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ABSTRACT

Exclusive contracts and market dominance*

We develop a theory of exclusive dealing that rehabilitates pre-Chicago-school analyses. Our theory rests on two realistic assumptions: that firms are imperfectly informed about demand, and that a dominant firm has a competitive advantage over its rivals. Under those assumptions, exclusive contracts tend to be pro-competitive when the dominant firm's competitive advantage is small, but are anti-competitive when it is more pronounced. In this latter case, the dominant firm uses exclusivity clauses as a means to increase its market share and profit, without necessarily driving its rivals out of the market, or impeding their entry. We discuss the implications of these results for competition policy.

JEL Classification: D42, D82 and L42
Keywords: dominant firm, exclusive dealing and non-linear pricing

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1 Introduction

Exclusive dealing has long been a controversial practice. In the past, the primary basis for condemnation was a naive theory of a firm that competes with rivals supplying substitute goods. The theory contended that by requiring buyers not to purchase from its competitors, the firm can boost the demand for its product. This raises its market share and profit, but harms its customers and rivals. In the last decades, however, this theory has fallen into disrepute as a result of heavy attacks by the Chicago school,[1] and it has been replaced as a reason for prohibiting exclusive contracts by entry deterrence arguments (discussed below).

The main purpose of this paper is to show that under realistic assumptions the old theory is substantially correct. To understand what assumptions are needed, consider the reformulation of the Chicago-school critique proposed by O'Brien and Shaffer (1997) and Bernheim and Whinston (1998). When dealing with the firm, a buyer has a reservation payoff equal to the benefit that he can obtain by trading exclusively with the firm's competitors. If the firm can price discriminate perfectly, it will then extract all the surplus in excess of said reservation payoff. Thus, it will offer a contract that maximizes the buyer's surplus – a property referred to as bilateral efficiency. Generally speaking, exclusivity clauses cannot be optimal as they would violate bilateral efficiency if buyers have a preference for variety, reducing the buyer's surplus and hence the firm's profit.

In reality, however, full extraction of surplus is almost impossible to achieve. One major obstacle is *incomplete information about demand*.[2] This is, therefore, the first key ingredient of our model.[3] Suppose, for example, that the buyer's demand can be either high or low, but the firm does not know which state is realized. The firm will then be unable to extract all the surplus when demand is high. Furthermore, in order to extract more surplus in the high-demand state, it must distort the contract that applies in the low-demand one.

[1] See, for instance, Bork (1978) and Posner (1976). A typical critique would run as follows:

> The theory of exclusionary tactics underlying the law appears to be that firm X, which already has ten percent of the market, can sign up more than ten percent of the retailers, perhaps twenty percent, and, by thus foreclosing rivals from retail outlets, obtain a larger share of the market. But one must then ask why so many retailers are willing to limit themselves to selling X's product. Why do not ninety percent of them turn to X's rivals? Because X has greater market acceptance? But then X's share of the market would grow for that reason and the requirements contracts have nothing to do with it. Because X offers them some extra inducement? But that sounds like competition. It is equivalent to a price cut, and surely X's competitors can be relied upon to meet competition. (Bork and Bowman, 1965, p. 366-7)

[2] Notice that bilateral efficiency must hold even if buyers have some bargaining power, as long as bargaining is efficient (O'Brien and Shaffer, 1997).

[3] Perfect price discrimination may also be impeded by the firm being restricted to linear pricing – a case analyzed by Mathewson and Winter (1987). With complete information, however, contracts that are relatively simple and widely used, such as two-part tariffs, would suffice for full extraction of surplus. For an excellent analysis of the role of contractual complexity in exclusive dealing arrangements, see Spector (2011).

What is the optimal distortion? If the firm is restricted to non-linear pricing, all it can do is to reduce its own quantity below the efficient level. With exclusive contracts, by contrast, the firm can set its rivals' volume to zero. This latter strategy will often be more profitable than the former, since the cost of creating the distortion is shifted onto the rivals.

These arguments imply that bilateral efficiency, and hence the non-optimality of exclusive dealing, holds only when demand is highest. In all other cases, the contract offered to the buyer must be distorted, and the optimal distortion may well involve exclusivity clauses. This simple observation is the starting point of our analysis.

To proceed, observe that when firms are symmetric, a firm's attempt to use exclusive contracts to implement the optimal distortion is destined to fail. The reason for this is that when all firms offer exclusive contracts because doing so is unilaterally profitable, they end up competing for exclusives. Now, competition for exclusives is competition in utility space, where firms' products are effectively homogeneous. As a result, competition with exclusive contracts becomes very intense and reduces both prices and profits. Symmetric firms are therefore caught in a prisoner's dilemma, as we show in a related article (Calzolari and Denicolò, 2013).

However, antitrust authorities and the courts are rarely concerned with equally sized competitors that compete vigorously; more often, they are concerned with dominant firms facing smaller rivals. The second key assumption of this paper is, therefore, *asymmetry of firms*: we assume that one firm has a competitive advantage over its rivals, in terms of lower cost, higher demand, or a combination of the two.

The competition-enhancing effect of exclusive contracts will still prevail if the dominant firm's competitive advantage is small. When it is more pronounced, however, the dominant firm can win the competition for exclusives easily. The competition-enhancing effect weakens, and it may even vanish altogether if the competitive advantage gets large enough. The reason for this is that the buyer obtains an information rent even if the dominant firm acts as a monopolist. If that rent is larger than the rent that the buyer can obtain by trading exclusively with the dominant firm's competitors, exclusive contracts effectively shelter the dominant firm from competition. As a result, there is no longer any prisoner's dilemma, and the dominant firm can simply increase its market share and profit, exactly as argued by the old theory.

Buyers are harmed by exclusive contracts, both in terms of higher prices and reduced variety. Rivals are harmed as well, but they need not be driven (or kept) out of the market for the strategy to be profitable. This marks a key difference with other post-Chicago theories that regard exclusive contracts as anti-competitive.

Why do buyers sign exclusive contracts if they get harmed by doing so? Because those are the best contracts that are actually offered to them. Buyers are harmed with respect to the hypothetical non-linear pricing equilibrium that would arise if exclusive contracts were prohibited. However, when exclusive contracts are permitted firms change their entire pricing strategies, and so non-exclusive contracts are no longer available at the same conditions.

Why do not the dominant firm's rivals resist being foreclosed? They actually do, both by offering exclusive contracts in their turn, and by reducing their non-exclusive prices as compared to the non-linear pricing equilibrium. However, the dominant firm may exploit its competitive advantage to offer exclusive deals so attractive that they cannot be matched by rivals.[4] Furthermore, the decrease in its competitors' non-exclusive prices limits, but does not eliminate, the profitability of exclusionary tactics.

Remarkably, our model can easily reproduce the key stylized facts of many real world antitrust cases.[5] These often involve two main actors: (i) a dominant firm that controls a substantial share of the market and has entered into some kind of exclusive arrangement with its customers, and (ii) a smaller competitor (or group of competitors) that has been active in the industry for some time and in principle could have used exclusive contracts, too, but apparently has not.[6] The equilibria generated by our model are consistent with this pattern. In particular, even if the dominant firm's competitors can offer exclusive contracts, only those offered by the dominant firm are accepted in equilibrium by some buyers.

We close the introduction by contrasting the theory proposed here with other post-Chicago theories that regard exclusive contracts as anti-competitive. All these theories rely on the absence from the contracting game of some of the agents that are affected by exclusivity clauses. Therefore, their practical relevance depends on the realism of this crucial assumption. For example, in Aghion and Bolton (1987) and Rasmusen, Ramseyer and Wiley (1991) the missing agent is the dominant firm's competitor – in those models, this is a potential entrant that initially cannot contract with the buyers. Thus, these theories cannot apply to the many antitrust cases in which the dominant firm's competitors are already active. In Bernheim and Whinston (1998, sect. IV), the missing agents are future buyers whose demand is essential for the dominant firm's rival to achieve economies of scale. Their explanation can therefore apply only to markets where a substantial increase in demand is expected in the not too distant future.

Furthermore, theories that rely on the absence of affected agents from the contracting stage explicitly or implicitly assume that signing a contract is a

[4] This is so even if efficiency requires that rivals stay active, as they supply a differentiated product for which there is buyers' demand.

[5] A prime instance is the case of Intel. Intel, whose main competitor, AMD, has been operating for years in the microchip sector, offered electronics manufacturers discounts that depended on their buying a minimum share of their total purchases of microchips from Intel. Some customers, to qualify for the discount, had to opt for an exclusive supply contract; that is, the minimum share in such cases was set at 100 percent. In 2009, the European Commission found these contracts to be abusive, and fined the company over a billion Euro, the largest fine ever seen in the history of European competition policy.

[6] This is often important in the antitrust evaluation of exclusive contracts. Efficiency-enhancing explanations may be plausible when exclusive contracts are used also by the dominant firm's competitors. When they are not, however, those explanations may be viewed with some skepticism.

form of commitment.[7] In fact, should the missing agents materialize, the parties would have an incentive to renegotiate – a point that has been forcefully made by Spier and Whinston (1995). In some cases, for exclusion to be prevented it may suffice that exclusivity clauses can be breached upon payment of reasonable damages (Simpson and Wickelgren, 2007).

Finally, theories that view exclusive dealing as a means to deprive a rival of economies of scale are faced with the difficulty that exclusivity clauses may not be necessary for that purpose. The same outcome can sometimes be achieved by simple non-linear pricing, e.g. *via* quantity forcing or quantity discounts. If this is so, then a prohibition of exclusive contracts might be easily overcome and could even be welfare reducing.

While these difficulties may not be insurmountable, they limit the applicability of existing anti-competitive theories in antitrust practice.[8] The pre-Chicago theory rehabilitated in this paper does not suffer from any of these drawbacks. It is consistent with the stylized facts mentioned above, including the fact that all involved parties can often participate in the contracting game. It produces an equilibrium in which exclusive contracts are (trivially) renegotiation proof and need not be long-term to be effective, as they play no commitment role. It does not rely on the dominant firm's rival being driven out of the market – so no proof of eviction or recoupment is needed. Finally, it uses as a benchmark the non-linear pricing equilibrium, which already allows for quantity discounts (including quantity forcing).

In sum, the theory developed in this paper seems in several ways more robust than other anti-competitive theories. Having said this, it must be stressed that the anti-competitive effects of exclusive contracts must be weighted not only against the well-known pro-efficiency rationales already discussed in the literature,[9] but also against the pro-competitive effects that arise in our model when firms are not too asymmetric.

The rest of the paper is structured as follows. Section 2 sets out the model. In section 3, we discuss how the possibility of offering exclusive contracts changes the formulation of the firm's pricing problem. Section 4 analyzes the case in which the dominant firm faces a competitive fringe. Section 5 studies the case of duopoly, where the dominant firm's competitor, too, may have some market power. Section 6 summarizes the paper's results and discusses their implications for competition policy. Proofs are in an Appendix.

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[7] This is reflected in the emphasis that antitrust authorities and the courts sometimes place on the duration of exclusive contracts.

[8] See Whinston (2008) for a discussion. Sometimes one difficulty can only addressed at the cost of exacerbating others. For example, Chen and Shaffer (2010) develop an interesting variant of the Rasmusen, Ramseyer and Wiley (1991) model, in which the incumbent uses as exclusionary devices market-share discounts rather than exclusivity clauses. They show that market-share discounts can be anti-competitive even if the entrant eventually enters, which makes their theory applicable to a broader set of cases. However, they assume not only that the entrant is missing at the contracting stage, but also that the incumbent can pre-committ to future prices – an assumption that is not made by Rasmusen, Ramseyer and Wiley (1991).

[9] See Whinston (2008) for an excellent discussion of these rationales.

2 The model

We consider a one-period model of price competition. There are two substitute goods, A and B. Good A is supplied by firm A, whereas good B may be supplied either by firm B (the *duopoly model*) or by a competitive fringe (the *competitive fringe model*). A buyer who buys q_A units of good A and q_B units of good B obtains a benefit, measured in monetary terms, of $u(q_A, q_B, \theta)$. We may think of buyers as downstream firms, and of u as their gross profits,[10] or as final consumers, with u as their utility function. The function u is symmetric and strictly concave in q_A and q_B, and it satisfies the single-crossing condition $u_{\theta q_i}(q_A, q_B, \theta) \geq 0$. The one-dimensional parameter θ is the buyer's private information; it is distributed over an interval $[\theta_{\min}, \theta_{\max}]$ according to a distribution function $F(\theta)$ with density $f(\theta)$. Notice that since heterogeneity is one-dimensional, the demand for the two products is correlated.

In the symmetric case, marginal production costs are normalized to zero. In the asymmetric case, firm A (the dominant firm) has a competitive advantage in terms of lower cost, higher demand, or both. With cost asymmetry, the unit production cost of product B is $c > 0$, whereas that of product A is still normalized to zero. With demand asymmetry, the utility function becomes $u(q_A, q_B, \theta) - cq_B$. The parameter c can now be interpreted as an index of vertical product differentiation, with product A being of better quality, and hence having a larger demand, than product B. These formulations are clearly equivalent; however, they entail different equilibrium prices. To fix ideas, in what follows we shall stick to the cost interpretation.

Buyers have no bargaining power, but are large enough so that firms can monitor whether they purchase from their competitors. Their reservation payoff, $u(0, 0, \theta)$, is normalized to zero. We abstract from fixed costs, and hence from economies of scale.[11]

Firms are expected profit maximizers. They compete by simultaneously and independently offering a menu of contracts. We distinguish two different modes of competition according to the type of contract that the firms may offer. With simple *non-linear pricing*, the payment to each firm depends only on its own quantity. A strategy for firm i then is a function $P_i(q_i)$ in which q_i is the quantity firm i is willing to supply and $P_i(q_i)$ is the corresponding total payment it asks. With *exclusive contracts*, by contrast, a strategy for firm i comprises two price schedules, $P_i^E(q_i)$ and $P_i^{NE}(q_i)$. The former applies to exclusive contracts $(q_{-i} = 0)$, the latter to non exclusive ones $(q_{-i} > 0)$.[12]

[10] In this interpretation, downstream firms must operate in separate markets and must not interact strategically with each other. This prevents the emergence of contractual externalities that would complicate the analysis: see, for instance, Fumagalli and Motta (2006), Simpson and Wickelgren (2007) and Wright (2009).

[11] As long as all firms remain active, this is with no loss of generality. Furthermore, in the competitive fringe model one can interpret c as the minimum average cost of a number of identical firms, thus allowing for economies of scale at the firm level.

[12] To guarantee that the buyer's maximization problem has a solution, we assume that each price schedule P_i must be non decreasing in q_i (a free disposal assumption which also implies that price schedules must be differentiable almost everywhere), that it satisfies $P_i(0) = 0$, and

Buyer θ observes the firms' offers and then chooses the quantities $\{q_A(\theta), q_B(\theta)\}$ that maximize his net payoff. Notice that the buyer may choose to purchase only one product even if he is faced with non-exclusive contracts. We shall call "common representation" the outcome in which the buyer buys positive quantities of both products (which of course can only arise when the buyer does not choose an exclusive contract). We denote[13]

$$U(\theta) = \max_{q_A, q_B \geq 0} \left[u(q_A, q_B, \theta) - P_A(q_A, q_B) - P_B(q_B, q_A) \right].$$

The full information, first-best quantities are

$$\left\{ q_A^{fb}(\theta), q_B^{fb}(\theta) \right\} = \arg\max_{q_A, q_B} \left[u\left(q_A, q_B, \theta\right) - c q_B \right].$$

To make the analysis interesting, we assume that $q_B^{fb}(\theta_{\max}) > 0$; if this condition is violated, good B should not (and would not) be produced in equilibrium.

3 The pricing problem

Before comparing the equilibrium with and without exclusive contracts, it may be instructive to discuss how the possibility of using exclusive contracts changes the formulation of the firms' optimal pricing problem. To fix ideas, we shall focus on the dominant firm, but the same approach applies, in the duopoly model, to its rival.

3.1 Non-linear pricing

When exclusive contracts are prohibited, the dominant firm must solve a fairly standard problem of non-linear pricing. The only twist is that buyers can also trade with the firm's rivals, so they effectively have an indirect utility function

$$v(q_A, \theta) = \max_{q_B \geq 0} \left[u\left(q_A, q_B, \theta\right) - P_B^{NE}(q_B) \right]. \tag{1}$$

In other words, $v(q_A, \theta)$ is the maximum rent that buyer θ can obtain by purchasing q_A and then trading optimally with the dominant firm's rivals.[14]

Let us briefly review the solution technique with non-linear pricing. The firm maximizes its profit $\int_{\theta_{\min}}^{\theta_{\max}} P_A(q_A(\theta)) f(\theta) d\theta$, where $q_A(\theta) = \arg\max_{q_A \geq 0} [v(q_A, \theta) - P_A(q_A)]$. By invoking the Revelation Principle, we can reformulate the problem as if the firm could control $q_A(\theta)$ directly, i.e. as a direct mechanism. Using the change of variables $U(\theta) = v(q_A(\theta), \theta) - P_A(q_A(\theta))$, the firm's problem becomes

that it is upper semi-continuous.

[13] Here $P_i(q_i, q_{-i})$ is used as a general notation that covers both exclusive and non-exclusive contracts.

[14] The notion of indirect utility function has been introduced by Martimort and Stole (2009). Its role is similar to that of residual demand in oligopoly models of linear pricing.

