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## UNDERSTANDING THE MATCHING FUNCTION: THE ROLE OF NEWSPAPERS AND JOB AGENCIES

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**HUMAN RESOURCES** 



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### **ABSTRACT**

Understanding the Matching Function: The Role of Newspapers and Job Agencies\*

This paper provides a microeconomic model of matching which implies that the standard, reduced form approach, is misspecified. A simple model is analysed (with help-wanted/employment-needed advertising) where the matching rate depends not only on the stocks of unemployed and vacancies in the market, but also on the flows of new vacancies and new job seekers. The model is consistent with the empirical fact that one-quarter of all new vacancies posted in a job centre are filled the same day.

JEL classification: J63, J64

Keywords: matching, job centres, returns to scale

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### NON-TECHNICAL SUMMARY

Job seekers do not search for work randomly, they check newspapers and job centres for information on vacancies, and contact suitable vacancies directly. This paper explicitly examines this contact mechanism and shows how such 'information channels' affect the matching process. In particular, it provides a new matching function which is based on microeconomic behaviour and which has directly testable implications. Moreover, not only does it suggest that there are increasing returns to matching, but it also shows why standard estimation procedures are misspecified and that such methodologies could 'estimate' decreasing returns, even when there are increasing returns. Clearly, this opens up new avenues for empirical work.

The basic idea is very straightforward and applies to many decentralized markets (not just the labour market). Consider a firm which decides to post a new vacancy. Its most likely strategy is to advertize it in the newspaper and/or post it in the local job centre. Similarly, the most sensible strategy of a job seeker is to check the situations vacant column in the local paper, and to visit the local job centre to see what jobs are available.

If the firm is lucky, there might already be a suitable job seeker in the unemployment pool who applies for the job, and the post is filled immediately. The vacancy is then withdrawn from the newspaper and the job centre. In fact, Coles and Smith (1994) find that one-quarter of all vacancies posted in UK job centres are filled on their first day. With some probability, however, the firm may be unlucky and cannot find a suitable worker to take the post. In that case, the vacancy may be left on the books at the job centre, or readvertized, hoping that a new entrant in the job seeking pool will be suitable for the post. In other words, if a new vacancy is not filled immediately, the vacancy has to wait for a suitable worker to enter the job seeking pool.

The process is similar for a newly unemployed worker. There is always a stock of vacancies in the newspapers and job centres. If the job seeker is lucky, there may be a suitable job available, in which case they immediately re-enter employment. If there is no such job, however, the job seeker is left waiting for a new vacancy to be advertized for which they are suitably trained, and which is not offered to another competing worker.

This paper formally models this matching procedure and arrives at the following conclusions:

(1) The standard matching function is inappropriate. It assumes that the matching flow is a function of the stocks of vacancies and unemployed workers. With newspapers/job centres and worker heterogeneity, a better description of the

matching process may be that the stock of unemployed is matching with the flow of new vacancies, as they have already sampled the stock of vacancies and a suitable match could not be made. Similarly, the stock of 'old' vacancies is matching with the flow of new job seekers.

- (2) New vacancies are fundamentally different from 'old' ones in the sense that they have not yet been sampled by the present stock of unemployed. In particular, new ones are much more likely to match. Coles and Smith (1994) show this is true empirically, where a vacancy matches with probability 0.25 on its first day, and with probability of less than 0.02 on each successive day.
- (3) Cobb-Douglas matching functions are misspecified. Furthermore, one can construct theoretical models where applied work would find 'decreasing returns to scale' using such a misspecification, even though there are increasing returns.
- (4) It provides an important direction for future empirical work on matching functions.
- (5) It implies there are increasing returns to matching. This potentially explains why industrialization causes rural to urban migration. Such migration ceases when the congestion costs of living in the cities exceeds the returns to greater matching (specialization) in the cities. In particular, such rural depopulation is not inefficient (unlike the standard Harris-Todaro type stories).

