DISCUSSION PAPER SERIES

No. 9402

THE MACROECONOMICS OF MODIGLIANI-MILLER

Hans Gersbach, Hans Haller and Jürg Müller

FINANCIAL ECONOMICS and INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

www.cepr.org

ISSN 0265-8003

THE MACROECONOMICS OF MODIGLIANI-MILLER

Hans Gersbach, CER-ETH Center of Economic Research and CEPR
Hans Haller, Virginia Polytechnic Institute and State University
Jürg Müller, CER-ETH Center of Economic Research

Discussion Paper No. 9402
March 2013

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 9402

March 2013

ABSTRACT

The Macroeconomics of Modigliani-Miller*

We examine the validity of a macroeconomic version of the Modigliani-Miller theorem. For this purpose, we develop a general equilibrium model with two production sectors, risk-averse households and financial intermediation by banks. Banks are funded by deposits and (outside) equity and monitor borrowers in lending. We impose favorable manifestations of the underlying frictions and distortions. We obtain two classes of equilibria. In the first class, the debt-equity ratio of banks is low. The first-best allocation obtains and banks' capital structure is irrelevant for welfare: a macroeconomic version of the Modigliani-Miller theorem. However, there exists a second class of equilibria with high debt-equity ratios. Banks are larger and invest more in risky technologies. Default and bailouts financed by lump sum taxation occur with positive probability and welfare is lower. Imposing minimum equity capital requirements eliminates all inefficient equilibria and guarantees the global validity of the macroeconomic version of the Modigliani-Miller theorem.

JEL Classification: D53, E44 and G2
Keywords: banking, capital requirements, capital structure, financial intermediation, general equilibrium and Modigliani-Miller

Hans Gersbach
CER-ETH Center of Economic Research at ETH Zürich
Zuerichbergstrasse 18
8092 Zurich
SWITZERLAND

Email: hgersbach@ethz.ch

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=119061

Hans Haller
Department of Economics
Virginia Polytechnic Institute and State University
3124 Pamplin Hall
Blacksburg, VA 24061-0316
USA

Email: haller@vt.edu

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=131589

Jürg Müller
CER-ETH Center of Economic
Research at ETH Zürich
Zürichbergstrasse 18
8032 Zürich
SWITZERLAND

Email: juergmueller@ethz.ch

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=176998

*We would like to thank Salomon Faure, Volker Hahn, Marina Iozef, Jean-Charles Rochet, O. Griffith Sexton, Eva Terberger-Stoy and Jan Wenzelburger for helpful comments. This work was supported by ETH Research Grant ETH-06 10-2.

Submitted 14 March 2013

1 Introduction

Motivation

The socially optimal capital structure of banks has become the focus of an extended debate among policy-makers and academics. New regulatory standards epitomized in Basel III aim at increasing bank capital requirements by moderate amounts. Some countries have considered a further strengthening of these requirements.[1] There is, however, no consensus among academics regarding the net effects of higher capital requirements on welfare. On the one hand, several studies point to higher and potentially significant welfare costs when capital requirements are substantially heightened (e.g. Van den Heuvel (2008), Angelini et al. (2011) or Bolton and Samama (2012)). On the other hand, a variety of papers stress that welfare costs of substantially heightened capital requirements are small or vanishing (see e.g. Brealey (2006)).[2] Recently, Admati et al. (2011) have set out and scrutinized the underlying logic for this line of reasoning.

It has long been recognized that the examination of bank capital regulation has to start with the Modigliani-Miller theorem.[3] Modigliani and Miller (1958) state that changes in the capital structure of a firm — and in particular changes of the ratio of debt and equity funding — only redistribute the total risk of the firm's asset returns among those who fund the firm. However, investment opportunities, total risk of the firm's (the bank's) asset returns and overall funding costs are not affected.[4] A large strand of literature has identified how deviations from the underlying assumptions of the Modigliani-Miller theorem in the form of distortions and market frictions can imply that particular capital structures are preferred over others from the perspective of an investor or from a social point of view (see Admati et al. (2011) for a comprehensive account). If a social perspective requires a lower debt-equity ratio than a private perspective, then

[1] See, for example, Siegenthaler et al. (2010) for Switzerland, where stricter capital requirements came into force on 1 March 2012.

[2] Furthermore, several studies acknowledge some costs of higher capital requirements over Basel II but conclude that the benefits exceed the costs (e.g. Basel Committee on Banking Supervision (2010), Hanson et al. (2011) or Miles et al. (2012)).

[3] See Schaefer (1990), King (1990), Berger et al. (1995) and Admati et al. (2011). Miller (1995) discussed whether the irrelevance result holds for banks and why enhanced capital requirements could protect depositors at comparatively low costs.

[4] See also the later succinct account in Miller (1977).

raising capital requirements above the level prevailing in unfettered markets is justified.

In the context of banking, we consider one friction and two potential distortions that have been at the center of the discussion on the foundations of capital requirements:

- Moral hazard of entrepreneurs,
- Deposit guarantees by governments,
- Bailout by governments in case of default, financed by taxes.

Alleviating moral hazard of entrepreneurs is standard in rationalizing the need for financial intermediaries. The guarantee of deposits and the associated bailout of banks in case of default are usually justified by high social costs of bank defaults, in particular when many banks fail simultaneously. They are also justified by protection of risk-averse depositors or the special role of deposits as a means of payment and the corresponding need to have a large amount of safe assets in the economy.

We are interested in types of bank capital structures that can occur in economies when there appears to be a rationale for making deposits safe. This may be because of social costs of bank failures or the protection of deposits of risk-averse households.[5]

Model

We adopt a general equilibrium perspective to investigate the validity of a macroeconomic version of the Modigliani-Miller theorem. We address this issue in the simplest model with the following characteristics:

- There is a homogeneous group of risk-averse households.
- Two technologies are available for real investments. In one technology, households can invest without frictions (henceforth called frictionless technology or FT). Funding of investments in the other technology is plagued by moral hazard and returns are risky (henceforth called risky technology or MT).

[5]In the final section we comment on how our findings may be applied to economies in which deposits function as a medium of exchange and should be safe for this reason.

- Banks alleviate moral hazard in lending to MT. Banks fund themselves by means of debt and outside equity (henceforth called equity).
- The government guarantees bank debt. If banks default they are bailed out and rescue funds are obtained via taxation.

On purpose, we make two assumptions that allow for the possibility for this economy to achieve Pareto efficient allocations that would occur in an Arrow-Debreu version of the economy:

- Banks can eliminate moral hazard in MT at no cost. There is no moral hazard on the part of bank managers monitoring entrepreneurs.
- Taxation to fund the bailout of defaulting banks is lump sum and thus non-distortionary.

Given these favorable manifestations of the underlying frictions and distortions, it is *a priori* unclear whether bank capital structures matter at the macroeconomic level. For instance, if the government guarantees deposits and bails out banks in case of default, then depositors are rescued; but they may be taxed by the same amount that they receive in rescue funds. Hence, when taxes are lump sum, such bailouts may not affect the total risk investors are facing and thus may not affect welfare.

Main Results

We first establish existence, uniqueness and Pareto efficiency in the Arrow-Debreu version of the economy. In this version, frictions and banks are absent and households can invest frictionlessly in both technologies via complete contingent commodity markets or complete security markets.

When frictions and distortions — in the favorable manifestation outlined above — as well as banks are present, two classes of equilibria occur. In the first class, a macroeconomic version of the Modigliani-Miller theorem holds: The first-best allocation obtains in all equilibria, and the capital structure of banks is irrelevant to investment, total risk and consumption allocation. Specifically, banks attract and channel the socially optimal amount of resources to the risky technology exposed to moral hazard. Banks are funded by a portfolio of debt and (outside) equity, with a sufficient amount of equity such that debt can be repaid in all states, and no bailout is necessary. Up to a critical debt-equity ratio, above which banks default with positive probability, every capital structure is an equilibrium outcome. The resulting allocation replicates the Arrow-Debreu solution.

In the second class of equilibria, debt-equity ratios are high. Banks attract more funds than in the efficient equilibria, over-invest in the risky technology and are financed considerably by debt. Banks generate high returns on equity in the good state and default in the bad state. In the case of default, banks receive funds from the government to pay out their debt holders. Those government expenditures are financed by lump sum taxes levied on households and thus on the debt holders themselves. *Ex post*, the bailout is neutral for households as they essentially finance their deposit claims with their own taxes. *Ex ante*, however, households are willing to hold large amounts of deposits. This is due to the fact that repayments of deposits are guaranteed and households have no influence on the risky investments of banks, the implied riskiness of bank equity, and on the ensuing tax burden when those banks default.

We conclude that the macroeconomic version of the Modigliani-Miller theorem fails to hold globally, i.e. for all debt-equity ratios. Ratios above a critical level cause changes in aggregate investment decisions and an increase in total risk of the assets held in the banking sector.

Furthermore, to avoid the inefficiencies associated with high debt-equity ratios, the regulator can impose bank equity capital requirements (henceforth "bank capital requirements"). Such requirements essentially prevent the occurrence of inefficient equilibria — so that only efficient equilibria emerge. With suitable capital requirements in place, the macroeconomic version of the Modigliani-Miller theorem holds for all permissible capital structures: all equilibria yield the same resource allocation and total risk for the economy. Finally, if bankruptcy costs were absent, the same welfare implications would prevail when the regulator could commit to forcing failing banks into bankruptcy. More specifically, equilibria with and without bank defaults can occur. However, all of them are efficient.

Broader Policy Implications

The concern here and in the contemporary policy debates is not risk taking by banks *per se*. Extending credit to firms with the risky technology and monitoring those firms is the key role of banks in our model. As long as these loans are primarily financed by bank equity, banks will not default and the equilibrium outcome is efficient.

But as soon as banks' debt-equity ratio exceeds a certain threshold, banks can find themselves in a situation where they can no longer keep their promises to depositors and default. Depositors are indemnified by deposit insurance. Banks obtain more total funds, which they channel into some sectors, thereby diverting funds from other sectors. An inefficient outcome results.

Our findings show that bank regulators do not necessarily have to face a trade-off between avoiding the adverse consequences of major bank failures and an efficient allocation of investment goods. In our model, capital requirements help prevent default of banks without tightening credit to businesses.

Organization of the Paper

Our paper is organized as follows. In the next section, we present the setup of our model in detail. The frictionless equilibrium as a benchmark case is established in Section 3. In Section 4, the implication of frictions and distortions on equilibria and welfare is analyzed. We also provide several examples. In Section 5, we examine how regulation can eliminate inefficient equilibria. Section 6 concludes.

2 Model Setup

We consider a two-period economy ($t = 1, 2$). At $t = 1$, there is a single physical good – called investment good – that can neither be stored nor consumed. Total endowment with this good in the economy is W ($W > 0$). Different technologies can transform this investment good into a consumption good in period $t = 2$. There are two different types of agents: households and entrepreneurs. All agents live for two periods.

2.1 Technologies

The model includes two different technologies that convert the investment good at $t = 1$ into a consumption good at $t = 2$. One is called the frictionless technology (FT) and is supposed to represent established businesses. There is no uncertainty about the returns in this sector. The other sector runs a risky technology that is plagued by moral or other hazards (MT). The returns from this technology are uncertain and subject to a sector-specific shock. This technology stands for innovative and risky new business ventures.

