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ABSTRACT

Providing financial education: a general equilibrium approach

Since the early 2000s, the importance of financial literacy for safe financial behaviors has increased in public debate and has been the motivation for several national and international institutions to launch and promote financial education initiatives. Although discussion on the effects of such education programs remains open, it is generally presumed that higher levels of financial literacy are associated with more stable financial markets. The present paper challenges this assumption and provides a model of heterogeneous agents which differ according to the level of their cognitive abilities. The model allows us to discuss the implications for asset pricing of policies aimed at increasing levels of financial literacy, and shows that general equilibrium effects cause market price volatility and the share of literate individuals to vary in a non-monotonic way with financial education.

JEL Classification: D82, G12, G14 and G18
Keywords: asset pricing, cognitive ability, financial literacy, heterogeneous agents and market stability

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1 Introduction

Standard asset pricing models assume rational investors trade in order to maximize the utility of their wealth. There is a variety of models which differ in their assumptions about preferences, beliefs, and market structure, and how information is shared among investors. These different assumptions deliver a wealth of asset pricing implications, which constitute a basis for empirical validation of such models. An important aspect of these models is how informed the investors are about the assets' return distribution.

There is an extensive literature documenting that a large fraction of the investor population lacks crucial information related to investing in financial markets and therefore is financially illiterate. Lusardi and Mitchell (2007) show the positive association between financial literacy, planning for retirement, McArdle, Smith, and Willis (2009) wealth, van Rooij, Lusardi, and Alessie (2011) stock market participation, and Guiso and Jappelli (2009) portfolio diversification.[1] Calvet, Campbell, and Sodini (2007) investigate the welfare costs of households' investment mistakes using comprehensive disaggregated Swedish data.[2] Policy makers around the world are encouraging the proliferation of financial education programs on the grounds that promoting financial education or facilitating access to financial information allows individuals to improve their financial behavior.[3]

While there is mixed evidence on the effects of programs on financial literacy and saving outcomes, nothing is known about the effects of financial education programs on the prices of assets.[4] Understanding the effect of financial programs on the price of assets is crucial to provide the answers to several important questions. For example, do financial education programs allow the individual a better understanding of the consequences of his/her financial behavior? Does this help to increase the stability of financial markets? Can large-scale financial education programs be used as a device to reduce the volatility of markets in the face of fast-paced financial innovation? To the best of our knowledge, the present paper is the first attempt to provide a model that addresses these questions. We argue that in addition to policy makers' effort to provide financial education programs, we need also to understand the incentives for investors to acquire financial literacy. The decision to invest in financial literacy has costs and

[1]A related work by Christelis, Jappelli, and Padula (2010) investigates the role of cognitive ability in shaping the stock market participation decision and shows that, other things being equal, including age and education, more cognitively able individuals are more likely to participate in the stock market. This suggests the existence of a channel through which cognitive ability affects the behavior of households in the financial market, as modeled by Jappelli and Padula (2013a).

[2]Calvet, Campbell, and Sodini (2009) work out an index of financial sophistication that captures behavioral biases: under-diversification, risk sharing inertia, and disposition effect.

[3]See Grifoni and Messy (2012) for a list of programs in the OECD countries. The aim of policy makers is to reduce the gap between what individuals are capable of handling/understanding and the complexity of financial markets.

[4]Gale and Levine (2011) point out that much of the empirical evidence suffers from econometric bias.

benefits, which individuals trade-off to obtain an optimal level of investment. While several studies take the benefits (and the costs) of financial education as given (see Jappelli and Padula, 2013a and 2013b), in the present paper we take the view that the return to the individual of his/her investment depends also on the investment and on the trading decisions of other individuals.

In our model of noise traders, and a risky and a riskless asset, financial literacy lowers the cost of buying a more precise signal of the risky asset payoff but entails a dis-utility cost, which varies among individuals depending on their cognitive abilities. Therefore, the heterogeneity of costs results in interesting general equilibrium implications that provide a base from which to evaluate policies aimed at increasing levels of financial literacy. In our setting, the effectiveness of financial education programs is a policy variable. We assume that more effective financial education programs increase the level of financial literacy to a larger extent.

We study the interactions between investment in financial literacy and in asset markets, through the returns on information choice. Investors can increase the payoff from their financial portfolios by acquiring information on the rates of return, an idea proposed by Arrow (1987) and developed in the literature on information acquisition and asymmetric information which originated in the 1980s with Grossman and Stiglitz (1980), Hellwig (1980) and Verrecchia (1982). In more recent studies, Peress (2004) and (2010) focus, respectively, on the increasing returns from information with respect to wealth, and on the trade-off between risk sharing and information production. The various models differ in the source of heterogeneity among investors. In Verrecchia (1982) risk-aversion differs among investors; in Peress (2004) wealth is the source of heterogeneity. Our paper departs from these related works by assuming that the cost to investors of acquiring information differs according to their levels of financial literacy. This assumption related to the cost of acquiring information distinguishes our work from a concurrent strand of literature, started by Wilson (1975) and developed by Sims (2005) and Van Nieuwerburgh and Veldkamp (2010), which emphasizes the role of learning technology. This literature models the economies of scale in learning technology and assumes that investors allocate their limited attention to the acquisition of information on asset returns.

Our paper emphasizes that individuals decide whether to improve upon a baseline level of financial literacy by undertaking financial education. We refer to investors who join a financial education program as literate and other investors as illiterate. In deciding whether to take financial education, individuals factor in feed-back effects on the informativeness of market prices. The incentive to acquire financial literacy increases with the effectiveness of financial education programs up to a point where illiterate investors prefer not to buy the signal, and consequently trade more conservatively. From a policy perspective, this means that increasing the effectiveness of financial education programs may even decrease the share of literate investors. Moreover, the variance in market prices, which we take as a proxy for market volatility, varies in a non-monotonic way with the effectiveness of the financial education programs for high levels of uncertainty of payoffs and noisy supply. This warrants some caution related to financial education policies which, by increasing the heterogeneity among investors might even contribute to fueling the instability of financial markets.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the model set up. Section 4 characterizes the equilibrium. Section 5 discusses the main implications for policy and Section 6 investigates the sensitivity of our results. Section 7 concludes and suggests some directions for further research. The proofs are all contained in the Appendix.

2 Financial education and market stability

Since early 2000s, policy makers around the world have been paying increased attention to financial education and since 2006, financial education has been a recurring topic in Ben Bernanke testimonies.[5] Bernanke has argued that "The recent crisis demonstrated the critical importance of financial literacy and good financial decision-making, both for the economic welfare of households and for the soundness and stability of the system as a whole."[6] Similar views have been expressed by Dan Iannicola, Jr. President and CEO of the Financial Education Group, who has claimed that "Consumer financial education is a valuable policy tool to promote economic stability".[7]

International institutions such as the World Bank and the OECD, have intensified their focus on financial education. The OECD published the "Recommendations on Principles and Good Practices for Financial Education and Awareness" which indicate that: (i) Governments and all stakeholders concerned should promote unbiased, fair, and coordinated financial education; (ii) Financial education should start in school so that people achieve financial literacy as early as possible; (iii) Financial education should be part of the good governance of financial institutions whose accountability and responsibility should be encouraged; (iv) Financial education should be clearly distinguished from commercial advice; codes of conduct for the staff of financial institutions should be developed; (v) Financial institutions should be encouraged to check that clients read and understand information, especially that related to long-term commitments or financial services with potentially significant financial consequences; small print and abstruse documentation should be discouraged.

[5]See http://www.federalreserve.gov/newsevents/testimony/Bernanke20060523a.htm.

[6]See http://www.federalreserve.gov/newsevents/testimony/bernanke20110420a.htm.

[7]From a presentation entitled "Financial Education and Financial Stability: A Case Study" made at the International Conference on Financial Education and Financial Awareness: Challenges, Opportunities and Strategies. Istanbul, Turkey, March 9, 2011.

The growing emphasis on the beneficial effects of financial education on the welfare of individuals and on the economy as a whole is being accompanied by burgeoning research on the relation between financial literacy and households' saving outcomes. The first generation of such studies documents a correlation between measures of financial literacy and saving, wealth, stock-market participation, portfolio choice, and household debt.[8] In all these studies the likely endogeneity of financial literacy with respect to households' saving decisions is an issue. There is a subsequent strand of work that investigates in more detail the gradient between financial literacy and saving outcomes, exploiting an instrumental variable approach, and confirming the correlation evidence. Christiansen, Joensen, and Rangvid (2008) sort out the dual causality between portfolio choice and the decision to become an economist, using a large register-based panel data set containing detailed information on Danish investors' education attainment, and financial and socioeconomic variables, and exploiting the establishment of a new university as an instrument for economics education. Behrman, Mitchell, Soo, and Bravo (2012) also use an IV approach and find that the ordinary least squares (OLS) estimate of the effect of financial literacy is potentially biased due to measurement error and unobserved factors.

A parallel strand of work investigates the effect of financial education on saving, adopting an experimental approach. However, these studies demonstrate that there is still considerable uncertainty related to the causal effect of financial education on various saving outcomes. Bernheim, Garrett, and Maki (2001) find a positive effect of financial education programs on saving, Gartner and Todd (2005) find no effect of on-line credit education, Duflo and Saez (2003) find that information does not seem to affect enrolment in a tax Deferred Account retirement plan, while Bayer, Bernheim, and Scholz (2009) find that retirement seminars are associated with higher saving.[9] More recently, Jappelli and Padula (2013a) proposed a model to study the joint decision to save and to acquire financial literacy. The model indicates that heterogeneity in preferences, in the cost of acquiring financial literacy, and in cognitive abilities can account for a positive correlation between financial literacy and wealth accumulation, without this implying a causal effect of financial literacy on saving. Therefore, it remains an open question as to whether the observed positive correlation between financial literacy and saving underestimates or overestimates the potential effect of financial education on saving outcomes.

The available literature focuses on the relation between financial literacy and individuals' welfare. However, while there is still some uncertainty about the effect of financial education on saving, nothing is known about the effect of financial education on the stability of markets.

[8] Lusardi and Mitchell (2007) examine the correlation between financial literacy and planning, McArdle, Smith, and Willis (2009) investigate financial literacy and wealth, van Rooij, Lusardi, and Alessie (2011) compare financial literacy and stock market participation, Guiso and Jappelli (2009) examine financial literacy and portfolio diversification, and Lusardi and Tufano (2009) look at the correlation financial literacy and debt.

[9] For a review of the literature on the effect of counseling and financial education see Collins and Rourke (2010).

Investigating the potential for financial education to act as a market stabilizer therefore is of prime importance, in light of the emphasis placed on lack of financial literacy as a threat to the smooth functioning of financial markets.

To address the issue, we propose a general equilibrium asset pricing model to study the effect of financial education on the volatility of financial markets. The model, featuring partially revealing prices and heterogeneous agents, is described in the next section.

3 Model

We distinguish between financial education and financial information, describing two different processes used to acquire them. Individuals are born with one unit of wealth and operate in two markets: the asset market and the information market. In the asset market, individuals trade a riskless and a risky security. In the information market, individuals can acquire the precision of a private signal on the risky asset payoff at a cost that depends on their financial literacy. Higher financial literacy guarantees a lower cost for acquisition of financial information, i.e. the precision of the private signal.

