DISCUSSION PAPER SERIES

No. 9451

STRATEGIC SEARCH DIVERSION, PRODUCT AFFILIATION AND PLATFORM COMPETITION

Andrei Hagiu and Bruno Jullien

INDUSTRIAL ORGANIZATION

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP9451.asp

ISSN 0265-8003

STRATEGIC SEARCH DIVERSION, PRODUCT AFFILIATION AND PLATFORM COMPETITION

Andrei Hagiu, Harvard University (HBS Strategy Unit)
Bruno Jullien, Toulouse School of Economics (IDEI & GREMAQ) and CEPR

Discussion Paper No. 9451
April 2013

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 9451

April 2013

ABSTRACT

Strategic Search Diversion, Product Affiliation and Platform Competition*

Platforms use search diversion in order to trade off total consumer traffic for higher revenues derived by exposing consumers to products other than the ones that best fit their preferences. Our analysis yields three key and novel insights regarding search diversion incentives, which have direct implications for platforms' strategies and empirical predictions.

First, platforms that charge positive access fees to consumers have weaker incentives to divert search relative to platforms that cannot (or choose not to) charge such fees. Second, endogenizing the affiliation of products that consumers are not interested in (advertising) leads to stronger incentives to divert search relative to the exogenous affiliation (vertical integration) benchmark, whenever the marginal product yields higher profits per consumer exposure relative to the average product. Third, the effect of platform competition on search diversion incentives depends on the nature of competition. Competition for advertising leads to more search diversion relative to competition for consumers. Both types of competition lead to at least as much search diversion as a monopoly platform. Nevertheless, in the case of competing platforms, the equilibrium level of search diversion increases with the degree of horizontal differentiation between platforms.

JEL Classification: L1, L2 and L8
Keywords: competition, search platforms and two-sided market

Andrei Hagiu
HBS Strategy Unit
Harvard University
Morgan Hall
Soldiers Field
Boston, MA 02163
USA

Email: ahagiu@hbs.edu

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=161729

Bruno Jullien
School of Economics
Manufacture des Tabacs,
Batîment F 2eme étage
21 Allée de Brienne, 31000 Toulouse
FRANCE

Email: bruno.jullien@tse-fr.eu

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=110975

*We are grateful to Ramon Casadesus-Masanell, David Henriques, Joao Montez and participants at the CRES Foundations of Business Strategy Conference (Olin Business School, Washington University in Saint Louis) and at the Third Annual Conference on Internet Search and Innovation (Northwestern Law School) for very helpful comments and feedback on a previous draft.

Submitted 22 April 2013

Strategic Search Diversion, Product Affiliation and Platform Competition*

Andrei Hagiu[†] and Bruno Jullien[‡]

February 2013

Abstract

Platforms use search diversion in order to trade off total consumer traffic for higher revenues derived by exposing consumers to products other than the ones that best fit their preferences. Our analysis yields three key and novel insights regarding search diversion incentives, which have direct implications for platforms' strategies and empirical predictions.

First, platforms that charge positive access fees to consumers have weaker incentives to divert search relative to platforms that cannot (or choose not to) charge such fees. Second, endogenizing the affiliation of products that consumers are not interested in (advertising) leads to stronger incentives to divert search relative to the exogenous affiliation (vertical integration) benchmark, whenever the marginal product yields higher profits per consumer exposure relative to the average product. Third, the effect of platform competition on search diversion incentives depends on the nature of competition. Competition for advertising leads to more search diversion relative to competition for consumers. Both types of competition lead to at least as much search diversion as a monoply platform. Nevertheless, in the case of competing platforms, the equilibrium level of search diversion increases with the degree of horizontal differentiation between platforms.

1 Introduction

Search diversion occurs when platforms giving consumers access to various products deliberately introduce noise in the search or browsing process through which consumers find the products they are most interested in. This practice is widespread among both offline and online platforms. All advertising-supported media (from magazines to online portals, news sites, and search engines) are purposefully designed to expose users to advertisements, even though they are primarily interested in content. Similarly, retailers often place the most sought-after items at the back or upper floors of their stores (e.g. bread and milk at supermarkets, iPods and iPhones at Apple stores), while shopping malls design their layout to maximize the distance travelled by visitors between anchor

*We are grateful to Ramon Casadesus-Masanell, David Henriques, Joao Montez and participants at the CRES Foundations of Business Strategy Conference (Olin Business School, Washington University in Saint Louis) and at the Third Annual Conference on Internet Search and Innovation (Northwestern Law School) for very helpful comments and feedback on a previous draft.

[†]Harvard University (HBS Strategy Unit), ahagiu@hbs.edu

[‡]Toulouse School of Economics (IDEI & GREMAQ), bjullien@cict.fr

stores (Petroski 2003). E-commerce sites (e.g. Amazon, Bing Shopping, eBay, Google Products) design their websites in order to divert users' attention from the products they were initially looking for, towards discovering products they might be interested in - and eventually buy.

On the one hand, search diversion may lead to higher platform revenues per consumer "visit" to the platform. On the other hand, it reduces the overall attractiveness of the platform to consumers and therefore also leads to lower consumer traffic (i.e. total mumber of visits). All platforms listed above face this fundamental tradeoff.

The basic economic logic of search diversion was first analyzed by Hagiu and Jullien (2011), using a model with a monopoly platform (intermediary) that offers consumers access to two products, whose affiliation with (availability through) the platform is exogenously given. There remain however several important issues raised by the use of search diversion as a strategic design tool. How does a platform's incentive to divert search change when it must attract both consumers and products (as opposed to just consumers)? What is the effect of competition between platforms on their optimal levels of search diversion? In particular, should increased competitive pressure - for consumers or for products - lead to more or less search diversion? The purpose of this paper is to present a formal model in order to address these questions.

1.1 Model overview and interpretation

In our model, each platform offers consumers access to two products, 1 and 2. Product 1 (content) offers consumers expected utility $u_H > 0$ and is assumed to be exogenously affiliated with each platform throughout the paper (in our model, exogenous affiliation is equivalent to vertical integration into product 1). Product 2 (advertising) offers consumers expected utility $u_L = 0$. We will focus on cases in which product 2 affiliates endogenously, i.e. it is supplied by a third-party seller (advertiser), who must be induced to affiliate by platforms' choices of fees and search diversion. Intermediaries may derive positive revenues from consumer exposure to both products. Each product exposure is costly to consumers (individual exposure cost c): it requires time and attention. The platforms' revenues per consumer exposure to product 1 (π_1) could be referral fees paid by an independent seller or the margin made on the sale of product 1 multiplied by the conversion rate (probability that a consumer who sees the product ends up buying it) if the platform supplies product 1 itself; or any type of fees directly tied to usage of product 1 (e.g. pay-per-view). Meanwhile, the platforms' revenues per consumer exposure to product 2 (π_2 or r) can be interpreted as "per-impression" or "per-click" fees paid by its seller. In addition, we also explore the effect of allowing the platform to charge consumers a fixed access fee (P).

The key decision made by the platform is the amount of search diversion to induce through its service, which we identify with the probability $s \in [0, 1]$ that it exposes consumers to product 2 *before* directing them to product 1. Indeed, although consumers always prefer being immediately exposed to product 1, the platform may find that first diverting them to product 2 maximizes total revenues. We use the term "search" because in a sense consumers are searching for product 1 and the platform chooses how efficient to make this search process. With probability $(1-s)$ a consumer finds product 1 after one "round of search" only, while with probability s she needs two rounds to get to product 1. More search diversion leads to higher total exposure costs incurred by consumers.

Our modelling set-up is best interpreted as a stylized representation of advertising-supported media, such as the ones listed in the following table:[1]

Platform	Product 1	Product 2	π_1	π_2	Access fee?
Search engines (e.g. Bing, Google)	Organic search results	Sponsored search results	=0	Advertising fees (mostly CPC)	No
Shopping portals (e.g. Bing Shopping, Google Shopping, Yahoo Shopping)	Products/information user is looking for	Other products advertised on the page	>0 (referral fees paid by 3rd party merchants of products listed)	Advertising fees	No
Content portals (e.g. Forbes, MSN, New York Times, Yahoo)	Editorial content	Display advertising	=0	Advertising fees (CPM and CPC)	Yes (NYT) No (Forbes, MSN, Yahoo)
E-commerce sites (e.g. Fandango, Zappos)	Products/information user is looking for (e.g. movie schedules; shoe descriptions)	Display advertising	>0 (booking fees paid by users buying movie tickets or margins made on shoes sold)	Advertising fees	No

Table 1

All of these platforms provide users with first-party content (cf. Hagiu and Spulber 2012), such as organic search results, information, editorial stories or products sold in their own name. All of them make positive revenues from user exposure to advertising or products users were not necesarily looking for ($\pi_2 > 0$). Some of them (search engines and content portals) make no revenues from first-party content, while others (shopping portals and e-commerce sites) derive positive revenues from exposing consumers to first-party content. For shopping portals, π_1 is equal to the click-through rate of listed products multiplied by the referral fees charged to the third-party merchants who sell those products. For e-commerce sites, π_1 is the conversion rate multiplied by the booking fees charged to users (Fandango) or margins made on shoes sold (Zappos).

The extent of search diversion varies across these platforms from minimal (small and unintrusive ads on Fandango.com, sponsored search results at the *bottom* of Google Shopping pages) to moderate (sponsored search results at the *top and right-hand side* of Google's search engine pages) to very high (in addition to showing several large ads on every content page, Forbes.com *requires* users to view a video ad prior to watching every piece of video content and oftentimes to click through a full-page display ad before reaching the desired content page).

Although highly stylized, our model contains the three ingredients necessary to capture the key tradeoffs associated with search diversion in these contexts. First, there are multiple products - 2 is enough -, some of which are more desired by consumers relative to others ($u_H > u_L = 0$). Second, platforms' profit incentives are imperfectly aligned with consumer preferences: platforms derive positive revenues from exposing consumers to products that they do not care about ($\pi_2 > 0$ and possibly $\pi_2 > \pi_1$). Third, exposure to individual products is costly for consumers (individual

[1] CPM is the standard industry term for cost per impression (literally, "cost per mille", i.e. a thousand impressions), while CPC stands for "cost per click".

cost c) and the platform can make design decisions (captured by s) that influence the degree to which consumers are exposed to one product relative to the other.

We have sought to build the simplest possible model that contains these three ingredients in order to keep the analysis of endogenous product affiliation and competing platforms as tractable as possible. Of course, in most real-world settings there are more than two products, multiple sellers and perhaps even complementarity or substitutability across products. Introducing any of these aspects would unnecessarily complicate our analysis, since the fundamental mechanics of search diversion remain unchanged. For the same reason, we treat π_1 and π_2 as exogenously given in our model, i.e. we do not endogenize price-setting by independent sellers. These extensions are treated by Hagiu and Jullien (2011) in the context of a monopoly platform choosing search diversion.