This matching paradigm generalizes to other market scenarios. Indeed, the idea for this mechanism arose while the author was looking for an antique farmhouse table. After several weeks/months of going to the same antique shops looking for a table, one Saturday, a new table arrived which was just what we were looking for. We immediately bought it. The shop owner said it had only been there a few hours. Once we'd rejected his original stock of tables, we could only match with the arrival of new, more suitable tables.

A second example is a person I once met who was trying to find rental accommodation in Coventry. He had a copy of the local newspaper in his hand. The newspaper had just been released. He had turned straight to the property for let section, ignored the properties he'd seen before, and immediately called a property for let which was being advertized for the first time. He then jumped on the bus to inspect it before anyone else had the chance to get there. Again he was trying to match with the flow of new properties. Conversely, a new person arriving in Coventry would inspect the current stock of properties available for let.

#### Introduction

A central component in constructing a theory of equilibrium unemployment is to describe how quickly the unemployed contact new vacancies. The standard assumption is that matches between U unemployed workers and V vacancies occur randomly, at a Poisson rate f(U,V). Using a microeconomic model of matching, this paper suggests that this reduced form function is misspecified. The proposed alternative specification is f(u,v,U,V), where u and v are the flows of new unemployed workers and of new vacancies. This reflects the empirical fact that a new vacancy (or a newly unemployed worker) is more likely to match in the immediate future than an old vacancy (or a long term unemployed worker).

The intuition for this alternative structure is straightforward. contacts between vacancies and the unemployed do not occur randomly. Many firms advertise their vacancies in newspapers, or contact job agencies for possible staff. On becoming unemployed, a job seeker checks these sources for possible employment. Now consider someone who has just lost their job. At first, their probability of immediately finding a job may be relatively high. There is usually a large stock of existing vacancies with which the worker might successfully match. But suppose none of these result in immediate re-employment. Getting a job then relies on waiting for a new vacancy to be created which is not only suitable for the worker in question, but is also not offered to somebody else. In other words, a long term unemployed worker has already rejected (or been rejected by) possible matches with the present stock of vacancies. Consequently, a newly unemployed worker has many more employment options and is therefore much more likely to match. The specification f(U,V) does not take this sorting/sampling effect into account.3

Finding the correct specification of the matching function is an

important research problem. It is well-known that if there increasing returns to matching, then an economy can sustain multiple search equilibria. If the economy is caught in the high unemployment 'trap', government policy should aim to switch the economy back to the good equilibrium (see Diamond (1982), Howitt and McAfee (1987), Mortensen (1989) for example). Recently, many researchers have estimated the matching function f(U,V), attempting to identify its returns to scale. Since intuition suggests that matching should be easier in 'thick' markets than in 'thin' ones, it is perhaps surprising that most empirical results find constant rather than increasing returns.

The matching model in this paper suggests that these estimates are affected by an aggregation bias. To formalise this bias, this paper considers the simpler model of a marriage market with heterogeneous men and women, where flows into the market are exogeneously specified. As in the labour market, the members of this marriage market wish to meet and match. They also use advertising columns in newspapers and dating agencies to speed up the contact process.

Suppose there is a dating agency (or a newspaper) which puts all of the unattached men and women into contact with each other. In a steady state with heterogeneous agents, men and women who continue to remain in the market, choose not to match with the present members of the market. They are waiting to match with new members. In this paper, where the arrival of new members is described by a Poisson process, the expected matching rate H(.) is given by

$$H(m,w,M,W) = m \phi_1(W) + w \phi_2(M)$$

where m,w are the arrival rates of new men and women and M,W are the stocks of members. In the symmetric example analysed,  $\phi_1 = \phi_2 = \phi$ , and  $\phi(.)$  is a positive, increasing and strictly concave function. The first term

describes the rate at which the flow of 'new' men matches with the stock of women, and similarly for the second term.

