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ABSTRACT

Bridging DSGE Models and the raw data*

A method to estimate DSGE models using the raw data is proposed. The approach links the observables to the model counterparts via a flexible specification which does not require the model-based component to be solely located at business cycle frequencies, allows the non-model-based component to take various time series patterns, and permits model misspecification. Applying standard data transformations induce biases in structural estimates and distortions in the policy conclusions. The proposed approach recovers important model-based features in selected experimental designs. Two widely discussed issues are used to illustrate its practical use.

JEL Classification: C3 and E3

Keywords: business cycles, DSGE models, filters and structural estimation

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1 Introduction

There have been considerable developments in the specification of DSGE models in the last few years. Steps forward have also been made in the estimation of these models. Despite recent efforts, structural estimation of DSGE models is conceptually and practically difficult. For example, classical estimation is asymptotically justified only when the model is the generating process (DGP) of the actual data, up to a set of serially uncorrelated measurement errors, and standard validation exercises are meaningless without such an assumption. Identification problems (see e.g. Canova and Sala, 2009) and numerical difficulties are widespread. Finally, while the majority of the models investigators use is intended to explain only the cyclical portion of observable fluctuations, both permanent and transitory shocks may produce cyclical fluctuations, and macroeconomic data contains many types of fluctuations, and some are hardly cyclical.

The generic mismatch between what models want to explain and what the data contains creates headaches for applied investigators. Over the last 10 years a number of approaches, reflecting different identification assumptions, have been used:

- Fit a model driven by transitory shocks to the observables filtered with an arbitrary statistical device (see Smets and Wouters, 2003, Ireland, 2004a, Rubio and Rabanal, 2005, among others). Such an approach is problematic for at least three reasons. First, since the majority of statistical filters can be represented as a symmetric, two-sided moving average of the raw data, the timing of the information is altered and dynamic responses hard to interpret. Second, while it is typical to filter each real variable separately and to demean nominal variables, there are consistency conditions that must hold - a resource constraint need not be satisfied if each variable is separately filtered - and situations when not all nominal fluctuations are relevant from the point of view of a model. Thus, specification errors can be important. Finally, contamination errors could be present. For example, a Band Pass (BP) filter only roughly captures the power of the spectrum at the frequencies

41 corresponding to cycles with 8-32 quarters average periodicity in small samples and taking
42 growth rates greatly amplifies the high frequency content of the data. In sum, rather than
43 solving the problem, the approach adds to the difficulties applied researchers face.

44 • Fit a model driven by transitory shocks to transformations of the observables which, in
45 theory, are likely to be void of non-cyclical fluctuations, e.g. consider real "great ratios" (as
46 suggested in Cogley, 2001, and McGrattan, 2010) or nominal " great ratios"(as suggested
47 in Whelan, 2005). As Figure 1 shows, such transformations need not resolve the problem
48 because many ratios still display low frequency movements. In addition, since the number
49 and the nature of the shocks driving non-cyclical fluctuations needs to be a-priori known,
50 specification errors may be produced.

51 • Construct a model driven by transitory and permanent shocks; scale the model by the
52 assumed permanent shocks; fit the transformed model to the observables transformed in the
53 same way (see e.g. Del Negro et al., 2006, Fernandez and Rubio, 2007, Justiniano, et al.,
54 2010, among others). Such an approach puts stronger faith in the model than previous ones,
55 explicitly imposes consistency between the theory and the observables, but it is not free of
56 problems. For example, since the choice of which shock is permanent is often driven by
57 computational rather than economic considerations, specification errors could be present. In
58 addition, structural parameter estimates may depend on nuisance features, such as the shock
59 which is assumed to be permanent and its time series characteristics. As Cogley (2001) and
60 Gorodnichenko and Ng (2010), have shown, misspecification of these nuisance features may
61 lead to biased estimates of the structural parameters.

62 • Construct a model driven by transitory and permanent shocks; fit the transformed
63 model to the transformed data in the frequency domain (see e.g. Diebold et. al, 1998, Chris-
64 tiano and Vigfusson, 2003) and select a particular frequency band over which to estimate
65 the structural parameters. This approach is also problematic since it inherits the misspeci-
66 fication problems of the previous approach and the filtering problems of statistically based
67 filtering approaches.

68 This paper provides an alternative method to estimate DSGE models. I show first that
69 the approach one takes to match the model to the data matters for structural parameter
70 estimation and for economic inference. Unless one has a strong view about what the model
71 is supposed to capture and with what type of shocks, it is difficult to credibly select among
72 various structural estimates (see Canova, 1998). In general, any preliminary data transfor-
73 mations (should these be statistical or model-based) should be avoided if the observed data
74 is assumed to be generated by rational agents maximizing under constraints in a stochastic
75 environment. Statistical filtering does not take into account that the data generated by a
76 DSGE model has power at all frequencies and that, if permanent and transitory shocks are
77 present, the permanent and the transitory component of the data will both appear at busi-
78 ness cycle frequencies. Model based transformations impose tight restrictions on the long
79 run properties of the data. Thus, any deviations from the imposed structure, being these
80 residual low frequency variations, unaccounted or idiosyncratic long run dynamics must be
81 captured by the shocks driving the transformed model. Hence, parameter estimates could
82 be distorted because estimates of income and substitution effects could be biased.

83 The paper proposes to estimate structural parameters by creating a flexible link between
84 the DSGE model and the raw data that allows model based and non-model based compo-
85 nents to have power at all frequencies. The methodology can be applied to models featuring
86 transitory or transitory and permanent shocks and only requires that interesting features of
87 the data are left out from the model - these could be low frequency movements of individual
88 series, different long run dynamics of groups of series, etc.. Since the non-model based com-
89 ponent can endogenously capture aspects of the data the model is not designed to explain,
90 researchers need not to take a stand on what is left out from the model, or on its time series
91 representation, and therefore shields the analysis from important specification errors. More-
92 over, because the information present at all frequencies is used in the estimation, filtering
93 distortions are eliminated and inefficiencies minimized. The setup has two other advantages
94 over competitors: structural estimates reflect the uncertainty present in the specification

95 of non-model based features; what the model leaves out at interesting frequencies is easily
96 quantifiable. Thus, R-squared type measures can be built to "test" the structure and to
97 evaluate the explanatory power of additional shocks.

98 The approach is related to work by Del Negro et al. (2006), in that certain cross equation
99 restrictions that the DGP may impose on the data are not used in estimation, and to the
100 work of Ireland (2004b), in that a non-structural part is added to a structural model prior to
101 estimation and, crucially, it does not substitute for theoretical efforts designed to strengthen
102 the ability of DSGE models to account for all observable fluctuations. But it can fill the gap
103 between what is nowadays available and such a worthy long run aspiration, giving researchers
104 a rigorous tool to address policy questions.

105 Using a simple experimental design and two practically relevant cases, the paper doc-
106 uments the biases that standard transformations produce, interprets them using the tools
107 developed in Hansen and Sargent (1993), and shows that crucial parameters are better esti-
108 mated with the proposed procedure. To highlight how the approach can be used in practice,
109 the paper finally examines two questions greatly discussed in macroeconomics: the time vari-
110 ations in the policy activism parameter and the sources of output and inflation fluctuations.

111 To focus attention on the issues of interest, two simplifying assumptions are made: (i) the
112 estimated DSGE model features no missing variables or omitted shocks and (ii) the number
113 of structural shocks equals the number of endogenous variables. While omitted variables
114 and singularity issues are important in practice, and the semi-structural methods suggested
115 in Canova and Paustian (2011) produce more robust inference when they are present, it
116 is useful to sidestep them because the problems discussed here occur regardless of whether
117 (i)-(ii) are present or not ¹.

118 The rest of the paper is organized as follows. The next section presents estimates of
119 the structural parameters of a simple model when number of statistical and model based

¹As a referee has pointed out the approach can be used to estimate singular structural models as long as the non-model based component has the same rank as the dimension of the observable variables. Such an extension is not pursued here.

transformations are employed. Section 3 discusses the alternative methodology. Section 4 compares approaches using a simple experimental design. Section 5 examines two economic questions. Section 6 concludes.

2 Estimation with transformed data

To show how estimates of the structural parameters of a DSGE model depend on the preliminary transformation employed, this section considers a textbook small scale New-Keynesian model, where agents face a labor-leisure choice, production is stochastic and requires labour, there is external habit in consumption, an exogenous probability of price adjustments, and monetary policy is conducted with a conventional Taylor rule. Details on the structure are in the on-line appendix.

The model features a technology disturbance z_t , a preference disturbance χ_t , a monetary policy disturbances ϵ_t , and a markup disturbance μ_t . The latter two shocks are assumed to be iid. Depending on the specification z_t, χ are either both transitory, with persistence ρ_z and ρ_χ respectively, or one of them is permanent. The structural parameters to be estimated are: σ_c , the risk aversion coefficient, σ_n , the inverse of the Frisch elasticity, h the coefficient of consumption habit, $1 - \alpha$, the share of labor in production, ρ_r , the degree of interest rate smoothing, ρ_π and ρ_y , the parameters of the monetary policy rule, $1 - \zeta_p$, the probability of changing prices. The auxiliary parameters to be estimated are: ρ_χ, ρ_z , the autoregressive parameters of transitory preference and technology shocks, and $\sigma_z, \sigma_\chi, \sigma_r, \sigma_\mu$ the standard deviations of the four structural shocks. The discount factor β and the elasticity among varieties θ are not estimated since they are very weakly identified from the data.

Depending on the properties of the technology and of the preference shocks, the optimality conditions will have a log-linear representation around the steady state or a growth path, driven either by the technology or by the preference shock, see table 1. Four observable variables are used in the estimation. When the model features transitory shocks, parameter estimates are obtained applying four statistical filters (linear detrending (LT), Hodrick and

146 Prescott filtering (HP), growth rate filtering (FOD) and band pass filtering (BP)) to output,
 147 the real wage, the nominal interest rate and inflation. Moreover, three data transformations
 148 are employed. In the first, the log of labour productivity, the log of real wages, the nominal
 149 rate and the inflation rate, all demeaned, are used as observables (Ratio 1). In the second
 150 the log ratio of output to the real wage, the log of hours worked, the nominal rate and the
 151 inflation rate, all demeaned, are used as observables (Ratio 2). In the third, the log of the
 152 labor share, the log ratio of real wages to output, the nominal interest rate and the inflation
 153 rate all demeaned, are used as observables (Ratio 3). When the model features a trending
 154 TFP (TFP trend), the linear stochastic specification $z_t = bt + \epsilon_t^z$, is used and the observables
 155 for the transformed model are linearly detrended output, linearly detrended wages, demeaned
 156 inflation and demeaned interest rates. When the model features trending preferences shocks
 157 (Preference trend), the unit root specification, $\chi_t = \chi_{t-1} + \epsilon_t^\chi$ is employed and the observables
 158 for the transformed model are the demeaned growth rate of output, demeaned log of real
 159 wages, demeaned inflation and demeaned interest rates. Finally, when the model feature
 160 a trending TFP, the likelihood function of the transformed model is approximated as in
 161 Hansen and Sargent (1993) and only the information at business cycle frequencies ($\frac{\pi}{32}, \frac{\pi}{8}$) is
 162 used in the estimation (TFP trend, frequency domain).

163 The data comes from the FRED database at the Federal Reserve Bank of St. Louis
 164 and Bayesian estimation is employed. Since some of the statistical filters are two-sided, a
 165 recursive LT filter and a one-sided version of the HP filter have also been considered. The
 166 qualitative features of the results are unchanged by this refinement.

167 Table 2 shows that the posterior distribution of several parameters depend on the prelim-
 168 inary transformation used (see e.g. the risk aversion coefficient σ_c , the Frisch elasticity σ_n^{-1} ,
 169 the interest smoothing coefficient ρ_r , and persistence and the volatility of the shocks). Since
 170 posterior standard deviations are tight, except when estimation is conducted in frequency
 171 domain, differences across columns are a-posteriori significant. Posterior differences are also
 172 economically relevant. For example, the volatility of markup shocks in the LT, the Ratio

173 1 and the Preference trend economies is considerably larger and, perhaps unsurprisingly,
174 risk aversion stronger. In addition, when a frequency domain approach is used, the Frisch
175 elasticity is estimated to be very small.

176 Differences in the location of the posterior of the parameters translate into important
177 differences in the transmission of shocks. As shown in Figure 2, the magnitude of the impact
178 coefficient and of the persistence of the responses vary with the preliminary transformation
179 employed and, for the first few horizons, differences are statistically significant. Furthermore,
180 in the case of technology shocks, the sign of some of the responses is affected.

