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ABSTRACT

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Keywords: advance selling, buying frenzy and customer desperation

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Product Launches and Buying Frenzies: A Dynamic Perspective

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Buying frenzies in which a firm intentionally undersupplies a product during its initial launch phase are a common practice within several industries such as electronics (cell phones, video games, game consoles), luxury cars, and fashion goods. We develop a dynamic model of buying frenzies that captures the production and sales of a product over time by the firm and then characterize the conditions under which frenzies are an optimal policy for the firm. We show that buying frenzies occur when customers are sufficiently uncertain about their product valuations and when customers discount the future but not excessively. Further, we propose a measure of "customer desperation" to measure the magnitude of frenzies and demonstrate that buying frenzies can have a significant impact on the firm's profit. This paper provides managerial insights on how firms can affect the market response to a new product through their pricing, production, and inventory decisions to induce profitable frenzies.

Key words: Advance Selling, Buying Frenzy, Customer Desperation, Strategic Customer Behavior

1. Introduction

Long queues of enthusiastic customers were common when Apple's iPad 2 hit the Hong Kong markets on April 29, 2010. Given the inclement weather, Apple stores were glad to provide umbrellas and raincoats (bearing Apple's logo) to waiting customers. Each store received a limited number of iPads and distributed them on a first-come, first-served basis. Some customers were willing to pay considerably higher prices to obtain the product in the "gray" market. Such shortage of supply is not confined to Apple's iPad 2. The shortage of Nintendo's game console Wii, for example, lasted from its initial introduction in 2006 until 2009 (Liu and Schiraldi 2012). Sony's PlayStation 2 faced the same problems after entering the US market eight months later than Japan's markets (Stock and Balachander 2005).

A *buying frenzy* is induced by a firm that intentionally undersupplies a market and excluded customers are strictly worse off. This practice is common in several industries such as luxury cars, fashion, and especially electronics (cell phones, video games, game consoles). Although shortages might be attributed to demand forecasting errors, issues in component supplies or production problems, their repeated occurrence—particularly during the launch phase of innovative products—suggests that a conscious marketing strategy is the true reason. The few studies investigating this issue have considered mainly static models. These models capture situations where firms sell the good only once; examples include tickets for sporting or music events, limited edition products, and one-off auctions. Yet, the predictions of static models collapse when the firm produces repeatedly over time, as is the case for most manufactured products. The reason is that the firm wants to serve the customers who were excluded from the early sales. But why should customers be desperate to buy early when products are available in the future? An important gap in the literature is the absence of a dynamic analysis that supports an initial buying frenzy followed by a period of regular sales without frenzies.

We develop a model in which production and sales occur in two periods and then characterize the dynamics of sales, prices, and scarcity. The two-period model that we employ is a stylized representation that features the classical dichotomy between the launch phase of a new product and its subsequent mature phase. We show that the firm's gains from inducing a buying frenzy (relative to matching supply and demand) can be economically substantial. We also investigate the conditions under which buying frenzies are optimal. Finally, we compute customers' loss from being excluded during the initial launch phase, which is a proxy for "customer desperation", and show that this loss can be significant. This explains why customers may invest resources to obtain the good early (e.g., wait in queues) and why prices can be significantly higher in resale markets.

There are several ingredients to our analysis. We assume that customers are initially uncertain about their preferences for the product (see e.g., Xie and Shugan 2001, Gallego and Şahin 2010 and Yu et al. 2011). This assumption applies to the innovative and fashion products that have been the object of buying frenzies. As DeGraba (1995) and Denicolò and Garella (1999), we assume that the firm cannot commit to future prices and quantities. This implies that the firm discounts prices when inventories build up and is consistent with the response of car and electronics manufacturers when products do not sell as expected. For example, HP had to slash prices to clear the unsold inventory of TouchPad only two months after its launch (*The Wall Street Journal* 2011). Finally, we use Pareto dominance as the selection criterion among the equilibria of the game (Cachon and Netessine 2004). In other words, given the firm policy, the equilibrium selected gives customers the highest payoff. The choice of the Pareto-dominant equilibrium is consistent with the interpretation that customers coordinate on the equilibrium that gives them the highest surplus which reflects the social dimension of buying frenzies.[1]

In a dynamic model, how does one account for uninformed customers rushing to buy early? Customers can always obtain the good upon waiting, and waiting benefits them because they can then incorporate the information they learn about their preferences in the decision to purchase the product. When a firm produces a large quantity in the first period, customers anticipate that they can get the product at a lower price in the second period in the equilibrium where customers wait. Thus, for a product to sell in the first period, its price must be sufficiently low to prevent such strategic customer waiting. This determines the maximum price a firm can charge in the first period. At that price, customers' expected utility from buying early is equal to the utility they obtain in the waiting equilibrium. But an individual customer is strictly worse-off waiting because the price the firm charges in the second period (on the equilibrium path) is higher than the price the customer could have obtained in the waiting equilibrium (off the equilibrium path). This explains why customers strictly prefer obtaining the good early.

Buying frenzies are more likely to happen when customers are initially uncertain about their preferences for the product. An increase in preference uncertainty increases customers' utility in the waiting equilibrium. Customer valuation uncertainty is likely to play a greater role in the case of an innovative product that will match the needs of some customers but not others. This may explain why frenzies are common for new electronic products (e.g., the iPhone) and not for similar "me too" products (e.g., Android phones) that are released later. It is reasonable to assume that customers have more uncertainty about new products than about knock offs produced later.

There are two main theories of buying frenzies. The first is based, as in our model, on inter-temporal price discrimination. Denicolò and Garella (1999) show that a monopolist may ration heterogenous customers to prevent strategic waiting. DeGraba (1995) considers a static model with individual preference uncertainty. In both of these models, buying frenzies exist only for specific rationing rules. Our model is dynamic and the existence of frenzies does not depend on the rationing rule used. Moreover, our analysis delivers a tractable model to study how preference uncertainty influences the existence of buying frenzies and to formally measure customer desperation.

Other papers on inter-temporal price discrimination and scarcity policies that come close to our paper include Liu and van Ryzin (2008), Cachon and Swinney (2009), and Liu and Schiraldi (2012). These papers, however, focus on different issues than we do. In particular, Liu and van Ryzin (2008) show that a stock-out possibility in the second period can prevent customers with known

[1] Coordination transpires through press reports and social media such as online networking sites (Facebook, Twitter, etc.). Through these channels, customers develop a sense of whether other customers are buying the product and whether they should do the same.

preferences from waiting. Cachon and Swinney (2009) suggest that by limiting the initial stocking level and offering optimal markdowns, the firm may constrain the strategic purchase behavior of its customers. Liu and Schiraldi (2012) show that the existence of resale markets can induce the firm to under-stock the product and increase its equilibrium price.

The second theory is based on asymmetric information. Stock and Balachnder (2005) argue that a high-quality firm employs scarcity to signal quality to uninformed customers. Rationing as a signal of quality has also been studied in Debo and van Ryzin (2009), and in Allen and Faulhaber (1991), among others. Papanastasiou et al. (2012) develop a model of "boundedly-rational social learning" and show that a firm may restrict the availability of a product to elicit favorable reviews from early adopters which positively affect the preferences of other customers. Our model is not based on information asymmetry and does not rely on irrationalities to explain frenzies. Neither do we assume that customers are myopic or the firm has to charge a fixed price over time.

Other models of buying frenzies are based on demand externality (Becker 1991) and psychological drive (Verhallen and Robben 1994). Becker's model accounts for social interactions and shows that an upward-sloping demand curve might result in an unstable equilibrium with arbitrarily small frenzies. Verhallen and Robben (1994) argue that scarcity itself can increase customers' willingness to pay—provided customers attribute that scarcity to demand-side variables (such as popularity) and not supply-side variables (such as the firm intentionally limiting supply). Finally, buying frenzies are distinct from herding (Debo and Veeraraghavan 2009), in which customers ignore their private noisy information about a product and merely follow what previous customers did. In contrast, imperfect information is absent from our model of frenzies.

Not much empirical work has been done on buying frenzies. An important exception is Balachander et al. (2009) who present a thorough analysis of buying frenzies in the context of the automobile market. Although their findings are not consistent with DeGraba's (1995) static model of buying frenzies, we argue that they are consistent with a dynamic model. This demonstrates the importance of distinguishing static and dynamic models of buying frenzies.

The rest of the paper is organized as follows. Section 2 outlines the basic model. Section 3 analyzes the dynamic model of buying frenzies when the firm can produce over time; and also defines measures of customer desperation and frenzy intensity. Section 4, extends the model to include second-period arrivals. Section 5 concludes the paper.

2. A Model of Buying Frenzies

A monopolist sells to a population of N_1 customers who arrive in the first period of a two-period horizon. Customer valuations are identically and independently distributed with density $f(v)$ and survival function $\bar{F}(v)$. We assume that f and F are continuous functions with support $[\underline{v}, \overline{v}] \subset$

$[0, \infty]$ and $E[\mathbf{v}] = \mu$. We also assume that the function $f(x) > 0$ is log-concave.[2] This demand specification approximates a large market in which customers are infinitesimal[3] and have idiosyncratic preferences that are discovered over time (Lewis and Sappington 1994). The two-period horizon is a stylized representation of a product launch phase followed by a mature phase of sales. It is therefore unnecessary for the two periods to have equal length; the second period can be arbitrarily longer than the first. What matters is that individual customer uncertainty is resolved by the end of the first period. Customers discount future utility by δ^c, and the monopolist discounts future profits by δ^m. Although we do not restrict the parameter space for (δ^c, δ^m), it is plausible that customers have a lower discount factor than the firm (Liu and van Ryzin 2008).

The monopolist can produce in both periods. We denote q_1 and q_2 as the production quantities in the first and second periods, respectively. To simplify the exposition, we assume that the marginal cost of production is zero. The firm announces (q_1, p_1) at the start of the first period. Customers are strategic and form expectations on what will happen in period 2. They buy or wait depending on which option maximizes their discounted utility given these expectations. As DeGraba (1995) and Denicolò and Garella (1999), we assume that all inventory available in the second period (period-2 production plus left-over inventory from period 1) is sold at the market-clearing price. In other words, the firm cannot commit to withhold inventory in the second period. It is a realistic assumption because product launches are rare and isolated in time, which helps keep the firm from developing a reputation for withholding or destroying excess inventory. As discussed in the Introduction, this assumption is consistent with practices observed for manufactured products. In contrast, firms selling perishable products (e.g., in the airline or hotel industry) sometimes commit to policies that do not necessarily discount excess inventories when sales are low; see Liu and van Ryzin (2008) for an extended discussion on this assumption.

We solve for the symmetric rational expectations equilibria (REE) once the firm announces (q_1, p_1).[4] Denote a customer's decision to buy or wait in an REE by the probability $x \in [0, 1]$. A customer waits if $x = 0$, buys if $x = 1$, and her strategy is mixed if $x \in (0, 1)$. We focus on symmetric equilibria—that is, we assume x is the same for all infinitesimal customers. The price that a customer expects to face in period 2 depends on the fraction of customers who have attempted to buy, x, and on the firm's initial production quantity q_1. Denote that expected period-2 price $p_2^e(x|q_1)$. The expected period-2 surplus is then $T_1(p_2^e(x|q_1)) = \delta^c E[\mathbf{v} - p_2^e(x|q_1)]^+$.[5]

[2] This is a standard technical assumption to ensure the uniqueness of the equilibrium. However, the results do not hinge on this assumption. Widely applied parametric families—including the uniform, exponential, normal, and logistic—have log-concave density (Bagnoli and Bergstrom 2005).

[3] This means that whether a customer decides to purchase or wait does not affect the payoff of other customers.

[4] Rational expectations equilibrium is a common concept applied in operations management; see e.g., Cachon and Swinney (2009), Liu and van Ryzin (2008), and Su and Zhang (2008); see the latter for a description.

[5] We use subscript '1' to denote that the expectation is with respect to valuation distribution of customers arriving in period 1. This emphasis will be important in Section 4 where we extend the model to include period-2 arrivals.