The amount K_F ($K_F \in [0, W]$) invested in FT at $t = 1$ yields $f(K_F)$ of the consumption good in period $t = 2$. This technology features decreasing returns to scale with $f'(K_F) > 0$ and $f''(K_F) < 0$. We assume that $f(\,\cdot\,)$ satisfies the Inada conditions $\lim\limits_{K_F \to 0} f'(K_F) = \infty$ and $\lim\limits_{K_F \to W} f'(K_F) = 0$. The return of investing in FT is given by

$$R_F := f'(K_F).$$

When $W = 1$, two explicit examples of such a production function are $f(K_F) = \sqrt{2K_F - K_F^2}$ and $f(K_F) = 2\sqrt{K_F} - K_F$ for $K_F \in [0, 1]$.

The amount invested in MT is denoted by K_M. Its return $\widetilde{R}$ is a binomially distributed random variable. There are two states of the world: good and bad. In the good state, occurring with probability σ, every unit invested in period one will turn into $\overline{R}$ units in period two. With probability $1 - \sigma$, we will end up in the bad state and the return will be $\underline{R}$ ($0 \leq \underline{R} < \overline{R}$). The expected return of investing one unit of the investment good in MT is given by

$$R_M := \mathbb{E}[\widetilde{R}] = \sigma\overline{R} + (1 - \sigma)\underline{R}.$$

2.2 Households

There is a continuum of households $h \in [0, 1]$ that derive utility from consumption in period $t = 2$. Preferences are represented by a utility function with constant relative risk aversion, $u(c) = \frac{c^{1-\theta}}{1-\theta}$ where $\theta > 0$ and $\theta \neq 1$. All households have the same preferences and own the same amount of the investment good in the first period. Furthermore, they are all equally endowed with property rights to the FT and MT technology. Property rights cannot be traded.[6] Under these assumptions, we can proceed as if there was a single representative household with utility function u and endowment bundle W.

2.3 Entrepreneurs

The technologies are operated by representative entrepreneurs that only play a passive role in our model. The representative entrepreneurs stand for a continuum of entrepreneurs and thus are assumed to behave competitively.[7] The entrepreneur operating FT is denoted by e_F and can be directly financed by households. The entrepreneur operating MT is denoted by e_M. She needs to be monitored and, hence, will be funded by banks. Related to the technology they run, one can interpret entrepreneur e_F as a manager of an established company while entrepreneur e_M can be taken as an innovator or a start-up founder.

3 Frictionless Economy – Arrow-Debreu Equilibrium

Before we introduce banks, we characterize the Arrow-Debreu equilibrium of the economy and assume that no frictions are present. Both entrepreneurs can be financed directly, and all agents can trade in markets with complete asset structures or contingent commodity markets. We follow first the latter approach and define the following variables:

- $(1, p_g, p_b)$ is the price vector, where the price of the investment good has been normalized to 1. The price at $t = 1$ for obtaining one unit of the consumption good in the good state and nothing in the bad state is denoted by p_g. The price for at $t = 1$ for obtaining one unit of the consumption good in the bad state and nothing in the good state is denoted by p_b.
- (c_g, c_b) denotes demand of households in states g and b, respectively.

[6] As asset markets are complete in all variants of the model, this assumption merely simplifies the analysis.

[7] In the case of FT, one typically assumes that each entrepreneur operates a project of size 1 with a specific productivity. The distribution of entrepreneurs' productivities generates the function $f(K_F)$.

- (y_F, y_M) denotes demand of entrepreneurs for investment goods for operating technology FT and MT, respectively.
- (Π_F, Π_M) denotes (aggregate) profits of firms in sector FT and MT, respectively.

We will next derive consumption and factor demand and state the market clearing conditions. Then, we establish existence and uniqueness of market equilibria.

3.1 Production

Consumption goods are produced in the production sector. Entrepreneurs operating the technologies maximize profits, taking prices of input and output goods as given. Entrepreneur e_F running FT solves

$$\max \left\{ \Pi_F(y_F) = (p_g + p_b) f(y_F) - y_F \right\}.$$

This yields the following factor demand function[8]

$$y_F(p_g, p_b) = f'^{-1}\left(\frac{1}{p_g + p_b}\right). \tag{1}$$

Operating MT, entrepreneur e_M solves

$$\max \left\{ \Pi_M(y_M) = y_M(p_g \overline{R} + p_b \underline{R} - 1) \right\}.$$

We observe that in any equilibrium,

$$p_g \overline{R} + p_b \underline{R} = 1. \tag{2}$$

Otherwise entrepreneur e_M would either demand an infinite amount of the investment good or none. Prices p_g and p_b adjust in equilibrium such that demand $y_F + y_M$ equals supply W. Therefore, infinite demand cannot occur in equilibrium. Furthermore, $y_M = 0$ would imply that $y_F = W$. But the first-order condition from profit maximization $(p_g + p_b) f'(W) - 1 = 0$ and the Inada condition $f'(W) = 0$ cannot hold simultaneously. As a result, $y_M = 0$ can be ruled out in equilibrium as well. Hence, $0 < y_M < W$, which requires (2). Equilibrium condition (2) implies that $\Pi_M = 0$.

[8] Note that this demand function is well defined as $f(\cdot)$ is concave and the Inada conditions hold.

3.2 Consumption

As households own all technologies and the total endowment, they are the only agents consuming. Risk-averse households face the following utility maximization problem:

$$\max \left\{ u_h(c_g, c_b) = \sigma \frac{c_g^{1-\theta}}{1-\theta} + (1-\sigma) \frac{c_b^{1-\theta}}{1-\theta} \right\}$$
$$\text{s.t. } W + \Pi_F + \Pi_M \geq p_g c_g + p_b c_b. \tag{3}$$

The corresponding demand functions for risk-averse households are given by

$$c_g(p_g, p_b) = \left[p_g + \left(\frac{p_g}{p_b} \frac{1-\sigma}{\sigma} \right)^{\frac{1}{\theta}} p_b \right]^{-1} (W + \Pi_F + \Pi_M), \tag{4}$$

$$c_b(p_g, p_b) = \left[\left(\frac{p_b}{p_g} \frac{\sigma}{1-\sigma} \right)^{\frac{1}{\theta}} p_g + p_b \right]^{-1} (W + \Pi_F + \Pi_M). \tag{5}$$

We note that for $p_g = p_b$ and $\sigma = \frac{1}{2}$, $c_g(p_g, p_b) = c_b(p_g, p_b) = \frac{W + \Pi_F + \Pi_M}{p_g + p_b}$. We also note that equilibrium prices p_g^* and p_b^* have to be positive as otherwise $c_g(p_g^*, p_b^*)$ or $c_b(p_g^*, p_b^*)$ would be infinite.

3.3 Market Clearing

We have derived factor and consumption demand functions. For markets to clear at prices $p_b > 0$, $p_g > 0$, the values of all excess demand functions must be equal to zero:

$$y_F(p_g, p_b) + y_M(p_g, p_b) - W = 0, \tag{6}$$

$$z_g := c_g(p_g, p_b) - f(y_F(p_g, p_b)) - y_M \overline{R} = 0, \tag{7}$$

$$z_b := c_b(p_g, p_b) - f(y_F(p_g, p_b)) - y_M \underline{R} = 0. \tag{8}$$

Next we show existence and uniqueness of market equilibria.

3.4 Equilibria

We first aim at an existence result. Assume first $\overline{R} > \underline{R} > 0$. Because of the Inada conditions, $y_F \in (0, W)$ and because of the equilibrium condition (6),

$$y_M = W - y_F. \tag{9}$$

It remains to clear the market at $t = 1$ in the good state and in the bad state. We consider the aggregate excess demand function $z(p)$ where $p = (p_g, p_b)$ and $z = (z_g, z_b)$. Using the equilibrium condition (2), we can restrict ourselves to price pairs in the simplex

$$\Delta = \{(p_g, p_b) \in \mathbb{R}^2_+ | \, p_g \overline{R} + p_b \underline{R} = 1\}.$$

z is well defined and continuous in the relative interior of Δ and satisfies a boundary condition. Therefore, we can apply the argument of the proof of Proposition 17.C.1 in Mas-Colell et al. (1995) to z and Δ and obtain existence of $p^* \in \mathsf{Relative\ Interior}(\Delta)$ with $z(p^*) = 0$.

Suppose next $\overline{R} > \underline{R} = 0$. Then the equilibrium condition (2) implies $p_g = 1/\overline{R}$, while it imposes no restriction on p_b. Now the formulas (1), (5) and (8) imply $\lim_{p_b \to 0} z_b(1/\overline{R}, p_b) > 0$ and $z_b(1/\overline{R}, p_b) < 0$ for p_b sufficiently large. For the second claim, consider

$$\left[\left(\frac{p_b}{p_g} \frac{\sigma}{1-\sigma} \right)^{\frac{1}{\theta}} p_g + p_b \right] \cdot z_b(1/\overline{R}, p_b) = f(y_F) \cdot \left[p_g - \left(\frac{p_b}{p_g} \frac{\sigma}{1-\sigma} \right)^{\frac{1}{\theta}} p_g \right] + W - y_F.$$

Hence by the intermediate value theorem, there exists $p_b^* > 0$ with $z_b(1/\overline{R}, p_b^*) = 0$. Because of Walras Law, the market for the consumption good in state g is cleared as well.

We have established:

Proposition 1

An equilibrium exists.

Since the only household is locally non-satiated, the first welfare theorem applies. Moreover, if $a' = (c_g', c_b', y_F', y_M')$ and $a'' = (c_g'', c_b'', y_F'', y_M'')$ denote two equilibrium outcomes, then, because of the first welfare theorem, the only household must attain the same utility level in both cases, that is $u(c_g', c_b') = u(c_g'', c_b'')$. Suppose $a' \neq a''$. Then they differ in all coordinates due to the particular features of the model. Because of the convexity of the consumption set and both technologies, the convex combination $\frac{1}{2}a' + \frac{1}{2}a''$ is also feasible. But because of the strict convexity of the household's preferences for bundles (c_g, c_b), $u(\frac{1}{2}(c_g', c_b') + \frac{1}{2}(c_g'', c_b'')) > \frac{1}{2}u(c_g', c_b') + \frac{1}{2}u(c_g'', c_b'') = u(c_g', c_b')$, contradicting the optimality of equilibrium consumption. Hence to the contrary, $a' = a''$. We have shown:

Proposition 2

Equilibrium allocations are efficient and unique.

The above uniqueness argument further shows:

Corollary 1

The Arrow-Debreu equilibrium allocation is the only Pareto optimal allocation.

It is useful to collect some of the findings obtained during the foregoing analysis:

Corollary 2

At equilibrium, (1)*,* (2)*,* $y_F \in (0, W)$*, and* (9) *have to hold.*

We conclude this section with some remarks on the role of the Inada conditions. First, one might be concerned that the second Inada condition at $K_F = W$ is not maintained if W changes — unless f would be altered as well. Imposing the Inada conditions on f helps simplify the analysis. But observe that if the economy has an equilibrium with $0 < y_F < W$, then it also has an equilibrium with the same outcomes $p_g, p_b, c_g, c_b, y_F, y_M$ when ceteris paribus f is replaced by another production function $\bar{f}$ satisfying decreasing returns to scale, $\bar{f}'(y_F) = f'(y_F)$, and $\bar{f}(y_F) = f(y_F)$. This observation does not require (or rule out) any Inada conditions for $\bar{f}$. An example is $\bar{f}(K_F) = f(K_F)$ for $K_F < y_F$ and $\bar{f}(K_F) = 2(\sqrt{K_F} - \sqrt{y_F})\sqrt{y_F} f'(y_F) + f(y_F)$ for $K_F \geq y_F$.

