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## ABSTRACT

### Aggregation and Labor Supply Elasticities

This paper develops a statistical aggregation procedure for the Frisch elasticity of labor supply. It allows for worker heterogeneity and is applicable to an individual labor supply function with non-employment as a possible outcome. Subjecting all offered or paid wages to an unanticipated temporary change we analytically derive the aggregate elasticity and its main components. We quantify each component using individual-specific data from the German SOEP for males at working-age. We measure the hours' adjustment along the intensive and the extensive margin with the help of observed wages and reservation wages, respectively. The estimated aggregate Frisch elasticity varies over time.

JEL Classification: C51, E10 and J22

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# 1 Introduction

The aggregate Frisch elasticity of labor supply has been at center stage in modern business cycle analysis for many years. It was first introduced into the literature by Ragnar Frisch and continues to be of interest from a theoretical as well as from an empirical perspective. At any point in time, it measures the reaction of total hours worked to a small change in the mean wage when wealth is held constant. The exact size of this particular elasticity matters a lot when macroeconomists try to assess the quantitative implications of certain types of policies on employment and hours worked. For example, changes in monetary or fiscal policy parameters which directly or indirectly impact a worker's net wage rate typically lead to a change in total labor supply. In spite of its relevance, the size of this aggregate change cannot easily be determined when worker heterogeneity is taken seriously. That is because the reaction of total labor supply is a highly complex object whose various components need to be accounted for. This object not only depends on the distribution of wage rates across employed workers and that of reservation wage rates across non-employed workers. It also depends on the hours' adjustment of existing workers (intensive margin) as well as of those who move between employment and non-employment following a wage change (extensive margin). Lastly, the overall reaction depends on the exact implementation of the underlying policy change.

In this paper, we develop a unified framework which allows us to simultaneously study the role that workers' participation and hours decisions play for the size of the aggregate Frisch elasticity. We depart from MaCurdy's (1985) standard intertemporal labor supply model that features complete markets, uncertainty and worker heterogeneity in observable and unobservable characteristics. We then modify the statistical aggregation approach developed by Paluch, Kneip and Hildenbrand (2012) to allow for a corner solution in a worker's labor supply decision. This procedure has the distinct advantage of being widely applicable, because it requires neither a particular preference structure nor specific distributional assumptions for explanatory variables. We use it to aggregate our individual labor supply functions and wage rates. In order to derive the aggregate Frisch elasticity of labor supply, we subject all offered or paid wages to an unanticipated

temporary increase. By eliminating wealth effects and taking account of the implied adjustment of labor supply, we derive an analytical expression for the aggregate elasticity and illustrate its components: (i) the intensive and extensive adjustment of hours worked, (ii) the extensive adjustment of wages, and (iii) the aggregate employment rate.

To empirically implement our aggregation approach, we rely on specific econometric models and estimate them using micro-level data from the German Socio-Economic Panel (SOEP). The SOEP is unique in that it provides evidence on non-employed workers' reservation wage rates. This variable is essential for estimating the adjustment of hours worked and wages paid of workers who change their participation decision – so-called movers. We estimate the adjustment of hours worked along the intensive margin, i.e., of stayers, with the help of a standard panel model. Our sample comprises German males who are between 25 and 64 years old and live in former West Germany, because their labor supply behavior is well captured by the intertemporal model. Our estimation results yield an average individual Frisch wage-elasticity of .29 – a value that stands in sharp contrast to our estimated aggregate values which vary between .63 and .70 over the period ranging from 2000 to 2008.

We are not the first ones to study the aggregate Frisch elasticity of labor supply in an environment with heterogeneous workers. Our work is related to two main strands of the literature. First, it relates to the many contributions in modern business cycle analysis where the aggregate Frisch elasticity enters as key entity that affects the reaction of total labor supply to a change in wages induced by policy or exogenous disturbances. The basic idea goes back to Lucas and Rapping (1969) which is considered as the origin of intertemporal labor supply in modern macroeconomics. Gary Hansen's (1985) employment lotteries have illustrated the importance of the extensive margin adjustment for the aggregate Frisch wage-elasticity, but except for the *ex post* status of a worker in the labor force it ignores worker heterogeneity. More closely related to our work are the papers by Chang and Kim (2005; 2006) who allow for worker heterogeneity and explore how the size of the aggregate Frisch elasticity of hours worked varies with incomplete markets. They focus on the intensive margin only. The work by Gourio and Noual (2009) is also relatively closely related to ours.

They use a complete market setup to explore the role of ‘marginal workers’ who by definition are indifferent between working and not working for adjustment along the extensive margin when wages change. All these contributions commonly use a parameterized version of a structural utility function which makes it possible to derive a functional relationship between the aggregate labor supply and aggregate wages. They differ with respect to the type and degree of worker heterogeneity, the assumed market structure, and whether they focus on the intensive or the extensive margin of adjusting labor supply. Another related piece is by Fiorito and Zanella (2012). They use the PSID to empirically explore the link between the micro and the macro Frisch wage-elasticity without deriving an exact analytical relationship between them. They nicely illustrate how the difference between the individual and the aggregate Frisch elasticity changes for various sub-populations, but they cannot measure the extensive margin. Second, our work relates to the growing micro literature that has produced estimates of the individual wealth-compensated wage elasticity of hours worked since the early work by MaCurdy (1981; 1985) and Altonji (1986). Their estimates for males range from .10 to .45, and from 0 to .35, respectively. The recent study by Chetty et al. (2012) provides quasi-experimental evidence on individual wage-elasticities. Its conclusion that the intensive margin of .5 is twice as large as the extensive one is juxtaposed to a central finding of the labor supply literature summarized in Blundell and MaCurdy (1999) that the extensive margin matters most for explaining variation in total person hours over the business cycle.

Our contribution to this literature is twofold. First, we develop a statistical aggregation approach which does not require specific assumptions about model parameters or distributions of explanatory variables. It is comprehensive enough to simultaneously capture adjustment along the intensive and the extensive margin when wage rates change unexpectedly in an environment where workers are heterogeneous. Secondly, we illustrate the importance of statistical aggregation by empirically implementing it using the German SOEP which contains as special feature information on reservation wage rates for non-employed workers.

This paper is organized as follows. Section 2 presents a dynamic model of individual labor supply under uncertainty. Section 3 develops a gen-

eral statistical aggregation procedure that features labor supply adjustment along the intensive and the extensive margin and is used to derive an analytical expression for the aggregate Frisch wage-elasticity of labor supply. Section 4 specifies the two econometric models used for empirical estimation, a panel data model on hours worked and a two-stage procedure to estimate conditional densities. Section 5 presents our database and introduces the main variables used for estimation. Section 6 reports all estimation results. Section 7 concludes.

## 2 A Dynamic Labor Supply Model

Underlying our aggregation exercise is an individual-specific labor supply function which relates the amount of labor that an individual supplies to the market in any given period  $t$  to a set of determinants. We view this function as the outcome of an intertemporal optimization problem under uncertainty.<sup>1</sup> In what follows we sketch this problem including the preferences, the constraints and the informational setting for each individual. For the sake of notational simplicity, we abstain from introducing a person-specific index until section 4.

Consider an infinitely-lived consumer. Her preferences are captured by a momentary utility function  $U$  which depends on private consumption  $c$ , leisure  $l$ , a vector of observable individual characteristics  $X$  and a vector of unobservable individual variables  $Z$ , including tastes and talents.  $U$  is assumed to be twice differentiable, separable over time and also in consumption  $c$  and market hours worked  $h$ . Furthermore,  $U$  is strictly increasing and concave in  $c$  and  $h$ . When choosing sequences of leisure, consumption and future asset holdings to maximize her expected life-time utility, the consumer takes the real wage rate  $w$  and the real market return on assets  $r$  as given and respects the following two constraints: First, the per-period time-constraint

$$\bar{T}_t \geq l_t + h_t \tag{1}$$

which equates the available time  $\bar{T}$  to the sum of leisure and market hours

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<sup>1</sup>Our model exposition closely follows that in MaCurdy (1985).

worked  $h$  in each period. Second, the budget constraint

$$c_t + a_{t+1} \leq w_t h_t + (1 + r_t) a_t \quad (2)$$

that sets the sum of consumption expenditures and the change in asset holdings  $a_{t+1} - a_t$  equal to total earnings plus interest income from current period asset holdings  $a_t$ . A consumer starts life with initial assets  $a_0$ .

Denoting by  $\mathbb{E}_t$  the mathematical expectation conditional on information known at the beginning of time  $t$  and by  $0 < \tilde{\beta} < 1$  the discount rate, the consumer's choice problem can be summarized as follows:

$$\max_{\{c_t, l_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t U(c_t, l_t; X_t, Z_t) \quad (3)$$

subject to equations (1), (2), the non-negativity constraints  $c_t > 0$ ,  $l_t \geq 0$ , and the initial condition  $a_0 > 0$ .<sup>2</sup> For any differentiable function  $f(x_1, \dots, x_n)$  let  $\partial_{x_i} f(x_1, \dots, x_n)$  denote the partial derivative with respect to the  $i$ -th component. Then, letting  $\lambda_t$  denote the Lagrange multiplier associated with the period  $t$  budget constraint, the first-order necessary conditions for utility maximization are given by:

$$\partial_c U(\cdot) - \lambda_t = 0 \quad (4a)$$

$$\partial_l U(\cdot) - \lambda_t w_t = 0 \quad (4b)$$

$$\lambda_t = \tilde{\beta} \mathbb{E}_t[(1 + r_{t+1})\lambda_{t+1}]. \quad (4c)$$

With the help of the implicit function theorem equations (4a) and (4b) can be solved for individual consumption and labor supply as functions of the form

$$c_t = d(w_t, \lambda_t, X_t, Z_t) \quad (5)$$

$$h_t = g(w_t, \lambda_t, X_t, Z_t). \quad (6)$$

The time-invariant functions  $d(\cdot)$  and  $g(\cdot)$  depend only on the specifics of the utility function  $U(\cdot)$  and on whether corner solutions are optimal for

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<sup>2</sup>A complete formulation of the consumer's dynamic decision problem also requires a transversality condition for wealth:  $\lim_{T \rightarrow \infty} \lambda_T a_T = 0$ .



hours worked in period  $t$ . These functions contain two types of arguments, namely those that capture what is going on in the current period –  $w_t$ ,  $X_t$  and  $Z_t$  – and  $\lambda_t$  which is a sufficient statistic for past and future information relevant for the individual's current choices. If we further assume consumption and leisure to be normal goods, the concavity of the utility function implies

$$\partial_\lambda d < 0, \partial_w g \geq 0, \partial_\lambda g \geq 0. \quad (7)$$