To increase their financial literacy, individuals can enroll in financial education programs. However, undertaking the program entails a dis-utility cost, which depends on the individuals' cognitive abilities. More able individuals face a lower cost. Therefore, the choice to take the program is based on the expected utility that individuals derive from being more financially literate when they trade risky assets with illiterate and noise traders. The benefit of being more literate depends on the composition of the traders: being the only literate trader is better than being one among many literate traders. Moreover, the magnitude of the benefit depends on the behavior of other agents.

Below, we discuss the elements of the main model. The model has three periods and we describe the decision problems individuals face in each period.

Assets market

Agents trade a riskless and a risky security. The riskless security is the numeraire, earns the gross return $R = (1 + r)$, and is supplied inelastically. The risky security has a risky payoff π with $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$ and its price, in terms of the numeraire, is p. Short selling is allowed and the per-capita supply, provided by noise traders, is $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$. Without loss of generality, we set $\mu_\theta = 0$. We assume that π and θ are mutually independent random variables and that their distribution is common knowledge.

Agents

A continuum of rational agents is distributed across the set $J = [0, 1]$. They all maximize the same utility function of their final wealth but differ in their cognitive ability. Agents are ordered on the set J by increasing the level of their cognitive ability. Those who take the financial education course incur a dis-utility cost which is inversely related to their cognitive ability, meaning that more able agents face a lower dis-utility cost of taking the program. Their dis-utility cost is indexed by the function $\gamma(j) : J \to \mathbb{R}^+$, with $\gamma(j)' < 0$. We assume that $\gamma(\cdot)$ decreases as cognitive ability increases and the cumulative distribution function $G(\gamma)$ represents the distribution of the agents on the set J. The agent's benefit from undertaking the program is a lower cost of acquiring financial information.[10]

We define as literate agents (L) those who take the program, and those who do not as illiterate (H) for whom acquisition of information comes at a higher cost. In this setting, as an equilibrium result we will provide the proportion of the agent population $L = \lambda J$ attending the program.

For tractability, we assume that individuals maximize a CARA utility function with absolute risk aversion equal to ρ, and that the dis-utility cost of attending the program is $e^{\gamma(j)}$ with $j \in J$.[11]

Information structure

Once π is realized but not revealed, each agent j can purchase financial information which we model as an unbiased signal s_j of the risky payoff, from which the agent can privately observe the realization:

$$s_j = \begin{cases} \pi + \sqrt{\dfrac{1}{x_j}}\epsilon & \text{if} \quad x_j > 0 \\ \emptyset & \text{if} \quad x_j = 0 \end{cases}$$

where ϵ is white noise, independent of π, θ, and across agents, and x_j is the precision of the non-negative signal. The signal is informative for the agent j if $x_j > 0$, and otherwise is uninformative. Signal precision is interpreted as the amount of private financial information about the risky asset payoff.

Agents can acquire private financial information incurring a monetary cost that depends on the level of financial literacy. Higher levels of illiteracy raise the cost of acquiring information. Acquiring the amount x of information entails a monetary cost $C(x, c_H)$ for the illiterate, and

[10] The outcomes of the model with rational agents are strengthened once we take account also of behavioral biases or some other types of friction.

[11] Despite its limitations, the CARA structure has been used to model features of investor behaviors: under diversification (Van Nieuwerburgh and Veldkamp, 2010), home bias puzzle (Van Nieuwerburgh and Veldkamp, 2009), risk sharing and information production (Peress, 2010).

$C(x, c_L)$ for the literate, where $C(x, c_H) > C(x, c_L)$, $c_H > c_L$ and $C(x, \cdot)$ is increasing, is convex, and differentiable at least twice in the first argument.[12]

The monetary cost of private financial information provides the metric to evaluate the quality of financial education programs. Becoming financially literate as the result of a financial education program, reduces the cost of acquiring the amount x of information, from $C(x, c_H)$ to $C(x, c_L)$.

Public financial information is freely available. We model this feature as common knowledge about the distribution of the risky asset payoff $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$ and assuming that all agents know that the other agents can purchase private financial information and that this is revealed by the market price to an extent.

Timing

There are three periods: training, planning, and trading. In the training period, the agents decide whether to attend a financial education program in order to improve their financial literacy above the baseline level, at a cost that is inversely related to their cognitive abilities. In the planning period, the agents purchase a private signal s and choose its precision x, which amounts to acquiring private financial information. In the trading period, the agents observe the private signal realization and the market price, and trade in a competitive market, choosing the optimal portfolio share α. Once uncertainty is revealed, agents consume the proceeds from their investment. Figure 1 depicts the model timeline.

To solve the model, we focus on a partially revealing noisy rational expectations equilibrium.[13] Agents conjecture a price function: $p = a + b\pi - d\theta$ where the coefficients a, b and d are determined in equilibrium.[14] Next we solve the model and describe the equilibrium.

4 Equilibrium

We solve the model by backward induction. In the terminal period, each agent j consumes all the wealth earned by his/her investments. Therefore, in the trading period, the agent chooses the share of wealth invested in the risky asset, α_j, to maximize the expected utility of

[12]The assumptions about the cost of acquiring private information ensure the existence of an optimal information choice.

[13]We assume that all agents have rational expectations and use the information revealed by market prices to form their posterior beliefs. The equilibrium price p contains some information about payoff value that comes from private signals provided by individual asset demands.

[14]We are aware that the exponential-normal framework allows for closed form solutions under special assumptions: normal joint distribution of the stochastic variables and normal distribution of the consumption, conditional on the informative set that implies a well defined expected utility function. Bernardo and Judd (2000) explores numerical solutions to overcome the limits of the framework, and allowing for general tastes, returns, and informational asymmetries.

his/her final wealth, given the precision purchased and the information cost paid. Each agent observes both a private and a public signal (the equilibrium price) and computes posterior beliefs regarding the final payoff: $E[\pi|\mathcal{F}_j]$ and $Var[\pi|\mathcal{F}_j]$, where $\mathcal{F}_j = \{s_j, p\}$ is the agent j's information set, respectively, the private signal and the public signal. Note that the private signal is informative only if agent j acquires some information precision. In the planning period, the agent chooses how much precision to acquire, x_j, and pays the monetary cost, which depends on undertaking the financial program in the training period. Since all agents face the same decision problem in each of the three periods, to ease the notation, here we drop the index j. This index is resumed in order to write the equilibrium conditions in the assets and information markets.

The trading period

In the trading period any agent maximizes:

$$\max_{\alpha} E[U(W_3)|\mathcal{F}], \tag{1}$$

subject to:

$$W_3 = W_2 R^p \,, \tag{2}$$
$$W_2 = W_1 - C \,, \tag{3}$$

where the budget constraint implies that the final wealth W_3 is the investment return of the agent's portfolio choice, α. The portfolio return $R^p = \alpha \frac{\pi - pR}{p} + R$ depends on the returns from the two assets, and on market prices. The amount of wealth invested, W_2, depends on the initial wealth W_1 and on the cost of acquiring information, C, which is sunk in the trading period and depends on the choices made in the training and planning periods.

The optimal share of wealth invested in risky assets varies among agents depending on the signal observed and the precision purchased. In the trading period, all private information choices have been made, and each trading agent transfers some of his/her purchased information to the market price through the corresponding assets demand. Thus, private information is partially revealed by the market price. In forming their posterior beliefs and formulating their assets demand, agents take account of the informativeness of p, which is transformed into an unbiased and informationally equivalent public signal, ξ.

The indirect utility of solving the trading problem is $E[U(W_3^*)|\mathcal{F}]$, which we note as $v(s, p; \Theta)$ where Θ is $\{j, c_L, c_H, R, W_1, \rho, \mu_\pi, \tau_\pi, \tau_\theta\}$, to emphasize its dependence on private and public signals.

The planning period

In the planning period, each agents maximizes the indirect utility with respect to the information choice:

$$\max_{x \geq 0} E[v(s, p; \Theta)] \, ,$$

subject to:

$$W_1 \geq C(x, c) \, ,$$

where the expected utility is computed over the joint distribution of s and p and with $c = \{c_L, c_H\}$. Recall that the signal precision x, by assumption, affects only the distribution of private signal s. In deciding how much precision to acquire, agents compare the costs and the benefits of private information. The former depend on the financial literacy level with higher levels of financial literacy allowing purchase of information at a lower cost. The latter depend on how much information is already present in the market, which in turn, depends on the amount of information purchased by all agents. To the extent that p reveals a lot of information, the benefits of private information are limited.

The indirect utility of solving the planning problem is $E[v(x^*)]$, which we denote $z(\Theta)$.

The training period

In the training period, the agent maximizes her/his utility with respect to the choice to attend the financial education program. The agent problem can be summarized as:

$$\max_{D} D \, z_L(\Theta) e^{\gamma} + (1 - D) z_H(\Theta) \, ,$$

where D is a binary variable that takes the value 1 for attending the program and 0 otherwise. Attending the program delivers a pay-off equal to $z_L(\Theta)e^{\gamma}$, while the payoff for not attending is $z_H(\Theta)$.[15] The notation makes it clear that the effectiveness of the program has a direct effect on the decision to attend the program. What is less clear is that the proportion of the agents who acquire financial literacy, λ, also affects the decision to acquire information.

The equilibrium

A rational expectations equilibrium is given by the sequence of training choices D_j, assets α_j, and demands for information x_j, and a price function p of π and θ such that $D_j = D^*(\Theta)$, $x_j = x^*(\Theta)$ and $\alpha_j = \alpha^*(s_j, p; \Theta)$ solve the training, planning, and trading problems; the price

[15] The dis-utility cost γ uniquely identifies the agent given the assumed monotonic relation with the cognitive abilities $\gamma(\cdot) : J \rightarrow \mathbb{R}^+$.

p clears the market for the risky asset:

$$\int_j \alpha_j \frac{W_1 - C(x_j, c_j)}{p} \, dG(j) = \theta,$$

and the following conditions hold:

1. The proportion of literate agents λ, implied by individual training choices is equal to that conjectured by each agent in order to solve their problem:

$$\lambda = \int_{j \in J} D_j \, dG(j)$$

2. The aggregate informativeness of the price I implied by aggregating individual precision choices, is equal to the informativeness conjectured by each of the agents to solve their problem:

$$I = \lambda x_L + (1 - \lambda) x_H \tag{4}$$

In noisy rational expectations equilibrium models, investors make self-fulfilling conjectures about prices, and the equilibrium is defined as the set of allocations such that utility is maximized, the markets clear, and the individual optimal choices are consistent with the aggregate variables. The agents conjecture that a proportion λ of them undertakes the financial education program and becomes literate. They conjecture also that the aggregate informativeness is given by a linear combination of the information choices of the financial literates and the illiterates as in (4), and that the market price is a linear function of the fundamentals. All these conjectures hold in equilibrium.

The following proposition shows the existence of an equilibrium within the family of noisy rational expectation equilibria.

Proposition 1. *There exists a noisy rational expectations equilibrium with linear price function.*

The proof provided in Appendix is constructive and proceeds in three steps. First, for a given proportion of literates λ and aggregate informativeness I, we compute the optimal assets demand and equilibrium market price function. The second step consists of computing the optimal information choices, checking for when the individual choices are consistent with the assumed aggregate informativeness. In the third step, we compute the expected utility for both types and derive individual participation choices, aggregating them to check their consistency with the assumed proportion of literates.