Second, existence, uniqueness and efficiency of equilibria still obtain if the Inada condition at 0 is violated while the one at W is satisfied. However, then $y_F = 0$ can but need not occur in equilibrium. In case both Inada conditions fail to hold, $y_F = 0$ and $y_F = W$ are possible equilibrium outcomes. Equilibrium is unique and efficient provided it exists. General existence in that case remains an open question. Next, we will provide several examples illustrating these facts.

3.5 Examples

We next provide several examples. We first present one example that satisfies all assumptions. Then, we present two examples in which the Inada condition at 0 is violated.

In Appendix A, we provide two further examples in which both Inada conditions are violated and which entail corner solutions. In all examples, we treat the investment good as numéraire.

■ **Example 1:** $0 < y_F < W$ **when both Inada conditions are satisfied.**
Let $W = 1$, $f(K_F) = 2\sqrt{K_F} - K_F$, $\theta = 2$, $\sigma = 2/3$, $\underline{R} = 1/2$ and $\overline{R} = 2$. We obtain the following results:

$$y_F = \left(\frac{q}{1+q}\right)^2, \tag{10}$$

$$f(y_F) = \frac{2q+q^2}{(1+q)^2}, \tag{11}$$

$$y_M = 1 - \left(\frac{q}{1+q}\right)^2, \tag{12}$$

$$\Pi_F = \frac{q^2}{1+q}, \tag{13}$$

where $q = p_g + p_b$. Furthermore, we can see that $c_g = [2p_g]^{-1}[1 + \Pi_F]$ from (4). Combining this with (10) to (13) and market clearing conditions (6) and (7) yields

$$\left(1 + \frac{q^2}{1+q}\right) = 2p_g\left[2 + 2\frac{q}{1+q} - 3\left(\frac{q}{1+q}\right)^2\right],$$

$$1 + 2q + 2q^2 + q^3 - 2p_g\left(2 + 6q + q^2\right) = 0. \tag{14}$$

Combining (2) with (14), we can assert that the markets for the investment good and the consumption good in the good state are cleared at prices $p_g = 1/3$ and $p_b = 2/3$. By Walras law, market clearing in the bad state obtains as well. Hence, $(p_g, p_b) = (1/3, 2/3)$ constitutes the equilibrium. The allocation of the investment good to the production sectors is given by $y_F = 1/4$ and $y_M = 3/4$. Profit in the FT sector is $\Pi_F = 1/2$ and households consume $c_g = 9/4$ and $c_b = 9/8$.

■ **Example 2:** $y_F = 0$ **when** $f'(0) < \infty$.

Let $W = 1$, $f(K_F) = (K_F - \frac{1}{2}K_F^2)$, $\theta = 1/2$, $\underline{R} = 2, \overline{R} = 3$. Then $p_g\overline{R} + p_b\underline{R} \leq 1$ implies $p_g + p_b < 1$. Suppose $p_g = \dfrac{1}{2\overline{R}}$, $p_b = \dfrac{1}{2\underline{R}}$. Then $p_g\overline{R} + p_b\underline{R} = 1$ and $p_g + p_b < 1$. Since $f'(0) = 1$, Π_F is maximized at $y_F = 0$. The market for the investment good is cleared when $y_M = W$. The resulting maximum profits are $\Pi_F = \Pi_M = 0$. Demand for consumption goods becomes

$$\begin{aligned} c_g(p_g, p_b) &= \left[p_g + \left(\frac{p_g}{p_b}\frac{1-\sigma}{\sigma}\right)^{\frac{1}{\theta}} p_b\right]^{-1} \cdot W; \\ c_b(p_g, p_b) &= \left[\left(\frac{p_b}{p_g}\frac{\sigma}{1-\sigma}\right)^{\frac{1}{\theta}} p_g + p_b\right]^{-1} \cdot W. \end{aligned}$$

Choose $\sigma \in (0,1)$ so that

$$\left(\frac{p_g}{p_b}\right)^{1-\theta} = \frac{\sigma}{1-\sigma}$$

and consequently,

$$\left(\frac{p_g}{p_b} \cdot \frac{1-\sigma}{\sigma}\right)^{1/\theta} = \frac{p_g}{p_b}.$$

Then $c_g = [2p_g]^{-1} \cdot W = W\overline{R}$, $c_b = [2p_b]^{-1} \cdot W = W\underline{R}$ and with $y_M = W$ all markets are cleared. In this example, returns on investments in FT are such that entrepreneurs prefer to invest in the risky MT technology only. This is possible because the Inada condition at zero is not satisfied.

■ **Example 3:** $0 < y_F < W$ **when** $f'(0) < \infty$.

We set $W = 1$, $\sigma = 1/2$, $\theta = 1/2$, $\underline{R} = 0$, $\overline{R} = 2$, and assume that $f(K_F) = 2(K_F - \frac{K_F^2}{2})$. We immediately obtain the following results:

$$y_F = 1 - \frac{1}{2q}, \tag{15}$$

$$f(y_F) = 1 - \frac{1}{4q^2}, \tag{16}$$

$$y_M = \frac{1}{2q}, \tag{17}$$

$$\Pi_F = q + \frac{1}{4q} - 1, \tag{18}$$

where $q = p_g + p_b$. Note that since this production function does not satisfy the Inada condition at 0, $f'(0) = 2 < \infty$,

$$p_g + p_b > \frac{1}{2} \tag{19}$$

must hold in order for y_F to be positive. For an equilibrium to exist, we need all three market clearing conditions to hold. If (15), (17) and (19) are satisfied, then the market for the investment good is cleared. It remains to clear the market for consumption in both states. Because of Walras law, it suffices to clear the market in the bad state. We set $p_g = 1/\overline{R}$ to meet (2). Substituting (5), $p_g = 1/\overline{R} = 1/2$, (16) and (18) in (8), we get

$$\begin{aligned}\left[2q^2 - q\right] \cdot [q^2 - \frac{1}{4}] = q^3 + \frac{1}{4}q \text{ or}\\ 4q^4 - 4q^3 - q^2 = 0,\end{aligned}$$

which has roots 0 and $\frac{1}{2}(1 \pm \sqrt{2})$. Hence $q = \frac{1}{2}(1 + \sqrt{2})$, $p_g = \frac{1}{2}$, $p_b = \frac{1}{2}\sqrt{2}$, $y_F = 2 - \sqrt{2}$, $y_M = \sqrt{2} - 1$ will do. Profit in the FT sector is $\Pi_F = \sqrt{2} - 1$, households consume $c_g = 4(\sqrt{2} - 1)$ and $c_b = 2(\sqrt{2} - 1)$.

3.6 Radner Equilibrium - Equivalence Result

We have derived a frictionless general equilibrium model under uncertainty in the spirit of Arrow and Debreu (see Debreu (1959)). We cannot readily compare the equilibrium prices for contingent goods with returns we will obtain in the financial intermediation case. It is therefore helpful to transform the contingent goods setup from above into one with assets (also called Radner equilibrium in reference to Radner (1982)). Let us introduce two assets with the following returns:

Asset	Price in $t = 1$	Return in state g	Return in state b	
a_1	q_1	R_S	R_S	(20)
a_2	q_2	$\overline{R}$	$\underline{R}$,	(21)

where R_S ($R_S > 0$) is an arbitrary safe return.

Remark: Throughout the paper, R_F denotes the marginal product of the FT technology and R_S is the return on safe assets.

Proposition 3

The Radner equilibrium defined by the asset structure given by (20) *and* (21) *is equivalent to the Arrow-Debreu equilibrium we have derived above.*

For a general proof of the equivalence of Arrow-Debreu and Radner equilibria, see Mas-Colell et al. (1995), Proposition 19.D.1.

Notice that in our model, there are no future spot markets. We can express the two assets as bundles $a_1 = (R_S, R_S)$ and $a_2 = (\overline{R}, \underline{R})$ in the Arrow-Debreu setting. Hence

$$q_1 = R_S p_g + R_S p_b, \ q_2 = \overline{R} p_g + \underline{R} p_b. \tag{22}$$

The matrix of coefficients

$$\mathbb{M} = \begin{pmatrix} R_S & R_S \\ \overline{R} & \underline{R} \end{pmatrix}$$

has inverse

$$\mathbb{M}^{-1} = \frac{1}{R_S(\underline{R} - \overline{R})} \cdot \begin{pmatrix} \underline{R} & -R_S \\ -\overline{R} & R_S \end{pmatrix} = \frac{1}{R_S(\overline{R} - \underline{R})} \cdot \begin{pmatrix} -\underline{R} & R_S \\ \overline{R} & -R_S \end{pmatrix}.$$

Hence

$$\begin{aligned} p_g &= \frac{1}{R_S(\overline{R} - \underline{R})} \cdot (-\underline{R} q_1 + R_S q_2), \\ p_b &= \frac{1}{R_S(\overline{R} - \underline{R})} \cdot (\overline{R} q_1 - R_S q_2). \end{aligned}$$

We conclude this section by the observation that the equilibrium asset price of a_2 in the Radner equilibrium is unity. This follows from (22) and (2). Hence,

Corollary 3

The Radner equilibrium with the asset structure given by (20) *and* (21) *involves* $q_2 = 1$ *and* $q_1 = \frac{R_S}{R_F^*}$, *where* $R_F^* := f'(y_F^*)$ *is the equilibrium return in FT in the Arrow-Debreu setting.*

4 Allocation with Financial Intermediation

Up to this point, we have assumed absence of any frictions and distortions in the economy and analyzed equilibria without considering financial intermediation. In this section, we are going to abandon the assumption of frictionless trade.

4.1 Frictions and Distortions

We assume that households cannot directly invest in MT as financing of entrepreneurs e_M is plagued by moral hazard.[9] Banks can alleviate this moral hazard problem by monitoring borrowers and enforcing contractual obligations. As for the production technologies, we assume that there is a continuum of banks that have access to a monitoring technology operated by bank managers. Bank managers play only a passive role in our model by operating the monitoring technology.

Banks are funded by households through equity acquisition and deposits. Banks take all funds they receive from households and lend to entrepreneurs e_M or invest in FT. Banks monitor entrepreneurs and maximize expected profits. The details of bank behavior are set out in Section 4.4. Furthermore, we assume that deposits are guaranteed by governments in case banks default and that the rescue funds needed for bailouts are raised via taxes. Hence there are two possible distortions — deposit guarantees and taxation — and one financial friction due to moral hazard present in our model.

We make three additional assumptions that promise to be favorable to the validity of a macroeconomic version of the Modigliani-Miller theorem, i.e. that the capital structure of banks may be irrelevant for welfare. First, we assume that banks can eliminate the moral hazard friction completely when investing in the MT technology and that monitoring costs are zero.[10] Consequently, MT becomes simply a risky technology banks can invest in. Second, there is no moral hazard associated with bank managers.[11] Third,

[9]See Freixas and Rochet (2008) for an overview of the microeconomic foundations.

[10]This assumption is more stringent than needed. For instance, our results can be extended to situations when entrepreneurs in MT can only pledge a fraction of the output to bankers even if they are monitored. In such circumstances, however, market equilibria have to be compared to appropriate second-best allocations. (Details are available upon request.)

[11]They perform the monitoring activities without compensation.

taxation is lump sum and thus non-distortionary. Figure 1 gives an overview of the model with frictions and distortions present and banks intermediating funds.