Unlike the standard matching function, the matching interaction in the steady state is between the stocks and the flows of the opposite sex, rather than between the two stocks. The dynamics are that when a man enters the dating agency,  $\phi(W)$  is the probability that he matches with one of the W existing women members (where an increase in W increases the probability that he matches immediately). If he is unsuccessful, he waits to match with the flow of new female members. A successful match then arrives at the Poisson rate  $w\phi(M)/M$ , w defines the rate at which he meets new female members, and  $\phi(M)/M$  is the probability the new woman chooses to match with him (where  $\phi(M)/M$  decreases with M). Similarly for a new female member.

Given this reduced form matching process, it is interesting to consider what would happen if we fitted the standard matching function f(M,W) to the resulting data. Since this function is misspecified, the results would depend on the time series properties of the flows m(t) and w(t). However, suppose m and w were constant. The expected matching rate would then be  $W = m \phi(W) + w \phi(W)$ , which exhibits decreasing returns (since  $\phi$  is strictly concave). Given a reasonable functional form approximation, the researcher should conclude that the matching function has decreasing returns to scale. But this would only identify the congestion effect. [From the viewpoint of an incumbent male, a larger stock of men in the dating agency reduces the probability that the new woman will match with him.] Although there are diminishing marginal returns to M and W, there are increasing returns to scale overall.

This specification also has important empirical implications concerning time series data. Consider the matching dynamics in the labour

market again. The following diagram is of matching flows in the U.S. manufacturing sector (taken from Blanchard and Diamond (1989)):

#### figure

Hiring flows are much more volatile than the stock variables U and V. Estimating a functional form f(U,V) cannot capture this volatility. More interestingly, notice that given this volatile hiring process, the stock of vacancies remains smooth over time - even though the hiring flow (per month) is greater than the stock of vacancies. Assuming no measurement error problems, this phenomenon can only be explained if new vacancies are filled with probability close to one (within a month), and the flow of new vacancies is relatively volatile (which creates a volatile hiring process while allowing the vacancy level to remain smooth). This view is consistent with Beaumont (1978) who found that in a Scottish job center, 90.3% of all new vacancies were filled within a month (and most were filled within 5 days). Clearly, the alternative matching function f(u,v,U,V) has greater potential to explain this matching data.

The paper is structured as follows. The first section describes the basic matching model and describes the resulting reduced form dynamics in the steady state. In this model, if the researcher fitted a standard Cobb-Douglas matching function to the resulting data, the conclusion would be that there are decreasing returns to scale, when in fact there are increasing returns. Section 2 shows that the welfare implications of the dating agency are very different to the standard matching framework with constant returns. As the arrival rates of new members increase, the value of joining the dating agency increases. This is not true in the standard model with constant returns. Section 3 then generalises the reduced form matching function with Poisson arrival rates of new members.

#### I. A Simple Model of Matching Flows in a Dating Agency

Consider a dating agency through which men and women meet and match. Time is discrete where each period is of length  $\Delta>0$ . For  $t\in\mathbb{N}^+$ , the agency has a stock of  $M_t$  men and a stock of  $W_t$  women. At the start of this period,  $m_t$  new men join the dating agency, as do  $w_t$  new women. Given H matches where  $H=H(m_t,w_t,M_t,W_t)$ , the next period stocks are given by

$$M_{t+1} = M_t + m_t - H(.)$$

$$W_{t+1} = W_t + w_t - H(.)$$

In the search literature, the matching function is usually specified as  $H(m_t+M_t, w_t+W_t)$ , which says that new entrants are equivalent to existing members. This assumption is not made here. The aim is to provide a structural model of matching behaviour with sorting, where new members are different to 'old' members in the sense that they have not yet sampled the members of the opposite sex.

Consider the following matching structure. Assume that if a man and woman are randomly drawn from some underlying population distribution, the probability that they find each other to be a jointly acceptable match is  $\lambda$ , where  $0 < \lambda < 1.^8$  All types are equally likely to join the dating agency (no selectivity bias).