$\max_{q_A(\theta)} \int_{\theta_{\min}}^{\theta_{\max}} [v(q_A(\theta),\theta) - U(\theta)]\, f(\theta) d\theta$. We can then replace the constraint that $q_A(\theta) = \arg\max_{q_A \geq 0}[v(q_A,\theta) - P_A(q_A)]$ with the familiar incentive compatibility and participation constraints. Provided that the indirect utility function satisfies the single-crossing condition $v_{\theta q_A}(q_A,\theta) \geq 0$, the incentive compatibility constraint is equivalent to the requirements that $U'(\theta) = v_\theta(q_A,\theta)$ and that $q_A(\theta)$ is non-decreasing. The participation constraint is $U(\theta) \geq 0$. Since $U'(\theta) \geq 0$, we can optimally set $U(\theta_{\min}) = 0$. The program then becomes

$$\begin{aligned} & \max_{q_A(\theta)} \int_{\theta_{\min}}^{\theta_{\max}} [v(q_A(\theta),\theta) - U(\theta)]\, f(\theta) d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= v_\theta(q_A(\theta),\theta) \qquad (2) \\ U(\theta_{\min}) &= 0 \end{aligned}$$

and $q_A(\theta)$ non-decreasing. This is an optimal control program with $q_A(\theta)$ as the control variable and $U(\theta)$ as the state variable. Once the optimal quantity has been found, one can then recover the tariff that supports it.

3.2 Exclusive contracts

When exclusive contracts are permitted, the dominant firm can control not only $q_A(\theta)$, but also whether $q_B(\theta)$ may be positive or must be nil.

When $q_B(\theta)$ may be positive, the firm's problem is similar to (2), except that the firm must guarantee to the buyer at least the type-dependent reservation utility

$$U_A^R(\theta) = \max_{q_B \geq 0} \left[u(0, q_B, \theta) - P_B^E(q_B)\right], \qquad (3)$$

which the buyer could obtain by trading with the dominant firm's rivals only.[15] The dominant firm's problem then becomes

$$\begin{aligned} & \max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} [v(q_A(\theta),\theta) - U(\theta)]\, f(\theta) d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= v_\theta(q_A(\theta),\theta) \qquad (4) \\ U(\theta) &\geq U_A^R(\theta) \end{aligned}$$

and $q_A(\theta)$ non-decreasing, where $\tilde{\theta}$ is the lowest demand buyer served by the firm. We denote by $q_A^{NE}(\theta)$ the solution to this problem.

[15] If $P_B^E(q_B) \geq P_B^{NE}(q_B)$, the constraint $U(\theta) \geq U_A^R(\theta)$ is subsumed into the indirect utility function. If $P_B^E(q_B) < P_B^{NE}(q_B)$, however, it must be dealt with separately. For an extensive treatment of type-dependent participation constraints in monopolistic screening problems see Jullien (2000).

If $q_B(\theta)$ is set to zero, however, the firm's problem is (following the same steps as above)

$$\begin{aligned} & \max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[u(q_A(\theta), 0, \theta) - U(\theta)\right] f(\theta) d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= u_\theta(q_A(\theta), 0, \theta) \\ U(\theta) &\geq U_A^R(\theta) \end{aligned} \tag{5}$$

and $q_A(\theta)$ non-decreasing. Compared with problem (4), the indirect utility function $v(q_A(\theta), \theta)$ is now replaced by $u(q_A(\theta), 0, \theta)$. In other words, the dominant firm can behave as a monopolist (except for the type-dependent reservation utility). Let us denote by $q_A^E(\theta)$ the solution to problem (5).

Since the firm can choose whether to impose an exclusivity clause or not,[16] it is faced with a *hybrid optimal control problem* involving two different control systems, (4) and (5), and the possibility of switching from one system to the other.[17] To solve this problem, one needs to choose a sequence of control systems, the switching points, and the control function $q_A(\theta)$ for each system that maximize the firm's profit. Clearly, the possibility of switching from one system to the other is generally valuable to the dominant firm.

Let us start from the choice of the optimal control function $q_A(\theta)$. A basic insight in the theory of hybrid optimal control problems is the following: if one particular control system is optimal over an interval of types, then the optimal control function over that interval must maximize the objective function for that system and that interval, given appropriate boundary conditions.[18] In general, the solution depends on the boundary conditions. For the particular non-linear pricing problem at hand, however, it does not: boundary conditions do not affect the optimal quantity $q_A(\theta)$, and hence the marginal prices that support it; they can only affect the fixed fee, or subsidy, of the optimal tariff.

To prove this, consider the choice of the optimal control function on a generic sub-interval $[\theta_1, \theta_2]$ of the support of the distribution of types. One boundary condition is the reservation payoff for type θ_1, which depends on the contracts offered to lower types, *via* the incentive compatibility constraints. However, this is optimally accommodated through a fixed fee or subsidy, which shifts the function $U(\theta)$ up or down in a parallel fashion. As for the upper bound of the interval, ours are free boundary problems. Furthermore, the value of the upper bound itself, i.e. θ_2, is irrelevant, since the optimal quantity for any type $\theta \in [\theta_1, \theta_2]$ depends only on how many higher types there are, both inside and outside the sub-interval. In other words, all that matters is $1 - F(\theta)$.

[16] Notice that the constraint $U(\theta) \geq U_A^R(\theta)$ guarantees that buyers would accept the exclusive contracts offered by the firm.

[17] Generally speaking, a hybrid optimal control problem is a problem involving both continuous and discrete control variables. Our problem is a special case of a hybrid system, called a *switched control system*, in which the discrete variable is a dummy that describes which control system actually applies: see Sun and Ge (2005) for an introduction.

[18] See, for instance, Sussmann (1999).

These properties imply that the optimal quantity can be found by pointwise maximization of a "virtual surplus function,"[19] which does not depend on the extremes of the interval.

This greatly simplifies the analysis. Taken together, the two properties above imply that the optimal control function $q_A(\theta)$ for the complete problem is formed by appropriately joining the control functions $q_A^{NE}(\theta)$ and $q_A^E(\theta)$ that are optimal for problems (4) and (5) separately. When the solution does not involve any exclusivity clause, so that the optimal quantity $q_A(\theta)$ results from the control system (4), it must therefore coincide with $q_A^{NE}(\theta)$. When instead the solution involves an exclusivity clause, the optimal quantity $q_A(\theta)$ results from the control system (5), so it will coincide with $q_A^E(\theta)$ (and of course $q_B(\theta)$ must be nil).

To proceed, we must determine the optimal sequence and the optimal switching points. Generally speaking, this is the most difficult part of a hybrid optimal control problem. At this point, it is therefore convenient to abandon the direct mechanism approach and address the optimal pricing problem straightaway. Let us denote by $P_A^{NE}(q_A)$ and $P_A^E(q_A)$ the tariffs that implement the control functions $q_A^{NE}(\theta)$ and $q_A^E(\theta)$, respectively. At this point, all that remains to be found are the constant terms of these schedules.

Consider the optimal sequence first. In all the problems that we shall analyze, the equilibrium rent function $U(\theta)$ is steeper under non-exclusivity than under exclusivity.[20] Since fixed fees or subsidies can only shift the rent functions up or down in a parallel fashion, the optimal sequence is necessarily from exclusivity (for low-demand types) to non-exclusivity (for high-demand ones). This makes intuitive sense: the no-distortion-at-the-top property implies that the solution for high-demand types must be nearly efficient. As argued by O'Brien and Shaffer (1997) and Bernheim and Whinston (1998), this rules out exclusivity clauses. But exclusivity clauses can be optimal for low-demand buyers, whose quantities are distorted more heavily.

The fact that the exclusive solution must apply, if ever, to low-demand types, and so also to the marginal buyer, implies that the constant term of the corresponding tariff must be nil.[21] We are therefore left with only one degree of freedom, i.e. the fixed fee for the non-exclusive price schedule $P_A^{NE}(q_A)$, Φ_A.

This fixed fee is pinned down by the optimal choice of the switching point

[19] This is the Hamiltonian of problem (4) or (5).

[20] By the envelope theorem, the slope of $U(\theta)$ is $u_\theta(q_A^E(\theta), 0, \theta)$ under exclusive dealing, and $u_\theta(q_A^{NE}(\theta), q_B^{NE}(\theta), \theta)$ under common representation. Since the goods are substitutes, $q_A^E(\theta)$ is generally lower than $q_A^{NE}(\theta) + q_B^{NE}(\theta)$. Given the single-crossing condition, this by itself implies that $U(\theta)$ tends to be steeper under non-exclusivity than under exclusivity. To guarantee the conclusion, however, some regularity conditions are generally needed. In the uniform-quadratic model that we shall focus on later, for instance, the conclusion always holds.

[21] The marginal buyer is the lowest type who purchases a positive amount of a good. When the market is uncovered, as we shall assume later, the marginal buyer purchases a negligible quantity of the goods. This implies that the price schedules which apply to the marginal buyer cannot involve any fixed fee or subsidy. This property was first noted by Wilson (1994) for the case of monopoly, and Martimort and Stole (2009) for the case of duopoly.

$\hat{\theta}$. This must maximize the dominant firm's profit

$$\int_{\theta_{\min}}^{\hat{\theta}} P_A^E(q_A^E(\theta))f(\theta)d\theta + \int_{\hat{\theta}}^{\theta_{\max}} P_A^{NE}(q_A^{NE}(\theta))f(\theta)d\theta,$$

under the constraint that at the switching point $\hat{\theta}$ the buyer must be indifferent between exclusive and non-exclusive contracts:

$$u(q_A^E(\hat{\theta}), 0, \hat{\theta}) - P_A^E(q_A^E(\hat{\theta})) = v(q_A^{NE}(\hat{\theta}), \hat{\theta}) - P_A^{NE}(q_A^{NE}(\hat{\theta})). \tag{6}$$

Implicit differentiation of the constraint (6) yields

$$\frac{d\hat{\theta}}{d\Phi_A} = \frac{1}{u_\theta(q_A^{NE}(\hat{\theta}), q_B^{NE}(\hat{\theta}), \hat{\theta}) - u_\theta(q_A^E(\hat{\theta}), 0, \hat{\theta})}$$

The first-order condition for the optimal switching point is therefore

$$\frac{P_A^{NE}(q_A^{NE}(\hat{\theta})) - P_A^E(q_A^E(\hat{\theta}))}{u_\theta(q_A^{NE}(\hat{\theta}), q_B^{NE}(\hat{\theta}), \hat{\theta}) - u_\theta(q_A^E(\hat{\theta}), 0, \hat{\theta})} = \frac{1 - F(\hat{\theta})}{f(\hat{\theta})}. \tag{7}$$

Taken together, (6) and (7) determine the optimal switching point $\hat{\theta}$ and the corresponding constant term of the non-exclusive price schedule, Φ_A (which turns out to be positive, i.e. a fixed fee). This completes the solution to the firm's optimal pricing problem with exclusive contracts.[22]

4 Competitive fringe

In this section, we focus on the case in which the dominant firm A faces a competitive fringe. This assumption serves two main purposes. First, it simplifies the analysis, as the competitive fringe will always price at cost (i.e., $P_B(q_B) = cq_B$), without imposing any exclusivity clause.[23] Given the passive behavior of the competitive fringe, finding the model's equilibrium is tantamount to finding the dominant firm's optimal pricing strategy.

Secondly, the dominant firm has no way to eliminate the competitive pressure from the fringe, which will always stand ready to supply product B at a unit price of c. This highlights the difference between our theory and other post-Chicago theories in which the role of exclusive contracts is to deter entry, or deprive a rival of economies of scale so as to drive it out of the market. If exclusive contracts are ever optimal in the competitive fringe model, their role must evidently be different.

[22] In the duopoly model, the equilibrium switching point is jointly determined by the pricing choices of the two firms. The corresponding equilibrium conditions will be derived in Section 5 below.

[23] Exclusivity is costly to the buyer, who must therefore be compensated to accept it. However, when prices are already competitive there is no room for compensating the buyer.

The downside of the competitive fringe assumption is that firms which just break even cannot really be harmed. Thus, if exclusive contracts are anti-competitive, they will harm only the buyers. However, in the next section we shall see that the main insights extend to the duopoly model, where exclusive contracts always decrease the dominant firm's rival's profit.

The uniform-quadratic model. In order to get explicit solutions, we shall henceforth focus on a uniform-quadratic specification of the model. The parameter θ is assumed to be uniformly distributed over the interval $[0, 1]$, and the function u is taken to be:

$$u\left(q_A, q_B, \theta\right) = \theta(q_A + q_B) - \frac{1-\gamma}{2}(q_A^2 + q_B^2) - \gamma q_A q_B. \tag{8}$$

The parameter γ captures the degree of substitutability among the products: it ranges from $\frac{1}{2}$ (perfect substitutes) to 0 (independent goods). The factor $\frac{1-\gamma}{2}$ in the middle term in (8) prevents changes in γ from affecting the size of the market.[24] The strict concavity of u implies that buyers have a preference for variety. For example, the extra benefit from buying $\frac{q}{2}$ units of both goods rather than q units of one is: $u(\frac{q}{2}, \frac{q}{2}, \theta) - u(q, 0, \theta) = \frac{1-\gamma}{4}q^2 > 0$.

The uniform-quadratic model has two properties that will be used repeatedly in what follows. First, the market is uncovered; in other words, in equilibrium the marginal buyer's demand is negligible, and so the price schedules which apply to the marginal buyer cannot involve any fixed fee or subsidy. Second, the envelope theorem directly implies that $\dfrac{dU}{d\theta} = q_A(\theta) + q_B(\theta)$, or

$$U(\theta) = \int_0^\theta \left[q_A(\theta) + q_B(\theta)\right] d\theta. \tag{9}$$

Thus, the equilibrium payoff of any buyer θ is fully determined by the quantities obtained in equilibrium by all lower demand buyers. Finally, notice that in the uniform-quadratic specification, the condition $q_B^{fb}(\theta_{\max}) > 0$ becomes

$$c < \frac{1-2\gamma}{1-\gamma}. \tag{10}$$

4.1 Equilibrium

Under the uniform-quadratic specification, the equilibrium of the competitive fringe model can be calculated explicitly by using the methods discussed in section 3. It involves several different sub-cases, depending on the size of the dominant firm's competitive advantage, c.

In particular, the size of c, together with the demand parameter θ, determines the form of the optimal pricing strategy for the dominant firm when it is restricted to simple non-linear pricing. When c is large, low-demand buyers are

[24] As argued by Shubik and Levitan (1980), this rules out spurious effects in the comparative statics analysis.

effectively captive, so the dominant firm can engage in monopoly pricing in the low-demand segment of the market. We denote by

$$P_A^m(q) = \frac{1}{2}q - \frac{1-\gamma}{4}q^2 \tag{11}$$

the monopoly price schedule, and by

$$q_A^m(\theta) = \frac{2\theta - 1}{1-\gamma} \tag{12}$$

the corresponding quantity.[25]

As the size of the competitive advantage decreases and/or demand increases, however, the buyer's temptation to purchase also product B increases. But if a buyer purchased a positive amount of product B, his demand for product A would decrease, as the products are substitutes. To prevent this, the dominant firm therefore engages in limit pricing, raising the sales of product A just up to the point where the buyer's marginal willingness to pay for product B falls below the competitive fringe's cost c. Let us denote by

$$P_A^{\lim}(q) = cq - \frac{1-2\gamma}{2}q^2 \tag{13}$$

this limit pricing price schedule, and by

$$q_A^{\lim}(\theta) = \frac{\theta - c}{\gamma}. \tag{14}$$

the corresponding quantity.[26]

Finally, when the buyer's demand gets still higher, foreclosing the competitive fringe becomes unprofitable. The dominant firm therefore accommodates, so that in equilibrium high-demand buyers purchase both goods. We show in the Appendix that the price schedule that supports such common representation outcome is:

$$P_A^{cr}(q) = \frac{1-(2-c)\gamma}{2(1-\gamma)}q - \frac{1-2\gamma}{4(1-\gamma)}q^2. \tag{15}$$

The corresponding quantities are

$$q_A^{cr}(\theta) = 2\theta - 1 + c\frac{\gamma}{1-2\gamma}; \qquad q_B^{cr}(\theta) = \theta\frac{1-2\gamma}{1-\gamma} + \frac{\gamma}{1-\gamma} - c\frac{1-\gamma}{1-2\gamma}. \tag{16}$$

Whether any buyers will be faced with monopoly or limit pricing depends on the size of the dominant firm's competitive advantage in the following way:

[25] The calculation of these functions is a standard exercise in optimal non-linear pricing.

[26] The function $q_A^{\lim}(\theta)$ is implicitly defined by the condition

$$u_{q_B}(q_A^{\lim}(\theta), 0, \theta) = c.$$

It is then easy to verify that this quantity is implemented by the price schedule (13).

Proposition 1 *In the competitive fringe model, there is a unique non-linear pricing equilibrium where $P_B = cq_B$ and:*

- *when $0 \leq c \leq \frac{1-2\gamma}{2-3\gamma}$, firm A offers the price schedule*

$$P_A(q) = P_A^{cr}(q);$$

- *when $\frac{1-2\gamma}{2-3\gamma} \leq c \leq \frac{1}{2}$, firm A offers the price schedule*

$$P_A(q) = \begin{cases} P_A^{\lim}(q) & \text{for } 0 \leq q \leq q_A^{\lim}(\breve{\theta}_B) \\ P_A^{cr}(q) + \text{ constant} & \text{for } q \geq q_A^{\lim}(\breve{\theta}_B), \end{cases}$$

where $\breve{\theta}_B$ is implicitly defined by the condition $q_B^{cr}(\breve{\theta}_B) = 0$ and the constant guarantees the continuity of the price schedule;

- *when $c \geq \frac{1}{2}$, firm A offers the price schedule*

$$P_A(q) = \begin{cases} P_A^m(q) & \text{for } 0 \leq q \leq q_A^m(\theta^m) \\ P_A^{\lim}(q) + \text{ constant} & \text{for } q_A^m(\theta^m) \leq q \leq q_A^{\lim}(\breve{\theta}_B) \\ P_A^{cr}(q) + \text{ constant} & \text{for } q \geq q_A^{\lim}(\breve{\theta}_B), \end{cases}$$

where θ^m is implicitly defined by the condition $q_A^m(\theta^m) = q_A^{\lim}(\theta^m)$ and the constants guarantee the continuity of the price schedule.