181 Why are parameter estimates so different? The first four transformations only approx-
182 imately isolate business cycle frequencies, leaving measurement errors in the transformed
183 data. In addition, different approaches spread the measurement error across different fre-
184 quencies: the LT transformation leaves both long and short cycles in the filtered data; the
185 HP transformation leaves high frequencies variability unchanged; the FOD transformation
186 emphasizes high frequency fluctuations and reduces the importance of cycles with business
187 cycle periodicity; and even a BP transformation induces significant small sample approxima-
188 tion errors (see e.g. Canova, 2007). Since the magnitude of the measurement error and its
189 frequency location is transformation dependent, differences in parameter estimates are likely
190 to be important. An approach which can reduce the problematic part of the measurement
191 error is in Canova and Ferroni (2011). More importantly, filtering approaches neglect the
192 fact that the spectral properties of a DSGE model are different from the output of a sta-
193 tistical filter. Data generated by a DSGE model driven by transitory shocks has power at
194 all frequencies of the spectrum and if shocks are persistent most of the power will be in the
195 low frequencies. Thus, concentrating on business cycles frequencies may lead to inefficien-
196 cies. Furthermore, when transitory and permanent shocks are present, the transitory and
197 the permanent components of the model will jointly appear in any frequency band and it
198 is not difficult to build examples where, e.g. permanent shocks dominate the variability at
199 business cycle frequencies (see Aguiar and Gopinath, 2007). Hence, the association between

200 the solution of the model and the filtered observables is generally incorrect and biases likely
201 to be generalized.

202 Implicit or explicit model-based transformations avoid these problems by specifying a
203 permanent and a transitory component of the data with power at all frequencies of the spec-
204 trum. However, since specification problems are present (should we use a unit root process
205 or a trend stationary process? Should we allow trending preferences or trending technol-
206 ogy?), particular choices lead to nuisance parameters problems (the model estimated with a
207 trending TFP has MA components which do not appear when the preferences are trending,
208 see table 1), and to particular cointegration relationships in the observables, inference de-
209 pends on the assumptions made and any deviation of the observed data from the assumed
210 structure leads to biases. Finally, frequency domain estimation is inefficient, since most of
211 the variability the model produces is in the low frequencies. Furthermore, while frequency
212 estimation can help to tone down the importance of aspects of the model researchers do not
213 trust, as suggested in Hansen and Sargent (1993), it can not de-emphasize the importance
214 of what the model leaves out at the frequencies of interest.

215 **3 The alternative methodology**

216 Start from the assumption that the observable data has been generated by rational expect-
217 ation agents, optimizing their objective functions under constraints in a stochastic environ-
218 ment. Assume that the econometrician knows the data generating process for a portion of
219 the data but she is unsure about the transmission produced by certain shocks (e.g. those in-
220 ducing permanent effects) or how to capture aspects of the data (e.g. those with medium-long
221 period of oscillation). Thus, she is aware that the model used for inference is misspecified.
222 Rather than trying to filter out from the data what the model is unsuited to explain or add
223 ad-hoc features to the model to reduce the misspecification, I will assume that the investi-
224 gator takes the misspecified structure as given, because it is unclear how to model all the
225 fluctuations present in the data or because the available short cuts are unlikely to satisfac-

226 torily account for its complexity. To estimate the parameters of the model she uses the raw
 227 data and disregards certain cross equations restrictions present in the DGP but builds a
 228 link between the misspecified structural model and the raw data which is sufficiently flexible
 229 to capture what the model is unsuited to explain and allows model and non-model based
 230 components to jointly appear at all frequencies of the spectrum.

231 Let the (log)-linearized stationary solution of a DSGE model be of the form:

$$x_{2t} = A(\theta)x_{1t-1} + B(\theta)\epsilon_t \quad (1)$$

$$x_{1t} = C(\theta)x_{1t-1} + D(\theta)\epsilon_t \quad (2)$$

232 where $A(\theta), B(\theta), C(\theta), D(\theta)$ depend on the structural parameters θ , $x_{1t} \equiv (\log \tilde{x}_{1t} - \log \bar{x}_{1t})$
 233 includes exogenous and endogenous states, $x_{2t} = (\log \tilde{x}_{2t} - \log \bar{x}_{2t})$ all other endogenous
 234 variables, ϵ_t the shocks and $\bar{x}_{2t}, \bar{x}_{1t}$ are the long run paths of \tilde{x}_{2t} and \tilde{x}_{1t} .

235 Let $x_t^m(\theta) = R[x_{1t}, x_{2t}]'$ be an $N \times 1$ vector, where R is a selection matrix picking out
 236 of x_{1t} and x_{2t} variables which are observable and/or interesting from the point of view of
 237 the researcher and let $\bar{x}_t^m(\theta) = R[\bar{x}_{1t}, \bar{x}_{2t}]'$. Let $x_t^d = \log \tilde{x}_t^d - E(\log \tilde{x}_t^d)$ be the log demeaned
 238 $N \times 1$ vector of observable data. The specification for the raw data is then:

$$x_t^d = c_t(\theta) + x_t^{nm} + x_t^m(\theta) + u_t \quad (3)$$

239 where $c_t(\theta) = \log \bar{x}_t^m(\theta) - E(\log \tilde{x}_t^d)$, u_t is a iid $(0, \Sigma_u)$ (proxy) noise, x_t^{nc}, x_t^m and u_t are
 240 mutually orthogonal and x_t^{nm} is given by:

$$\begin{aligned} x_t^{nm} &= \rho_1 x_{t-1}^{nm} + w_{t-1} + e_t & e_t &\sim iid(0, \Sigma_e) \\ w_t &= \rho_2 w_{t-1} + v_t & v_t &\sim iid(0, \Sigma_v) \end{aligned} \quad (4)$$

241 where $\rho_1 = diag(\rho_{11}, \dots, \rho_{1N}), \rho_2 = diag(\rho_{21}, \dots, \rho_{2N}), 0 < \rho_{1i}, \rho_{2i} \leq 1, i = 1, \dots, N$. To under-
 242 stand what the specification for x_t^{nm} implies, notice that when $\rho_1 = \rho_2 = I$, and e_t, v_t are
 243 uncorrelated (4) is the locally linear trend specification used in state space models, see e.g.
 244 Gomez (1999). In addition, if $\rho_1 = \rho_2 = I, \Sigma_e$ and Σ_v are diagonal, $\Sigma_{v_i} = 0$, and $\Sigma_{e_i} > 0, \forall i$,

245 x_t^{nm} is a vector of I(1) processes while if $\Sigma_{v_i} = \Sigma_{e_i} = 0$, $\forall i$, x_t^{nm} is deterministic. When
 246 instead $\rho_1 = \rho_2 = I$, and Σ_{v_i} and Σ_{e_i} are functions of Σ_ϵ , (4) approximates the double ex-
 247 ponential smoothing setup used in discounted least square estimation of state space models,
 248 see e.g. Delle Monache and Harvey (2010). Thus, if $\bar{x}_t^m(\theta) = \bar{x}^m(\theta)$, $\forall t$, the observable x_t^d can
 249 display any of the typical structures that motivates the use of the statistical filters. Further-
 250 more, as Delle Monache and Harvey (2010) have emphasized, (4) is robust against several
 251 types of misspecification of the time series properties of what the model does not explain.
 252 Note also, whenever Σ_v is not constrained to be zero, the growth rates of the endogenous
 253 variables may display persistent deviations from their mean, a feature that characterizes
 254 many real macroeconomic variables, see e.g. Ireland (2010). Finally, when $\bar{x}_t^m(\theta)$ is not
 255 constant, and ρ_{1i} and ρ_{2i} are complex conjugates for some i , the specification can capture
 256 residual low frequency variations with power at frequency ω . To see this notice that when
 257 $N=1$, (4) implies that $(1 - \rho_2 L)(1 - \rho_1 L)x_t^{nm} = (1 - \rho_2 L)e_t + v_{t-1} \equiv (1 - \psi L)\eta_t$. If the roots
 258 $\lambda_1^{-1}, \lambda_2^{-1}$ of the polynomial $1 - (\rho_1 + \rho_2)z + \rho_1\rho_2z^2 = 0$ are complex, they can be written as
 259 $\lambda_1^{-1} = r(\cos \omega + i \sin \omega)$, $\lambda_2^{-1} = r(\cos \omega - i \sin \omega)$, where $r = \sqrt{\rho_1\rho_2}$ and $\omega = \cos^{-1}[\frac{\rho_1 + \rho_2}{2\sqrt{\rho_1\rho_2}}]$ and
 260 (4) is $x_t^{nm} = \sum_j r \frac{\sin \omega(j+1)}{\sin \omega} (1 - \psi L)\eta_t$, whose period of oscillation is $p = \frac{2\pi}{\omega} = \frac{2\pi}{\cos^{-1}[\frac{\rho_1 + \rho_2}{2\sqrt{\rho_1\rho_2}}]}$.
 261 Thus, given r and p , there exists ρ_1, ρ_2 that produce x_t^{nm} with the required properties.

262 Given (1)-(4), the data will endogenously select the specification for the non-model based
 263 component which is more appropriate for each series and this will be done jointly with the
 264 estimation of the structural parameters θ . Identification of the structural parameters is
 265 achieved via the cross equation restrictions that the model imposes on the data. Estimates
 266 of the non-structural parameters are implicitly obtained from the portion of the data the
 267 model can not explain.

268 The specification has a number of advantages over existing approaches. One does not
 269 need to take a stand on the time series properties of the non-model based component and on
 270 the choice of filter to tone down its importance and this shields researchers from important
 271 specification and filtering errors. As shown in Ferroni (2011), the setup can be used to find

272 the most appropriate specification of the non-model based component and, if a researcher
 273 is interested in doing so, to perform Bayesian averaging over different types of non-model
 274 based specifications, which is not possible in standard setups. Furthermore, as shown below,
 275 all components in (3) may have power at all frequency. Finally, since joint estimation is
 276 performed, structural parameter estimates reflect the uncertainty present in the specification
 277 of the non-model based component.

278 3.1 Two special cases

279 It is useful to consider two special cases of the setup to give a sense of what the approach
 280 does. Suppose first that the model features only transitory shocks while the data may
 281 display common or idiosyncratic long run drifts, low frequency movements and business
 282 cycle fluctuations. Here $\bar{x}_t^m(\theta) = \bar{x}^m(\theta), \forall t$, are the steady states of the model and, if the
 283 model is correctly specified on average, $c_t(\theta) = 0$. Assume that no proxy errors are present.
 284 Then (3) is

$$x_t^d = x_t^{nm} + x_t^m(\theta) \quad (5)$$

285 and x_t^{nm} captures the features of x_t^d that the stationary model does not explain. Depending
 286 on the specification of ρ_1 and ρ_2 , these include long run drifts, both of common and idio-
 287 syncratic types, and those idiosyncratic low and business cycle movements the model leaves
 288 unexplained. In this setup, x_t^{nm} has two interpretations. As in Altug (1989), McGrattan
 289 (1994) and Ireland (2004b), it can be thought of as a measurement error added to the struc-
 290 tural model. However, rather than being iid or AR(1), it has the richer representation (4).
 291 Alternatively, it can be thought as a reduced form representation for the components of the
 292 data the investigator is unsure how to model. Thus, as in Del Negro et al. (2006), x_t^{nm}
 293 relaxes the cross equations restrictions that the DGP implies and captures what the model
 294 can not explain via the flexible parameterization (4).

295 Suppose, alternatively, that the model features transitory shocks and one or more per-
 296 manent shocks. In this case $x_t^m(\theta)$ represents the (stationary) solution in deviation from

297 the permanent shocks and $\bar{x}_t^m(\theta)$ the model based component generated by the permanent
 298 shocks. Suppose again that there are no proxy errors. In that case (3) reduces to

$$x_t^d = c_t(\theta) + x_t^{*,nm} + x_t^m(\theta) \quad (6)$$

299 where $x_t^{*,nm}$ captures the features of x_t^d which neither the transitory portion $x_t^m(\theta)$ nor the
 300 permanent portion $c_t(\theta)$ of the model explains. These may include, idiosyncratic long run
 301 patterns (such as diverging trends), idiosyncratic low frequency movements, or unaccounted
 302 cyclical fluctuations. Comparing (5) and (6), one can see that $x_t^{nm} = c_t(\theta) + x_t^{*,nm}$. Thus,
 303 the setup can be used to measure how much of the data the model leaves unexplained and
 304 to evaluate whether certain shocks may reduce the discrepancy. For example, one could
 305 start from a model featuring a few transitory shocks and measure the relative importance
 306 of x_t^{nm} at a particular set of frequencies. If it is large, one could add additional transitory
 307 shocks and see how much the relative importance of x_t^{nm} has fallen at those frequencies.
 308 Alternatively, one could add a permanent shock and compare the magnitude of $x_t^{*,nm}$ and
 309 x_t^{nm} at a particular set of frequencies. By comparing the outcomes of the two exercises, one
 310 can also assess whether the addition of a permanent or a transitory shock is more beneficial.