Equation (4c) summarizes the stochastic process governing  $\lambda_t$ . Assuming interest rates do not vary stochastically, this process can alternatively be expressed as an expectational difference equation:

$$\lambda_t = \tilde{\beta}(1 + r_{t+1})\mathbb{E}_t \lambda_{t+1}.$$

Recall that any variable can be rewritten as the sum of what was expected and an expectational error  $\varepsilon$ :

$$\lambda_t = \mathbb{E}_{t-1} \lambda_t + \varepsilon_t.$$

Combining the last two expressions and solving backward yields

$$\lambda_t = \tilde{\beta}^{-t} R_t \lambda_0 + \sum_{j=0}^{t-1} \varepsilon_{t-j} \equiv \tilde{\beta}^{-t} R_t \lambda_0 + \eta_t, \quad (8)$$

where  $R_t \equiv 1/[(1 + r_1)(1 + r_2) \cdot \dots \cdot (1 + r_t)]$  is the common discount rate. Equation (8) nicely illustrates that apart from the sum of past expectational errors,  $\eta_t$ , the time-varying individual marginal utility of wealth consists of a fixed individual component  $\lambda_0$  and a common time-varying component. When inserting this expression together with the consumption and labor supply function (5) and (6) into the individual life-time budget constraint which results from solving equation (2) forward we get

$$a_0 \geq \sum_{t=0}^{\infty} R_t [d(w_t, \lambda_t, X_t, Z_t) - w_t g(w_t, \lambda_t, X_t, Z_t)]. \quad (9)$$

Equation (9) implicitly defines  $\lambda_t$ . It shows that the marginal utility of

consumption is a highly complex variable that depends on the initial assets, life-time wages, the market interest rate, observable and unobservable individual characteristics and preferences. For the purpose of our analysis it matters that the assumed concavity of preferences implies

$$\frac{\partial \lambda_t}{\partial a_0} < 0, \frac{\partial \lambda_t}{\partial w_t} \leq 0. \quad (10)$$

Taken together the inequalities in (7) and (10) indicate that there exists a direct and an indirect effect of wages on hours worked. A rise in the current period's wage rate directly leads to an increase in hours worked. The indirect link exists, because a rising wage rate contributes to a rise in wealth which tends to reduce labor supply. Hence, in the intertemporal framework laid out the net effect of a change in wages on individual labor supply is unclear from a theoretical point of view.

Summing up, we can express the individual labor supply function as follows:

$$h_t = \begin{cases} g(w_t, \lambda(w_t, \eta_t), X_t, Z_t) > 0 & \text{if } w_t \geq w_t^R \\ 0 & \text{if } w_t < w_t^R \end{cases} \quad (11)$$

where the individual reservation wage rate in period  $t$  is derived from expression (4b):

$$w_t^R = \frac{\partial_l U[c_t, T, X_t, Z_t]}{\partial_c U[c_t, T, X_t, Z_t]}$$

with  $(1 + r_t)a_t \geq a_{t+1}$ . Equation (11) implies that the individual wage rate  $w_t$  is observed only if it is greater than or equal to the individual's reservation wage  $w_t^R$ . In general, we can think of  $w_t$  as the maximal wage rate *offered* to an individual in period  $t$ . We introduce the wage rate as a possibly hypothetical quantity so that we can later define a suitable population model.

We finally use the labor supply function to define the Frisch wage-elasticity of an individual's labor supply:

$$\epsilon_t = \left. \frac{\partial \log g(w, \lambda_t, X_t, Z_t)}{\partial \log w} \right|_{w=w_t} = \partial_w g(w_t, \lambda_t, X_t, Z_t) \frac{w_t}{h_t}. \quad (12)$$

Since we focus on an individual Frisch wage-elasticity, we only consider the **direct** effects of the wage change. We compensate indirect effects due to a rise in wealth by keeping  $\lambda_t = \lambda(w_t, \eta_t)$  fixed at their individual levels, instead of allowing  $\lambda_t$  to change with changes in  $w_t$ . Since the Frisch wage-elasticity abstracts from the wealth effect of a wage change, this elasticity by definition cannot become negative.

### 3 Aggregation and the Frisch Elasticity

We modify the statistical aggregation approach developed by Paluch et al. (2012) (see also Hildenbrand and Kneip (2005)) to allow for corner solutions in individual labor supply decisions in order to arrive at an aggregate labor supply function. Recall that for each individual working hours  $h_t$  in period  $t$  are given by

$$h_t = g(w_t, \lambda_t, Y_t)I(w_t \geq w_t^R), \quad (13)$$

where  $I(\cdot)$  denotes the indicator function. The function  $g$  is derived from utility maximization and depends on the individual wage rate  $w_t$ , the marginal utility of wealth  $\lambda_t$ , observable individual characteristics  $X_t$  and unobservable random factors  $Z_t$  such as preferences or talents, which we summarize using the variable  $Y_t = (X_t, Z_t)$ . For each period  $t$ , working hours  $h_t$ , wages rates  $w_t$ , reservation wage rates  $w_t^R$ , as well as  $\lambda_t$  and  $Y_t$  are random variables with means depending on the corresponding distributions within the given population. More precisely the mean labor supply as well as the mean wage rate received by all working individuals are given by the following two expressions:

$$\bar{H}_t = \mathbb{E}(h_t) = \int g(w, \lambda, Y)I(w \geq w^R)d\pi_{w, w^R, \lambda, Y}^t, \quad (14a)$$

$$\bar{W}_t = \mathbb{E}(w_t) = \int wI(w \geq w^R)d\pi_{w, w^R}^t, \quad (14b)$$

where  $\pi_{w, w^R, \lambda, Y}^t$  denotes the joint distribution of the variables  $(w_t, w_t^R, \lambda_t, Y_t)$  over the population and  $\pi_{w, w^R}^t$  stands for the marginal distribution of  $(w_t, w_t^R)$ . All other marginal distributions are written analogously. More-

over, we denote the conditional distribution of some random variable  $V$  given a random variable  $W$  by  $\pi_{V|W}^t$  and its density, if existent, by  $f_{V|W}^t(\cdot)$ . In particular, we will assume that the conditional distribution  $\pi_{w^R|w}^t$  of  $w_t^R$  given  $w_t = w$  has a continuous density  $f_{w^R|w}^t(\cdot)$ . Finally, we require that the marginal distribution  $\pi_w^t$  of  $w_t$  also possesses a continuous density  $f_w^t(\cdot)$ .

We are interested in determining the aggregate Frisch wage-elasticity of labor supply – an entity which receives much attention in macroeconomics. The goal is to quantify changes in mean working hours  $\bar{H}_t$  in reaction to a small change of the mean wage rate,  $\bar{W}_t$ . As equation (14a) nicely illustrates, mean hours worked depend among others on the distribution of wages  $w_t$  across individuals. Since the shape of the wage distribution can change in many different ways if individual wages change, several alternative definitions of an aggregate elasticity are feasible.

For the sake of determining the aggregate elasticity, we perform the following thought experiment. Consider the situation that each individual faces an unanticipated temporary fixed change  $\Delta > 0$  of her wage rate  $w_t$ , so that  $w_t$  is transformed into  $w_t + \Delta$  for some  $\Delta$  close to zero.<sup>3</sup> All other variables remain unchanged including the reservation wage rate  $w_t^R$ . That is because we consider iid wage shocks. Using this transformation we obtain a higher new mean wage,  $\bar{W}_t(\Delta)$ , and a new mean of working hours,  $\bar{H}_t(\Delta)$ :

$$\begin{aligned}\bar{H}_t(\Delta) &:= \mathbb{E} (g(w_t + \Delta, \lambda_t, Y_t)I(w_t + \Delta \geq w_t^R)) \\ &= \int g(w + \Delta, \lambda, Y)I(w + \Delta \geq w^R)d\pi_{w, w^R, \lambda, Y}^t,\end{aligned}\tag{15a}$$

$$\begin{aligned}\bar{W}_t(\Delta) &:= \mathbb{E} ((w_t + \Delta)I(w_t + \Delta \geq w_t^R)) \\ &= \int (w + \Delta)I(w + \Delta \geq w^R)d\pi_{w, w^R}^t.\end{aligned}\tag{15b}$$

Since we focus on an aggregate Frisch wage-elasticity, we only consider the **direct** effects of the wage transformation in (15a). We compensate indirect effects due to a rise in wealth by keeping  $\lambda_t = \lambda(w_t, \eta_t)$  fixed at their individual levels. If we allowed  $\lambda_t$  to change to  $\lambda(w_t + \Delta, \eta_t)$ , our approach could generate an aggregate Marshallian wage-elasticity of labor

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<sup>3</sup>Note that all arguments hold analogously in case of marginal wage cuts, where  $\Delta < 0$ .

supply.<sup>4</sup>

Obviously  $\overline{H}_t(0) = \overline{H}_t$  and  $\overline{W}_t(0) = \overline{W}_t$ . Our model does not specify a direct functional relationship between  $\overline{W}_t(\Delta)$  and  $\overline{H}_t(\Delta)$ , because we abstain from a particular utility specification and account for worker heterogeneity. Recall that elasticities are defined via logarithmic derivatives. Since derivatives in turn are limits of incremental changes, an aggregate Frisch elasticity with respect to our particular change of individual wage rates is defined as follows:

$$\begin{aligned}
e_t &:= \lim_{\log \overline{W}_t(\Delta) \rightarrow \log \overline{W}_t} \frac{\log \overline{H}_t(\Delta) - \log \overline{H}_t}{\log \overline{W}_t(\Delta) - \log \overline{W}_t} \\
&= \lim_{\Delta \rightarrow 0} \frac{\Delta(\log \overline{H}_t(\Delta) - \log \overline{H}_t)}{\Delta(\log \overline{W}_t(\Delta) - \log \overline{W}_t)} \\
&= \frac{\frac{\partial}{\partial \Delta} \log \overline{H}_t(\Delta)|_{\Delta=0}}{\frac{\partial}{\partial \Delta} \log \overline{W}_t(\Delta)|_{\Delta=0}} = \frac{\overline{W}_t \frac{\partial}{\partial \Delta} \overline{H}_t(\Delta)|_{\Delta=0}}{\overline{H}_t \frac{\partial}{\partial \Delta} \overline{W}_t(\Delta)|_{\Delta=0}}. \tag{16}
\end{aligned}$$

The aggregate quantities  $\overline{W}_t$  and  $\overline{H}_t$  can be determined from observed data so that we only have to analyze the expressions  $\frac{\partial}{\partial \Delta} \overline{H}_t(\Delta)|_{\Delta=0}$  and  $\frac{\partial}{\partial \Delta} \overline{W}_t(\Delta)|_{\Delta=0}$ .

