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ABSTRACT

Risk-Adjusting the Returns to Venture Capital*

Performance evaluation of venture-capital (VC) payoffs is challenging because payoffs are infrequent, skewed, realized over endogenously varying time horizons, and cross-sectionally dependent. We show that standard stochastic discount factor (SDF) methods can be adapted to handle these issues. Our approach generalizes the Public Market Equivalent (PME) measure commonly used in the private-equity literature. We find that the abnormal returns from both VC funds and VC start-up investments are robust to relaxing the strong distributional assumptions and implicit SDF restrictions from the prior literature: VC start-up investments earn substantial positive abnormal returns, and VC fund abnormal returns are close to zero. We further show that the systematic component of start-up company and VC fund payoffs resembles the negatively skewed payoffs from selling index put options, which contrasts with the call option-like positive skewness of the idiosyncratic payoffs. Motivated by this finding, we explore an SDF that includes index put option returns. This results in negative abnormal returns to VC funds, while the abnormal returns to start-up investments remain large and positive.

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Venture capital (VC) investments play an important role in supporting the formation of young enterprises in the economy. Assessing the risk and return from this type of investments has thus far proven difficult for a number of reasons: Payoffs are infrequent, realized over multiple periods with varying and possibly endogenous time horizons, highly skewed, and cross-sectionally dependent. Furthermore, available data sets are plagued by selection biases and other problems.

Leaving aside selection bias issues for a moment, risk-adjustment of such payoffs is conceptually straightforward. Arbitrage-free stochastic discount factors (SDF) implied by standard asset-pricing models price highly skewed (“option-like”) payoffs just like any other payoff. For example, a large class of models implies an SDF in exponential-affine form,

$$M_{t+1} = \exp(a - bf_{t+1}). \tag{1}$$

One special case is the log-utility Capital Asset Pricing Model (CAPM) with $a = 0$, $b = 1$, and f the log return on wealth. A more general version of the CAPM arises with power utility and IID consumption growth in an endowment economy, where f is the log return on wealth, b equals the relative risk aversion coefficient, and a is a function of relative risk aversion, the time preference parameter, and mean consumption growth. Additional risk factors, such as those of Fama and French (1993), could be added as well. Multi-period valuation is also straightforward as these single-period SDFs compound to multi-period SDFs in a natural way.

We use this approach to value payoffs of VC-backed start-up companies and VC funds (collectively referred to as VC payoffs). In our baseline specification, we use the public equity market portfolio log return as the risk factor, f , and we choose the SDF parameters a and b to exactly fit the average public equity market and Treasury bill returns during our sample period. We then apply this SDF to VC payoffs. In doing so, we are effectively asking whether the VC payoffs are comparable—when matched by their systematic risk—to those available in public equity and Treasury bill markets as summarized by the SDF. More precisely, if each

VC payoff could be replicated with a levered portfolio in the public equity market (where the degree of leverage pins down the level of systematic risk), the application of this method would deliver a pricing error of zero.

We show that this approach generalizes the *Public Market Equivalent (PME)* measure of Kaplan and Schoar (2005). While Kaplan and Schoar (2005) provide a heuristic motivation of their measure as one that compares private equity payoffs with those in public markets under the assumption of a beta coefficient of one, we show that their measure is actually an implementation of a standard SDF valuation in the special case of log-utility ($a = 0$ and $b = 1$) that does not imply any assumptions about beta. Thus, with regards to how it accounts for systematic risk, the *PME* measure is actually more general than the literature has assumed. However, the *PME* method is restrictive on a different dimension: It implicitly assumes that the equity premium (and hence the incremental risk premium for an additional unit of beta exposure) is equal to the variance of the market return. Our approach, which we label *Generalized Public Market Equivalent (GPME)*, avoids this restriction by relaxing the assumption that $a = 0$ and $b = 1$. This allows for a more accurate risk-matched comparison of start-up company payoffs with the investment opportunities available in the public markets during the same time period.

In our approach, the endogenous payoff horizon problem is not an issue as long as final outcomes are observed. The applicability of the SDF valuation formula does not depend on *how* the timing of the payoff was generated—all that matters is *when* the payoff occurs. However, in our start-up company data set we do have to address the problem that some final outcomes are unobserved, depending on the start-up’s performance (Cochrane (2005) and Korteweg and Sorensen (2010)). We deal with this problem by showing results for a variety of assumptions about the unobserved end-of-round firm values.

To empirically implement our *GPME* measure, we propose a GMM method to conduct statistical inference in a way that is robust to cross-correlation between VC payoffs. While it is common in the literature to assume that payoffs are uncorrelated between start-up

companies or funds (or, in the case of *PME*, to not report standard errors), changes in start-up company valuations between funding rounds or the cash flows of VC funds over the fund life are overlapping in time to varying degrees, and are likely subject to similar common factor shocks and, hence, cross-sectional correlation. To account for this cross-sectional correlation, we borrow methods from the spatial GMM literature and treat the degree of overlap between a pair of observations as analogous to a spatial distance that determines the level of correlation.

We apply our method to a data set of VC funds from 1985 to 2012 and a data set of start-up company financing rounds from 1987 to 2005. The VC fund payoffs are measured net of fees, whereas the payoffs in the start-up company data reflect the gross returns from VC investments, before any fees and carried interest. In the VC fund data, we find (equally-weighted) abnormal returns close to zero when we use the public equity market return as the single risk factor in equation (1). In contrast, we find strongly positive abnormal returns of VC investments in start-up companies. Both findings are broadly in line with prior work in this area.¹

To some extent, however, the similarity of our findings and prior work is a coincidence. For example, in the start-up company sample, it turns out not to make much difference whether one uses the *PME*, i.e., with the SDF restriction $a = 0$ and $b = 1$, or the *GPME* that leaves these parameters unrestricted. The reason is that an exact matching of the average public market equity and T-Bill returns in our sample period yields parameter estimates that are very close to $a = 0$ and $b = 1$. This is good news for existing work in this area, because it means that results based on *PME* in the literature that are based on data over a similar time period, may not have been affected all that much by the restriction that the *PME* calculation implicitly imposes on the equity premium. However, it is important to keep in

¹Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), Woodward (2009), Robinson and Sensoy (2011), Stucke (2011), and Ewens, Jones, and Rhodes-Kropf (2013) report VC fund returns that are equal to or slightly above the return on the market portfolio. Conversely, Phalippou and Zollo (2005), Phalippou and Gottschalg (2009), and Driessen, Lin, and Phalippou (2012) find below-market returns. Harris, Jenkinson, and Kaplan (2013) report that VC funds outperformed public equities in the 1990s but underperformed in the 2000s. Cochrane (2005), Hall and Woodward (2007), and Korteweg and Sorensen (2010) find positive abnormal round-to-round returns (gross of fees) from VC investments in start-up companies.

mind that the similarity is sample-specific. In the VC funds sample, which is more heavily concentrated in a later time period than the start-up company sample, we find that the SDF parameters are significantly different from log-utility. The wedge between the *PME* and *GPME* estimates for VC funds is consequently larger than in the start-up company sample. This underscores the importance of allowing for more general utility functions than log-utility in the SDF specification.

With regards to the skewness of VC payoffs, the skewness typically emphasized in the literature is their *call*-option-like nature: Returns to individual start-up company investments lead to a small number of enormous successes and a large number of failures. Yet, as we show, the *systematic* risk profile of VC payoffs exhibits a very different type of optionality: VC payoffs are far more sensitive to market downturns than to market upturns. As a result, the systematic risk of venture capital investments to some extent resembles the systematic risk from selling stock market index *put* options.

This short index-put option feature of VC payoffs raises the question whether public-market investment opportunities that we summarize in the SDF should perhaps also include the payoffs available in equity index option markets. Option payoffs may offer risk premia that are not captured in public equity market returns (see, e.g., Bollerslev and Todorov (2011)). For this reason, we also explore an expanded specification of the SDF in which we add an index put option return series as a risk factor to the SDF.² Doing so has little effect on our conclusions about the *GPME*, and hence abnormal returns, of start-up company investments. However, abnormal returns of VC funds turn negative once we include the index put option returns in the SDF. In other words, VC fund returns look less attractive once the short-put option feature of their payoffs is taken into account.

In comparison to the prior literature, it is also useful to emphasize that our approach

²The skewness of VC payoffs also implies that it may be undesirable to work with linearized versions of (1) of the form $M_{t+1} \approx \tilde{a} - \tilde{b} \exp(f_{t+1})$. Approximating the SDF in this fashion yields a convenient beta-pricing expression for expected returns, but there is a concern that the approximation error could be substantial when such a linearized SDF is applied to highly skewed payoffs. This is one of several reasons (multi-period compounding is another) why we prefer to work directly with (1) rather than its linear approximation.

avoids the strong distributional assumptions in some earlier work. For example, to deal with the multi-period nature of payoffs and the endogenous selection of payoff horizons by venture capitalists or start-up firms, Cochrane (2005) and Korteweg and Sorensen (2010) assume a log-normal process for start-up company values combined with a selection model. An implicit assumption in this log-normal model is that the variance of the firm value changes grows proportionally with the time horizon over which the return is measured (between funding rounds, or from funding round to an exit event). In contrast, we find in the data that the relationship is much flatter: Returns over short horizons are more volatile, and returns over long horizons are much less volatile than implied by the log-normal model. This is an important issue because volatility determines the magnitude of the Jensen's inequality adjustment in calculations of implied arithmetic average abnormal returns within the log-normal model. The selection models in Cochrane (2005) and Korteweg and Sorensen (2010) help match this rather flat variance-horizon relationship through endogenous timing of the financing rounds or exit events. The approach we take here avoids the need to take a stand on a highly parameterized selection model.

In independent and contemporaneous work, Sorensen and Jagannathan (2013) also point out the log-utility assumption behind *PME*. Compared to their paper, we generalize *PME* to allow for utility functions other than log-utility, we define *GPME* in a way that avoids a Jensen's Inequality problem with ratios of present values in small samples, we develop a GMM estimator that allows for cross-sectional correlation, and we take the method to the data.

The methodology that we present in this paper can be extended to allow for other risk factors, such as size, book-to-market and liquidity factors (for the importance of liquidity in private equity, see Longstaff (2009), Franzoni, Nowak, and Phalippou (2012), Sorensen, Wang, and Yang (2013)). Furthermore, our method can be applied to other infrequently traded asset classes with highly levered or option-like returns, such as leveraged buyouts or real estate. Our approach also has implications for the risk and reward to entrepreneurs

(Moskowitz and Vissing-Jorgensen (2002)), and the performance of different types of limited partners (Lerner, Schoar, and Wongsunwai (2007)) or general partners (Ewens and Rhodes-Kropf (2013)) in private equity investments.

