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ABSTRACT

Acquisitions, Entry, and Innovation in Network Industries*

In industries with network effects, incumbents' installed bases create barriers to entry that discourage entrepreneurs from developing new innovations. Yet, entry is not the only commercialization route for entrepreneurs. We show that the option of selling to an incumbent increases innovation incentives for entrepreneurs when network effects are strong and incumbents compete to preemptively acquire innovations. We thus establish that network effects and installed bases do not necessarily restrict innovation incentives. We also show that network effects promote acquisitions over entry and that the entrepreneur has strong incentives to invest in the initial user base of the innovation.

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1 Introduction

In industries with network effects, incompatibility with incumbents' installed bases create barriers to entry (Farrell and Klemperer (2007)). These entry barriers can discourage entrepreneurs from developing new innovations; particularly when network effects are strong. Yet, entry is not the only commercialization route for entrepreneurs. Selling to established firms can prove to be extremely lucrative, in particular if the entrepreneur has managed to gather a sufficient initial user base for the innovation. The business press contains numerous accounts of preemptive bidding between incumbents for young firms. One example is Apple's acquisition of the music site Lala.com. BusinessWeek (2010) described the acquisition as follows: "Late last year, Apple entered the bidding for the online music site Lala.com, after Google and several other potential acquirers had gotten involved. The company moved unusually quickly, closing the deal in a few weeks, rather than the more typical two to three months. It was clear that Apple didn't want to lose out again, and especially not to Google". Another example is the battle between Google and Facebook over Skype in 2011. A Reuters article notes: "Indeed, some speculate that Google could be bidding for Skype just to keep it out of the hands of other companies. Any deal that takes a great asset away from Facebook is a win for Google".[1]

In this paper, we study how innovation incentives in industries with installed bases and network effects are affected by bidding competition between incumbents for new entrepreneurial firms. We do this by introducing an endogenous acquisition auction into a canonical model of competition in an industry with network effects. We show how the option of selling to an incumbent increases innovation incentives for entrepreneurs, in particular when network effects are strong and entry is hindered by incumbent installed bases. Our paper thus establishes that network effects and installed bases need not restrict innovation incentives for entrepreneurial firms.

The model looks as follows. In Stage 1, an entrepreneur exerts effort to discover an innovation. If the effort is successful, the entrepreneur attracts an initial user base which is smaller than the installed base of incumbents (locked-in consumers). In Stage 2, then entrepreneur decides on the commercialization route by arranging a first price auction for the innovation and the initial user base. The bidders are incumbents firms with established installed bases of locked-in users. The reservation price is the value of entry into the market, so the entrepreneur enters the market if no incumbent submits

[1] Source: http://www.reuters.com/article/2011/05/06/us-facebook-google-skype-idUSTRE74505A20110506 Accessed April 2013.

a bid high enough. In Stage 3, product market competition for new consumers takes place. New consumers first form expectations of network size for each firm. Network size consist of locked-in consumers and of the expected number of new consumers. They then decide which firm to patronize given their expectations. Finally, firms compete in quantities taking expectations as given.

The model shows how the option to sell mitigates the negative effect of installed bases on innovation incentives. To see the intuition, consider the effects of network effects on the two commercialization routes.

Entry. The value for the entrepreneur of entering the industry is inversely U-shaped in network effects. At low levels of network effects, the entrant–as well as the incumbents– benefit from stronger network effects as consumers' willingness to pay increase. When network effects become strong enough, however, the larger installed base of incumbents are more attractive to consumers so that the entrant starts losing consumers to incumbents. The entry profit decreases, and eventually entry is not profitable and innovation incentives are non-existent.

Sale. The value of selling to incumbents is determined endogenously in the acquisition auction. Two equilibrium prices exist. One is an entry-deterring acquisition price. This price is paid when an incumbent makes the acquisition to prevent entry. The equilibrium price is the entrants reservation price (the entry profit). The second equilibrium price is a pre-emptive acquisition price. This price is paid when incumbents compete to acquire the innovation. This price is determined by the difference in profits of acquiring the innovation relative to the profit when a rival incumbent acquires it. The preemptive acquisition price is strictly increasing in network effects when network effects are strong, since the profit as an acquirer increases in network effects and the profit as a non-acquirer decreases in network effects. The reason is that acquiring the innovation generates an asymmetry in expected network size between incumbents. This asymmetry is amplified by network effects and by installed bases, creating a *demand side synergy*: by adding the initial user base of the entrepreneur to its own installed base, the acquirer obtains a strong market position as new consumers are attracted to firm with the largest expected network. Network effects thus have a dual positive effect on the acquisition price.

When both commercialization routes are available, three regions exist. For weak network effects, entry takes place. For medium network effects, one incumbent will deter entry by acquiring the entrepreneurial firm at her reservation price. For strong network effects, however, incumbents compete fiercely to prevent rivals from obtaining the innovation and the acquisition price is driven up to the preemptive value. Since the entry value in this region is decreasing in network effects while the preemptive value is increasing in network effects, a sale to an incumbent not only avoids the entry barriers created by installed bases—it also allows the entrepreneur to exploit the rivalry between incumbents to secure a high reward to innovating. Paradoxically, the incentive to innovate for sale is then the greatest when network effects and large incumbent installed bases create the most harmful innovation incentives under entry.

The model generates additional novel insights. First, network effects promote acquisitions over entry: we should expect substantial takeover activity in markets with network effects as incumbents fight to obtain innovations and the initial user base attached to them. Entrepreneurs should have incentives to innovate for sale, rather than to innovate for entry. Second, preemptive bidding competition when network effects are strong gives entrepreneurs incentives to invest in acquiring a large initial user base. As with network effects, increasing the initial user base has a dual effect on the equilibrium acquisition price under preemptive bidding. It increases the asymmetry in networks between incumbents, and they pay both to obtain the initial user base and to prevent rivals from obtaining it.

This paper contributes to the literature on network effects by developing a model of competition in network industries that allows for innovation efforts by an independent entrepreneur and that endogenously determines whether an acquisition or entry into the industry takes place.[2] We depart from the network model in Katz and Shapiro (1985), bring in installed bases and asymmetries as in Cremer et al. (2000) or Malueg and Schwartz (2006), and embed an innovation and commercialization stage from Norbäck et al. (2011) and Norbäck and Persson (2012).

In the literature on network effects, papers such as Farrell and Saloner (1985), Farrell and Saloner (1986), and Katz and Shapiro (1992) have studied how expectations and installed bases can lead to too fast or too slow movement to a new technology.[3] In particular, as emphasized in Farrell and Klemperer (2007), installed bases can create a barrier to entry. Our setting is separate from these papers by allowing incumbents to acquire the startup as a way of deterring entry. We also allow the startup to innovate prior to entry. This makes it possible for us to show that network effects and installed bases need not restrict innovation incentives for entrepreneurial firms due to a demand side synergy between the initial user base of the entrepreneur and the installed base of incumbents.

The literature on the commercialization of innovations has shown that commercializa-

[2] For an overview of the literature on network effects, see Economides (1996a) or Farrell and Klemperer (2007).

[3] In Section 4.1 we also show that our results also hold when the entrepeneur brings an innovation which reduced marginal cost.

tion by sale (or by licensing) is more likely when entry costs are high, the entrepreneurial firm lacks complementary assets, brokers facilitating trade are available, the expropriation problem associated with asset transfers is low, and the intensity of product market competition is high (Anton and Yao (1994), Gans and Stern (2000), Gans et al. (2002) and Gans and Stern (2003)). Two papers close to this one are Norbäck et al. (2011) and Norbäck and Persson (2012).[4] Norbäck et al. (2011) develop a theory of commercialization of entrepreneurial innovations into oligopoly, and show that an innovation of higher quality is more likely to be sold to incumbents. Norbäck and Persson (2012) describe how competition policy affects entry and acquisition incentives when preemptive bidding for entrepreneurs is possible. We add to this literature by examining how network effects affect the commercialization route and equilibrium acquisition prices. This allows us to show how network effects create and strengthen a demand synergy present in acquisitions in networks industries. This synergy is created by the combination of the entrepreneur's initial user base and the incumbent's installed base; a combination which increases the expectation of the firm's network size and thereby attracts new consumers to the merged firm.

Finally, and more generally, our paper relates to the work on auctions with externalities by Jehiel et al. (1996), Jehiel and Moldovanu (1996) and Jehiel and Moldovanu (2000). A seminal analysis of this mechanism in the context of preemptive patenting is given in Gilbert and Newbery (1982) and Katz and Shapiro (1986), but we are not aware of work that ties this literature to acquisitions of startups in network industries.

We have organized the paper as follows. The next section describes and solves the model. Section 3 presents the central propositions of the paper by undertaking comparative statics on the strength of network effects. We present extensions in Section 4 and offer concluding remarks in Section 5.

2 The model

We start by describing and solving the model using backwards induction. The structure of the game is illustrated in Figure 1 and the game proceeds as follows.

1. In Stage 1, an entrepreneur undertakes an effort to increase the probability ρ of discovering an innovation of quality k. If the innovation effort succeeds, the entrepreneur is able to attract an initial base of "locked-in" consumer who prefer

[4] See also related work combining downstream oligopoly interaction with an auction with externalities in Norbäck and Persson (2009) and Norbäck et al. (2010).

the entrepreneur's product. We assume that a higher quality of the innovation will attract more initial consumers. To simplify the exposition, we normalize so that the initial user base is equal to the quality of the innovation, k. Incumbents have installed bases of size b. The installed base of an incumbent consists of consumers who are tied to the incumbents' product by habit or customized equipment. Locked-in consumers cannot switch to other firms.

2. In Stage 2, the entrepreneur decides whether to sell the innovation–and the associated initial user base k–to an incumbent firm or enter the market. An acquisition of the entrepreneurial firm will add the initial user base of the entrepreneur to the acquiring incumbent's installed base. The commercialization stage is modelled as a first price perfect information auction with externalities, with the value of entering the market as the reservation price.

3. In Stage 3, product market competition takes place. Firms compete for *new* consumers who put value on a firm's network size in terms of its locked-in consumers and the expected number of attracted new consumers. Consumers first form expectations of network sizes, then decide which firm to patronize given their expectations, and finally firms compete in quantities taking expectations as given.

The quality of the innovation k stems primarily from its ability to attract initial locked-in consumers, which, in turn, enables the possessor of the innovation to gain an advantage in attracting new consumers since new consumers are attracted to a larger network. Our results remain the same if we where instead to assume that the quality k represents a true improvement in technology, say, generating a reduced marginal cost, and where the entrepreneur is unable to create an initial user base. We discuss this in detail in Section 4.1.

2.1 Stage 3: Product market competition

We start by analyzing product market competition in Stage 3. We first establish that a fulfilled expectations equilibrium exists and is unique (Lemma 1). We then characterize how profits of acquirers, non-acquirers, and the entrant are affected by the commercialization decision and the strength of network effects (Lemma 3 and 4).

Let the set of firms in the industry be $\mathcal{J} = e \times \mathcal{I}$, where $\mathcal{I} = \{i_1, i_2, ..., i_n\}$ is the set of n incumbent firms and e is the entrant. Let the set of potential ownerships of the innovation k be $l \in \mathcal{L}$, where $\mathcal{L} = \{e, i_1, i_2, ..., i_n\}$. Denote the number of firms in the product market by $n(l)$. The number of firms is $n(i_1) = ... = n(i_n) = n$ under an

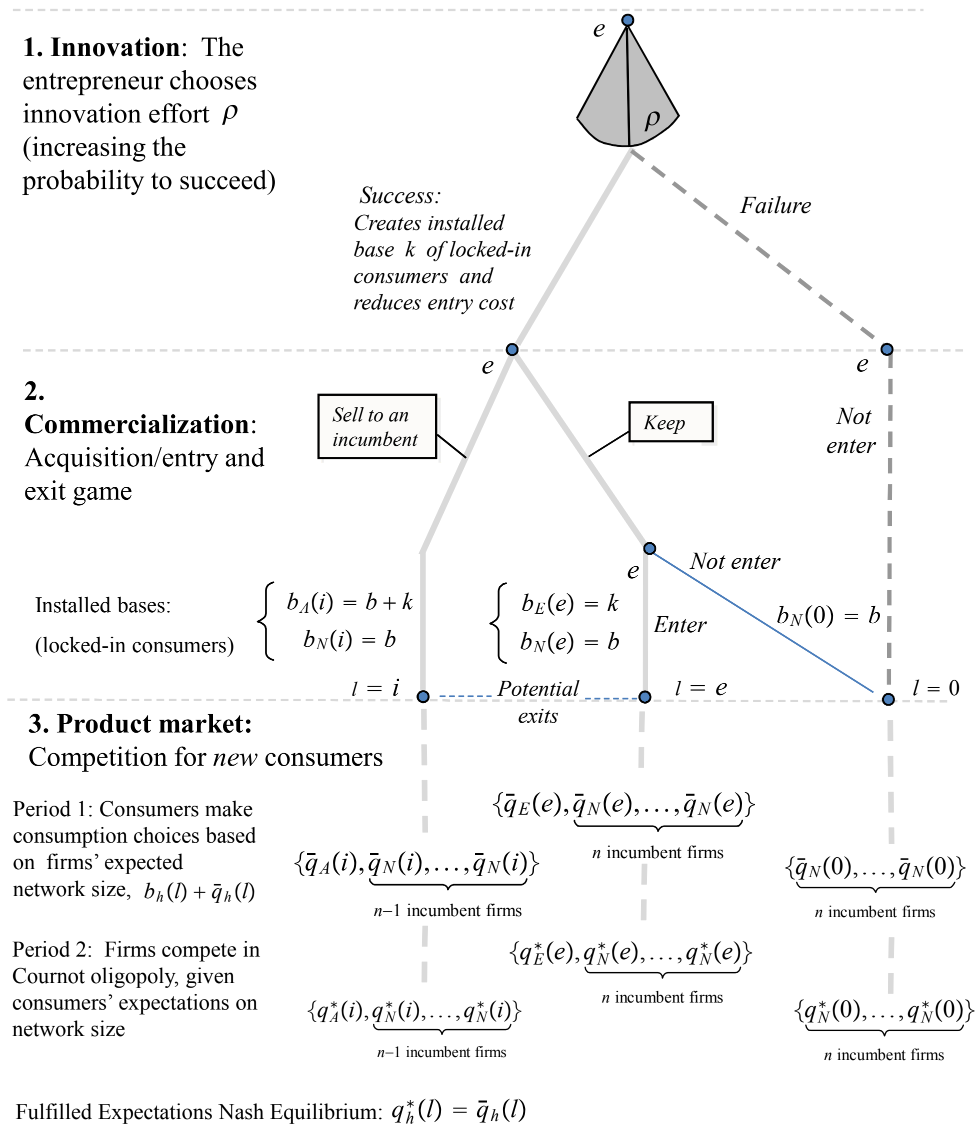

Figure 1: The structure of the game we analyze.

acquisition and $n(e) = n + 1$ firms under entry. As shown in Figure 1, in a first period of Stage 3, new consumers make their consumption choices given their expectation of a firm's network size. A firm's expected network size depends on the size of its installed base in terms of old locked-in consumers, $b_j(l)$, where j indicates the identity of a firm and l indicates the ownership of k, and the expected number of new consumers (specified below). In the second period of Stage 3, taking consumers expectations as given, firms compete in setting quantities a la Cournot.

2.1.1 The Fulfilled Expectations Nash-Equilibrium

We first show that there exists a unique and stable Fulfilled Expectations Nash-Equilibrium in the product market. Let the net value for a new consumer of buying from firm j be

$$t + z\left[b_j(l) + \bar{q}_j(l)\right] - p_j \tag{1}$$

where t is a consumer's stand-alone value for the product, which is *specific* for each consumer but *symmetric* across all products for a consumer. The price of firm j's product is p_j and $b_j(l) + \bar{q}_j(l)$ is the network size of firm j consisting of the sum of the number of locked-in consumers $b_j(l)$ and the expected number of new consumers, which is simply firm j's expected output, $\bar{q}_j(l)$. The parameter z in (1) is key to our analysis. We label $z \in [0, 1)$ as the "*network effect*". This parameter is common to all consumers and measures how intensely new consumers value a firm's network. That is, z reflects the increase in value of a product j for a new consumer when the sum of old and new consumers buying product j, $b_j(l) + \bar{q}_j(l)$, increases.