Let us first consider the simpler term  $\overline{W}_t(\Delta)$  which, for  $\Delta > 0$ , quantifies the new mean wage rate paid by employers. Note that for a working individual her new wage rate simply is  $w_t + \Delta$ , and hence  $\frac{\partial}{\partial \Delta}(w_t + \Delta)|_{\Delta=0} = 1$ . This is not generally true at the aggregate level. The point is that for  $\Delta > 0$  we consider the increase in the mean wage rate for the entire labor force and not only for the subpopulation of employed workers. The transformation implies that a wage rate  $w_t + \Delta$  is offered to an unemployed individual, but the actual wage rate paid will remain zero if  $w_t + \Delta < w_t^R$ . On the other hand, there exist marginal workers who do not work at a wage rate  $w_t$ , but may decide to work at a higher wage rate  $w_t + \Delta$ . More precisely, by (15b) we have

$$\overline{W}_t(\Delta) = \int (w + \Delta) I(w \geq w^R) d\pi_{w, w^R}^t + \int (w + \Delta) I(w^R \in [w, w + \Delta]) d\pi_{w, w^R}^t \tag{17}$$

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<sup>4</sup>Our approach can be generalized to any other strictly monotone transformation  $w_t \rightarrow T(\Delta, w_t)$  with  $T(0, w_t) = w_t$ . For example, instead of considering  $T(\Delta, w_t) = w_t + \Delta$ , one may be interested in looking at proportional changes  $T(\Delta, w_t) = w_t(1 + \Delta)$  or more complex transformations. The formulae for aggregate elasticities derived below then have to be modified. They will depend on  $\frac{\partial}{\partial \Delta} T(\Delta, w)|_{\Delta=0}$ . Details are given in Appendix A.

$$= \int (w + \Delta) I(w \geq w^R) d\pi_{w,w^R}^t + \int (\nu + \Delta) \left( \int_{\nu}^{\nu+\Delta} f_{w^R|\nu}^t(\tilde{\nu}) d\tilde{\nu} \right) f_w^t(\nu) d\nu.$$

Taking derivatives yields

$$\frac{\partial}{\partial \Delta} \overline{W}_t(\Delta)|_{\Delta=0} = \underbrace{\int I(w \geq w^R) d\pi_{w,w^R}^t}_{EPR_t} + \underbrace{\int \nu f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu}_{\tau_{w,t}^{ext}}. \quad (18)$$

The first term  $EPR_t$  corresponds to the employment ratio in period  $t$ , i.e., the fraction of the labor force employed. The second term is due to changes in mean earnings with respect to employment adjustment along the extensive margin. For any small  $\Delta > 0$  and any wage rate  $w$  the conditional probability given  $w$  that an arbitrary individual has a reservation wage rate which lies in the interval  $[w, w + \Delta]$  is approximately equal to  $\Delta f_{w^R|w}^t(w)$ . Therefore,  $\Delta w f_{w^R|w}^t(w)$  approximates the amount of additional wages to be paid to marginal workers transiting from non-employment to employment. For a given wage rate  $w$  the term  $w f_{w^R|w}^t(w)$  thus quantifies the rate of increase of wages to be paid to marginal workers if  $w$  increases by  $\Delta > 0$ .  $\tau_{w,t}^{ext}$  is the mean of these rates over all wages,  $\tau_{w,t}^{ext} = \mathbb{E}(w f_{w^R|w}^t(w))$ .

Necessarily  $\tau_{w,t}^{ext} \geq 0$ , and one typically expects that  $\tau_{w,t}^{ext} > 0$ . To simplify the argument consider the case that  $w_t^R$  and  $w_t$  are independent such that  $f_{w^R|w}^t \equiv f_{w^R}^t$  does not depend on  $w$  and is equal to the marginal density of reservation wages.<sup>5</sup> Then  $\tau_{w,t}^{ext} > 0$  if for some wage rate  $\nu$  with  $f_w^t(\nu) > 0$  we also have  $f_{w^R}^t(\nu) > 0$ . In other words,  $\tau_{w,t}^{ext} > 0$  if there exists some overlap between the support of the distributions of wages  $w_t$  and the support of the distribution of reservation wages  $w_t^R$ . This will typically be fulfilled for any real economy.

Let us now analyze the term  $\overline{H}_t(\Delta)$  which, for  $\Delta > 0$ , quantifies the

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<sup>5</sup>The micro model implies that reservation wages are variables which do not depend on actual wages paid or offered. Therefore it does not seem implausible to assume that the random variables  $w_t^R$  and  $w_t$  are independent. However, there may exist an indirect link due to correlations with common explanatory variables such as education, for example. Highly educated individuals tend to have higher reservation wages than others and they are likely to receive higher wage offers. This may introduce a correlation between  $w_t^R$  and  $w_t$  over the population. Our procedure for estimating  $\tau_{w,t}^{ext}$  described in section 4 takes such effects into account.

new mean working hours. Similar to (17) we obtain

$$\begin{aligned}\bar{H}_t(\Delta) &= \int g(w + \Delta, \lambda, Y) I(w \geq w^R) d\pi_{w, w^R, \lambda, Y}^t \\ &\quad + \int g(w + \Delta, \lambda, Y) I(w^R \in [w, w + \Delta]) d\pi_{w, w^R, \lambda, Y}^t,\end{aligned}\tag{19}$$

where the second term quantifies the part of the change of  $\bar{H}_t$  which is due to the fact that if wage rates rise from  $w_t$  to  $w_t + \Delta$ , then the subpopulation of all individuals with reservation wage rates  $w_t^R \in [w_t, w_t + \Delta]$  will contribute non-zero working hours. Using  $\partial_w g(w, \lambda, Y)$  to denote the partial derivative of  $g$  with respect to  $w$ , the derivative of the first term simply is  $\mathbb{E}(\partial_w g(w_t, \lambda_t, Y_t))$ . Calculating the derivative of the second term is slightly more complicated. A rigorous analysis can be found in Appendix B. We then arrive at the following expression:

$$\begin{aligned}\frac{\partial \bar{H}_t(\Delta)}{\partial \Delta} \Big|_{\Delta=0} &= \underbrace{\int \partial_w g(w, \lambda, Y) I(w \geq w^R) d\pi_{w, w^R, \lambda, Y}^t}_{\tau_{h,t}^{int}} \\ &\quad + \underbrace{\int \mathbb{E} \left( h_t \mid w_t^R = w_t = \nu \right) f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu}_{\tau_{h,t}^{ext}}.\end{aligned}\tag{20}$$

The first term  $\tau_{h,t}^{int}$  quantifies the average derivatives of the individual functions  $g$  for the subpopulation  $\mathcal{E}_t$  of all individuals already working at wage rate  $w_t$ . Put differently,  $\tau_{h,t}^{int}$  measures the total labor supply adjustment along the intensive margin. It can also be interpreted as a weighted mean of individual Frisch elasticities for the subpopulation  $\mathcal{E}_t$ . Recall that individual Frisch elasticities are given by  $\epsilon_t = \frac{\partial \log g(w, \lambda_t, Y_t)}{\partial \log w} \Big|_{w=w_t} = \partial_w g(w_t, \lambda, Y) \frac{w_t}{h_t}$ . Therefore,

$$\tau_{h,t}^{int} = \int_{\mathcal{E}_t} \partial_w g(w, \lambda, Y) d\pi_{w, w^R, \lambda, Y}^t = \mathbb{E}_{\mathcal{E}_t}(\partial_w g(w_t, \lambda_t, Y_t)) = \mathbb{E}_{\mathcal{E}_t} \left( \epsilon_t \frac{h_t}{w_t} \right),\tag{21}$$

where  $\mathbb{E}_{\mathcal{E}_t}(\cdot)$  is used to denote expected values over all individuals in  $\mathcal{E}_t$ . Note that usually  $\mathbb{E}_{\mathcal{E}_t}(\epsilon_t \frac{h_t}{w_t}) \neq \mathbb{E}_{\mathcal{E}_t}(\epsilon_t) \frac{\bar{H}_t}{\bar{W}_t}$  which means that even  $\frac{\bar{W}_t}{\bar{H}_t} \tau_{h,t}^{int}$  does not correspond to a simple mean of individual elasticities over  $\mathcal{E}_t$ .

The second term  $\tau_{h,t}^{ext} \geq 0$  captures all adjustments of working hours along the extensive margin, i.e., all changes due to transitions between non-employment and employment. Its interpretation is analogous to that of  $\tau_{w,t}^{ext}$  already discussed above. Note that  $\mathbb{E}(h_t | w_t^R = w_t = w)$  is the average number of hours a marginal worker with reservation wage rate  $w_t^R = w$  intends to work if she is offered the wage rate  $w_t = w$ . For any small  $\Delta > 0$  and any wage rate  $w$  the term  $\Delta \mathbb{E}(h_t | w_t^R = w_t = w) f_{w^R|w}^t(w)$  thus approximates the average amount of working hours provided by marginal workers transiting between non-employment and employment if the wage rate offer rises from  $w$  to  $w + \Delta$ . Hence, for a given wage rate  $w$  the term  $\mathbb{E}(h_t | w_t^R = w_t = w) f_{w^R|w}^t(w)$  quantifies the rate of change of hours worked by marginal workers if  $w$  changes by  $\Delta > 0$ .  $\tau_{h,t}^{ext}$  is the mean of these rates over all wages,  $\tau_{h,t}^{ext} = \mathbb{E} \left( \mathbb{E}(h_t | w_t^R = w_t) f_{w^R|w}^t(w_t) \right) \geq 0$ .

Summarizing our discussion, the aggregate Frisch wage-elasticity is given by

$$e_t = \frac{\overline{W}_t}{\overline{H}_t} \left( \frac{\tau_{h,t}^{int} + \tau_{h,t}^{ext}}{EPR_t + \tau_{w,t}^{ext}} \right). \quad (22)$$

Most existing work in business cycle analysis is based on models which assume time-invariant wage elasticities of labor supply. At a first glance it may come as a surprise that aggregate elasticities determined by (22) explicitly depend on time. Time dependence of  $e_t$  is an inevitable consequence of the fact that all major determinants vary over time, albeit at a high degree of persistence.

## 4 Econometric Modeling

In what follows, we will describe an econometric approach to estimate the total labor supply adjustment along the intensive margin as well as the adjustments along the extensive margin in our general effort to quantify the aggregate Frisch wage-elasticity  $e_t$ . Note that the quantities  $\overline{W}_t$ ,  $\overline{H}_t$  and  $EPR_t$  can be determined directly from observed data, so that we have to find estimates for  $\tau_{h,t}^{int}$ ,  $\tau_{h,t}^{ext}$  and  $\tau_{w,t}^{ext}$ .