Below, we provide an example to illustrate the main intuition of the model.

Example. *Each agent $j \in J = [0,1]$ has a dis-utility cost $\gamma = -\log(j)$. The distribution of agents with respect to γ is denoted by a density function $G(\gamma)$. We also specify a linear cost function: $C(x,c) = c\,x$. The domain of private signal precision is $X = [0, \overline{x}]$ where $\overline{x} = \frac{W_1}{c}$ is the maximum amount of information that an agent can purchase. The threshold triggering the choice to be informed is: $\overline{c} = [2R\rho(\tau_\pi + \tau_\xi)]^{-1}$ and the optimal information choice is:*

$$x^*(\Theta) = \begin{cases} \frac{1}{2c\rho R} - \tau_\pi - \tau_\xi & \text{if} \quad c < \overline{c}, \\ 0 & \text{if} \quad c \geq \overline{c}. \end{cases}$$

In equilibrium, the aggregate informativeness is given by $I = \frac{1}{2\rho R}\tilde{c} - \tau_\pi - \tau_\xi$ where $\tilde{c} = \lambda \frac{1}{c_L} + (1-\lambda)\frac{1}{c_H}$. The expected indirect utility is:

$$z(\Theta) = -\tfrac{1}{\rho}\left(1 + \sigma_f^2 k^*\right)^{-1/2} e^{-\rho R[W_1 - cx^*]}.$$

We distinguish between two cases. Case A where both types optimally prefer to be informed: $c_L < c_H < \overline{c}$, and case B where the illiterates prefer to remain uniformed, $c_L < \overline{c} < c_H$. In case A, the agent attends the program if:

$$\frac{z_L(\Theta)e^\gamma}{z_H(\Theta)} > 1,$$

which holds $\sqrt{\frac{c_L}{c_H}} e^{-\rho R(c_L - c_H)(\tau_\pi + \tau_\xi) + \gamma} < 1$ for any $\gamma < \overline{\gamma}$ such that:

$$\overline{\gamma} = \rho R(c_L - c_H)(\tau_\pi + \tau_\xi) - \tfrac{1}{2}\log\tfrac{c_L}{c_H}.$$

In case B, the agent takes the program for any $\gamma < \overline{\gamma}$ such that:

$$\overline{\gamma} = \rho R c_L(\tau_\pi + \tau_\xi) - \tfrac{1}{2}[1 + \log\tfrac{2\rho R c_L}{\tau_\pi + \tau_\xi}].$$

The proportion of literates is given by:

$$\lambda = \int D_j^*(\Theta) dG(j) = \int_0^{\overline{\gamma}} dG(j) = G(\overline{\gamma})$$

We use the model to evaluate the effects of a policy aimed at increasing the effectiveness of financial education programs. In this setting, this means reducing c_L, which amplifies the difference in the cost of acquiring financial information between those who do and do not attend the financial education program. The effects of such a policy are described in the next section.

5 The effect of increasing financial literacy on market stability

In this section we examine the effects of reducing c_L on agents' behavior, price informativeness, and market volatility. Policies that reduce c_L but leave c_H unchanged result in an increase in the effectiveness of financial education which, however, contributes to amplifying the disparities in financial literacy levels between those who decide to take financial education program and those who do not. Whether this fuels wealth inequality and market stability is the subject of the following numerical analysis.[16]

Optimal information choice

We start by analyzing the effect of financial education on the decision to acquire private financial information. Understanding the factors leading individuals to acquire private information provides the basis for discussing the effect of financial education on the informativeness of market prices, and ultimately, on the volatility of markets.

Increasing the effectiveness of the financial education leads to an increase in the amount of financial information acquired by those agents who attend the program due to the lower costs incurred. On the other hand, for the agents who do not attend the program, the amount of information acquired is optimally reduced because they substitute costly private information for information freely available from the market price, which becomes more revealing, see Figure 2. The horizontal axis records $c_H - c_L$, and the vertical axis shows the optimal amount of financial information acquired. We focus on two possible situations: a high uncertainty ($\tau_\pi = \tau_\theta = 0.2$) scenario, and a low uncertainty ($\tau_\pi = \tau_\theta = 2$) scenario. In Figure 2 the continuous line refers to the optimal information choice of those who attend the program, and the dotted line is the optimal information choice of those who do not attend the program, for any given level of c_L. We take account of the endogeneity of the attendance choice and compute, for any given level of c_L, the lowest level of cognitive ability ($\overline{\gamma}$) for which attending the program would be worthwhile, see Figure 3.

If $c_H = c_L$ no one takes the program and the two lines eventually merge. Agents still optimally acquire some financial information, which is revealed by the market price. If c_L decreases, some individuals - depending on their cognitive abilities - will find it optimal to take the program, while others will not. In both the high and the low uncertainty scenarios, the financial information acquired increases for the literates and decreases for the illiterates. Agents who become literate acquire more information than those who remain illiterate.

[16] The vector of parameters Θ is set as follow: zero return for the riskless asset $R = 1$; normalized initial wealth: $W_1 = 1$; risk aversion of the agents $\rho = 1$; the prior mean of the risky asset is equal to 1: $\mu_\pi = 1$ and zero denote noisy supply $\mu_\theta = 0$. We present two scenarios, with high and low uncertainty in the fundamentals. The financial literacy cost c_H equals 0.04, while we let c_L take the values $[0.005, 0.04]$. The cost function is linear: $C(x, c) = c\,x$. The assumed cumulative distribution function over $G(\gamma)$ is a $\Gamma(1.1, 1)$.

The information is revealed by the market price, and the more informative the market price, the lower the incentive for the illiterate to acquire private information. When c_L becomes too low, the illiterates stay out of the information market and rely solely on the public signal. The fact that the illiterates do not acquire financial information increases the incentive for the literates to acquire financial information even more, which explains why the continuous line becomes steeper when the dotted line crosses the horizontal axis.

When the economy is in a state of low uncertainty, we observe the same dynamics. Ceteris paribus, agents need to acquire less information in order to trade optimally. In the low uncertainty scenario, the illiterates prefer to remain uninformed for a higher value of c_L, compared to the case of high uncertainty.

Aggregate informativeness and financial literacy distribution

Next, we analyze the effect of reducing c_L on aggregate variables. We focus on the informativeness of the market price, distinguishing between a high (continuous line) and low (dotted line) uncertainty scenario (left hand panel in Figure 3). How informative is the market price depends on the agents' decisions about acquiring private information. In both scenarios, for a high level of c_L, all the agents decide to acquire some financial information. As c_L decreases, the private information acquired by the literates increases, while the private information acquired by the illiterates decreases to a point where the illiterates prefer to remain uninformed. At that point, the literates are the sole contributors to the informativeness of the market price, which continues to increase since the financial information acquired by the literates explodes when the illiterates decide to remain uninformed and c_L decreases.

The effect of reducing c_L means the decision to take the financial education program is also affected. This in turn affects the fraction of agents in the population that prefer to remain illiterate. While it is intuitive that lowering c_L cannot reduce the informativeness of market price, the effect on λ^* is less obvious, as the central panel in Figure 3 shows. The optimal proportion of literates is zero if there is no benefit from attending the program, i.e. $c_H = c_L$, and is larger in the high uncertainty scenario than in the low uncertainty one. As c_L decreases, the proportion of agents investing in financial literacy through attendance at the financial education program also increases. The maximum attendance rate is given by the level of c_L such that the illiterates still acquire some information: $\overline{c}(\Theta) > c_H$. When the illiterates prefer to be uninformed, λ^* drops as the incentive to be financially literate in a market with uninformed and noise traders decreases. Figure 4 shows that when c_L is too small, the illiterates have no incentive to acquire information. They realize that, in addition to themselves, there are well informed agents and noise traders in the market, and they react by reducing their exposure to risk. Their utility decreases (dotted line), but they limit eventual losses by trading more conservatively. This reduces the gains for the literates. Comparing the left hand and right hand panels in Figure 4 shows that the financial program premium (the wedge between the continuous and the dotted lines) is larger in the high uncertainty than in the low uncertainty scenario. This premium is fully offset by the dis-utility cost of the marginal agent, who has the lowest level of cognitive ability, making it worthwhile to attend the program.

Financial market volatility

We can now analyze the effect of decreasing c_L on market volatility. We consider two proxies: the market price variance reported in Figure 5, and the excess return variance reported in Figure 6 which takes account of the covariance between the payoff and the market price.

For a low level of uncertainty (dotted line in Figure 5(a)), the market price variance is increasing with inequality in agents' literacy levels. A more effective program implies greater price informativeness, through the higher amount of information privately acquired by literate agents. The illiterates have increasingly fewer incentives to acquire information and rely more on market price, and trade more conservatively. The increasing presence of aggressive literate traders reduces the positive effect, due to risk sharing, and increases the market price variance. On the other hand, for a high level of uncertainty (continuous line in Figure 5(b)), the market price variance is a non-monotonic function of c_L. Improving the effectiveness of the program has two effects. With low inequality and high price informativeness, implied by the huge presence of noise traders, a lower c_L means slightly higher price informativeness which leads to lower market price variance. The illiterates are still trading and the risk sharing has a positive effect on the stability of the market. With high inequality, and either low or high uncertainty, price informativeness is high enough to clear the uncertainty due to noise traders, who mainly trade against the literates. Therefore, the market price variance reflects only the prior volatility of the risky asset payoff.

If we focus on the variance of excess return, we can check investors' incentives to trade risky assets. When c_L decreases and only the literates contribute to aggregate informativeness, the market excess return variance decreases, as shown in Figure 6 for the high (left panel) and low (right panel) uncertainty cases. This is because higher price informativeness implies higher covariance between the market price and the asset payoff.

6 Sensitivity checks

In this section we investigate the sensitivity of our results to alternative parameterizations of the model. First, we explore how the results change if we allow τ_π and τ_θ to be equal to .3, .5, and 1, respectively, and leave the other parameters unchanged, as in the baseline model parameterization.

Financial education affects the decisions of two types of agents to acquire private financial information, in the same way, for different scenarios of uncertainty in the fundamentals. For a given amount of financial education, higher uncertainty leads to acquisition of more private information, as in Figure 7. As a consequence, price informativeness is shifted upward for higher uncertainty and for a given level of financial education, as shown in Figure 8.

We also study the effect of financial education on the fraction of literates and on the gap in the expected utility between literate and illiterate investors. The fraction and the expected utility function of the literate investors remain a non-monotonic function of the effectiveness of financial education programs under these alternative parameterizations. Figure 9 and 10 show, respectively, that higher uncertainty increases the incentives to attend the program and the expected utility of both types because of the opportunity to exploit the noise traders more.

The sensitivity analysis shows that the market price variance also remains an increasing function of the effectiveness of financial education programs. For low uncertainty in the risky asset payoff, a regime of lower uncertainty in the noisy supply increases the effect of financial education on the market price variance, see Figure 11(a). Similar implications can be derived for high uncertainty on the risky market asset, see Figure 11(b). The non-monotonicity of the market price variance in the inequality of agents' literacy is smoothed by the reduction of the uncertainty in the fundamentals.

Second, we explore how the optimal information choices, aggregate informativeness, fraction of literates, and the variance of market prices change with risk aversion.