The paper is organized as follows. Section I outlines our SDF pricing approach to risk-adjusting VC returns. Section II presents our VC fund results. Section III presents our start-up company results, and Section IV concludes.

I. Risk-Adjusting Venture Capital Returns

The fundamental challenge in VC performance evaluation is the valuation of the cash flows between individual start-up companies and their investors, or, alternatively, the cash flows into and out of VC funds. Evaluation of these cash flows involves discounting. Various approaches to this problem exist in the literature, often with rather strong assumptions on the benchmark model or on the distribution of payoffs. Our objective here is to develop a more general and robust approach.

We work with a standard asset-pricing approach. Throughout the paper we use lower case letters for logs. For example R_t is an arithmetic return, and r_t is a log return. The time- t value of an asset, V_t , that pays a single cash flow at time $t + 1$, is the expected value of the cash flow, C_{t+1} , discounted with a stochastic discount factor (SDF), M_{t+1} ,

$$V_t = E_t[M_{t+1} \cdot C_{t+1}]. \quad (2)$$

If the asset has a continuation value, V_{t+1} , at time $t + 1$,

$$V_t = E_t [M_{t+1} \cdot (C_{t+1} + V_{t+1})]. \quad (3)$$

Dividing both sides of the above equation by V_t yields the pricing relation in returns space,

$$1 = E_t[M_{t+1} \cdot R_{t+1}], \quad (4)$$

where $R_{t+1} \equiv (C_{t+1} + V_{t+1})/V_t$.

Our specification of the single-period SDF is exponentially-affine,

$$M_{t+1} = \exp(a - bf_{t+1}). \quad (5)$$

This specification subsumes many standard asset-pricing models as special cases. For example, the log-utility Capital Asset Pricing Model (CAPM) is the special case with $a = 0$, $b = 1$, and f_{t+1} equal to the log return on aggregate wealth from t to $t + 1$. A more general version of the CAPM arises with power utility and IID consumption growth in an endowment economy, where $a = 0$, $b = \gamma$ (i.e., relative risk aversion), and f_{t+1} is the log return on wealth. Additional risk factors, such as the case of Epstein and Zin (1989) preferences or the Fama and French (1993) risk factors, can be added easily, but to keep notation simple, we discuss the single-factor case here.

One particularly pressing problem with payoffs on non-public equity and illiquid asset classes more generally is the multi-period nature of the payoffs. Cash flows of VC funds occur at irregularly spaced time points throughout the life of the fund. The timing of funding rounds and exit events of VC-backed start-up companies (at which valuation estimates are available) is similarly irregular. The exponentially-affine SDF setup is particularly well-suited for multi-period payoffs measured over varying time horizons, because the pricing relation holds for longer horizons

$$V_t = E_t[M_{t+h}^h \cdot C_{t+h}], \quad (6)$$

where M_{t+h}^h , the multi-period SDF from t to $t + h$, simply compounds the single-period discount factors

$$M_{t+h}^h \equiv \prod_{i=1}^h M_{t+i} \quad (7)$$

$$= \exp(ah - bf_{t+h}^h), \quad (8)$$

where

$$f_{t+h}^h \equiv \sum_{i=1}^h f_{t+i}. \quad (9)$$

A. The Generalized Public Market Equivalent Measure

Currently, the most widely used measures of private equity performance are the internal rate of return (IRR), total value to paid-in capital (TVPI), and the public market equivalent (*PME*) measure developed by Kaplan and Schoar (2005). IRR and TVPI are problematic as they do not account for risk (nor for timing in the case of TVPI). The *PME* measure is traditionally defined as the sum of a private equity fund’s cash outflows (i.e., distributions to Limited Partners (LPs)) discounted at the realized public market return, divided by the sum of all cash inflows (i.e., takedowns: contributions from LPs to the fund), also discounted at the realized public market return.

We slightly modify the classic *PME* definition. We work with the *difference* between discounted values of inflows and outflows, rather than their *ratio* as in the original Kaplan and Schoar definition. Asset pricing theory implies that the expected difference (the net present value) is zero, but when inflows are stochastic there are no clear predictions about the expected ratio because of a Jensen’s inequality effect.³ Working with the difference also simplifies the econometrics. From here on, when we write *PME* we mean this redefined *PME*. For fund i the (redefined) *PME* is thus the sum of all discounted cash flows to and from the fund

$$PME_i \equiv \sum_{j=1}^J \frac{1}{R_{m,t+h(j)}^{h(j)}} \cdot C_{i,t+h(j)}, \quad (10)$$

where t is the date of the first cash flow, and $C_{i,t+h(j)}$ is the net cash flow (distributions minus takedowns) for fund i at date $t+h(j)$. The number of cash flows, J , and the initial cash flow date, t , vary by fund, but we suppress dependence on i for notational simplicity.

³If there is only one initial investment that occurs at the time of the valuation (as in the case of round-to-round returns of start-up companies) the initial inflow is non-stochastic, and hence the Jensen’s inequality problem does not arise. In the case of VC funds, however, investors typically make multiple capital contributions during the life of the fund, and the amounts and timing are initially uncertain, making this a relevant problem.

The common interpretation is that a fund with a PME greater (less) than zero has outperformed (underperformed) the market, and the PME therefore imposes a CAPM β equal to one.⁴ Robinson and Sensoy (2011) explore the robustness of PME by leveraging up the public market return used to discount fund cash flows using beta assumptions different from one. However, assuming $\beta = 1$ corresponds to discounting the expected fund cash flows at the *expected* market return, which is different from discounting realized cash flows with *realized* market returns as in the PME calculation. We show that in fact the PME of Kaplan and Schoar (2005) does not restrict beta, but rather assumes log-utility preferences.

To see the intuition, consider the expected PME :

$$E[PME_i] = \sum_{j=1}^J E \left[\frac{1}{R_{m,t+h(j)}^{h(j)}} \cdot C_{i,t+h(j)} \right], \quad (11)$$

which is estimated by averaging PME_i across funds.⁵ Equation (11) shows that the PME calculation is the special case of SDF pricing under log-utility, i.e. $M_{t+h}^h = 1/R_{m,t+h}^h$, and expected PME should equal zero if this is the correct specification of the SDF, such that the net present value of the fund is zero. The PME thus fully accounts for the systematic risk of the cash flows being valued, and funds can even have varying degrees of systematic risk exposure. The PME approach is therefore more general than the literature has thus far assumed.

The true restriction implicitly imposed by the PME calculation is a different one. The log-utility CAPM assumes a relative risk aversion coefficient of one and it implies that the (instantaneous) equity premium is equal to the variance of the market return. We can illustrate this assumption most clearly in the case of round-to-round returns of start-up-company investments. Applying the PME calculation to returns, i.e, the $t+h$ value divided

⁴Kaplan and Schoar (2005), for example, describe their PME analysis as follows (p. 1797): “If private equity returns have a beta greater (less) than 1, PME will overstate (understate) the true risk-adjusted returns to private equity.”

⁵As we discuss in detail further below, for this average to be well behaved, there must be some variation across funds in the time periods during which they are alive.

by the value at the time of the prior funding round at t , yields

$$E[PME_i] = E\left[\frac{1}{R_{m,t+h}^h} \cdot R_{i,t+h}^h\right] - 1 \quad (12)$$

$$= E\left[\frac{1}{R_{m,t+h}^h}\right] E[R_{i,t+h}^h] + \text{Cov}\left(\frac{1}{R_{m,t+h}^h}, R_{i,t+h}^h\right) - 1 \quad (13)$$

$$= \frac{E[R_{i,t+h}^h]}{R_F} + \text{Cov}\left(\frac{1}{R_{m,t+h}^h}, R_{i,t+h}^h\right) - 1, \quad (14)$$

since under log-utility the unconditional h -period risk-free rate is $R_F = 1/E\left[\frac{1}{R_{m,t+h}^h}\right]$. Defining $\beta \equiv -\frac{\text{Cov}(1/R_{m,t+h}^h, R_{i,t+h}^h)}{\text{Var}(1/R_{m,t+h}^h)}$, we get⁶

$$\frac{E[R_{i,t+h}^h]}{R_F} - 1 = E[PME_i] + \beta \text{Var}\left(1/R_{m,t+h}^h\right). \quad (15)$$

Under the null hypothesis that $PME = 0$, the first term drops out and we obtain the familiar CAPM-type pricing model. If PME is larger (smaller) than zero, then we realize positive (negative) *risk-adjusted* excess returns. Thus, the calculation of PME takes systematic risk into account (β is not restricted) but it restricts the equity premium. In the continuous-time limit, the equity premium in this model is equal to the instantaneous variance of the market return. This implied assumption is restrictive, and we relax it with our more general SDF specification (5), which we use to compute a Generalized PME ($GPME$) as

$$GPME_i \equiv \sum_{j=1}^J M_{t+h(j)}^{h(j)} \cdot C_{i,t+h(j)}. \quad (16)$$

If there is no mispricing, i.e., venture investment payoffs are not abnormal relative to the public markets benchmarks captured by the SDF, then expected $GPME = 0$.

⁶Note the negative sign in β because we are looking at covariance with the inverse of the market here and the negative sign ensures that this beta has the same interpretation as the traditional beta in a linearized CAPM.

B. Non-linear Payoffs and Linearized Factor Models

A common approach in the VC and Private Equity area is to work with linearized versions of the SDF, which imply a linear beta-pricing relationship (see, for example, Ljunqvist and Richardson (2003), Hall and Woodward (2007), Driessen, Lin, and Phalippou (2012), Ewens, Jones, and Rhodes-Kropf (2013)). Performing a first-order Taylor approximation of (8) around $F_{t+h}^h = 1$ and $h = 1$ yields

$$\widetilde{M}_{t+h}^h \approx \exp(a) \cdot [1 - b(F_{t+h}^h - 1) + a(h - 1)]. \quad (17)$$

Redefining parameters,

$$\widetilde{M}_{t+h}^h \approx c + \tilde{a} \cdot h - \tilde{b}(F_{t+h}^h - 1), \quad (18)$$

which implies a linear beta-pricing specification for expected returns,

$$E[R_{i,t+h}^h] - R_F^h = \beta^h \left[b \text{Var}(F_{t+h}^h) R_F^h \right], \quad (19)$$

where $\beta^h \equiv \frac{\text{Cov}(R_{i,t+h}^h, F_{t+h}^h)}{\text{Var}(F_{t+h}^h)}$.

