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José-Luis Moraga-González and
Vaiva Petrikaitė

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José-Luis Moraga-González, VU University Amsterdam, University of Groningen and CEPR
Vaiva Petrikaitė, University of Groningen

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Search Costs, Demand-Side Economies and the Incentives to Merge under Bertrand Competition*

We study the incentives to merge in a Bertrand competition model where firms sell differentiated products and consumers search sequentially for satisfactory deals. In the pre-merger symmetric equilibrium, consumers visit firms randomly. However, after a merger, because insiders raise their prices more than the outsiders, consumers start searching for good deals at the non-merging stores, and only when they do not find a satisfactory product there they visit the merging firms. As search costs go up, consumer traffic from the non-merging firms to the merged ones decreases and eventually mergers become unprofitable. This new merger paradox can be overcome if the merged entity chooses to stock each of its stores with all the products of the constituent firms, which generates sizable search economies. We show that such demand-side economies can confer the merging firms a prominent position in the marketplace, in which case their price may even be lower than the price of the non-merging firms. In that situation, consumers start searching for a satisfactory good at the merged entity and the firms outside the merger lose out. When search economies are sufficiently large, a merger is beneficial for consumers too, and overall welfare increases.

JEL Classification: D40, D83 and L13
Keywords: consumer search, demand-side economies, economies of search, insiders, long-run, mergers, orders of search, outsiders, prominence, sequential search and short-run

José-Luis Moraga-González
Department of Economics
VU University Amsterdam
De Boelelaan 1105
1081HV Amsterdam
NETHERLANDS

Email: j.l.moragagonzalez@vu.nl

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Vaiva Petrikaitė
University of Groningen
Broerstraat 5
9712 CP Groningen
NETHERLANDS

Email: v.petrikaite@rug.nl

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1 Introduction

While no one would deny that searching for price and product fit can be quite costly in real-world markets –think for example about the time we spend test-driving new cars, acquiring new furniture, touching digital tablets, trying on new clothes, etc.– there has been little work in the industrial organization literature about the influence of search costs on the incentives to merge and on the aggregate implications of mergers. In this paper, we demonstrate that when search costs are sizable, the predictions obtained in an otherwise standard model of price competition and differentiated products about the effects of mergers differ markedly from the current state of knowledge.

We study mergers in a model that could well be referred to as the workhorse model of consumer search for differentiated products. The model was introduced by Wolinsky (1986) and was further studied by Anderson and Renault (1999).[1] A finite number of firms sell horizontally differentiated products and compete in prices. Consumers search for satisfactory deals sequentially and can recall the offers at previously visited firms costlessly. In the unique symmetric equilibrium of the pre-merger market, all firms are equally attractive and consumers randomly pick a first shop to visit. Those consumers who fail to find a satisfactory product at the first shop they visit continue searching and randomly pick a second shop to visit; and so on.

When a number of firms merge, consumers no longer search for good deals in a random way. The order in which consumers visit merging and non-merging firms depends on the prices these firms are expected to charge and on the amount of variety they carry. We distinguish between the short-run and the long-run effects of mergers. In the short-run, firms that merge coordinate their prices and everything else is kept constant. In the long-run, by contrast, the merged entity may choose to undertake a business reorganization consisting in stocking all the products of the parent firms.

In the short-run, price coordination among the merging firms leads them to charge higher prices than the non-merging firms. Given this, consumers optimally start their search for a satisfactory product at the non-merging firms and then, in the event they fail to find a product to their liking at those firms, they continue searching at the merging stores. The merging firms, by internalizing the pricing externalities they impose on one another, thus confer the non-merging firms a *prominent* position in the marketplace. This puts the merging firms at a market disadvantage vis-à-vis the non-merging firms. In fact, in equilibrium, as search costs increase, consumer traffic from the non-merging stores to the shops of the merged entity diminishes, which makes merging less profitable. We show that any 2-firm merger is unprofitable if search costs are sufficiently high. Moreover, any arbitrary k-firm merger becomes unprofitable if search costs and the number of non-merging firms are sufficiently high.

[1] More recently, this model has been used to explain incentives to invest in quality (Wolinsky, 2005), product-design differentiation (Bar-Isaac et al., 2011) and the emergence and effects of market *prominence* (Armstrong *et al.*, 2009; Armstrong and Zhou, 2011; Haan and Moraga-González, 2011; Zhou, 2009).

With these results we establish *a new merger paradox.* What is interesting about this paradox is that it arises when firms sell horizontally differentiated products and compete in prices.

In the long-run, however, the merged entity can counter the detrimental effects of price coordination by choosing to stock in each of its shops all the products of the constituent firms.[2] By stocking a wider range of products, the merged entity effectively lowers the costs of searching for a satisfactory good at the merging firms. This generates *demand-side economies* which, when significant, may confer the merged entity a prominent position in the marketplace. We show that, when search costs are sufficiently high, in the unique symmetric equilibrium of the post-merger market the merged entity gains prominence in the marketplace and attracts all consumer first-visits. If unsatisfied with the products available at the merged entity, consumers continue searching at the non-merging stores.

In contrast to the paradoxical result found in the short-run where the merging firms just coordinate their prices, we show that when search costs are sufficiently high and the merged entity offers all the products of the parent firms, merging becomes profitable. In addition, and in contrast to most papers on mergers, we find that the outsiders' profits decrease after a merger takes place, which helps us understand why the outsiders to a merger sometimes oppose consolidation processes of rival firms. We finally show that in the long-run consumers may even benefit from consolidation in the marketplace because of the benefits arising from lower search frictions.

The literature on the incentives to merge and the aggregate implications of mergers is quite extensive. For a recent survey of the main theoretical and empirical insights see Whinston (2006). A seminal paper in the literature is Salant *et al.* (1983), which demonstrated that mergers are not profitable when firms compete in quantities and offer similar products. This result is referred to as the *merger paradox.* Deneckere and Davidson (1985) showed that price-setting firms selling horizontally differentiated products, other things equal, always have an incentive to merge. In contrast to the Cournot case analyzed by Salant *et al.* (1983), this result arises because price increases of the merging firms, by strategic complementarity, are accompanied by price increases of the non-merging firms. Our paper puts forward *a new merger paradox*, which surprisingly arises under price competition with differentiated products. The underlying reason is based on the impact of price coordination on optimal consumer search, something quite different from the merger paradox of Salant *et al.* (1983), which concerns competition with decision variables that are strategic substitutes.

Since the seminal paper of Williamson (1968), the role of mergers at generating supply-side economies (or cost-synergies) that more than offset the market power effects of consolidation has been

[2] Hewlett Packard and Compaq, whose merger was cleared in 2002, soon started selling each other products in their separate online shops; in recent days, the merged entity has chosen to downplay the Compaq name in its products. KLM, which merged with Air France in a transaction that was cleared subject to conditions in 2004, sells flights operated by Air France in its online shop, and viceversa. Daimler-Benz and Chrysler merged in 1998 but their retail sales largely remained separate. This likely hindered Chrysler's market penetration in Europe and added to the difficulties experienced by the automobile giant short after merging.

the focus of a considerable amount of research. Perry and Porter (1985), Farrell and Shapiro (1990) and McAfee and Williams (1992) explicitly modelled the cost efficiencies that arise from economies of sharing assets in product markets and stated conditions for the so-called *efficiency defense* of mergers. Our paper brings out *a new efficiency argument* in favor of mergers, but based on demand-side rather than on supply-side economies. We show that the economies of search that unfold when the merging firms stock a wider range of products can result in the merging firms becoming prominent in the marketplace, thereby weakening (and sometimes even more than offsetting) their incentives to raise prices above the outsider firms. Altogether, these effects may make a merger welfare-improving.

To the best of our knowledge, the US and EU guidelines do not mention demand-side economies arising from merger activity. By contrast, Section 5.7 of the 2010 Merger Assessment Guidelines of the UK Competition Commission and the Office of Fair Trading acknowledges the importance of demand-side efficiencies in merger control. However, the guidelines mainly focus on cases where consumers buy multiple items and product complementarities are significant: "*Demand-side efficiencies arise if the attractiveness to customers of the merged firm's products increases as a result of the merger. Common examples of demand-side efficiencies include: network effects, pricing effects and 'one-stop shopping'.*[3] The argument in our paper is clearly different. Sizable demand-side efficiencies can *also* arise when products are substitutes and consumers buy a single product, savings in search costs being at the heart of such efficiencies. We hope that this will add to the design of future merger guidelines.

Since in the post-merger market consumers visit merging and non-merging firms in an order that maximizes expected utility, our paper is also related to the recent literature on ordered search. Arbatskaya (2007) studies a market for homogeneous products where the order in which firms are visited is exogenously given. In equilibrium prices must fall as the consumer walks away from the firms visited first. Zhou (2011) considers the case of differentiated products and finds the opposite result. Armstrong *et al.* (2009) study the implications of *prominence* in consumer search markets. In their model, there is a firm that is always visited first and this firm charges lower prices and derives greater profits than the rest of the firms, which are visited randomly after consumers have visited the prominent firm. Zhou (2009) studies the case in which a set of firms, rather than just one, is prominent. In Haan and Moraga-González (2011) firms gain prominence by investing in advertising; they find that an increase in consumer search costs may result in higher advertising efforts and lower firm profits. Armstrong and Zhou (2011) present alternative ways in which firms can become prominent. Our paper shows one other way to gain prominence: merging and stocking the shelves of the merged entity with all the

[3] *Merger Assessment Guidelines* (OFT1254, p.57, 2010). When *network effects* are present, users value a product more highly when it is used by a greater number of other customers. A merger may make the networks of the merging firms compatible with one another and in this way the welfare of consumers will rise. *Pricing effects* arise when complementary products are brought under common ownership, which may result in a decrease in the prices of all products. Gains from *one-stop shopping* arise when consumers have a strong preference for buying a range of products from a single supplier.

products of the parent firms. Interestingly, we show that this business reorganization is only profitable if search costs are relatively high.

Our paper is also related to a strand of the consumer search literature dealing with firm entry and choice of location, where consumer search economies also play a central role. In Stahl (1982) and Wolinsky (1983) savings in search costs can explain the observed geographical concentration of stores selling differentiated products. Fischer and Harrington (1996) investigate the role of product heterogeneity in explaining interindustry variation in firm agglomeration. Schulz and Stahl (1996) show that economies of scope in search costs can lead to excessive (price-increasing) entry.