Proof. See the Appendix. ■

In equilibrium, the highest type, i.e. $\theta = 1$, purchases both goods under common representation. In particular, it can be easily verified that he obtains the efficient quantities: $q_i^{cr}(1) = q_i^{fb}(1)$. As we shall see, this no-distortion-at-the-top property holds in all equilibria, with and without exclusive contracts.[27] It implies that in the absence of economies of scale the dominant firm's rivals would never be driven out of the market.

When exclusive contracts are permitted, the dominant firm can still offer non-exclusive contracts similar to those described above. In addition, it can also offer exclusive contracts. In this case, it can either undercut the competitive fringe, pricing just below c and selling an amount

$$q_c^e(\theta) = \arg\max_q \left[u(0, q, \theta) - cq\right] \tag{17}$$

of its product, or engage in monopoly pricing. Obviously, the latter strategy is always more profitable, but it may not be feasible because of the pressure from the competitive fringe.

While the equilibrium outcome is still unique, it can be supported by different price schedules. The reason for this is that when the dominant firm offers both exclusive and non-exclusive contracts, some contracts are destined not to be accepted and may therefore be specified arbitrarily, at least to some extent. Accordingly, the following proposition specifies only the relevant parts of the equilibrium price schedules.

[27] The property must hold even under monopoly, and is preserved under competition.

Proposition 2 *With exclusive contracts, in the competitive fringe model there is a unique equilibrium outcome where $P_B^E(q_B) = P_B^{NE}(q_B) = cq_B$ for all $q_B \geq 0$. Furthermore :*

- *when $c \leq \bar{c}$, firm A offers the price schedules:*

$$\begin{array}{ll} P_A^E(q) = cq & \text{for } 0 \leq q \leq q_c^e(\hat{\theta}) \\ P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A & \text{for } q \geq q_c^e(\hat{\theta}), \end{array}$$

  where $\hat{\theta}$, Φ_A and $\bar{c}$ are defined in the Appendix;

- *when $\bar{c} \leq c \leq \frac{1}{2}$, firm A offers the price schedules:*

$$P_A^E(q) = \begin{cases} cq & \text{for } 0 \leq q \leq q_A^m(\bar{\theta}) \\ P_A^m(q) + \text{ constant} & \text{for } q_A^m(\bar{\theta}) \leq q \leq q_A^m(\hat{\theta}) \end{cases}$$

  where $\bar{\theta} = 1 - c$ is the solution to $q_c^e(\bar{\theta}) = q_A^m(\bar{\theta})$, and the constant guarantees the continuity of the price schedule, and

$$P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A \qquad \text{for } q \geq q_A^m(\hat{\theta})$$

  where $\hat{\theta}$ and Φ_A are defined in the Appendix;

- *when $c \geq \frac{1}{2}$, firm A offers the price schedules:*

$$\begin{array}{ll} P_A^E(q) = P_A^m(q) & \text{for } 0 \leq q \leq q_A^m(\hat{\theta}) \\ P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A & \text{for } q \geq q_A^m(\hat{\theta}) \end{array}$$

  where $\hat{\theta}$ and Φ_A are as in the previous case.

Proof. See the Appendix. ■

Figure 1 shows, in the parameter space (γ, c), the various regions where different equilibrium patterns arise.

4.2 Discussion

We start our discussion from the extremes cases, in which the dominant firm's competitive advantage is either quite small or quite large. The role of exclusive contracts turns out to be quite different in these two cases. Intermediate cases are just a combination of the extreme ones and will be briefly considered later.

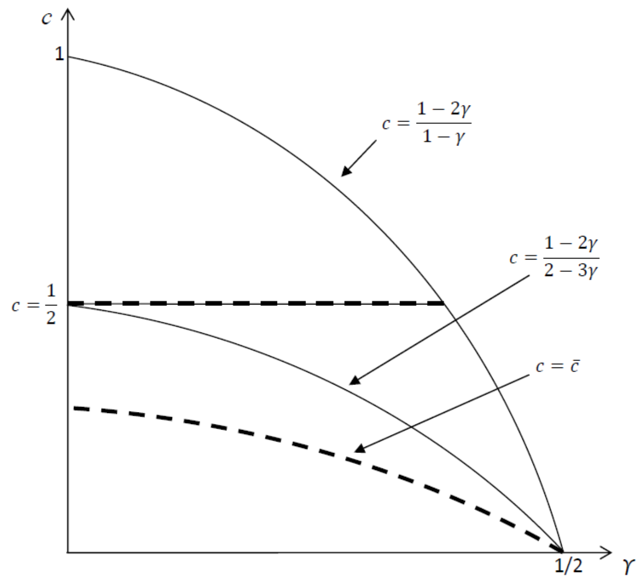

Figure 1: Critical thresholds for the competitive fringe model.

4.2.1 Small competitive advantage ($c \leq \bar{c}$)

When the dominant firm's competitive advantage is small, in the non-linear pricing equilibrium the marginal buyer purchases product B only. The intuition for this is simple. On the one hand, product A is less costly to produce, or of higher quality. On the other hand, product B is supplied competitively whereas the dominant firm exercises its market power in the market for product A. This latter effect prevails when the competitive advantage is small (the precise condition for this is, in fact, $c \leq \frac{1-2\gamma}{2-3\gamma}$).

In equilibrium, therefore, while low-demand types ($\theta \leq c$) do not purchase any good, intermediate demand types ($c \leq \theta \leq \breve{\theta}_A$) purchase an amount $q_c^e(\theta)$ of good B only. High demand types ($\breve{\theta}_A \leq \theta \leq 1$) purchase both goods, in which case the equilibrium quantities are the common representation quantities $q_A^{cr}(\theta)$ and $q_B^{cr}(\theta)$. The threshold $\breve{\theta}_A$ is the solution to $q_A^{cr}(\theta) = 0$. The non-linear pricing equilibrium quantities are depicted as the solid lines in Figure 2.

Starting from this equilibrium, consider what role exclusive contracts may play. The fact that some buyers purchase only product B is disappointing from the point of view of the dominant firm (as well as being inefficient from the social viewpoint). If only the dominant firm could replace the competitive fringe in the low-demand segment of the market, it would save the production cost cq_B and increase its profits by the same amount. However, this would require undercutting the competitive fringe's price c. With simple non-linear

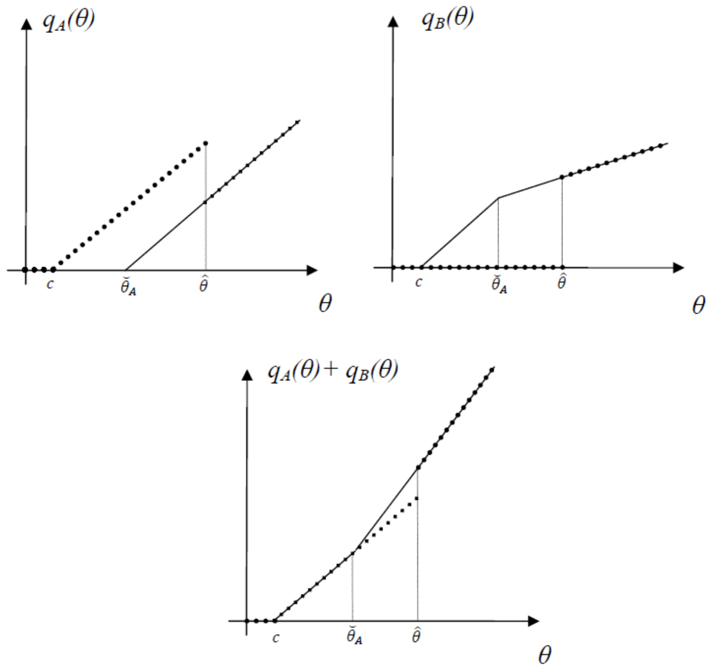

Figure 2: Equilibrium quantities in the competitive fringe model (small competitive advantage).

pricing, this would give high-demand buyers the opportunity of purchasing a certain amount of product A at a unit price slightly below c, in addition to that of purchasing product B at a unit price of c. This would be so attractive an option for them that the dominant firm's profits would actually decrease.

With exclusive contracts, however, the dominant firm can undercut the competitive fringe conditioning on exclusivity. This leaves the high-demand buyers' outside option substantially unchanged, as they can already purchase any amount of just one product, namely B, at a price only nominally higher than that charged by firm A. It follows that by imposing an exclusivity clause the dominant firm can replace the competitive fringe in the low-demand segment of the market, without losing any profit on the high-demand segment.

Notice that the dominant firm would ideally want to restrict its supply to low-demand buyers to the monopoly quantity. However, buyers still have the option of refusing the exclusivity clause and trading with the competitive fringe only. Therefore, under exclusivity the dominant firm must offer a quantity of at least $q_c^e(\theta)$. When c is sufficiently low, as we assume here, this is greater than the monopoly quantity, so the constraint is binding. [28]

[28] Technically speaking, the constraint $U(\theta) \geq U_A^R(\theta)$ in problem (5) is binding.

Under the exclusivity clause, then, the dominant firm sells $q_c^e(\theta)$ units of product A and makes a profit of c per unit of output. This is greater than the profit it would make, without imposing exclusive dealing, not only on types $\theta \leq \breve{\theta}_A$, but also on buyers of type θ just above $\breve{\theta}_A$, who would purchase a small amount of good A. Therefore, the dominant firm will impose exclusivity also on those buyers. As a result, their quantities will be distorted more heavily than in the non-linear pricing equilibrium, and they will also suffer a loss in terms of reduced variety. Only for θ sufficiently high will the dominant firm stop imposing the exclusivity clause. At that point, the equilibrium quantities must coincide with those of the non-linear pricing equilibrium.[29] The optimal switching point, $\hat{\theta}$, will therefore exceed $\breve{\theta}_A$. Notice that since $q_A^{cr}(\hat{\theta}) + q_B^{cr}(\hat{\theta}) > q_A^E(\hat{\theta})$, the denominator of left hand side of (7) is strictly positive. This implies $P_A^{NE}(q_A^{cr}(\hat{\theta})) > P_A^E(q_A^E(\hat{\theta}))$, so the dominant firm extracts more rents, at the margin, from buyers who accept non-exclusive contracts than from those who accept exclusive ones. The equilibrium quantities with exclusive contracts are depicted as the dotted lines in Figure 2.

Let us now compare the equilibrium with and without exclusive contracts. By revealed preferences, it is clear that the dominant firm's profit must increase. It is also clear that the competitive fringe will just break even anyway. The interesting question then is how exclusive contracts impact on buyers. To answer this question, remember that a buyer's equilibrium rent must be equal to the sum of the equilibrium quantities of all lower-demand buyers (equation (9)). The equilibrium aggregate quantities are depicted in panel c of Figure 2.

Clearly, low-demand buyers (i.e., types $\theta \leq \breve{\theta}_A$) are unaffected by exclusive contracts. However, all higher types are harmed. Some intermediate demand buyers (i.e., types $\breve{\theta}_A \leq \theta \leq \hat{\theta}$) suffer from both lower volumes and reduced variety. Higher demand buyers obtain the same quantities as in the non-linear pricing equilibrium. However, the dominant firm can now extract more rents from them, thanks to the fact that they have less attractive alternatives (since the quantities for lower types are distorted more heavily) than in the non-linear pricing equilibrium. The additional rents are extracted by adding a positive fixed fee to the non-linear pricing equilibrium tariff.[30] [31]

The effect of exclusive contracts on social welfare is ambiguous, though. On the one hand, aggregate equilibrium quantities, which are already inefficiently low, are further reduced. This is bad for efficiency. A countervailing effect, however, is the replacement of the competitive fringe with the more efficient dominant firm in the low-demand segment of the market. This reduces total production costs, with a positive production-efficiency effect. In general, either

[29] This follows from the fact that since $P_B^E(q_B) = P_B^{NE}(q_B)$, problem (4) coincides with problem (2).

[30] This follows from the fact that marginal prices cannot differ from those prevailing under non-linear pricing, as they must support the same quantities.

[31] When buyers are downstream firms, the extent to which their gains or losses are shifted onto final consumers may depend on how prices exactly change. Generally speaking, higher upstream prices will translate into higher downstream prices, so final consumers should also suffer from exclusive contracts when downstream firms do. However, if the only change is an increase in a fixed fee, there may be no effect on final consumers.

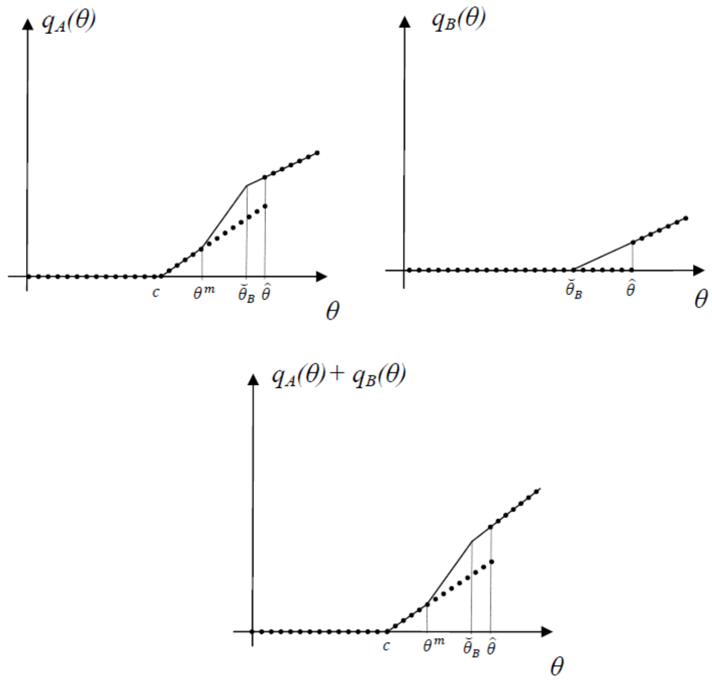

Figure 3: Equilibrium quantities in the competitive fringe model (large competitive advantage).

effect may prevail.

4.2.2 Large competitive advantage ($c \geq \frac{1}{2}$)

Next, consider the case in which the competitive advantage of the dominant firm is so large that (i) in the non-linear pricing equilibrium the marginal buyer purchases product A only and (ii) under exclusivity, the competitive fringe does not exert any competitive pressure on the dominant firm, which can therefore behave as an unconstrained monopolist. In the uniform-quadratic model, the exact condition for this is $c \geq \frac{1}{2}$.

In this case, in the non-linear pricing equilibrium the dominant firm can sell the monopoly quantity in the low-demand segment of the market (i.e. $\frac{1}{2} \leq \theta \leq \theta^m$), must sell the limit-pricing quantity to intermediate-demand buyers, and eventually accommodates, selling the common representation quantity, in the high-demand segment. The high demand segment comprises types $\breve{\theta}_B \leq \theta \leq 1$, where $\breve{\theta}_B$ is the highest θ such that $q_B^{cr}(\theta) = 0$. The non-linear pricing equilibrium quantities are depicted as the solid lines in Figure 3.

Here, the problem for the dominant firm is that the only way to foreclose its rivals is to engage in limit pricing. This is costly, as prices must be reduced relative to the monopoly solution. By imposing an exclusivity clause, however, the dominant firm can keep selling the profit-maximizing monopoly quantity $q_A^m(\theta)$ to a larger set of types, without having to resort to limit pricing. In other words, exclusive contracts now allow the dominant firm to foreclose its competitors more efficiently.

Of course, buyers have the option of refusing the exclusivity clause and trading with the competitive fringe only. When $c \geq \frac{1}{2}$, however, the dominant firm's competitive advantage is so large that this option does not really constrain its pricing strategy. This is so because the buyer obtains an information rent even under exclusive dealing. When $c \geq \frac{1}{2}$, this is greater than the rent that he could obtain by trading with the competitive fringe only, so exclusive dealing effectively shelters the dominant firm from competition. [32]

Since the foreclosure strategy *via* exclusivity clauses is more profitable than limit pricing, the dominant firm will want to use it more extensively. Therefore, we must have $\hat{\theta} > \breve{\theta}_B$. In other words, the marginal buyer who switches to a non-exclusive contract will be a higher type than the one who switches to the common representation quantities $q_A^{cr}(\theta)$ and $q_B^{cr}(\theta)$ under non-linear pricing. Eventually, however, no exclusivity clause will be imposed. The reason for this is that the distortion due to information asymmetry must optimally vanish at the top of the distribution of types. Intuitively, it is profitable for the dominant firm to allow high-demand types to purchase quantities that are nearly efficient and then extract the surplus by adding a fixed fee Φ_A. This fixed fee can be interpreted as a "tax" that the dominant firm levies to allow buyers to benefit from product variety. Thanks to this fixed fee, the dominant firm can extract more profits under common representation than under exclusivity, even if in the latter case it charges monopoly prices.

When no exclusivity clause is imposed, buyers will purchase exactly the same quantities as in the non-linear pricing equilibrium, i.e. $q_A^{cr}(\theta)$ and $q_B^{cr}(\theta)$. The equilibrium quantities with exclusive contracts are depicted as the dotted lines in Figure 3.

Comparing the equilibrium with and without exclusive contracts, it is clear that buyers are once again harmed. For some intermediate-demand buyers (i.e. $\theta^m \leq \theta \leq \breve{\theta}_B$), equilibrium quantities are reduced. Hence, so are the equilibrium rents for them and also all higher types. The latter obtain the same quantities as in the non-linear pricing equilibrium, but are left with lower rents as they now have less attractive alternatives. As in the previous case, the dominant firm extracts the extra rents by adding a positive fixed fee to the non-linear pricing equilibrium tariff.

Unlike the previous case, however, the impact of exclusive contracts on social welfare is now unambiguously negative. This follows immediately from the fact that equilibrium quantities decrease and there is no countervailing cost-saving effect, since the dominant firm no longer replaces the competitive fringe's output.

[32] Formally, the constraint $U(\theta) \geq U_A^R(\theta)$ in problem (5) is never binding.