311 The same logic can be used to evaluate the model when, e.g. the permanent shock takes
 312 the form of a stochastic deterministic trend (as in the case of labor augmenting technological
 313 progress), when it is represented with a unit root, or when all long run paths are left unmod-
 314 elled. Hence, the approach naturally provides a setup to judge the goodness of fit of a model
 315 and to evaluate the contribution of certain features to the understanding of economic phe-
 316 nomena. It does so by giving researchers a constructive criteria to increase the complexity
 317 of models; and an integrated framework to examine the sensitivity of the estimation results
 318 to the specification of nuisance features, both of which are absent from existing methods.

3.2 Estimation

Estimation of the structural parameters can be carried out with both classical and Bayesian methods. (1)-(4) can be cast into the linear state space system:

$$s_{t+1} = F s_t + G \omega_{t+1} \quad \omega_t \sim (0, \Sigma_\omega) \quad (7)$$

$$x_t^d = c_t(\theta) + H s_t \quad (8)$$

where $s_t = (x_t^{nm} \quad w_t \quad x_t^m(\theta) \quad u_t)'$, $\omega_{t+1} = (e_{t+1}, v_{t+1}, u_{t+1}, \epsilon_{t+1})'$, $H = (I \quad 0 \quad I \quad I)$,
 $F = \begin{pmatrix} \rho_1 & I & 0 & 0 \\ 0 & \rho_2 & 0 & 0 \\ 0 & 0 & R[A \ C] & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $G = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & R[B \ D]' \\ 0 & 0 & I & 0 \end{pmatrix}$. Hence, the likelihood can
be computed with a modified Kalman filter (accounting for the possibility of diffuse initial observations) for a given $\vartheta = (\theta, \rho_1, \rho_2, \Sigma_e, \Sigma_v, \Sigma_u)$ and maximized using standard tools.

When a Bayesian approach is preferred, one can obtain the non-normalized posterior of ϑ , using standard MCMC tools. For example, the estimates presented in this paper are obtained with a Metropolis algorithm where, given initial ϑ_{-1} and a prior $g(\vartheta)$, candidate draws are obtained from $\vartheta_* = \vartheta_{-1} + v$, where v is distributed $t(0, \kappa * \Omega, 5)$ and κ is a tuning parameter, and the draw accepted if the ratio $\frac{\check{g}(\vartheta_*|y)}{\check{g}(\vartheta_{-1}|y)}$ exceeds a uniform random variable, where $\check{g}(\vartheta_i|y) = g(\vartheta_i) \mathcal{L}(y|\vartheta_i)$, $i = *, -1$, and $\mathcal{L}(y|\vartheta_i)$ is the likelihood of ϑ_i . Iterated a large number of times, for κ appropriately chosen, the algorithm ensures that the limiting distribution of ϑ is the target distribution (see e.g. Canova, 2007).

3.3 The relationship with the existing literature

Apart from the work of Del Negro et al. (2006) and of Altug (1989), McGrattan (1994) and Ireland (2004b) already mentioned, the procedure is related to a number of existing works.

First, the state space setup (7)-(8) is similar in spirit to the one suggested by Harvey and Jeager (1993), even though these authors consider only univariate processes and do not use a structural model to explain the observables. It also shares important similarities with the one employed by Cayen et al. (2009), who are interested in forecasting trends. Two

341 are the most noticeable differences. First, these authors use a two-step estimation approach,
342 conditioning on filtered estimates of the parameters of the DSGE model, while here a one
343 step approach is employed. Second, all the deviations from the model are bundled up in the
344 non-model specification while here it is possible to split them into model interpretable and
345 model non-interpretable parts.

346 The contribution of the paper is also related to two distinct branches of the macroeco-
347 nomic and macroeconometric literature. The first attempts to robustify inference when the
348 trend properties of data are misspecified (see Cogley, 2001, and Gorodnichenko and Ng,
349 2010). I share with the first author the idea that economic theory may not have much to say
350 about certain types of fluctuations but rather than distinguishing between trend stationary
351 and difference stationary cycles, the paper wants to design an estimation procedure which
352 deals with the mismatch between theoretical and empirical concepts of fluctuations without
353 taking a stand on the time series properties of what the model leaves unexplained. The idea
354 of jointly estimating structural and auxiliary parameters without fully specifying the DGP
355 is also present in Gorodnichenko and Ng. However, a likelihood based estimator, as opposed
356 to a minimum distance estimator, is used here because it works regardless of the time series
357 properties of the raw data. In addition, rather than assuming that the model is the DGP, the
358 procedure assumes that the DSGE model is misspecified - a much more useful assumption
359 in practice.

360 The second branch points out that variations in trend growth are as important as cyclical
361 fluctuations in explaining the dynamics of macroeconomic variables in emerging markets
362 (see e.g. Aguiar and Gopinath, 2007, and Andrlé, 2008). While the first paper characterizes
363 differences between emerging and developing economies, the latter is concerned with the
364 misuse of models driven by transitory shocks in policy analyses for developing countries.
365 This paper shows that the problems they highlight are generic and that policy analyses with
366 misspecified models are possible without imposing controversial assumptions about what the
367 model is not designed to explain.

3.4 Setting the priors for Σ_e and Σ_v

If the number of observable variables is small and the number of data points large, one can easily obtain estimates of θ from(7)-(8). If the number of observables is large or the sample size limited, weak identification problems and small sample biases may become relevant. Note, in fact, that in (4) there are $2N + 2N^2$ non-structural parameters to be estimated and that it may be difficult to distinguish variations in the level from variations in the growth rates of the variables. Thus, it may be worth to impose some structure on Σ_v and Σ_e , if information about what the model leaves out is available, and shrewdly cut down on the dimensionality of the non-structural parameter space. For example, one may want to assume that Σ_v and Σ_e are diagonal (so that the non-model based component is series specific), and of reduced rank (the non-model based component is common across (groups of) series); that they have only sparse non-zero elements on the diagonal (the non-model based component exists only in a number of observables) or that they are proportional to each other (shocks to the level and the growth rate are related). Some a-priori restrictions appear to be necessary also because given a DSGE structure, the decomposition of the data in model based and non-model based components depends on the strength of the shock signals. Thus, the procedure defines a family of decompositions, indexed by the relative intensity of the shocks driving the model and the non-model based components. Given that it is typically difficult to estimate this intensity parameter unrestrictedly in small samples, and that unrestricted estimates may imply non-model based components with undesirable high frequency variability, a sensible smoothness prior for Σ_e and Σ_v is needed.

The restrictions which we recommend to be used, and are employed in the two applications described below, involves making Σ_e and Σ_v diagonal, of reduced rank, sparse, and function of the structural shocks. As mentioned, it is possible to approximate the double exponential smoothing restrictions used in discounted least square estimation of state space models by selecting e.g. $\Sigma_{e_i} = \sqrt{\frac{\sigma_\epsilon^2}{\lambda}}$ and $\Sigma_{v_i} = \sqrt{\frac{\sigma_\epsilon^2}{(4\lambda)^2}}$, where i indicates the non-zero elements of the matrices, ϵ_t is one of structural shocks and λ a smoothing parameter. Thus,

395 given a prior for ϵ_t and λ , a prior for all non-zero elements of Σ_e and Σ_v is automatically gen-
 396 erated. The specification is attractive because it is parsimonious and considerably reduces
 397 the number of non-structural parameters to be estimated. Since λ has the same interpreta-
 398 tion as in the HP filter, an agnostic prior for λ could be centered at 400 with uniform range
 399 over [4,6400], which allows for very smooth as well as relatively jagged non-model based com-
 400 ponents². When the likelihood for this parameter is flat, one could alternatively calibrate λ
 401 to different values and, in models driven by transitory shocks, eliminate candidates produc-
 402 ing non-model based estimates which are not sufficiently smooth. Since one of the structural
 403 shocks needs be selected to form the prior for Σ_e and Σ_v , one could also experiment choosing
 404 the disturbance with, potentially, the largest or the smallest variance to calibrate the prior.
 405 For the applications in section 5, which structural disturbance is employed to calibrate the
 406 prior is irrelevant.

407 In sum, the approach is easy to implement - it requires only a few additional lines in
 408 an existing computer code, requires some ingenuity to decrease the dimensionality of the
 409 parameter space when the sample is small, but it is otherwise fully operational in practice
 410 and, as shown below, it has nice properties in a simple experimental design.

411 4 The procedure in a controlled experiment

412 To examine the properties of the procedure and to compare them to those of standard
 413 transformations, I use the same setup employed in section 2 and simulate 150 data points
 414 assuming that the preference shock has a transitory and a permanent component. Thus,
 415 $\chi_t = \chi_{1t} + \chi_{2t}$, $\chi_{1t} = \rho_\chi \chi_{1t-1} + \epsilon_t^{X^T}$ and $\chi_{2t} = \chi_{2t-1} + \epsilon_t^{X^P}$. This specification is chosen since
 416 Chang et al. (2007) have indicated that a model with permanent preference shocks can
 417 capture well low frequency variations in hours worked. In this setup, the data will display
 418 stationary fluctuations driven by four transitory shocks (which we correctly capture with

²It is worth noting that selecting the signal to noise ratio λ is much less demanding than assuming a particular format for the drifts the data displays or selecting a shock which drives them.

419 a model) and important non-stationary fluctuations driven the permanent preference shock
420 (which we will either try to filter out, eliminate with certain data transformations, or account
421 with a non-model based component) making the design relevant for practical purposes. The
422 estimated model is misspecified relative to the DGP in that the permanent component due
423 to the preference shock is left out, but all the other features are correctly represented.
424 Furthermore, since the permanent component of the preference shock is orthogonal to all
425 transitory shocks, the design fits the setup of section 3.

426 The structural parameters will be estimated using the proposed approach and the same
427 transformations used of section 2 in the most ideal situations one could consider - these
428 include priors centered at the true parameter vector and initial conditions equal to the true
429 parameter vector. When the approach of section 3 is used, the non-model based component
430 is restricted to have a double exponential smoothing format and, consistently with the DGP
431 (see appendix) is allowed to enter only in output and the real wage. The true values of
432 the structural parameters are in table 3. In the estimation the same prior distributions
433 for the structural parameters displayed in table 2 are used. Two cases are examined: one
434 where the permanent disturbance has relative high variability $\sigma_\chi^p = 1.50$ and one where it
435 has relative low variability $\sigma_\chi^p = 0.15$. In the first case, the contribution of the permanent
436 component to the spectrum of the series is of the same order of magnitude as the contribution
437 of the transitory component at almost all frequencies. Thus, both filtering and specification
438 errors are present with standard transformations. In the second case, the contribution of
439 the permanent component to the spectrum of the series is everywhere small. Here, standard
440 transformations will only produce filtering errors and, in a large sample, the BP filter provides
441 a consistent although inefficient estimator of model based fluctuations.

442 As table 3 shows, the distortions produced by standard approaches are important. Apart
443 from producing estimates of utility and technology parameters which are biased and very
444 much filter dependent, the persistence of the preference and of the technology shocks ρ_χ, ρ_z
445 and the standard deviations of the preference and the markup shocks σ_χ and σ_μ are gen-

446 erally distorted. In comparison, estimates of utility and technology parameters reported in
 447 the column labelled "Flexible" are closer (in a MSE sense) to the true values and both the
 448 persistence and the standard deviations of the shocks are better captured. Matching the
 449 persistence and the volatility of the shocks is important since conditional and unconditional
 450 moments crucially depend on these parameters. Note also that while with standard transfor-
 451 mations, estimates depend on the relative intensity of the permanent and transitory signals,
 452 this is much less the case for the procedure this paper suggests.

453 To understand the nature of the distortions produced by standard transformations,
 454 note that the log-likelihood of the data can be represented as $L(\theta|y_t) = [A_1(\theta) + A_2(\theta) +$
 455 $A_3(\theta)|y]$, see Hansen and Sargent (1993), where $A_1(\theta) = \frac{1}{\pi} \sum_{\omega_j} \log \det G_\theta(\omega_j)$, $A_2(\theta) =$
 456 $\frac{1}{\pi} \sum_{\omega_j} \text{trace} [G_\theta(\omega_j)^{-1} F(\omega_j)]$, $A_3(\theta) = (E(y) - \mu(\theta))G_\theta(\omega_0)^{-1}(E(y) - \mu(\theta))$, $\omega_j = \frac{\pi j}{T}$, $j =$
 457 $0, 1, \dots, T-1$. $G_\theta(\omega_j)$ is the model based spectral density matrix of y_t , $\mu(\theta)$ the model based
 458 mean of y_t , $F(\omega_j)$ is the data based spectral density and $E(y)$ the unconditional mean of y_t .
 459 $A_2(\theta)$ and $A_3(\theta)$ are penalty functions: $A_2(\theta)$ sums deviations of the model-based from the
 460 data-based spectral density over frequencies; $A_3(\theta)$ weights deviations of model-based from
 461 data-based means with the spectral density matrix of the model at frequency zero.