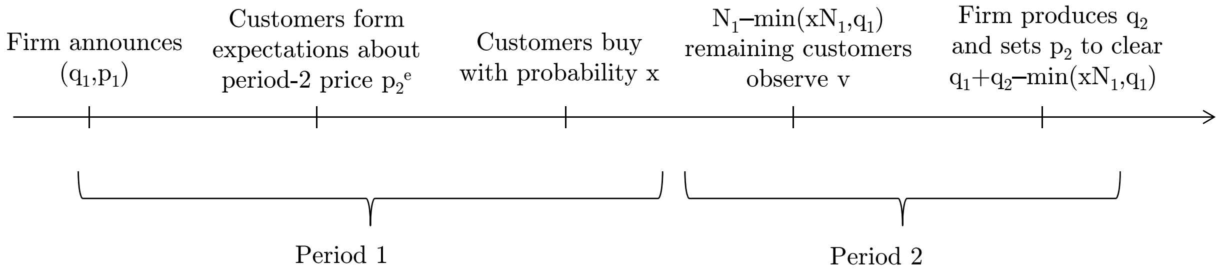


Figure 1 The sequence of events

Definition 1 *Assume the firm announces* (q_1,p_1) *in period 1. (a)* $x=0$ *is a pure strategy REE if and only if* $\mu-p_1\le\delta^cT_1(p_2^e(0|q_1))$*. (b)* $x=1$ *is a pure strategy REE if and only if* $\mu-p_1\ge\delta^cT_1(p_2^e(1|q_1))$*. (c)* $x\in(0,1)$ *is a mixed strategy REE if and only if* $\mu-p_1=\delta^cT_1(p_2^e(x|q_1))$*.*

There are typically multiple symmetric REE for a given announcement (q_1,p_1). To resolve the problem of equilibrium multiplicity, we select the Pareto-dominant REE and denote it PDREE. If there are multiple PDREE—that is, customers' expected surplus is the same, we select the PDREE that maximizes the firm's profits. This is consistent with other works in the literature (see Cachon and Netessine (2004) and references therein) and with the notion of social coordination although we do not formally model the mechanism behind coordination.

At the beginning of period 2, the firm observes sales in period 1, $Q_1=\min(xN_1,q_1)$, and sets q_2. The second-period price p_2 is such that the second-period supply $q_2+q_1-\min(xN_1,q_1)$ equals second-period demand. The rational expectations assumption implies that $p_2=p_2^e(x|q_1)$. In period 2, customers buy if their valuation is greater than the market-clearing price. Figure 1 shows the sequence of events.

3. Model Analysis

Profit maximization in period 2 subject to market clearing implies that

$$\begin{aligned} & p_2^e(x|q_1)=\operatorname{argmax} p\bar{F}(p) \qquad (1)\\ \text{s.t.}\qquad & N_1-\min(xN_1,q_1)\bar{F}(p)\ge q_1-\min(xN_1,q_1). \end{aligned}$$

This price is unique for a given announcement (q_1,p_1) because f is log-concave. Define $p_2^m\triangleq\operatorname{argmax}_p p\bar{F}(p)$ as the unconstrained period-2 profit-maximizing price. Our first result establishes the properties of $p_2^e(x|q_1)$.

Lemma 1 *Assume* $x\in[0,1]$ *is part of an REE. (a) If* $q_1\le N_1\bar{F}(p_2^m)$*, then* $p_2^e(x|q_1)=p_2^m$ *for all* $x\in[0,1]$*. (b) If* $q_1>N_1\bar{F}(p_2^m)$*, then* $p_2^e(x|q_1)=\bar{F}^{-1}(\frac{q_1-xN_1}{N_1(1-x)})$ *for* $x\in[0,\frac{q_1-N_1\bar{F}(p_2^m)}{N_1F(p_2^m)}]$ *and is strictly increasing in* x *and* $p_2^e(x|q_1)=p_2^m$ *for* $x\in[\frac{q_1-N_1\bar{F}(p_2^m)}{N_1F(p_2^m)},1]$ *and is constant in* x*.*

Define $p_1^b(q_1) \triangleq \mu - \delta^c T_1(p_2^e(1|q_1))$ and $p_1^w(q_1) \triangleq \mu - \delta^c T_1(p_2^e(0|q_1))$. Buying, $x = 1$, is an REE for any price $p_1 \leq p_1^b(q_1)$ and waiting, $x = 0$, is an REE for any price $p_1 \geq p_1^w(q_1)$. Because $T_1(p)$ is decreasing in p, Lemma 1 implies that $p_1^w(q_1) \leq p_1^b(q_1)$. This establishes that an REE exists for any announcement (q_1, p_1). Our next result shows that whenever $p_2^e(x|q_1)$ is strictly increasing at $x = 0$, we can ignore all mixed strategy equilibria.

Lemma 2 *Any $x \in (0,1)$ such that $p_2^e(x|q_1) > p_2^e(0|q_1)$ cannot be part of a PDREE.*

To see why, assume $x \in (0,1)$ is a mixed strategy PDREE for a firm announcement (q_1, p_1). Therefore, customers are indifferent between buying and waiting, i.e., $\mu - p_1 = \delta^c T_1(p_2^e(x|q_1))$. Because $\mu - p_1 = \delta^c T_1(p_2^e(x|q_1)) < \delta^c T_1(p_2^e(0|q_1))$, we conclude that the pure strategy waiting REE ($x = 0$) exists and gives higher expected surplus than PDREE x; a contradiction.

For any announcement (q_1, p_1), we show in the Appendix that any mixed strategy REE is either strictly Pareto-dominated by the waiting pure strategy equilibrium $x = 0$ (by Lemma 2) or Pareto-equivalent to the pure strategy buying REE ($x = 1$) which is weakly preferred by the firm. We prove the latter by comparing the firm's profits in mixed strategy REE x and REE $x = 1$. We subsequently ignore all mixed strategy REE and derive the set of pure strategy PDREE which could be $x = 0$, $x = 1$ or both. We then fix q_1 and solve for the price p_1 that maximizes firm profits. That is, we consider all possible prices p_1, compute the profits in all PDREE associated with (q_1, p_1), and select the price associated with the highest profits. Denote that price $p_1(q_1)$.

Proposition 1 *Assume the period-1 production is q_1. The profit maximizing period-1 price is $p_1(q_1) = p_1^w(q_1)$ and the associated PDREE is $x = 1$.*

The firm cannot produce q_1 and charge $p_1^b(q_1)$ because waiting is the Pareto-dominant equilibrium. In order to sell q_1, the firm can charge at most $p_1^w(q_1)$. At that price, the two REE (buying and waiting) are Pareto-equivalent.

Proposition 1 implies that the period-1 price has to be less than customers' expected valuation, μ, which is the profit per customer when the firm can credibly commit not to produce and sell in period 2. Hence, $\delta^c T_1(p_2^e(0|q_1))$ is a measure of the cost to the firm associated with not being able to commit. Further, Proposition 1 shows that the firm maximization problem is well-defined. In particular, the firm chooses q_1 to maximize

$$\pi(q_1) = q_1 p_1(q_1) + \delta^m (N_1 - q_1) p_2^m \bar{F}(p_2^m). \tag{2}$$

This expression covers the case where the firm sells only in period 2. This happens when $q_1 = 0$ and customers have to wait. A frenzy occurs if customers are worse off when they do not obtain the good early. Because customers can always buy at p_2^m in period 2 if they wait, this condition is satisfied only if $p_2^e(0|q_1) < p_2^e(1|q_1) = p_2^m$. Therefore, customers' expected loss from being rationed out is

$$L(q_1) = p_1^b(q_1) - p_1^w(q_1) = \delta^c(T_1(p_2^e(0|q_1)) - T_1(p_2^m)). \tag{3}$$

We distinguish two scarcity strategies. A frenzy takes place when customers are strictly worse off being rationed out, i.e., $L(q_1) > 0$, and there is excess demand, i.e., $N_1 > q_1$. If, on the other hand, $L(q_1) = 0$ and $N_1 > q_1$, we say the firm employs a 'rationing policy'. Note that our definitions of frenzy and rationing policy differ from previous works that have not made a clear distinction between the two concepts. Here, $L(q_1)$ is a measure of the *intensity* of the frenzy (customer desperation). Conditional on this measure being positive, a measure of the *extent* of the frenzy is $N_1 - q_1$. Denote the maximum profit per customer obtained by selling in period 2, $\bar{\pi} = p_2^m \bar{F}(p_2^m)$. The following proposition characterizes the conditions under which a buying frenzy occurs.

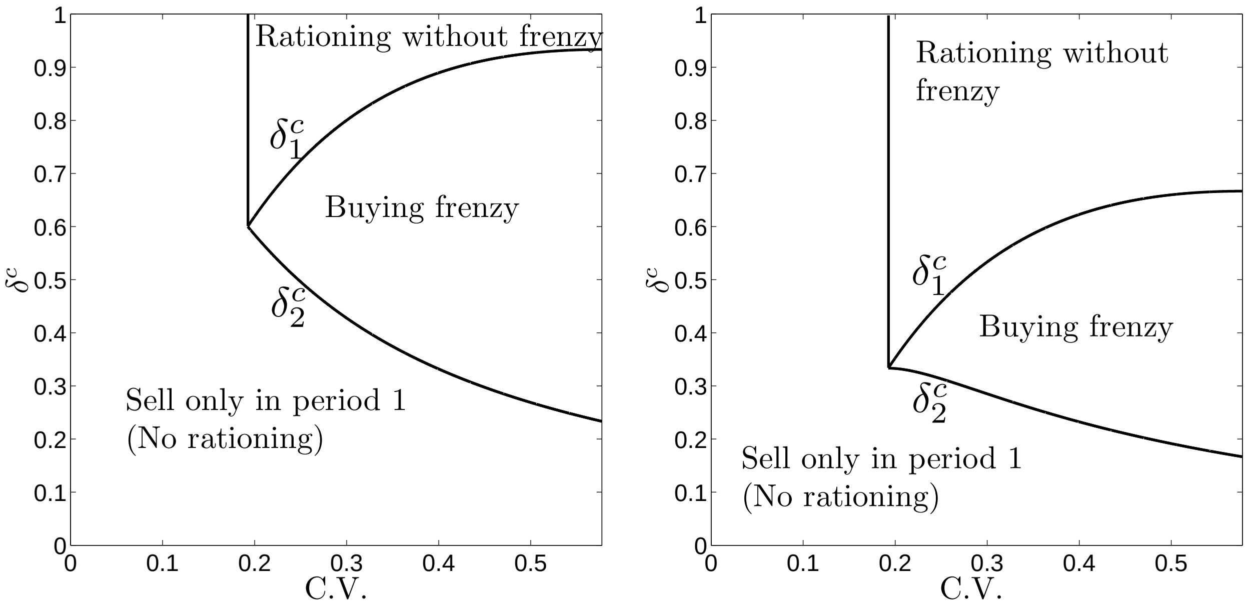
Proposition 2 *The firm's optimal policy induces a unique buying frenzy if* $\bar{F}(p_2^m) < 1$ *and* $\delta_2^c < \delta^c < \delta_1^c$*, where* $\delta_1^c = \dfrac{\mu - \delta^m \bar{\pi}}{T_1(p_2^m) + \frac{[\bar{F}(p_2^m)]^2}{f(p_2^m)}}$ *and* $\delta_2^c = \dfrac{\mu - \delta^m \bar{\pi}}{\mu - \underline{v} + \frac{1}{f(\underline{v})}}$.

If customers are sufficiently patient ($\delta^c > \delta_2^c$), the firm sells in both periods. If customers are excessively patient ($\delta^c \geq \delta_1^c$), then the firm rations supply in the early period without frenzy. In this case, the optimal production quantity is such that customers are indifferent between buying early and waiting as individuals. Finally, if customers are sufficiently but not excessively patient ($\delta_2^c < \delta^c < \delta_1^c$), the firm's optimal policy is to induce a buying frenzy in the early market. The equilibrium selection rule says that customers are indifferent between buying early and waiting as a group. But an individual customer who waits is strictly worse-off ($L_1(q_1) > 0$). This is because $p_2^m > p_2^e(0|q_1)$ and so $\mu - p_1(q_1) > \delta^c T_1(p_2^m)$.

To demonstrate the relevance of our dynamic framework, we revisit the finding by Balachander et al. (2009) that the introductory price of a new car is positively correlated with its scarcity (H3B, p. 1627). The concept of scarcity corresponds to $N_1 - q_1$ in our framework. From Proposition 1, it follows that the introductory price decreases with q_1. Yet the extent of a frenzy, $N_1 - q_1$, also decreases with q_1. Therefore, a decrease in q_1 (as might be caused by a shift in one of the model's primitives) will increase both the extent of a frenzy and the introductory price.