The inverse demand facing firms in the industry can now be derived as follows. Since the willingness to pay t is the same across products, two products must have the same network adjusted price if positive amounts of these goods are to be sold. From (1), a consumer t is indifferent from buying product j and w if and only if $t + z\left[b_j(l) + \bar{q}_j(l)\right] - p_j = t + z\left[b_w(l) + \bar{q}_w(l)\right] - p_w$, or

$$p_j - z\left[b_j(l) + \bar{q}_j(l)\right] = p_w - z\left[b_w(l) + \bar{q}_w(l)\right] \equiv \phi. \tag{2}$$

In Equation (2), ϕ is the network adjusted price common to all products sold on the market. Following Katz and Shapiro (1985), we then assume that consumers (stand-alone) willingness to pay t is uniformly distributed with support $(-\infty, T]$ and density of unity. It then follows that only consumers with willingness to pay $t \in [\phi, T]$ will buy a product. Thus, with a uniform distribution, $T - \phi$ consumers will buy a product. Define

$Q(l) = \sum_j^{n(l)} q_j(l)$ as aggregate output. In a consistent expectations equilibrium firms must then be able to supply these purchases, that is

$$T - \phi = Q(l). \tag{3}$$

From (2) and (3), it follows that the (inverse) demand for product firm j by new consumers is

$$p_j(l) = T + z\left[\bar{q}_j(l) + b_j(l)\right] - Q(l). \tag{4}$$

Firms compete in attracting new consumers. The profit of firm j is then

$$\pi_j(q_j, q_{-j}, l) = \left[p_j(l) - c\right] q_j, \tag{5}$$

where c is a symmetric marginal cost, $p_j(l) = p_j(q_j, q_{-j}, l)$ is the price of firm j's product given from (4) noting that $Q(l) = q_j(l) + q_{-j}(l)$, where $q_j(l)$ is the sales of firm j and $q_{-j}(l)$ is the sales of its rivals. Taking consumers expectations of the number of new consumers buying firm j's product $\bar{q}_j(l)$ as given, the optimal outputs are determined from the first-order conditions

$$\frac{\partial \pi_j}{\partial q_j} = p_j(l) - c - q_j^*(l) = 0, \forall j \tag{6}$$

where we assume that usual stability and second-order conditions are fulfilled.

We now show that there exists a Fulfilled Expectations Cournot Equilibrium (FECE) where consumers' expected network size corresponds to firms' optimal output decisions. Following Economides (1996b), we prove the following Lemma:

Lemma 1 *There exists a unique and stable Fulfilled Expectations Cournot Equilibrium (FECE) where consumers expectations are fulfilled:* $Q^*(l) = \sum_j^{n(l)} q_j^*(l) = \bar{Q}(l)$.

To see this, substitute (4) into (6) and sum over all $n(l)$ firms. Solve for total output $Q^*(l) = \sum_j^{n(l)} q_j^*(l)$ as a function of total expected output $\bar{Q}(l) = \sum_j^{n(l)} \bar{q}_j(l)$. Denote this solution

$$\check{Q}(l, \bar{Q}(l)) = \frac{\Lambda + zB(l) + z\bar{Q}(l)}{n(l) + 1}, \tag{7}$$

where $\Lambda = T - c$, and $B(l) = \sum_j^{n(l)} b_j(l)$ is the aggregate installed base. In Figure 2, we depict the locus of $\check{Q}(l, \bar{Q}(l))$. The figure also plots the 45 degree line where firms' aggregate output is equal to the expected aggregate output, $Q(l) = \bar{Q}(l)$. It follows that the locus $\check{Q}(l, \bar{Q}(l))$ only intersect the 45 degree line once since $\check{Q}(l, 0) > 0$ and

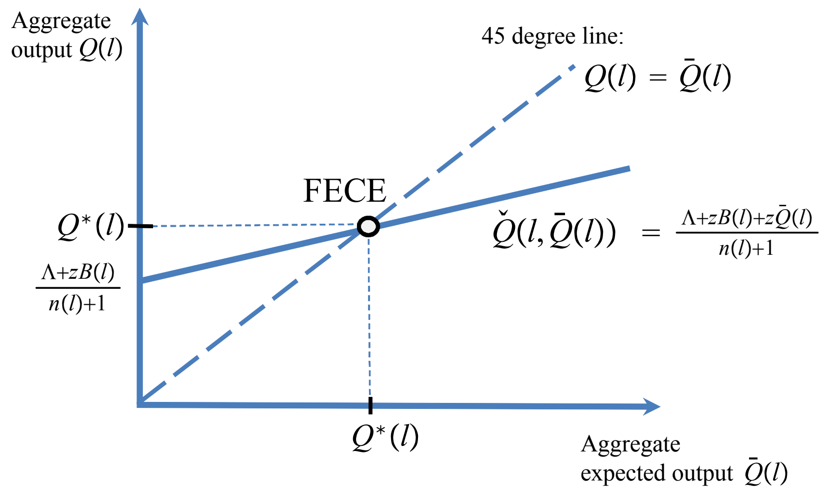

Figure 2: Illustrating the Fulfilled Expectations Cournot Equilibrium (FECE).

$d\check{Q}/d\bar{Q} = z/(n+1) < 1$. Hence, there exists a unique Fulfilled Expectations Cournot Equilibrium (FECE) where consumers expectations are fulfilled, $Q^*(l) = \sum_j^{n(l)} q_j^*(l) = \bar{Q}(l)$. Furthermore, the Fulfilled Expectations Cournot Equilibrium (FECE) is stable. To see this, note that an increase in firms' aggregate output $\check{Q}(l, \bar{Q}(l))$, from say an increase in aggregate installed base $B(l)$, shifts up the locus of $\check{Q}(l, \bar{Q}(l))$ for any positive expectation by consumers $\bar{Q}(l)$. This increase in aggregate output by firms is then accompanied by an increase in expected aggregate output by consumers, $\bar{Q}(l)$.

With fulfilled expectations of consumers from Lemma 1, $\bar{Q}(l) = Q^*(l)$, we can now use (7) to solve for total output in a Fulfilled Expectations Cournot Equilibrium,

$$Q^*(l) = \frac{n(l)\Lambda + zB(l)}{n(l) + 1 - z}. \tag{8}$$

From (4) and (6) again applying fulfilled expectations from Lemma 1, $\bar{q}_j(l) = q_j^*(l)$, it can finally be shown that outputs and profits (consistent with fulfilled consumer expectations) are

$$q_j^*(l) = \frac{\Lambda + zb_j(l) - Q^*(l)}{1 - z} \quad \text{and} \quad \pi_j^*(l) = [q_j^*(l)]^2, \tag{9}$$

where total output $Q^*(l)$ is given from (8). A fulfilled consumer expectations equilibrium exists even if not all firms are active. Indeed, below we will show that an acquisition may lead to a monopoly for the acquiring incumbent when network effects are very strong. In such a case, non-acquiring incumbents will exit the competition for new consumers in the end of Stage 2 (and only serve locked-in consumers) as they would make losses participating in the competition for new consumers in Stage 3. In the beginning of Stage 3, new consumers then know that the market will be a monopoly. A monopoly equilibrium is thus consistent with fulfilled consumer expectations.

2.1.2 Firms profits and network effects

Having established that a Fulfilled Expectations Cournot Equilibrium exists and is stable for any l, we next study how firms reduced-form profit functions $\pi_j^*(l)$ respond to changes in z. This information will be key when we examine how network effects and installed bases affect the entrepreneur's commercialization choice in Stage 2 and the incentive to innovate in Stage 1.

Differentiate the reduced-form profit $\pi_j^*(l)$ with respect to the network effect z. Using the envelope theorem and applying Lemma 1 and fulfilled consumer expectations $\bar{q}_j(l) = q_j^*(l)$, we obtain

$$\frac{d\pi_j^*(l)}{dz} = \left[\underbrace{q_j^*(l) + b_j(l)}_{\text{Direct price effect}} + \underbrace{z\frac{dq_j^*(l)}{dz}}_{\text{Network effect}} - \underbrace{\frac{dq_{-j}^*(l)}{dz}}_{\text{Strategic effect}} \right] q_j^*(l). \tag{10}$$

The first terms within the bracket, $\frac{\partial P_j(l)}{\partial z} = q_j^*(l)+b_j(l)$ represents a *direct price effect:* for given network size, stronger network effects increase consumers' willingness to pay from (4). The term $z\frac{dq_j^*(l)}{dz}$ is *a direct network effect* which arises as consumers willingness to pay also changes from a *changing expected network size.* The sign of the direct network effect is ambiguous. It will depend on whether consumers expect that the firm's network is going to expand or contract. The last term within the bracket $-\frac{dq_{-j}^*(l)}{dz} = \frac{dP_j(l)}{dq_{-j}}\frac{dq_{-j}^*(l)}{dz}$ represents the *strategic price effect* arising from the change in price generated by the induced change in the output of competitors. The sign of the latter effect is also ambiguous—it will depends on the outcome in the interaction in the product market.

To evaluate the sign of (10), first define $\varphi_j^*(l)$ as the relative network size of a firm j,

$$\varphi_j^*(l) = \frac{q_j^*(l) + b_j(l)}{Q^*(l) + B(l)} \tag{11}$$

where $q_j^*(l) + b_j(l)$ is the network size of a firm j and the total network size of the industry is $Q^*(l)+B(l)$. Since profits are quadratic in outputs from (9), we have $\frac{d\pi_j^*(l)}{dz} =$

$2\frac{dq_j^*(l)}{dz}q_j^*(l)$. From (8), (9) and (11), we have

$$\frac{d\pi_j^*(l)}{dz} = 2\left(\frac{Q^*(l)+B(l)}{1-z}\right)\left(\varphi_j^*(l) - \frac{1}{n(l)} + \frac{1-z}{n(l)[n(l)+1-z]}\right)q_j^*(l). \qquad (12)$$

Equation (12) reveals that the reduced-form product market profit of a firm will increase in network effect z, if it has a share of the total industry network $\varphi_j^*(l)$ which is not significantly *smaller* than $1/n(l)$, where the latter is the relative network share in a symmetric industry (where all firms have the same installed base). As long as this condition is fulfilled, the direct increase in consumers willingness to pay (the direct price effect) in (10) will dominate, and the reduced-form product market profit $\pi_j^*(l)$ will increase in z. However, if a firm has a relative network size $\varphi_j^*(l)$ which is significantly smaller than $1/n(l)$, the direct price effect will be dominated by the direct network effect and the strategic effect and the profit will decrease when network effects become stronger, $\frac{d\pi_j^*(l)}{dz} < 0$. The reason is that other firms with a larger network size will steal new consumers as these are attracted to a larger network from (4). The associated expansion by the firm's competitors is then met by a reduction in own output in order to mitigate a fall the firm's product price.

Due to the symmetry of incumbents, we need only to explore how the profits of different *types of firms* react to network effect. If the innovation succeeds, and it is commercialized, there are two types of ownership: entry into the market ($l = e$) and sale to an incumbent ($l = i$). For these two types of ownerships, there are three types of firms h: the entering entrepreneurial firm ($h = E$), the acquiring incumbent ($h = A$) and non-acquiring incumbents ($h = N$). If the entrepreneur fails or does not commercialize ($l = 0$), only non-acquiring incumbents are present in the market ($h = N$). Firm types only differ in their installed bases or in terms of their loyal or "locked-in" consumers, $b_h(l)$

$$b_h(l) = \begin{cases} b_A(i) = b + k \\ b_N(l) = b, \ l = \{i, e, 0\} \\ b_E(e) = k \end{cases} . \qquad (13)$$

As shown in Figure 1, if the entrepreneur fails or does not commercialize ($l = 0$), all incumbents remain symmetric with installed bases $b_N(0) = b$. The aggregate installed base is then $B(0) = nb$. If the entrepreneur succeeds with her innovation, she generates an installed base $b_E(e) = k$ of initial loyal consumers if she enters. If she sells the acquiring incumbent will have an installed base of $b_A(i) = b + k$ locked-in consumers. Non-acquiring incumbents always have the installed base $b_N(l) = b$. If the entrepreneur succeeds and commercializes the innovation, the *aggregate* installed base is independent of ownership: $B(l) = nb + k$ for $l = \{e, i\}$.

To proceed, we make the following assumption:

Assumption 1 $0 < k < b$

The assumption says that the installed base of the incumbents b is larger (or much much larger) than the initial user base of the entrepreneur, k. We will maintain Assumption 1 in the rest of the paper. The reason is that we want to avoid a situation of innovating *for* the market (to replace incumbents) and instead focus on a situation of innovating for entry *into* the market. When the initial user base generated by the innovation is substantially larger than the installed base of incumbents, it trivially follows that entry will always occur and installed bases of incumbents does not affect innovation incentives.

2.1.3 Profits when the entrepreneur fails

As a benchmark, let us first examine firms' profits when the entrepreneur fails with her innovation (or if the innovation is not commercialized upon success).

Lemma 2 *Incumbents profits always increase in network effects when the innovation fails,* $\frac{d\pi_N^*(0)}{dz} > 0$.

If the entrepreneur fails, entry does not occur and all incumbents are symmetric from (5) and (13). Thus, incumbents have a symmetric network share, $\varphi_N^*(0) = \frac{1}{n}$. It then follows from (12) that incumbents profits must increase in network effect, $\frac{d\pi_N^*(0)}{dz} > 0$. Thus, in absence of the entrepreneur's innovation, the direct increase in consumers willingness to pay (the direct price effect) in (10) will dominate and incumbents always benefit from network effects. With this bench benchmark result in hand, let us now examine the effects when the entrepreneur succeeds and the innovation is commercialized.

2.1.4 Profits under commercialization by sale

Start with case when the entrepreneur's installed base is sold to an incumbent. We have the following result.

Lemma 3 *Assume that the entrepreneur has succeeded with the innovation in Stage 1 and that an incumbent has acquired the innovation,* $l = i$, *in Stage 2. Then:*

(i) the acquiring incumbent's reduced-form product market profit $\pi_A^*(i)$ is strictly increasing in network effects, z.
(ii) A non-acquirer's reduced-form product market profit $\pi_N^*(i)$ is strictly concave in network effect, z, with a unique maximum $z^N(i) \in (0,1)$ and $\pi_N^*(i) = 0$ for $z = z_{\max}^N(i) \in (z^N(i), 1)$.

Proof. See the Appendix. ■

To see this, note that the initial user base k of the entrepreneur creates an asymmetric market structure when network effects are present in (4). When a sale occurs, new consumers infer that the acquiring incumbent will have an installed base of $b_A(i) = b + k > b_N(i) = b$ of locked-in consumers. This gives the acquirer an advantage in attracting new consumers and the largest network share, $\varphi_A^*(i) > 1/n > \varphi_N^*(i)$. As illustrated in Figure 3(i), stronger network effects then always benefits the acquirer, $\frac{d\pi_A^*(i)}{dz} > 0$. As also shown in Figure 3(i), non-acquiring rivals will see their profits increase in network effects when the network effect is of limited size, $z \in [0, z^N(i))$. For low network effects, the impact of the asymmetry in installed bases is limited from (4). The direct increase in consumer's willingness to pay then dominates the network effect and the strategic effect, and profits increase: $\frac{d\pi_N^*(i)}{dz} > 0$.[5] However, at stronger network effects, consumers attraction to the larger network of the acquirer increases. When network effects become sufficiently large, $z > z^N(i)$, consumers correctly infer that the networks of non-acquiring incumbents will decrease as the acquiring incumbent's network expands. The profit of non-acquiring incumbents then decreases in network effect, $\frac{d\pi_N^*(i)}{dz} < 0$. Figure 3(ii) reveals that this will, in general, occur when the quality of the innovation (as measured by the initial user base) is higher. Intuitively, the larger is the initial user base obtained by the acquirer, the larger is the advantage over rivals when networks effects increase.

Figure 3(iii) finally depicts the combination of initial user base k and network effect z at which the market tips to the acquiring incumbent and a monopoly is established, $\pi_N^*(i) = 0$ and $\pi_A^*(i) = \pi^m$, where π^m is the monopoly profit. In this region, non-acquiring rivals will not compete for new consumers in Stage 2. Only the acquiring incumbent from Stage 2 remains.

[5] Note also that if network effects are initially absent, $z = 0$, all firms become symmetric with market shares, $\varphi_h^*(l) = \frac{1}{n(l)}$. Then, from (12), all firms gain in profits for a small increase in network strength, $\frac{d\pi_h^*(l)}{dz} > 0$.