By paying a fixed fee  $F \ge 0$ , a new member joins the dating agency. The agency publishes a newspaper which describes full (and true) details of each member (though as will be seen in the steady state, it is sufficient that the newspaper simply describes details of the new members). All members costlessly observe the newspaper and immediately contact suitable matches. For simplicity, assume that the subsequent selection period takes no time, so that there is no time delay between a successful contact and a consumated match. If a match is consumated, both parties benefit by b>0 and leave the agency for good. If a member does not match, the member must wait

another time interval for a possible match. If one member receives several offers from members of the opposite sex, assume that the member randomly chooses a match amongst those who have the fewest alternative offers (the reason for this will be made clear later). Finally, all members have the same discount factor  $1/(1+r\Delta)$ , where r > 0 is the discount rate.

For the moment, consider the symmetric case where a new man and a new woman always arrive together, and assume that the stock of men always equals the stock of women;  $M_{\rm t}=W_{\rm t}$ . Anticipating the generalisation to Poisson arrival rates in section III, suppose that at the start of each period, one new man and one new woman join the club.

In the steady state, a man who is presently an unmatched member of the dating agency, cannot match with any of the women who are also unmatched members (since search is costless and takes no time within the club). Hence, he is waiting to match with a new female member (and similarly for an unmatched woman).

When a new woman enters the agency at time t, her immediate potential number of matches is with  $M_t$  resident 'old' men and with the 'new' man. The crucial point is that each old man in the club is never contacted by more than one acceptable offer at any point in time (since new women arrive one at a time). Hence if the new woman proposes an acceptable match to an old man, the old man's best response is to accept.

With this insight, it is now possible to describe the steady state matching dynamics in the market. Given an equal stock of men and women, so that  $M_t = W_t = N_t$ , the transition probabilities between the stocks  $N \in \mathbb{N}^+$  are given by :

#### Proposition 1

The stock dynamics  $\{N_{\mbox{\scriptsize t}}\}$  follow a stationary Markov process where for N  $\succeq$  1

$$P(N \Rightarrow N+1) = (1-\lambda)^{2N+1}$$

$$P(N \Rightarrow N) = 1 - [1-(1-\lambda)^{N}]^{2} - (1-\lambda)^{2N+1}$$

$$P(N \Rightarrow N - 1) = [1 - (1 - \lambda)^{N}]^{2}$$

and  $P(0\Rightarrow0) = \lambda$ ,  $P(0\Rightarrow1) = 1-\lambda$ .

#### Proof

Suppose N  $\geq$  1 and consider the transition N  $\Rightarrow$  N - 1. In this state, both new members must be able to match with an old member. Now the probability that the new woman can form an acceptable match with at least one old man is  $1-(1-\lambda)^N$ . Similarly for the new man. The probability that both can match with an old member must be  $[1-(1-\lambda)^N]^2$ . Given this outcome, it does not matter whether the new man and woman can match with each other (since each would then have at least 2 alternatives and the allocation rule requires that each choose an old member instead). Hence the stated transition probability for N  $\Rightarrow$  N - 1.

Now consider N  $\Rightarrow$  N + 1. In this case, neither must be able to match with an old member, which occurs with probability  $(1-\lambda)^{2N}$ , nor can they find each other acceptable, which occurs with probability  $(1-\lambda)$ . Hence the stated transition probability.

The only other outcome is  $N \Rightarrow N$ , where only one match is made.

The remaining case is for N=0. In this state, the new members either match with probability  $\lambda$ , or do not. ||

Proposition 1 implies that the size of the dating agency will vary over time, described by a stationary Markov process. Moreover, as N changes, the matching probabilities change. Indeed, given N, the expected number of matches per period, denoted by  $H(N;\lambda)$ , is defined by

$$H(N;\lambda) = 2 P(N \Rightarrow N-1) + P(N \Rightarrow N)$$
$$= 2 [1-(1-\lambda)^{N}] + \lambda(1-\lambda)^{2N}$$