Finally, while the assumption $u_L = 0$ best fits contexts in which product 2 is advertising (as in the examples above), the general implications we derive hold for any platforms that have incentives to divert consumers away from the products that best suit their preferences and towards products they are *less* - but still positively - interested in ($u_L > 0$). For instance, Netflix uses its recommender system in an attempt to steer users towards less popular movies, which entail lower licensing costs and are less likely to run out of stock, which in turn means they generate higher margins for Netflix (Shih et al. 2007). Similarly for Amazon.com. Indeed, the difference between diverting consumers to advertising and diverting them to products that they find less desirable is simply one of degree. In most cases, consumers derive 0 expected utility from being exposed to advertisements, whereas they might perceive a (small) positive expected utility from being exposed to products other than the ones that they initially came to the platform for. The only thing that matters is that the platform derives positive margins from products that are *less* valuable to consumers. In a previous draft version, we worked with $u_L > 0$: the analysis turned out to be more complex than the one presented here but the main results were the same. This is why we have opted to work with $u_L = 0$ throughout.

1.2 Related literature

Our paper builds upon the model of search diversion introduced by Hagiu and Jullien (2011). That paper established that search diversion allows platforms to: (i) trade off higher total consumer traffic for higher revenues per consumer visit; and (ii) influence independent product sellers' choices of strategic variables (e.g., pricing). It also showed that search diversion is a strategic instrument that cannot be easily replaced by contractual extensions and that it can be socially desirable because consumers do not internalize the benefits of their search activities for product sellers. We complement Hagiu and Jullien (2011)'s analysis with two important and novel elements: endogenous product and consumer affiliation decisions and competition among platforms (Hagiu and Jullien 2011 focus exclusively on a monopoly platform with exogenously given consumer and seller affiliation).

We contribute to the strategy and economics literature on two-sided platforms by introducing a key design decision (search diversion) that many platforms have to make, but has not been formally studied. Indeed, most of the existing work on two-sided platforms focuses on pricing strategies (Armstrong (2006), Economides and Katsamakas (2006), Parker and Van Alstyne (2005), Rochet and Tirole (2006), Spulber (2006), Weyl (2010)) and market outcomes (Caillaud and Jullien (2003), Hossain et al. (2011), Zhu and Iansiti (2011)) in the presence of indirect network effects. Our paper is aligned with an emerging body of work aiming to expand the formal study of platforms to design decisions (e.g. Boudreau (2010), Parker and Van Alstyne (2008) who study openness choices; Eisenmann et al. (2011) who study envelopment strategies; Hagiu and Spulber (2012) who study platforms' strategic incentives to provide first-party content).

At a broader level, several management articles have pointed out that platforms have to make design compromises between the interests of their two sides (e.g. Kaplan and Sawhney (2000) in a survey of B2B business models; Evans and Schmalensee (2007) in an overview of markets featuring two-sided platforms). To the best of our knowledge however, this issue has not received formal modelling treatment.

Finally, our research is also connected to the literature on advertising-supported platforms: Anderson and Coate (2005), Casadesus-Masanell and Zhu (2010), Crampes et al. (2009), Gabszewicz et al. (2006), Peitz and Valletti (2008). These papers study product positioning and product quality choices by ad-sponsored platforms. In contrast, our focus is on the design of platforms' product exposure mechanism. A recent strand of this literature studies the effect of consumers' and advertisers' "multihoming" behavior (i.e. their presence on multiple online outlets) on firm profits. Athey Calvano and Gans (2012) show that the value of advertising in one outlet depends on consumer multihoming and focus on tracking technologies, while Athey and Gans (2010) study targeted advertising. In our competition section (4), we analyze the impact of consumer and advertiser multihoming on the endogenous rate of exposure to advertising (search diversion in our model). Ambrus Calvano and Reisinger (2012) discuss the nature of price competition when all agents can multihome. We share with them the conclusion that competition does not restore efficiency, but their paper focuses on the volume of advertising, whereas we focus on the design of the exposure service. Broadly, targeting, tracking and diversion can all be viewed as various instances of platform service design, thus our work is complementary to this stream of literature.

The remainder of the paper is organized as follows. Section 2 lays out the modeling set-up and analyzes the monopoly platform case, taking product affiliation as exogenously given. Section 3 endogenizes the affiliation of product 2 (advertising) in a monopoly setting. Section 4 introduces competition between platforms and analyzes three scenarios: a) platforms compete for consumers only (product affiliations are exogenously given); b) platforms compete only for the exclusive affiliation of product 2 (consumer affiliations are exogenously given); c) platforms compete for both consumers and the exclusive affiliation of product 2. We conclude in section 5.

2 Monopoly platform - exogenous product affiliation

In this section we lay out the foundation for our analysis using a variant of the model in Hagiu and Jullien (2011). There is a monopoly platform which allows a unit mass of consumers to access two products, 1 and 2, which are *already* available through (or affiliated with) the platform. This corresponds to settings in which there exist long-standing affiliation contracts between independent product sellers and the platform or in which the platform supplies the products itself (vertical integration). To access either product, a consumer must first affiliate with (i.e. visit) the platform and then be exposed to the product through a search process described below.

For the sake of concision, we work with two products throughout the paper. Consumers are interested in product 1 only, which can be interpreted as content, e.g. editorial stories, videos, organic search results. They are not interested in product 2, which can be interpreted as advertising.

2.1 Consumers

All consumers derive *net* expected utility u_H from being exposed to product 1 and $u_L = 0$ from being exposed to product 2, where $0 < u_H < 1$ is exogenously given. These utilities should be interpreted as encompassing the utility of just viewing the product plus the expected utility of actually consuming it, net of the price paid.

Ex-ante, i.e. before affiliating with the platform, consumers only differ in their location x, uniformly distributed on $[0, 1]$. The monopoly platform is located at 0. When consumer x affiliates with the platform, she incurs transportation costs tx, where $t > 0$. We assume t is large enough so that the consumer market is never fully covered by the monopoly platform.

Ex-post, i.e. after deciding whether or not to affiliate with the platform, consumers learn their unitary cost of exposure (or cost of search) c, which they incur whenever they are exposed to a product. When both products are affiliated with the platform, consumers can only view them *sequentially* and therefore are subject to one or two product exposures. The search cost c can be interpreted as the cost of consumer attention; it is distributed on $[0, 1]$ according to a twice continuously differentiable cumulative distribution function F. From an *ex-ante* perspective, a consumer located at any position x perceives the same *ex-post* distribution of search costs $F(.)$.

Thus, consumers make two decisions: whether or not to visit a given platform, and whether or not to engage in product search on the platform(ies) they have decided to visit.

2.2 The platform

The platform derives expected profits $\pi_1 \geq 0$ for each consumer exposure to product 1 and $\pi_2 \geq 0$ for each consumer exposure to product 2. In this section, π_1 and π_2 are known by all players and exogenously fixed; in the sections that follow, we endogenize π_2.

Consistent with the content-advertising intepretation, π_1 could be a referral fee paid by the seller of product 1 (content) to the platform, or the margin made on the sale of product 1 multiplied by the conversion rate if the platform supplies it. Meanwhile, π_2 could be a per-impression (or per-click) advertising fee paid by the seller of product 2.

The platform may also be able to charge consumers a fixed access fee, denoted by P. A priori, P can be positive or negative. A negative access fee should be interpreted as a monetary subsidy (e.g. cash or redeemable points) or first-party content (beyond product 1) that consumers value at more than the price being charged. In what follows, we derive results under two scenarios: access fees are not feasible (i.e. $P = 0$, known by all players ex-ante) and access fees are feasible (i.e. P can be set by the platform at any value).

2.3 Search diversion

The platform has a design technology that allows it to choose a probability $s \in [0, 1]$ with which it first exposes *any* given consumer to product 2, before directing her to product 1. The probability s represents the level of *search diversion* induced by the platform. Once a consumer has been exposed to product 2, she knows that she will next be exposed for sure to product 1, but she will then need to incur her search cost c again. The focus of our paper is on the platforms' choice of s. We assume that s can be costlessly set to any value between 0 and 1.

One can think of $(1 - s)$ as a measure of how efficient the design of the platform is from consumers' perspective. Does the platform provide quick and clear access to the products and content that consumers are searching for (low s)? Or does it try to expose consumers to various forms of advertising before providing the service they came for in the first place (high s)?

2.4 Timing

The timing of the game we consider in this section is as follows:

1. The platform announces s publicly and credibly
2. If access fees are feasible then the platform announces P publicly and credibly
3. Consumers decide whether or not to visit (affiliate with) the platform
4. Affiliated consumers learn their individual cost c, then engage in product search.

Two aspects of this set-up deserve mention. First, separating the choices of s and P has no impact here: the solution of the game would be identical if we assumed (s, P) are chosen simultaneously. This timing assumption reflects the different time horizons associated with various decisions in practice. Design-related decisions tend to be longer-term than pricing decisions in most real-world settings. Thus, the design parameter s is observed by consumers before deciding whether or not to visit the platform (for instance, through reviews or word of mouth) and is not subject to *ex-post* opportunism, i.e. cannot be adjusted once consumers have affiliated.[2] Furthermore, as will become clear in the next sections, it is in the platform's best interest to credibly announce s upfront because s affects not only consumer utility, but also expected payoffs for a possibly independent seller of product 2 (advertiser) that the platform might be courting. Credibly announcing s allows the platform to commit to the effectiveness of its "diversion service" for the product 2 seller, who benefits directly from a higher s.

Second, affiliation decisions by users typically involve longer time horizons than activity (search) decisions. Moreover, affiliation is usually based on less information than activity, as agents learn about the platform gradually. This is captured by our assumption that affiliation is based on the level of search diversion and *expectation* of search costs, while activity (search) is based on the *realized* individual search cost. This assumption simplifies the analysis without loss of substance: all we need is that *total* consumer demand is decreasing in s.

2.5 The consumer search process and affiliation decision

In stage 4, consumers affiliated with the platform must decide whether to search or not. When the platform has chosen $s > 0$, consumers know that they may be diverted. A consumer with search cost $c \leq u_H$ who is first diverted to product 2 will still proceed to product 1, because she knows with certainty that she will obtain net utility $u_H - c \geq 0$. If the consumer is not diverted, i.e. if she is directly exposed to product 1, then she stops searching immediately and will not be exposed to product 2 (which would yield negative net utility $-c$). To fix ideas, it is useful to think of an advertising-supported news website. If a user is first shown an ad, she will still click or scroll to find the news content. If she is shown the content right away, she will never go on searching for ads.

The consumer's net expected utility from searching is thus $u_H - (1 + s)\, c$ and is positive for $c \leq u_H/(1 + s)$. Consumers with search cost above $u_H/(1 + s)$ do not engage in the search process at all. Using the news website example: the expected utility provided by the site to such consumers is not sufficient to justify the time wasted clicking through or scrolling over ads.