The effect of financial education on the acquisition of financial information is similar to the baseline model. Although risk aversion may make private information more valuable per unit of asset traded, the reduced volume being traded drives the literate agents to reduce their acquisition of private information. Figure 12 shows that the gap between the financial information acquired by the literate and illiterate investors is wider if the agents are less risk averse.

While the literate investors reduce their information acquisition, price informativeness is higher for higher levels of risk aversion (see Figure 13). This result is due to the larger fraction of agents who decide to attend the program and to became literate, as shown in Figure 14.

Sensitivity analysis of the effect of financial education on market price variance highlights the role played by the level of risk aversion of the agents. In a low uncertainty scenario, the market price variance is increasing in financial education for different degrees of risk aversion. We conclude that the positive effects of price informativeness on the behavior of the market are lower than the negative effect of the exit of the less literate agents from the information market which leads them to trade more conservatively, as shown in Figure 15(a).

In the high uncertainty scenario, the presence of more risk averse agents (dotted line in Figure 15(b)) implies that financial education lowers the market price variance due to the increase in price informativeness. Less risk averse agents (continuous line in Figure 15(b)) are less responsive to the change in price informativeness because their greater acceptance of risk has already reduced the uncertainty thereby reducing the instability of the market.

7 Conclusions

The ongoing demographic transition and the recent financial crisis have added the issue of whether individuals are equipped with the tools needed to make complex financial decisions to the agendas of policy makers around the world. A consensus is emerging on the need to increase the financial literacy of the general population, i.e. to equip people with the minimum knowledge on finance required to make decisions that have long-term welfare consequences, such as choices related to saving for retirement or taking out a mortgage to purchase a main residence - see Lusardi (2008) for more details.[17] Many authors have highlighted the importance of financial programs for improving financial literacy, and have emphasized the need to begin financial education early in life as part of normal school education.[18] It may not be straightforward to understand the effect of financial education on individual behavior, but it is considered vital by those who advocate financial education. Data show that shrewd financial behavior is associated with high levels of financial literacy, which raises the issue of the endogeneity of financial literacy. To understand the effect of financial education on individuals' welfare, we need to take account of the possible general equilibrium effects of financial education programs; so far, this has not been tackled in the literature.

The present paper contributes to filling this gap by endogenizing the individual decision to undertake a financial education program, as in Jappelli and Padula (2013a). We focus on the effect of financial education on market stability, and challenge the view that increasing financial literacy causes markets to be more stable. We assume that financial education programs increase financial literacy. However, individuals differ in their cognitive ability, which in turn affect the dis-utility of taking financial education. More able individuals face a lower dis-utility cost, and therefore, other things being equal, are more willing to take financial education. In the information market, individuals can acquire the precision of a private signal on a risky assets payoff at a cost that depends on their financial literacy. Financial education grants a

[17] A concurrent line of enquiry, see e.g. Willis (2008), suggests that financial products, whether dedicated to households' assets or liabilities, should be designed in such a way as to induce optimal behavior from the individual. While we do not address the implications of changing the financial architecture, our results go towards achieving a balance between the financial architecture and financial education by depicting the general equilibrium effect of financial education programs.

[18] See PISA 2012, a large-scale international assessment of the financial literacy of young people OECD (2013).

lower cost for acquiring financial information, and the precision of a private signal, providing informational advantages with respect to those agents who do not attend such a program. Therefore, in deciding whether to become financially literate by attending a financial education program the individual factors in the decisions of the other individuals to the informativeness of market prices.

Our main results show that reducing the cost of acquiring financial information fosters market volatility and inequality among the agents. Promoting financial education leads to higher aggregate information, provided mainly by the literate agents. The share of literates increases up to the point where the illiterates decide not to acquire private financial information. If the cost of acquiring financial information for the literate agents is too low, the illiterate agents prefer to trade conservatively, and to acquire no private information. The expected utility of the literates drops and only the agents with the lower dis-utility costs attend the program. As a result, the share of literates decreases. Our general equilibrium model indicates non-monotonic effects of financial education. Policy makers should be aware of the possible feed-back effects on market volatility from a decision to provide financial education programs.

Our model abstracts from the financing aspects of financial education policies. The implicit assumption is that financial education is provided for free, and that individuals incur a dis-utility cost for undertaking a financial education program. Assuming that financial education policies are funded through general taxation cannot do better for all agents and this raises two questions. The first relates to the welfare effects of financial education and to the optimal way of funding financial education programs. The second concerns the winners and losers from financial education policies. The current setting does not allow us to address these extremely important questions, and they are left for future research.

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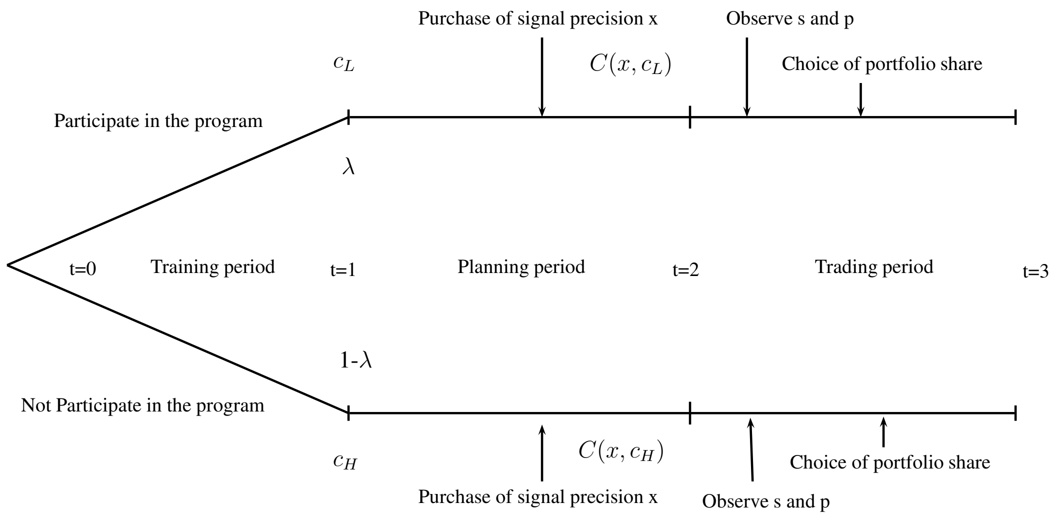


Figure 1: Time-line of the model

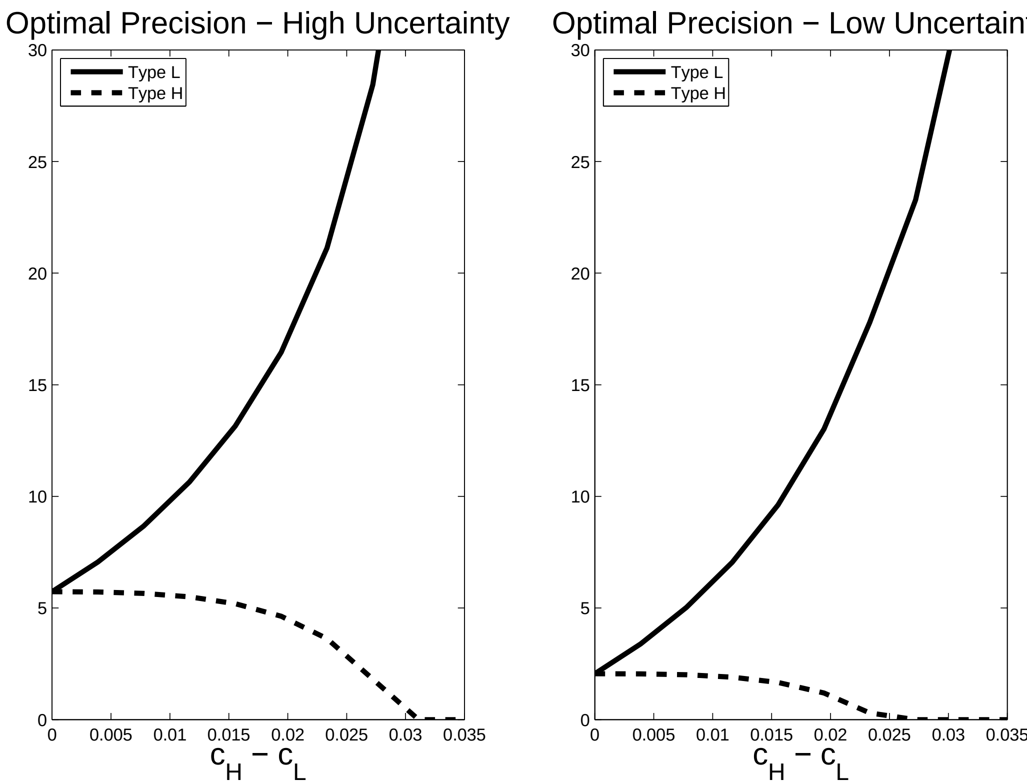


Figure 2: Optimal information choice for both types in high ($\tau_\pi = 0.2$, $\tau_\theta = 0.2$) and low ($\tau_\pi = 2$, $\tau_\theta = 2$) regime of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

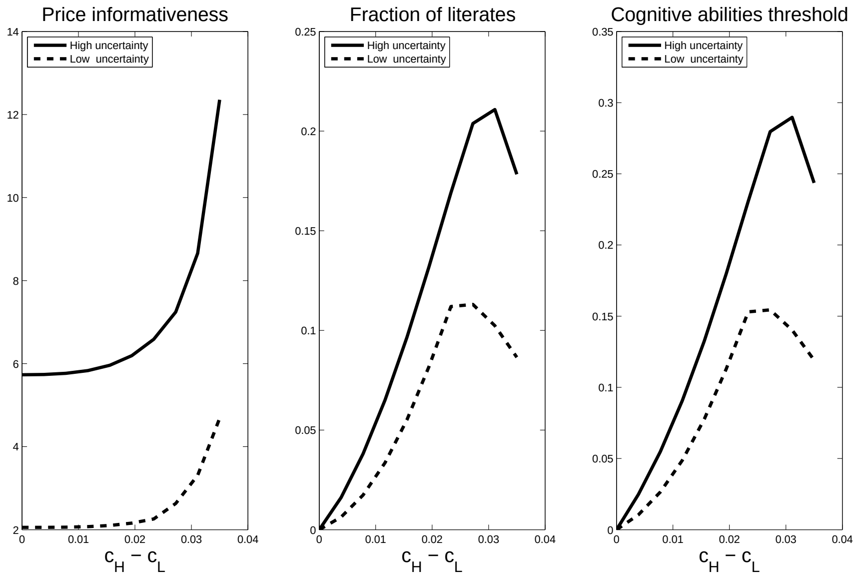


Figure 3: Price informativeness, optimal proportion of literate agents and threshold over cognitive abilities domain in high ($\tau_\pi = 0.2$, $\tau_\theta = 0.2$) and low ($\tau_\pi = 2$, $\tau_\theta = 2$) regime of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