Finally, our paper also contributes to the literature on the nature of *multiproduct* firm pricing in the presence of search frictions. While our paper focuses on situations where consumers buy one of the products only, this literature has centered around models where consumers buy various products and prefer to concentrate their purchases within a single supplier. Klemperer (1992) shows that in these situations firms may prefer head-to-head competition over product-line differentiation and Klemperer and Padilla (1997) demonstrate that search cost economies can lead to excessive product-line variety. Rhodes (2012) studies the pricing strategy of a monopolist selling an array of independent products. He demonstrates that when a retailer sells enough products, the Diamond's (1971) hold-up problem disappears. Zhou (2012) also examines pricing by multiproduct firms selling independent products. Interestingly, he finds that equilibrium prices are lower than the prices that single-product firms would set; moreover, he shows that prices can decrease with search costs. [4]

The remainder of the paper is organized as follows. Section 2 describes the consumer search model and the benchmark pre-merger market equilibrium. Section 3 focuses on the effects of price coordination between the merging parties. Section 4 extends the analysis by allowing the merged entity to stock all the products of the constituent firms. Section 5 discusses the main results and studies conditions under which the merged entity prefers to keep selling its products in separate shops. Section 6 concludes. The main proofs are placed in an appendix to ease the reading of the paper.[5]

2 The model and the pre-merger symmetric equilibrium

We study mergers in Wolinsky's (1986) model of consumer search for differentiated products. On the supply side of the market there are n firms selling horizontally differentiated products. All firms use the same constant returns to scale technology of production and we normalize unit production costs to zero. Firms compete in prices and they choose them simultaneously. On the demand side of the market, there is a unit mass of consumers. A consumer has tastes described by the following

[4] See also Shelegia (2012), which studies a model where some consumers exogenously visit all shops and others visit only one. In equilibrium prices are dispersed and when the products are substitutes like in our model their prices are uncorrelated.

[5] For the rest of the proofs, we refer the reader to our working paper Moraga-González and Petrikaitė (2013).

indirect utility function: $u_i = \varepsilon_i - p_i$, if she buys product i at price p_i. The parameter ε_i can be thought of as a match value between the consumer and product i. We assume that the match value ε_i is the realization of a random variable uniformly distributed on $[0, 1]$.[6] Match values are independently distributed across consumers and products. Moreover, they are private information of consumers so personalized pricing is not possible. For later reference, it is useful to calculate the optimal price of a multi-product monopolist selling k varieties, which we denote by p_k^m. This price maximizes the expression $p(\Pr[z_k \geq p])$, where $z_k \equiv \max\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_k\}$, and is given by $p_k^m = (1+k)^{-\frac{1}{k}}$. Setting $k = 1$ we get the price of the single-product monopolist, which we simply denote by p^m.

Consumers search sequentially with costless recall. At all times, consumers have correct beliefs about the equilibrium prices.[7] Each search costs the consumer s. To avoid that a market equilibrium fails to exist (Diamond, 1971), we assume throughout that the first search is not prohibitively costly. When $s = 0$ the model is similar to Perloff and Salop (1985) and the effects of mergers in that case are similar to Deneckere and Davidson (1985).

The pre-merger market equilibrium

As a benchmark case, we characterize here the pre-merger market symmetric equilibrium.[8] Assume that search cost $s \in [0, 1/8]$.[9] Let p^* denote the symmetric equilibrium price charged by firms other than firm i and consider the (expected) payoff to a firm i that deviates by charging a price $p_i \neq p^*$, with $p_i < 1 - \overline{x} + p^*$. In order to compute firm i's demand, we need to characterize consumer search behavior. Since consumers do not observe deviations before searching, we can rely on Kohn and Shavell (1974), who study the search problem of a consumer who faces a set of independently and identically distributed options with known distribution. Kohn and Shavell show that the optimal search rule is static in nature and has the stationary reservation utility property. Accordingly, denote the solution to

$$\int_x^1 (\varepsilon - x) d\varepsilon = s \tag{1}$$

by $\overline{x}(= 1 - \sqrt{2s})$. The left-hand-side (LHS) of (1), which is equal to $(1-x)^2/2$, is the expected benefit in symmetric equilibrium from searching one more time for a consumer whose best option so far is

[6] The uniform distribution is adopted for simplicity, specially in the subsequent analysis of mergers. In an earlier working paper that contains part of the analysis here, Moraga-González and Petrikaitė (2011), we show that alternative distributions give similar results.

[7] Though we can view this assumption as a standard Nash assumption, its rationalization requires some sophistication on the part of consumers. In fact, we shall assume that consumers do know the ownership structure of the firms, their equilibrium prices and the number of products sold in each of the establishments.

[8] We note that asymmetric equilibria may be sustained in this model. The idea is that if consumers believe that firms' prices follow a given ranking, say, $p_1 < p_2 < ... < p_n$, then it is optimal for consumers to start their search at firm 1, continue at firm 2, and so on, and for firms to price in such a way so as to make consumer beliefs consistent with equilibrium. The unattractive feature of these equilibria is that they are not determined by the underlying characteristics of the market, but by an indeterminacy of consumer beliefs. In general, we will ignore asymmetric equilibria in our paper.

[9] In order to make sure that the first search is always worthwhile, we take the worst case scenario where consumers expect the firms to charge the monopoly price. Therefore, we require that $s \leq \Pr[\varepsilon \geq p^m] E[\varepsilon - p^m | \varepsilon \geq p^m]$, where E denotes the expectation operator. This is equivalent to requiring $s \leq \overline{s} \equiv (1 - p^m)^2/2$. Since $p^m = 1/2$, $s \leq 1/8$ suffices.

x. Its right-hand-side (RHS) is the cost of search. Therefore, $\overline{x}$ represents the threshold match value above which a consumer will optimally decide not to continue searching for another product. The number $\overline{x} - p^*$ is referred to as the reservation utility for visiting a firm. Since $s \in [0, 1/8]$, we have that $\overline{x} \in [1/2, 1]$ and, correspondingly, $\overline{x} - p^* \geq 0$.

In order to compute firm i's demand, consider a consumer who visits firm i in her h^{th} search (after having walked away from $h-1$ other firms), $h = 1, 2, ..., n$. Since consumers expect all firms to charge the same price p^*, the probability firm i is in h^{th} position is $1/n$. Let $\varepsilon_i - p_i$ denote the utility the consumer derives from the product of firm i. Suppose $\varepsilon_i - p_i \geq \max\{z_{h-1} - p^*, 0\}$ for otherwise the consumer would not buy product i. The expected gains from searching one more firm, say j, are equal to $\int_{\varepsilon_i - p_i + p^*}^{1} [\varepsilon_j - (\varepsilon_i - p_i + p^*)] d\varepsilon_j$. Comparing this to (1), it follows that, conditional on the deviant firm being in h^{th} position, the probability that the buyer visits firm i and stops searching at firm i is equal to $\Pr[\varepsilon_i - p_i > \overline{x} - p^* > z_{h-1} - p^*] = \overline{x}^{h-1}(1 - \overline{x} - p_i + p^*)$. Summing the unconditional probability for all h, we obtain a demand equal to $\frac{1-\overline{x}^n}{n(1-\overline{x})}(1 - \overline{x} - p_i + p^*)$.

The consumer also buys the product of firm i when she walks away from it, walks away from the rest of the firms in the market and happens to return to firm i because such a firm offers her the best deal after all. Conditional on the deviant firm being in h^{th} position, this occurs with probability $\Pr[\max\{0, z_{n-1} - p^*\} < \varepsilon_i - p_i < \overline{x} - p^*]$.[10] Summing the unconditional probability for all h, we obtain a demand from returning consumers equal to

$$r_a(p_i; p^*) \equiv \int_{p_i}^{\overline{x}+p_i-p^*} (\varepsilon - p_i + p^*)^{n-1} d\varepsilon = \int_0^{\overline{x}-p^*} (\varepsilon + p^*)^{n-1} d\varepsilon = \frac{1}{n}(\overline{x}^n - p^{*n}), \tag{2}$$

where the notation $r_a(p_i; p^*)$ is to indicate that these sales originate from consumers who buy from firm i after having visited *all* the firms in the market.

We can now write firm i's expected profits:

$$\pi_i(p_i; p^*) = p_i \left[\frac{1 - \overline{x}^n}{n(1 - \overline{x})}(1 - \overline{x} - p_i + p^*) + r_a(p_i; p^*) \right]. \tag{3}$$

We look for a symmetric Nash equilibrium in prices. Since the payoff in (3) is strictly concave, the first-order condition (FOC) suffices for a maximum. After applying symmetry, i.e., $p_i = p^*$, the FOC is:

$$1 - p^{*n} - p^* \frac{1 - \overline{x}^n}{1 - \overline{x}} = 0. \tag{4}$$

It is easy to check that (4) has a unique solution that satisfies $\overline{x} \geq p^* \geq 1 - \overline{x}$. To ensure that p^* is indeed an equilibrium, we need to check that firms do not have an incentive to deviate from it. Because of the strict concavity of (3), deviations such that $p_i < 1 - \overline{x} + p^*$ are clearly not profitable. Suppose now that the deviant firm charges a price $p_i \geq 1 - \overline{x} + p^*$. In such a case, consumers always

[10] Because the reservation utility is stationary, no consumer who walks away from a firm will return to such a firm without first having visited all the firms in the market.

walk away from the deviant firm no matter the position in which they visit it for the first time. As a result, firm i only sells to those consumers who have visited all firms. The deviant profits become $\pi_i(p_i;p^*) = p_i \int_{p_i}^{1} (\varepsilon - p_i + p^*)^{n-1} d\varepsilon$ and the deviation is not profitable either.[11]

The profits of a typical firm in the pre-merger situation are

$$\pi^* = \frac{1}{n} p^*(1 - p^{*n}). \tag{5}$$

It is readily seen that the equilibrium price and profits increase in search cost s.

Next we study the impact of a merger of an arbitrary number of firms. We first focus on the effects of a merger on the prices of insiders and outsiders and optimal consumer search. Then we examine whether the merging firms have indeed an incentive to merge and the welfare implications of mergers. The analysis is divided into two parts. In section 3, we study the effects of mergers abstracting from any source of efficiency gains. In this sense, the focus in section 3 is on the effects of joint (price) decision-making, exactly as in Deneckere and Davidson (1985). In section 4, we study the effects of mergers from a medium- to long-run perspective; there we let the merged entity stock the shelves of its shops with all the products of the parent firms. Finally, in Section 5 we examine the merged entity's incentives to undertake such a business reorganization.

3 Effects of mergers in the short-run

Consider that k firms merge, with $2 \leq k \leq n-1$. Let us continue to focus on symmetric equilibria in the sense that all non-merging firms charge a price denoted by $\tilde{p}^*$ and all merging firms a price denoted by $\hat{p}^*$. Since a non-merging firm controls the price of a single variety, we let $\tilde{p}^* \in [0, p^m]$; the merged entity, by contrast, controls the price of k varieties and, correspondingly, we let $\hat{p}^* \in [0, p_k^m]$. We maintain the assumption that $s \in [0, (1-(1+k)^{-1/k})^2/2]$.[12]

We start by assuming that the merging firms charge a higher price than the non-merging firms, i.e., $\tilde{p}^* < \hat{p}^*$. A priori, this is a reasonable conjecture because the merging firms internalize the pricing externalities they impose upon one another. In Section 5, however, we study whether an alternative symmetric equilibrium exists where the merging firms charge a *lower* price than the non-merging firms.