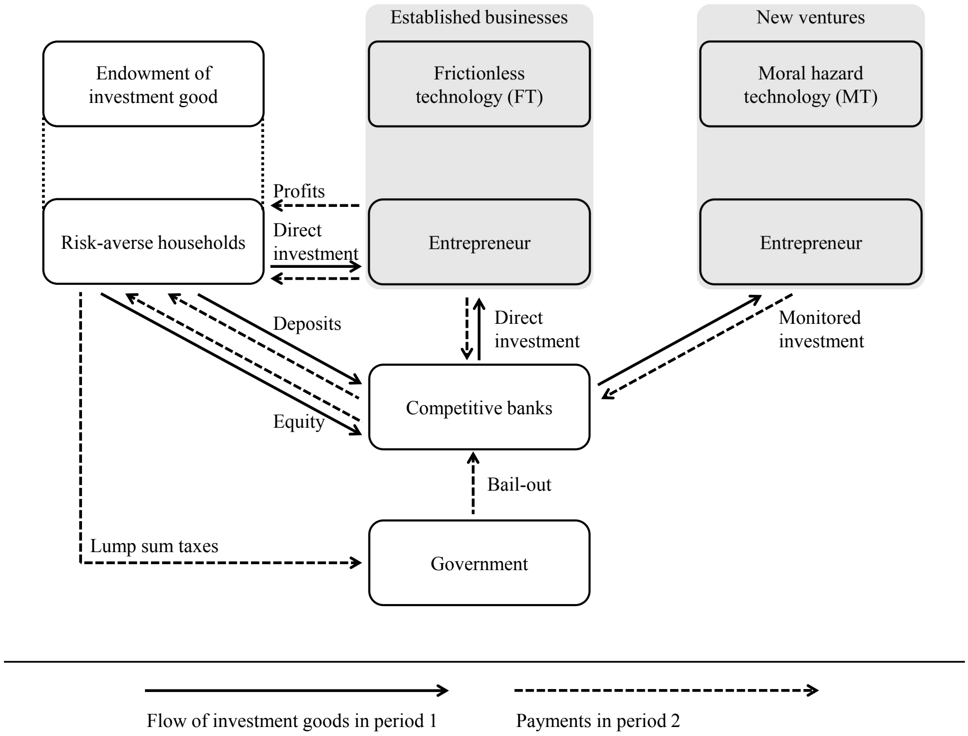

Figure 1: Model setup with financial intermediation.

All agents in our economy are price (or contract) takers and thus perfect competition prevails in all markets. Next we characterize the optimal choices of all three agents — households, entrepreneurs, and banks — given the aforementioned friction and distortions.

4.2 Optimal Choices of Households

Due to the moral hazard friction, households cannot or would not directly finance entrepreneur e_M. They can, however, lend to entrepreneur e_F if investment returns are more attractive than those of bank deposits.[12]

[12] If households are indifferent between depositing money at a bank or investing in FT, the allocation of funds to these two risk-free assets is determined by equilibrium requirements.

In this subsection, we investigate the portfolio choice of households. For this purpose, we denote by R_F the return on investment $K_{F,h}$ in FT, by R_D the return on deposits D and by $\overline{R}_E$ and $\underline{R}_E$ the returns on bank equity E in the good and bad state, respectively.[13] Hence part of initial wealth W can be saved risk-free, paying out $R_S = \max\{R_F, R_D\}$ per unit of investment. Alternatively, a household can purchase bank equity with return $\overline{R}_E$ or $\underline{R}_E$. In period two, it consumes its returns from both investments after having received its share of profits from the firm operating the FT technology and paid the tax T (if any).[14]

The solution to the households' optimal portfolio choice problem can be expressed by means of a variable γ, the optimal share of wealth held in risk-free assets. A fraction of risk-free assets consists of bank deposits, while the rest accounts for direct investment in FT. The first-order conditions for the households' optimal portfolio choice yield (see Appendix B):

Lemma 1

$$\gamma = \frac{(1/W)[\Pi_F(1-A_1) + TA_1] + \overline{R}_E - A_1\underline{R}_E}{\overline{R}_E - R_S + A_1(R_S - \underline{R}_E)}, \tag{23}$$
$$\text{where } A_1 := \left[\frac{\sigma(\overline{R}_E - R_S)}{(1-\sigma)(R_S - \underline{R}_E)}\right]^{\frac{1}{\theta}}.$$

Observe that $A_1 > 0$ holds if $\overline{R}_E > R_S > \underline{R}_E$ and $A_1 > 1$ holds if $\sigma\overline{R}_E + (1-\sigma)\underline{R}_E > R_S$. The variables Π_F, T, $\overline{R}_E$, $\underline{R}_E$ and R_S are determined in equilibrium. In turn, they determine A_1 and γ. We will show that in equilibrium $0 < \gamma \leq 1$.

[13] D and E denote the amount of investment goods households supply to banks in the form of safe deposits and equity, respectively. $K_{F,h}$ denotes the amount of investment goods households supply to firms in FT.

[14] As profits of entrepreneurs e_M will be zero in equilibrium, we neglect them in this subsection already.

Households will supply their wealth as follows:

$$D = \lambda \gamma W,$$

$$K_{F,h} = (1 - \lambda)\gamma W,$$

$$E = (1 - \gamma)W.$$

Here λ denotes the share of risk-free assets held in the form of deposits and $1-\lambda$ denotes the share held in the form of direct investment in FT. We have

$$\lambda = \begin{cases} 0 & \text{if } R_F > R_D, \\ 1 & \text{if } R_F < R_D, \\ \in [0,1] & \text{if } R_F = R_D. \end{cases} \tag{24}$$

4.3 Optimal Choices of Entrepreneurs

As in the Arrow-Debreu case, entrepreneurs are passive in the sense that they only run the technologies. Again, entrepreneurs – as agents of their firm – maximize profits. But instead of maximizing present value, they are going to maximize future value in this section.[15] In order to do so, they borrow in period one.

Entrepreneur e_F solves the following problem:

$$\max \left\{ \Pi_F(K_F) = f(K_F) - R_F K_F \right\}, \tag{25}$$

where R_F is the repayment obligation (principal plus interest) per unit at which she can borrow funds from banks and households (see below). The entrepreneur optimally raises the amount K_F of funds in period one. In lieu of (1) we get

$$K_F = f'^{-1}(R_F). \tag{26}$$

[15]For convenience, we use the same symbols Π_F and Π_M to denote the future value of profits.

Entrepreneur e_M solves the following problem:

$$\max \left\{ \Pi_M(K_M) = \left[\sigma(\overline{R} - \overline{R}_M) + (1-\sigma)(\underline{R} - \underline{R}_M) \right] K_M \right\}$$
$$\text{s.t.} \quad \sigma(\overline{R} - \overline{R}_M)K_M \geq 0,$$
$$(1-\sigma)(\underline{R} - \underline{R}_M)K_M \geq 0.$$

Due to moral hazard (see Section 4.1), banks monitor this entrepreneur. Therefore, they are able to offer state contingent repayment rates $\overline{R}_M = 1 + \overline{r}_M$ and $\underline{R}_M = 1 + \underline{r}_M$, where $\overline{r}_M$ and $\underline{r}_M$ are the state contingent interest rates. The two constraints mean that the entrepreneur is able to fulfill his repayment obligation in both states. Moreover, we assume that perfect monitoring prevails and banks can enforce the terms of the loan contract: The entrepreneur cannot cheat, threaten to voluntarily default or renegotiate the credit terms in period 2. The production function is linear and, therefore, profits will be zero in equilibrium. Otherwise, entrepreneur e_M will either demand no funds at all from banks or an infinite amount. As a result, we need $\overline{R}_M = \overline{R}$ and $\underline{R}_M = \underline{R}$ in any potential equilibrium. Consequently, the optimal choice of funds raised in period one is $K_M \in [0, \infty]$. In equilibrium, we have again (9).

4.4 Optimal Choices of Banks

There is a continuum of banks $v \in [0,1]$ that are financed by equity e_v and interest bearing deposits d_v.[16] They can lend $l_{F,v}$ to entrepreneur e_F and $l_{M,v}$ to entrepreneur e_M. The typical balance sheet of a bank v in period $t = 1$ looks like:

$l_{F,v}$ $l_{M,v}$	d_v e_v
a_v	o_v

Table 1. Bank balance sheet

Here, a_v and o_v stand for total assets (*activa*) and total liabilities (*passiva*, obligations), respectively. Assets a_v equal liabilities o_v in period $t = 1$. Initially, banks are only a label

[16] Equity is outside equity in our model as consumers are passive shareholders. As stressed in Section 4.1, there is no moral hazard of bank managers and thus no need to have inside equity. For foundations of counter-cyclical capital ratios in the presence of inside equity and moral hazard of bankers, see Gersbach and Rochet (2012).

or index. After they have received equity, the objective of banks is to maximize expected profits or, equivalently, return on equity (ROE).[17] The objective of a bank without equity is questionable. However, bank equity is positive in the equilibria depicted in Propositions 4, 5 and 6. In Proposition 7, we assume that bank equity is non-zero so that maximization of the expected return on equity is a meaningful objective.

Since banks are equal, we may assume that they all receive the same amount of equity E and deposits D. Hence, total assets a_v are the same for all banks and equal to $D+E$. Otherwise, we would just have banks of different scale, but the same relative size of various assets and liabilities in equilibrium. Therefore, we proceed as if there is only one representative bank and drop the subscript v in the following. Then a_v becomes simply a etc. and all variables in banking are understood as aggregate quantities.

Let us denote by $\alpha \in [0,1]$ the share of risky loans a bank has granted to entrepreneur e_M. Now we can express the amount l_M of loans to entrepreneur e_M as $\alpha(D+E)$ and the amount l_F of funds provided to entrepreneur e_F as $(1-\alpha)(D+E)$.

An important remark is in order. D and E are the amount of investment goods banks receive in the form of deposits and equity, respectively. D and E are also the amount of deposits and equity contracts. Of course, we have to check in any equilibrium that the prices of deposits and equity are 1, justifying the use of D and E in these two meanings. In Proposition 6 and Proposition 7, we will have to differentiate explicitly the amounts of contracts and amounts of investment goods banks receive. Then D and E denote the amount of contracts and $p_d D$ and $p_e E$ are the amount of investment goods banks receive when p_d and p_e are the prices of deposits and equity in units of the investment good, respectively.

Perfect competition in the banking sector ensures that banks lend to e_F at the same rate R_F as households.[18] For entrepreneur e_M, repayment rates are $\overline{R}$ and $\underline{R}$ respectively

[17] We stress that there are two equivalent ways to specify the objective of banks when banks are bailed out in case of default. First, banks maximize expected profits. Since deposits are guaranteed by the government, banks anticipate that profits in case of default are zero. Second, banks maximize the expected return on equity (ROE) and are subject to limited liability and thus return on equity in case of default is zero as well.

[18] More precisely, there exists no equilibrium in which those returns can be different and households and banks invest a positive amount in FT.

(see above). Furthermore, perfect competition leads to

$$R_D = R_F, \tag{27}$$

that is, banks borrow and lend at the same (endogenous) risk-free rate.[19]

Banks' objective is

$$\max_{\alpha} \mathbf{E}\left[ROE(\alpha)\right] = \mathbf{E}\left[\max\left\{0, \frac{\left[\alpha\tilde{R} + (1-\alpha)R_F\right](D+E) - R_F D}{E}\right\}\right].$$

Recall that $\tilde{R}$ is the random return in MT. Let us define $B_1 := \underline{R}(D+E) - R_F D$, $B_2 := \sigma\overline{R} + (1-\sigma)\underline{R} - R_F$ and $B_3 := [\sigma\overline{R} - R_F](D+E) + (1-\sigma)R_F D$. We obtain the following optimal values for α:[20]

$$\begin{aligned}
&\text{If } B_1 \geq 0 \text{ and } B_2 < 0, \text{ then } \alpha = 0.\\
&\text{If } B_1 \geq 0 \text{ and } B_2 > 0, \text{ then } \alpha = 1.\\
&\text{If } B_1 \geq 0 \text{ and } B_2 = 0, \text{ then } \alpha \in [0,1].\\
&\text{If } B_1 < 0 \text{ and } B_3 > 0, \text{ then } \alpha = 1.\\
&\text{If } B_1 < 0 \text{ and } B_3 < 0, \text{ then } \alpha = 0.\\
&\text{If } B_1 < 0 \text{ and } B_3 = 0, \text{ then } \alpha = \{0,1\}.\\
&\text{If } B_1 < 0 \text{ and } B_2 \geq 0, \text{ then } B_3 > 0.
\end{aligned}$$

Total supply of funds is

$$L_M = \alpha(D+E) \text{ and} \tag{28}$$

$$L_F = (1-\alpha)(D+E) \tag{29}$$

for the MT and the FT sector, respectively. Next, we will prove existence of equilibria when frictions and distortions are present and financial intermediation by banks is needed.