It can be shown that  $H(N;\lambda)$  is strictly increasing and strictly concave in N for all  $\lambda \in (0,1)$  and for all  $N \geq 0.9$  Moreover, the ergodic distribution of N is centered around  $H(N;\lambda)=1$ , which for  $\lambda$  small, implies  $(1-\lambda)^N=0.5+0(\lambda)$ . Calculating the matching elasticity  $\epsilon=(N/H)\partial H/\partial N$  at this point, gives  $\epsilon=\log(2)+0(\lambda)=0.69$ , for  $\lambda$  small. Thus if the researcher fitted a standard Cobb-Douglas form to this matching data,  $\log(H)=\alpha+\beta\log(N)$ , we would expect a ball-park estimate of  $\beta\cong 0.7$ , and the standard conclusion would be that there are decreasing returns to matching.

This result is misleading. The estimate identifies diminishing marginal returns to N, basically because of congestion effects. The exit probability of an 'old' member is decreasing in N. As N increases, the new member is more likely to be able to form several possible matches, which reduces the probability that one particular 'old' member will match, even if mutually acceptable.

The distinction between decreasing returns to scale and diminishing marginal returns is crucial. The next section shows that in this dating agency, the value of being a member increases as the flow of new members increases (in the steady state). This reflects the fact that there are increasing returns to scale overall. This result does not hold in the standard random matching framework if there are constant (or decreasing) returns to matching.

# II. Comparing the Welfare Implications of the Dating Agency with a Standard Random Matching Model with Constant Returns

Suppose in the absence of a dating agency, members of the opposite sex match at a Poisson rate H(M,W). The standard assumption in the random matching framework is that H(.) is increasing, concave and homogeneous of degree 1. In that case, let k=H(1,1), and so H(M,M)=kM. Again suppose the population of men and women are equal, so that  $M_t=W_t$  for all t. Then the exit probability of one particular member at time t is  $H(M_t,M_t)\Delta/M_t=k\Delta$  (for  $\Delta$  small), which is independent of  $M_t$ . If  $V_{RM}$  is the value of being unmatched in this market, then

$$V_{RM} = b \sum_{t=0}^{\infty} [k\Delta/(1+r\Delta)] [(1-k\Delta)/(1+r\Delta)]^{t}$$
$$= kb/(r+k)$$

which is independent of  $M_t$ . Thus the member is indifferent to searching in a 'thick' market or a 'thin' one. In particular, an increase in the flow of new members does not lead to a faster exit rate for 'old' members. This is not true in the dating agency.

Suppose at t=0, the dating agency has N members of both sexes and consider the value of joining this club. Given the description of the matching process, the probability that the new member matches in period t  $\geq$  0 can be denoted by  $f(N,\lambda,t)$ , which must be strictly positive for all  $N,t \geq$  0,  $\lambda > 0$ . The crucial result is that this probability is <u>independent</u> of  $\Delta$ , the assumed period interval. Moreover, the ergodic distribution of N must also be independent of  $\Delta$ .

Now consider an increase in the flow rate of new members. Within the discrete time framework developed above, this can be considered as a reduction in  $\Delta$ , the period interval between arrivals. If  $\phi(N,\lambda)$  denotes the ergodic distribution of N,  $V(\lambda,\Delta)$  denotes the expected value of joining this agency, then in the steady state :

$$V(\lambda,\Delta) = \sum_{N=0}^{\infty} \left[ \phi(N,\lambda) \sum_{t=0}^{\infty} \{ b f(N,\lambda,t) / (1+r\Delta)^{t+1} \} \right]$$

As  $\Delta$  decreases, the value of being a member of this club strictly increases. Unlike the standard model with constant returns, an increase in the arrival rate of new members increases the value of joining the agency. Moreover, as  $\Delta \Rightarrow 0$ , it can be shown that  $V(.) \Rightarrow b$ . With an arbitrarily large flow of new members, an old member expects to match within an arbitrarily short period of time (holding  $\lambda$  fixed).