For a given period  $t$ , the expression for the total labor supply adjustment



along the intensive margin from equation (20) can be estimated via its sample equivalent

$$\hat{\tau}_{h,t}^{int} = \frac{1}{N_t^w} \sum_{i: h_{it} > 0} \partial_w \hat{g}(w_{it}, \lambda_{it}, Y_{it}) \quad (23)$$

where  $N_t^w$  denotes the employed workers in period  $t$  in our sample. The determinants of the individual labor supply  $h_{it} = g(w_{it}, \lambda_{it}, Y_{it}) I(w_{it} \geq w_{it}^R)$  are given by the wage rate  $w_{it}$ , the marginal utility of wealth  $\lambda_{it}$ , observable individual characteristics  $X_{it}$  and unobservable random factors  $Z_{it}$  with  $Y_{it} = (X_{it}, Z_{it})$ . Recall that we assume that there are no systematic differences between wages paid to working individuals and those unobserved wages declined by non-working individuals, and that all wage rates are drawn from the same distribution. Thus, we do not need to consider selection.<sup>6</sup> We closely follow the empirical literature on male labor supply analysis where hours worked are treated as a continuous variable. Assuming that all determinants have a linear effect on the individual labor supply we get the following panel data model:<sup>7</sup>

$$\log h_{it} = \gamma_0 + \gamma_1 \log w_{it} + (X_{it})' \beta + \lambda_{it} + z_{it}, \quad (24)$$

where  $X_{it}$  is a vector of  $p$  different observable attributes and the  $p$ -dimensional parameter vector  $\beta$  captures their influence on the individual labor supply.

<sup>6</sup>For each year we test for sample selection bias by estimating a Probit model for the labor force participation equation over the whole sample and including the resulting inverse Mills ratio in the structural equation. For most years, the coefficient on the inverse Mills ratio is not significant and hence the null hypothesis of having no selection bias cannot be rejected. Details of the estimation and test procedure as well as results are available from the authors upon request.

<sup>7</sup>Note that if we assumed the utility function to be separable between leisure and consumption, linearity would directly follow. Let  $U = f(c_{it}, Z_{it}) - \exp(-X_{it}'\beta^* - z_{it}^*)(T - l_{it})^\sigma$  as in MaCurdy (1985) where  $\beta^*$  is a vector of parameters associated with the observable individual characteristics  $X_{it}$ ,  $z_{it}^*$  is the contribution of the unmeasured characteristics and  $\sigma > 1$  is a preference parameter common to all individuals. Then, the first order condition (4b) reads as follows and can be reformulated further:

$$\begin{aligned} \lambda_{it} w_{it} &= \exp(-X_{it}'\beta^* - z_{it}^*) \sigma h_{it}^{\sigma-1} \\ \log \lambda_{it} + \log w_{it} &= -X_{it}'\beta^* - z_{it}^* + \log \sigma + (\sigma - 1) \log h_{it} \\ \log h_{it} &= (\sigma - 1)^{-1} (-\log \sigma + \log w_{it}) + X_{it}'\beta + \tilde{\lambda}_{it} + \tilde{z}_{it}, \end{aligned}$$

with  $\beta = (\sigma - 1)^{-1} \beta^*$ ,  $\tilde{\lambda}_{it} = (\sigma - 1)^{-1} \log \lambda_{it}$  and  $\tilde{z}_{it} = (\sigma - 1)^{-1} z_{it}^*$ .

The term  $z_{it}$  measures the influence of unobservable individual characteristics.

In order to retrieve the individual fixed components of  $\lambda_{it}$  and  $z_{it}$  we decompose their sum into their respective time averages and a time-varying residual:

$$\lambda_{it} + z_{it} = \underbrace{\lambda_i + z_i}_{\mu_i} + \underbrace{\lambda_{it} - \lambda_i + z_{it} - z_i}_{\xi_{it}}. \quad (25)$$

This yields

$$\log h_{it} = \gamma_0 + \gamma_1 \log w_{it} + (X_{it})' \beta + \mu_i + \xi_{it}, \quad (26)$$

where we now assume  $\xi_{it}$  to be i.i.d. idiosyncratic errors with zero mean and common variance. Since the individual wage rate is correlated with the marginal utility of wealth  $\lambda_{it}$  which enters the error term, we instrument for wage rates. The structure of the panel model above as well as the instrumental variable (IV) approach are in accordance with the setup commonly used in the literature estimating the individual labor supply of males (cf. for example Blundell and MaCurdy (1999), Fiorito and Zanella (2012)). The instruments must be uncorrelated with the time-varying wealth and preference component of the error, i.e.,  $\lambda_{it} - \lambda_i$  and  $z_{it} - z_i$ . However, they may correlate with the individual fixed effects. We estimate equation (26) using a fixed-effect estimator. In order to guarantee identification of  $\beta$ , there may not be a constant in  $X$  and none of the observable attributes may be determined by the wage rate, so that the matrix  $\mathbb{E}\{[X - \mathbb{E}[X | \log w]][X - \mathbb{E}[X | \log w]]'\}$  be positive definite. As is common in this literature, the sum over all individual effects is standardized to equal zero.

The panel data model implies that an estimate of the derivative of the individual labor supply function with respect to the wage rate is given by

$$\partial_w \hat{g}(w_{it}, \lambda_{it}, Y_{it}) = \frac{h_{it}}{w_{it}} \hat{\gamma}_1,$$

so that for each period  $t$  the total labor supply adjustment along the in-

tensive margin can be estimated by

$$\hat{\tau}_{h,t}^{int} = \frac{1}{N_t^w} \sum_{i: h_{it} > 0} \frac{h_{it}}{w_{it}} \hat{\gamma}_1. \quad (27)$$

Let us now consider the adjustments along the extensive margin. To maintain a high degree of generality, we take a non-parametric estimation approach. Recall from equations (18) and (20) that  $\tau_{w,t}^{ext}$  and  $\tau_{h,t}^{ext}$  are given by

$$\tau_{w,t}^{ext} = \int \nu f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu \quad (28)$$

and

$$\tau_{h,t}^{ext} = \int \mathbb{E}\left(h_t \mid w_t^R = w_t = \nu\right) f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu, \quad (29)$$

respectively. Therefore, for given  $\nu$  we have to find estimates for the product of densities  $f_{w^R|\nu}^t(\nu) f_w^t(\nu) = f_{w^R,w}^t(\nu, \nu)$  and the conditional expectation  $\mathbb{E}(h_t \mid w_t^R = w_t = \nu)$ . As the joint distribution of reservation wages and hourly wage rates is unknown, we condition on observable individual characteristics,  $X$ , to estimate the product of densities

$$\begin{aligned} f_{w^R,w}^t(w_1, w_2) &= \int f_{w^R,w|X}^t(w_1, w_2) d\pi_X^t \\ &= \int f_{w^R|X}^t(w_1) f_{w|X}^t(w_2) d\pi_X^t \end{aligned} \quad (30)$$

and assume that the joint density of the wage and the reservation wage can be factorized conditional on individual characteristics. This assumption is comparable to what Hall (2012) calls proportionality hypothesis which states that individual reservation wage rates and actual wage rates are proportional to the individual productivity. Both densities as well as the conditional expectation are estimated nonparametrically, resulting in  $\hat{f}_{w^R|X}^t(\cdot)$ ,  $\hat{f}_{w|X}^t(\cdot)$  and  $\hat{\mathbb{E}}(h_t \mid w_t^R = w_t = \cdot)$ , respectively. We employ a two-step conditional density estimator and consider first two simple regression models, followed by a nonparametric kernel density estimator to determine an estimate from the residuals of the regression models. For the estimation of the conditional expectation we employ a local constant kernel estimator,

also referred to as the Nadaraya-Watson kernel estimator.<sup>8</sup> For each period  $t$ ,  $\tau_{w,t}^{ext}$  and  $\tau_{h,t}^{ext}$  can then be approximated by

$$\hat{\tau}_{w,t}^{ext} = \int \nu \left( \frac{1}{N_t} \sum_i \hat{f}_{w^R|X=X_{it}}^t(\nu) \hat{f}_{w|X=X_{it}}^t(\nu) \right) d\nu \quad (31)$$

and

$$\hat{\tau}_{h,t}^{ext} = \int \hat{\mathbb{E}}(h_t | w_t^R = w_t = \nu) \left( \frac{1}{N_t} \sum_i \hat{f}_{w^R|X=X_{it}}^t(\nu) \hat{f}_{w|X=X_{it}}^t(\nu) \right) d\nu \quad (32)$$

where  $N_t$  denotes the sum of working and non-working individuals in period  $t$  in our sample. This allows us to estimate the aggregate Frisch wage-elasticity as specified in equation (22) for any period  $t$ .

## 5 Data

Our empirical work is based on data from the German Socio-Economic Panel (SOEP), a representative sample of private households and individuals living in Germany. The panel was started in 1984 (wave A) and has been updated annually through 2011 (wave BB). The panel design closely follows that of the Panel Study of Income Dynamics (PSID) – a representative sample of US households and individuals – but also takes idiosyncrasies of the German legal and socio-economic framework into account.<sup>9</sup> Since 2000, the SOEP covers on average 12,000 households and 20,000 individuals per year. A set of core questions is asked every year, including questions on education and training, labor market behavior, earnings, taxes and social security, etc.

We use the SOEP, because we consider it particularly well suited for the purpose of our analysis. To our knowledge it is the only micro panel currently available that contains indirect information on reservation wage

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<sup>8</sup>The nonparametric estimation procedure for  $\hat{f}_{w^R|X}^t(\cdot)$ ,  $\hat{f}_{w|X}^t(\cdot)$  and  $\hat{\mathbb{E}}(h_t | w_t^R = w_t = \cdot)$  is described in Appendix C (see e.g. Li and Racine (2006)).

<sup>9</sup>A detailed description of the panel's design, its coverage, the main questions asked, etc. is contained in the *Desktop Companion* to the SOEP, which is accessible online at [www.diw.de](http://www.diw.de).

rates of non-employed workers. This variable is essential for our effort to quantify changes in a worker's participation decision. Apart from detailed information on individual characteristics, the SOEP also reports an employed individual's market hours worked and earnings. We can thus compute an individual's hourly wage rate.

## 5.1 Sample

For the sake of our empirical analysis we need consistent data on individual labor market behavior over a rather long time horizon. Therefore, we focus on the working age population of German males living in former West Germany who are between 25 and 64 years old. We do so, because we are neither interested in the peculiarities of women's working behavior nor in the institutional differences between former East and West Germany. Including females in a relatively long panel study would be problematic because in Germany, unlike in many other countries, females have undergone severe changes in their labor market behavior during the past decades and are less attached to the workforce than elsewhere. Since we want to focus on those who actively participate in the labor market, we exclude retirees, individuals in military service under conscription or in community service which can serve as substitute for compulsory military service, and individuals currently undergoing education. We also exclude individuals with missing information on unemployment experience or the amount of education or training. A maximum of 56 individuals is affected. Our sample ranges from 2000 to 2009. That is because in 2000 a refreshment sample was added to the SOEP which effectively doubled the number of observations.

