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# RESIDUAL-BASED RANK SPECIFICATION TESTS FOR AR-GARCH TYPE MODELS

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# ABSTRACT

## Residual-based Rank Specification Tests for AR-GARCH type models

This paper derives the asymptotic distribution for a number of rank-based and classical residual specification tests in AR-GARCH type models. We consider tests for the null hypotheses of no linear and quadratic serial residual autocorrelation, residual symmetry, and no structural breaks. For these tests we show that, generally, no size correction is needed in the asymptotic test distribution when applied to AR-GARCH type residuals obtained through QMLE estimation. To be precise, we give exact expressions for the limiting null distribution of the test statistics applied to residuals, and find that standard critical values often lead to conservative tests. For this result, we give simple sufficient conditions. Simulations show that our asymptotic approximations work well for a large number of AR-GARCH models and parameter values. We also show that the rank-based tests often, though not always, have superior power properties over the classical tests, even if they are conservative. We thereby provide a useful extension to the econometrician's toolkit. An empirical application illustrates the relevance of these tests to the AR-GARCH models for the weekly stock market return indices of some major and emerging countries.

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# 1 Introduction

Given the large literature on AR-GARCH type models, there has been much interest in specification testing for location-scale time-series models. In this paper we provide a unifying approach to derive the asymptotic distribution of (rank-based) residual specification tests. We focus on the assumptions of independence, symmetry, and stability of innovations in AR-GARCH type models. More precisely, we consider the size-correction needed when applying existing and new rank-based tests for: (1) null hypotheses of no linear and quadratic correlation in the standardized residuals of AR-GARCH type models; (2) null hypothesis of symmetry of the innovations; and (3) the null hypothesis of structural stability. The motivation of our special focus on rank-based statistics for these specifications tests lies in the fact that most applications of AR-GARCH models show non-normal innovation distributions (e.g., when applied to financial asset returns which are often leptokurtic and/or asymmetric or exhibit infinite fourth moment). Rank-based tests deal well with these features and complement recent tests in the literature.

Specification tests for AR-GARCH type models have a long history. We mention a few examples. Li and Mak (1994) propose a test based on the sample autocorrelation of squared residuals under conditional normality. Berkes, Horváth and Kokoszka (2003) extend this result, dropping the conditional normality condition and using mild non-explosiveness assumpions. Tse (2002) also deals with residual-based specification tests for GARCH type models assuming asymptotically efficient estimators. Lundbergh and Teräsvirta (2002) contributed an important approach that unifies the standard LM-type tests for remaining volatility clustering, as implemented by Bollerslev (1986), and the LM tests of Engle and Ng (1993) for volatility asymmetry. Their approach is quite flexible for checking conditional variance specifications. Halunga and Orme (2009) extend the Lundbergh and Teräsvirta approach to take into account conditional mean estimation uncertainty. Our rank tests for linear and quadratic residual autocorrelation are robust to different innovation distributions and account for the estimation uncertainty. The same holds for the rank-based tests for symmetry and the absence of structural breaks. Our tests turn out to have standard limiting distributions and better power for leptokurtic AR-GARCH models compared to, e.g., recently proposed tests in the literature by Kulperger and Yu (2005). The rank-based tests for the null of symmetry of the innovation distribution in AR-GARCH type models are easy to implement and in some cases compare favorably to other recent tests in the literature including those in Bai and Ng (2001) and Lambert et al (2012). Overall we find that our rank residual-based tests complement various tests in the literature, are never oversized and enjoy good power properties especially for leptokurtic error innovations of AR-GARCH models.

The analysis of residual-based tests is generally complicated by its two-step nature. The use of residuals (calculated using *estimated* parameter values) in the test statistics instead of actual innovations may lead to a change in the null limiting distribution and, thereby, to a size-distortion. A well-known example of this phenomenon is the degrees-of-freedom correction when applying the standard Ljung-Box test to residuals of an ARMA-type model. Without this correction, the residual-based Ljung-Box test is undersized. In general, applying a standard test to residuals of some model may lead to both over- and undersizing. Note that in a GMM-type framework, residual-based tests (in the form of a $J$-test for overidentifying restrictions) will almost automatically always have a limiting null distribution that is less spread-out than the innovation-based tests. We, however, consider a (quasi) likelihood framework that is often applied in the context of AR-GARCH models. In this setting, we show that generally a size correction is needed and, when it is, we provide it explicitly. Also, often, but unlike the GMM framework not always, ignoring such a correction leads to a conservative, thus still valid, test. We state simple sufficient conditions for the specifications tests we consider to be conservative when applied to residuals.

In this respect, the present paper has two important theoretical contributions. We give explicit expressions for the limiting null distribution of (rank-based) statistics used in specification testing. We precisely identify situations where the critical values of the tests need not be adjusted, i.e., the two-step nature of the procedure does not lead to a size distortion. Also, in relevant special situations, our results show that the test statistic applied to residuals, but using uncorrected critical values, leads to a conservative test. Thus, in applied work, such tests can be used without adjustment. Our second contribution is that, for the rank-based tests we consider, this conservativeness does not come at the cost of low power. That is, the power of the rank-based tests applied to residuals still makes these tests a competitor for more classical ones. Extensive simulations in Section 4 confirm these asymptotic claims for finite samples.

For our theoretical results, we rely on the Hájek-Le Cam framework of Locally Asymptotically Normal (LAN) experiments, following more abstract results in Andreou and Werker (2012). This framework, discussed, e.g., in Bickel, Klaassen, Ritov and Wellner (1993), Le Cam and Yang (1990), Pollard (2004), and van der Vaart (1998), provides high-level assumptions under which likelihood based inference procedures lead to Gaussian limiting distributions. This method is especially suited for deriving the asymptotic distribution of rank-based statistics since it does not require (asymptotic) smoothness conditions on the statistic of interest with respect to the nuisance parameters.[1] The method also applies to general model specifications, as long as they satisfy the LAN condition. For instance, IGARCH models are not ruled out. For instance, they would also be valid for multivariate GARCH type models which are also LAN and for which our rank-based tests can be readily extended thereby addressing one of the open research questions in such models mentioned in Bawens et al. (2006). As such, the limiting distributions need not be derived case-by-case in a model specific way.[2]

The rest of the paper is organized as follows. In Section 2 we formally introduce the GARCH type models we consider, including the regularity conditions needed. Essentially, these regularity conditions induce the model to be Locally Asymptotically Normal (LAN) so that existing results on asymptotic statistics can be invoked immediately. We also introduce the conditions needed to derive the limiting distribution of a statistic when applied to some model's residuals, where we restrict attention to the case the model is estimated using Gaussian QMLE which is still the most commonly used estimator for GARCH models. This leads to our main theoretical result in Theorem 1 and several propositions for special cases. Section 3 derives the asymptotic distribution of three broad categories of specification tests when applied to standardized residuals, namely tests for (1) temporal dependence, (2) symmetry, and (3) stability. We use our main theorem to derive the corrections to critical values needed to obtain tests with an exact asymptotic size, and indicate when precisely ignoring such a correction leads to a conservative test. In Section 4 we present a comprehensive simulation study corroborating our theoretical results and showing that the rank-based tests considered generally have strong power properties compared to more classical tests widely applied in the literature. Section 5 provides an illustration of our results for modelling some of the major and emerging stock market returns indices. Section 6 concludes the paper.

# 2 Model and theory

As explained in the introduction, we rely on Andreou and Werker (2012) for the formal analysis of our residual-based specification tests in AR-GARCH models. That

[1] Recent developments in the econometrics literature that involve ranks are found, for instance, in transformation models (e.g., Cavanagh and Sherman (1998) and Sherman (1993)), in two-step rank regression (Honoré and Hu (2004)), for robust testing in linear models (Rothenberg and Thompson (2003)), in variance-ratio tests (Wright, 2000) and the Kendall's tau test for the residuals of binary choice models (Andreou and Werker, 2011).

[2] Other advances in econometric theory using LAN can be found in Abadir and Distaso (2007), Jeganathan (1995), Ploberger (2004), and Ploberger and Phillips (2012).

paper provides an analysis which is especially useful for non-pointwise differentiable statistics, for instance, those involving ranks or runs. Their main theorem is based on two assumptions, called ULAN and AN. The ULAN (Uniform Local Asymptotic Normality) condition imposes the model of interest to be sufficiently "regular". For GARCH-type models this condition is well-studied and we, thus, refer to existing results. The AN (Asymptotic Normality) condition describes the joint asymptotic behavior of the model's score, the estimator of the unknown model parameters, and the test statistic of interest. This joint, trivariate, limiting variance matrix determines whether a size correction in the residual-based statistic is needed, or not. Both conditions are discussed below in the context of location-scale time-series models.

Consider a time series $Y_1, \ldots, Y_T$ modeled as

$$Y_t = \mu_{t-1}(\eta) + \sigma_{t-1}(\theta)\varepsilon_t,\ t = 1, \ldots, T, \tag{2.1}$$

where both $\mu_{t-1}(\theta)$ and $\sigma_{t-1}(\theta)$ may depend on past observed values $Y_{t-1}, Y_{t-2}, \ldots$. Moreover, $(\varepsilon_t)$ is a sequence of i.i.d. innovations. We assume throughout that these innovations have an absolutely continuous density $f$ with finite Fisher information for location and scale, i.e.,

$$\mathcal{I}_\mu = \mathcal{I}_{\mu,f} := \int (f'(x)/f(x))^2 f(x)\mathrm{d}x < \infty, \tag{2.2}$$

$$\mathcal{I}_\sigma = \mathcal{I}_{\sigma,f} := \int (1 + xf'(x)/f(x))^2 f(x)\mathrm{d}x < \infty. \tag{2.3}$$

We also introduce the notation $\mathcal{I}_{\mu\sigma} = \int (f'(x)/f(x))(1 + xf'(x)/f(x))f(x)\mathrm{d}x$, which equals zero in case $f$ is symmetric. Finally, we impose the identification restrictions $\mathrm{E}\varepsilon_t = 0$, $\mathrm{E}\varepsilon_t^2 = 1$, and also $\kappa_\varepsilon := \mathrm{E}\varepsilon_t^4 < \infty$. The unknown parameters $\eta$ and $\theta$ are assumed to belong to open Euclidean sets.

Under suitable regularity conditions this model is regular in the ULAN sense. We formulate this as a high-level assumption.

**Assumption 1** *The model (2.1) generating $Y_1, \ldots, Y_T$ is Uniformly Asymptotically Normal (ULAN), as $T \to \infty$, with scores (Central Sequences) for $\eta$ and $\theta$ given by*

$$\begin{aligned}
\Delta(\eta) &= \frac{1}{\sqrt{T}} \sum_{t=1}^{T} -\frac{f'(\varepsilon_t(\eta,\theta))}{f(\varepsilon_t(\eta,\theta))} \sigma_{t-1}^{-1}(\theta) \frac{\partial}{\partial \eta} \mu_{t-1}(\eta), \qquad (2.4) \\
\Delta(\theta) &= \frac{1}{\sqrt{T}} \sum_{t=1}^{T} -\left(1 + \varepsilon_t(\eta,\theta) \frac{f'(\varepsilon_t(\eta,\theta))}{f(\varepsilon_t(\eta,\theta))}\right) \sigma_{t-1}^{-1}(\theta) \frac{\partial}{\partial \theta} \sigma_{t-1}(\theta),
\end{aligned}$$

*where $\varepsilon_t(\eta,\theta) := (Y_t - \mu_{t-1}(\eta))/\sigma_{t-1}(\theta)$. The Fisher Information matrix consists of four blocks determined by*

$$I_{\eta\eta'} = \mathcal{I}_\mu W_{\mu\mu'} \text{ with } W_{\mu\mu'} = E\left[\sigma_{t-1}^{-2}(\theta) \frac{\partial}{\partial \eta} \mu_{t-1}(\eta) \frac{\partial}{\partial \eta'} \mu_{t-1}(\eta)\right], \qquad (2.5)$$

$$I_{\eta\theta'} = \mathcal{I}_{\mu\sigma} W_{\mu\sigma'} \text{ with } W_{\mu\sigma'} = E\left[\sigma_{t-1}^{-2}(\theta) \frac{\partial}{\partial \eta} \mu_{t-1}(\eta) \frac{\partial}{\partial \theta'} \sigma_{t-1}(\theta)\right], \qquad (2.6)$$

$$I_{\theta\theta'} = \mathcal{I}_\sigma W_{\sigma\sigma'} \text{ with } W_{\sigma\sigma'} = E\left[\sigma_{t-1}^{-2}(\theta) \frac{\partial}{\partial \theta} \sigma_{t-1}(\theta) \frac{\partial}{\partial \theta'} \sigma_{t-1}(\theta)\right]. \qquad (2.7)$$

By symmetry, we have $I_{\theta\eta'} = I'_{\eta\theta'}$ and $W_{\sigma\mu'} = W'_{\mu\sigma'}$.

Under sufficient regularity conditions, the model (2.1) indeed generally satisfies the ULAN condition. For instance, consider the pure GARCH(1,1) specification $\mu_{t-1}(\eta) = 0$ and

$$\sigma_t^2(\theta) = \omega + \alpha Y_{t-1}^2 + \beta \sigma_{t-1}^2(\theta), \qquad (2.8)$$

with $\theta = (\omega, \alpha, \beta) \in \mathbb{R}_+^3$ and some fixed initial value $\sigma_0^2(\theta)$. Theorem 2.1 in Drost and Klaassen (1997) then implies that the ULAN condition is satisfied under the Nelson (1990) strict stationarity condition $\mathrm{E} \log(\beta + \alpha \varepsilon_t^2) < 0$. Note in particular that IGARCH(1,1) models are not ruled out. The first ULAN result for GARCH-type models was established by Linton (1993). Asymmetric GARCH-type models have been analyzed in Sun and Stengos (2006). Pure ARMA models, i.e. $\sigma_{t-1}(\theta) = 1$, have been considered by Kreiss (1987a) and Kreiss (1987b). Dynamic location-scale models of the general form (2.1) are studied in Drost, Klaassen and Werker (1997). Ling and McAleer (2003) study non-stationary ARMA-GARCH models, but a special case of their Theorem 3.1 provides a LAN result for non-explosive models that is our focus of interest. Also, note that other models for positive observations, like durations, are sometimes of the form (2.1). This is, for instance, the case for Autoregressive Conditional Duration (ACD) models of Engle and Russell (1998). Their asymptotic structure has been analyzed in Drost and Werker (2004).

In order to estimate the model parameters $(\eta, \theta)$, empirical work almost invariantly uses a Gaussian Quasi Maximum Likelihood (QML) estimator $(\widehat{\eta}, \widehat{\theta})$. This estimator is based on an imposed Gaussian distribution for the innovations $\varepsilon_t$, which, generally, leads to consistent and asymptotically normal estimates. We formalize this in the following assumption.

**Assumption 2** *The Gaussian QML estimator for $(\eta, \theta)$ in (2.1) satisfies*

$$\begin{aligned}
\sqrt{T}\begin{bmatrix} \widehat{\eta}-\eta \\ \widehat{\theta}-\theta \end{bmatrix} &= \begin{bmatrix} W_{\mu\mu'} & 0 \\ 0 & 2W_{\sigma\sigma'} \end{bmatrix}^{-1} \\
&\quad \times \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \begin{bmatrix} \varepsilon_t(\eta,\theta)\sigma_{t-1}^{-1}(\theta)\frac{\partial}{\partial\eta}\mu_{t-1}(\eta) \\ \left(\varepsilon_t(\eta,\theta)^2 - 1\right)\sigma_{t-1}^{-1}(\theta)\frac{\partial}{\partial\theta}\sigma_{t-1}(\theta) \end{bmatrix} + o_P(1) \qquad (2.9) \\
&\xrightarrow{\mathcal{L}} N\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} W_{\mu\mu'}^{-1} & \frac{1}{2}E\varepsilon^3 W_{\mu\mu'}^{-1} W_{\mu\sigma'} W_{\sigma\sigma'}^{-1} \\ \frac{1}{2}E\varepsilon^3 W_{\mu\mu'}^{-1} W_{\mu\sigma'} W_{\sigma\sigma'}^{-1} & \frac{1}{4}(\kappa_\varepsilon - 1) W_{\sigma\sigma'}^{-1} \end{bmatrix} \right).
\end{aligned}$$

The validity of Assumption 2 follows informally from standard Taylor expansions of the QMLE estimator and standard martingale difference CLTs. Formal results are often available under regularity conditions. For pure ARMA models, an overview, e.g., can be found in Brockwell and Davis (1991). For scale models, the analysis is generally more involved but some strong results exist. For instance, for the GARCH(1,1) model, Lumsdaine (1996) establishes consistency and asymptotic normality of the Gaussian QMLE estimator. Berkes and Horváth (2004) have improved upon these results showing that, still under (2.8), (2.9) holds. More precisely, (2.9) follows from their result (4.18) which, as noted in the proof of their Theorem 2.1, is also valid for $\widehat{\theta}$ applied to their Example 2.1. Note that the results in Berkes and Horváth (2004) are established for GARCH($p$,$q$) processes. Theorem 2 in Bardet and Wintenberger (2009) establishes Asymptotic Normality and consistency of the QMLE estimator for general causal time-series models precisely of the form (2.1).

We are interested in the present paper in specification testing, based on a residual-based statistic. We therefore assume to be given an innovation-based statistic of interest $S(\eta, \theta)$. To be able to derive the limiting distribution of the associated residual-based statistic $S(\widehat{\eta}, \widehat{\theta})$, we introduce a last assumption. Note that this is a multivariate extension of Andreou and Werker (2012).

**Assumption 3** *The innovation-based version of the test statistic of interest $S(\eta, \theta)$, the central sequence, and the estimator $(\widehat{\eta}, \widehat{\theta})$ are jointly asymptotically normal, that is*

$$\begin{bmatrix} S(\eta,\theta) \\ \Delta(\eta) \\ \Delta(\theta) \\ \sqrt{T}(\widehat{\eta}-\eta) \\ \sqrt{T}(\widehat{\theta}-\theta) \end{bmatrix} \xrightarrow{\mathcal{L}} N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix};\right. \tag{2.10}$$
$$\left.\begin{bmatrix} \Sigma & c_\eta' & c_\theta' & \alpha_\eta' & \alpha_\theta' \\ c_\eta & \mathcal{I}_\mu W_{\mu\mu'} & \mathcal{I}_{\mu\sigma} W_{\mu\sigma'} & I & 0 \\ c_\theta & \mathcal{I}_{\mu\sigma} W_{\sigma\mu'} & \mathcal{I}_\sigma W_{\sigma\sigma'} & 0 & I \\ \alpha_\eta & I & 0 & W_{\mu\mu'}^{-1} & \frac{1}{2} E\varepsilon_t^3 W_{\mu\mu'}^{-1} W_{\mu\sigma'} W_{\sigma\sigma'}^{-1} \\ \alpha_\theta & 0 & I & \frac{1}{2} E\varepsilon_t^3 W_{\sigma\sigma'}^{-1} W_{\sigma\mu'} W_{\mu\mu'}^{-1} & \frac{1}{4}(\kappa_\varepsilon - 1) W_{\sigma\sigma'}^{-1} \end{bmatrix}\right),$$

*where $I$ and $0$ denote an identity and zero matrix, of appropriate dimensions, respectively.*

**Remark 1** *(AN) in Andreou and Werker (2012) is formulated using convergence under local alternatives to the parameters. As explained in their Appendix B it is, for residual-based statistics, sufficient to verify the condition under fixed alternatives only.*

Assumption 3 describes the joint behavior of all ingredients that determine whether a size correction is needed for a residual-based test, namely the model (represented by the central sequences $\Delta(\eta)$ and $\Delta(\theta)$), the estimators $\widehat{\theta}$ and $\widehat{\theta}$ used, and the test statistic of interest (whose innovation-based version is $S(\eta, \theta)$).

This is a convenient place to mention some equalities that we will use throughout the paper.

$$\mathrm{E}\frac{-f'(\varepsilon_t)}{f(\varepsilon_t)}\varepsilon_t = -\int f'(x)x\mathrm{d}x = 1, \tag{2.11}$$

$$-\mathrm{E}\left(1+\varepsilon_t\frac{f'(\varepsilon_t)}{f(\varepsilon_t)}\right)\varepsilon_t = -\int f'(x)x^2\mathrm{d}x = 0, \tag{2.12}$$

$$\mathrm{E}\frac{-f'(\varepsilon_t)}{f(\varepsilon_t)}\left(\varepsilon_t^2-1\right) = \int f'(x)x^2\mathrm{d}x = 0, \tag{2.13}$$

$$-\mathrm{E}\left(1+\varepsilon_t\frac{f'(\varepsilon_t)}{f(\varepsilon_t)}\right)\left(\varepsilon_t^2-1\right) = -1-\int f'(x)x^3\mathrm{d}x = 2. \tag{2.14}$$

In particular these results corroborate Assumption 3 as far as it concerns the asymptotic covariance between the scores and the QML estimator.

We can now state the asymptotic null distribution of the residual-based test statistic $S(\widehat{\eta}, \widehat{\theta})$.

**Theorem 1** *Under Assumptions 1-3, the residual-based statistic $S(\widehat{\eta}, \widehat{\theta})$ is asymptotically normally distributed with zero mean and variance*

$$\begin{aligned}\Sigma + c_\eta' W_{\mu\mu'}^{-1} c_\eta + \frac{1}{2} E\varepsilon^3 \left( c_\eta' W_{\mu\mu'}^{-1} W_{\mu\sigma'} W_{\sigma\sigma'}^{-1} c_\theta + c_\theta' W_{\sigma\sigma'}^{-1} W_{\sigma\mu'} W_{\mu\mu'}^{-1} c_\eta \right) \\ + \frac{\kappa_\varepsilon - 1}{4} c_\theta' W_{\sigma\sigma'}^{-1} c_\theta - \alpha_\eta' c_\eta - \alpha_\theta' c_\theta - c_\eta' \alpha_\eta - c_\theta' \alpha_\theta.\end{aligned} \tag{2.15}$$

Proof From Theorem 1 in Andreou and Werker (2012) we find the asymptotic variance of the residual-based statistic as $\Sigma + c'\Gamma c - \alpha' c - c'\alpha$, where $\Gamma$ denotes the limiting variance of the Gaussian QML estimator $(\hat{\eta}, \hat{\theta})$. Using the partitioning of the variance-covariances as in Assumption 3, the result follows. □

Theorem 1 shows that replacing innovations by residuals may leave the asymptotic variance of the test statistic $S$ unchanged, increase it, or decrease it, depending on the values of $c_\eta$, $c_\theta$, $\alpha_\eta$, and $\alpha_\theta$. In particular, unlike what is sometimes considered to be common wisdom, it's not the case that the asymptotic variance of residual-based statistics is always smaller than that of the innovation-based statistic. Such a situation is of particular interest as applied researchers could forego a detailed analysis of the residual-based statistic and just use critical values based on the innovation-based version that are generally available. If indeed the residual-based asymptotic variance is smaller, such an approach will lead to a conservative test, thus not invalidate the analysis in terms of size. If needed, the results in the present paper can be used to get critical values that lead to an (asymptotically) size closer to the desired level.

We give two results under which applying an innovation-based test to residuals indeed leads to a conservative test.

**Proposition 1** *Impose the conditions of Theorem 1 and assume $c_\theta = 0$ and $\alpha_\eta = a W_{\mu\mu'}^{-1} c_\eta$ for some scalar $a$. Then, the non-size-corrected residual-based test is conservative if*

$$a \geq \frac{1}{2}. \tag{2.16}$$

Proof In view of Theorem 1, the non-size-corrected test based on $S(\widehat{\eta}, \widehat{\theta})$ is conservative when $c_\theta = 0$ in case

$$c_\eta' W_{\mu\mu'}^{-1} c_\eta - 2\alpha_\eta' c_\eta = (1 - 2a)\, c_\eta' W_{\mu\mu'}^{-1} c_\eta \leq 0.$$

As $c_\eta' W_{\mu\mu'}^{-1} c_\eta \geq 0$, the result follows. □

**Proposition 2** *Impose the conditions of Theorem 1 and assume $c_\eta = 0$ and $\alpha_\theta = a W_{\sigma\sigma'}^{-1} c_\theta$ for some scalar $a$. Then, the non-size-corrected residual-based test is conservative if*

$$a \geq \frac{\kappa_\varepsilon - 1}{8}. \tag{2.17}$$

Proof In view of Theorem 1, the non-size-corrected test based on $S(\widehat{\eta}, \widehat{\theta})$ is conservative when $c_\eta = 0$ in case

$$\frac{\kappa_\varepsilon - 1}{4} c_\theta' W_{\sigma\sigma'}^{-1} c_\theta - 2\alpha_\theta' c_\theta = \left( \frac{\kappa_\varepsilon - 1}{4} - 2a \right) c_\theta' W_{\sigma\sigma'}^{-1} c_\theta \leq 0.$$

As $c_\theta' W_{\sigma\sigma'}^{-1} c_\theta \geq 0$, the result follows. □

It's also worth stating the following proposition explicitly. It gives a simple condition under which the residual-based statistic has exactly the same (asymptotic) size as the innovation-based statistic.

**Proposition 3** *Impose the conditions of Theorem 1 and assume $c_\eta = 0$ and $c_\theta = 0$. Then, the limiting null distribution of the residual-based statistic equals that of the innovation-based statistic.*

Proof This follows immediately from Theorem 1. □

A question that arises naturally at this point is the effect on the (local) *power* of tests that are applied to residuals instead of actual innovations. Similar techniques as above can be used to assess this power, whether the tests are rank-based or not. In the present paper we will assess the actual power of the various tests by extensive simulations in Section 4. We only remark that it is possible, maybe somewhat surprisingly, for residual-based statistics to be more powerful against certain alternatives than the same statistic applied to actual innovations.

# 3 Specification tests

We consider a panoply of specification tests for the location-scale time-series model introduced above. We group these tests in various sections.

## 3.1 Linear residual autocorrelation tests

We are interested in testing for serial correlation in the residuals of the model. Based on the innovations $\varepsilon_t(\eta, \theta)$, the classical standard $s$-th order autocorrelation statistic satisfies, under$(\eta, \theta)$,

$$\begin{aligned} S_{C1}^{(s)}(\eta,\theta) &:= \sqrt{T}\frac{(T-s+1)^{-1}\sum_{t=s+1}^{T}\varepsilon_t(\eta,\theta)\varepsilon_{t-s}(\eta,\theta) - \left(T^{-1}\sum_{t=1}^{T}\varepsilon_t(\eta,\theta)\right)^2}{T^{-1}\sum_{t=1}^{T}\varepsilon_t(\eta,\theta)^2 - \left(T^{-1}\sum_{t=1}^{T}\varepsilon_t(\eta,\theta)\right)^2} \\ &= \frac{1}{\sqrt{T}}\sum_{t=s+1}^{T}\varepsilon_t(\eta,\theta)\varepsilon_{t-s}(\eta,\theta) + o_P(1). \end{aligned} \tag{3.1}$$

In order to use Theorem 1, we apply a suitable martingale difference limit theorem to $S_{C1}(\eta,\theta) = \left[S_{C1}^{(1)}(\eta,\theta), \ldots, S_{C1}^{(m)}(\eta,\theta)\right]'$. First, $\Sigma = \mathrm{I}_m$. Moreover, in view of (2.13), we have $c_\theta = 0$ and, as a result, the value of $\alpha_\theta$ is irrelevant. Also, we find that $c_\eta$ is determined row wise by

$$c_{\eta,s} = \mathrm{E}\varepsilon_{t-s}\frac{\partial}{\partial\eta'}\frac{\mu_{t-1}(\eta)}{\sigma_{t-1}(\theta)}, \tag{3.2}$$

for $l = 1, \ldots, m$. Finally, in view of $\mathrm{E}\varepsilon^2 = 1$, we have $\alpha_\eta = W_{\mu\mu'}^{-1}c_\eta$. Taking all these together, the limiting null variance of the residual-based test statistic $S_{C1}(\hat{\eta}, \hat{\theta})$ becomes

$$\mathrm{I}_m - c_\eta' W_{\mu\mu'}^{-1} c_\eta. \tag{3.3}$$

A few remarks are in place. First, note that this result is valid even if $\mathrm{E}\varepsilon^3 \neq 0$. Second, when applied to pure $AR$ models of order $p < m$, the result reduces to the standard Ljung-Box degrees of freedom correction as, in that case, $c_\eta = [\mathrm{I}_p; 0_{m-p,p}]'$ and $W_{\mu\mu'} = \mathrm{I}_p$. Finally, in case of a pure scale model, i.e., no $\eta$ parameter, we find that no size correction is needed. In line with Proposition 2, applying no size correction, i.e., using $m$ degrees of freedom in Ljung and Box (1978) test applied to residuals, always leads to a conservative test for linear residual autocorrelation in location-scale time series. Using $m - \dim(\eta)$ degrees of freedom only leads to a valid test for pure AR specifications. These results are corroborated by the simulations in Section 4.