The source of the increasing return to scale in the dating agency is the assumption that the newspaper puts all of the members of each sex into contact with each member of the opposite sex. With one new entrant per period, the total number of male-female contacts at the start of period t is  $(M_t+1)(W_t+1)$ . This quadratic contact technology has increasing returns.

The quadratic contact technology has played a central role in many microeconomic models of search. In a random search framework, it implies that the rate at which an agent contacts trading opportunities is proportional to the number of agents on the other side of the market (see Howitt and McAfee (1987) for a fuller discussion). But here, this is not the observed matching rate because of sorting and congestion effects. In particular, the matching rate will depend on the flow rate of new arrivals. The next section now generalises this matching process to random entry by new members.

# III. The Dating Agency with Poisson Arrival of New Members

Having derived the basic intuition for the dating agency dynamics, men and women are now assumed to arrive independently of each other, where both arrival processes are Poisson with finite arrival rates m > 0 for men and w > 0 for women. Of course, in an equilibrium matching model, these arrival rates would be functions of other economic variables.

Again consider the time framework described in section I, except this time  $\Delta > 0$  is small, and we shall consider the limit as  $\Delta \Rightarrow 0$ . For  $\Delta$  small, the probability that a new woman arrives at time t is given by w $\Delta$ , and the probability that a new man arrives is m $\Delta$ . Repeating the analysis as before, suppose that at time t,  $M_t = M$  and that a new woman has arrived. Then it follows that

P(new woman immediately matches ) = 1 -  $(1-\lambda)^M$  +  $O(\Delta)$ 

With probability [1-0( $\Delta$ )], no other new members arrive at time t, and the probability that she matches with an old member is  $1-(1-\lambda)^M$  (who accepts her offer as he has no alternatives). With probability  $O(\Delta)$ , at least one more new member arrives at t, whereupon the probability that our original new member matches is bounded between zero and one.

Given this, consider the matching probability of an 'old' man in period t, again with  $M_{\mbox{\scriptsize t}}=M.$  Matching over this interval requires that at least one new woman member arrives at t. Hence :

P(old man matches) =  $[w\Delta] [1-(1-\lambda)^{M}]/M + O(\Delta^{2})$ 

w $\Delta$  + O( $\Delta^2$ ) is the probability that one new woman and no new men arrived at t, whereupon  $[1-(1-\lambda)^M]/M$  is the probability that she matches with the old man in question. The O( $\Delta^2$ ) term captures the matching probability if additional new members arrived at t.

In the limit  $\Delta \Rightarrow 0$ , it follows that there is a strictly positive probability that a new member will match immediately, where :

P(new woman immediately matches) =  $1 - (1-\lambda)^{M}$ 

Conversely, the exit probability of an 'old' member is described by a Poisson process, where the exit rate (hazard) h of an 'old' man is given by

$$h = (w/M)[1-(1-\lambda)^{H}]$$

Notice that this exit rate can be decomposed into two parts. w describes the rate at which an 'old' male member of the club contacts new female

members (which corresponds to a quadratic contact technology assumption). Given a contact has occured,  $[1-(1-\lambda)^M]/M$  is then the probability that she will match with our particular 'old' male member. That this probability is decreasing in M corresponds to the congestion effect described earlier. The two effects together define the hazard rate for an 'old' male member.

Symmetry implies that similar conditions hold for a new man and an old woman. The implication is that a new member has a strictly positive probability of instantaneously matching. If that member is unlucky and does not match, finding a match relies on waiting for the arrival of new members, where the exit rate depends positively on the flow of new opportunities (members of the opposite sex), and negatively on the stock of members of the same sex (the congestion effect). The reduced form expected matching function is then given by

$$H(m, w, M, W) = w [1-(1-\lambda)^{M}] + m [1-(1-\lambda)^{W}]$$

which is symmetric across (m,M) and (w,W). The first term is the expected number of new female members who match immediately, while the second term is the expected flow out of old female members who match with new male members (and similarly for men).