462 Suppose the data is transformed so that the zero frequency is eliminated and the low
 463 frequencies de-emphasized. Then, the log-likelihood consists of $A_1(\theta)$ and of $A_2(\theta)^* =$
 464 $\frac{1}{\pi} \sum_{\omega_j} \text{trace} [G_\theta(\omega_j)]^{-1} F(\omega_j)^*$, where $F(\omega_j)^* = F(\omega_j)I_{\omega_j}$ and I_{ω_j} is a function describing
 465 the effect of the filter at frequency ω_j . Suppose that $I_\omega = I_{[\omega_1, \omega_2]}$, i.e. an indicator function
 466 for the business cycle frequencies, as in an ideal BP filter. Then $A_2(\theta)^*$ matters only at
 467 business cycle frequencies. Since at these frequencies $[G_\theta(\omega_j)] < F(\omega_j)^*$, $A_2(\theta)^*$ and $A_1(\theta)$
 468 enter additively $L(\theta|y_t)$, two types of biases will be present. Since estimates $\hat{F}(\omega_j)^*$ only
 469 approximately capture the features of $F(\omega_j)^*$, $\hat{A}_2(\theta)^*$ has smaller values at business cycle
 470 frequencies and a nonzero value at non-business cycle ones. Moreover, in order to reduce the
 471 contribution of the penalty function to the log-likelihood, parameters are adjusted so that
 472 $[G_\theta(\omega_j)]$ is close to $\hat{F}(\omega_j)^*$ at those frequencies where $\hat{F}(\omega_j)^*$ is not zero. This is done by

473 allowing fitting errors, (a larger $A_1(\theta)$), at frequencies where $\hat{F}(\omega_j)^*$ is zero - in particular,
 474 the low frequencies. Hence, the volatility of the structural shocks will be overestimated (this
 475 makes $G_\theta(\omega_j)$ close to $\hat{F}(\omega_j)^*$ at the relevant frequencies), in exchange for misspecifying
 476 their persistence. These distortions affect agents' decision rules. Higher perceived volatility,
 477 for example, implies distortions in the risk aversion coefficient. Inappropriate persistence
 478 estimates, on the other hand, imply that perceived substitution and income effects are dis-
 479 torted with the latter typically underestimated. When I_ω is not the indicator function, the
 480 derivation of the size and the direction of the distortions is more complicated but the same
 481 logic applies. Clearly, different I_ω produce different $\hat{F}(\omega_j)$ and thus different distortions.

482 Since estimates of $F(\omega_j)^*$ are imprecise, even for large T , there are only two situations
 483 when estimation biases are small. First, the permanent component has low power at business
 484 cycle frequencies - in this case, the distortions induced by the penalty function are limited.
 485 This occurs when transitory volatility dominates (as in the second panel of table 3). Second,
 486 when Bayesian estimation is performed, the prior is selected to limit the distortions induced
 487 by the penalty function. This is very unlikely, however, since priors are not elicited with
 488 such a scope in mind.

489 If instead one fits a transformed version of the model to transformed data, as it is done
 490 in model based approaches, the log-likelihood is composed of $A_1(\theta)^* = \frac{1}{\pi} \sum_{\omega_j} \log |G_\theta(\omega_j) I_{\omega_j}|$
 491 and $A_2(\theta)$ - since the actual and model data are filtered in the same way, the filter does not
 492 affect the penalty function. Suppose that $I_\omega = I_{[\omega_1, \omega_2]}$. Then $A_1(\theta)^*$ matters only at business
 493 cycle frequencies while the penalty function is present at all frequencies. Therefore, parame-
 494 ter estimates are adjusted so as to reduce the misspecification at all frequencies. Since the
 495 penalty function is generally more important at the low frequencies, parameters are selected
 496 to make $[G_\theta(\omega_j)]$ close to $\hat{F}(\omega_j)$ at those frequencies and large fitting errors are permitted
 497 at medium and high frequencies. Consequently, the volatility of the shocks will be generally
 498 underestimated in exchange for overestimating their persistence - somewhat paradoxically,
 499 this procedure implies that the low frequency components of the data are those that matter

500 most for estimation. Cross frequency distortions imply that the econometrician recovers
501 an economy which differs substantially from the true one. For example, since less noise is
502 perceived, agents decision rules imply a higher degree of data predictability, and higher per-
503 ceived persistence implies that perceived substitution and income effects are distorted with
504 the latter overestimated.

505 To further highlight the properties of the proposed approach, the top row of figure 3
506 reports estimates of the permanent and transitory components of output obtained with the
507 Kalman filter and either the true parameters or the median estimates presented in the top
508 panel of table 3. The bottom two rows of figure 3 compare the autocorrelation function and
509 the spectral density of the true and estimated components of output.

510 The true and the estimated components of output display similar volatility properties.
511 In addition, the rate of decay of the autocorrelation functions of the true and the estimated
512 components is practically identical. Finally, as anticipated, the two estimated components
513 have power at all frequencies of the spectrum, and at business cycle frequencies (indicated
514 by the vertical bars in the last row of graphs) the permanent component is more important
515 than the transitory component.

516 The conditional dynamics in response to transitory shocks are also well captured. Figure
517 4, which presents impulse responses obtained with true and estimated parameters, indicates
518 that the sign and the persistence of the responses are well matched. Magnitudes are occa-
519 sionally imprecisely estimated - this problem would remain even if we double the sample size
520 but overall, the approach does a good job in reproducing the main qualitative features of
521 the DGP. Thus, economic inference is less prone to "mismatch" distortions.

522 **5 Two applications**

523 This section shows how the proposed approach can be used to inform researchers about two
524 questions which have received a lot of attention in the literature: the time variations in the
525 policy activism parameter and the sources of output and inflation fluctuations. The first

526 question is analyzed with the model presented in section 2. The second, with a medium
 527 scale model widely used in academic and policy circles.

528 **5.1 The policy activism parameter**

529 What are the features of the monetary policy rule in place during the "Great Inflation" of
 530 the 1970s and the return to norm of the 1980s and 1990s? This question has been extensively
 531 studied in the literature following Clarida et al. (2000). One synthetic way to summarize
 532 the information contained in the data is to compute the policy activism parameter $\frac{\rho_y}{\rho_\pi - 1}$,
 533 which gives a sense of the relative importance of the output and the inflation stabilization
 534 objectives of the Central Bank. The conventional wisdom suggests that the absolute value of
 535 this parameter has declined over time, reflecting changes in the preferences of the monetary
 536 authorities, but most of the available evidence is obtained either with reduced form methods
 537 or, when structural method are used, with filtered data. Are the results to be trusted? Is the
 538 characterization offered by the approach of this paper different? Figure 5 plots the posterior
 539 density of the policy activism parameter obtained when the data is linearly detrended (top
 540 left box) or HP filtered (top right box) before estimation and when the approach of this
 541 paper is employed (lower left box) for the samples 1964:1-1979:4 and 1984:1-2007:4. The
 542 prior for the structural and auxiliary parameters is the same as in table 1. In the flexible
 543 approach, and given the short subsamples, Σ_e and Σ_v are assumed to be diagonal, a common
 544 non-model based component is assumed for all the variables, the signal-to-noise ratio in the
 545 four series is captured by a single parameter λ , a-priori uniformly distributed over [100,
 546 6400], $\rho_1 = \rho_2 = I$ and the proxy error is set to zero.

547 The posterior density of the policy activism parameter shifts to the left in the second
 548 sample when HP filtered data is used and, for example, the posterior median moves from
 549 -0.23 in the first sample to -0.33 in the second. This left shift of the posterior density is
 550 absent when LT data is used and the median of the posterior in the second sample moves
 551 closer to zero (from -0.38 to 0.12) - care should be exercised here since the median is not a

552 good estimator of the central tendency of the posterior for the 1984-2007 sample. In both
553 cases, the Kolmogorov-Smirnov statistic rejects the null that the posterior distributions are
554 the same in the two samples. Thus, standard approaches confirm the existence of a break
555 in the conduct of monetary policy, although it is not clear in which direction the movement
556 is: with HP filtered data, output gap considerations have become relatively more important;
557 with LT filtered data, the opposite appears to be true.

558 When the approach of section 3 is used, the posterior density of $\frac{\rho_y}{\rho_\pi - 1}$ in the two samples
559 overlaps considerably. Interestingly, both the location and the shape of the density in the
560 two samples are very similar and the Kolmogorov-Smirnov statistic does not reject the null
561 that the posterior distributions in the two samples are the same. Thus, evidence in favor of
562 a structural break in the conduct of monetary policy is much weaker in this case.

563 Why are the results different? As mentioned, the non-model based component soaks
564 up all the features that the model is not designed to explain. Thus, in principle, it could
565 absorb the changes present in the endogenous variables. This, however, does not seem to
566 be the case: the median estimate of λ is around 3200 in both samples, making the non-
567 model based component quite smooth relative to the model based component (see on-line
568 appendix for plots of the two components of the four variables) and essentially time invariant.
569 Thus, variations in the time series properties of the endogenous variables are not captured
570 by the non-model based component. What instead happens, is that structural non-policy
571 parameters change to accommodate for the changes in the time series properties of inflation
572 and interest rate over time. Interestingly, the explanatory power of the model increases in
573 the second sub-sample: on average, at business cycle frequencies, the model explains 40 per
574 cent of output variations in the first sample and 55 per cent in the second sample. For
575 inflation and interest rates, the increase is smaller (from 40 to 50 percent).

576 Since about 50 percent of the variability observables at business cycle frequencies is not
577 captured by the model in both samples, it is worth investigating how the fit can be improved
578 by altering its structure, keeping the number of observables and the estimation approach

579 unchanged. One device that the literature has employed to improve the fit of this kind of
580 models is to allow for a time varying inflation target in the policy rule, see e.g. Ireland
581 (2007). The target is assumed to be driven by a permanent shock and enters only in the
582 interest rate equation. Thus, the estimated specification moves from (5) to (6), where now
583 $c_t(\theta)$ appears only in the interest rate equation. What would this modification do to the
584 posterior distribution of the policy activism parameter?

585 The last box of figure 5 indicates that adding a time varying inflation target reduces the
586 spread of the posterior distributions. Hence, the shift to the right in the posterior in the
587 second sub-sample becomes statistically significant even though ,e.g., the median value of
588 the two distributions is close in absolute value. Adding an inflation target improves the fit
589 for the interest rate at business cycle frequencies (the proportion of the variance explained
590 increase to 57 percent in the first sample and to 68 percent in the second); for inflation,
591 instead, the explanatory power of the model is unchanged in the first sub-sample and worsen
592 considerably in the second (the variance share explained at business cycle frequencies is now
593 only 28 percent). Hence, adding a time varying inflation target does not seem to be a very
594 promising way to improve our understanding of how inflation fluctuations are generated.

595 **5.2 Sources of output and inflation fluctuations**

596 The question of what drives output and inflation fluctuations has a long history in macro-
597 economics. In standard medium scale DSGE models, like the one employed by Smets and
598 Wouters (2003) and (2007), output and inflation fluctuations tend to be primarily explained
599 by markup shocks. Since these shocks are an unlikely source of cyclical fluctuations, Chari et
600 al (2009) have argued that misspecification is likely to be present (see Justiniano et al., 2010,
601 for an alternative interpretation). Researchers working in the area use filtering devices to fit
602 the model to the data (as in Smets and Wouters (2003)), arbitrarily data transformations (as
603 in Smets and Wouters, 2007) or build a permanent component in the model (as in Justiniano
604 et al., 2010) and use model-consistent data transformations to estimate the structural para-

605 meters. What would the approach of this paper tell us about sources of cyclical fluctuations
606 in output and inflation? How much of the variability of the observables at business cycle
607 frequencies is explained by the model? To answer this question, the same model and the
608 same data set used in Smets and Wouters (2007) are employed but a more standard setup
609 is employed. In particular, no MA terms for the price and wage markup disturbances are
610 assumed - all shocks have a standard AR(1) structure; the model is solved in deviations from
611 the steady state, rather than in deviation from the flexible price equilibrium; and the policy
612 rule does not include a term concerning output growth.