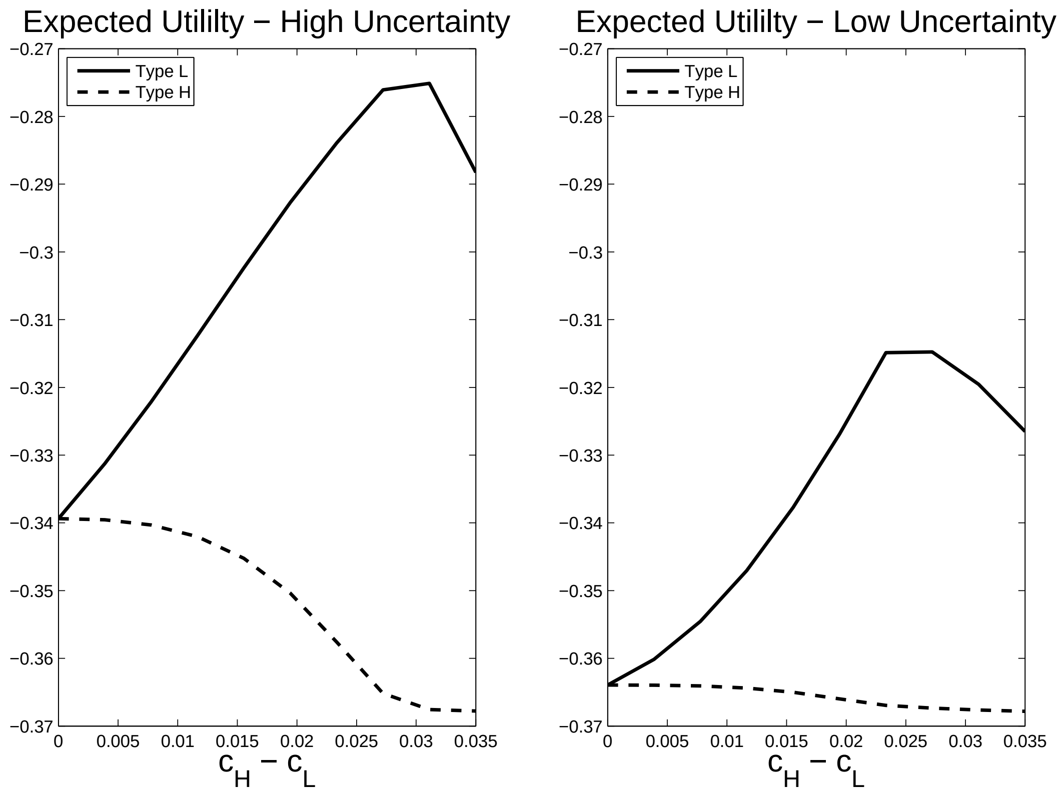
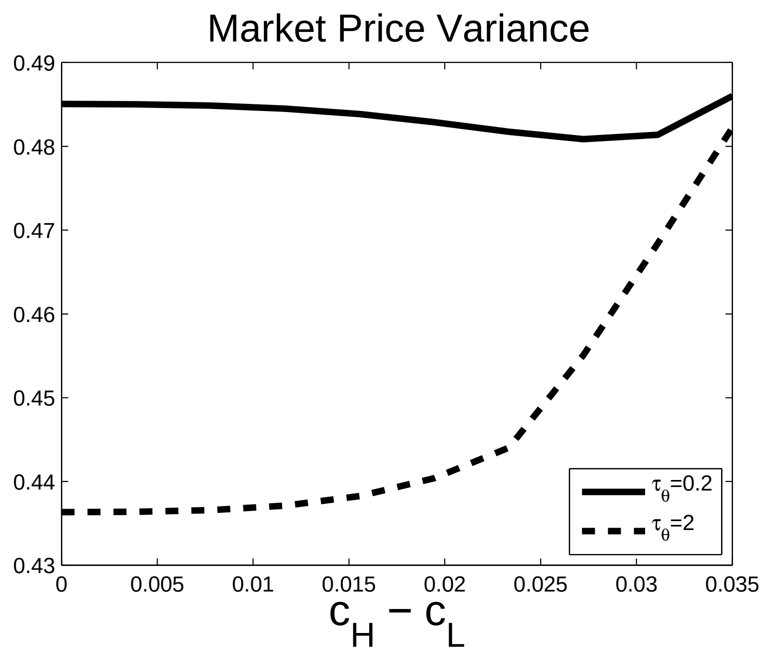
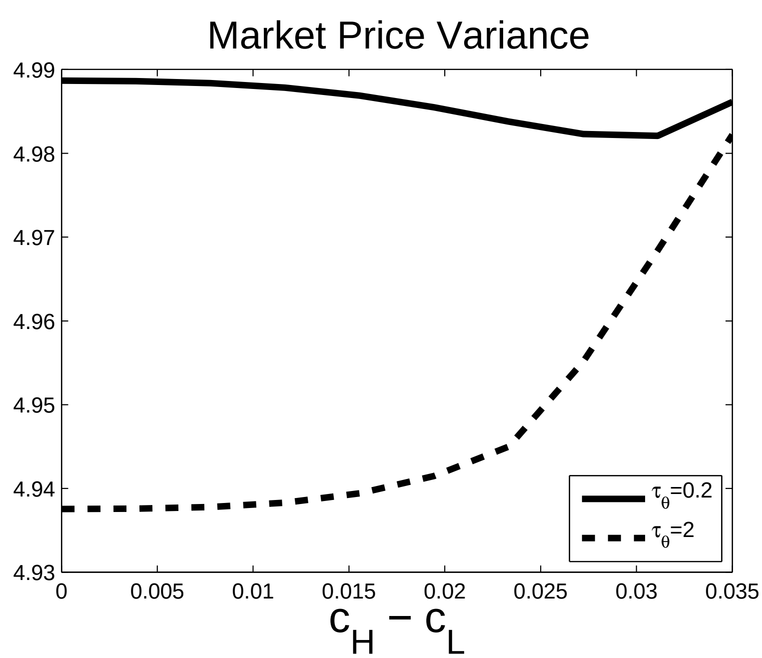


Figure 4: Expected utility for both types in high ($\tau_\pi = 0.2$, $\tau_\theta = 0.2$) and low ($\tau_\pi = 2$, $\tau_\theta = 2$) regime of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.



(a) Prior precision: $\tau_\pi = 2$



(b) Prior precision: $\tau_\pi = .2$

Figure 5: We report market price variance, computed in equilibrium. We set c_H equal to 0.04 and we let c_L decreases up to 0.005. Moreover, $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

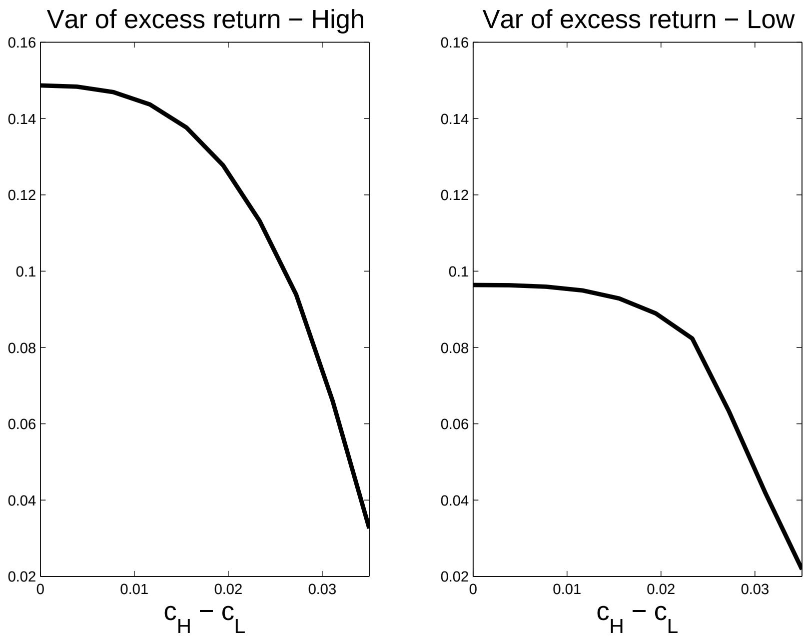


Figure 6: Variance of the excess return in high ($\tau_\pi = 0.2$, $\tau_\theta = 0.2$) and low ($\tau_\pi = 2$, $\tau_\theta = 2$) regime of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

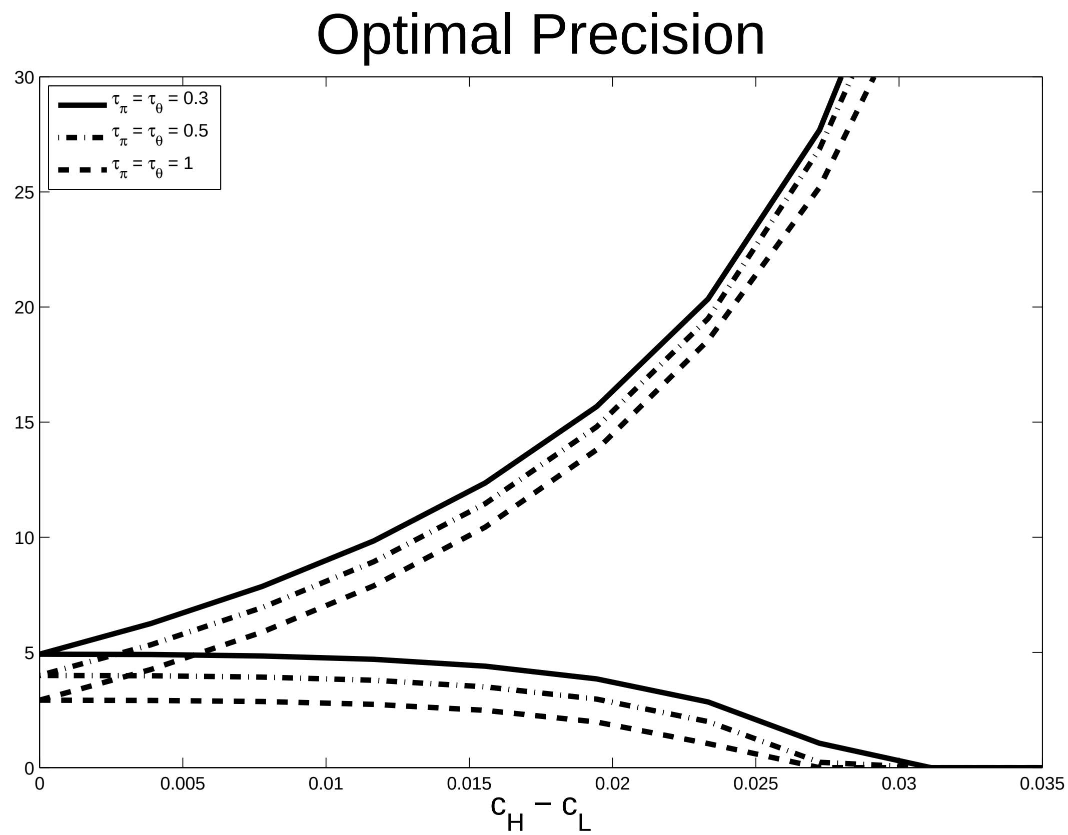


Figure 7: Optimal information choice for both types for several regimes of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