One of the advantages of our approach is that we do not require point-wise differentiability of our test statistic with respect to the parameters $\eta$ and $\theta$. This is particularly helpful when considering rank-based statistics since they are, by definition, not smooth in the parameters. While some form of asymptotic smoothness, as required by more traditional approaches, often holds for rank-based statistics, at least under additional regularity conditions, the verification of such results may be nontrivial. In such cases, Theorem 1 is useful. We introduce the notation $R_t(\eta,\theta)$ for the rank of the $t$-th innovation, $\varepsilon_t(\eta,\theta)$, among all $\varepsilon_1(\eta,\theta),\ldots,\varepsilon_T(\eta,\theta)$.

Rank-based test statistics are known for their robustness properties. In particular, rank-based statistics are often distribution-free, i.e., their asymptotic behavior is the same irrespective of the actual innovation distribution $f$. In order to introduce the rank-based statistic for linear residual autocorrelation, we consider a so-called *reference density* $g$. We assume throughout that $g$ admits finite Fisher information for location $\mathcal{I}_{\mu,g} = \int (g'/g)^2 g < \infty$, has mean zero and unit variance. Moreover, we assume that $g$ is strongly unimodal (i.e., $-g'/g$ is monotone increasing). Finally, we denote by $G$ the distribution function associated to $g$.

Now, the rank-based test for $l$-th order autocorrelation is based on

$$S_{R1}^{(s)}(\eta,\theta;g) = \frac{1}{T-s}\sum_{t=s+1}^{T} \frac{-g'}{g}\left(G^{-1}\left(\frac{R_t(\eta,\theta)}{T+1}\right)\right) G^{-1}\left(\frac{R_{t-s}(\eta,\theta)}{T+1}\right)/\sqrt{\mathcal{I}_{\mu,g}}, \quad (3.4)$$

Using for instance the results mentioned in Hallin and Werker (1999), one may show, under the stated assumptions on the reference density $g$,

$$\begin{aligned} S_{R1}^{(s)}(\eta,\theta;g) &= \frac{1}{T-s}\sum_{t=s+1}^{T} \frac{-g'}{g}\left(G^{-1}\left(F(\varepsilon_t(\eta,\theta))\right)\right) \qquad (3.5)\\ &\qquad \times G^{-1}\left(F(\varepsilon_{t-s}(\eta,\theta))\right)/\sqrt{\mathcal{I}_{\mu,g}} + o_P(T^{-1/2}) \\ &\xrightarrow{\mathcal{L}} N(0,1), \end{aligned}$$

as $T\to\infty$. Note that $F(\varepsilon_t(\eta,\theta))$ follows a standard uniform distribution. The rank-based autocorrelations are asymptotically normally distributed with unit variance even if $G \neq F$. Actually, the representation (3.5) would need no assumption on $F$ other than continuity to avoid possible complications due to ties in the ranks. Finally, observe that the unit variance assumption on the reference density $g$ is innocuous as any scale factor would cancel in $S_{R1}^{(s)}(\eta,\theta)$. Popular choices for $G$ are the so-called van der Waerden autocorrelations obtained by taking the standard normal distribution, while the logistic distribution leads to the Wilcoxon autocorrelations; see Section 4 for details. Both satisfy Assumption G and many more examples can be found in the overview of Hallin and Werker (1999).

The prime advantage of using rank-based autocorrelations is that they are insensitive to misspecification of the innovation distribution (since they are distribution free), while they still may lead to semiparametrically efficient inference procedures

(Hallin and Werker (2003)). As a result, any reference density $G$ satisfying the assumptions leads to a valid innovation-based test (in terms of size), while the power depends on the actual choice of $G$ relative to the true distribution $F$. Moreover, while tests directly based on the powers of the innovations often require the existence of appropriate moments, rank-based tests do not.

Using the asymptotically linear representation (3.5), we easily find the matrices $c$ and $\alpha$. To that end, define matrices $A_\mu$ and $A_\sigma$ by their rows, for $s = 1, \ldots, m$,

$$\begin{aligned} A_{R1,\mu,s} &= \mathrm{E}\left[G^{-1}(F(\varepsilon_{t-s}))\sigma_{t-1}(\theta)^{-1}\frac{\partial}{\partial\eta'}\mu_{t-1}(\eta)\right], \\ A_{R1,\sigma,s} &= \mathrm{E}\left[G^{-1}(F(\varepsilon_{t-s}))\sigma_{t-1}(\theta)^{-1}\frac{\partial}{\partial\theta'}\sigma_{t-1}(\theta)\right]. \end{aligned}$$

Then

$$\begin{aligned} c_\eta &= \mathcal{I}_{\mu,g}^{-1/2}\mathrm{E}\left[\frac{f'(\varepsilon)}{f(\varepsilon)}\frac{g'(G^{-1}(F(\varepsilon)))}{g(G^{-1}(F(\varepsilon)))}\right]A_{R1,\mu}, \\ c_\theta &= \mathcal{I}_{\mu,g}^{-1/2}\mathrm{E}\left[\left(1+\varepsilon\frac{f'(\varepsilon)}{f(\varepsilon)}\right)\frac{g'(G^{-1}(F(\varepsilon)))}{g(G^{-1}(F(\varepsilon)))}\right]A_{R1,\sigma}, \\ \alpha_\eta &= \mathcal{I}_{\mu,g}^{-1/2}\mathrm{E}\left[\varepsilon\frac{-g'(G^{-1}(F(\varepsilon)))}{g(G^{-1}(F(\varepsilon)))}\right]W_{\mu\mu'}^{-1}A_{R1,\mu}, \\ \alpha_\theta &= \frac{1}{2}\mathcal{I}_{\mu,g}^{-1/2}\mathrm{E}\left[\left(\varepsilon^2-1\right)\frac{-g'(G^{-1}(F(\varepsilon)))}{g(G^{-1}(F(\varepsilon)))}\right]W_{\sigma\sigma'}^{-1}A_{R1,\sigma}. \end{aligned}$$

With the above expressions, Theorem 1 can be applied directly so that exact (asymptotic) critical values are available for rank-based linear residual autocorrelation tests. In applied work, one may wish to settle for a conservative test so that $c_\eta$, $c_\theta$, $\alpha_\eta$, and $\alpha_\theta$ need not be estimated. One easily checks the following two special cases.

1. In case we test for linear residual autocorrelation in pure location models, i.e., $\sigma_t = \theta$ is constant ($c_\theta = 0$), a conservative test is obtained as long as $2\varepsilon + (f'/f)(\varepsilon)$ is increasing in $\varepsilon$. Indeed, this follows immediately from Proposition 1.

2. In case we test for linear residual autocorrelation in pure scale models, i.e., $\mu_t = \eta$ ($c_\eta = 0$), no size-correction is needed in case $f$ and $g$ are both symmetric about zero. Indeed, we then have $c_\theta = 0$ as the expectation of an anti-symmetric function under a symmetric distribution and apply Proposition 3. Thus, for an asymptotically correctly sized test on $m$ rank-based autocorrelations, one should use $\chi^2_m$ quantiles.

### 3.2 Quadratic residual autocorrelation tests

In order to test for possible autocorrelation in the squared residuals, we consider both the Li and Mak (1994) and a rank-based test on the autocorrelation of squared innovations. First, the Li and Mak (1994) test is based on the classical $s$-th order autocorrelation of squared residuals, i.e.,

$$
\begin{aligned}
S_{C2}^{(s)}(\eta,\theta) &:= \sqrt{T}\frac{\sum_{t=s+1}^{T}\left(\varepsilon_t^2(\eta,\theta)-T^{-1}\sum_{t=1}^{T}\varepsilon_t^2(\eta,\theta)\right)\left(\varepsilon_{t-s}^2(\eta,\theta)-T^{-1}\sum_{t=1}^{T}\varepsilon_t^2(\eta,\theta)\right)}{\sum_{t=1}^{T}\left(\varepsilon_t^2(\eta,\theta)-T^{-1}\sum_{t=1}^{T}\varepsilon_t^2(\eta,\theta)\right)^2} \\
&= \frac{1}{\sqrt{T}}\sum_{t=s+1}^{T}\frac{(\varepsilon_t^2(\eta,\theta)-1)(\varepsilon_{t-s}^2(\eta,\theta)-1)}{\kappa_\varepsilon-1}+o_P(1). \qquad (3.6)
\end{aligned}
$$

In order to verify Assumption 3 for $S_{C2}(\eta,\theta)=\left[S_{C2}^{(1)}(\eta,\theta),\dots,S_{C2}^{(m)}(\eta,\theta)\right]'$ observe again $\Sigma=\mathrm{I}_m$. Furthermore, from (2.13) we find $c_\eta=0$ so that the value of $\alpha_\eta$ is no longer relevant in (2.15). Observe that this holds even when $\mathrm{E}\varepsilon^3\neq 0$. Introduce the matrix $A_{C2,\sigma}$ by its rows, $s=1,\dots,m$,

$$
A'_{C2,\sigma,s}=\mathrm{E}\left[\left(\varepsilon_{t-s}^2-1\right)\sigma_{t-1}(\theta)^{-1}\frac{\partial}{\partial\theta}\sigma_{t-1}(\theta)\right]. \qquad (3.7)
$$

As, using (2.14),

$$
\begin{aligned}
c_{\theta,s} &= \frac{2}{\kappa_\varepsilon-1}A_{C2,\sigma}, \\
\alpha_{\theta,s} &= \frac{1}{2}W_{\sigma\sigma'}^{-1}A_{C2,\sigma},
\end{aligned}
$$

we apply Theorem 1 to find the limiting variance of the residual-based statistic $S_{C2}(\hat\eta,\hat\theta)$ as

$$
\mathrm{I}_m-\frac{1}{\kappa_\varepsilon-1}A_{C2,\sigma}W_{\sigma\sigma'}^{-1}A'_{C2,\sigma}. \qquad (3.8)
$$

The above limit is also derived in Berkes, Horváth and Kokoszka (2003) for residuals of the GARCH($p$,$q$) model (compare also Horváth and Kokoszka (2001)). Their Theorem 2.2 is the counterpart of (3.8) with the notation $d_0^2=\kappa_\varepsilon-1$, $i_k=s$, $c_{i_k}=\mathrm{E}\left[(\varepsilon_{t-s}^2-1)\frac{\partial}{\partial\theta'}\log\sigma_{t-1}^2(\theta)\right]=2A'_{C2,\sigma,s}$, $A_0=\frac{1}{4}(\kappa_\varepsilon-1)^2\Gamma^{-1}$, and $B_0=-\frac{1}{2}(\kappa_\varepsilon-1)\Gamma^{-1}$, where $\Gamma$ is the limiting variance matrix of the QMLE estimator in Assumption 2. Note that their Theorem 2.2 gives the limiting distribution of $(\kappa_\varepsilon-1)\widehat{\rho}_2(\widehat{\theta};s)$. Our results extend to the case where a mean term $\mu_{t-1}$ is present (without affecting the limiting distribution), as long as our high-level assumptions remain satisfied. Moreover, the results apparently generalize to non-symmetric GARCH($p$,$q$) models like Asymmetric GARCH (compare Sun and Stengos (2006)).

As (3.8) is obviously smaller than $\mathrm{I}_m$, the Li and Mak (1994) test for remaining autocorrelation in squared residuals in (2.1) based on $\chi^2_m$ quantiles, will always be conservative. For applications where $\kappa_\varepsilon$ is large (like financial applications), the undersizing may actually be fairly small. This is a convenient result for empirical analysis.

Second, we consider rank-based tests of exactly the same form as (3.4), but with $R_t(\eta,\theta)$ the rank of the squared innovation $\varepsilon_t^2$ among $\varepsilon_1^2,\ldots,\varepsilon_T^2$. If we denote by $F^{(2)}$ the cumulative distribution function of $\varepsilon^2$, we find

$$
\begin{aligned}
c_\eta &= I_g^{-1/2}\mathrm{E}\left[\frac{f'(\varepsilon)}{f(\varepsilon)}\frac{g'(G^{-1}(F^{(2)}(\varepsilon^2)))}{g(G^{-1}(F^{(2)}(\varepsilon^2)))}\right]\mathrm{E}\left[G^{-1}(F^{(2)}(\varepsilon_{t-s}^2))\sigma_{t-1}(\theta)^{-1}\frac{\partial}{\partial\eta}\mu_{t-1}(\eta)\right],\\
c_\theta &= I_g^{-1/2}\mathrm{E}\left[\left(1+\varepsilon\frac{f'(\varepsilon)}{f(\varepsilon)}\right)\frac{g'(G^{-1}(F^{(2)}(\varepsilon^2)))}{g(G^{-1}(F^{(2)}(\varepsilon^2)))}\right]\mathrm{E}\left[G^{-1}(F^{(2)}(\varepsilon_{t-s}^2))\sigma_{t-1}(\theta)^{-1}\frac{\partial}{\partial\theta}\sigma_{t-1}(\theta)\right],\\
\alpha_\eta &= I_g^{-1/2}\mathrm{E}\left[\varepsilon\frac{-g'(G^{-1}(F^{(2)}(\varepsilon^2)))}{g(G^{-1}(F^{(2)}(\varepsilon^2)))}\right]W_{\mu\mu'}^{-1}\mathrm{E}\left[G^{-1}(F^{(2)}(\varepsilon_{t-s}^2))\sigma_{t-1}(\theta)^{-1}\frac{\partial}{\partial\eta}\mu_{t-1}(\eta)\right],\\
\alpha_\theta &= \frac{1}{2}I_g^{-1/2}\mathrm{E}\left[\left(\varepsilon^2-1\right)\frac{-g'(G^{-1}(F^{(2)}(\varepsilon^2)))}{g(G^{-1}(F^{(2)}(\varepsilon^2)))}\right]W_{\sigma\sigma'}^{-1}\mathrm{E}\left[G^{-1}(F^{(2)}(\varepsilon_{t-s}^2))\sigma_{t-1}(\theta)^{-1}\frac{\partial}{\partial\theta}\sigma_{t-1}(\theta)\right].
\end{aligned}
$$

Again, Theorem 1 gives a precise and detailed answer to the effect of using residuals in this rank-based autocorrelation test for squared innovations. But, as before, for applied work two special cases may be convenient.

1. In case we test for quadratic residual autocorrelation in pure location models, i.e., $\sigma_t = \theta$ is constant ($c_\theta = 0$), no size-correction is needed in case $f$ is symmetric as then $c_\eta = 0$ as well.

2. In case we test for quadratic residual autocorrelation in pure scale models, i.e., $\mu_t = \eta$ ($c_\eta = 0$), a conservative test is obtained, using Proposition 2, in case

$$
\frac{\mathrm{E}\left[\left(\varepsilon^2-1\right)\frac{-g'(G^{-1}(F^{(2)}(\varepsilon^2)))}{g(G^{-1}(F^{(2)}(\varepsilon^2)))}\right]}{\mathrm{E}\left[\left(1+\varepsilon\frac{f'(\varepsilon)}{f(\varepsilon)}\right)\frac{g'(G^{-1}(F^{(2)}(\varepsilon^2)))}{g(G^{-1}(F^{(2)}(\varepsilon^2)))}\right]} \geq \frac{\kappa_\varepsilon - 1}{4}. \tag{3.9}
$$

Condition (3.9) looks somewhat involved, but is easily verified for given specifications of $f$ and $g$. We provide in Table 3.1 the ratio of the left-hand and right-hand side in some standard cases. With the exception of the $t(5)$ distribution, this condition

| | $N(0,1)$ | $t_5$ | $t_8$ | $t_{10}$ |
|---|---|---|---|---|
| vd Waerden | 2 | 0.86 | 1.64 | 1.80 |
| Wilcoxon | 2 | 0.78 | 1.55 | 1.72 |

Table 3.1: Ratio of the left-hand and right-hand side of (3.9) for some standard cases.

is always satisfied. A more comprehensive, finite-sample, analysis is provided in the simulations section.

## 3.3 Residual symmetry tests

Tests for symmetry of the innovations $\varepsilon_t$ are often based on marginal properties of the individual innovations, i.e., they satisfy

$$S_{Sym}(\eta, \theta) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \psi(\varepsilon_t) + o_P(1),$$

for some antisymmetric function $\psi$. This holds, in particular, for the sign test and the Wilcoxon signed-rank test.

**Remark 2** *The results in this section, formally, easily extend to testing other marginal properties of the innovation distribution, like testing for excess kurtosis. However, tests for symmetry lead to significant simplifications in the size-corrections and thus are particularly interesting to study.*

Observe that, under the null of symmetry, $\mathrm{E}\left[\left(1 + \varepsilon \frac{f'(\varepsilon)}{f(\varepsilon)}\right) \psi(\varepsilon)\right] = \mathrm{E}\left[\left(1 - \varepsilon^2\right) \psi(\varepsilon)\right] = 0$ so that $c_\theta = \alpha_\theta = 0$. In order to apply Theorem 1 we further note

$$\begin{aligned} c_\eta &= \mathrm{E}\left[-\frac{f'(\varepsilon)}{f(\varepsilon)} \psi(\varepsilon)\right] \mathrm{E}\left[\sigma_{t-1}(\theta)^{-1} \frac{\partial}{\partial \eta} \mu_{t-1}(\eta)\right], \\ \alpha_\eta &= \mathrm{E}\left[\varepsilon \psi(\varepsilon)\right] W_{\mu\mu'}^{-1} \mathrm{E}\left[\sigma_{t-1}(\theta)^{-1} \frac{\partial}{\partial \eta} \mu_{t-1}(\eta)\right], \end{aligned}$$

so that the residual-based statistic $S_{Sym}(\eta, \theta)$ has a limiting variance of

$$\begin{aligned} &\mathrm{Var}\left\{\psi(\varepsilon)\right\} + \mathrm{E}\left[\left(-\frac{f'(\varepsilon)}{f(\varepsilon)} - 2\varepsilon\right) \psi(\varepsilon)\right] \times \\ &\mathrm{E}\left[\sigma_{t-1}(\theta)^{-1} \frac{\partial}{\partial \eta} \mu_{t-1}(\eta)\right]' W_{\mu\mu'}^{-1} \mathrm{E}\left[\sigma_{t-1}(\theta)^{-1} \frac{\partial}{\partial \eta} \mu_{t-1}(\eta)\right]. \end{aligned} \tag{3.10}$$

This result gives again rise to two important special cases.

1. In case we test for symmetry of the innovation distribution in zero-mean scale models, i.e., $\mu_t = 0$, then $c_\theta = 0$ so that no size-correction is needed in the residual-based test.

2. In case we test for symmetry of the innovation distribution in general location-scale models, using critical values of the innovation-based test, i.e., based on $\mathrm{Var}\{\psi(\varepsilon)\}$, leads to a conservative test when applied to residuals in case both $\psi$ and $2\varepsilon + (f'/f)(\varepsilon)$ are increasing in $\varepsilon$. Indeed, this also follows directly from Proposition 1.

## 3.4 Structural break tests in the residual distribution

Finally, we consider testing for a break point at the $s$-th quantile of the sample, in either the innovation distribution or the autocorrelation structure of (squared) innovations. Tests for break points at an unknown location in the sample are, generally, rooted in tests for a known break point by integrating out the possible break locations or by taking a supremum over those locations. While our theory, at the expense of added technical complexity, can be extended to such integral- of sup-based statistics, we do not discuss the details here. We focus on tests with a known break location, but the simulations in Section 4 indicate that the extension to integral- or sup-based tests works indeed as expected.

As the analysis for the various possible tests for breaks is very similar, we focus here on testing for a structural break in the conditional variance using the squared standardized innovations. To that extent, let, in line with Section 3.2, $R_t(\eta,\theta)$ again denote the rank of the squared residual $\varepsilon_t^2(\eta,\theta)$ among $\varepsilon_1^2(\eta,\theta),\ldots,\varepsilon_T^2(\eta,\theta)$. An often-used statistic is the CUSUM type rank statistic

$$S_B(\eta,\theta) = \frac{1}{\sqrt{T}} \sum_{t=1}^{\lfloor Ts \rfloor} \left( \frac{R_t(\eta,\theta)}{T+1} - \frac{1}{2} \right),$$

where $\lfloor \cdot \rfloor$ denotes the entire function. A standard theorem on the asymptotically linear representation of rank statistics (e.g.,Hájek, Šidák and Sen (1999), Chapter 6, or van der Vaart (1998), Theorem 13.5) shows

$$S_B(\theta) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \left( I\{t \le \lfloor Ts \rfloor\} - s \right) \left( F^{(2)}(\varepsilon_t^2(\eta,\theta)) - \frac{1}{2} \right) + o_P(1),$$

as $T \to \infty$. Here, as before, $F^{(2)}$ denotes the distribution function of $\varepsilon_t^2$. Theorem 1 now shows that no correction in critical values is needed when this statistic is applied to residuals as

$$c_\eta = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}(I\{t \leq \lfloor Ts \rfloor\} - s)\,\mathrm{E}\left[-\frac{f'(\varepsilon)}{f(\varepsilon)}F^{(2)}(\varepsilon^2)\right]\mathrm{E}\left[\sigma_{t-1}(\theta)^{-1}\frac{\partial}{\partial\theta}\sigma_{t-1}(\theta)\right] = 0,$$

$$c_\theta = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}(I\{t \leq \lfloor Ts \rfloor\} - s)\,\mathrm{E}\left[-\left(1+\varepsilon\frac{f'(\varepsilon)}{f(\varepsilon)}\right)F^{(2)}(\varepsilon^2)\right]\mathrm{E}\left[\sigma_{t-1}(\theta)^{-1}\frac{\partial}{\partial\theta}\sigma_{t-1}(\theta)\right] = 0.$$

In view of Proposition 3, a test using critical values of the innovation-based statistic has the desired (asymptotic) size, even when applied to residuals of location-scale time-series models. As this result is based on the CUSUM centering, $(I\{t \leq \lfloor Ts \rfloor\} - s)$, the same idea applies to, e.g., similar tests for breaks in the innovation distribution or in linear standardized residuals.

# 4 Simulations

This section presents simulation evidence for the performance of the rank residual-based specification tests proposed in the previous section for AR-GARCH type processes. The objective of this section is twofold: First, we study whether the asymptotic results presented in the previous section approximate well the finite-sample behaviour of residual-based statistics for representative AR-GARCH type processes encountered in financial time-series applications. Hence, we evaluate the size of both the classical tests and the more recent rank-based tests to examine their degree of conservativeness for AR-GARCH processes. Second, we compare the power of the rank residual-based tests with some traditional specification tests applied to AR-GARCH models. This is important as it may be convenient that generally specification tests are conservative when applied to residuals rather than innovations, but if this comes at the expense of little power, the tests are not very useful. So, to be precise, in all simulations below, we do *not* apply any size correction to the asymptotic critical values of the tests and, thus, take the simplest empiricist approach who would use the same critical values as if the distribution of the innovations were known and not estimated. Each section below refers to a particular null hypothesis tested and follows the structure of Section 3.[3]

[3]All simulations were performed in Matlab version R2011b. The properties of the tests were evaluated for 5000 simulations.

## 4.1 Linear residual autocorrelation tests simulations

The objective of this section is to evaluate the performance of tests that examine the null hypothesis of no (further) linear autocorrelation in the standardized residuals of AR-GARCH type models. We evaluate the performance of the proposed rank-based statistics vis-à-vis the traditional portmanteau Ljung-Box test (Ljung and Box (1978)).

The AR($m$)-GARCH($p$,$q$) DGP with autoregressive coefficients ($c_0$,$\phi_1$,...,$\phi_m$) and GARCH($p$,$q$) coefficients $(\omega, \alpha_1, ..., \alpha_p, \beta_1, ..., \beta_q)$ is simulated for the sequence $\{Y_t\}$ given by:

$$\begin{aligned} Y_t &= c_0 + \sum_{i=1}^{m} \phi_i Y_{t-i} + v_t, \\ v_t &= \sigma_t \varepsilon_t, \\ \sigma_t^2 &= \omega + \sum_{i=1}^{p} \alpha_i v_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \end{aligned} \tag{4.1}$$

where $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $|\phi_i| < 1$. The DGPs under the null exhibit no AR structure and refer to a highly persistent GARCH(1,1) process with $(\omega, \alpha_1, \beta_1) = (0.1, 0.1, 0.8)$ driven by normal, Student's $t(3)$, $t(5)$, and $t(8)$ innovations. The DGPs under the alternative exhibit an AR structure in the conditional mean of $Y_t$ generated by an AR(1)-GARCH(1,1) with $(c_0, \phi)$-$(\omega, \alpha_1, \beta_1) = (0, \phi)$-$(0.1, 0.1, 0.8)$ with $\phi =$ 0.1, 0.2, or 0.5. We consider various sample sizes $T = 300$, 500, 1000, and 2000. The residual-based serial correlation tests for AR-GARCH models are based on the standardized residuals $\hat{\varepsilon}_t = \left( Y_t - \hat{c}_0 - \sum_{i=1}^{m} \hat{\phi}_i Y_{t-i} \right) \Big/ \hat{\sigma}_t$.

We consider the traditional Ljung-Box (LB) test statistic given by

$$\mathrm{LB} = T(T+2) \sum_{s=1}^{K} \frac{r_s^2}{T-s}, \tag{4.2}$$

based on the sample autocorrelation function $r_s$ defined as

$$r_s = \frac{\sum_{t=s+1}^{T} (\hat{\varepsilon}_t - \bar{\hat{\varepsilon}})(\hat{\varepsilon}_{t-s} - \bar{\hat{\varepsilon}})}{\sum_{t=1}^{T} (\hat{\varepsilon}_t - \bar{\hat{\varepsilon}})^2}, \quad s = 1, 2, \ldots, K, \tag{4.3}$$

where $\bar{\hat{\varepsilon}} = \sum_{t=1}^{T} \hat{\varepsilon}_t / T$. We follow Ljung and Box (1978) and set $s = 0$ for all the residual serial correlation tests. We also consider the following two residual-based rank tests. The van der Waerden rank autocorrelation test (vdW) (e.g., in Hallin, Vermandele and Werker (2006)) defined by

$$\mathrm{vdW} = T(T+2)\sum_{s=1}^{K}\frac{r_{vdW,s}^2}{T-s}, \tag{4.4}$$

where the autocorrelation function $r_{vdW,s}$ is given by

$$\begin{aligned} r_{vdW,s} = \Bigg\{ &(T-s)^{-1}\sum_{t=s+1}^{T}\Phi^{-1}\Big(\frac{R_t}{T+1}\Big)\Phi^{-1}\Big(\frac{R_{t-s}}{T+1}\Big) \\ &- [T(T-1)]^{-1}\sum\sum_{1\le i\ne j\le T}\Phi^{-1}\Big(\frac{i}{T+1}\Big)\Phi^{-1}\Big(\frac{j}{T+1}\Big)\Bigg\}\Big/\sigma_{vdW}. \end{aligned} \tag{4.5}$$

In addition, we apply the Wilcoxon rank autocorrelation test (Wilc)

$$\mathrm{Wilc} = T(T+2)\sum_{s=1}^{K}\frac{r_{Wilc,s}^2}{T-s}, \tag{4.6}$$

which is based on the Wilcoxon autocorrelations given by

$$\begin{aligned} r_{Wilc,s} = \Bigg\{ &(T-s)^{-1}\sum_{t=s+1}^{T}\phi_{log}\Big(\frac{R_t}{T+1}\Big)\psi_{log}\Big(\frac{R_{t-s}}{T+1}\Big) \\ &- [T(T-1)]^{-1}\sum_{1\le i}\sum_{\ne j\le T}\phi_{log}\Big(\frac{i}{T+1}\Big)\psi_{log}\Big(\frac{j}{T+1}\Big)\Bigg\}\Big/\sigma_{Wilc}, \end{aligned} \tag{4.7}$$

where $\phi_{log}(u) := 2u-1$, $\psi_{log}(u) := \ln\big(\frac{u}{1-u}\big)$ and $\sigma_{Wilc} = 1$. The ranks in (4.5) and (4.7) are those of the standardized residuals $\hat{\varepsilon}_t$.[4] Both vdW and Wilc adhere to the general form of rank-based statistics for linear residual autocorrelation in (3.4). In the simulations we use the 5% critical value of the $\chi^2_K$-distribution for the test statistics in (4.2), (4.4), and (4.6). Following the asymptotic analysis in Section 3.1, under the null, the aforementioned statistics are asymptotically nuisance parameter distribution free for the pure scale models with symmetric innovation distributions.