Corollary 1 *(a) The firm's optimal profit is increasing in* δ^m *and decreasing in* δ^c*. (b) The optimal period-1 production* q_1^* *is decreasing in both* δ^c *and* δ^m*.*

Figure 2 shows the optimal policy for uniform valuation distributions with mean μ and standard deviation σ. A uniform distribution is indexed by its coefficient of variation, σ/μ. The optimal



(a) $\delta^m = 0.6$ (b) $\delta^m = 1$

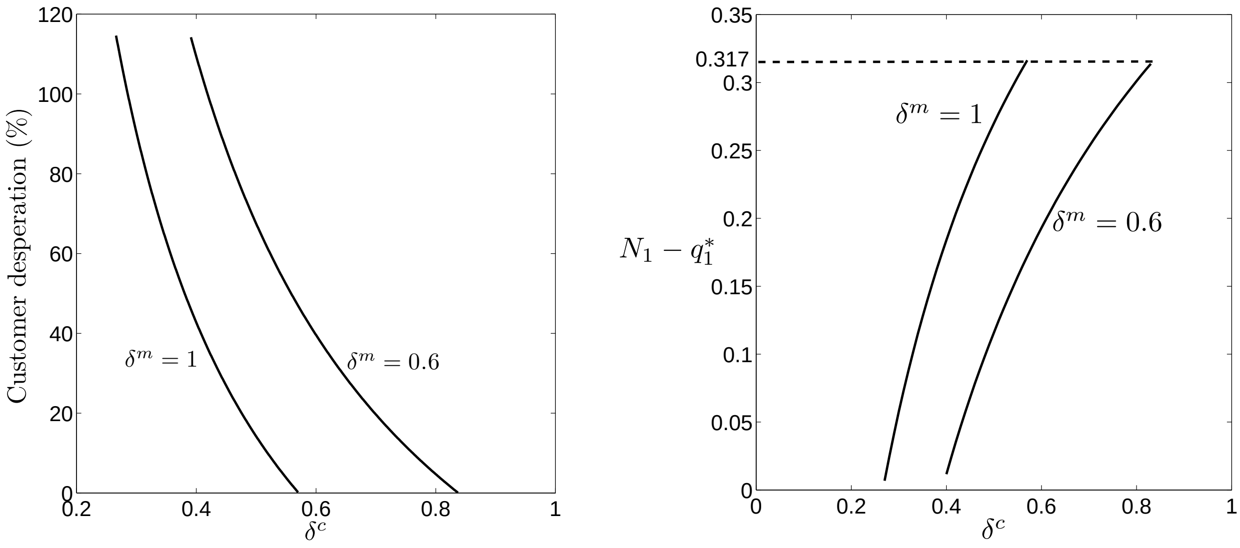
Figure 2 The firm's optimal policy when the firm can produce in both periods and when customers' valuation is uniform with coefficient of variation of C.V. C.V. $\leq \sqrt{3}/3$ to ensure that the support of v is non-negative.

policy does not induce a buying frenzy if $\sigma/\mu \leq \sqrt{3}/9$. The reason is that if $\sigma/\mu \leq \sqrt{3}/9$, then $p_2^m = \underline{v}$ and Proposition 2 implies that the optimal policy is $q_1^* = N_1$, i.e., to serve the entire market early. Therefore, a buying frenzy can occur only when customers are sufficiently uncertain about their valuations, which is a typical state with respect to new or innovative products.

As δ^m increases, the range of customer impatience that supports a buying frenzy shrinks.[6] In particular, it is never optimal to induce a buying frenzy if $\delta^c = \delta^m = 1$. Our model distinguishes rationing from frenzy policies. Although for $\delta^c > \delta_1^c$ it is not optimal to induce a frenzy, the firm does ration customers (i.e., $q_1^* < N_1$). Customers are indifferent between buying early and waiting and customer desperation, as defined in (3), is zero.

Figure 3(a) shows customer desperation, normalized as $L_1(q_1^*)/(\delta_c T_1(p_2^m))$ and Figure 3(b) plots the extent of the frenzy under the optimal policy. Customer desperation can be economically significant. The loss from not obtaining a good early is 20% of the expected value of consumption when, for example, $\delta^m = 1$ and $\delta^c = 0.5$. For a given value of δ^m, customers' relative desperation decreases as they become more patient (Figure 3(a)) whereas the extent of the frenzy $N_1 - q_1^*$ increases (Figure 3(b)) because q_1^* is decreasing in δ^c (see Corollary 1). Moreover, for a given δ^c, customer desperation also decreases in δ^m whereas the extent of the frenzy increases because q_1^* is decreasing in δ^m (Corollary 1).

[6] This is because $\delta_1^c - \delta_2^c$ is decreasing in δ^m (note that $T_1(p) + [\bar{F}(p)]^2/f(p)$ is decreasing in p and that $p_2^m > \underline{v}$).



(a) Customer desperation (b) Frenzy extent

Figure 3 Customer desperation $L(q_1^*)$ (normalized as $\frac{L(q_1^*)}{\delta^c T_1(p_2^m)} \times 100$) and the frenzy intensity $N_1 - q_1^*$ in a buying frenzy when customers' valuation is uniform with mean $\mu = 12$ and standard deviation $\sigma = 4$ (see Appendix B for details). In this figure, $N_1 = 1$. For $\delta^c \geq \delta_1^c$ and $\delta^c \leq \delta_2^c$, we have $L(q_1^*) = 0$.

Figure 4 compares the firm's optimal profits with two benchmark cases. One corresponds to the firm's profit under period-2 sales, $N_1 \bar{F}(p_2^m) p_2^m$, and the other corresponds to the commitment profit—that is charging μ in period 1 to N_1 customers (and committing, credibly, not to sell in period 2). Figure 4(b) shows that, at $\sigma/\mu = 0.5$, the loss due to noncommitment is almost 100% whereas inducing a frenzy recovers 66.7% to 36.2% of the profits (the more patient customers are the lower the percentage recovered). The equivalent profit recovered when $\sigma/\mu = 0.25$ is 66.9% to 55.2%; that is, the lower the coefficient of variation the higher the percentage recovered. Furthermore, the percentage of profits recovered by inducing a frenzy decreases with increasing firm impatience (lower δ^m).

4. Second-Period Arrivals

A limitation of the analysis in Section 3 is that customers arrive only in period 1. A natural way to generalize the model is to distinguish *early adopters*, who arrive in period 1 and face valuation uncertainty, and *followers*, who arrive in period 2 and know their valuations. In this section, we demonstrate that the results are robust to the arrival of followers.

Assume that N_2 new customers arrive in the second period. These customers have independently and identically distributed valuations with density f_2 and survival function $\bar{F}_2$. The period-2 demand is a regular downward-sloping demand $N_2 \bar{F}_2(p)$. This setup in consistent with other models in the literature (see e.g., Gallego and Şahin 2010 and Xie and Shugan 2001).

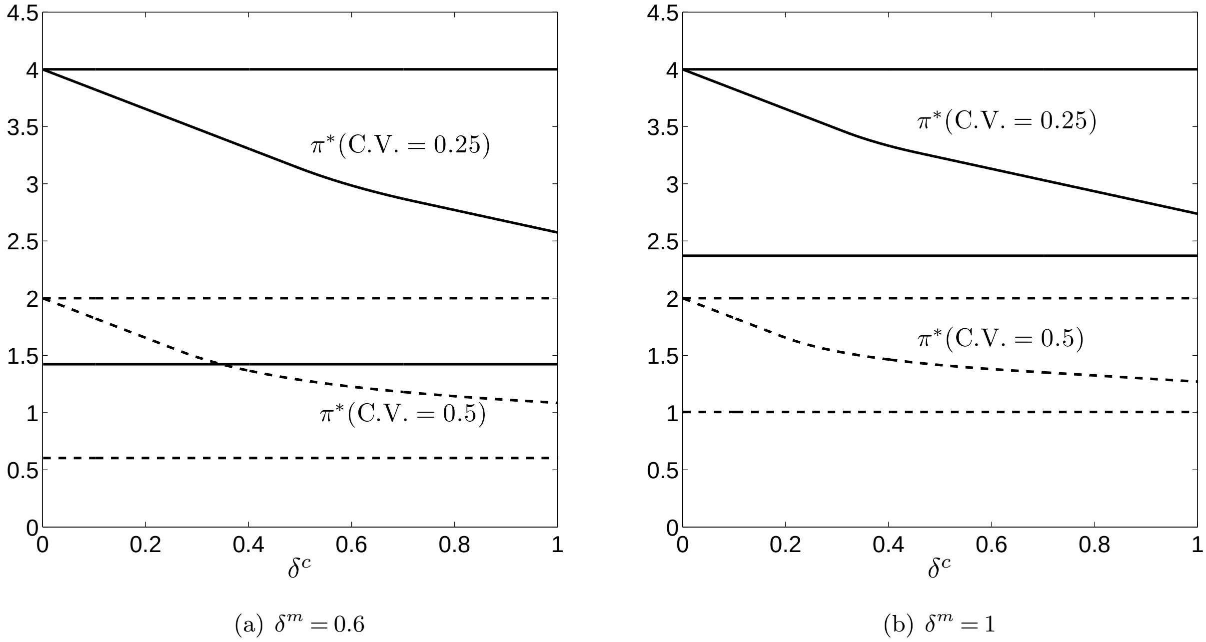


Figure 4 The optimal profit is bounded from above by commitment profits and from below by profits from selling only in period 2. Dashed and solid lines show the corresponding profits for C.V. $= 0.5$ **and C.V.** $= 0.25$**, respectively. In panel (a), frenzy profits when C.V.** $= 0.5$ **correspond to the interval** $\delta^c \in (0.27, 0.93)$**; when C.V.** $= 0.25$ **they correspond to the interval** $\delta^c \in (0.50, 0.72)$**. In panel (b), frenzy profits when** $C.V. = 0.5$ **correspond to the interval** $\delta^c \in (0.19, 0.66)$**; when C.V.** $= 0.25$ **they correspond to the interval** $\delta^c \in (0.31, 0.46)$**. In this figure,** $N_1 = 1$**.**

We need additional notation to facilitate the exposition in this section. We use subscripts to denote period and superscripts to denote the arrival cohort. Thus, $p_t^{i,m}$ denotes the monopoly price in period t for customer cohort i, i.e., $p_2^{1,m} \triangleq \text{argmax}_p\, N_1 p \bar{F}_1(p)$ and $p_2^{2,m} \triangleq \text{argmax}_p\, N_2 p \bar{F}_2(p)$. We assume that these two optimization problems are strictly concave (e.g., f_1 and f_2 are both log-concave). When Q_1 customers buy in period 1, we let $Q_2^b(p, Q_1) \triangleq (N_1 - Q_1)\bar{F}_1(p) + N_2 \bar{F}_2(p)$ denote the demand in period 2. Consistent with Section 3, we let $p_2^m \triangleq \text{argmax}_p\, p Q_2^b(p, 0)$ be the monopoly price when the firm sells only in period 2, and $Q_2^m = Q_2^b(p_2^m, 0)$ be the corresponding quantity.

Suppose the firm offers q_1 units for sale in period 1 at a price p_1. The period-2 price $p_2^e(x|q_1)$ maximizes the period-2 profits subject to market clearing constraint. In other words,

$$\begin{aligned} p_2^e(x|q_1) &= \text{argmax}_p\, p\, Q_2^b(p, \min(xN_1, q_1)) \\ \text{s.t.} \qquad Q_2^b(p, \min(xN_1, q_1)) &\geq q_1 - \min(xN_1, q_1) \end{aligned} \tag{4}$$

Define the unconstrained maximizer of (4) as $p_2^b(y) = \text{argmax}_p\, p\, Q_2^b(p, y)$.

Lemma 3 *$p_2^b(y)$ is increasing in y if $p_2^{1,m} < p_2^{2,m}$, constant if $p_2^{1,m} = p_2^{2,m}$, and decreasing otherwise.*

Observe that the constraint in (4) is not binding for $x \in [\frac{q_1}{N_1}, 1]$. For $x < \frac{q_1}{N_1}$, define $\hat{p}_2(x, q_1)$ as the solution to

$$N_1(1-x)\bar{F}_1(p) + N_2\bar{F}_2(p) = q_1 - xN_1. \tag{5}$$

There is a unique solution to (5) because its left-hand side is decreasing in p, takes a maximum of $N_1(1-x) + N_2 \geq q_1 - xN_1$ at $p = 0$ and a minimum of $0 \leq q_1 - xN_1$ at $p = v_{\max}$. The following result characterizes $p_2^e(x|q_1)$.

Lemma 4 *Assume $x \in [0,1]$ is part of an REE. If $x \in [0, \frac{q_1}{N_1}]$, then $p_2^e(x|q_1) = \min(p_2^b(xN_1), \hat{p}_2(x, q_1))$. Otherwise, $p_2^e(x|q_1) = p_2^b(q_1)$.*

We can show that $p_2^e(x|q_1)$ is (weakly) increasing only when $p_2^{1,m} \leq p_2^{2,m}$. Thus, we distinguish this case and the case $p_2^{1,m} > p_2^{2,m}$.

4.1. Buying frenzies when $\mathbf{p_2^{1,m} \leq p_2^{2,m}}$

We first show that, similar to Section 3, $p_2^e(x|q_1)$ is monotone in x. To do so, we define $\hat{x}(q_1)$ as the solution to

$$Q_2^b(p_2^b(xN_1), xN_1) = q_1 - xN_1. \tag{6}$$

We show in the Appendix that (6) has a unique solution when $Q_2^m \leq q_1 \leq N_1 + N_2\bar{F}_2(p_2^{2,m})$.