Working backwards to stage 3, a consumer located at x affiliates with the platform if and only if $V(s) - P - tx \geq 0$, where:

$$V(s) \equiv \int_0^{u_H/(1+s)} \left(u_H - (1 + s)\, c\right) f(c)\, dc \qquad (1)$$

is the expected consumer utility from the perspective of stage 3, gross of access price and trans-

[2] If ex-post opportunism was possible, then the unique equilibrium with consistent expectations would be $s^* = 1$. Thus, the platform has every interest to credibly commit to (or develop a reputation for) a level of search diversion $s^* < 1$ ex-ante.

portation costs. Note that $V(s)$ is decreasing.

For the sake of completeness, suppose that only one product is affiliated with the platform (this scenario is not relevant in this section but will be in the next sections). In this case, consumers who visit the platform find the affiliated product with probability 1 in just one round of search. If only product 1 is affiliated then expected consumer utility from the perspective of stage 3 is $\int_0^{u_H} (u_H - c) f(c)\, dc = V(0)$. If only product 2 is affiliated then expected consumer utility is 0.

2.6 Intermediary profits

Total consumer demand for (or traffic to) the platform is then $[V(s) - P]/t$, decreasing both in the level of search diversion s and in the price P. The platform's profits as a function of s and P are then:

$$[P + X(s, \pi_1, \pi_2)] \frac{[V(s) - P]}{t} \tag{2}$$

where:

$$X(s, \pi_1, \pi_2) \equiv [\pi_1 + s\pi_2] F\left(\frac{u_H}{1+s}\right)$$

denotes the revenues derived by the platform from the product exposures of each affiliated consumer (i.e. excluding access fees).

The optimization of (2) over s involves a trade-off between *total* consumer traffic and participation in the search process on the one hand, and the average number of product exposures *per consumer* on the other hand. Indeed, an increase in s induces a consumer to see two products with probability s, which yields revenues $\pi_1 + s\pi_2$ to the platform, but it also reduces the proportion of consumers who engage in search, $F(u_H/(1+s))$, as well as total consumer traffic to the platform, $[V(s) - P]/t$. Variations of this tradeoff are analyzed at length in Hagiu and Jullien (2011). The key novelty here is the term $[V(s) - P]/t$: indeed, Hagiu and Jullien (2011) treat total consumer affiliation with the platform as exogenously given, equal to 1.

2.7 Optimal search diversion

Suppose first that the platform cannot (or is committed not to) charge access fees to consumers. Then its profits are:

$$X(s, \pi_1, \pi_2) \frac{V(s)}{t}$$

The platform maximizes these profits over s, resulting in $s^M(\pi_1, \pi_2)$. The following lemma, proven in the appendix, characterizes the optimal level of search diversion:

Lemma 1 *When the platform does not charge access fees, the optimal level of search diversion is:*

$$s^M(\pi_1, \pi_2) = \arg\max_s \{X(s, \pi_1, \pi_2) V(s)\} \tag{3}$$

It is decreasing in π_1 *and increasing in* π_2*.*

The second part of the lemma states that the platform diverts search more when it derives higher revenues from the product that consumers are *not* interested in (advertising) relative to the product that they are interested in (content). The reason is straightforward: when the platform derives more revenues from content (advertising), its interests are more (less) aligned with those of consumers, therefore the optimal level of search diversion is lower (higher). Recall indeed that consumers always prefer less search diversion.

This result is consistent with the examples discussed in the introduction. Conditional on not charging access fees, platforms that derive no revenues from first-party content (e.g. search engines, Forbes.com) clearly engage in more search diversion relative to platforms with $\pi_1 > 0$ (e.g. Google Shopping, Fandango). Note in particular the stark contrast between Google search, on which sponsored search results are at the top and on the right hand side of the page, and Google Shopping, where sponsored search results are at the bottom of the page.

Suppose now that the platform has full flexibility in charging access fees. In this case, it optimizes expression (2) over P and s. Relegating details to the appendix, we obtain:

Lemma 2 *When the platform can charge access fees, the optimal level of search quality is:*

$$\widehat{s}^M(\pi_1, \pi_2) = \arg\max_s \{X(s, \pi_1, \pi_2) + V(s)\} \tag{4}$$

It is weakly decreasing in π_1 *and weakly increasing in* π_2*.*

The first part of the lemma says that when the platform can monetize consumer participation, it chooses search diversion to maximize the joint surplus (platform + consumers). The second part is the same as Lemma 1 and has the same interpretation.

It is interesting to compare $s^M(\pi_1, \pi_2)$ with $\widehat{s}^M(\pi_1, \pi_2)$, i.e. to ask whether allowing the platform to charge access fees to consumers increases or decreases the optimal level of search diversion. In the appendix, we prove the following proposition:

Proposition 1 *Allowing the platform to charge access fees results in less search diversion (*$\widehat{s}^M \leq s^M$*) if and only if the profit-maximizing access fee is non-negative (*$P_M \geq 0$*).*

Thus, the ability to monetize consumer participation generally reduces search diversion incentives. This result is intuitive: if the platform wishes to charge consumers a positive access fee, it must increase their willingness-to-pay, which means it needs to *decrease* s (recall from expression 1 that consumers always prefer less search diversion). Conversely, if the platform decides to subsidize the participation of consumers, it needs to make up for the loss by increasing advertising revenues, which it can achieve by inducing more search diversion.

This result is also consistent with some of the examples discussed in the introduction. Compare the websites of Forbes and the New York Times. Neither derives any revenues from first-party content and both use advertising. The New York Times charges users a subscription fee, while access to Forbes is entirely free for users. Advertising on the New York Times web page is moderate; in contrast, advertising is highly intrusive on Forbes.com.

2.8 Example

Suppose $F(x) = x$; $0 \leq \pi_1 < \pi_2$; and t large enough so that the platform never covers the consumer market. It is then easily verified that:

$$s^M(\pi_1, \pi_2) = \max\left(1 - \frac{2\pi_1}{\pi_2}, 0\right) \quad \text{and} \quad \widehat{s}^M(\pi_1, \pi_2) = \begin{cases} 1 & \text{if} \quad u_H \leq 2(\pi_2 - \pi_1) \\ 0 & \text{if} \quad u_H \geq 2(\pi_2 - \pi_1) \end{cases}$$

Both $\widehat{s}^M$ and s^M are weakly decreasing in π_1 and weakly increasing in π_2 as expected. Furthermore, $\widehat{s}^M \leq s^M$ if and only if $u_H \geq 2(\pi_2 - \pi_1)$. Thus, allowing the platform to charge a access fee leads to less search diversion when advertising revenue π_2 is smaller relative to content revenue π_1 and when consumer willingness to pay for content (u_H) is larger.

3 Monopoly platform - endogenous seller affiliation

The monopoly platform from the previous section remains vertically integrated into product 1, but we now assume that product 2 is supplied by an independent seller (advertiser), who must be induced to affiliate. The platform's expected profit per consumer exposure to product 1 is still π_1. The advertiser derives profit π_2 for every consumer exposure to its product. In order to affiliate with the platform and thereby gain access to its consumers, the advertiser must pay the platform a per-impression (per-click) fee r. We assume the platform has all the bargaining power when setting r. The timing is now as follows:

1. The platform commits to s publicly and credibly
2. The platform sets the fee r to be paid by the advertiser
3. The advertiser decides whether or not to affiliate with the platform
4. If access fees are feasible, the platform announces P publicly and credibly
5. Consumers observe (P, s) and the advertiser's affiliation decision and decide whether or not to affiliate with the platform

6. Affiliated consumers learn their individual cost c, then engage in product search.

Separating the choices of s and r has no effect on the solution of this game: we did it solely for the sake of consistency with the competing platforms case, where this assumption does make a difference as we discuss in section 4 below.

Allowing the platform to choose P *after* the advertiser's affiliation decision may create a form of *ex-post* opportunism by the platform. If the platform does not observe π_2 when setting r (incomplete information) then, once the advertiser decides to affiliate in stage 3, the platform sets P too high from a joint perspective in stage 4, because it does not fully internalize the advertiser's profits. This hold-up issue arises more generally in two-sided settings whenever one side makes participation decisions before the other and the platform (platform) cannot commit to the price it will charge to the second side *ex-ante*. The implications of commitment in such settings are analyzed in depth in Hagiu and Spulber (2012). We chose the timing laid out above because it is the most realistic: the advertiser's participation determines to a large extent the platform's business model (in the sense of Casadesus-Masanell and Zhu 2010). It seems unrealistic for the platform to commit to a specific consumer access fee *before* knowing whether it will derive advertising revenues from the independent seller of product 2. Furthermore, none of our generalizable results below would change if we allowed the platform to commit to P prior to the advertiser's affiliation decision. In fact, results would be identical if we allowed the platform to commit to two different P's, contingent on whether the advertiser affiliates or not.

3.1 Full information

Suppose first the advertiser's per impression profit π_2 is common knowledge. In this case, the platform sets $r = \pi_2$ (slightly below) in stage 2, which ensures the advertiser affiliates and extracts its entire profit. In stage 4, the platform's profits are then $[P + X(s, \pi_1, \pi_2)][V(s) - P]/t$, which is the exact same expression as (2) in the previous section. Consequently, the platform's optimal choices of search diversion are the same as with exogenous advertiser affiliation: $s^M(\pi_1, \pi_2)$ without access fees and $\widehat{s}^M(\pi_1, \pi_2)$ with access fees. These are also the same levels that the platform would choose if it were fully vertically integrated, i.e. if it owned both products.

If the advertiser does not affiliate then the platform only offers consumers product 1, which affiliated consumers would find without being diverted. In this case, platform profits are $[P + \pi_1 F(u_H)][V(0) - P]/$ which is also equal to $[P + X(0, \pi_1, \pi_2)][V(0) - P]/t$. Comparing with the expression of profits in the previous paragraph, it is apparent that the platform always finds it profitable to induce affiliation of the advertiser.

Thus, under full information, endogenous affiliation of the advertiser does not create any additional search diversion incentives - we obtain the same results as with exogenously given affiliation.

3.2 Incomplete information

Let us now turn to the more interesting case, in which the advertiser's profit π_2 is unobservable to the platform. In particular, we assume throughout that π_2 is drawn from a cummulative distribution function $G(.)$, which is common knowledge.[3] In stage 3, the independent advertiser affiliates if and only if $\pi_2 \geq r$.

