovation may occur in all three (sub)sectors (clean, dirty and non-polluting). If innovation were limited to clean and dirty technologies within the polluting sector, then the North could not build a comparative advantage in a specific sector. With clean innovation in the polluting sector only (as in Di Maria and Smulders, 2004; and Di Maria and van der Werf, 2008), the model would falsely assume that all innovations in the polluting sector decrease emissions. On the contrary, with only dirty innovations in the polluting sector, no innovations could replace existing polluting technologies since the polluting sector and the nonpolluting sector are complements, $\sigma \leq 1$.[25]

Importantly, note that dirty innovations generally include not only innovations in the energy sector that make fossil fuel energy cheaper, but also innovations in components that are complements to fossil fuel energy and thus increase its demand,[26] or the introduction of new goods or inputs that rely on fossil fuel energy. In practice, some innovations in the polluting sector may complement both fossil fuel energy and alternative forms of energy; one could represent such innovations in this model as improving the productivity of an additional input in the polluting sector complement to both the clean and dirty inputs. This would not affect the economic intuitions developed and my results could be extended to this scenario.

The South's behavior. The paper assumes that the South does not implement any policy. Regarding environmental policy today, this seems a reasonable assumption: several countries seem willing to move forward, while others are opposing a global agreement while often undertaking very limited domestic policies (Barrett, 1994, explains why designing a self-enforcing international agreement on climate change is difficult). A reason why these divisions—which do not necessarily follow the classic North/South lines—may persist in reality is the significant delay between emissions and damages that climate models predict, an aspect that I abstract from here: as a result, it may be too late before skeptic countries get convinced that they should start undertaking significant policy actions. Even if one expects that these divisions will eventually end, the results of the paper are still useful for countries who are

[25] Here clean innovations allow to develop an input which substites for the dirty one, and the polluting sector's productivity can grow at the same rate whether it relies mostly on the clean or the dirty inputs. If the clean alternative had some growth's costs, then preventing a disaster with unilateral policies would be more difficult (this could be the case in a different model where the clean alternative refers to energy efficiency improvements, and where energy is complement to the other inputs in the polluting sector).

[26] Aghion et al. (2012) for instance show that the majority of innovation for fossil fuel engines in the automotive industry are of this type.

willing to intervene before the rest of the world.

Yet, even if the South does not implement any environmental policy, it may still want to implement trade policies, particularly if the North's trade policy hurts the South. Yet, South's consumption is not necessarily negatively affected by the North's unilateral policies, and the South benefits from better environmental quality. For instance, if the North's temporary policy reverses the pattern of comparative advantages, both countries fully specialize in the long run. In the Cobb–Douglas case ($\sigma = 1$), income shares are linked to the consumption share of the good that the country exports; therefore, if the income share for the polluting good is smaller than for the nonpolluting one ($\nu < 1 - \nu$), then the South's income share will be larger under the North unilateral policy than under laissez-faire.[27]

Although a full analysis of the strategic interactions between two governments is beyond the scope of this paper, a case which can be considered is one where the South government is myopic and maximizes only current consumption. This government implements its own trade tax to improve its terms of trade. As long as the South retains an initial comparative advantage in the polluting good, this trade tax moves both countries closer to autarky and thus does not prevent the North from reversing the pattern of comparative advantage. Once the North exports the polluting good, the South implements its own tariff. This tariff slows down the South's specialization in the nonpolluting sector. Yet, once the North has acquired a sufficiently large comparative advantage, it does not prevent the South from fully specializing in the nonpolluting sector. Therefore, a disaster can still be avoided for sufficiently high initial environmental quality.

4 Optimal policy and numerical illustration

I now turn to the normative part of the paper, characterizing the first-best policy and the second-best policy under the constraint that the social planner cannot intervene in the South. I use a numerical example to illustrate both policies and compute their welfare costs, and to show that both trade and directed technical change act as double-edge swords.

4.1 Parameter Choices

This subsection briefly describes the calibration; details are given in Appendix B.5. A period corresponds to 5 years, and initial values are based on the 2003–2007 world economy while assuming laissez-faire in both countries. Final good consumption is Cobb-Douglas in the polluting and nonpolluting goods ($\sigma = 1$) and the elasticity of intertemporal substitution is

[27] Even in the short run, the South might benefit: a tariff implemented by the North hurts the South when the South exports the polluting good, but a trade tax high enough to reverse the pattern of trade immediately may benefit the South (this trade tax is then an export subsidy).

unity ($\eta = 1$). The annual time discount rate is 0.015, as in Nordhaus (2008). The North comprises 33 countries in Annex I of the Kyoto protocol (i.e., the group of countries that are subject to binding constraints on their emissions) and the South to 18 major countries in the rest of the world. Restricting attention to manufacturing, I compute the world rate of emissions per dollar of value-added in each sector at the available aggregation level, here using data on sectoral emissions of CO_2 from fossil fuel combustion given by the International Energy Agency, IEA, 2010a, and data on sectoral value added by the United Nations Industrial Development Organization, UNIDO, 2011. The sectors with the highest rate are identified with sector E—namely the manufacture of chemicals and chemical products, ISIC code 24, of other nonmetallic mineral products, 26, and of basic metals, 27—and the others with sector F.[28] Southern production is tilted toward sector E relative to Northern production ($Y_{E0}^N / Y_{E0}^S \times Y_{F0}^S / Y_{F0}^N = 0.77$), so that the South has a small initial comparative advantage in the polluting sector E. The consumption share of good E is computed using world production of both sectors: $\nu = 0.257$.

The capital shares are $\alpha = 0.5$ for sector E and $\beta = 0.3$ for sector F, here using the ratio of capital to labor compensations in both sectors in the United States according to the EU KLEMS dataset, Timmer et al. (2008), and the share of intermediates $\gamma = 1/3$, a common value in endogenous growth models. The elasticity of substitution between the clean and the dirty input, ε is fixed at 5, but Appendix B.11 considers the cases of $\varepsilon = 3$ and 10. The innovation size κ is adjusted so that the long-run annual growth rate is 2 percent, and the concavity of the innovation function is fixed by choosing $\iota = 0.55$ (≥ 0.5 so that the equilibrium is unique for a small κ).

The quality of the environment S_t is linearly and negatively related to the atmospheric concentration of CO_2; here the previous assumption that $S_0 = \overline{S}$ is relaxed, and the initial environmental quality S_0 is set to the current atmospheric concentration of 379 ppm. Δ is chosen such that, at current levels, half of CO_2 emissions are absorbed and do not add to atmospheric concentrations. Changes in atmospheric CO_2 concentrations are then mapped against changes in temperature, and $S = 0$ is chosen to correspond to a disaster temperature level of 6°C. The function $\nu(S_t)$ is the same as in AABH and mimics the cost function of Nordhaus (2008) for temperature increases up to 3°C. I identify the ratio Y_{c0}^X / Y_{d0}^X with the ratio of nonfossil to fossil fuel energy produced for country X's primary energy supply (following IEA, 2010b). From this I derive the ratio A_{c0}^X / A_{d0}^X. This, together with the emission rates in sector E in both countries, gives me the emission rates per unit of dirty input ξ^X.[29]

[28] According to the model, I ignore emissions from sector F. Sector F corresponds to the other sectors in manufacturing except 23, 25, 33, 36, and 37, for which data are not available.

[29] Overall the emission rates in the polluting sector in the South is nearly 4 times that of the North's, so that, even though $A_{d0}^N / A_{c0}^N < A_{d0}^S / A_{c0}^S$, I have $\xi^S > \xi^N$.

4.2 First-Best

In the first-best, the social planner maximizes (1) or (2) subject to the following constraints: the production function equations (3), (4), (5), (7), (8), and (9); the factor market–clearing equations (11) and (15); the goods market–clearing equation (12); the environmental degradation equation (16); and the knowledge accumulation equation (13). The first-best is characterized as follows:

Proposition 3 *The first-best policy can be decentralized by combining a carbon tax in both North and South (with the same price for carbon), research subsidies/taxes (in North and South) in both sectors, and a subsidy for the use of all intermediates. When the social planner maximizes (2), international transfers are also required.*

Proof. See Appendix A.4. ■

Each instrument allows the social planner to correct for one distortion. The subsidy $1-\gamma$ to all intermediates corrects the monopoly distortion. The environmental externality is corrected by a carbon tax in both countries that equalizes the marginal cost of the tax (lower current consumption) with the marginal benefit (higher environmental quality in all subsequent periods). Carbon taxes in the North and the South differ in *ad valorem* values across countries but are identical as a tax per unit of CO_2. The social planner corrects for the myopia of monopolists in their innovation decisions by allocating scientists in accordance with the discounted value of the entire stream of additional revenues generated by their innovation. More specifically: contra (21) and (22), scientists are now allocated across the dirty, clean, and nonpolluting (sub)sectors according to

$$\frac{\left(s_{Ft}^{X}\right)^{\iota-1}}{1+\kappa\left(s_{Ft}^{X}\right)^{\iota}}\sum_{s=t}^{\infty}B_{s,t}\widehat{p}_{Fs}Y_{Fs}=\frac{\left(s_{ct}^{X}\right)^{\iota-1}}{1+\kappa\left(s_{ct}^{X}\right)^{\iota}}\sum_{s=t}^{\infty}B_{s,t}\widehat{p}_{cs}^{X}Y_{cs}=\frac{\left(s_{dt}^{X}\right)^{\iota-1}}{1+\kappa\left(s_{dt}^{X}\right)^{\iota}}\sum_{s=t}^{\infty}B_{s,t}\widehat{p}_{ds}^{X}Y_{ds}, \qquad (24)$$

where $\widehat{p}_{cs}^{X}$ and $\widehat{p}_{ds}^{X}$ denote the shadow price of (respectively) the clean and dirty inputs in country X, and $\widehat{p}_{Fs}$ is the shadow price of good F. $B_{s,t}$ is the effective discount factor between periods s and t, given by $\frac{1}{(1+\rho)^{s-t}}\frac{\frac{\partial u}{\partial C}(C_{Ws},S_s)}{\frac{\partial u}{\partial C}\left(C_t^W,S_t\right)}$, where $u\left(C_t^W,S_t\right)=\frac{\left(\nu(S_t)C_t^W\right)^{1-\eta}}{1-\eta}$ and $C_t^W\equiv C_t^N+C_t^S$. When the social planner cares about the distribution of consumption, then transfers are used to equalize the marginal social value of consumption in each country (i.e., $\Psi\left(C_t^N\right)^{-\eta}=(1-\Psi)\left(C_t^S\right)^{-\eta}$).

Since utility flow is minimized during a disaster and since the social planner can always reduce world emissions, the optimal policy always avoids a disaster. Moreover, as demonstrated in Appendix B.6, if the discount rate ρ is sufficiently small and the inverse elasticity of intertemporal substitution $\eta\leq 1$, then innovation in sector E switches to mostly clean innovation in

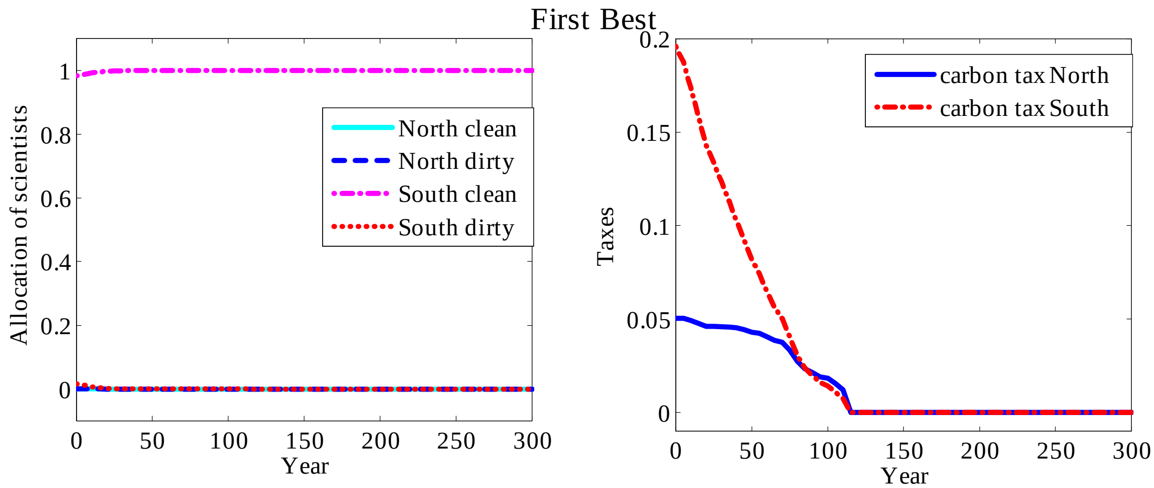

Figure 1: First-best policy. From left to right, figures: 1.A and 1.B.

finite time.[30] Indeed, in this case, the optimal policy maximizes the long-run growth rate. A switch to clean innovation allows the polluting sector to grow at a positive rate, while still avoiding a disaster. Moreover, long-run growth is maximized if each country innovates only in its own sector so that there is no overlap in the innovations they undertake. The difference in comparative advantage then becomes so large that both countries end up fully specializing. Since the dirty input becomes a negligible part of the production process, emissions vanish. With the law of motion (16), the quality of the environment reverts to $\overline{S}$—and the carbon tax reaches zero—in finite time.[31,32]

These results are illustrated in the numerical example (which satisfies $\eta = 1$). Figure 1.A shows that sector-E innovation switches to clean technologies (here immediately), and is rapidly only carried out in the South, since both countries rapidly fully specialize. This rapid full specialization results from a relatively large growth rate (2% a year), combined with a small difference in capital shares between the two sectors ($\alpha - \beta = 0.2$) and a small initial comparative advantage. Either imperfect mobility of factors, cross-sector or cross-country knowledge spillovers, or imperfect substitutability between domestic and foreign goods would have the effect of slowing down the specialization process. As shown in Figure 1.B, in both

[30] These are only sufficient conditions, and the optimal policy is likely to feature a switch to clean innovations also when $\eta > 1$.

[31] For an alternative law of motion where environmental regeneration decreases as the quality of the environment S_t approaches $\overline{S}$, then S_t reaches $\overline{S}$ only asymptotically. The optimal carbon tax may then not converge to 0 but it becomes irrelevant in the sense that a 0 carbon tax would only have a negligible effect on welfare.

[32] Interestingly, reducing the production of the polluting good creates a terms-of-trade effect which is beneficial to the country exporting the polluting good. Therefore, in the absence of redistribution concerns, this country is not necessarily the one where consumption is reduced the most by environmental policy.

countries, the *ad valorem* carbon tax declines and eventually reaches 0 as the environment recovers; it declines faster in the South where clean technologies catch up with dirty ones.

4.3 Second-Best

I now turn to the case where the social planner cannot implement any policy in the South, whose economy is in laissez-faire, and cannot transfer income from one country to another. Trade balance must be maintained at every point in time. The second-best policy is defined by the social planner maximizing (1) or (2) subject to the following constraints: (3) for the North and the South; constraints (4), (5), (7), (8), (9), (11), (15) and (13) for the North only; the environmental degradation constraint (16); the goods market–clearing constraints in both countries, which are now written as

$$C_{Yt}^{N} = Y_{Yt}^{N} + M_{Yt} \text{ and } C_{Yt}^{S} = Y_{Yt}^{S} - M_{Yt}, \text{ for } Y \in \{E, F\}, \tag{25}$$

where M_{Yt} denotes net imports of the North of good Y; the trade balance constraint

$$p_t M_{Et} + M_{Ft} = 0, \tag{26}$$

where $p_t \equiv p_{Et}/p_{Ft}$ is the international price ratio; and constraints describing the South's laissez-faire economy. These latter constraints (detailed in Appendix A.5) are: a consumer demand equation

$$\frac{\frac{\partial C^S}{\partial C_E^S}}{\frac{\partial C^S}{\partial C_F^S}} = \frac{\nu}{1-\nu}\left(\frac{C_{Ft}^S}{C_{Et}^S}\right)^{\frac{1}{\sigma}} = p_t; \tag{27}$$

offers equations in the South of the type

$$Y_{Et}^S = y_E^S\left(p_t, A_{Et}^S, A_{Ft}^S\right) \text{ and } Y_{Ft}^S = y_F^S\left(p_t, A_{Et}^S, A_{Ft}^S\right); \tag{28}$$

an emissions equation $Y_{dt}^S = \left(A_{dt}^S/A_{Et}^S\right)^{\varepsilon} Y_{Et}^S$; an equation that specifies the mass of scientists allocated to sector E,

$$s_{Et}^S = s_E^S\left(p_t, A_{dt}^S, A_{ct}^S, A_{Ft}^S\right); \tag{29}$$

and the resulting law of motion of aggregate productivity in the South:

$$\begin{aligned} A_{Ft}^S &= \left(1 + \kappa\left(1 - s_{Et}^S\right)^{\iota}\right)^{1-\gamma} A_{F(t-1)}^S, \\ A_{zt}^S &= \left(1 + \kappa\left(s_{zt}^S\left(s_{Et}^S, a_{t-1}^S\right)\right)^{\iota}\right)^{1-\gamma} A_{z(t-1)}^S, \text{ for } z \in \{c, d\}. \end{aligned} \tag{30}$$

The allocation between clean and dirty innovation s_{ct}^S, s_{dt}^S is uniquely determined by the total mass s_{Et}^S and the ratio $a_{t-1}^S \equiv \left(A_{c(t-1)}^S/A_{d(t-1)}^S\right)^{\varepsilon-1}$. For the problem to be well-defined, the South's equilibrium must be unique given the North's allocation. An argument similar to that of Appendix B.1 shows that it is the case when κ is sufficiently small and $\iota \geq 1/2$. (This is where the Ricardian case would pose a technical difficulty, with $\alpha = \beta$, even for a small κ, the South's equilibrium may not be uniquely defined.) This leads to the following result.