Given that consumers correctly expect equilibrium prices to satisfy the inequality $\tilde{p}^* < \hat{p}^*$, they now face a problem of search where the set of available options have known, independent but non-identical utility distributions. Weitzman (1979) shows that also in such a case the optimal decision

[11] Because of log-concavity of the uniform density function, this profits expression is quasi-concave in own price (Caplin and Nalebuff, 1991). Taking the derivative of the deviating profits with respect to p_i and setting $p_i = p^*$, we get $d\pi_i/dp_i|_{p_i=p^*} = (1-p^{*n}-np^*)/n < 0$, where the inequality follows from the fact that p^* solves (4). Since deviating profits are quasi-concave and they decrease at $p_i = p^*$, we conclude they are even lower at prices p_i such that $p_i \geq 1 - \overline{x} + p^*$.

[12] Again, we assume that consumers find it worthwhile to make a first search even if they expect firms to charge the monopoly price. Since $p_k^m > p^m$ for all k, in order to ensure that the merged entity has a positive market share we require that $s \leq \Pr[\varepsilon \geq p_k^m] E[\varepsilon - p_k^m | \varepsilon \geq p_k^m]$.

rule is static in nature and has the reservation utility property. At every step of the optimal search process, a consumer should consider visiting next the (not-yet-visited) shop for which her reservation utility is the highest; moreover, a consumer should terminate her search whenever the maximum utility obtained so far is higher than the reservation utility at the shop to be visited next. Because the utility distributions of the various options are non-identical, the reservation utility is non-stationary and, consequently, the probability consumers return to one of the non-merging firms without checking the products of the merging ones is strictly positive.

Let $\overline{x}$ be given by (1). The number $\overline{x} - \hat{p}^*$ defines the reservation utility for searching the product of a merging firm. Likewise, $\overline{x} - \tilde{p}^*$ is the reservation utility for searching the product of a non-merging store. Since $\tilde{p}^* < \hat{p}^*$, Weitzman's (1979) results prescribe consumers to start searching for a satisfactory product at the non-merging firms. If no alternative is found to be good enough in those firms, buyers should continue searching for a fine product at the merging firms. This implies that the post-merger demands of the two types of stores (merging and non-merging) are related to the demands derived by Zhou (2009) in his paper on prominent and non-prominent firms.[13]

To calculate the post-merger equilibrium prices, we proceed by computing the payoff that merging and non-merging firms would obtain when deviating from the equilibrium prices. While deriving the payoffs of the two types of (deviating) firms, we require consumer expectations about the prices charged by firms not yet visited to be equal to the equilibrium prices.

Payoff to a deviant non-merging store.

Consider a non-merging store j that deviates by charging a price $\tilde{p} \neq \tilde{p}^*$, with $\tilde{p} < 1 - \overline{x} + \tilde{p}^*$. As consumers expect all non-merging firms to charge $\tilde{p}^*$, they visit them randomly. The deviant firm may thus be visited in first place, second place and so on till the $(n-k)^{th}$ place, each position occurring with probability $1/(n-k)$. Because of the non-stationarity of the reservation utility, it is convenient to distinguish among consumers who visit the deviant in their 1st, 2nd, ..., or $(n-k-1)^{th}$ search and consumers who visit it in their $(n-k)^{th}$ search.

Consider then first the case in which a consumer visits the deviant non-merging firm j in her h^{th} search, with $h = 1, 2, ..., n-k-1$. Suppose the deal the consumer observes upon entering the deviant's shop is $\varepsilon_j - \tilde{p}$. There are three circumstances in which the consumer will buy the product of the deviant. First, the consumer may stop searching at this shop and buy there right away. Conditional on the deviant being in the h^{th} position, this occurs with probability $\Pr[z_{h-1} - \tilde{p}^* < \overline{x} - \tilde{p}^* < \varepsilon_j - \tilde{p}]$, which is equal to $\overline{x}^{h-1}(1 - \overline{x} + \tilde{p}^* - \tilde{p})$. Summing the unconditional probability for all h, we get a

[13] In Zhou's (2009) paper consumers search for a satisfactory product first at the prominent firms; if they do not find there a product they like enough, they proceed by searching at the non-prominent firms. The payoff of a non-merging firm here is exactly identical to the payoff of a prominent firm in his paper. However, the payoff of the merged entity is clearly different from the payoff of a non-prominent firm in Zhou's paper.

demand equal to $\frac{1-\overline{x}^{n-k-1}}{(n-k)(1-\overline{x})}\left(1-\overline{x}+\tilde{p}^{*}-\tilde{p}\right)$.

Second, the consumer may walk away from the deviant non-merging firm and come back to it after having visited the rest of the non-merging stores. Conditional on the deviant being in h^{th} place, this occurs with probability $\Pr[\max\{z_{n-k-1}-\tilde{p}^{*},\overline{x}-\hat{p}^{*}\}<\varepsilon_j-\tilde{p}<\overline{x}-\tilde{p}^{*}]$. To indicate that these sales originate from consumers who buy from the deviant non-merging firm j after having visited all the non-merging stores, we denote this conditional probability by

$$\tilde{r}_{nm}(\tilde{p};\tilde{p}^{*},\hat{p}^{*})\equiv\int_{\overline{x}-\hat{p}^{*}+\tilde{p}}^{\overline{x}-\tilde{p}^{*}+\tilde{p}}\left(\varepsilon-\tilde{p}+\tilde{p}^{*}\right)^{n-k-1}d\varepsilon=\frac{1}{n-k}\left[\overline{x}^{n-k}-\left(\overline{x}-\hat{p}^{*}+\tilde{p}^{*}\right)^{n-k}\right].$$

Summing the unconditional probability for all h, we get a demand equal to $\frac{n-k-1}{n-k}\tilde{r}_{nm}(\tilde{p};\tilde{p}^{*},\hat{p}^{*})$.

Finally, the consumer may walk away from the deviant non-merging firm and come back to it after having visited the rest of firms in the market. Conditional on the deviant being in h^{th} position, this occurs with probability $\Pr[\max\{z_{n-k-1}-\tilde{p}^{*},z_k-\hat{p}^{*},0\}<\tilde{\varepsilon}_j-\tilde{p}<\overline{x}-\hat{p}^{*}]$. To indicate that these are the sales to consumers who buy from the deviant non-merging firm j after having visited *all* the stores in the market, we denote this conditional probability by

$$\tilde{r}_{a}(\tilde{p};\tilde{p}^{*},\hat{p}^{*})\equiv\int_{\tilde{p}}^{\overline{x}-\hat{p}^{*}+\tilde{p}}\left(\varepsilon+\tilde{p}^{*}-\tilde{p}\right)^{n-k-1}\left(\varepsilon+\hat{p}^{*}-\tilde{p}\right)^{k}d\varepsilon=\int_{0}^{\overline{x}-\hat{p}^{*}}\left(\varepsilon+\tilde{p}^{*}\right)^{n-k-1}\left(\varepsilon+\hat{p}^{*}\right)^{k}d\varepsilon. \quad (6)$$

Summing the unconditional probability for all h, we get a demand equal to $\frac{n-k-1}{n-k}\tilde{r}_{a}(\tilde{p};\tilde{p}^{*},\hat{p}^{*})$.

We now consider the case in which the consumer visits the deviant non-merging firm in her $(n-k)^{th}$ search. There are two situations in which the consumer will buy the deviant's product. First, the consumer may stop searching at the deviant's shop and buy there right away. Conditional on visiting the deviant in $(n-k)^{th}$ place, this occurs with probability $\Pr[\varepsilon_j-\tilde{p}\geq\max\{z_{n-k-1}-\tilde{p}^{*},\overline{x}-\hat{p}^{*}\}$ and $z_{n-k-1}<\overline{x}]$, which gives a demand equal to:

$$\overline{x}^{n-k-1}\left(1-\overline{x}+\tilde{p}^{*}-\tilde{p}\right)+\frac{1}{n-k}\left[\overline{x}^{n-k}-\left(\overline{x}-\hat{p}^{*}+\tilde{p}^{*}\right)^{n-k}\right].$$

Second, the consumer may walk away from the deviant firm and come back to it after having visited the rest of firms in the market. In this second case we have exactly the same expression for returning consumers as in (6).

Adding the demands above for $h=1,2,...,n-k-1$ to the demand for the case in which the consumer visits the deviant in her $(n-k)^{th}$ search and simplifying, we get the profits of a deviant non-merging firm:

$$\tilde{\pi}(\tilde{p};\tilde{p}^{*},\hat{p}^{*})=\tilde{p}\left[\frac{1}{n-k}\frac{1-\overline{x}^{n-k}}{1-\overline{x}}\left(1-\overline{x}+\tilde{p}^{*}-\tilde{p}\right)+\tilde{r}_{nm}(\tilde{p};\tilde{p}^{*},\hat{p}^{*})+\tilde{r}_{a}(\tilde{p};\tilde{p}^{*},\hat{p}^{*})\right]. \quad (7)$$

Payoff to a deviant merged entity.

The merged entity chooses its prices to maximize the joint profit of the k partner firms. Therefore, the vector of prices $(\hat{p}^{*},\hat{p}^{*},...,\hat{p}^{*})$ is part of a symmetric equilibrium if the merged entity does not have an incentive to deviate by choosing a different set of prices $(p_1, p_2, ..., p_k)$. It can readily be seen that for the purpose of writing the FOC at a symmetric equilibrium, it is enough to write down the payoff of a merged entity that deviates by charging a different price for one of its products only. We next compute such a payoff and relegate the issue of existence and uniqueness of symmetric equilibrium to the appendix.

Consider that the merged entity deviates by charging a price $\hat{p} \neq \hat{p}^*$ for its product i, with $\hat{p} < 1 - \overline{x} + \hat{p}^*$. The deviation affects not only the demand for product i but also the demand for the other $k-1$ products controlled by the merged entity. Let us start by computing the demand for product i. Since $\tilde{p}^* < \hat{p}^*$ in equilibrium, consumers only contemplate visiting the stores of the merged entity after having visited all the non-merging firms. The probability consumers search product i in position $h = 1, 2, ..., k$ (after the non-merging firms) is $1/k$. Take now a consumer who searches product i in her h-th search and denote the deal she gets there by $\varepsilon_i - \hat{p}$. There are two cases in which the consumer will buy product i. First, the consumer may stop searching and buy product i right away. Conditional on firm i being in h^{th} position, this occurs with probability $\Pr[\max\{z_{n-k} - \tilde{p}^*, z_{h-1} - \hat{p}^*\} < \overline{x} - \hat{p}^* < \varepsilon_i - \hat{p}]$, which gives a demand equal to $\overline{x}^{h-1} (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} (1 - \overline{x} + \hat{p}^* - \hat{p})$. Summing the unconditional probability for all h, we obtain a demand equal to $\frac{1-\overline{x}^k}{k(1-\overline{x})} (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} (1 - \overline{x} + \hat{p}^* - \hat{p})$.

Second, the consumer may walk away from the firm selling product i and come back to it after visiting the rest of the merging firms. Conditional on visiting firm i in h^{th} place, this happens with probability $\Pr[\max\{z_{n-k} - \tilde{p}^*, z_{k-1} - \hat{p}^*, 0\} < \varepsilon_i - \hat{p} < \overline{x} - \hat{p}^*]$. Using a similar notation as above, we denote this probability by

$$\hat{r}_{ia}(\hat{p}; \tilde{p}^*, \hat{p}^*) \equiv \int_{\hat{p}}^{\overline{x} - \hat{p}^* + \hat{p}} (\varepsilon - \hat{p} + \tilde{p}^*)^{n-k} (\varepsilon - \hat{p} + \hat{p}^*)^{k-1} d\varepsilon = \int_0^{\overline{x} - \hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k} (\varepsilon + \hat{p}^*)^{k-1} d\varepsilon.$$

Summing the unconditional probability for all h, we get a demand equal to $\hat{r}_{ia}(\hat{p}; \tilde{p}^*, \hat{p}^*)$.