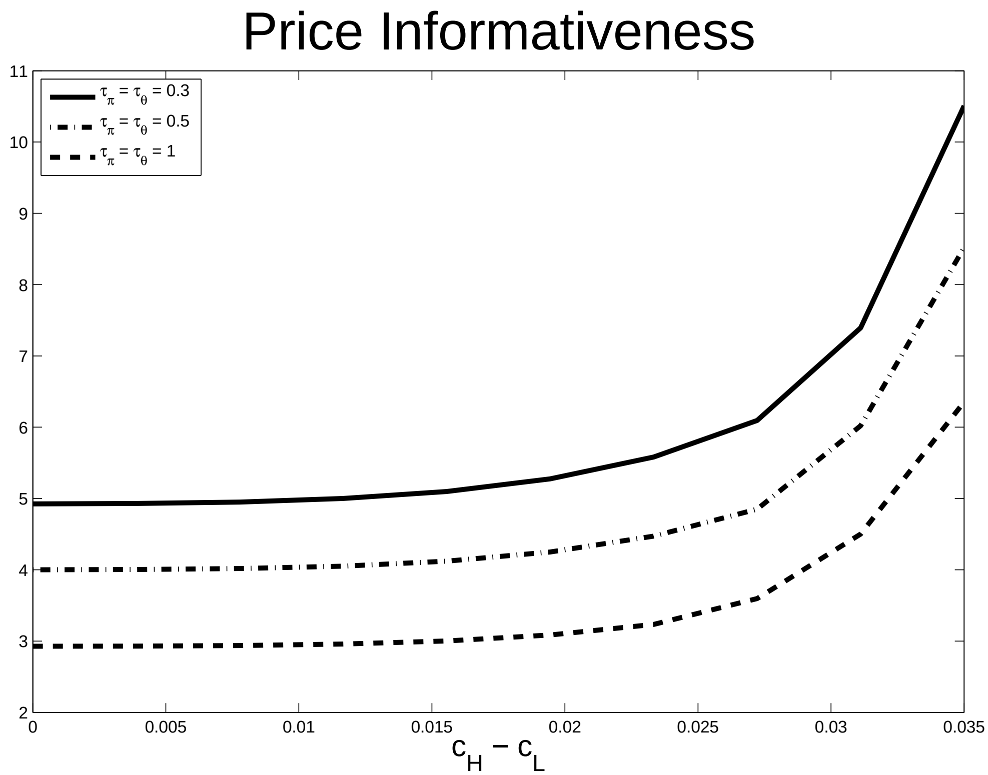


Figure 8: Price informativeness for several regimes of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

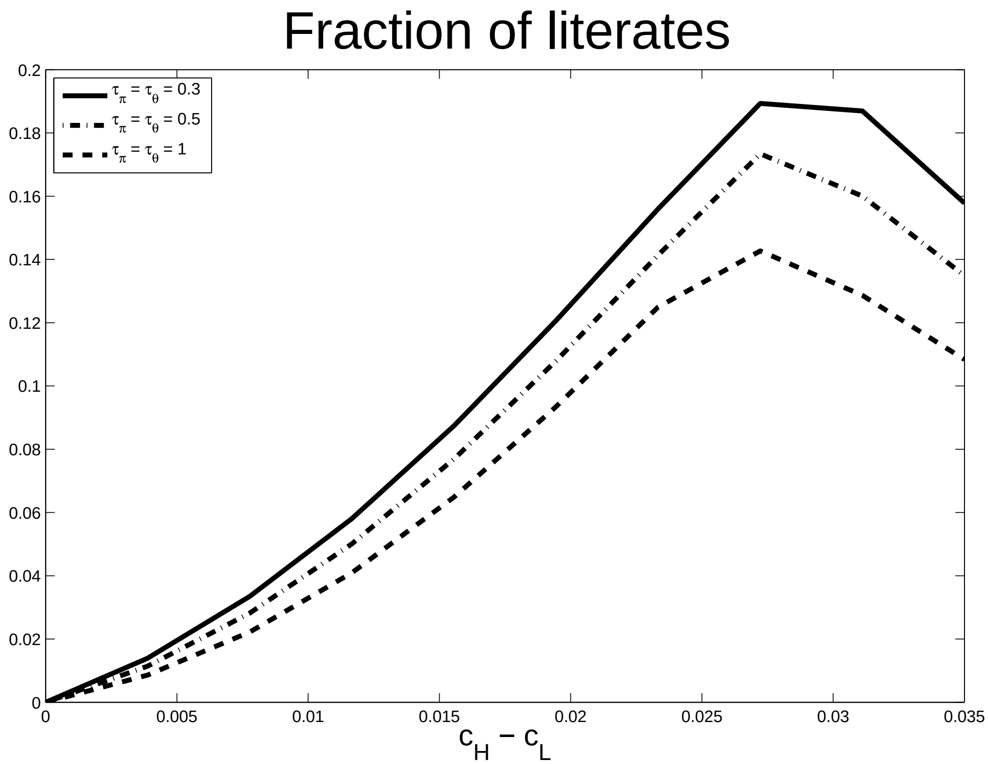


Figure 9: Fraction of the literate agents for several regimes of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

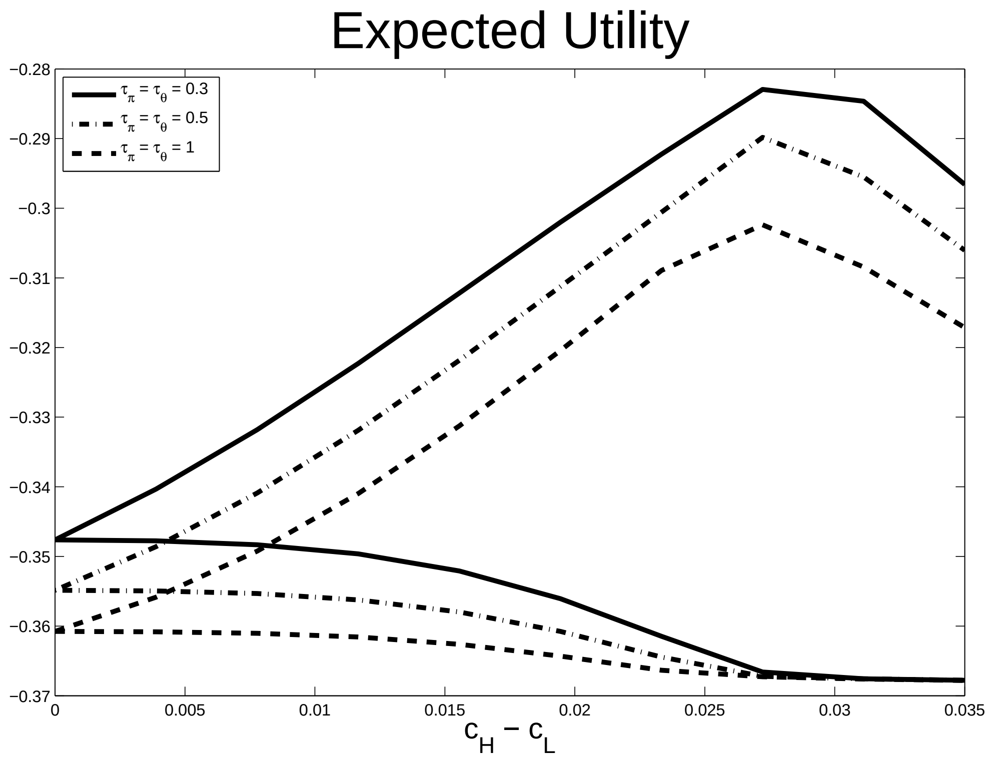
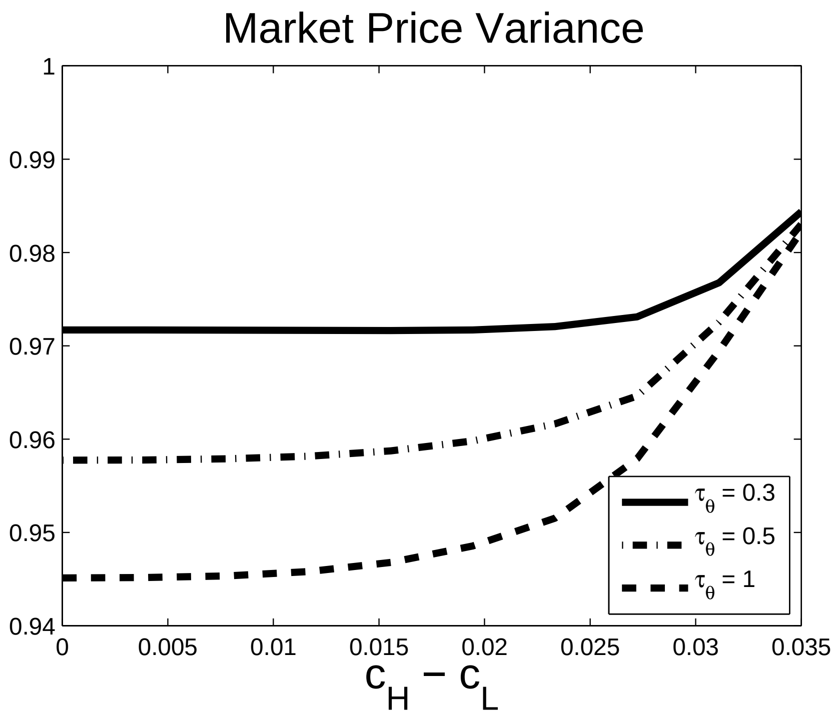
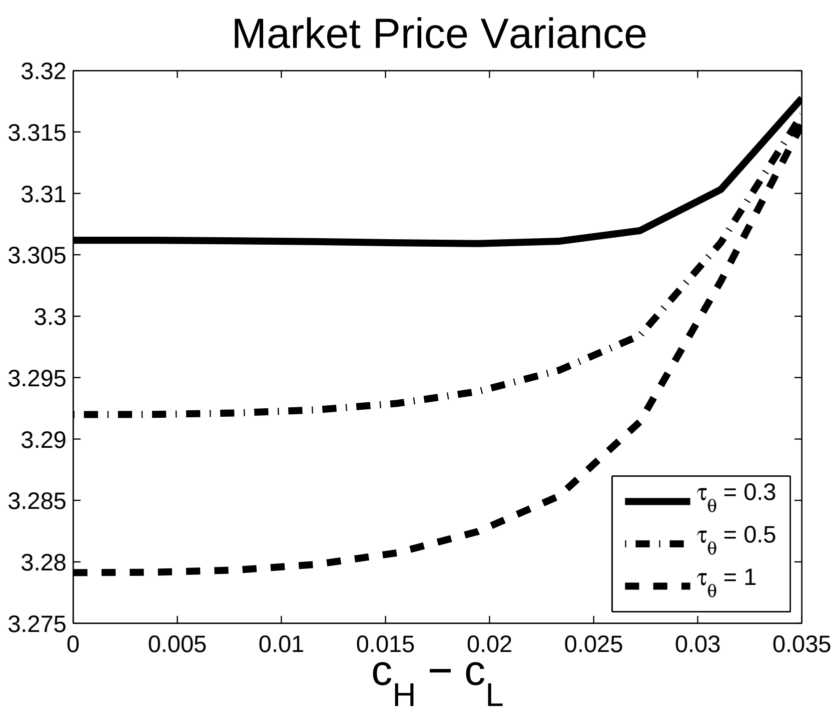


Figure 10: Expected Utility of the agents for several regimes of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.