Proposition 4 *The second-best policy can be decentralized through a carbon tax in the North, research subsidies/taxes in the North, a subsidy for the use of all intermediates, and a trade tax.*

Proof. See Appendix A.5. ■

In this second-best scenario, the social planner uses the same instruments as before to address the inefficiencies in the North's economy: the environmental externality, the knowledge externality and the monopoly distortion. The trade tax,[33] b_t, allows the social planner to distort prices in the South thereby affecting the allocation of factors there. When the social planner maximizes (1), the optimal allocation satisfies:

$$\begin{aligned} & b_t \frac{\partial C^N}{\partial C_F^N} \left(p_t \frac{\partial y_E^S}{\partial p_t} + \frac{(1-\sigma) C_{Et}^S + Y_{Et}^S}{p_t \frac{C_{Et}^S}{C_{Ft}^S} + 1} \right) + \frac{M_{Et}}{p_t} \left(\frac{\partial C^S}{\partial C_{Et}^S} - \frac{\partial C^N}{\partial C_E^N} \right), \qquad (31) \\ = \ & \widehat{\omega}_t \xi^S \left(\frac{A_{dt}^S}{A_{Et}^S} \right)^{\varepsilon} \frac{\partial y_E^S}{\partial p_t} - \widehat{\phi}_t \frac{\partial s_{Et}^S}{\partial p_t} \end{aligned}$$

where $\widehat{\omega}_t$ is the shadow value of a unit of environmental quality at time t (in units of consumption at time t) and $\widehat{\phi}_t$ is the shadow value of moving an additional scientist in the South from sector F to sector E. This expression shows that the social planner imposes a wedge between relative prices in the North and in the South. Since $\partial C^N / \partial C_E^N = \partial C^S / \partial C_{Et}^S$ at equal relative prices, this wedge is generated by an environmental motive (the first term on the right-hand side) and an innovation motive (the second term). The first term is always positive. A positive trade tax on the polluting good E imposed by the North reduces its relative price in the South, which decreases its production there and hence emissions (which are not directly taxed). The second term is generally also positive as there is typically too much innovation in the polluting sector in the South ($\widehat{\phi}_t < 0$) for two reasons. First, more innovation in the polluting sector in the South leads to more emissions. Second, to avoid a disaster—which the social planner typically does—the South must at least asymptotically fully specialize in the nonpolluting sector (see Lemma 2), so that current innovations in the polluting sector will be of little use in the future. Because of their myopia, Southern innovators do not internalize this and their innovation efforts are tilted too much toward the polluting good. By reducing the production of the polluting good in the South, a positive trade tax moves Southern scientists

[33] For $\sigma < 1/2$, a trade tax may not lead to a single decentralized equilibrium, the North must then be able to pick the level of imports or exports that corresponds to the allocation it wishes to implement.

from sector E to sector F. Therefore, the trade tax is generally positive; it takes the form of a tariff when the North imports the polluting good and of an export subsidy otherwise.

For the maximization of (2), terms-of-trade matter and the optimal trade tax is modified in order to favor the country with the largest social marginal value of consumption. If the social planner cares only about the North ($\Psi = 1$), then this motive pushes toward a tariff when the North imports the polluting good and toward an export tax otherwise. If the social planner cares equally about both countries ($\Psi = 1/2$) but the South is poorer, then it pushes toward an import or an export subsidy.[34]

The next proposition further characterizes the optimal policy.

Proposition 5 *(i) Whenever doing so is feasible, the social planner avoids a disaster if the inverse elasticity of intertemporal substitution $\eta \geq 1$; or if $\eta < 1$ and the discount rate ρ is sufficiently low. The South must asymptotically be fully specialized in the nonpolluting sector F if initially clean technologies are less developed than dirty ones there ($A^S_{c0} \leq A^S_{d0}$).*
(ii) If $A^S_{c0} \leq A^S_{d0}$, if avoiding a disaster is feasible, if ρ is sufficiently small, and if either the inverse elasticity of intertemporal substitution $\eta \leq 1$ or the polluting and nonpolluting goods are strict complements ($\sigma < 1$), then there is a switch toward clean innovation in the North. The mass of scientists allocated to the dirty subsector tends to 0, and the mass of scientists allocated to clean technologies in the North is positive (asymptotically 1 for $\eta \leq 1$).

Proof. See Appendix B.9. ■

First, since the North cannot fully control the Southern economy, avoiding a disaster may not be feasible when S_0 is low. Yet, when it is feasible, a social planner will do it, if the elasticity of intertemporal substitution $\eta \geq 1$ (as then a disaster brings a utility of $-\infty$), or if $\eta < 1$ and the discount rate is sufficiently low (as then the social planner maximizes long-run utility growth).[35] The rest of statement (i) in the proposition is a direct consequence of Lemma 2. Avoiding a disaster can be done by either restricting the production of the polluting good or by switching to clean technologies. Statement (ii) specifies conditions under which the switch to clean innovation occurs. Importantly, these are only sufficient conditions. A switch to clean innovation, with the North asymptotically innovating only in clean technologies, maximizes long-run growth, which explains why this occurs for $\eta \leq 1$ and a low discount rate.[36] When

[34] Equation (31) is modified in the following way: $\widehat{\omega}_t$ and $\widehat{\phi}_t$ are shadow values in units of consumption in the North and the term $\frac{\partial C^S}{\partial C^S_{Et}}$ is now preceded by the shadow value of one unit of consumption in the South expressed in units of consumption in the North. It is now less straightforward to sign $\widehat{\phi}_t$, because the social value of moving a Southern scientist from one sector to the other now also reflects how it affects terms of trade.

[35] This is the only statement which depends on the assumption that $S = 0$ is an absorbing state. If it is not the case then a temporary disaster could be part of the optimal policy when $\eta < 1$.

[36] In that case, one can further shows that, in finite time, both countries fully specialize, the optimal trade tax reaches 0 and—since environmental quality can fully recover—the optimal carbon tax reaches 0.

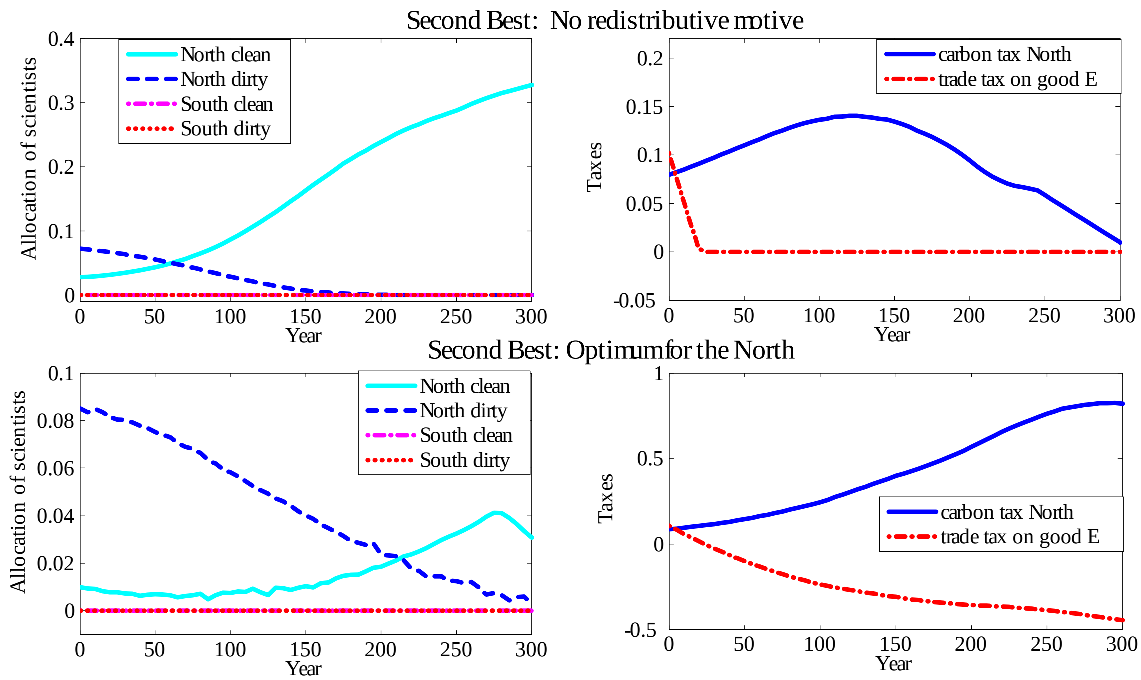

Figure 2: Second-best policies, when the social planner has no redistributive motive and when she cares only about the North. From left to right, top to bottom, figures 2.A, 2.B, 2.C and 2.D.

the two goods are strict complements ($\sigma < 1$), the mass of scientists innovating in clean technologies must be positive in the long-run to achieve positive utility growth, which a patient social planner always prefers to no utility growth.

These results are illustrated in Figure 2 which shows the second-best policies for the cases where the social planner maximizes (1) and (2) with $\Psi = 1$ (so that the North only cares about the welfare of its representative agent). Contrary to the first-best case, the North must now export the polluting good E in the long run. For these parameter values, a large trade tax on good E (see Figures 2.B and 2.D) ensures that, right from the first period, the South specializes in the nonpolluting sector F, and thus does not innovate at all in the polluting sector (see Figures 2.A and 2.C). Several factors explain this feature: the high emission rate in the South means that the South should specialize rapidly in sector F, the low initial comparative advantage of the South that the pattern of trade is easily reversed, and the smaller size of the South together with a small difference $\alpha - \beta$ in factor shares between the two sectors imply that full specialization in the South is reached quickly. With no redistributive motive, the switch from predominantly dirty to clean innovation in the polluting sector occurs after 65 years (Figure 2.A). The switch is delayed relative to the first-best because the North starts with a lower emission rate in the polluting sector, so that the initial temperature increase is lower, and because continuing to invest in dirty technologies helps the North build a large comparative advantage in the polluting sector. It occurs even later (after 215 years) when the North cares only about its own consumption: less innovation in the polluting sector improves the North's terms of trade, as it exports the polluting good, and in return allows for a delayed switch toward clean innovation. The amount of clean innovation increases over time and, beyond the time frame of the simulation, eventually reaches one when the North fully specializes in the polluting sector (in line with Proposition 5). In the optimum for the North case, the trade tax eventually becomes negative (it reaches -1 asymptotically) as the North eventually acquires the comparative advantage in the polluting sector, and a negative trade tax—an export tax here—improves the North's terms of trade. The later the switch to clean technologies, the more temperature eventually increases, so that the carbon tax is higher than in the first-best and even higher when the social planner cares more about consumption in the North than in the South (see Figures 2.B and 2.D).

4.4 Welfare costs

Table 1: Disaster and welfare cost (with no redistributive motive)

	Welfare cost
First-best	6.36%
Second-best	24.64%
Third-best	24.75%

Table 1 reports the welfare costs of the different policies to avoid climate change in the case where the social planner has no redistributive motive. The welfare cost is computed as the equivalent percentage loss of world consumption every period relative to the first-best case in a "miracle" scenario under which the dirty input would cease to pollute (i.e. $\xi^N = \xi^S = 0$ from the first period). The inability to intervene in the South sharply increases the welfare costs of climate change policy (they are 4 times as large). The reason is that reversing the pattern of comparative advantages leads to significant static costs in the first periods and to lower productivity levels in subsequent periods. Therefore unilateral intervention is possible here but a global one is much preferred.[37]

Table 1 also presents the case of a "third" best in which the North can implement a positive carbon tax and research subsidies/taxes but cannot implement trade, consumption, or production taxes. With the calibrated parameter values it is still possible to avoid disaster under such a policy. As stipulated in Remark 1, this is not always true. In fact, the welfare costs of dispensing with the trade tax are not large. Since the difference in initial comparative

[37] This increase in cost is almost entirely due to the environmental externality. In the miracle case there are also be some welfare costs from not being able to intervene in the South, since innovation there is not allocated optimally, but the costs are very small: 0.03 percent.

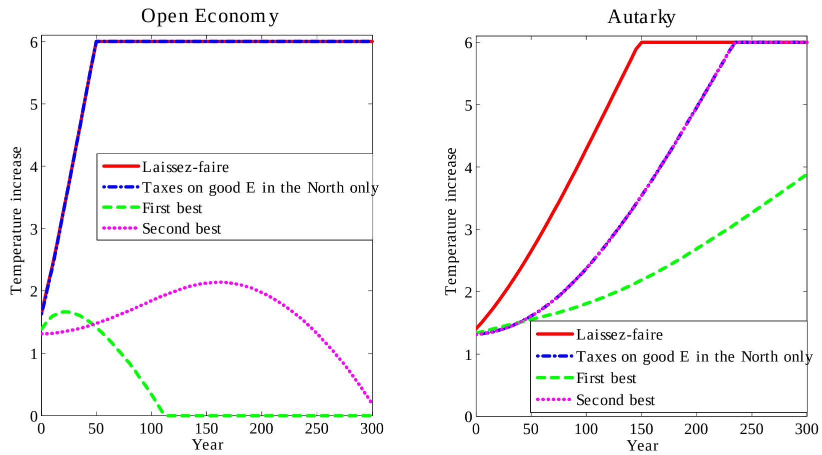

Figure 3: Temperature increase in open economy and in autarky (no redistribution concerns for the social planner). From left to right: figures 2.A and 2.B.

advantages is small, and since innovation is very effective in affecting technological levels, the North can quickly acquire a comparative advantage in the polluting sector without the help of a trade tax by innovating more in the polluting sector than in the second-best and implementing a low carbon tax initially. For the reasons explained above, the South quickly specializes in the non-polluting sector. Importantly though, this third-best policy still bears protectionist aspects since it indirectly subsidizes the production of the polluting good, which the North initially imported.

4.5 Trade and Directed Technical Change, Two Double-Edged Swords

Figure 3 shows the temperature increase for different policies when trade is allowed for and when the two countries are in autarky in laissez-faire and under various policies. Laissez-faire leads to an environmental disaster after 50 years for the open economy case—with log-utility this is a 100% welfare cost—but occurs later in autarky, since economic growth is lower in that case. Under free-trade and following proposition 1, no combination of a positive carbon tax or a tax on dirty research in the North can prevent an environmental disaster. Figures 3.A depicts the combination that minimizes CO_2 emissions ("Taxes on Good E in the North Only"), the curve is indistinguishable from the laissez-faire one, as it is not even possible to delay a disaster with such a policy when trade is allowed.[38] On the contrary, in autarky, such a policy can postpone the disaster for 85 years, as there is no pollution haven effect. The second-best curve in Figure 3.A shows how the appropriate unilateral intervention avoids an

[38]This is still the case if we change parameters and fix $A^S_{c0}/A^S_{d0} = A^N_{c0}/A^N_{d0}$ and $\xi^S = \xi^N$, such that North and South have the same initial emission rate.