4.2.3 Intermediate cases

We have seen that in the competitive fringe model exclusive contracts can serve two purposes: when c is small, they allow the dominant firm to replace the competitive fringe in the low-demand segment of the market; when c is large, they allow the dominant firm to foreclose its rivals more efficiently. When $\bar{c} < c < \frac{1}{2}$, exclusive contracts serve both purposes. For low-demand buyers, the dominant firm replaces the competitive fringe, pricing just below c and selling $q_c^e(\theta)$ units of product A. For higher demand buyers, the dominant firm uses exclusive contracts as a substitute for limit pricing.

In any case, buyers are always harmed by exclusive contracts. The welfare effects, however, depend on whether the $q_c^e(\theta)$ units of product A replace an equal amount of product B (which happens when $\bar{c} < c < \frac{1-2\gamma}{2-3\gamma}$), or the limit quantity $q_A^{\lim}(\theta)$ (which happens when $\frac{1-2\gamma}{2-3\gamma} < c < \frac{1}{2}$). In the former case, there is a positive cost-saving effect that may make the total welfare effect ambiguous. In the latter case, exclusive contracts unambiguously decrease social welfare, for the same reasons as in the large competitive advantage case.

5 Duopoly

In this section, we turn to the case in which there is only one supplier of product B (firm B), which will then have some market power. Unlike the competitive fringe, firm B can therefore respond actively to the dominant firm's attempt at foreclosing it. In particular, it may offer exclusive contracts in its turn, lower its non-exclusive prices, or both.

There are also two other important differences with the competitive fringe model. Firstly, since firm B may reap positive profits in equilibrium, it can be hurt by the dominant firm's exclusionary strategy. Secondly, the duopoly model allows to endogenize the choice of who offers exclusive contracts, and whose exclusive contracts are accepted in equilibrium.

5.1 Non-linear pricing equilibrium

The analysis of the duopoly model is more complex than that of the competitive fringe model, as the solution to a firm's pricing problem (discussed in section 3) does not yield directly the equilibrium, but only its best response to its rival's strategy. The non-linear pricing equilibrium must be found by adapting to the asymmetric case the solution procedure proposed by Martimort and Stole (2009) for the symmetric case (i.e. $c = 0$).[33] This is a non-trivial exercise; although in

[33] This is a "guess and check" procedure that starts from the conjecture that the equilibrium price schedules are quadratic and then verifies it by identifying the coefficients of the price schedules. It is important to stress that this procedure makes a guess on the structure of the equilibrium, but does not restrict firms to quadratic price schedules. The drawback of the guess and check procedure is that it cannot find equilibria in which the price schedules do not

this paper it serves mainly to provide a benchmark for comparison, it may also be of some interest in its own right.

Like in the competitive fringe model, the dominant firm can engage in monopoly or limit pricing, or it can accommodate its rival. The monopoly schedules are exactly the same as in the competitive fringe model. The limit pricing schedules are also similar, except that now the unit cost c is replaced by the marginal price that firm B charges for the first unit it offers, $P_B'^{cr}(0)$.

The new price and quantity schedules under common representation are[34]

$$P_A^{cr}(q) = \alpha q + c\frac{\alpha\gamma}{1-2\gamma}q - \frac{\alpha}{2}q^2; \quad P_B^{cr}(q) = \alpha q + c\left[1 - \frac{\alpha(1-\gamma)}{1-2\gamma}\right]q - \frac{\alpha}{2}q^2 \quad (18)$$

and

$$q_A^{cr}(\theta) = \frac{\theta-\alpha}{1-\alpha} + c\frac{\gamma}{1-2\gamma}; \qquad q_B^{cr}(\theta) = \frac{\theta-\alpha}{1-\alpha} - c\frac{1-\gamma}{1-2\gamma}, \qquad (19)$$

respectively, where $\alpha = \frac{1}{4}\left[3(1-\gamma) - \sqrt{1-2\gamma+9\gamma^2}\right] \geq 0$ is a decreasing function of γ that vanishes when $\gamma = \frac{1}{2}$. In the limiting case of perfect substitutes ($\gamma = \frac{1}{2}$), the solution therefore converges to the standard limit pricing equilibrium in which the dominant firm undercuts its rival pricing just below its unit cost c. As the degree of substitutability γ decreases, competition becomes less intense, prices increase, and quantities decrease. Equilibrium quantities are distorted downward; as it turns out, the size of the distortion is the same for both goods and is independent of c.[35] With independent goods ($\gamma = 0$), one obtains the monopoly solution. When $c = 0$, the Martimort and Stole solution returns.

In the non-linear pricing equilibrium, when demand is high buyers always obtain the common representation quantities. However, some low-demand buyers (i.e. types $\theta \leq \breve{\theta}_B$, where $\breve{\theta}_B$ is implicitly defined by $q_B^{cr}(\breve{\theta}_B) = 0$) must purchase product A only.[36] Whether these buyers are contested (in which case limit pricing applies) or captive (which would allow the dominant firm to engage in monopoly pricing) depends on the size of the competitive advantage c. The equilibrium is described in the following:

Proposition 3 *In the duopoly model, the following is a non-linear pricing equilibrium. Firm B offers the price schedule*

$$P_B(q) = P_B^{cr}(q)$$

and:

conform to the guess, if there are any. However, this is not a serious problem for our purposes. If there were multiple non-linear pricing equilibria, for each there would exist a corresponding equilibrium with exclusive contracts, with the same comparative statics properties.

[34] Notice that the guess and check procedure is used only to find the common representation price schedules. The monopoly and limit pricing schedules are pinned down uniquely.

[35] This property, however, does not play any special role in what follows.

[36] Unlike the competitive fringe model, the case in which the marginal buyer purchases product B only can no longer arise, as both firms now have market power.

- *when* $c \leq \tilde{c} \equiv \frac{1}{2}\frac{(1-2\alpha)(1-2\gamma)}{1-\alpha(1-\gamma)-2\gamma}$, *firm A offers the price schedule*

$$P_A(q) = \begin{cases} P^{\lim}(q) & \text{for } 0 \leq q \leq q_A^{\lim}(\breve{\theta}_B) \\ P_A^{cr}(q) + \ \textit{constant} & \text{for } q \geq q_A^{\lim}(\breve{\theta}_B) \end{cases}$$

  where $\breve{\theta}_B$ *is implicitly defined by the condition* $q_B^{cr}(\breve{\theta}_B) = 0$ *and the constant guarantees the continuity of the price schedule*;

- *when* $c \geq \tilde{c}$, *firm A offers the price schedule*

$$P_A(q) = \begin{cases} P^m(q) & \text{for } 0 \leq q \leq q_A^m(\theta^m) \\ P^{\lim}(q) + \ \textit{constant} & \text{for } q_A^m(\theta^m) \leq q \leq q_A^{\lim}(\breve{\theta}_B) \\ P^{cr}(q) + \ \textit{constant} & \text{for } q \geq q_A^{\lim}(\breve{\theta}_B), \end{cases}$$

  where θ^m *is implicitly defined by the condition* $q_A^m(\theta^m) = q_A^{\lim}(\theta^m)$ *and the constants guarantee the continuity of the price schedule.*

Proof. See the Appendix. ■

5.2 Exclusive contracts

With exclusive contracts, the nature of the equilibrium depends on the size of the dominant firm's competitive advantage even more profoundly than in previous cases. In particular, when c is small the equilibrium outcome is no longer unique, as the firms are faced with coordination problems that may have multiple solutions. In all equilibria, however, the effect of exclusive contracts is to reduce prices and profits. When the dominant firm's competitive advantage is large, by contrast, there is a unique equilibrium outcome, and exclusive contracts tend to be anti-competitive.

Because of these differences, it is convenient to deal with the two cases separately. Figure 4 shows the parameter values for which the different equilibrium patterns may arise.

5.2.1 Small asymmetry ($c \leq \breve{c}$)

When the dominant firm's competitive advantage is relatively small, firm B can compete for exclusives effectively. Competition for exclusives, though, is competition in utility space, where the firms' products effectively become homogeneous. The familiar Bertrand undercutting process then drives exclusive prices towards marginal costs.[37] In particular, there is always an equilibrium in which (i) exclusive prices are equal to marginal cost, with firm A just undercutting its rival, and (ii) non-exclusive prices are exorbitantly high, so that no buyer purchases both products. In this equilibrium, firm B's profit vanishes, and firm A's is reduced to a minimum.

[37] This is well known from models of one-stop shopping, where exclusive dealing is assumed from the outset: see, for instance, Armstrong and Vickers (2001).

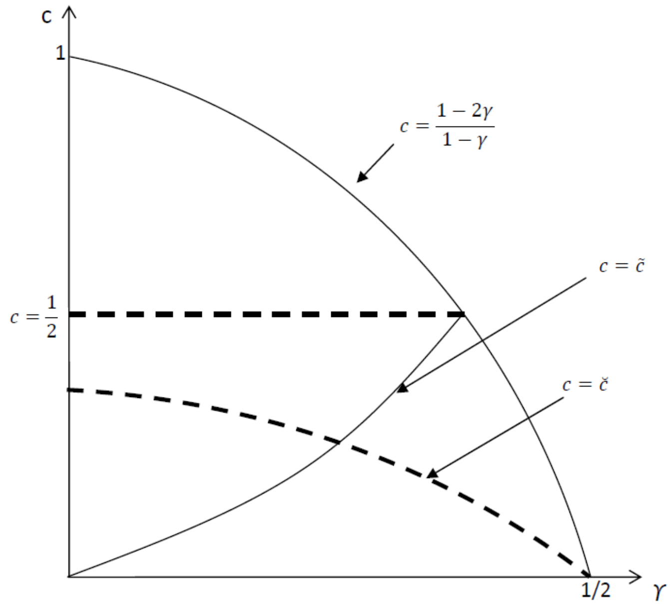

Figure 4: Critical thresholds for the duopoly model.

However, this equilibrium arises only in the case of very serious failures of coordination between the firms. Both firms can obtain larger profits, and buyers larger surpluses, but this requires some coordination among the firms. Specifically, they must lower their non-exclusive prices, and raise their exclusive prices, in coordinated fashion. The first move induces some buyers to purchase both products, allowing firms to extract the buyers' preference for variety; the second reduces the intensity of competition.

The degree to which firms can coordinate their strategies may vary, though. When firms lower their non-exclusive prices in order to extract the buyers' preference for variety, they may do so in an asymmetric fashion. If one firm reduces its prices excessively, however, buyers will purchase a disproportionate amount of its product, and so the benefit from variety that can be extracted is reduced. This is the first source of multiplicity of equilibria. Secondly, if buyers are purchasing both products, firms have no definite incentive to undercut one another's exclusive prices. Thus, exclusive prices may be raised above marginal cost. This, too, can be done to a variable extent, providing another source of multiple equilibria.

Since our aim is to show that when c is small exclusive contracts increase the intensity of competition, we focus on the equilibrium in which firms coordinate their strategies as best as they can. This is the "most cooperative" equilibrium, in which profits are largest.[38]

This equilibrium can be obtained as follows. Let $U^E(\theta)$ be the (type-dependent) reservation utility that buyers could obtain by choosing their most preferred exclusive contract. To extract the buyers' preference for variety, firms must introduce non-exclusive price schedules implicitly defined by the condition:

$$\max_{q_A, q_B} \left[u(q_A, q_B, \theta) - P_A^{NE}(q_A) - P_B^{NE}(q_B) \right] = U^E(\theta), \tag{20}$$

with a small tie-breaking discount if necessary. However, high-demand buyers will actually obtain more than $U^E(\theta)$ simply thanks to competition with non-exclusive contracts. Therefore, each of the non-exclusive price schedules comprises two branches: a lower branch that is intended for buyers who obtain exactly the reservation utility $U^E(\theta)$; and an upper branch that is intended for high-demand buyers who obtain strictly more. The latter must coincide with the equilibrium price schedules under non-linear pricing, except possibly for constant terms that serve to guarantee the continuity of equilibrium prices.[39]

Since we look for the equilibrium in which firms' profits are largest, we posit that firms maximize the rents that they extract from low-demand buyers. This requires maximization of the total surplus $u(q_A, q_B, \theta) - cq_B$, subject to the constraint that these buyers must obtain $U^E(\theta)$. Using the envelope theorem, the constraint can be rewritten as

$$q_A(\theta) + q_B(\theta) = q^E(\theta), \tag{21}$$

where $q^E(\theta)$ is the optimal quantity under exclusivity (to be determined). Generally speaking, the more efficient firm must produce more than the less efficient one. In particular, the problem of total-surplus maximization may have a corner solution in which some low-demand types must buy good A only. In this case, exclusive contracts must be accepted in equilibrium by those types, and so Bertrand competition in utility space implies that exclusive prices must fall to marginal costs. Therefore, $q_A(\theta)$ must coincide with $q_c^e(\theta)$ as defined by condition (17), and $q_B(\theta)$ must vanish.

When instead the total-surplus maximization problem has an interior solution, which is

$$q_A(\theta) = \frac{1}{2}q^E(\theta) + \frac{c}{2(1-2\gamma)}; \quad q_B(\theta) = \frac{1}{2}q^E(\theta) - \frac{c}{2(1-2\gamma)}, \tag{22}$$

buyers purchase both products. In this case exclusive contracts are not accepted, and hence there may be room for coordinating the exclusive prices. Thus, $q^E(\theta)$ depends on what exclusive prices are sustainable in the most cooperative

[38] In Calzolari and Denicolò (2013) we provide a complete characterization of the set of equilibria in the symmetric case. The structure of the set of equilibria does not change when asymmetry is small.

[39] This follows from the property of "type consistency" discussed in Calzolari and Denicolò (2013).

equilibrium.[40] Let us denote by an upper bar the highest exclusive prices that are consistent with playing the game non-cooperatively (i.e. the exclusive prices in the "most cooperative" equilibrium). To find them, we can assume, with no loss of generality, that both firms offer the same exclusive price schedule $\bar{P}^E(q)$.[41] By construction, low-type buyers must be just indifferent between an exclusive contract and a non-exclusive one (equation (20)). Thus, any arbitrarily small discount would trigger a switch to an exclusive contract. In equilibrium, no such deviation can be profitable. This implies the following no undercutting conditions:

$$\begin{aligned} P^E(q^E(\theta)) &\leq P_A^{NE}(q_A^{cr}(\theta)); \\ P^E(q^E(\theta)) - cq^E(\theta) &\leq P_B^{NE}(q_B^{cr}(\theta)) - cq_B^{cr}(\theta), \end{aligned} \tag{23}$$

which in the most cooperative equilibrium must hold as equalities.

Denote by $\bar{q}^E(\theta)$ the optimal quantity associated with the exclusive prices $\bar{P}^E(q)$, and by $\bar{q}_i^{cr}(\theta)$ the values of $q_i(\theta)$ given by (22) when $q^E(\theta) = \bar{q}^E(\theta)$. Rewrite (20) as

$$u(\bar{q}_A^{cr}(\theta), \bar{q}_B^{cr}(\theta), \theta) - \bar{P}_A^{NE}(\bar{q}_A^{cr}(\theta)) - \bar{P}_B^{NE}(\bar{q}_B^{cr}(\theta)) = u(0, \bar{q}^E(\theta), \theta) - \bar{P}^E\left(\bar{q}^E(\theta)\right)$$

and use the no-undercutting conditions (23) to get

$$\bar{P}^E\left(q^E(\theta)\right) = \left[u(\bar{q}_A^{cr}(\theta), \bar{q}_B^{cr}(\theta), \theta) - u(0, \bar{q}^E(\theta), \theta)\right] + c\left[\bar{q}^E(\theta) - \bar{q}_B^{cr}(\theta)\right].$$

From (21), the term inside the first square brackets on right-hand side can be interpreted as the preference for variety, while the term inside the second square bracket is the cost saving. Using (22), we finally get

$$\bar{P}^E(q) = \frac{c^2}{2(1-2\gamma)} + \frac{c}{2}q + \frac{1-2\gamma}{4}q^2. \tag{24}$$

The corresponding quantity is

$$\bar{q}^E(\theta) = \frac{2\theta - c}{3 - 4\gamma}. \tag{25}$$

The non-exclusive prices that generate the same net utility and maximize the preference for variety that the firms can extract, thus supporting the quantities $\bar{q}_A^{cr}(\theta)$ and $\bar{q}_B^{cr}(\theta)$, are

$$\bar{P}_A^{cr}(q_A) = -cq + (1-2\gamma)q^2 + cq_c^e(\hat{\theta}); \quad \bar{P}_B^{cr}(q_B) = 2cq + (1-2\gamma)q^2, \tag{26}$$

[40] Notice that while exclusive contracts are not accepted, they do affect the equilibrium outcome. The less aggressively firms bid for exclusivity, the greater the payments firms can obtain for non-exclusive contracts.

[41] The proof that this does not entail any loss of generality is by contradiction. Suppose to the contrary that one firm offered more attractive exclusive contracts than its rival. Since these contracts are not accepted in equilibrium, the firm could increase its exclusive prices without losing any profits on its exclusive contracts. In fact, the buyers' reservation utility would decrease, allowing both firms to increase their profits from non-exclusive contracts.

where $\hat{\theta}$ is now the solution to $q_c^e(\hat{\theta}) = \bar{q}_A^{cr}(\hat{\theta})$ and the constant term in $\bar{P}_A^{cr}(q_A)$ guarantees a smooth-pasting condition from exclusive to non-exclusive contracts.

We are now ready to provide the characterization of the most cooperative equilibrium.