613 Table 4 reports results obtained eliminating a linear trend from the variables; taking
614 growth rates of the real variables and demeaning nominal ones; and using the approach
615 suggested in this paper. When a linear trend is removed, the forecast error variance decom-
616 position of output at the five years horizon is indeed primarily driven by price markup shocks,
617 with a considerably smaller contribution of investment specific and preference shocks. For
618 inflation, price markup shocks account for almost 90 percent of the forecast error variability
619 at the five years horizon. When the model is instead fitted to growth rates, price markup
620 shocks account for over 90 percent of the variability of both output and inflation at the five
621 years horizon. Thus, even without some of the standard bells and whistles, the conclusion
622 that markup shocks dominate remains. Why are price markup shocks important? Since,
623 compared to other shocks, they are relatively unrestricted in the model, they tend to absorb
624 any misspecification the model has and any measurement error that the filters leave in the
625 transformed data. Furthermore, since the combined specification and measurement errors
626 are unlikely to be iid, the role of markup shocks is overestimated. When the bridge suggested
627 in this paper is used, the non-model based component of real variables is restricted to have a
628 common structure (there are only two parameters simultaneously controlling the non-model
629 based component of output, consumption, investment), $\rho_1 = \rho_2 = I$, and a proxy error is
630 allowed in each equation, the picture is quite different. Output fluctuations at the five year
631 horizon are driven almost entirely by preference disturbances, while inflation fluctuations are

632 jointly accounted for by wage markup, TFP and price markup disturbances. More interest-
633 ingly, the model explains only 20 percent of the output and inflation fluctuations at business
634 cycle frequencies. Thus, it seems premature to use it to evaluate policy alternatives.

635 It is useful to characterize the properties of the non-model based component to evaluate
636 the theoretical modifications that are needed to capture what the current model leaves out.
637 The non-model component is well represented by the specification employed and restrictions
638 on the representation used assuming, for example, no or only one unit root are all rejected
639 in formal testing (log Bayes factor exceeding 10 in both cases). Thus, if shocks are to be
640 added to the model, it is important that they have permanent features and display persistent
641 deviations from a balanced growth path. Ireland (2010) has suggested one such specification.
642 Others, which allow both TFP and investment shocks to have these features, are also possible.

643 **6 Conclusions**

644 Estimating DSGE models with data that is model-based transformed or statistically filtered
645 may lead researchers astray because the association between the output of the filter and the
646 stationary solution of the model is generally incorrect and because model-based transforma-
647 tions impose tight restrictions which are, more likely than not, violated in the data. The
648 consequences of filtering and specification errors could be economically important because
649 income and substitution effects could be distorted, the volatilities and persistence of the
650 shocks over or underestimated and, thus, the decision rules of the agents, as perceived by
651 the econometrician, altered.

652 The alternative methodology this paper proposes avoids these errors by building a flexible
653 bridge between the DSGE models and the raw data. The procedure is applicable to a large
654 class of models and i) it takes into account the uncertainty in the specification of the non-
655 model component when deriving estimates of the structural parameters; ii) it provides a
656 natural environment to judge the goodness of fit of a model and to evaluate the contribution
657 of certain shocks to the understanding of economic phenomena; iii) it gives researchers an

658 integrated framework to examine the sensitivity of the estimation results to the specification
659 of nuisance features, and iv) it is easy to implement and requires minor modifications of
660 existing routines.

661 Unaccounted low frequency movements, such as those appearing in hours or labor pro-
662 ductivity, or idiosyncratic trends, such as those present in certain relative prices, are typically
663 hard to handle in standard DSGE models. Hence, certain shocks which are left somewhat
664 unrestricted in the model end up capturing these features in standard frameworks. The
665 approach this paper suggests is likely to be very useful in these difficult situations because
666 it helps researchers to distinguish what the model can explain and what it can not, thus
667 avoiding important policy distortions. In general, applications of the methodology appear
668 to be numerous.

669 Extensions of the setup used in the paper are easy to conceive. For example, structural
670 breaks in the time series features of the observables could be handled either within the
671 model-based (as in Eklund et al., 2008) or the non model-based components and the impli-
672 cations for structural parameters could be compared. Similarly, stochastic volatility could
673 be captured in the model-based or non model-based components and differences evaluated.
674 The unified framework that the approach provides requires very little changes to allow for
675 these situations.

676 **7 Tables and Figures**

Model with transitory shocks	
w_t	$= (\frac{\sigma_n}{1-\alpha} + \frac{\sigma_c}{1-h})y_t - \frac{h\sigma_c}{1-h}y_{t-1} - \frac{\sigma_n}{1-\alpha}z_t - \chi_t$
y_t	$= E_t[\frac{1}{1+h}y_{t+1} + \frac{h}{1+h}y_{t-1} - \frac{1-h}{(1+h)\sigma_c}(\chi_{t+1} - \chi_t + r_t - \pi_{t+1})]$
π_t	$= \beta E_t\pi_{t+1} + \frac{1-\alpha}{1-\alpha+\alpha\theta} \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p} (\epsilon_t^\mu + w_t + \frac{\alpha}{1-\alpha}y_t - \frac{1}{1-\alpha}z_t)$
r_t	$= \rho_r r_{t-1} + (1 - \rho_r)(\rho_y y_t + \rho_\pi \pi_t) + \epsilon_t^r$
n_t	$= \frac{1}{1-\alpha}(y_t - z_t)$
Model with stochastically trending TFP	
w_t	$= (\frac{\sigma_n}{1-\alpha} + \frac{1}{1-h})y_t - \frac{\bar{h}}{1-h}y_{t-1} - \chi_t - \frac{\bar{h}}{1-h}(\epsilon_{t-1}^z - \epsilon_t^z)$
y_t	$= \frac{1}{1+h}E_t(y_{t+1} + hy_{t-1} - (1 - \bar{h})(\chi_{t+1} - \chi_t + r_t - \pi_{t+1})) + \bar{h}\epsilon_{t-1}^z + \epsilon_{t+1}^z - (1 - \bar{h})\epsilon_t^z$
π_t	$= \beta E_t\pi_{t+1} + \frac{1-\alpha}{1-\alpha+\alpha\theta} \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p} (\epsilon_t^\mu + w_t + \frac{\alpha}{1-\alpha}y_t)$
r_t	$= \rho_r r_{t-1} + (1 - \rho_r)(\rho_y y_t + \rho_\pi \pi_t) + \epsilon_t^r$
n_t	$= \frac{1}{1-\alpha}y_t$
Model with unit roots in preferences	
w_t	$= (\sigma_n + \frac{1}{1-h})y_t - \frac{h}{1-h}y_{t-1} - \sigma_n z_t + \frac{h}{1-h}\epsilon_t^X$
y_t	$= \frac{1}{1+h}E_t(y_{t+1} + hy_{t-1} - (1 - h)(r_t - \pi_{t+1})) - (h\epsilon_t^X + ((1 - h)\sigma_n - h)\epsilon_{t+1}^X)$
π_t	$= \beta E_t\pi_{t+1} + \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p} (\epsilon_t^\mu + w_t - z_t)$
r_t	$= \rho_r r_{t-1} + (1 - \rho_r)(\rho_y y_t + \rho_\pi \pi_t) + \epsilon_t^r$
n_t	$= y_t - z_t$

678 Table 1: Optimality conditions of the log-linear stationary model. All variables are expressed
679 in percentage deviation from the steady state (balanced growth path). $\bar{h} = e^b h$ and b is the slope
680 of the stochastic trend. With trends $\sigma_c = 1$ and with unit roots in preferences also $\alpha = 0$. z_t is a
681 technology shock, χ_t a preference shock, ϵ_t^r a monetary policy shock and ϵ_t^μ a markup shock. If z_t
682 and χ_t are transitory, $z_t = \rho_z z_{t-1} + \epsilon_t^z$, $\chi_t = \rho_\chi \chi_{t-1} + \epsilon_t^X$. When TFP is trending, $z_t = bt + \epsilon_t^z$, when
683 preferences are trending $\chi_t = \chi_{t-1} + \epsilon_t^X$. In each block the first equation defines the equilibrium
684 real wage, the second is an Euler equation, the third a Phillips curve, the fourth a Taylor rule and
685 the fifth a labor demand function.

	Prior	LT	HP	FOD	BP	Ratio 1	Ratio2
		Median (s.e.)	Median (s.e.)	Median (s.e.)	Median (s.e.)	Median(s.e.)	Median (s.e.)
σ_c	$\Gamma(20, 0.1)$	1.90 (0.25)	1.41 (0.21)	0.04 (0.01)	0.96 (0.11)	2.33 (0.27)	0.81 (0.15)
σ_n	$\Gamma(20, 0.1)$	1.75 (0.16)	1.37 (0.13)	5.23 (0.08)	1.19 (0.09)	3.02 (0.24)	2.68 (0.19)
h	$B(6, 8)$	0.83 (0.02)	0.88 (0.02)	0.45 (0.01)	0.96 (0.01)	0.72 (0.05)	0.88 (0.02)
α	$B(3, 8)$	0.07 (0.04)	0.09 (0.05)	0.42 (0.01)	0.07 (0.03)	0.05 (0.04)	0.03 (0.01)
ρ_r	$B(6, 6)$	0.19 (0.05)	0.11 (0.04)	0.62 (0.01)	0.09 (0.02)	0.38 (0.06)	0.28 (0.04)
ρ_π	$N(1.5, 0.1)$	1.33 (0.08)	1.37 (0.05)	1.53 (0.02)	1.51(0.06)	1.92 (0.06)	1.80 (0.05)
ρ_y	$N(0.4, 0.1)$	-0.16 (0.03)	-0.18 (0.03)	0.06 (0.00)	-0.22 (0.03)	0.16 (0.02)	-0.03 (0.02)
ζ_p	$B(6, 6)$	0.82 (0.02)	0.80 (0.03)	0.63 (0.01)	0.86 (0.01)	0.82 (0.02)	0.80 (0.02)
ρ_χ	$B(18, 8)$	0.69 (0.04)	0.40 (0.05)	0.52 (0.01)	0.70(0.02)	0.67 (0.03)	0.66 (0.02)
ρ_z	$B(18, 8)$	0.96 (0.02)	0.95 (0.02)	0.99 (0.01)	0.97(0.01)	0.97 (0.01)	0.96 (0.01)
σ_χ	$\Gamma^{-1}(10, 20)$	0.53 (0.19)	0.47 (0.11)	4.96(0.13)	0.23 (0.05)	3.41 (0.74)	0.97 (0.13)
σ_z	$\Gamma^{-1}(10, 20)$	0.20 (0.04)	0.23 (0.04)	2.00 (0.22)	0.19 (0.03)	0.06 (0.01)	0.06 (0.01)
σ_r	$\Gamma^{-1}(10, 20)$	0.11 (0.01)	0.08 (0.01)	2.30(0.23)	0.07 (0.01)	0.10 (0.01)	0.11 (0.18)
σ_μ	$\Gamma^{-1}(10, 20)$	25.06 (0.97)	14.25 (0.93)	7.17 (0.13)	18.19 (0.66)	22.89 (1.91)	15.94 (0.49)
	Prior	Ratio 3	TFP	Preferences	TFP FD		
		Median (s.e.)	Median (s.e.)	Median (s.e.)	Median (s.e.)		
σ_c	$\Gamma(20, 0.1)$	0.12 (0.03)	1.0	1.0	1.0		
σ_n	$\Gamma(20, 0.1)$	2.09 (0.14)	2.24 (0.26)	2.43 (0.20)	43.17 (23.32)		
h	$B(6, 8)$	0.10 (0.03)	0.08 (0.04)	0.78 (0.03)	0.49 (0.28)		
α	$B(3, 8)$	0.03 (0.02)	0.17 (0.03)	1.0	0.51 (0.28)		
ρ_r	$B(6, 6)$	0.20 (0.06)	0.30 (0.04)	0.61 (0.02)	0.49 (0.28)		
ρ_π	$N(1.5, 0.1)$	1.51 (0.07)	1.74 (0.06)	1.69 (0.05)	1.82 (2.09)		
ρ_y	$N(0.4, 0.1)$	0.77 (0.04)	0.49 (0.03)	0.38 (0.07)	0.09 (2.16)		
ζ_p	$B(6, 6)$	0.81 (0.01)	0.41 (0.03)	0.84 (0.01)	0.48 (0.29)		
ρ_χ	$B(18, 8)$	0.75 (0.03)	0.63 (0.03)		0.48 (0.28)		
ρ_z	$B(18, 8)$	0.62 (0.03)		0.59 (0.02)			
σ_χ	$\Gamma^{-1}(10, 20)$	0.26 (0.04)	0.21 (0.03)	0.06 (0.008)	828.3(81.1)		
σ_z	$\Gamma^{-1}(10, 20)$	0.08 (0.01)	0.05 (0.006)	0.15 (0.02)	284.2 (144.8)		
σ_r	$\Gamma^{-1}(10, 20)$	2.68 (0.27)	0.10 (0.01)	0.07 (0.007)	679.7(232.2)		
σ_μ	$\Gamma^{-1}(10, 20)$	15.98 (1.09)	0.25 (0.04)	36.68 (1.42)	666.9(139.2)		

686

687 Table 2: Posterior estimates. LT refers to linearly detrended data, HP to Hodrick and Prescott
688 filtered data, FOD to demeaned growth rates, BP to band pass filtered data. For Ratio 1 the ob-
689 servables are $\log(y_t/n_t)$, $\log(w_t)$, π_t , r_t , all demeaned, for Ratio 2 they are $\log(y_t/w_t)$, $\log(n_t)$, π_t , r_t ,
690 all demeaned, For Ratio 3, the observables are $\log((w_t n_t)/y_t)$, $\log(w_t/y_t)$, π_t , r_t , all demeaned. For
691 TFP trending, the observable are linearly detrending output and real wages and demeaned inflation
692 and interest rates. For Preference trending, the observable are demeaned growth rate of output,
693 demeaned log real wages, demeaned inflation and demeaned interest rates. When frequency domain
694 estimation is used, only information in the band $(\frac{\pi}{32}, \frac{\pi}{8})$ is employed. The sample is 1980:1-2007:4.