(a) Prior precision: $\tau_\pi = 1$



(b) Prior precision: $\tau_\pi = .3$

Figure 11: Market price variance for several regimes of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

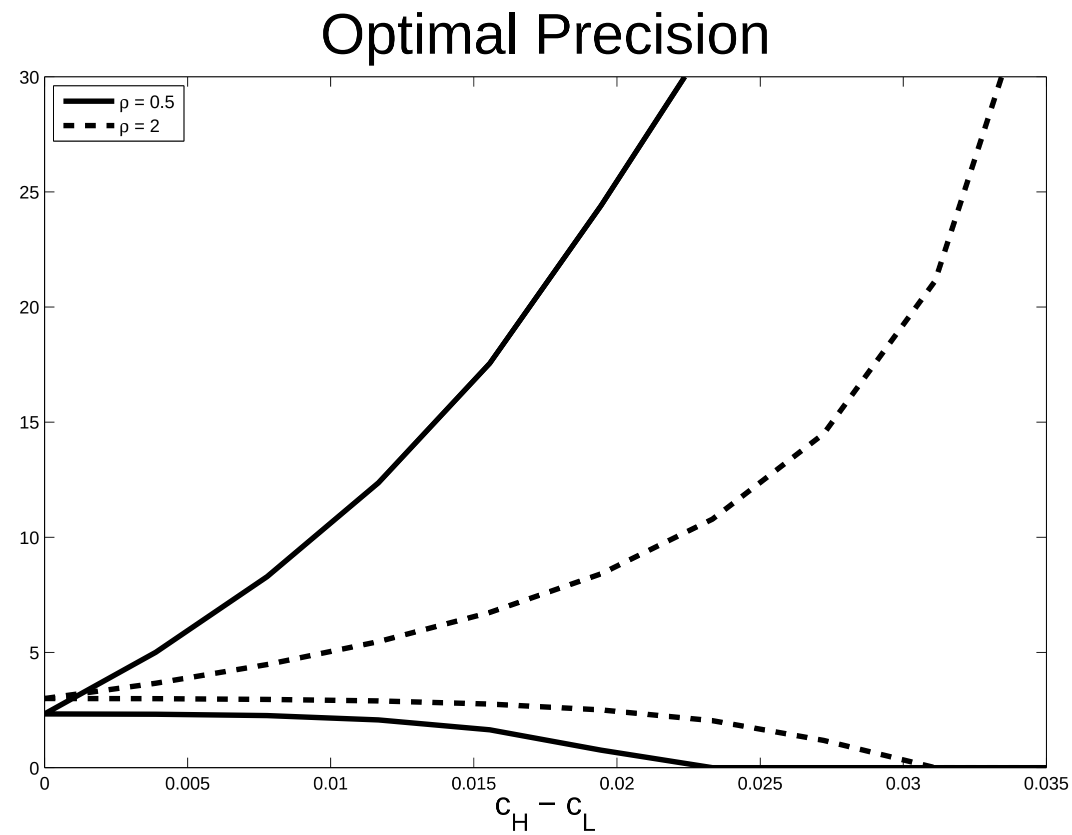


Figure 12: Optimal information choice for both types for different degrees of risk aversion of the agents: $R = 1$, $W_1 = 1$, $\tau_\pi = 1$, $\tau_\theta = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

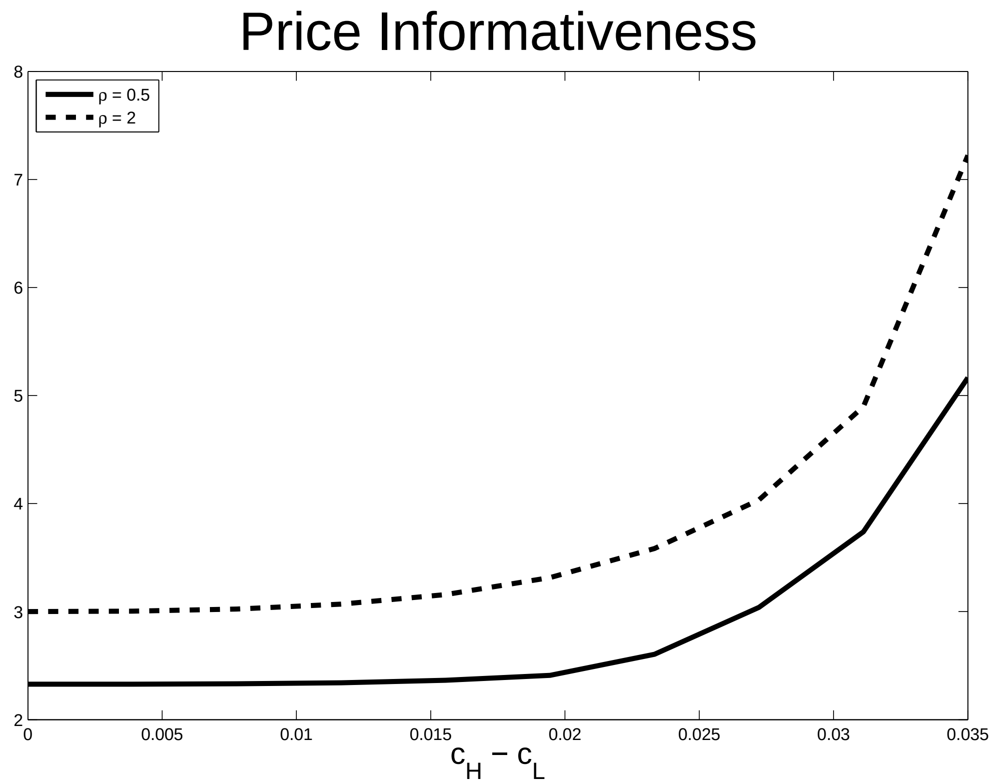


Figure 13: Price informativeness for different degrees of risk aversion of the agents: $R = 1$, $W_1 = 1$, $\tau_\pi = 1$, $\tau_\theta = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

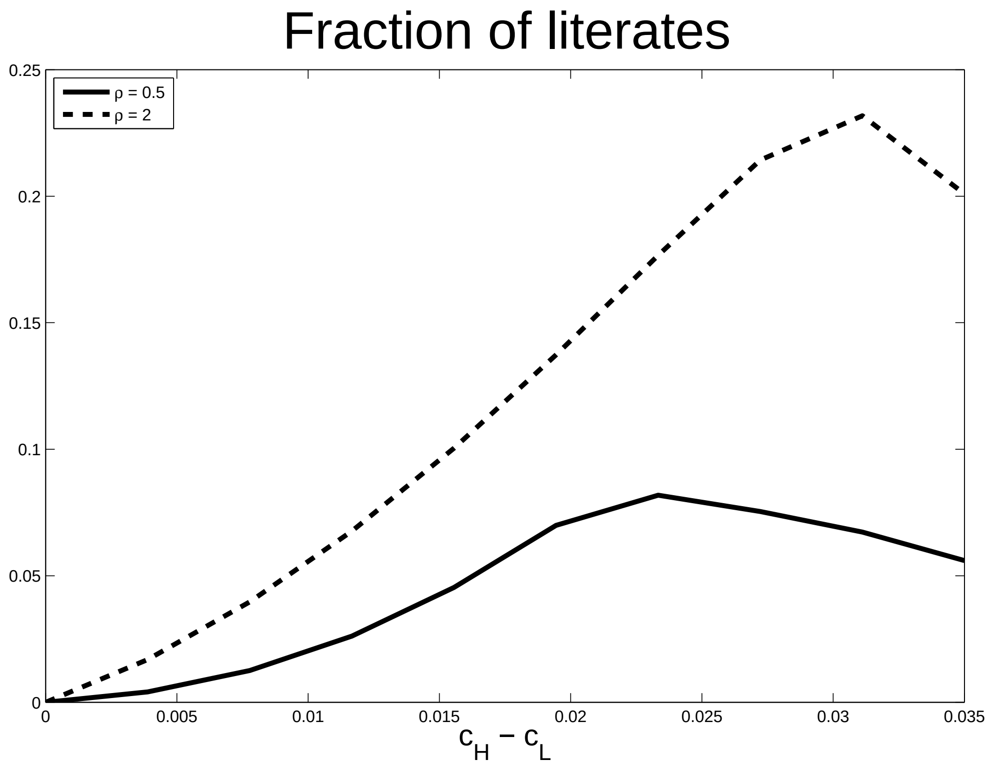
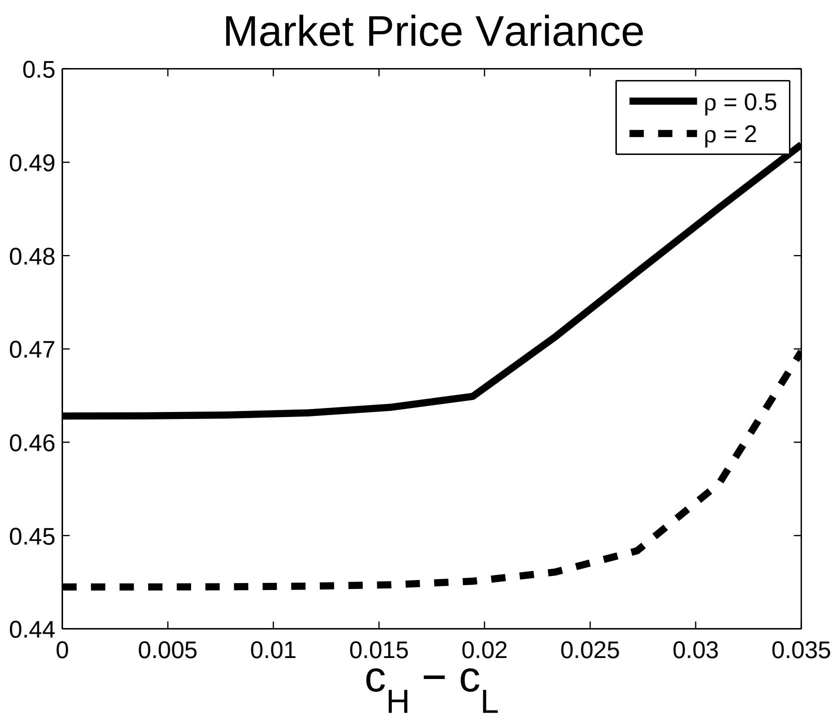
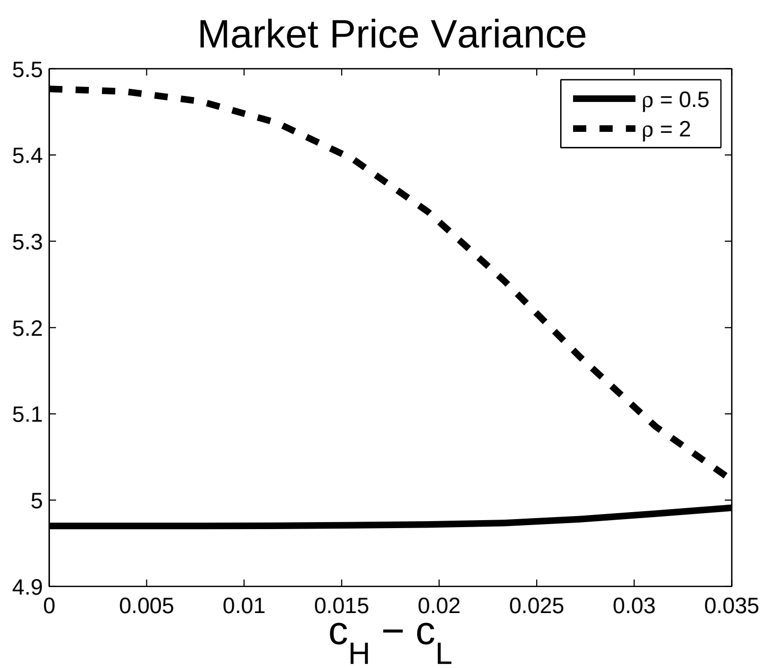


Figure 14: Fraction of the literate agents for different degrees of risk aversion of the agents: $R = 1$, $W_1 = 1$, $\tau_\pi = 1$, $\tau_\theta = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.