Moreover, our fixed-effect estimation procedure requires the time index  $t$  to converge to infinity to ensure consistent estimates of the individual fixed effects. Therefore, we create a balanced panel from our sample which includes those working males who are continuously employed over the sample period. Our balanced panel comprises 1,296 individuals. We use these individuals whenever we compute measures related to employed workers. For all questions related to non-employment we consider individuals who are not employed and have answered the question on reservation wages.

This leaves us with 91 to 140 individuals between 2000 and 2009.<sup>10</sup>

## 5.2 Variables

Our key variables of interest are the hourly wage rate and actual working hours for the employed, the reservation wage rate for the unemployed, and individual characteristics.<sup>11</sup> A person’s total hours worked,  $h_{it}$ , are given by the average actual weekly working hours. There is a wide range of answers to the question “And how much on average does your actual working week amount to, with possible overtime?” – answers range from 5.5 to 80 hours per week. In fact, the distribution of  $h_{it}$  is not discrete in nature, but quite dispersed, in particular during the last 15 to 20 years. It seems that the traditional 40 hours workweek gradually loses its prevalence as there are increasing possibilities of part-time work, higher skilled workers are asked to work more, and more flexible work options have become available.<sup>12</sup>

The hourly wage rate is calculated by dividing the current net monthly earnings by the product of 4.3 and contractual weekly working hours. We use net earnings, since information on the reservation wage is only available in net terms and we need the wage rate,  $w_{it}$ , and the reservation wage rate to be comparable. We convert all nominal values into real ones by dividing all nominal expressions by the consumer price index which uses 2005 as base year.

The reservation wage is generated from answers to the question “How much would the net pay have to be for you to consider taking the job?” which is posed to all individuals who are not in gainful employment or in military service and who intend to take up a job in the future. The associated working hours are deduced from the variable “Interest in full or part-time work”. We assume persons answering the question “Are you interested in full- or part-time employment?” with “Full-time employment”, “Either“ or “Don’t know” to be interested in 40 hours of work per week. We assign 20 hours of work per week to those who indicate an interest in

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<sup>10</sup>A detailed description of our sample is given in Appendix D. In particular, Table 6 shows summary statistics and we list all refinements to the original data.

<sup>11</sup>A list of all SOEP variables with respective names as well as a list of all generated variables with description is given in Appendix D.

<sup>12</sup>Histograms of actual hours worked for the years 2000, 2005 and 2009 are available in Appendix D.

Table 1: Preferred Working Hours Linked to Reservation Net Income [%]

Wave	Full-time, Either, Don't know				Part-time			
	Obs.	[0,35)	[35,45]	(45,70]	Obs.	[0,15)	[15,25]	(25,40]
2007	107	0.05	0.88	0.07	11	0.00	0.64	0.36
2008	86	0.08	0.87	0.05	5	0.00	0.60	0.40
2009	112	0.05	0.88	0.06	7	0.00	0.43	0.57

Notes: Obs. denotes the number of observations for West German males aged 25 to 64 with answers “Full-time”, “Either”, “Don't know” and “Part-time”, respectively, to the question “Are you interested in full- or part-time employment?”.

“Part-time employment”. The reservation wage rate corresponds to the ratio of the monthly net reservation earnings to the product of 4.3 and desired weekly working hours. Since the year 2007 the SOEP contains detailed information on desired weekly working hours. If available we use the answer to the question “In your opinion how many hours a week would you have to work to earn this net income?” to calculate the reservation wage. In fact, we can use this more detailed information to check whether attributing 20 and 40 hours work per week is reasonable. Table 1 shows that for individuals who are indifferent or those interested in full-time work the assumed 40 hours of work per week for the years 1984 to 2006 are a reasonable choice. For the years 2007, 2008 and 2009, around 88 % of those individuals believe that they would have to work between 35 and 45 hours to earn the desired reservation net income. For individuals interested in part-time work the picture is not as clear. Part-time work is usually any work with less than 30 to 35 hours per week, but in a legal sense is defined as employment with fewer hours than a comparable full-time job. This vague definition is reflected in the relative frequencies of the number of working hours associated with the reservation net earnings in Table 1. However, note that for all years few individuals fall into this category, in fact at most 11 individuals. Therefore, we stick to the assumption of 20 working hours per week for individuals interested in part-time work.

We use different individual characteristics for the employed and the non-employed. For the sake of estimating our panel model, we consider as individual characteristics of the employed a dummy for the family status (1 if married or currently living in dwelling with steady partner, 0 otherwise),

work experience in full-time employment, and three dummy variables on the occupational group. Each working individual belongs to one out of the following four occupational groups. The first group comprises employees in agriculture, animal husbandry, forestry, horticulture or in mining. The second group comprises employees in manufacturing or technical occupations (e.g. engineers, chemists, technicians). All employees in the service industry belong to the third group. The fourth group comprises all other workers, in particular persons who do not report an established profession or workers without any further specification of their professional activity.

As mentioned in section 4 we use an IV approach to account for the possible endogeneity of wages. Following the ideas of Mincer (1974) who viewed wages as predominantly determined by accumulated human capital, we include as instruments schooling, work experience in full-time employment, and work experience squared. The schooling variable is based on the number of years of education or training undergone and exhibits some variation over time. It includes secondary vocational education and ranges from 7 to 18 years.<sup>13</sup>

The determinants of the reservation wage which are needed for the estimation of the conditional density  $f_{w^R|X}^t(\cdot)$  are given by unemployment experience in years, a dummy on whether or not information for unemployment benefits is provided, the size of unemployment benefits, and a dummy for highly qualified individuals. The latter group has obtained a college or university degree.<sup>14</sup> Note that in each year individuals are asked about the size of the unemployment benefits in the previous year so that the information about unemployment benefits is not available for the last wave, i.e. 2009. For estimating  $f_{w|X}^t(\cdot)$  we use schooling, work experience in full-time employment, and work experience squared.

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<sup>13</sup>There exist alternative instruments, e.g., a regionally varying unemployment rate which is available from IAB (German Bureau of Labor Statistics), Nuremberg.

<sup>14</sup>These determinants of the reservation wage rate are in line with the literature as Prasad (2004) and Addison et al. (2009), among others, find that duration of joblessness, availability and level of unemployment compensation and observables of education or skill level are the most important determinants of reservation wages.



## 6 Results

We start this section by presenting results from the panel, density and conditional expectation estimation needed for the determination of the total adjustments along the intensive and extensive margin, respectively. Then, we provide results for the aggregate Frisch wage-elasticity of labor supply.

### 6.1 Panel model estimation

For calculating the total labor supply adjustment along the intensive margin  $\tau_{h,t}^{int}$ , we first have to estimate the panel data model for the working population. Results for the first stage of the panel model estimation are given in Table 7 in Appendix E. All instruments and the constant are highly significant. Wage rates rise in the years of schooling and in work experience gathered. However, the coefficient on work experience squared is negative, so that each further increase in experience conveys a progressively smaller increase in the wage rate.

Table 2 shows results for the panel model estimation, equation (26). For the benchmark specification, i.e. the IV approach, the coefficient on the family status dummy variable is significantly negative at the 5 percent level showing that married or cohabiting individuals have a lower engagement in the workforce. The constant and the coefficient on the logarithm of the wage rate are highly significantly positive. The parameter estimate of the latter variable equals .29. This estimate corresponds to the wealth-compensated individual wage elasticity of labor supply which has received a lot of attention in the empirical labor literature. Our estimate for working age males in Germany is in line with what is commonly reported in that literature. Table 2b shows that neglecting the endogeneity of wage rates leads to negative point estimates on the logarithm of the hourly wage rate as is also discussed in Reynaga and Rendon (2012).

An important issue when using an IV approach is the strength of instruments. The first stage F-statistic which is equivalent to the Cragg-Donald-statistic in a linear IV regression in the case of one endogenous regressor is 52.09 (cf. Cragg and Donald (1993)). In the case of an IV regression with a single endogenous regressor and iid errors, instruments are considered to be strong, if the first stage F-statistic exceeds 10 (cf. rule

Table 2: Results for the Panel Model Estimation

(a) With IVs (Benchmark)		(b) Without IVs	
$\log h$	Coef.	$\log h$	Coef.
$\log w$	0.2879269***	$\log w$	-0.1233714***
FAMILY	-0.0201949**	FAMILY	0.0092642
EXPFT	-0.0006551	EXPFT	0.0019046***
O1	0.0242284	O1	0.028944
O3	0.0114577*	O3	0.0103813*
O4	0.0281898	O4	0.0367002**
CONST	3.072701***	CONST	4.029012***

Notes: \*\*\*, \*\*, and \* denote significance at the 1, 5 and 10 percent level, respectively. FAMILY, EXPFT, O1, O3, O4 and CONST represent the family status dummy variable, work experience in years, dummy variables on occupational group and a constant, respectively. The sample underlying the estimation is described in section 5.

of thumb by Staiger and Stock (1997)). For linear IV regressions Stock and Yogo (2005) provide critical values to test for weak instruments based on two alternative definitions of weak instruments. The first test is based on the maximum IV estimator bias, the second one is based on the maximum Wald test size distortion. The critical values for one endogenous regressor and seven instruments at the 5% significance level are 19.86 and 31.5, respectively. Whenever the Cragg-Donald-statistic exceeds the critical values, one can reject the null hypothesis of weak instruments. We consider this as evidence of strong instruments.

## 6.2 Conditional Density and Expectation Estimation

As described in section 4 and in Appendix C we have to first estimate the wage and reservation wage regression, equation (33) and (34), to get the conditional densities  $\hat{f}_{w|X}^t(\cdot)$  and  $\hat{f}_{w^R|X}^t(\cdot)$ , respectively. Regression results are shown in Table 3 and 4.

As is the case for the first stage of the panel model estimation, for all years except for 2001 the coefficients on the individual characteristics as well as the constant are highly significant. Wage rates rise in the years of schooling and in work experience gathered. However, the coefficient on work experience squared is negative, so that each further increase in

Table 3: Results for Wage Regression, Equation (C.33)

Wave	CONST	SCHOOL	EXPFT	EXPFT2
2000	-5.356504***	0.9735708***	0.6026383***	-0.0122329***
2001	-4.24767	1.260115***	0.1101167	-0.0004187
2002	-4.408469***	0.9711685***	0.5200087***	-0.010236***
2003	-5.467164***	1.062251***	0.4847722***	-0.0083184***
2004	-4.966449***	1.027526***	0.4441191***	-0.0072727***
2005	-7.751894***	1.152485***	0.5540573***	-0.0092175***
2006	-7.050421***	1.070592***	0.5755334***	-0.0098765***
2007	-8.159443***	1.134918***	0.5571551***	-0.0089508***
2008	-5.125946***	1.05965***	0.3536003***	-0.0050957***
2009	-6.227629***	1.047241***	0.4771336***	-0.0074261***

Notes: See Table 2. CONST, SCHOOL, EXPFT and EXPFT2 denote a constant, the schooling variable, work experience, and work experience squared, respectively.