The results for testing the null hypothesis of no AR in AR-GARCH models are summarized in Table 1. The Ljung-Box (LB), van der Waerden (vdW), and Wilcoxon (Wilc) tests yield sizes that are close to the nominal 5% size. For normal errors, it is

[4] By symmetry, the second term in (4.5) and in (4.7) is zero and can be omitted as in (3.4).

known that the LB and vdW statistic have identical asymptotic behavior. This result carries over to the residual-based versions, which implies that the asymptotic power of both tests is the same. The evidence in Table 1 corroborates this result. Moreover, the classical Chernoff-Savage result is seen to hold as well: For the Student's $t$-distribution the vdW test performs strictly better than LB. Also, for $t$-distributed errors, the Wilcoxon test shows better power than vdW. Again this result is in line with the simulation results in Table 1 and holds for alternative degrees of AR persistence. This result is also robust to the sample sizes considered as well as the GARCH parameter values, kurtosis, and the number of autocorrelations of the standardized residuals in the sum of the test statistics, $K$.

From an applied point of view, the accuracy that asymptotic approximations provide to finite sample behavior may even be more important than the exact power. It is this approximation that governs the validity of using asymptotic critical values. From that perspective, it is convenient that Table 1 corroborates the result in Section 3.1 that no size correction is needed when testing for linear residual autocorrelation in pure scale models with symmetric innovation distributions.

In Table 2 we evaluate the size of these tests for AR-GARCH models for which our asymptotic analysis in Section 3.1 suggests that the $\chi^2_K$ critical values will yield conservative tests. Indeed this asymptotic result is evident from Table 2. Yet it is interesting that the degree of undersizing does not compromise power for alternative levels of AR persistence, especially as $T$ increases.

## 4.2 Quadratic residual autocorrelation tests simulations

In this section we examine the performance of tests for the null hypothesis of no (remaining) quadratic autocorrelation in the standardized residuals of AR-GARCH type processes. We first focus on a pure GARCH(1,1)-$(\omega, \alpha_1, \beta_1)$ DGP with no AR component for the sequence $\{Y_t\}$. We consider the following DGPs for obtaining the size of the tests given by: (i) ARCH(1) with low persistence $(\omega, \alpha_1)$:= (0.01,0.1) and with high persistence $(\omega, \alpha_1)$:= (0.01,0.8), (ii) GARCH(1,1) with high persistence $(\omega,\alpha_1,\beta_1)$:= (0.01,0.1,0.8) and IGARCH(1,1) with $(\omega,\alpha_1,\beta_1)$:= (0.01,0.1,0.9). The simulated DGPs for evaluating the power of the tests are given by: (i) ARCH(2) processes with low persistence $(\omega, \alpha_1, \alpha_2)$:= (0.01,0.1,0.1) and with high persistence $(\omega, \alpha_1, \alpha_2)$:= (0.01,0.1,0.4), (ii) GARCH(1,1) with high persistence $(\omega,\alpha_1,\beta_1)$ := (0.01,0.1,0.8) and IGARCH(1,1) with $(\omega,\alpha_1,\beta_1)$:= (0.01,0.1,0.9), where, under the null, we estimate an ARCH(1) model in line with Li and Mak (1994). Some of the parameter choices for the above GARCH DGPs are considered in Andreou and Ghysels (2002).

The innovation distributions used for the above DGPs are the following: $N(0, 1)$, Student's $t(3)$, $t(5)$, $t(8)$ as well as the asymmetric Hansen's Skewed Student's $t$-family proposed by Hansen (1994) for GARCH type processes. This Hansen SkSt$(\lambda, \eta)$ distribution is given by its density:

$$g(z) = \begin{cases} bd\Big(1 + \frac{1}{\eta-2}\big(\frac{bz+a}{1-\lambda}\big)^2\Big)^{-(\eta+1)/2}, & \text{if } z < -\frac{a}{b}, \\ bd\Big(1 + \frac{1}{\eta-2}\big(\frac{bz+a}{1+\lambda}\big)^2\Big)^{-(\eta+1)/2}, & \text{otherwise,} \end{cases} \tag{4.8}$$

where $2 < \eta < \infty$ and $-1 < \lambda < 1$. The constants $a$, $b$, and $d$ are given by

$$a = 4\lambda d\left(\frac{\eta-2}{\eta-1}\right), \quad b^2 = 1 + 3\lambda^2 - a^2, \quad \text{and} \quad d = \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta-2)}\Gamma(\frac{\eta}{2})}.$$

We consider the set of parameter estimates found in the empirical application of Hansen (1994) for the weekly US Dollar vis-à-vis the Swiss Franc exchange rate given by SkSt(-0.09,8.1). In this case, the sample skewness and kurtosis of the standardized residuals is around -0.24 and 4, respectively, compared to 0.01 and 4 for the Student's t(8) distribution for $T = 500$. On the other hand, the parameter choice given by the Hansen SkSt(0.99,8.1) represents a highly skewed distribution, given that the upper bounds of skewness are $-1 < \lambda < 1$. For this case, sample skewness is around 1.6 and kurtosis is around 7 for the standardized residuals of the GARCH model when $T = 500$ to 1000. In the same vein as above, we estimate the ARCH(1) model with Hansen SkSt errors to obtain the power of the tests for a number of parameter choices of the Hansen SkSt$(\lambda, \eta)$:=(-0.09,8.1), (0.99,8.1), (0.9,3), (-0.09,5) and (0.99,5).[5] The objective is to examine the properties of the residual-based ranks tests for a highly-skewed distribution that may affect the asymptotic approximation.

We evaluate the size and power of the traditional Li and Mak (1994) test as well as the proposed rank-based quadratic residual autocorrelation tests discussed in Section 3.2. The Li and Mak (1994) test statistic is given by

$$\mathrm{LM}(r, K) = T \sum_{i=r+1}^{K} \hat{r}_i^2, \tag{4.9}$$

[5] Another asymmetric distribution considered is the Standardized Fernández and Steel (1998) Skewed-$t$ with parameters $(\xi, \nu)$=(10,3) and $(\exp(0.08), 8)$. Lambert, Laurent and Veredas (2012) use the Std FS SkSt distribution to evaluate the performance of dynamic skewness in GARCH type processes. Comparing the FS SkSt with Hansen SkSt, when $\lambda = 0$ (in the Hansen SkSt) and when $\xi = 1$ (in FS SkSt) we get the Student's $t$-distribution, where $\lambda = \frac{\xi^2-1}{\xi^2+1}$. When FS SkSt is standardized, the distribution is the same with Hansen's SkSt distribution and, therefore, the results for the Std FS SkSt distribution are not presented here.

where $\hat{r}_s$ are the residual autocorrelations

$$\hat{r}_s = \frac{\sum_{t=s+1}^{T}\left(\hat{\varepsilon}_t^2 - \bar{\hat{\varepsilon}}\right)\left(\hat{\varepsilon}_{t-s}^2 - \bar{\hat{\varepsilon}}\right)}{\sum_{t=1}^{T}\left(\hat{\varepsilon}_t^2 - \bar{\hat{\varepsilon}}\right)^2}, \quad s = r+1, ..., K \tag{4.10}$$

and $\hat{r}_s =$ is the $s^{\text{th}}$ lag squared (standardized) residual autocorrelation, $\bar{\hat{\varepsilon}} = \sum_{t=1}^{T} \hat{\varepsilon}_t^2/T$, $\hat{\varepsilon}_t = Y_t/\hat{\sigma}_t$ and $\hat{\sigma}_t^2$ the estimated variance from the GARCH model.[6]

We also consider a set of rank-based statistics. The van der Waerden (vdW) test for evaluating quadratic autocorrelation in the standardized residuals of GARCH type processes is given by

$$\text{vdW} = T(T+2)\sum_{s=r+1}^{K}\frac{r_{vdW,s}^2}{T-s}, \tag{4.11}$$

where the autocorrelation function $r_{vdW,s}$ is given in (4.5), where the ranks are now those of the squared residuals. In addition, we propose the following Absolute van der Waerden (AvdW) scores which can lead to improved power for testing quadratic dependence in the conditional variance. The AvdW rank autocorrelations are given by

$$\begin{aligned} r_{AvdW,s} = \Bigg\{ &(T-s)^{-1}\sum_{t=s+1}^{T}\Phi^{-1}\left(\frac{1+\frac{R_t}{T+1}}{2}\right)\Phi^{-1}\left(\frac{1+\frac{R_{t-s}}{T+1}}{2}\right) \\ &- [T(T-1)]^{-1}\sum\sum_{1\le i\neq j\le T}\Phi^{-1}\left(\frac{1+\frac{i}{T+1}}{2}\right)\Phi^{-1}\left(\frac{1+\frac{j}{T+1}}{2}\right)\Bigg\}\Bigg/\sigma_{AvdW}, \end{aligned} \tag{4.12}$$

where $\sigma_{AvdW}$ is given by

$$\sigma_{AvdW}^2 = \left[\text{E}\left(V_T^2\right) - \left(\text{E}\left(V_T\right)\right)^2\right](T-s), \tag{4.13}$$

with

$$\text{E}\left(V_T\right) = \frac{1}{T(T-1)}\sum_{i\neq j}\Phi^{-1}\left(\frac{1+\frac{i}{T+1}}{2}\right)\Phi^{-1}\left(\frac{1+\frac{j}{T+1}}{2}\right) \tag{4.14}$$

[6] Note that Lundbergh and Teräsvirta (2002) give a different version of the LM test compared to that in equations (4.9)-(4.10) with a different covariance matrix. However, the two covariance matrices are asymptotically equal. In the simulations we use $r = 1$ following Li and Mak (1994).

and

$$\begin{aligned}
\mathrm{E}\left(V_T^2\right) = {} & \frac{1}{T-s}\frac{1}{T(T-1)}\sum_{i\neq j}\Phi^{-1}\left(\frac{1+\frac{i}{T+1}}{2}\right)^2\Phi^{-1}\left(\frac{1+\frac{j}{T+1}}{2}\right)^2 \\
& + 2\frac{T-2s}{(T-s)^2}\frac{1}{T(T-1)(T-2)}\sum_{i\neq j\neq k}\Phi^{-1}\left(\frac{1+\frac{i}{T+1}}{2}\right)\Phi^{-1}\left(\frac{1+\frac{j}{T+1}}{2}\right)^2\Phi^{-1}\left(\frac{1+\frac{k}{T+1}}{2}\right) \\
& + \left[1-\frac{1}{T-s}-2\frac{T-2s}{(T-s)^2}\right]\frac{1}{T(T-1)(T-2)(T-3)} \\
& \sum_{i\neq j\neq k\neq z}\Phi^{-1}\left(\frac{1+\frac{i}{T+1}}{2}\right)\Phi^{-1}\left(\frac{1+\frac{j}{T+1}}{2}\right)\Phi^{-1}\left(\frac{1+\frac{k}{T+1}}{2}\right)\Phi^{-1}\left(\frac{1+\frac{z}{T+1}}{2}\right).
\end{aligned} \tag{4.15}$$

The expectations $\mathrm{E}\left(V_T\right)$ and $\mathrm{E}\left(V_T^2\right)$ follow along the lines of standard rank statistic analysis using the observation that, under the null, the ranks are uniformly distributed over all permutations of $1,\ldots,T$. Now, the test statistic based on the AvdW rank autocorrelations is given by

$$\mathrm{AvdW}(r,K) = T\sum_{i=r+1}^{K}\hat{r}^2_{AvdW,i}, \tag{4.16}$$

where $r=1$ is used in the simulations. Finally, the Wilcoxon rank autocorrelation test (Wilc) in equations (4.6) and (4.7) is used with the squared standardized residuals where the summation in (4.6) starts from $s=r+1$ instead of $s=1$. In the simulations we use the 5% $\chi^2_{K-r}$ critical value for the (4.6), (4.9), (4.11) and (4.16) statistics.

The results for the size of the quadratic residual autocorrelations tests are summarized in Tables 3, 5, and 6 for (G)ARCH processes. The corresponding power results for symmetric error distributions appear in Table 4 and for asymmetric error distributions in Table 7. The results reported in Table 3 for symmetric error innovations show that some undersizing occurs for the test statistics. This evidence is consistent with the results in Section 3.2 according to which for pure (G)ARCH type models with symmetric error distributions the asymptotic distribution of residual-based tests is more concentrated that those based on innovations. The simulation results suggest that the degree of conservativeness of these tests is small and, in general, the size is closer to the nominal 5% value for large $T\geq 1000$ for most statistics. In particular we find that the AvdW and Wilcoxon tests yield size closest to the nominal 5% level for the different (G)ARCH processes driven by alternative symmetric error distributions as $T$ increases in Table 3. The results in Table 4 show that the power of all the quadratic residual dependence tests approaches unity as $T$ increases when the GARCH processes are driven by an error with a symmetric distribution. We find that the AvdW usually provides the best power among the tests considered for very leptokurtic error distributions such as the $t(3)$, $t(5)$, and even for the $t(8)$, a result which is consistent with the properties of rank-based statistics and the Chernoff-Savage theorem. For DGPs with normal innovations, the Li Mak and AvdW tests have the highest power across all the DGPs and sample sizes $T$ considered, a result which is also consistent with asymptotic theory. Finally, it's worth mentioning that for the rank-based tests for remaining quadratic dependence in AR-GARCH residuals (with symmetric error innovations), there is no serious undersizing effect.[7]

We now turn to Tables 5 and 6 to consider the size and to Table 7 to consider the power of quadratic residual autocorrelation tests for GARCH type processes with asymmetric innovation distributions. In particular, we consider the parameter values of Hansen (1994) given by Hansen SkSt(-0.09,8.1) for the innovations of a GARCH type model, as well as the parameter set Hansen SkSt(0.99,8.1) in order to examine the case of excess skewness. The simulation analysis shows that, even if the innovation distribution is asymmetric as in the cases considered here, the tests still enjoy good size properties. We also consider the highly persistent GARCH DGPs with Hansen SkSt(-0.09,5) and the Hansen SkSt(0.99,5) distributions. For these processes, we have the same skewness as mentioned above but now the kurtosis is higher. The results in Tables 5 and 6 show that there is very mild undersizing mainly for the Li Mak test in the presence of either high skewness and/or high kurtosis. Table 7 shows that all tests considered have good power for detecting remaining quadratic residual dependence in asymmetric GARCH processes. In particular, in the presence of high skewness and/or kurtosis in highly persistent (G)ARCH processes, the power of the AvdW test outperforms considerably that of the LiMak test. For instance, for $T = 1000$ we find that the difference in power between the AvdW and LiMak test can reach up to 0.80 for the IGARCH (and highly persistent GARCH) processes driven by skewed (Hansen SkSt(0.9,3) and Hansen SkSt(0.99,8.1)) and leptokurtic distributed errors.[8] The relatively higher power of the AvdW and other rank-based tests, especially for IGARCH processes, is also consistent with the fact that the rank-

[7] For conciseness these results are available upon request from the authors.

[8] Overall the power of the tests for the Std FS SkSt(exp(0.08),8) is very similar to the power of the tests for the Hansen SkSt(-0.09,8.1) and the power for the Std FS SkSt(10,3) is very similar to the power for the Hansen SkSt(0.9,3).

based tests do not require finite moments. For the GARCH processes with Hansen SkSt(-0.09,8.1) errors, which are characterized by relatively low skewness and kurtosis, the LiMak and AvdW produce similar power and they outperform the vdW and Wilc tests.

Given that most empirical applications of GARCH processes yield highly persistent estimates, we consider the newly proposed AvdW ranks tests as a useful test to the applied econometrician. Moreover, given that most financial time series that are used to estimate GARCH processes may exhibit excess kurtosis and/or skewness, the proposed AvdW test appears to be a robust test that enjoys the relatively highest power among the tests considered here.

## 4.3 Residual symmetry tests simulations

We present simulation evidence for the properties of tests based on ranks and signs of standardized residuals for symmetry in the innovation distribution of GARCH type processes. We compare these tests with that proposed in Bai and Ng (2001). First, we simulate the GARCH(1,1) in (4.2) with and without the AR component under the null with $(\omega,\alpha_1,\beta_1)$:=(20,0.05,0.9) and $(\omega,\alpha_1,\beta_1)$:=(20,0.3,0.5) with normal, $t(3)$, and $t(5)$ innovations, following the parameter values and innovation distributions in Bai and Ng (2001). The objective here is to evaluate the size properties of the tests for pure GARCH models and the degree of conservativeness of these tests due to the AR component in AR-GARCH processes as discussed in Section 3.3.

Under the alternative hypothesis, the DGP is a GARCH(1,1) with $(\omega,\alpha_1,\beta_1)$:= (20,0.05,0.9) and $(\omega,\alpha_1,\beta_1)$:=(20,0.3,0.5) driven by innovations with the following asymmetric distributions: (i) Chi-square with 2 degrees of freedom, $\chi^2_2$ (as in Bai and Ng (2001)); (ii) Lognormal(0,1) (as in Bai and Ng (2001)); (iii) Lambda $(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$ (as in Bai and Ng (2001)) given by

$$F^{-1}(u) = \lambda_1 + [u^{\lambda_3} - (1-u)^{\lambda_4}]/\lambda_2, \tag{4.17}$$

with parameter values $(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$:=(0,-1,-0.001,-0.13); (iv) Hansen Skewed $t(\lambda,\eta)$ (Hansen (1994)), Hansen SkSt given by (4.8). The parameter values used are $(\lambda,\eta)$:= (0.9,3) and $(\lambda,\eta)$:=(0.5,3) as highly skewed alternatives, as well as $(\lambda,\eta)$:=(0.99,8.1) and $(\lambda,\eta)$:=(-0.09,8.1) following Hansen (1994), with standardized residuals' skewness and kurtosis given by 4.63 and 51.51, 3.81 and 46.02, 1.61 and 7.57, -0.26 and 4.45, respectively.[9]

[9] When FS SkSt is standardized the distribution is the same with Hansen's SkSt distribution and as expected, power for the Std FS SkSt ($(\xi,v)$:=$(\sqrt{3},3)$ and $(\sqrt{0.8349},8.1)$) and Hansen SkSt ($(\lambda,\eta)$:=(0.5,3) and (-0.09,8.1)) innovations is very similar.

In Table 8 we examine the size of the test for symmetry in the standardized residuals' distribution of GARCH and AR-GARCH processes. The following tests are employed to examine the null hypothesis of symmetry in the standardized residuals: (i) The Sign test statistic given by

$$\text{Sign} = \frac{2x \pm 1 - T}{\sqrt{T}}, \tag{4.18}$$

where $x$ is the number of positive signs. We use $+1$ when $x > T/2$, 0 when $x = T/2$ and $-1$ when $x < T/2$. (ii) The Wilcoxon signed rank test (Wilc) statistic is given by

$$\text{Wilc} = \min\left\{T^{-}, \frac{T(T+1)}{2} - T^{-}\right\}, \tag{4.19}$$

where $T$ is the sample size and $T^{-}$ is the sum of the ranks of the negative differences. Under the null the distribution of the test statistics in (4.18) and (4.19) follows asymptotically the Binomial distribution ($B(x, 0.5)$ and $B(T^{-}, 0.5)$ respectively). (iii) The Bai and Ng (2001) (BN) test is given by

$$BN = \max\{|S_T(x)|\}, \tag{4.20}$$

where $S_T(x) = \hat{W}_T(x) - \hat{W}_T(0) + \int_x^0 h_T^{-}(y)\,\mathrm{d}y$ for $x \leq 0$, $S_T(x) = \hat{W}_T(x) - \hat{W}_T(0) + \int_0^x h_T^{-}(y)\,\mathrm{d}y$ for $x > 0$ and $W_T(x) = \frac{1}{\sqrt{T}}\sum_{t=1}^{T}[I(e_t \leq x) - I(-e_t \leq x)]$ with $e_t = (Y_t - E(Y))/\sigma_t$. The asymptotic critical value of the test is 2.20 at the 5% level and the asymptotic distribution of $S_T(x)$ is $\max_{0 \leq s \leq 1} |B(s)|$, where $B(r)$ is a standard Brownian motion on $[0, 1]$.

The results in Table 8 show that all the tests considered above have size close to 5% for GARCH processes, as opposed to the AR-GARCH processes where serious undersizing is evident for the Sign and Wilcoxon tests. This simulation evidence is consistent with the theoretical results in Section 3.3 and Proposition 1. However, compared to the rest of the tests evaluated so far in the simulations for AR-GARCH DGPs, we find that only for the rank-based residual symmetry tests there is serious undersizing compared to the quadratic residual autocorrelation tests. In terms of power, we focus on the alternative of a pure GARCH DGP with asymmetric error distributions. The third panel of Table 8 shows that for the very skewed alternative distributions, namely the $\chi_2^2$, the Lambda, and the Lognormal(0,1) distributions, considered in Bai and Ng (2001), all tests enjoy power close to one. However, some interesting differences arise across the alternative tests when the innovations of the GARCH are driven by the Hansen SkSt distribution with parameter values as shown in Table 8. As expected, generally power improves with the degree of skewness for the Hansen SkSt distributions. For the Hansen SkSt cases that exhibit both high skewness and kurtosis, namely the SkSt(0.9,3) and SkSt(0.5,3), the Bai and Ng (2001) (BN) test and Sign test have power equal to 1 for $T \geq 500$. For the Hansen SkSt(-0.09, 8.1) which exhibits the relatively lowest degree of skewness the power is lower, as expected, except for the BN test and the Sign test for $T \geq 1000$.[10]

## 4.4 Structural break tests simulations

The objective of this final simulation section is to examine the properties of structural breaks or change-points applied to the squared standardized residuals of GARCH processes. Under the null hypothesis there is no change in the GARCH parameters given by

$$H_0 : \begin{cases} Y_t = \sigma_t \varepsilon_t, & t = 0, 1, ..., T \\ \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i Y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \end{cases} \tag{4.21}$$

against the alternative hypothesis of one permanent change in the GARCH parameters

$$H_1 : \begin{cases} Y_t = \sigma_t \varepsilon_t, \\ \sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^{p} \alpha_i Y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, & \text{if } t = 0, ..., \pi T \\ \omega' + \sum_{i=1}^{p} \alpha_i' Y_{t-i}^2 + \sum_{j=1}^{q} \beta_j' \sigma_{t-j}^2, & \text{if } t = \pi T + 1, ..., T \end{cases} \end{cases} \tag{4.22}$$

where

$$(\omega, \alpha_1, ..., \alpha_p, \beta_1, ..., \beta_q) \neq (\omega', \alpha_1', ..., \alpha_p', \beta_1', ..., \beta_q'),$$

and $0 < \pi < 1$. We consider the following DGPs to examine the size and power of the tests:
(i) GARCH(1,1) with ($\omega$,$\alpha_1$,$\beta_1$):=(0.0002,0.1,0.7) under the null and (0.0003,0.1,0.7), (0.0002,0.167,0.7) and (0.0002,0.1,0.767) under the alternatives. Similar DGPs and parameters are considered in Kulperger and Yu (2005).
(ii) GARCH(1,1) with ($\omega$,$\alpha_1$,$\beta_1$):=(0.0002,0.1,0.8) under the null and (0.0003,0.1,0.8), (0.0002,0.167,0.8) and (0.0002,0.1,0.867) under the alternatives. The distributions considered for the innovations are the N(0,1) and the Student's t(3), t(5) and t(8) and break points at $\pi = 0.5$ and 0.7.

We consider the following test statistics to evaluate the null hypothesis of stability in the squared standardized residuals of GARCH processes: the Kulperger and

[10] We find that the power of these tests for the Std FS SkSt(10,3) is similar to the power for the Hansen SkSt(0.9,3), but slightly larger for all tests.

Yu (2005) CUSUM (KY), a corresponding rank residual-based CUSUM (RBC) test proposed here, the Pettitt (1979) test (Pet) and the Lombard (1987) test (Lomb). Kulperger and Yu (2005) recently proposed the (KY) $CUSUM$ of squared residuals tests.

The KY statistic is given by

$$KY\ CUSUM_1 = \max_{1\leq i<T} \frac{\left| \sum_{t=1}^{i} \hat{\varepsilon}_t^2 - i \sum_{t=1}^{T} \hat{\varepsilon}_t^2 / T \right|}{\hat{\nu}_2 \sqrt{T}}, \tag{4.23}$$

where

$$\hat{\nu}_2^2 = \frac{1}{T} \sum_{t=1}^{T} ((\hat{\varepsilon}_t - \bar{\hat{\varepsilon}})^2 - \hat{\sigma}_{(T)}^2)^2.$$

The second statistic is defined as

$$KY\ CUSUM_2 = \max_{1\leq i<T} \frac{\left| \sum_{t=1}^{i} (\hat{\varepsilon}_t - \bar{\hat{\varepsilon}})^2 - i \hat{\sigma}_{(T)}^2 \right|}{\hat{\nu}_2 \sqrt{T}}, \tag{4.24}$$

where $\bar{\hat{\varepsilon}}$ is the residual sample mean and $\hat{\sigma}_{(T)}^2$ the residual sample variance. The limiting null distribution of the above statistics is that of the supremum of a Brownian Bridge. The $KYCUSUM_2$ is centred about the residual sample mean $\bar{\hat{\varepsilon}}$ in contrast to the non centred $KYCUSUM_1$. The two statistics give very similar results. In the simulation results we focus on the $KY\ CUSUM_2$ (denoted by KY in the Tables) given that Kulperger and Yu (2005) also report results for this statistic only.

The rank-based CUSUM (RBC) of squared GARCH standardized residual test is given by

$$\text{RBC} = \max \left| T^{-\frac{1}{2}} \sum_{i=1}^{\lfloor Ts \rfloor} s(R_i) \right|, \tag{4.25}$$

where

$$s(R_i) = A^{-1} \left[ \left( \frac{R_i}{T+1} \right) - \frac{1}{2} \right], \tag{4.26}$$

and

$$A^2 = (T-1)^{-1} \sum_{i=1}^{T} \left[ \left( \frac{R_i}{T+1} \right) - \frac{1}{2} \right]^2.$$

The asymptotic distribution of $s(R_i)$ is that of a Brownian Motion (BM) and of RBC that of a Brownian Bridge (under the null hypothesis of no breaks and when these tests are applied to the true innovations). Given the results in Section 3.4, we expect that no size correction is required in the asymptotic distribution of the test when applied to estimated squared standardized residuals of pure GARCH processes. Hence, we use 5% critical value (of 1.36) in the simulations. Similarly, Lombard (1987) proposed a rank-based score test for change-points in independent random variables. Following our results in Section 3.4 we can also apply this test to the squared standardized residuals given in (4.26) and the statistic given by

$$\text{Lomb} = \sum_{t=1}^{T-1} \sum_{i=1}^{t} s(R_i)^2. \tag{4.27}$$

follows the Cramer-von Mises asymptotic distribution. We apply the 5% critical value (of 0.461) in simulations, thus again adopting no size correction in the asymptotic distribution of the test.

Finally, the Pettitt (Pet) test is based on testing the null hypothesis that a sequence of random variables has a common distribution function against the alternative of having a change point. The statistic is given by

$$\text{Pet} = \sup_{t=1,\ldots,T} |\hat{\varepsilon}_t^2|, \tag{4.28}$$

where

$$\hat{\varepsilon}_t^2 = \left( T^{-1} \sqrt{\frac{3}{T+1}} \right) U_{t,T} \tag{4.29}$$

and

$$U_{t,T} = 2W_t - t(T+1) \text{ with } W_t = \sum_{j=1}^{t} R_j.$$

The limiting distribution of the Pettitt statistic, under the null, is again the supremum of a Brownian Bridge.

The results in Table 9 show that for most tests size approaches 5% as $T$ increases, which is consistent with the results in Section 3.4 according to which no size correction is expected for the residual-based tests. We find that the Lombard test yields size closer to the nominal 5%, whereas the KY test is often undersized (which agrees with the findings of Kulperger and Yu (2005)). In terms of power the KY performs best for detecting breaks in the standardized residuals of GARCH processes driven by $N(0,1)$ errors, for all $T$ considered. In general, as the sample size $T$ increases, all tests have similar power for the alternative innovations' distributions. These results are consistent with the asymptotic theory and hold for different levels of persistence in the GARCH, different break sizes and different locations of the break point. A notable exception is found for the heavy tailed distributed $t(3)$ GARCH processes where the power of the RBC test outperforms that of the KY test even for large $T$. The findings in Table 9 show that, for $t(3)$ driven GARCH processes and for all the cases considered under the alternative, the KY test has no power for $T = 500$ and 1000 and can only reach a power of 0.27 for $T = 3000$. In contrast, the RBC test for the $t(3)$ GARCH model enjoys high power already for $T \geq 1000$ and the maximum difference between the power of the KY and RBC tests is found to be a sizeable 0.6. Note that the Pettitt and Lombard tests give very similar size and power to the RBC. This simulation evidence is consistent with the properties of rank statistics which are robust to heavy-tailed distributions. Finally it is worth mentioning that the size of the rank quadratic residual-based tests for structural breaks also approximates the 5% for AR-GARCH DGPs with symmetric error innovations and there is no undersizing effect compared to previous tests.[11]

The size and power of the tests were also evaluated for the Hansen Skewed Student's $t$-distribution. The choice of the parameters was based on Hansen (1994), namely SkSt(-0.09,8.1), SkSt(-0.15,5.57), and SkSt(0.05,4.23).[12] The results in Table 9 show that for the Hansen SkSt distribution with parameters as in Hansen (1994) (which have low kurtosis and skewness) the size and power are very similar to those of the normal distribution. In contrast, when the Hansen distribution function becomes more skewed and leptokurtic the power of the rank-based tests is higher than that of the KY test for $T \geq 1000$.