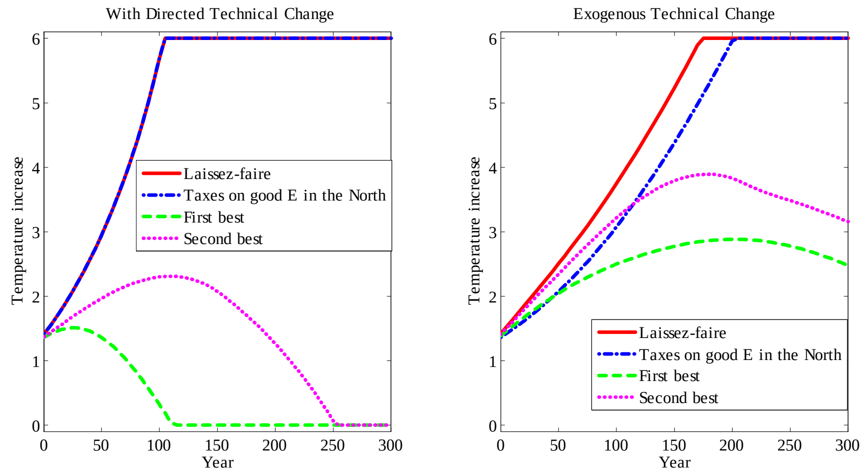

Figure 4: Temperature increase with and without directed technical change (no redistribution concerns for the social planner, different capital shares than in the baseline scenario: $\alpha = 0.7$, $\beta = 0.1$). From left to right: figures 3.A and 3.B.

environmental disaster, while adding the same instrument (research subsidies) does not affect emissions much in autarky (in Figure 3.A, the second-best refers to the maximization of (1), while in Figure 3.B, it is the combination of research subsidies and positive carbon tax which minimizes CO_2 emissions). Even in the first-best case temperatures increase more in autarky because the growth rate of clean technologies is lower than in the open economy scenario.[39] Overall, Figure 2 illustrates the double-edged nature of trade: without it, unilateral policies cannot prevent a disaster; but opening up to trade accelerates environmental degradation if the North does not undertake the appropriate policy.

Directed technical change (DTC) plays a similar role. To study it, I compare the current scenario with DTC to one in which the allocation of innovation is exogenous and equal in all subsectors ($s_{ct}^X = s_{dt}^X = s_{Ft}^X = 1/3$). With the calibrated values, however, Northern taxes on the polluting good cannot postpone the disaster even in the exogenous growth case. So as to better illustrate the impact of DTC, I perform the same exercise (still with no redistributive motive) but now assume that $\alpha = 0.7$ and $\beta = 0.1$. (A larger difference in capital shares limits the pollution haven effect in a static model and therefore better illustrates how it is amplified by the innovation response.) Figure 4 shows that DTC accelerates the disaster under laissez-faire because it accelerates the economy's growth rate. With DTC, a disaster cannot be postponed with a combination of positive carbon tax and tax on dirty research in the North: in fact the

[39] Comparing the increase in temperature between the first-best and the second-best in the open economy case is interesting. The temperature is initially higher in the first-best because the South's emission rate is higher, but since the switch to clean innovation occurs sooner, temperatures decrease faster.

combination that minimizes CO_2 emissions is no taxes. Without DTC, it is possible to delay an environmental disaster for up to 30 years with this policy: the reason is that without DTC, only the classic pollution haven effect exists but not the dynamic pollution haven effect that this paper emphasizes. The second-best policy can avoid a disaster both with and without DTC in this case, but without DTC, the increase in temperature is much larger—despite a much lower growth rate—and a large trade tax must be permanently maintained in order to reverse the pattern of trade.

In fact, there are parameters for which a disaster cannot be avoided *without* DTC with unilateral policies, regardless of initial environmental quality. To avoid a disaster, the North should be able to produce the polluting good relying mostly on clean technologies and to force the South to asymptotically fully specialize in the non-polluting sector. Therefore, the most extreme way for the North to do this is to produce only the non-polluting good (with nearly only the clean input) and to give it for free to the South. Yet, without DTC, the ratio of relative productivities stay the same over time, so if initially the South has a large comparative advantage in the non-polluting sector, or if clean technologies in the North are sufficiently backward, this is not enough to push the South towards specialization and to avoid a disaster. This thought experiment demonstrates that innovation's ability to affect comparative advantage is essential to deriving the previous results.

5 Knowledge Diffusion

I now relax the assumption that productivity improvements are entirely country specific. In reality, some productivity improvements cross borders, mitigating the amplification of comparative advantage effect, which partly drove the previous results.[40] This brings into question the robustness of the previous analysis. Here I consider an extension of the original model whereby the lagging country can benefit from the diffusion of innovations produced in the leading country. Appendix B.13 provides a full analysis of a different extension of the model where innovating firms are global. In both cases the main lessons from Section 3 still hold.

To model knowledge diffusion in a simple way, I assume that, at the beginning of every period, the country with the less advanced average productivity in a given sector can partially catch up exogenously. That is, before any innovation occurs, the producer of intermediate i in

[40] One should not expect all productivity improvement to cross borders easily, because some may be embedded in capital or may depend on local know-how. Dechezleprêtre et al. (2011) suggest that clean technology transfers between developing and developed countries exist but are limited: for the period 2000–2005, only 15 percent of the clean innovations were patented in more than one country; this is slightly less than the share (17 percent) of all innovations patented in more than one country.

sector $z \in \{c, d, F\}$ gains access to the technology:

$$\overline{A_{zit}^{X}} = \max\left(\left(\frac{A_{z(t-1)}^{(-X)}}{A_{z(t-1)}^{X}}\right)^{\delta}, 1\right) A_{zi(t-1)}^{X},$$

where $\delta \in [0, 1]$ measures the strength of the technological diffusion. This equality then delivers the following law of motion for aggregate productivity:

$$A_{zt}^{X} = \left(1 + \kappa\left(s_{zt}^{X}\right)^{\iota}\right)^{1-\gamma} \max\left(\left(\frac{A_{z(t-1)}^{(-X)}}{A_{z(t-1)}^{X}}\right)^{\delta}, 1\right) A_{z(t-1)}^{X}$$

for $z \in \{c, d, F\}$. Under this formulation, the ratio of the technological levels across countries cannot diverge: as soon as one country acquires a strong advantage over the other, the catching-up process ensures that this difference is reduced in the next period.

In particular, Northern policies that foster clean innovation in the North now also increase the productivity of clean Southern technologies. In fact, they may even put the South on a clean innovation track: if, in some period, pre-innovation clean Southern technologies become more advanced than dirty ones (i.e., for some t, $\overline{A_{ct}^{S}} > \overline{A_{dt}^{S}}$), market forces will induce more clean than dirty innovations in the South from that period onwards. Preventing a disaster does not necessarily involve pushing the South toward specializing in the nonpolluting sector any more; it can also be achieved by ensuring a switch to clean innovation there. That transition will occur as soon as more scientists are allocated to clean technologies in the North than to dirty technologies in the South for a sufficient amount of time. Clean innovation in the North and dirty innovation in the South enter a horse race, which determines whether or not the polluting sector will be produced in a clean way in the long-run. Who wins depends on the policies that the North allows for and on the pattern of comparative advantage, much as in Section 3. Hence the intuitions developed there still apply, and the broad results are less different than one might expect. In particular, I can show the following.

Proposition 6 *Assume that initially: (i) technologies are sufficiently close to each other across countries, that κ is sufficiently small, and that the spillovers δ are sufficiently strong; (ii) the South is relatively well-endowed in capital, $K^S/L^S > K^N/L^N$; and (iii) clean technologies are sufficiently less advanced than dirty ones (A_{c0}^{S}/A_{d0}^{S} sufficiently small). Then no combination of a carbon tax and a tax on dirty research in the North can prevent a disaster irrespective of how high $\overline{S}$ is.*

Proof. See Appendix B.12. ■

This proposition mirrors Proposition 1. Assumptions (i) imply that technological levels remain sufficiently close to each other across countries. When combined with assumption (ii), this ensures that the South maintains its comparative advantage in the polluting sector. Assumption (iii) plays the same role as in Proposition 1, ensuring that, when the South has the comparative advantage in the polluting sector, it innovates there more than does the North. As a result, the South keeps its comparative advantage in the polluting sector, and since a carbon tax in the North can only reinforce this comparative advantage, there are more Southern scientists innovating in dirty technologies than Northern scientists innovating in clean ones. Hence Southern clean productivity $\overline{A_{ct}^S}$ never catches up, so a switch in the South to clean innovation never occurs. The Northern market for the polluting good is too small to generate enough clean innovations.

As before, a temporary combination of clean research subsidies and a tariff can prevent a disaster for sufficiently large initial environmental quality (i.e., Proposition 2 still holds). Clean research subsidies can reallocate Northern innovation to clean technologies, and a tariff can limit Southern innovation in dirty technologies. Then $\overline{A_{ct}^S}$ grows faster than $\overline{A_{dt}^S}$, and a switch to clean innovation eventually occurs in the South.

Remark 1 is not robust when clean and dirty inputs are imperfect substitutes ($\varepsilon < \infty$). Sufficiently large clean research subsidies in the North are now enough to avoid a disaster if the initial environmental quality is sufficiently high—even when final consumption is Cobb–Douglas in the polluting and the nonpolluting goods ($\sigma = 1$). Because clean technologies in the South grow at the same rate as in the North, the Southern ratio of clean to dirty technologies cannot approach zero if the North allocates all its scientists to clean technologies. In that case, the mass of Southern scientists allocated to dirty technologies remains bounded away from one even if the South specializes in the polluting sector. Eventually, the North wins the horse race, $\overline{A_{ct}^S}$ becomes greater than $\overline{A_{dt}^S}$ at some t, and a switch to clean innovation must occur.[41]

The structures of the first-best and second-best policies are broadly similar, but the trade tax and subsidies for research must take knowledge spillovers into account, and the second-best policy may prevent a disaster with a South exporting the polluting good in the long-run. In addition, the welfare costs of unilateral intervention are typically lower than in the absence of knowledge spillovers. Indeed, the reversal in comparative advantages, which generated the large welfare cost in the no-spillover case, may not happen, and even if it does, is much less costly since the South ends up benefiting from the technologies that the North had developed. Accordingly, Table 2 shows the welfare costs in the first-best and the second-best cases in the presence of knowledge spillovers ($\delta = 0.4$ and $\delta = 0.8$, and the social planner maximizes (1)):

[41] This analysis relies on the innovation function κs^{ι} satisfying the Inada condition. If, the innovation function were instead $\kappa\left((s+\Upsilon)^{\iota} - \Upsilon^{\iota}\right)$ with $\Upsilon > 0$ then, for clean technologies initially sufficiently less advanced than dirty ones, all Southern innovation would be devoted to dirty technologies when the South is fully specialized in the polluting sector, and Remark 1 would hold. Similarly, when $\varepsilon = \infty$, and initial endowments are sufficiently far apart, the South's full specialization in the nonpolluting sector can continue maintained indefinitely, so that a disaster cannot be avoided without a trade tax.

the welfare costs of the first-best policy are very similar to those in Table 1, but those of the second-best policy are now much lower.[42]

Table 2: Welfare cost in the presence of knowledge spillovers

	$\delta = 0.4$	$\delta = 0.8$
First-best	5.71%	5.95%
Second-best	6.92%	6.58%

To some extent, technological diffusion itself is a parameter that can be affected by policy: laxer intellectual property rights, direct financing of projects abroad, or migrations of skilled workers could all contribute to a faster diffusion of technology. Therefore, according to the analysis presented here, the diffusion of clean technologies from North to South renders a tariff less necessary, and significantly reduces the costs of a unilateral policy intervention.

With the inclusion of knowledge spillovers, one can now add a nontradeable sector to the economy without changing the results. Assume that final consumption is a Cobb-Douglas aggregate of nontradeable and tradeable goods. Both are produced according to (3), with the associated goods E and F (and the associated subsector c and d), but for the nontradeable good, the polluting and non-polluting inputs must be sourced locally. The same intermediates are used whether the good is produced for the tradeable or nontradeable sector. In the no-spillovers case, it is impossible to prevent a disaster because Southern emissions from nontradeables will increase unboundedly regardless of Northern policy. In the spillover case, however, the same results as before still apply: if Northern clean technologies win the horse race over Southern dirty technologies, then nontradeables in the South will also begin using clean inputs more intensively, so that emissions can decrease in both countries.

6 Conclusion

This paper shows that when evaluating the long-term consequences of unilateral environmental policies, it is essential to consider their impact on the allocation of innovation within the polluting sector between technologies (clean/dirty) and between countries (intervening/non-intervening). The stylized theoretical model allows to develop two main intuitions, which hold whether knowledge spillovers are present or not. First, the pollution haven effect becomes worse in a dynamic setting. Positive taxes on the polluting sector in the North risk placing the economy on a path that leads to the South having a comparative advantage in the polluting sector. This leads to the relocation of not only the production of the polluting good but also of innovation in the polluting sector, which dramatically hampers the benefits of such a policy on worldwide emissions. The South innovates more in dirty technologies, while innovation in clean

[42] Here, the reversal of comparative advantage still takes place in the presence of knowledge spillovers because the difference in factor endowments is small.

technologies in the North does not take off because the market share for the polluting sector is reduced. Second, sustainable growth can be achieved without cooperation from the South, but this requires a somewhat protectionist industrial policy (with clean research subsidies and perhaps a trade tax) in order to ensure that there is more clean than dirty innovation worldwide. Such a policy can guarantee that either the North acquires a long-run comparative advantage in the polluting sector, or, with knowledge spillovers, that a switch towards clean innovation occurs in the South.

Therefore, in practice, the paper argues that unilateral environmental policies should be devoted to developing clean technologies, which have the potential to reduce emissions in the North, but also in the South either through technology diffusion or by slowing down the move of polluting industries there. These policies should be thought of as transitory until a satisfactory global agreement is reached. The paper aims at analyzing what "well-intentioned" countries should do until then, and therefore, as a first step, it has taken as given the absence of such an agreement. The next logical step is to analyze why some countries are willing to participate and others are not, and how unilateral policies shape their intentions in the long-run. This is, however, a complex issue as the incentive to sign a global agreement depends on the benefit that the reluctant country would get from it. Unilateral policies can affect this potential benefit in at least three dimensions: by decreasing environmental damages which discourages a reluctant country from joining (the free-rider problem), by developing clean technologies which can diffuse and therefore reduce the costs of an environmental policy for the reluctant country, and by affecting comparative advantages and therefore the impact of a potential environmental policy on the reluctant country's terms of trade.

Another aspect left for future research is to study policies that directly boost technological diffusion. Such policies (e.g., "clean development mechanism") are already part of climate negotiations. Studying technological diffusion would, however, require a proper model of intellectual property rights (IPR), whose impact on emissions is a priori ambiguous. On the one hand, laxer IPR could lead to more rapid diffusion of clean technologies to the South, which would facilitate the switch to a clean path there. On the other hand, they might reduce the incentives to develop Northern clean technologies in the first place. Finally, the paper's results suggest that directed technical change renders Southern emissions much more responsive to Northern policies in the long run. This finding calls into question existing estimates of the carbon leakage rate obtained from static models. Therefore, in order to properly evaluate the impact of local carbon taxes and carbon tariffs, it would be useful to integrate directed technical change into numerical models of the world economy.

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A Appendix A: Main Proofs of the Paper

A.1 Characterization of the equilibrium in a given period

For this subsection I consider the economy at a given period. To avoid future repetition, I allow for a carbon tax τ^X and do not generally impose free-trade. First I derive the aggregate production functions in each sector given local good prices. Second I solve for good prices in autarky and free trade, and then I characterize the pattern of specialization in free trade. Finally, I derive the allocation of innovation in function of local good prices.