As will become clear, there is not really any need to linearize the model, and the linearized model is cumbersome for multi-period payoffs, as it loses the nice compounding properties of the exponential-affine model. Linearized approximations like (18) (and hence linear beta-pricing formulations for expected returns) could also lead to specification errors because it is possible to have negative realizations of the SDF, implying that some states of nature have negative state prices, which is inconsistent with the absence of arbitrage opportunities. This is especially problematic for assets with highly non-linear payoffs such as options.

Payoffs of venture investments are known to have option-like features. The optionality that is typically emphasized is the out-of-the money call option nature of venture investments: Individual start-ups fail with a high probability, but when they succeed, the returns can be

astronomical. As a result, most of the return generated by a portfolio of VC investments comes from a few spectacular successes while the large majority of start-up companies fail.

In our empirical section below, we highlight a second type of payoff optionality that has not yet been noted in the literature: the relationship between public stock market returns and VC payoffs is concave, somewhat akin to the return from selling index put options.

The highly nonlinear nature of venture investments therefore suggests that linear approximations of the SDF are best avoided. The SDF approach behind the valuation in (16) can handle arbitrary payoff non-linearities, and the exponentially-affine specifications of SDFs that we examine here are strictly positive and hence consistent with the absence of arbitrage opportunities. Thus, these SDFs can price any payoff, including option payoffs.

C. Endogenous Payoff Horizons and Sample Selection

Thus far we have treated the payoff horizon, h , as given. In reality, start-ups endogenously decide when to raise new financing, and venture funds have the option to extend their life for several years beyond the ten-year limit, making h an endogenous variable. For example, projects that are more successful may come back to investors for a new financing round sooner, in order to scale up the business model. Thus, the return horizon is endogenous in the sense that it is correlated with the unexpected degree of success of the project or the fund.

For valuation purposes within the SDF framework, this endogeneity does not present a problem, as long as the realized payoffs are ultimately observed. What matters for valuation in the SDF framework is (a) the payoff in each state of the world (b) the value of the SDF in those states of the world. The endogeneity of h can generate a particular state-dependence through the actual timing of the cash flow, but this is not a problem as the appropriate valuation (state price) for the endogenously generated payoff in a certain state is the product of the state's probability and the SDF in that state. The endogeneity concerns that remain relevant in our approach are those that may cause: i) a right-censoring problem, and; ii)

a sample selection problem. We discuss in the data section below how we deal with these remaining problems.

D. GMM estimation

For the start-up companies data, we pool the data across firms and funding rounds. For each round-to-round observation $i = 1, \dots, N$ we observe returns, $R_{i,t+h}^h$, between t and $t+h$, where t and h can be different for each i . We match each return with factor returns, F_{t+h}^h , and the return of Treasury Bills, $R_{f,t+h}^h$, over the valuation horizon from t to $t+h$.⁷ Thus, we work with N observations of $Y_i = (R_{i,t+h}^h, F_{t+h}^h, R_{f,t+h}^h)'$. Let θ denote the parameters of the SDF, and define the vector

$$u_i(\theta) \equiv M_{t+h}^h(\theta) \cdot Y_i - 1. \quad (20)$$

The first element of u_i is the $GPM E_i$ of equation (16) for funding round i . In other words, it is the net present value of investing \$1 in round i .

In the VC funds data, we observe J cash flows to and from each fund $i = 1, \dots, N$, and possibly a final net asset value (NAV) if the fund is not yet liquidated. The first cash flow of the fund occurs at date t . As before, we suppress the dependence of t and J on i for notational simplicity. We match each cash flow with with factor returns, $F_{t+h(j)}^{h(j)}$, and the return of Treasury Bills, $R_{f,t+h(j)}^{h(j)}$ over the valuation horizon from t to $t+h(j)$. For each matched observation, we construct the vector $Y_{i,t+h(j)} = (C_{i,t+h(j)}, [F_{t+h(j)}^{h(j)} - 1]/J, [R_{f,t+h(j)}^{h(j)} - 1]/J)'$, and we define

$$u_i(\theta) \equiv \sum_{j=1}^J M_{t+h(j)}^{h(j)}(\theta) \cdot Y_{i,t+h(j)}. \quad (21)$$

We take care to construct the elements of $Y_{i,t+h(j)}$ in a way that ensures that estimates are well-behaved and interpretable. The first element of $Y_{i,t+h(j)}$ is the fund net cash flow

⁷We use subscript $t+h$ for the Treasury Bill return, not t , as one typically would with a conditionally risk-free asset, because it is the return on rolling over short-term Treasury Bills from t to $t+h$, and hence the return is not known until $t+h$.

$C_{i,t+h(j)}$, normalized by the fund’s size. This normalization gives each fund equal weight when we average across the first elements of u_i . The first element of u_i is the $GPME_i$ of equation (16) (i.e., the net present value) for an investment with a total commitment of \$1.

The division by J of the factor returns and the T-Bill return in $Y_{i,t+h(j)}$ means that for each fund i , the series of J discounted factor returns and T-Bill returns add up to a price of \$1. Thus, similar to the VC fund cash flows, the factor and T-Bill payoff series matched with each fund i represent a total \$1 “commitment” to investing in each of these public market assets. As a consequence, when we estimate the SDF parameters to set the averages of the second and third element of $u_i(\theta)$ to exactly zero, we give equal weight to the factor and T-Bill return series corresponding to each fund.⁸ This particular weighting helps with the interpretability of the results, because if the VC fund payoffs were perfectly replicable as linear combinations of the factor and T-Bill returns, the $GPME$ estimate would be exactly zero, even in a finite sample. Of course, as the size of the dataset grows, these weighting issues become irrelevant as large-sample estimates should be the same regardless of the weighting scheme.

For both the VC fund and start-up company data, we employ the GMM estimator

$$\hat{\theta} = \arg \min_{\theta} \left(\frac{1}{N} \sum_i u_i(\theta) \right)' W \left(\frac{1}{N} \sum_i u_i(\theta) \right). \quad (22)$$

Our objective is to evaluate how venture investment payoffs compare, on a risk-adjusted basis, to investments in publicly traded securities. Consistent with this objective, we estimate the SDF parameters, θ , from public capital market data alone. More precisely, we choose as a weighting matrix a diagonal matrix with entries of zero for the element corresponding to VC payoffs (the first element in u_i), and ones for the remaining elements. This leads to exact identification and parameter estimates that allow the SDF to exactly match the unconditional average of the factor and T-Bill returns in our sample with zero sample pricing errors.

⁸Without dividing factor and T-Bill returns by J , the estimation of SDF parameters would give more weight to the factor and T-Bill series matched to funds with a greater number of cash flows

For inference, we want to assess the magnitudes of the pricing errors, i.e., the *GPME*, associated with the venture investment payoffs. A key ingredient of the standard GMM formulas is the spectral density matrix

$$S = \sum_{k=-\infty}^{+\infty} E[u_i u'_k] \quad (23)$$

The complication here is that there is likely to be substantial correlation between u_i and u_k if they are measured over fully or partly overlapping time periods. Earlier work in the existing literature either does not report *PME* standard errors, or assumes uncorrelatedness across funds, or across start-up firms. In our framework, standard GMM techniques allow us to avoid this assumption in a relatively simple way. We assume that the correlation of payoffs i and k depends on the degree of overlap of the time windows over which these payoffs are measured. In analogy to spatial GMM methods (Conley (1999)), we treat the degree of overlap as a measure of distance, assuming that correlation declines with distance. Let $t(i)$ and $t(k)$ be the start, and $t(i) + h(i)$ and $t(k) + h(k)$ be the end of the time windows for observation i and k , respectively. Define the distance as

$$d(i, k) \equiv 1 - \frac{\min[t(i) + h(i), t(k) + h(k)] - \max[t(i), t(k)]}{\max[t(i) + h(i), t(k) + h(k)] - \min[t(i), t(k)]}. \quad (24)$$

If the return measurement windows exactly overlap, then $d(i, k) = 0$, if they are adjacent, but non-overlapping then $d(i, k) = 1$, and $d(i, k) > 1$ if there is a gap between them. Our estimator for S uses Bartlett-type weights for observations that decline with greater distance,

$$\hat{S} = \frac{1}{N} \sum_{k=1}^N \sum_{i=1}^N \max(1 - d(i, k)/\bar{d}, 0) u_i u'_k, \quad (25)$$

where \bar{d} is a constant. In our empirical work, we set $\bar{d} = 1.5$, which means that observation-pairs with a gap between return measurement time windows still get some weight in the calculation of \hat{S} .

With \hat{S} in hand, we can use standard GMM formulas to compute a J -statistic for a test that the pricing error for venture investment payoffs (the first element of u_i) is zero, which is equivalent to a test that $GPME = 0$, taking into account the estimation error in the SDF parameters. Since the SDF parameters are estimated using only information on pricing factor and T-Bill returns, but not venture investment payoffs, this J -statistic has a $\chi^2(1)$ distribution.

Finally, it is important to point out that identification of the SDF parameters, θ , and estimation of $GPME$ requires time variation. For example, if we were to observe many round-to-round returns over exactly the same time window, there would be no variation in the T-Bill and factor returns to identify the SDF parameters. Moreover, even with the SDF parameters given, if all returns were observed over the same time-window, the realization of the SDF would be exactly the same for each cross-sectional unit. Asset-pricing theory does not predict that the cross-sectional average of returns multiplied by this single realization of the SDF value equals one; instead it predicts that the *expected* cross-product of the SDF and the return equals one. Estimation of this expected value requires variation over time. Thus, for the estimation of $GPME$ to work well, the data set needs to have sufficient variation in the time-windows over which the payoffs of the cross-sectional units, i , are realized. More formally, application of the usual asymptotic results for GMM estimation requires large T ; a large number of cross-sectional units alone is not sufficient.

E. Factor returns

The estimation of $GPME$ requires data on a risk-free rate proxy and risk factor returns. We obtain one-month Treasury bill returns and excess returns on the CRSP value-weighted market index from Ken French' website.

To the extent that VC returns indeed resemble option-like payoffs, they may carry risk premia that are not captured by the public market returns (see, e.g., Bollerslev and Todorov (2011)). Therefore we also compute a return series of a strategy that buys short-maturity,

at-the-money put options on the S&P 500. This put strategy invests in a portfolio with a 10% weight on (long) S&P500 index futures put options, and a 90% weight on one-month Treasury bills at the beginning of each month, with a one-month holding period. For the most part, the put option return series is from Broadie, Chernov, and Johannes (2009), supplemented with Optionmetrics data where observations are missing, and is constructed from a rolled-over position in one-month at-the-money put options, held until expiration.⁹ In a small number of cases, Optionmetrics returns are missing, and we use returns of the closest moneyness category (the average of the next highest and lowest if equidistant) as a substitute.