Overall, the improved power results of rank-based quadratic standardized residual tests are consistent with the properties of rank test statistics which are more robust to different innovation distributions.

# 5 Empirical Application

## 5.1 Data

We consider two groups of stock markets returns indices. The first refers to 12 emerging market stock returns indices located in Asia and Latin America, namely Argentina (ARG), Brazil (BRA), Chile (CHI), Colombia (COL), India (IND), Korea (KOR), Malaysia (MAL), Mexico (MEX), Pakistan (PAK), Philippines (PHIL), Thailand (THAI), and Venezuela (VEN). We consider weekly data for the sample

[11] For conciseness we do not report the size of the breaks tests for the AR-GARCH DGPs but these are available upon request from the authors.

[12] This choice matches the parameters of two emerging stock market returns considered in the empirical section below, namely PAK and VEN.

period 06/01/1989-15/08/2008 with sample size $T = 1024$ for all emerging countries except Venezuela which is 06/01/1989-06/04/2007 with $T = 953$, due to data availability. The data source for the emerging stock market index returns is the emerging markets database (EMDB) of the Standard and Poor's except for Pakistan which is the Global Financial Data database, which has the updated data for this series. The second group refers to major stock markets represented by the indices of the S&P500 (US), the FTSE100 (UK), the NIKKEI225 (Japan), the DAX (Germany), the CAC 40 (France), the S&P/TSX (Canada) and the HANG SENG (Hong Kong). The series were taken from Datastream and the Global Financial Data database. For comparison purposes we also consider the sample period from 07/01/1989-16/08/2008 for these series. For each price index, we study the sample ending at the end of August 2008 marking the period before the collapse of the Lehman Brothers and the recent financial crisis and we obtain the weekly returns defined as $r_t = 100\left(\ln\left(p_t\right) - \ln\left(p_{t-1}\right)\right)$, where $p_t$ is the price at time $t$ in local currency.

Some descriptive statistics for the stock market indices are reported in Table 10. In general, the emerging markets stock returns indices exhibit similar skewness compared to some of the major stock market indices, except Argentina which exhibits the relatively highest skewness. The emerging stock market returns indices are relatively more leptokurtic compared to some of the major stock market returns indices with Argentina and Malaysia showing considerably large kurtosis. This motivates our use of rank-based statistics below.

The estimation results reported in Table 11 show that the GARCH coefficients are significant for all series and the null hypothesis of IGARCH effects is not rejected in around half of the countries considered here. It is important to note that the theoretical results for GARCH processes, are also valid for (stationary) IGARCH models, i.e., finite variance of the observed process is not needed in the QMLE and LAN results referred to before. On the other hand, the estimated AR coefficients are relatively small, although they are (statistically) significant for the S&P500 and most of the emerging markets. For the emerging stock markets, there is relatively higher kurtosis in the standardized residuals compared to the major markets as shown in the last two columns of Table 10.

## 5.2 Empirical results

The empirical results in Tables 12 and 13 lead to some interesting conclusions related to the normal AR(1)-GARCH(1,1) benchmark model of volatility for some of the major and emerging stock market returns indices. We allow for an AR(1) given that misspecifying the conditional mean can lead to spurious GARCH effects (e.g., Lumsdaine and Ng (1999)).

The residual specification tests in Table 12 show that for the major stock market weekly returns indices considered, the AR(1)-GARCH(1,1) specification captures the dynamics of these series given that almost all the linear and quadratic autocorrelation tests for the standardized residuals do not provide evidence against the null hypotheses. In contrast, the evidence in Table 13 for the linear dynamics of the weekly emerging stock market returns indices is different from that of most of the major stock market indices. Namely, in seven of the twelve emerging markets (Argentina, Brazil, Colombia, Malasia, Pakistan, Philippines and Thailand) the null hypothesis of no linear autocorrelation in the standardized residuals of the AR(1)-GARCH(1,1) model is rejected by almost all tests. Interestingly, though the GARCH model captures the quadratic dynamics of both the emerging and major weekly stock market returns indices. There are however some sporadic rejections of the null hypothesis of no remaining quadratic standardised residual autocorrelation by some of the rank tests.

Testing the null hypothesis that the conditional distribution of the standardized residuals of the normal AR(1)-GARCH(1,1) model is symmetric, we find empirical support in most developed and emerging stock markets returns except the US, Germany, Brazil, Chile and Pakistan. In most of these cases Bai and Ng (2001) test provides evidence against the null hypothesis of symmetry.

In testing for the null hypothesis of no structural breaks in the GARCH coefficients we find an interesting difference between rank residual-based tests vis-à-vis other recent tests proposed for GARCH type models. While the KY test provides no evidence of instability in the standardized residuals, the rank tests detect structural breaks in the squared standardized residuals of this model in three emerging markets, Brazil, Pakistan and Venezuela, which are associated with some major economic and political events.[13] The descriptive statistics results in Table 10 also show that Brazil and Venezuela exhibit relatively high skewness and kurtosis in the standardized residuals (as shown in the last two columns). This is consistent with the theoretical and simulation evidence reported in Table 8 which shows that the power of the rank statistics is superior than the KY test statistic for non-normal distributions.

[13]"Real plan" to tackle inflation implemented in Brazil with new currency introduced on 01/07/1994, Pakistan elections for national assembly were held on 06/10/1993 where Benazir Bhutto was elected prime minister and on 25/05/2000 the Venezuelan authorities declared the postponement of the presidential elections that were scheduled on 28/05/2000.

# 6 Concluding remarks and future work

We consider specification testing for location-scale dynamic models. In particular, we focus on testing for linear and quadratic residual autocorrelation, testing for symmetry of the innovation distribution, and testing for structural breaks. Many specification tests for these hypotheses have been proposed. We provide exact asymptotic size corrections needed when applying general innovation-based tests to residuals. We show that, in many cases though not always, *ignoring* such size correction makes the residual-based test conservative. We provide precise conditions under which this occurs.

We present a number of rank-based tests for the above-mentioned hypotheses. We show that their strong power properties often outweigh the conservativeness when applied to residuals. Thus, these rank-based tests are more powerful than their classical counterparts. Therefore, an empiricist can, in a first-step, rely on the readily available standard critical values ignoring any adjustment and, only if desired, our theory can be used to adjust critical values to further improve power. These results are corroborated by extensive simulations. An application to some of the major and emerging market stock return indices illustrated this empirical strategy.

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Table 1: Simulation results for the size (using GARCH models) and power (using AR(1)-GARCH models) of alternative tests for remaining linear residual autocorrelation in the standardized residuals of AR-GARCH type models

| | | | Max lag used in tests K=2 | | | | Max lag used in tests K=4 | | | | Max lag used in tests K=6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Model | T | Test | Size | P1 | P2 | P3 | Size | P1 | P2 | P3 | Size | P1 | P2 | P3 |
| AR(1)-GARCH(1,1) | 300 | LB | 0.039 | 0.249 | 0.772 | 1.000 | 0.053 | 0.192 | 0.691 | 1.000 | 0.051 | 0.157 | 0.618 | 1.000 |
| $(c_0, \phi)$-(0.1,0.1,0.8) | | vdW | 0.035 | 0.247 | 0.766 | 1.000 | 0.045 | 0.180 | 0.673 | 1.000 | 0.041 | 0.144 | 0.595 | 1.000 |
| | | Wilc | 0.048 | 0.239 | 0.798 | 1.000 | 0.048 | 0.208 | 0.700 | 1.000 | 0.049 | 0.151 | 0.637 | 1.000 |
| N(0,1) | 500 | LB | 0.052 | 0.394 | 0.961 | 1.000 | 0.055 | 0.336 | 0.924 | 1.000 | 0.060 | 0.237 | 0.886 | 1.000 |
| | | vdW | 0.048 | 0.379 | 0.957 | 1.000 | 0.050 | 0.320 | 0.917 | 1.000 | 0.052 | 0.231 | 0.880 | 1.000 |
| | | Wilc | 0.067 | 0.441 | 0.961 | 1.000 | 0.073 | 0.369 | 0.929 | 1.000 | 0.065 | 0.304 | 0.882 | 1.000 |
| | 1000 | LB | 0.052 | 0.761 | 1.000 | 1.000 | 0.052 | 0.619 | 0.998 | 1.000 | 0.052 | 0.561 | 0.998 | 1.000 |
| | | vdW | 0.051 | 0.760 | 1.000 | 1.000 | 0.053 | 0.616 | 0.998 | 1.000 | 0.047 | 0.554 | 0.998 | 1.000 |
| | | Wilc | 0.070 | 0.752 | 1.000 | 1.000 | 0.073 | 0.641 | 1.000 | 1.000 | 0.062 | 0.589 | 0.999 | 1.000 |
| | 2000 | LB | 0.044 | 0.966 | 1.000 | 1.000 | 0.045 | 0.924 | 1.000 | 1.000 | 0.039 | 0.901 | 1.000 | 1.000 |
| | | vdW | 0.040 | 0.967 | 1.000 | 1.000 | 0.044 | 0.923 | 1.000 | 1.000 | 0.035 | 0.902 | 1.000 | 1.000 |
| | | Wilc | 0.049 | 0.970 | 1.000 | 1.000 | 0.067 | 0.920 | 1.000 | 1.000 | 0.076 | 0.909 | 1.000 | 1.000 |
| AR(1)-GARCH(1,1) | 300 | LB | 0.046 | 0.187 | 0.651 | 0.992 | 0.058 | 0.136 | 0.495 | 0.990 | 0.051 | 0.132 | 0.419 | 0.971 |
| $(c_0, \phi)$-(0.1,0.1,0.8) | | vdW | 0.057 | 0.354 | 0.868 | 1.000 | 0.077 | 0.255 | 0.761 | 0.999 | 0.096 | 0.222 | 0.707 | 1.000 |
| | | Wilc | 0.040 | 0.376 | 0.920 | 1.000 | 0.058 | 0.288 | 0.852 | 1.000 | 0.041 | 0.257 | 0.820 | 1.000 |
| t(3) | 500 | LB | 0.036 | 0.287 | 0.849 | 0.996 | 0.041 | 0.227 | 0.741 | 0.991 | 0.050 | 0.179 | 0.672 | 0.992 |
| | | vdW | 0.053 | 0.511 | 0.978 | 1.000 | 0.067 | 0.407 | 0.937 | 1.000 | 0.075 | 0.394 | 0.919 | 0.999 |
| | | Wilc | 0.061 | 0.592 | 0.996 | 1.000 | 0.054 | 0.512 | 0.981 | 1.000 | 0.068 | 0.422 | 0.966 | 1.000 |
| | 1000 | LB | 0.050 | 0.546 | 0.974 | 0.999 | 0.037 | 0.411 | 0.957 | 1.000 | 0.046 | 0.377 | 0.936 | 0.996 |
| | | vdW | 0.057 | 0.812 | 0.998 | 1.000 | 0.059 | 0.696 | 0.998 | 1.000 | 0.064 | 0.665 | 0.997 | 1.000 |
| | | Wilc | 0.053 | 0.887 | 1.000 | 1.000 | 0.058 | 0.798 | 1.000 | 1.000 | 0.065 | 0.754 | 1.000 | 1.000 |
| | 2000 | LB | 0.054 | 0.832 | 0.998 | 1.000 | 0.043 | 0.762 | 0.993 | 0.997 | 0.051 | 0.711 | 0.995 | 1.000 |
| | | vdW | 0.059 | 0.980 | 1.000 | 1.000 | 0.044 | 0.943 | 1.000 | 1.000 | 0.046 | 0.923 | 1.000 | 1.000 |
| | | Wilc | 0.063 | 0.995 | 1.000 | 1.000 | 0.076 | 0.985 | 1.000 | 1.000 | 0.060 | 0.975 | 1.000 | 1.000 |
| AR(1)-GARCH(1,1) | 300 | LB | 0.044 | 0.192 | 0.725 | 1.000 | 0.045 | 0.160 | 0.635 | 0.999 | 0.056 | 0.110 | 0.543 | 1.000 |
| $(c_0, \phi)$-(0.1,0.1,0.8) | | vdW | 0.041 | 0.218 | 0.800 | 1.000 | 0.051 | 0.198 | 0.705 | 1.000 | 0.052 | 0.233 | 0.596 | 1.000 |
| | | Wilc | 0.050 | 0.289 | 0.855 | 1.000 | 0.060 | 0.260 | 0.793 | 1.000 | 0.074 | 0.246 | 0.727 | 1.000 |
| t(5) | 500 | LB | 0.051 | 0.351 | 0.927 | 1.000 | 0.038 | 0.272 | 0.860 | 1.000 | 0.037 | 0.227 | 0.825 | 1.000 |
| | | vdW | 0.052 | 0.437 | 0.959 | 1.000 | 0.046 | 0.334 | 0.918 | 1.000 | 0.034 | 0.272 | 0.878 | 1.000 |
| | | Wilc | 0.062 | 0.508 | 0.981 | 1.000 | 0.067 | 0.400 | 0.954 | 1.000 | 0.056 | 0.350 | 0.937 | 1.000 |
| | 1000 | LB | 0.040 | 0.663 | 1.000 | 1.000 | 0.054 | 0.563 | 0.994 | 1.000 | 0.051 | 0.482 | 0.993 | 1.000 |
| | | vdW | 0.043 | 0.761 | 1.000 | 1.000 | 0.052 | 0.665 | 0.998 | 1.000 | 0.079 | 0.557 | 0.998 | 1.000 |
| | | Wilc | 0.064 | 0.815 | 1.000 | 1.000 | 0.073 | 0.744 | 1.000 | 1.000 | 0.084 | 0.646 | 0.999 | 1.000 |
| | 2000 | LB | 0.043 | 0.935 | 1.000 | 1.000 | 0.049 | 0.869 | 1.000 | 1.000 | 0.043 | 0.809 | 1.000 | 1.000 |
| | | vdW | 0.050 | 0.973 | 1.000 | 1.000 | 0.056 | 0.937 | 1.000 | 1.000 | 0.046 | 0.888 | 1.000 | 1.000 |
| | | Wilc | 0.063 | 0.988 | 1.000 | 1.000 | 0.080 | 0.959 | 1.000 | 1.000 | 0.056 | 0.929 | 1.000 | 1.000 |
| AR(1)-GARCH(1,1) | 300 | LB | 0.058 | 0.251 | 0.755 | 1.000 | 0.059 | 0.194 | 0.656 | 1.000 | 0.048 | 0.151 | 0.574 | 1.000 |
| $(c_0, \phi)$-(0.1,0.1,0.8) | | vdW | 0.049 | 0.264 | 0.781 | 1.000 | 0.048 | 0.197 | 0.677 | 1.000 | 0.039 | 0.155 | 0.586 | 1.000 |
| | | Wilc | 0.056 | 0.308 | 0.835 | 1.000 | 0.073 | 0.257 | 0.741 | 1.000 | 0.061 | 0.210 | 0.683 | 1.000 |
| t(8) | 500 | LB | 0.055 | 0.385 | 0.950 | 1.000 | 0.058 | 0.287 | 0.907 | 1.000 | 0.056 | 0.255 | 0.850 | 1.000 |
| | | vdW | 0.045 | 0.400 | 0.965 | 1.000 | 0.057 | 0.299 | 0.928 | 1.000 | 0.051 | 0.252 | 0.869 | 1.000 |
| | | Wilc | 0.059 | 0.477 | 0.977 | 1.000 | 0.081 | 0.382 | 0.951 | 1.000 | 0.068 | 0.319 | 0.911 | 1.000 |
| | 1000 | LB | 0.049 | 0.687 | 1.000 | 1.000 | 0.043 | 0.585 | 0.998 | 1.000 | 0.050 | 0.538 | 0.994 | 1.000 |
| | | vdW | 0.046 | 0.734 | 1.000 | 1.000 | 0.042 | 0.615 | 0.998 | 1.000 | 0.045 | 0.565 | 0.996 | 1.000 |
| | | Wilc | 0.057 | 0.765 | 1.000 | 1.000 | 0.055 | 0.673 | 1.000 | 1.000 | 0.068 | 0.647 | 0.999 | 1.000 |
| | 2000 | LB | 0.050 | 0.948 | 1.000 | 1.000 | 0.040 | 0.922 | 1.000 | 1.000 | 0.046 | 0.854 | 1.000 | 1.000 |
| | | vdW | 0.050 | 0.967 | 1.000 | 1.000 | 0.041 | 0.930 | 1.000 | 1.000 | 0.044 | 0.873 | 1.000 | 1.000 |
| | | Wilc | 0.069 | 0.975 | 1.000 | 1.000 | 0.062 | 0.949 | 1.000 | 1.000 | 0.071 | 0.904 | 1.000 | 1.000 |

**Notes:** The table shows the size/power for the Ljung-Box test (LB), the van der Waerden test (vdW) and the Wilcoxon test (Wilc). For the size calculations a GARCH(1,1) model was generated and estimated. For the power calculations an AR(1)-GARCH(1,1) model was generated and a GARCH(1,1) was estimated. The AR(1)-GARCH(1,1) model with AR coefficients ($c_0$,$\phi$) and GARCH(1,1) coefficients ($\omega$,$\alpha_1$,$\beta_1$) is given by $Y_t = c_0 + \phi Y_{t-1} + \sigma_t \varepsilon_t$ with $\sigma_t^2 = \omega + \alpha_1 v_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ and $v_t = \sigma_t \varepsilon_t$. The models used in the simulations are the AR(1)-GARCH(1,1) model with GARCH(1,1) coefficients ($\omega$,$\alpha_1$,$\beta_1$):=(0.1,0.1,0.8) and AR(1) coefficients ($c_0, \phi$):=(0,0) for size and (0,0.1) for P1, (0,0.2) for P2 and (0,0.5) for P3, where P refers to the cases for evaluating the power of the tests. The innovation's distribution is the normal, the Student's t(3), t(5) and t(8). The maximum lag order, K, used in the tests indicate that the first K standardized residual autocorrelations are used in the test statistic, i.e. $s = 1, ..., K$. The critical value of the tests is based on the 5% significance level of the $\chi_K^2$. The above results refer to 5000 simulations.

Table 2: Simulation results for the size (using AR(1)-GARCH) and power (using AR(2)-GARCH) of alternative tests for remaining linear residual autocorrelation in the standardised residuals of AR-GARCH models

| | | | Max lag used in tests K=2 | | | | Max lag used in tests K=4 | | | | Max lag used in tests K=6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Model | T | Test | S1 | S2 | P1 | P2 | S1 | S2 | P1 | P2 | S1 | S2 | P1 | P2 |
| AR(1)-GARCH(1,1) | 300 | LB | 0.010 | 0.022 | 0.173 | 1.000 | 0.013 | 0.019 | 0.162 | 1.000 | 0.024 | 0.025 | 0.104 | 0.998 |
| $(c_0, \phi)$-(0.1,0.1,0.8) | | vdW | 0.009 | 0.020 | 0.163 | 1.000 | 0.011 | 0.016 | 0.140 | 1.000 | 0.021 | 0.024 | 0.102 | 0.997 |
| | | Wilc | 0.013 | 0.029 | 0.186 | 0.999 | 0.022 | 0.026 | 0.170 | 1.000 | 0.040 | 0.040 | 0.118 | 0.999 |
| N(0,1) | 500 | LB | 0.013 | 0.018 | 0.303 | 1.000 | 0.024 | 0.025 | 0.258 | 1.000 | 0.032 | 0.028 | 0.225 | 1.000 |
| | | vdW | 0.012 | 0.017 | 0.282 | 1.000 | 0.021 | 0.021 | 0.247 | 1.000 | 0.023 | 0.023 | 0.214 | 1.000 |
| | | Wilc | 0.015 | 0.023 | 0.309 | 1.000 | 0.030 | 0.047 | 0.281 | 1.000 | 0.047 | 0.042 | 0.267 | 1.000 |
| | 1000 | LB | 0.016 | 0.024 | 0.610 | 1.000 | 0.027 | 0.020 | 0.564 | 1.000 | 0.031 | 0.025 | 0.500 | 1.000 |
| | | vdW | 0.015 | 0.022 | 0.605 | 1.000 | 0.024 | 0.020 | 0.553 | 1.000 | 0.028 | 0.024 | 0.499 | 1.000 |
| | | Wilc | 0.024 | 0.031 | 0.620 | 1.000 | 0.038 | 0.035 | 0.577 | 1.000 | 0.046 | 0.035 | 0.542 | 1.000 |
| | 2000 | LB | 0.015 | 0.022 | 0.913 | 1.000 | 0.025 | 0.023 | 0.907 | 1.000 | 0.034 | 0.023 | 0.875 | 1.000 |
| | | vdW | 0.015 | 0.022 | 0.911 | 1.000 | 0.024 | 0.022 | 0.913 | 1.000 | 0.029 | 0.020 | 0.875 | 1.000 |
| | | Wilc | 0.016 | 0.028 | 0.908 | 1.000 | 0.035 | 0.033 | 0.916 | 1.000 | 0.054 | 0.037 | 0.887 | 1.000 |
| AR(1)-GARCH(1,1) | 300 | LB | 0.021 | 0.029 | 0.141 | 0.981 | 0.021 | 0.034 | 0.113 | 0.966 | 0.037 | 0.032 | 0.098 | 0.974 |
| $(c_0, \phi)$-(0.1,0.1,0.8) | | vdW | 0.024 | 0.036 | 0.232 | 0.999 | 0.040 | 0.055 | 0.212 | 0.995 | 0.061 | 0.062 | 0.227 | 0.998 |
| | | Wilc | 0.019 | 0.028 | 0.254 | 1.000 | 0.030 | 0.038 | 0.213 | 1.000 | 0.049 | 0.028 | 0.223 | 1.000 |
| t(3) | 500 | LB | 0.022 | 0.033 | 0.229 | 0.993 | 0.029 | 0.039 | 0.205 | 0.998 | 0.025 | 0.034 | 0.151 | 0.987 |
| | | vdW | 0.024 | 0.032 | 0.382 | 0.999 | 0.043 | 0.045 | 0.365 | 1.000 | 0.038 | 0.036 | 0.290 | 1.000 |
| | | Wilc | 0.013 | 0.030 | 0.428 | 1.000 | 0.036 | 0.036 | 0.443 | 1.000 | 0.032 | 0.032 | 0.362 | 1.000 |
| | 1000 | LB | 0.024 | 0.020 | 0.450 | 0.997 | 0.038 | 0.042 | 0.393 | 0.999 | 0.037 | 0.037 | 0.356 | 0.997 |
| | | vdW | 0.028 | 0.024 | 0.709 | 1.000 | 0.035 | 0.040 | 0.676 | 1.000 | 0.047 | 0.039 | 0.622 | 1.000 |
| | | Wilc | 0.019 | 0.022 | 0.781 | 1.000 | 0.030 | 0.036 | 0.773 | 1.000 | 0.042 | 0.049 | 0.743 | 1.000 |
| | 2000 | LB | 0.018 | 0.022 | 0.769 | 0.998 | 0.027 | 0.039 | 0.757 | 0.999 | 0.044 | 0.039 | 0.704 | 0.999 |
| | | vdW | 0.017 | 0.021 | 0.951 | 1.000 | 0.036 | 0.036 | 0.948 | 1.000 | 0.049 | 0.028 | 0.924 | 1.000 |
| | | Wilc | 0.023 | 0.021 | 0.986 | 1.000 | 0.040 | 0.042 | 0.980 | 1.000 | 0.046 | 0.035 | 0.983 | 1.000 |
| AR(1)-GARCH(1,1) | 300 | LB | 0.012 | 0.024 | 0.144 | 1.000 | 0.018 | 0.032 | 0.135 | 0.998 | 0.027 | 0.038 | 0.104 | 0.995 |
| $(c_0, \phi)$-(0.1,0.1,0.8) | | vdW | 0.008 | 0.022 | 0.151 | 1.000 | 0.011 | 0.024 | 0.148 | 1.000 | 0.027 | 0.024 | 0.122 | 0.998 |
| | | Wilc | 0.010 | 0.025 | 0.186 | 1.000 | 0.027 | 0.033 | 0.187 | 1.000 | 0.036 | 0.031 | 0.163 | 1.000 |
| t(5) | 500 | LB | 0.021 | 0.029 | 0.279 | 1.000 | 0.021 | 0.041 | 0.239 | 1.000 | 0.033 | 0.033 | 0.194 | 0.999 |
| | | vdW | 0.017 | 0.031 | 0.305 | 1.000 | 0.022 | 0.032 | 0.273 | 1.000 | 0.022 | 0.026 | 0.227 | 1.000 |
| | | Wilc | 0.019 | 0.041 | 0.353 | 1.000 | 0.028 | 0.040 | 0.341 | 1.000 | 0.047 | 0.039 | 0.302 | 1.000 |
| | 1000 | LB | 0.010 | 0.026 | 0.580 | 1.000 | 0.023 | 0.033 | 0.499 | 1.000 | 0.036 | 0.032 | 0.431 | 1.000 |
| | | vdW | 0.006 | 0.025 | 0.658 | 1.000 | 0.020 | 0.032 | 0.588 | 1.000 | 0.031 | 0.033 | 0.496 | 1.000 |
| | | Wilc | 0.010 | 0.031 | 0.719 | 1.000 | 0.028 | 0.040 | 0.656 | 1.000 | 0.039 | 0.046 | 0.605 | 1.000 |
| | 2000 | LB | 0.018 | 0.023 | 0.898 | 1.000 | 0.039 | 0.039 | 0.843 | 1.000 | 0.021 | 0.033 | 0.806 | 1.000 |
| | | vdW | 0.021 | 0.018 | 0.940 | 1.000 | 0.032 | 0.025 | 0.894 | 1.000 | 0.025 | 0.032 | 0.877 | 1.000 |
| | | Wilc | 0.024 | 0.031 | 0.958 | 1.000 | 0.037 | 0.028 | 0.941 | 1.000 | 0.039 | 0.051 | 0.925 | 1.000 |
| AR(1)-GARCH(1,1) | 300 | LB | 0.015 | 0.017 | 0.149 | 0.998 | 0.029 | 0.034 | 0.148 | 0.999 | 0.032 | 0.024 | 0.101 | 0.997 |
| $(c_0, \phi)$-(0.1,0.1,0.8) | | vdW | 0.010 | 0.020 | 0.143 | 1.000 | 0.024 | 0.026 | 0.140 | 0.999 | 0.025 | 0.023 | 0.106 | 0.998 |
| | | Wilc | 0.013 | 0.031 | 0.184 | 1.000 | 0.031 | 0.038 | 0.174 | 0.999 | 0.039 | 0.037 | 0.146 | 0.998 |
| t(8) | 500 | LB | 0.017 | 0.034 | 0.284 | 1.000 | 0.020 | 0.029 | 0.228 | 1.000 | 0.029 | 0.024 | 0.204 | 1.000 |
| | | vdW | 0.014 | 0.022 | 0.290 | 1.000 | 0.018 | 0.023 | 0.239 | 1.000 | 0.024 | 0.023 | 0.221 | 1.000 |
| | | Wilc | 0.018 | 0.030 | 0.334 | 1.000 | 0.021 | 0.032 | 0.318 | 1.000 | 0.035 | 0.034 | 0.276 | 1.000 |
| | 1000 | LB | 0.013 | 0.017 | 0.556 | 1.000 | 0.024 | 0.040 | 0.541 | 1.000 | 0.027 | 0.033 | 0.463 | 1.000 |
| | | vdW | 0.011 | 0.016 | 0.575 | 1.000 | 0.020 | 0.036 | 0.564 | 1.000 | 0.023 | 0.030 | 0.486 | 1.000 |
| | | Wilc | 0.016 | 0.022 | 0.646 | 1.000 | 0.032 | 0.041 | 0.647 | 1.000 | 0.038 | 0.046 | 0.555 | 1.000 |
| | 2000 | LB | 0.013 | 0.019 | 0.906 | 1.000 | 0.022 | 0.024 | 0.888 | 1.000 | 0.022 | 0.035 | 0.855 | 1.000 |
| | | vdW | 0.016 | 0.021 | 0.916 | 1.000 | 0.019 | 0.023 | 0.905 | 1.000 | 0.021 | 0.028 | 0.870 | 1.000 |
| | | Wilc | 0.016 | 0.028 | 0.939 | 1.000 | 0.027 | 0.026 | 0.930 | 1.000 | 0.030 | 0.053 | 0.897 | 1.000 |

**Notes:** The table shows the size/power for the Ljung-Box test (LB), the van der Waerden test (vdW) and the Wilcoxon test (Wilc). For the size calculations an AR(1)-GARCH(1,1) model was generated and estimated. For the power calculations an AR(2)-GARCH(1,1) model was generated and an AR(1)-GARCH(1,1) was estimated. The AR(1)-GARCH(1,1) model with AR coefficients ($c_0$,$\phi$) and GARCH(1,1) coefficients ($\omega$,$\alpha_1$,$\beta_1$) is given by $Y_t = c_0+\phi Y_{t-1}+\sigma_t\varepsilon_t$ with $\sigma_t^2 = \omega+\alpha_1 v_{t-1}^2+\beta_1\sigma_{t-1}^2$ and $v_t = \sigma_t\varepsilon_t$. The models used in the simulations are the AR(1)-GARCH(1,1) model with GARCH(1,1) coefficients ($\omega$,$\alpha_1$,$\beta_1$):=(0.1,0.1,0.8) and AR(1) coefficients $(c_0, \phi)$:=(0,0.4) for S1 and (0,0.8) for S2, where S refers to the cases for evaluating the size of the tests, and (0,0.4,0.1) for P1 and (0,0.4,0.4) for P2, where P refers to the cases for evaluating the power of the tests. The innovation's distribution is the normal and the Student's t with 3, 5 and 8 degrees of freedom. The maximum lag order, K, used in the tests indicate that the first K standardized residual autocorrelations are used in the test statistic, i.e. $s = 1, ..., K$. The critical value of the tests is based on the 5% significance level of the $\chi_K^2$. The above results refer to 5000 simulations.