Table 4: Results for Reservation Wage Regression, Equation (C.34)

Wave	CONST	EXPUE	UEBEN	HQD	UEBEND
2000	9.287464***	-0.074731	0.0018317	1.579977*	-2.46782**
2001	8.632496***	-0.170788**	0.003354***	2.905584***	-2.710164***
2002	9.934094***	-0.188054**	0.0029888***	0.8428682	-3.273079***
2003	9.488645***	-0.05987	0.0035565***	3.540291***	-4.117936***
2004	10.4398***	-0.191361**	0.0019663***	1.274672	-3.644075***
2005	8.584524***	-0.11929	0.005742***	3.807951***	-5.233218***
2006	8.935665***	-0.235793**	0.0051221***	4.686461***	-4.434229***
2007	8.3453***	-0.097721	0.0037337***	3.437334***	-4.102274***
2008	8.747728***	0.098416	0.0053015	0.9004927	-5.430932

Notes: See Table 2. CONST, EXPUE, UEBEN, HQD and UEBEND denote a constant, unemployment experience in years, unemployment benefits in 100 euros, a dummy for highly qualified individuals and one on whether information on unemployment benefits is provided, respectively. We do not provide results for the year 2009 as data for the size of unemployment benefits are not available for this year.

experience conveys a progressively smaller increase in the wage rate.

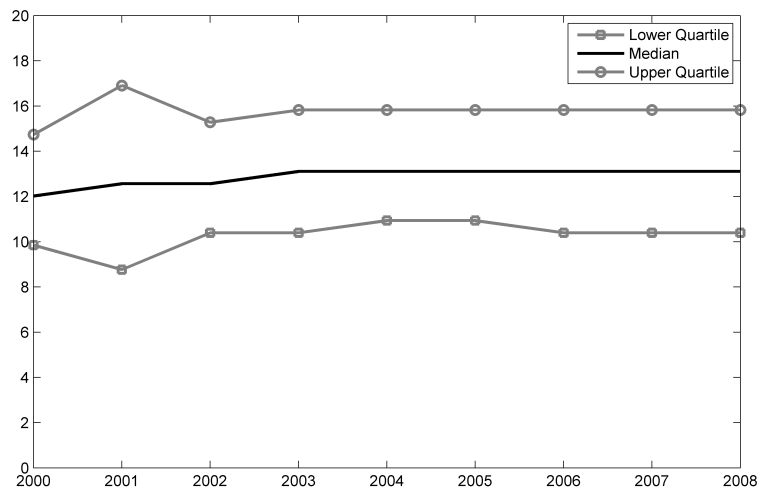
For the estimation of equation (C.34) we have between 91 and 140 observations and the constant is highly significant between 8.35 and 10.44. The coefficient on the unemployment duration is mostly negative and not significant. The predominant sign of the coefficient is in line with predictions from theoretical models and empirical evidence that the reservation wage decreases with waiting time for a new job. The reservation wage rate significantly decreases if non-employed individuals receive unemployment benefits, but they increase in the level of those benefits. Being a highly qualified individual, i.e. having obtained a college or university degree, increases the reservation wage, in most cases (highly) significantly.

The resulting conditional densities  $f_{w|X}^t(\cdot)$  and  $f_{w^R|X}^t(\cdot)$  vary with individual characteristics  $X = X_{it}$ . Therefore, we restrict our analysis to the densities conditional on mean individual characteristics, i.e.  $X_{it} = \bar{X}_t$ . Note that this choice is rather arbitrary. One could also consider results for median or prespecified individual characteristics. Figure 1 shows the lower quartile, the median and the upper quartile for the wage as well as the reservation wage distribution conditional on mean individual characteristics. It does not come as a surprise that the distribution of the reservation wage is left of the wage distribution for all years as individuals are only working if the offered wage exceeds the reservation wage. For the wage distribution, the lower quartile, the median and the upper quartile vary around 10.3, 12.9 and 15.8, respectively. For 2001 the distribution is more dispersed which is possibly also one reason for the less accurate regression results in this year. On the other hand, for the reservation wage distribution, the lower quartile, the median and the upper quartile vary around 7.7, 9.5 and 11.4, respectively. In 2008, the distribution shifts slightly to the left which probably is the result of a decrease in the mean size of unemployment benefits which have a negative influence on reservation wage rates.

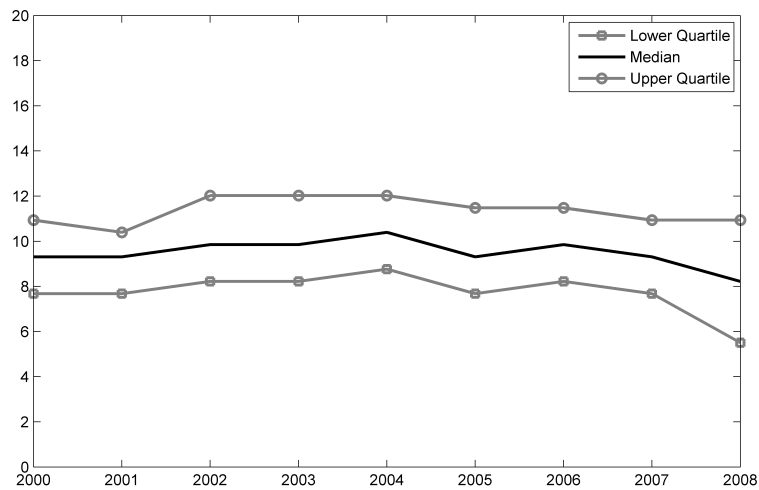
In the following, we consider results from the conditional expectation estimation generated by considering the reservation wage  $w^R$  and associated hours data  $h^R$  for each year. Figure 2 shows the nonparametric regression results for all years. The expectation corresponds to the hours a marginal worker would work at her reservation wage. Therefore, the estimated values

Figure 1: Quartiles of the densities conditional on  $X = \bar{X}$

(a)  $\hat{f}_{w|\bar{X}}^t(\cdot)$

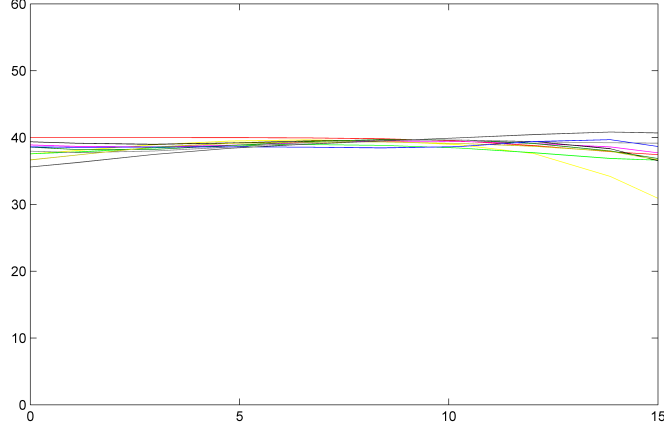


(b)  $\hat{f}_{w^R|\bar{X}}^t(\cdot)$



Notes: The horizontal axes measure years and the vertical axes represent the wage rate (a) and the reservation wage rate (b), respectively. This figure shows the lower quartile, the median and the upper quartile of the conditional densities  $\hat{f}_{w|\bar{X}}^t(\cdot)$  and  $\hat{f}_{w^R|\bar{X}}^t(\cdot)$ , respectively.

Figure 2: Expectation of weekly working hours conditional on  $w = w^R$



Notes: The horizontal axis measures the real hourly wage rate and the vertical axis represents working hours. This figure shows the regression functions for the conditional expectation  $\hat{\mathbb{E}}(h_t | w_t^R = w_t)$  for the years 2000 to 2009.

of around 40 working hours per week seem plausible.

### 6.3 The Aggregate Frisch Wage-Elasticity of Labor Supply

For the calculation of the aggregate Frisch elasticity we determine the employment ratio  $EPR_t$ , the mean labor supply  $\bar{H}_t$  as well as the mean wage rate  $\bar{W}_t$  received by all working individuals directly from observed data (see Table 8 in Appendix E). Results for the estimated determinants of the aggregate Frisch wage-elasticity, i.e.  $\hat{\tau}_{h,t}^{int}$ ,  $\hat{\tau}_{h,t}^{ext}$  and  $\hat{\tau}_{w,t}^{ext}$  are shown in Table 9 in Appendix E whereas results for the aggregate Frisch wage-elasticity

$$\begin{aligned} \hat{e}_t &= \frac{\bar{W}_t}{\bar{H}_t} \left( \frac{\hat{\tau}_{h,t}^{int} + \hat{\tau}_{h,t}^{ext}}{EPR_t + \hat{\tau}_{w,t}^{ext}} \right) \\ &= \underbrace{\frac{\bar{W}_t}{\bar{H}_t} \frac{1}{EPR_t + \hat{\tau}_{w,t}^{ext}} \cdot \hat{\tau}_{h,t}^{int}}_{\tilde{\tau}_{h,t}^{int}} + \underbrace{\frac{\bar{W}_t}{\bar{H}_t} \frac{1}{EPR_t + \hat{\tau}_{w,t}^{ext}} \cdot \hat{\tau}_{h,t}^{ext}}_{\tilde{\tau}_{h,t}^{ext}}. \end{aligned}$$

and its weighted components  $\tilde{\tau}_{h,t}^{int}$  and  $\tilde{\tau}_{h,t}^{ext}$  are shown in Table 5. The aggregate Frisch elasticity ranges from .63 in 2008 to .70 in 2003 and 2004. Considering only the first eight years from 2000 to 2007, the aggregate

Frisch elasticity varies very little between .68 and .70. The slightly lower value of 0.63 in 2008 is caused by the lower hours adjustment along the extensive margin, i.e. a lower value of  $\hat{\tau}_{h,t}^{ext}$  in 2008 compared to the other years. Our estimates of the aggregate elasticity is very close to what Fiorito and Zanella (2012) report for continuously employed men in the US. Table 5 also shows that about one third of the aggregate adjustment is due to hours adjustment of stayers and the remaining two-thirds are due to hours worked by new entrants into the labor market.

Table 5: The Aggregate Frisch Wage-Elasticity and Weighted Components

Wave	$\hat{e}_t$	$\tilde{\tau}_{h,t}^{int}$	$\tilde{\tau}_{h,t}^{ext}$
2000	0.69	0.22	0.47
2001	0.68	0.27	0.42
2002	0.68	0.22	0.46
2003	0.70	0.22	0.48
2004	0.70	0.21	0.49
2005	0.69	0.24	0.45
2006	0.69	0.23	0.46
2007	0.68	0.24	0.45
2008	0.63	0.25	0.38

Notes: For the determination of the aggregate Frisch wage-elasticity  $\hat{e}_t$  we consider the sample as described in section 5.