695

$\sigma_\chi^p = 1.50$							
	True	LT	HP	FOD	BP	Ratio1	Flexible
		Median (s.e.)	Median (s.e.)	Median (s.e.)	Median(s.e.)	Median(s.e.)	Median(s.e.)
σ_n	0.50	0.12(0.02)	0.21(0.02)	1.30(0.05)	0.08(0.01)	1.00(0.04)	0.24(0.03)
h	0.70	0.91(0.03)	0.74(0.03)	0.71(0.03)	0.88(0.03)	0.11(0.04)	0.76(0.05)
α	0.30	0.07(0.02)	0.06(0.02)	0.04(0.02)	0.16(0.02)	0.04(0.02)	0.20(0.05)
ρ_r	0.70	0.39(0.04)	0.46(0.04)	0.74(0.03)	0.36(0.02)	0.47(0.05)	0.34(0.03)
ρ_π	1.50	1.41(0.06)	1.60(0.06)	1.63(0.06)	1.36(0.05)	1.50(0.08)	1.59(0.08)
ρ_y	0.40	0.01(0.00)	0.01(0.01)	-0.01(0.00)	-0.01(0.00)	0.55(0.07)	-0.01(0.01)
ζ_p	0.75	0.88(0.03)	0.85(0.03)	0.88(0.03)	0.90(0.03)	0.89(0.03)	0.83(0.03)
ρ_χ	0.50	0.40(0.03)	0.36(0.03)	0.69(0.03)	0.73(0.03)	0.37(0.03)	0.51(0.04)
ρ_z	0.80	0.68(0.04)	0.69(0.04)	0.99(0.03)	0.80(0.03)	0.64(0.03)	0.79(0.04)
σ_χ	1.20	3.38(0.41)	0.35(0.06)	0.26(0.05)	0.33(0.12)	0.24(0.04)	0.27(0.07)
σ_z	0.50	0.50(0.11)	0.21(0.04)	0.62(0.11)	0.32(0.06)	0.09(0.01)	0.22(0.04)
σ_r	0.10	0.06(0.01)	0.06(0.01)	0.07(0.01)	0.06(0.01)	0.07(0.01)	0.05(0.00)
σ_μ	1.60	5.97(0.42)	0.80(0.28)	5.60(0.34)	6.62(0.25)	12.33(0.73)	1.56(0.53)
σ_χ	1.20	3.38(0.41)	0.35(0.06)	0.26(0.05)	0.33(0.12)	0.24(0.04)	0.27(0.07)
$\sigma_\chi^p = 0.15$							
	True	LT	HP	FOD	BP	Ratio1	Flexible
		Median (s.e.)	Median (s.e.)	Median (s.e.)	Median(s.e.)	Median(s.e.)	Median(s.e.)
σ_n	0.50	0.18(0.03)	0.35(0.06)	0.89(0.03)	0.31(0.04)	0.95(0.04)	0.14(0.01)
h	0.70	0.92(0.03)	0.91(0.03)	0.90(0.03)	0.97(0.03)	0.13(0.04)	0.79(0.03)
α	0.30	0.05(0.02)	0.07(0.04)	0.23(0.01)	0.14(0.02)	0.03(0.02)	0.15(0.01)
ρ_r	0.70	0.53(0.03)	0.51(0.02)	0.58(0.02)	0.50(0.02)	0.36(0.04)	0.50(0.02)
ρ_π	1.50	1.75(0.07)	1.67(0.06)	1.59(0.05)	1.77(0.06)	1.53(0.07)	1.57(0.05)
ρ_y	0.40	-0.01(0.01)	-0.03(0.01)	-0.03(0.00)	-0.03(0.00)	0.67(0.09)	0.34(0.02)
ζ_p	0.75	0.86(0.03)	0.89(0.03)	0.86(0.03)	0.93(0.03)	0.87(0.03)	0.83(0.03)
ρ_χ	0.50	0.27(0.04)	0.22(0.04)	0.66(0.02)	0.60(0.03)	0.27(0.05)	0.60(0.03)
ρ_z	0.80	0.68(0.04)	0.87(0.03)	0.98(0.03)	0.92(0.03)	0.59(0.05)	0.67(0.03)
σ_χ	1.20	0.39(0.11)	0.31(0.08)	4.23(0.18)	0.30(0.06)	0.18(0.03)	0.85(0.16)
σ_z	0.50	0.23(0.05)	0.22(0.04)	3.37(0.22)	0.17(0.02)	0.06(0.01)	0.22(0.04)
σ_r	0.10	0.06(0.01)	0.06(0.01)	2.61(0.17)	0.06(0.01)	0.07(0.01)	0.07(0.01)
σ_μ	1.60	0.93(0.29)	1.97(0.50)	5.13(0.18)	6.11(0.28)	3.60(0.37)	0.93(0.11)

696

697

698 Table 3: Parameters estimates, simulated data, T=150. LT refers to linearly detrended data,
699 HP to Hodrick and Prescott filtered data, FOD to demeaned growth rates, BP to band pass filtered
700 data, Ratio1 to output scaled by hours, and Flexible to the approach suggested in the paper.

	LT		FOD		Flexible	
	Output	Inflation	Output	Inflation	Output	Inflation
TFP shocks	0.01	0.04	0.00	0.01	0.01	0.21
Gov. expenditure shocks	0.00	0.00	0.00	0.00	0.00	0.02
Investment shocks	0.08	0.00	0.00	0.00	0.00	0.05
Monetary policy shocks	0.01	0.00	0.00	0.00	0.00	0.01
Price markup shocks	0.75(*)	0.88(*)	0.91(*)	0.90(*)	0.00	0.19
Wage markup shocks	0.00	0.01	0.08	0.08	0.03	0.49(*)
Preference shocks	0.11	0.04	0.00	0.00	0.94(*)	0.00

701

702 Table 4: Variance decomposition at the 5 years horizon. Estimates are obtained using the median
703 of the posterior of the parameters. A (*) indicates that the 68 percent highest credible set is
704 entirely above 0.10. The model and the data set are the same as in Smets and Wouters, 2007. LT
705 refers to linearly detrended data, FOD to growth rates and Flexible to the approach this paper
706 suggests.

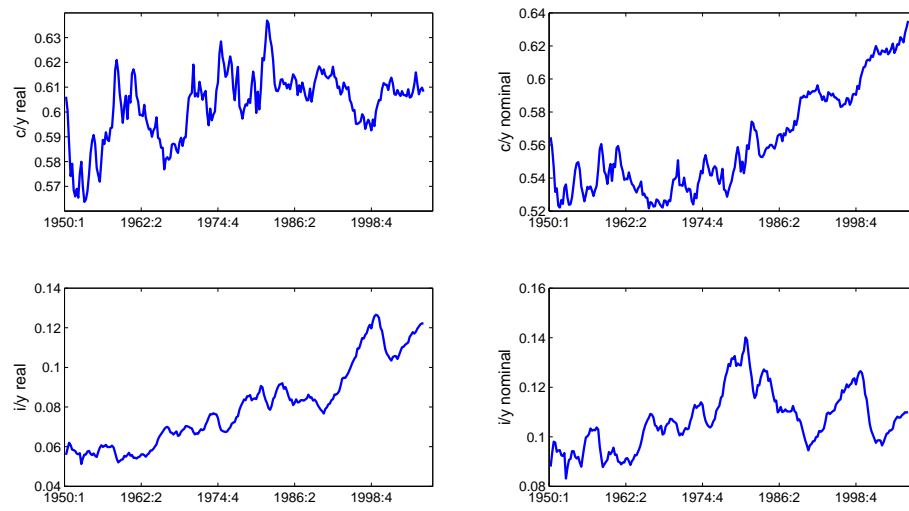


Figure 1: US real and nominal great ratios

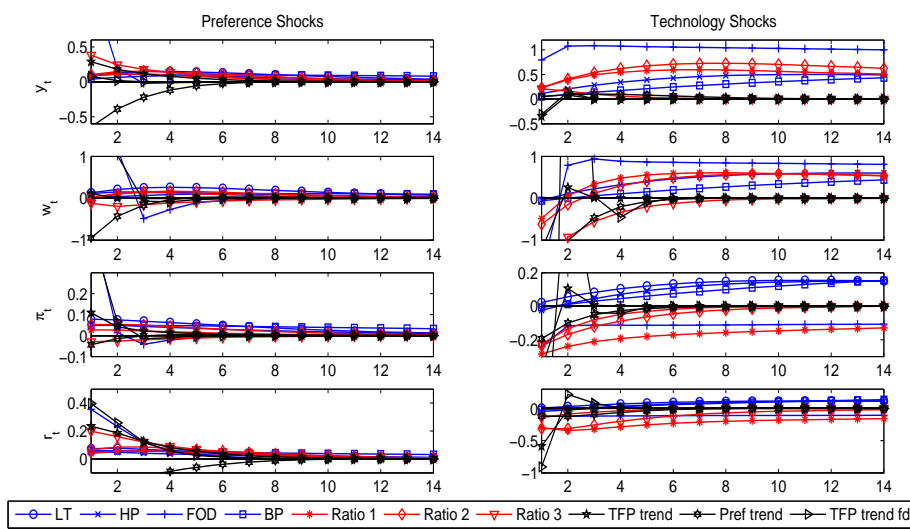


Figure 2: Impulse responses, sample 1980:1-2007:4

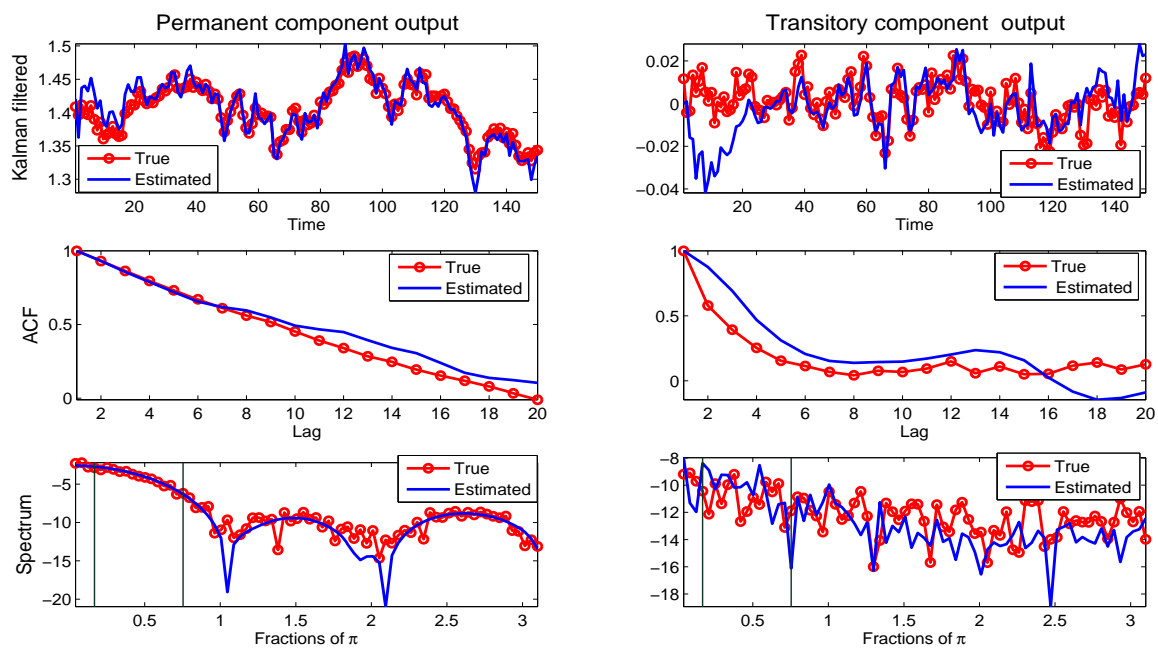


Figure 3: Output decompositions, true and estimated with a flexible approach. Vertical bars indicate cycles with 8-32 quarters periodicity.