Proposition 4 *Suppose that* $c \leq \breve{c} \equiv \frac{2(1-2\gamma)}{5(1-\gamma)+\sqrt{1-2\gamma+9\gamma^2}}$. *Then, in the duopoly model the most cooperative equilibrium with exclusive contracts is as follows.*[42] *Both firms offer the exclusive price schedules*

$$P_A^E(q) = P_B^E(q) = \begin{cases} cq & \text{for } q \leq q_c^e(\hat{\theta}) \\ \bar{P}^E(q) & \text{for } q > q_c^e(\hat{\theta}) \end{cases}$$

with firm A slightly undercutting firm B, though. Furthermore:

$$P_A^{NE}(q) = \begin{cases} \bar{P}_A^{cr}(q) & \text{for } q \leq \bar{q}_A^{cr}(\bar{\theta}) \\ P_A^{cr}(q) + \text{constant} & \text{for } q \geq \bar{q}_A^{cr}(\bar{\theta}) \end{cases}$$
$$P_B^{NE}(q) = \begin{cases} \bar{P}_B^{cr}(q) & \text{for } q \leq \bar{q}_B^{cr}(\bar{\theta}) \\ P_B^{cr}(q) + \text{constant} & \text{for } q \geq \bar{q}_B^{cr}(\bar{\theta}) \end{cases}$$

where $\hat{\theta}$ is the solution to $q_c^e(\hat{\theta}) = \bar{q}_A^{cr}(\hat{\theta})$ and $\bar{\theta}$ the solution to $\bar{q}_A^{cr}(\bar{\theta}) = q_A^{cr}(\bar{\theta})$ (and to $\bar{q}_B^{cr}(\bar{\theta}) = q_B^{cr}(\bar{\theta})$), and the constants guarantee the continuity of the price schedules.

Notice that the only exclusive contracts that are accepted in equilibrium are those offered by the dominant firm. While firm B offers exclusive contracts as well, these contracts are not accepted by any buyer. The equilibrium is thus broadly consistent with the observation that only the dominant firm uses exclusive contracts: for one thing, contracts that are offered but not accepted are difficult to observe; for another thing, to sustain the equilibrium it suffices, in practice, that firm B just stands ready to offer its prescribed exclusive contracts.

We can now compare the equilibrium with and without exclusive contracts. In the non-linear pricing equilibrium, low-demand buyers purchase only product A, while high-demand buyers buy both products. The non-linear pricing equilibrium quantities are depicted as the solid lines in Figure 5.

In the most cooperative equilibrium with exclusive contracts, buyers are divided into four groups. For $\theta \leq c$, buyers do not buy any product; for $c < \theta \leq \hat{\theta}$, buyers purchase $q_c^e(\theta)$ units of product A only, under an exclusivity clause and at a price just below c; for $\hat{\theta} < \theta \leq \bar{\theta}$, buyers purchase $\bar{q}_A^{cr}(\theta)$ units of good A and $\bar{q}_B^{cr}(\theta)$ units of good B, and so obtain the same net surplus as if they accepted an exclusive contract; finally, for $\bar{\theta} < \theta \leq 1$, buyers buy $q_A^{cr}(\theta)$ units of good A and $q_B^{cr}(\theta)$ units of good B, and strictly prefer their non-exclusive contract to any exclusive one. These equilibrium quantities are depicted as the dotted lines in Figure 5.

[42] If there were different equilibrium price schedules under common representation, $P_i^{cr}(q)$, for each of them there would be corresponding equilibria with exclusive contracts. This remark applies also to Proposition 5 below.

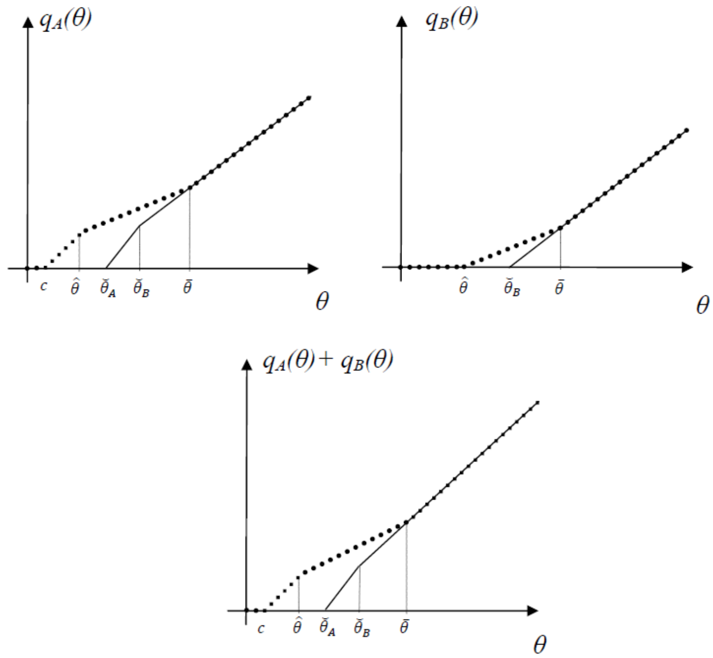

Figure 5: Equilibrium quantities in the duopoly model (small competitive advantage).

Clearly, exclusive contracts are unambiguously pro-competitive. To see why, notice that equilibrium quantities are larger than under non-linear pricing, and are everywhere closer to the first best. Since the social surplus (i.e., the sum of buyers' surplus and firms' profits) is concave in q_A and q_B, it is clear that exclusive contracts increase social welfare.

In particular, buyers now benefit from exclusive contracts. Low-demand buyers ($\breve{\theta}_A < \theta < \bar{\theta}$) increase their purchases. High-demand buyers ($\theta \geq \bar{\theta}$) purchase the same quantities as in the non-linear pricing equilibrium, but they too are better off as they now have more attractive alternatives. The benefit now is obtained *via* fixed subsidies that in the equilibrium with exclusive contracts are added to the non-linear pricing equilibrium price schedules.

As for firms, prices are lower than in the non-linear pricing equilibrium (to support higher quantities). Since prices are already lower than under monopoly, the fact that they are further reduced means that exclusive contracts decrease firms' profits. Thus, firms are caught in a prisoner's dilemma: each has a unilateral incentive to offer exclusive contracts if it can, but both would gain if exclusive contracts were prohibited.

All of these conclusions agree with those obtained in Calzolari and Denicolò (2013) for the symmetric case. This is natural, as the asymmetry is small.

5.2.2 Large asymmetry ($c > \check{c}$)

When the dominant firm's competitive advantage is relatively large, things are different. The dominant firm can engage in monopoly pricing on a segment of the market. If c is not too large, however, firm B can still compete for exclusives, and in any case it can resist being foreclosed by reducing its non-exclusive prices.

The competition for exclusives has now a unique outcome. When $c > \check{c}$, there is no longer any room for extracting the buyers' preference for variety.[43] The exclusive contracts offered by the dominant firm are therefore accepted. This implies that there is no scope for coordinating the exclusive prices. As a result, firm B will always price at cost, whereas firm A will either undercut firm B or engage in monopoly pricing – whichever leads to lower prices.

Next, let us focus on the non-exclusive prices. Even though it cannot win the competition for exclusives, firm B can at least try to induce high-demand buyers, who value product variety more, to reject the exclusive contracts offered by firm A and buy both products. To get buyers to purchase both products, firm B must be willing to lower its non-exclusive prices, as compared to the non-linear pricing equilibrium. Firm A, by contrast, would prefer to serve buyers under an exclusivity clause. However, it can now raise its non-exclusive prices – again, taking the non-linear pricing equilibrium as a benchmark – which directly increases its profit and also makes the exclusive deals more appealing, comparatively speaking.

In other words, both firms try to affect the critical buyer $\hat{\theta}$ who is just indifferent between accepting exclusive and non-exclusive contracts: firm A tries to push $\hat{\theta}$ up, firm B to pull it down. In order to do so, firms modify the constant terms of their non-exclusive price schedules.[44]

Consider, then, the optimal choice of these constant terms, which we denote by Φ_A and Φ_B. Each firm maximizes its profit. This is

$$\int_{\theta_{\min}}^{\hat{\theta}} P_A^m(q_A^m(\theta))d\theta + \int_{\hat{\theta}}^{\theta_{\max}} P_A^{NE}(q_A^{NE}(\theta))d\theta$$

for firm A,[45] and

$$\int_{\hat{\theta}}^{\theta_{\max}} \left[P_B^{NE}(q_B^{NE}(\theta)) - cq_B^{NE}(\theta)\right] d\theta$$

for firm B. The critical buyer $\hat{\theta}$ is the one who is just indifferent between exclusive

[43] Intuitively, the dominant firm's competitive advantage is so large that if B's output was positive, the benefits from greater product variety would be offset by the increase in total production costs.

[44] The reason why they use only those constant terms as their strategic weapons at this stage is that marginal prices are pinned down by the fact that once buyers start purchasing both goods, the equilibrium quantities must be given by (19).

[45] This formula accounts for the fact that under an exclusivity clause the dominant firm now enforces the monopoly solution.

and non-exclusive contracts:

$$u(q_A^m(\hat{\theta}), 0, \hat{\theta}) - P_A^m(q_A^m(\hat{\theta})) = u(q_A^{NE}(\hat{\theta}), q_B^{NE}(\hat{\theta}), \hat{\theta}) - P_A^{NE}(q_A^{NE}(\hat{\theta})) - P_B^{NE}(q_B^{NE}(\hat{\theta}). \tag{27}$$

It follows that the equilibrium conditions are:

$$\frac{P_A^{NE}(q_A^{NE}(\hat{\theta})) - P_A^m(q_A^m(\hat{\theta}))}{q_A^{NE}(\hat{\theta}) + q_B^{NE}(\hat{\theta}) - q_A^m(\hat{\theta})} = 1 - \hat{\theta}, \tag{28}$$

$$\frac{P_B^{NE}(q_B^{NE}(\hat{\theta})) - cq_B^{NE}(\hat{\theta})}{q_A^{NE}(\hat{\theta}) + q_B^{NE}(\hat{\theta}) - q_A^m(\hat{\theta})} = 1 - \hat{\theta}. \tag{29}$$

The first condition implies that the dominant firm charges a fixed fee: $\Phi_A > 0$. The proof is very simple: since the right-hand side and the denominator of the left-hand side are both positive, it must be $P_A^{NE}(q_A^{NE}(\hat{\theta})) > P_A^m(q_A^m(\hat{\theta}))$. However, we know that $q_A^m(\hat{\theta}) > q_A^{NE}(\hat{\theta})$ as the goods are substitutes, and on the other hand in the absence of a fixed fee we would have $P_A^{NE}(q) < P_A^m(q)$ as competition reduces the equilibrium prices. Therefore, the inequality $P_A^{NE}(q_A^{NE}(\hat{\theta})) > P_A^m(q_A^m(\hat{\theta}))$ can hold only if the non-exclusive price schedule involves also a fixed fee.

Likewise, the second equilibrium condition implies that less efficient firm must charge a fixed subsidy (the argument is similar). Intuitively, firm B trades off market share and profitability.

Given that $c > \check{c}$, we still must distinguish between two sub-cases. If $c < \frac{1}{2}$, firm B exerts a competitive pressure on firm A's exclusive prices. This implies that for small volumes, firm A must just undercut firm B; only for higher volumes (and hence higher demand buyers) can it engage in monopoly pricing. When $c \geq \frac{1}{2}$, by contrast, the information rent under exclusive dealing and monopoly pricing is greater than the rent that the buyer could obtain by trading with firm B only. Thus, exclusive dealing effectively shelters the dominant firm from competition.

Proposition 5 *Suppose that $c > \check{c}$.*

- *When $\check{c} < c < \frac{1}{2}$ the two firms offer the following exclusive price schedules*

$$P_B^E(q) = cq$$
$$P_A^E(q) = \begin{cases} cq & \text{for } q \leq q_c^e(\theta^m) \\ P_A^m(q) + \text{ constant} & \text{for } q > q_c^e(\theta^m) \end{cases}$$

where θ^m is such that $q_c^e(\theta^m) = q_A^m(\theta^m)$ and the constant guarantees the continuity of the price schedule, and the following non-exclusive price schedules

$$P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A \qquad \text{for } q \geq q_A^{cr}(\hat{\theta})$$
$$P_B^{NE}(q) = P_B^{cr}(q) + \Phi_B \qquad \text{for } q \geq q_B^{cr}(\hat{\theta})$$

where $\hat{\theta}$, Φ_A and Φ_B are the solution to the system (27)-(29)

- *When $c \geq \frac{1}{2}$ the two firms offer the following price schedules*

$$\begin{aligned} P_B^E(q) = &\quad cq \\ P_A^E(q) = &\quad P^m(q) \end{aligned}$$

$$\begin{aligned} P_A^{NL}(q) = &\quad P_A^{cr}(q) + \Phi_A \quad \text{for } q \geq q_A^{cr}(\hat{\theta}) \\ P_B^{NL}(q) = &\quad P_B^{cr}(q) + \Phi_B \quad \text{for } q \geq q_B^{cr}(\hat{\theta}) \end{aligned}$$

where $\hat{\theta}$, Φ_A and Φ_B are defined as in the previous case.

The role of exclusive contracts in this case is similar to the competitive fringe model: by imposing an exclusivity clause, the dominant firm can keep selling the profit-maximizing monopoly quantity $q_A^m(\theta)$ without having to resort to limit pricing. Thus, exclusive contracts allow the dominant firm to foreclose its competitor more efficiently. Like in the small asymmetry case, however, the only exclusive contracts that are accepted in equilibrium are those offered by the dominant firm.

Let us now compare the equilibrium with and without exclusive contracts. We start from the case $c \geq \frac{1}{2}$. In this case, with simple non-linear pricing the dominant firm serves some low-demand buyers under monopoly but then must engage in limit pricing (the equilibrium quantities are the solid lines in Figure 6). With exclusive contracts, by contrast, the dominant firm can engage in monopoly pricing straightaway. Since the foreclosure strategy *via* exclusivity clauses is more profitable than limit pricing, the dominant firm will also want to use it more extensively. Therefore, we must have $\hat{\theta} > \breve{\theta}_B$. In other words, the marginal buyer who switches to a non-exclusive contract will be a higher type than the one who switches to the common representation quantities $q_A^{cr}(\theta)$ and $q_B^{cr}(\theta)$ under non-linear pricing. The equilibrium quantities under exclusive contracts are depicted as the dotted lines in Figure 6.

Comparing the equilibrium with and without exclusive contracts, one finds that exclusive contracts are now unambiguously anti-competitive, exactly as in the corresponding case of the competitive fringe model. Buyers are harmed, as the equilibrium quantities for some intermediate demand buyers are decreased. High-demand buyers obtain the same quantities as in the non-linear pricing equilibrium, but they are left with lower rents as they now have less attractive alternatives.

The main difference with the competitive fringe model is that now the less efficient firm is also harmed, both in terms of market share and profit margins. The dominant firm gains, as exclusive contracts have no competition-enhancing effect. Remarkably, however, the dominant firm's gain does not rest on firm B being driven out of the market.

Social welfare decreases, since equilibrium quantities, which are already distorted downward in the non-linear pricing equilibrium, further decrease when exclusive contracts are permitted. Thus, the dominant firm's gain from exclusive contracts does not compensate for the losses that it inflicts to its customers and competitors.

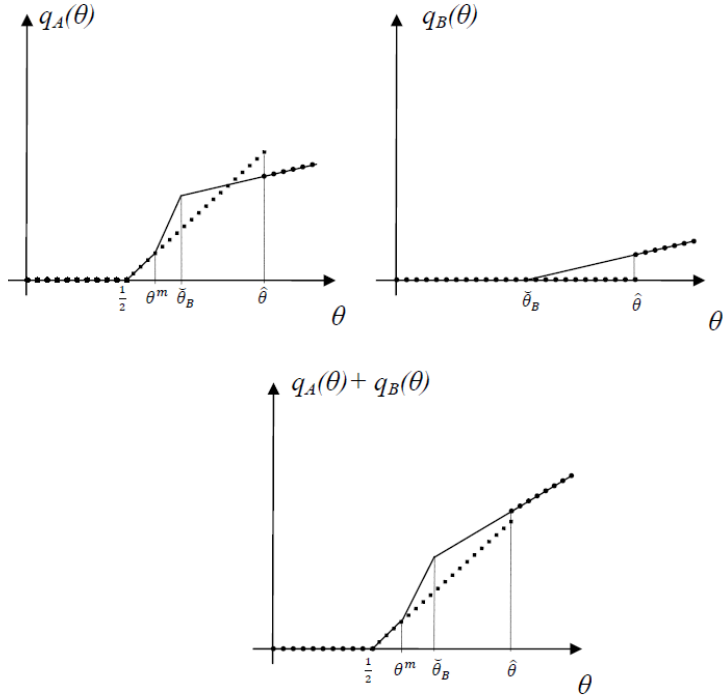

Figure 6: Equilibrium quantities in the duopoly model (large competitive advantage).

When $\breve{c} < c < \frac{1}{2}$, the equilibrium is similar to the case $c \geq \frac{1}{2}$, except for the fact that now low-demand types are faced with exclusive prices just below c and so buy $q_c^e(\theta)$. This is higher than the limit pricing quantity that they would have purchased under non-linear pricing; furthermore, less buyers are excluded under exclusive contracts. Therefore, low demand buyers benefit from exclusive contracts. High-demand buyers, by contrast, are harmed, for the same reasons as above. The dominant firm gains and its rival loses, but the total effect on social welfare is now ambiguous. When c is close to $\breve{c}$, the pro-competitive effect on the low-demand segment of the market must prevail. When c is close to $\frac{1}{2}$, by contrast, what prevails is the anti-competitive effect on the high-demand segment.

6 Conclusion

In this paper, we have developed a modern, consistent version of the pre-Chicago theory of exclusive dealing. The theory maintains that a dominant firm may find it profitable to use exclusive contracts just to increase its market share, without necessarily driving its rivals out of the market or impeding entry. This theory is valid under two assumptions. First, firms are imperfectly informed about demand. Second, the dominant firm has a competitive advantage over its rivals, in terms of lower cost, higher demand, or a combination of the two.

Not only are these assumptions realistic, but the model's predictions are also consistent with the facts of many antitrust cases. In addition to a dominant firm that controls a substantial share of the market and has entered into some kind of exclusive arrangement with its customers, these often involve one or more smaller competitors, which have been active in the industry for some time and in principle could themselves use exclusive contracts, but apparently have not. Existing theories have found it difficult to explain this recurrent situation without making *ad hoc* assumptions. Ours, by contrast, can reproduce these stylized facts naturally.