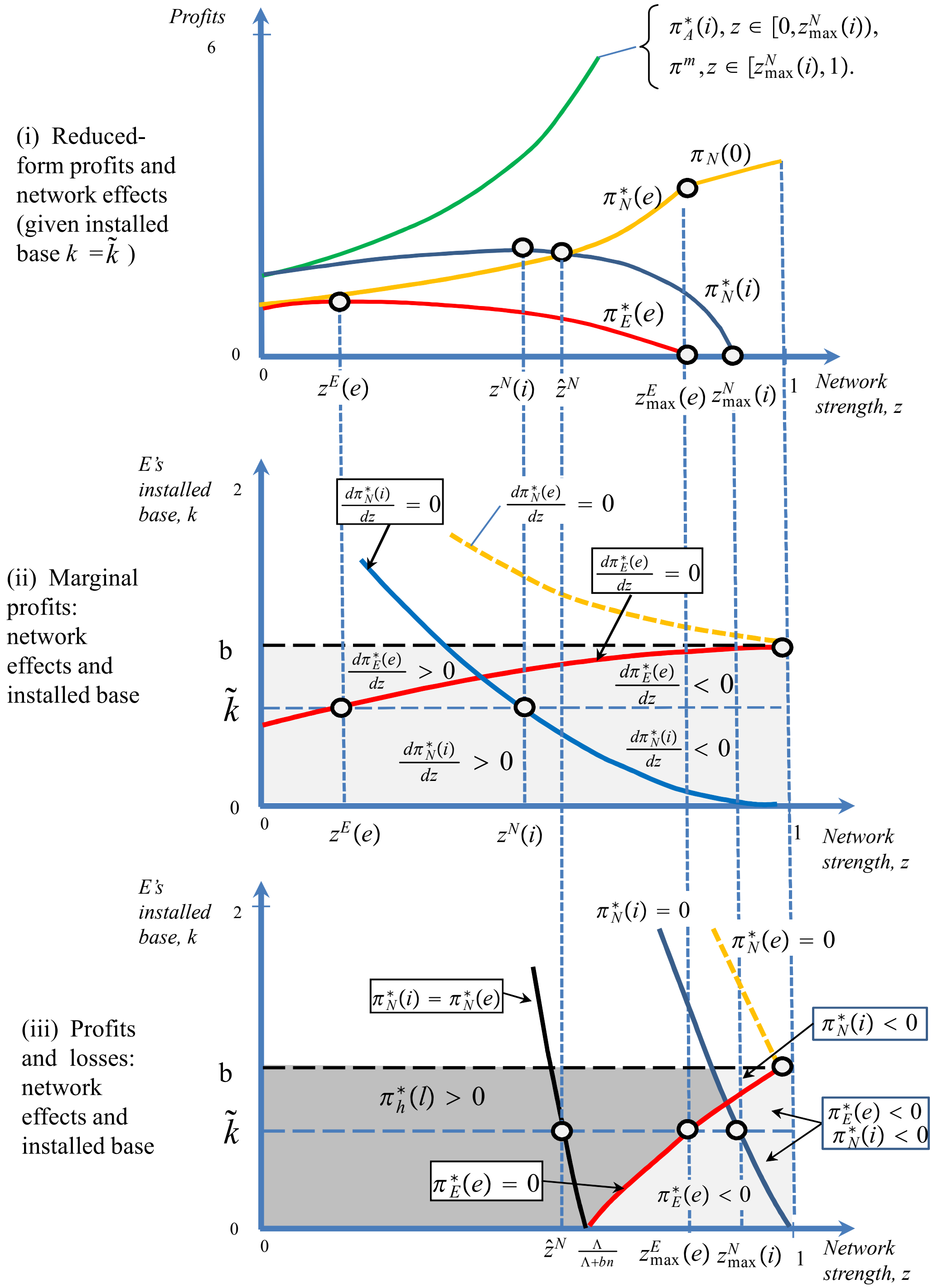

Figure 3: Illustrating reduced profit functions, $\pi_h^*(l)$. Parameter values at $n = 3, b = 1$, $\Lambda = T - c = 5$. In panel (i), $\tilde{k} = 0.6$.

2.1.5 Profits under commercialization by entry

How, then, do firms profit react to network effects under commercialization by entry? We have the following Lemma.

Lemma 4 *Assume that the entrepreneur has succeeded with the innovation in Stage 1, and that the entrepreneur has entered with the innovation, $l = e$, in Stage 2. Then:*
(i) a non-acquiring incumbent's reduced-form product market profit $\pi_N(e)$ is strictly increasing in network effect, z.
(ii) The entrepreneur's reduced-form product market profit $\pi_E(e)$ is strictly concave in network effects, z, with a unique maximum $z^E(e) \in (0,1)$ and $\pi_E^*(e) = 0$ for $z = z_{\max}^E(e) \in (z^E(e), 1)$.

Proof. See the Appendix. ■

Entry also creates an asymmetric market, since the entrepreneur will hold a smaller network share than its incumbent rivals, $\varphi_E^*(e) < 1/(n+1) < \varphi_N^*(e)$. Since the entrepreneur holds a smaller installed base, $b_E(e) = k < b = b_N(e)$, she is less able attract new consumers who are drawn towards the incumbents' larger installed bases. Due to their larger networks, incumbent rivals will always see their profits increase in network effects, $\frac{d\pi_N^*(e)}{dz} > 0$, as shown in Figure 3(i). When network effects are weak, $z \in [0, z^E(e))$, the asymmetry in installed bases is less important for new consumers from (4) and the profit increases for the entrant, $\frac{d\pi_E^*(e)}{dz} > 0$. At stronger networks effects, $z > z^E(e)$, new consumers attraction to the larger networks of the incumbents implies that the entrepreneur's profit decreases in network effects, $\frac{d\pi_E^*(e)}{dz} < 0$.

Figure 3(ii) reveals that the entry profit will decrease in network effects when the initial user base k is not too high. Intuitively, the larger is the installed base of the entrepreneur the smaller is the disadvantage vis-a-vis incumbent rivals when networks effects increase. Figure 3(iii) then depicts the combination of installed base and network effects at which the market tips to incumbents, $\pi_E^*(e) = 0$ and $\pi_N^*(e) > 0$. This illustrates that entry with new technologies or new products can be difficult in network industries due to incumbent advantages in terms of large installed bases. As shown in Figure 3(iii), in the south east region, the entrepreneur would abstain from entry if she has not sold the innovation in Stage 2. Note finally that there also exists a region with strong network effects where the entrepreneur cannot make profitable entry $\pi_E^*(e) < 0$ and a sale in Stage 2 generates a monopoly for the acquiring incumbent, $\pi_N^*(i) < 0$. The profit of the non-acquirer under entry is always strictly positive, $\pi_E^*(e) > 0$ under Assumption 1.

2.1.6 Non-acquiring incumbents' profits under sale and entry

Let us end this section by comparing the profits of a non-acquiring incumbent under commercialization by sale and entry. The following corollary holds:

Corollary 1 *There exists a $\hat{z}^N \in (0,1)$ such that the profit as non-acquirer is lower under entry $\pi_N^*(e) < \pi_N^*(i)$ for low network effects, $z \in [0, \hat{z}^N)$, whereas the profit as non-acquirer is higher under entry $\pi_N^*(e) > \pi_N^*(i)$ for strong network effects, $z > \hat{z}^N$.*

As shown in Figure 3(i), network effects may imply that the profits for a non-acquirer under a sale may be lower than that under entry, $\pi_N^*(i) < \pi_N^*(i)$. This is surprising since entry increases competition in the market. However, it is intuitive noting that the asymmetry in installed bases under a sale $b_A(i) = b + k > b_N(i) = b$ implies that network effects z acts like a *synergy* for the acquirer: by *adding* the installed base of the entrepreneur k to its incumbent base b, the acquirer obtains a very strong market position as new consumers are attracted to the firm with the largest network. In contrast, since the entrant has a smaller installed base $b_E(e) = k < b = b_N(e)$, incumbents can attract consumers from the entrepreneur when network effects increases. This latter effect mitigates the increased competition from entry.

2.2 Stage 2: the acquisition/entry decision

Having solved for how network effects and the commercialization route affects product market profits, we now turn to the commercialization decision in Stage 2 to characterize the equilibrium ownership structure, acquisition price, and entrepreneurial reward given product market profits (Lemma 5). Whereas Stage 3 closely followed Katz and Shapiro (1985), extended to include installed bases as in Cremer et al. (2000) and Malueg and Schwartz (2006), Stage 2 and Stage 1 closely follow Norbäck et al. (2011) and Norbäck and Persson (2012).

Consider Stage 2 in Figure 1. Given a successful innovation, there is *first* an entry-acquisition game where the entrepreneur can decide either to sell the innovation to one of the incumbents or to keep the innovation. If she decides to keep the innovation she can enter the market at an entry cost $F_E(l)$, or choose not to commercialize. In our analysis we want to examine how entry barriers, emerging from the *combination of network effects and large installed bases of incumbents*, affect the incentive for an entrepreneur to innovate and commercialize new innovations. Without loss of generality, we then assume that succeeding with the innovation also lowers the fixed cost of entry. We normalize the fixed of entry for a successful innovation to zero: $F_E(e) = 0$ and suppose that the fixed cost for the entrepreneur when failing $F_E(0) = F$ is sufficiently high to prevent entry.[6]

The acquisition game, which occurs in the early period in Stage 2, is an auction where n incumbents simultaneously post bids, and the entrepreneur then either accepts or rejects these bids. Each incumbent announces a bid, b_i, for the innovation. $\mathbf{b} = (b_1, ..b_i.., b_n) \in R^n$ is the vector of these bids. Following the announcement of $\mathbf{b}$, the innovation may be sold to one of the incumbents at the bid price, or remain in the ownership of entrepreneur. If more than one bid is accepted, the bidder with the highest bid obtains the innovation. If there is more than one incumbent with such a bid, each obtains the innovation with equal probability. The acquisition auction is solved by finding Nash equilibria in undominated pure strategies. There is a smallest amount ε chosen such that all inequalities are preserved if ε is added or subtracted.

Given the acquisition game, exits can occur in the late period in Stage 2. If the entrepreneur sells, Lemma 3 shows that tipping towards the acquiring incumbent will arise in the product market in Stage 3 when network effects become sufficiently large, $z \in [z^N_{\max}(i), 1)$. Thus, non-acquiring incumbents know that engaging in competition for new consumers in Stage 3 will generate losses and exit the market for new consumers (and only supply locked-in consumers). In the early period of Stage 3, new consumers know that the only supplier will be the acquiring incumbent, so the monopoly equilibrium is consistent with expectations. Likewise, if the entrepreneur rejects the incumbents' offers, Lemma 4 implies that she cannot make a positive profit when network effects are strong, $z \in [z^E_{\max}(i), 1)$.

We can now specify firms' valuations in the acquisition game in the early period of Stage 2.

- v_{ii} in (14) is the value of obtaining the entrepreneur's initial user base k for an incumbent, when otherwise a rival incumbent would obtain it.

$$v_{ii} = \tilde{\pi}_A(i) - \tilde{\pi}_N(i). \tag{14}$$

The first term $\tilde{\pi}_A(i)$ shows the profit when possessing k. The second term $\tilde{\pi}_N(i)$

[6] With $n+1$ firms on the market, we thus assume that $\pi^*_E(0) < F$. Since incumbents have already sunk their entry cost, they have strictly positive profits, $\pi^*_N(0) > 0$. It is tedious but straightforward to relax the assumption of fixed cost savings to have $F_E(l) = F$. This does *not* however change our main results but requires more cases to keep track of as profitable entry might require a sufficiently large base of initial users, k.

shows the profit if a rival incumbent obtains k. From Lemma 3 these profits are:

$$\tilde{\pi}_A(i) = \begin{cases} \pi_A^*(i), \ z \in [0, z_{\max}^N(i)) \\ \pi^m, \ z \in [z_{\max}^N(i), 1) \end{cases} \quad \text{and} \quad \tilde{\pi}_N(i) = \begin{cases} \pi_N^*(i), \ z \in [0, z_{\max}^N(i)) \\ 0, \ z \in [z_{\max}^N(i), 1) \end{cases} \tag{15}$$

- v_{ie} in (16) is the value of obtaining k for an incumbent, when otherwise the entrepreneur would enter the market with k:

$$v_{ie} = \tilde{\pi}_A(i) - \pi_N^*(e). \tag{16}$$

  where $\tilde{\pi}_A(i)$ is given from (15) and $\pi_N^*(e) > 0$ from Assumption 1. The profit for an incumbent of not obtaining k is different in this case, due to the change of identity of the firm that otherwise would possess it.

- v_e in (17) is the value for the entrepreneur of keeping k and entering the market.[7] Using Lemma 4, we obtain:

$$v_e = \begin{cases} \pi_E^*(e), \ z \in [0, z_{\max}^E(i)) \\ 0, \ z \in [z_{\max}^E(i), 1) \end{cases} . \tag{17}$$

We can now proceed to solve for the Equilibrium Ownership Structure (EOS). Since incumbents are symmetric, valuations v_{ii}, v_{ie} and v_e can be ordered in six different ways, as shown in Table 1. These inequalities are useful for solving the model and illustrating the results.

Lemma 5 *Equilibrium ownership l^*, acquisition price S^* and entrepreneurial reward $R_E(l^*)$ are described in Table 1:*

Table 1: The equilibrium ownership structure and the acquisition price.

Inequality:	Definition:	Ownership l^*:	Acquisition price, S^*:	Reward, $R_E(l^*)$:
$I1:$	$v_{ii} > v_{ie} > v_e$	i	v_{ii}	v_{ii}
$I2:$	$v_{ii} > v_e > v_{ie}$	i *or* e	v_{ii}	v_{ii} or v_e
$I3:$	$v_{ie} > v_{ii} > v_e$	i	v_{ii}	v_{ii}
$I4:$	$v_{ie} > v_e > v_{ii}$	i	v_e	v_e
$I5:$	$v_e > v_{ii} > v_{ie}$	e	.	v_e
$I6:$	$v_e > v_{ie} > v_{ii}$	e	.	v_e

[7] Note we assume that $\pi_E^*(i) = 0$, so the entrepreneur cannot enter the market without an innovation (in that case we assume that new consumers would not purchase the good produced by the entrepreneur).

Proof. See the Appendix. ∎

Lemma 5 shows that when one of the inequalities $I1$, $I3$, or $I4$ holds, k is obtained by one of the incumbents. Under $I1$ and $I3$, the acquiring incumbent pays the acquisition price $S = v_{ii}$, and $S = v_e$ under $I4$. When I_5 or I_6 holds, the entrepreneur retains its assets. When $I2$ holds, there exist multiple equilibria. The last column summarizes the reward R_E accruing to the entrepreneur.

2.3 Stage 1: innovation by the entrepreneurial firm

In Stage 1, the entrepreneur decides on innovation intensity given the reward determined in Stage 2. The entrepreneur undertakes effort to discover the innovation by selecting the probability $\rho \in [0, 1]$ of discovering the innovation. If she succeeds her innovation will attract an initial user base of k consumers (which are locked-in in Stage 3, where firms compete for new consumers). Let the effort cost $y(\rho)$ be an increasingly increasing function of the success probability: $y'(\rho) > 0$ and $y''(\rho) > 0$. The expected net profit of undertaking effort to discover an innovation is thus $\bar{\Pi}_E = \rho R_E(l) - y(\rho)$. The optimal success probability as a function of the reward $\rho^*(R_E)$ is implicitly given from the first-order condition

$$\frac{d\bar{\Pi}_E}{d\rho} = R_E(l) - y'(\rho^*(l)) = 0, \tag{18}$$

with the associated second-order condition (omitting the ownership variable l) equal to $\frac{d_E^2\bar{\Pi}}{d\rho^2} = -y''(\rho) < 0$. We have the following Lemma.

Lemma 6 *The equilibrium probability of successfully innovating in Stage 1 increases with the reward:* $d\rho^*(l)^*/dR_E(l) > 0$.

This Lemma, obtained by using the implicit function theorem, simply states that the entrepreneur's innovation incentives (the optimal success probability) is increasing in the reward to innovation determined in the commercialization stage (Stage 2).

3 Comparative statics

Having set up and solved the model, we now perform comparative statics with respect to network effects z. We will show that increases in network effects promotes acquisitions over entry (Proposition 1), increases the acquisition premium (Proposition 2), and thereby promotes innovation incentives (Proposition 3). We also show how stronger network effects give incentives to entrepreneurs to invest in attaining a larger mass of initial users (Proposition 4).