Lemma 5 *(a) Assume $p_2^{1,m} < p_2^{2,m}$. It follows that $p_2^e(x|q_1)$ is continuous and weakly increasing for $x \in [0,1]$ with $\frac{\partial}{\partial x}p_2^e(x|q_1) > 0$ at $x = 0$. (b) Assume $p_2^{1,m} = p_2^{2,m}$. (b1) If $q_1 \leq Q_2^m$ then $p_2^e(x|q_1) = p_2^m$. (b2) If $Q_2^m \leq q_1 \leq N_1 + N_2\bar{F}_2(p_2^{2,m})$, then $p_2^e(x|q_1) = p_2^m$ for $x \geq \hat{x}(q_1)$, and $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ otherwise. (b3) If $q_1 > N_1 + N_2\bar{F}_2(p_2^{2,m})$, then $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$. (b4) $p_2^e(x|q_1)$ is weakly increasing in x.*

Lemma 5 shows that $p_2^e(x|q_1)$ is weakly increasing in x. We leverage this property and apply Lemma 2 to eliminate all mixed strategies such that $p_2^e(x|q_1) > p_2^e(0|q_1)$. We then show that a similar result to that in Proposition 1 holds, i.e., the profit maximizing price for quantity q_1 is $p_1(q_1) = p_1^w(q_1)$ (see Appendix for proofs). Moreover, an early production quantity larger than the period-1 market size is never strictly profitable for the firm. This is because excess capacity in period 1 lowers the period-2 expected price ($p_2^e(x|q_1)$ is decreasing in q_1) and therefore lowers the price the firm can charge in the first period while not increasing sales.

In summary, the monopolist chooses the early production quantity $q_1 \leq N_1$ to maximize

$$\pi(q_1) = q_1 p_1(q_1) + \delta^m p_2^e(1|q_1) Q_2^b(p_2^e(1|q_1), q_1). \tag{7}$$

The measure of customer desperation derived in Section 3 generalizes as $L(q_1) = \delta^c(T_1(p_2^e(0|q_1)) - T_1(p_2^e(1|q_1)))^+$. The inequality $p_2^e(0|q_1) \leq p_2^e(1|q_1)$ holds as p_2^e is increasing in x when $p_2^{1,m} \leq p_2^{2,m}$.

We next investigate the conditions under which a buying frenzy equilibrium exists and is an optimal policy for the firm.

Lemma 6 *There is no buying frenzy equilibrium if* $\dfrac{N_2}{N_1} \geq \dfrac{F_1(p_2^m)}{\bar{F}_2(p_2^m)}$.

One implication of Lemma 6 is that buying frenzies are less likely to be optimal when the customer cohort arriving in period 2 is large relative to the one arriving in period 1 (i.e., when $N_2 \gg N_1$). In this case, the waiting equilibrium involves a relatively small number of period-1 customers who do not have much of an impact on the second-period price. For the rest of the section, we assume that $\frac{N_2}{N_1} < \frac{F_1(p_2^m)}{\bar{F}_2(p_2^m)}$; this assumption is equivalent to $Q_2^m < N_1$, which implies that the period-2 price *can* change when the waiting equilibrium is selected.[7] The following proposition characterizes when a frenzy occurs.

Proposition 3 *Assume that* $\frac{N_2}{N_1} < \frac{F_1(p_2^m)}{\bar{F}_2(p_2^m)}$. *Sufficient conditions for the firm's optimal policy to induce a buying frenzy are: (i)* $p_2^{1,m} < p_2^{2,m}$ *and* $\delta^c > \delta_2^c$*; or (ii)* $p_2^{1,m} = p_2^{2,m}$ *and* $\delta_2^c < \delta^c < \delta_1^c$ *where*

$$\delta_1^c = \frac{\mu - \delta^m p_2^m \bar{F}_1(p_2^m)}{T_1(p_2^m) + \frac{Q_2^m \bar{F}_1(p_2^m)}{N_1 f_1(p_2^m) + N_2 f_2(p_2^m)}}, \delta_2^c = \frac{\mu - \delta^m p_2^{2,m} \bar{F}_1(p_2^{2,m})}{T_1(p_2^w(N_1)) + \frac{N_1 \bar{F}_1(p_2^w(N_1))}{N_1 f_1(p_2^w(N_1)) + N_2 f_2(p_2^w(N_1))}}. \tag{8}$$

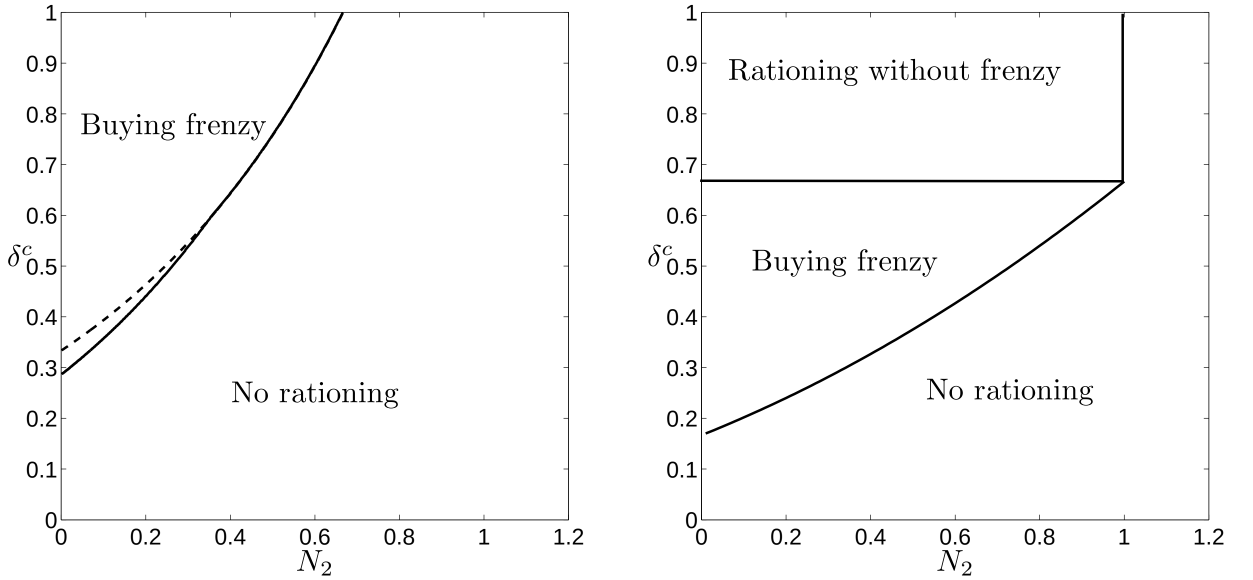
Proposition 3 establishes sufficient conditions under which $q_1^* < N_1$ and $L(q_1^*) > 0$ whenever $p_2^{1,m} < p_2^{2,m}$ or $p_2^{1,m} = p_2^{2,m}$.

Figure 5 generalizes Figure 2(b) when new customers arrive in period 2. Overall, the main insights from Section 3 carry over. The shape of the parameter space where frenzies occur does not change much relative to Figure 2(b) with one exception. When $p_2^{1,m} < p_2^{2,m}$, Figure 5(a) reveals such a parameter space that is larger than its counterpart in Figures 2(b) and 5(b). The areas labeled "rationing without frenzy", become "buying frenzy" in Figure 5(a). The area "rationing without frenzy" corresponds to the corner solution at Q_2^m with $p_2^e(1|Q_2^m) \leq p_2^e(0|Q_2^m)$. In Figure 5(a), this area disappears because we have $p_2^e(1|Q_2^m) > p_2^e(0|Q_2^m)$. Figure 5(b) considers the special case where $f_1(\cdot) = f_2(\cdot) = f(\cdot)$. In that case, we can derive a closed-form solution for the parameter space that generates frenzies. Moreover, the frenzy equilibrium is unique (see the proof of Proposition 3 in Appendix A).

4.2. Buying frenzies when $\mathbf{p_2^{1,m} > p_2^{2,m}}$

We first characterize the behavior of $p_2^e(x|q_1)$ which is significantly different in this case.

[7] This assumption is similar to the assumption in Section 3 that $p_2^b > \underline{v}$ (see Proposition 2). The reason is that if $p_2^b = \underline{v}$ then the size of the period 2 market when the firm sells only in period 2 (i.e., $N_1 \bar{F}(\underline{v}) = N_1$) is as large as the period-1 market size.



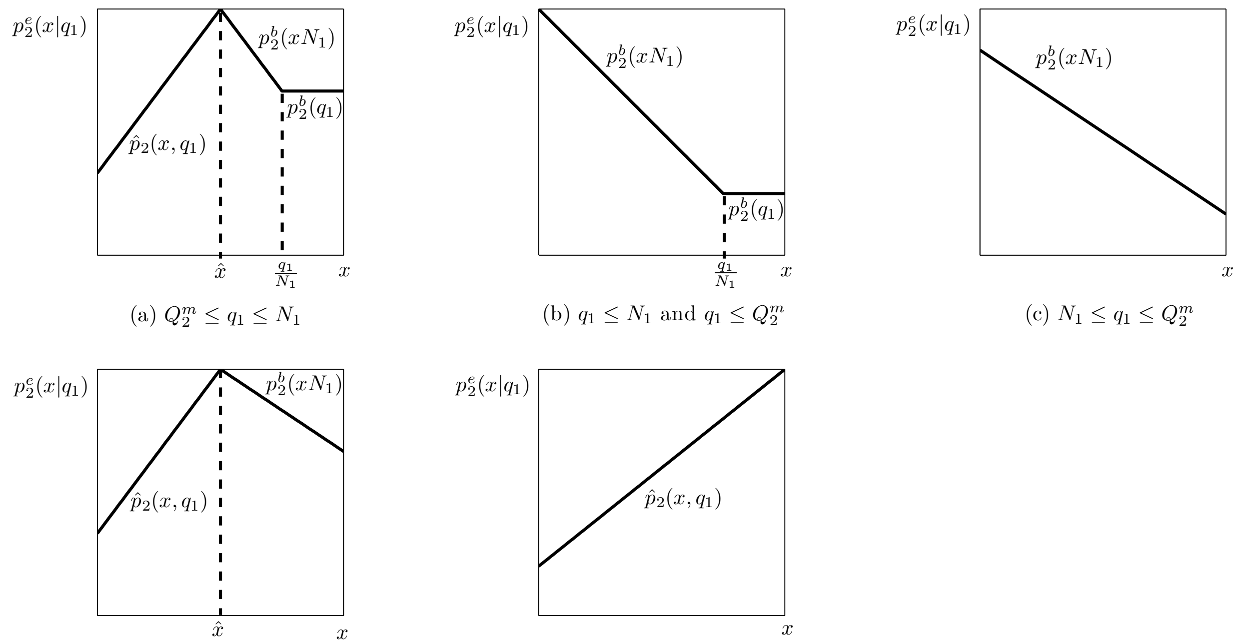
(a) $p_2^{1,m} < p_2^{2,m}$ (b) $p_2^{1,m} = p_2^{2,m}$

Figure 5 The firm's optimal policy when the valuation distribution of early arrivals is $U[0, b_1]$ and that of late arrivals is $U[0, b_2]$. In this figure, $\delta^m = 1$, $N_1 = 1$. Further, $(b_1, b_2) = (10, 20)$ in panel (a) and $(b_1, b_2) = (10, 10)$ in panel (b). The dashed line in panel (a) shows the sufficient conditions characterized in Proposition 3. The frenzy parameter space is slightly larger than the one implied by these conditions. Panel (b) shows the necessary and sufficient conditions for $f_1(\cdot) = f_2(\cdot)$.

Lemma 7 *Assume $p_2^{1,m} > p_2^{2,m}$. (i) If $q_1 \le Q_2^m$, then $p_2^e(x|q_1) = p_2^b(xN_1)$ with $\frac{dp_2^e(x|q_1)}{dx} < 0$ for $x \in [0, \frac{q_1}{N_1}]$ and $p_2^e(x|q_1) = p_2^b(q_1)$ for $x \in [\frac{q_1}{N_1}, 1]$. (ii) If $Q_2^m \le q_1 \le N_1 + N_2\bar{F}_2(p_2^{2,m})$, then $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ with $\frac{dp_2^e(x|q_1)}{dx} > 0$ for $x \in [0, \hat{x}(q_1)]$, $p_2^e(x|q_1) = p_2^b(xN_1)$ with $\frac{dp_2^e(x|q_1)}{dx} < 0$ for $x \in [\hat{x}(q_1), \frac{q_1}{N_1}]$ and $p_2^e(x|q_1) = p_2^b(q_1)$ for $x \in [\frac{q_1}{N_1}, 1]$. (iii) If $q_1 \ge N_1 + N_2\bar{F}_2(p_2^{2,m})$, then $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ with $\frac{dp_2^e(x|q_1)}{dx} > 0$ for $x \in [0, 1]$.*

In Lemma 7, $\hat{x}(q_1)$ and the conditions under which it exists and is unique are as in Section 4.1. Figure 6 schematically shows $p_2^e(x|q_1)$ in the five possible cases implied by Lemma 7. In effect, $p_2^e(x|q_1)$ can take three shapes. When case (i) in Lemma 7 applies, $p_2^e(x|q_1)$ is weakly decreasing as in Panels (b) and (c). Panels (a) and (d) correspond to case (ii) where $p_2^e(x|q_1)$ is increasing up to a threshold and then weakly decrease. Finally, in Panel (e), $p_2^e(x|q_1)$ is increasing in x.