Our theory offers new insights for competition policy. Since exclusive contracts may be either pro or anti-competitive, the theory does not call for a radical change in the current policy, which is based on the rule of reason. However, it may suggest that different factors should be considered for the purposes of antitrust evaluation.

In our model, the key factor is the size of the dominant firm's competitive advantage. This determines whether the dominant firm's rival can compete for exclusives effectively or not. If it can, exclusive contracts tend to be pro-competitive, reducing prices and profits and benefiting buyers. If it cannot, exclusive contracts are anti-competitive. The dominant firm gains, but both its rival and buyers are harmed, and social welfare goes down.

While the size of the dominant firm's competitive advantage can not be observed precisely, it is correlated with two variables that often can. One is the dominant firm's market share, the other is the fraction of the market foreclosed. When both these two variables are large, an anticompetitive effect is more likely.

Factors other than the dominant firm's competitive advantage, which are often emphasized by antitrust authorities and the courts, turn out to be less important in our analysis. For example, the length of exclusive contracts is irrelevant, since contracts are not used for commitment purposes. Another factor that should be reconsidered is economies of scale. Their existence would be crucial if a negative impact of exclusive contracts on competition could only arise because rivals are driven, or kept, out of the market. However, our analysis clarifies that exclusive contracts may have anti-competitive effects even if the dominant firm's rivals are and stay active. This is so because exclusive contracts may be profitable even in the short run; in other words, without entailing a sacrifice for the dominant firm. This implies that no proof of eviction and recoupment may be needed.

If, however, economies of scale do matter, anti-competitive effects may arise under even broader circumstances than when they do not. In our duopoly model, exclusive contracts always reduce the less efficient firm's profit. Thus, they have the potential to foreclose a rival, or deter its entry, even if the dominant firm's competitive advantage is small. In this case, the fraction of the market that is foreclosed may be small as well. However, the effects of a small segment of the market being foreclosed may reverberate throughout the entire market thus having a significant impact on the less efficient firm's profits. [46]

[46] This observation may cast doubts on policies that provide a safe harbour when exclusive dealing arrangements foreclose 30% of the market or less, such as those adopted in the US Department of Justice's guidelines. Such an high threshold reflects the view that exclusive contracts can only be anti-competitive if they deprive a rival of economies of scale. From that perspective, it may seem reasonable that if 70% of the market remains contestable, it should suffice for a rival to prosper. But we have shown that the anti-competitive effects of exclusive contracts may extend well beyond the segment of the market that is actually foreclosed.

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Appendix

This Appendix contains the proofs of Propositions 1-5.

Proof of Proposition 1. Because the proof is based on the use of direct mechanisms, it is convenient to report the equilibrium quantities first. They are:

- when $c \leq \frac{1-2\gamma}{2-3\gamma}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_A \\ q_A^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_A \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q_c^{e}(\theta) & \text{for } c \leq \theta \leq \breve{\theta}_A \\ q_B^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_A, \end{cases}$$

- when $\frac{1-2\gamma}{2-3\gamma} \leq c \leq \frac{1}{2}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q_A^{\lim}(\theta) & \text{for } c \leq \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_B \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_B, \end{cases}$$

- when $c \geq \frac{1}{2}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \frac{1}{2} \\ q_A^{m}(\theta) & \text{for } \frac{1}{2} \leq \theta \leq \theta^m \\ q_A^{\lim}(\theta) & \text{for } \theta^m \leq \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_B \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_B, \end{cases}$$

where the thresholds $\breve{\theta}_B$, θ^m and $\breve{\theta}_A$ are calculated explicitly below. The marginal buyer is $\theta = c$ when $c \leq \frac{1}{2}$, and $\theta = \frac{1}{2}$ when $c \geq \frac{1}{2}$.

Obviously, the competitive fringe will always price at cost: $P_B(q_B) = cq_B$. To prove the proposition, it then suffices to show that the dominant firm's equilibrium pricing strategy is indeed optimal. To do so, we shall focus on direct mechanisms and hence find the optimal quantity $q_A(\theta)$, showing that it coincides with the equilibrium quantity reported above. It is then straightforward to conclude that the price schedules that support these quantities, which are the equilibrium price schedules, are indeed optimal.

To begin with, we calculate the indirect utility function $v(q_A, \theta)$ when $P_B(q_B) = cq_B$. This is piecewise quadratic, with two branches corresponding to the cases in which the quantity

$$\begin{aligned} \tilde{q}_B(q_A, \theta) &= \arg\max_{q_B \geq 0} \left[u\left(q_A, q_B, \theta\right) - cq_B\right] \\ &= \max\left[0, \frac{\theta - c - \gamma q_A}{1-\gamma}\right] \end{aligned}$$

is 0 or is strictly positive, and a kink between the two branches. That is:

$$v(q_A,\theta) = \begin{cases} \theta q_A - \dfrac{1-\gamma}{2} q_A^2 & \text{if } \tilde{q}_B(q_A,\theta) = 0 \text{ or, equivalently, } q_A \geq q_A^{\lim}(\theta) \\ A_0 + A_1 q_A + A_2 q_A^2 & \text{if } \tilde{q}_B(q_A,\theta) > 0 \text{ or, equivalently, } q_A < q_A^{\lim}(\theta), \end{cases}$$

where

$$A_0 = \frac{(\theta - c)^2}{2(1-\gamma)}, \quad A_1 = \frac{c\gamma + \theta(1-2\gamma)}{1-\gamma}, \quad \text{and } A_2 = -\frac{1-2\gamma}{2(1-\gamma)}.$$

On both branches, the coefficients of the quadratic terms are negative. Furthermore,

$$\left.\frac{\partial^2 v(q_A,\theta)}{\partial q_A^2}\right|_{q_A \leq q_A^{\lim}(\theta)} = -\frac{1-2\gamma}{1-\gamma} \geq \left.\frac{\partial^2 v(q_A,\theta)}{\partial q_A^2}\right|_{q_A > q_A^{\lim}(\theta)} = -(1-\gamma),$$

so the function v is globally concave.

It can also be easily checked that the single-crossing condition $v_{\theta q_A}(q_A,\theta) \geq 0$ is satisfied since:

$$v_{\theta q_A}(q_A,\theta) = \begin{cases} 1 & \text{if } q_A \geq q_A^{\lim}(\theta) \\ \frac{1-2\gamma}{1-\gamma} & \text{if } q_A < q_A^{\lim}(\theta). \end{cases}$$

The single-crossing condition guarantees that the participation constraint binds only for the marginal buyer, whom we indicate here as $\tilde{\theta}$, so that firm A's optimization program (2) becomes

$$\begin{aligned} &\max_{q_A(\theta)} \int_{\tilde{\theta}}^{1} \left[v(q_A(\theta),\theta) - U(\theta)\right] d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= v_\theta(q_A,\theta) \\ U(\tilde{\theta}) &= 0 \end{aligned}$$

By a standard integration by parts, the problem reduces to finding the function $q_A(\theta)$ that pointwise maximizes the indirect virtual surplus:

$$s(q_A,\theta) = v(q_A,\theta) - (1-\theta)v_\theta(q_A,\theta).$$

Like the indirect utility function, the indirect virtual surplus is a piecewise quadratic function, with two branches and a kink at $q_A = q_A^{\lim}(\theta)$. Since the additional term $(1-\theta)v_\theta(q_A,\theta)$ is linear in q_A and $v(q_A,\theta)$ is globally concave, $s(q_A,\theta)$ is also globally concave in q_A.

Generally speaking, for any θ the maximum can occur in either one of the two quadratic branches, or at the kink. Let

$$\begin{aligned} q^m(\theta) &= \arg\max_{q_A} \left[\theta q_A - \frac{1-\gamma}{2} q_A^2\right], \\ q_A^{cr}(\theta) &= \arg\max_{q_A} \left[A_0 + A_1 q_A + A_2 q_A^2\right], \end{aligned}$$

and recall that the kink $q_A^{\lim}(\theta)$ is implicitly defined by the condition $\theta - c - \gamma q_A^{\lim} = 0$. It is easy to verify that $q^m(\theta)$, $q_A^{\lim}(\theta)$ and $q_A^{cr}(\theta)$ are given precisely by the expressions (12), (14) and (16), respectively. We can then conclude that $q_A(\theta) = q_A^m(\theta)$ if the maximum is achieved on the upper branch, $q_A(\theta) = q_A^{cr}(\theta)$ if the maximum is achieved on the lower branch, and $q_A(\theta) = q_A^{\lim}(\theta)$ if the maximum is achieved at the kink.

Global concavity of $s(q_A, \theta)$ implies that if $q_A^m(\theta) > q_A^{\lim}(\theta)$, then $s(q_A, \theta)$ is increasing at the kink and the maximum is achieved at $q_A^m(\theta)$. If instead $q_A^m(\theta) < q_A^{\lim}(\theta)$, then $s(q_A, \theta)$ is decreasing to the right of the kink, and one must further distinguish between two cases. If $q_A^{cr}(\theta) > q_A^{\lim}(\theta)$, then $s(q_A, \theta)$ is increasing to the left of the kink and so the maximum is achieved at the kink $q_A^{\lim}(\theta)$. If instead $q_A^{cr}(\theta) < q_A^{\lim}(\theta)$, the maximum is achieved to the left of the kink.

It remains to find out when each type of solution applies. Provided that $\gamma < \frac{1}{3}$, the condition $q_A^m(\theta) > q_A^{\lim}(\theta)$ is equivalent to

$$\theta < \frac{c(1-\gamma) - \gamma}{1 - 3\gamma} (\equiv \theta^m).$$

(If $\gamma > \frac{1}{3}$, the condition $q_A^m(\theta) > q_A^{\lim}(\theta)$ is never met provided that (10) holds.) Since $q_A^m(\theta)$ is positive only for $\theta > \frac{1}{2}$, the monopoly solution is obtained if and only if the interval $\frac{1}{2} \leq \theta \leq \theta^m$ is not empty. This is true if only if $\theta^m > \frac{1}{2}$, which is equivalent to $c > \frac{1}{2}$. In this case, then, we have $q_A(\theta) = q_A^m(\theta)$ for $\frac{1}{2} \leq \theta \leq \theta^m$. Of course, the corresponding equilibrium quantity of good B must be nil.

Now suppose that $\theta > \theta^m$, so that $q_A^m(\theta) < q_A^{\lim}(\theta)$. In this case, the solution depends on whether $q_A^{cr}(\theta)$ is larger or smaller than $q_A^{\lim}(\theta)$. The limit pricing solution can emerge only if $q_A^{cr}(\theta) > q_A^{\lim}(\theta)$. The condition $q_A^{cr}(\theta) > q_A^{\lim}(\theta)$ is equivalent to $q_B^{cr}(\theta) < 0$, or

$$\theta < c\frac{(1-\gamma)^2}{(1-2\gamma)^2} - \frac{\gamma}{1-2\gamma} (\equiv \breve{\theta}_B).$$

Since $q_A^{\lim}(\theta)$ is positive only for $\theta > c$, the limit pricing solution is obtained if and only if $\max[c, \theta^m] \leq \theta \leq \breve{\theta}_B$. The condition $\theta^m \leq \breve{\theta}_B$ is always met. The condition $c \leq \breve{\theta}_B$ is equivalent to $c \geq \frac{1-2\gamma}{2-3\gamma}$. When this condition holds, there exists an interval of types, $\theta \in \left[c, \breve{\theta}_B\right]$, to whom the limit pricing solution applies. Again, the corresponding equilibrium quantity of good B must be nil.

Finally, consider the case in which $\theta \geq \breve{\theta}_B$, so that $q_A^{cr}(\theta) \leq q_A^{\lim}(\theta)$ and the maximum is achieved on the lower branch of the virtual surplus function. Here, we must distinguish between two sub-cases, depending on whether the solution is interior, or is a corner solution at $q_A(\theta) = 0$. Clearly, the solution is interior, and is $q_A^{cr}(\theta)$, when $\theta \geq \breve{\theta}_A$. In this case, the corresponding equilibrium quantity of good B is $q_B^{cr}(\theta) = \tilde{q}_B(q_A^{cr}(\theta), \theta)$. Now, notice that $\frac{1-2\gamma}{2-3\gamma}$ is also the critical threshold for c such that when $c < \frac{1-2\gamma}{2-3\gamma}$ we have $\breve{\theta}_B < \breve{\theta}_A$, whereas the inequality is reversed when $c \geq \frac{1-2\gamma}{2-3\gamma}$. This means that if $c \geq \frac{1-2\gamma}{2-3\gamma}$ and the maximum is achieved in the lower branch, it must necessarily be an interior solution. However, when $c < \frac{1-2\gamma}{2-3\gamma}$ we have $\breve{\theta}_B < \breve{\theta}_A$. In this case, for $c \leq \theta \leq \breve{\theta}_A$, we have a corner solution for q_A, and the corresponding equilibrium quantity of good B is $q_c^e(\theta)$; for $\theta \geq \breve{\theta}_A$, the solution is again interior.

This completes the derivation of the optimal quantities in all possible cases. It is then easy to check that they coincide with the equilibrium quantities reported above, and that they are implemented by the equilibrium price schedules. This completes the proof of the Proposition.

Notice that since equilibrium quantities are everywhere continuos, the equilibrium price schedules must be continuous. The constant terms that guarantee continuity are all negative, i.e. fixed subsidies. In fact, it can be verified that the equilibrium price schedules are also everywhere smooth. ■

Proof of Proposition 2. Since the proof will use direct mechanisms, we start again by reporting the equilibrium quantities. They are:

- when $c \leq \bar{c}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q_c^e(\theta) & \text{for } c \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases}$$

- when $\bar{c} \leq c \leq \frac{1}{2}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q_c^e(\theta) & \text{for } c \leq \theta \leq 1-c \\ q_A^m(\theta) & \text{for } 1-c \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases}$$

when $c \geq \frac{1}{2}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \frac{1}{2} \\ q_A^m(\theta) & \text{for } \frac{1}{2} \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases}$$

where $\hat{\theta}$ is defined in the text of the Proposition.

Obviously, the competitive fringe will always price at cost, i.e. $P_B^E(q_B) = P_B^{NE}(q_B) = cq_B$. To prove the proposition, it then suffices to show that the dominant firm's equilibrium pricing strategy is indeed optimal. We shall focus on direct mechanisms and hence look for the optimal quantity $q_A(\theta)$, showing that it coincides with the equilibrium quantity reported above. It is then straightforward to conclude that the price schedules that support these quantities, which are the equilibrium price schedules, are indeed optimal. From the arguments in section 3, we know that the solution to the dominant firm's problem is formed by appropriately joining the solution to the maximization program (5), which applies to low-demand buyers ($\theta < \hat{\theta}$), and that to the maximization program (4), which applies to high-demand buyer ($\theta > \hat{\theta}$). The solution to problem (4) has been characterized in the proof of Proposition 1. (Notice that since $P_B^E(q_B) = P_B^{NE}(q_B)$, the constraint $U(\theta) \geq U^R(\theta)$ is already subsumed into the indirect utility function.) We therefore start by focusing on problem (5).

Problem (5) is a standard monopolistic non-linear pricing problem with a utility function

$$u(q_A, 0, \theta) = \theta q_A - \frac{1-\gamma}{2} q_A^2,$$

except that buyers now have a type-dependent reservation utility

$$\begin{aligned} U_A^R(\theta) &= \arg\max \left[u(0, q, \theta) - cq\right] \\ &= \frac{(\theta - c)^2}{2(1-\gamma)}. \end{aligned}$$

Thus, the problem becomes

$$\begin{aligned} & \max_{q_A(\theta)} \int_0^1 \left[\theta q_A - \frac{1-\gamma}{2} q_A^2 - U(\theta)\right] d\theta \\ \text{s.t.} \quad \frac{dU}{d\theta} &= q_A \qquad\qquad\qquad\qquad\qquad\qquad \text{(A.1)} \\ U(\theta) &\geq \frac{(\theta - c)^2}{2(1-\gamma)}. \end{aligned}$$

Its solution is given in the following

Lemma 6 *When $c \geq \frac{1}{2}$, the solution to problem (A.1) is*

$$q_A(\theta) = \begin{cases} 0 & \text{for } 0 \leq \theta \leq \frac{1}{2} \\ q_A^m(\theta) & \text{for } \theta \geq \frac{1}{2}. \end{cases}$$

When instead $c \leq \frac{1}{2}$, the solution is

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q_c^e(\theta) & \text{for } c \leq \theta \leq 1 - c \\ q_A^m(\theta) & \text{for } \theta \geq 1 - c. \end{cases}$$

Proof. Consider first the unconstrained problem. The solution is $q_A(\theta) = \max[0, q_A^m(\theta)]$, and the corresponding utility is

$$\begin{aligned} U^m(\theta) &= u(q_A^m(\theta), 0, \theta) - P_A^m(q_A^m(\theta)) \\ &= \frac{(2\theta - 1)^2}{4(1-\gamma)} \end{aligned}$$

When $c \geq \frac{1}{2}$, we have $U^m(\theta) \geq U_A^R(\theta)$ for all θ, so the unconstrained solution applies.