If the platform does not charge any access fees, its expected profits in stage 1 are:

$$\begin{aligned} &\max_{s,r}\left\{[1-G(r)]\cdot X(s,\pi_1,r)\frac{V(s)}{t}+G(r)\cdot\pi_1 F(u_H)\frac{V(0)}{t}\right\} \\ =\;&\max_{r}\left\{[1-G(r)]\max_{s}\left\{\frac{X(s,\pi_1,r)V(s)}{t}\right\}+G(r)\frac{\pi_1 F(u_H)V(0)}{t}\right\} \end{aligned} \tag{5}$$

Suppose now that access fees are feasible. If the seller decides to affiliate in stage 3 then the platform's stage 4 profit is $[P+X(s,\pi_1,r)][V(s)-P]/t$. If the seller does not affiliate, the platform's stage 4 profit is $[P+\pi_1 F(u_H)][V(0)-P]/t$. The platform's expected profits from the perspective of stage 1 are then:

$$\begin{aligned} &\max_{s,r}\left\{[1-G(r)]\max_{P}\{[P+X(s,\pi_1,r)][V(s)-P]/t\}+G(r)\max_{P}\{[P+\pi_1F(u_H)][V(0)-P]/t\}\right\} \\ =\;&\max_{r}\left\{[1-G(r)]\max_{s}\left\{\frac{[V(s)+X(s,\pi_1,r)]^2}{4t}\right\}+G(r)\frac{[V(0)+\pi_1F(u_H)]^2}{4t}\right\} \end{aligned} \tag{6}$$

In both cases, with probability $G(r)$ the platform only has product 1 to offer, hence there is no search diversion and the platform's revenues come from consumer exposure to product 1 and access fees. With probability $[1-G(r)]$, the independent advertiser affiliates and the platform derives higher revenues, which come from consumer exposures to both products and access fees.

For the sake of concision, we omit the first-order conditions of (5) and (6) with respect to the advertiser fee r. In both cases, the optimal fee ($r^M(\pi_1)$ and $\widehat{r}^M(\pi_1)$ respectively) is set such that the marginal net gain from a small reduction in r (higher probability of advertiser affiliation multiplied by the revenue differential) is equal to the marginal net loss (lower advertising revenues whenever the advertiser affiliates). We focus directly on the optimal levels of search diversion, $s^M_{end}(\pi_1)$ and $\widehat{s}^M_{end}(\pi_1)$ respectively, which are given by:

$$\begin{aligned} s^M_{end}(\pi_1) &= \arg\max_s\left\{X\left(s,\pi_1,r^M(\pi_1)\right)V(s)\right\}=s^M\left(\pi_1,r^M\right) \\ \widehat{s}^M_{end}(\pi_1) &= \arg\max_s\left\{X\left(s,\pi_1,\widehat{r}^M(\pi_1)\right)+V(s)\right\}=\widehat{s}^M\left(\pi_1,\widehat{r}^M(\pi_1)\right) \end{aligned}$$

Thus, in both cases (without or with access fees), the optimal level of search diversion under incomplete information is equal to the monopoly level under exogenous affiliation and complete

[3] This set-up is meant to capture a more realistic scenario in which there are many advertisers and many advertising "slots", characterized by different π_2's. We opted for the simplest possible analytical formulation here (one advertiser with uncertain π_2) in order to sidestep coordination issues which would distract from the main point (contained in Proposition 2 below).

information (cf. previous section) when profit per exposure to advertising is equal to the profit derived by the *marginal advertiser* (i.e. $\pi_2 = r^M(\pi_1)$ or $\pi_2 = \widehat{r}^M(\pi_1)$ respectively). Of course, the marginal advertisers are different in the two cases, but the general principle is the same.

Let us now compare this outcome to the corresponding exogenous affiliation under incomplete information benchmark. Specifically, product 2 is now exogenously affiliated with the platform (stages 2 and 3 are eliminated), but we maintain the incomplete information setting by assuming the platform does not observe π_2 prior to setting s and P (when access fees are feasible). Alternatively, one could assume that, when access fees are feasible, the platform observes π_2 prior to setting P - we treat this case in the appendix and show that the analysis that follows is not materially different.

When access fees are not feasible, the platform's expected profits in stage 1 are:

$$\max_s \left\{ \int_{\pi_2} X(s, \pi_1, \pi_2) \frac{V(s)}{t} dG(\pi_2) \right\} = \max_s \left\{ X(s, \pi_1, E\pi_2) \frac{V(s)}{t} \right\}$$

where $E\pi_2 \equiv \int_{\pi_2} \pi_2 dG(\pi_2)$ is the expected value of π_2 and the equality follows from the linearity of $X(s, \pi_1, \pi_2)$ in π_2.

When access fees are feasible, the platform's expected profits in stage 1 are:

$$\max_{s,P} \left\{ \int_{\pi_2} [P + X(s, \pi_1, \pi_2)] \frac{[V(s) - P]}{t} dG(\pi_2) \right\} = \max_s \left\{ \frac{[V(s) + X(s, \pi_1, E\pi_2)]^2}{4t} \right\}$$

The optimal level of search diversion is therefore $s^M(\pi_1, E\pi_2)$ without access fees and $\widehat{s}^M(\pi_1, E\pi_2)$ with access fees (cf. expressions 3 and 4 above). In other words, the optimal level of search diversion in the benchmark exogenous affiliation case is determined by the *average* advertiser $E\pi_2$, both with and without access fees.

Combining the previous results and using Lemma 1, we have:

Proposition 2 *With endogenous affiliation of product 2, the platform's optimal level of search diversion is determined by the marginal advertiser who affiliates ($\pi_2 = r^M(\pi_1)$ or $\pi_2 = \widehat{r}^M(\pi_1)$). Endogenous advertiser affiliation results in more search diversion relative to the exogenous affiliation benchmark if and only if the marginal advertiser under endogenous affiliation derives higher profits per consumer exposure relative to the average advertiser, i.e. if and only if $r^M(\pi_1) > E\pi_2$ without access fees and $\widehat{r}^M(\pi_1) > E\pi_2$ with access fees.*

This result is reminiscent of Spence (1975)'s insight that a monopolist's incentives to invest in product quality are driven by the marginal customer. In our model, things are more complicated because there are two sides (or two customer groups): consumers and the advertiser. Since consumers all have the same marginal disutility from search diversion, the relevant "customer" for determining search diversion (the inverse of "quality") is the advertiser.

An important implication of this result is that endogenous seller affiliation creates a novel potential source of search diversion incentives, relative to the two basic sources of incentives identified in Hagiu and Jullien (2011). Specifically, endogenous affiliation of the advertiser leads to more search diversion relative to exogenous affiliation if and only if the marginal advertiser derives higher profits relative to the average advertiser. This is because consumers are *not* interested in advertising, therefore higher advertising revenues make the platform's profit incentives run counter to consumer preferences (cf. Lemmas 1 and 2).

If we endogenized the affiliation of product 1 (content) instead of that of product 2 (advertising), we would expect to obtain a very similar result, but with a reversed inequality in Proposition 2. Indeed, suppose that product 2 is exogenously affiliated (with π_2 paid to the platform fixed and known), while an independent seller of product 1 must be convinced to affiliate, with π_1 unobservable by the platform, distributed with cdf. $G(.)$. A very similar analysis to the one above (details are relegated to the appendix) shows that endogenous affiliation of product 1 leads to *more* search diversion if and only if the marginal seller of product 1 derives *lower* profits per consumer exposure than the average seller of product 1.

3.3 Example

Suppose $F(x) = x$; $G(\pi_2) = \pi_2$; $0 \leq \pi_1 < 1$. Straightforward calculations (cf. appendix) yield:

- $E\pi_2 = 1/2$ and:

$$s^M(\pi_1, 1/2) = \max\{1 - 4\pi_1, 0\} \quad \text{and} \quad \widehat{s}^M(\pi_1, 1/2) = \begin{cases} 1 & \text{if} \quad \pi_1 < 1/2 - u_H/2 \\ 0 & \text{if} \quad \pi_1 \geq 1/2 - u_H/2 \end{cases}$$

- When consumer access fees are not feasible:

$$s^M_{end}(\pi_1) = \begin{cases} \frac{\sqrt{(1-\pi_1)(1+7\pi_1)} - \pi_1 - 1}{2\pi_1} & \text{if} \quad \pi_1 \leq 1/2 \\ 0 & \text{if} \quad \pi_1 \geq 1/2 \end{cases}$$

- When consumer access fees are feasible:

$$\widehat{s}^M_{end}(\pi_1) = \begin{cases} 1 & \text{if} \quad \pi_1 < 1 - u_H/2 \\ 0 & \text{if} \quad \pi_1 \geq 1 - u_H/2 \end{cases}$$

With these expressions, it is easily verified that $s^M_{end}(\pi_1) \geq s^M(\pi_1, 1/2)$ and $\widehat{s}^M_{end}(\pi_1) \geq \widehat{s}^M(\pi_1, 1/2)$ $\blacksquare$ for all π_1, with strict inequalities for $\pi_1 \leq 1/2$ and $\pi_1 < 1 - u_H/2$ respectively. Thus, in this example, endogenizing the affiliation of product 2 always leads to more search diversion.

4 Competition

We maintain the same structure of consumer preferences, but there are now two competing platforms, A and B, one at each end of the Hotelling $[0, 1]$ segment. Each platform is vertically integrated into product 1. We consider three competition scenarios:

- Competition for consumers: both products are exogenously affiliated with both platforms and the latter compete solely for the exclusive affiliation of consumers
- Competition for product 2 (advertising): all consumers are exogenously affiliated with both platforms and the platforms compete for the exclusive affiliation of a unique, independent seller of product 2 (advertiser)
- Two-sided competition: the platforms compete for the exclusive affiliation of consumers and of an independent seller of product 2.

In the following three subsections we derive the equilibrium outcomes for these scenarios. In the fourth subsection we compare the resulting equilibrium levels of search diversion.

4.1 Competition for consumers

In this scenario, products 1 and 2 are exogenously affiliated with both platforms. Each platform derives profits π_1 per consumer exposure to its product 1 and π_2 per consumer exposure to its product 2.[4] Intermediary $i \in \{A, B\}$ chooses a level of search diversion $s_i \in [0, 1]$ and, if feasible, an access fee P_i charged to consumers. The timing is as follows:

1. Intermediaries announce s_A and s_B simultaneously
2. If access fees are feasible, platforms announce P_A and P_B simultaneously
3. Consumers decide which platform to affiliate with (visit)
4. Consumers learn their search costs and search on the platforms they are affiliated with.

We separated the choices of s_i and P_i between the first two stages of the game in order to better reflect reality: design choices (s_i) are longer-term decisions for platforms relative to pricing choices, which can be modified more frequently. From a technical standpoint, it turns out that in this model the equilibrium level of search diversion would be exactly the same if instead we assumed that platforms choose (s_i, P_i) simultaneously (the space of deviations to consider would be, however, more complex).

[4]This scenario is also equivalent to assuming that each platform is vertically integrated into product 1 and that there is a seller of product 2 (advertiser) who multihomes. As discussed in section 3.1, each platform would then charge the advertiser $r = \pi_2$, leading to the same setup.