Table 3: Simulation results for the size of alternative tests for remaining quadratic residual autocorrelation in the standardized residuals of GARCH models

| | | | Max lag used in tests K=2 | | | | Max lag used in tests K=4 | | | | Max lag used in tests K=6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DGP | Distr. | T | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc |
| ARCH(1) (0.01,0.1) | N(0,1) | 300 | 0.036 | 0.050 | 0.038 | 0.049 | 0.047 | 0.050 | 0.037 | 0.056 | 0.042 | 0.046 | 0.038 | 0.049 |
| | | 500 | 0.037 | 0.046 | 0.050 | 0.064 | 0.045 | 0.047 | 0.040 | 0.059 | 0.051 | 0.046 | 0.040 | 0.060 |
| | | 1000 | 0.040 | 0.051 | 0.053 | 0.068 | 0.049 | 0.039 | 0.044 | 0.060 | 0.053 | 0.053 | 0.056 | 0.075 |
| | | 2000 | 0.051 | 0.054 | 0.060 | 0.064 | 0.051 | 0.059 | 0.057 | 0.066 | 0.054 | 0.054 | 0.052 | 0.077 |
| | t(3) | 300 | 0.028 | 0.045 | 0.048 | 0.056 | 0.030 | 0.048 | 0.038 | 0.051 | 0.066 | 0.058 | 0.049 | 0.069 |
| | | 500 | 0.024 | 0.038 | 0.057 | 0.062 | 0.048 | 0.047 | 0.048 | 0.066 | 0.041 | 0.052 | 0.035 | 0.055 |
| | | 1000 | 0.032 | 0.061 | 0.055 | 0.059 | 0.055 | 0.050 | 0.047 | 0.048 | 0.060 | 0.052 | 0.048 | 0.071 |
| | | 2000 | 0.015 | 0.045 | 0.044 | 0.053 | 0.043 | 0.038 | 0.045 | 0.063 | 0.050 | 0.062 | 0.043 | 0.066 |
| | t(5) | 300 | 0.049 | 0.055 | 0.045 | 0.062 | 0.044 | 0.046 | 0.039 | 0.054 | 0.054 | 0.040 | 0.034 | 0.069 |
| | | 500 | 0.025 | 0.052 | 0.044 | 0.061 | 0.054 | 0.042 | 0.048 | 0.053 | 0.058 | 0.035 | 0.041 | 0.067 |
| | | 1000 | 0.030 | 0.043 | 0.054 | 0.065 | 0.056 | 0.062 | 0.043 | 0.061 | 0.061 | 0.049 | 0.044 | 0.058 |
| | | 2000 | 0.039 | 0.035 | 0.051 | 0.051 | 0.056 | 0.047 | 0.049 | 0.057 | 0.070 | 0.054 | 0.048 | 0.069 |
| | t(8) | 300 | 0.029 | 0.044 | 0.047 | 0.059 | 0.036 | 0.036 | 0.044 | 0.050 | 0.034 | 0.043 | 0.023 | 0.063 |
| | | 500 | 0.032 | 0.060 | 0.046 | 0.047 | 0.046 | 0.051 | 0.031 | 0.058 | 0.051 | 0.050 | 0.041 | 0.064 |
| | | 1000 | 0.037 | 0.047 | 0.048 | 0.051 | 0.056 | 0.051 | 0.046 | 0.073 | 0.048 | 0.047 | 0.043 | 0.075 |
| | | 2000 | 0.054 | 0.051 | 0.045 | 0.074 | 0.044 | 0.050 | 0.047 | 0.061 | 0.056 | 0.045 | 0.048 | 0.066 |
| ARCH(1) (0.01,0.8) | N(0,1) | 300 | 0.032 | 0.050 | 0.039 | 0.046 | 0.047 | 0.049 | 0.036 | 0.052 | 0.047 | 0.044 | 0.042 | 0.054 |
| | | 500 | 0.034 | 0.040 | 0.041 | 0.055 | 0.043 | 0.044 | 0.033 | 0.050 | 0.047 | 0.045 | 0.039 | 0.060 |
| | | 1000 | 0.031 | 0.045 | 0.048 | 0.055 | 0.044 | 0.039 | 0.038 | 0.062 | 0.051 | 0.045 | 0.051 | 0.063 |
| | | 2000 | 0.039 | 0.047 | 0.050 | 0.060 | 0.048 | 0.050 | 0.056 | 0.062 | 0.041 | 0.049 | 0.048 | 0.073 |
| | t(3) | 300 | 0.029 | 0.042 | 0.038 | 0.045 | 0.041 | 0.040 | 0.029 | 0.049 | 0.061 | 0.047 | 0.036 | 0.046 |
| | | 500 | 0.024 | 0.032 | 0.039 | 0.056 | 0.042 | 0.052 | 0.038 | 0.053 | 0.055 | 0.055 | 0.035 | 0.062 |
| | | 1000 | 0.024 | 0.047 | 0.048 | 0.054 | 0.044 | 0.032 | 0.037 | 0.056 | 0.056 | 0.047 | 0.038 | 0.060 |
| | | 2000 | 0.026 | 0.047 | 0.053 | 0.042 | 0.037 | 0.040 | 0.043 | 0.060 | 0.057 | 0.038 | 0.045 | 0.067 |
| | t(5) | 300 | 0.029 | 0.038 | 0.033 | 0.036 | 0.049 | 0.033 | 0.037 | 0.057 | 0.058 | 0.050 | 0.035 | 0.061 |
| | | 500 | 0.028 | 0.039 | 0.040 | 0.042 | 0.056 | 0.037 | 0.044 | 0.057 | 0.060 | 0.048 | 0.035 | 0.051 |
| | | 1000 | 0.026 | 0.041 | 0.038 | 0.045 | 0.041 | 0.049 | 0.033 | 0.058 | 0.059 | 0.051 | 0.050 | 0.064 |
| | | 2000 | 0.035 | 0.036 | 0.036 | 0.043 | 0.055 | 0.031 | 0.043 | 0.062 | 0.070 | 0.050 | 0.040 | 0.067 |
| | t(8) | 300 | 0.028 | 0.044 | 0.037 | 0.056 | 0.038 | 0.040 | 0.037 | 0.052 | 0.037 | 0.039 | 0.039 | 0.057 |
| | | 500 | 0.037 | 0.042 | 0.034 | 0.058 | 0.051 | 0.037 | 0.037 | 0.043 | 0.055 | 0.047 | 0.036 | 0.071 |
| | | 1000 | 0.055 | 0.034 | 0.038 | 0.035 | 0.054 | 0.058 | 0.044 | 0.055 | 0.055 | 0.050 | 0.036 | 0.060 |
| | | 2000 | 0.038 | 0.047 | 0.055 | 0.047 | 0.054 | 0.042 | 0.042 | 0.066 | 0.043 | 0.038 | 0.043 | 0.063 |
| GARCH(1,1) (0.01,0.1,0.8) | N(0,1) | 300 | 0.020 | 0.035 | 0.033 | 0.041 | 0.028 | 0.030 | 0.028 | 0.046 | 0.036 | 0.033 | 0.025 | 0.043 |
| | | 500 | 0.018 | 0.028 | 0.031 | 0.040 | 0.030 | 0.029 | 0.031 | 0.050 | 0.033 | 0.036 | 0.031 | 0.045 |
| | | 1000 | 0.024 | 0.032 | 0.033 | 0.045 | 0.033 | 0.031 | 0.037 | 0.048 | 0.038 | 0.037 | 0.041 | 0.055 |
| | | 2000 | 0.024 | 0.038 | 0.044 | 0.054 | 0.035 | 0.032 | 0.041 | 0.053 | 0.032 | 0.033 | 0.036 | 0.058 |
| | t(3) | 300 | 0.037 | 0.033 | 0.031 | 0.034 | 0.044 | 0.038 | 0.031 | 0.048 | 0.060 | 0.023 | 0.024 | 0.035 |
| | | 500 | 0.026 | 0.034 | 0.034 | 0.040 | 0.059 | 0.031 | 0.035 | 0.045 | 0.052 | 0.030 | 0.044 | 0.060 |
| | | 1000 | 0.025 | 0.030 | 0.028 | 0.030 | 0.041 | 0.039 | 0.039 | 0.048 | 0.064 | 0.038 | 0.038 | 0.051 |
| | | 2000 | 0.022 | 0.035 | 0.025 | 0.042 | 0.046 | 0.030 | 0.028 | 0.044 | 0.057 | 0.035 | 0.036 | 0.055 |
| | t(5) | 300 | 0.021 | 0.036 | 0.034 | 0.038 | 0.056 | 0.034 | 0.032 | 0.042 | 0.054 | 0.027 | 0.033 | 0.050 |
| | | 500 | 0.026 | 0.033 | 0.033 | 0.038 | 0.042 | 0.023 | 0.036 | 0.052 | 0.060 | 0.044 | 0.032 | 0.045 |
| | | 1000 | 0.024 | 0.027 | 0.032 | 0.053 | 0.044 | 0.022 | 0.029 | 0.047 | 0.059 | 0.033 | 0.027 | 0.046 |
| | | 2000 | 0.022 | 0.030 | 0.034 | 0.040 | 0.051 | 0.032 | 0.033 | 0.040 | 0.062 | 0.036 | 0.035 | 0.049 |
| | t(8) | 300 | 0.029 | 0.032 | 0.027 | 0.050 | 0.041 | 0.024 | 0.031 | 0.037 | 0.036 | 0.024 | 0.036 | 0.052 |
| | | 500 | 0.026 | 0.039 | 0.037 | 0.036 | 0.028 | 0.018 | 0.027 | 0.042 | 0.046 | 0.033 | 0.040 | 0.049 |
| | | 1000 | 0.033 | 0.026 | 0.032 | 0.044 | 0.034 | 0.036 | 0.031 | 0.053 | 0.041 | 0.027 | 0.033 | 0.059 |
| | | 2000 | 0.041 | 0.043 | 0.035 | 0.055 | 0.032 | 0.032 | 0.038 | 0.046 | 0.051 | 0.030 | 0.043 | 0.048 |
| IGARCH(1,1) (0.01,0.1,0.9) | N(0,1) | 300 | 0.026 | 0.040 | 0.034 | 0.045 | 0.028 | 0.034 | 0.027 | 0.042 | 0.036 | 0.033 | 0.022 | 0.044 |
| | | 500 | 0.025 | 0.037 | 0.038 | 0.046 | 0.033 | 0.041 | 0.031 | 0.054 | 0.038 | 0.036 | 0.037 | 0.053 |
| | | 1000 | 0.031 | 0.045 | 0.043 | 0.058 | 0.037 | 0.034 | 0.039 | 0.057 | 0.046 | 0.040 | 0.048 | 0.057 |
| | | 2000 | 0.035 | 0.037 | 0.054 | 0.060 | 0.039 | 0.043 | 0.046 | 0.058 | 0.040 | 0.043 | 0.045 | 0.067 |
| | t(3) | 300 | 0.038 | 0.037 | 0.031 | 0.044 | 0.057 | 0.039 | 0.037 | 0.052 | 0.054 | 0.039 | 0.037 | 0.057 |
| | | 500 | 0.035 | 0.047 | 0.045 | 0.044 | 0.048 | 0.039 | 0.032 | 0.048 | 0.062 | 0.042 | 0.037 | 0.054 |
| | | 1000 | 0.029 | 0.052 | 0.044 | 0.049 | 0.045 | 0.034 | 0.045 | 0.054 | 0.063 | 0.050 | 0.044 | 0.052 |
| | | 2000 | 0.026 | 0.041 | 0.044 | 0.052 | 0.044 | 0.043 | 0.043 | 0.052 | 0.049 | 0.036 | 0.046 | 0.056 |
| | t(5) | 300 | 0.037 | 0.031 | 0.029 | 0.042 | 0.049 | 0.039 | 0.025 | 0.045 | 0.048 | 0.038 | 0.022 | 0.040 |
| | | 500 | 0.028 | 0.033 | 0.036 | 0.055 | 0.035 | 0.041 | 0.031 | 0.043 | 0.050 | 0.038 | 0.038 | 0.063 |
| | | 1000 | 0.034 | 0.046 | 0.051 | 0.046 | 0.066 | 0.037 | 0.046 | 0.052 | 0.056 | 0.031 | 0.039 | 0.071 |
| | | 2000 | 0.040 | 0.044 | 0.040 | 0.050 | 0.049 | 0.032 | 0.033 | 0.043 | 0.073 | 0.038 | 0.044 | 0.046 |
| | t(8) | 300 | 0.020 | 0.035 | 0.032 | 0.035 | 0.049 | 0.033 | 0.027 | 0.038 | 0.046 | 0.043 | 0.036 | 0.054 |
| | | 500 | 0.032 | 0.029 | 0.044 | 0.044 | 0.041 | 0.036 | 0.032 | 0.041 | 0.050 | 0.023 | 0.036 | 0.045 |
| | | 1000 | 0.031 | 0.046 | 0.038 | 0.057 | 0.042 | 0.049 | 0.055 | 0.053 | 0.060 | 0.042 | 0.036 | 0.050 |
| | | 2000 | 0.035 | 0.031 | 0.041 | 0.055 | 0.033 | 0.041 | 0.033 | 0.054 | 0.054 | 0.035 | 0.035 | 0.051 |

**Notes:** The tests applied in the table are the LiMak test (LM), the AvdW, the van der Waerden test (vdW) and the Wilcoxon test (Wilc). Size was calculated by generating and estimating a (G)ARCH model. The GARCH(1,1) model with coefficients ($\omega$,$\alpha_1$,$\beta_1$) is given by $Y_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = \omega + \alpha_1 Y_{t-1}^2 + \beta \sigma_{t-1}^2$. The DGPs used in this table are the low persistence ARCH(1) with parameters $(\omega, \alpha_1)$:=(0.01,0.1) and the high persistence ARCH(1) with parameters $(\omega, \alpha_1)$:=(0.01,0.8), the high persistence GARCH(1,1) with parameters $(\omega, \alpha_1, \beta)$:=(0.01,0.1,0.8) and the IGARCH(1,1) with parameters $(\omega, \alpha_1, \beta)$:=(0.01,0.1,0.9). The distributions used for the innovations are the standard normal N(0,1) and the Student's t with 3, 5 and 8 degrees of freedom. The maximum lag order, K, used in the tests indicate that the maximum standardized residual autocorrelations lag used in the test statistic is K, i.e. $s = 2, ..., K$. The critical value of the tests is based on the 5% significance level of the $\chi^2_{K-1}$. The above results refer to 5000 simulations.

Table 4: Simulation results for the power of alternative tests for remaining quadratic residual autocorrelation in the standardized residuals of GARCH models

| | | | Max lag used in tests K=2 | | | | Max lag used in tests K=4 | | | | Max lag used in tests K=6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DGP | Distr. | T | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc |
| ARCH(2) (0.01,0.1,0.1) | N(0,1) | 300 | 0.320 | 0.282 | 0.182 | 0.202 | 0.234 | 0.207 | 0.107 | 0.126 | 0.207 | 0.183 | 0.082 | 0.121 |
| | | 500 | 0.503 | 0.435 | 0.278 | 0.295 | 0.381 | 0.306 | 0.189 | 0.212 | 0.318 | 0.278 | 0.147 | 0.174 |
| | | 1000 | 0.765 | 0.710 | 0.501 | 0.487 | 0.644 | 0.562 | 0.352 | 0.378 | 0.562 | 0.473 | 0.274 | 0.297 |
| | | 2000 | 0.963 | 0.929 | 0.801 | 0.774 | 0.912 | 0.848 | 0.657 | 0.627 | 0.870 | 0.785 | 0.563 | 0.535 |
| | t(3) | 300 | 0.154 | 0.173 | 0.103 | 0.136 | 0.137 | 0.122 | 0.068 | 0.106 | 0.109 | 0.129 | 0.073 | 0.104 |
| | | 500 | 0.204 | 0.295 | 0.164 | 0.206 | 0.163 | 0.205 | 0.098 | 0.131 | 0.164 | 0.179 | 0.083 | 0.122 |
| | | 1000 | 0.341 | 0.526 | 0.333 | 0.390 | 0.241 | 0.385 | 0.199 | 0.249 | 0.244 | 0.333 | 0.201 | 0.266 |
| | | 2000 | 0.463 | 0.829 | 0.603 | 0.651 | 0.352 | 0.664 | 0.461 | 0.524 | 0.309 | 0.597 | 0.369 | 0.444 |
| | t(5) | 300 | 0.239 | 0.231 | 0.152 | 0.170 | 0.195 | 0.166 | 0.093 | 0.114 | 0.171 | 0.132 | 0.071 | 0.117 |
| | | 500 | 0.319 | 0.370 | 0.239 | 0.259 | 0.266 | 0.241 | 0.173 | 0.178 | 0.248 | 0.184 | 0.126 | 0.155 |
| | | 1000 | 0.532 | 0.649 | 0.451 | 0.480 | 0.412 | 0.450 | 0.310 | 0.328 | 0.385 | 0.421 | 0.249 | 0.281 |
| | | 2000 | 0.743 | 0.913 | 0.709 | 0.739 | 0.666 | 0.802 | 0.579 | 0.588 | 0.611 | 0.747 | 0.489 | 0.547 |
| | t(8) | 300 | 0.265 | 0.244 | 0.179 | 0.168 | 0.216 | 0.171 | 0.086 | 0.117 | 0.172 | 0.161 | 0.063 | 0.099 |
| | | 500 | 0.411 | 0.404 | 0.240 | 0.272 | 0.330 | 0.295 | 0.148 | 0.192 | 0.243 | 0.217 | 0.137 | 0.155 |
| | | 1000 | 0.646 | 0.650 | 0.464 | 0.497 | 0.554 | 0.509 | 0.327 | 0.352 | 0.478 | 0.420 | 0.265 | 0.296 |
| | | 2000 | 0.867 | 0.911 | 0.749 | 0.718 | 0.808 | 0.819 | 0.619 | 0.616 | 0.782 | 0.739 | 0.520 | 0.537 |
| ARCH(2) (0.01,0.1,0.4) | N(0,1) | 300 | 0.978 | 0.981 | 0.932 | 0.907 | 0.951 | 0.949 | 0.853 | 0.837 | 0.929 | 0.928 | 0.775 | 0.767 |
| | | 500 | 0.998 | 1.000 | 0.994 | 0.990 | 0.997 | 0.998 | 0.983 | 0.981 | 0.993 | 0.996 | 0.971 | 0.963 |
| | | 1000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 2000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | t(3) | 300 | 0.535 | 0.846 | 0.656 | 0.684 | 0.464 | 0.719 | 0.509 | 0.583 | 0.420 | 0.677 | 0.402 | 0.467 |
| | | 500 | 0.671 | 0.975 | 0.874 | 0.887 | 0.606 | 0.931 | 0.755 | 0.797 | 0.568 | 0.878 | 0.679 | 0.716 |
| | | 1000 | 0.819 | 0.999 | 0.993 | 0.996 | 0.747 | 0.996 | 0.977 | 0.983 | 0.729 | 0.996 | 0.956 | 0.961 |
| | | 2000 | 0.896 | 1.000 | 1.000 | 1.000 | 0.860 | 1.000 | 0.999 | 1.000 | 0.819 | 1.000 | 1.000 | 1.000 |
| | t(5) | 300 | 0.805 | 0.942 | 0.848 | 0.829 | 0.705 | 0.873 | 0.726 | 0.746 | 0.673 | 0.825 | 0.624 | 0.657 |
| | | 500 | 0.910 | 0.994 | 0.969 | 0.970 | 0.881 | 0.983 | 0.933 | 0.921 | 0.840 | 0.982 | 0.874 | 0.885 |
| | | 1000 | 0.982 | 1.000 | 1.000 | 1.000 | 0.967 | 1.000 | 1.000 | 0.999 | 0.959 | 1.000 | 0.998 | 0.998 |
| | | 2000 | 0.990 | 1.000 | 1.000 | 1.000 | 0.983 | 1.000 | 1.000 | 1.000 | 0.986 | 1.000 | 1.000 | 1.000 |
| | t(8) | 300 | 0.880 | 0.950 | 0.868 | 0.876 | 0.843 | 0.925 | 0.784 | 0.744 | 0.792 | 0.880 | 0.680 | 0.674 |
| | | 500 | 0.979 | 0.997 | 0.986 | 0.977 | 0.953 | 0.989 | 0.964 | 0.942 | 0.944 | 0.986 | 0.926 | 0.911 |
| | | 1000 | 0.999 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 0.997 | 0.997 | 1.000 | 1.000 | 0.998 |
| | | 2000 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 |
| GARCH(1,1) (0.01,0.1,0.8) | N(0,1) | 300 | 0.278 | 0.255 | 0.162 | 0.186 | 0.420 | 0.359 | 0.198 | 0.233 | 0.456 | 0.400 | 0.225 | 0.265 |
| | | 500 | 0.426 | 0.389 | 0.257 | 0.271 | 0.630 | 0.549 | 0.358 | 0.374 | 0.683 | 0.600 | 0.361 | 0.406 |
| | | 1000 | 0.677 | 0.637 | 0.457 | 0.460 | 0.891 | 0.844 | 0.644 | 0.640 | 0.938 | 0.903 | 0.715 | 0.723 |
| | | 2000 | 0.934 | 0.900 | 0.757 | 0.727 | 0.996 | 0.987 | 0.934 | 0.924 | 0.999 | 0.998 | 0.971 | 0.962 |
| | t(3) | 300 | 0.137 | 0.223 | 0.148 | 0.174 | 0.209 | 0.312 | 0.136 | 0.200 | 0.236 | 0.328 | 0.167 | 0.240 |
| | | 500 | 0.172 | 0.314 | 0.220 | 0.261 | 0.294 | 0.460 | 0.286 | 0.356 | 0.314 | 0.496 | 0.302 | 0.383 |
| | | 1000 | 0.254 | 0.562 | 0.410 | 0.454 | 0.421 | 0.788 | 0.582 | 0.653 | 0.495 | 0.784 | 0.590 | 0.668 |
| | | 2000 | 0.399 | 0.854 | 0.691 | 0.692 | 0.618 | 0.969 | 0.875 | 0.887 | 0.688 | 0.984 | 0.913 | 0.927 |
| | t(5) | 300 | 0.207 | 0.219 | 0.160 | 0.159 | 0.324 | 0.339 | 0.196 | 0.235 | 0.368 | 0.377 | 0.227 | 0.266 |
| | | 500 | 0.269 | 0.357 | 0.225 | 0.280 | 0.465 | 0.512 | 0.332 | 0.373 | 0.522 | 0.591 | 0.414 | 0.422 |
| | | 1000 | 0.479 | 0.644 | 0.455 | 0.471 | 0.715 | 0.812 | 0.650 | 0.652 | 0.786 | 0.898 | 0.705 | 0.747 |
| | | 2000 | 0.684 | 0.881 | 0.737 | 0.719 | 0.919 | 0.991 | 0.925 | 0.921 | 0.950 | 0.990 | 0.962 | 0.956 |
| | t(8) | 300 | 0.244 | 0.214 | 0.159 | 0.156 | 0.357 | 0.327 | 0.199 | 0.260 | 0.408 | 0.346 | 0.203 | 0.261 |
| | | 500 | 0.336 | 0.356 | 0.218 | 0.244 | 0.533 | 0.537 | 0.326 | 0.353 | 0.586 | 0.555 | 0.383 | 0.424 |
| | | 1000 | 0.569 | 0.600 | 0.464 | 0.451 | 0.826 | 0.819 | 0.650 | 0.650 | 0.862 | 0.881 | 0.669 | 0.726 |
| | | 2000 | 0.825 | 0.887 | 0.730 | 0.742 | 0.985 | 0.985 | 0.928 | 0.922 | 0.994 | 0.989 | 0.954 | 0.952 |
| IGARCH(1,1) (0.01,0.1,0.9) | N(0,1) | 300 | 0.438 | 0.513 | 0.395 | 0.411 | 0.714 | 0.700 | 0.578 | 0.585 | 0.798 | 0.758 | 0.663 | 0.668 |
| | | 500 | 0.669 | 0.774 | 0.687 | 0.688 | 0.929 | 0.925 | 0.862 | 0.860 | 0.968 | 0.954 | 0.908 | 0.911 |
| | | 1000 | 0.910 | 0.975 | 0.934 | 0.931 | 0.997 | 0.999 | 0.991 | 0.990 | 0.999 | 1.000 | 0.998 | 0.996 |
| | | 2000 | 0.981 | 0.999 | 0.998 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | t(3) | 300 | 0.156 | 0.410 | 0.321 | 0.330 | 0.274 | 0.588 | 0.490 | 0.525 | 0.345 | 0.646 | 0.553 | 0.555 |
| | | 500 | 0.217 | 0.641 | 0.513 | 0.545 | 0.372 | 0.828 | 0.737 | 0.759 | 0.469 | 0.842 | 0.792 | 0.818 |
| | | 1000 | 0.318 | 0.887 | 0.803 | 0.824 | 0.566 | 0.981 | 0.950 | 0.954 | 0.676 | 0.991 | 0.975 | 0.978 |
| | | 2000 | 0.461 | 0.995 | 0.981 | 0.979 | 0.740 | 1.000 | 1.000 | 1.000 | 0.801 | 1.000 | 1.000 | 1.000 |
| | t(5) | 300 | 0.277 | 0.450 | 0.385 | 0.384 | 0.474 | 0.627 | 0.585 | 0.607 | 0.577 | 0.726 | 0.641 | 0.655 |
| | | 500 | 0.389 | 0.751 | 0.653 | 0.643 | 0.661 | 0.920 | 0.855 | 0.843 | 0.757 | 0.943 | 0.889 | 0.890 |
| | | 1000 | 0.609 | 0.952 | 0.910 | 0.926 | 0.892 | 0.996 | 0.979 | 0.991 | 0.942 | 0.998 | 0.991 | 0.993 |
| | | 2000 | 0.791 | 1.000 | 0.999 | 0.997 | 0.951 | 1.000 | 1.000 | 1.000 | 0.968 | 1.000 | 1.000 | 1.000 |
| | t(8) | 300 | 0.320 | 0.472 | 0.397 | 0.402 | 0.572 | 0.674 | 0.580 | 0.589 | 0.669 | 0.764 | 0.632 | 0.675 |
| | | 500 | 0.524 | 0.730 | 0.631 | 0.677 | 0.793 | 0.921 | 0.867 | 0.847 | 0.896 | 0.950 | 0.891 | 0.906 |
| | | 1000 | 0.749 | 0.957 | 0.928 | 0.919 | 0.957 | 0.999 | 0.992 | 0.994 | 0.984 | 0.999 | 0.993 | 0.996 |
| | | 2000 | 0.897 | 1.000 | 0.996 | 0.999 | 0.973 | 1.000 | 1.000 | 1.000 | 0.989 | 1.000 | 1.000 | 1.000 |

**Notes:** The tests applied in the table are the LiMak test (LM), the AvdW, the van der Waerden test (vdW) and the Wilcoxon test (Wilc). The ARCH(2) model with coefficients ($\omega$,$\alpha_1$,$\alpha_2$) is given by $Y_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = \omega + \alpha_1 Y_{t-1}^2 + \alpha_2 Y_{t-2}^2$. The GARCH(1,1) model with coefficients ($\omega$,$\alpha_1$,$\beta_1$) is given by $Y_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = \omega + \alpha_1 Y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. An ARCH(1) model was estimated to obtain power. The DGPs used are the low persistence ARCH(2) with coefficients $(\omega, \alpha_1, \alpha_2)$:=(0.01,0.1,0.1), the high persistence ARCH(2) with coefficients $(\omega, \alpha_1, \alpha_2)$:=(0.01,0.1,0.4), the high persistence GARCH(1,1) with coefficients $(\omega, \alpha_1, \beta_1)$:=(0.01,0.1,0.8), used by Berkes, Horváth and Kokoszka (2003), and the IGARCH(1,1) with parameters $(\omega, \alpha_1, \beta)$:=(0.01,0.1,0.9). The distributions used for the innovations are the standard normal N(0,1) and the Student's t with 3, 5 and 8 degrees of freedom. The maximum lag order, K, used in the tests indicate that the maximum standardized residual autocorrelations lag used in the test statistic is K, i.e. $s = 2, ..., K$. The critical value of the tests is based on the 5% significance level of the $\chi^2_{K-1}$. The above results refer to 5000 simulations.