A.1.1 Deriving aggregate production function

Assume that good E is produced in country X, then both subsectors c and d must be active. The maximization problem for producers in subsector z leads to the demand function for capital and labor in assembly of good z:

$$r^X K_z^X = (1-\gamma)\, \alpha p_z^X Y_z^X \text{ and } w^X L_z^X = (1-\gamma)(1-\alpha)\, p_z^X Y_z^X \tag{A.1}$$

and the demand for intermediates:

$$\varphi_{zi}^X = \gamma p_z^X A_{zi}^X x_{zi}^{\gamma-1} \left(\left(K_z^X\right)^\alpha \left(L_z^X\right)^{1-\alpha} \right)^{1-\gamma}, \tag{A.2}$$

with φ_{zi}^X the consumer price of intermediate i. From (9), the cost of producing one unit of intermediate is given by $\psi \left(\frac{r^X}{\alpha}\right)^\alpha \left(\frac{w^X}{1-\alpha}\right)^{1-\alpha}$. Monopolists maximizes profits by imposing a mark-up $1/\gamma$ on their costs, but the consumption of intermediates is subsidized at a rate $1-\gamma$, therefore intermediates are priced at marginal costs:

$$\varphi_{zi}^X = \psi \left(\frac{r^X}{\alpha}\right)^\alpha \left(\frac{w^X}{1-\alpha}\right)^{1-\alpha}. \tag{A.3}$$

The production of intermediates is then given by:

$$x_{zi}^X = \left(\frac{p_z^X \gamma}{\psi} \left(\frac{\alpha}{r^X}\right)^\alpha \left(\frac{1-\alpha}{w^X}\right)^{1-\alpha} \right)^{\frac{1}{1-\gamma}} \left(A_{zi}^X\right)^{\frac{1}{1-\gamma}} \left(K_z^X\right)^\alpha \left(L_z^X\right)^{1-\alpha}, \tag{A.4}$$

and factor demands in the production of intermediate i in sector z follows:

$$K_{zi}^X = \left(\frac{\alpha}{r^X} \frac{w^X}{1-\alpha} \right)^{1-\alpha} \psi x_{zi}^X \text{ and } L_{zi}^X = \left(\frac{r^X}{\alpha} \frac{1-\alpha}{w^X} \right)^\alpha \psi x_{zi}^X. \tag{A.5}$$

Plugging (A.1) and (A.4) into (8), I get the price of good z as:

$$p_z^X = \frac{1}{\zeta A_z^X} \left(\frac{r^X}{\alpha}\right)^\alpha \left(\frac{w^X}{1-\alpha}\right)^{1-\alpha}, \tag{A.6}$$

Now, profit maximization by producers of good E leads to the demand function:

$$\frac{Y_c^X}{Y_d^X} = \left(\frac{p_c^X}{(1+\tau^X)\,p_d^X}\right)^{-\varepsilon}, \tag{A.7}$$

and the price of good E is given by $p_E^X = \left(\left(p_c^X\right)^{1-\varepsilon} + \left(1+\tau^X\right)^{1-\varepsilon}\left(p_d^X\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$. Define $A_E^X \equiv \left(\left(A_c^X\right)^{\varepsilon-1} + \left(\left(1+\tau^X\right)^{-1}A_d^X\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}$ (which generalizes the notation given in the text for the case $\tau^X \neq 0$), then using (A.6), one obtains:

$$p_E^X = \frac{1}{\zeta A_E^X}\left(\frac{r^X}{\alpha}\right)^{\alpha}\left(\frac{w^X}{1-\alpha}\right)^{1-\alpha}. \tag{A.8}$$

This relationship holds if country X produces good E, otherwise the equality is replaced by:

$$p_E^X \leq \frac{1}{\zeta A_E^X}\left(\frac{r^X}{\alpha}\right)^{\alpha}\left(\frac{w^X}{1-\alpha}\right)^{1-\alpha}.$$

Similarly in sector F,

$$p_F^X \leq \frac{1}{\zeta A_F^X}\left(\frac{r^X}{\alpha}\right)^{\beta}\left(\frac{w^X}{1-\alpha}\right)^{1-\beta},$$

with equality if good F is produced in country X.

Note that (A.7) gives:

$$Y_d^X = \left(\frac{\left(1+\tau^X\right)^{-1}A_d^X}{A_E^X}\right)^{\varepsilon} Y_E^X, \tag{A.9}$$

which directly leads to the expression for the emission rate. Combining (A.1), (A.4), (A.5), (A.7), and (A.8), one gets that total factor employment in sector E satisfies:

$$K_E^X = \left(\frac{\alpha}{r^X}\frac{w^X}{1-\alpha}\right)^{1-\alpha}\frac{1-\delta^X}{\zeta A_E^X}Y_E^X, \tag{A.10}$$

$$L_E^X = \left(\frac{r^X}{\alpha}\frac{1-\alpha}{w^X}\right)^{\alpha}\frac{1-\delta^X}{\zeta A_E^X}Y_E^X, \tag{A.11}$$

with $\zeta \equiv \frac{\gamma^\gamma(1-\gamma)^{1-\gamma}}{\psi^\gamma}$ and $\delta^X \equiv \tau^X\left(\frac{A_d^X}{A_E^X}\right)^{\varepsilon-1}\left(1+\tau^X\right)^{-\varepsilon} \in [0,1)$. Combining these two expressions and following the same strategy in sector F, one gets:

$$Y_{Et}^X = \frac{\zeta A_E^X}{1-\delta^X}\left(K_{Et}^X\right)^{\alpha}\left(L_{Et}^X\right)^{1-\alpha} \text{ and } Y_{Ft}^X = \zeta A_F^X\left(K_{Ft}^X\right)^{\beta}L\left(_{Ft}^X\right)^{1-\beta}, \tag{A.12}$$

so that $A_E^X/\left(1-\delta^X\right)$ measures the effective average productivity of sector E in country X in presence of a tax τ^X. These equation translates into (18) in laissez-faire. When both sectors are active, taking the ratio of (A.8) and the equivalent expression for p_F^X, one can express the capital rent to wage ratio as

$$\frac{r^X}{w^X} = \left(\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{\beta^\beta (1-\beta)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{A_E^X}{A_F^X}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{p_E^X}{p_F^X}\right)^{\frac{1}{\alpha-\beta}}.$$

Plugging this expression into (A.10) and (A.11) and the equivalent equations in sector F, and using factor market clearing (11), one gets a system of two equations with two unknowns $\left(Y_E^X, Y_F^X\right)$ that can be solved as:

$$\begin{aligned} Y_E^X &= \frac{\zeta}{\alpha-\beta} \left(\frac{\beta^{\beta\alpha} (1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha} (1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{A_E^X}{1-\delta^X} \\ &\quad \times \left(\left(\frac{\alpha^\alpha (1-\alpha)^{(1-\alpha)}}{\beta^\beta (1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} (1-\beta) \left(\frac{p_E^X}{p_F^X}\frac{A_E^X}{A_F^X}\right)^{\frac{1-\alpha}{\alpha-\beta}} K^X - \beta L^X \left(\frac{p_E^X}{p_F^X}\frac{A_E^X}{A_F^X}\right)^{\frac{-\alpha}{\alpha-\beta}} \right) \end{aligned} \tag{A.13}$$

$$\begin{aligned} Y_F^X &= \frac{\zeta}{\alpha-\beta} \left(\frac{\beta^{\beta\alpha} (1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha} (1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}} A_F^X \\ &\quad \times \left(\alpha \left(\frac{p_E^X}{p_F^X}\frac{A_E^X}{A_F^X}\right)^{\frac{-\beta}{\alpha-\beta}} L^X - \left(\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{\beta^\beta (1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} (1-\alpha) \left(\frac{p_E^X}{p_F^X}\frac{A_E^X}{A_F^X}\right)^{\frac{1-\beta}{\alpha-\beta}} K^X \right). \end{aligned} \tag{A.14}$$

A.1.2 Equilibrium price

Consumer maximization leads to $\frac{p_E^X}{p_F^X} = \frac{\nu}{1-\nu}\left(\frac{C_F^X}{C_E^X}\right)^{\frac{1}{\sigma}}$. In autarky this translates into: $\frac{Y_E^X}{Y_F^X} = \left(\frac{\nu}{1-\nu}\right)^\sigma \left(\frac{p_E^X}{p_F^X}\right)^{-\sigma}$, which combined with (A.13) and (A.14), defines the equilibrium autarky price uniquely (given technologies) since $\frac{Y_E^X}{Y_F^X}$ is increasing in $\frac{p_E^X}{p_F^X}$, and the right-hand side decreasing. More specifically, one gets that the autarky price must satisfy:

$$\begin{aligned} &\left(\frac{p_E^X}{p_F^X}\right)^\sigma \frac{A_E^X}{1-\delta^X} \begin{pmatrix} \left(\frac{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} (1-\beta) \left(\frac{p_E^X}{p_F^X}\frac{A_E^X}{A_F^X}\right)^{\frac{1-\alpha}{\alpha-\beta}} K^X \\ -\beta L^X \left(\frac{p_E^X}{p_F^X}\frac{A_E^X}{A_F^X}\right)^{\frac{-\alpha}{\alpha-\beta}} \end{pmatrix} \\ = &\left(\frac{\nu}{1-\nu}\right)^\sigma A_F^X \begin{pmatrix} \alpha \left(\frac{p_E^X}{p_F^X}\frac{A_E^X}{A_F^X}\right)^{\frac{-\beta}{\alpha-\beta}} L^X \\ -\left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} (1-\alpha) \left(\frac{p_E^X}{p_F^X}\frac{A_E^X}{A_F^X}\right)^{\frac{1-\beta}{\alpha-\beta}} K^X \end{pmatrix} \end{aligned} \tag{A.15}$$

It is direct to check that in the absence of any tax, the relative autarky price of good E over good F is higher in the North than in the South if and only if (19) is satisfied, so that in this case the North imports good E (as claimed in the text).

Under free-trade, the equilibrium price ratio is the same in both countries and satisfies

$$\frac{p_E}{p_F} = \frac{\nu}{1-\nu}\left(\frac{C_F^X}{C_E^X}\right)^{\frac{1}{\sigma}} = \frac{\nu}{1-\nu}\left(\frac{Y_F^N + Y_F^S}{Y_E^N + Y_E^S}\right)^{\frac{1}{\sigma}}, \tag{A.16}$$

which similarly defines uniquely the price ratio given technologies (as Y_F^X is decreasing in the price ratio and Y_E^X increasing). When both countries produce both goods, one can use (A.13) and (A.14) to get:

$$\left(\frac{p_E}{p_F}\right)^{\sigma}\left(\begin{array}{c} \frac{A_E^N(\tau^N)}{1-\delta^N(\tau^N)}\left(\left(\frac{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_E}{p_F}\frac{A_E^N}{A_F^N}\right)^{\frac{1-\alpha}{\alpha-\beta}}K^N - \beta\left(\frac{p_E}{p_F}\frac{A_E^N}{A_F^N}\right)^{\frac{-\alpha}{\alpha-\beta}}L^N\right) \\ +\frac{A_E^S(\tau^S)}{1-\delta^S(\tau^S)}\left(\left(\frac{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_E}{p_F}\frac{A_E^S}{A_F^S}\right)^{\frac{1-\alpha}{\alpha-\beta}}K^S - \beta\left(\frac{p_E}{p_F}\frac{A_E^S}{A_F^S}\right)^{\frac{-\alpha}{\alpha-\beta}}L^S\right) \end{array}\right) \tag{A.17}$$

$$= \left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\begin{array}{c} A_F^N\left(\alpha\left(\frac{p_E}{p_F}\frac{A_E^N}{A_F^N}\right)^{\frac{-\beta}{\alpha-\beta}}L^N - \left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_E}{p_F}\frac{A_E^N(\tau^N)}{A_F^N}\right)^{\frac{1-\beta}{\alpha-\beta}}K^N\right) \\ +A_F^S\left(\alpha\left(\frac{p_E}{p_F}\frac{A_E^S}{A_F^S}\right)^{\frac{-\beta}{\alpha-\beta}}L^S - \left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_E}{p_F}\frac{A_E^S}{A_F^S}\right)^{\frac{1-\beta}{\alpha-\beta}}K^S\right) \end{array}\right)$$

A.1.3 Pattern of specialization in free trade

I now derive the full pattern of specialization in free trade. I introduce the notations $\widetilde{K^X} \equiv \frac{(A_E^X)^{\frac{1-\beta}{\alpha-\beta}}}{(A_F^X)^{\frac{1-\alpha}{\alpha-\beta}}}K^X$ and $\widetilde{L^X} \equiv \frac{(A_F^X)^{\frac{\alpha}{\alpha-\beta}}}{(A_E^X)^{\frac{\beta}{\alpha-\beta}}}L^X$, which are a measure of "effective endowments". Using (A.13), (A.14) and (A.17), assuming that both countries produce both goods, the condition $Y_E^X > 0$ translates into

$$\frac{\widetilde{K^X}}{\widetilde{L^X}} > \frac{\beta}{1-\beta}\frac{\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L^X}}{\widetilde{K^X}}\right)^{\alpha-\beta}\right)^{1-\sigma}(1-\alpha)\left(\widetilde{K^N}+\widetilde{K^S}\right)+(1-\beta)\left(\frac{\widetilde{K^N}}{1-\delta^N}+\frac{\widetilde{K^S}}{1-\delta^S}\right)}{\left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L^X}}{\widetilde{K^X}}\right)^{\alpha-\beta}\right)^{1-\sigma}\alpha\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\widetilde{L^N}+\widetilde{L^S}\right)+\beta\left(\frac{\widetilde{L^N}}{1-\delta_N}+\frac{\widetilde{L^S}}{1-\delta^S}\right)}, \tag{A.18}$$

and $Y_F^X > 0$ into:

$$\frac{\widetilde{K^X}}{\widetilde{L^X}} < \frac{\alpha}{1-\alpha}\frac{\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L^X}}{\widetilde{K^X}}\right)^{\alpha-\beta}\right)^{1-\sigma}(1-\alpha)\left(\widetilde{K^N}+\widetilde{K^S}\right)+(1-\beta)\left(\frac{\widetilde{K^N}}{1-\delta^N}+\frac{\widetilde{K^S}}{1-\delta^S}\right)}{\left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L^X}}{\widetilde{K^X}}\right)^{\alpha-\beta}\right)^{1-\sigma}\alpha\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\widetilde{L^N}+\widetilde{L^S}\right)+\beta\left(\frac{\widetilde{L^N}}{1-\delta^N}+\frac{\widetilde{L^S}}{1-\delta^S}\right)}, \tag{A.19}$$

Therefore, conditions (A.18) and (A.19) define the set of endowments, productivity and taxes for which there is incomplete specialization in both countries.

Assume now that country X fully specializes in sector E, but country $-X$ does not. Using (A.12), production of good E in X is given by: $Y_E^X = \frac{\zeta}{1-\delta^X}\widetilde{K^X}^{\alpha}\widetilde{L^X}^{1-\alpha}$. Combining this expression with (A.16) and (A.13) and (A.14) for country $-X$, delivers an implicit equation for the price ratio, which can be used to show that the condition $Y_E^{(-X)} > 0$ is equivalent to:

$$\left(\frac{\widetilde{K^{-X}}}{\widetilde{L^{-X}}}\right)^{(\alpha-\beta)\sigma} \widetilde{K^{-X}}^{\beta}\widetilde{L^{-X}}^{1-\beta} > \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma}\left(\frac{1-\nu}{\nu}\right)^{\sigma}\frac{\widetilde{K^X}^{\alpha}\widetilde{L^X}^{1-\alpha}}{1-\delta^X}. \tag{A.20}$$

Therefore country X fully specializes in sector E and country $-X$ produces both goods when the opposite of (A.18) and (A.20) hold.

Similarly if country X specializes in sector F, one gets $Y_F^X = \zeta\widetilde{K^X}^{\beta}\widetilde{L^X}^{1-\beta}$ and the condition $Y_{F(-X)} > 0$ can be written as:

$$\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\sigma}\widetilde{K^X}^{\beta}\widetilde{L^X}^{1-\beta} < \left(\frac{1-\nu}{\nu}\right)^{\sigma}\left(\frac{\widetilde{K^{-X}}}{\widetilde{L^{-X}}}\right)^{(\alpha-\beta)(1-\sigma)}\frac{\widetilde{K^{-X}}^{\beta}\widetilde{L^{-X}}^{1-\beta}}{1-\delta^{-X}}. \tag{A.21}$$

Country X fully specializes in sector F and country $-X$ produces both goods when the opposite of (A.19) and (A.21) hold.

Finally the case where country X fully specializes in E while country $-X$ fully specializes in F corresponds to the opposite of (A.21).and the opposite of (A.20). For future use, it is convenient to express these two conditions with the actual endowments and productivities as:

$$\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\sigma} A_F^{-X}\left(K^{-X}\right)^{\beta}\left(L^{-X}\right)^{1-\beta} \geq \left(\frac{1-\nu}{\nu}\right)^{\sigma}\left(\frac{K^X}{L^X}\right)^{(\alpha-\beta)(1-\sigma)}\frac{\left(K^X\right)^{\beta}\left(L^X\right)^{1-\beta}}{1-\delta^X}\left(A_F^X\right)^{\sigma}\left(A_E^X\right)^{1-\sigma} \tag{A.22}$$

$$\left(A_F^{-X}\right)^{1-\sigma}\left(A_E^{-X}\right)^{\sigma}\left(\frac{L^{-X}}{K^{-X}}\right)^{(\alpha-\beta)(1-\sigma)}\left(K^{-X}\right)^{\alpha}\left(L^{-X}\right)^{1-\alpha} \leq \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\frac{1-\nu}{\nu}\right)^{\sigma}\frac{A_E^X\left(K^X\right)^{\alpha}\left(L^X\right)^{1-\alpha}}{1-\delta^X}. \tag{A.23}$$

These endowment sets have no overlap. Moreover, in each case the relative price of good E over good F is uniquely defined, therefore in free trade and for given technologies, the equilibrium is unique.