Finally, to construct a potentially closer public-market analogue to VC returns than the value-weighted market index, we compute the daily return on a value-weighted portfolio of micro-cap stocks. On June 30 of each year, we select the smallest 1 percent of stocks in the CRSP universe of stocks, and compute the daily return of a portfolio of these stocks over the next year, with weights proportional to the market capitalization at the close of the prior trading day.

II. VC Funds

A. Data

We use a large sample of international VC fund cash flows between 1985 and 2012, obtained from Preqin.¹⁰ The data contain capital takedowns by the fund from LPs (i.e., cash flows into the private equity partnership), cash distributions from the fund to LPs, and quarterly Net Asset Values (NAVs). Following Kaplan and Schoar (2005), we eliminate funds with committed capital below \$5 million, in 1990 dollars. Table I shows descriptive statistics for

⁹We thank Mark Broadie, Michael Chernov, and Michael Johannes for generously sharing their options data

¹⁰Results in Harris, Jenkinson, and Kaplan (2013) suggest that Preqin is the best publicly available dataset for private equity funds, especially when compared to VentureXpert (from Thomson Venture Economics), which has important updating problems in its post-2001 funds data (see also Stucke (2011)).

Table I
Summary Statistics: VC Fund Data

Descriptive statistics for the sample of VC funds from Preqin over the period 1985 to 2012, eliminating funds with committed capital below \$5 million in 1990 dollars. *Fund size* is the total commitment to the fund, in millions of dollars. *Fund vintage year* is the calendar year in which the fund is raised. *Fund effective years* is the time between the first and the last observed cash flow of a fund. The fund *IRR* is computed using the final observed Net Asset Value (NAV) of the fund. *TVPI* stands for total value to paid-in capital, and is computed as the sum of cash distributions to Limited Partners plus final NAV divided by the sum of cash takedowns by the fund from LPs.

	mean	st.dev	quantile		
			10	50	90
# Funds	692				
# VC firms	356				
# Funds / VC firm	1.94	1.48	1.00	1.00	4.00
Fund size (\$m)	374.86	477.59	65.00	235.00	777.00
Fund vintage year	2001.76	6.46	1993.00	2002.00	2009.00
Fund effective years	8.68	4.80	2.13	8.63	14.63
# Cash flows / fund	10.52	11.11	0.00	8.00	24.00
IRR (%)	0.07	0.35	-0.15	0.04	0.28
TVPI	1.46	2.02	0.55	1.11	2.25

the sample of funds.

The median VC firm in our sample raised one fund, with a total commitment of \$235 million. There is a long right tail of older VC firms that have raised more than one fund, and these firms tend to raise funds that are increasingly larger. The median fund has 8 unique cash flows (both capital calls and distributions) during the sample period. The time between the first and last observed cash flow for the median (average) fund is 8.63 (8.68) years. With respect to performance, the median (average) fund has an IRR of 4% (7%) and a total value to paid-in capital (TVPI) of 1.11 (1.46), where TVPI is computed as the cumulative cash distributions plus final NAV divided by cumulative takedowns. Untabulated results for the subsample of 88 fully liquidated funds (funds that Preqin reports as liquidated, or with a reported end-of-sample NAV of zero) shows better performance, with a median (average) IRR of 10% (23%) and a TVPI of 1.44 (2.56). This is in part because most distributions arrive at the end of a fund's life, but also because the fully liquidated funds tend to have

been raised before the turn of the new millennium, when times were relatively good for VC. This also suggests that it is important to check the robustness of our results in the sample of fully liquidated funds.

Our performance statistics are generally in the ballpark of other studies. For comparison, Kaplan and Schoar (2005) report a median (mean) IRR for 577 liquidated VC funds between 1980 and 1994 of 11% (17%), and TVPI of 1.75 (2.42), close to our liquidated funds sample. Harris, Jenkinson, and Kaplan (2013) report a median (mean) IRR for 775 VC funds between 1984 and 2008 of 11.1% (16.8%), higher than our full sample but also close to our liquidated funds sample. Our performance statistics are also close to Robinson and Sensoy (2011), who report median (mean) IRR of 1% (8%), and TVPI of 1.03 (1.38) for 295 VC funds from a large institutional LP between 1984 and 2009. For 192 fully liquidated funds in their sample the IRR is 2% (9%) and the TVPI is 1.05 (1.44). Finally, Ewens, Jones, and Rhodes-Kropf (2013) use a sample of 1,040 VC funds between 1980 and 2007, and report median (mean) IRR of 6.4% (15.3%).

B. A First Look at the Systematic Risk of Venture Capital Funds

Before analyzing the GMM estimates in detail, we first take an informal look at the systematic risk profile of VC fund payoffs. Figure 1 presents a scatterplot of the VC fund payoffs and the public equity market returns measured, roughly speaking, over the same horizon. As each fund's history features a series of cash flows, summarizing the payoffs and market returns over the fund life time in one number requires some further assumptions. We make these assumptions only here for the purposes of illustrating the systematic risk profile, not in the GMM estimation that follows.

We define fund payoffs as the present value of cash distributions (including the final NAV if non-zero), discounted at the risk-free rate (approximated by compounded T-Bill returns). To bring payoffs of different funds to the same scale, we divide this number by the present value of capital calls. For the purpose of making the fund horizons somewhat similar for this

Figure, we limit the sample to funds with life-times of at least 5 years but less than 15 years (our GMM estimation does not impose this restriction). To summarize the public equity market payoffs during the fund's life time, we take the stock market gross returns matched to each fund cash flow (measured from the start date of the fund to the date of the cash flow), and discount them, also, at the risk-free rate. When summing the discounted stock market returns, we weigh each return by the proportion of each matched fund cash disbursement in the total present value of the fund's cash disbursements. For example, suppose a fund started in December 1995 and made cash disbursements in many months, including December 2005. Suppose further that the cash disbursement made in December 2005 accounts for 15% of the present value of all cash disbursements. Then we assign a weight of 0.15 to the discounted stock market return from December 1995 to December 2005 when we compute the matched public equity market payoff. This way, we construct a public market equity payoff that is approximately duration-matched to the fund's cash disbursements.

In addition to the scatterplot of fund payoffs against public equity market returns, Figure 1 also shows a nonparameteric regression estimate of their relationship, estimated with local linear regression that is robust to outliers. A rather surprising pattern in Figure 1 is that the VC fund payoffs resemble the returns to a short index put option (plus a bond position) rather than a long call option. Common wisdom dictates that start-up companies behave much like real call options, where the investor loses most or all of the investment the majority of the time, but when successful, the payoff can be very large. It is often one such "home-run" investment that makes a fund stand out. But this call option type feature does not show up in Figure 1. The reason is that this call-option like risk is likely not systematic, but rather idiosyncratic to the start-up company: The chance that a new technology works, or that demand for a certain product exists, is relatively independent from the state of the market.

One possible explanation for the concave, short put option-like systematic component of the payoffs with regards to public equity market movements relates to the endogeneity of financing rounds, and in particular exits. Suppose that the investors in a start-up seek an exit

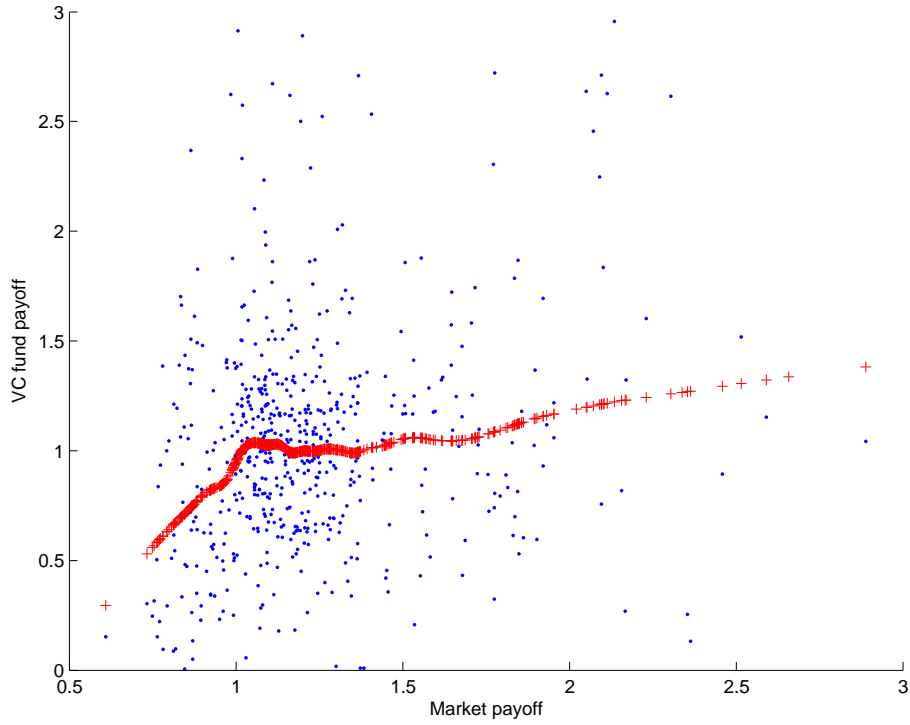


Figure 1. Relationship Between VC Fund Payoffs and Stock Market Returns. Scatterplot of the VC fund payoffs on the vertical axis versus the stock market return on the horizontal axis. VC fund payoffs are measured as the present value (discounting with R_f) of cash disbursements to investors divided by the present value of capital calls. For funds that are not yet liquidated by the end of the sample, the final NAV is treated as a cash disbursement. Stock market returns are measured from the start date of the fund to the date of each cash disbursement, and their present values (discounted by R_f), weighed by the proportion of the present value of the matched fund cash disbursement in the total present value of disbursements. The sample is limited to funds with at least 5 years of data and life-times less than 15 years. The points labeled with ‘+’ show a robust local linear regression fit.

(going public or looking for an acquirer) as soon as they reach a target return. The observed returns are thus capped when the market is doing well, and exits are easy to realize. However, when times are bad, exits are more difficult to achieve, valuations are lower, and investors prefer to wait, if possible. The returns that do realize when the market is performing poorly are then very low, and as a result, VC returns are concave in the market return.

The SDF approach that we take in this paper can price option-like payoffs like the one in Figure 1. Still, to capture any risk premia of option returns that are not captured by public equity market returns (see for example, Bollerslev and Todorov (2011)), we also consider a specification that includes option strategy returns as a risk factor in the SDF.