Suppose first that access fees are not feasible, i.e. stage 2 is eliminated. In this case, platform i' profits from the perspective of stage 1 are:

$$\Pi_i(s_i) = X(s_i, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s_i) - V(s_j)}{2t}\right] \tag{7}$$

Suppose now access fees are possible. In this case, stage 4 profits for platform i are:

$$\Pi_i(s_i, P_i) = [P_i + X(s_i, \pi_1, \pi_2)]\left[\frac{1}{2} + \frac{V(s_i) - P_i - V(s_j) + P_j}{2t}\right]$$

Given (s_i, s_j) chosen in stage 1, the stage 2 pricing equilibrium is easily verified to be $P_i = t + [V(s_i) - V(s_j) - X(s_j, \pi_1, \pi_2) - 2X(s_i, \pi_1, \pi_2)]/3$, leading to stage 1 platform profits:

$$\Pi_i(s_i) = \frac{t}{2}\left[1 + \frac{V(s_i) + X(s_i, \pi_1, \pi_2) - V(s_j) - X(s_j, \pi_1, \pi_2)}{3t}\right]^2 \tag{8}$$

From expressions (7) and (8) we can directly derive:

Proposition 3 *When seller affiliation is exogenously given and platforms compete for consumers only, the symmetric equilibrium level of search diversion solves:*

$$s^C_{cons}(\pi_1, \pi_2) = \arg\max_s \left\{X(s, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s) - V\left(s^C_{cons}\right)}{2t}\right]\right\} \tag{9}$$

when access fees are not feasible and:

$$\widehat{s}^C_{cons}(\pi_1, \pi_2) = \arg\max_s \{X(s, \pi_1, \pi_2) + V(s)\} = \widehat{s}^M(\pi_1, \pi_2) \tag{10}$$

when access fees are feasible.

In this equilibrium, the platforms' choices of search quality are entirely driven by competition for users. When access fees are feasible, the equilibrium level of search diversion is identical to the one chosen by a monopoly platform when both products are exogenously affiliated. In this case, the platform maximizes the joint surplus of the relationship with consumers and uses the access fee to share this surplus with consumers. Competition only affects the price level.

Furthermore, comparing the equilibrium levels of search diversion with and without access fees, we obtain a similar result to the monopoly case (cf. Proposition 1):

Corollary 1 *When platforms compete for consumers only, the equilibrium level of search diversion is lower with access fees ($\widehat{s}^C_{cons} < s^C_{cons}$) if and only if the equilibrium access fee is positive.*

The intepretation is the same as for Proposition 1. If the platforms charge positive access fees to consumers, they must offer them more value, i.e. lower s. Conversely, if the platforms subsidize the participation of consumers, they need to make up for the loss by increasing advertising revenues, which they can achieve by increasing s.

4.2 Competition for product 2 (advertising)

The polar opposite scenario relative to the previous subsection occurs when consumer affiliations with the two platforms are exogenously given and the latter compete for the exclusive affiliation of an independent seller of product 2 (advertiser). Specifically, we assume that all consumers (mass 1) are affiliated with both platforms. Thus, access fees are irrelevant here and platform i's choice of s_i affects consumer activity (search decisions) but not total consumer affiliation.

The independent seller derives a profit π_2 for each consumer impression on either platform, where π_2 is known to all players throughout (full information). This scenario occurs in mature markets, where consumers have long-standing and stable affiliation decisions, but platforms must attract new advertisers. We assume the seller affiliates exclusively with one platform.[5] Intermediaries compete for the seller by setting per impression fees (r_A, r_B). The timing is as follows:

1. Intermediaries announce s_A and s_B simultaneously
2. Intermediaries announce r_A and r_B simultaneously
3. The advertiser decides which platform to affiliate with
4. Consumers observe the advertiser's decision, learn their individual search costs and search on the platforms they are affiliated with. Revenues are realized.

We have separated the choices of s_i and r_i between the first two stages of the game in order to better reflect reality: design choices (s_i) are typicaly longer-term decisions for platforms relative to pricing choices (r_i here), which can be modified more frequently. Our equilibrium characterization below would be the same if we worked with the entire space of (s_i, r_i) deviations. The difference is that the set of equilibrium conditions to satisfy would be significantly more complicated: this is another reason for adopting our simpler set-up.

If the advertiser affiliates with platform A in stage 3, then platform A's profits are $X(s_A, \pi_1, r_A)$, while platform B's are $\pi_1 F(u_H)$. The advertiser's payoff is:

$$(\pi_2 - r_A)\, s_A F\left(\frac{u_H}{1+s_A}\right) = X(s_A, \pi_1, \pi_2) - X(s_A, \pi_1, r_A)$$

[5] Exclusive affiliation could be obtained endogenously by assuming that, for each consumer, only the first product exposure matters and that there is a sufficiently large probability that the same consumers are exposed to product 2 (advertising) on both platforms (see Athey Calvano and Gans 2012).

Note therefore that platform A necessarily sets (s_A, r_A) such that $X(s_A, \pi_1, r_A) \geq \pi_1 F(u_H)$. Relegating the remaining details to the appendix, we obtain:

Proposition 4 *When consumer affiliation is exogenously given and platforms compete for the exclusive affiliation of an advertiser characterized by π_2, the equilibrium level of search diversion is:*

$$s_{adv}^{C}(\pi_1, \pi_2) = \arg\max_{s} \{X(s, \pi_1, \pi_2)\} \tag{11}$$

This result is driven by Bertrand competition for the advertiser. Each platform i sets its search diversion level s_i to maximize its joint profits with the advertiser, $X(s_i, \pi_1, \pi_2)$, when the latter affiliates with i exclusively. Then both platforms compete in fees r_i so that the advertiser ends up capturing all of the joint profits in excess of each platform's outside option, $\pi_1 F(u_H)$.

4.3 Competition for both consumers and advertising

Let us now turn to the case in which the two platforms compete for exclusive affiliation on both sides of the market: consumers and an independent advertiser characterized by π_2, which is once again common knowledge. To keep things as simple and concise as possible, we assume consumer access fees are not feasible. Thus, each platform $i \in \{A, B\}$ chooses a level of search diversion $s_i \in [0, 1]$ and a per impression fee r_i that the advertiser would have to pay if it affiliated with platform i. The timing is now as follows:

1. Intermediaries announce s_A and s_B simultaneously
2. Intermediaries announce r_A and r_B simultaneously
3. The advertiser decides which platform to affiliate with
4. Consumers observe (s_A, s_B) and the advertiser's affiliation decision and decide which platform to affiliate with
5. Consumers learn their search costs and search on the platforms they are affiliated with.

The expressions of platforms' profits and of the advertiser's payoff when the latter affiliates exclusively with platform A are:

$$\Pi_A = X(s_A, \pi_1, r_A)\left[\frac{1}{2} + \frac{V(s_A) - V(0)}{2t}\right] \quad \text{and} \quad \Pi_B = \pi_1 F(u_H)\left[\frac{1}{2} + \frac{V(0) - V(s_A)}{2t}\right]$$

$$\Pi^{adv}(A) = (\pi_2 - r_A)\, s_A F\left(\frac{u_H}{1 + s_A}\right)\left[\frac{1}{2} + \frac{V(s_A) - V(0)}{2t}\right]$$

Relegating the proof to the appendix, we obtain:

Proposition 5 *When both consumers and the independent seller singlehome, the equilibrium level of search diversion $s^C(\pi_1, \pi_2)$ is given by:*

$$s^C(\pi_1,\pi_2) = \arg\max_s \left\{ X(s,\pi_1,\pi_2)\left[\frac{1}{2} + \frac{V(s) - V(0)}{2t}\right] + \pi_1 F(u_H)\left[\frac{1}{2} + \frac{V(0) - V(s)}{2t}\right]\right\} \quad (12)$$

The first term in the expression maximized by s^C is the joint profit of the advertiser and the platform with which the advertiser affiliates in equilibrium. The second term might seem odd at first glance. To understand its origin, note that it is equal to the profit of the platform who "loses" the advertiser affiliation game (say, B) and that it depends on the search diversion level chosen by the winning platform (say, A). The presence of this term can be interpreted as follows: A must offer the advertiser a payoff just above the largest payoff that can be offered by B, who in turn can only offer the advertiser the difference between joint profits and B's outside option, i.e. what B gets when the advertiser affiliates with A. This term is *increasing* in the search diversion level chosen by A: a higher s_A increases B's outside option (more consumers go to B instead of A) and thereby decreases the hurdle that A needs to overcome in order to attract the advertiser.

4.4 Comparison

First, does competition among platforms lead to less search diversion relative to a monopolist? Not in this model, not even when competition is for consumers. Indeed, in the appendix we prove:[6]

Corollary 2 *The equilibrium level of search diversion with competing platforms is always at least as high as that chosen by a monopoly platform:*

$$\begin{aligned} \widehat{s}^C_{cons} &= \widehat{s}^M \text{ and } s^C_{cons} > s^M \\ s^C_{adv} &\geq \max\left(s^M, \widehat{s}^M\right) \\ s^C &> s^M \end{aligned}$$

Furthermore, s^C_{cons} and s^C are increasing in t, while $\widehat{s}^C_{cons}$ and s^C_{adv} do not depend on t.

Competition for product 2 (advertising) increases search diversion relative to a monopolist because the platforms focus their competitive efforts on the advertiser and ignore the effect of increased search diversion on consumer affiliation, which they take as given.[7]

[6] We have dropped functional arguments (π_1, π_2) for ease of reading.

[7] Note, however, that even from seller 2's perspective, the optimal level of search diversion is not necessarily 1. Indeed, suppose seller 2 is affiliated with a monopoly platform that does not charge membership fees to consumers, has set the level of search diversion at s and charges per impression fee $r < \pi$. In this case, seller 2's revenues are $s(\pi - r)F(u(s))V(s)/t$, so that seller 2 perceives a tradeoff between more impressions and lower overall consumer traffic to the platform, which ultimately also hurts the advertiser. Clearly, the revenue-maximizing s is smaller than 1.

Perhaps more surprisingly, even when platforms compete for consumers, they still end up diverting search more than a monopolist would. This is true both when product 2 is exogenously affiliated with the platforms (s^C_{cons} and $\widehat{s}^C_{cons}$) and when the platforms compete for an independent product 2 seller (s^C). The reason is that a monopoly platform is able to extract larger rents from consumers and from the independent seller relative to competing platforms. Thus, the monopolist has stronger incentives to reduce search diversion in this model. It is straightforward to verify that this result would be reversed if the monopoly platform covered the entire consumer market, so that its choice of search diversion ignored the impact on consumer affiliation decisions, which competing platforms can never ignore.

Finally, note that *more intense* competition for consumers (*lower* t) has the expected effect, i.e. it leads to (weakly) *less* search diversion in all 3 competition scenarios. This is unsurprising given that consumers prefer less search diversion.