The important difference, in this section, with the our analysis in Sections 3 and 4.1 is that the period-2 expected price, $p_e^e(x|q_1)$, is increasing in x only in Panel (e). Otherwise, the period-2 expected price can be decreasing in x implying that $p_2^e(0|q_1)$ is not always smaller than $p_2^e(x|q_1)$. Thus, the waiting equilibrium does not necessarily Pareto-dominate the mixed strategy REE. We can still rule out all mixed strategy REE such that $p_2^e(x|q_1) > p_2^e(0|q_1)$ by applying Lemma 2. We further show in the Appendix that the remaining mixed strategy REE are equivalent to a pure



(a) $Q_2^m \leq q_1 \leq N_1$ (b) $q_1 \leq N_1$ and $q_1 \leq Q_2^m$ (c) $N_1 \leq q_1 \leq Q_2^m$

(d) $N_1 \leq q_1$ and $Q_2^m \leq q_1 \leq N_1 + N_2\bar{F}_2(p_2^{2,m})$ (e) $N_1 + N_2\bar{F}(p_2^{2,m}) \leq q_1$

Figure 6 $p_2^e(x|q_1)$ **when** $p_2^{1,m} > p_2^{2,m}$**.**

strategy PDREE associated with a different firm announcement. Therefore, all mixed strategy REE can still be ignored without loss of generality. These results are summarized in the following proposition.

Proposition 4 *Assume the period 1 production is q_1. We can restrict the analysis, without loss of generality, to the announcement $\tilde{p}_1(q_1) = \min(p_1^b(q_1), p_1^w(q_1))$ and associated PDREE $x = 1$.*

The interpretation of $\tilde{p}_1(q_1)$ differs from that of $p_1(q_1)$ in Section 4.1. $\tilde{p}_1(q_1)$ is the profit maximizing price conditional on selling q_1 while $p_1(q_1)$ was the profit-maximizing price. The difference is that in the latter, selling q_1 was always optimal. However, when $p_2^{1,m} > p_2^{2,m}$, not selling at all, i.e., $x = 0$, may dominate selling at $\tilde{p}_1(q_1)$.[8] But the profits under $x = 0$ are weakly dominated by profits under alternative announcement $q_1 = 0$ and $p_1 > v_{\max}$ implying that such early production volumes will never be the firm's choice in equilibrium.

The firm optimization problem to characterize the production level in period 1 is

$$\max_{q_1} \tilde{p}_1(q_1)q_1 + \delta^m p_2^e(1|q_1) Q_2^b(p_2^e(1|q_1), q_1). \tag{9}$$

We now focus on the conditions that induce a frenzy in the market. Figure 6(a,b) are the only equilibrium period-2 prices that can be part of a frenzy because in a frenzy it must be that $q_1^* < N_1$.

[8] This will be the case when the profits from selling $\min(N_1, q_1)$ at $\tilde{p}_1(q_1)$, i.e., $\tilde{p}_1(q_1)q_1 + \delta_m p_2^b(q_1) Q_2^b(p_2^b(q_1), q_1)$, are lower than the profits under $x = 0$, i.e., $\delta^m Q_2^m p_2^m$ if $q_1 \leq Q_2^m$ and $\min(N_1, q_1)\hat{p}_2(q_1)$ otherwise.

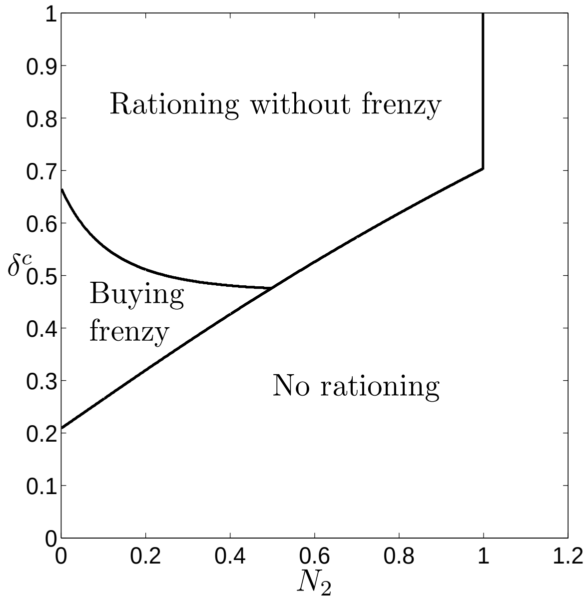


Figure 7 **The firm's optimal policy when the valuation distribution of early arrivals is $U[0,b_1]$ and that of late arrivals is $U[0,b_2]$. In this figure, $\delta^m = 1$ and $N_1 = 1$ and $(b_1, b_2) = (10, 5)$.**

However, a buying frenzy cannot happen when $p_2^e(x|q_1)$ is as in Figure 6(b), i.e., when $q_1 \le N_1$ and $q_1 < Q_2^m$. In this case, $p_2^e(1|q_1) < p_2^e(0|q_1)$ which implies $\tilde{p}_1(q_1) = p_1^b(q_1)$ (Proposition 4). Therefore, an individual customer is indifferent between buying and waiting. As a result, a buying frenzy may happen only in Figure 6(a). If $p_2^b(q_1) \le p_2^e(0|q_1)$, then an individual customer is indifferent between buying and waiting as both yield the surplus $\mu - p_1 = \delta^c T_1(p_2^b(q_1))$. Therefore, a frenzy may happen only if $\hat{p}_2(0, q_1) < p_2^b(q_1)$. For general distributions f_i, $i \in \{1, 2\}$ this inequality does not yield a threshold value for q_1 and hence we cannot provide sufficient conditions under which a frenzy happens. One needs to directly check $q_1^* < N_1$ and $\hat{p}_2(0, q_1^*) < p_2^b(q_1^*)$.

Figure 7 shows the firm optimal policy when customer valuations are uniform distributions.

To demonstrate the relevance of our analysis in Section 4.1 and 4.2, we revisit one of the two key results of Balachander et al. (2009). They find a positive association between buying frenzies and intrinsic preferences for a product that lasts beyond the introductory period of the product (H2A, p. 1626). One needs a dynamic model of sales—such as the one presented here—in order to interpret this finding properly. Assume that customers do not discount excessively. An increase in the strength of the second-period demand can be interpreted as an increase in p_2^m. When p_2^m is low, the equilibrium is likely to be a "rationing without frenzy" (upper left area in Figure 7). For p_2^m sufficiently large, the equilibrium becomes a "buying frenzy" (upper left area in Figure 5(a)). Buying frenzies are therefore associated with a high level of p_2^m, which can be interpreted as strong and persistent intrinsic preferences.

5. Conclusion

This paper offers a tractable and convenient framework to analyze buying frenzies in a dynamic context. In contrast with other works in the literature, the existence of buying frenzies in our model does not depend on the rule applied by the firm to allocate goods under rationing, or information asymmetry among customers. Buying frenzies occur when customers are sufficiently uncertain about their product valuations and when customers discount the future but not excessively. Overly patient customers wait until they learn their preferences for the product while very impatient customers are served early. The utility loss from not obtaining the good in a frenzy can be substantial. Similarly, the firm's gain from a frenzy policy can be economically large.

The analysis presented here offers a rich framework to interpret stylized facts and empirical findings about buying frenzies. We have illustrated this point by revisiting the two key results presented in Balachander et al. (2009), i.e., the introductory price positively correlated with product scarcity and a positive association between buying frenzies and intrinsic preferences for a product that lasts beyond the introductory period. These finding are difficult to rationalize within static models, but, both are consistent with our analysis.

References

Allen, F., G.R. Faulhaber. 1991. Rational rationing. *Economica* 58(230) 189-198.

Bagnoli, M., T. Bergstrom. 2005. Log-concave probability and its applications. *Economic Theory* 26(2) 445-469.

Balachander, S., Y. Liu, A. Stock. 2009. An empirical analysis of scarcity strategies in the automobile industry. *Management Science* 55(10) 1623-1637.

Balachander, S., A. Stock. 2009. Limited edition products: When and when not to offer them. *Marketing Science* 28(2) 336-355.

Becker, G.S. 1991. A note on restaurant pricing and other examples of social influences on price. *Journal of Political Economy* 99(5) 1109-1116.

Cachon, G.P., S. Netessine. 2004. Game theory in supply chain analysis. Chapter 2 in "Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era", Eds. D. Simchi-Levi, S.D. Wu, M. Shen. Kluwer Academic Publishers.

Cachon, G.P., R. Swinney. 2009. Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science* 55(3) 497-511.

Debo, L., G.J. van Ryzin. 2009. Creating sales with stock-outs. Working Paper.

Debo, L., S. Veeraraghavan. 2009. Models of herding behavior in operations management. Chapter in Consumer-Driven Demand and Operations Management Models, pp. 81-114, (eds. S. Netessine and C. Tang). Springer Series in Operations Research and Management Science.

DeGraba, P. 1995. Buying frenzies and seller-induces excess demand. *RAND Journal of Economics* 26(2) 331-342.

Denicolò, V., P.G. Garella. 1999. Rationing in a durable goods monopoly. *RAND Journal of Economics* 30(1) 44-55.

Gallego, G., Ö. Şahin. 2010. Revenue management with partially refundable fares. Operations Research 58(4) 817-833.

Lewis, T., and D. Sappington. 1994. Supplying Information to Facilitate Price Discrimination. *International Economic Review* 35(2) 309-327.

Liu, Q., G.J. van Ryzin. 2008. Strategic capacity rationing to induce early purchases. *Management Science* 54(6) 1115-1131.

Papanastasiou, Y., N. Bakshi, N. Savva. 2012. Strategic stock-outs with boundedly rational social learning. London Business Shcool Working Paper.

Liu, T., P. Schiraldi. 2012. Buying frenzies in durable-goods markets. Working paper.

Stock, A., S. Balachander. 2005. The making of a "hot product": A signaling explanation of marketers' scarcity strategy. *Management Science* 51(8) 1181-1192.

Su, X., F. Zhang. 2008. Strategic customer Behavior, commitment, and supply chain performance. *Management Science* 54(10) 1759-1773.

The Wall Street Journal. 2011. Losing $207 a Pop, H-P Brings Back Its iPad Rival. August 31.

Verhallen, T.M.M., H.S.J. Robben. 1994. Scarcity and preference: An experiment on unavailability and product evaluation. *Journal of Economic Psychology* 15(2) 315-331.

Xie, J., S. M. Shugan. 2001. Electronic tickets, smart cards, and online prepayments: When and how to advance sell. *Marketing Science* 20(3) 219-243.

Yu, M., R. Kapuscinski, H. Ahn. 2011. Advance selling with interdependent customer valuations. Working Paper.

Appendix A: Proofs

Proof of Lemma 1: The function $p\bar{F}(p)$ is unimodal and $\operatorname{argmax} p\bar{F}(p) = p_2^m$. (a) In this case, the constraint in (1) is non-binding. This is because $N_1 - \min(xN_1, q_1)\bar{F}(p) \geq q_1 - \min(xN_1, q_1)$ implies $N_1\bar{F}(p) + \min(xN_1, q_1)F(p) \geq q_1$. This inequality holds for $p = p_2^m$ because $q_1 \leq N_1\bar{F}(p_2^m)$ and hence $p_2^e(x|q_1) = p_2^m$.

(b) Assume first that $x \in [0, \frac{q_1 - N_1\bar{F}(p_2^m)}{N_1F(p_2^m)}]$. We have

$$
\begin{aligned}
x &\leq \frac{q_1 - N_1\bar{F}(p_2^m)}{N_1F(p_2^m)}, \\
xN_1F(p_2^m) &\leq q_1 - N_1\bar{F}(p_2^m),
\end{aligned}
$$

$$xN_1 \leq q_1 - N_1\bar{F}(p_2^m)(1-x),$$
$$xN_1 \leq q_1.$$

Therefore, $p_2^e(x|q_1) = \text{argmax}\, p\bar{F}(p)$ subject to the constraint $(N_1 - xN_1)\bar{F}(p) \geq q_1 - xN_1$. Because $(N_1 - xN_1)\bar{F}(p_2^m) \leq q_1 - xN_1$, it follows that the constraint is binding. We conclude that $p_2^e(x|q_1)$ is the unique solution to $(N_1 - xN_1)\bar{F}(p) = q_1 - xN_1$ (uniqueness follows because f is log-concave), i.e., $p_2^e(x|q_1) = \bar{F}^{-1}(\frac{q_1 - xN_1}{N_1 - xN_1})$. Finally, because $\frac{q_1 - xN_1}{N_1 - xN_1}$ is strictly decreasing in x, it follows that $p_2^e(x|q_1)$ is strictly increasing in x_1.