C. GMM Results

Table II shows the *GPME* estimates for various specifications of the SDF. First, we consider the log-utility model, i.e., the standard *PME* measure, but expressed as a difference between in and outflows rather than a ratio. The *PME* estimate is 0.031, which is statistically significantly different from zero, as indicated by the *p*-value of the *J*-test. This means that on a \$1 fund commitment, the abnormal profit is 3.1 cents, or 3.1%. Note that this is the present value of the abnormal cash flow for the fund, i.e., the net present value, and not an annualized number. Our estimate is comparable to reported *PMEs* in the literature: the mean *PME* is 0.96 in Kaplan and Schoar (2005), 1.36 in Harris, Jenkinson, and Kaplan (2013) (0.91 for their 2000s subsample of 423 funds), and 1.06 in Robinson and Sensoy (2011) (1.03 for their subsample of fully liquidated funds). Remember that these traditional *PMEs* should be benchmarked to one, so numbers slightly above one are comparable to our estimate that is slightly above zero.

The second column in Table II shows the estimates for the more general SDF in equation (1). The *GPME* estimate of -0.036 is statistically not distinguishable from zero, but this is driven by higher standard errors: the point estimate is similar in absolute magnitude to the *PME* in column (i). The reason for the higher standard errors is that the more general

specification requires estimation of the SDF parameters, whereas in the log-utility case of the traditional *PME* they are exogenously fixed. The estimated SDF parameters in column (ii) are statistically significantly different from log-utility. In particular, the coefficient on the market return is -1.638 (with a standard error of 0.437). The negative sign results from the fact that the VC fund payoffs in our sample are concentrated in the post-2000 period in which the realized equity premium was negative. This implies that during the sample the realized compensation for systematic exposure to equity market risk was negative. Our *GPME* measure compares the realized VC fund payoffs to the realized public equity market payoffs in a way that accounts for the negative realized systematic risk compensation in public markets during the same time period. In contrast, the *PME* approach, by fixing $b_1 = 1$ imposes a positive systematic risk compensation. This would tend to push the abnormal payoff to be lower (negative) in the *PME* approach in our sample. There is, however, a second difference. The log-utility model underlying the *PME* approach also imposes a restriction on the risk-free rate, while the *GPME* approach in column (ii) uses an SDF that perfectly fits the average T-Bill payoff in the sample. A higher b under log-utility compared to our estimate of -1.638 from *GPME* implies a lower risk-free rate under log-utility. Both effects combine to give rise to a lower discount rate under log-utility, compared to our more general exponential-affine SDF. With capital calls occurring early in the fund's life while distributions occur later, a lower discount rate results in a higher *PME* in column (i), compared to the *GPME* in column (ii).

In economic terms, generalizing the utility function from log-utility reduces the *GPME* by 0.067 (from 0.031 to -0.036), or 6.7 cents on a $\$1$ commitment. This non-negligible difference underscores the importance of allowing for more general utility functions.

In the third column of Table II we add long put option returns to the specification of the SDF, to capture risk premia that are not present in public market returns (see, e.g., Bollerslev and Todorov (2011)). This lowers the *GPME* to -0.223 , a statistically significant difference from zero. The SDF loads significantly on option returns with a coefficient of 5.042 .

Table II
Generalized Public Market Equivalents for VC Funds

For a VC fund with cash flows starting in period t , we match each subsequent cash flow in $t+h$ with the return of the CRSP value-weighted index ($r_{m,t+h}^h$ in logs) from t to $t+h$, the return from rolling over 1-month Treasury Bills, the return on a portfolio that buys at-the-money put options on the S&P500 index, and the return on a microcap portfolio. We estimate the Generalized Public Market Equivalent ($GPME$) by discounting the fund cash flows with the stochastic discount factor

$$M_{t+h}^h = \exp(ah - b_1 r_{m,t+h}^h - b_2 r_{x,t+h}^h),$$

summing each fund's discounted cash flows, and averaging across all funds. The log-utility CAPM special case in column (i) with $a = 0$, $b_1 = 1$, and $b_2 = 0$ corresponds to the Public Market Equivalent of Kaplan and Schoar (2005). In columns (iii) and (iv), the second factor, $r_{x,t+h}^h$, is the log return of the put option or microcap portfolio, respectively. The SDF parameters in columns (ii) to (iv) are estimated, with exact identification, to fit the average return of T-Bills and the returns on the SDF risk factors. The J -statistic tests the null hypothesis $GPME = 0$. The spectral density matrix used in the computation of the J -statistic takes into account error dependence arising from overlapping fund life times as described in the text. Standard errors of the SDF parameter estimates are in parentheses.

	(i)	(ii)	(iii)	(iv)
	Log-utility		CAPM	CAPM
	CAPM	CAPM	augm. w/ puts	augm. w/ micaps
Panel A: Evaluation of VC fund returns				
<i>GPME</i>	0.031	-0.036	-0.223	0.005
<i>J</i> -test <i>p</i> -value	0.000	0.187	0.000	0.953
Panel B: SDF parameters				
<i>a</i>	0	0.032	0.558	0.019
		(0.026)	(0.028)	(0.027)
<i>b</i> ₁	1	-1.638	11.935	-1.480
		(0.437)	(1.101)	(0.438)
<i>b</i> ₂	-	-	5.042	-0.097
			(0.339)	(0.191)

The implied risk premia for systematic exposure to option-like risks cannot easily be read off from the SDF coefficient estimates, though, because they depend on the covariance of the put option and market factors, and the introduction of the put option factor also changes the coefficient on the market factor. But the results clearly show that including put option returns in the SDF substantially lowers the attractiveness of VC fund payoffs vis-a-vis the public market investment opportunities summarized by the SDF.

In the last specification of Table II we add microcap returns to the SDF. The motivation for doing so is that VC-backed start-up companies are most similar to this segment of public equity markets. Hence, to the extent that microcaps earn risk premia that are not well captured by our SDFs in columns (ii) and (iii), the inclusion of a microcap return in the SDF might help price VC fund payoffs. The *GPME* is 0.005, and the estimate is not significantly different from zero. Since the loading on the microcap factor is economically small and statistically insignificant, it is not surprising that these results are close to the specification in column (ii) that does not include the microcap factor.

D. Robustness

Our VC fund sample includes many funds that are not liquidated yet at the end of the sample period. These funds self-report a Net Asset Value (NAV) at the end of the sample period. These NAVs are known to be conservative and stale (e.g., Woodward (2009) and Phalippou and Gottschalg (2009)). In our baseline estimates we followed Kaplan and Schoar (2005) and treated the NAVs as a final cash flow. We check the robustness of our results to using the subsample of 88 fully liquidated funds, as well as funds with end-of-sample NAVs less than 5% (135 funds) and 10% (159 funds) of the initial commitment. To eliminate funds that have recently been raised but not yet invested, we require that the funds have been at least 50% invested at some earlier date in order to be included in these subsamples.¹¹

¹¹Other approaches in the literature have been to write off the final observed NAVs for funds that are more than 10 years old and for funds that show no signs of recent activity (Phalippou and Gottschalg (2009)), or to model the NAVs as a function of fund size and other covariates Driessen, Lin, and Phalippou (2012).

Re-estimating the SDF with the market factor as in column (ii) of Table II for the fully liquidated funds sample yields a *GPME* estimate of 0.160 with a *J*-test *p*-value of 0.120. These estimates are fairly close to those in Table II. In contrast, if one applies the log-utility model as in column (ii) of Table II to the fully liquidated funds sample, one obtains a *PME* estimate of 0.405 with a zero *p*-value, which is substantially higher than the *PME* estimate in Table II. This illustrates again the advantage of allowing the SDF parameters to be estimated over the sample corresponding to the fund cash flows rather than fixing them at log-utility values. This helps obtain more stable estimates of abnormal returns.

The *GPME* with the put-option and microcap factors in the SDF for the fully liquidated funds sample are -0.246 and 0.154 , respectively, and both are not statistically different from zero at a 5 percent significance level. Using the samples with end-of-sample NAVs less than 5% and 10% of the initial commitment instead of the fully liquidated fund sample produces estimates that are in between the fully liquidated sample ones and those in Table II.

Overall, restricting the fund sample to those that are fully, or almost fully liquidated does not produce substantially different *GPME* estimates as long as one applies our more general method and estimates the SDF parameters from public markets data.

III. VC-backed Start-up Companies

A. Data

We use data of financing rounds for VC-backed start-up companies, provided to us by Sand Hill Econometrics (SHE). SHE has combined two commercially available databases, VentureXpert (from Thomson Venture Economics) and VentureSource (formerly Venture One), and invested substantial time and effort to fill in missing financing rounds¹², and to ensure accuracy of the data by removing duplicate investment rounds, adding missing rounds, and consolidating rounds, such that each round corresponds to a single investment by one or more

¹²Gompers and Lerner (1999) and Kaplan, Sensoy, and Strömberg (2002) find that missing investments in VentureXpert are predominantly smaller and more idiosyncratic ones.

VCS.

The full SHE dataset contains 61,356 financing rounds for 18,237 unique start-ups between 1987 and 2005. Of these start-ups, 1,891 (10.4%) ultimately went public in an IPO, 4,271 (23.4%) were acquired, and 2,892 (15.9%) were liquidated. The ultimate outcome for the remaining 9,183 firms (50.4%) was unknown by the end of the sample. Some of these firms were still operating as private firms, but many of them have likely been liquidated (the so-called “zombie” firms). For these firms, the average (median) time since the last financing round was 57 (41) months by the end of the sample. Of the 52,302 rounds in which new venture capital was raised (i.e., non-IPO, non-acquisition, non-liquidation rounds), 1,393 (2.7%) were seed rounds, 34,066 (65.1%) were early rounds, 16,466 (31.5%) were late rounds, and 377 (0.7%) were designated as mezzanine rounds.¹³

We construct round-to-round returns as the change in valuation of the start-up from the post-money valuation in a given round to the pre-money valuation in the subsequent round. Post-money is a term used in the VC industry to denote the value of the start-up including the new investment. Pre-money is defined as the post-money valuation minus the amount invested in that round. With consecutive financing rounds at time t and $t + h$, the return is,

$$R_{t+h}^h = \frac{V_{t+h}^{PRE}}{V_t^{POST}}, \quad (26)$$

where V_t^{PRE} is the pre-money valuation for a financing round that takes place at time t , and V_t^{POST} is the post-money valuation of that same round. The return to a buy-and-hold investor who holds on to her initial investment, and does not invest any additional money in future rounds, is simply the compounded return across rounds.¹⁴

¹³The label “early” versus “late” is somewhat subjective, and the mezzanine round designation is even more fuzzy. Typically a mezzanine round is the financing round that bridges the 6 to 12 month gap to a liquidity event—IPO, or sometimes acquisition—but sometimes refers to the round between the early and late stage.