Second, how does the nature of competition affect the equilibrium level of search diversion? Competition for product 2 (advertising) only always leads to the highest level of search diversion (the proof is contained in that of Corollary 2):

Corollary 3 $s^C_{adv} > \max\left\{\widehat{s}^C_{cons}, s^C_{cons}, s^C\right\}$

This result further strengthens the intuition above: competition for product 2 goes against users' interests, since seller 2 directly benefits from search diversion.

More interesting and complex is the comparison between the scenario when platforms compete for consumers only and the scenario in which platforms compete for product 2 as well as for consumers. In both scenarios, let us focus on the case when access fees are not feasible (s^C_{cons} vs. s^C). It is useful to write the first-order conditions determining s^C_{cons} and s^C respectively, derived from equations (9) and (12) (we denote $X_s \equiv \partial X/\partial s$):

$$X_s\left(s^C_{cons}, \pi_1, \pi_2\right) + X\left(s^C_{cons}, \pi_1, \pi_2\right)\frac{V'\left(s^C_{cons}\right)}{t} = 0$$

$$X_s\left(s^C, \pi_1, \pi_2\right)\cdot\left[1 - \frac{V(0) - V\left(s^C\right)}{t}\right] + X\left(s^C, \pi_1, \pi_2\right)\frac{V'\left(s^C\right)}{t} - \pi_1 F\left(u_H\right)\frac{V'\left(s^C\right)}{t} = 0$$

Note first that both $X_s\left(s^C_{cons}, \pi_1, \pi_2\right)$ and $X_s\left(s^C, \pi_1, \pi_2\right)$ are positive. Comparing the two first-order conditions term by term, the following tradeoff becomes apparent. On the one hand, the *positive* term $-\pi_1 F\left(u_H\right) V'\left(s^C\right)/t$ tends to make s^C larger than s^C_{cons}. This term is the derivative of the profit obtained by platform B when A attracts the advertiser exclusively. By *increasing* search diversion, A increases B's outside option and thereby makes it more likely that B will be content not to fight too hard for the advertiser, so that A can attract the advertiser more cheaply. When the advertiser multihomes, this effect disappears.

On the other hand, the positive term $\left\{1-\left[V(0)-V\left(s^C\right)\right]/t\right\}$ tends to make s^C smaller than s^C_{cons}. This is because the corresponding term in the first-order condition for s^C_{cons} is 1 (which is larger) and in both first-order conditions they multiply the *positive* term $X_s(.,\pi_1,\pi_2)$. The difference is that, when the advertiser singlehomes, equilibrium consumer traffic at the platform who attracts the advertiser is $1/2-\left[V(0)-V\left(s^C\right)\right]/2t$, strictly smaller than 1/2 and *decreasing* in the level of search diversion. In contrast, when the seller multihomes, equilibrium consumer traffic is always 1/2 and insensitive to the equilibrium level of search diversion. This difference creates an incentive for the platform who attracts the advertiser exclusively to *decrease* s in order to make up for the advertiser's presence. The balance of these two countervailing effects can go either way, as we illustrate with the example below.

Third and finally, it is easily seen that, just like in the monopoly case studied in section 2, the equilibrium level of search diversion in all competition scenarios is decreasing in π_1 (the value of a consumer exposure to product 1) and increasing in π_2 (value of a consumer exposure to the advertiser). In other words, there is less search diversion when the advertising monetization opportunity is more attractive relative to the standalone monetization opportunity.

4.5 Example

Let us now consider a specific example with closed form solutions. Let $F(x)=x$ and assume $2>4t/u_H^2>\pi_2/(\pi_2-\pi_1)>3t/u_H^2$ (these conditions are necessary to ensure all solutions are well-defined and belong to the interval $(0,1)$). In this case, we have:

$$s^C_{cons} = \frac{1}{\frac{\pi_2}{\pi_2-\pi_1}-\frac{2t}{u_H^2}}-1 \quad \text{and} \quad \widehat{s}^C_{cons}=\begin{cases} 0 & \text{if} \quad u_H \geq 2(\pi_2-\pi_1) \\ 1 & \text{if} \quad u_H \leq 2(\pi_2-\pi_1) \end{cases}$$

$$s^C_{adv} = 1$$

$$s^C = \frac{1}{1-\frac{t}{u_H^2}}-1$$

Several observations are in order. First, s^C_{cons}, $\widehat{s}^C_{cons}$ and s^C are all decreasing in u_H, the utility derived by consumers from product 1 (content). In other words, the more valuable content is, the lower the incentives to divert search.

Second, s^C_{cons}, $\widehat{s}^C_{cons}$ and s^C are decreasing in π_1 and increasing in π_2 as expected.

Third:

$$s^C \geq s^C_{cons} \text{ if and only if } \frac{\pi_2}{\pi_2-\pi_1} \geq 1+\frac{t}{u_H^2}$$

Thus, introducing competition for the advertiser is more likely to *increase* the equilibrium level of search diversion (i.e. s^C is *larger* than s^C_{cons}) when: i) competition for consumers is more intense (t is smaller); ii) content value to consumers (u_H) is larger; iii) advertising revenues are less attractive relative to standalone revenues (π_1/π_2 is larger). The counterintuitive effect of π_1/π_2 is understood as follows. When π_1/π_2 increases, platforms have an incentive to shift their revenue composition (reflected in $X(s)$) towards higher total consumer participation by reducing search diversion s (cf. Lemma 1). This effect turns out to be weaker when the advertiser singlehomes because the platform who attracts the advertiser exclusively also ends up attracting fewer total users relative to the case when the advertiser multihomes, so the weight placed on maximizing $X(s)$ is smaller.

5 Conclusions: managerial implications and empirical predictions

Our study of search diversion by platforms has yielded several important and novel insights (relative to Hagiu and Jullien 2011) that should be of managerial interest. The first concerns the relationship between access fees charged to consumers and search diversion. Intermediaries that charge positive access fees to consumers have weaker incentives to divert search relative to platforms that cannot (or choose not to) charge such fees. On the other hand, platforms that subsidize consumer participation have stronger incentives to divert search. This prediction of the model can be tested empirically: one could examine the design practices of various platforms, create an objective measure of search diversion and estimate the correlation between this measure and the magnitude of access fees charged by the platforms.

Second, endogenous affiliation of content sellers and advertisers can create an additional reason to divert search. Specifically, platforms' incentives to strategically divert consumer search are stronger relative to the exogenous affiliation benchmark if the marginal independent advertiser (respectively, content seller) derives higher (respectively, lower) profits per consumer exposure than the average advertiser (respectively, content seller). The reason is that search diversion reduces the variance of revenues among sellers or advertisers, which enables platforms to extract larger fees from them. A relevant empirical study would compare search diversion measures between platforms that are vertically integrated into one or more of the products offered to consumers (vertical integration would be tantamount to exogenous affiliation) and platforms that connect consumers to third-party, independent sellers. Our model suggests that the extent of vertical integration should be an important predictor of the magnitude of search diversion.

Third, the effect of competition between platforms on the equilibrium level of search diversion is determined by the nature of competition. When horizontal differentiation between competing platforms in the eyes of consumers is reduced, the equilibrium level of search quality increases, as expected. On the other hand, we found that competition among platforms leads to at least as much search diversion as monopoly. More specifically, competition for advertising leads to the highest level of search diversion, which is always higher than the level induced by a monoply platform. These results suggest straightforward strategy recommendations and empirical tests: platforms' search diversion incentives increase with competitive pressure on the advertising side of the market and decrease with competitive pressure on the consumer side.

A broader implication of our analysis is that design decisions by two-sided platforms and platforms (search diversion being but one specific example) are fundamentally driven by their quest to improve rent-extraction power and can favor one side of the market or the other. One should then not be surprised that intensified competition may lead to design decisions which go against consumers' preferences - but instead favor third-party sellers, advertisers, or whatever the relevant second side might be. A striking example was the privacy compromise made by Microsoft with the design of its Internet Explorer 8 web browser in favor of advertisers and against users' interests. The episode was reported by the Wall Street Journal: "*As the leading maker of Web browsers, the gateway software to the Internet, Microsoft must balance conflicting interests: helping people surf the Web with its browser to keep their mouse clicks private, and helping advertisers who want to see those clicks. In the end, the product planners lost a key part of the debate. The winners: executives who argued that giving automatic privacy to consumers would make it tougher for Microsoft to profit from selling online ads. Microsoft built its browser so that users must deliberately turn on privacy settings every time they start up the software.*" (Wingfield (2010))

References

[1] Anderson S. and Coate S. (2005), "Market Provision of Broadcasting: Welfare Analysis", *Review of Economic Studies*, 72(4), 947-972.

[2] Ambrus A., Calvano E. and Reisinger M. (2012) "A Two-Sided Theory of Advertising With Overlapping Viewership" working paper.

[3] Armstrong, M. (2006) "Competition in Two-Sided Markets," *Rand Journal of Economics* 37 (3), 669-691.

[4] Athey S., Calvano E. and Gans J. S. (2012) "The Impact of the Internet on Advertising Markets for News Media," *Rotman School of Management Working Paper* No. 2180851.

[5] Athey S., Gans J. S. (2010) "The Impact of Targeting Technology on Advertising Markets and Media Competition", *American Economic Review* 100(2), 608-613.

[6] Bagwell K. (2007), "The Economic Analysis of Advertising", in Armstrong R. and R. Porter eds.,*Handbook of Industrial Organization*, Amsterdam: Elsevier, 2007, pp. 1702-1844.

[7] Baye M. R. and Morgan J. (2001): "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets", *American Economic Review* 91(3), 454-474.

[8] Boudreau, K.J. (2010) "Open Platform Strategies and Innovation: Granting Access vs. Devolving Control," *Management Science* 56(10), 1849-1872.

[9] Caillaud, B., and B. Jullien (2003) “Chicken and Egg: Competition Among Intermediation Service Providers,” *Rand Journal of Economics* 34(2), 309-328.

[10] Casadesus-Masanell, R. and Zhu F. (2010) "Strategies to Fight Ad-Sponsored Rivals," *Management Science* 56(9), 1484–1499.

[11] Crampes, C., C. Haritchabalet and B. Jullien (2009) "Advertising, Competition and Entry in Media Industries," *Journal of Industrial Economics*, 57(1), 7–31.

[12] Economides, N. and E. Katsamakas (2006) "Two-Sided Competition of Proprietary vs. Open Source Technology Platforms and the Implications for the Software Industry," *Management Science* 52(7), 1057-1071.

[13] Eisenmann, T., G. Parker and M. Van Alstyne (2011) "Platform Envelopment," *Strategic Management Journal*, 32(12), 1270–1285.

[14] Evans, D. and R. Schmalensee (2007) "The Industrial Organization of Markets with Two-Sided Platforms," *Competition Policy International* 3(1).