[19]Again, there is no equilibrium in which households invest in FT and in bank deposits in which $R_D \neq R_F$.

[20]For a complete characterization of optimal α, see Appendix C.

4.5 Equilibria

We distinguish between different equilibria depending on whether defaults happen with positive probability or not. Note that $K_F = K_{F,h} + L_F$ and $K_M = L_M$ in the financial intermediation case are equivalent to the variables y_F and y_M, respectively, in the Arrow-Debreu case. Throughout this section, we will use the equilibrium values obtained in the Arrow-Debreu setting. To avoid confusion, these equilibrium values are denoted by p_g^*, p_b^*, c_g^*, c_b^*, y_F^*, y_M^* and R_F^*.

Equilibria without default

We first consider equilibria with financial intermediation, yet without defaults and $T = 0$. This corresponds to the frictionless case except for the condition that investment in the risky technology can only take place through banks. The next proposition establishes existence of an equilibrium with financial intermediation that is equivalent to the Arrow-Debreu equilibrium derived in Section 3.

Proposition 4
Suppose the Arrow-Debreu equilibrium allocation satisfies $0 < y_F^* < W$. Then there exists an equilibrium with financial intermediation where the investment in FT is y_F^*, the investment in the risky technology is y_M^*, $D = 0$, banks only invest in the risky technology and never default.

The proof of Proposition 4 can be found in Appendix D. This equilibrium corresponds to a banking system in which all banks are funded by equity only.[21]

Proposition 4 means that the efficient Arrow-Debreu equilibrium allocation is still an equilibrium outcome when households cannot directly invest in the risky technology and banks are maximizing return on equity. When banks are funded by equity only, the return on equity is solely determined by the return of the risky technology. Intuitively, this can be interpreted as the removal of financial frictions without changing households' investment options from the Arrow-Debreu setup (i.e., the variant with assets; see Section

[21] The deposit return is indeterminate. Apart from $R_D = R_F^*$, any values $R_D < R_F^*$ can occur in equilibrium. They do not affect the allocation.

3.6). We stress that the result hinges on the fact that banks can perfectly monitor e_M at no cost, can charge e_M state-contingent interest rates and are perfectly competitive. Next we investigate to what extent this particular equilibrium is unique.

Proposition 5

Suppose the Arrow-Debreu equilibrium allocation satisfies $0 < y_F^* < W$. Then the equilibrium with financial intermediation and no default in Proposition 4 is unique if $\underline{R} = 0$. In case $\underline{R} > 0$, there exists an equilibrium with financial intermediation for each $D \in [0, y_M^* \cdot \underline{R}/R_F^*]$ where the investment in FT is y_F^*, the investment in the risky technology is $y_M^* = E + D$, banks only invest in the risky technology and never default.

The proof of Proposition 5 is given in Appendix D. These equilibria correspond to a stable banking system in which banks are funded by equity and deposits.

Proposition 5 states that up to a critical debt-equity ratio, the capital structure of banks is irrelevant to aggregate investment and risk. Banks' debt-equity ratio is low enough so that losses are absorbed by equity holders and there are no defaults. Banks raise and invest the socially optimal amount of resources. Intuitively, the more a bank is financed with debt, the riskier its equity becomes. Facing riskier investment choices, risk-averse households desire more risk-free debt and less risky equity. The extra amount of risk-free debt required due to increased risk equals exactly the amount of deposits that have increased the risk of equity in the first place. Thus, under moderate debt-equity ratios, a macroeconomic version of the Modigliani-Miller theorem emerges. We next investigate the case where the debt-equity ratio exceeds the critical level.

Equilibria with default

In the following, we explore equilibria where banks default in the bad state and consequently $T > 0$ has to hold. We consider again the case where the Arrow-Debreu equilibrium allocation satisfies $0 < y_F^* < W$.

Proposition 6

Suppose the Arrow-Debreu equilibrium allocation satisfies $0 < y_F^* < W$*. Then there exist equilibria with financial intermediation where the investment in FT is strictly smaller than* y_F^**, the investment in the risky technology is* $E + D$*, banks only invest in the risky technology and default in the bad state. The resulting equilibrium allocation is inefficient.*

The proof of Proposition 6 can be found in Appendix D. These equilibria describe a fragile banking system. The debt-equity ratio of banks is too high in the sense that equity does not suffice to absorb losses in the bad state.

Proposition 6 means that above a certain debt-equity ratio, inefficient equilibria arise. While too little is invested in the FT sector, there is over-investment in the risky technology. Within this class of equilibria, banks raise too much funds. They achieve high returns on equity in the good state and default in the bad state. The macroeconomic version of the Modigliani-Miller theorem fails to hold in this setting as the capital structure of banks alters aggregate investment and risk: welfare is lower than in the Arrow-Debreu allocation. This is remarkable insofar as we assume no monitoring costs and taxes are lump sum.

4.6 Examples

Next, we reconsider the examples from Section 3 and adopt them to the case with financial intermediation. We focus on Example 1 and 3, which we re-label Example 1' and 3'. This allows to illustrate all main findings.

■ Example 1'

Recall that $W = 1$, $f(K_F) = 2\sqrt{K_F} - K_F$, $\sigma = 2/3$, $\theta = 2$, $\underline{R} = 1/2$ and $\overline{R} = 2$. We obtain

$$K_F = \left(\frac{1}{1+R_F}\right)^2, \tag{30}$$

$$f(K_F) = \frac{1}{1+R_F}\left[2 - \frac{1}{1+R_F}\right], \tag{31}$$

$$K_M = 1 - \left(\frac{1}{1+R_F}\right)^2, \tag{32}$$

$$\Pi_F = \frac{1}{1+R_F}. \tag{33}$$

Efficient Equilibria

We first consider the case without any deposits. We set $R_F = R_F^* = 1/(p_g^* + p_b^*)$ and thus $R_F = 1$. Entrepreneur e_F optimally chooses $K_F = 1/4$, implying $\Pi_F = 1/2$. $K_M = 3/4$ is an optimal choice for entrepreneur e_M. Provided $D = 0$, we can assert that $T = 0$ and returns on equity are $\overline{R}_E = \overline{R}$ and $\underline{R}_E = \underline{R}$. Thus we are given $B_1 = 3/8 \geq 0$ and $B_2 = 1/2 > 0$. Banks optimally choose $\alpha = 1$ and invest in the risky technology only. Applying (23), we conclude that households optimally choose $\gamma = 1/4$, i.e. they optimally invest one forth of their wealth in riskless assets. For $D = 0$, they invest $K_F = 1/4$ directly in FT and $E = 3/4$ in bank equity. The resulting equilibrium yields the same allocation as in the Arrow-Debreu case. We obtain $y_F = 1/4$ and $y_M = 3/4$. Households consume $c_g = 9/4$ and $c_b = 9/8$.

Next we investigate an efficient equilibrium with deposits. Proposition 5 implies that for allocations with total amount of deposits below a certain threshold, the efficient Arrow-Debreu allocation is attained. Here we need $D \leq 3/8$. Again, we set $R_F = R_F^* = 1$, implying $K_F = 1/4$ and $\Pi_F = 1/2$. We set $D = 1/4$. As $B_2 = 1/2 > 0$, banks invest in the risky technology only. Return on equity is given by $\overline{R}_E = 5/2$ and $\underline{R}_E = 1/4$. Applying (23), we obtain that households optimally choose $\gamma = 1/2$. Thus they invest $E = 1/2$ in bank equity, $D = 1/4$ in deposits and $K_F = 1/4$ directly in the frictionless technology. The allocation of investment goods to the production sectors is still the same as in the Arrow-Debreu case and given by $y_F = 1/4$ and $y_M = 3/4$. Households consume $c_g = 9/4$ and $c_b = 9/8$.

Inefficient Equilibria

Proposition 6 states that there exist inefficient equilibria in which banks attract too many resources and default in the bad state. We illustrate this class of equilibria by setting $R_F = 5/4 > R_F^*$. Entrepreneur e_F optimally chooses $K_F = 16/81$, implying

$\Pi_F = 4/9$. Since $B_2 = 1/4 > 0$, banks only invest in the risky technology. Return on equity is given by $\overline{R}_E = 2 + \frac{3D}{4E}$ and $\underline{R}_E = 0$, as banks default in the bad state. Taxes are given by $T = \frac{3}{4}D - \frac{1}{2}E$. Applying (23), we can write the market clearing condition for equity as

$$E = 1 - \frac{\left[\frac{27D-18E-16}{36}\right]\left[\frac{6}{5}\frac{D+E}{E}\right]^{1/2} + \frac{88E+27D}{36E}}{\frac{27}{36}\left[1+\frac{D}{E}\right] + \frac{45}{36}\left[\frac{6}{5}\frac{D+E}{E}\right]^{1/2}}. \tag{34}$$

Since banks invest in the risky technology only, we can assert that $E = 65/81 - D$. Combining this with (34) and simplifying, we obtain

$$\left(59\sqrt{\frac{78}{65-81D}} - 124\right)(81D - 65) = 0.$$

An inefficient equilibrium exists in which households invest $E = \frac{45253}{207576} \approx 0.22$ in bank equity, $D = \frac{363961}{622728} \approx 0.58$ in deposits and $K_F = \frac{16}{81} \approx 0.20$ directly in the frictionless technology. Bank bailouts in the bad state are financed via lump sum taxes $T = \frac{273455}{830304} \approx 0.33$. We obtain $y_F = \frac{16}{81} \approx 0.20$ and $y_M = \frac{65}{81} \approx 0.80$ and households consume $c_g = \frac{62}{27} \approx 2.30$ and $c_b = \frac{59}{54} \approx 1.09$.

■ **Example 3'**

Recall that $W = 1$, $\sigma = 1/2$, $\theta = 1/2$, $\underline{R} = 0$, $\overline{R} = 2$, and $f(K_F) = 2(K_F - \frac{K_F^2}{2})$. As $f'(0) = 2 < \infty$, for K_F to be positive

$$R_F < 2 \tag{35}$$

must hold. We obtain the following results:

$$K_F = 1 - \frac{1}{2}R_F, \tag{36}$$

$$f(K_F) = 1 - \frac{1}{4}R_F^2, \tag{37}$$

$$\Pi_F = 1 + \frac{1}{4}R_F^2 - R_F. \tag{38}$$

Efficient Equilibrium

We set $R_F = R_F^* = 1/(p_g^* + p_b^*)$ and thus $R_F = 2\sqrt{2} - 2$. Entrepreneur e_F optimally chooses $K_F = 2 - \sqrt{2}$, implying $\Pi_F = 6 - 4\sqrt{2}$.[22] $K_M = \sqrt{2} - 1$ is an optimal choice for entrepreneur e_M. Provided $D = 0$, we can assert that $T = 0$ and returns on equity are $\overline{R}_E = \overline{R}$ and $\underline{R}_E = \underline{R}$. Since $B_1 = 0 \geq 0$ and $B_2 = 3 - 2\sqrt{2} > 0$, banks optimally choose $\alpha = 1$, i.e. invest in the risky technology only. Applying (23), we obtain that households optimally choose $\gamma = 2 - \sqrt{2}$. For $D = 0$, they invest $K_F = 2 - \sqrt{2}$ directly in FT and $E = \sqrt{2} - 1$ in bank equity. The resulting allocation is the same as in the Arrow-Debreu case. We obtain $y_F = 2 - \sqrt{2}$ and $y_M = \sqrt{2} - 1$. Households consume $c_g = 4(\sqrt{2} - 1)$ and $c_b = 2(\sqrt{2} - 1)$. This is the only efficient equilibrium in this example as banks will default in the bad state with any positive amount of deposits.