3.1 Effects on commercialization route

We start with showing that network effects promote acquisitions over entry. From Lemma 5, under $I3$, $I4$, or $I6$ commercialization by sale occurs as a unique equilibrium if and only if $v_{ie} - v_e > 0$. To explore the commercialization pattern and how this relates to network effects z, the following Lemma is useful:

Lemma 7 *There exists a $z^{ED} \in (0,1)$ such that $v_{ie}(z^{ED}) = v_e(z^{ED})$ and a $z^{PE} \in (0,1)$ such that $v_{ie}(z^{PE}) = v_e(z^{PE})$*

The existence of a $z^{ED} \in (0,1)$ such that $v_{ie}(z^{ED}) = v_e(z^{ED})$ and a $z^{PE} \in (0,1)$ such that $v_{ii}(z^{PE}) = v_e(z^{PE})$ follows directly from the proof of Proposition 1. For illustrational purposes, we then make the following assumption, which is relaxed below.

Assumption 2 *z^{ED} and z^{PE} are unique with $z^{ED} < z^{PE} \leq z^E_{\max}(e)$ and it holds that $z^E_{\max}(e) < z^N_{\max}(i)$.*

The final part implies that tipping towards the acquiring incumbent does not occur, $\pi^*_N(i) > 0$. Using Lemma 7 and Assumption 2, we then have the following Proposition relating the commercialization decision of a successful innovation to network effects.

Proposition 1 *An increase in network effects z makes an acquisition more likely. Under Assumption 2:*
(i) Entry takes place if $z \in [0, z^{ED})$,
(ii) an entry deterring acquisition at price $S^* = v_e$ takes place for $z \in [z^{ED}, z^{PE})$, and
(iii) a preemptive acquisition at price $S^* = v_{ii}$ occurs for $z \in [z^{PE}, z^E_{\max}(e))$.

Proof. See Appendix. ■

The proof of Proposition 1 is illustrated in Figure 4. First note that Lemma 4 implies the reservation price v_e is strictly concave in network effects z. v_e increases for weak network effects but then decreases when network effects are strong as consumers are increasingly attracted to the incumbents' larger installed bases of locked-in consumers:

$$v'_{e,z} = \begin{cases} \frac{d\pi^*_E(e)}{dz} > 0, \ z \in [0, z^E(e)) \\ \frac{d\pi^*_E(e)}{dz} < 0, \ z \in (z^E, z^E_{\max}(e)) \end{cases}, \tag{19}$$

where we use the short-hand v'_z for the derivative, dv/dz.

The inverse U-shape of v_e is shown in Figure 4(ii), where we maintain the parameter values from Figure 3. As shown in Figure 4(i), we also note there will exist a network

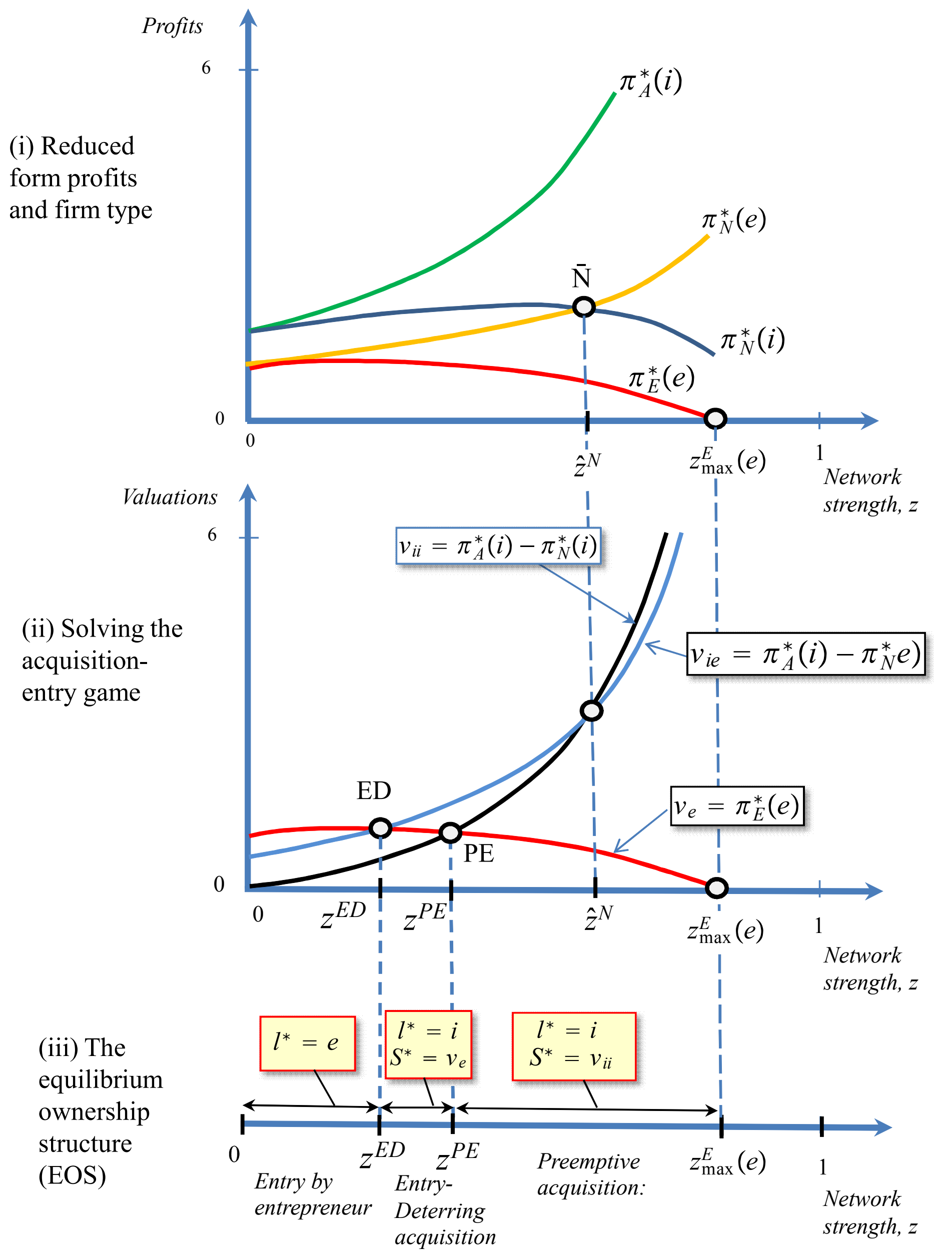

Figure 4: Deriving the equilibrium ownership structure. Parameter values at $n = 3, b = 1, \Lambda = T - c = 5$ and $k = 0.6$.

effect such that the entrepreneur no longer makes a positive profit, i.e. $v_e = \pi_E^*(e) = 0$ for $z = z_{\max}^E(e) < 1$.

Then, turn to incumbents' valuations v_{il}. Starting with the *preemptive* valuation in (14), we have

$$v'_{ii,z} = \begin{cases} \underset{(+)}{\frac{d\pi_A^*(i)}{dz}} - \underset{(+)}{\frac{d\pi_N^*(i)}{dz}} > 0, \text{ for } z \in [0, z^N(i)) \\ \underset{(+)}{\frac{d\pi_A^*(i)}{dz}} - \underset{(-)}{\frac{d\pi_N^*(i)}{dz}} > 0, \text{ for } z \in (z^N(i), z_{\max}^E(e)) \end{cases}. \tag{20}$$

For $z > z^N(i)$, Lemma 3 implies that incumbents' preemptive willingness to pay for k increases in network effects z from the incentive to obtain a strong position as acquirer (as consumers are attracted to the larger network), $\frac{d\pi_A^*(i)}{dz} > 0$, but also from the incentive to avoid a weak position as non-acquirer (as consumers shy away from a smaller network), $\frac{d\pi_N^*(i)}{dz} < 0$. In the Appendix where we provide the proof of Proposition 1, we also show that an incumbent's preemptive valuation v_{ii} is strictly increasing in network effects z for weaker network effects, $z \in [0, z^N(i))$, as the increase in profits for the acquirer is larger than the increase in profits as non-acquirer. Thus, an incumbent's preemptive valuation is strictly increasing in networks effects $\frac{dv_{ii}}{dz} > 0$, as shown in Figure 4(ii). But then since the reservation price v_e is decreasing in network effects for strong network effects, the preemptive valuation must exceed the reservation price $v_{ii} > v_e$ at sufficiently strong network effects. As shown in the Appendix where we provide the proof of Proposition 1, the entry deterring valuation will also be strictly positive for strong network effects, i.e. $v_{ie} = \pi_A^*(i) - \pi_N^*(e) > 0$ will hold when z approaches $z_E^{\max}(e)$. Intuitively, while the entrepreneur's output decreases in network effects benefitting symmetric incumbents under entry, network effects create an even larger advantage for the acquiring incumbent since this firm holds the largest installed base.

From Corollary 1, we know that the preemptive valuation v_{ii} will exceed the entry-deterring valuation v_{ie} when network effects become sufficiently strong, $z > \hat{z}^N$. Again, the reason is that the ownership of the initial user base of the entrepreneur k creates demand side synergy with the acquirer's installed base b through consumers' willingness to pay for a larger network. As the network effects increases, the competitive situation will be worse for a non-acquirer under an acquisition than under entry, despite the fact that entry increases the number of firms. As illustrated in Figure 4(ii), Assumption 2 then implies that $v_{ii} > v_{ie} > v_e$ holds when networks effects are very strong, $z > z^{PE}$. But then since $v_{ie} > v_e$, incumbents always have an incentive to bid v_e for k. However, if an incumbent bids v_e, rivals always have an incentive to bid more since $v_{ii} > v_e$.

Indeed, as incumbents desire to have the largest network of locked-in consumers in order to attract new consumers and steal consumers from rivals, the bidding competition will drive the acquisition price all the way to the preemptive valuation v_{ii}. Thus, as shown in Figure 4(iii), for $z \in [z^{PE}, z^{E}_{\max}(e))$ a *preemptive acquisition* occurs ($l^* = i$) at the acquisition price $S^* = v_{ii}$ (inequality I1 in Table 1).

Then, turn to the opposite case when network effect are weak–or even absent. When $z = 0$, Equation (4) implies that installed bases do not affect consumers willingness to pay and all firms are symmetric. From (14) it then follows that the preemptive valuation is zero, $v_{ii} = 0$. In the Appendix where we provide the proof of Proposition 1, we also show that $v_e > v_{ie} > 0$ holds at $z = 0$. Incumbents then have an incentive to acquire the entrepreneurial firm to prevent entry, but without synergies from network effects, an acquisition is not profitable (the latter results is known from Salant et al. (1983)). Assumption 2(i) then implies that there must exist a region $z \in [0, z^{ED})$ where $v_e > v_{ie} > v_{ii}$ holds. Thus, as illustrated in Figures 4 (ii) and (iii), the entrepreneur will then enter the market in this region (inequality I6 in Table 1).

Finally, under Assumption 2(ii) there will also exist a region $z \in [z^{ED}, z^{PE})$ where $v_{ie} > v_e > v_{ii}$ holds. An *entry-deterring acquisition* at the acquisition price $S^* = v_e$ now occurs at $z = z^{ED}$ since $v_{ie} > v_e$. Other incumbents will not preempt a rival acquisition since the net value of preemption is negative, $v_{ii} - v_e < 0$. Thus, as shown in Figure 4(iii), the entrepreneurial firm will be acquired ($l^* = i$) at the price $S^* = v_e$ in the region $z \in [z^{ED}, z^{PE})$ (inequality I4 in Table 1).

3.1.1 A note on generalization

While Assumption 2 is useful for explaining the mechanisms through which stronger network effects promote commercialization by sale under bidding competition, it is not always fulfilled. When relaxing Assumption 2, we have the following Corollary.

Corollary 2 *Let $z_A, z_B \in \{z^{ED}, z^{PE}\}$. Then,*
(i) stronger network effect lead to commercialization by sale: there exists a $z_A \in (0, 1)$ such that $z > z_A$ implies $l^* = i$.
(ii) when network effects become sufficiently strong, commercialization by sale under bidding competition emerge: there exists a $z_B \in (0, 1)$ such that $z > z_B \geq z_A$ implies $l^* = i$ and $S^* = v_{ii}$.

Proof. See the Appendix. ■

First, note that if $z^{PE} < \hat{z}^N$ holds, the inequality $v_{ii} > v_e > v_{ie}$ may arise for medium sized network effects. We then see that under this inequality I2 in Table 1 multiple equilibria arise: If incumbents believe that a rival will acquire the installed base k, they will try to preempt a rival acquisition; however, if they believe that entry instead will occur they will not bid in order to prevent entry. Note, however, that Lemmas 1-3 will still imply that inequality I1 in Table 1, $v_{ii} > v_{ie} > v_e$, will hold as network effects become very strong. Indeed, in Figure 4(iii), in the region where entry is not profitable ($\pi^*_N(e) < 0$) and tipping occurs to an incumbent acquirer ($\pi^*_N(i) < 0$), valuations fulfill $v_{ii} = v_{ie} = \pi^m > v_e = 0$.

Second, it can also be shown that when Assumption 2(i) is relaxed, the region with an acquisition without bidding competition at the reservation price $S^* = v_e$ can disappear, as inequality I4 $v_{ie} > v_e > v_{ii}$ may vanish at medium sized network effects.

In sum, even if ambiguities may arise for medium sized network effects when Assumption 2 is relaxed, it will still be true that entry occurs at low network effects and commercialization by sale under bidding competition occurs at strong network effects. To keep the presentation simple, we will therefore use Assumption 2 in the remainder of the paper.

3.2 Effects on the acquisition price

Given that an acquisition takes place, how do network effects affect the equilibrium acquisition price and premium? Define the acquisition premium as the acquisition price net of the reservation price, $S^* - v_e$. We then have the following proposition.

Proposition 2 *The acquisition premium increases in network effects: under Assumption 2 and $z^{PE} > z_E(e)$ we have $d(v_{ii} - v_e)/dz > 0$.*

As shown in (19), we know that when network effects become sufficiently strong, a further increase in network effects worsens the competitive situation for the entrepreneur in the market (reducing v_e). Again, the reason is that new consumers find it more attractive to belong to the incumbents larger networks originating from the latter firms larger installed base, $b_N(e) = b > b_E(e) = k$.

On the other hand, when one incumbent obtains the entrepreneur's installed base and adds it to its own installed base, the acquirer holds the largest installed base, $b_A(i) = b+k > b_N(i) = b$. Incumbents realize that consumers will be drawn towards the acquiring incumbent's larger network while shedding a non-acquiring incumbent's smaller network. As shown in (20), v_{ii} increases as these opposing forces are amplified under stronger network effects, and incumbents perceive an improved competitive situation as acquirer and a deteriorated competitive position as non-acquirer. As illustrated in Figure 4(iii), when network effects become sufficiently strong, the preemptive valuation $v_{ii} = \pi_A^*(i) - \pi_N^*(i)$ increases strongly in z, while the entrepreneur's reservations price $v_e = \pi_E^*(e)$ decreases in z, increasing the acquisition premium $v_{ii} - v_e$.

To sum up, Proposition 2 gives an intuition for why acquisition prices can be high in network industries: acquiring an innovation gives an incumbent a larger lead over rivals at the same time as preventing a rival from acquiring the innovation.

3.3 Effects on innovation incentives

We now turn to the main message of the paper: the option of selling out to an incumbent increases innovation incentives for entrepreneurs, in particular when network effects are strong and incumbents preemptively outbid each other. From Lemma 4(ii), strong network effects dampen or even eliminate the entrepreneur's incentive to innovate under entry.

Lemma 8 *When innovating for entry, innovation incentives will decrease in network effects $d\rho^*(e)/dz < 0$ when network effect become sufficiently large, $z > z^E(e)$ and are absent $\rho^*(e) = 0$ when $z > z^E_{\max}(e)$.*

That network effects create barriers to entry has previously been noted in the literature (see, for example, the survey by Farrell and Klemperer (2007)). However, our model suggests while innovation incentives for entry are reduced by strong network effects, network effects may significantly boost the entrepreneur's incentive to innovate when commercialization takes place through a sale. We have the following proposition concerning the entrepreneurial firm's innovation incentives under sale.

Proposition 3 *Assume that Assumption 2 holds.*
(i) Under preemptive acquisitions ($z \in [z^{PE}, z^{\max})$), entrepreneurial firms face stronger innovation incentives than under entry-deterring acquisitions or under entry: $\rho^*(i) > \rho^*(e)$.
(ii) When network effects become sufficiently large $z > z^E(e)$, network effects have a positive effect on innovation incentives under sale but a negative effect under entry: $d\rho^*(i)/dz > 0 < d\rho^*(e)/dz$.