[15] Gabszewicz, J. J., D. Laussel and N. Sonnac (2006) "Competition in the Media and Advertising Markets," *Manchester School,* 74(1), 1–22.

[16] Hagiu, A. and B. Jullien (2011) "Why Do Intermediaries Divert Search?" *Rand Journal of Economics* 42(2), 337-362.

[17] Hagiu, A. and D. Spulber (2012) "First-Party Content and Coordination in Two-Sided Markets," *Management Science*, forthcoming.

[18] Hossain, T., D. Minor and J. Morgan (2011) "Competing Matchmakers: An Experimental Analysis," *Management Science*, 57, 1913-1925.

[19] Kaplan, S. and M. Sawhney (2000) "E-Hubs: The New B2B Marketplaces," *Harvard Business Review*, May-June 2000, R00306, 97-103.

[20] Parker, G. and M. W. Van Alstyne (2005) “Two-Sided Network Effects: A Theory of Information Product Design,” *Management Science,* 51: 1494-1504.

[21] Parker, G. and M. W. Van Alstyne (2008) "Innovation, Openness and Platform Control," working paper, Tulane University, Boston University and MIT.

[22] Peitz, M. and T. M. Valletti (2008) "Content and Advertising in the Media: Pay-TV Versus Free-to-Air," *International Journal Industrial Organization,* 26(4), 949–965.

[23] Petroski, H. (2003) *Small Things Considered: Why There Is No Perfect Design*, New York: Knopf.

[24] Rochet, J.-C., and J. Tirole (2006) "Two-Sided Markets: Where We Stand," *Rand Journal of Economics*, Vol. 37 (3), 645-66.

[25] Shih, W., S. Kaufman and D. Spinola (2007) "Netflix," Harvard Business School case study 9-607-138.

[26] Spence, M. (1975) "Monopoly, Quality and Regulation," *Bell Journal of Economics*, 6, 417-429.

[27] Spulber, D. F. (1999) *Market Microstructure: Intermediaries and the Theory of the Firm*, Cambridge: Cambridge University Press.

[28] Spulber, D. F. (2006) Firms and Networks in Two-Sided Markets, in Terry Hendershott ed., *Handbook of Economics and Information Systems*, 1, Amsterdam: Elsevier, 2006, pp. 137-200.

[29] Weyl, E. G. (2010). "A Price Theory of Multi-sided Platforms." *American Economic Review*, 100(4): 1642–72.

[30] Wingfield, N. (2010) "Microsoft Quashed Effort to Boost Online Privacy," *The Wall Street Journal*, August 2 2010.

[31] Zhu, F. and M. Iansiti (2011) "Entry Into Platform-Based Markets," *Strategic Management Journal*.

6 Appendix

6.1 Proof of Lemma 1

The first-order condition determining $s^M(\pi_1, \pi_2) = \arg\max_s \{X(s, \pi_1, \pi_2) V(s)\}$ can be written:

$$(\pi_1/\pi_2 + s) \cdot \left[\frac{-u_H}{(1+s)^2} F'\left(\frac{u_H}{1+s}\right) V(s) + F\left(\frac{u_H}{1+s}\right) V'(s) \right] + F\left(\frac{u_H}{1+s}\right) V(s) = 0$$

Since $V'(s) < 0$, the left-hand side is decreasing in π_1/π_2, which implies (assuming second-order conditions are satisfied) that $s^M(\pi_1, \pi_2)$ is decreasing in π_1/π_2.

6.2 Proof of Lemma 2

We have:

$$\begin{aligned} \widehat{s}^M(\pi_1, \pi_2) &= \arg\max_s \left\{ \max_P \{[P + X(s, \pi_1, \pi_2)][V(s) - P]\} \right\} \\ &= \arg\max_s \{X(s, \pi_1, \pi_2) + V(s)\} \end{aligned}$$

The first-order condition determining $\widehat{s}^M(\pi_1, \pi_2)$ can then be written:

$$(\pi_1/\pi_2 + s) \cdot \frac{-u_H}{(1+s)^2} F'\left(\frac{u_H}{1+s}\right) + F\left(\frac{u_H}{1+s}\right) + V'(s)/\pi_2 = 0$$

The left-hand side is decreasing in π_1 and increasing in π_2, which implies (assuming the second-order condition is satisfied) that $\widehat{s}^M$ is decreasing in π_1 and increasing in π_2.

6.3 Proof of Proposition 1

The first order condition determining s^M is:

$$X_s(s, \pi_1, \pi_2) V(s) + X(s, \pi_1, \pi_2) V'(s) = 0$$

while the first order condition determining $\widehat{s}^M$ is:

$$X_s(s, \pi_1, \pi_2) + V'(s) = 0$$

Comparing these two first-order conditions and assuming second-order conditions hold in both cases, we have $\widehat{s}^M \leq s^M$ if and only if $V\left(\widehat{s}^M\right) \geq X\left(\widehat{s}^M, \pi_1, \pi_2\right)$, which is equivalent to $P_M \geq 0$, since the profit-maximizing access fee is $P_M = \left[V\left(\widehat{s}^M\right) - X\left(\widehat{s}^M, \pi_1, \pi_2\right)\right]/2$.

6.4 Monopoly platform - endogenous affiliation of product 2

In section 3, we compared the optimal choices of search diversion under endogenous affiliation relative to a benchmark scenario with exogenous affiliation and incomplete information. Specifically, for the benchmark case we assumed that the platform only observes the advertiser's profit π_2 after it sets s and P.

Let us briefly analyze an alternative benchmark, in which the platform observes π_2 after setting s but prior to setting P. Obviously, this alternative is only relevant for the case when access fees are feasible. The analysis of the case with no access fees remains unchanged.

The platform's expected profits given its choice of s are now:

$$\int_{\pi_2} \max_P \left\{ [P + X(s, \pi_1, \pi_2)] \frac{[V(s) - P]}{t} \right\} dG(\pi_2) = \int_{\pi_2} \frac{[V(s) + X(s, \pi_1, \pi_2)]^2}{4t} dG(\pi_2)$$

$$= \frac{[V(s) + X(s, \pi_1, E\pi_2)]^2}{4t} + \frac{Var(\pi_2)}{4t} \left[sF\left(\frac{u_H}{1+s}\right) \right]^2$$

where $Var(\pi_2) \equiv \int_{\pi_2} (\pi_2 - E\pi_2)^2 dG(\pi_2)$ is the variance of π_2. The last equality is obtained by using the well-known property $E(X^2) = (EX)^2 + Var(X)$ and recalling that $X(s, \pi_1, \pi_2)$ is linear in π_2. In stage 1, the platform chooses s to maximize the expression above.

Thus, if we used this benchmark for the case with access fees instead of the one used in the main text, there would be one additional factor driving a wedge between optimal search diversion levels with endogenous and exogenous affiliation. The first factor is unchanged: endogenizing advertiser affiliation leads the platform to make its search diversion decision based on the marginal as opposed to the average advertiser. The second factor is captured by the second term in the last expression above. This term is positive and reflects the information rent that the platform can extract by adjusting P after observing π_2. As the variance of π_2 increases (at constant mean $E\pi_2$), the platform's optimal choice of search diversion places a higher weight on maximizing pure advertising revenues $sF\left(u_H/\left(1+s\right)\right)$, which leads to a higher optimal s, i.e. more search diversion.

6.5 Monopoly platform - endogenous affiliation of product 1

A very similar analysis to the one with endogenous affiliation of product 2 leads to the following expression of platform profits when the platform can charge access fees P:

$$\max_{s,r}\left\{\left[1-G\left(r\right)\right]\cdot\max_{P}\left\{\left[P+X\left(s,\pi_1,r\right)\right]\frac{\left[V\left(s\right)-P\right]}{t}\right\}\right\}$$

The key difference is that without affiliation of product 1, platform profits are 0 because it offers no value to consumers. Thus, if access fees are feasible then the optimal choices $\widehat{s}^M_{end}$ and $\widehat{r}^M$ verify:

$$\widehat{s}^M_{end}=\widehat{s}^M\left(\widehat{r}^M,\pi_2\right)$$

Without access fees ($P=0$), platform profits are:

$$\max_{s,r}\left\{\left[1-G\left(r\right)\right]X\left(s,r,\pi_2\right)\frac{V\left(s\right)}{t}\right\}$$

so that the optimal choices s^M_{end} and r^M verify:

$$s^M_{end}=s^M\left(r^M,\pi_2\right)$$

Thus, we can conclude once again that the optimal search quality is determined by the marginal seller of product 1 who affiliates. Endogenous product 1 affiliation results in more search diversion relative to vertical integration (i.e. $s^M\left(r^M,\pi_2\right)>s^M\left(E\pi_1,\pi_2\right)$ without access fees or $\widehat{s}^M\left(\widehat{r}^M,\pi_2\right)>\widehat{s}^M\left(E\pi_1,\pi_2\right)$ with access fees) if and only if the marginal seller of product 1 under endogenous affiliation derives lower profits from search diversion relative to the average seller of product 1, i.e. $r^M<E\pi_1$ without access fees or $\widehat{r}^M<E\pi_1$ with access fees.

6.6 Example for monopoly with endogenous affiliation of product 2

Suppose $F\left(x\right)=x$; $G\left(\pi_2\right)=\pi_2$; $0\leq\pi_1<1$ and t large enough so that the platform never covers the consumer market. We have:

$$V(s) = \frac{u_H^2}{2(1+s)} \quad \text{and} \quad X(s,\pi_1,\pi_2) = \frac{(\pi_1 + s\pi_2)\, u_H}{1+s}$$

so that:

$$\begin{aligned} s^M(\pi_1,\pi_2) &= \arg\max_s \{X(s,\pi_1,\pi_2)\, V(s)\} = \max(0, 1 - 2\pi_1/\pi_2) \\ \widehat{s}^M(\pi_1,\pi_2) &= \arg\max_s \{X(s,\pi_1,\pi_2) + V(s)\} = \begin{cases} 1 & \text{if} \quad u_H \le 2(\pi_2 - \pi_1) \\ 0 & \text{if} \quad u_H \ge 2(\pi_2 - \pi_1) \end{cases} \end{aligned}$$

When access fees are not feasible, the platform solves the optimization problem (5). Plugging in the functional forms determined above and taking the first-order conditions in s and r respectively, we obtain the following two equations:

$$\begin{aligned} s_{end}^M &= \max\left(0, 1 - 2\pi_1/r^M\right) \\ 2r^M - 1 &= \pi_1\left(2 + s_{end}^M\right) \end{aligned}$$

Solving for $\left(s_{end}^M, r^M\right)$, we obtain:

- if $\pi_1 \le 1/2$ then $s_{end}^M(\pi_1) = \frac{-1-\pi_1+\sqrt{(1-\pi_1)(1+7\pi_1)}}{2\pi_1}$ and $r^M(\pi_1) = \frac{3\pi_1+1+\sqrt{(1-\pi_1)(1+7\pi_1)}}{4}$
- if $\pi_1 \ge 1/2$ then $s_{end}^M(\pi_1) = 0$ and $r^M(\pi_1) = \pi_1 + 1/2$

When access fees are feasible, the platform solves the optimization problem (6). Plugging in the functional forms determined above and taking the first-order conditions in s and r respectively, we obtain the following two equations:

$$\begin{aligned} \widehat{s}_{end}^M &= \begin{cases} 1 & \text{if} \quad u_H \le 2\left(\widehat{r}^M - \pi_1\right) \\ 0 & \text{if} \quad u_H \ge 2\left(\widehat{r}^M - \pi_1\right) \end{cases} \\ (u_H + 2\pi_1)^2\left(2 + \widehat{s}_{end}^M\right) &= 4(u_H + 2\pi_1)\left(2\widehat{r}^M - 1\right) + 4\widehat{r}^M\left(3\widehat{r}^M - 2\right)\widehat{s}_{end}^M \end{aligned}$$

Solving for $\left(\widehat{s}_{end}^M, \widehat{r}^M\right)$, we obtain:

- if $\pi_1 < 1 - u_H/2$ then $\widehat{s}_{end}^M(\pi_1) = 1$ and $\widehat{r}^M(\pi_1) = \frac{2(1-u_H-2\pi_1)+\sqrt{4+4(u_H+2\pi_1)+13(u_H+2\pi_1)^2}}{6}$
- if $\pi_1 \ge 1 - u_H/2$ then $\widehat{s}_{end}^M(\pi_1) = 0$ and $\widehat{r}^M(\pi_1) = \frac{u_H+2\pi_1+2}{4}$

6.7 Proof of Corollary 1

The first-order condition determining s_{cons}^C is:

$$X_s(s,\pi_1,\pi_2) + X(s,\pi_1,\pi_2)\frac{V'(s)}{t} = 0$$

while the first-order condition determining $\widehat{s}^C_{cons}$ is:

$$X_s(s,\pi_1,\pi_2)+V'(s)=0$$

Comparing these two first-order conditions and assuming second-order conditions hold in both cases, we have $\widehat{s}^C_{cons}\leq s^C_{cons}$ if and only if $X\left(\widehat{s}^C_{cons},\pi_1,\pi_2\right)\leq t$. The result follows by recalling that the equilibrium access fee is $P^C=t-X\left(\widehat{s}^C_{cons},\pi_1,\pi_2\right)$.

6.8 Proof of Proposition 4

Consider stage 3. The advertiser's payoff from affiliating exclusively with platform i is:

$$(\pi_2-r_i)\,s_iF\left(\frac{u_H}{1+s_i}\right)=X(s_i,\pi_1,\pi_2)-X(s_i,\pi_1,r_i),$$

while platform i's payoff is $X(s_i,\pi_1,r_i)$. In stage 2, platform i is prepared to lower its fee r_i until its payoff is equal to its outside option, $\pi_1F(u_H)$. Consequently, in the equilibrium of the game starting at stage 2, the advertiser affiliates with the platform that has the highest $X(s_i,\pi_1,\pi_2)$. The fees in the stage 2 equilibrium are determined by $X(s_i,\pi_1,\pi_2)-X(s_i,\pi_1,r_i)=X(s_j,\pi_1,\pi_2)-\pi_1F(u_H)$ for the winning platform i and $X(s_j,\pi_1,r_j)=\pi_1F(u_H)$ for the other platform (j).

Consider now stage 1. If $X(s_i,\pi_1,\pi_2)>X(s_j,\pi_1,\pi_2)$ then platform i attracts the advertiser with probability 1 and obtains profits $\pi_1F(u_H)+X(s_i,\pi_1,\pi_2)-X(s_j,\pi_1,\pi_2)$. If $X(s_i,\pi_1,\pi_2)=X(s_j,\pi_1,\pi_2)$ then the advertiser is indifferent between affiliating with either platform and both platforms' profits are equal to $\pi_1F(u_H)$. This implies that any allocation in which platforms set $s_i=s_j=\arg\max_s\{X(s,\pi_1,\pi_2)\}$ and the advertiser affiliates exclusively with platform A is an equilibrium. Equilibrium payoffs for the platforms are then $\pi_1F(u_H)$. Finally, the equilibrium fee charged by platform A to the advertiser is determined by $X(\arg\max_s\{X(s,\pi_1,\pi_2)\},\pi_1,r^*)=\pi_1F(u_H)$.

6.9 Proof of Proposition 5

Let us start by characterizing the singlehoming equilibrium (this is the unique equilibrium) starting in stage 2. Denote by $\Pi^{adv}(A)$ (respectively, $\Pi^{adv}(B)$) the payoffs obtained by the advertiser when it affiliates exclusively with platform A (respectively, platform B) and by Π^A (respectively Π^B) the profits derived by platform A (respectively, platform B).

Suppose the advertiser affiliates exclusively with platform A in equilibrium. Then the equilibrium fees

(r_A, r_B) must be such that:

$$\begin{aligned}
\Pi^{adv}(A) &= \Pi^{adv}(B) \\
\Pi^{adv}(B) &= X(s_B, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s_B) - V(0)}{2t}\right] - \pi_1 F(u_H)\left[\frac{1}{2} + \frac{V(0) - V(s_A)}{2t}\right] \\
\Pi^A &= X(s_A, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s_A) - V(0)}{2t}\right] - \Pi^{adv}(A) \\
\Pi^A &\geq \pi_1 F(u_H)\left[\frac{1}{2} + \frac{V(0) - V(s_B)}{2t}\right]
\end{aligned}$$

The first condition ensures that the advertiser weakly prefers to affiliate with A and that platform A cannot increase its profits by raising r_A.

The second condition requires platform B to offer the advertiser all the surplus in excess of B's outside option that would be created if the advertiser were to affiliate with B instead of A. If B was offering any less in equilibrium, it could then profitably deviate by slightly decreasing r_B and getting the advertiser to affiliate with it (because of the first condition)

The third condition is a simple accounting equality.

The fourth and final condition ensures that platform A prefers the equilibrium to a deviation in which it would let the advertiser affiliate with B.

Using these 4 conditions, we obtain:

$$\begin{aligned}
\Pi^A &= X(s_A, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s_A) - V(0)}{2t}\right] \\
&\quad - X(s_B, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s_B) - V(0)}{2t}\right] + \pi_1 F(u_H)\left[\frac{1}{2} + \frac{V(0) - V(s_A)}{2t}\right] \\
&\geq \pi_1 F(u_H)\left[\frac{1}{2} + \frac{V(0) - V(s_B)}{2t}\right]
\end{aligned}$$

which implies:

$$\begin{aligned}
& X(s_A, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s_A) - V(0)}{2t}\right] + \pi_1 F(u_H)\left[\frac{1}{2} + \frac{V(0) - V(s_A)}{2t}\right] \\
\geq\; & X(s_B, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s_B) - V(0)}{2t}\right] + \pi_1 F(u_H)\left[\frac{1}{2} + \frac{V(0) - V(s_B)}{2t}\right]
\end{aligned}$$

Conversely, if this condition holds, it is easily seen that the advertiser affiliates exclusively with A in equilibrium. Otherwise, the advertiser affiliates exclusively with B.

Stepping back to stage 1, platform A's choice of s_A must maximize:

$$X(s_A, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s_A) - V(0)}{2t}\right] - X(s_B, \pi_1, \pi_2)\left[\frac{1}{2} + \frac{V(s_B) - V(0)}{2t}\right] + \pi_1 F(u_H)\left[\frac{1}{2} + \frac{V(0) - V(s_A)}{2t}\right] \blacksquare$$

which leads to:

$$s^C = \arg\max_s \left\{ X(s,\pi_1,\pi_2) \left[\frac{1}{2} + \frac{V(s) - V(0)}{2t} \right] + \pi_1 F(u_H) \left[\frac{1}{2} + \frac{V(0) - V(s)}{2t} \right] \right\}$$

6.10 Proof of Corollary 2

Recall:

$$\begin{aligned} s^C_{adv}(\pi_1,\pi_2) &= \arg\max_s X(s,\pi_1,\pi_2) \\ s^M(\pi_1,\pi_2) &= \arg\max_s \{X(s,\pi_1,\pi_2) V(s)\} \\ \widehat{s}^M(\pi_1,\pi_2) &= \arg\max_s \{X(s,\pi_1,\pi_2) + V(s)\} \end{aligned}$$

Since $V(s)$ is decreasing, the second result follows easily: $s^C_{adv} \geq \max\left(s^M, \widehat{s}^M\right)$.

We already know that $\widehat{s}^C_{cons}(\pi_1,\pi_2) = \widehat{s}^M(\pi_1,\pi_2)$.

From (9) and (12), the first-order conditions determining $s^C_{cons}(\pi_1,\pi_2)$ and $s^C(\pi_1,\pi_2)$ are:

$$X_s\left(s^C_{cons},\pi_1,\pi_2\right) + X\left(s^C_{cons},\pi_1,\pi_2\right)\frac{V'\left(s^C_{cons}\right)}{t} = 0 \tag{13}$$

$$X_s\left(s^C,\pi_1,\pi_2\right)\cdot\left[1 - \frac{V(0) - V\left(s^C\right)}{t}\right] + X\left(s^C,\pi_1,\pi_2\right)\frac{V'\left(s^C\right)}{t} - \pi_1 F(u_H)\frac{V'\left(s^C\right)}{t} = 0 \tag{14}$$

First, note that both s^C_{cons} and s^C are smaller than $\arg\max_s \{X(s,\pi_1,\pi_2)\} = s^C_{adv}$. Indeed, $V'(s) < 0$ for all s and $\max_s \{X(s,\pi_1,\pi_2)\} \geq \pi_1 F(u_H)$. Therefore, $X_s\left(s^C_{cons},\pi_1,\pi_2\right) > 0$ and $X_s\left(s^C,\pi_1,\pi_2\right) > 0$. Consequently, the left-hand sides in both equations above are increasing in t, implying that $s^C_{cons}(\pi_1,\pi_2)$ and $s^C(\pi_1,\pi_2)$ are both increasing in t.

Recall that $s^M(\pi_1,\pi_2)$ is determined by:

$$X_s\left(s^M,\pi_1,\pi_2\right) V\left(s^M\right) + X\left(s^M,\pi_1,\pi_2\right) V'\left(s^M\right) = 0$$

Compare this equation with (13) and (14): assuming second-order conditions are satisfied (i.e. the left-hand sides in all three equations are decreasing in s) and using $X_s\left(s^M,\pi_1,\pi_2\right) > 0$ and $V\left(s^M\right) \leq V(0) < t$, it follows that $s^C_{cons} > s^M$ and $s^C > s^M$.