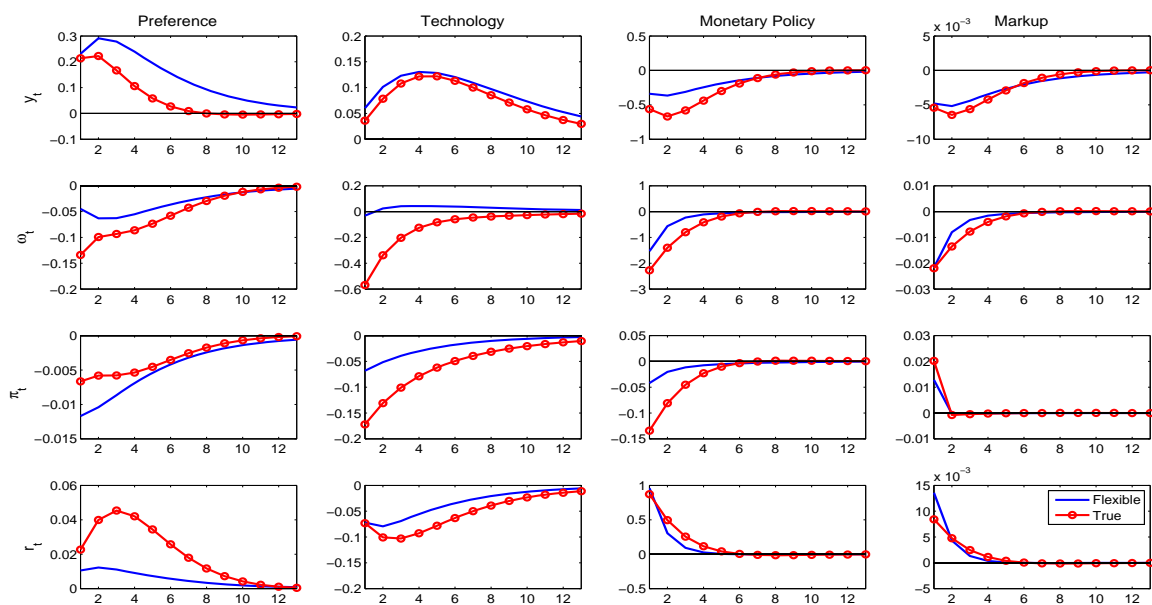


Figure 4: Impulse responses to transitory shocks, true and estimated with flexible approach.

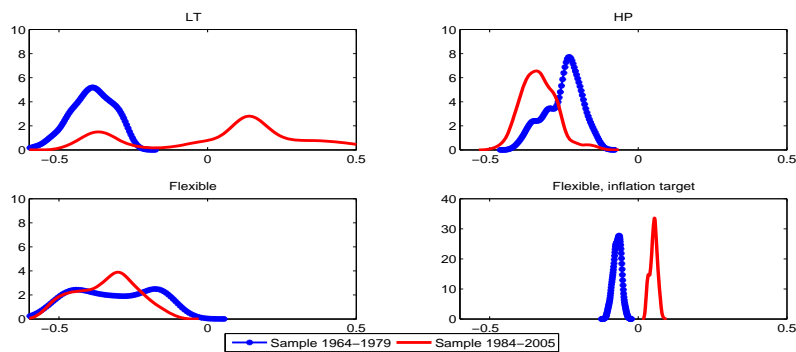


Figure 5: Posterior distributions of the policy activism parameter, samples 1964:1-1979:4 and 1984:1-2007:4. LT refers to linearly detrended data, HP to Hodrick and Prescott filtered data and Flexible to the approach the paper suggests

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785 **On-line Appendix (not intended for publication)**

786 **A. The basic DSGE model of section 2**

787 The bundle of goods consumed by the representative household is

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon_t-1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t-1}} \quad (9)$$

788 where $C_t(j)$ is the consumption of the good produced by firm j and ϵ_t the elasticity of substi-
789 tution between varieties. Maximization of the consumption bundle, given total expenditure,
790 leads to

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_t} C_t \quad (10)$$

791 where $P_t(j)$ is the price of the good produced by firm j . Consequently, the price deflator is
792 $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_t} dj \right)^{\frac{1}{1-\epsilon_t}}$ and $P_t C_t = \left[\int_0^1 P_t(j) C_t(j) dj \right]$.

793 The representative household chooses sequences for consumption and leisure to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[X_t \frac{1}{1-\sigma_c} (C_t - hC_{t-1})^{1-\sigma_c} - \frac{1}{1+\sigma_n} N_t^{1+\sigma_n} \right] \quad (11)$$

794 where X_t is an exogenous utility shifter following an AR(1) in logs:

$$\chi_t = \rho_\chi \chi_{t-1} + \epsilon_t^\chi \quad (12)$$

795 where $\chi_t = \ln X_t$ and $\epsilon_t^\chi \sim N(0, \sigma_\chi^2)$. The household budget constraint is

$$P_t C_t + b_t B_t = B_{t-1} + W_t N_t \quad (13)$$

796 where B_t are one-period bonds with price b_t , W_t is nominal wage and N_t is hours worked.

797 There is a continuum of firms, indexed by $j \in [0, 1]$, each of which produces a differenti-
798 ated good. The common technology is:

$$Y_t(j) = Z_t N_t(j)^{1-\alpha} \quad (14)$$

799 where Z_t is an exogenous productivity disturbance following an AR(1) in log,

$$z_t = \rho_z z_{t-1} + \epsilon_t^z \quad (15)$$

800 where $z_t = \ln Z_t$ and $\epsilon_t^z \sim N(0, \sigma_z^2)$. Each firm resets its price with probability $1 - \zeta_p$ in
 801 any t , independently of time elapsed since the last adjustment. Therefore, aggregate price
 802 dynamics are

$$\Pi_t^{1-\epsilon_t} = \zeta_p + (1 - \zeta_p)(P_t^*/P_{t-1})^{1-\epsilon_t} \quad (16)$$

803 A reoptimizing firm chooses the P_t^* that maximizes the current value of discounted profits

$$\max_{P_t^*} \sum_{k=0}^{\infty} \zeta_p^k E_t Q_{t,t+k} [P_t^* Y_{t+k|t} - TC_{t+k}(Y_{t+k|t})] \quad (17)$$

804 subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_{t+k}} Y_{t+k} \quad (18)$$

805 $k = 0, 1, 2, \dots$ where $Q_{t,t+k} \equiv \beta^k (C_{t+k}/C_t)(P_t/P_{t+k})$, $TC(\cdot)$ is the total cost function, and
 806 $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that reset its price at t .

807 Finally, the monetary authority sets the nominal interest rate according to

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\rho_\pi \pi_t + \rho_y gdp_t) + \epsilon_t^r \quad (19)$$

808 where $\epsilon_t^r \sim N(0, \sigma_{ms}^2)$.

809 The first order conditions of the optimization problems are:

$$0 = X_t (C_t - hC_{t-1})^{-\sigma_c} - \lambda_t \quad (20)$$

$$0 = -N_t^{-\sigma_n} - \lambda_t \frac{W_t}{P_t} \quad (21)$$

$$1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{t+1}}{P_t} R_t \right] \quad (22)$$

$$0 = \sum_{k=0}^{\infty} \zeta_p^k E_t Q_{t,t+k} Y_{t+k|t} [P_t^* - \mathcal{M}_{t+k} MC_{t+k|t}^m] \quad (23)$$

810 where λ_t is the Lagrangian multiplier associated with the consumer budget constraint, $R_t \equiv$
 811 $1 + i_t = 1/b_t$ is the gross nominal rate of return on bonds, $MC^m(\cdot)$ are nominal marginal cost
 812 and

$$\mathcal{M}_t = \mu e^{\epsilon_t^\mu} \quad (24)$$

813 where $\epsilon_t^\mu \sim N(0, \sigma_\mu^2)$ and μ is the steady state markup.

814 Market clearing requires

$$Y_t(j) = C_t(j) \quad (25)$$

$$N_t = \int_0^1 N_t(j) dj \quad (26)$$

815 and letting the aggregate output be $GDP_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\epsilon_t-1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t-1}}$ we have $C_t = GDP_t$.

816 The shocks driving the dynamics of the model are: a preference disturbance χ_t , a tech-
817 nology disturbance z_t , a markup shock ϵ_t^μ and a monetary shock ϵ_t^r .

818 B. The solution with transitory shocks

819 When all the shocks are transitory, the log-linearized equilibrium conditions are:

$$w_t = \left(\frac{\sigma_n}{1-\alpha} + \frac{\sigma_c}{1-h} \right) y_t - \frac{h\sigma_c}{1-h} y_{t-1} - \frac{\sigma_n}{1-\alpha} z_t - \chi_t \quad (27)$$

$$y_t = E_t \left[\frac{1}{1+h} y_{t+1} - \frac{h}{1+h} y_{t-1} + \frac{1-h}{(1+h)\sigma_c} (\chi_{t+1} - \chi_t + r_t - \pi_{t+1}) \right] \quad (28)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p (\epsilon_t^\mu + w_t + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} z_t) \quad (29)$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r) (\rho_y y_t + \rho_\pi \pi_t) + \epsilon_t^r \quad (30)$$

$$n_t = \frac{1}{1-\alpha} (y_t - z_t) \quad (31)$$

820 where all variables are expressed in deviation from the (constant) steady state, $k_p = \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p} \frac{1-\alpha}{1-\alpha+\psi\alpha}$,

821 $z_t = \rho_z z_{t-1} + \epsilon_t^z$, $\chi_t = \rho_\chi \chi_{t-1} + \epsilon_t^\chi$, ϵ_t^r and ϵ_t^μ are iid. Equation (27) defines the equilibrium
822 real wage, (28) is an Euler equation, (29) a Phillips curve, (30) a Taylor rule and (31) a labor
823 demand function.

824 This is the model fitted to filtered data (first four columns on the top part of table 2)
825 and to transformed data (the next three columns of table 2).

826 C. The solution with a stochastic trend in the technology

827 Assume that the technology has a stochastic linear trend, i.e. $z_t = bt + \epsilon_t^z$, while the other
828 three shocks are assume to be transitory. A log-linearized solution can be found only setting

829 $\sigma_c = 1$. Defining $\bar{h} = \exp(b)h$, the equations in this case are

$$w_t = \left(\frac{\sigma_n}{1-\alpha} + \frac{1}{1-\bar{h}} \right) y_t - \frac{\bar{h}}{1-\bar{h}} y_{t-1} - \chi_t + \frac{\bar{h}}{1-\bar{h}} (\epsilon_{t-1}^{z,p} - \epsilon_t^{z,p}) \quad (32)$$

$$y_t = \frac{1}{1+\bar{h}} E_t(y_t + h y_{t-1} - (1-\bar{h})(\chi_{t+1} - \chi_t + r_t - \pi_{t+1}) + \bar{h} \epsilon_{t-1}^{z,p} + \epsilon_{t+1}^{z,p} - (1-\bar{h}) \epsilon_t^{z,p}) \quad (33)$$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{1-\alpha}{1-\alpha-\alpha\theta} \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p} (\epsilon_t^\mu + w_t + \frac{\alpha}{1-\alpha} y_t) \quad (34)$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\rho_y y_t + \rho_\pi \pi_t) + \epsilon_t^r \quad (35)$$

$$n_t = \frac{1}{1-\alpha} (y_t - z_t) \quad (36)$$

830 where all variables are expressed in deviation from the (constant) steady state, $k_p = \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p} \frac{1-\alpha}{1-\alpha+\psi\alpha}$,

831 $\chi_t = \rho_\chi \chi_{t-1} + \epsilon_t^\chi$, ϵ_t^r and ϵ_t^μ are iid. Then

$$\ln Y_t - c_y - bt = y_t + \epsilon_t^z \quad (37)$$

$$\ln W_t - c_w - bt = w_t + \epsilon_t^z \quad (38)$$

$$\Pi_t - c_\pi = \pi_t \quad (39)$$

$$R_t - c_r = r_t \quad (40)$$

832 where capital letters indicate the observable variables, lower case letters the model variables
 833 and c_j are constants (the mean of each process). This is the model fitted to the data in
 834 column 8 and column 10 of the bottom part of table 2.