A.1.4 Equilibrium profits and innovation decision

Using (8) and (A.4), I can express intermediates production in sector $z \in \{c, d\}$ as: $x_{zi}^X = \frac{p_z^X\gamma}{\psi}\left(\frac{\alpha}{r^X}\right)^{\alpha}\left(\frac{1-\alpha}{w^X}\right)^{1-\alpha}\left(\frac{A_{zi}^X}{A_z^X}\right)^{\frac{1}{1-\gamma}} Y_{zt}^X$, combining this with (A.6) and (A.3) gives (20). Using (A.7), this translates into:

$$\pi_{cit}^X = (1-\gamma)\left(\frac{A_{cit}^X}{A_{ct}^X}\right)^{\frac{1}{1-\gamma}}\frac{\left(A_{ct}^X\right)^{\varepsilon-1}}{\left(A_{ct}^X\right)^{\varepsilon-1}+\left(\left(1+\tau_t^X\right)^{-1}A_{dt}^X\right)^{\varepsilon-1}}p_{Et}Y_{Et}^X, \tag{A.24}$$

$$\pi_{dit}^X = (1-\gamma)\left(\frac{A_{dit}^X}{A_{dt}^X}\right)^{\frac{1}{1-\gamma}}\frac{\left(1+\tau_t^X\right)^{-\varepsilon}\left(A_{dt}^X\right)^{\varepsilon-1}}{\left(A_{ct}^X\right)^{\varepsilon-1}+\left(\left(1+\tau_t^X\right)^{-1}A_{dt}^X\right)^{\varepsilon-1}}p_{Et}Y_{Et}^X. \tag{A.25}$$

The same reasoning in sector F gives:

$$\pi_{Fit}^{X}=(1-\gamma)\left(\frac{A_{Fit}^{X}}{A_{Ft}^{X}}\right)^{\frac{1}{1-\gamma}}p_{Ft}^{X}Y_{Ft}^{X}. \tag{A.26}$$

To avoid repetition, I let both countries implement a tax q_t^X on the wages of scientists in the dirty subsector. Combining the first order conditions with respect to the number of scientists in the clean and dirty subsector (and assuming that some production takes place in sector E in country X) delivers the allocation of scientists within sector E as:

$$\frac{\left(s_{ct}^{X}\right)^{1-\iota}\left(1+\kappa\left(s_{ct}^{X}\right)^{\iota}\right)}{\left(s_{dt}^{X}\right)^{1-\iota}\left(1+\kappa\left(s_{dt}^{X}\right)^{\iota}\right)}=\frac{p_{ct}^{X}Y_{ct}^{X}}{p_{dt}^{X}Y_{dt}^{X}}=\frac{\left(1-q_t^X\right)\left(1+\tau_t^X\right)^{\varepsilon}\left(A_{ct}^{X}\right)^{\varepsilon-1}}{\left(A_{dt}^{X}\right)^{\varepsilon-1}}, \tag{A.27}$$

where the second equality arises from (A.7) and (A.9). Similarly, combining the first order condition with respect to the number of scientists in sector F and subsector d, I get:

$$\frac{\left(s_{dt}^{X}\right)^{1-\iota}\left(1+\kappa\left(s_{dt}^{X}\right)^{\iota}\right)}{\left(s_{Ft}^{X}\right)^{1-\iota}\left(1+\kappa\left(s_{Ft}^{X}\right)^{\iota}\right)}=\frac{\left(1-q_t^X\right)\left(1+\tau_t^X\right)^{-\varepsilon}\left(A_{dt}^{X}\right)^{\varepsilon-1}}{\left(A_{ct}^{X}\right)^{\varepsilon-1}+\left(\left(1+\tau_t^X\right)^{-1}A_{dt}^{X}\right)^{\varepsilon-1}}\frac{p_{Et}^{X}Y_{Et}^{X}}{p_{Ft}^{X}Y_{Ft}^{X}}. \tag{A.28}$$

With $\tau_t^X=0$, these two last equations give (21) and (22).

A.2 Proof of Lemma 2

Since the emission rate per unit of the polluting good cannot decrease in the South in the absence of policy when $A_{c0}^S\leq A_{d0}^S$, the South's production of the polluting good must remain bounded in order to avoid a disaster. Therefore a disaster cannot be avoided if the South fully specializes in sector E, but it may be avoided (with sufficiently large initial environmental quality) if the South fully specializes in sector F in finite time.

Assume now that there is not full specialization in the South, so that there are an infinite number of periods where the South produces both goods (and in the following I restrict attention to those periods). Rewriting (A.13) with $\tau_t^S=0$ and $p_t=p_{Et}^S/p_{Ft}$, production in sector E is given by

$$\begin{aligned} Y_{Et}^{S} &= \frac{\zeta A_{Et}^{S}}{(\alpha-\beta)}\left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}}\left(p_t\frac{A_{Et}^{S}}{A_{Ft}^{S}}\right)^{\frac{-\alpha}{\alpha-\beta}} \\ &\times\left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(p_t\frac{A_{Et}^{S}}{A_{Ft}^{S}}\right)^{\frac{1}{\alpha-\beta}}K^S-\beta L^S\right). \end{aligned} \tag{A.29}$$

Therefore to keep Y_{Et}^S bounded, $\left(p_t\frac{A_{Et}^{S}}{A_{Ft}^{S}}\right)^{\frac{1}{\alpha-\beta}}$ must be bounded, with either $\lim\left(p_t\frac{A_{Et}^{S}}{A_{Ft}^{S}}\right)^{\frac{1}{\alpha-\beta}}=$

$\left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}}\frac{\beta L^S}{(1-\beta)K^S}$ or with A_{Et}^S bounded. Using (A.14), Y_{Ft}^S can be rewritten as:

$$\begin{aligned} Y_{Ft}^S &= \frac{\zeta A_{Ft}^S}{(\alpha-\beta)}\left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}}\left(p_t\frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{-\beta}{\alpha-\beta}} \\ &\quad\times\left(\alpha L^S-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(p_t\frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}}K^S\right). \end{aligned} \tag{A.30}$$

Combining these two expressions with (A.28) implies that the allocation of innovation in the South must satisfy

$$\frac{\kappa'\left(s_{dt}^S\right)}{\left(1+\kappa\left(s_{dt}^S\right)\right)}\frac{1+\kappa\left(1-s_{dt}^S\right)}{\kappa'\left(1-s_{dt}^S\right)}\frac{\left(A_{dt}^S\right)^{\varepsilon-1}}{\left(A_{ct}^S\right)^{\varepsilon-1}+\left(A_{dt}^S\right)^{\varepsilon-1}}=\frac{\alpha L^S-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(p_t\frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}}K^S}{\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(p_t\frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}}K^S-\beta L^S}. \tag{A.31}$$

If $\lim\left(p_t\frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}}\neq\left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}}\frac{\beta L^S}{(1-\beta)K^S}$, s_{dt}^S cannot tend toward 0 therefore A_{Et}^S must become unbounded, which leads to a disaster. Therefore avoiding a disaster in this case requires that $\lim\left(p_t\frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}}=\left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}}\frac{\beta L^S}{(1-\beta)K^S}$, so that asymptotically all factors in the South (scientists, capital, labor) must be allocated to sector F.

Moreover, denoting M_{Et} and M_{Ft} net imports from the North, (A.16) leads to:

$$\frac{p_{Et}^S Y_{Et}^S}{p_{Ft}^S Y_{Ft}^S}=\frac{\nu}{1-\nu}\left(\frac{Y_{Et}^S-M_{Et}}{Y_{Ft}^S-M_{Ft}}\right)^{-\frac{1}{\sigma}}\frac{Y_{Et}^S}{Y_{Ft}^S}.$$

If the North does not export the polluting good ($M_{Et}\geq 0$), then the right-hand side (RHS) of the above expression is greater than $\frac{\nu}{1-\nu}\left(\frac{Y_{Ft}^S}{Y_{Et}^S}\right)^{\frac{1-\sigma}{\sigma}}$. But avoiding a disaster requires that the LHS tends toward 0, while $\frac{Y_{Ft}^S}{Y_{Et}^S}$ becomes unbounded: this yields a contradiction, so the North must export the polluting good E.

A.3 Proof of Proposition 1

This proofs proceeds in three steps. First I introduce some notations that are useful for the remainder of the proof. Second, I show that innovation amplifies comparative advantage in laissez-faire. Finally, I consider the case where the North implements a carbon tax and/or a tax on dirty research and prove Proposition 1.

A.3.1 Notation and properties

As in Appendix B.1, I will use the notation $\widetilde{\kappa}(s)\equiv\kappa s^{\iota}$. Following equations (A.27) and (A.28), the equilibrium allocation of innovation (when there may be taxes on the polluting good) obeys:

$$f\left(s_{Et}^X,\left(\left(1+\tau_t^X\right)\frac{A_{c(t-1)}^X}{A_{d(t-1)}^X}\right)^{\varepsilon-1},\frac{1-q_t^X}{1+\tau_t^X}\right)=\frac{p_{Ft}Y_{Ft}^X}{p_{Et}Y_{Et}^X}, \tag{A.32}$$

where the function f is defined as:

$$f\left(s_E,a,\widetilde{q}\right)\equiv\frac{1+\widetilde{\kappa}\left(1-s_E\right)}{\widetilde{\kappa}'\left(1-s_E\right)}\frac{\widetilde{\kappa}'\left(s_c\right)\left(1+\widetilde{\kappa}\left(s_c\right)\right)^{(\varepsilon-1)(1-\gamma)-1}a+\widetilde{q}\widetilde{\kappa}'\left(s_d\right)\left(1+\widetilde{\kappa}\left(s_d\right)\right)^{(\varepsilon-1)(1-\gamma)-1}}{2\left(\left(1+\widetilde{\kappa}\left(s_c\right)\right)^{(\varepsilon-1)(1-\gamma)}a+\left(1+\widetilde{\kappa}\left(s_d\right)\right)^{(\varepsilon-1)(1-\gamma)}\right)},$$

where s_c and s_d depend on $\left(s_E,a,\widetilde{q}\right)$ and are defined by $s_c+s_d=s_E$ and

$$\widetilde{\kappa}'\left(s_c\right)\left(1+\widetilde{\kappa}\left(s_c\right)\right)^{(\varepsilon-1)(1-\gamma)-1}a=\widetilde{q}\widetilde{\kappa}'\left(s_d\right)\left(1+\widetilde{\kappa}\left(s_d\right)\right)^{(\varepsilon-1)(1-\gamma)-1}.$$

The function f represents the ratio of the marginal benefit of an additional scientist in sector F scaled by sector F revenues over the same quantity in sector E.

I also define $g\left(s_E,a,\widetilde{q}\right)\equiv\left(\left(1+\widetilde{\kappa}\left(s_c\right)\right)^{(\varepsilon-1)(1-\gamma)}a+\left(1+\widetilde{\kappa}\left(s_d\right)\right)^{(\varepsilon-1)(1-\gamma)}\right)/\left(a+1\right)$, where s_c and s_d are functions of $\left(s_E,a,\widetilde{q}\right)$ as above. $g^{\frac{1}{\varepsilon-1}}$ represents the growth rate of average productivity in sector E. f and g satisfy the following properties:

Lemma A.1 *For κ small enough, $f\left(s_E,a,1\right)=f\left(s_E,a^{-1},1\right)$, $f|_{\widetilde{q}=1}$ is decreasing in a on $(0,1)$ and $f\left(s_E,a,\widetilde{q}\right)<f\left(s_E,a,1\right)$ if $\widetilde{q}\in(0,1)$. $g\left(s_E,a,1\right)=g\left(s_E,a^{-1},1\right)$, g is increasing in s_E and increasing in $\widetilde{q}$ on $(0,1)$ and $g|_{\widetilde{q}=1}$ is decreasing in a on $(0,1)$.*

Proof. The symmetry properties are obvious. It is also direct to show that for κ sufficiently small (so that $\widetilde{\kappa}'\left(s\right)\left(1+\widetilde{\kappa}\left(s\right)\right)^{(\varepsilon-1)(1-\gamma)-1}$ is decreasing) g is increasing in s_E and $\widetilde{q}$ on $(0,1)$. Furthermore, $\frac{\partial g}{\partial a}=\frac{\left(1+\widetilde{\kappa}(s_c)\right)^{(\varepsilon-1)(1-\gamma)}-\left(1+\widetilde{\kappa}(s_d)\right)^{(\varepsilon-1)(1-\gamma)}}{(a+1)^2}<0$ for $a<1$ since then $s_d>s_c$. In other words, for a given amount of scientists in sector E, average productivity grows faster when the gap between the two subsectors is large. As for f, one can derive

$$\frac{\partial f}{\partial a}|_{\widetilde{q}=1}=\frac{\left(1-\iota\right)\left(1+\widetilde{\kappa}\left(1-s_E\right)\right)}{\widetilde{\kappa}'\left(1-s_E\right)\kappa\iota\left(\frac{1+\widetilde{\kappa}(s_c)}{\widetilde{\kappa}'(s_c)}+\frac{(1+\widetilde{\kappa}(s_d))}{\widetilde{\kappa}'(s_d)}\right)^2}\left(\frac{1}{s_c^\iota}-\frac{1}{s_d^\iota}\right)\frac{\partial s_d}{\partial a},$$

$\frac{\partial s_d}{\partial a}<0$ and $s_d>s_c$ if and only if $a>1$ (for κ small). Therefore, in this case, f is decreasing in a on $(0,1)$. Moreover, when $\widetilde{\kappa}'\left(s\right)\left(1+\widetilde{\kappa}\left(s\right)\right)^{(\varepsilon-1)(1-\gamma)-1}$ is decreasing in s, the numerator of f is increasing in $\widetilde{q}$ (for $\widetilde{q}\in(0,1)$), yet the denominator is also increasing in $\widetilde{q}$ (as g increases in $\widetilde{q}$), but for κ small, the variations in the denominator are negligible, so that $f\left(s_E,a,\widetilde{q}\right)<f\left(s_E,a,1\right)$ if $\widetilde{q}\in(0,1)$. ■

A.3.2 Amplification of comparative advantages in laissez-faire

This subsection derives conditions under which the laissez-faire equilibrium leads to the amplification of comparative advantages and full specialization in both countries.