Now assume that $x \geq \frac{q_1 - N_1\bar{F}(p_2^m)}{N_1 F(p_2^m)}$ or equivalently $xN_1 \geq q_1 - N_1\bar{F}(p_2^m)(1-x)$. Two cases are possible: either $xN_1 \geq q_1$ or $q_1 \geq xN_1 \geq q_1 - N_1\bar{F}(p_2^m)(1-x)$. In either case, the constraint in (1) is non-binding and so $p_2^e(x|q_1) = p_2^m$ which is constant in x. $\square$

Proof of Proposition 1: We first derive the PDREE preferred by the firm for a given (q_1, p_1) and then select the profit maximizing price p_1 for a given q_1. We proceed by doing the analysis in two possible cases.

CASE 1: Assume period-1 production is $q_1 > N_1\bar{F}(p_2^m)$. We characterize the PDREE in the next result.

Claim 1 *(a) $p_1^w(q_1) < p_1^b(q_1)$. (b) Any mixed strategy REE is Pareto-dominated by REE $x = 0$. (c1) If $p_1 < p_1^w(q_1)$, then $x = 1$ is the unique PDREE. (c2)If $p_1 = p_1^w(q_1)$, then there are two PDREE $x = 0$ and $x = 1$ and the latter is preferred by the firm. (c3) Finally, if $p_1^w(q_1) < p_1$, then $x = 0$ is the unique PDREE.*

Proof: (a) follows directly from Lemma 1. (b) The condition in Lemma 2 holds for any $x \in (0,1)$. (c1) If $p_1 < p_1^w(q_1)$, then $\mu - p_1 > \mu - p_1^w(q_1) = \delta_c T_1(p_2^e(0|q_1)) \geq \delta_c T_1(p_2^e(x|q_1))$ for all x. Hence $x = 1$ is the unique PDREE.

(c2) If $p_1 = p_1^w(q_1)$, then $x = 0$ and $x = 1$ are REE and PDREE because $\mu - p_1 = \delta_c T_1(p_2^e(0|q_1))$. We show that the PDREE $x = 1$ is preferred by the firm. The firm profit under the equilibrium $x = 0$ is $\pi(x=0) = \delta^m N_1 p_2^e(0|q_1)\bar{F}(p_2^e(0|q_1))$ and that under the equilibrium $x = 1$ is $\pi(x=1) = p_1 q_1 + \delta^m (N_1 - q_1) p_2^e(1|q_1)\bar{F}(p_2^e(1|q_1)) = (p_1 - \delta^m p_2^m \bar{F}(p_2^m))q_1 + \delta^m N_1 p_2^m \bar{F}(p_2^m)$. The result then follows because $p_1 - p_2^m\bar{F}(p_2^m) \geq 0$ and $p_2^m\bar{F}(p_2^m) > p_2^e(0|q_1)\bar{F}(p_2^e(0|q_1))$. The former inequality follows because $p_1 = p_1^w(q_1)$ and

$$\begin{aligned}
p_1^w(q_1) - p_2^m\bar{F}(p_2^m) &= \mu - \int_{p_2^e(0|q_1)} (v - p_2^e(0|q_1))dF - \int_{p_2^m} p_2^m dF \\
&> \mu - \int_{p_2^m} (v - p_2^e(0|q_1) + p_2^m)dF \quad \text{because } p_2^e(0|q_1) > p_2^m \text{ (Lemma 1)} \\
&= \int_0^{p_2^m} v dF + \int_{p_2^m} (p_2^e(0|q_1) - p_2^m)dF \\
&> 0.
\end{aligned}$$

(c3) $x=0$ is an REE because $\delta_c T_1(p_2^e(0|q_1)) > \mu - p_1$. Further, $x=1$ is not an REE because $\mu - p_1 < \delta^c T_1(p_2^e(1|q_1))$. Therefore $x=0$ is the only PDREE. $\square$

CASE 2: Assume period-1 production is $q_1 \le N_1 \bar{F}(p_2^m)$. We characterize the PDREE in the next result.

Claim 2 *(a) $p_1^w(q_1) = p_1^b(q_1)$. (b1) If $p_1 < p_1^w(q_1)$, then $x=1$ is the unique PDREE. (b2) If $p_1 = p_1^w(q_1)$, then all $x \in [0,1]$ are Pareto-equivalent PDREE and $x=1$ is the PDREE preferred by the firm. (b3) If $p_1 > p_1^w(q_1)$, then $x=0$ is the unique PDREE.*

Proof: (a) This follows from Lemma 1 because $p_2^e(0|q_1) = p_2^e(1|q_1) = p_2^m$ for $q_1 \le N_1 \bar{F}(p_2^m)$.

(b1) We have $\mu - p_1 > \mu - p_1^w(q_1) = \delta^c T_1(p_2^e(x|q_1)) = \delta^c T_1(p_2^m)$ for all $x \in [0,1]$, hence a customer is better off buying the product and $x=1$ is the unique PDREE.

(b2) Because $\mu - p_1 = \mu - p_1^w(q_1) = \delta^c T_1(p_2^e(x|q_1)) = \delta^c T_1(p_2^m)$ for all $x \in [0,1]$, it follows that all $x \in [0,1]$ are PDREE because the customer's expected surplus is the same. We show that $x=1$ in the PDREE that maximizes the firm profits. The firm profit for a given $x \in [0,1]$ is $\pi(x) = Q_1 p_1^w(q_1) + (N_1 - Q_1) p_2^m \bar{F}(p_2^m) = Q_1(p_1^w(q_1) - p_2^m \bar{F}(p_2^m)) + N_1 p_2^m \bar{F}(p_2^m)$ where $Q_1 = \min(xN_1, q_1)$ is the sales in period 1. The profit $\pi(x)$ is increasing in x because $p_1^w(q_1) = \mu - \delta^c T_1(p_2^m)$ and

$$p_1^w(q_1) - p_2^m \bar{F}(p_2^m) = \mu - \int_{p_2^m} (v - p_2^m) dF - \int_{p_2^m} p_2^m dF = \int_0^{p_2^m} v dF > 0.$$

Therefore, $x=1$ is the PDREE that maximizes the firm profits.

(b3) $\mu - p_1 < \delta^c T_1(p_2^e(x|q_1))$, hence $x=0$ is the unique PDREE. $\square$

We can now conclude by collecting the PDREE preferred by the firm for an announcement (q_1, p_1) from the two cases above. For a period-1 price such that $p_1 > p_1^w(q_1)$, customers wait in any PDREE and hence the firm profit is $\pi(x=0) = \delta^m p_2^e(0|q_1) \bar{F}(p_2^e(0|q_1))$. On the other hand, the firm profit, if it sets $p_1 = p_1^w(q_1)$, is $\pi(x=1) = p_1^w(q_1) q_1 + \delta^m (N_1 - q_1) p_2^m \bar{F}(p_2^m)$. Because $\pi(x=1) > \pi(x=0)$, it follows that any price $p_1 > p_1^w(q_1)$ is dominated by $p_1(q) = p_1^w(q_1)$. For a period-1 price such that $p_1 < p_1^w(q_1)$, customers buy in any PDREE and the firm profit is $\pi(x=1) = p_1 q_1 + \delta^m (N_1 - q_1) p_2^m \bar{F}(p_2^m)$. This profit is increasing in p_1 and it follows that $p_1 = p_1^w(q_1)$ dominates $p_1 < p_1^w(q_1)$. Therefore, the firm profits are maximized at $p_1(q_1) = p_1^w(q_1)$. $\square$

Proof of Proposition 2: If $\bar{F}(p_2^m) = 1$ then, by Lemma 1, $p_2^e(0|q_1) = p_2^m = \underline{v}$ and $p_1^w(q_1) = \mu - \delta^c T_1(p_2^m)$. It follows from (2) that $\pi(q_1) = q_1(\mu - \delta^c T_1(p_2^m)) + \delta^m (N_1 - q_1) p_2^m \bar{F}(p_2^m)$, which is increasing in q_1. This is because $\mu - \delta^c T_1(p_2^m) - \delta^m p_2^m \bar{F}(p_2^m) \ge \mu - T_1(p_2^m) - p_2^m \bar{F}(p_2^m) = 0$[9].

[9] For the particular case $\delta^c = \delta^m = 1$, the profit function is independent of q_1 and so all $q_1 \in [0, N_1]$ are optimal. In this case, customers are still indifferent between buying and waiting and a frenzy does not happen.

Hence $q_1^* = N_1$; in other words, the firm serves the entire market in period 1 and customers are indifferent between buying and waiting. Thus, a buying frenzy cannot occur.

Assume for the remainder of the proof that $\bar{F}(p_2^m) < 1$. Observe that if $q_1^* = 0$ then the firm's optimal policy is to sell only in period 2. We prove next that the optimal production is strictly positive, which implies that selling only in period 2 is never optimal when the firm has the option of producing over time.

If $q_1 \leq N_1\bar{F}(p_2^m)$ then $p_2^e(x|q_1) = p_2^m$ and $p_1(q_1) = \mu - \delta^c T_1(p_2^m)$, i.e., $p_1(q_1)$ becomes independent of q_1 (Lemma 1 and Proposition 1). The profit function in (2) is then linear and increasing in q_1 because

$$\begin{aligned}\frac{d\pi(q_1)}{dq_1} &= \mu - \delta^c T_1(p_2^m) - \delta^m p_2^m \bar{F}(p_2^m)\\ &\geq \mu - T_1(p_2^m) - p_2^m \bar{F}(p_2^m)\\ &= \int \min(v, p_2^m)\, dF - p_2^m \bar{F}(p_2^m)\\ &= \int_{\underline{v}}^{p_2^m} v dF > 0.\end{aligned}$$

The last inequality follows because $\bar{F}(p_2^m) < 1$. Therefore, $q_1 = N_1\bar{F}(p_2^m)$ yields greater profits than any lower q_1. However, the production quantity $q_1 = N_1\bar{F}(p_2^m)$ cannot induce a buying frenzy because customers are indifferent between buying early and waiting. It follows that a necessary condition for a buying frenzy is that $q_1 > N_1\bar{F}(p_2^b)$. We next characterize the sufficient conditions under which it is optimal for the firm to induce a buying frenzy.

If $N_1\bar{F}(p_2^m) \leq q_1$, then $p_2^e(0|q_1) = \bar{F}^{-1}(\frac{q_1}{N_1})$ (Lemma 1). The firm's profit function is

$$\pi(q_1) = q_1(\mu - \delta^c T_1(p_2^e(0|q_1))) + \delta^m (N_1 - q_1) p_2^m \bar{F}(p_2^m),$$

which is strictly concave in q_1 because (we suppress the argument in $p_2^e(0|q_1)$ for brevity)

$$\frac{d\pi(q_1)}{dq_1} = \mu - \delta^c T_1(p_2^e) - \delta^c \frac{q_1^2}{N_1^2 f(p_2^e)} - \delta^m q_1 p_2^m \bar{F}(p_2^m),$$

and

$$\frac{d^2\pi(q_1)}{dq_1^2} = -\delta^c \frac{q_1}{N_1^2 f(p_2^e)} \Big(1 + \frac{2[f(p_2^e)]^2 + \bar{F}(p_2^e) f'(p_2^e)}{[f(p_2^e)]^2}\Big) < 0.$$

The inequality holds because f is log-concave and so $[f(p)]^2 + \bar{F}(p)f'(p) > 0$ (Bagnoli and Bergstrom 2005). The first-order conditions then characterize the optimal q_1; that is, q_1^* solves

$$\frac{d\pi}{dq_1} = \mu - \delta^c T_1(p_2^e(0|q_1)) - \delta^c \frac{q_1^2}{N_1^2} \frac{1}{f(p_2^e(0|q_1))} - \delta^m p_2^m \bar{F}(p_2^m) = 0, \tag{10}$$

provided that $N_1\bar{F}(p_2^m) \leq q_1 \leq N_1$. Since the left-hand side of (10) is strictly decreasing in q_1 (because π is strictly concave), it follows that if $\frac{d\pi}{dq_1}|_{q_1=N_1} < 0$ and $\frac{d\pi}{dq_1}|_{q_1=N_1\bar{F}(p_2^m)} > 0$ then there is a unique interior solution in $(N_1\bar{F}(p_2^m), N_1)$ and otherwise we obtain a boundary solution. To summarize (1) if $\delta^c \leq \frac{\mu-\delta^m \bar{p}i}{\mu-\underline{v}+\frac{1}{f(\underline{v})}}$, then $q_1^* = N_1$, (2) if $\frac{\mu-\delta^m \bar{\pi}}{\mu-\underline{v}+\frac{1}{f(\underline{v})}} < \delta^c < \frac{\mu-\delta^m \bar{\pi}}{T_1(p_2^m)+\frac{[\bar{F}(p_2^m)]^2}{f(p_2^m)}}$, then $q_1^* \in (N_1\bar{F}(p_2^m), N_1)$ and is characterized by (10), and (3) otherwise, $q_1^* = N_1\bar{F}(p_2^m)$.