Inefficient Equilibria

Proposition 6 states that there exist inefficient equilibria in which banks attract too many resources and default in the bad state. We illustrate this class of equilibria by setting $R_F = 1 > R_F^*$. Entrepreneurs optimally choose $K_F = 1/2$, implying $\Pi_F = 1/4$. Since $B_2 = 0 \geq 0$ and $B_1 < 0$ for every positive amount of D, banks invest in the risky technology only. Return on equity is given by $\overline{R}_E = 2 + D/E$ and $\underline{R}_E = 0$, as banks default in the bad state. Taxes are given by $T = D$. Applying (23), we can write the market clearing condition for equity as

$$E = 1 - \frac{\frac{1}{4} + \left[D - \frac{1}{4}\right]\left[\frac{D+E}{E}\right]^2 + \frac{D+2E}{E}}{\left[\frac{D+E}{E}\right]\left[\frac{D+2E}{E}\right]}. \tag{39}$$

Since banks invest in the risky technology only, we can assert that $E = 1/2 - D$. Combining this with (39) and simplifying, we obtain

$$7D^2 - 7D + 1 = 0.$$

An inefficient equilibrium exists in which households invest $D = \frac{1}{2}(1 - \sqrt{\frac{3}{7}}) \approx 0.17$

[22]Note that this profit is denoted in period two consumption goods. Hence, it cannot be readily compared to the present value profit obtained in the Arrow-Debreu case, which is denoted in period one investment goods.

in deposits, $E = \frac{1}{2}\sqrt{\frac{3}{7}} \approx 0.33$ in bank equity and $K_F = \frac{1}{2}$ directly in the frictionless technology. Bank bailouts in the bad state are financed via lump sum taxes $T = \frac{1}{2}(1 - \sqrt{\frac{3}{7}}) \approx 0.17$. We obtain $y_F = \frac{1}{2}$ and $y_M = \frac{1}{2}$. Households consume $c_g = \frac{7}{4}$ and $c_b = \frac{3}{4}$.

5 Bank Regulation

We next introduce and discuss two regulatory measures that can eliminate all inefficient equilibria.[23]

5.1 Bank Capital Regulation

We have shown that the macroeconomic version of the Modigliani-Miller theorem fails to hold globally. Above a certain debt-equity ratio in the banking sector, banks' capital structure is relevant as aggregate investment can deviate from the socially optimal level. From Propositions 5 and 6 we obtain:

Corollary 4

The class of inefficient equilibria can be eliminated by imposing a minimum bank capital requirement in the form of an upper bound on the debt-equity ratio:

$$\frac{D}{E} \leq \varphi := \frac{\underline{R}}{R_F^* - \underline{R}}. \tag{40}$$

We note that at the regulatory debt-equity ratio we have

$$\frac{D}{E} = \frac{\underline{R}}{R_F^* - \underline{R}} = \frac{\frac{y_M^* \underline{R}}{R_F^*}}{y_M^* - \frac{y_M^* \underline{R}}{R_F^*}}.$$

In this case, D and E correspond to the critical values of the maximal debt-equity ratio that can be supported in an equilibrium that yields the same allocations as the Arrow-Debreu equilibrium. Hence, first-best allocations are guaranteed if banks are required to operate with debt-equity ratios below $\frac{\underline{R}}{R_F^* - \underline{R}}$. Given this constraint, the

[23] A third regulatory measure could be private insurance along the lines of Gersbach (2009), in which banks need to buy insurance against default in the private market place. An entirely different policy approach would be to tax risky investments made by banks and to use these revenues to bail out banks in case of default.

macroeconomic version of the Modigliani-Miller theorem holds. The capital structure of banks is irrelevant and equilibria are efficient. We further note that the regulatory capital requirement can also be expressed in terms of a minimum requirement for the ratio of equity to assets:

$$\frac{E}{y_M^*} \geq \frac{y_M^* - \frac{y_M^* \underline{R}}{R_F^*}}{y_M^*} = \frac{R_F^* - \underline{R}}{R_F^*}.$$

An important consequence of Corollary 4 is that for a given risk-free interest rate and a given amount of deposits, banks have to be funded the more by equity, the lower are the returns of the risky technology in the bad state. This is intuitive as the interest on deposits determines banks' repayment obligations and the returns of the risky technology determines how much funds banks have available to meet those obligations.

We conclude the discussion of optimal bank capital regulation with two remarks. First, bank capital regulation in the spirit of the one displayed in Corollary 4 can be interpreted as a risk-sensitive capital requirement as it is calculated using the returns of the asset in the bad state. Second, an important consequence of our results is that more stringent capital requirements do not impose costs on the economy. Thus, stricter capital requirements than φ, which do not rely on precise measurements of the asset risks, will also implement first-best allocations.[24]

5.2 Commitment to Bankruptcy

We conclude the regulatory section with the observation that Pareto-efficiency could also be achieved by forcing failing banks into bankruptcy when bankruptcy costs are zero. This means that default of banks does not impede efficiency if bankruptcy costs are zero and the government can commit itself not to intervene.

[24] In our model, even extremely low bounds on debt-equity ratios are consistent with Pareto-efficiency.

Proposition 7

Suppose costs of bank bankruptcy are zero independently of how many banks fail. Suppose further that bank equity is non-zero so that maximization of the expected return on equity is a meaningful objective. Then the allocation in any equilibrium is efficient if the regulator forces all failing banks into bankruptcy.

The proof of Proposition 7 is given in Appendix D. Here we provide the intuition. First, all equilibria without default continue to be equilibria under the conditions of the proposition. Second, all equilibria with default are ultimately equivalent to Radner equilibria characterized in Proposition 3.

6 Conclusion

We have analyzed the socially optimal capital structure of banks in a macroeconomic setting. It turned out that the macroeconomic version of the Modigliani-Miller theorem fails to hold globally. Even in a basic general equilibrium model, the capital structure of the banking sector can distort the optimal risk allocation in the economy and inefficient equilibria can arise. Regulatory measures such as minimal capital requirements can eliminate these inefficient equilibria. There are several potential extensions to our model. First, we could introduce heterogeneous households with respect to risk aversion. While this extension promises valuable insights into distributional effects of different capital structures, they will not fundamentally alter the core insight of the paper. Second, the special role of bank deposits as a medium of exchange is well known and could be introduced. When deposits provide payment services above their role as a store of value, the debt-equity ratio in unregulated markets will likely remain above some level, i.e. banks funded by equity only will not emerge in equilibrium. However, this does not alter the policy guideline for keeping the debt-equity ratio in the banking sector below a certain threshold in order to avoid an inefficient investment allocation in the economy.

References

Admati, A. R., DeMarzo, P. M., Hellwig, M. F., and Pfleiderer, P. C. (2011). Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity Is Not Expensive. *Stanford GSB Research Paper*, 2063.

Angelini, P., Clerc, L., Curdia, V., Gambacorta, L., Gerali, A., Locarno, A., Motto, R., Röger, W., Heuvel, S. V. d., and Vlček, J. (2011). Basel III: Long-Term Impact on Economic Performance and Fluctuations. *SSRN Working Paper*.

Basel Committee on Banking Supervision (2010). An assessment of the long-term economic impact of stronger capital and liquidity requirements. *BIS Report*, Basel.

Berger, A. N., Herring, R. J., and Szegö, G. P. (1995). The Role of Capital in Financial Institutions. *Journal of Banking and Finance*, 19(3–4):393–430.

Bolton, P. and Samama, F. (2012). Capital Access Bonds: Contingent Capital with an Option to Convert. *Economic Policy*, 27(70):275–317.

Brealey, R. (2006). Basel II: The Route Ahead or Cul-de-Sac? *Journal of Applied Corporate Finance*, 18(4):34–43.

Debreu, G. (1959). *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*. Yale University Press, New Haven and London.

Freixas, X. and Rochet, J.-C. (2008). *Microeconomics of Banking*. MIT Press, Cambridge, Mass., 2 edition.

Gersbach, H. (2009). Private Insurance Against Systemic Crises? *CEPR Discussion Paper*, 7342.

Gersbach, H. and Rochet, J.-C. (2012). Capital Regulation and Credit Fluctuations. *CEPR Discussion Paper*, 9077.

Hanson, S. G., Kashyap, A. K., and Stein, J. C. (2011). A Macroprudential Approach to Financial Regulation. *Journal of Economic Perspectives*, 25(1):3–28.

King, M. (1990). International Harmonisation of the Regulation of Capital Markets: An Introduction. *European Economic Review*, 34(2–3):569–577.

Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press, New York.

Miles, D., Yang, J., and Marcheggiano, G. (2012). Optimal Bank Capital. *The Economic Journal*, Article in Press.

Miller, M. H. (1977). Debt and Taxes. *The Journal of Finance*, 32(2):261–275.

Miller, M. H. (1995). Do the M & M Propositions Apply to Banks? *Journal of Banking and Finance*, 19(3-4):483–489.

Modigliani, F. and Miller, M. H. (1958). The Cost of Capital, Corporation Finance and the Theory of Investment. *The American Economic Review*, 48(3):261–297.

Radner, R. (1982). Equilibrium under Uncertainty. In Arrow, K. J. and Intriligator, M. D., editors, *Handbook of Mathematical Economics*, volume 2, page 923–1006. North-Holland, Amsterdam.

Schaefer, S. M. (1990). The Regulation of Banks and Securities Firms. *European Economic Review*, 34(2–3):587–597.

Siegenthaler, P., Jordan, T., Raaflaub, P., and Brunetti, A. (2010). Schlussbericht der Expertenkommission zur Limitierung von volkswirtschaftlichen Risiken durch Grossunternehmen. *Bericht des Eidgenoessischen Finanzdepartements (EFD)*, Bern.

Van den Heuvel, S. J. (2008). The Welfare Cost of Bank Capital Requirements. *Journal of Monetary Economics*, 55(2):298–320.

A Appendix - Additional Examples

In the following two examples both Inada conditions are violated and corner solutions emerge.

■ **Example A1:** $y_F = W$ **when** $f'(0) < \infty$ **and** $f'(W) > 0$**.**
We set $W = 1$, $\sigma = 0.5$, $\theta \in (0, 1)$, $\underline{R} = 0$, $\overline{R} \in (0, 3]$, and assume that $f(K_F) = K_F + \ln(1 + K_F)$. Setting $p_g = p_b = 1/3$ will clear the market with $y_F = W$, $y_M = 0$. In this example, the returns in the risky sector are too low in order to attract any funds from households. Hence households invest in the safe technology FT only.