Putting terms together and simplifying, we obtain the demand for product i:

$$d_i(\hat{p}; \tilde{p}^*, \hat{p}^*) = \frac{(1 - \overline{x}^k)}{k(1 - \overline{x})} (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} (1 - \overline{x} + \hat{p}^* - \hat{p}) + \hat{r}_{ia}(\tilde{p}^*, \hat{p}^*).$$

The deviation also affects the merged entity's demand for products other than i. Let us compute next the demand for one of the other products, say product m. Suppose firm m is visited by a consumer in her h^{th} search, $h = 1, 2, ..., k$, which happens with probability $1/k$. Note that the probability that the deviant's product i has not yet been inspected by the consumer is $(k-h)/(k-1)$. Conditional on the consumer visiting firm m in h^{th} place and on not having yet visited firm i, the consumer will stop searching at firm m and buy there right away with probability $\Pr[\varepsilon_m - \hat{p}^* > \overline{x} - \hat{p}^* > \max\{z_{n-k} - \tilde{p}^*, z_{h-1} - \hat{p}^*\}]$, which is equal to $\frac{k-h}{k-1} (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} (1 - \overline{x}) \overline{x}^{h-1}$. Summing the unconditional probability for $h = 1, 2, ..., k$ we obtain a demand for product m equal to

$$(\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} (1 - \overline{x}) \sum_{h=1}^{k-1} \frac{k-h}{k(k-1)} \overline{x}^{h-1}.$$

With probability $(h-1)/(k-1)$, the deviant's product i has already been checked by the consumer. Conditional on visiting firm m in h^{th} place and on having already inspected product i, the consumer will stop searching at firm m and buy there right away with probability $\Pr[\varepsilon_m - \hat{p}^* > \overline{x} - \hat{p}^* > \max\{z_{n-k} - \tilde{p}^*, z_{h-2} - \hat{p}^*, \varepsilon_i - \hat{p}\}]$, which equals $(\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k}(\overline{x} - \hat{p}^* + \hat{p})(1-\overline{x})\overline{x}^{h-2}$. Summing the unconditional probability for $h = 1, 2, ..., k-1$ we obtain a demand for product m equal to

$$(\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k}(\overline{x} - \hat{p}^* + \hat{p})(1-\overline{x})\sum_{h=1}^{k}\frac{h-1}{k(k-1)}\overline{x}^{h-2}.$$

The consumer may also buy at firm m if she walks away from it and happens to return to it after having visited the rest of the merging firms. Conditional on firm m being visited in h^{th} position, this happens with probability $\Pr[\max\{z_{n-k} - \tilde{p}^*, z_{k-2} - \hat{p}^*, \varepsilon_i - \hat{p}, 0\} < \varepsilon_m - \hat{p}^* < \overline{x} - \hat{p}^*]$. We will denote this probability by

$$\hat{r}_{ma}(\hat{p}; \tilde{p}^*, \hat{p}^*) \equiv \int_{\hat{p}^*}^{\overline{x}}(\varepsilon - \hat{p}^* + \tilde{p}^*)^{n-k}(\varepsilon - \hat{p}^* + \hat{p})\varepsilon^{k-2}d\varepsilon = \int_0^{\overline{x}-\hat{p}^*}(\varepsilon + \tilde{p}^*)^{n-k}(\varepsilon + \hat{p}^*)^{k-2}(\varepsilon + \hat{p})\,d\varepsilon.$$

Summing the unconditional probability for $h = 1, 2, ..., k$ gives $\hat{r}_{ma}(\hat{p}; \tilde{p}^*, \hat{p}^*)$.

Putting together the above demands gives the following total demand for product m:

$$\begin{aligned} d_m(\hat{p}; \tilde{p}^*, \hat{p}^*) = &\frac{k(1-\overline{x}) - (1-\overline{x}^k)}{k(k-1)(1-\overline{x})}(\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} + \\ &\frac{1-\overline{x}^k - k\overline{x}^{k-1}(1-\overline{x})}{k(k-1)(1-\overline{x})}(\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k}(\overline{x} - \hat{p}^* + \hat{p}) + \hat{r}_{ma}(\hat{p}; \tilde{p}^*, \hat{p}^*). \end{aligned}$$

Since the demands for the other products of the merged entity are the same, the payoff function of the deviant merged entity is

$$\hat{\pi}(\hat{p}; \tilde{p}^*, \hat{p}^*) = \hat{p}d_i(\hat{p}; \tilde{p}^*, \hat{p}^*) + (k-1)\hat{p}^* d_m(\hat{p}; \tilde{p}^*, \hat{p}^*) \tag{8}$$

Results

Taking the first order derivative of the payoff in (7) with respect to the deviation price $\tilde{p}$ and applying symmetry yields the following FOC for the non-merging firms:

$$1 - \frac{1-\overline{x}^{n-k}}{1-\overline{x}}\tilde{p}^* - (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} + (n-k)\int_0^{\overline{x}-\hat{p}^*}(\varepsilon + \tilde{p}^*)^{n-k-1}(\varepsilon + \hat{p}^*)^k\,d\varepsilon = 0 \tag{9}$$

Likewise, the FOC for the merged entity is:

$$(\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k}\left(1 - \overline{x}^k - k\hat{p}^*\overline{x}^{k-1}\right) + k\int_0^{\overline{x}-\hat{p}^*}(\varepsilon + \tilde{p}^*)^{n-k}(\varepsilon + \hat{p}^*)^{k-2}(\varepsilon + k\hat{p}^*)\,d\varepsilon = 0 \tag{10}$$

Proposition 1 *Assume that k firms merge. Then, for any s, there exists a unique symmetric Nash equilibrium in the short-run post-merger market where:*

- *Consumers start searching at the non-merging stores and then, if they wish so, they proceed by searching at the merged ones.*

- *Merging firms charge a price $\hat{p}^*$ and the non-merging stores charge a price $\tilde{p}^*$; these prices are given by the unique solution to the system of FOCs (9)-(10) and the price ranking is consistent with consumer search behavior, that is, $\hat{p}^* > \tilde{p}^*$.*

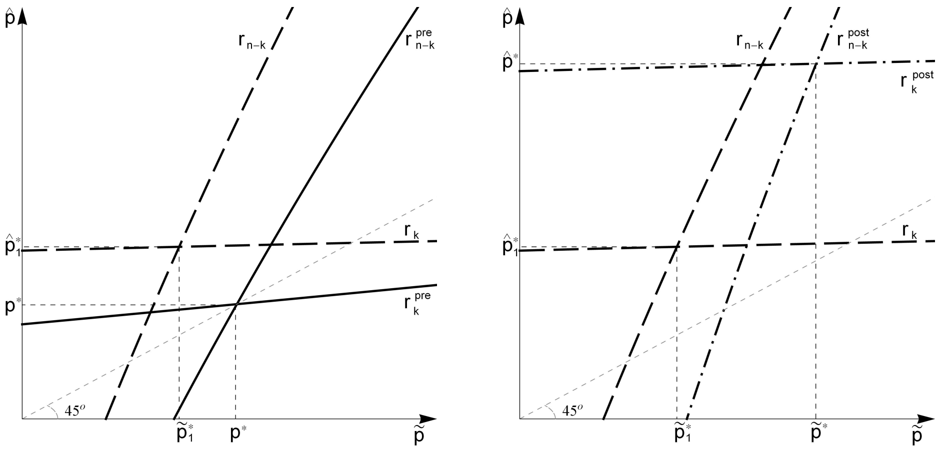
The proof of this Proposition has the following steps. We first show that there exists a unique pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies the FOCs (9)-(10). We then show that these prices satisfy the inequality $\hat{p}^* > \tilde{p}^*$, which immediately implies that the prescribed consumer search behavior is optimal. We finally check that no firm gains by deviating from the symmetric equilibrium prices.

When some firms merge, two effects take place. On the one hand, since consumers expect the insiders to charge higher prices than the outsiders, consumers place the merging firms all the way back in the queue they follow when they search for satisfactory products. On the other hand, as usual when firms merge, there is an internalization-of-pricing-externalities effect. These two effects take place simultaneously and it is illustrative to separate them in a graph.

Following Deneckere and Davidson (1985), because the prices of similar firms are identical, the effects of a merger can be illustrated in a two-dimensional graph. In Figure 1a, the crossing point between the solid curves gives the pre-merger equilibrium. The line r_k^{pre} (r_{n-k}^{pre}) is the joint reaction of the *potential* insiders (outsiders) to a price $\tilde{p}$ ($\hat{p}$) charged by the outsiders (insiders), given consumer beliefs that the equilibrium price is p^*. The curves cross the 45 degrees line at p^* so both types of firms charge p^* and consumers' expectations are fulfilled.

When a merger takes place, by the *search-order effect*, consumers start their search for satisfactory products at the shops of the outsiders. Equilibrium pricing in a similar market where some firms are visited first by consumers has been studied by Zhou (2009). In his paper, consumers search first the products of the so-called "prominent" firms and if they do not find a satisfactory product there they continue by searching the products of the so-called "non-prominent" firms. The line r_k (r_{n-k}) in Figure 1a is the joint reaction of the non-prominent (prominent) firms to a price $\tilde{p}$ ($\hat{p}$) charged by the prominent (non-prominent) ones, given consumer beliefs that the equilibrium prices are Zhou's equilibrium ones. As we can see, by the search-order effect, the joint reaction curve of the insiders (outsiders) shifts upwards (leftwards) from r_k^{pre} (r_{n-k}^{pre}) to r_k (r_{n-k}). These moves capture the fact that, relative to the pre-merger situation, the insiders' demand becomes more inelastic while the outsiders' demand turns more elastic. The crossing point between the dashed curves r_k and r_{n-k}, denoted $\{\tilde{p}_1^*, \hat{p}_1^*\}$, gives Zhou's equilibrium.