Table 5: Simulation results for the size of alternative tests for remaining quadratic residual autocorrelation in the standardized residuals of asymmetric distributions of GARCH models

| | | | Max lag used in tests K=2 | | | | Max lag used in tests K=4 | | | | Max lag used in tests K=6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DGP | Distr. | T | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc |
| ARCH(1) | Hansen | 300 | 0.030 | 0.051 | 0.047 | 0.058 | 0.051 | 0.042 | 0.047 | 0.046 | 0.046 | 0.048 | 0.042 | 0.058 |
| (0.01,0.1) | SkSt | 500 | 0.045 | 0.055 | 0.048 | 0.052 | 0.043 | 0.040 | 0.039 | 0.066 | 0.052 | 0.043 | 0.040 | 0.067 |
| | (-0.09,8.1) | 1000 | 0.060 | 0.040 | 0.043 | 0.052 | 0.050 | 0.054 | 0.044 | 0.081 | 0.056 | 0.042 | 0.041 | 0.058 |
| | | 2000 | 0.039 | 0.050 | 0.047 | 0.059 | 0.057 | 0.067 | 0.060 | 0.059 | 0.051 | 0.054 | 0.041 | 0.067 |
| ARCH(1) | Hansen | 300 | 0.040 | 0.033 | 0.035 | 0.053 | 0.041 | 0.045 | 0.042 | 0.052 | 0.053 | 0.047 | 0.041 | 0.055 |
| (0.01,0.8) | SkSt | 500 | 0.029 | 0.053 | 0.043 | 0.053 | 0.044 | 0.041 | 0.037 | 0.059 | 0.051 | 0.045 | 0.048 | 0.057 |
| | (-0.09,8.1) | 1000 | 0.042 | 0.035 | 0.044 | 0.046 | 0.047 | 0.040 | 0.046 | 0.057 | 0.050 | 0.052 | 0.041 | 0.067 |
| | | 2000 | 0.040 | 0.054 | 0.042 | 0.040 | 0.042 | 0.039 | 0.035 | 0.053 | 0.057 | 0.041 | 0.049 | 0.063 |
| GARCH(1,1) | Hansen | 300 | 0.038 | 0.031 | 0.029 | 0.030 | 0.036 | 0.039 | 0.026 | 0.043 | 0.037 | 0.025 | 0.026 | 0.040 |
| (0.01,0.1,0.8) | SkSt | 500 | 0.023 | 0.022 | 0.029 | 0.043 | 0.037 | 0.033 | 0.023 | 0.042 | 0.051 | 0.033 | 0.037 | 0.046 |
| | (-0.09,8.1) | 1000 | 0.033 | 0.029 | 0.042 | 0.041 | 0.036 | 0.029 | 0.031 | 0.043 | 0.039 | 0.040 | 0.029 | 0.040 |
| | | 2000 | 0.035 | 0.030 | 0.033 | 0.037 | 0.041 | 0.030 | 0.022 | 0.044 | 0.038 | 0.033 | 0.028 | 0.057 |
| IGARCH(1,1) | Hansen | 300 | 0.035 | 0.036 | 0.030 | 0.039 | 0.041 | 0.044 | 0.028 | 0.054 | 0.051 | 0.040 | 0.036 | 0.069 |
| (0.01,0.1,0.9) | SkSt | 500 | 0.033 | 0.044 | 0.046 | 0.046 | 0.055 | 0.035 | 0.038 | 0.053 | 0.049 | 0.033 | 0.043 | 0.040 |
| | (-0.09,8.1) | 1000 | 0.039 | 0.040 | 0.032 | 0.050 | 0.052 | 0.039 | 0.030 | 0.056 | 0.051 | 0.038 | 0.038 | 0.049 |
| | | 2000 | 0.042 | 0.034 | 0.038 | 0.052 | 0.055 | 0.040 | 0.045 | 0.058 | 0.058 | 0.042 | 0.046 | 0.051 |
| ARCH(1) | Hansen | 300 | 0.036 | 0.054 | 0.038 | 0.061 | 0.051 | 0.044 | 0.033 | 0.054 | 0.055 | 0.044 | 0.035 | 0.048 |
| (0.01,0.1) | SkSt | 500 | 0.039 | 0.048 | 0.056 | 0.054 | 0.056 | 0.049 | 0.049 | 0.055 | 0.061 | 0.053 | 0.038 | 0.069 |
| | (0.99,8.1) | 1000 | 0.032 | 0.044 | 0.044 | 0.049 | 0.058 | 0.041 | 0.051 | 0.070 | 0.067 | 0.047 | 0.050 | 0.070 |
| | | 2000 | 0.031 | 0.045 | 0.050 | 0.058 | 0.056 | 0.053 | 0.044 | 0.064 | 0.065 | 0.051 | 0.046 | 0.080 |
| ARCH(1) | Hansen | 300 | 0.037 | 0.057 | 0.035 | 0.060 | 0.053 | 0.040 | 0.032 | 0.054 | 0.054 | 0.043 | 0.032 | 0.049 |
| (0.01,0.8) | SkSt | 500 | 0.037 | 0.045 | 0.055 | 0.053 | 0.057 | 0.049 | 0.050 | 0.056 | 0.060 | 0.058 | 0.036 | 0.067 |
| | (0.99,8.1) | 1000 | 0.032 | 0.047 | 0.039 | 0.048 | 0.058 | 0.042 | 0.050 | 0.070 | 0.066 | 0.046 | 0.052 | 0.070 |
| | | 2000 | 0.037 | 0.038 | 0.056 | 0.054 | 0.055 | 0.063 | 0.042 | 0.057 | 0.068 | 0.057 | 0.053 | 0.074 |
| GARCH(1,1) | Hansen | 300 | 0.038 | 0.049 | 0.038 | 0.060 | 0.047 | 0.047 | 0.036 | 0.057 | 0.074 | 0.040 | 0.044 | 0.060 |
| (0.01,0.1,0.8) | SkSt | 500 | 0.034 | 0.048 | 0.043 | 0.063 | 0.060 | 0.048 | 0.036 | 0.062 | 0.070 | 0.038 | 0.040 | 0.071 |
| | (0.99,8.1) | 1000 | 0.037 | 0.052 | 0.052 | 0.055 | 0.053 | 0.045 | 0.050 | 0.068 | 0.069 | 0.048 | 0.058 | 0.058 |
| | | 2000 | 0.045 | 0.048 | 0.060 | 0.061 | 0.053 | 0.055 | 0.035 | 0.056 | 0.080 | 0.052 | 0.043 | 0.074 |
| IGARCH(1,1) | Hansen | 300 | 0.036 | 0.057 | 0.055 | 0.051 | 0.057 | 0.060 | 0.040 | 0.050 | 0.061 | 0.046 | 0.045 | 0.060 |
| (0.01,0.1,0.9) | SkSt | 500 | 0.035 | 0.063 | 0.039 | 0.044 | 0.050 | 0.052 | 0.038 | 0.067 | 0.068 | 0.056 | 0.036 | 0.064 |
| | (0.99,8.1) | 1000 | 0.038 | 0.034 | 0.057 | 0.061 | 0.040 | 0.056 | 0.046 | 0.065 | 0.056 | 0.047 | 0.046 | 0.060 |
| | | 2000 | 0.035 | 0.050 | 0.033 | 0.058 | 0.058 | 0.048 | 0.046 | 0.067 | 0.052 | 0.051 | 0.044 | 0.073 |

**Notes:** The tests applied in the table are the LiMak test (LM), the AvdW, the van der Waerden test (vdW) and the Wilcoxon test (Wilc). The DGPs used are the low persistence ARCH(1) with parameters $(\omega, \alpha_1)$:=(0.01,0.1), the high persistence ARCH(1) with parameters $(\omega, \alpha_1)$:=(0.01,0.8), the high persistence GARCH(1,1) with parameters $(\omega, \alpha_1, \beta_1)$:=(0.01,0.1,0.8) used by Berkes, Horváth and Kokoszka (2003) and the IGARCH(1,1) with parameters $(\omega, \alpha_1, \beta_1)$:=(0.01,0.1,0.9). The distribution used for the innovations in every generated and estimated model is the Hansen Skewed t $(\lambda,\eta)$ distribution, where $\lambda$ is the skewness parameter and $\eta$ is the degrees of freedom. The parameters used for the Hansen SkSt are $(\lambda,\eta)$:=(-0.09,8.1) (skewness is -0.24 and kurtosis is 4.39) and $(\lambda,\eta)$:=(0.99,8.1) (skewness is 1.52 and kurtosis is 6.73). The maximum lag order, K, used in the tests indicate that the maximum standardized residual autocorrelations lag used in the test statistic is K, i.e. $s = 2, ..., K$. The critical value of the tests is based on the 5% significance level of the $\chi^2_{K-1}$. The above results refer to 5000 simulations.

Table 6: Simulation results for the size of alternative tests for remaining quadratic residual autocorrelation in the standardized residuals of asymmetric distributions of GARCH models

| | | | Max lag used in tests K=2 | | | | Max lag used in tests K=4 | | | | Max lag used in tests K=6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DGP | Distr. | T | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc |
| ARCH(1) | Hansen | 300 | 0.028 | 0.048 | 0.041 | 0.049 | 0.050 | 0.049 | 0.042 | 0.061 | 0.042 | 0.057 | 0.034 | 0.056 |
| (0.01,0.1) | SkSt | 500 | 0.016 | 0.047 | 0.035 | 0.069 | 0.044 | 0.052 | 0.037 | 0.049 | 0.057 | 0.052 | 0.033 | 0.073 |
| | (0.9,3) | 1000 | 0.021 | 0.052 | 0.049 | 0.061 | 0.048 | 0.067 | 0.050 | 0.057 | 0.049 | 0.050 | 0.037 | 0.066 |
| | | 2000 | 0.015 | 0.069 | 0.052 | 0.052 | 0.038 | 0.052 | 0.057 | 0.071 | 0.051 | 0.052 | 0.044 | 0.068 |
| ARCH(1) | Hansen | 300 | 0.033 | 0.067 | 0.041 | 0.052 | 0.034 | 0.058 | 0.045 | 0.047 | 0.064 | 0.049 | 0.045 | 0.062 |
| (0.01,0.8) | SkSt | 500 | 0.033 | 0.055 | 0.041 | 0.058 | 0.039 | 0.052 | 0.037 | 0.061 | 0.047 | 0.046 | 0.044 | 0.049 |
| | (0.9,3) | 1000 | 0.028 | 0.048 | 0.045 | 0.062 | 0.047 | 0.047 | 0.046 | 0.066 | 0.053 | 0.053 | 0.045 | 0.068 |
| | | 2000 | 0.022 | 0.050 | 0.048 | 0.061 | 0.050 | 0.043 | 0.043 | 0.068 | 0.057 | 0.044 | 0.047 | 0.063 |
| GARCH(1,1) | Hansen | 300 | 0.027 | 0.044 | 0.050 | 0.050 | 0.044 | 0.047 | 0.039 | 0.055 | 0.055 | 0.047 | 0.040 | 0.070 |
| (0.01,0.1,0.8) | SkSt | 500 | 0.019 | 0.050 | 0.045 | 0.055 | 0.043 | 0.050 | 0.047 | 0.047 | 0.061 | 0.042 | 0.043 | 0.066 |
| | (0.9,3) | 1000 | 0.026 | 0.049 | 0.048 | 0.051 | 0.054 | 0.045 | 0.046 | 0.063 | 0.053 | 0.049 | 0.045 | 0.068 |
| | | 2000 | 0.022 | 0.038 | 0.038 | 0.070 | 0.043 | 0.053 | 0.046 | 0.056 | 0.047 | 0.043 | 0.049 | 0.058 |
| IGARCH(1,1) | Hansen | 300 | 0.023 | 0.054 | 0.051 | 0.058 | 0.041 | 0.049 | 0.046 | 0.057 | 0.065 | 0.055 | 0.030 | 0.077 |
| (0.01,0.1,0.9) | SkSt | 500 | 0.025 | 0.054 | 0.047 | 0.060 | 0.047 | 0.047 | 0.051 | 0.060 | 0.058 | 0.048 | 0.050 | 0.075 |
| | (0.9,3) | 1000 | 0.029 | 0.048 | 0.048 | 0.062 | 0.039 | 0.057 | 0.037 | 0.079 | 0.058 | 0.043 | 0.045 | 0.062 |
| | | 2000 | 0.021 | 0.056 | 0.047 | 0.080 | 0.038 | 0.047 | 0.040 | 0.063 | 0.057 | 0.047 | 0.061 | 0.076 |
| GARCH(1,1) | Hansen | 300 | 0.024 | 0.030 | 0.031 | 0.044 | 0.046 | 0.031 | 0.030 | 0.041 | 0.046 | 0.039 | 0.041 | 0.050 |
| (0.01,0.1,0.8) | SkSt | 500 | 0.036 | 0.022 | 0.031 | 0.047 | 0.049 | 0.035 | 0.028 | 0.043 | 0.060 | 0.030 | 0.030 | 0.050 |
| | (-0.09,5) | 1000 | 0.036 | 0.040 | 0.052 | 0.046 | 0.050 | 0.029 | 0.042 | 0.048 | 0.059 | 0.040 | 0.032 | 0.050 |
| | | 2000 | 0.023 | 0.025 | 0.037 | 0.060 | 0.058 | 0.023 | 0.028 | 0.055 | 0.049 | 0.032 | 0.033 | 0.060 |
| IGARCH(1,1) | Hansen | 300 | 0.035 | 0.043 | 0.029 | 0.039 | 0.031 | 0.041 | 0.031 | 0.051 | 0.049 | 0.033 | 0.028 | 0.051 |
| (0.01,0.1,0.9) | SkSt | 500 | 0.032 | 0.036 | 0.034 | 0.031 | 0.051 | 0.042 | 0.035 | 0.049 | 0.058 | 0.038 | 0.038 | 0.050 |
| | (-0.09,5) | 1000 | 0.034 | 0.020 | 0.038 | 0.047 | 0.050 | 0.048 | 0.039 | 0.054 | 0.061 | 0.041 | 0.028 | 0.044 |
| | | 2000 | 0.040 | 0.047 | 0.051 | 0.046 | 0.053 | 0.036 | 0.039 | 0.054 | 0.053 | 0.041 | 0.033 | 0.056 |
| GARCH(1,1) | Hansen | 300 | 0.042 | 0.033 | 0.044 | 0.049 | 0.055 | 0.066 | 0.036 | 0.064 | 0.073 | 0.052 | 0.032 | 0.045 |
| (0.01,0.1,0.8) | SkSt | 500 | 0.031 | 0.058 | 0.037 | 0.051 | 0.061 | 0.047 | 0.042 | 0.060 | 0.063 | 0.056 | 0.046 | 0.070 |
| | (0.99,5) | 1000 | 0.033 | 0.035 | 0.045 | 0.058 | 0.050 | 0.054 | 0.046 | 0.059 | 0.076 | 0.048 | 0.047 | 0.063 |
| | | 2000 | 0.027 | 0.064 | 0.058 | 0.066 | 0.054 | 0.052 | 0.052 | 0.068 | 0.065 | 0.046 | 0.048 | 0.071 |
| IGARCH(1,1) | Hansen | 300 | 0.031 | 0.049 | 0.052 | 0.055 | 0.042 | 0.058 | 0.039 | 0.040 | 0.054 | 0.065 | 0.035 | 0.065 |
| (0.01,0.1,0.9) | SkSt | 500 | 0.036 | 0.051 | 0.044 | 0.056 | 0.045 | 0.039 | 0.049 | 0.063 | 0.074 | 0.057 | 0.044 | 0.064 |
| | (0.99,5) | 1000 | 0.027 | 0.053 | 0.052 | 0.047 | 0.056 | 0.058 | 0.049 | 0.055 | 0.077 | 0.060 | 0.048 | 0.054 |
| | | 2000 | 0.035 | 0.052 | 0.053 | 0.070 | 0.061 | 0.051 | 0.038 | 0.065 | 0.075 | 0.042 | 0.045 | 0.071 |

**Notes:** The tests applied in the table are the LiMak test (LM), the AvdW, the van der Waerden test (vdW) and the Wilcoxon test (Wilc). The DGPs used are the low persistence ARCH(1) with parameters $(\omega, \alpha_1)$:=(0.01,0.1), the high persistence ARCH(1) with parameters $(\omega, \alpha_1)$:=(0.01,0.8), the high persistence GARCH(1,1) with parameters $(\omega, \alpha_1, \beta_1)$:=(0.01,0.1,0.8) used by Berkes, Horváth and Kokoszka (2003) and the IGARCH(1,1) with parameters $(\omega, \alpha_1, \beta_1)$:=(0.01,0.1,0.9). The distribution used for the innovations in every generated and estimated model is the Hansen Skewed t $(\lambda,\eta)$ distribution, where $\lambda$ is the skewness parameter and $\eta$ is the degrees of freedom. The parameters used for the Hansen SkSt are $(\lambda,\eta)$:=(-0.09,5) (skewness is -0.28 and kurtosis is 6.35), $(\lambda,\eta)$:=(0.99,5) (skewness is 2.07 and kurtosis is 10.96) and $(\lambda,\eta)$:=(0.9,3). The maximum lag order, K, used in the tests indicate that the maximum standardized residual autocorrelations lag used in the test statistic is K, i.e. $s = 2, ..., K$. The critical value of the tests is based on the 5% significance level of the $\chi^2_{K-1}$. The above results refer to 5000 simulations.

Table 7: Simulation results for the power of alternative tests for remaining quadratic residual autocorrelation in the standardized residuals of asymmetric distributions of GARCH models

| | | | Max lag used in tests K=2 | | | | Max lag used in tests K=4 | | | | Max lag used in tests K=6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DGP | Distr. | T | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc | LM | AvdW | vdW | Wilc |
| ARCH(2) | Hansen | 300 | 0.287 | 0.242 | 0.189 | 0.163 | 0.213 | 0.157 | 0.102 | 0.137 | 0.164 | 0.123 | 0.074 | 0.104 |
| (0.01,0.1,0.1) | SkSt | 500 | 0.413 | 0.366 | 0.287 | 0.252 | 0.321 | 0.270 | 0.142 | 0.180 | 0.303 | 0.220 | 0.137 | 0.181 |
| | (-0.09,8.1) | 1000 | 0.654 | 0.642 | 0.451 | 0.461 | 0.565 | 0.509 | 0.327 | 0.345 | 0.458 | 0.393 | 0.240 | 0.303 |
| | | 2000 | 0.917 | 0.921 | 0.786 | 0.742 | 0.813 | 0.844 | 0.618 | 0.602 | 0.768 | 0.766 | 0.521 | 0.560 |
| ARCH(2) | Hansen | 300 | 0.906 | 0.966 | 0.872 | 0.873 | 0.830 | 0.923 | 0.762 | 0.759 | 0.797 | 0.879 | 0.736 | 0.687 |
| (0.01,0.1,0.4) | SkSt | 500 | 0.978 | 0.998 | 0.979 | 0.984 | 0.957 | 0.993 | 0.965 | 0.956 | 0.932 | 0.981 | 0.925 | 0.925 |
| | (-0.09,8.1) | 1000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 | 0.996 | 1.000 | 0.998 | 1.000 |
| | | 2000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 |
| GARCH(1,1) | Hansen | 300 | 0.220 | 0.224 | 0.146 | 0.155 | 0.360 | 0.322 | 0.205 | 0.238 | 0.416 | 0.335 | 0.205 | 0.255 |
| (0.01,0.1,0.8) | SkSt | 500 | 0.343 | 0.369 | 0.246 | 0.243 | 0.539 | 0.510 | 0.370 | 0.372 | 0.605 | 0.525 | 0.397 | 0.437 |
| | (-0.09,8.1) | 1000 | 0.559 | 0.637 | 0.439 | 0.433 | 0.793 | 0.825 | 0.653 | 0.668 | 0.885 | 0.879 | 0.706 | 0.748 |
| | | 2000 | 0.845 | 0.882 | 0.725 | 0.747 | 0.973 | 0.987 | 0.925 | 0.927 | 0.989 | 0.991 | 0.964 | 0.946 |
| IGARCH(1,1) | Hansen | 300 | 0.319 | 0.464 | 0.373 | 0.410 | 0.543 | 0.697 | 0.579 | 0.590 | 0.651 | 0.741 | 0.664 | 0.688 |
| (0.01,0.1,0.9) | SkSt | 500 | 0.506 | 0.748 | 0.623 | 0.635 | 0.806 | 0.921 | 0.844 | 0.825 | 0.893 | 0.949 | 0.893 | 0.912 |
| | (-0.09,8.1) | 1000 | 0.717 | 0.970 | 0.916 | 0.916 | 0.963 | 1.000 | 0.991 | 0.994 | 0.978 | 0.998 | 0.996 | 0.995 |
| | | 2000 | 0.885 | 1.000 | 0.998 | 1.000 | 0.976 | 1.000 | 1.000 | 1.000 | 0.983 | 1.000 | 1.000 | 1.000 |
| ARCH(2) | Hansen | 300 | 0.219 | 0.335 | 0.195 | 0.239 | 0.173 | 0.230 | 0.127 | 0.169 | 0.163 | 0.191 | 0.099 | 0.132 |
| (0.01,0.1,0.1) | SkSt | 500 | 0.315 | 0.486 | 0.316 | 0.378 | 0.262 | 0.346 | 0.199 | 0.252 | 0.250 | 0.282 | 0.162 | 0.239 |
| | (0.99,8.1) | 1000 | 0.522 | 0.781 | 0.549 | 0.623 | 0.422 | 0.648 | 0.423 | 0.505 | 0.364 | 0.585 | 0.330 | 0.408 |
| | | 2000 | 0.770 | 0.979 | 0.858 | 0.899 | 0.662 | 0.926 | 0.744 | 0.785 | 0.599 | 0.893 | 0.671 | 0.709 |
| ARCH(2) | Hansen | 300 | 0.804 | 0.978 | 0.912 | 0.928 | 0.712 | 0.933 | 0.836 | 0.840 | 0.657 | 0.915 | 0.761 | 0.794 |
| (0.01,0.1,0.4) | SkSt | 500 | 0.927 | 1.000 | 0.993 | 0.993 | 0.865 | 0.996 | 0.974 | 0.985 | 0.839 | 0.993 | 0.947 | 0.957 |
| | (0.99,8.1) | 1000 | 0.986 | 1.000 | 1.000 | 1.000 | 0.971 | 1.000 | 1.000 | 1.000 | 0.965 | 1.000 | 1.000 | 1.000 |
| | | 2000 | 0.998 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 | 0.996 | 1.000 | 1.000 | 1.000 |
| GARCH(1,1) | Hansen | 300 | 0.183 | 0.292 | 0.187 | 0.213 | 0.268 | 0.449 | 0.287 | 0.321 | 0.327 | 0.469 | 0.326 | 0.374 |
| (0.01,0.1,0.8) | SkSt | 500 | 0.250 | 0.482 | 0.316 | 0.354 | 0.410 | 0.693 | 0.489 | 0.551 | 0.480 | 0.706 | 0.521 | 0.583 |
| | (0.99,8.1) | 1000 | 0.409 | 0.766 | 0.571 | 0.623 | 0.657 | 0.944 | 0.797 | 0.851 | 0.739 | 0.957 | 0.836 | 0.866 |
| | | 2000 | 0.673 | 0.975 | 0.859 | 0.893 | 0.907 | 0.999 | 0.981 | 0.994 | 0.935 | 0.999 | 0.987 | 0.991 |
| IGARCH(1,1) | Hansen | 300 | 0.218 | 0.566 | 0.419 | 0.458 | 0.341 | 0.786 | 0.663 | 0.757 | 0.434 | 0.842 | 0.701 | 0.772 |
| (0.01,0.1,0.9) | SkSt | 500 | 0.303 | 0.794 | 0.664 | 0.730 | 0.554 | 0.952 | 0.899 | 0.912 | 0.677 | 0.976 | 0.744 | 0.956 |
| | (0.99,8.1) | 1000 | 0.517 | 0.988 | 0.935 | 0.945 | 0.801 | 1.000 | 0.998 | 1.000 | 0.875 | 1.000 | 0.999 | 1.000 |
| | | 2000 | 0.673 | 1.000 | 0.998 | 0.997 | 0.913 | 1.000 | 1.000 | 1.000 | 0.948 | 1.000 | 1.000 | 1.000 |
| ARCH(2) | Hansen | 300 | 0.110 | 0.272 | 0.151 | 0.180 | 0.102 | 0.159 | 0.089 | 0.133 | 0.124 | 0.144 | 0.068 | 0.101 |
| (0.01,0.1,0.1) | SkSt | 500 | 0.134 | 0.419 | 0.244 | 0.281 | 0.128 | 0.288 | 0.134 | 0.219 | 0.133 | 0.217 | 0.112 | 0.161 |
| | (0.9,3) | 1000 | 0.182 | 0.667 | 0.429 | 0.533 | 0.153 | 0.529 | 0.304 | 0.376 | 0.190 | 0.471 | 0.254 | 0.325 |
| | | 2000 | 0.317 | 0.930 | 0.751 | 0.789 | 0.245 | 0.816 | 0.588 | 0.661 | 0.197 | 0.798 | 0.489 | 0.611 |
| ARCH(2) | Hansen | 300 | 0.446 | 0.866 | 0.726 | 0.756 | 0.312 | 0.801 | 0.561 | 0.635 | 0.297 | 0.690 | 0.477 | 0.565 |
| (0.01,0.1,0.4) | SkSt | 500 | 0.577 | 0.985 | 0.915 | 0.932 | 0.451 | 0.951 | 0.836 | 0.852 | 0.426 | 0.917 | 0.745 | 0.799 |
| | (0.9,3) | 1000 | 0.716 | 0.999 | 0.995 | 0.997 | 0.657 | 0.999 | 0.990 | 0.996 | 0.542 | 0.998 | 0.970 | 0.989 |
| | | 2000 | 0.825 | 1.000 | 1.000 | 1.000 | 0.775 | 1.000 | 1.000 | 1.000 | 0.737 | 1.000 | 1.000 | 1.000 |
| GARCH(1,1) | Hansen | 300 | 0.078 | 0.315 | 0.170 | 0.219 | 0.155 | 0.420 | 0.278 | 0.327 | 0.180 | 0.445 | 0.288 | 0.335 |
| (0.01,0.1,0.8) | SkSt | 500 | 0.123 | 0.470 | 0.288 | 0.337 | 0.204 | 0.670 | 0.424 | 0.513 | 0.249 | 0.678 | 0.462 | 0.594 |
| | (0.9,3) | 1000 | 0.151 | 0.771 | 0.546 | 0.623 | 0.245 | 0.915 | 0.754 | 0.819 | 0.325 | 0.941 | 0.803 | 0.862 |
| | | 2000 | 0.228 | 0.965 | 0.870 | 0.884 | 0.356 | 0.999 | 0.981 | 0.988 | 0.444 | 0.997 | 0.983 | 0.993 |
| IGARCH(1,1) | Hansen | 300 | 0.074 | 0.512 | 0.348 | 0.445 | 0.165 | 0.677 | 0.579 | 0.599 | 0.175 | 0.740 | 0.609 | 0.685 |
| (0.01,0.1,0.9) | SkSt | 500 | 0.115 | 0.720 | 0.534 | 0.636 | 0.226 | 0.918 | 0.809 | 0.937 | 0.290 | 0.947 | 0.854 | 0.916 |
| | (0.9,3) | 1000 | 0.146 | 0.950 | 0.847 | 0.892 | 0.282 | 0.995 | 0.967 | 0.991 | 0.376 | 0.999 | 0.989 | 0.997 |
| | | 2000 | 0.217 | 1.000 | 0.992 | 0.995 | 0.420 | 1.000 | 1.000 | 1.000 | 0.476 | 1.000 | 1.000 | 1.000 |

**Notes:** The tests applied in the table are the LiMak test (LM), the AvdW, the van der Waerden test (vdW) and the Wilcoxon test (Wilc). The DGPs used are the low persistence ARCH(1) with parameters $(\omega, \alpha_1)$:=(0.01,0.1), the high persistence ARCH(1) with parameters $(\omega, \alpha_1)$:=(0.01,0.8), the low persistence ARCH(2) with parameters $(\omega, \alpha_1, \alpha_2)$:=(0.01,0.1,0.1), the high persistence ARCH(2) with parameters $(\omega, \alpha_1, \alpha_2)$:=(0.01,0.1,0.4), the high persistence GARCH(1,1) with parameters $(\omega, \alpha_1, \beta_1)$:=(0.01,0.1,0.8) used by Berkes, Horváth and Kokoszka (2003) and the IGARCH(1,1) with parameters $(\omega, \alpha_1, \beta_1)$:=(0.01,0.1,0.9). The estimated distribution in the power calculations is an ARCH(1) model with Hansen SkSt errors. The distribution used for the innovations in every generated and estimated model is the Hansen Skewed t $(\lambda,\eta)$ distribution, where $\lambda$ is the skewness parameter and $\eta$ is the degrees of freedom. The parameters used for the Hansen SkSt are $(\lambda,\eta)$:=(-0.09,8.1) (skewness is -0.24 and kurtosis is 4.39), $(\lambda,\eta)$:=(0.99,8.1) (skewness is 1.52 and kurtosis is 6.73), $(\lambda,\eta)$:=(-0.09,5) (skewness is -0.28 and kurtosis is 6.35), $(\lambda,\eta)$:=(0.99,5) (skewness is 2.07 and kurtosis is 10.96) and $(\lambda,\eta)$:=(0.9,3). The maximum lag order, K, used in the tests indicate that the maximum standardized residual autocorrelations lag used in the test statistic is K, i.e. $s = 2, ..., K$. The critical value of the tests is based on the 5% significance level of the $\chi^2_{K-1}$. The above results refer to 5000 simulations.