¹⁴Consider a simple example of a buy-and-hold investor. Suppose an investor invests \$40 into a series A round, for 100 shares of stock, representing 40% of the equity of the firm, implying a post-money valuation of \$100. In the next, series B round, another investor purchases 150 shares of new stock (i.e., a stake of $150/(250+150) = 37.5\%$) at \$1 per share, for a total investment of \$150. The post-money B-round valuation is \$400 (400 shares at \$1 each), and the pre-money round B valuation is $\$400 - \$150 = \$250$. The return to our buy-and-hold investor from round A to B is $\$250/\$100 = 2.5$ (of course we could more easily compute the

Not all rounds have valuations filled in, so we can compute round-to-round returns for only a subset of the data. We observe 6,861 round-to-round returns (after dropping one return that was less than -100%) for 3,497 unique firms. Table III reports summary statistics for this sample.

This sample suffers from two potential forms of selection bias, which are both ultimately related to the endogeneity of financing rounds. First, there could be a right-censoring problem. For firms started towards the end of our sample, we only observe financing rounds for a relatively short time period until the end of the sample. Since the most successful firms are likely those that proceed most quickly through the VC funding process towards exit, we are more likely to observe the eventual outcomes for these successful firms. Less successful firms might not have a final valuation available, and the one from the last available financing round before the end of the sample might be out of date. To mitigate the censoring problem, we treat unobserved final valuations as liquidations, and we explore how our results depend on the assumed liquidation return. As this assumption may understate VC payoffs, we also do robustness checks where we include only round-to-round returns where the initial round occurs early enough so that we are confident that we have seen most of the horizons by the end of the sample.

Second, data is missing for reasons that are likely to be related to performance. The first row in Panel B in Table III shows observed frequencies of different investment and outcome stages (Seed, Early, Late, Mezzanine, Acquisition, IPO, or Liquidation) without correcting for the missing outcomes. However, for a substantial number of firms, their eventual fate is unknown (these firms are also known as “zombie” firms). These tend to be companies with bad outcomes: unsuccessful start-ups do not return for new financing rounds. As a first step to deal with the missing data problem, we treat zombie rounds (where the final observed round for a firm is not an exit, i.e., an IPO, acquisition, or known liquidation) as liquidations.

return from the share prices, but these are usually not reported in the SHE data). Now suppose the start-up is acquired a few months later, at a share price of \$2. The return for round B to IPO is $\$2 \cdot 400 / \$400 = 2$. Our buy-and-hold investor initially bought her shares at \$0.40/share, and she realized a total return on her investment of $\$2/\$0.40 = 5$. This exactly equals the compounded return over the rounds, 2.5 times 2.

Table III
Summary Statistics: VC Rounds Data

Descriptive statistics for the sample of VC financing rounds of start-up companies from Sand Hill Econometrics over the period 1987 to 2005. Panel A reports statistics for the sample of round-to-round returns. The top row of Panel B shows the sampling frequencies by funding stage for the sample of round-to-round returns. The second and third rows show the frequencies for the sample where companies with unobserved exits are assumed to be liquidated, where the second row uses the full sample regardless of the availability of returns, and the third row uses the sample of round-to-round returns. Panel C shows the mean gross round-to-round returns in the resampled sample that matches the frequencies in the second row of panel B (see the text for details), where the unobserved liquidation returns are assumed to be -90% (first row), -70% (second row), and -50% (third row). Panel D reports the mean gross return on investments in the CRSP value-weighted index that are matched in time to the resampled round-to-round returns, and Panel E shows the mean return horizons (in years), by funding stage.

	mean	st.dev	quantile				
			10	50	90		
Panel A: Raw VC financing rounds data (rounds with a return)							
# Rounds	6,861						
# Start-up companies	3,497						
# Rounds / company	2.96	1.30	2.00	2.00	5.00		
Time between rounds (yrs)	1.01	0.83	0.25	0.83	1.92		
Panel B: Sampling frequencies							
	Seed	Early	Late	Mezz	Acq	IPO	Liq
Observed end-of-round event	0.005	0.414	0.363	0.008	0.099	0.044	0.067
Unobserved end-of-round event treated as liq.	0.004	0.341	0.299	0.007	0.082	0.036	0.231
Observed return (incl. assumed liq. return)	0.001	0.325	0.217	0.017	0.048	0.094	0.297
Panel C: Mean gross round-to-round returns (in resampled sample)							
Ass. liq.ret. = -90%	1.256	2.389	1.710	2.410	3.190	3.331	0.101
Ass. liq.ret. = -70%	1.256	2.389	1.710	2.410	3.190	3.331	0.301
Ass. liq.ret. = -50%	1.256	2.389	1.710	2.410	3.190	3.331	0.501
Panel D: Time-matched payoffs on CRSP value-weighted index (in resampled sample)							
Mean gross return	1.110	1.138	1.125	1.203	1.221	1.189	1.191
Panel E: Round-to-round time horizon (in resampled sample)							
Mean (in years)	0.589	0.955	0.916	1.076	1.675	1.056	2.166

The observed frequencies for the resulting sample are shown in the second row of Panel B in Table III.

Furthermore, 14,738 companies in the raw data do not have enough valuation data to calculate even one round-to-round return. In fact, out of the 3,497 companies for which we observe one or more returns, we have the full return history (i.e., where we observe valuations for every single round) for only 742 firms. Again, the missing outcomes tend to be the bad ones: of the 742 firms with a full return history, 524 ultimately went public, and 218 were ultimately acquired, and none were liquidated. This is due to the fact that IPO and acquisitions data tend to be more widely publicized and thus easier to find and backfill (for example in S-1 registration statements). As shown in the third row of Panel B of Table III, the observed frequencies for the subsample of rounds where we observe a round-to-round return are not quite the same as those in the second row. We correct for this selection bias by resampling (with replacement) round-to-round returns (including their time-matched risk-factor payoffs) to match the distribution across investment stages of the full sample that includes the missing liquidation returns in the second row of Panel B of Table III.

The mean gross returns per funding round category are shown in Panel C of Table III. For robustness, we estimate the *GPME* in our analysis below assuming that liquidations with unknown returns (including the zombie rounds) can have a -90%, -70% or -50% return. The final column of Panel C in Table III shows how this affects the average return in the liquidation category.

B. A First Look at Systematic Risk of VC-backed Start-Up Companies

As we did for the VC funds data, we take a first look at the systematic risk profile of start-up company investments with some simple scatterplots before proceeding to the GMM estimation. Figure 2 plots VC round-to-round returns against the stock market return measured over the same horizon, both in excess of the risk-free rate (approximated by discounting by the compounded T-Bill returns). To compare investments with roughly comparable invest-

ment horizons, we group all round-to-round returns into four categories based on percentiles of the return horizon. Round-to-round returns measured over the shortest (longest) horizon are shown in the top left (bottom right) quadrant. In addition to the scatterplot of start-up company payoffs against public equity market returns, Figure 1 also shows a nonparametric regression estimate of their relationship, estimated with local linear regression that is robust to outliers.

The plots show a broadly similar pattern to the VC fund data: The investment payoffs from VC-backed start-up companies in all four quadrants resemble the returns from a short index put option position (plus a bond position). As in the funds sample, we therefore also apply an SDF with an index put option returns factor to the start-up company data.

The round-to-round returns in these scatterplots are based on an assumed liquidation return of -90% . The clustering of liquidation returns is visible just above the x-axis in these plots. The payoff concavity result is, however, robust to using -70% or -50% as the assumed liquidation return.

C. GMM Results

Table IV reports the *GPME* estimates for a variety of models. The standard *PME* estimate from the log-utility model is 0.526 under the assumption that liquidation returns are -90% . For higher liquidation returns of -70% and -50% , we find *PME* estimates of 0.572 and 0.617, respectively. One reason why these estimates are higher than the *PMEs* for VC funds is that we are looking at the returns gross of fees to General Partners (management fees and carried interest), whereas fund-level *PMEs* are net of fees. Another reason is that the start-up company sample is weighed more towards earlier years, when VC performed better, compared to the funds data, which has a higher proportion of observations after the year 2000 when VC returns were poor (see for example, Korteweg and Sorensen (2010), Harris, Jenkinson, and Kaplan (2013)).

Relaxing the log-utility assumption to the more general utility model of equation (1)