Lemma A.2 *Denote* $a_t^X \equiv \left(A_{ct}^X/A_{dt}^X\right)^{\varepsilon-1}$*. Consider a laissez-faire economy and assume, that country* X *initially has a (weak) comparative advantage in sector* E $\left(A_{E0}^X/A_{F0}^X\right)^{\frac{1}{\alpha-\beta}} K^X/L^X \geq \left(A_{E0}^{-X}/A_{F0}^{-X}\right)^{\frac{1}{\alpha-\beta}} K^{-X}/L^{-X}$ *and that (i)* $\min\left(a_0^X, \left(a_0^X\right)^{-1}\right)$ *and* $\min\left(a_0^{-X}, \left(a_t^{-X}\right)^{-1}\right)$ *are sufficiently small and the previous inequality is strict or (ii)* $\min\left(a_t^X, \left(a_t^X\right)^{-1}\right) < \min\left(a_t^{-X}, \left(a_t^{-X}\right)^{-1}\right)$*; then at all points in time:* $s_{Et}^X > s_{Et}^{-X}$*. Furthermore,* A_{Et}^S/A_{Et}^N *and* A_{Ft}^N/A_{Ft}^S *tend toward infinity, and both countries eventually fully specialize.*

Proof. Without loss of generality, assume that at time $t \geq 1$, $A_{c(t-1)}^X \leq A_{d(t-1)}^X$ in both countries, $\left(A_{E(t-1)}^S/A_{F(t-1)}^S\right)^{\frac{1}{\alpha-\beta}} K^S/L^S \geq \left(A_{E(t-1)}^N/A_{F(t-1)}^N\right)^{\frac{1}{\alpha-\beta}} K^N/L^N$ and either a_{t-1}^S and a_{t-1}^N are both negligible—and the previous inequality is strict— or $a_{t-1}^S < a_{t-1}^N$. Then (A.27) and (A.28) imply:

$$f\left(s_{Et}^X, a_{t-1}^X, 1\right) = \frac{p_{Ft}Y_{Ft}^X}{p_{Et}Y_{Et}^X}.$$

Using equations (A.13) and (A.14), when neither country is fully specialized, the equilibrium can be summarized by three equations:

$$f\left(s_{Et}^X, a_{t-1}^X, 1\right) \tag{A.33}$$
$$= \frac{\alpha - (1-\alpha)\left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\beta^\beta(1-\beta)^{(1-\beta)}}\frac{p_{Et}}{p_{Ft}}\right)^{\frac{1}{\alpha-\beta}}\left(\frac{\left(g\left(a_{t-1}^X, s_E^X, 1\right)\right)^{\frac{1}{\varepsilon-1}}}{\left(1+\widetilde{\kappa}\left(s_{Ft}^X\right)\right)^{1-\gamma}}\right)^{\frac{1}{(\alpha-\beta)}}\left(\frac{A_{E(t-1)}^X}{A_{F(t-1)}^X}\right)^{\frac{1}{\alpha-\beta}}\frac{K^X}{L^X}}{(1-\beta)\left(\frac{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}{\beta^\beta(1-\beta)^{(1-\beta)}}\frac{p_{Et}}{p_{Ft}}\right)^{\frac{1}{\alpha-\beta}}\left(\frac{\left(g\left(a_{t-1}^X, s_E^X, 1\right)\right)^{\frac{1}{\varepsilon-1}}}{\left(1+\widetilde{\kappa}\left(s_{Ft}^X\right)\right)^{1-\gamma}}\right)^{\frac{1}{(\alpha-\beta)}}\left(\frac{A_{E(t-1)}^X}{A_{F(t-1)}^X}\right)^{\frac{1}{\alpha-\beta}}\frac{K^X}{L^X} - \beta},$$

for $X \in \{N, S\}$ and the equation determining the price ratio $\frac{p_{Ft}}{p_{Et}}$.

If $a_{t-1}^S < a_{t-1}^N$, the left-hand side (LHS) of (A.33) is lower for the North than for the South at equal s_{Et}^X (see Lemma A.1). Moreover, at equal s_{Et}^X, $\left(\frac{\left(g\left(a_{t-1}^X, s_E^X, 1\right)\right)^{\frac{1}{\varepsilon-1}}}{\left(1+\widetilde{\kappa}\left(s_{Ft}^X\right)\right)^{1-\gamma}}\right)^{\frac{1}{(\alpha-\beta)}}\left(\frac{A_{E(t-1)}^X}{A_{F(t-1)}^X}\right)^{\frac{1}{\alpha-\beta}}\frac{K^X}{L^X}$ would be higher for the South than for the North, since $\left(A_{E(t-1)}^S/A_{F(t-1)}^S\right)^{\frac{1}{\alpha-\beta}} K^S/L^S \geq \left(A_{E(t-1)}^N/A_{F(t-1)}^N\right)^{\frac{1}{\alpha-\beta}} K^N/L^N$ and g is decreasing in a. For given prices, both the LHS and the right-hand side (RHS) are decreasing in s_{Et}^X, but for sufficiently small κ, the LHS decreases faster, therefore $s_{Et}^S > s_{Et}^N$.

Similarly if both a_{t-1}^S and a_{t-1}^N are negligible (relative to the difference in comparative advantage), $s_{dt}^X \simeq s_{Et}^X$, $f\left(a_{t-1}^X, s_{Et}^X, 1\right) \simeq \frac{1+\widetilde{\kappa}\left(s_{Ft}^X\right)}{\widetilde{\kappa}'\left(s_{Ft}^X\right)} \frac{\widetilde{\kappa}'\left(s_{Et}^X\right)}{1+\widetilde{\kappa}\left(s_{Et}^X\right)}$ and

$$
\begin{aligned}
& \frac{\left(\left(1+\widetilde{\kappa}\left(s_{ct}^X\right)\right)^{(\varepsilon-1)(1-\gamma)}\left(A_{c(t-1)}^X\right)^{\varepsilon-1}+\left(1+\widetilde{\kappa}\left(s_{dt}^X\right)\right)^{(\varepsilon-1)(1-\gamma)}\left(A_{d(t-1)}^X\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{\left(1+\widetilde{\kappa}\left(s_{Ft}^X\right)\right)^{1-\gamma} A_{F(t-1)}^X} \\
\simeq \quad & \frac{\left(1+\widetilde{\kappa}\left(s_{Et}^X\right)\right)^{(1-\gamma)} A_{E(t-1)}^X}{\left(1+\widetilde{\kappa}\left(s_{Ft}^X\right)\right)^{1-\gamma} A_{F(t-1)}^X},
\end{aligned}
$$

so that, following a similar reasoning, $\left(\frac{A_{E(t-1)}^S}{A_{F(t-1)}^S}\right)^{\frac{1}{\alpha-\beta}} \frac{K^S}{L^S} > \left(\frac{A_{E(t-1)}^N}{A_{F(t-1)}^N}\right)^{\frac{1}{\alpha-\beta}} \frac{K^N}{L^N}$ leads to $s_{Et}^S > s_{Et}^N$.

Therefore, in both cases $A_{Et}^S/A_{Et}^N > A_{E(t-1)}^S/A_{E(t-1)}^N$ and $A_{Ft}^N/A_{Ft}^S > A_{F(t-1)}^N/A_{F(t-1)}^S$. Note that $a_{t-1}^X < a_t^X$, so if both a_{t-1}^N an a_{t-1}^S are negligible, a_t^N and a_t^S will be negligible too. Moreover, $\frac{1+\widetilde{\kappa}(s_A(a,s_E))}{1+\widetilde{\kappa}(s_a(a,s_E))}$ is increasing in s_E and decreasing in a, so if $a_{t-1}^N \geq a_{t-1}^S$ and $s_{Et}^S \geq s_{Et}^N$ then $a_t^N \geq a_t^S$. The analysis extends directly to the case where one country specializes. By induction, this is enough to show that $s_{Et}^S > s_{Et}^N$ and, that A_{Et}^S/A_{Et}^N and A_{Ft}^N/A_{Ft}^S are increasing.

To conclude that A_{Et}^S/A_{Et}^N and A_{Ft}^N/A_{Ft}^S tend to infinity, I further need to show that s_{Et}^N and s_{Et}^S do not converge towards each other. Suppose they do. Then, either $\left(\frac{A_{E(t-1)}^S}{A_{F(t-1)}^S}\right)^{\frac{1}{\alpha-\beta}} \frac{K^S}{L^S}$ and $\left(\frac{A_{E(t-1)}^N}{A_{F(t-1)}^N}\right)^{\frac{1}{\alpha-\beta}} \frac{K^N}{L^N}$ also converge toward each other (which is impossible as the ratio of this term is initially weakly greater than 1 and strictly increasing); or both s_{Et}^N and s_{Et}^S tend toward the same corner solution. This implies that, in both countries, $\frac{p_{Ft}}{p_{Et}} \frac{Y_{Ft}^X}{Y_{Et}^X}$ either tend toward 0 or toward infinity. Both can be ruled out: in the Cobb-Douglas case $\frac{p_{Ft}}{p_{Et}} \frac{Y_{Ft}^N+Y_{Ft}^S}{Y_{Et}^N+Y_{Et}^S} = \frac{1-\nu}{\nu}$, and when $\sigma < 1$, innovation favors the most backward sector preventing all scientists from innovating in the same sector in both countries asymptotically. This establishes that A_{Et}^S/A_{Et}^N and A_{Ft}^N/A_{Ft}^S tend to infinity.

Finally, I show that, full specialization must occur. Using the expressions (A.13) and (A.14), avoiding full specialization in both countries asymptotically requires that $\frac{p_{Ft}}{p_{Et}} \frac{A_{Ft}^N}{A_{Et}^N}$ remains bounded (from $Y_{Et}^N \geq 0$) and similarly $\frac{p_{Et}}{p_{Ft}} \frac{A_{Et}^S}{A_{Ft}^S}$ remain bounded. Taking the product of the two, this leads toward $\frac{A_{Ft}^N}{A_{Et}^N} \frac{A_{Et}^S}{A_{Ft}^S}$ bounded which is a contradiction. Therefore at least one country fully specializes. For the sake of the argument assumes that the South fully specializes in sector E. If this is the case, note that asymptotically A_{Et}^S must grow at the rate $(1+\kappa)^{1-\gamma} - 1$ (since eventually all scientists are in the dirty sector there). Then to avoid full specialization in the North in finite time, one must keep (from (A.20)):

$$\left(\frac{A_{Ft}^N}{A_{Et}^S}\right)^{1-\sigma}\left(\frac{K^N}{L^N}\right)^{(\alpha-\beta)\sigma}\left(K^N\right)^{\beta}\left(L^N\right)^{1-\beta} > \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma}\left(\frac{1-\nu}{\nu}\right)^{\sigma}\left(\frac{A_{Et}^S}{A_{Et}^N}\right)^{\sigma}\left(K^S\right)^{\alpha}\left(L^S\right)^{1-\alpha},$$

where $0<\sigma\leq 1$. A_{Et}^S/A_{Et}^N grows exponentially, while A_{Ft}^N/A_{Et}^S cannot grow asymptotically since A_{Et}^S asymptotically grows at the fastest rate. Therefore, satisfying this inequality in the long-run is impossible and the North must also fully specialize. A similar reasoning applies to the case where the North specializes first. Overall this shows that full specialization is reached in finite time.[43] ■

A.3.3 Case where the North implements taxes on the polluting good.

First period. Consider that the initial situation satisfies the assumptions of Proposition 1, here I show that $s_{E1}^N < s_{E1}^S$. Recall that the allocation of innovation in the North is given by (A.32). Define now $a_{t-1}^N \equiv \left(\left(1+\tau_t^N\right)A_{c(t-1)}^N/A_{d(t-1)}^N\right)^{\varepsilon-1}$. Following Lemma A.1, $f\left(s_{E1}, a_0^N, \frac{1-q_1^N}{1+\tau_1^N}\right) < f\left(s_{E1}, a_0^N, 1\right)$, further, using (A.13) and (A.14), $p_{Ft}Y_{Ft}^N/\left(p_{Et}Y_{Et}^N\right)$ is increasing in τ at given price ratio p_{Ft}/p_{Et} and technological levels. As a result, the logic of the proof of Lemma A.2 fully applies provided that $\min\left(a_0^N,\left(a_0^N\right)^{-1}\right) < a_0^S$. Since $A_{c0}^N \leq A_{d0}^N$, this is the case unless $\left(1+\tau_1^N\right) > \left(A_{d0}^N/A_{c0}^N\right)/\left(A_{d0}^S/A_{c0}^S\right)$. Yet, for A_{c0}^S/A_{d0}^S sufficiently small, a carbon tax that satisfies such an inequality must be very large large. This results in a large difference in comparative advantage between the North and the South, so that the logic of Lemma A.2 still applies (but with a_0^S negligible relative to the difference in comparative advantage). Therefore $s_{E1}^N < s_{E1}^S$. A tax on dirty research and a carbon tax further distort the allocation of innovation for a given mass of scientists in sector E, so that: $\frac{A_{E1}^S}{A_{F1}^S}\frac{A_{F1}^N}{\left(\left(A_{c1}^N\right)^{\varepsilon-1}+\left(A_{d1}^N\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}} > \frac{A_{E0}^S}{A_{F0}^S}\left(\frac{A_{E0}^N}{A_{E0}^N}\right)$.

Following periods. To establish that $s_{Et}^S > s_{Et}^N$ in all periods by induction, I need to show that if for $\tau\in[1,t-1]$, $s_{E\tau}^S > s_{E\tau}^N$ then $\min\left(a_{t-1}^N, a_{t-1}^N\right) > a_{t-1}^S$ or a_{t-1}^S is negligible relative to the difference in comparative advantages. First, note that as long as $A_{d(t-1)}^N \geq A_{c(t-1)}^N$, then every period $A_{c(t-1)}^N/A_{d(t-1)}^N \geq A_{c(t-1)}^S/A_{d(t-1)}^S$, since less scientists are allocated to sector E in the North and the allocation is tilted toward the clean subsector. As above, a large carbon

[43] In principle, each country may also specialize in its sector in turn, but this situation can only happen if during some periods $\left(\frac{A_{Ft}^N}{A_{Et}^S}\right)^{1-\sigma}\left(\frac{K^N}{L^N}\right)^{(\alpha-\beta)\sigma}\left(K^N\right)^{\beta}\left(L^N\right)^{1-\beta} > \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma}\left(\frac{1-\nu}{\nu}\right)^{\sigma}\left(\frac{A_{Et}^S}{A_{Et}^N}\right)^{\sigma}\left(K^S\right)^{\alpha}\left(L^S\right)^{1-\alpha}$ and during others $\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\sigma}\left(\frac{A_{Ft}^N}{A_{Ft}^S}\right)^{\sigma}\left(K^N\right)^{\beta}\left(L^N\right)^{1-\beta} < \left(\frac{1-\nu}{\nu}\right)^{\sigma}\left(\frac{A_{Et}^S}{A_{Ft}^N}\right)^{1-\sigma}\left(K^S\right)^{\alpha}\left(L^S\right)^{1-\alpha}\left(\frac{L^S}{K^S}\right)^{\sigma(\alpha-\beta)}$. Since these inequalities must be flipped for certain periods, it must be the case that all the terms grow at the same rate, which contradicts the result that $\frac{A_{Et}^S}{A_{Et}^N}/\frac{A_{Ft}^S}{A_{Ft}^N}$ must tend towards infinity. Therefore this case is ruled out too.

tax would be necessary to induce $\min\left(a_{t-1}^N, a_{t-1}^N\right) < a_{t-1}^S$, but then it would have a significant impact on the pattern of comparative advantage.

Now assume that $A_{c(t-1)}^N \geq A_{d(t-1)}^N$, and that $a_{t-1}^N < a_{t-1}^S$. The latter can be achieved if either $\left(A_{d(t-1)}^N / A_{c(t-1)}^N\right)^{\varepsilon-1}$ and a_{t-1}^S are close to each other, or, if τ is large. Now with a_0^S is sufficiently small, and since $s_{E\tau}^N < s_{E\tau}^S$ for every $\tau < t$, it would take a large number of periods for $\left(A_{d(t-1)}^N / A_{c(t-1)}^N\right)^{\varepsilon-1}$ and a_{t-1}^S to become close to each other. Over such a time period, the difference in comparative advantage would increase and a_{t-1}^S would be small relative to it. Moreover, as before, a large τ would have a direct impact on the pattern of comparative advantages, unless $\left(A_{d(t-1)}^N / A_{c(t-1)}^N\right)^{\varepsilon-1}$ is small which can only be achieved after a large number of periods, at which point the difference in comparative advantages would be large relative to a_{t-1}^S. In all cases, a_{t-1}^S is small relative to the difference in comparative advantage. Overall, this establishes that $s_{Et}^S > s_{Et}^N$ every period.

Reaching full specialization. Here as well, A_{Et}^S / A_{Et}^N and A_{Ft}^N / A_{Ft}^S grow unboundedly. From (A.13) and (A.14), this necessarily leads to specialization in at least one country. Assume that there is full specialization in sector E in the South, so that asymptotically A_{Et}^S must grow at the rate $(1+\kappa)^{1-\gamma} - 1$. Then avoiding full specialization in the North in finite time requires to keep (from (A.23)):

$$\left(A_{Ft}^N\right)^{1-\sigma} \left(A_{Et}^N\right)^{\sigma} \left(K^N\right)^{\alpha} \left(L^N\right)^{1-\alpha} \geq \left(\frac{\beta^\alpha (1-\beta)^{(1-\alpha)}}{\alpha^\alpha (1-\alpha)^{(1-\alpha)}}\right)^{\sigma} \left(\frac{1-\nu}{\nu}\right)^{\sigma} A_{Et}^S \left(K^S\right)^{\alpha} \left(L^S\right)^{1-\alpha},$$

which is impossible. Similarly if the North fully specializes in sector F, avoiding specialization in the South is also impossible. Therefore, both countries fully specialize, the emissions in the South necessarily grow unbounded and a disaster occurs.