Table 8: Simulated size and power of alternative test statistics for testing the null hypothesis of symmetry in the standardized residuals of AR-GARCH and GARCH models

| | DGP | GARCH(1,1)-(20,0.05,0.9) | | | | GARCH(1,1)-(20,0.3,0.5) | | | |
|---|---|---|---|---|---|---|---|---|---|
| Distr. | T | 200 | 500 | 1000 | 2000 | 200 | 500 | 1000 | 2000 |
| | | Size | | | | | | | |
| N(0,1) | BN | 0.043 | 0.041 | 0.035 | 0.032 | 0.041 | 0.040 | 0.036 | 0.032 |
| | Sign | 0.046 | 0.040 | 0.039 | 0.041 | 0.046 | 0.040 | 0.039 | 0.041 |
| | Wilc | 0.048 | 0.040 | 0.045 | 0.055 | 0.044 | 0.039 | 0.043 | 0.055 |
| t(3) | BN | 0.083 | 0.071 | 0.068 | 0.056 | 0.091 | 0.080 | 0.080 | 0.072 |
| | Sign | 0.044 | 0.049 | 0.038 | 0.047 | 0.053 | 0.043 | 0.046 | 0.042 |
| | Wilc | 0.039 | 0.057 | 0.039 | 0.046 | 0.037 | 0.058 | 0.043 | 0.050 |
| t(5) | BN | 0.054 | 0.046 | 0.052 | 0.045 | 0.049 | 0.041 | 0.057 | 0.041 |
| | Sign | 0.036 | 0.036 | 0.050 | 0.052 | 0.032 | 0.040 | 0.048 | 0.050 |
| | Wilc | 0.048 | 0.042 | 0.060 | 0.055 | 0.055 | 0.052 | 0.053 | 0.050 |
| | DGP | AR-GARCH(1,1)-(0,$\phi$,20,0.05,0.9) | | | | AR-GARCH(1,1)-(0,$\phi$,20,0.3,0.5) | | | |
| | | Size | | | | | | | |
| N(0,1) | BN | 0.036 | 0.049 | 0.037 | 0.040 | 0.033 | 0.040 | 0.033 | 0.039 |
| $\phi = 0.1$ | Sign | 0.003 | 0.000 | 0.002 | 0.004 | 0.004 | 0.007 | 0.008 | 0.005 |
| | Wilc | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| N(0,1) | BN | 0.026 | 0.041 | 0.038 | 0.039 | 0.035 | 0.033 | 0.041 | 0.040 |
| $\phi = 0.4$ | Sign | 0.001 | 0.003 | 0.000 | 0.002 | 0.004 | 0.010 | 0.001 | 0.002 |
| | Wilc | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| N(0,1) | BN | 0.038 | 0.037 | 0.040 | 0.032 | 0.039 | 0.033 | 0.032 | 0.037 |
| $\phi = 0.8$ | Sign | 0.000 | 0.002 | 0.002 | 0.002 | 0.006 | 0.007 | 0.004 | 0.005 |
| | Wilc | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | Power | | | | | | | |
| | DGP | GARCH(1,1)-(20,0.05,0.9) | | | | GARCH(1,1)-(20,0.3,0.5) | | | |
| Hansen SkSt (0.9, 3) | BN | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | Sign | 0.989 | 1.000 | 1.000 | 1.000 | 0.977 | 1.000 | 1.000 | 1.000 |
| | Wilc | 0.838 | 0.996 | 1.000 | 1.000 | 0.850 | 0.998 | 1.000 | 1.000 |
| Hansen SkSt (0.5, 3) | BN | 0.970 | 1.000 | 1.000 | 1.000 | 0.971 | 1.000 | 1.000 | 1.000 |
| | Sign | 0.883 | 1.000 | 1.000 | 1.000 | 0.893 | 0.999 | 1.000 | 1.000 |
| | Wilc | 0.628 | 0.948 | 0.999 | 1.000 | 0.666 | 0.949 | 0.999 | 1.000 |
| Hansen SkSt (0.99, 8.1) | BN | 0.995 | 1.000 | 1.000 | 1.000 | 0.994 | 1.000 | 1.000 | 1.000 |
| | Sign | 0.756 | 0.987 | 1.000 | 1.000 | 0.745 | 0.989 | 1.000 | 1.000 |
| | Wilc | 0.433 | 0.760 | 0.971 | 0.999 | 0.410 | 0.762 | 0.965 | 1.000 |
| Hansen SkSt (-0.09, 8.1) | BN | 0.086 | 0.131 | 0.231 | 0.418 | 0.092 | 0.137 | 0.257 | 0.424 |
| | Sign | 0.069 | 0.091 | 0.145 | 0.306 | 0.064 | 0.120 | 0.148 | 0.269 |
| | Wilc | 0.060 | 0.063 | 0.086 | 0.158 | 0.065 | 0.068 | 0.098 | 0.144 |
| $\chi^2_2$ | BN | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | Sign | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | Wilc | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Lambda 1 (0, -1, -0.001, -0.13) | BN | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | Sign | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | Wilc | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Lognormal (0,1) | BN | 0.998 | 1.000 | 1.000 | 1.000 | 0.998 | 1.000 | 1.000 | 1.000 |
| | Sign | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | Wilc | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

**Notes:** The tests used in the table are the Bai and Ng (BN), the Sign test (Sign) and the Wilcoxon test (Wilc). The AR(1)-GARCH(1,1) model with coefficients ($c_0$,$\phi$,$\omega$,$\alpha_1$,$\beta_1$) is given by $Y_t = c_0 + \phi Y_{t-1} + v_t$, $v_t = \sigma_t \varepsilon_t$ and $\sigma_t^2 = \omega + \alpha_1 Y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. The DGPs used are the AR(1)-GARCH(1,1) with coefficients $(c_0, \phi)$ :=(0,0.1), (0,0.4) and (0,0.8) and $(\omega, \alpha_1, \beta_1)$:=(20,0.05,0.9) and (20,0.3,0.5). The distribution used for the residuals under the null is the normal, N(0,1). The GARCH(1,1) model with coefficients ($\omega$,$\alpha_1$,$\beta_1$) is given by $Y_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = \omega + \alpha_1 Y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. The DGPs used are the GARCH(1,1) with coefficients $(\omega, \alpha_1, \beta_1)$:=(20,0.05,0.9) and the GARCH(1,1) with coefficients $(\omega, \alpha_1, \beta_1)$:=(20,0.3,0.5), both used by Bai and Ng (2001). The distribution used for the residuals under the null is the normal, N(0,1), and the Student's t($v$) distribution with $v$:=3 and 5 degrees of freedom. Under the alternative the distributions are the Hansen's Skewed t($\lambda$,$\eta$) by Hansen (1994) with skewness parameter $\lambda$ and $\eta$ degrees of freedom, $\chi_2^2$ is the Chi-Squared distribution with 2 degrees of freedom, Lambda 1($\lambda_1$,$\lambda_2$,$\lambda_3$,$\lambda_4$) is the Lambda distribution used by Bai and Ng (2001) given by $F^{-1}(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$ and the Lognormal(0,1) distribution is the Lognormal distribution with mean 0 and variance 1. Hansen SkSt $(0.9, 3)$ has skewness 4.63 and kurtosis 51.51, Hansen SkSt $(0.5, 3)$ has skewness 3.81 and kurtosis 46.02, Hansen SkSt $(0.99, 8.1)$ has skewness 1.61 and kurtosis 7.57 and lastly, Hansen SkSt $(-0.09, 8.1)$ skewness -0.26 and kurtosis 4.45. The critical values for the BN, the Sign and the Wilcoxon tests are 2.20, 3.84, 1.96 and 1.96 respectively.The above results refer to 5000 simulations.

Table 9: Simulation results of the breaks tests on GARCH type models

| | | DGP | GARCH(1,1) (0.0002,0.1,0.7) | | | | | | | | GARCH(1,1) (0.0002,0.1,0.8) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Break point $\pi$ | | 0.5 | | | | 0.7 | | | | 0.5 | | | | 0.7 | | | |
| Distr. | T | Test | Size | P1 | P2 | P3 | Size | P1 | P2 | P3 | Size | P1 | P2 | P3 | Size | P1 | P2 | P3 |
| N(0,1) | 500 | KY | 0.021 | 0.237 | 0.070 | 0.165 | 0.021 | 0.147 | 0.055 | 0.108 | 0.017 | 0.122 | 0.184 | 0.118 | 0.017 | 0.065 | 0.188 | 0.191 |
| | | RBC | 0.034 | 0.061 | 0.017 | 0.043 | 0.034 | 0.040 | 0.020 | 0.033 | 0.031 | 0.028 | 0.057 | 0.029 | 0.031 | 0.016 | 0.047 | 0.049 |
| | | Pet | 0.034 | 0.061 | 0.017 | 0.043 | 0.034 | 0.044 | 0.020 | 0.033 | 0.031 | 0.028 | 0.057 | 0.029 | 0.031 | 0.016 | 0.047 | 0.049 |
| | | Lomb | 0.042 | 0.116 | 0.047 | 0.094 | 0.042 | 0.023 | 0.005 | 0.018 | 0.047 | 0.067 | 0.103 | 0.063 | 0.047 | 0.009 | 0.031 | 0.040 |
| | 1000 | KY | 0.035 | 0.708 | 0.282 | 0.600 | 0.035 | 0.536 | 0.262 | 0.441 | 0.026 | 0.398 | 0.628 | 0.484 | 0.026 | 0.246 | 0.544 | 0.569 |
| | | RBC | 0.031 | 0.336 | 0.092 | 0.248 | 0.031 | 0.181 | 0.072 | 0.141 | 0.028 | 0.126 | 0.249 | 0.139 | 0.028 | 0.080 | 0.179 | 0.148 |
| | | Pet | 0.031 | 0.336 | 0.092 | 0.248 | 0.031 | 0.181 | 0.072 | 0.141 | 0.028 | 0.126 | 0.249 | 0.139 | 0.028 | 0.080 | 0.179 | 0.148 |
| | | Lomb | 0.048 | 0.351 | 0.172 | 0.284 | 0.048 | 0.156 | 0.042 | 0.107 | 0.039 | 0.195 | 0.312 | 0.207 | 0.039 | 0.047 | 0.144 | 0.137 |
| | 3000 | KY | 0.044 | 0.989 | 0.999 | 0.998 | 0.044 | 0.994 | 0.937 | 0.991 | 0.046 | 0.999 | 0.999 | 1.000 | 0.046 | 0.970 | 0.999 | 0.998 |
| | | RBC | 0.044 | 0.976 | 0.889 | 0.984 | 0.044 | 0.915 | 0.614 | 0.865 | 0.041 | 0.896 | 0.993 | 0.960 | 0.041 | 0.600 | 0.899 | 0.879 |
| | | Pet | 0.044 | 0.976 | 0.889 | 0.984 | 0.044 | 0.915 | 0.614 | 0.865 | 0.041 | 0.896 | 0.993 | 0.960 | 0.041 | 0.600 | 0.899 | 0.879 |
| | | Lomb | 0.046 | 0.943 | 0.704 | 0.920 | 0.046 | 0.892 | 0.583 | 0.851 | 0.046 | 0.753 | 0.929 | 0.890 | 0.046 | 0.549 | 0.906 | 0.870 |
| t(3) | 500 | KY | 0.011 | 0.041 | 0.018 | 0.022 | 0.011 | 0.034 | 0.019 | 0.025 | 0.019 | 0.019 | 0.022 | 0.026 | 0.019 | 0.020 | 0.018 | 0.042 |
| | | RBC | 0.022 | 0.182 | 0.066 | 0.133 | 0.022 | 0.148 | 0.063 | 0.098 | 0.034 | 0.124 | 0.092 | 0.240 | 0.034 | 0.068 | 0.073 | 0.179 |
| | | Pet | 0.022 | 0.182 | 0.066 | 0.133 | 0.022 | 0.148 | 0.063 | 0.098 | 0.034 | 0.124 | 0.092 | 0.240 | 0.034 | 0.068 | 0.073 | 0.179 |
| | | Lomb | 0.037 | 0.206 | 0.075 | 0.144 | 0.037 | 0.163 | 0.066 | 0.114 | 0.040 | 0.138 | 0.111 | 0.248 | 0.040 | 0.081 | 0.098 | 0.196 |
| | 1000 | KY | 0.017 | 0.084 | 0.026 | 0.056 | 0.017 | 0.079 | 0.033 | 0.059 | 0.015 | 0.059 | 0.043 | 0.067 | 0.015 | 0.040 | 0.048 | 0.088 |
| | | RBC | 0.044 | 0.432 | 0.142 | 0.321 | 0.044 | 0.306 | 0.103 | 0.219 | 0.039 | 0.304 | 0.200 | 0.594 | 0.039 | 0.204 | 0.174 | 0.469 |
| | | Pet | 0.044 | 0.432 | 0.142 | 0.321 | 0.044 | 0.306 | 0.103 | 0.219 | 0.039 | 0.304 | 0.200 | 0.594 | 0.039 | 0.204 | 0.174 | 0.469 |
| | | Lomb | 0.050 | 0.425 | 0.147 | 0.326 | 0.050 | 0.304 | 0.115 | 0.227 | 0.048 | 0.282 | 0.211 | 0.572 | 0.048 | 0.223 | 0.177 | 0.455 |
| | 3000 | KY | 0.026 | 0.269 | 0.069 | 0.188 | 0.026 | 0.245 | 0.065 | 0.157 | 0.020 | 0.188 | 0.116 | 0.296 | 0.020 | 0.163 | 0.108 | 0.300 |
| | | RBC | 0.043 | 0.951 | 0.378 | 0.846 | 0.043 | 0.827 | 0.320 | 0.695 | 0.039 | 0.842 | 0.679 | 0.989 | 0.039 | 0.673 | 0.596 | 0.976 |
| | | Pet | 0.043 | 0.951 | 0.378 | 0.846 | 0.043 | 0.827 | 0.320 | 0.695 | 0.039 | 0.842 | 0.679 | 0.989 | 0.039 | 0.673 | 0.596 | 0.976 |
| | | Lomb | 0.041 | 0.933 | 0.372 | 0.817 | 0.041 | 0.799 | 0.312 | 0.665 | 0.040 | 0.807 | 0.658 | 0.983 | 0.040 | 0.636 | 0.575 | 0.952 |
| t(5) | 500 | KY | 0.018 | 0.061 | 0.042 | 0.067 | 0.018 | 0.074 | 0.048 | 0.061 | 0.018 | 0.046 | 0.051 | 0.036 | 0.018 | 0.034 | 0.064 | 0.055 |
| | | RBC | 0.027 | 0.157 | 0.088 | 0.129 | 0.027 | 0.096 | 0.062 | 0.102 | 0.028 | 0.115 | 0.136 | 0.168 | 0.028 | 0.052 | 0.095 | 0.159 |
| | | Pet | 0.027 | 0.157 | 0.088 | 0.129 | 0.027 | 0.096 | 0.062 | 0.102 | 0.028 | 0.115 | 0.136 | 0.168 | 0.028 | 0.052 | 0.095 | 0.159 |
| | | Lomb | 0.039 | 0.161 | 0.095 | 0.144 | 0.039 | 0.114 | 0.076 | 0.123 | 0.039 | 0.124 | 0.141 | 0.180 | 0.039 | 0.074 | 0.115 | 0.175 |
| | 1000 | KY | 0.029 | 0.237 | 0.116 | 0.192 | 0.029 | 0.193 | 0.088 | 0.150 | 0.012 | 0.136 | 0.168 | 0.176 | 0.012 | 0.118 | 0.138 | 0.205 |
| | | RBC | 0.041 | 0.380 | 0.182 | 0.296 | 0.041 | 0.287 | 0.158 | 0.240 | 0.045 | 0.225 | 0.324 | 0.407 | 0.045 | 0.162 | 0.219 | 0.351 |
| | | Pet | 0.041 | 0.380 | 0.182 | 0.296 | 0.041 | 0.287 | 0.158 | 0.240 | 0.045 | 0.225 | 0.324 | 0.407 | 0.045 | 0.162 | 0.219 | 0.351 |
| | | Lomb | 0.032 | 0.387 | 0.187 | 0.310 | 0.032 | 0.304 | 0.171 | 0.258 | 0.040 | 0.247 | 0.342 | 0.412 | 0.040 | 0.161 | 0.253 | 0.357 |
| | 3000 | KY | 0.035 | 0.760 | 0.360 | 0.655 | 0.035 | 0.637 | 0.309 | 0.567 | 0.029 | 0.557 | 0.645 | 0.818 | 0.029 | 0.400 | 0.553 | 0.817 |
| | | RBC | 0.056 | 0.928 | 0.544 | 0.862 | 0.056 | 0.827 | 0.424 | 0.741 | 0.040 | 0.750 | 0.864 | 0.979 | 0.040 | 0.589 | 0.777 | 0.926 |
| | | Pet | 0.056 | 0.928 | 0.544 | 0.862 | 0.056 | 0.827 | 0.424 | 0.741 | 0.040 | 0.750 | 0.864 | 0.979 | 0.040 | 0.589 | 0.777 | 0.926 |
| | | Lomb | 0.059 | 0.902 | 0.535 | 0.842 | 0.059 | 0.785 | 0.413 | 0.694 | 0.047 | 0.717 | 0.833 | 0.961 | 0.047 | 0.543 | 0.742 | 0.900 |
| t(8) | 500 | KY | 0.019 | 0.135 | 0.072 | 0.091 | 0.019 | 0.106 | 0.057 | 0.075 | 0.016 | 0.059 | 0.065 | 0.037 | 0.016 | 0.064 | 0.081 | 0.094 |
| | | RBC | 0.024 | 0.152 | 0.092 | 0.123 | 0.024 | 0.102 | 0.060 | 0.086 | 0.031 | 0.100 | 0.135 | 0.135 | 0.031 | 0.053 | 0.109 | 0.125 |
| | | Pet | 0.024 | 0.152 | 0.092 | 0.123 | 0.024 | 0.102 | 0.060 | 0.086 | 0.031 | 0.100 | 0.135 | 0.135 | 0.031 | 0.053 | 0.109 | 0.125 |
| | | Lomb | 0.036 | 0.172 | 0.113 | 0.150 | 0.036 | 0.107 | 0.093 | 0.124 | 0.036 | 0.121 | 0.149 | 0.151 | 0.036 | 0.068 | 0.122 | 0.147 |
| | 1000 | KY | 0.019 | 0.409 | 0.183 | 0.305 | 0.019 | 0.306 | 0.149 | 0.244 | 0.035 | 0.223 | 0.299 | 0.293 | 0.035 | 0.176 | 0.256 | 0.349 |
| | | RBC | 0.039 | 0.387 | 0.194 | 0.295 | 0.039 | 0.265 | 0.158 | 0.221 | 0.037 | 0.211 | 0.318 | 0.355 | 0.037 | 0.146 | 0.244 | 0.324 |
| | | Pet | 0.039 | 0.387 | 0.194 | 0.295 | 0.039 | 0.265 | 0.158 | 0.221 | 0.037 | 0.211 | 0.318 | 0.355 | 0.037 | 0.146 | 0.244 | 0.324 |
| | | Lomb | 0.050 | 0.386 | 0.209 | 0.298 | 0.050 | 0.284 | 0.176 | 0.242 | 0.053 | 0.239 | 0.336 | 0.345 | 0.053 | 0.164 | 0.257 | 0.341 |
| | 3000 | KY | 0.039 | 0.956 | 0.675 | 0.904 | 0.039 | 0.878 | 0.543 | 0.805 | 0.044 | 0.813 | 0.918 | 0.981 | 0.044 | 0.682 | 0.891 | 0.976 |
| | | RBC | 0.051 | 0.925 | 0.634 | 0.883 | 0.051 | 0.810 | 0.492 | 0.705 | 0.056 | 0.748 | 0.884 | 0.937 | 0.056 | 0.588 | 0.797 | 0.900 |
| | | Pet | 0.051 | 0.925 | 0.634 | 0.883 | 0.051 | 0.810 | 0.492 | 0.705 | 0.056 | 0.748 | 0.884 | 0.937 | 0.056 | 0.588 | 0.797 | 0.900 |
| | | Lomb | 0.060 | 0.912 | 0.599 | 0.860 | 0.060 | 0.768 | 0.480 | 0.688 | 0.061 | 0.699 | 0.847 | 0.912 | 0.061 | 0.556 | 0.756 | 0.842 |

**Notes:** The tests used in the breaks tests are the KY CUSUM test (KY), the AW CUSUM test (RBC), the Pettitt test (Pet) and the Lombard test (Lomb). The GARCH(1,1) model with coefficients ($\omega$,$\alpha_1$,$\beta_1$) is given by $Y_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = \omega + \alpha_1 Y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. The generated DGPs are the GARCH(1,1) models used by Kulperger and Yu (2005) with coefficients $(\omega, \alpha_1, \beta_1)$:=(0.0002,0.1,0.7) and (0.0002,0.1,0.8). For size calculations a GARCH(1,1) model was estimated. The distributions used for the residuals are the standard normal N(0,1) and the Student's t(3), t(5) and t(8). For the power calculations the following alternatives were generated: a GARCH(1,1) with coefficients $(\omega, \alpha_1, \beta_1)$:=(0.0003,0.1,0.7) as P1, (0.0002,0.167,0.7) as P2 and (0.0002,0.1,0.767) as P3, for the first DGP, and (0.0003,0.1,0.8) as P1, (0.0002,0.167,0.8) as P2 and (0.0002,0.1,0.867) as P3, for the second DGP. The break location $\pi$ is set at $0.5T$ and at $0.7T$. The RBC, KY and Pettitt tests have critical values 1.358 and the Lombard test has 0.461. The above results refer to 5000 simulations.