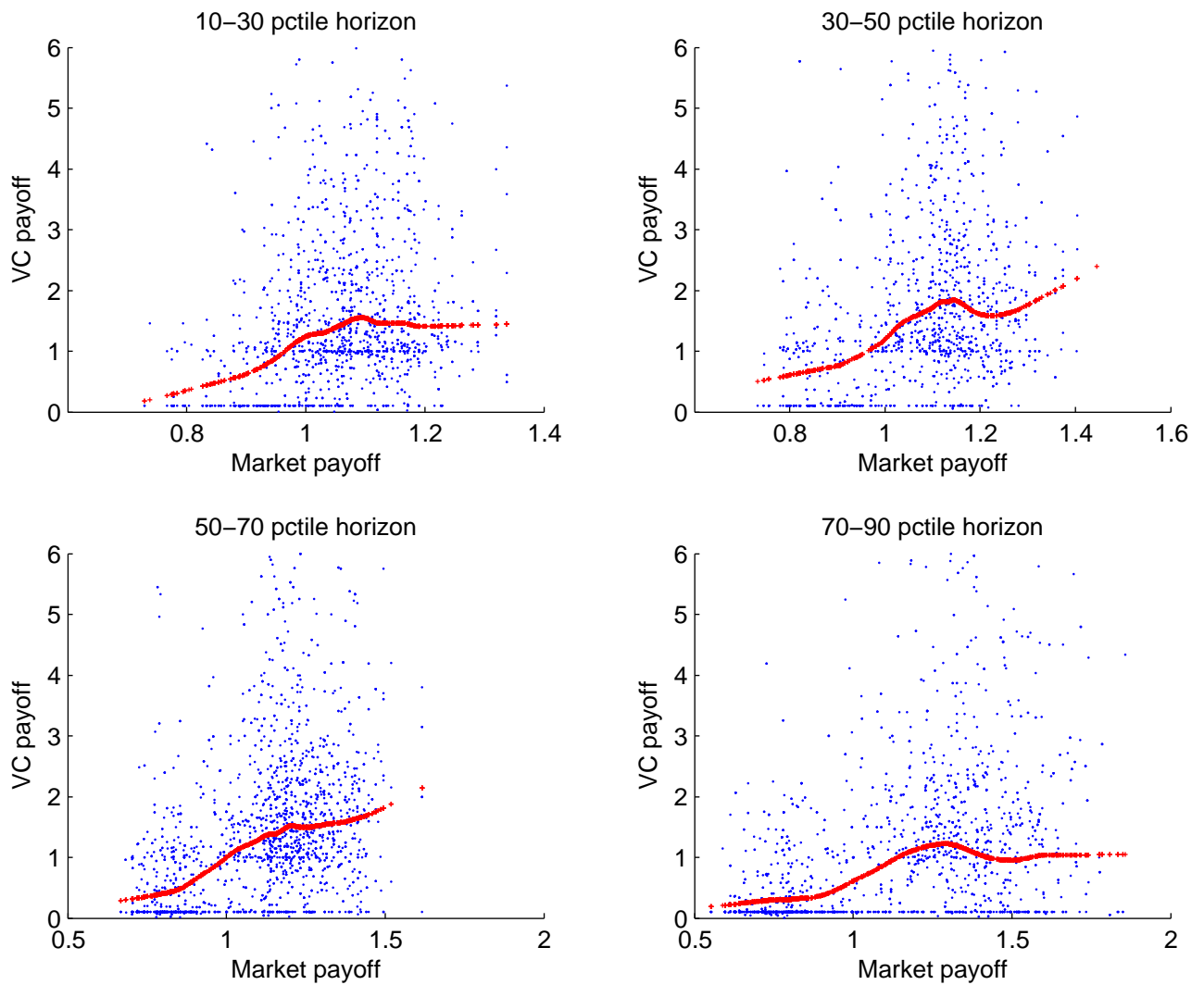


Figure 2. Relationship Between Start-Up Company Round-to-Round Returns and Stock Market Returns. Scatterplot of round-to-round gross return from the resampled sample (assuming that the return upon liquidation is -90%, see the text for details) on the vertical axis, and the time-matched gross market return on the horizontal axis. Both returns are divided by the time-matched gross compounded T-Bill return. The round-to-round returns are grouped into four categories by percentiles of the return horizon, with each panel of the Figure representing one of the groups. The points labeled with ‘+’ show a robust local linear regression fit.

Table IV
Generalized Public Market Equivalents for VC Round-to-Round Returns

We match each start-up company round-to-round gross return, measured over horizon h from time t to $t+h$, with the return of the CRSP value-weighted index ($r_{m,t+h}^h$ in logs) from t to $t+h$, the return from rolling over 1-month Treasury Bills, the return on a strategy that buys at-the-money put options on the S&P500 index, and the return on a microcap portfolio. We estimate the Generalized Public Market Equivalent ($GPME$) by discounting the round-to-round gross returns with the stochastic discount factor

$$M_{t+h}^h = \exp(ah - b_1 r_{m,t+h}^h - b_2 r_{x,t+h}^h)$$

and averaging across all observations. The log-utility CAPM special case in column (i) with $a = 0$, $b_1 = 1$, and $b_2 = 0$ corresponds to the Public Market Equivalent of Kaplan and Schoar (2005). In columns (iii) and (iv), the second factor, $r_{x,t+h}^h$, is the log return of the put option or microcap portfolio, respectively. The SDF parameters in columns (ii) to (iv) are estimated, with exact identification, to fit the average return of T-Bills and the returns on the SDF risk factors. The J -statistic tests the null hypothesis $GPME = 0$. The spectral density matrix used in the computation of the J -statistic takes into account error dependence arising from overlapping round-to-round return measurement periods as described in the text. Standard errors of the SDF parameter estimates are in parentheses.

	(i)	(ii)	(iii)	(iv)
	Log-utility		CAPM	CAPM
	CAPM	CAPM	augm. w/ puts	augm. w/ micaps
Panel A: Evaluation of VC round-to-round returns				
Ass.: Unobserved liquidation returns = -90%				
$GPME$	0.526	0.499	0.561	0.548
J -test p -value	0.000	0.001	0.005	0.000
Ass.: Unobserved liquidation returns = -70%				
$GPME$	0.572	0.548	0.610	0.595
J -test p -value	0.000	0.000	0.002	0.000
Ass.: Unobserved liquidation returns = -50%				
$GPME$	0.617	0.598	0.659	0.642
J -test p -value	0.000	0.000	0.001	0.000
Panel B: SDF parameters				
a	0	0.027 (0.096)	0.133 (0.151)	0.031 (0.104)
b_1	1	1.384 (1.216)	-1.624 (2.288)	0.292 (1.451)
b_2	-	-	-1.069 (0.886)	1.075 (0.492)

lowers the *GPME* by about 0.02. Relative to the magnitude of the *GPME* point estimate, this is a small decrease. However, in terms of absolute economic magnitudes this is actually a significant reduction. A reduction of the pricing error by 0.02 for investments that have a roughly one-year horizon on average corresponds roughly to a reduction of the annual abnormal return by about 2 percentage points.

The reason why the *GPME* and *PME* results are not all that different is that an exact matching of the average public market equity and T-Bill returns in our sample period yields parameter estimates that are very close to $a = 0$ and $b = 1$. This is in contrast to the VC funds sample, where we find significantly different SDF parameters and consequently a larger difference between *GPME* and *PME* estimates. The difference between the two samples is primarily due to the fact that they are concentrated in different time periods. It is therefore important to stress that one cannot always rely on the log-utility model of column (i) to produce similar estimates as the more general approach in column (ii) of Table IV.

Adding index put option returns to the SDF in column (iii) raises the *GPME* by roughly 0.06 compared with column (ii). Thus, after allowing for exposure to the option risk factor, the start-up company payoffs look even more abnormally positive. The impact of put option returns is somewhat smaller here (in absolute value) compared to the VC funds, because the loading on put options in the SDF is closer to zero (-1.069 here versus 5.042 for the VC funds sample)

The effect on the *GPME* estimate of adding the microcap returns to the SDF in column (iv) is roughly comparable to the effect of adding the put option returns in column (iii). Across all specifications, the *J*-test rejects the null hypothesis that the *GPME* is zero in all cases at a 1% significance level.

To explore robustness with regards to our treatment of the right-censoring issue, we re-run these tests on a sample that includes only round-to-round returns where the initial round occurs earlier than 2003, so that it is highly likely that we would get an observation of a subsequent financing round or exit event if the firm remains a viable project (rather than a

“zombie”). Imposing this requirement on the sample raises the $GPME$, but only slightly, and hence it does not lead to substantially different conclusions.

Overall, the positive abnormal returns from VC-backed start-up company investments found in the literature (Cochrane (2005) and Korteweg and Sorensen (2010)) seem robust. Relaxing the implausible restrictions of the log-utility model on the equity premium, or introducing put option and microcap returns as factors in the SDF does not change the basic conclusion that the payoffs from start-up company investments are substantially higher than risk-matched payoffs from public market investments.

These magnitudes of the $GPME$ estimates are big, but they are roughly consistent with the arithmetic alphas that Cochrane (2005) (32% annualized) and Korteweg and Sorensen (2010) (3.5% monthly) obtain by combining a log-normal model with a selection model for endogenous funding and exit events. The approach that we propose here, however, obtains these results in a much simpler and more robust fashion without the need for the specific distributional assumptions and the rather cumbersome estimation involved with a selection model. Without correcting for selection, Cochrane (2005) and Korteweg and Sorensen (2010) obtain much higher estimates of arithmetic alphas. We obtain a similar result below if we use a standard log-normal model in which variance grows proportionally with horizon.

D. Comparison with Log-Normal Model

In order to deal with both the multi-period nature of payoffs and the sample selection problem in the round-to-round returns, an alternative approach in the earlier literature (e.g., Cochrane (2005) and Korteweg and Sorensen (2010)) makes strong distributional assumptions that lead to a linear factor model in logs that can be paired with a selection model, and estimated with maximum likelihood (ML) methods. We now turn to a brief comparison of our approach with this alternative method.

If asset returns, R_t^h , and the risk factor, F_t , are IID jointly log-normal, the pricing restriction (16) with SDF (1), under the null hypothesis $GPME = 0$, and applied to multi-period

returns, yields

$$E[r_{t+h}^h] - r_f^h + \frac{h}{2}\sigma^2 = \beta \left(E[f_{t+h}^h] - r_f^h + \frac{h}{2}\sigma_f^2 \right), \quad (27)$$

where $\beta \equiv \text{Cov}(r_{t+1}, f_{t+1})/\sigma_f^2$ and $\sigma^2 \equiv \text{Var}(r_{t+1})$. Allowing for a pricing error, β can be estimated with the regression

$$r_{t+h}^h - r_f^h = g(h) + \beta(f_{t+h}^h - r_f^h) + \varepsilon_{t+h}^h, \quad (28)$$

where comparison with (27) shows that the abnormal return due to the pricing error is

$$\alpha^h = g(h) + \frac{h}{2}(\sigma^2 - \beta\sigma_f^2) \quad (29)$$

$$= g(h) + \frac{h}{2}(\sigma_\varepsilon^2 + \beta(\beta - 1)\sigma_f^2). \quad (30)$$

However, the null hypothesis gives no guidance about the specification of the intercept term $g(h)$ in (28). Cochrane (2005) and Korteweg and Sorensen (2010) assume $g(h) = \gamma h$, which accumulates nicely over time as longer horizon returns are simply summations of shorter-horizon returns, but it is not obvious that this is the correct specification.¹⁵ A perhaps equally plausible alternative specification of randomness in start-up company investments is that nature initially draws whether a project is a success or not. This initial draw fixes the (initially unobserved) abnormal return of the project. Projects differ in the amount of time it takes for the project value to be revealed, but a longer horizon does not imply that more abnormal return will be accumulated, nor that idiosyncratic variance is higher, let alone that it grows linearly in the horizon. Instead, $g(h) = \gamma$ and the variance of ε_{t+h} is constant, σ_ε^2 (which is why we drop the h -superscript), yielding

$$r_{t+h}^h - r_f^h = \gamma + \beta(f_{t+h}^h - r_f^h) + \varepsilon_{t+h}, \quad (31)$$

¹⁵Driessen, Lin, and Phalippou (2012) do not assume log-normality, but use a similar compounding of the intercept in a linear factor model.

and

$$\alpha^h = \gamma + \frac{1}{2}(\sigma_\varepsilon^2 + \beta(\beta - 1)h\sigma_f^2). \quad (32)$$