■ **Example A2:** $y_F = 0$ **when** $f'(0) < \infty$ **and** $f'(W) > 0$**.**
Let us use $f(K_F) = \frac{K_F}{1+K_F}$ instead of $f(K_F) = (K_F - \frac{1}{2}K_F^2)$ in Example 2. In this case, markets are cleared with $y_F = 0$, $y_M = W$, i.e. no investment in the safe technology. This example is similar to Example 2. Again, returns on investments in FT are so low that people prefer to invest in the risky technology only.

B Appendix - Households' Optimal Portfolio Choice Problem

Household utility is given by

$$\begin{aligned} U(\gamma) &= \frac{1}{1-\theta}\left\{\sigma c_g^{1-\theta} + (1-\sigma)c_b^{1-\theta}\right\}, \\ \text{where } c_g &= W((1-\gamma)\overline{R}_E + \gamma R_S) + \Pi_F, \\ c_b &= W((1-\gamma)\underline{R}_E + \gamma R_S) + \Pi_F - T. \end{aligned}$$

The representative household solves the following problem:

$$\begin{aligned} \max_{\gamma} U(\gamma) = \frac{1}{1-\theta}\Big\{&\sigma\left[W((1-\gamma)\overline{R}_E + \gamma R_S) + \Pi_F\right]^{1-\theta} \\ &+(1-\sigma)\left[W((1-\gamma)\underline{R}_E + \gamma R_S) + \Pi_F - T\right]^{1-\theta}\Big\}. \end{aligned}$$

The first-order condition for γ is

$$\frac{\sigma(R_S - \overline{R}_E)}{c_g^{\theta}} + \frac{(1-\sigma)(R_S - \underline{R}_E)}{c_b^{\theta}} = 0. \tag{41}$$

Rearranging this expression, we obtain

$$\frac{c_g}{c_b} = A_1, \tag{42}$$

$$\text{where } A_1 := \left[\frac{\sigma(\overline{R}_E - R_S)}{(1-\sigma)(R_S - \underline{R}_E)}\right]^{\frac{1}{\theta}}.$$

Using c_b and c_g to get an expression for γ, we obtain

$$\left[W((1-\gamma)\overline{R}_E + \gamma R_S) + \Pi_F\right] = \left[W((1-\gamma)\underline{R}_E + \gamma R_S) + \Pi_F - T\right] A_1 \text{ or}$$

$$\gamma = \frac{(1/W)[\Pi_F(1-A_1) + TA_1)] + \overline{R}_E - A_1\underline{R}_E}{\overline{R}_E - R_S + A_1(R_S - \underline{R}_E)}.$$

C Appendix - Banks' Return on Equity Maximization Problem

We ignore the denominator E in $\mathbf{E}\left[ROE(\alpha)\right]$ and work with the bank's expected profit instead. Without limited liability, bank profits would be

$\alpha[\overline{R} - R_F](D+E) + R_F E$ in the good state and

$\alpha[\underline{R} - R_F](D+E) + R_F E$ in the bad state.
In case $R_F \leq \underline{R}$, $\alpha = 1$ is optimal.
In case $R_F \geq \overline{R}$, $\alpha = 0$ is optimal.

From hereon, we consider the case $\underline{R} < R_F < \overline{R}$. In that case, there is never a default in the good state. Let us focus on the bad state. At $\alpha = 0$, the bank makes a positive profit. At $\alpha = 1$, the bank's profit is $B_1 = \underline{R}(D+E) - R_F D$.

If $B_1 \geq 0$, the bank never defaults in either state and its expected profit is

$$\begin{aligned} & [\alpha\sigma\overline{R} + \alpha(1-\sigma)\underline{R} + (1-\alpha)R_F](D+E) - R_F D \\ = \; & \alpha[\sigma\overline{R} + (1-\sigma)\underline{R} - R_F](D+E) + R_F E \\ = \; & \alpha B_2(D+E) + R_F E. \end{aligned}$$

Hence,

$$\begin{array}{lll} \alpha = 0 & \text{is optimal if} & B_2 < 0, \\ \alpha = 1 & \text{is optimal if} & B_2 > 0, \\ \alpha \in [0,1] & \text{is optimal if} & B_2 = 0. \end{array}$$

Next we consider the case $B_1 < 0$. The bank breaks even in the bad state at $\widehat{\alpha} \in (0,1)$ satisfying $\widehat{\alpha}[\underline{R} - R_F](D+E) + R_F E = 0$.

- If $B_2 > 0$, then the bank's expected profit is increasing in α in the entire interval $[0,1]$ and $\alpha = 1$ is optimal.
- If $B_2 = 0$, then the bank's expected profit is constant in the interval $[0, \widehat{\alpha}]$ and increasing in the interval $[\widehat{\alpha}, 1]$ and, hence, $\alpha = 1$ is optimal.
- If $B_2 < 0$, then the bank's expected profit is decreasing in the interval $[0, \widehat{\alpha}]$ and increasing in the interval $[\widehat{\alpha}, 1]$. Therefore, the optimal α is obtained from comparing $\sigma \cdot \{[\overline{R} - R_F](D+E) + R_F E\}$, the expected profit when $\alpha = 1$, and $R_F E$, the expected profit when $\alpha = 0$.

The comparison of $\sigma \cdot \{[\overline{R} - R_F](D+E) + R_F E\}$ and $R_F E$ also provides the answer if $B_2 \geq 0$. Denote the difference

$$\begin{aligned} B_3 &= \sigma \cdot \{[\overline{R} - R_F](D+E) + R_F E\} - R_F E \\ &= [\sigma\overline{R} - R_F](D+E) + (1-\sigma)R_F D. \end{aligned}$$

Then $B_3 > 0$ means that $\alpha = 1$ is optimal, $B_3 < 0$ means that $\alpha = 0$ is optimal while $B_3 = 0$ means that both $\alpha = 0$ and $\alpha = 1$ are optimal. Further note that $B_1 < 0$ and $B_2 \geq 0$ imply $B_3 > 0$.

D Appendix - Proofs

Proof of Proposition 4

Let p_g^* and p_b^* be the Arrow-Debreu equilibrium prices. We set $R_F = \dfrac{1}{p_g^* + p_b^*}$. Since $f'(y_F^*) = \dfrac{1}{p_g^* + p_b^*}$, it follows that $R_F = R_F^*$.

■ *Firms*

The two problems

$$\max_{K_F} (p_g^* + p_b^*) \cdot f(K_F) - K_F \text{ and}$$
$$\max_{K_F} \; f(K_F) - R_F^* K_F$$

have the same solution y_F^*. In the first problem, the firm maximizes the present value of profits whereas in the second problem, it maximizes the future value. Again, $\Pi_M = 0$ and $y_M^* = W - y_F^*$ is an optimal choice.

■ *Households*

For the household, investing in FT amounts to buying bonds with return $a_1 = (R_F^*, R_F^*)$ at price $q_1 = 1$. Buying equity in the bank amounts to buying shares in the risky asset with returns $a_2 = (\overline{R}, \underline{R})$ at price $q_2 = 1$. But by (22), the market clearing asset prices in the corresponding Radner equilibrium are

$q_1 = R_F^*(p_g^* + p_b^*) = 1$ because of $R_F^* = 1/(p_g^* + p_b^*)$ and

$q_2 = \overline{R}p_g^* + \underline{R}p_b^* = 1$ because of (2).

Hence for the household, investing y_F^* in FT and y_M^* in equity is optimal and yields the same consumption bundle as in the Arrow-Debreu case, provided $D = 0$ and $l_F = 0$.

■ *Banks*

For the bank, $\overline{R}p_g^* + \underline{R}p_b^* = 1$ and $R_F^*(p_g^* + p_b^*) = 1$ imply $\underline{R} < R_F^* < \overline{R}$. If $D = 0$, then $B_1 \geq 0$ and $\alpha = 1$ provided that $B_2 > 0$, i.e.,

$$\sigma\overline{R} + (1-\sigma)\underline{R} > R_F^* \text{ or}$$
$$\sigma(\overline{R} - R_F^*) + (1-\sigma)(\underline{R} - R_F^*) > 0. \tag{43}$$

To prove (43), note that (2) and $p_g^* R_F^* + p_b^* R_F^* = 1$ further imply $p_g^*(\overline{R} - R_F^*) + p_b^*(\underline{R} - R_F^*) = 0$. (43) follows if we can demonstrate that $\dfrac{\sigma}{1-\sigma} > \dfrac{p_g^*}{p_b^*}$. To see the latter, notice that $y_M^* > 0$ implies $c_g^* > c_b^*$ and, hence,

$$\left(\frac{p_b^*}{p_g^*} \cdot \frac{\sigma}{1-\sigma}\right)^{\frac{1}{\theta}} p_g^* + p_b^* > \left(\frac{p_g^*}{p_b^*} \cdot \frac{1-\sigma}{\sigma}\right)^{\frac{1}{\theta}} p_b^* + p_g^* \text{ or}$$

$$\left(\frac{p_b^*}{p_g^*} \cdot \frac{\sigma}{1-\sigma}\right)^{\frac{1}{\theta}} \frac{p_g^*}{p_b^*} + 1 > \left(\frac{p_g^*}{p_b^*} \cdot \frac{1-\sigma}{\sigma}\right)^{\frac{1}{\theta}} + \frac{p_g^*}{p_b^*}$$

or, with $\pi = \frac{p_g^*}{p_b^*}, \tau = \frac{\sigma}{1-\sigma}$,

$$(\tau/\pi)^{\frac{1}{\theta}} \pi + 1 > (\pi/\tau)^{\frac{1}{\theta}} + \pi.$$

Suppose $\pi/\tau \geq 1$. Then lhs $\leq 1 + \pi$ and rhs $\geq 1 + \pi$, a contradiction. Hence to the contrary, $\pi/\tau < 1$ or $\pi < \tau$ or $\dfrac{\sigma}{1-\sigma} > \dfrac{p_g^*}{p_b^*}$. Consequently, (43) has to hold. It follows that $D = 0$ implies that $\alpha = 1$ is optimal. This completes the proof. $\square$

Proof of Proposition 5

In order for y_F^* to solve $\max f(K_F) - R_F K_F$, we set again $R_F = \dfrac{1}{p_g^* + p_b^*}$, which is equal to R_F^*. Then the bank will only invest in the risky technology, since $B_2 > 0$ by the argument given in the proof of Proposition 4. Suppose $D > 0$. Without default, the bank's profit would become

$(E + D)\overline{R} - DR_F^*$ in the good state and

$(E + D)\underline{R} - DR_F^*$ in the bad state.

If $\underline{R} = 0$, the bank defaults in the bad state.

Otherwise, there is no default as long as $DR_F^* \leq (E + D)\underline{R} = y_M^* \underline{R}$ or $D \leq \dfrac{\underline{R}}{R_F^*} y_M^*$. In this case, the return on equity is

$[(E + D)\overline{R} - DR_F^*]/E = [y_M^* \overline{R} - DR_F^*]/E$ in the good state;

$[(E + D)\underline{R} - DR_F^*]/E = [y_M^* \underline{R} - DR_F^*]/E$ in the bad state.