## 7 Conclusion

This paper illustrates the power and the importance of taking statistical aggregation seriously when thinking about possible links between individual and aggregate Frisch wage-elasticities of labor supply in an environment where workers are heterogenous. Statistical aggregation introduces non-linearities which drive a wedge between the mean of individual elasticities and their aggregate counterpart. Moreover, it allows for simultaneous treatment of hours adjustment along the intensive and the extensive margin. When illustrating the method's quantitative implications using information on males at working-age living in former West-Germany we find that adjustment along the extensive margin is twice as important as that along the intensive margin for the total variation in hours work. We

thus corroborate a key result from the literature which reports that ca. two-thirds of all person hours variation is due to workers moving in or out of employment.

The aggregation method developed in this paper is very general and flexible. Adapting it to alternative models of labor supply or using it to compute static elasticities such as Marshallian or Hicks is a straightforward exercise.



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## Appendix A Aggregate Frisch elasticities under monotone transformations

An aggregate Frisch wage-elasticity can be derived for any smooth, strictly monotone transformation  $w_t \rightarrow T(\Delta, w_t)$  with  $T(0, w_t) = w_t$  of wages. Given such a transformation, (15a) and (15b) generalize to

$$\begin{aligned}\bar{H}_{t;T}(\Delta) &:= \int g(T(\Delta, w), \lambda, Y) I(T(\Delta, w) \geq w^R) d\pi_{w, w^R, \lambda, Y}^t, \\ \bar{W}_{t;T}(\Delta) &:= \int T(\Delta, w) I(T(\Delta, w) \geq w^R) d\pi_{w, w^R}^t.\end{aligned}$$

By (16), determining a Frisch wage-elasticity  $e_{t;T}$  then requires to calculate the derivatives of  $\bar{W}_{t;T}(\Delta)$  and  $\bar{H}_{t;T}(\Delta)$  at  $\Delta = 0$ . Let  $T^*(w) = \frac{\partial T(\Delta, w)}{\partial \Delta} \Big|_{\Delta=0}$ . If  $T(\Delta, w) = w + \Delta$  then  $T^*(w) \equiv 1$ . For proportional changes  $T(\Delta, w) = w(1 + \Delta)$  we have  $T^*(w) = w$ . Straightforward generalizations of the arguments leading to (18) and (20) then yield

$$\begin{aligned}\frac{\partial \bar{W}_{t;T}(\Delta)}{\partial \Delta} \Big|_{\Delta=0} &= \underbrace{\int T^*(w) I(w \geq w^R) d\pi_{w, w^R}^t}_{\tau_{w,t;T}^{int}} + \underbrace{\int \nu T^*(\nu) f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu}_{\tau_{w,t;T}^{ext}}, \\ \frac{\partial \bar{H}_{t;T}(\Delta)}{\partial \Delta} \Big|_{\Delta=0} &= \underbrace{\int T^*(w) \partial_w g(w, \lambda, Y) I(w \geq w^R) d\pi_{w, w^R, \lambda, Y}^t}_{\tau_{h,t;T}^{int}} \\ &\quad + \underbrace{\int \mathbb{E} \left( h_t \mid w_t^R = w_t = \nu \right) T^*(\nu) f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu}_{\tau_{h,t;T}^{ext}},\end{aligned}$$

The aggregate Frisch wage-elasticity  $e_{t;T}$  with respect to the transformation  $T$  is given by

$$e_{t;T} = \frac{\bar{W}_t}{\bar{H}_t} \left( \frac{\tau_{h,t;T}^{int} + \tau_{h,t;T}^{ext}}{\tau_{w,t;T}^{int} + \tau_{w,t;T}^{ext}} \right).$$

## Appendix B Formal derivation of the derivative of equation (19), second term

We obtain

$$\begin{aligned}
& \int g(w + \Delta, \lambda, Y) I(w^R \in [w, w + \Delta]) d\pi_{w, w^R, \lambda, Y}^t \\
&= \int \int \int g(w + \Delta, \lambda, Y) d\pi_{(\lambda, Y)|(w^R, w)}^t I(w^R \in [w, w + \Delta]) d\pi_{w^R|w}^t d\pi_w^t \\
&= \int \left( \int_{\nu}^{\nu + \Delta} \mathbb{E} \left( g(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \tilde{\nu}, w_t = \nu \right) f_{w^R|\nu}^t(\tilde{\nu}) d\tilde{\nu} \right) f_w^t(\nu) d\nu.
\end{aligned}$$

In what follows we assume the conditional expectation  $\mathbb{E} \left( g(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \tilde{\nu}, w_t = \nu \right)$  as well as  $f_{w^R|\nu}^t(\tilde{\nu})$  to be continuous functions of  $\nu$  and  $\tilde{\nu}$ . Also note that  $\mathbb{E} \left( g(w_t, \lambda_t, Y_t) \mid w_t^R = w_t = \nu \right) = \mathbb{E} \left( h_t \mid w_t^R = w_t = \nu \right)$ . The mean value theorem then implies that for all  $\nu$  there exist a  $\xi_\nu \in [\nu, \nu + \Delta]$  such that

$$\begin{aligned}
& \int \left( \int_{\nu}^{\nu + \Delta} \mathbb{E} \left( g(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \tilde{\nu}, w_t = \nu \right) f_{w^R|\nu}^t(\tilde{\nu}) d\tilde{\nu} \right) f_w^t(\nu) d\nu \\
&= \int \Delta \mathbb{E} \left( g(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \xi_\nu, w_t = \nu \right) f_{w^R|\nu}^t(\xi_\nu) f_w^t(\nu) d\nu \\
&= \Delta \int \mathbb{E} \left( h_t \mid w_t^R = w_t = \nu \right) f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu \\
&\quad + \Delta \int \left( \mathbb{E} \left( g(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \xi_\nu, w_t = \nu \right) f_{w^R|\nu}^t(\xi_\nu) \right. \\
&\quad \quad \left. - \mathbb{E} \left( g(w_t, \lambda_t, Y_t) \mid w_t^R = w_t = \nu \right) f_{w^R|\nu}^t(\nu) \right) f_w^t(\nu) d\nu.
\end{aligned}$$

Obviously, for all  $\nu$ ,

$$\left| \mathbb{E} \left( g(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \xi_\nu, w_t = \nu \right) f_{w^R|\nu}^t(\xi_\nu) - \mathbb{E} \left( g(w_t, \lambda_t, Y_t) \mid w_t^R = w_t = \nu \right) f_{w^R|\nu}^t(\nu) \right| \rightarrow 0$$

as  $\Delta \rightarrow 0$ . Therefore,

$$\frac{\partial}{\partial \Delta} \int g(w + \Delta, \lambda_t, Y) I(w^R \in [w, w + \Delta]) d\pi_{w, w^R, \lambda, Y}^t \Big|_{\Delta=0}$$

$$\begin{aligned}
&= \lim_{\Delta \rightarrow 0} \frac{\int g(w + \Delta, \lambda_t, Y) I(w^R \in [w, w + \Delta]) d\pi_{w, w^R, \lambda, Y}^t}{\Delta} \\
&= \int \mathbb{E} \left( h_t \mid w_t^R = w_t = \nu \right) f_{w^R | \nu}^t(\nu) f_w^t(\nu) d\nu.
\end{aligned}$$

## Appendix C Conditional density and expected hours estimation

In order to approximate  $\tau_{h,t}^{ext}$  and  $\tau_{w,t}^{ext}$  we need to first estimate the conditional densities  $f_{w|X}^t(\cdot)$  and  $f_{w^R|X}^t(\cdot)$  as well as the conditional expectation  $\mathbb{E}(h_t \mid w_t^R = w_t = \cdot)$ .

For the density estimation, we employ a two-step conditional density estimator and consider first the following two simple regression models for each period  $t$  and individuals  $i$  with positive (reservation) wage rate

$$w_{it} = \alpha_{t0} + \sum_{j=1}^p \alpha_{tj} X_{it,j} + \delta_{it}, \quad i = 1, \dots, N_t^w, \quad (33)$$

$$w_{it}^R = \alpha_{t0}^R + \sum_{j=1}^p \alpha_{tj}^R X_{it,j} + \delta_{it}^R, \quad i = 1, \dots, N_t^R \quad (34)$$

where  $N_t^w$  denotes the number of wage observations in period  $t$ ,  $N_t^R$  denotes the number of reservation wage observations in period  $t$ ,  $\alpha_t = (\alpha_{t0}, \dots, \alpha_{tp})'$  and  $\alpha_t^R = (\alpha_{t0}^R, \dots, \alpha_{tp}^R)'$  are of dimension  $(p+1 \times 1)$  and  $X_{it}$  is a vector of  $p$  different observable attributes. We assume that the distributions of the random terms  $\delta_{it}$  and  $\delta_{it}^R$  are independent of  $X_{it}$  and calculate estimates  $\hat{\alpha}_t$  as well as residuals  $\hat{\delta}_{it} = w_{it} - \hat{\alpha}_{t0} - \sum_{j=1}^p \hat{\alpha}_{tj} X_{it,j}$  and  $\hat{\alpha}_t^R$  as well as  $\hat{\delta}_{it}^R = w_{it}^R - \hat{\alpha}_{t0}^R - \sum_{j=1}^p \hat{\alpha}_{tj}^R X_{it,j}$ , respectively.

Let  $f_{\delta}^t$  ( $f_{\delta^R}^t$ ) denote the density of the error terms  $\delta_{it}$  ( $\delta_{it}^R$ ) over the population. Then, on the one hand  $f_{w|X=X_{it}}^t(w_2) = f_{\delta}^t(w_2 - \alpha_{t0} - \sum_{j=1}^p \alpha_{tj} X_{it,j})$  and we use a nonparametric kernel density estimator to determine an estimate  $\hat{f}_{\delta}$  from the residuals  $\{\hat{\delta}_{it}\}_{i=1}^{N_t^w}$  of regression model (34), on the other hand  $f_{w^R|X=X_{it}}^t(w_1) = f_{\delta^R}^t(w_1 - \alpha_{t0}^R - \sum_{j=1}^p \alpha_{tj}^R X_{it,j})$  and we use a nonparametric kernel density estimator to determine an estimate  $\hat{f}_{\delta^R}$  from the

residuals  $\{\hat{\delta}_{it}^R\}_{i=1}^{N_t^R}$  of regression model (33):

$$\begin{aligned}\hat{f}_{w|X=X_{it}}^t(\cdot) &= \frac{1}{N_t^w b w_t^w} \sum_{j=1}^{N_t^w} k\left(\frac{\hat{\delta}_{jt} - (\cdot - \hat{\alpha}_{t0} - \sum_{l=1}^p \hat{\alpha}_{tl} X_{it,l})}{b w_t^w}\right) \\ \hat{f}_{w^R|X=X_{it}}^t(\cdot) &= \frac{1}{N_t^R b w_t^{w^R}} \sum_{j=1}^{N_t^R} k\left(\frac{\hat{\delta}_{jt}^R - (\cdot - \hat{\alpha}_{t0}^R - \sum_{l=1}^p \hat{\alpha}_{tl}^R X_{it,l})}{b w_t^{w^R}}\right)\end{aligned}$$

where  $k(\cdot)$  is a standard normal kernel and the bandwidths  $b w_t^{w^R}$  and  $b w_t^w$  are chosen according to the normal reference rule-of thumb, i.e.

$$\begin{aligned}k(v) &= \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}v^2\right), \\ b w_t^w &= 1.06 \cdot \sigma_{\delta_t} \cdot (N_t^w)^{-1/5} \quad \text{and} \quad b w_t^{w^R} = 1.06 \cdot \sigma_{\delta_t^R} \cdot (N_t^R)^{-1/5},\end{aligned}$$

with  $\sigma_{\delta_t}$  ( $\sigma_{\delta_t^R}$ ) being the standard deviation of the error terms  $\delta_{it}$  ( $\delta_{it}^R$ ) in period  $t$ .