The effects of price coordination among the insiders are shown in Figure 1b. The line r_k^{post} (r_{n-k}^{post}) is the joint reaction of the insiders (outsiders) to a price $\tilde{p}$ ($\hat{p}$) charged by the outsiders (insiders), given consumer beliefs that the equilibrium prices are $\hat{p}^*$ and $\tilde{p}^*$. The internalization-of-pricing-externalities effect is captured by the shift from r_k (r_{n-k}) to r_k^{post} (r_{n-k}^{post}). The post-merger equilibrium is given by



(a) Search-order effect (b) Internalization of pricing externalities

Figure 1: The price effects of mergers in the short-run ($n = 3, k = 2$).

the crossing point of the two dotted-dashed curves, where consumer expectations are also fulfilled.[14]

Whether the post-merger prices are higher or lower than the pre-merger price is a priori ambiguous. The merger confers the non-merging stores a prominent position in the marketplace. This provides the non-merging firms with incentives to lower their prices because when a firm becomes prominent its pool of consumers becomes more elastic. The merging firms, by contrast, tend to raise their prices because consumers postpone visiting them and, correspondingly, their demands become more inelastic. This, by strategic complementarity, gives the non-merging firms incentives to raise their prices too. In addition, because the merging firms internalize the pricing externalities they impose on one another, they raise their price even further, which, again by strategic complementarity, pushes the prices of the non-merging firms up. Our next proposition shows that when the search-order effects are not very strong then all prices increase after a merger.[15]

Proposition 2 *In the short-run post-merger equilibrium of Proposition 1, the ranking of pre- and post-merger equilibrium prices is* $p^* < \tilde{p}^* < \hat{p}^*$ *whenever one of the following conditions holds: (a) the search cost is sufficiently low, (b) the search cost is sufficiently high, (c) the number of firms* $n = 3$.

As expected, the case in which search cost is small reproduces naturally the situation in Deneckere and Davidson (1985). However, as search costs increase, fewer consumers walk away from the non-merging stores and visit the merged ones. This fall in consumer traffic from the outsiders to the insiders has important consequences for merger profitability.

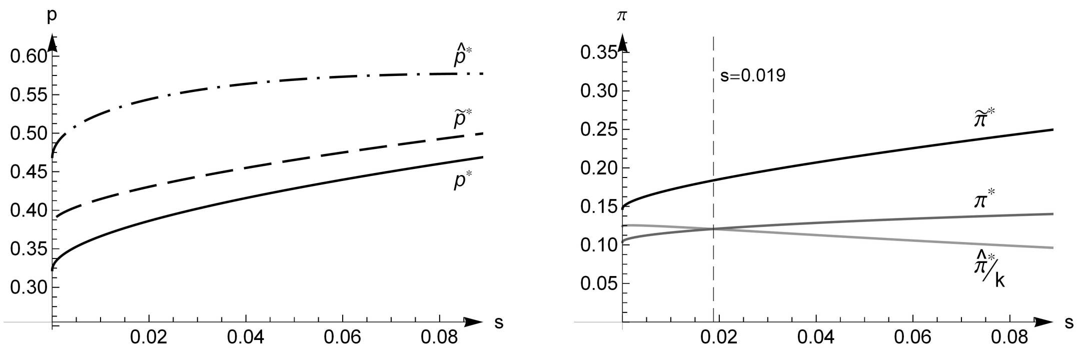
[14]Normally the internalization-of-pricing-externalities effect only changes the reaction curve of the insiders. Here, the reaction function of the outsiders also moves because of the change in consumer expectations.

[15]We note, however, that solving numerically the model we have found no instance in which this does not happen.

Proposition 3 *In the short-run post-merger equilibrium of Proposition 1: (a) Any 2-firm merger is not profitable if the search cost is sufficiently high. (b) Any k-firm merger is not profitable if the search cost and the number of competitors are sufficiently high. (c) If the search cost is sufficiently small, any k-firm merger is profitable.*

Proposition 3 shows that, unless there are many firms in the industry and the merger comprises almost all of them, eventually as the search cost becomes relatively high merging is not profitable for the merging firms. The interest of this Proposition is that it puts forward a new *merger paradox*, which arises under price competition with differentiated products. The underlying reason is based on consumer search costs, something quite different from the merger paradox of Salant *et al.* (1983), which concerns competition with decision variables that are strategic substitutes.[16]

Propositions 2 and 3 are illustrated in Figure 2, where we plot the post-merger prices and profits against search costs. For comparison purposes, we also plot the pre-merger price and profits. Figure 2a shows that all prices are increasing in search costs because when searching becomes more costly, an individual firm has more market power over the consumers who visit it.[17] As the graph reveals, post-merger prices, whether from merging or non-merging firms, are *higher* than the pre-merger price.



(a) Prices (b) Profits

Figure 2: Pre- and post-merger prices, and merger profitability ($n = 3, k = 2$).

Figure 2b shows that the profits of a typical merging firm, $\hat{\pi}^*/k$, decline as search cost goes up. The reason is that, as the search cost increases, fewer consumers walk away from the non-merging firm in order to check the products of the merged entity. This has a major implication on merger profitability: for search costs approximately above 0.019 (about 3.8% of the average value of a firm's product), merging is not profitable. The graph also reveals that the non-merging firm "gets a free

[16]We are implicitly assuming that the non-merging firms can absorb the (possibly large) post-merger increase in consumer traffic towards their stores. If firms were capacity constrained, our result would have to be qualified. Moreover, we are abstracting from cost-synergies. If, as in Farrel and Shapiro (1990), the costs of the insiders decreased and their prices fell below those of the outsiders as a result of the merger, then the situation would be quite the opposite.

[17]For a proof of this fact, see our working paper Moraga-González and Petrikaitė (2011).

ride" and that "this ride is freer" the higher the search cost.[18]

4 Effects of mergers in the long-run

In this section we take a medium- to long-term view of mergers and assume that, after a more or less complex business reorganization, the merged entity starts selling the k products of the mother firms in a single shop.[19] For the moment, we take this reorganization as exogenous and study its effects. Later in Section 5 we examine the circumstances under which such a reorganization is profitable for the merging firms.

In this section we maintain the assumption that[20]

$$s \in \left[0, \min\left\{\frac{1}{8}, \frac{k}{k+1}\left[1-(k+2)(k+1)^{-\frac{k+1}{k}}\right]\right\}\right] \tag{11}$$

As before, let $\tilde{p}^*$ and $\hat{p}^*$ denote the symmetric equilibrium prices of the non-merging and merging firms, respectively. Since consumers can try k products at the merged entity, the trade-off they face is clear: relative to the deal offered by a non-merging firm, at the merged entity consumers encounter more variety but probably, though not surely as we will see later, at higher prices.[21]

To characterize the order of search, we invoke again Weitzman's (1979) results. Let $\overline{\overline{x}}$ be the solution to

$$\int_x^1 (\varepsilon - x)k\varepsilon^{k-1}d\varepsilon - s = 0 \tag{12}$$

As in (1), $\overline{\overline{x}}$ represents a threshold match value above which a consumer will decide not to continue searching at the merged entity. Correspondingly, the number $\overline{\overline{x}} - \hat{p}^*$ defines the reservation utility for searching the k products of the merged entity. As before, $\overline{x} - \tilde{p}^*$ is the reservation utility for searching the product of a non-merging store.

Momentarily, assume $\overline{\overline{x}} - \hat{p}^* > \overline{x} - \tilde{p}^*$. Given this assumption, which according to Weitzman's optimal search rule prescribes consumers to start searching at the merged entity, we next calculate the post-merger equilibrium prices. To do this, we proceed by computing the payoffs the merging and non-merging firms would obtain when deviating from the equilibrium prices. After taking the FOCs

[18] The reader may think that our result that mergers are unprofitable for high search costs is driven by the fact that search is random in the pre-merger market while it is directed in the post-merger market. This is not so. If we assumed for example that the potentially merging firms are searched last in the pre-merger market, our result would also hold. For further details, see our working paper Moraga-González and Petrikaitė (2013).

[19] Alternatively, we can assume that the merged entity keeps all the shops open but stocks each of them with the k varieties stemming from the k original merging firms. If there are positive fixed costs of keeping shops open, the first type of business reorganization is more economical. Nevertheless, up to the fixed costs, it is easy to see that both alternatives yield exactly the same equilibrium payoff.

[20] Once again we need to make sure that both types of firm obtain a positive market share. For a consumer to visit a non-merging firm, the condition $s \leq 1/8$ suffices. However, if a consumer contemplates to visit the merged entity, it must be the case $s \leq \Pr[z_k \geq p_k^m]E[z_k - p_k^m \mid z_k \geq p_k^m]$. Using the facts that $p_k^m = (k+1)^{-1/k}$ and the distribution of z_k is ε^k, we obtain the RHS expression inside the curly brackets in (11).

[21] The prices set by the merged entity need not be higher than the prices of the non-merging firms. While the merged entity internalizes the pricing externalities between its products and this tends to raise its prices, the fact that this firm is being visited first in the marketplace tends to lower them.

and applying symmetry, we check if the inequality $\overline{\overline{x}} - \hat{p}^* > \overline{x} - \tilde{p}^*$ indeed holds at the solution of the system of FOCs. Later in Section 5 we prove that the symmetric equilibrium we derive here is the unique symmetric equilibrium provided that search costs are sufficiently large.

Payoff to a deviant merged entity.

Consider that the merged entity deviates from equilibrium by setting a vector of prices $(p_1, p_2, ..., p_k) \neq (\hat{p}^*, \hat{p}^*, ..., \hat{p}^*)$. As before, for the purpose of writing out the FOC in a symmetric equilibrium, it is sufficient to consider the payoff of a merged entity that deviates by picking a different price for just one of its varieties, say i. We now write such a payoff for the case where the deviation price $\hat{p} < \min\{\hat{p}^*, 1 - \overline{x} + \tilde{p}^*\}$.[22]

Take a consumer who visits the merged entity. The consumer stops searching and buys product i right away with probability $\Pr[\varepsilon_i - \hat{p} > \max\{z_{k-1} - \hat{p}^*, \overline{x} - \tilde{p}^*\}]$. This gives the merged entity a demand for product i equal to

$$\hat{p}^* - \hat{p} + \int_{\overline{x}-\tilde{p}^*+\hat{p}}^{1-\hat{p}^*+\hat{p}} (\varepsilon - \hat{p} + \hat{p}^*)^{k-1} \, d\varepsilon = \hat{p}^* - \hat{p} + \int_{\overline{x}-\tilde{p}^*}^{1-\hat{p}^*} (\varepsilon + \hat{p}^*)^{k-1} \, d\varepsilon.$$

The merged entity also receives demand for product i from consumers who decide to walk away from it, visit all the non-merging firms and finally return to it because product i turns out to be their best match in the market. This happens with probability $\Pr\left[\max\{z_{n-k} - \tilde{p}^*, z_{k-1} - \hat{p}^*, 0\} < \varepsilon_i - \hat{p} < \overline{x} - \tilde{p}^*\right]$, which gives the merged entity an additional demand for product i equal to

$$\hat{c}_{ia}(\hat{p}; \tilde{p}^*, \hat{p}^*) \equiv \int_{\hat{p}}^{\overline{x}-\tilde{p}^*+\hat{p}} (\varepsilon - \hat{p} + \tilde{p}^*)^{n-k} (\varepsilon - \hat{p} + \hat{p}^*)^{k-1} \, d\varepsilon = \int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k} (\varepsilon + \hat{p}^*)^{k-1} \, d\varepsilon, \quad (13)$$

where we use a notation similar to that above.