As before, $q_1 = 1$ whereas the asset with unit returns

$a_3 = ([(E + D)\overline{R} - DR_F^*]/E, [(E + D)\underline{R} - DR_F^*]/E)$ has the arbitrage-free unit price

$$q_3 = \frac{1}{E}[(y_M^* \overline{R} - DR_F^*)p_g^* + (y_M^* \underline{R} - DR_F^*)p_b^*] = \frac{1}{E}[y_M^* - D] = 1.$$

At these prices, the household invests y_F^* in FT, makes the amount D of deposits, and purchases $E = y_M^* - D$ units of bank equity. $\square$

Proof of Proposition 6

If $\underline{R} = 0$, let us take as a reference point the Arrow-Debreu equilibrium and the corresponding unique equilibrium with financial intermediation and no default. In that equilibrium, bank deposits have the value $D^o = 0$ and equity assumes the value $E^o = W - y_F^* = y_M^*$. If $\underline{R} > 0$, let us take as a reference point the equilibrium with financial intermediation and no default where the bank is on the brink of defaulting in the bad state. In that equilibrium, as shown in the proof of Proposition 5, deposits assume the value $D^o = \frac{\underline{R}}{R_F^*} y_M^*$ and equity assumes the value $E^o = (1 - \frac{\underline{R}}{R_F^*}) y_M^*$. Moreover, we found in the proof of Proposition 4 that the equilibrium bond return R_F^* satisfies $\sigma \overline{R} + (1-\sigma)\underline{R} > R_F^*$.

Next let us fix a bond return (denoted by $\widehat{R}_F$) slightly above R_F^* such that $\sigma\overline{R}+(1-\sigma)\underline{R} > \widehat{R}_F$. Then the analogue of $B_2 > 0$ holds and the bank still chooses $\alpha = 1$. Given the higher bond return $\widehat{R}_F$, entrepreneur e_F will respond by choosing a profit maximizing input denoted by $\widehat{K}_F$, with $\widehat{K}_F < y_F^*$. The resulting profit is denoted by $\widehat{\Pi}_F$ and satisfies $\widehat{\Pi}_F < \Pi_F^*$.

At the reference equilibrium, the demand for equity is E^o when $T = 0$, the return on bonds is R_F^* and a unit of equity pays $\overline{R}(1 + \frac{D^o}{E^o}) - R_F^* \frac{D^o}{E^o}$ in the good state and zero in the bad state. If one replaced R_F^* by $\widehat{R}_F > R_F^*$, Π_F^* by $\widehat{\Pi}_F$, and $T = 0$ by $\widehat{T} = \widehat{R}_F D^o - \underline{R}(E^o + D^o) > 0$, then the household would demand more of the risk-free asset.[25] Now assume $E \in (0, E^o]$ and $D = W - E - \widehat{K}_F$. Consider the household's portfolio choice when the profit distributed is $\widehat{\Pi}_F$, $T = \widehat{R}_F D - \underline{R}(E + D)$, the return

[25]Observe that $c_g > c_b$ and homothetic preferences of the household (together with standard properties) imply that $|MRS|$ is smaller at the consumption bundle $(\hat{c}_g, \hat{c}_b) = (c_g - (\Pi_F - \widehat{\Pi}_F), c_b - (\Pi_F - \widehat{\Pi}_F) - \widehat{T})$ than at (c_g, c_b). To see this, consider normalized gradients of the form $(|MRS|, 1)$. Denote by ∇ the household's normalized gradient at (c_g, c_b) and by $\hat{\nabla}$ its normalized gradient at $(\hat{c}_g, \hat{c}_b)$. If in the reference equilibrium situation, the household replaces one unit of the bond by one unit of equity, then consumption is changed in the direction $\upsilon = (\overline{R}(1 + \frac{D^\circ}{E^\circ}) - R_F \frac{D^\circ}{E^\circ} - R_F, -R_F)$ and at equilibrium, portfolio choice is optimal, that is $\nabla \cdot \upsilon = 0$. If in the new situation, the household replaces one unit of the bond by one unit of equity, then consumption is changed in the direction $\hat{\upsilon} = (\overline{R}(1 + \frac{D^\circ}{E^\circ}) - \hat{R}_F \frac{D^\circ}{E^\circ} - \hat{R}_F, -\hat{R}_F)$. It follows that $0 = \nabla \cdot \upsilon > \nabla \cdot \hat{\upsilon} > \hat{\nabla} \cdot \hat{\upsilon}$. But $\hat{\nabla} \cdot \hat{\upsilon} < 0$ means that the household benefits from reducing its equity holding and increasing its bond holding by the same amount.

on bonds is $\widehat{R}_F$ and a unit of equity pays $\overline{R}(1+\frac{D}{E})-\widehat{R}_F\frac{D}{E}$ in the good state and zero in the bad state. There is a unique optimal $\gamma(E)\in[0,1]$ so that the household invests $\gamma(E)W$ in bonds and $[1-\gamma(E)]W$ in equity. By Berge's maximum theorem (or because of (23)), $\gamma(E)$ is a continuous function of E. Set $\eta(E)=[1-\gamma(E)]W$. As reasoned above, $\eta(E^o)<E^o$. If $E\to 0$, then $D\to W-\widehat{K}_F$, $T\to(\widehat{R}_F-\underline{R})(W-\widehat{K}_F)$, and $\overline{R}(1+\frac{D}{E})-\widehat{R}_F\frac{D}{E}\to\infty$. Hence there exists $E_o\in(0,E^o)$ with $\eta(E_o)>E_o$.[26] By the intermediate value theorem, there exists $E\in(E_o,E^o)$ with $\eta(E)=E$. At this E and the corresponding value for T, the asset market is cleared — as well as the consumption good market in both states — while the bond return is $\widehat{R}_F$ and FT production is less than at the Arrow-Debreu equilibrium.

It remains to check whether the bank is actually going to default in the bad state. In the reference equilibrium, $\underline{R}(E^o+D^o)-R_FD^o=0$. Let $\Delta=y^*_F-\widehat{K}_F>0$. Then $\underline{R}(E^o+D^o+\Delta)-\widehat{R}_F(D^o+\Delta)<0$. Further $E^o+D^o=W-y^*_F$, $E+D=W-\widehat{K}_F$ and $E<E^o$. Hence $E^o+D^o+\Delta=W-\widehat{K}_F=E+D$ and $D^o+\Delta=W-E^o-y^*_F+y^*_F-\widehat{K}_F=W-E^o-\widehat{K}_F<W-E-\widehat{K}_F=D$. It follows that $\underline{R}(E+D)-\widehat{R}_FD<0$ which means that the bank is going to default in the bad state, indeed.

Since $\widehat{K}_F\neq y^*_F$, the equilibrium allocation is inefficient, by Corollary 1. □

Proof of Proposition 7

First of all, all equilibria without default continue to be equilibria under the conditions of the proposition. Second, there are equilibria with default that are equivalent to Radner equilibria explored in Proposition 3.

Namely, the economy has an Arrow-Debreu equilibrium with quantities p^*_g, p^*_b, c^*_g, c^*_b, y^*_F, y^*_M such that $0<y^*_F<W$. Consider the following three securities: A bond with price $p_f=1$ and return $R^*_F=(p^*_g+p^*_b)^{-1}$, risky bank deposits that promise $R_D\in(\underline{R},\overline{R})$

[26] Observe that $\overline{R}_E+(A_1-1)\widehat{R}_F>0$. Hence, $\eta(E_o)>E_o$ can be rewritten as

$$(A_1-1)\widehat{R}_FW-[\widehat{\Pi}_F(1-A_1)+TA_1]>E_o[\overline{R}_E+(A_1-1)\widehat{R}_F].$$

Substituting T, D, $\overline{R}_E$ and simplifying the expression, we obtain

$$(A_1-1)[\widehat{R}_F\widehat{K}_F+\widehat{\Pi}_F]+[A_1\underline{R}-\overline{R}](W-\widehat{K}_F)>0.$$

If $E\to 0$, then $A_1\to\infty$, which establishes the existence of $E_o\in(0,E^o)$ with $\eta(E_o)>E_o$.

in both states, but actually pay R_D in the good state and $\underline{R}$ in the bad state, and bank equity that pays $\overline{R} - R_D$ in the good state and nothing in the bad state. Bank deposits have the price $p_d = p_g^* R_D + p_b^* \underline{R}$ and a bank share costs $p_e = p_g^*(\overline{R} - R_D)$. A unit of bank deposit together with one bank share constitutes one unit of asset a_2 at the price $q_2 = 1$. Hence the household obtains its first-best consumption bundle by purchasing y_F^* bonds and providing y_M^* units of capital to the bank, by investing $p_d y_M^*$ in bank deposits and $p_e y_M^*$ in bank equity. Funds of size y_F^* are used by e_F while the bank invests its capital y_M^* in the MT sector and all markets are cleared. In the bad state, the bank has revenue $y_M^* \underline{R}$ which falls short of its promised payment to depositors, $y_M^* R_D$.

It remains to be shown that under the assumptions made, these are the only equilibria with default.

Step 1:

Let us consider an arbitrary equilibrium with default in the bad state only.[27] The price of the asset a_f for investment in FT yielding return R_F is denoted by p_f. Suppose next that the bank has obtained the amount D of deposits at price p_d and E equity contracts at price p_e and thus $p_d D + p_e E$ units of the investment good. The promised return on deposits is R_D. Suppose that the bank invests a fraction α into MT and $1 - \alpha$ into FT with $0 \leq \alpha \leq 1$. The realized returns on bank debt and equity are thus as follows:

	Equity	Deposit
good state	R_E	R_D
bad state	0	$\underline{R}_D$

$$\text{where } R_E := \frac{[\alpha \overline{R} + (1-\alpha)\frac{R_F}{p_f}](p_d D + p_e E) - D R_D}{E}$$
$$\text{and } \underline{R}_D := \frac{[\alpha \underline{R} + (1-\alpha)\frac{R_F}{p_f}](p_d D + p_e E)}{D}.$$

[27] There exist equilibria in which the bank defaults in both states. The reasoning of the proof can be readily adapted to such cases as well.

The condition that the bank defaults in the bad state and thus the matrix applies is

$$[\alpha\underline{R} + (1-\alpha)\frac{R_F}{p_f}](p_d D + p_e E) - R_D D < 0$$

and consequently,

$$D > \frac{[\alpha\underline{R} + (1-\alpha)\frac{R_F}{p_f}]p_e E}{R_D - [\alpha\underline{R} + (1-\alpha)\frac{R_F}{p_f}]p_d}. \tag{44}$$

Step 2:

We next show that $\frac{R_F}{p_f} < \overline{R}$. Suppose $\frac{R_F}{p_f} \geq \overline{R}$. Then it would be profitable for the bank to invest all resources in FT. Then either there is default in both states or no default at all, contrary to the assumption made. Hence $\frac{R_F}{p_f} < \overline{R}$ must hold.

Step 3:

Given that $\frac{R_F}{p_f} < \overline{R}$, the bank invests only in MT. That is, $\alpha = 1$. For otherwise, by putting more of its funds into MT, the bank could increase its return on equity in the good state while the return in the bad state would remain zero or become positive.

Step 4:

Since $\alpha = 1$, by buying μ_1 units of deposit contracts and μ_2 units of equity contracts with

$$\mu_1 = \frac{D}{(p_d D + p_e E)} \text{ and } \mu_2 = \frac{E}{(p_d D + p_e E)},$$

the household can create a new asset $\tilde{a}_2$ with the following characteristics:

Asset	Price in $t=1$	Return in state g	Return in state b
$\tilde{a}_2$	$\tilde{p}_2$	$\overline{R}$	$\underline{R}$

Note that

$$\tilde{p}_2 = \frac{p_d D}{(p_d D + p_e E)} + \frac{p_e E}{(p_d D + p_e E)} = 1.$$

Asset $\tilde{a}_2$ is identical with asset a_2. Therefore, the household faces the following portfolio choice. It can invest a fraction of its wealth at price 1 into the asset with return $\overline{R}$ and $\underline{R}$ and the remaining part into a safe asset at price p_f with return R_F. Except for potential rescaling the units of the safe asset, this is essentially the same situation as in Section 3.6 and we can use Proposition 3 which establishes the equivalence of the ensuing equilibrium to the Arrow-Debreu equilibrium. $\square$