(a) Low uncertainty: $\tau_\pi = \tau_\theta = 2$



(b) High uncertainty: $\tau_\pi = \tau_\theta = .2$

Figure 15: Market price variance for different degrees of risk aversion of the agents: $R = 1$, $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

Appendix Proof of proposition 1

To prove the existence of the equilibrium, we need to prove the existence and the uniqueness of an equilibrium market price that depends linearly on the fundamentals, π and θ, and clears the market. In equilibrium, all agents maximize their utility. We compute optimal asset demands and the equilibrium market price for given aggregate informativeness I. We then derive optimal information choices and work out the equilibrium in the information market, for a given fraction of literates λ in the population. The last step involves proof of the existence of the equilibrium for the training choices.

Lemma 1 (Individual asset demands and market price)**.** *For given aggregate informativeness I, the optimal portfolio share for agent $j \in J$ is given by:*

$$\alpha_j^* = \frac{k_j\, p}{\rho[W_1 - C(x_j, c_j)]}(E[\pi|\mathcal{F}_j] - pR),$$

and the equilibrium market price is $pR = a + b\pi - d\theta$ where the coefficients a, b and d are given by:

$$a = \frac{\mu_\pi \tau_\pi + \frac{I}{\rho}\mu_\theta \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2}\tau_\theta}, \quad b = \frac{I + \frac{I^2}{\rho^2}\tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2}\tau_\theta}, \quad d = \frac{\rho + \frac{I}{\rho}\tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2}\tau_\theta}. \tag{A-1}$$

Proof. The proof consists of five steps. In the first step, we guess a linear price function and derive the informationally equivalent public signal ξ from the price function. In the second step, we compute the mean and the variance of posterior beliefs, given the two unbiased signals, ξ and s. In the third step, we derive optimal asset demands. In the fourth step, we derive the market clearing conditions, and in the fifth step we impose rationality and determine the coefficients of the guessed linear price function. For more details, see the proof of lemma 1 in Pettinicchi (2012). □

To simplify the notation, we rewrite the precision of the public signal ξ:

$$\tau_\xi = \frac{I^2}{\rho^2}\tau_\theta. \tag{A-2}$$

Lemma 2 (Optimal information choice)**.** *For given aggregate informativeness I, for all the agents with $c < \overline{c}(\Theta)$, the optimal information choice x^* solves the following equation:*

$$2\rho R C_x(x^*, c)\,(\tau_\pi + x^* + \tau_\xi) = 1, \tag{A-3}$$

where $\overline{c}(\Theta)$ is given implicitly by:

$$[2\rho R(\tau_\pi + \tau_\xi)]^{-1} = C_x(0; \overline{c}). \tag{A-4}$$

Proof. In order to solve for the information choice x^*, we need to compute the indirect utility: $v(s_j, p; \Theta) = E[U(W_2(\alpha_j^*))|s_j, \xi]$, and its expected value. The latter depends on the first two moments of the expected return of trading a unit of risky asset. Having derived the expected indirect utility function, we apply the concave maximum theorem to obtain the optimal information choice x^* and to identify the threshold $\overline{c}(\Theta)$ above which it becomes worthwhile to be informed. See lemma 2 in Pettinicchi (2012). □

Recall that we denote the indirect utility of solving the planning problem:

$$z(\Theta) = -\tfrac{1}{\rho}\left(1 + \sigma_f^2 k^*\right)^{-1/2} e^{-\rho R[W_1 - C(x^*, c)]}$$

and we use the subscript to refer to a literate agent, $z_L(\Theta)$, or to an illiterate one, $z_H(\Theta)$.

Lemma 3. *The optimal training choice is:*

$$D = \begin{cases} 1 & \textit{if} \quad \gamma < \overline{\gamma}, \\ 0 & \textit{otherwise}, \end{cases}$$

where $\overline{\gamma} = \log z_H(\Theta) - \log z_L(\Theta)$.

Proof. Let the function $f : \mathbb{R}^+ \to \mathbb{R}$ be defined as follows:

$$f(\gamma) = z_L(\Theta) e^{\gamma} - z_H(\Theta).$$

It is continuous and monotonic ($f'_\gamma < 0$). For given λ, we can show that the optimal attendance choice is:

$$D^*(\gamma, \Theta) = \begin{cases} 1 & \text{if } f(\gamma) > 0, \\ 0 & \text{if } f(\gamma) < 0. \end{cases}$$

We check the behavior of the function for $\gamma = 0$ and $\gamma \to +\infty$. We distinguish three cases of states of being informed: both types, only literates, neither types. In the first case, $c_L < c_H < \overline{c}$, $f(0) = z_L(\Theta) - z_H(\Theta) > 0$ given that:

$$\frac{e^{C(x_L^*, c_L)}}{e^{C(x_H^*, c_H)}} < 1 < \sqrt{\frac{k_L^*}{k_H^*}},$$

by monotonicity of the cost function $C_c' > 0$ and of the optimal information choice $x_c' > 0$. In the second case, $c_L < \overline{c} < c_H$, $f(0) > 0$ given that:

$$e^{\rho R C(x_L^*, c_L)} < \sqrt{\frac{k_L^*}{I + \tau_\xi}},$$

the dis-utility cost of acquiring information is lower than the benefit obtainable from the reduction in uncertainty of the posterior beliefs with respect to the uninformed agent. In the third case, $\overline{c} < c_L < c_H$, both types are uninformed and have the same expected utility: $f(0) = 0$.

For γ going to infinity, $\lim_{\gamma \to +\infty} f(\gamma) < 0$, for all c_L, c_H and given $\lambda \in [0, 1]$. Therefore, applying Bolzano's theorem, there exists a $\overline{\gamma} \in \mathbb{R}^+$ such that $f(\overline{\gamma}) = 0$, by continuity of the function $f(\cdot)$. Therefore, we can rewrite the optimal training choice as:

$$D^*(\gamma, \Theta) = \begin{cases} 1 & \text{if } \gamma \leq \overline{\gamma}, \\ 0 & \text{if } \gamma > \overline{\gamma}, \end{cases}$$

where $\overline{\gamma} = \log z_H(\Theta) - \log z_L(\Theta)$. Moreover, $\overline{\gamma}$ is continuous with respect to λ, given the continuity of the optimal information choice x^*. Recall that

$$z(\Theta) = E[v(x^*)] = -\frac{1}{\rho}\left(1 + \sigma_f^2 k^*\right)^{-1/2} e^{-\rho R[W_1 - C(x^*, c)]}.$$

For any given λ, we can show that $z(c, \Theta)$ is negative and non-increasing in c: $\forall c_L, c_H$ with

$c_L < c_H$, $z(c_L, \Theta) \geq z(c_H, \Theta)$. Formally, monotonicity is satisfied when

$$\frac{\partial z(c,\Theta)}{\partial c} = -\phi \left[\frac{\partial x^*}{\partial c}\left(1 + k^* \rho R C_x\right) + k^* \rho R C_c\right] < 0$$

where $\phi = \dfrac{e^{-\rho R[W_1 - C(x^*,c)]}}{\rho\sqrt{\frac{1}{k^*} + \sigma_f^2}} > 0$ and $\frac{\partial x^*}{\partial c} = -\dfrac{k^* C_{xc} + C_x \frac{\partial k}{\partial I}\frac{\partial I}{\partial c}}{k^* C_{xx} + C_x}$. This holds when:

$$\frac{\partial x^*}{\partial c} > -\frac{k^* \rho R C_c}{1 + k^* \rho R C_x}$$

□

To prove the existence of a noisy rational expectations equilibrium, we need to check that individual optimal choices are consistent with the aggregate variables. For a given λ, the information market is in equilibrium if the optimal individual information choices are consistent with aggregate informativeness. Then, we check that optimal training choices are consistent with the proportion of agents who become literate.

Lemma 4. *Given λ, the following condition holds in equilibrium:*

$$I = \lambda x_L^* + (1 - \lambda) x_H^*.$$

Proof. Recall that $\underline{c}$ is derived implicitly from: $C_x(\overline{x}(\underline{c}), \underline{c}) = \frac{1}{2\rho R(\overline{x}(\underline{c}) + \tau_\pi)}$ where $\overline{x}$ is the maximum amount of information that an individual can acquire.

For any $\lambda \in [0, 1]$ and for any $c_L > c_H > \underline{c}$, let the compact set $\mathcal{I} = [0, \frac{1}{2\rho R C_x'(0,\underline{c})}] \subset \mathbb{R}^+$ the domain of the following mapping operator $F : \mathcal{I} \to \mathbb{R}^+$:

$$F(I) = \lambda x_L^*(I; \Theta) + (1 - \lambda) x_H^*(I; \Theta),$$

where I is an element of $\mathcal{I}$

Continuity of C_x' assures the continuity of $F(I)$. We need to prove that F maps onto itself to apply the fixed-point Brouwer's theorem ($F(I) = I$).

For all $c > \underline{c}$ and $I \in \mathcal{I}$, $x^*(I; \Theta) \geq 0$. Moreover, given the strict convexity of the cost function, $C_x'(0, \underline{c}) \leq C_x'(x^*, c)$. This implies that:

$$0 \leq x^* = \frac{1}{2\rho C_x'(x^*,c)} - (\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta) \leq \frac{1}{2\rho C_x'(x^*,c)} \leq \frac{1}{2\rho C_x'(0,\underline{c})}.$$

The convex combination of two terms belonging to $\mathcal{I}$ still belongs to $\mathcal{I}$:

$$0 \leq F(I) = \lambda x^*(c_L, I; \Theta) + (1-\lambda)x^*(c_H, I; \Theta) \leq \frac{1}{2\rho C'_x(0,\underline{c})}.$$

Thus, $F(I)$ maps onto itself and we prove the existence of the equilibrium condition in the information market for any given λ. □

Lemma 5. *The proportion λ is given by:*

$$\lambda = \int_j D^*(\Theta) dG(j).$$

Proof. Let the compact set $\Lambda = [0, 1]$ be the domain of the following mapping operator $S : \Lambda \to \mathbb{R}^+$:

$$S(\lambda) = \int_j D_j^*(\Theta) dG(j).$$

Recall that $D_j^*(\Theta)$ takes the value 1 if $\gamma(j) < \overline{\gamma}$ and recall that $\gamma(\cdot)$ is a continuous and monotonic function. We can show that $\int_j D_j^*(\Theta) dG(j) = G(\overline{\gamma})$ where $G(\cdot)$ is the cumulative distribution function, derived by the distribution of agents over the cognitive ability space. Given that $G(\gamma) \in [0, 1]$ through the properties of cumulative distribution functions, and that $\overline{\gamma}(\lambda)$ is continuous with respect to λ, the function $S(\cdot)$ is continuous and maps onto itself. Thus, the Brouwer's theorem applies and we prove the existence of a rational expectations equilibrium for the model.

□