A.4 Proof of Proposition 3

I first solve for the problem of maximizing (1), I define the Lagrange parameters (with the corresponding constraints in parentheses): λ_t^X (3), λ_{Ft}^X (4), λ_{Et}^X (7), λ_{zt}^X (8), φ_{zit}^X (9), φ_{Fit}^X (5), η_{Kt}^X (11) for capital, η_{Lt}^X (11) for labor, θ_{Et} (12) in sector E, θ_{Ft} (12) in sector F, ω_t (16), μ_{zit}^X (13), υ_t^X (15), in addition the social planner faces the constraints: $0 \leq Y_{Et}^X$ and $0 \leq Y_{Ft}^X$, with Lagrange parameters: ι_{Et}^X, ι_{Ft}^X. Taking the first order condition with respect to Y_{Ft}^X and Y_{Et}^X gives:

$$\lambda_{Ft}^X = \theta_{Ft} + \iota_{Ft}^X \text{ and } \lambda_{Et}^X = \theta_{Et} + \iota_{Et}^X.$$

Defining $u\left(C_t^W, S_t\right) \equiv \frac{\left(\nu(S_t) C_t^W\right)^{1-\eta}}{1-\eta}$ with $C_t^W \equiv C_t^N + C_t^S$, the first order conditions with respect to C_t^N and C_t^S lead to:

$$\frac{1}{(1+\rho)^t} \frac{\partial u}{\partial C}\left(C_t^W, S_t\right) = \frac{\nu\left(S_t\right)^{1-\eta}}{(1+\rho)^t} \left(C_t^W\right)^{-\eta} = \lambda_t^X \equiv \lambda_t.$$

First order conditions with respect to C_{Et}^X and C_{Ft}^X give:

$$\lambda_t \nu \left(C_{Et}^X\right)^{-\frac{1}{\sigma}} \left(\nu \left(C_{Et}^X\right)^{\frac{\sigma-1}{\sigma}} + (1-\nu)\left(C_{Ft}^X\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \theta_{Et}, \tag{A.34}$$

$$\lambda_t (1-\nu) \left(C_{Ft}^X\right)^{-\frac{1}{\sigma}} \left(\nu \left(C_{Et}^X\right)^{\frac{\sigma-1}{\sigma}} + (1-\nu)\left(C_{Ft}^X\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \theta_{Ft}. \tag{A.35}$$

θ_{Et}/λ_t and θ_{Ft}/λ_t can be interpreted as consumer prices in terms of units of welfare. To emphasize this interpretation, I denote $\widehat{p}_{Et} = \theta_{Et}/\lambda_t$ and $\widehat{p}_{Ft} = \theta_{Et}/\lambda_t$. I then get:

$$\frac{\widehat{p}_{Et}}{\widehat{p}_{Ft}} = \frac{\nu}{1-\nu}\left(\frac{C_{Ft}^X}{C_{Et}^X}\right)^{\frac{1}{\sigma}} = \frac{\nu}{1-\nu}\left(\frac{C_{Ft}^N + C_{Ft}^S}{C_{Et}^N + C_{Et}^S}\right)^{\frac{1}{\sigma}},$$

which is equivalent to the equilibrium condition (A.16). Taking the first order condition with respect to Y_{Ft}^X and Y_{Et}^X gives:

$$\lambda_{Ft}^X = \theta_{Ft} + \iota_{Ft}^X \text{ and } \lambda_{Et}^X = \theta_{Et} + \iota_{Et}^X,$$

so that when production of good $Y \in \{E, F\}$ takes place: $\lambda_{Yt}^X = \theta_{Yt}$. Defining $\widehat{\varphi}_{zit}^X \equiv \varphi_{zit}^X/\lambda_t$ and $\widehat{p}_{zt}^X \equiv \lambda_{zt}^X/\lambda_t$, which can be interpreted as the price of intermediate x_{zi}^X and of input Y_z^X, the first order condition with respect to x_{zit}^X gives:

$$\widehat{\varphi}_{zit}^X = \gamma \widehat{p}_{zt}^X A_{zit}^X \left(x_{zit}^X\right)^{\gamma-1}\left(\left(K_{zt}^X\right)^{\alpha}\left(L_{zt}^X\right)^{1-\alpha}\right)^{1-\gamma},$$

which is the same as (A.2). Combining the first order conditions with respect to K_{zit}^X and L_{zit}^X further gives

$$\widehat{\varphi}_{zit}^X = \frac{\psi\left(\widehat{r}_t^X\right)^{\alpha}\left(\widehat{w}_t^X\right)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}},$$

where $\widehat{r}_t^X \equiv \eta_{Kt}^X/\lambda_t$ and $\widehat{w}_t^X \equiv \eta_{Lt}^X/\lambda_t$ are the prices of capital and labor in country X. This last equation is identical to (A.3), so that the optimal subsidy is indeed $1-\gamma$. Recovering the equations equivalent to (A.5) is direct. First order conditions with respect to K_{zt}^X and L_{zt}^X allow to recover the equations equivalent to (A.1). Now taking the first order condition with respect to Y_{dt}^X and Y_{ct}^X, one gets (when $Y_{Et}^X \neq 0$):

$$\begin{aligned}
\widehat{p}_{Et}\left(Y_{dt}^X\right)^{-\frac{1}{\varepsilon}}\left(\left(Y_{ct}^X\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(Y_{dt}^X\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} &= \widehat{p}_{dt}^X + \xi^X \frac{\omega_t}{\lambda_t}, \\
\widehat{p}_{Et}\left(Y_{ct}^X\right)^{-\frac{1}{\varepsilon}}\left(\left(Y_{ct}^X\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(Y_{dt}^X\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} &= \widehat{p}_{ct}^X
\end{aligned}$$

this is equivalent to (A.7) with a tax

$$\tau_t^X = \xi^X \frac{\omega_t}{\lambda_{dt}^X} = \xi^X \frac{(1+\rho)^t \omega_t}{\widehat{p}_{dt}^X \frac{\partial u}{\partial C}\left(C_t^W, S_t\right)}. \tag{A.36}$$

Therefore:

$$\lambda_{Et}^X = \frac{\psi^\gamma \left(\eta_{Kt}^X\right)^\alpha \left(\eta_{Lt}^X\right)^{1-\alpha}}{\left(\left(A_{ct}^X\right)^{\varepsilon-1} + \left(\left(1+\tau_t^X\right)^{-1} A_{dt}^X\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} (1-\gamma)^{1-\gamma}\gamma^\gamma\alpha^\alpha (1-\alpha)^{1-\alpha}},$$

so that, as in the laissez-faire case, country X specializes in good F if

$$\widehat{p}_{Et}^X < \frac{\psi^\gamma \left(\widehat{r}_t^X\right)^\alpha \left(\widehat{w}_t^X\right)^{1-\alpha}}{\left(\left(A_{ct}^X\right)^{\varepsilon-1} + \left(\left(1+\tau_t^X\right)^{-1} A_{dt}^X\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} (1-\gamma)^{1-\gamma}\gamma^\gamma\alpha^\alpha (1-\alpha)^{1-\alpha}}.$$

The analysis of sector F is identical except that there is no tax there.

Taking the first order condition with respect to S_t (knowing that S_t is bounded by $\overline{S}$) gives:

$$\omega_t = \frac{1}{(1+\rho)^t}\frac{\partial u}{\partial S}\left(C_s^N + C_s^S, S_s\right) + (1+\Delta)\, I_{S_t<\overline{S}}\omega_{t+1}, \tag{A.37}$$

which achieves the description of the optimal carbon tax.

I now turn to the optimal solution for the innovation part. First as in the equilibrium case, only the average level of technologies (defined in (14)) matters. Since the law of motion can be written as

$$\left(A_{zit}^X\right)^{\frac{1}{1-\gamma}} = \left(A_{zi(t-1)}^X\right)^{\frac{1}{1-\gamma}} + \widetilde{\kappa}\left(s_{zit}^X\right)\left(A_{z(t-1)}^X\right)^{\frac{1}{1-\gamma}}, \text{ for } z \in \{c, d, F\}, \tag{A.38}$$

the solution is also symmetric: $s_{zit}^X = s_{zt}^X$ for $z \in \{c, d, F\}$. Now taking the first order condition with respect to A_{zt}^X, gives:

$$\begin{aligned}
& \mu_{zit}^X \\
= \; & \lambda_{Ft}^X \left(x_{zit}^X\right)^\gamma \left(\left(K_{zt}^X\right)^{\overline{\alpha\beta}} \left(L_{zt}^X\right)^{1-\overline{\alpha\beta}}\right)^{1-\gamma} \\
& + \mu_{zi(t+1)}^X \left(\left(1 + \widetilde{\kappa}\left(s_{zt}^X\right)\left(\frac{A_{zt}^X}{A_{zit}^X}\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma} - \widetilde{\kappa}\left(s_{zt}^X\right)\left(\frac{A_{zt}^X}{A_{zit}^X}\right)^{\frac{1}{1-\gamma}}\left(1 + \widetilde{\kappa}\left(s_{zt}^X\right)\left(\frac{A_{zt}^X}{A_{zit}^X}\right)^{\frac{1}{1-\gamma}}\right)^{-\gamma}\right) \\
& + \int_0^1 \widetilde{\kappa}\left(s_{zt}^X\right) \frac{\left(A_{zit}^X\right)^{\frac{1}{1-\gamma}-1}}{\left(A_{zjt}^X\right)^{\frac{1}{1-\gamma}}}\left(1 + \widetilde{\kappa}\left(s_{zt}^X\right)\left(\frac{A_{zt}^X}{A_{zjt}^X}\right)^{\frac{1}{1-\gamma}}\right)^{-\gamma} A_{zjt}^X \mu_{zj(t+1)}^X dj,
\end{aligned}$$

(with $\overline{\alpha\beta} = \alpha$ if $z \in \{c, d\}$ and $\overline{\alpha\beta} = \beta$ if $z = F$), multiplying both sides by $\left(A_{zit}^X\right)^{\frac{-\gamma}{1-\gamma}}$, one gets:

$$\begin{aligned}
\mu_{zit}^X \left(A_{zit}^X\right)^{\frac{-\gamma}{1-\gamma}} \;=\; & \lambda_{zt}^X \left(x_{zit}^X\right)^\gamma \left(A_{zit}^X\right)^{\frac{-\gamma}{1-\gamma}}\left(\left(K_{zt}^X\right)^\beta \left(L_{zt}^X\right)^{1-\beta}\right)^{1-\gamma} + \mu_{zi(t+1)}^X \left(A_{zi(t+1)}^X\right)^{\frac{-\gamma}{1-\gamma}} \\
& + \widetilde{\kappa}\left(s_{zt}^X\right)\int \left(A_{zj(t+1)}^X\right)^{\frac{-\gamma}{1-\gamma}} \mu_{zj(t+1)}^X dj.
\end{aligned}$$

Since the equivalent of (A.4) also holds for sector F, $\lambda_{zt}^{X}\left(x_{zit}^{X}\right)^{\gamma}\left(A_{zit}^{X}\right)^{\frac{-\gamma}{1-\gamma}}\left(\left(K_{zt}^{X}\right)^{\overline{\alpha\beta}}\left(L_{zt}^{X}\right)^{1-\overline{\alpha\beta}}\right)^{1-\gamma}$ is a constant across varieties i. Therefore, $\mu_{zit}^{X}\left(A_{zit}^{X}\right)^{\frac{-\gamma}{1-\gamma}}$ is constant across varieties and one can define:

$$\mu_{zt}^{X}\equiv\left(\frac{A_{zt}^{X}}{A_{zit}^{X}}\right)^{\frac{\gamma}{1-\gamma}}\mu_{zit}^{X},$$

which represents the shadow value of one unit of average productivity in sector z, in country X at time t. I can then show that μ_{zt}^{X} follows the law of motion:

$$\mu_{zt}^{X}A_{zt}^{X}=\lambda_{zt}^{X}Y_{zt}^{X}+\mu_{z(t+1)}^{X}A_{z(t+1)}^{X}. \tag{A.39}$$

Taking the first order condition with respect to s_{zit}^{X} one gets:

$$\upsilon_{t}^{X}=\mu_{zit}^{X}\left(1-\gamma\right)\widetilde{\kappa}'\left(s_{zit}^{X}\right)\left(\frac{A_{z(t-1)}^{X}}{A_{zi(t-1)}^{X}}\right)^{\frac{1}{1-\gamma}}\left(1+\widetilde{\kappa}\left(s_{zit}^{X}\right)\left(\frac{A_{z(t-1)}^{X}}{A_{zi(t-1)}^{X}}\right)^{\frac{1}{1-\gamma}}\right)^{-\gamma}A_{zi(t-1)}^{X},$$

which can then be rewritten as:

$$\upsilon_{t}^{X}=\frac{\left(1-\gamma\right)\widetilde{\kappa}'\left(s_{zt}^{X}\right)}{1+\widetilde{\kappa}\left(s_{zt}^{X}\right)}\mu_{zt}^{X}A_{zt}^{X}.$$

Defining $\widehat{\upsilon}_{t}^{X}=\upsilon_{t}^{X}/\lambda_{t}^{X}$, the wage of scientists in terms of utility units, I can rewrite the last equality as

$$\widehat{\upsilon}_{t}^{X}=\frac{\left(1-\gamma\right)\widetilde{\kappa}'\left(s_{zt}^{X}\right)}{1+\widetilde{\kappa}\left(s_{zt}^{X}\right)}\sum_{s=t}^{\infty}\frac{\lambda_{s}}{\lambda_{t}}\widehat{p}_{zs}Y_{zs}. \tag{A.40}$$

Using (A.6), (A.7), (A.8), (A.9) gives:

$$\frac{p_{ct}^{X}Y_{ct}^{X}}{p_{Et}^{X}Y_{Et}^{X}}=\frac{\left(A_{ct}^{X}\right)^{\varepsilon-1}}{\left(A_{ct}^{X}\right)^{\varepsilon-1}+\left(\left(1+\tau_{t}^{X}\right)^{-1}A_{dt}^{X}\right)^{\varepsilon-1}},\ \frac{p_{dt}^{X}Y_{dt}^{X}}{p_{Et}^{X}Y_{Et}^{X}}=\frac{\left(1+\tau_{t}^{X}\right)^{-\varepsilon}\left(A_{dt}^{X}\right)^{\varepsilon-1}}{\left(A_{ct}^{X}\right)^{\varepsilon-1}+\left(\left(1+\tau_{t}^{X}\right)^{-1}A_{dt}^{X}\right)^{\varepsilon-1}}.$$

Combining these last two equations with (A.40) and (A.39), I get (24).

Solving for the maximization of (2) can be done in a very similar way. One gets:

$$\lambda_{t}=\frac{\Psi\nu\left(S_{t}\right)^{1-\eta}\left(C_{t}^{N}\right)^{-\eta}}{\left(1+\rho\right)^{t}}=\frac{\left(1-\Psi\right)\nu\left(S_{t}\right)^{1-\eta}\left(C_{t}^{S}\right)^{-\eta}}{\left(1+\rho\right)^{t}}=\frac{\left(\Psi^{\frac{1}{\eta}}+\left(1-\Psi\right)^{\frac{1}{\eta}}\right)^{\eta}}{\left(1+\rho\right)^{t}}\frac{\partial u}{\partial C}\left(C_{t}^{W},S_{t}\right),$$

and all results carry through provided that one replaces u by $\left(\Psi^{\frac{1}{\eta}}+\left(1-\Psi\right)^{\frac{1}{\eta}}\right)^{\eta}u$ (which does not affect the optimal allocation).

A.5 Proof of Proposition 4

This proof has two steps, first I specify the equilibrium constraints for the South, second I derive the social optimum for the case of the maximization of (1) - the maximization of (2) is treated in Appendix B.8.