Now suppose that $c < \frac{1}{2}$, so that the type-dependent participation constraint must bind for a non-empty set of types. To deal with this constraint, we use the results of Jullien (2000), and in particular his Proposition 3. To apply that proposition, we must show that our problem satisfies the conditions of Weak Convexity, Potential Separation, Homogeneity, and Full Participation. Following Jullien (2000), define the virtual surplus function

$$\begin{aligned} s^E(g, q_A, \theta) &= u(q_A, 0, \theta) + (\theta - g)\, u_\theta(q_A, 0, \theta) \\ &= (2\theta - g)q_A - \frac{1-\gamma}{2} q_A^2 \end{aligned}$$

where the "weight" $g \in [0, 1]$ accounts for the possibility that the participation constraint may bind over any subset of the support of the distribution of types. Pointwise maximization of the virtual surplus function yields

$$\ell^E(g, \theta) = \arg\max_{q_A} s^E(g, q_A, \theta) = \frac{2\theta - g}{1 - \gamma}.$$

The condition of Weak Convexity requires that

$$\frac{\partial \ell^E(\hat{g}(\theta), \theta)}{\partial \theta} \geq \frac{dq_c^e(\theta)}{d\theta},$$

where $\hat{g}(\theta)$ is implicitly defined by

$$q_c^e(\theta) = \ell^E(\hat{g}(\theta), \theta).$$

Straightforward calculations show that this inequality always holds. Potential Separation is also satisfied, since it requires simply that $\ell^E(g, \theta)$ is non-decreasing in θ, which is obviously true. Homogeneity is obvious, as it requires that $U_A^R(\theta)$ can be implemented by a continuous and non decreasing quantity. which in our case is, by construction, $q_c^e(\theta)$. Finally, the condition of Full Participation requires that in equilibrium all types $\theta > c$ obtain positive quantities, which is obvious given that their reservation utility is strictly positive.

Proposition 3 in Jullien (2000) then implies that the solution to problem (5) is

$$q_A(\theta) = \begin{cases} q_c^e(\theta) & \text{for } c \leq \theta \leq 1 - c \\ q_A^m(\theta) & \text{for } \theta \geq 1 - c, \end{cases}$$

and obviously $q_A(\theta) = 0$ for $\theta \leq c$. ■

Next, we proceed to the characterization of the optimal switching point, $\hat{\theta}$. To begin with, we show that the equilibrium rent function $U(\theta)$ is steeper under non-exclusivity than under exclusivity. By (9), the slope of $U(\theta)$ is the sum of the equilibrium quantities. It is easy to verify that the sum of equilibrium quantities in the non-linear pricing equilibrium is always at least as large as in the solution to problem (A.1): $q_A^{NE}(\theta) + q_B^{NE}(\theta) \geq q_A^E(\theta)$, with a strict inequality whenever $q_B^{NE}(\theta) > 0$. This implies that the solution to the hybrid optimal control problem involves a unique switch from problem (A.1) (which applies to low-demand types) to problem (4) (which applies to high-demand types).

The next lemma says that the switch must be from exclusive dealing to a common representation equilibrium. In other words, at the switching point the solution to problem (4) is given by the common representation quantities $q_A^{cr}(\theta), q_B^{cr}(\theta) > 0$. This rules out the possibility that the switch occurs for types who obtain the monopoly or limit pricing quantity of product A.

Lemma 7 *When $\theta > \hat{\theta}$, both $q_A(\theta)$ and $q_B(\theta)$ are strictly positive.*

Proof. In the uniform-quadratic model, condition (7) becomes

$$\frac{P_A^{NE}(q_A^{NE}(\hat{\theta})) - P_A^E(q_A^E(\hat{\theta}))}{q_A^{NE}(\hat{\theta}) + q_A^{NE}(\hat{\theta}) - q_A^E(\hat{\theta})} = 1 - \hat{\theta}.$$

Since $q_A^{NE}(\hat{\theta}) + q_A^{NE}(\hat{\theta}) > q_A^E(\hat{\theta})$, it must be $P_A^{NE}(q_A^{cr}(\hat{\theta})) > P_A^E(q_A^E(\hat{\theta}))$, so the dominant firm extracts more rents, at the margin, from buyers who accept non-exclusive contracts than from those who accept exclusive ones. From this, it follows immediately that that $q_A^{NE}(\hat{\theta}) > 0$ (otherwise, $P_A^{NE}(q_A^{NE}(\hat{\theta}))$ must be nil). The proof that also $q_B^{NE}(\hat{\theta}) > 0$ is equally simple. If the solution to problem (4) entails $q_B(\theta) = 0$, it must be either $\max[q_A^m(\theta), q_c^e(\theta)]$ or $q_A^{\lim}(\theta)$. In the former case, the dominant firm would obtain the same rent from buyers who accept non-exclusive contracts as from those who accept the exclusive one; in the latter, it would actually obtain less. Since we have just shown that it must obtain more, these two cases are not possible. ■

Lemma 7 implies that at the switching point the total quantity $q_A(\theta) + q_B(\theta)$ is discontinuous, and in particular that it jumps upward (thus preserving monotonicity). To the right of $\hat{\theta}$ we have $q_A(\theta) = q_A^{cr}(\theta)$ and $q_B(\theta) = q_B^{cr}(\theta)$. When $c \geq \frac{1}{2}$, to the left of $\hat{\theta}$ we must necessarily have $q_A(\theta) = q_A^m(\theta)$. Therefore, conditions (6) and (7) become

$$\begin{aligned} u(q_A^m(\hat{\theta}), 0, \hat{\theta}) - P_A^m(q_A^m(\hat{\theta})) &= v(q_A^{cr}(\hat{\theta}), \hat{\theta}) - P_A^{cr}(q_A^{cr}(\hat{\theta})) - \Phi_A \\ \frac{P_A^{cr}(q_A^{cr}(\hat{\theta})) + \Phi_A - P_A^m(q_A^m(\hat{\theta}))}{q_A^{cr}(\hat{\theta}) + q_B^{cr}(\hat{\theta}) - q_A^m(\hat{\theta})} &= 1 - \hat{\theta}, \end{aligned}$$

where the indirect utility function $v(q_A(\theta), \theta)$ is that derived in the proof of Proposition 1. The explicit solutions for $\hat{\theta}$ and Φ_A are complicated and are reported in a Mathematica file that is available upon request from the authors.

When instead $c < \frac{1}{2}$, to the left of $\hat{\theta}$ we can have either $q_A(\theta) = q_A^m(\theta)$ or $q_A(\theta) = q_c^e(\theta)$. The former case applies when $\hat{\theta} > 1-c$, the latter when $\hat{\theta} < 1-c$. In the former case, the switching point $\hat{\theta}$ is determined by the same conditions as above. In the latter case, conditions (6) and (7) become

$$\begin{aligned} u(q_c^e(\hat{\theta}), 0, \hat{\theta}) - cq_c^e(\hat{\theta}) &= v(q_A^{cr}(\hat{\theta}), \hat{\theta}) - P_A^{cr}(q_A^{cr}(\hat{\theta})) - \Phi_A \\ \frac{P_A^{cr}(q_A^{cr}(\hat{\theta})) + \Phi_A - cq_c^e(\hat{\theta})}{q_A^{cr}(\hat{\theta}) + q_B^{cr}(\hat{\theta}) - q_c^e(\hat{\theta})} &= 1 - \hat{\theta}, \end{aligned}$$

Once again, the explicit solutions for $\hat{\theta}$ and Φ_A are reported in a Mathematica file that is available upon request from the authors.

The threshold $\bar{c}$ is implicitly defined as the solution to $\hat{\theta} = 1-\bar{c}$. The explicit expression for $\bar{c}$ is reported in the Mathematica file mentioned above. In any case, it can be shown that $\bar{c} < \frac{1-2\gamma}{2-3\gamma} \leq \frac{1}{2}$ and that $\hat{\theta} > \max[\breve{\theta}_A, \breve{\theta}_B]$.

This completes the derivation of the equilibrium quantities in all possible cases. It is then easy to check that these equilibrium quantities are implemented by the price schedules reported in the statement of the Proposition.

Although the explicit expressions for Φ_A are complicated, it can be shown that in any case we have $\Phi_A > 0$. In other words, while high-demand buyers are faced with the same marginal non-exclusive prices as in the non-linear pricing equilibrium, they now pay a fixed fee instead of receiving a fixed subsidy. ■

Proof of Proposition 3. As usual, we start by reporting the equilibrium quantities, which are

- when $c \leq \tilde{c}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq P_B'^{cr}(0) \\ q_A^{\lim}(\theta) & \text{for } \breve{\theta}_A \leq \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq 1. \end{cases}$$

- when $c > \tilde{c}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \frac{1}{2} \\ q_A^{m}(\theta) & \text{for } \frac{1}{2} < \theta \leq \theta^m \\ q_A^{\lim}(\theta) & \text{for } \theta^m < \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \theta > \breve{\theta}_B \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq 1. \end{cases}$$

where

$$P_B'^{cr}(0) = \alpha + c\left[1 - \frac{\alpha(1-\gamma)}{1-2\gamma}\right].$$

Like in Section 4, $\breve{\theta}_B$ is implicitly defined by the condition $q_B^{cr}(\breve{\theta}_B) = 0$ and θ^m by the condition $q_A^m(\theta^m) = q_A^{\lim}(\theta^m)$; now, however, the explicit expressions are different as $q_B^{cr}(\breve{\theta}_B)$ and $q_A^{\lim}(\theta^m)$ in the duopoly model differ from the competitive fringe model. The explicit solutions are

$$\breve{\theta}_B = \alpha + c\frac{(1-\gamma)(1-\alpha)}{1-2\gamma}$$

and

$$\theta^m = \frac{(1-\gamma)}{1-3\gamma}P_B'^{cr}(0) - \frac{\gamma}{1-3\gamma}.$$

To prove the proposition, we must show that the equilibrium price schedules satisfy the best response property. Given its rival's price schedule, a firm is faced with an optimal non-linear pricing problem that can be solved by invoking the Revelation Principle and thus focusing on direct mechanisms. The strategy of the proof is to show that for each firm $i = A, B$ the optimal quantities $q_i(\theta)$, given $P_{-i}(q_{-i})$, coincide with the equilibrium quantities reported above. It is then straightforward to conclude that the price schedules that support these quantities must be equilibrium price schedules.

Given $P_{-i}(q_{-i})$, firm i faces a monopolistic screening problem where type θ has an indirect utility function

$$v^i(q_i, \theta) = \max_{q_{-i} \geq 0} \left[u(q_i, q_{-i}, \theta) - P_{-i}(q_{-i}) \right],$$

which accounts for any benefit he can obtain by optimally trading with its rival. Since u is quadratic and $P_{-i}(q_{-i})$ is piecewise quadratic, v_i is also piecewise quadratic. It may have kinks, but we shall show that any such kink preserve concavity, so the indirect utility function is globally concave.

Provided that the single-crossing condition holds, firm i's problem reduces to finding a function that pointwise maximizes the "indirect virtual surplus"

$$s^i(q_i, \theta) = v^i(q_i, \theta) - c_i q_i - (1 - \theta) v^i_\theta,$$

where c_i is zero for $i = A$ and c for $i = B$. It is easy to verify ex post that the maximizer $q_i(\theta)$ satisfies the monotonicity condition.

Consider, then, firm A's best response to the equilibrium price schedule of firm B, $P_B(q_B)$. The indirect utility function is piecewise quadratic, with two branches corresponding to the case in which $\arg\max_{q_B \geq 0} \left[u(q_A, q_B, \theta) - P_B(q_B) \right]$ is 0 or is strictly positive, and a kink between the two branches:

$$v^A(q_A, \theta) = \begin{cases} \theta q_A - \dfrac{1-\gamma}{2} q_A^2 & \text{if } q_B = 0 \text{ or, equivalently, } q_A \geq q_A^{\lim}(\theta) \\ A_0 + A_1 q_A + A_2 q_A^2 & \text{if } q_B > 0 \text{ or, equivalently, } q_A < q_A^{\lim}(\theta). \end{cases}$$

The coefficients A_0, A_1 and A_2 can be calculated as

$$\begin{aligned} A_0 &= \frac{\left[(\theta - c)(1 - 2\gamma) - \alpha(1 - c(1-\gamma) - 2\gamma) \right]^2}{2(1 - \gamma - \alpha)(1 - 2\gamma)^2}, \\ A_1 &= \gamma \frac{c(1 - 2\gamma) + \alpha(1 - c(1 - \gamma) - 2\gamma)}{(1 - \gamma - \alpha)(1 - 2\gamma)} + \theta \frac{1 - 2\gamma - \alpha}{1 - \gamma - \alpha} \\ A_2 &= -\frac{1 - 2\gamma + \alpha(1 - \gamma)}{2(1 - \gamma - \alpha)} < 0. \end{aligned}$$

On both branches of the indirect utility function, the coefficients of the quadratic terms are negative. In addition, it can be checked that

$$\left. \frac{\partial^2 v^A(q_A, \theta)}{\partial q_A^2} \right|_{q_A \leq q_A^{\lim}(\theta)} = A_2 \geq \left. \frac{\partial^2 v^A(q_A, \theta)}{\partial q_A^2} \right|_{q_A > q_A^{\lim}(\theta)} = -(1 - \gamma),$$

so the function $v^A(q_A, \theta)$ is globally concave in q_A. It can also be checked that the sorting condition $\frac{\partial^2 v^A}{\partial\theta\partial q_A} > 0$ is satisfied as

$$\frac{\partial^2 v^A}{\partial\theta\partial q_A} = \begin{cases} 1 & \text{if } q_A \geq q_A^{\lim}(\theta) \\ \dfrac{1-2\gamma-\alpha}{1-\gamma-\alpha} > 0 & \text{if } q_A < q_A^{\lim}(\theta). \end{cases}$$

We can therefore obtain A's best response by pointwise maximizing the virtual surplus function $s^A(q_A, \theta)$. Like the indirect utility function, the virtual surplus function is piecewise quadratic with a kink. The maximum can occur in either one of the two quadratic branches, or at the kink. To be precise:

$$\arg\max_{q_A(\theta)}[\sigma^A(q_A,\theta)] = \begin{cases} \dfrac{2\theta-1}{1-\gamma} & \text{if } \gamma < \frac{1}{3} \text{ and } \frac{1}{2} \leq \theta \leq \theta^m \\ \dfrac{\theta - P_B'^{cr}(0)\alpha}{\gamma} & \begin{array}{l}\text{if } \gamma < \frac{1}{3} \text{ and } \theta^m \leq \theta \leq \breve{\theta}_B \\ \text{or if } \gamma \geq \frac{1}{3} \text{ and } P_B'^{cr}(0) \leq \theta \leq \breve{\theta}_B\end{array} \\ \dfrac{\theta-\alpha}{1-\alpha} + \dfrac{c\gamma}{1-2\gamma} & \text{if } \theta \geq \breve{\theta}_B. \end{cases}$$

But these are precisely the monopoly, limit-pricing and common representation quantities defined in the main text. Note also that the case in which $\gamma < \frac{1}{3}$ and $\frac{1}{2} \leq \theta \leq \theta^m$ cannot arise if $c < \tilde{c}$. In this case, the optimum is never achieved on the upper branch of the indirect utility function; in other words, firm A's best response never involves setting the quantity at the monopoly level. It is therefore apparent that firm A's best response is to offer precisely the equilibrium quantities. This can be achieved by offering the equilibrium price schedules. This verifies that firm A's equilibrium price schedule satisfies the best response property.

Consider now firm B. The procedure is the same as for firm A, but now we must distinguish between two cases, depending on whether A's price schedule comprises the lowest monopoly branch or not.

Consider first the case in which there is no monopoly branch in A's price schedule. The indirect utility function of a buyer when trading with firm B then is

$$v^B(q_B,\theta) = \begin{cases} \theta q_B - \frac{1-\gamma}{2} q_B^2 & \text{if } q_B \geq q_B^{\lim}(\theta) \\ \hat{B}_0 + \hat{B}_1 q_B + \hat{B}_2 q_B^2 & \text{if } \check{q}_B(\theta) \leq q_B < q_B^{\lim}(\theta) \\ B_0 + B_1 q_B + B_2 q_B^2 & \text{if } 0 < q_B \leq \check{q}_B(\theta) \end{cases}$$

where

$$\begin{aligned} q_B^{\lim}(\theta) &= \frac{\theta-\alpha}{\gamma} - \frac{\alpha c}{1-2\gamma} \\ \check{q}_B(\theta) &= \frac{\theta-\alpha-c(1-\alpha)}{\gamma} + \frac{\alpha c}{1-2\gamma}. \end{aligned}$$

The first branch corresponds to firm B acting as a monopolist. Along the second branch, firm B competes with firm A's limit-pricing price schedule. Clearly, neither case can occur in equilibrium, but these may be out of equilibrium outcomes which therefore must be considered. Finally, the third branch corresponds to the case in which firm A accommodates.

The coefficients of the lower branches of the indirect utility functions are

$$\hat{B}_0 = \frac{(\theta - c)^2}{2\gamma}; \quad \hat{B}_1 = c; \quad \hat{B}_2 = -\frac{1 - 2\gamma}{2}$$

and

$$B_0 = \frac{2\theta - 1}{2(1-\gamma)}; \quad B_1 = \theta - \frac{\gamma}{1-\gamma}; \quad B_2 = -\frac{1-\gamma}{2}.$$

All branches are concave, and global concavity can be checked by comparing the left and right derivatives of $v^B(q_B, \theta)$ at the kinks. The sorting condition can also be checked as for firm A. We can therefore find B's best response by pointwise maximization of the virtual surplus function.

It is easy to verify that there is never an interior maximum on the upper or intermediate branch of the virtual surplus function. This is equivalent to saying that firm B is active only when firm A supplies the common representation quantity $q_A^{cr}(\theta)$. Pointwise maximization of the relevant branch of virtual surplus function then leads to

$$\arg\max[\sigma^B(q_B, \theta)] = \frac{\theta - \alpha}{1 - \alpha} - c\frac{1-\gamma}{1-2\gamma}.$$

This coincides with $q_B^{cr}(\theta)$, thereby confirming that the equilibrium price schedule $P_B(q_B)$ is indeed its best response to firm A's strategy.