A frenzy can happen only in the second case. It remains to show that is this case $L_1(q_1^*) > 0$. This follows because $N_1\bar{F}(p_2^m) < q_1^* < N_1$ and so $p_2^e(0|q_1^*) < p_2^m$ (Lemma 1). Therefore, $L_1(q_1^*) > 0$, because $T_1(p)$ is strictly decreasing in p. $\square$

Proof of Corollary 1: (a) This part follows from the Envelope Theorem given that, by (2), we have $\frac{d\pi^*}{d\delta^c} = -q_1^* T_1(p_2^e(0|Q_1^*)) < 0$ and $\frac{d\pi^*}{d\delta^m} = (N_1 - q_1^*)p_2^m \bar{F}(p_2^m) \geq 0$.

(b) q_1^* solves (10). Differentiating with respect to δ^c and simplifying yields (we suppress the argument in $p_2^e(0|q_1^*)$ for brevity)

$$\frac{dq_1^*}{d\delta^c} = \frac{T_1(p_2^e) + \frac{q_1^{*2}}{N_1^2}\frac{1}{f(p_2^e)}}{\delta^c \bar{F}(p_2^e)\frac{dp_2^e}{dq_1} - \frac{\delta^c q_1^*}{N_1^2}\frac{2f(p_2^e)-f'(p_2^e)\frac{dp_2^e}{dq_1}q_1^*}{[f(p_2^e)]^2}} < 0.$$

The inequality follows because $p_2^w(0|q_1) = \bar{F}^{-1}(\frac{q_1}{N_1})$ and so $\frac{dp_2^e(0|q_1)}{dq_1} = \frac{-1}{N_1 f(p_2^e)} < 0$. Moreover, since $\bar{F}(p_2^e(0|q_1)) = \frac{q_1}{N_1}$, we have $2f(p_2^e) - f'(p_2^e)\frac{dp_2^e}{dq_1}q_1^* = \frac{2[f(p_2^e)]^2 + f'(p_2^e)\bar{F}(p_2^e)}{[f(p_2^e)]^2} > 0$ (because f is log-concave; see the proof of Proposition 2).

Similarly, it is straightforward to observe that

$$\frac{dq_1^*}{d\delta^m} = \frac{p_2^m \bar{F}(p_2^m)}{\delta^c \bar{F}(p_2^e)\frac{dp_2^e}{dq_1} - \frac{\delta^c q_1^*}{N_1^2}\frac{2f(p_2^e)-f'(p_2^e)\frac{dp_2^e}{dq_1}q_1^*}{[f(p_2^e)]^2}} < 0. \qquad \square$$

Proof of Lemma 3: $p_2^b(y)$ solves the first-order condition

$$(N_1 - y)\bar{F}_1(p) - p(N_1 - y)f_1(p) + N_2\bar{F}_2(p) - pN_2 f_2(p) = 0.$$

Differentiating both sides with respect to y, we obtain

$$-[\bar{F}_1(p_2^b) - p_2^b f_1(p_2^b)] + (N_1 - y)[-2f_1(p_2^b)\frac{dp_2^b}{dy} - pf_1'(p_2^b)\frac{dp_2^b}{dy}] + N_2[-2f_2(p_2^b)\frac{dp_2^b}{dy} - pf_2'(p_2^b)\frac{dp_2^b}{dy}] = 0,$$

or

$$\frac{dp_2^b}{dy} = \frac{\bar{F}_1(p_2^b) - p_2^b f_1(p_2^b)}{(N_1 - y)[-2f_1(p_2^b) - pf_1'(p_2^b)] + N_2[-2f_2(p_2^b) - pf_2'(p_2^b)]}.$$

The denominator is negative as f_i, $i \in \{1, 2\}$ is log-concave and hence $p\bar{F}_i(p)$ is strictly concave. Further, the numerator is negative if $p_2^{1,m} < p_2^{2,m}$, positive if $p_2^{1,m} > p_2^{2,m}$, and 0 otherwise. $\square$

Proof of Lemma 4: If $x \in [0, \frac{q_1}{N_1}]$, then the constraint is not binding if $Q_2^b(p_2^b(xN_1), xN_1) \geq q_1 - xN_1$ and $p_2^e(x|q_1) = p_2^b(xN_1)$. Otherwise, because the objective function is strictly concave, $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$. In other words, the price that clears the inventory remaining from period 1 is optimal. The second part follows because if $x \in [\frac{q_1}{N_1}, 1]$, the constraint in (4) is not binding. $\square$

Proof of Lemma 5: (a) If $x \in [\frac{q_1}{N_1}, 1]$, then $p_2^e(x|q_1) = p_2^b(q_1)$ (Lemma 4) which is strictly increasing in q_1 (Lemma 3) and constant in x. If, on the other hand, $x \in [0, \frac{q_1}{N_1}]$, then $p_2^e(x|q_1) = \min(p_2^b(xN_1), \hat{p}_2(x, q_1))$ which is continuous in x. Further, $p_2^b(xN_1)$ and $\hat{p}_2(x, q_1)$ are strictly increasing in x. The former follows from Lemma 3. To prove the latter, recall that $\hat{p}_2(x, q_1)$ solves (5). Upon differentiating, we obtain

$$\frac{d\hat{p}_2}{dx} = \frac{N_1 F_1(\hat{p}_2)}{(N_1 - xN_1) f_1(\hat{p}_2) + N_2 f_2(\hat{p}_2)} > 0.$$

It follows that $p_2^e(x|q_1)$ is strictly increasing at $x = 0$ and further weakly increasing in $(0, 1]$.

(b) We first establish the conditions under which (6) has a unique solution. We then characterize $p_2^e(x|q_1)$ and show that it is (weakly) increasing in x.

Claim 3 *There exists a unique solution $x \in [0, 1]$ to (6) if and only if $Q_2^m \leq q_1 \leq N_1 + N_2 \bar{F}_2(p_2^{2,m})$.*

Proof: Define

$$G(x) = Q_2^b(p_2^b(xN_1); xN_1) - (q_1 - xN_1). \tag{11}$$

$G(x)$ is increasing in x because $\frac{dG(x)}{dx} = \frac{\partial Q_2^b}{\partial p} \frac{\partial p_2^b}{\partial x} + \frac{\partial Q_2^b}{\partial Q_1} \frac{\partial x N_1}{\partial x} + N_1 > 0$. The inequality follows because $\frac{\partial Q_2^b}{\partial p} \leq 0$ and $\frac{\partial p_2^b}{\partial x} \leq 0$ (Lemma 3). Further, $\frac{\partial Q_2^b}{\partial Q_1} \frac{\partial x N_1}{\partial x} + N_1 = N_1(-\bar{F}_1(p_2^b) + 1) > 0$. Therefore, if (6) has a solution, it is unique.

To guarantee the existence of a solution it must be that $G(0) \leq 0 \leq G(1)$ or $Q_2^m \leq q_1 \leq N_1 + N_2 \bar{F}_2(p_2^{2,m})$. $\square$

We can now prove the results in part (b) of the lemma. First, note that from the proof of Claim 3, the function $G(x)$ is increasing in x. We use this property to characterize $p_2^e(x|q_1)$.

(b1) If $q_1 \leq Q_2^m$, then $G(x) \geq 0$ for all x which implies $p_2^b(xN_1) \leq \hat{p}_2(x, q_1)$. Therefore, from Lemma 4, we have $p_2^e(x|q_1) = p_2^b(xN_1)$ if $x \leq \frac{q_1}{N_1}$ and else $p_2^b(x|q_1) = p_2^b(q_1)$. Because $p_2^{1,m} = p_2^{2,m}$, then $p_2^{1,m} = p_2^{2,m} = p_2^b(xN_1) = p_2^b(q_1) = p_2^m$; see (4), i.e., $p_2^e(x|q_1) = p_2^m$.

(b2) If $Q_2^m \leq q_1 \leq N_1 + N_2 \bar{F}_2(p_2^{2,m})$, then (6) has a unique solution $\hat{x}(q_1) \in [0, 1]$ (Claim 3). Hence, if $x \geq \hat{x}(q_1)$, then $p_2^e(x|q_1) = p_2^m$. Otherwise, $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$. As a caveat, note that if $q_1 < N_1$, then for $x \geq \frac{q_1}{N_1}$, we have $p_2^e(x|q_1) = p_2^b(q_1)$ (Lemma 4). However, when $p_2^{1,m} = p_2^{2,m}$, we have $p_2^b(q_1) = p_2^m$.

(b3) If $q_1 > N_1 + N_2 \bar{F}_2(p_2^{2,m})$, then $p_2^e(x|q_1) = \hat{p}_2(x, q_1)$ for all $x \in [0, 1]$ because $G(x) \leq 0$ for all x and hence $\hat{p}_2(x, q_1) \leq p_2^b(xN_1) = p_2^m$.

(b4) This result is immediate from the proof of parts (b1)-(b3). $\square$

Derivation of $\mathbf{p_1(q_1)}$ when $\mathbf{p_2^{1,m} \leq p_2^{2,m}}$. The derivation is similar to that in Section 3 and Proposition 1. We first show that mixed strategy REE are never PDREE. If $p_1^{1,m} < p_2^{2,m}$, then $p_2^e(x|q_1)$ is increasing in x and Lemma 2 applies. If $p_1^{1,m} = p_2^{2,m}$, then the proof of Lemma 5 shows that we should distinguish three subcases (b1)-(b3). In cases (b2) and (b3), we have $p_2^e(0|q_1) < p_2^e(x|q_1)$ for any $x \in (0,1)$ and Lemma 2 applies again. In case (b1), $p_2^e(x|q_1)$ is constant. Any $x \in (0,1)$ is an REE and a PDREE if and only if $p_1 = p_2^m$. A proof similar to Proposition 1, omitted here for brevity, shows that $x = 1$ is the weakly preferred PDREE by the firm. □

Claim 4 *The profit maximizing price is* $p_1(q_1) = p_1^w(q_1)$ *and the associated PDREE is* $x = 1$.

Proof: If $p_1 > p_1^w(q_1)$ then $x = 0$ is the unique PDREE. If $p_1 = p_1^w(q_1)$ then $x = 1$ is the PDREE preferred by the firm. If $p_1 < p_1^w(q_1)$ then $x = 1$ is the unique PDREE. The profits are highest for $p_1 = p_1^w(q_1)$. □

Claim 5 $q_1 > N_1$ *is never optimal.*

Proof: $p_2^e(x|q_1)$ is weakly decreasing in q_1. Therefore $p_1(q_1)$ is also weakly decreasing in q_1. Increasing q_1 above N_1 does not increase sales $Q_1 = \min(xN_1, q_1)$ and reduces $p_1(q_1)$ and hence suboptimal. □

Proof of Lemma 6: The condition in the lemma is equivalent to $N_2\bar{F}_2(p_2^m) + N_1\bar{F}_1(p_2^m) \geq N_1$ or $Q_2^m \geq N_1$. To prove the result, it suffices to show that $q_1^* \geq Q_2^m$ because this inequality implies that $q_1^* \geq N_1$, in other words, no excess demand in equilibrium and hence no frenzy.