To see this, note that Lemma 6 showed that the research effort of the entrepreneur and hence the probability of success $\rho^*(l)$ increases in the reward $R_E(l)$. Figure 5 (iii) depicts the reward $R_E(l)$ as a function of the network effect z. The equilibrium reward $R_E(l)$ is S-shaped. When the network effect is low $z \in (0, z^{ED})$, entry will take place

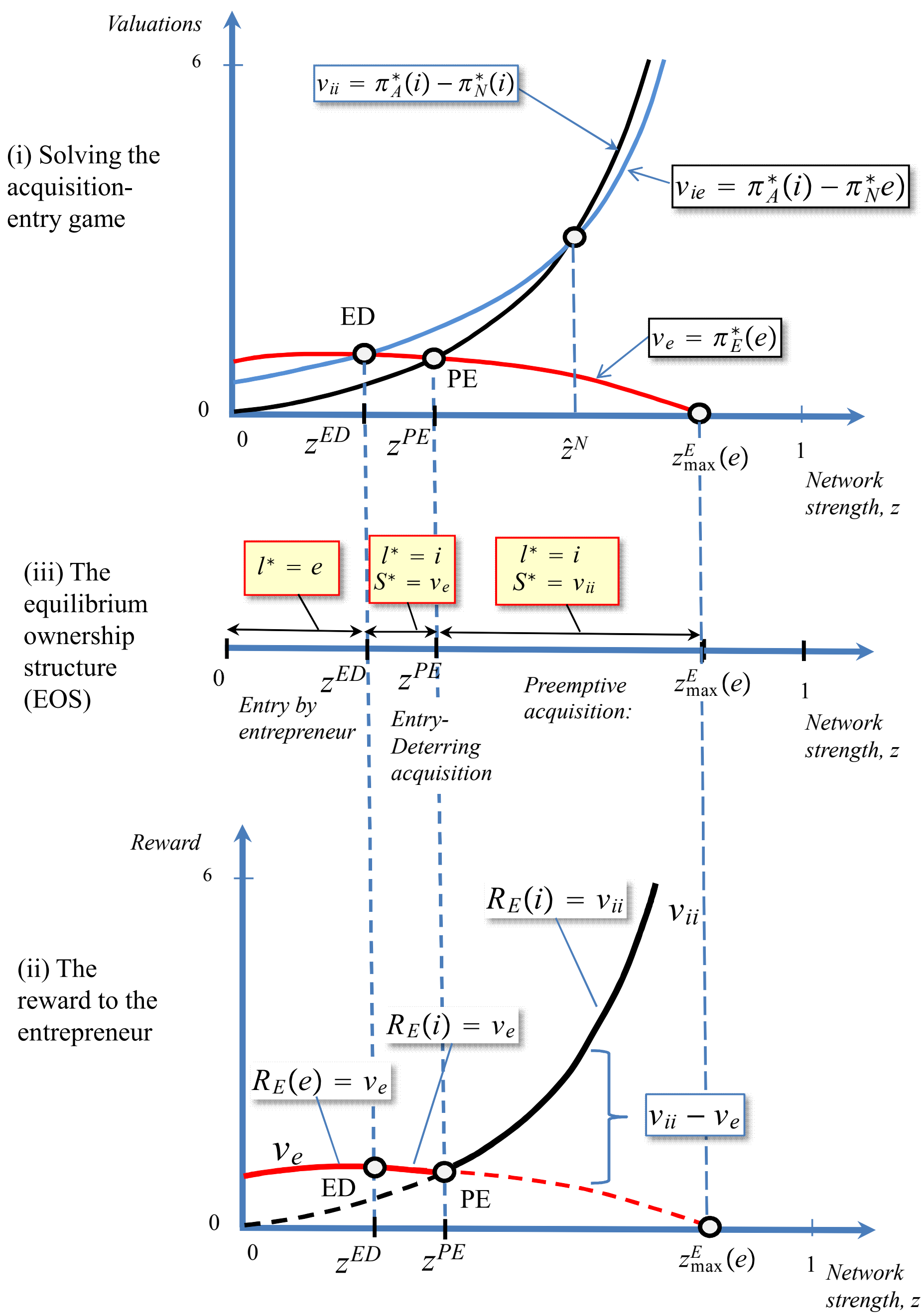

Figure 5: Deriving the reward to innovation. Parameter values at $n = 3, b = 1, \Lambda = T - c = 5$ and $k = 0.6$.

and the reward is $R_E(e) = v_e = \pi_E^*(e)$. This is also the reward if an entry deterring acquisition occurs in region $z \in [z^{ED}, z^{PE})$ since $R_E(i) = S^* = v_e$. From Lemma 4 we know that v_e increases for weak network effects but then decreases when network effects become sufficiently large. As illustrated in Figure 5(iii), the incentive to innovate (and hence the probability to succeed) is inversely U-shaped in the network effect, mirroring the fact that when the network effect becomes sufficiently strong consumers are drawn to the incumbents larger networks, reducing the entry profit. However, when network effects are further increased, $z \in [z^{PE}, z^{\max})$, a preemptive acquisition occurs. In this region, bidding competition among incumbents for the entrepreneurial firm causes the reward for innovation to be strictly greater than the reward for innovation under entry or an entry-deterring acquisition: $R_E(i) = v_{ii} > v_e = R_E(e)$. Since the probability of success $\rho^*(l)$ is increasing in the reward $R_E(l)$, it directly follows from Equation (20) that there will be a higher probability of success if there is bidding competition for the entrepreneurial firm and a preemptive acquisition occurs, $\rho^*(i) > \rho^*(e)$. As illustrated in Figure 5 (iii), strong network effects create bidding competition, where the reward under a sale $R_E(i) = v_{ii}$ increases strongly in z while the reward to entry $R_E(e) = v_e$ decreases in network effects.

3.4 Investing in initial user base

Finally, let us show that the option of selling to an incumbent will not only increase innovation incentives, it will also give incentives to the entrepreneur to invest in attaining a larger initial base of users, Lemma 8 shows that strong network effects in combination with large installed bases of incumbents hinder entrepreneurial entry and, by that, reduce the incentive for an entrepreneur to innovate. This result, however, assumes that the size of the initial installed base created is given. The entrepreneur could be more successful if she could improve the innovation. For instance, by actively investing into gathering a larger following of initial users (after learning that the innovation has succeeded in Stage 1), the entrepreneur might be able to attract a sufficient number of new consumers in Stage 3 to ensure profitable entry in Stage 2. Another way would be to offer the product for free initially aiming to cash out by initiating preemptive bidding competition in Stage 2 among incumbents for the initial installed base k.

This argument is illustrated in panel (i) of Figure 6, which depicts the reward to the entrepreneur from a successful project on vertical axis, where the "floor" of diagram is spanned by the network effect, z, and the size of the initial user base, k. When network effects are very strong the entrepreneur cannot compete with the installed bases of the incumbents unless she has a sufficiently large installed base of her own, k. However, investing into a larger initial user base k, would enable the entrepreneur to enter at a profit. This can also be seen from (13) and (9) which give $v'_{e,k} = \frac{d\pi^*_E(e)}{dk} > 0$.

Increasing the initial base of users will however also increase the innovation's value for the incumbents. A larger k makes an incumbent acquirer a stronger competitor to its rivals: combining an even larger installed base of locked-in consumers with the incumbents own installed base, makes it easier to attract more new consumers and steal from rivals. Likewise, entry from an entrepreneur equipped with a larger installed base makes it harder to attract new consumers. From (9) and (13), it is easy to show that $\frac{d\pi^*_A(i)}{dk} > 0 > \frac{d\pi^*_N(l)}{dk}$.

Since non-acquiring incumbents see their profits decrease when an rival incumbent– or the entrepreneur–obtains a larger installed base, incumbent valuations will increase faster in the size of the initial user base k than the entrepreneur's reservation price. As proved in the Appendix:

$$v'_{il,k} - v'_{e,k} = \underset{(+)}{\frac{d\pi^*_A(i)}{dk}} - \underset{(+)}{\frac{d\pi^*_E(e)}{dk}} - \underset{(-)}{\frac{d\pi^*_N(l)}{dk}} > 0,\ l = \{i, e\} \tag{21}$$

Since incumbents valuations increase more than the entry profit in k, the entrepreneur will invest more in initial user base under a sale under bidding competition than under entry. To see this, let $R_E(l) - C(k)$ be the *net* reward for a successful innovation. From the solution to the commercialization decision in Stage 2 in Lemma 5 and Table 1, it follows that:

$$R_E(l) - C(k) = \begin{cases} v_e - C(k), \text{ under I5 or I6} \\ v_e - C(k), \text{ under I4} \\ v_{ii} - C(k), \text{ under I1, I2 or I3.} \end{cases} \tag{22}$$

Assume that the cost function $C(k)$ is strictly convex, with with $C(0) = 0$ and $C'(k) > 0$. Suppose further that $C(k)$ sufficiently convex to have $R_E(l) - C(k)$ strictly concave, $R''_{E,kk}(l) - C''(k) < 0$ with $k_E^{opt} = \arg\max_k[R_E(l) - C(k)] < b$ maintaining Assumption 1. If the entrepreneur enters or sells without bidding competition in Stage 2, the optimal initial user base k^*_E is given from

$$v'_{e,k} = C'(k^*_E). \tag{23}$$

If the entrepreneur would sell under bidding competition between incumbents in Stage 2, the optimal initial user base k^*_S is given from

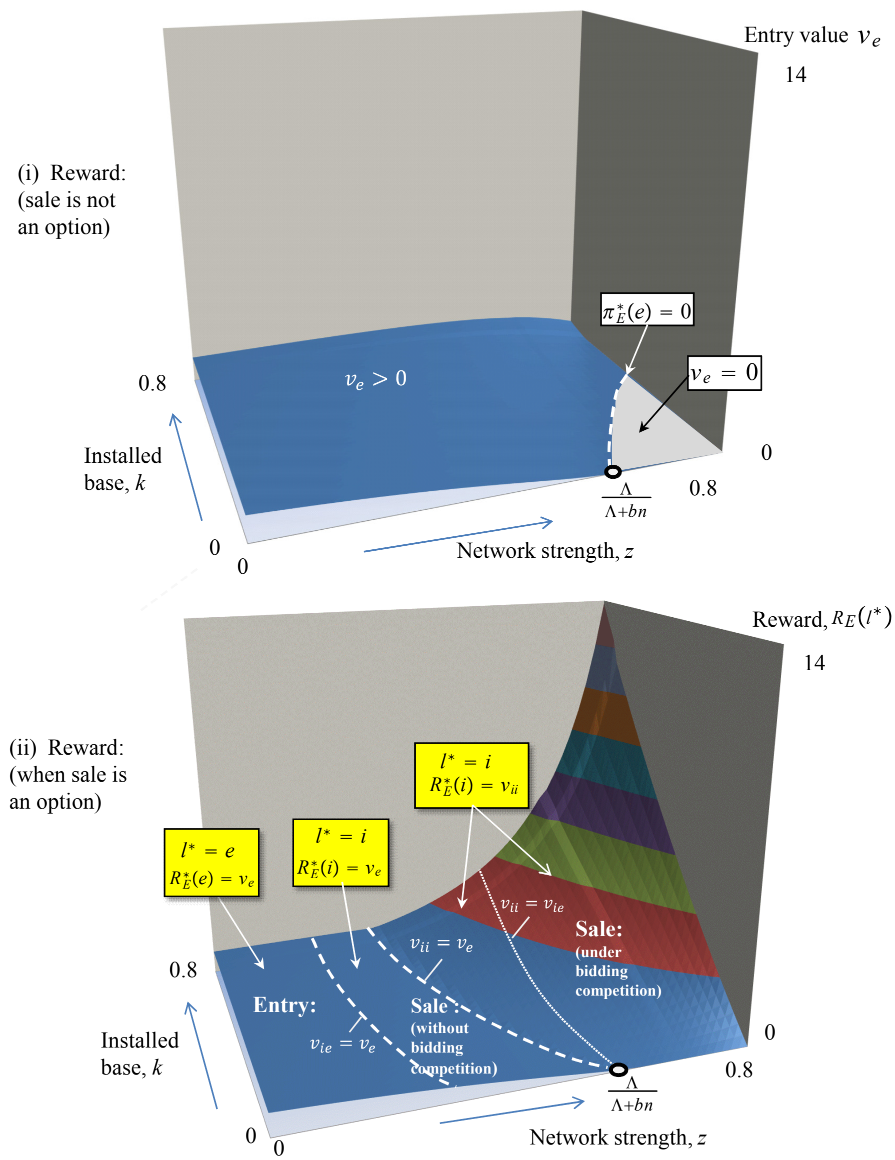

Figure 6: Illustrating reduced the reward to innovation, $R_E^*(l)$. Parameter values at $n = 3, b = 1,\ \Lambda = T - c = 5$.

$$v'_{ii,k} = C'(k_S^*). \tag{24}$$

From (21), (23) and (24), we have the following result.

Lemma 9 *For $z > 0$,*
(i) optimal initial user base is larger under sale than entry: $k_S^* > k_E^*$.
(ii) optimal initial user base is increasing in network effects under sale: $dk_S^*/dz > 0$.
(iii) optimal initial user base is increasing in network effects under entry, $dk_E^*/dz > 0$, when network effects are not too strong. They are decreasing, $dk_E^*/dz < 0$, when network effects are strong (z approaches $z_E^{\max}$).

Proof. See the Appendix. ■

Part (i) follows directly from Equation (21), since when selling under bidding competition the entrepreneur can also exploit the negative effect on a non-acquiring incumbent from a larger installed base ($\frac{d\pi_N^*(i)}{dk} < 0$). Panel (i) in Figure 7 illustrates the optimal k under entry and sale under bidding competition as functions of the network effect, $k_E^*(z)$ and $k_S^*(z)$, where $k_S^*(z) > k_E^*(z)$. Panel (i) also illustrates part (ii), where the investment will increase in network effects under a sale with bidding competition. As shown in the appendix, this occurs since the marginal revenue for investing in k under bidding competition increases in network effects, $v''_{ii,kz} > 0$. Intuitively, stronger network effects increase the reward for the acquirer of getting a larger network as consumers are willing to pay more for a large network: at the same time, this effect makes it even harder for non-acquiring incumbents to attract new consumers. In the Appendix, we also show that marginal revenue of investing in k under entry will decrease in network effects when network effects are strong, that is, $v''_{e,kz} < 0$ when z approaches $z_E^{\max}(e)$. The strong competition from the larger installed bases of the incumbents forces the entrepreneur to choose a smaller k.