The deviation also affects the merged entity's demand for products other than i. Let us compute now the demand for one of the other products, say m. A consumer who visits the merged entity stops searching and buys product m right away with probability $\Pr[\varepsilon_m - \hat{p}^* > \max\{z_{k-2} - \hat{p}^*, \varepsilon_i - \hat{p}, \overline{x} - \tilde{p}^*\}]$, which gives the merged entity a demand for product m equal to

$$\int_{\overline{x}-\tilde{p}^*+\hat{p}^*}^{1} (\varepsilon - \hat{p}^* + \hat{p}) \, \varepsilon^{k-2} d\varepsilon.$$

Product m is also bought by consumers who walk away from the merged entity, visit all the non-merging firms and return to the former because product m is the best for them. This occurs with probability $\Pr\left[\max\{z_{n-k} - \tilde{p}^*, z_{k-2} - \hat{p}^*, \varepsilon_i - \hat{p}, 0\} < \varepsilon_m - \hat{p}^* < \overline{x} - \tilde{p}^*\right]$, which gives the merged entity a demand for product m equal to

$$\hat{c}_{ma}(\hat{p}; \tilde{p}^*, \hat{p}^*) \equiv \int_{\hat{p}^*}^{\overline{x}-\tilde{p}^*+\hat{p}^*} (\varepsilon - \hat{p}^* + \tilde{p}^*)^{n-k} (\varepsilon - \hat{p}^* + \hat{p}) \, \varepsilon^{k-2} d\varepsilon = \int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k} (\varepsilon + \hat{p}^*)^{k-2} (\varepsilon + \hat{p}) \, d\varepsilon.$$

[22] When the deviation price $\hat{p} > \hat{p}^*$ (and still $\hat{p} < 1 - \overline{x} + \tilde{p}^*$) the payoff function is slightly different but the FOC in symmetric equilibrium is exactly the same.

The total profit of the deviant merged entity therefore equals

$$\hat{\pi}(\hat{p};\tilde{p}^*,\hat{p}^*) = \hat{p}\left[\hat{p}^* - \hat{p} + \int_{\overline{x}-\tilde{p}^*}^{1-\hat{p}^*} (\varepsilon + \hat{p}^*)^{k-1}\, d\varepsilon + \hat{c}_{ia}(\hat{p};\tilde{p}^*,\hat{p}^*)\right]$$
$$+ (k-1)\,\hat{p}^*\left[\int_{\overline{x}-\tilde{p}^*+\hat{p}^*}^{1} (\varepsilon - \hat{p}^* + \hat{p})\,\varepsilon^{k-2} d\varepsilon + \hat{c}_{ma}(\hat{p};\tilde{p}^*,\hat{p}^*)\right]. \tag{14}$$

Payoff to a deviant non-merging store.

Consider a non-merging firm j that deviates to a price $\tilde{p} \neq \tilde{p}^*$, with $\tilde{p} < 1-\overline{x}+\tilde{p}^*$. As all non-merging firms are expected to charge $\tilde{p}^*$, consumers visit them randomly. The deviant non-merging firm j may be visited in first place (after the merged entity), second place and so on till the $(n-k)^{th}$ place. Each of these positions occurs with probability $1/(n-k)$.

Consider that the deviant's firm is visited by a consumer in her h^{th} search, with $h = 1, 2, ..., n-k$.[23] Denote the deal the consumer observes upon entering its shop by $\varepsilon_j - \tilde{p}$. There are two situations in which the deviant's firm sells to this consumer. First, the consumer may stop searching at the deviant's shop and buy there right away. Conditional on firm j being in h^{th} place, this occurs with probability $\Pr[\max\{z_{h-1} - \tilde{p}^*, z_k - \hat{p}^*\} < \overline{x} - \tilde{p}^* < \varepsilon_j - \tilde{p}]$, which gives the deviant's firm a demand equal to $\overline{x}^{h-1}(\overline{x} - \tilde{p}^* + \hat{p}^*)^k\,(1 - \overline{x} + \tilde{p}^* - \tilde{p})$. Summing the unconditional probability for all h gives a demand equal to

$$\frac{1}{n-k}\frac{1-\overline{x}^{n-k}}{1-\overline{x}}(\overline{x} - \tilde{p}^* + \hat{p}^*)^k\,(1 - \overline{x} + \tilde{p}^* - \tilde{p})\,. \tag{15}$$

Second, the consumer may walk away from the deviant's firm and come back to it after checking the products of the rest of the non-merging stores. Conditional on the consumer visiting firm j in her h^{th} search, this occurs with probability $\Pr[\max\{z_k - \hat{p}^*, z_{n-k-1} - \tilde{p}^*, 0\} < \varepsilon_j - \tilde{p} < \overline{x} - \tilde{p}^*]$, which gives a demand from returning consumers equal to

$$\tilde{c}_a(\tilde{p};\tilde{p}^*\hat{p}^*) \equiv \int_{\tilde{p}}^{\overline{x}-\tilde{p}^*+\tilde{p}} (\varepsilon + \hat{p}^* - \tilde{p})^k(\varepsilon + \tilde{p}^* - \tilde{p})^{n-k-1} d\varepsilon = \int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \hat{p}^*)^k(\varepsilon + \tilde{p}^*)^{n-k-1} d\varepsilon. \tag{16}$$

Summing the unconditional probability for all h gives a demand equal to $\tilde{c}_a(\tilde{p};\tilde{p}^*\hat{p}^*)$.

The total profits of a deviating non-merging firm are

$$\tilde{\pi}(\tilde{p},\tilde{p}^*\hat{p}^*) = \tilde{p}\left[\frac{1}{n-k}\frac{1-\overline{x}^{n-k}}{1-\overline{x}}(\overline{x} - \tilde{p}^* + \hat{p}^*)^k\,(1 - \overline{x} + \tilde{p}^* - \tilde{p}) + \tilde{c}_a(\tilde{p};\tilde{p}^*\hat{p}^*)\right]. \tag{17}$$

Results

Taking the first order derivative of (14) with respect to $\hat{p}$ and setting $\hat{p} = \hat{p}^*$, we obtain the following FOC:

$$1 - (\overline{x} - \tilde{p}^* + \hat{p}^*)^{k-1}\,(\overline{x} - \tilde{p}^* + (k+1)\,\hat{p}^*) + k\int_0^{\overline{x}-\tilde{p}^*} (\varepsilon + \tilde{p}^*)^{n-k}\,(\varepsilon + \hat{p}^*)^{k-2}\,(\varepsilon + k\hat{p}^*)\, d\varepsilon = 0 \tag{18}$$

[23] We note that when the consumer visits the deviant immediately after leaving the merged firm, $h = 1$, the consumer, even if surprised by a deviation, will never return to the merged entity without searching further. In fact, this event has probability $\Pr[\varepsilon - \tilde{p} < z_k - \hat{p}^* < \overline{x} - \tilde{p}^* \text{and } \varepsilon - \tilde{p} > \overline{x} - \tilde{p}^*] = 0$.

Likewise, taking the FOC in (17) and imposing symmetry among the prices of the non-merging firms gives:

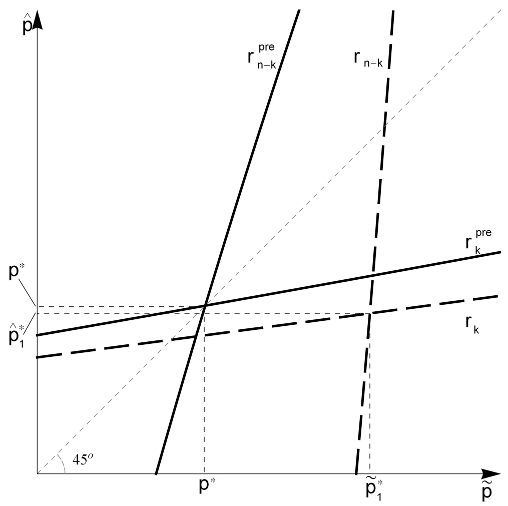
$$\frac{1}{n-k}\left(\overline{x}-\tilde{p}^{*}+\hat{p}^{*}\right)^{k}\frac{1-\overline{x}^{n-k}}{1-\overline{x}}\left(1-\overline{x}-\tilde{p}^{*}\right)+\int_{0}^{\overline{x}-\tilde{p}^{*}}\left(\varepsilon+\tilde{p}^{*}\right)^{n-k-1}\left(\varepsilon+\hat{p}^{*}\right)^{k}d\varepsilon=0 \tag{19}$$

Proposition 4 *Assume that $k < 10$ firms merge. Then, in the long-run after the merged entity stocks all the products of the parent firms, there exists a Nash equilibrium in the post-merger market where:*

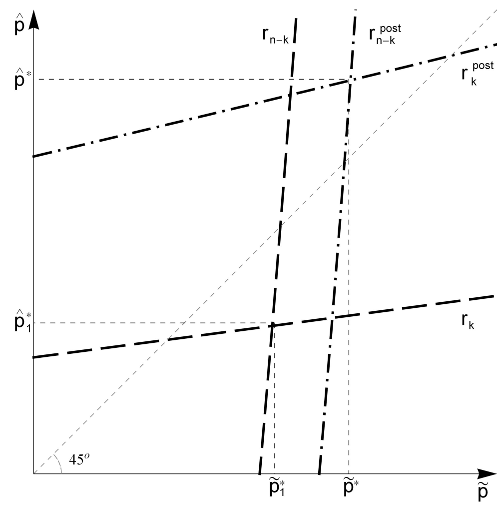
- *Consumers prefer start searching at the merged entity and then, if they wish so, continue searching at the non-merging firms, i.e. $\overline{\overline{x}}-\hat{p}^{*}>\overline{x}-\tilde{p}^{*}$.*
- *The merged entity charges a price $\hat{p}^{*}$ and the non-merging stores charge a price $\tilde{p}^{*}$; these prices solve the system of FOCs (18)-(19).*

This equilibrium exists if the search cost s is sufficiently large, in which case $\hat{p}^{*}>\tilde{p}^{*}$.

The proof is organized in the same way as the proof of Proposition 1.[24] As we did in the previous section, it is illustrative to look at the behavior of the reaction functions of the different types of firm once a merger occurs. We illustrate the main effects in Figure 3. As in Figure 1, the crossing point between the two solid reaction functions gives the pre-merger equilibrium. When the potentially merging firms merge, *a search-order effect* and an *internalization-of-pricing-externalities effect* take place.



(a) Search-order effect



(b) Internalization of pricing externalities

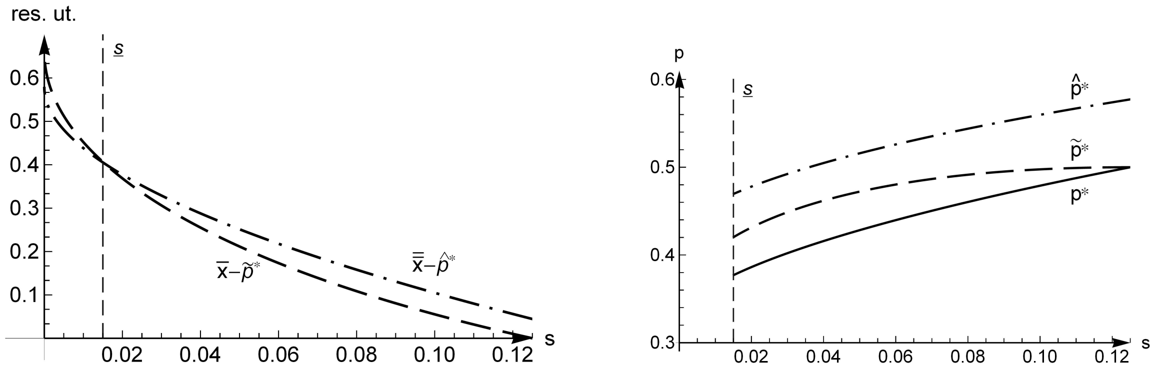
Figure 3: Long-run pre- and post-merger equilibria ($n = 3, k = 2$).