Assume to the contrary that $q_1^* < Q_2^m$. Then $p_2^e(0|q_1) = p_2^m$ and $p_1(q_1) = \mu - \delta^c T_1(p_2^m)$, i.e., $p_1(q_1)$ becomes independent of q_1. The firm's profit function,

$$\pi(q_1) = q_1(\mu - \delta^c T_1(p_2^m)) + \delta^m p_2^e(1|q_1)\Big[(N_1 - q_1)\bar{F}_1(p_2^e(1|q_1)) + N_2\bar{F}_2(p_2^e(1|q_1))\Big],$$

is increasing in q_1. To see this, observe that

$$\frac{d\pi(q_1)}{dq_1} = \mu - \delta^c T_1(p_2^m) - \delta^m p_2^e(1|q_1)\bar{F}_1(p_2^e(1|q_1)) > 0.$$

The inequality follows because

$$\begin{aligned}\mu - \delta^c T_1(p_2^m) &> p_2^m \bar{F}_1(p_2^m) \\ &\geq p_2^e(1|q_1)\bar{F}_1(p_2^e(1|q_1)) \\ &\geq \delta^m p_2^e(1|q_1)\bar{F}_1(p_2^e(1|q_1)).\end{aligned}$$

The second inequality follows because $p\bar{F}_1(p)$ is single-peaked at $p_2^{1,m}$, and further (i) $p_2^{1,m} < p_2^m < p_2^e(1|q_1)$ if $p_2^{2,m} > p_2^{1,m}$ and (ii) $p_2^e(1|q_1) \leq p_2^m \leq p_2^{1,m}$ if $p_2^{2,m} \leq p_2^{1,m}$. Therefore, $q_1 = Q_2^m$; a contradiction. □

Proof of Proposition 3: (i) $p_2^{1,m} < p_2^{2,m}$. Therefore, $p_2^e(0|q_1) < p_2^e(1|q_1)$ (Lemma 5). Thus, a frenzy occurs as long as $q_1 < N_1$. In other words, a sufficient condition is

$$\frac{d\pi(q_1)}{dq_1} = \mu - \delta^c T_1(p_2^e(0|q_1)) + \delta^c q_1 \bar{F}_1(p_2^e(0|q_1)) \frac{dp_2^e(0|q_1)}{dq_1} - \delta^m p_2^e(1|q_1)\bar{F}_1(p_2^e(1|q_1)) \bigg|_{q_1=N_1} < 0, \quad (12)$$

or, equivalently, $\delta^c > \delta_2^c$. Note that the coefficient of δ^c in (12) is negative because $p_2^e(0|q_1)$ is decreasing in q_1. To see this, note that by Lemma 4, $p_2^e(0|q_1) = \min(p_2^b(xN_1), \hat{p}_2(x,q_1))$. Because $p_2^b(xN_1)$ is constant in q_1 and $\hat{p}_2(x,q_1)$ is decreasing in q_1 (see (5)), we conclude that $\frac{dp_2^e(0|q_1)}{dq_1} \leq 0$.

(ii) $p_2^{1,m} = p_2^{2,m}$. Because the optimal production level is at most N_1 (Claim 5), it follows from Lemma 5 that $p_2^e(1|q_1) = p_2^m = p_2^{1,m} = p_2^{2,m}$ and so is constant in q_1. Unlike case (i), in this case, a frenzy occurs only if $q_1^* \in (Q_2^m, N_1)$. That is, we must also eliminate the corner at Q_2^m in which no frenzy occurs because $p_2^e(1|Q_2^m) = p_2^e(0|Q_2^m)$ (see Lemma 5). Therefore, a set of sufficient conditions for a buying frenzy equilibrium to exist is $\frac{d\pi}{dq_1}\,|_{q_1=N_1} < 0$ and $\frac{d\pi}{dq_1}\,|_{q_1=Q_2^m} > 0$ or $\delta_2^c < \delta^c < \delta_1^c$. Similar to case (i), these conditions are not necessary and do not guarantee the *uniqueness* of the frenzy equilibrium (unlike the model in Section 3 and Proposition 2).

Finally, consider the case $f_2(\cdot) = f_1(\cdot) = f(\cdot)$ discussed in the text. In this case, the firm's optimal policy is to induce a unique buying frenzy when $F(p_2^m) > \dfrac{N_2}{N_1+N_2}$ and $\delta_2^c < \delta^c < \delta_1^c$ where $\delta_1^c = \dfrac{\mu - \delta^m p_2^m \bar{F}(p_2^m)}{T_1(p_2^m) + \frac{Q_2^m \bar{F}_1(p_2^m)}{(N_1+N_2)f(p_2^m)}}$ and $\delta_2^c = \dfrac{\mu - \delta^m p_2^{2,m} \bar{F}(p_2^{2,m})}{T_1(p_2^w(N_1)) + \frac{N_1 \bar{F}(p_2^w(N_1))}{(N_1+N_2)f(p_2^w(N_1))}}$. Uniqueness of the equilibrium follows because the objective function over the relevant values of q_1 is strictly concave (cf. the proof of Proposition 2). $\square$

Proof of Lemma 7: A similar result as Claim 3 holds when $p_2^{1,m} > p_2^{2,m}$ (the same proof applies). We define the function $G(x)$ as in (11).

(i) From Lemma 4, if $x \in [0, \frac{q_1}{N_1}]$, then $p_2^e(x|q_1) = \min(p_2^b(xN_1), \hat{p}_2(x,q_1))$. Because $q_1 \leq Q_2^m$, it follows that $G(x) \geq 0$ for all $x \in [0,1]$; see (11), hence $p_2^e(x|q_1) = p_2^b(xN_1)$ which is decreasing in x because $p_2^{1,m} > p_2^{2,m}$ (Lemma 3). If $x \in [\frac{q_1}{N_1}, 1]$, then $p_2^e(x|q_1) = p_2^b(q_1)$ (Lemma 3) which is constant in x.

(ii) If $Q_2^m \leq q_1 \leq N_1 + N_2\bar{F}_2(p_2^{2,m})$, then $G(x)$ has a solution $\hat{x}(q_1) \in [0,1]$ and consequently $p_2^e(x|q_1) = \hat{p}_2(x,q_1)$ for $x \in [0, \hat{x}(q_1)]$ and $p_2^e(x|q_1) = p_2^b(xN_1)$ for $x \in [\hat{x}(q_1), \frac{q_1}{N_1}]$. From Lemma 3, $p_2^b(xN_1)$ is decreasing in x while $\hat{p}_2(x,q_1)$ is increasing in x (see proof of Lemma 5). Finally, from Lemma 4 if $x \in [\frac{q_1}{N_1}, 1]$, then $p_2^e(x|q_1) = p_2^b(q_1)$ which is constant in x. $\square$

(iii) If $q_1 \geq N_1 + N_2\bar{F}_2(p_2^{2,m})$, then $G(x) \leq 0$ for all $x \in [0,1]$ implying that $p_2^e(x|q_1) = \hat{p}_2(x,q_1)$ which is increasing in x (see proof of Lemma 5).

Proof of Proposition 4: We first show that we can ignore mixed strategy REE without loss of generality. If $p_2^e(1|q_1) \leq p_2^e(0|q_1)$, define $\tilde{x}(q_1) = \min\{x \in (0,1) \text{ s.t. } p_2^e(x|q_1) = p_2^e(0|q_1)\}$. Lemma 2 implies that we can ignore mixed strategy REE when $p_2^e(1|q_1) > p_2^e(0|q_1)$ and when $p_2^e(1|q_1) \leq$

$p_2^e(0|q_1)$ and $x < \tilde{x}(q_1)$. The only case left to consider occurs when $p_2^e(1|q_1) \le p_2^e(0|q_1)$ and mixed strategy $x \ge \tilde{x}(q_1)$. We now focus on this case by dividing the interval into $x \in [\tilde{x}(q_1), \min(\frac{q_1}{N_1}, 1))$ and $x \ge \min(\frac{q_1}{N_1}, 1)$.

Claim 6 *We can ignore mixed strategy $x \ge \min(\frac{q_1}{N_1}, 1)$ without loss of generality.*

Proof: Assume $x_0 \ge \min(\frac{q_1}{N_1}, 1)$ is an REE. Because $p_2^e(x|q_1)$ is constant for $x \in [\min(\frac{q_1}{N_1}, 1), 1]$, $x = 1$ is also an REE. The two strategies x_0 and $x = 1$ are Pareto-equivalent and yield the same firm revenue. It is then without loss of generality that we ignore x_0. □

The only mixed REE left are $x \in [\tilde{x}(q_1), \min(\frac{q_1}{N_1}, 1))$. We cannot eliminate these mixed strategy REE by showing that they are Pareto-equivalent to the pure strategy REE $x = 1$, as we did in the proof of Proposition 1, because we cannot compare the profits under the two REE anymore. Instead, we show that for any REE $x_0 \in [\tilde{x}(q_1), \min(\frac{q_1}{N_1}, 1))$ there exists a different firm announcement that induces a PDREE with the same firm revenue. Thus, x_0 can be ignored without loss of generality.

Claim 7 *Assume $p_2^e(1|q_1) \le p_2^e(0|q_1)$ and $x_0 \in [\tilde{x}(q_1), \min(\frac{q_1}{N_1}, 1))$ is an REE for firm announcement (q_1, p_1). Then, $x' = 1$ is a PDREE weakly preferred by the firm for announcement $(q_1' = x_0 N_1, p_1' = p_1)$ and it gives the same revenue as REE x_0.*

Proof: We first derive the period-2 expected price for firm announcement (q_1', p_1'). Because $q_1' = x_0 N_1 < N_1$, Lemma 7 implies that $p_2^e(x|q_1') = \hat{p}_2(x|q_1')$ for $x \le \hat{x}(q_1')$, $p_2^e(x|q_1') = p_2^b(xN_1)$ for $x \in [\hat{x}(q_1'), x_0]$, and $p_2^e(x|q_1') = p_2^b(x_0 N_1)$ for $x \ge x_0$. Note that $\hat{x}(q_1')$ might not exist in which case $p_2^e(x|q_1') = p_2^b(xN_1)$ for $x \in [0, x_0]$ and $p_2^e(x|q_1') = p_2^b(x_0 N_1)$ for $x \in [x_0, 1]$. Our proof below applies in both cases.

The next step is to show that $x \in [x_0, 1]$ are the only PDREE for the announcement $(q_1' = x_0 N_1, p_1' = p_1)$. For that, we must rule out $x \in [0, x_0)$. For any $x \in [0, x_0)$. We have $\mu - p_1' = \mu - p_1 = \delta_c T_1(p_2^e(x_0|q_1)) = \delta_c T_1(p_2^e(x_0|q_1')) > \delta_c T_1(p_2^e(x|q_1'))$. The second equality is because x_0 is a mixed strategy for the announcement (q_1, p_1). The third equality holds because $p_2^e(x_0|q_1) = p_2^b(x_0 N_1) = p_2^e(x_0|q_1')$. Finally, the inequality follows because $p_2^e(x_0|q_1') = p_2^b(x_0 N_1)$ and $p_2^e(x|q_1') = \min(\hat{p}_2(x, q_1'), p_2^b(xN_1))$. Moreover, because $\hat{p}_2(x, q_1)$ is decreasing in q_1, we have $\hat{p}_2(x, q_1') \ge \hat{p}_2(x, q_1) > p_2^b(x_0 N_1)$. Finally, $p_2^b(xN_1)$ is decreasing in x (Lemma 3) and hence $p_2^b(xN_1) > p_2^b(x_0 N_1)$. We conclude that $p_2^e(x_0|q_1') < p_2^e(x|q_1')$. Therefore, buying dominates waiting for $x \in [0, x_0)$ and x cannot be a REE.

Consider next $x \ge x_0$. We have $\mu - p_1' = \mu - p_1 = \delta_c T_1(p_2^e(x_0|q_1)) = \delta_c T_1(p_2^e(x_0|q_1')) = \delta_c T_1(p_2^e(x|q_1'))$ (because $x \ge x_0 = \frac{q_1'}{N_1}$). Any $x \in [x_0, 1]$ is then an REE and also a PDREE because the consumer surplus is constant and equal to $\mu - p_1$.

We conclude by comparing the profits under (q_1, p_1, x_0) and (q_1', p_1', x) for $x \in [x_0, 1]$. The prices are equal $p_1 = p_1'$ and so are sales $\min(xN_1, q_1') = x_0 N_1 = \min(x_0 N_1, q_1)$. Firm's revenue are equal in period 1 and also in period 2. Therefore, the firm is indifferent between REE x_0 for announcement (q_1, p_1) and PDREE $x' = 1$ for the announcement (q_1', p_1'). $\square$

We only have to consider pure strategy PDREE $x = 0$ and $x = 1$. We distinguish two cases. If $p_1^w(q_1) \le p_1^b(q_1)$, then $p_1 = p_1^w(q_1)$ is the highest period-1 price such that $x = 1$ is a PDREE. Otherwise, $p_1 = p_1^b(q_1)$ is the highest price such that $x = 1$ is a PDREE. The function $\tilde{p}_1(q_1) = \min(p_1^w(q_1), p_1^b(q_1))$ matches the highest period-1 price in each case.

It is important to observe that, for a given q_1, $x = 0$ is also a PDREE for $(q_1, \tilde{p}_1(q_1))$. For $x = 1$, the firm profits are $\tilde{p}_1(q_1)q_1 + \delta_m p_2^b(q_1) Q_2^b(p_2^b(q_1), q_1)$. For $x = 0$, the firm profits are $\delta^m Q_2^m p_2^m$ if $q_1 \le Q_2^m$ and $q_1 \hat{p}_2(0, q_1)$ otherwise. It is possible that the firm prefers PDREE $x = 0$. But the profits under $x = 0$ are weakly dominated by the profits under firm announcement $(q_1 = 0, p_1 = \infty)$. In other words, at optimality the firm will not choose such values for q_1. Therefore, we can assume, without loss of generality, that PDREE $x = 1$ is associated to announcement $\tilde{p}_1(q_1)$ even though it is not necessarily the preferred PDREE. $\square$