How, then, does the optimal commercialization pattern and optimal investment in k interact when network effects increase? We now show that strong network effects induce the entrepreneur to invest in k in order to exploit the rivalry among incumbents over obtaining the largest network. We have the following proposition:

Proposition 4 *Suppose that the investment cost $C(k)$ is sufficiently convex and that the installed base of the incumbents b is sufficiently large. Then, when the network effect z becomes sufficiently strong,*
(i) the entrepreneur will invest into k to be able to sell the innovation under bidding competition,

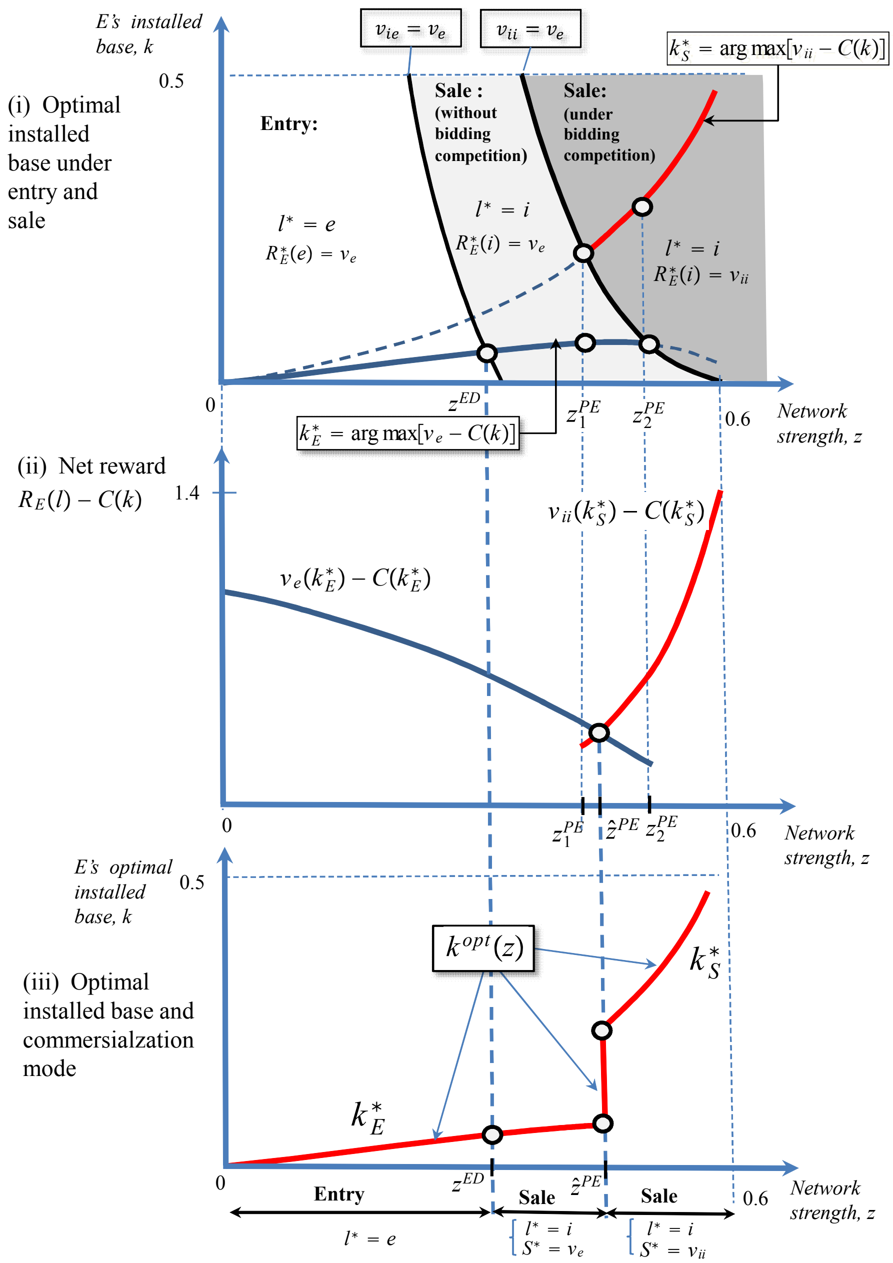

Figure 7: Illustrating endogenous investment in k and commersialzation, $R_E^*(l)$. Parameter values at $n = 3, b = 1$, $\Lambda = T - c = 5$.

(ii) the installed base sold to the acquiring incumbent will be strictly larger than the installed base she would have employed herself under entry.

The proof is illustrated in Figure 7. First, note that when the commercialization mode is decided in Stage 2 k is fixed. Figure 7(i) then shows the combination of initial user base k and network effect z such that the entry-deterring valuation equals the reservation price, $v_{ie} = v_e$, which we label the *ED-condition*. Figure 7(i) also shows the combination of z and k such that the preemptive valuation equals the reservation price, $v_{ii} = v_e$, which we label the *PE-condition*.

From Assumption 2 and from the inequality (21), the ED- and PE condition are downward-sloping. It follows directly from Proposition 1 that the entrepreneur will commercialize by entry ($l^* = e$) receiving the reward $R_E(e) = v_e$ to the left of the ED-condition, whereas she will sell k without bidding competition ($l^* = i$) receiving the reward $R_E(i) = v_e$ in the middle region between the ED and the PE conditions. Finally, she will sell k under bidding competition ($l^* = i$) with reward $R_E(i) = v_{ii}$ to the right of the PE condition.

In Stage 1, (23) implies that the entrepreneur will choose k_E^* when network effects are small, $z \in (0, z^{ED})$. This will also be her choice in the region of medium network effects, $z \in [z^{ED}, z_1^{PE})$, since the reward is still v_e under sale without bidding competition between incumbents. When a sale under bidding competition arises under strong network effects, $z \in [z_2^{PE}, 1)$, the entrepreneur optimally chooses k_S^* from (24). An ambiguity however arises in the region $z \in (z_1^{PE}, z_2^{PE})$. Here, the Stage 2 auction produces either a sale without bidding competition at price $S^* = v_e$ at the lower k_E^*, or a sale at $S^* = v_{ii}$ when the entrepreneur invests into $k_S^* > k_E^*$ in Stage 1 to induce bidding competition.

To sort out which alternative is chosen, panel (ii) of Figure 7 depicts the optimal net reward under entry and a sale under bidding competition, $v_e(k_E^*) - C(k_E^*)$ and $v_{ii}(k_S^*) - C(k_S^*)$ respectively. First, note that $v_{ii}(k_S^*) - C(k_S^*) < v_e(k_E^*) - C(k_E^*)$ holds at $z = z_1^{PE}$.[8] Second, by using the envelope theorem, we note that

$$\frac{d[v_{ii}(k_S^*) - C(k_S^*)]}{dz} = \frac{\partial v_{ii}(k_S^*)}{\partial z} \tag{25}$$

and

$$\frac{d[v_e(k_E^*) - C(k_E^*)]}{dz} = \frac{\partial v_e(k_E^*)}{\partial z}. \tag{26}$$

[8] To see this, note that $v_{ii}(k_S^*) - C(k_S^*) - [v_e(k_E^*) - C(k_E^*)]$ can be rewritten as $v_{ii}(k_S^*) - v_e(k_S^*) - C(k_S^*) - [v_e(k_E^*) - v_e(k_S^*) - C(k_E^*)]$. At z_1^{PE}, however, $v_{ii}(k_S^*) = v_e(k_S^*)$, so we have at z_1^{PE} $v_{ii}(k_S^*) - C(k_S^*) - [v_e(k_E^*) - C(k_E^*)] = v_e(k_S^*) - C(k_S^*) - [v_e(k_E^*) - C(k_E^*)] < 0$ since $k_E^* = \arg\max_k [v_e - C(k)]$.

From Equation (20), the net reward from a sale under bidding competition $v_{ii}(k_S^*) - C(k_S^*)$ must then unambiguously increase in network effects at $z = z_1^{PE}$. When new consumers attach a larger value to belonging to a large network, the value for an incumbent increases both from the incentive to attain the largest network and the incentive to prevent rivals from achieving this. From Equation (19), we also know that the net reward to entry $v_e(k_E^*) - C(k_E^*)$ will decrease in network effects when network effects are sufficiently strong as consumers are attracted to the incumbents larger installed bases. As we show in the Appendix, if the installed base of the incumbents b is sufficiently large and the investment cost function $C(k)$ is sufficiently convex, the net reward to entry $v_e(k_E^*) - C(k_E^*)$ will unambiguously decrease in network effects. The initial user base k_E^* will then be too small as compared to the incumbents' installed bases b and new consumers are increasingly drawn to the incumbents' networks.[9] The net reward under entry and sale without bidding competition $v_e(k_E^*) - C(k_E^*)$ will thus fall in network effects z at $z = z_1^{PE}$. As shown in panel (ii), there will then exist a unique $\hat{z}^{PE} \in (z_1^{PE} z_1^{PE})$ at which the entrepreneur will choose the higher initial user base to induce a sale under bidding competition.

Panel (iii) of Figure 7 finally summarizes the commercialization decision and the investment decision in k. Thus, when network effects are weak, the entrepreneur enters the market and invests in k to maximize the net entry profit, $k_E^{opt} = k_E^* = \arg\max_k [v_e(k) - C(k)]$. However, when network effects increase the entrepreneur internalizes the fact that she can induce bidding competition between incumbents by aggressively investing into a larger k, $k_E^{opt} = k_S^* = \arg\max_k [v_{ii}(k) - C(k)]$. This is shown in Figure 7(iii), where we see the discrete jump in k, $k_S^* > k_E^*$, at $z = \hat{z}^{PE}$.

4 Extensions

This section provides a few extensions to our model. First, we show that the model can easily be recast as the innovation reducing marginal cost of the entrant, rather than generating an initial mass of users. Second, we show how larger installed bases of incumbents, given the size of the network effect, in fact themselves act to increase the acquisition price under preemptive bidding competition. Third, we point out that with strong network effects, consumers are better off when the innovation is commercialized

[9] This is illustrated in Figure 7(ii), where the locus combination of k and z at which $\frac{\partial \pi_E(e)}{dz} = 0$ is above the origin. Comparing with the optimal installed base under entry in Figure 7(i), where $k_E^* = 0$ when network effects are absent $z = 0$ and where $k_E^*(z)$ will be of limited size if investment is sufficiently costly, the locus $k(z)|_{d\pi_E(e)/dz=0}$ in Figure 3 will always be above the locus $k_E^*(z)$ in Figure 7.

through a sale to an incumbent.

4.1 Innovations reducing marginal cost

An alternative set up is that the innovation leads to a reduction of the marginal cost and the entrepreneur is unable to create an installed base of locked-in consumers. Then only incumbents have installed bases, say from historical sales. In such a setting;

$$b_h(l) = \begin{cases} b_A(i) = b, \\ b_N(l) = b, \ l = \{i, e, 0\}, \\ b_E(e) = 0. \end{cases} c_h(l) = \begin{cases} c_A(i) = c - k, \\ c_N(l) = c, \ l = \{i, e, 0\}, \\ c_E(e) = c - k. \end{cases} \tag{27}$$

Then, it is straightforward to show that the change in the profit of a firm j when network effects increase is:

$$\frac{d\pi_j(l)}{dz} = 2\left(\frac{Q^*(l) + B(l)}{1-z}\right)\left(\varphi_j^*(l) - \frac{1}{n(l)} + \frac{1-z}{n(l)[n(l)+1-z]}\right) q_j^*(l). \tag{28}$$

where we now have $B(l) = nb$ in (11).

Note that the same trade-offs still apply. In particular, it might be that even though the entrepreneur creates an innovation that reduces its marginal cost, this may not help in the competition for new consumers if less efficient incumbents hold large installed bases. Unless the innovation brings a very large reduction in marginal cost, so that the incumbents' advantages in terms of their larger installed bases are swept away, Lemma 4 still applies and the profit under entry decrease in network effect when network effects become large. Thus, innovations incentives under entry will then decrease in network effects as suggested by Lemma 8. However, as shown in Proposition 2, when selling the innovation the entrepreneur can benefit as the innovation creates an asymmetry in the market: at a lower marginal cost new consumers will infer that the acquiring incumbent will have the largest network. As suggested by Propositions 2 and 3, when network effects increase, this will increase the bidding competition and increase premium from selling which, in turn, increases the incentive to innovate. Figure 8 illustrates also how all results extend to a setting with marginal cost reductions, where we again use the same parameter value as in previous sections.

4.2 Incumbents' installed bases and innovation incentives

Larger installed bases of incumbents, given the size of the network effects, in fact themselves act to increase the acquisition price under preemptive bidding competition. This

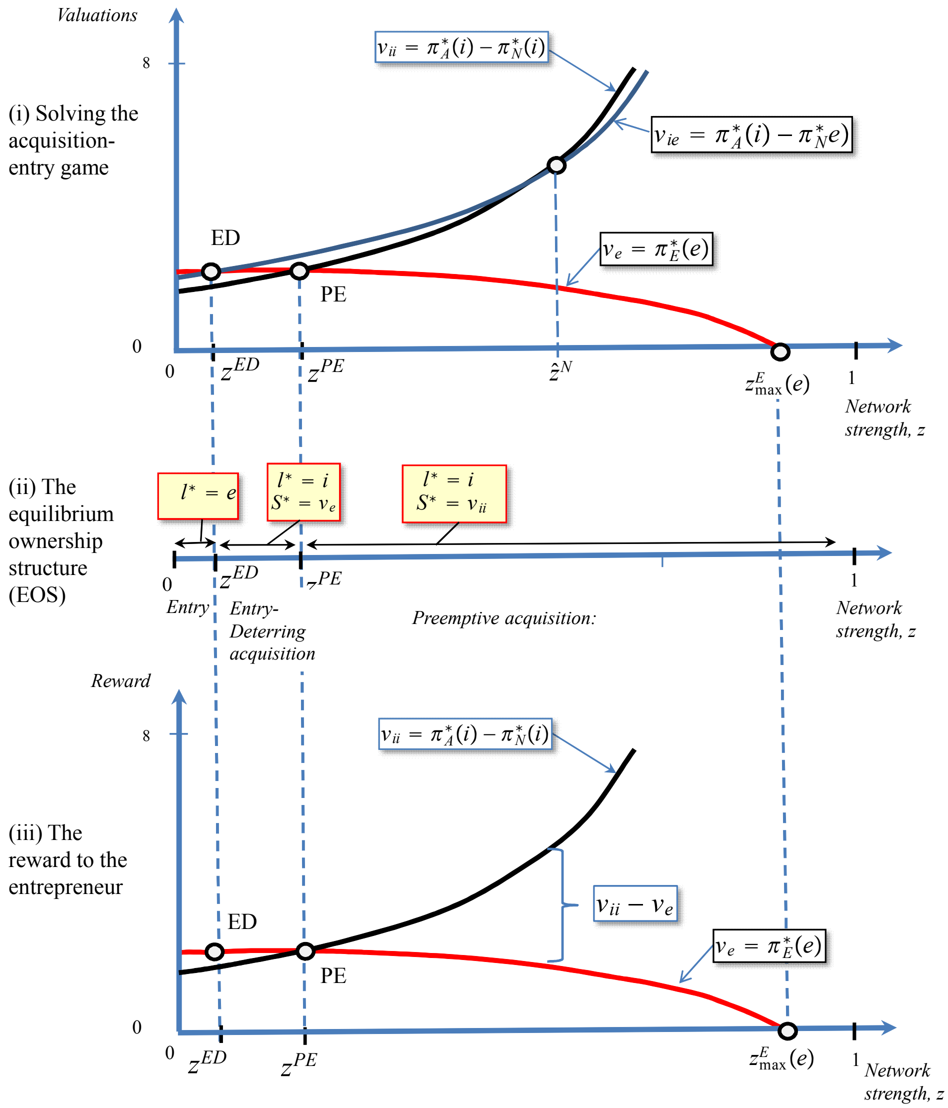

Figure 8: Illustrating commersialization equilibrium mode and innovation incentives when innovations reduce marginal costs. Parameter values at $n = 3, b = 1, \Lambda = T - c = 5$ and $k = 0.6$.

is shown in the following proposition.

Proposition 5 *The entrepreneur's innovation incentives decreases in the size of incumbents installed bases under entry, $v'_{e,b} < 0$, but increases in incumbents' installed bases under sale with bidding competition, $v'_{ii,b} > 0$.*

Larger installed bases of the incumbents will amplify the asymmetry both under entry and under sale. In the former this increase in asymmetry harms the smaller entrant as consumers are drawn to the larger incumbent networks. Under a sale, the entrepreneur can instead benefit from that rivalry over having the largest network between increases among incumbents.

4.3 Consumer welfare

With strong network effects, consumers are better of when the innovation is commercialized through a sale to an incumbent than if the entrepreneur would have entered the market. From Katz and Shapiro (1985), the consumer surplus is:

$$CS(l) = [Q^*(l)]^2 /2 \tag{29}$$

Corollary 1 directly implies the following result:

Proposition 6 *Consumer gain from commercialization by sale: (i) $CS(i) > CS(0)$, (ii) If $z > \hat{z}^N$, $CS(i) > CS(e)$*

This results follow directly from (9) and (29), since $CS(i) > CS(0)$ implies that the profit of a non-acquirer under sale must be lower than the profit of a non-acquirer under entry. In turn, this implies that total output must be higher under a sale than under entry, $Q^*(i) > Q^*(e)$. Hence, under the asymmetric market structure when an incumbent acquires the installed base if the entrepreneur the acquirer's is so aggressive in attracting new consumers that, new consumers will be better off from a sale if $z > \hat{z}^N$. Making use of (9), (14) and (16), we note that $Q^*(i) > Q^*(e)$ is fulfilled whenever the preemptive valuation exceeds the entry-deterring valuation, $v_{ii} > v_{ie}$. Figure 6(ii) then illustrates consumers are better off under commercialization by sale when network effects are sufficiently strong. This proposition illustrates that avoiding entry barriers created by installed bases through selling the innovation does not harm consumer welfare.

5 Concluding remarks

In this paper, we have studied how innovation incentives in industries with installed bases and network effects are affected by bidding competition between incumbents for new entrepreneurial firms. We showed how the option of selling out to an incumbent increases innovation incentives for entrepreneurs when network effects are strong and incumbents compete to preemptively acquire innovations. Hence, network effects and installed bases do not necessarily restrict innovation incentives. We also showed that network effects promote acquisitions over entry and that entrepreneur's have incentives to invest in acquiring a large mass of initial users.