For the estimation of the conditional expectation  $\mathbb{E}(h_t | w_t^R = w_t = \cdot)$  we employ a local constant kernel estimator, also referred to as the Nadaraya-Watson kernel estimator (cf. Nadaraya (1964) and Watson (1964)). We use the reservation wage  $w^R$  as explanatory variable and associated desired working hours  $h^R$  as dependent variable to account for the condition  $w_t^R = w_t$ . This leads to

$$\hat{\mathbb{E}}\left(h_t | w_t^R = w_t = \nu\right) = \frac{\int h^R \hat{f}_{h^R, w^R}^t(\nu, h^R) dh^R}{\hat{f}^t(\nu)} = \frac{\sum_{i=1}^{N_t^R} h_{it}^R \cdot k\left(\frac{w_{it}^R - \nu}{b w^{\mathbb{E}}}\right)}{\sum_{i=1}^{N_t^R} k\left(\frac{w_{it}^R - \nu}{b w^{\mathbb{E}}}\right)}, \quad (35)$$

where  $b w^{\mathbb{E}}$  denotes the bandwidth and is calculated as follows. We use local constant least squares cross-validation with leave-one-out kernel estimator to calculate the smoothing parameter for each year. Then, the bandwidth  $b w^{\mathbb{E}}$  is the average over all smoothing parameters.

# Appendix D Data

## D.1 SOEP Samples

Each household and thereby each individual in the SOEP is part of one of the following samples:

- Sample A: ‘Residents in the FRG’, started 1984
- Sample B: ‘Foreigners in the FRG’, started 1984
- Sample C: ‘German Residents in the GDR’, started 1990
- Sample D: ‘Immigrants’, started 1994/95
- Sample E: ‘Refreshment’, started 1998
- Sample F: ‘Innovation’, started 2000
- Sample G: ‘Oversampling of High Income’, started 2002
- Sample H: ‘Extension’, started 2006
- Sample I: ‘Incentivation’, started 2009



## D.2 SOEP Variables

Variable Name	Variable Label
\$SAMREG	Current wave sample region
PSAMPLE	Sample member
SEX	Gender
GEBJAHR	Year of birth
\$POP	Sample membership
\$NETTO	Current wave survey status
LABNET\$\$	Monthly net labor income
\$TATZEIT	Actual weekly working hours
\$VEBZEIT	Agreed weekly working hours
\$UEBSTD	Overtime per week
STIB\$\$	Occupational Position
Y11101\$\$	Consumer price index
e.g. DP170	Amount of necessary net income
e.g. AP20	Interest in full or part-time work
e.g. XP19	Number of hours for net income
EXPFT\$\$	Working experience full-time employment
EXPUE\$\$	Unemployment experience
KLAS\$\$	StaBuA 1992 Job Classification
ISCED\$\$	Highest degree/diploma attained
\$FAMSTD	Marital status in survey year
e.g. DP9201	Currently have steady partner
e.g. HP10202	Partner lives in household
\$BILZEIT	Amount of education or training (in years)
\$P2F03	Amount of monthly unemployment insurance
\$P2G03	Amount of monthly unemployment assistance

### D.3 SOEP Variable Refinements

- Actual weekly working hours: When the value for the variable actual weekly working hours is missing, we use instead, if available, agreed weekly working hours and, if available, add overtime per week.
- Agreed weekly working hours: When the value for the variable agreed weekly working hours is missing, we use instead, if available, actual weekly working hours and, if available, subtract overtime per week.
- Amount of necessary net income: For the years 1984 to 2001 DM-values are converted to euros by dividing the respective DM-values by 1.95583.

### D.4 Sample

Sample Definition	Condition
Only private households	keep if POP=1 $\vee$ POP=2
Only successful interviews	keep if NETTO $\in$ {10, 12, 13, 14, 15, 16, 18, 19}
No first time interviewed persons aged 17	drop if NETTO=16
Male population	drop if SEX=2
West Germany	drop if SAMPREG=2
Age	drop if AGE < 25 $\vee$ AGE > 64
Exclusion of retirees	drop if STIB=13
Exclusion of individuals in military service under conscription or in community service as substitute for compulsory military service	drop if STIB=15
Exclusion of individuals that are currently in education	drop if STIB=11
Individuals from sample A, E, F, H, I	drop if PSAMPLE $\in$ {2, 3, 4, 7}
No individuals with missing information	drop if BILZEIT < 0
	drop if EXPUE < 0 and $h = 0$

## D.5 Descriptive Statistics

Table 6: Summary Statistics of Our Sample

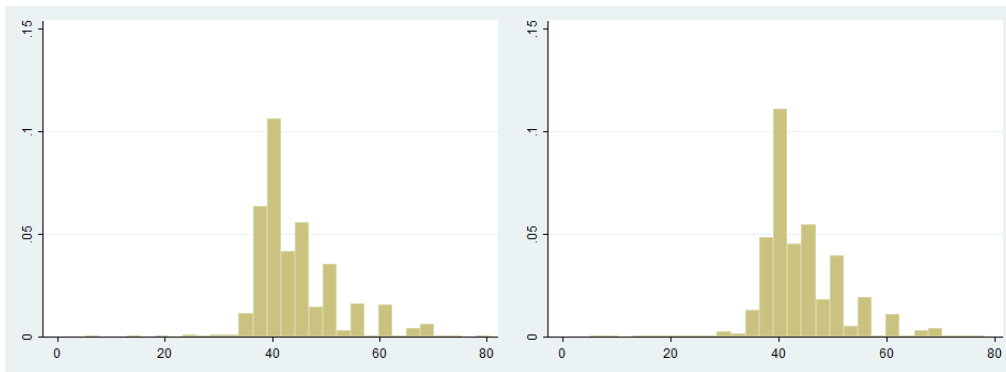
Wave	Employees			Non-Employees		
	2000	2005	2009	2000	2005	2009
Observations	1,296	1,296	1,296	121	126	119
Age [yrs.]	39.35	44.35	48.35	41.73	42.09	42.29
Schooling completed [yrs.]	12.54	12.57	12.59	11.15	11.11	10.83
Work experience [yrs.]	16.60	21.51	25.43	16.62	16.51	15.60
Married or cohabiting [%]	0.81	0.84	0.84	0.68	0.78	0.66
High-skilled [%]	0.24	0.24	0.25	0.12	0.10	0.08
Employed in O1	0.01	0.02	0.01	-	-	-
Employed in O2	0.45	0.44	0.41	-	-	-
Employed in O3	0.54	0.54	0.57	-	-	-
Employed in O4	0.00	0.00	0.01	-	-	-
Duration of non-employment [yrs.]	-	-	-	2.70	3.34	3.88
Entitled to unemployment benefits [%]	-	-	-	0.60	0.29	-

Notes: O1 represents workers employed in agriculture and related fields. O2 stands for employment in manufacture or technical occupations. O3 measures employment in services. O4 comprises all other workers. A detailed description of all variables is given in section 5.

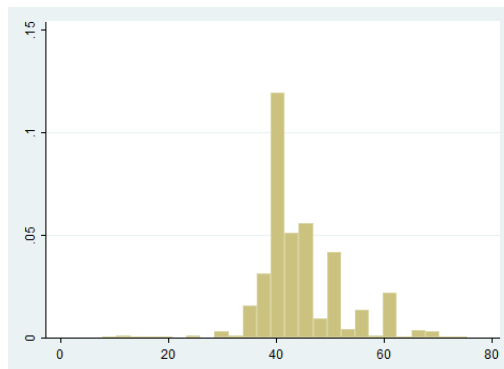
Figure 3: Histograms of Actual Weekly Hours Worked

(a) 2000

(b) 2005



(c) 2009



## Appendix E Results

Table 7: Results for the First Stage of the Panel Model Estimation

$\log w$	Coef.
SCHOOL	0.0700249***
EXPFT	0.0250717***
EXPFT2	-0.0004517***
FAMILY	0.0556724***
O1	0.0103846
O3	-0.0039625
O4	0.0198174
CONST	1.298827***

Notes: See Table 2. SCHOOL, EXPFT, EXPFT2, FAMILY, O1, O3, O4 and CONST denote the instruments for the wage rate, i.e., the schooling variable, work experience, and work experience squared, as well as the exogenous variables such as the family status, the three occupational groups, and a constant, respectively.

Table 8: Means of Hours Worked, Wages, and Employment Ratios

Wave	$\bar{H}_t$	$\bar{W}_t$	$EPR_t$
2000	44.19	12.61	0.91
2001	44.01	13.35	0.92
2002	44.05	13.18	0.91
2003	43.88	13.57	0.90
2004	43.93	13.47	0.91
2005	43.92	13.72	0.91
2006	44.16	13.64	0.91
2007	44.66	13.61	0.92
2008	44.48	13.44	0.93
2009	44.25	13.74	0.92

Notes: The employment ratio  $EPR_t$  is computed by dividing the number of working individuals by the total sample size in each period  $t$ .

Table 9: Estimated Components of the Aggregate Frisch Elasticity

Wave	$\hat{\tau}_{h,t}^{int}$	$\hat{\tau}_{h,t}^{ext}$	$\hat{\tau}_{w,t}^{ext}$
2000	1.23	2.56	0.64
2001	1.21	1.88	0.45
2002	1.14	2.43	0.66
2003	1.12	2.41	0.65
2004	1.12	2.58	0.71
2005	1.12	2.12	0.55
2006	1.12	2.25	0.60
2007	1.13	2.13	0.54
2008	1.15	1.74	0.44