835 **D. The solution with non-stationary preference shocks**

836 Assume that $\chi_t = \chi_{t-1} + \epsilon_t^\chi$. A log linearized solution can be found only setting $\sigma_c = 1.0$
 837 and $\alpha = 0$. The log-linearized equilibrium conditions are

$$w_t = \left(\sigma_n + \frac{1}{1-h}\right)y_t - \frac{h}{1-h}y_{t-1} - \sigma_n z_t + \frac{h}{1-h}\epsilon_t^{\chi:p} \quad (41)$$

$$y_t = \frac{1}{1+h}E_t(y_{t+1} + hy_{t-1} - (1-h)(r_t - \pi_{t+1}) - (h\epsilon_t^{\chi:p} + ((1-h)\sigma_n - h)\epsilon_{t+1}^{\chi:p})) \quad (42)$$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p}(\epsilon_t^\mu + w_t - z_t) \quad (43)$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\rho_y y_t + \rho_\pi \pi_t) + \epsilon_t^r \quad (44)$$

$$n_t = y_t - z_t \quad (45)$$

838 where all variables are expressed in deviation from the (constant) steady state, $k_p = \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p}$,

839 $z_t = \rho_z z_{t-1} + \epsilon_t^z$, ϵ_t^r and ϵ_t^μ are iid. Then

$$\ln \Delta Y_t - c_y = y_t + \epsilon_t^\chi \quad (46)$$

$$\ln W_t - c_w = w_t \quad (47)$$

$$\Pi_t - c_\pi = \pi_t \quad (48)$$

$$R_t - c_r = r_t \quad (49)$$

840 where capital letters indicate the observable variables, lower case letters the model variables
 841 and c_j are constants (the mean of the process). This is the model fitted to the data in column
 842 9 of table 2.

843 **E. Simulating data from a model with non-stationary preference** 844 **shocks**

845 Let Y_t^o be a $N \times 1$ vector of observables and let:

$$Y_t^o = \nu(\theta^*, \vartheta^*) + H^{ns} x_t^{ns} + H^s x_t^s \quad (50)$$

846 where x_t^s is $N_s \times 1$ vector containing the variables rescaled by the non-stationary preference
 847 shock in log deviations from the steady state, $\nu(\theta^*, \vartheta^*)$ is a $N \times 1$ vector of the logarithm of
 848 the (rescaled) variables at the steady state, and x_t^{ns} is $N_{ns} \times 1$ vector containing the logarithm

849 of the non-stationary preference shock. H^{ns} is a $N \times N_{ns}$ a selection matrix and H^s is a
 850 $N \times N_s$ selection matrix. Finally, $\theta \in \Theta_s$ is the vector of structural parameters describing the
 851 stationary dynamics of the DSGE model and $\vartheta \in \Theta_{ns}$ is the vector of parameters that define
 852 the non-stationary dynamics. Moreover, $\theta^* \in \Theta_s^* \subset \Theta_s$ and $\vartheta^* \in \Theta_{ns}^* \subset \Theta_{ns}$ are the vectors
 853 of parameters that affect the steady state values. Rescaled variables, x_t^s , evolve according to

$$x_{t+1}^s = \Phi(\theta, \vartheta)x_t^s + \Psi(\theta, \vartheta)\eta_{t+1} \quad \eta_t \sim N(0, \Sigma(\theta, \vartheta)) \quad (51)$$

854 where η_t is the vector of the structural innovations of the shock processes, $\eta_t = [\eta_t^{ns}, \eta_t^s]'$. It
 855 turns out that, for the particular model we have chosen, these equations are given (41)-(45)
 856 The vector of non-stationary shock processes $\log X_t^P$ is assumed to follows

$$\ln X_t^P = \ln X_{t-1}^P + e_t^{X,P} \quad (52)$$

857 while the vector of transitory shock processes is

$$\log z_t = \rho_z \log z_{t-1} + e_t^z \quad (53)$$

$$\log \chi_t = \rho_\chi \log \chi_{t-1} + e_t^\chi \quad (54)$$

$$v_t = e_t^v \quad (55)$$

$$\mu_t = e_t^\mu \quad (56)$$

858 Thus:

$$x_t^s = [y_t, w_t, \pi_t, r_t, z_t, \chi_t]' \quad (57)$$

$$x_t^{ns} = \ln X_t^P \quad (58)$$

$$\eta_t^s = [e_t^z, e_t^x, v_t, \mu_t]' \quad (59)$$

$$\eta_t^{ns} = e_t^{X,P} \quad (60)$$

$$\nu(\theta^*, \vartheta^*) = [\ln y_s, \ln W_s, \ln \Pi_s, \ln R_s]' \quad (61)$$

$$H^{ns} = [1, 1, 0, 0]' \quad (62)$$

$$H^s = \begin{pmatrix} I_{4 \times 4} & 0_{4 \times 2} \end{pmatrix} \quad (63)$$

$$\theta = [h, \sigma_n, \rho_r, \rho_y, \rho_\pi, k_p, \rho_z, \rho_\chi, \sigma_z, \sigma_x, \sigma_r, \sigma_\mu] \quad (64)$$

$$\vartheta = \sigma_{X,P} \quad (65)$$

859 F. The medium scale DSGE model used in section 5

860

(a): The variables of the model

861

Label	Definition
y_t	: output
c_t	: consumption
i_t	: investment
q_t	: Tobin's q
k_t^s	: capital services
k_t	: capital
z_t	: capacity utilization
r_t	: real rate
μ_t^p	: price markup
π_t	: inflation rate
μ_t^w	: wage markup
N_t	: total hours
w_t	: real wage rate
R_t	: nominal rate

862 (b): The parameters of the model

Label	Definition
σ_c	elasticity of intertemporal substitution
σ_l	elasticity of labor supply with respect to real wages
h	habit persistence parameter
δ	depreciation rate
$\phi_p - 1$	share of fixed costs in production
χ	steady state elasticity of capital adjustment cost function
ψ	positive function of the elasticity of capital utilization adjustment costs function.
α	share of capital services in production
γ_p	price indexation parameter
ζ_p	price stickiness parameter
ϵ_p	curvature of good market aggregator
863 γ_w	wage indexation parameter
ζ_w	wage stickiness parameter
ϵ_w	curvature of labor market aggregator
Label	Definition
λ_r	interest smoothing parameter
λ_π	inflation parameter
λ_y	output parameter
gy	government expenditure to output ratio
ky	steady state capital output ratio
$r_* = \beta^{-1}$	steady state rental rate
w_*	steady state real wage rate
N_*/C_*	steady state hours to consumption ratio

864

(c): The equations of the model (in deviation from steady states)

$y_t = (1 - gy - \delta ky)c_t + \delta ky i_t + r_* ky z_t + g_t$	(C.1)
$c_t = \frac{h}{1+h} E_t c_{t+1} + \frac{h}{1+h} c_{t-1} - \frac{(\sigma_c - 1)w_* N_* / C_*}{(1+h)\sigma_c} (N_t - E_t N_{t+1}) - \frac{1-h}{(1+h)\sigma_c} (R_t - E_t \pi_{t+1} + e_t^b)$	(C.2)
$i_t = \frac{\beta}{1+\beta} E_t i_{t+1} + \frac{1}{1+\beta} x_{t-1} + \frac{\chi^{-1}}{1+\beta} q_t + e_t^i$	(C.3)
$q_t = \beta(1 - \delta) E_t q_{t+1} + (1 - \beta(1 - \delta)) E_t r_{t+1} - (R_t - E_t \pi_{t+1} + e_t^b)$	(C.4)
$y_t = \phi_p (\alpha k_t^s + (1 - \alpha) N_t + e_t^a)$	(C.5)
$k_t^s = k_{t-1} + z_t$	(C.6)
$z_t = \frac{1-\psi}{\psi} r_t$	(C.7)
$k_{t+1} = (1 - \delta) k_t + \delta i_t + \delta (1 + \beta) \psi e_t^i$	(C.8)
$\mu_t^p = \alpha(k_t^s - N_t) + e_t^a - w_t$	(C.9)
$\pi_t = \frac{\beta}{1+\beta\gamma_p} E_t \pi_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \pi_{t-1} - T_p \mu_t^p + e_t^p$	(C.10)
$r_t = -(k_t - N_t) + w_t$	(E.11)
$\mu_t^w = w_t - (\sigma_l N_t + (1 - h)^{-1} (c_t - h c_{t-1}))$	(C.12)
$w_t = \frac{1}{1+\beta} w_{t-1} + \frac{\beta}{1+\beta} (E_t \pi_{t+1} + E_t w_{t+1}) - \frac{1+\beta\gamma_w}{1+\beta} \pi_t + \frac{\gamma_w}{1+\beta} \pi_{t-1} - T_w \mu_t^w + e_t^w$	(C.13)
$R_t = \lambda_r R_{t-1} + (1 - \lambda_r) (\lambda_\pi \pi_t + \lambda_y y_t) + e_t^r$	(C.14)

865

866

The seven disturbances are: TFP shock (e_t^a); monetary policy shock (e_t^r); investment

867

shock (e_t^i); price markup shock (e_t^p); wage markup shock (e_t^w); risk premium shock (e_t^b);

868

government expenditure shock (e_t^g). The compound parameters in equation (C.11) and

869

(C.13) are defined as: $T_p \equiv \frac{1}{1+\gamma_p} \frac{(1-\beta\zeta_p)(1-\zeta_p)}{((\phi_p-1)\epsilon_p)\zeta_p}$ and $T_w \equiv \frac{1}{1+\beta} \frac{(1-\beta\zeta_w)(1-\zeta_w)}{((\phi_w-1)\epsilon_w)\zeta_w}$.

870

(d): The process for the shocks

871

$$\begin{array}{l} e_t = (e_t^a, e_t^r, e_t^i, e_t^p, e_t^w, e_t^b, e_t^g) \\ e_t = \rho e_{t-1} + \eta_t \end{array}$$

872

where both ρ and $\Sigma = E_t \eta_t \eta_t'$ are diagonal.

873 **G. Additional Tables and Graphs**

	LT	HP	FOD	BP
	Median (s.e.)	Median (s.e.)	Median (s.e.)	Median (s.e.)
σ_c	1.68 (0.30)	1.53 (0.26)	0.04 (0.01)	2.98 (0.49)
σ_n	1.73 (0.15)	1.62 (0.12)	5.28 (0.07)	0.55 (0.06)
h	0.85 (0.03)	0.87 (0.03)	0.40 (0.01)	0.89 (0.02)
α	0.05 (0.02)	0.08 (0.03)	0.41 (0.01)	0.04 (0.02)
ρ_r	0.18 (0.06)	0.16 (0.05)	0.64 (0.01)	0.13 (0.03)
ρ_π	1.36 (0.07)	1.36 (0.08)	1.48 (0.02)	1.42 (0.06)
ρ_y	-0.17 (0.03)	-0.17 (0.04)	0.05 (0.00)	-0.11 (0.03)
ζ_p	0.82 (0.01)	0.82 (0.02)	0.64 (0.01)	0.83 (0.01)
ρ_χ	0.66 (0.04)	0.67 (0.04)	0.54 (0.01)	0.81 (0.03)
ρ_z	0.97 (0.02)	0.97 (0.01)	0.99 (0.01)	0.76 (0.02)
σ_χ	0.63 (0.18)	0.65 (0.21)	4.63 (0.07)	0.45 (0.12)
σ_z	0.19 (0.04)	0.23 (0.05)	2.89 (0.19)	0.14 (0.02)
σ_{mp}	0.11 (0.01)	0.11 (0.01)	2.69 (0.14)	0.12 (0.01)
σ_μ	23.13 (1.99)	29.07 (0.94)	7.63 (0.10)	30.22 (1.12)

874
875 Table G.1 Parameters estimates obtained with standard transformations; real variables filtered,
876 nominal variables demeaned.

877

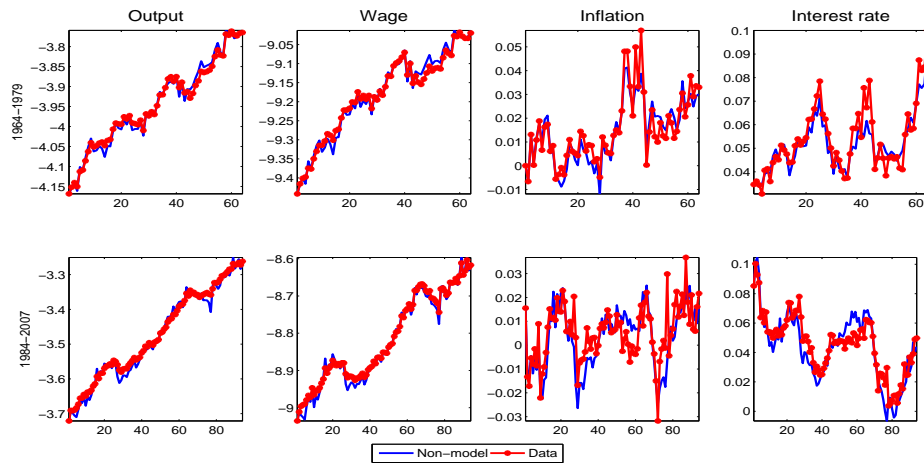


Figure G.2: Data and estimated non-model based components, samples 1964:1-1979:4 and 1984:1-2007:4, flexible approach

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879