The model gives rise to empirically testable predictions. Both the ratio of acquisitions to entry in network industries and the total innovation output (e.g. patents) by potential innovative entrants should be higher the stronger are network effects. Testing these predictions is a fruitful avenue for further research. The model could also be extended in several directions. Though not trivial and certainly cumbersome, one useful extension would be to study compatibility decisions of incumbents, entrants, and governments. Another extension would be to allow incumbents to innovate. Using data on Belgian manufacturing firms, Cassiman and Veugelers (2006) provide econometric evidence consistent with complementarity between acquiring inventions and internal R&D. Escribano et al. (2009) uses data on Spanish firms to show that this absorptive capacity is an important source of competitive advantage.

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A Appendix

A.1 Proof of Lemma 3

To show the properties of the profit functions, we make use of the quadratic profits, $\frac{d\pi_h^*(l)}{dz} = 2\frac{dq_h^*(l)}{dz}q_h^*(l)$. Since $q_h^*(l) > 0$, it follows that

$$sign\left[\frac{d\pi_h^*(l)}{dz}\right] = sign\left[\frac{dq_h^*(l)}{dz}\right] \tag{30}$$

A.1.1 (i) Profit of the acquirer: $\pi_A^*(i)$

Let us start with the acquirer's profits, $\pi_A^*(i)$. To simplify notation, write $\frac{dq_h^*(l)}{dz} = q_{h,z}^*(l)$. Solve for k as a function of z such that $q_{A,z}^*(i) = 0$ and denote this $k(z)|_{q_{A,z}^*(i)=0}$. Then, by calculation

$$k(z)|_{q_{A,z}^*(i)=0} = -\frac{(1-z)^2(b+\Lambda+bn)}{n-2z-2nz+n^2+2z^2} < 0 \tag{31}$$

Note that (30) and Lemma 2 implies that $q_{A,z}^*(i) > 0$ for $z = 0$. Then, since $k \in [0, b)$, $q_{A,z}^*(i) > 0$ for $z \in [0, 1)$. Hence, from (30) we must have $\frac{d\pi_A^*(i)}{dz} > 0$ for $z \in [0, 1)$. Thus, the acquiring incumbent's reduced-form product market profit $\pi_A^*(i)$ is strictly increasing in network effects, z.

A.1.2 (ii) Profit of a non-acquirer: $\pi_N^*(i)$

Then, turn to the profit of a non-acquirer, $\pi_N^*(i)$. This is illustrated in Figure 9(i). Solving $k(z)|_{q_{N,z}^*(i)=0}$, we obtain:

$$k(z)|_{q_{N,z}^*(i)=0} = (1-z)^2\frac{b+\Lambda+bn}{n+1-z^2} \geq 0 \tag{32}$$

Note that $k'(z)|_{q_{N,z}^*(i)=0} = -2(1-z)(n-z+1)\frac{b+\Lambda+bn}{(n-z^2+1)^2} < 0$. Also, note that we have $k(0)|_{q_{N,z}^*(i)=0} = b + \frac{\Lambda}{n+1}$ and that $lim_{z\to 1}\left[k(z)|_{q_{N,z}^*(i)=0}\right] = 0$. Thus, as shown in Figure 9(i), for any $k \in (0, b)$, there will exist a unique $z_N(i) = k^{-1}(z)\big|_{q_{N,z}^*(i)=0} \in (0, 1)$ such that $q_{N,z}^*(i) > 0$ for $z \in [0, z_N(i))$, $q_{N,z}^*(i) = 0$ for $z = z_N(i)$ and $q_{N,z}^*(i) < 0$ for $z \in (z_N(i), 1)$. Again, from (30), it then follows that $\pi_N^*(i)$ is strictly concave in network effect, z, with a unique maximum $z_N(i) \in (0, 1)$.

In Figure 9(i), we also illustrate the line $k(z)|_{q_N^*(i)=0}$, that is, the combination of k and z at which tipping occurs and where only the acquiring incumbent becomes a

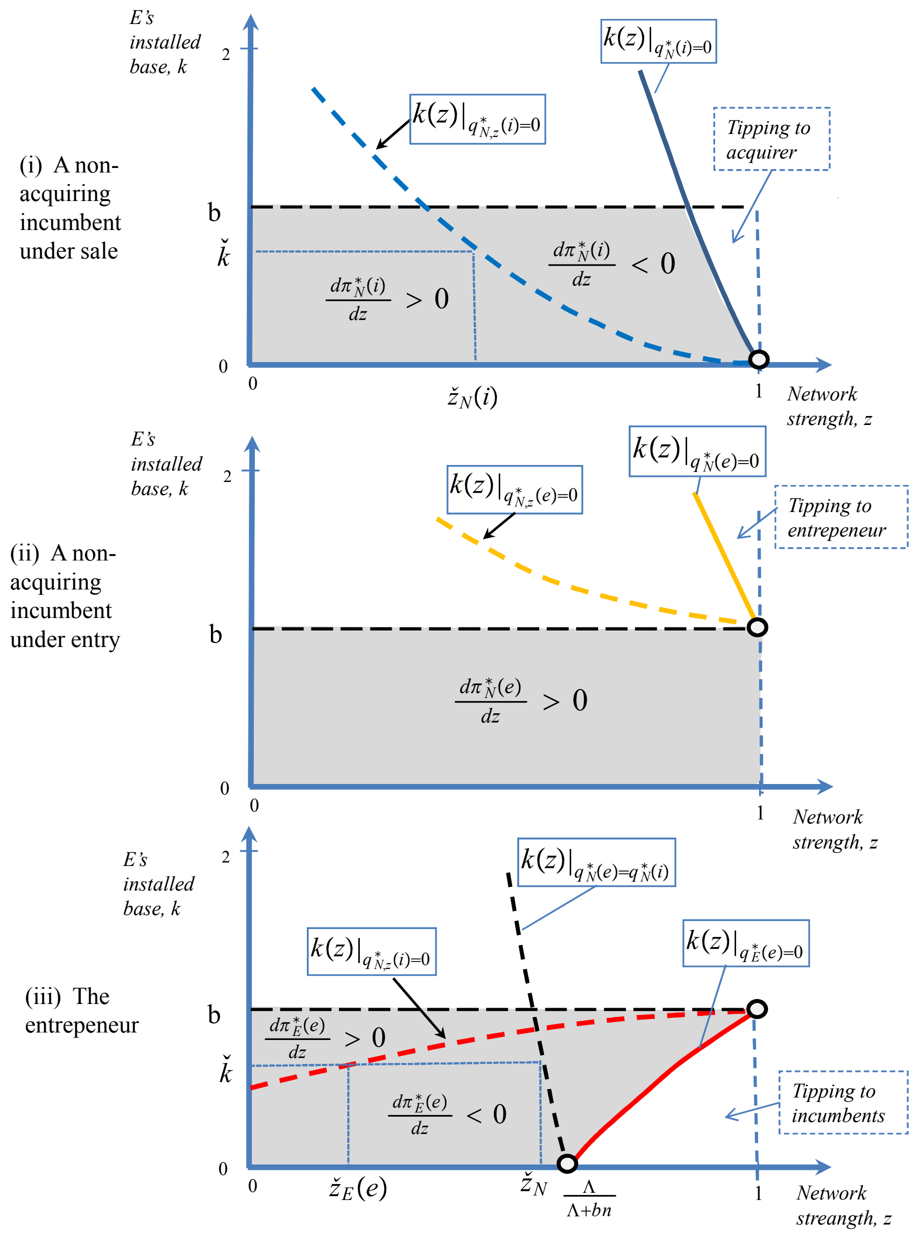

Figure 9: Illustrating the properties of profits , $\pi^*_h(l)$. Parameter values at $n = 3, b = 1$, $\Lambda = T - c = 5$.

monopolist. By calculation:

$$k(z)|_{q_N^*(i)=0} = (1-z)\,\frac{\Lambda + bz}{z}, \tag{33}$$

where $k'(z)|_{q_{N,}^*(i)=0} = -\frac{\Lambda+bz^2}{z^2} < 0$, $lim_{z\to 0}\left[k(z)|_{q_N^*(i)=0}\right] = \infty$ and $lim_{z\to 1}\left[k(z)|_{q_N^*(i)=0}\right] = 0$. Then, tipping to the acquiring incumbent occurs for $z > z_N^{\max}(i) = k^{-1}(z)\big|_{q_N^*(i)=0}$. Thus, to the right of the locus $k(z)|_{q_N^*(i)=0}$ in Figure 9(i) non-acquiring incumbents will exit, where

$$k(z)|_{q_{N,z}^*(i)=0} - k(z)|_{q_N^*(i)=0} = -(1-z)\left(\Lambda + bz^2\right)\frac{n-z+1}{z\left(n-z^2+1\right)} < 0. \tag{34}$$

A.2 Proof of Lemma 4

A.2.1 (i) Profit of a non-acquirer: $\pi_N^*(e)$

Consider the profit of a non-acquirer under entry, $\pi_N^*(e)$. This is illustrated in Figure 9(ii). Solving for $k(z)|_{q_{N,z}^*(e)=0}$, we obtain:

$$k(z)|_{q_{N,z}^*(e)=0} = \frac{\left(z^2-2z+1\right)\Lambda + b\left(4+z^2-4z+2n+nz^2-2nz\right)}{n-z^2+2} > 0 \tag{35}$$

since $n \geq 2$ and $z \in [0,1)$.

Note that $k'(z)|_{q_{N,z}^*(e)=0} = -2(1-z)(2b+\Lambda+bn)\frac{n-z+2}{(n-z^2+2)^2} < 0$, and that $k(0)|_{q_{N,z}^*(i)=0} = 2b+\frac{\Lambda}{n+2}$ and $lim_{z\to 1}\left[k(z)|_{q_{N,z}^*(i)=0}\right] = b$. Note also that Lemma 2 implies that $q_{N,z}^*(e) > 0$ for $z=0$. Thus, as shown in Figure 9(ii), since $k \in [0,b)$ from Assumption 1, we must have $q_{N,z}^*(e) > 0$ for $z \in [0,1)$. Hence, from (30), $\frac{d\pi_N^*(e)}{dz} > 0$ for $z \in [0,1)$. Thus, a non-acquiring incumbent's reduced-form product market profit $\pi_N^*(e)$ is strictly increasing in network effects, z.

In Figure 9(i), we also illustrate the line $k(z)|_{q_N^*(e)=0}$, that is, the combination of k and z at which tipping occurs and where only the entrepreneur remains. By calculation:

$$k(z)|_{q_N^*(e)=0} = \frac{\Lambda - z\Lambda - bz^2 + 2bz}{z} \tag{36}$$

where $k'(z)|_{q_{N,}^*(e)=0} = -\frac{\Lambda+bz^2}{z^2} < 0$, $lim_{z\to 0}\left[k(z)|_{q_N^*(e)=0}\right] = \infty$ and $lim_{z\to 1}\left[k(z)|_{q_N^*(e)=0}\right] = b$. Thus, since $k \in [0,b)$, $q_N^*(e) > 0$, and hence $\pi_N^*(e) > 0$. Due to the larger installed base of incumbents, tipping in terms of incumbents being driven out by the entrepreneur can never occur under Assumption 1.

A.2.2 (i) Profit of the entrepreneur: $\pi_E^*(e)$

Now, turn to the profit of the entrepreneur, $\pi_E(e)$. Solving $k(z)|_{q_{E,z}^*(e)=0}$, we obtain:

$$k(z)|_{q_{E,z}^*(e)=0} = \frac{-\Lambda + 2z\Lambda + bn^2 - z^2\Lambda + 2bn - bnz^2}{3n - 4z - 2nz + n^2 + 2z^2 + 2} \tag{37}$$

By calculation, we have $k'(z)|_{q_{E,z}^*(e)=0} = 2n\frac{(1-z)(2b+\Lambda+bn)(n-z+2)}{(3n-4z-2nz+n^2+2z^2+2)^2} > 0$, and $k(0)|_{q_{E,z}^*(e)=0} = \frac{-\Lambda+bn(n+2)}{(n+2)(n+1)}$. Note also that finally, $lim_{z\to 1}\left[k(z)|_{q_{E,z}^*(e)=0}\right] = b$. There will then exist a unique $z_E(e) = k^{-1}(z)|_{q_{E,z}^*(e)=0} \in (0,1)$ such that $q_{E,z}^*(e) > 0$ for $z \in [0, z_E(e))$, $q_{E,z}^*(e) = 0$ for $z = z_E(e)$ and $q_{E,z}^*(e) < 0$ for $z \in (z_E(e), 1)$. From (30), it then follows that $\pi_E^*(e)$ is strictly concave in network effect, z, with a unique maximum $z_E(e) \in (0,1)$.

In Figure 9(i), we also illustrate the line $k(z)|_{q_E^*(e)=0}$, that is, the combination of k and z at which tipping towards incumbents occurs and where only the incumbent would remain (so the entrepreneur would not enter). By calculation:

$$k(z)|_{q_E^*(e)=0} = \frac{(z-1)\,\Lambda + bnz}{z\,(n-z+1)} \tag{38}$$

We then have $k'(z)|_{q_E^*(e)=0} = \frac{\Lambda+n\Lambda-2z\Lambda+z^2\Lambda+bnz^2}{z^2(n-z+1)^2} > 0$, $lim_{z\to 0}\left[k(z)|_{q_E^*(e)=0}\right] = -\infty$ and also $lim_{z\to 1}\left[k(z)|_{q_E^*(e)=0}\right] = b$.

Then, tipping to incumbents occurs for $z > z_E^{\max}(e) = k^{-1}(z)_{q_E^*(e)=0}$. Note also that $k(z)|_{q_E^*(e)=0} = 0$ for $z = \frac{\Lambda}{\Lambda+bn} \in (0,1)$. Thus, to the right of the locus upward-sloping $k(z)|_{q_N^*(i)=0}$ in Figure 9(iii), the entrepreneur will never enter the market. Note, finally, that

$$k(z)|_{q_{N,z}^*(i)=0} - k(z)|_{q_E^*(e)=0} = \frac{(1-z)(n-z+2)\left(\Lambda+n\Lambda-2z\Lambda+z^2\Lambda+bnz^2\right)}{z(n-z+1)(3n-4z-2nz+n^2+2z^2+2)} > 0. \tag{39}$$

A.3 Proof of Corollary 1

Solving for the combination of k and z at which non-acquirer's profits are the same under entry and sale $k(z)|_{q_N^*(e)=q_N^*(i)}$, we obtain:

$$k(z)|_{q_N^*(e)=q_N^*(i)} = \frac{\Lambda(1-z) - bnz}{z} \tag{40}$$

By calculation, $k'(z)|_{q_N^*(e)=q_N^*(i)} = -\frac{\Lambda}{z^2} < 0$ and $lim_{z\to 0}\left[k(z)|_{q_N^*(e)=q_N^*(i)}\right] = \infty$. Note also that $lim_{z\to 1}\left[k(z)|_{q_N^*(e)=q_N^*(i)}\right] = -bn < 0$. Thus, as shown in Figure 9(iii) there will exist a unique $\hat{z}_N \in (0,1)$ such that $q_N^*(e) < q_N^*(i)$ for $z \in [0, \hat{z}_N)$, $q_N^*(e) = q_N^*(i)$ for

$z = \hat{z}_N$ and $q_N^*(e) > q_N^*(i)$ for $z \in (\hat{z}_N, 1)$.

Note finally that $k(z)|_{q_N^*(e)=q_N^*(e)} = 0$ for $z = \frac{\Lambda}{\Lambda+bn} \in (0,1)$, that is, we have $k(z)|_{q_N^*(e)=q_N^*(e)} = 0 = k(z)|_{q_E^*(e)=0}$ for $z = \frac{\Lambda}{\Lambda+bn}$.

A.4 Proof of Lemma 5

Note that $b_i \geq \max v_{ij}$, $, j = \{e,i\}$, is a weakly dominated strategy, since no firm will post a bid equal to or above its maximum valuation of obtaining the entrepreneurial firm. The entrepreneurial firm e will accept a bid iff $b_i \geq v_e$.

Inequality I1 Consider equilibrium candidate $\mathbf{b}^* = (b_1^*, b_2^*, ..., yes)$. Let us assume incumbent w is the incumbent that has posted the highest bid and obtains the assets, and that firm ς is the incumbent with the second highest bid.