A.5.1 Step 1: Laissez-faire constraints in the South

Y_{Et}^S and Y_{Ft}^S are given by (A.29) and (A.30) if $\left(p_t \frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}} \in \left(\left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L^S}{K^S}, \left(\frac{\beta^\beta(1-\beta)^{(1-\beta)}}{\alpha^\beta(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L^S}{K^S}\right)$,

$$Y_{Et}^S = 0 \text{ and } Y_{Ft}^S = \zeta A_{Ft}^S \left(K^S\right)^\beta \left(L^S\right)^{1-\beta}$$

if $\left(p_t \frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}} \leq \left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L^S}{K^S}$ and

$$Y_{Et}^S = \zeta A_{Et}^S \left(K^S\right)^\alpha \left(L^S\right)^{1-\alpha}$$

if $\left(p_t \frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}} \geq \left(\frac{\beta^\beta(1-\beta)^{(1-\beta)}}{\alpha^\beta(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L^S}{K^S}$. This overall delivers the constraint (28) with the function y_E^S increasing in p_t (weakly) and A_{Et}^S, and decreasing in A_{Ft}^S (weakly), and the function y_F^S decreasing in p_t (weakly) and A_{Et}^S (weakly) but increasing in A_{Ft}^S. y_E^S and y_F^S are only piecewise smooth (at the corner of full specialization, the functions are not differentiable). Note that since the South economy maximizes GDP:

$$p_t \frac{\partial y_E^S}{\partial p} + \frac{\partial y_F^S}{\partial p} = 0. \tag{A.41}$$

When $\left(p_t \frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}} > \left(\frac{\beta^\beta(1-\beta)^{(1-\beta)}}{\alpha^\beta(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L^S}{K^S}$, the allocation of scientists is trivially given by $s_{dt}^S = 1$ and when $\left(p_t \frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}} \leq \left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L^S}{K^S}$, by $s_{dt}^S = 0$. When $\left(p_t \frac{A_{Et}^S}{A_{Ft}^S}\right)^{\frac{1}{\alpha-\beta}} \in \left(\left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L^S}{K^S}, \left(\frac{\beta^\beta(1-\beta)^{(1-\beta)}}{\alpha^\beta(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L^S}{K^S}\right)$, the allocation of scientists is given by (A.31), that is:

$$\begin{aligned} & \frac{1+\widetilde{\kappa}\left(1-s_{Et}^S\right)}{\widetilde{\kappa}'\left(1-s_{Et}^S\right)} \frac{\widetilde{\kappa}'\left(\widetilde{s_{dt}^S}\left(s_{Et}^S, \frac{A_{dt}^S}{A_{ct}^S}\right)\right)}{\left(1+\widetilde{\kappa}\left(\widetilde{s_{dt}^S}\left(s_{Et}^S, \frac{A_{dt}^S}{A_{ct}^S}\right)\right)\right)} \left(\frac{A_{dt}^S}{A_{Et}^S}\right)^{\varepsilon-1} \\ = & \frac{\alpha \frac{\left(A_{Ft}^S\right)^{\frac{\alpha}{\alpha-\beta}}}{\left(A_{Et}^S\right)^{\frac{\beta}{\alpha-\beta}}} L^S - (1-\alpha)\left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} \frac{\left(A_{Et}^S\right)^{\frac{1-\beta}{\alpha-\beta}}}{\left(A_{Ft}^S\right)^{\frac{1-\alpha}{\alpha-\beta}}} p_t^{\frac{1}{\alpha-\beta}} K^S}{(1-\beta)\left(\frac{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} p_t^{\frac{1}{\alpha-\beta}} \frac{\left(A_{Et}^S\right)^{\frac{1-\beta}{\alpha-\beta}}}{\left(A_{Ft}^S\right)^{\frac{1-\alpha}{\alpha-\beta}}} K^S - \beta \frac{\left(A_{Ft}^S\right)^{\frac{\alpha}{\alpha-\beta}}}{\left(A_{Et}^S\right)^{\frac{\beta}{\alpha-\beta}}} L^S}. \end{aligned} \tag{A.42}$$

where $\widetilde{s_{dt}^S}$ is itself define through:

$$\frac{\widetilde{\kappa}'\left(s_{Et}^S - \widetilde{s_{dt}^S}\left(s_{Et}^S, \frac{A_{ct}^S}{A_{dt}^S}\right)\right)}{1+\widetilde{\kappa}\left(s_{Et}^S - \widetilde{s_{dt}^S}\left(s_{Et}^S, \frac{A_{ct}^S}{A_{dt}^S}\right)\right)} = \frac{\widetilde{\kappa}'\left(\widetilde{s_{dt}^S}\left(s_{Et}^S, \frac{A_{ct}^S}{A_{dt}^S}\right)\right)}{1+\widetilde{\kappa}\left(\widetilde{s_{dt}^S}\left(s_{Et}^S, \frac{A_{ct}^S}{A_{dt}^S}\right)\right)} \left(\frac{A_{dt}^S}{A_{ct}^S}\right)^{\varepsilon-1}. \tag{A.43}$$

This corresponds to the constraint (29). Note that I defined $\widetilde{s_{dt}^S}$ as a function of s_{Et}^S and $\frac{A_{dt}^S}{A_{ct}^S}$ *not* of s_{Et}^S and $\frac{A_{d(t-1)}^S}{A_{c(t-1)}^S}$ as in Appendices A.3 and B.1 (or in equations (30)), this allows to express

s_{Et}^{S} as a function of the current productivity levels, which simplifies considerably the expression of the optimal tariff. I use the tilde to ensure that the difference between the two functions is explicit. Yet, following the same reasoning as in Appendix (B.1), (A.42) also implicitly define s_{Et}^{S} as a unique function of p_t and the previous period technology levels, this function (weakly) increases in p_t, and (weakly) decrease in A_{Ft}^{S} (moreover one can show that s_{Et}^{S} is continuously differentiable). Note that (A.42) can be rewritten as:

$$\left(p_t\frac{\partial y_E^S}{\partial A_{Ft}^S}+\frac{\partial y_F^S}{\partial A_{Ft}^S}\right)\frac{\widetilde{\kappa}'\left(s_{Ft}^S\right)}{\left(1+\widetilde{\kappa}\left(s_{Ft}^S\right)\right)}A_{Ft}^S \tag{A.44}$$
$$= \frac{\widetilde{\kappa}'\left(\widetilde{s_{dt}^S}\left(s_{Et}^S,\frac{A_{dt}^S}{A_{ct}^S}\right)\right)}{\left(1+\widetilde{\kappa}\left(\widetilde{s_{dt}^S}\left(s_{Et}^S,\frac{A_{dt}^S}{A_{ct}^S}\right)\right)\right)}\left(\frac{A_{dt}^S}{A_{Et}^S}\right)^{\varepsilon-1}A_{Et}^S\left(p_t\frac{\partial y_E^S}{\partial A_{Et}^S}+\frac{\partial y_F^S}{\partial A_{Et}^S}\right),$$

so that that for given prices, innovation in the South maximizes current GDP $p_tY_{Et}^S+Y_{Ft}^S$.

A.5.2 Step 2: Deriving the social optimum

To simplify a bit the exposition, I combine (16) and the emission equation for the South $Y_{dt}^S=\left(A_{dt}^S/A_{Et}^S\right)^{\varepsilon}Y_{Et}^S$ into:

$$S_t=\max\left(\min\left((1+\Delta)\,S_{t-1}-\xi^N Y_{dt}^N-\xi^S\left(\frac{A_{dt}^S}{A_{Et}^S}\right)^{\varepsilon}Y_{Et}^S,\overline{S}\right),0\right), \tag{A.45}$$

I then use the following notations for the Lagrange parameters (the corresponding constraints are in parentheses): λ_t^X for (3) - both in North and South -; for the North only: λ_{Ft}^N (4), λ_{Et}^N (7), λ_{zt}^N (8), φ_{zit}^N (9), φ_{Fit}^N (5), η_{Kt}^N (11) for capital, η_{Lt}^N (11) for labor, μ_{zit}^N (13), υ_t^N (15); ω_t (A.45), θ_{Et}^N, θ_{Ft}^N, θ_{Et}^S and θ_{Ft}^S -with obvious superscripts- for the equations in (25), χ_t (26), $\varkappa_t$ (27), λ_{Et}^S and λ_{Ft}^S (28), ϕ_t (29), μ_{Ft}^S, μ_{dt}^S and μ_{ct}^S (30), in addition, the social planner faces the constraints: $0\le Y_{Et}^N$, $0\le Y_{Ft}^N$, with Lagrange parameters: ι_{Et}^N, ι_{Ft}^N.

As specified above, the functions y_E^S and y_F^S are not everywhere differentiable, in the following I use generalized Karush Kuhn Tucker conditions: at a point of non differentiability the notation $\frac{\partial y_E^S}{\partial p_t}$, $\frac{\partial y_E^S}{\partial A_{Et}^S}$, $\frac{\partial y_E^S}{\partial A_{Ft}^S}$ refers to elements of a vector $\left(\frac{\partial y_E^S}{\partial p_t},\frac{\partial y_E^S}{\partial A_{Et}^S},\frac{\partial y_E^S}{\partial A_{Ft}^S}\right)$ belonging to the Clarke generalized gradient of y_E^S. Therefore, it is still the case at these points that $\frac{\partial y_E^S}{\partial p_t}\ge 0$, $\frac{\partial y_E^S}{\partial A_{Et}^S}>0$, $\frac{\partial y_E^S}{\partial A_{Ft}^S}\le 0$.

First order conditions with respect to all the "North" variables, and S_t allow us to recover exactly the same equations as in the first-best for the North part of the economy (with θ_{Et}^N and θ_{Ft}^N replacing θ_{Et} and θ_{Ft}). This shows that the economy in the North is similar to the first-best case (with a carbon tax, subsidy to the use of intermediates, and research taxes/subsidies). Taking first order condition with respect to C_t^S gives:

$$\frac{1}{(1+\rho)^t}\frac{\partial u}{\partial C}\left(C_t^N+C_t^S,S_t\right)=\frac{\nu\left(S_t\right)^{1-\eta}}{(1+\rho)^t}\left(C_t^N+C_t^S\right)^{-\eta}=\lambda_t^S=\lambda_t^N\equiv\lambda_t.$$

Taking the first order condition with respect to C^S_{Ft}, I get:

$$\theta^S_{Ft} + \varkappa_t \frac{\partial}{\partial C^S_{Ft}} \frac{\frac{\partial C^S}{\partial C^S_{Et}}}{\frac{\partial C^S}{\partial C^S_{Ft}}} = \lambda_t \frac{\partial C^S}{\partial C^S_{Ft}} = \lambda_t (1-\nu) \left(C^S_{Ft}\right)^{-\frac{1}{\sigma}} \left(\nu \left(C^S_{Et}\right)^{\frac{\sigma-1}{\sigma}} + (1-\nu) \left(C^S_{Ft}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}, \tag{A.46}$$

and with respect to C^S_{Et}:

$$\theta^S_{Et} + \varkappa_t \frac{\partial}{\partial C^S_{Et}} \frac{\frac{\partial C^S}{\partial C^S_{Et}}}{\frac{\partial C^S}{\partial C^S_{Ft}}} = \lambda_t \frac{\partial C^S}{\partial C^S_{Et}} = \lambda_t \nu \left(C^S_{Et}\right)^{-\frac{1}{\sigma}} \left(\nu \left(C^S_{Et}\right)^{\frac{\sigma-1}{\sigma}} + (1-\nu) \left(C^S_{Ft}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}. \tag{A.47}$$

Therefore combining the two:

$$\left(\theta^S_{Ft} + \varkappa_t \frac{\partial}{\partial C^S_{Ft}} \frac{\frac{\partial C^S}{\partial C^S_{Et}}}{\frac{\partial C^S}{\partial C^S_{Ft}}}\right) \left(\theta^S_{Et} + \varkappa_t \frac{\partial}{\partial C^S_{Et}} \frac{\frac{\partial C^S}{\partial C^S_{Et}}}{\frac{\partial C^S}{\partial C^S_{Ft}}}\right)^{-1} = \frac{\frac{\partial C^S}{\partial C^S_{Ft}}}{\frac{\partial C^S}{\partial C^S_{Et}}} = \frac{1}{p_t},$$

so that:

$$\varkappa_t = \frac{\frac{\theta^S_{Et}}{p_t} - \theta^S_{Ft}}{\frac{\partial}{\partial C^S_{Ft}} \frac{\frac{\partial C^S}{\partial C^S_{Et}}}{\frac{\partial C^S}{\partial C^S_{Ft}}} - \frac{1}{p_t} \frac{\partial}{\partial C^S_{Et}} \frac{\frac{\partial C^S}{\partial C^S_{Et}}}{\frac{\partial C^S}{\partial C^S_{Ft}}}}. \tag{A.48}$$

First order conditions with respect to Y^S_{Ft} and Y^S_{Et} give:

$$\lambda^S_{Et} = \theta^S_{Et} - \omega_t \xi^S \left(\frac{A^S_{dt}}{A^S_{Et}}\right)^{\varepsilon} \text{ and } \lambda^S_{Ft} = \theta^S_{Ft}, \tag{A.49}$$

First order conditions with respect to M_{Ft} and M_{Et} give:

$$p_t \chi_t = \theta^N_{Et} - \theta^S_{Et} \text{ and } \chi_t = \theta^N_{Ft} - \theta^S_{Ft}, \tag{A.50}$$

so that

$$\frac{\theta^S_{Et}}{p_t} - \theta^S_{Ft} = \frac{\theta^N_{Et}}{p_t} - \theta^N_{Ft}. \tag{A.51}$$

Finally the first order condition with respect to p_t gives:

$$M_{Et} \chi_t = \lambda^S_{Et} \frac{\partial y^S_E}{\partial p_t} + \lambda^S_{Ft} \frac{\partial y^S_F}{\partial p_t} + \varkappa_t + \phi_t \frac{\partial s^S_{Et}.}{\partial p_t} \tag{A.52}$$

Let us denote by $(1+b_t)$ an *ad valorem* trade tax on good E, using (A.34) and (A.35) in the North. One gets

$$\frac{\frac{\partial C^N}{\partial C^N_E}}{\frac{\partial C^N}{\partial C^N_F}} = \frac{\nu}{1-\nu} \left(\frac{C^N_{Ft}}{C^N_{Et}}\right)^{\frac{1}{\sigma}} = \frac{\theta^N_{Et}}{\theta^N_{Ft}} = \frac{\widehat{p}_{NEt}}{\widehat{p}_{Ft}} = p_t (1+b_t). \tag{A.53}$$

Now plugging (A.49), (A.48), (A.50) and (A.51) in (A.52), I get:

$$M_{Et}\frac{(\theta_{Et}^N-\theta_{Et}^S)}{p_t} \tag{A.54}$$
$$= \left(\theta_{Et}^S-\omega_t\xi\left(\frac{A_{dt}^S}{A_{Et}^S}\right)^{\varepsilon}\right)\frac{\partial y_E^S}{\partial p_t}+\theta_{St}\frac{\partial y_F^S}{\partial p_t}+\frac{\frac{\theta_{Et}^S}{p_t}-\theta_{Ft}^S}{\frac{\partial}{\partial C_{Ft}^S}\frac{\frac{\partial C^S}{\partial C_{Et}^S}}{\frac{\partial C^S}{\partial C_{Ft}^S}}-\frac{1}{p_t}\frac{\partial}{\partial C_{Et}^S}\frac{\frac{\partial C^S}{\partial C_{Et}^S}}{\frac{\partial C^S}{\partial C_{Ft}^S}}}+\phi_t\frac{\partial s_{Et}^S}{\partial p_t}.$$

Further, using (A.41), (A.34) and (A.35) for the North - replacing θ_{Et} by θ_{Et}^N-, (A.47), (A.46), (A.48) and (A.53):

$$b_t\frac{\partial C^N}{\partial C_F^N}\left(p_t\frac{\partial y_E^S}{\partial p_t}+\frac{1-\frac{M_{Et}}{p_t}\frac{\partial}{\partial C_{Et}^S}\frac{\frac{\partial C^S}{\partial C_{Et}^S}}{\frac{\partial C^S}{\partial C_{Ft}^S}}}{\frac{\partial}{\partial C_{Ft}^S}\frac{\frac{\partial C^S}{\partial C_{Et}^S}}{\frac{\partial C^S}{\partial C_{Ft}^S}}-\frac{1}{p_t}\frac{\partial}{\partial C_{Et}^S}\frac{\frac{\partial C^S}{\partial C_{Et}^S}}{\frac{\partial C^S}{\partial C_{Ft}^S}}}\right)-\frac{M_{Et}}{p_t}\left(\frac{\partial C^N}{\partial C_E^N}-\frac{\partial C^S}{\partial C_{Et}^S}\right), \tag{A.55}$$
$$= \frac{\omega_t}{\lambda_t}\xi^S\left(\frac{A_{dt}^S}{A_{Et}^S}\right)^{\varepsilon}\frac{\partial y_E^S}{\partial p_t}-\frac{\phi_t}{\lambda_t}\frac{\partial s_{Et}^S}{\partial p_t}.$$

Defining $\widehat{\omega}_t \equiv \omega_t/\lambda_t$ and $\widehat{\phi}_t \equiv \phi_t/\lambda_t$ and some algebra (described in Appendix B.7) delivers (31). Appendix B.7 also gives more details on the sign of $\widehat{\phi}_t$ and b_t.