The ad-hoc assumptions about $g(h)$ and σ_ε can have a large impact on the empirical estimates. For example, if the process for a start-up's valuation is closer to (31) than to (28) with $g(h) = \gamma h$, this could lead to a large upward bias in the estimate of the arithmetic alpha in a sample with heterogeneous payoff horizons. To see this, suppose the alternative log-normal model (31) is the true model. For a simple illustration, consider the special case where $\beta = 0$ and $r_f = 0$ so that $r_{t+h}^h = \gamma + \varepsilon_t$. We are interested in the arithmetic alpha for an investment with horizon $h = 1$, which, in this $\beta = 0$ case, is simply

$$\alpha^1 = \gamma + \frac{1}{2}\sigma_\varepsilon^2. \quad (33)$$

If the log-normal model (28) with the (false) assumption $g(h) = \gamma h$ is employed, the ML estimator of γ^{16} in a large sample is $\hat{\gamma} = \gamma/E[h]$, which implies an arithmetic annualized alpha of

$$\hat{\alpha}^1 = \hat{\gamma} + \frac{1}{2}E \left[\left(\frac{r_{t+h}^h}{\sqrt{h}} - \hat{\gamma}\sqrt{h} \right)^2 \right] \quad (34)$$

$$= \frac{\gamma}{E[h]} + \frac{1}{2}\sigma_\varepsilon^2 E \left[\frac{1}{h} \right] + \gamma^2 \left(E \left[\frac{1}{h} \right] - \frac{1}{E[h]} \right), \quad (35)$$

which is not generally equal to α^1 . The multiplication of γ and σ_ε^2 in the first two terms by $1/E[h]$ and $E[1/h]$, respectively, arises because application of the false model (28) with $g(h) = \gamma h$ scales the alpha by horizon, which is not consistent with the true model (31). For example, if all payoffs have $h > 1$, the application of the false model scales the arithmetic alpha towards zero, because the model assumes that $g(h)$ and volatility would be smaller in magnitude at the shorter horizon of $h = 1$. Thus, the effect of the first two terms in (35)

¹⁶In this case, the ML estimator is equivalent to the OLS slope estimator in a regression of r_{t+h}^h/\sqrt{h} on \sqrt{h} without intercept.

depends on the characteristics of the data. In our data we have $1/E[h] < 1$, $\gamma < 0$, and $E[1/h] > 1$ in terms of sample equivalents, which suggests a positive inconsistency. The third term is always positive, which adds to the positive inconsistency, and it is bigger the greater the dispersion of h in the data. Taken together, if (31) is true, but the econometrician applies (28) with the assumption $g(h) = \gamma h$ in estimation, the arithmetic alpha estimates could be substantially inconsistent.

In the log-normal model, the payoff endogeneity discussed above leads to an additional inconsistency because of the additional restrictive assumptions about the process of a venture's value. The endogeneity implies that ε_{t+h}^h in (28) is negatively correlated with h , as more successful firms are more likely to proceed quickly to the next funding round or exit. As a consequence, if estimation proceeds under the assumption $g(h) = \gamma h$, the estimate of γ will be inconsistent (but not in the alternative log-normal model (31) where the intercept term does not depend on h). The selection models in Cochrane (2005) and Korteweg and Sorensen (2010) help ameliorate this potential shortcoming of the log-normal model by specifying the probability of a new funding round or an exit event as a function of the value of the start-up company. As the firm value rises, the probability of obtaining a new funding round or a successful exit rises as well. This limits the extent to which the firm value within a funding round can grow with h , which gets the model closer to (31). These selection models, however, require rather strong assumptions.

In comparison, our SDF approach circumvents the need to make strong distributional assumptions. This avoids the inconsistency that can arise from misspecification of these assumptions. Nevertheless, to facilitate comparison with the prior literature, we estimate the log-normal model (28) on our data. We estimate both the specification with $g(h) = \gamma h$, and the alternative model (31) where $g(h) = \gamma$. Comparing these specifications helps us assess which stochastic process best describes start-up company project values and how to think about the economics of entrepreneurial ventures. In each case, we convert the alpha to a *GPME* measure as $GPME = \exp(\alpha^h) - 1$, averaged across all round-to-round observations

(which differ in h).

Table V shows the results for the log-normal model. The first three specifications use the assumption $g(h) = \gamma h$ from Cochrane (2005) and Korteweg and Sorensen (2010), while the last three specifications use the alternative assumption, $g(h) = \gamma$.

Across in columns (i) to (iii) in all panels, the β estimates are between 1.8 and 3.5, and this range partly overlaps with recent estimates in the literature (Gompers and Lerner (1997) estimate betas from 1.1 to 1.4, Peng (2001) finds 1.3 to 2.4, Woodward (2009) finds 2.2, Korteweg and Sorensen (2010) find 2.8, Driessen, Lin, and Phalippou (2012) find 2.7, and Ewens, Jones, and Rhodes-Kropf (2013) find 1.2). The implied *GPMEs* are considerably higher than what we find with the SDF approach that does not make strong distributional assumptions. Evidently, relying on the log-normal model without combining it with a selection model leads to implausibly large positive abnormal returns.

Changing the assumption about the horizon-dependence of abnormal return and volatility leads to more plausible estimates. In the alternative log-normal model in columns (iv) to (vi), the *GPMEs* for liquidation returns of -70% and -50% are comparable to those from the SDF model. This result suggests that the alternative model does a better job describing the data compared to the distributional assumptions in columns (i) to (iii).

Note that in Table V the arithmetic alpha shrinks as we increase the assumed liquidation return. This counter-intuitive feature appears to be driven by the effect that a higher liquidation return lowers the volatility (and hence the Jensen's inequality adjustment term) and dominates the positive effect of the higher liquidation return on γ .

The results in this section demonstrate some of the difficulties that can arise when applying an approach with strong distributional assumptions. How to model the randomness in start-up companies' value processes is an interesting question for further research. At this point, however, without extensive evidence on this issue, it is difficult to set up the log-normal model in a way that is consistent with important features of the data. Our SDF approach circumvents the need to take a stand on these issues.

Table V
Results from a Log-Normal Model for VC Round-to-Round Returns

We match each start-up company round-to-round gross return, measured over horizon h from t to $t + h$, with the return of the CRSP value-weighted index ($r_{m,t+h}^h$ in logs) from t to $t + h$, the return from rolling over 1-month Treasury Bills, the return on a strategy that buys at-the-money put options on the S&P500 index, and the return on a microcap portfolio. We estimate a linear regression in logs

$$r_{it+h}^h - r_{ft+h}^h = g(h) + \beta_m(r_{m,t+h}^h - r_{f,t+h}^h) + \beta_p(r_{p,t+h}^h - r_{f,t+h}^h) + \varepsilon_{t+h}^h,$$

with OLS, where $g(h) = \gamma h$ in columns (i) to (iii) and $g(h) = \gamma$ in columns (iv) to (vi). The standard errors in parentheses are calculated assuming independent errors, in line with common practice based on this model in the existing literature. The annualized arithmetic alpha is calculated from γ with a Jensen's inequality adjustment. The calculation of variance for this adjustment assumes that the variance of ε_{t+h}^h is proportional to h in columns (i) to (iii), and constant in columns (iv) to (vi). The number reported in the table is the average of these arithmetic alphas across the whole sample. The implied *GPME* is calculated for each observation as the arithmetic alpha for the observation's horizon, α^h , as $GPME = \exp(\alpha^h) - 1$, and then averaged across the whole sample.

	$g(h) = \gamma h$			$g(h) = \gamma$		
	(i)	w/ puts (ii)	w/ micaps (iii)	(iv)	w/ puts (v)	w/ micaps (vi)
Assumed unobserved liquidation returns = -90%						
γ	-0.355 (0.010)	-0.340 (0.021)	-0.339 (0.011)	-0.373 (0.014)	0.022 (0.016)	-0.318 (0.014)
β_m	3.123 (0.079)	3.257 (0.202)	3.423 (0.092)	1.313 (0.090)	5.530 (0.114)	2.248 (0.090)
β_p		0.052 (0.064)	-0.264 (0.044)		1.163 (0.029)	-0.746 (0.039)
Annualized arithmetic alpha	0.666	0.682	0.678	0.591	0.697	0.687
Implied <i>GPME</i>	1.362	1.410	1.399	0.809	0.937	1.041
Assumed unobserved liquidation returns = -70%						
γ	-0.130 (0.008)	-0.079 (0.018)	-0.116 (0.009)	-0.109 (0.011)	0.166 (0.012)	-0.071 (0.011)
β_m	2.236 (0.061)	2.669 (0.172)	2.497 (0.073)	0.997 (0.064)	3.928 (0.089)	1.653 (0.067)
β_p		0.168 (0.054)	-0.229 (0.038)		0.809 (0.023)	-0.523 (0.029)
Annualized arithmetic alpha	0.563	0.613	0.574	0.455	0.536	0.524
Implied <i>GPME</i>	1.068	1.206	1.098	0.577	0.653	0.715
Assumed unobserved liquidation returns = -50%						
γ	-0.025 (0.008)	0.042 (0.017)	-0.012 (0.008)	0.014 (0.010)	0.232 (0.011)	0.044 (0.010)
β_m	1.824 (0.056)	2.396 (0.163)	2.066 (0.067)	0.850 (0.054)	3.183 (0.083)	1.376 (0.059)
β_p		0.222 (0.052)	-0.212 (0.036)		0.644 (0.020)	-0.420 (0.025)
Annualized arithmetic alpha	0.582	0.647	0.592	0.464	0.531	0.520
Implied <i>GPME</i>	1.119	1.306	1.148	0.589	0.649	0.700

IV. Conclusions

We propose a stochastic discount factor approach to valuation of payoffs in private equity markets. This approach is an alternative to existing methods in this area that are either heuristically motivated or heavily parameterized. Our approach

- is robust in that it avoids strong distributional assumptions.
- compares payoffs in private equity markets with payoffs in public equity markets that are matched by systematic risk.
- is suited for multi-period, skewed, and endogenously timed payoffs.
- generalizes existing approaches based on the Public Market Equivalent (*PME*) by leaving the equity premium unrestricted.
- allows for statistical inference that takes into account cross-sectional dependence.

In our empirical application with an exponentially-affine version of the CAPM, we find substantial positive abnormal returns from VC investments in start-up companies and close to zero abnormal returns for VC funds. We also find that the relationship between VC investment payoffs and public equity market returns exhibits a pronounced concavity, similar to the payoff from selling index put options. To account for this feature, we expand the set of public market instruments that we use for comparison with VC payoffs to include put option returns. Doing so raises the positive abnormal return of start-up company investments, while it pushes the abnormal return from VC funds into negative territory.

In this paper we have focused on the risk and return to Venture Capital. Our methodology can also be applied to other infrequently traded assets, in particular those with highly levered or option-like returns, such as real estate or leveraged buyouts.

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