Then, $b_w^* \geq v_{ii}$ is a weakly dominated strategy. $b_w^* < v_{ii} - \varepsilon$ is not an equilibrium, since firm $j \neq w, e$ then benefits from deviating to $b_j = b_w^* + \varepsilon$, since it will then obtain the assets and pay a price lower than its valuation of obtaining them. If $b_w^* = v_{ii} - \varepsilon$, and $b_\varsigma^* \in [v_{ii} - 2\varepsilon, v_{ii} - \varepsilon]$, then no incumbent has an incentive to deviate. By deviating to no firm e loses since $b_w^* = v_{ii} - \varepsilon > v_e$. Accordingly, the entrepreneur has no incentive to deviate and thus, $\mathbf{b}^*$ is a Nash equilibrium.

Let $\mathbf{b} = (b_1, ..., b_n, no)$ be a Nash equilibrium. Let incumbent h be the incumbent with the highest bid. The entrepreneur will then say *no* iff $b_h \leq v_e$. But incumbent $j \neq e$ will have the incentive to deviate to $b' = v_e + \varepsilon$ in period 1, since $v_{ie} > v_e$. This contradicts the assumption that $\mathbf{b}$ is a Nash equilibrium.

Inequality I2 Consider equilibrium candidate $\mathbf{b}^* = (b_1^*, b_2^*, ..., yes)$. Let us assume incumbent w is the incumbent that has posted the highest bid and obtains the assets, and that firm ς is the incumbent with the second highest bid. Then, $b_w^* \geq v_{ii}$ is a weakly dominated strategy. $b_w^* < v_{ii} - \varepsilon$ is not an equilibrium since firm $j \neq w, \varsigma, e$ then benefits from deviating to $b_j = b_w^* + \varepsilon$, since it will then obtain the assets and pay a price lower than its valuation of obtaining them. If $b_w^* = v_{ii} - \varepsilon$, and $b_\varsigma^* \in [v_{ii} - 2\varepsilon, v_{ii} - \varepsilon]$, then no incumbent has an incentive to deviate. By deviating to no firm e loses since $b_w^* = v_{ii} - \varepsilon > v_e$. Accordingly, the entrepreneur has no incentive to deviate and thus, $\mathbf{b}^*$ is a Nash equilibrium.

Consider the equilibrium candidate $\mathbf{b}^{**} = (b_1^{**}, b_2^{**}, ..., no)$. Then, $b_w^* \geq v_e$ is not an equilibrium since the entrepreneur would benefit by deviating to *yes*. If $b_w^* \leq v_e$, then no incumbent has an incentive to deviate. By deviating to *yes*, the entrepreneur's payoff decreases since it then sells its assets at a price below its valuation, v_e. The entrepreneur has no incentive to deviate and thus, $\mathbf{b}^{**}$ is a Nash equilibrium.

Inequality I3 Consider equilibrium candidate $\mathbf{b}^* = (b_1^*, b_2^*, ..., yes)$. Let us assume incumbent w is the incumbent that has posted the highest bid and obtains the assets, and that firm ς is the incumbent with the second highest bid. Then, $b_w^* \geq v_{ii}$ is not an equilibrium since firm w would then benefit from deviating. $b_w^* < v_{ii} - \varepsilon$ is not an equilibrium since firm $j \neq w$ then benefits from deviating to $b_j = b_w^* + \varepsilon$, since it will then obtain the assets and pay a price lower than its valuation of obtaining them. If $b_w^* = v_{ii} - \varepsilon$, and $b_\varsigma^* \in [v_{ii} - 2\varepsilon, v_{ii} - \varepsilon]$, then no incumbent has an incentive to deviate. By deviating to no firm e loses since $b_w^* = v_{ii} - \varepsilon > v_e$. Accordingly, the entrepreneur has no incentive to deviate and thus, $\mathbf{b}^*$ is a Nash equilibrium.

Let $b = (b_1, ..., b_n, no)$ be a Nash equilibrium. The entrepreneur will then say *no* iff $b_h < v_e$. But incumbent $j \neq e$ will then have the incentive to deviate to $b' = v_e + \varepsilon$ in Stage 1, since $v_{ie} > v_e$. This contradicts the assumption that $\mathbf{b}$ is a Nash equilibrium.

Inequality I4 Consider equilibrium candidate $b^* = (b_1^*, b_2^*, ..., yes)$. Then, $b_w^* > v_e$ is not an equilibrium since firm w would then benefit from deviating to $b_w = v_e$. $b_w^* < v_e$ is not an equilibrium, since the entrepreneur would then not accept any bid. If $b_w^* = v_e - \varepsilon$, then firm w has no incentive to deviate. By deviating to $b_j' \leq b_w^*$, firm j's, $j \neq w, e$, payoff does not change. By deviating to $b_j' > b_w^*$, firm j's payoff decreases since it must pay a price above its willingness to pay v_{ii}. Accordingly, firm j has no incentive to deviate. By deviating to *no*, the entrepreneur's payoff decreases since it foregoes a selling price above its valuation v_e. Accordingly, the entrepreneur has no incentive to deviate and thus, b^* is a Nash equilibrium.

Let $b = (b_1, ..., b_n, no)$ be a Nash equilibrium. The entrepreneur will then say *no* iff $b_h < v_e$. But incumbent j will have the incentive to deviate to $b' = v_e + \varepsilon$ in Stage 1 since $v_{ie} > v_e$, which contradicts the assumption that b is a Nash equilibrium.

Inequalities I5 or I6 Consider equilibrium candidate $b^* = (b_1^*, b_2^*, ..., no)$, where $b_j^* < v_e$ $\forall j$. It then directly follows that no firm has an incentive to deviate and thus, b^* is a Nash equilibrium.

Then, note that the entrepreneur will accept a bid iff $b_j \geq v_e$. But $b_j \geq v_e$ is a weakly dominating bid in these intervals, since $v_e > \max\{v_{ii}, v_{ie}\}$. Thus, the assets will not be sold in these intervals.

A.5 Proof of Proposition 1

We first examine $v_{ii}(z)$. Note that $v'_{ii,z}(0) = 2k\frac{\Lambda}{n+1} > 0$. Then solve for the combination of k and z at which the derivative of the preemptive valuation in z is zero, $k(z)|_{v_{ii,z}=0}$. We obtain

$$k(z)|_{v'_{ii,z}=0} = (z-1)\,\frac{\left(n-z^2+1\right)\Lambda + bz\left(2-2z+2n-nz\right)}{z\left(-z-2nz+n^2+2z^2-1\right)} < 0. \tag{41}$$

But then $n \geq 2$ and since $k \in [0, b)$ from Assumption 1, we must have $v'_{ii,z} > 0$ for $z \in [0, 1)$. Thus, the acquiring incumbent's preemptive valuation v_{ii} is strictly increasing in network effects, z.

To proceed, we find that

$$v_{ie}(0) = \frac{(2n+3)\Lambda^2}{(n+2)^2(n+1)^2} > 0 = v_{ii}(0), \tag{42}$$

and that

$$v_e(0) - v_{ie}(0) = \Lambda^2\frac{n^2-2}{(n+2)^2(n+1)^2} > 0. \tag{43}$$

Thus, we have shown that, $v_e > v_{ie} > v_{ii}$ at $z = 0$. From Table 1 and inequality I6, there is commercialization by entry in equilibrium $l^* = e$ when network effects is sufficiently weak.

From Proposition 4, $v_e = \pi_E(e)$ is strictly concave in network effects, z, with a unique maximum $z_E(e) \in (0, 1)$. But then since v_{ii} is strictly increasing in network effects, z, $v_{ii} > v_e$ must hold when z becomes sufficiently large, i.e. when it approaches $z_E(e) = k^{-1}(z)\big|_{q^*_{E,z}(e)=0}$.

Also, note that if evaluate v_{ie} at $k(z)|_{q^*_E(e)=0}$we obtain

$$v_{ie}\big|_{k(z)|_{q^*_E(e)=0}} = \frac{(n-z)(-\Lambda(1-z)+bnz)[\left(n(1-z)+2-3z+z^2\right)\Lambda+bz\left(n^2-3nz+2n+2z^2-4z+2\right)]}{(z-1)^2(n-z+1)^4}. \tag{44}$$

To evaluate (44), first note that $-\Lambda + z\Lambda + bnz > 0$, if $z > \frac{\Lambda}{\Lambda+bn}$. Then note that

$$\left(n(1-z)+2-3z+z^2\right)\Lambda + bz\left(n^2-3nz+2n+2z^2-4z+2\right) > 0 \tag{45}$$

since $z \in [0, 1)$ and $n \geq 2$.To proceed, note that (38) implies for any $k \in (0, b)$, we must have $z_E^{\max}(e) = k^{-1}(z)\big|_{q^*_E(e)=0} > \frac{\Lambda}{\Lambda+bn}$. Corollary 1, (14) and (16) then imply that for $z \in (z_N, 1)$, $v_{ii}(z) > v_{ie}(z)$. From (40), we have $k(\frac{\Lambda}{\Lambda+bn})\Big|_{q^*_N(e)=q^*_N(i)} = 0$. Since $k'(z)|_{q^*_N(e)=q^*_N(i)} < 0$, it now follows that $v_{ii} > v_{ie}$ holds at $k(z)|_{q^*_E(e)=0}$ for $k > 0$. This can be seen in Figure 9(iii) where the locus of $k(z)|_{q_N^*(e)=q_N^*(i)} = 0$ is always to the left of the locus $k(z)|_{q_E^*(e)} = 0$ if $k \in (0,b)$. Thus, we have shown that the inequality $v_{ii} > v_{ie} > v_e = 0$ holds at $k(z)|_{q_E^*(e)=0}$. From Table 1 this implies that inequality I1 holds and the innovation is sold at bidding competition, $l^* = i$ and $S^* = v_{ii}$ when network effects become sufficiently strong.

A.6 Proof of Lemma 9

A.6.1 Part (i): Proof of inequality (21) and $k_S^* > k_E^*$

By calculation, we obtain

$$v'_{ii,k} - v'_{e,k} = 2z\frac{\left(2n-5z-2nz+2z^2+3\right)\Lambda+\left(n^3-3n^2z+3n^2+3nz^2-8nz+5n-z^3+5z^2-8z+4\right)zb+(3z-3n-5)zk}{(z-1)^2(n-z+1)(n-z+2)^2} \tag{46}$$

Since $n \geq 2$ and $z \in [0,1)$, $3z-3n-5 < 0$ and $2n-5z-2nz+2z^2+3 > 0$. Then, Assumption 1 implies $\max k = b$, given that $k(z)|_{q_N^*(i)=0} \geq b$. Then, evaluate the two last terms in the numerator of $v_{ii,k} - v_{e,k}$ at $k = b$. This gives:

$$bz\left(2n-5z+3nz^2-3n^2z-8nz+3n^2+n^3+5z^2-z^3-1\right) \geq 0 \tag{47}$$

from $n \geq 2$ and $z \in [0,1)$. Thus, $v'_{ii,k} > v'_{e,k}$ holds for $z \in [0, \frac{\sqrt{4b\Lambda+\Lambda^2}-\Lambda}{2b})$ where $z = \frac{\sqrt{4b\Lambda+\Lambda^2}-\Lambda}{2b} > 0$ for $b > 0$ is the solution to $b = k(z)|_{q_N^*(i)=0}$. From (24) and (23), expression (46) immediately gives $k_S^* > k_E^*$.

Now turn to:

$$\begin{aligned} v'_{ie,k} - v'_{e,k} \quad = \quad & 2z\frac{\left(2n^2-2n^2z+4nz^2-7nz+3n-2z^3+5z^2-3z\right)\Lambda-kz\left(6n-6z-6nz+3n^2+3z^2+2\right)}{(z-1)^2(n-z+1)^2(n-z+2)^2}+ \\ & \frac{\left(9n-9z+16nz^2-14n^2z-4nz^3-4n^3z+6n^2z^2-21nz+9n^2+4n^3+n^4+12z^2-6z^3+z^4+2\right)bz}{(z-1)^2(n-z+1)^2(n-z+2)^2} \end{aligned} \tag{48}$$

where $2n^2-2n^2z+4nz^2-7nz+3n-2z^3+5z^2-3z > 0$ and $6n-6z-6nz+3n^2+3z^2+2 > 0$ since since $n \geq 2$ and $z \in [0,1)$.Then, note that $\max k = b$, given that $k(z)|_{q_N^*(i)=0} \geq b$. Then, evaluate the two last terms in the numerator at $k = b$. This gives

$$bz\,(n-z)\left(6n-9z+3nz^2-3n^2z-10nz+4n^2+n^3+6z^2-z^3+3\right) > 0 \tag{49}$$

since $n \geq 2$ and $z \in [0,1)$.Thus, $v'_{ie,k} > v'_{e,k}$ holds for $z \in [0, \frac{\sqrt{4b\Lambda+\Lambda^2}-\Lambda}{2b})$ where $z = \frac{\sqrt{4b\Lambda+\Lambda^2}-\Lambda}{2b} > 0$ for $b > 0$ is the solution to $b = k(z)|_{q_N^*(i)=0}$.

A.6.2 Part (ii): $\frac{dk_S^*}{dz}$

Totally differentiate (24) in k and z

$$\frac{dk_S^*}{dz} = -\frac{v''_{ii,kz}}{v''_{ii,kk} - C''(k_S^*)} \tag{50}$$

where $v''_{ii,kk} - C''(k_S^*) < 0$ by the second-order condition.

Then, by direct calculation:

$$v''_{ii,kz} = 2\frac{\left(n-z-nz-z^2+z^3+1\right)\Lambda+\left(2z-3nz^2+nz^3+2nz-4z^2+2z^3\right)b+2kz\left(-z-2nz+n^2+2z^2-1\right)}{(1-z)^3(n-z+1)^2} > 0 \tag{51}$$

since $n \geq 2$ and $z \in [0,1)$. Thus, from (50) and (51) it follows that $\frac{dk_S^*}{dz} > 0$.

A.6.3 Part (iii): $\frac{dk_E^*}{dz}$

Then, totally differentiate (23) to get:

$$\frac{dk_E^*}{dz} = -\frac{v''_{e,kz}}{v''_{e,kk} - C''(k_E^*)} \tag{52}$$

where $v''_{e,kk} - C''(k_E^*) < 0$ by the second-order condition. Using that profits are quadratic in output and that $v_e =$ we have

$$v''_{e,kz} = \frac{d^2\pi_E^*(e)}{dkdz} = 2\left[\frac{dq_E^*(e)}{dz}\frac{dq_E^*(e)}{dk} + q_E^*(e)\frac{d^2q_E^*(e)}{dkdz}\right] \tag{53}$$

From (9) and (13), we have $\frac{d\pi_E^*(e)}{dk} > 0$. From Proposition 4 we know that whereas $\frac{dq_E^*(e)}{dz} < 0$ when $z > z^E(e)$ and that $q_E^*(e)$ must go to zero when z approaches $z_{\max}^E(e)$. Therefore when z becomes sufficiently large the term $q_E^*(e)\frac{d^2q_E^*(e)}{dkdz}$ goes to zero, whereas term $\frac{dq_E^*(e)}{dz}\frac{dq_E^*(e)}{dk}$ is negative. Hence, for large network effects, $v''_{e,kz} < 0$ and, from (52) and (53), $\frac{dk_E^*}{dz} < 0$.

By calculation

$$\begin{aligned} v''_{e,kz} \;&=\; 2\frac{\left(n^2z-n^2-nz^3+4nz-3n+z^4-z^3-3z^2+5z-2\right)\Lambda+bnz\left(6n-6z-nz^2-3nz+2n^2+z^2+z^3+4\right)}{(z-1)^3(n-z+2)^3} - \\ &\frac{2kz(n-z+1)\left(3n-4z-2nz+n^2+2z^2+2\right)}{(z-1)^3(n-z+2)^3} \end{aligned} \tag{54}$$

Evaluating at $z = 0$, we obtain $v''_{e,kz}\Big|_{z=0} = 2\frac{\Lambda}{(n+2)^2}(n+1) > 0$. Thus, when network effects are of limited size, it follows from (54) and (52), $\frac{dk_E^*}{dz} > 0$ for small network effects.

A.7 Proof of Proposition 5

By calculation,

$$v'_{ii,b} = 2k \frac{z^2}{(1-z)(n-z+1)} > 0 \tag{55}$$

and

$$v'_{e,b} = -2nz \frac{\Lambda - z\Lambda - kz^2 + kz - bnz + knz}{(z-1)^2 (n-z+2)^2} < 0 \tag{56}$$

since $n \geq 2$ and $z \in [0, 1)$.