First notice that there is no cross derivative between M and W in H(.). Unlike the standard matching function, the matching rate does not involve any interaction between the two stocks of members. The stock of men is matching with the flow of new women, while the stock of women is matching with the flow of new men. Second, this matching function, while not being homogeneous, does exhibit increasing returns to scale. A market with larger flows will decrease expected waiting times to match and so make members better off through reduced discounting.

#### Summary

This paper has provided a simple microeconomic model of matching which questions the present interpretation placed on recent empirical results. It suggests that when estimating the matching function, it is important to augment the matching function with flows of new vacancies and of newly unemployed workers. As has been argued, omitting this effect can give highly misleading results.

It is also easy to show that this model can have multiple equilibria. In the absence of the dating agency, matching might be described by the random matching model, which gives some expected utility  $V_0$  to being unmatched. Now the dating agency clearly speeds up the matching process if everyone is a member. But if few people are members of the dating agency, the value of joining will also be small (since introductions will be few). If the added value by joining does not cover the per person operating cost of the agency, then the agency is not sustainable and closes down. But of course, if everyone joins, the value of membership will be much higher and the agency might then be sustainable. This suggests why governments might publicly fund job agencies.

#### <u>Footnotes</u>

See Pissarides (1990) for an in-depth survey of how the matching function can be used in simple macroeconomic models. Also see Blanchard and Diamond (1992) for a more recent interpretation of this framework. Pissarides (1979) and Hall (1977) provide a structural model which gives a reduced form matching process of this type with constant returns to scale. There is no random search in these models, in the sense that workers know where the vacancies are. Also, there is no sorting, so that  $\lambda = 1$ , and time is discrete. Workers are restricted to making a single application at a time and must wait to be accepted or rejected before sending off a second application.

- <sup>2</sup> See Beaumont (1978) for data which describes declining hazard rates for vacancies. Alternatively, Ours and Ridder (1992) use vacancy duration data to examine search and selection effects by employers in the Netherlands.
- It is well-known that the exit rate (hazard rate) of an unemployed worker decreases with the duration of unemployment. The aggregate matching function f = f(U,V) must be an incomplete description of the reduced form dynamics (it implicitly assumes all workers and vacancies are identical). One way to explain declining hazard rates is to assume some form of unobserved heterogeneity (see for example Lancaster and Nickell (1980)). Blanchard and Diamond (1990) argue that this heterogeneity may be caused by the depreciation of skills while unemployed. Ceteris paribus, a firm offers the job to the person who has been unemployed for the shorter period.
- <sup>4</sup> Some papers have extended this result to show that increasing returns to matching can support cycles see Boldrin, Kiyotaki and Wright (1991), Howitt and McAfee (1992), Coles (1992) and the related work of Diamond and Fudenberg (1989), Mortensen (1991) who assume increasing returns to aggregate production.
- Recent empirical work on estimating the matching function (or the associated hazard rate) include Pissarides (1986), Blanchard and Diamond (1989), Lindeboom, Ours and Renes (1992), Gorter and Ours (1992), Coles and Smith (1992), who each find evidence for constant returns to scale, while Warren (1992), using a transcendental logarithmic specification for matching, finds increasing returns.
- <sup>6</sup> Following Blanchard and Diamond (1989), R.E. Hall comments that finding constant returns to matching "may reflect the much higher level of specialisation in dense markets. The benefit of better matching is taken in the form of moderate matching rates for highly specialised workers. There must be some reason why so many workers choose to locate in New York given the congestion costs and location rents there."
- In an unemployment/vacancy model, we should model vacancy creation rules. This would unnecessarily complicate the central issue here of understanding labour market sorting effects.
- This can be relaxed by assuming some underlying distribution of idiosynchratic matching payoffs. For example, see Burdett and Wright (1992) who model  $\lambda$  as an equilibrium outcome to a two-sided matching problem with sequential search.
- Since N must be an integer, the suitable definition of strict concavity is that  $(N_2-N_1) H(N) > (N-N_1)H(N_1) + (N_2-N)H(N_2)$  for all  $0 \le N_1 < N < N_2$ .

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