Table 9 Continued: Simulation results of the breaks tests on GARCH type models

| | | DGP | GARCH(1,1) (0.0002,0.1,0.7) | | | | | | | | GARCH(1,1) (0.0002,0.1,0.8) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Break point $\pi$ | | 0.5 | | | | 0.7 | | | | 0.5 | | | | 0.7 | | | |
| Distr. | T | Test | Size | P1 | P2 | P3 | Size | P1 | P2 | P3 | Size | P1 | P2 | P3 | Size | P1 | P2 | P3 |
| Hansen | 500 | KY | 0.013 | 0.114 | 0.084 | 0.086 | 0.013 | 0.107 | 0.057 | 0.089 | 0.019 | 0.053 | 0.069 | 0.047 | 0.019 | 0.057 | 0.092 | 0.094 |
| SkSt | | RBC | 0.034 | 0.137 | 0.095 | 0.109 | 0.034 | 0.102 | 0.076 | 0.093 | 0.031 | 0.095 | 0.130 | 0.135 | 0.031 | 0.070 | 0.098 | 0.142 |
| (-0.09,8.1) | | Pet | 0.034 | 0.137 | 0.095 | 0.109 | 0.034 | 0.102 | 0.076 | 0.093 | 0.031 | 0.095 | 0.130 | 0.135 | 0.031 | 0.070 | 0.098 | 0.142 |
| | | Lomb | 0.035 | 0.151 | 0.114 | 0.125 | 0.035 | 0.126 | 0.087 | 0.098 | 0.034 | 0.108 | 0.159 | 0.150 | 0.034 | 0.079 | 0.115 | 0.165 |
| | 1000 | KY | 0.033 | 0.385 | 0.189 | 0.304 | 0.033 | 0.301 | 0.152 | 0.276 | 0.027 | 0.221 | 0.279 | 0.317 | 0.027 | 0.148 | 0.271 | 0.341 |
| | | RBC | 0.036 | 0.372 | 0.209 | 0.320 | 0.036 | 0.242 | 0.155 | 0.225 | 0.034 | 0.208 | 0.314 | 0.350 | 0.034 | 0.134 | 0.253 | 0.287 |
| | | Pet | 0.036 | 0.372 | 0.209 | 0.320 | 0.036 | 0.242 | 0.155 | 0.225 | 0.034 | 0.208 | 0.314 | 0.350 | 0.034 | 0.134 | 0.253 | 0.287 |
| | | Lomb | 0.046 | 0.381 | 0.223 | 0.318 | 0.046 | 0.239 | 0.165 | 0.246 | 0.043 | 0.232 | 0.322 | 0.372 | 0.043 | 0.148 | 0.266 | 0.309 |
| | 3000 | KY | 0.037 | 0.951 | 0.650 | 0.902 | 0.037 | 0.892 | 0.546 | 0.833 | 0.033 | 0.780 | 0.916 | 0.979 | 0.033 | 0.648 | 0.848 | 0.970 |
| | | RBC | 0.042 | 0.936 | 0.619 | 0.870 | 0.042 | 0.822 | 0.500 | 0.781 | 0.045 | 0.737 | 0.899 | 0.933 | 0.045 | 0.562 | 0.791 | 0.894 |
| | | Pet | 0.042 | 0.936 | 0.619 | 0.870 | 0.042 | 0.822 | 0.500 | 0.781 | 0.045 | 0.737 | 0.899 | 0.933 | 0.045 | 0.562 | 0.791 | 0.894 |
| | | Lomb | 0.033 | 0.915 | 0.601 | 0.853 | 0.033 | 0.784 | 0.479 | 0.727 | 0.047 | 0.715 | 0.860 | 0.902 | 0.047 | 0.524 | 0.745 | 0.856 |
| Hansen | 500 | KY | 0.008 | 0.069 | 0.039 | 0.071 | 0.008 | 0.069 | 0.044 | 0.047 | 0.015 | 0.033 | 0.044 | 0.034 | 0.015 | 0.031 | 0.051 | 0.090 |
| SkSt | | RBC | 0.034 | 0.150 | 0.102 | 0.136 | 0.034 | 0.110 | 0.071 | 0.106 | 0.035 | 0.086 | 0.127 | 0.159 | 0.035 | 0.075 | 0.098 | 0.136 |
| (-0.15,5.57) | | Pet | 0.034 | 0.150 | 0.102 | 0.136 | 0.034 | 0.110 | 0.071 | 0.106 | 0.035 | 0.086 | 0.127 | 0.159 | 0.035 | 0.075 | 0.098 | 0.136 |
| | | Lomb | 0.037 | 0.141 | 0.099 | 0.132 | 0.037 | 0.111 | 0.070 | 0.105 | 0.036 | 0.086 | 0.125 | 0.153 | 0.036 | 0.078 | 0.099 | 0.131 |
| | 1000 | KY | 0.022 | 0.282 | 0.103 | 0.201 | 0.022 | 0.230 | 0.131 | 0.158 | 0.027 | 0.143 | 0.196 | 0.199 | 0.027 | 0.116 | 0.169 | 0.215 |
| | | RBC | 0.041 | 0.420 | 0.188 | 0.325 | 0.041 | 0.261 | 0.141 | 0.238 | 0.032 | 0.275 | 0.333 | 0.434 | 0.032 | 0.177 | 0.228 | 0.357 |
| | | Pet | 0.041 | 0.420 | 0.188 | 0.325 | 0.041 | 0.261 | 0.141 | 0.238 | 0.032 | 0.275 | 0.333 | 0.434 | 0.032 | 0.177 | 0.228 | 0.357 |
| | | Lomb | 0.040 | 0.395 | 0.181 | 0.308 | 0.040 | 0.236 | 0.139 | 0.231 | 0.035 | 0.258 | 0.308 | 0.413 | 0.035 | 0.164 | 0.214 | 0.320 |
| | 3000 | KY | 0.028 | 0.811 | 0.440 | 0.735 | 0.028 | 0.710 | 0.348 | 0.621 | 0.036 | 0.587 | 0.713 | 0.866 | 0.036 | 0.476 | 0.628 | 0.855 |
| | | RBC | 0.040 | 0.941 | 0.620 | 0.864 | 0.040 | 0.835 | 0.480 | 0.717 | 0.039 | 0.765 | 0.880 | 0.966 | 0.039 | 0.621 | 0.771 | 0.940 |
| | | Pet | 0.040 | 0.941 | 0.620 | 0.864 | 0.040 | 0.835 | 0.480 | 0.717 | 0.039 | 0.765 | 0.880 | 0.966 | 0.039 | 0.621 | 0.771 | 0.940 |
| | | Lomb | 0.043 | 0.923 | 0.587 | 0.832 | 0.043 | 0.805 | 0.448 | 0.675 | 0.042 | 0.741 | 0.857 | 0.954 | 0.042 | 0.573 | 0.729 | 0.914 |
| Hansen | 500 | KY | 0.016 | 0.049 | 0.025 | 0.036 | 0.016 | 0.055 | 0.026 | 0.030 | 0.020 | 0.040 | 0.034 | 0.031 | 0.020 | 0.029 | 0.040 | 0.076 |
| SkSt | | RBC | 0.034 | 0.148 | 0.092 | 0.133 | 0.034 | 0.120 | 0.066 | 0.096 | 0.025 | 0.098 | 0.126 | 0.186 | 0.025 | 0.067 | 0.101 | 0.165 |
| (0.05,4.23) | | Pet | 0.034 | 0.148 | 0.092 | 0.133 | 0.034 | 0.120 | 0.066 | 0.096 | 0.025 | 0.098 | 0.126 | 0.186 | 0.025 | 0.067 | 0.101 | 0.160 |
| | | Lomb | 0.039 | 0.144 | 0.089 | 0.131 | 0.039 | 0.117 | 0.063 | 0.092 | 0.027 | 0.096 | 0.124 | 0.184 | 0.027 | 0.067 | 0.101 | 0.153 |
| | 1000 | KY | 0.024 | 0.191 | 0.068 | 0.136 | 0.024 | 0.151 | 0.085 | 0.103 | 0.024 | 0.100 | 0.143 | 0.137 | 0.024 | 0.084 | 0.110 | 0.174 |
| | | RBC | 0.037 | 0.416 | 0.164 | 0.315 | 0.037 | 0.270 | 0.116 | 0.231 | 0.034 | 0.278 | 0.295 | 0.473 | 0.034 | 0.181 | 0.203 | 0.395 |
| | | Pet | 0.037 | 0.416 | 0.164 | 0.315 | 0.037 | 0.270 | 0.116 | 0.231 | 0.034 | 0.278 | 0.295 | 0.473 | 0.034 | 0.181 | 0.203 | 0.395 |
| | | Lomb | 0.040 | 0.388 | 0.151 | 0.303 | 0.040 | 0.256 | 0.109 | 0.218 | 0.036 | 0.260 | 0.280 | 0.456 | 0.036 | 0.172 | 0.196 | 0.372 |
| | 3000 | KY | 0.033 | 0.597 | 0.240 | 0.473 | 0.033 | 0.514 | 0.218 | 0.419 | 0.033 | 0.449 | 0.443 | 0.694 | 0.033 | 0.325 | 0.390 | 0.693 |
| | | RBC | 0.050 | 0.942 | 0.535 | 0.851 | 0.050 | 0.838 | 0.424 | 0.721 | 0.037 | 0.767 | 0.853 | 0.984 | 0.037 | 0.612 | 0.715 | 0.950 |
| | | Pet | 0.050 | 0.942 | 0.535 | 0.851 | 0.050 | 0.838 | 0.424 | 0.721 | 0.037 | 0.767 | 0.853 | 0.984 | 0.037 | 0.612 | 0.715 | 0.950 |
| | | Lomb | 0.053 | 0.926 | 0.505 | 0.826 | 0.053 | 0.804 | 0.397 | 0.674 | 0.038 | 0.739 | 0.822 | 0.972 | 0.038 | 0.567 | 0.686 | 0.937 |

**Notes:** The tests used in the breaks tests are the KY CUSUM test (KY), the AW CUSUM test (RBC), the Pettitt test (Pet) and the Lombard test (Lomb). The GARCH(1,1) model with coefficients ($\omega$,$\alpha_1$,$\beta_1$) is given by $Y_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = \omega + \alpha_1 Y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. The generated DGPs are the GARCH(1,1) models used by Kulperger and Yu (2005) with coefficients $(\omega, \alpha_1, \beta_1)$:=(0.0002,0.1,0.7) and (0.0002,0.1,0.8). For size calculations a GARCH(1,1) model was estimated. The distribution used for the residuals is the Hansen's Skewed t($\lambda$,$\eta$) by Hansen (1994) with skewness parameter $\lambda$ and $\eta$ degrees of freedom. The choice of the Hansen SkSt parameters was based on those of Hansen (1994) and to match the parameters of two emerging markets, PAK and VEN. For the power calculations the following alternatives were generated: a GARCH(1,1) with coefficients $(\omega, \alpha_1, \beta_1)$:=(0.0003,0.1,0.7) as P1, (0.0002,0.167,0.7) as P2 and (0.0002,0.1,0.767) as P3, for the first DGP, and (0.0003,0.1,0.8) as P1, (0.0002,0.167,0.8) as P2 and (0.0002,0.1,0.867) as P3, for the second DGP. The break location $\pi$ is set at $0.5T$ and at $0.7T$. The break location $\pi$ is set at $0.5T$ and at $0.7T$. The RBC, KY and Pettitt tests have critical values 1.358 and the Lombard test has 0.461. The above results refer to 5000 simulations.

Table 10: Descriptive statistics of the stock market returns indices and the standardised residuals of GARCH type models of the major and emerging stock markets

| | Returns | | | | Standardised Residuals | |
|---|---|---|---|---|---|---|
| Series | Mean | Std Dev | Skewness | Kurtosis | N-AR-GARCH Skewness | N-AR-GARCH Kurtosis |
| NIKKEI225 | -0.085 | 2.881 | -0.108 | 4.257 | -0.226 | 4.109 |
| FTSE100 | 0.109 | 2.082 | -0.059 | 4.448 | -0.135 | 3.899 |
| SP500 | 0.150 | 2.073 | -0.455 | 5.585 | -0.416 | 3.900 |
| DAX | 0.112 | 2.895 | -0.314 | 4.941 | -0.310 | 3.413 |
| CAC40 | 0.102 | 2.667 | -0.153 | 3.966 | -0.239 | 3.233 |
| SP/TSX | 0.132 | 1.993 | -0.620 | 6.615 | -0.437 | 4.225 |
| HSI | 0.202 | 3.444 | -0.501 | 6.080 | -0.215 | 4.148 |
| ARG | 1.051 | 7.459 | 2.515 | 22.536 | 0.215 | 5.449 |
| BRA | 1.884 | 6.384 | 0.172 | 7.810 | -0.532 | 5.131 |
| CHI | 0.401 | 2.644 | 0.007 | 4.600 | 0.069 | 4.125 |
| COL | 0.534 | 3.325 | 0.397 | 9.865 | 0.312 | 4.827 |
| INDIA | 0.318 | 3.712 | -0.122 | 4.941 | -0.208 | 3.867 |
| KOR | 0.104 | 4.297 | -0.060 | 5.393 | -0.048 | 3.860 |
| MAL | 0.137 | 3.372 | 0.172 | 11.776 | -0.430 | 5.485 |
| MEX | 0.469 | 3.242 | -0.286 | 4.814 | -0.316 | 3.830 |
| PAK | 0.279 | 3.617 | -0.430 | 5.708 | -0.158 | 4.335 |
| PHIL | 0.140 | 3.536 | -0.609 | 8.179 | -0.654 | 8.023 |
| THAI | 0.121 | 4.525 | -0.043 | 6.814 | -0.230 | 4.963 |
| VEN | 0.681 | 4.741 | 0.552 | 6.735 | 0.738 | 6.932 |

**Notes:** The first seven series are some of the major stock market returns indices and the remaining series are the stock market index returns for the 12 emerging countries that are used in the analysis. The returns series are weekly, measured in local currency and span from 06/01/1989 up to 15/08/2008 (1024 sample size) for the emerging market return indices, with the exception of Venezuela which spans from 06/01/1989 up to 06/04/2007 (953 sample size), due to data availability. The major stock market returns indices span from 07/01/1989-16/08/2008 (FTSE100 and S&P500 span from 03/01/1989-18/08/2008 and S&P/TSX spans from 07/01/1989 up to 18/08/2008). The table shows the descriptive statistics for the returns and the descriptive statistics of the standardized residuals obtained from fitting a normal-AR(1)-GARCH(1,1) (N-AR-GARCH) model to the returns series.

Table 11: Normal AR(1)-GARCH(1,1) parameter estimates and tests for stock market returns indices of some of the major stock markets and emerging markets in Asia and Latin America

| Series | $c$ | $\rho$ | $\omega$ | $\alpha$ | $\beta$ | $\alpha+\beta$ | Wald test for IGARCH |
|---|---|---|---|---|---|---|---|
| NIKKEI225 | 0.043 | -0.024 | 0.545 | 0.101 | 0.836 | 0.937 | 7.552** |
| | (0.080) | (0.034) | (0.201)** | (0.031)** | (0.044)** | | |
| FTSE100 | 0.210 | -0.032 | 0.116 | 0.086 | 0.889 | 0.975 | 4.557** |
| | (0.061)** | (0.034) | (0.041)** | (0.021)** | (0.023)** | | |
| SP500 | 0.229 | -0.125 | 0.038 | 0.064 | 0.929 | 0.993 | 0.848 |
| | (0.055)** | (0.034)** | (0.023) | (0.015)** | (0.015)** | | |
| DAX | 0.266 | -0.027 | 0.240 | 0.113 | 0.859 | 0.972 | 4.161** |
| | (0.075)** | (0.035) | (0.086)** | (0.028)** | (0.030)** | | |
| CAC40 | 0.194 | -0.025 | 0.209 | 0.091 | 0.881 | 0.972 | 4.278** |
| | (0.072)** | (0.032) | (0.085)** | (0.025)** | (0.029)** | | |
| SP/TSX | 0.181 | 0.022 | 0.064 | 0.060 | 0.924 | 0.984 | 1.584 |
| | (0.055)** | (0.033) | (0.039) | (0.017)** | (0.022)** | | |
| HSI | 0.365 | 0.012 | 0.202 | 0.100 | 0.888 | 0.988 | 1.203 |
| | (0.093)** | (0.035) | (0.100)** | (0.032)** | (0.032)** | | |
| ARG | 0.429 | 0.092 | 1.116 | 0.210 | 0.779 | 0.989 | 0.133 |
| | (0.134)** | (0.038)** | (0.404)** | (0.056)** | (0.040)** | | |
| BRA | 0.727 | 0.095 | 0.428 | 0.096 | 0.894 | 0.990 | 0.876 |
| | (0.155)** | (0.037)** | (0.244) | (0.019)** | (0.020)** | | |
| CHI | 0.287 | 0.185 | 0.157 | 0.087 | 0.891 | 0.978 | 2.358 |
| | (0.073)** | (0.034)** | (0.082) | (0.027)** | (0.035)** | | |
| COL | 0.447 | 0.199 | 1.307 | 0.286 | 0.606 | 0.892 | 6.395** |
| | (0.075)** | (0.038)** | (0.349)** | (0.065)** | (0.070)** | | |
| INDIA | 0.352 | 0.114 | 0.697 | 0.125 | 0.829 | 0.954 | 4.720** |
| | (0.100)** | (0.033)** | (0.252)** | (0.029)** | (0.037)** | | |
| KOR | 0.172 | -0.039 | 0.205 | 0.070 | 0.920 | 0.990 | 1.407 |
| | (0.110) | (0.034) | (0.112) | (0.019)** | (0.021)** | | |
| MAL | 0.186 | 0.077 | 0.086 | 0.103 | 0.896 | 0.999 | 0.020 |
| | (0.083**) | (0.041) | (0.050) | (0.030)** | (0.026)** | | |
| MEX | 0.524 | 0.089 | 0.375 | 0.084 | 0.882 | 0.966 | 4.231** |
| | (0.094)** | (0.033)** | (0.155)** | (0.023)** | (0.030)** | | |
| PAK | 0.186 | 0.190 | 0.489 | 0.173 | 0.806 | 0.979 | 1.653 |
| | (0.090)** | (0.037)** | (0.153)** | (0.032)** | (0.033)** | | |
| PHIL | 0.165 | 0.110 | 0.288 | 0.064 | 0.916 | 0.980 | 1.333 |
| | (0.111) | (0.034)** | (0.168) | (0.020)** | (0.030)** | | |
| THAI | 0.254 | 0.048 | 0.526 | 0.092 | 0.882 | 0.974 | 3.755 |
| | (0.114)** | (0.032) | (0.235)** | (0.036)** | (0.038)** | | |
| VEN | 0.348 | 0.149 | 5.188 | 0.318 | 0.484 | 0.802 | 15.281** |
| | (0.121)** | (0.038)** | (1.291)** | (0.083)** | (0.087)** | | |

**Notes:** The first seven series are some of the major stock market returns indices and the remaining series are the stock market index returns for the 12 emerging countries that were used in the analysis with the AR(1)-GARCH(1,1)-normal model. The estimates and standard errors of the estimates using the Bollerslev-Wooldridge method for the parameters of the AR(1)-GARCH(1,1) model are reported. Results with (**) indicate that the parameter estimates are significant using the 5% critical value. The Wald test statistic for testing the null hypothesis of the IGARCH $\alpha + \beta = 1$ is also reported. The returns series are weekly, measured in local currency and span from 06/01/1989 up to 15/08/2008 (1024 sample size) for the emerging market return indices, with the exception of Venezuela stock market index returns which spans from 06/01/1989 up to 06/04/2007 (953 sample size), due to data availability. The major stock market returns indices span from 07/01/1989-16/08/2008 (FTSE100 and S&P500 span from 03/01/1989-18/08/2008 and S&P/TSX spans from 07/01/1989 up to 18/08/2008).

Table 12: Residual-based specification tests of the normal AR(1)-GARCH(1,1) model applied to some of the major stock market returns indices

| Null Hypothesis | K | Test | NIKKEI225 | FTSE100 | SP500 | DAX | CAC40 | SP/TSX | HSI |
|---|---|---|---|---|---|---|---|---|---|
| No Linear Residual Autocorrelation | 2 | LB | 4.781 | 1.156 | 2.131 | 3.170 | 0.454 | 4.502 | 5.103 |
| | | vdW | 4.256 | 0.987 | 1.756 | 1.474 | 0.042 | 3.961 | 2.347 |
| | | Wilc | 4.216 | 1.752 | 2.864 | 2.714 | 0.153 | 4.196 | 3.587 |
| | 4 | LB | 5.673 | 2.534 | 3.614 | 5.596 | 1.403 | 4.506 | 5.389 |
| | | vdW | 4.807 | 1.998 | 3.266 | 4.279 | 1.028 | 4.048 | 2.568 |
| | | Wilc | 5.081 | 2.404 | 4.836 | 5.514 | 1.939 | 4.529 | 3.840 |
| | 6 | LB | 6.747 | 2.769 | 6.773 | 8.128 | 5.305 | 5.074 | 9.061 |
| | | vdW | 5.638 | 2.142 | 7.946 | 6.907 | 5.560 | 5.114 | 6.516 |
| | | Wilc | 6.043 | 2.579 | 12.744** | 9.934 | 6.883 | 6.624 | 7.977 |
| No Quadratic Residual Autocorrelation | 2 | LM | 0.859 | 1.256 | 0.028 | 0.891 | 3.363 | 1.446 | 5.710** |
| | | vdW | 0.026 | 2.288 | 0.455 | 2.531 | 0.789 | 0.014 | 0.245 |
| | | AvdW | 0.258 | 3.872** | 0.531 | 1.706 | 2.246 | 0.735 | 0.150 |
| | | Wilc | 0.003 | 2.617 | 0.084 | 3.492 | 0.871 | 0.031 | 0.173 |
| | 4 | LM | 3.650 | 6.480 | 1.188 | 0.960 | 3.651 | 1.620 | 6.543 |
| | | vdW | 2.233 | 2.609 | 5.837 | 2.779 | 2.322 | 0.032 | 0.840 |
| | | AvdW | 3.812 | 4.958 | 6.032 | 1.819 | 4.006 | 0.739 | 0.806 |
| | | Wilc | 3.043 | 3.279 | 5.242 | 3.774 | 2.419 | 0.098 | 0.498 |
| | 6 | LM | 4.075 | 7.229 | 2.394 | 1.548 | 3.729 | 1.924 | 8.306 |
| | | vdW | 3.276 | 4.739 | 6.043 | 4.609 | 3.289 | 1.664 | 1.989 |
| | | AvdW | 5.819 | 6.503 | 6.288 | 3.165 | 4.439 | 1.531 | 1.246 |
| | | Wilc | 5.266 | 5.729 | 6.321 | 6.138 | 2.965 | 2.013 | 1.128 |
| Symmetry of Conditional Distribution | | BN CS | 1.896 | 1.472 | 2.424** | 2.511** | 1.205 | 1.796 | 1.004 |
| | | BN CS- | 1.817 | 1.371 | 2.154 | 2.511** | 1.148 | 1.452 | 0.859 |
| | | BN CS+ | 1.896 | 1.472 | 2.424** | 2.459** | 1.205 | 1.796 | 1.004 |
| | | Sign | 0.439 | 0.281 | 1.563 | 0.563 | -0.063 | 1.376 | -0.375 |
| | | Wilc | -0.620 | -0.752 | -0.368 | -0.437 | -0.390 | -0.811 | -0.364 |
| No Breaks in GARCH coeff.s | | RBC | 1.046 | 0.611 | 1.029 | 1.190 | 0.979 | 0.716 | 0.795 |
| | | KY | 0.663 | 0.767 | 0.840 | 0.923 | 0.977 | 1.045 | 0.845 |
| | | Pet | 1.046 | 0.611 | 1.029 | 1.190 | 0.979 | 0.716 | 0.795 |
| | | Lomb | 0.373 | 0.052 | 0.146 | 0.219 | 0.139 | 0.080 | 0.173 |

**Notes:** The residual-based specification tests are applied to the weekly stock market returns, measured in local currency, of some of the major stock market returns indices. The indices span from 07/01/1989-16/08/2008 (FTSE100 and S&P500 span from 03/01/1989-18/08/2008 and S&P/TSX spans from 07/01/1989 up to 18/08/2008) and they are comprised of weekly returns to match the sample period examined for emerging markets. Results with (**) indicate that the parameter estimates are significant using the 5% critical values. The critical value of the linear residual autocorrelation tests at the 5% level is 5.992 for $K = 2$, 9.488 for $K = 4$ and 12.592 for $K = 6$. The critical value of the quadratic residual autocorrelation tests is 3.84 for $K = 2$, 7.815 for $K = 4$ and 11.071 for $K = 6$. For the symmetry tests, the critical values of the BN, the Sign and the Wilcoxon tests are 2.21, 1.96 and 1.96 respectively. For the GARCH coefficient breaks tests the RBC, KY and Pettitt tests have critical values 1.358 and the Lombard test has 0.4614.

Table 13: Residual-based specification tests of the normal AR(1)-GARCH(1,1) model applied to the stock market returns indices of 12 emerging stock markets in Asia and Latin America

| Null Hypothesis | K | Test | ARG | BRA | CHI | COL | INDIA | KOR | MAL | MEX | PAK | PHIL | THAI | VEN |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No Linear Residual Autocorrelation | 2 | LB | 11.317** | 20.623** | 1.460 | 15.729** | 1.284 | 3.160 | 7.301** | 2.561 | 8.588** | 11.094** | 12.068** | 6.377** |
| | | vdW | 8.071** | 22.743** | 2.327 | 14.759** | 1.311 | 3.146 | 6.873** | 2.283 | 9.310** | 11.611** | 10.305** | 6.208** |
| | | Wilc | 7.522** | 24.524** | 1.926 | 12.331** | 1.213 | 2.745 | 6.086** | 2.680 | 8.550** | 8.946** | 9.736** | 4.757 |
| | 4 | LB | 12.290** | 38.415** | 5.001 | 23.238** | 5.408 | 8.247 | 8.260 | 6.951 | 13.270** | 12.744** | 12.121** | 8.063 |
| | | vdW | 8.858 | 43.799** | 6.471 | 21.917** | 4.789 | 8.125 | 8.370 | 7.013 | 16.085** | 14.025** | 10.423** | 9.043 |
| | | Wilc | 8.169 | 48.998** | 4.438 | 19.049** | 3.828 | 7.085 | 7.559 | 7.012 | 16.538** | 10.984** | 9.860** | 8.981 |
| | 6 | LB | 14.280** | 47.256** | 6.249 | 24.583** | 6.156 | 8.347 | 10.382 | 7.155 | 15.553** | 12.867** | 12.727** | 12.413 |
| | | vdW | 9.951 | 55.438** | 7.512 | 22.934** | 5.273 | 8.224 | 12.021 | 7.619 | 17.759** | 14.315** | 11.345 | 13.712** |
| | | Wilc | 9.663 | 60.861** | 5.225 | 20.629** | 4.348 | 7.144 | 11.177 | 8.632 | 18.594** | 11.948 | 10.466 | 13.685** |
| No Quadratic Residual Autocorrelation | 2 | LM | 0.319 | 0.433 | 0.292 | 0.006 | 0.730 | 0.178 | 0.430 | 0.341 | 1.263 | 0.047 | 3.038 | 1.429 |
| | | vdW | 0.121 | 4.193** | 0.619 | 2.750 | 0.154 | 0.661 | 0.000 | 1.156 | 2.868 | 0.585 | 0.551 | 0.000 |
| | | AvdW | 0.003 | 0.614 | 1.039 | 0.479 | 0.121 | 0.678 | 0.155 | 0.801 | 2.415 | 1.110 | 0.085 | 0.494 |
| | | Wilc | 0.069 | 3.885** | 0.490 | 2.125 | 0.005 | 0.347 | 0.026 | 1.574 | 2.331 | 0.145 | 1.557 | 0.068 |
| | 4 | LM | 0.864 | 0.839 | 0.527 | 1.331 | 4.463 | 0.445 | 0.565 | 0.651 | 2.724 | 0.464 | 5.755 | 1.614 |
| | | vdW | 1.557 | 5.893 | 1.187 | 3.055 | 5.508 | 1.460 | 0.979 | 2.243 | 5.413 | 2.134 | 1.071 | 8.411** |
| | | AvdW | 1.380 | 2.354 | 1.745 | 1.475 | 6.373 | 0.682 | 1.051 | 2.156 | 5.417 | 4.187 | 0.899 | 7.220 |
| | | Wilc | 2.529 | 5.621 | 1.106 | 2.233 | 6.200 | 1.758 | 0.713 | 2.377 | 4.291 | 2.285 | 2.130 | 10.697** |
| | 6 | LM | 4.259 | 1.196 | 1.641 | 1.794 | 5.529 | 2.450 | 2.335 | 1.620 | 3.166 | 1.121 | 6.453 | 4.023 |
| | | vdW | 4.275 | 6.101 | 1.308 | 6.441 | 7.700 | 2.569 | 3.031 | 3.251 | 6.983 | 2.175 | 1.217 | 10.488 |
| | | AvdW | 3.416 | 2.689 | 2.318 | 3.826 | 9.550 | 2.809 | 2.954 | 3.952 | 6.342 | 4.566 | 0.951 | 10.298 |
| | | Wilc | 5.925 | 6.163 | 1.549 | 3.880 | 9.534 | 2.774 | 2.249 | 3.205 | 6.639 | 2.326 | 2.565 | 13.494** |
| Symmetry of Conditional Distribution | | BN CS | 0.856 | 1.324 | 2.367** | 0.886 | 1.367 | 1.011 | 1.171 | 2.051 | 1.778 | 1.327 | 0.910 | 2.808** |
| | | BN CS- | 0.668 | 1.324 | 2.367** | 0.886 | 1.329 | 1.011 | 1.171 | 1.930 | 1.778 | 1.327 | 0.910 | 2.808** |
| | | BN CS+ | 0.856 | 0.709 | 2.216** | 0.502 | 1.367 | 0.847 | 1.021 | 2.051 | 1.529 | 1.215 | 0.899 | 2.038 |
| | | Sign | 0.875 | 3.252** | -1.313 | -1.501 | 0.563 | -0.688 | 0.000 | 0.563 | 1.688 | -0.375 | -0.563 | -0.810 |
| | | Wilc | -0.583 | -3.489** | -0.280 | -1.221 | -0.082 | -0.704 | -0.350 | -0.107 | -2.129** | -0.152 | -0.637 | -0.287 |
| No Breaks in GARCH coeff.s | | KY | 1.180 | 0.812 | 0.904 | 1.209 | 0.928 | 0.757 | 0.610 | 0.884 | 0.855 | 0.912 | 0.944 | 1.143 |
| | | RBC | 0.666 | 1.820** | 0.828 | 1.210 | 0.600 | 0.884 | 0.911 | 0.832 | 1.645** | 0.880 | 0.727 | 2.554** |
| | | | | 08/07/1994 | | | | | | | 01/10/1993 | | | 26/05/2000 |
| | | Pet | 0.666 | 1.820** | 0.828 | 1.210 | 0.600 | 0.884 | 0.911 | 0.832 | 1.645** | 0.880 | 0.727 | 2.554** |
| | | | | 08/07/1994 | | | | | | | 01/10/1993 | | | 26/05/2000 |
| | | Lomb | 0.060 | 0.734** | 0.128 | 0.367 | 0.057 | 0.122 | 0.143 | 0.135 | 0.996** | 0.117 | 0.139 | 2.197** |
| | | | | 08/07/1994 | | | | | | | 01/10/1993 | | | 26/05/2000 |

**Notes:** The residual-based specification tests are applied to the weekly stock market returns of 12 emerging markets for the sample period 06/01/1989 up to 15/08/2008 (1024 sample size), with the exception of Venezuela which spans from 06/01/1989 up to 06/04/2007 (953 sample size). The table reports the results for some of the traditional tests and the new tests developed in this paper. Results with (**) indicate that the parameter estimates are significant using the 5% critical values. The critical value of the linear residual autocorrelation tests at the 5% level is 5.992 for $K = 2$, 9.488 for $K = 4$ and 12.592 for $K = 6$. The critical value of the quadratic residual autocorrelation tests is 3.84 for $K = 2$, 7.815 for $K = 4$ and 11.071 for $K = 6$. For the symmetry tests, the critical values of the BN, the Sign and the Wilcoxon tests are 2.21, 1.96 and 1.96 respectively. For the GARCH coefficient breaks tests the RBC, KY and Pettitt tests have critical values 1.358 and the Lombard test has 0.4614.