The search-order effect stems from the *demand-side economies* that unfold after the potentially merging stores merge and start carrying all the products of the parent firms. By this effect, the reaction

[24]The restriction $k < 10$ (or, alternatively, $n \leq 10$) is adopted for convenience. If $k \geq 10$ the search cost bound in (11) is a complicated function of k and this makes the calculations cumbersome. Since mergers are relevant in relatively concentrated markets and often take place between 2 firms at most, the restriction $k < 10$ implies little loss of generality.

curve of the outsiders (insiders) shifts rightwards (downwards) from r_{n-k}^{pre} (r_k^{pre}) to r_{n-k} (r_k). These moves are driven by the changes in the demand elasticity of non-merging and merging firms after the merger affects the search-order. The crossing point between the dashed curves r_{n-k} and r_k determines the price implications of the search-order effect. The usual internalization-of-pricing-externalities effect shifts the joint reaction function of the insiders (outsiders) further from r_k (r_{n-k}) to r_k^{post} (r_{n-k}^{post}). At the post-merger equilibrium (crossing point between the dotted-dashed reaction functions), all prices, whether from outsiders or insiders, increase. Proposition 4 shows that when search costs are sufficiently large, the price of the merged entity is higher than the price of the non-merging firms. This means that the internalization-of-pricing-externalities effect dominates the search-order effect. Still, the trade-off consumers face turns out to be favorable for the merging firms: consumers prefer to start searching at the merged entity despite the fact that this firm has a higher price. Economies of search are at the heart of this result.

Before turning to a discussion of the aggregate implications of mergers in the long-run, we make two remarks in connection with Proposition 4. The first observation is that, even though the proposition is proven for the case where the search cost converges to its maximum value, the result is true for much lower search costs. This can be seen in Figure 4a, where we plot the reservation utilities $\overline{\overline{x}} - \hat{p}^*$ and $\overline{x} - \tilde{p}^*$ against search costs for the $n = 3$ case. The equilibrium of Proposition 4 exists for search costs to the right of the point where the two reservation utility curves intersect (approximately 0.015, i.e., 3% of the average value of a firm's good). In Figure 4b we see that the price of the merged entity is higher than the price of the non-merging firm no matter the level of search costs.

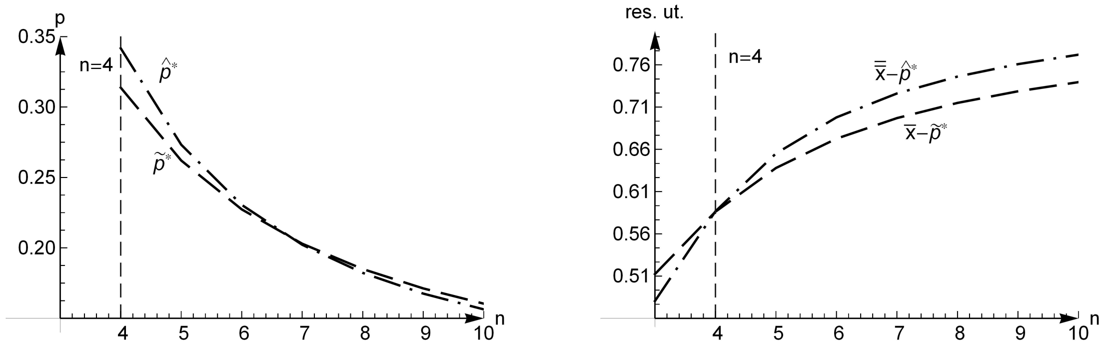


(a) Reservation utilities

(b) Prices

Figure 4: Reservation utilities, prices and search costs ($n = 3$, $k = 2$).

However, our second observation is that the ranking of merging and non-merging firm prices given in Proposition 4 need not hold for all parameters. In fact, it is possible that the search-order effect more than offsets the internalization-of-pricing-externalities effect, in which case the price of the merged entity is *lower* than the price of the non-merging stores. This occurs when the search cost is relatively small and the number of merging firms relative to the total number of firms in the market is also small. In the graphs of Figure 5, the number of merging firms is set equal to 2 and the search cost is very small ($s = 0.005$). Figure 5a plots the post-merger equilibrium prices and shows that the merged entity charges a price lower than that of the non-merging firms when $n \geq 7$. Figure 5b plots consumer reservation utilities for searching the two types of firms and shows that consumer search-order is consistent with equilibrium pricing for all $n \geq 4$.



(a) Prices (b) Reservation utilities

Figure 5: Post-merger prices and reservation utilities ($\overline{x} = 0.9, k = 2$).

We now turn to the impact of merging activity on the profits of insiders and outsiders.

Proposition 5 *In the long-run post-merger equilibrium of Proposition 4: (a) Any k-firm merger is profitable for the merging firms, that is, $\hat{\pi}^*/k > \pi^*$. (b) If search cost is sufficiently large, in any k-firm merger the non-merging firms obtain lower profits than the merging firms, that is, $\hat{\pi}^*/k > \tilde{\pi}^*$.*

In the short-run equilibrium of Proposition 1 firms did not have an incentive to merge when the search costs are high. The reason is that, everything else equal, the merging firms are placed in the end of consumers' search-order. When search costs are high, in the long-run firms that merge gain a prominent position in the marketplace because their shops are stocked with a larger array of products. This clearly makes merging profitable and, in addition, it has a serious impact on the profits of the non-merging firms. In fact, Proposition 5 shows that, when search frictions are high, the non-merging firms obtain *lower* profits than the merged entity. The non-merging firms, being relegated to the end of the optimal consumer search-order, receive little demand and, correspondingly, lose out relative to the merging firms. This result is in contrast with the standard "free-riding effect" by which outsiders to a merger benefit more than the insiders. To the extent that the free-riding effect is at odds with observed merger waves, our result is more comforting.

Figure 6 illustrates the results in Proposition 5. The merged entity's profits (dotted-dashed curve) are clearly above pre-merger levels (solid curve). This is the outcome of two forces: one the one hand, the merged entity benefits from the market prominence it gains by stocking all the products of the

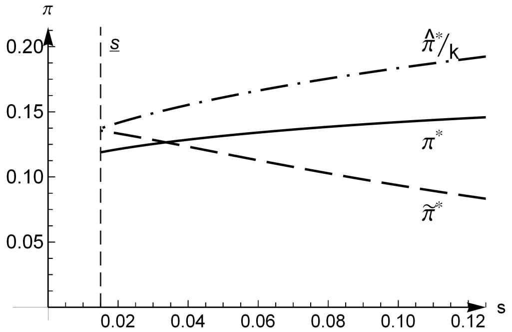


Figure 6: Long-run pre- and post-merger equilibrium profits ($n = 3, k = 2$).

parent firms; on the other hand, the merged entity profits from increased market power. The figure also shows that, unless search costs are very low, outsiders lose out (dashed curve). Finally, it is also worth mentioning the asymmetry in the way search costs affect the profits of the different firms after a merger. As search costs increase, the profits of the merged entity go up while the profits of the non-merging firms fall. This is due to the fact that consumer traffic from the merged entity to the non-merging firms decreases as search costs rise.

Our final result in this section pertains to the aggregate implications of mergers. As usual, we evaluate the effects of a merger on welfare grounds by comparing the pre- and post-merger sum of consumer surplus and firms' profits minus search costs. We now compute the expected surplus consumers derive in the post-merger market. Consider first those consumers who buy from the merged entity. These consumers either buy there directly upon arrival or after having visited all the non-merging firms. Their expected consumer surplus is

$$\widehat{CS} = \int_{\overline{x}-\tilde{p}^*+\hat{p}^*}^{1} (\varepsilon - \hat{p}^*)\, d\varepsilon^k + \int_{\hat{p}^*}^{\overline{x}-\tilde{p}^*+\hat{p}^*} (\varepsilon - \hat{p}^* + \tilde{p}^*)^{n-k} (\varepsilon - \hat{p}^*)\, d\varepsilon^k. \tag{20}$$

Consider now those consumers who buy from the non-merging firms. Again, these consumers may buy directly upon arrival or after visiting all the firms in the market. Their expected consumer surplus is

$$\widetilde{CS} = (\overline{x} - \tilde{p}^* + \hat{p}^*)^k \,\frac{1-\overline{x}^{n-k}}{1-\overline{x}} \int_{\overline{x}}^{1} (\varepsilon - \tilde{p}^*)\, d\varepsilon + (n-k) \int_{\tilde{p}^*}^{\overline{x}} \varepsilon^{n-k-1} (\varepsilon - \tilde{p}^* + \hat{p}^*)^k (\varepsilon - \tilde{p}^*)\, d\varepsilon. \tag{21}$$

In the long-run post-merger equilibrium, search economies play a crucial role. Consumers who buy directly at the merged entity search only one time. Consumers who walk away from the merged entity and stop searching at the first non-merging store they enter search only two times. And so on and so forth. The total number of searches is denoted

$$NS = 1 - (\overline{x} - \tilde{p}^* + \hat{p}^*)^k + (\overline{x} - \tilde{p}^* + \hat{p}^*)^k (1-\overline{x}) \sum_{j=1}^{n-k} (j+1)\overline{x}^{j-1} + (\overline{x} - \tilde{p}^* + \hat{p}^*)^k\, \overline{x}^{n-k}(n-k+1).$$

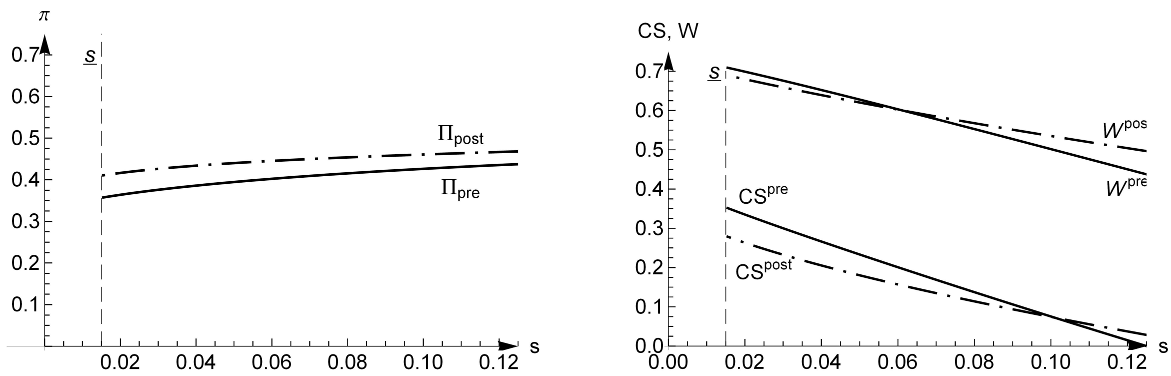
Let Sc be the total search costs incurred by consumers. After simplification,

$$Sc = \frac{1}{2}(1-\overline{x}) \left[1 - \overline{x} + (\overline{x} - \tilde{p}^* + \hat{p}^*)^k (1 - \overline{x}^{n-k})\right], \tag{22}$$

which, keeping prices fixed, clearly decreases in k.[25] Taking into account the costs of searching, net consumer surplus is therefore $CS = \widehat{CS} + \widetilde{CS} - Sc$. Adding the profits of the firms, we obtain a measure of expected social welfare $SW = CS + \hat{\pi} + (n-k)\tilde{\pi}$.