The case where firm A's price schedule comprises also the monopoly branch is similar. The indirect utility function $v^B(q_B, \theta)$, and hence the virtual surplus $s^B(q_B, \theta)$, now comprise four branches (all quadratic). The equation of the fourth branch, which corresponds to $0 < q_A < q_A^m(\theta)$, is

$$v^B(q_B, \theta) = \tilde{B}_0 + \tilde{B}_1 q_B + \tilde{B}_2 q_B^2$$

where

$$\tilde{B}_0 = \frac{(2\theta - 1)^2}{4(1-\gamma)}; \quad \tilde{B}_1 = \frac{\theta + \gamma(1 - 3\gamma)}{1-\gamma}; \quad \tilde{B}_2 = -\frac{1 - \gamma(2+\gamma)}{2(1-\gamma)}.$$

However, it turns out that the optimum still lies on the same branch as before and that it therefore entails a quantity equal to $q_B^{cr}(\theta)$. This observation completes the proof of the Proposition. ■

Proof of Proposition 4. The equilibrium quantities are:

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \le c \\ q_c^e(\theta) & \text{for } c \le \theta \le \hat{\theta} \\ \bar{q}_A^{cr}(\theta) & \text{for } \hat{\theta} \le \theta \le \bar{\theta} \\ q_A^{cr}(\theta) & \text{for } \bar{\theta} \le \theta \le 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \le \hat{\theta} \\ \bar{q}_B^{cr}(\theta) & \text{for } \hat{\theta} \le \theta \le \bar{\theta} \\ q_B^{cr}(\theta) & \text{for } \bar{\theta} \le \theta \le 1, \end{cases}$$

where $\hat{\theta}$ and $\bar{\theta}$, which are defined in the text of the Proposition, are given by

$$\begin{aligned}\hat{\theta} &= \frac{c(2-3\gamma)}{1-2\gamma},\\ \bar{\theta} &= \frac{c(1-2\gamma)+\alpha[3-c-2(2-c)\gamma]}{\alpha+2(1-2\gamma)}.\end{aligned}$$

The claim that this is the most cooperative equilibrium is justified in the text above the Proposition. Here, we just verify that this is indeed an equilibrium of the game. The logic of the proof is the same as for Proposition 3. We must show that for each firm the equilibrium price schedules satisfy the best response property. When calculating the best response, we take $\{P^E_{-i}(q), P^{NE}_{-i}(q)\}$ as given and hence can invoke the Revelation Principle and focus on direct mechanisms. We must therefore show that for each firm $i = A, B$ the optimal quantities $q_i(\theta)$ coincide with the equilibrium quantities reported above. It is then straightforward to conclude that the price schedules $P^E_i(q), P^{NE}_i(q)$ that support these quantities must be equilibrium price schedules.

Given its rival's exclusive and non exclusive price schedules, a firm must solve a monopolistic screening problem in which the buyer has an indirect utility function

$$v^i(q_i,\theta) = \max_{q_{-i}\geq 0}\left[u\left(q_i, q_{-i}, \theta\right) - P^{NE}_{-i}(q_{-i})\right],$$

and a reservation utility

$$U^R_i(\theta) = \max_{q_{-i}}\left[u(0, q_{-i}, \theta) - P^E_{-i}(q_{-i})\right].$$

Since firm i can impose exclusivity clauses, it must solve a hybrid optimal control problem in which the two control systems are

$$\begin{aligned} &\max_{q_i}\int\left[v^i(q_i,\theta) - U(\theta) - c_i q_i\right] d\theta \\ \text{s.t.}\quad \frac{dU}{d\theta} &= v^i_\theta(q_i,\theta) \\ U(\theta) &\geq U^R_i(\theta)\end{aligned} \tag{A.2}$$

if $q_{-i}(\theta) > 0$, and

$$\begin{aligned} &\max_{q_i}\int\left[u(q_i,0,\theta) - U(\theta) - c_i q_i\right] d\theta \\ \text{s.t.}\quad \frac{dU}{d\theta} &= u_\theta(q_i,0,\theta) \\ U(\theta) &\geq U^R_i(\theta)\end{aligned} \tag{A.3}$$

if $q_{-i}(\theta) = 0$. In both cases, $q_i(\theta)$ must be non-decreasing.

Problem (A.3) is relevant only for the dominant firm. When it sets $q_B(\theta) = 0$, noting that problem (A.3) coincides with problem (A.1) in the proof of Proposition 2, we can apply Lemma 6 and conclude that

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \le c \\ q_c^e(\theta) & \text{for } c \le \theta \le 1-c \\ q_A^m(\theta) & \text{for } \theta \ge 1-c. \end{cases}$$

It is then easy to verify that $\hat{\theta}$ is now lower than $1-c$, so the only relevant part of the solution is $q_c^e(\theta)$.

Consider now problem (A.2). Several properties of this problem must hold for both firms. By construction, the indirect utility functions $v^i(q_i,\theta)$ are almost everywhere differentiable. At any point where the derivatives exist, by the envelope theorem we have

$$v_\theta^i(q_i,\theta) = q_i + \tilde{q}_{-i}(q_i,\theta),$$

where

$$\tilde{q}_{-i}(q_i,\theta) = \arg\max_{q_{-i}>0} \left[u\left(q_i, q_{-i}, \theta\right) - P_{-i}^{NE}(q_{-i})\right]$$

Generally speaking, the indirect utility functions $v^i(q_i,\theta)$ have two branches, according to whether $\tilde{q}_{-i}(q_i,\theta) \le \bar{q}_{-i}(\bar{\theta})$ or $\tilde{q}_{-i}(q_i,\theta) \ge \bar{q}_{-i}(\bar{\theta})$ respectively. When $\tilde{q}_{-i}(q_i,\theta) \le \bar{q}_{-i}(\bar{\theta})$, we have $P_{-i}^{NE}(q_{-i}) = \bar{P}_{-i}^{cr}(q_{-i})$. When $\tilde{q}_{-i}(q_i,\theta) \ge \bar{q}_{-i}(\bar{\theta})$, we have $P_{-i}^{NE}(q_{-i}) = P_{-i}^{cr}(q_{-i})$ (plus a constant).

The indirect utility functions $v^i(q_i,\theta)$ are continuous, almost everywhere differentiable, and satisfy $v_{\theta q_i}^i(q_i,\theta) > 0$. Continuity and a.e. differentiability follows directly from the definition of $v^i(q_i,\theta)$. To prove the sorting condition, observe that

$$v_{\theta q_i}^i(q_i,\theta) = 1 - \gamma \frac{\partial \tilde{q}_{-i}(q_i,\theta)}{\partial q_i} \ge 0.$$

Consider the two branches of the indirect utility function in turn. When $\tilde{q}_{-i}(q_i,\theta) \le \bar{q}_{-i}(\bar{\theta})$,

$$v_{\theta q_i}^i(q_i,\theta) = 1 + \frac{\partial \tilde{q}_{-i}(q_i,\theta)}{\partial \theta}(-\gamma)^2 = \frac{3-6\gamma}{3-5\gamma} > 0.$$

When instead $\tilde{q}_{-i}(q_i,\theta) \ge \bar{q}_{-i}(\bar{\theta})$ the sorting condition is immediately verified since

$$v_{\theta q_i}(q_i,\theta) = \frac{1-\alpha-2\gamma}{1-\alpha-\gamma} \ge 0.$$

Now consider problem (A.2). Because of the type-dependent participation constraint, following Jullien (2000) we define the virtual surplus function:

$$\sigma^i(g, q_i, \theta) = v^i(q_i,\theta) - (g-\theta)\, v_\theta^i(q_i,\theta)$$

where the "weight" $g \in [0,1]$ accounts for the possibility that the participation constraint may bind for a whole set of types. Let

$$\ell_i(g,\theta) = \arg\max_{q_i \ge 0} \sigma^i(g, q_i, \theta)$$

be the maximizer of the virtual surplus function. This solution is still in implicit form, as it depends on the value of g, which is still to be determined. This can be done by exploiting Proposition 5.5 of Jullien (2000).

To apply that Proposition, we first prove the following lemma.

Lemma 8 *Problem (A.2) satisfies the conditions of Potential Separation, Homogeneity and Weak Convexity.*

Proof. Potential Separation requires that $\ell_i(g,\theta)$ is non-decreasing in θ. This follows from the fact that the virtual surplus function has increasing differences. To show this, consider each branch of the indirect utility function separately. First, when $\tilde{q}_{-i}(q_i,\theta) \leq \bar{q}_{-i}(\bar{\theta})$ we have

$$\sigma^i_{q_i\theta}(q_i,\theta) = v^i_{q_i\theta}(q_i,\theta) - \left[1 + \frac{\partial \tilde{q}_{-i}(q_i,\theta)}{\partial q_i}\right] \frac{d}{d\theta}(g-\theta).$$

The first term is positive, as we have just shown. The second term is positive because $\frac{d}{d\theta}(g-\theta) < 0$ and

$$1 + \frac{\partial \tilde{q}_{-i}}{\partial q_i} = \frac{1-2\gamma}{3-5\gamma} > 0.$$

Second, when $\tilde{q}_{-i}(q_i,\theta) \geq \bar{q}_{-i}(\bar{\theta})$ the indirect utility function coincides, modulo a constant, with the one arising in the equilibrium with non-linear pricing. In this case, it is immediate to show that $\sigma^i_{q_i\theta}(q_i,\theta) > 0$. This completes the proof that problem (A.2) satisfies the condition of Potential Separation.

Homogeneity requires that $U^R_i(\theta)$ can be implemented by a continuous and non decreasing quantity. This is obvious, since $U^R_i(\theta)$ is implemented by $q^E(\theta)$, where $q^E(\theta)$ is the optimal quantity given the exclusive price schedule $P^E_{-i}(q)$:

$$q^E(\theta) = \begin{cases} q^e_c(\theta) & \text{if } \theta \leq \hat{\theta} \\ \bar{q}^E(\theta) & \text{if } \theta > \hat{\theta}. \end{cases}$$

To prove Weak Convexity, we first show that $\ell_i(0,\theta)+\tilde{q}_{-i}(\ell_i(0,\theta),\theta) \geq q^E(\theta)$ for all $\theta \in [0,1]$. By definition,

$$\ell_i(0,\theta) = \arg\max_{q_i} \left[v^i(q_i,\theta) + \theta v^i_\theta(q_i,\theta)\right].$$

Thus, $\ell_i(0,\theta)$ is implicitly defined by the first order condition

$$v^i_{q_i}(q_i,\theta) + \theta v^i_{\theta q_i}(q_i,\theta) = 0.$$

Since $v^i_{\theta q_i}(q_i,\theta) > 0$, this implies that $v^i_{q_i}(q_i,\theta) < 0$, or $u_{q_i}(q_i,\tilde{q}_{-i}(q_i,\theta),\theta) < 0$. In other words, $\ell_i(0,\theta)$ exceeds the satiation consumption $u_{q_i}(q_i,\tilde{q}_{-i}(q_i,\theta),\theta) = 0$. The quantity $q^E(\theta)$, on the contrary, is lower than the satiation consumption. It follows that $\ell_i(0,\theta) + \tilde{q}_{-i}(\ell_i(0,\theta),\theta) \geq q^E(\theta)$.

In addition, Weak Convexity requires that the curve $q^E(\theta)$ cuts the curve $\ell_i(1,\theta)+\tilde{q}_{-i}(\ell_i(1,\theta),\theta) = q_A^{cr}(\theta)+q_B^{cr}(\theta)$ from above. Noting that $\ell_i(1,\theta) = q_i^{cr}(\theta)$, the fact that $q^E(\theta)$ can only cut the curve $q_A^{cr}(\theta)+q_B^{cr}(\theta)$ from above as

$$\frac{d[q_A^{cr}(\theta)+q_B^{cr}(\theta)]}{d\theta} \geq \frac{dq^E(\theta)}{d\theta},$$

irrespective of whether $q^E(\theta)$ is $q_c^e(\theta)$ or $\bar{q}^E(\theta)$. This finally proves Weak Convexity and hence the lemma. ■

With these preliminary results at hand, let us now consider the dominant firm's problem. The solution when $q_B(\theta)=0$ has been already characterized. If $q_B(\theta)>0$, Proposition 5.5 in Jullien (2000) guarantees that generally speaking the solution partitions the set of types into three sets: buyers who are excluded, buyers who obtain their reservation utility $U_A^R(\theta)$, and buyers whose payoff is strictly greater than $U_A^R(\theta)$. Clearly, the first set is always empty: if $q_B(\theta)>0$, we always have $q_A(\theta)>0$.

Next consider the second group of buyers. When the participation constraint binds, firm A can guarantee to each low type consumer his reservation utility $U_A^R(\theta)$ in two ways. First, it can offer an exclusive price schedule that just undercuts that of firm B. Alternatively, it can implement *via* non-exclusive prices the quantities that satisfy the condition

$$\bar{q}_A^{cr}(\theta)+\bar{q}_B^{cr}(\theta) = \bar{q}^E(\theta),$$

which by the envelope theorem guarantees that the participation constraint is met as an equality. The maximum payment that firm A can requested for $\bar{q}_A^{cr}(\theta)$ is

$$\bar{P}_A^{cr}\left(\bar{q}_A^{cr}(\theta)\right) = -c\bar{q}_A^{cr}(\theta)+(1-2\gamma)\left[\bar{q}_A^{cr}(\theta)\right]^2+cq_c^e(\hat{\theta}).$$

The second strategy is at least as profitable as the first one if

$$\bar{P}_A^{cr}\left(\bar{q}_A^{cr}(\theta)\right) \geq \bar{P}^E(\bar{q}^E(\theta)),$$

which is precisely the no-undercutting condition (23) which holds by construction. This shows that offering $\bar{P}_A^{cr}(q_A)$ is indeed a best response for firm A when the participation constraint is binding.

Finally, when the participation constraint does not bind, the solution to firm A's program is obtained simply by setting $g=1$. Assume that $\ell_A(1,\theta) \geq \bar{q}_A^{cr}(\bar{\theta})$ when $\theta > \hat{\theta}$ (this will be proven shortly). Since the virtual surplus function $\sigma_A(1,q_A,\theta)$ is exactly the same as in the non-linear pricing equilibrium, modulo a constant, the maximizers of the virtual surplus functions must coincide and the optimal quantity is

$$\ell_A(1,\theta) = q_A^{cr}(\theta).$$

Finally, the cutoff $\bar{\theta}$ is implicitly given by the condition

$$\bar{q}^E(\bar{\theta}) = \ell_A(1,\bar{\theta})+\tilde{q}_B(\ell_A(1,\bar{\theta}),\bar{\theta}).$$

This also establishes that $\ell_A(1,\theta) \geq \bar{q}_A^{cr}(\bar{\theta})$ when $\theta > \hat{\theta}$.

To complete the verification of the best response property for firm A, it remains to consider the switch from exclusive to non-exclusive contracts. By the no-deviation condition (23), which in the most cooperative equilibrium holds as an equality, firm A is just indifferent between imposing an exclusivity clause or not for all $\theta \leq \bar{\theta}$. Exclusive dealing arises just when $\bar{q}_B^{cr}(\theta) \leq 0$, which is equivalent to $\theta \leq \hat{\theta}$. Because firm A is indifferent between the exclusive and non-exclusive regimes, at the switching point a smooth-pasting condition must now hold, which implies that aggregate quantities must be continuous, and hence that $P_A^{NE}(\bar{q}_A^{cr}(\hat{\theta})) = P_A^E(\bar{q}_A^E(\hat{\theta}))$.

The problem faced by firm B is similar, except that firm B can never make a profit by selling under an exclusivity clause. Thus, we can focus on problem (A.2). Proceeding as for firm A, one can show that the optimal quantity is $\bar{q}_B^{cr}(\theta)$ when the participation constraint $U(\theta) \geq U_A^R(\theta)$ is binding, and $q_B^{cr}(\theta)$ when it is not.

These arguments complete the proof that the solution to the problem of firm i coincides with $q_i(\theta)$ as shown in the text of the proposition. By construction, this solution can be implemented by firm i using the equilibrium price schedules $\left(P_i^E(q_i), P_i^{NE}(q_i)\right)$.

This solution is well defined when the three intervals $[c,\hat{\theta})$, $[\hat{\theta},\bar{\theta}]$ and $(\bar{\theta},1]$ are non-empty. This requires $c \leq \hat{\theta}$, $\hat{\theta} \leq \bar{\theta}$ and $\bar{\theta} \leq 1$. It is immediate to show that the first and the last of these inequality always hold. Thus, the solution is well defined if and only is $\hat{\theta} \leq \bar{\theta}$, which is equivalent to

$$c \leq \breve{c} \equiv \frac{2(1-2\gamma)}{5(1-\gamma)+\sqrt{1-2\gamma+9\gamma^2}}. \blacksquare$$

Proof of Proposition 5. As usual, we start by reporting the equilibrium quantities, which are:

- when $\tilde{c} \leq c \leq \frac{1}{2}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q_c^e(\theta) & \text{for } c \leq \theta \leq \theta^m \\ q_A^m(\theta) & \text{for } \theta^m \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1, \end{cases}$$

- when $c > \frac{1}{2}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \frac{1}{2} \\ q_A^m(\theta) & \text{for } \frac{1}{2} \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1. \end{cases}$$

The strategy of the proof is the same as for Proposition 4. Many of the arguments are indeed the same as in previous proofs and so need not be repeated here. In particular, notice that:

- first, when $c > \check{c}$, there is no longer any scope for coordinating exclusive prices (this was shown in the proof of Proposition 4). Hence, firm B always sets exclusive prices at the competitive level $P_B^E(q_B) = cq_B$. This implies that when firm A imposes an exclusivity clause, the buyers' reservation utility is exactly the same as in the competitive fringe model. It follows that the solution to problem (5) is still given by Lemma 6;

- second, without exclusivity the problems that are faced by the firms are exactly the same as in the proof of Proposition 4 when the participation constraint does not bind.

These remarks imply that Proposition 5 can be proved simply by combining arguments already presented in the proofs of Proposition 2 and Proposition 4. The only difference is that now the switch from the exclusive to the non-exclusive regime is the result of the interaction between the pricing choices of firm A and firm B. This point, however, has already been discussed in the main text, which shows that the equilibrium switching point must satisfy conditions (27)-(29). The explicit expressions for Φ_A and Φ_B are complicated and are reported in a Mathematica file that is available upon request from the authors. In any case, it can be verified that $\Phi_A > 0$ and $\Phi_B < 0$, as argued in the main text. ■