Proposition 6 *In the long-run post-merger equilibrium of Proposition 4, if search cost is high enough: (a) Any k-firm merger results in an increase in industry profits. (b) Consumer surplus increases after a k-firm merger. As a result, a merger increases social welfare.*

The aggregate implications of a merger are illustrated in Figure 7. In Figure 7a we compare pre- and post-merger industry profits. Collectively firms obtain greater profits post-merger (dotted-dashed curve) than pre-merger (solid curve).



(a) Industry profits

(b) Consumer surplus

Figure 7: Long-run pre- and post-merger profits and consumer surplus ($n = 3, k = 2$).

Figure 7b depicts pre- and post-merger consumer surplus and social welfare. The graph illustrates our result in Proposition 6 that when search cost is relatively high, the search economies consumers experience after a merger takes place more than offset the negative price effects of consolidation. When search costs are intermediate, the price effects are stronger than the search economies and consumers lose out; however, their loss is not so large because of the savings in search costs and therefore overall welfare increases. When search costs are small, the negative price effects associated to consolidation have a dominating influence and a merger results in a welfare loss, as in Deneckere and Davidson (1985).

5 Alternative symmetric equilibria and the decision to stock all the products of the parent firms

The most important results of the paper arise in situations where search costs are relatively high for otherwise the model is similar to the perfect information case of Deneckere and Davidson (1985).

[25] In fact, numerical calculations show that the total search costs also decrease in k when we take into account how prices change with k.

In particular, in Section 3 on the effects of mergers purely arising from price-coordination, we have shown that mergers are not profitable when search costs are relatively high. Likewise, in Section 4 we have proven that demand-side economies can make mergers welfare-improving when search costs are relatively high. The purpose of this Section is twofold. We first demonstrate that when search costs are sufficiently high, the equilibria reported in Propositions 1 and 4 are the only symmetric equilibria that can exist; in this situation, it is then clear that the merged entity wishes to undertake the reorganization we have assumed, namely, stocking the shelves of the merged entity with all the products of the parent firms. Second, we show that when search costs are low, it is not profitable for the merged entity to undertake such a business reorganization.

Opposite consumer search-order in the short-run

The discussion in Section 3 centered around a symmetric equilibrium with the merging firms charging a higher price than the non-merging firms and, correspondingly, with consumers starting their search for satisfactory products at the non-merging stores. This equilibrium was portrayed as a natural extension of the equilibrium that arises under perfect information and we showed in Proposition 1 that it exists for all admissible levels of the search cost.

However, another *symmetric* equilibrium can be proposed. Suppose consumers hold the belief that the merging stores charge lower prices than the non-merging firms and, consequently, they start searching for satisfactory products at the former; given this, firms respond by setting prices in such a way that consumer beliefs are fulfilled. This could very well occur if the merger process looms so large in consumer minds that the merging firms capture consumer attention and become prominent in the marketplace. In that case, the power of consumer beliefs at dictating the prices of the firms must be sufficiently strong so as to more than offset the internalization-of-pricing externalities effect. In this Section we characterize such an equilibrium and prove that it does not exist when search costs are high.

To do this, we compute the payoff functions of (deviating) merging and non-merging firms, derive the FOCs and study whether the solution to the system of FOCs satisfies the above mentioned price ranking. Consider first the merged entity's problem. It is a matter of proceeding similarly as in Sections 3 and 4 to arrive to the following payoff for a deviating merged firm that changes the price of one of its products, say i, to $\hat{p} \neq \hat{p}^*$, with $\hat{p} < 1 - \overline{x} + \tilde{p}^*$:[26]

$$\hat{\pi}(\hat{p}; \hat{p}^*, \tilde{p}^*) = p_i d_i(\hat{p}; \hat{p}^*, \tilde{p}^*) + (k-1)\, \hat{p}^* d_m(\hat{p}; \hat{p}^*, \tilde{p}^*),$$

[26]For details see our working paper Moraga-González and Petrikaitė (2013).

where

$$d_i(\hat{p};\hat{p}^*,\tilde{p}^*) = \frac{1}{k}\frac{1-\bar{x}^k}{1-\overline{x}}(\hat{p}^*-\hat{p}) + \int_{\bar{x}-\hat{p}^*}^{1-\hat{p}^*}(\varepsilon+\hat{p}^*)^{k-1}d\varepsilon + \int_0^{\overline{x}-\tilde{p}^*}(\varepsilon+\tilde{p}^*)^{n-k}(\varepsilon+\hat{p}^*)^{k-1}d\varepsilon$$

$$d_m(\hat{p};\hat{p}^*,\tilde{p}^*) = \frac{1-k+k\overline{x}-\bar{x}^k}{k(k-1)(1-\overline{x})}(\hat{p}-\hat{p}^*)$$
$$+\int_{\overline{x}-\tilde{p}^*}^{1-\hat{p}^*}(\varepsilon+\hat{p}^*)^{k-2}(\varepsilon+\hat{p})\,d\varepsilon + \int_0^{\overline{x}-\tilde{p}^*}(\varepsilon+\tilde{p}^*)^{n-k}(\varepsilon+\hat{p})(\varepsilon+\hat{p}^*)^{k-2}d\varepsilon.$$

Consider now the problem of a non-merging firm j. Its demand is made of consumers who walk away from all the merging stores, happen to stop by firm j and buy right away there; in addition, some consumers return to firm j after having visited all shops. The decision to walk away from the last merging store is based on the comparison between $z_k-\hat{p}^*$ and $\overline{x}-\tilde{p}^*$. This condition is exactly the same as that in Section 4 for consumers to leave the merged entity and visit one of the non-merging stores. As a result, the payoff of a (deviant) non-merging firm here is exactly the same as (17).

Proposition 7 *Assume that k firms merge. Then, in the short-run, a symmetric equilibrium where $\hat{p}^* < \tilde{p}^*$ so that consumers start searching at the stores of the merged entity and then proceed by searching at the non-merging stores does not exist whenever one of the following conditions holds: (a) the search cost is sufficiently low, (b) the search cost is sufficiently high, (c) $n=3$, (d) the number of competitors is sufficiently large.*[27]

The intuition behind this result is as follows. The price ranking of the firms is the outcome of the tension between the search-order effect, which, being visited first, gives merging firms incentives to lower prices, and the internalization-of-pricing-externalities effect, which works in the opposite direction. The magnitude of the search cost and the number of non-merging firms affect the outcome of this tension. For example, we know that when the search cost is exactly equal to zero, the price ranking of Proposition 7 is impossible. By continuity, we expect this alternative equilibrium to fail to exist when the search cost is positive but small and this is what the first part of Proposition 7 shows. What happens is that when the search cost is arbitrarily close to zero, the search-order effect is practically non-existent and the internalization-of-pricing-externalities effect is the strongest. When the search cost increases, the search-order effect gains importance, while the internalization-of-pricing-externalities effect weakens. For intermediate levels of the search cost, the alternative equilibrium where the merging firms charge lower prices and are visited first may exist (though not necessarily as demonstrated for the case $n = 3$). Finally, when search costs are very high, prices, whether from merging or not merging firms, are close to monopoly prices and the search-order effect is again weaker than the internalization-of-pricing-externalities effect.[28] The number of firms affects the tension between the search-order effect and the internalization-of-pricing-externalities effect in a similar way.

[27] The proof of this result is in our working paper Moraga-González and Petrikaitė (2013).

[28] We have explored alternative ways to affect the trade-off between the search-order effect and the internalization-

Proposition 7 implies that the alternative equilibrium where merging firms charge lower prices and consumers start their search for fine products at the merging firms can only exist for intermediate levels of the search cost and the number of firms. This casts doubts about the appeal of such an equilibrium. At the very least, taking such an alternative equilibrium seriously requires consumer beliefs to be discontinuous in parameters such as the search cost and the number of firms. We find such a requirement on beliefs difficult to justify.

Opposite consumer search-order in the long-run

In Section 4 we have characterized an equilibrium where the potentially merging firms gain market prominence if they indeed merge. This gain in prominence arises because the merged entity is assumed to put all its products on display at each of its stores. If this is so, and provided search costs are sufficiently large, consumers find it optimal to first search for a satisfactory product at the merged entity and, if desired, continue searching later at the non-merging firms.

In this section we argue that the equilibrium in Proposition 4 is the unique symmetric equilibrium when search costs are relatively high. We prove this by contradiction. Suppose that consumers find it optimal to start searching for a satisfactory good at the non-merging firms. If this is so, then the reservation utility at the merged entity, $\overline{\overline{x}} - \hat{p}^*$, must be lower than the reservation utility at the non-merging firms, $\overline{x} - \tilde{p}^*$ (where $\overline{x}$ and $\overline{\overline{x}}$, as before, solve (1) and (12), respectively). We now show that this is not possible when search costs are sufficiently large.

To do this, we compute the payoff functions of (deviating) merging and non-merging firms, derive the FOCs and show that the reservation utility ranking above mentioned is impossible when the search cost is sufficiently high. Consider first the merged entity's problem. The payoff to a merged entity that deviates by charging a price $\hat{p} < \hat{p}^*$ for its product i can be shown to be[29]

$$\hat{\pi}(\hat{p};\hat{p}^*,\tilde{p}^*) = \hat{p}d_i(\hat{p};\hat{p}^*,\tilde{p}^*) + (k-1)\,\hat{p}^*d_m(\hat{p};\hat{p}^*,\tilde{p}^*)$$

where

$$d_i(\hat{p};\hat{p}^*,\tilde{p}^*) = \frac{\left(\overline{\overline{x}} - \hat{p}^* + \tilde{p}^*\right)^{n-k}}{k}\left(k\hat{p}^* - k\hat{p} + 1 - \overline{\overline{x}}^k\right) + \int_0^{\overline{\overline{x}}-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-1}\left(\varepsilon + \tilde{p}^*\right)^{n-k} d\varepsilon, \text{ and}$$

$$d_m(\hat{p};\hat{p}^*,\tilde{p}^*) = \left(\overline{\overline{x}} - \hat{p}^* + \tilde{p}^*\right)^{n-k}\left(\int_{\overline{\overline{x}}}^1 \left(\varepsilon - \hat{p}^* + \hat{p}\right)\varepsilon^{k-2} d\varepsilon\right) + \int_0^{\overline{\overline{x}}-\hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-2}\left(\varepsilon + \hat{p}\right)\left(\varepsilon + \tilde{p}^*\right)^{n-k} d\varepsilon.$$

To compute the payoff of a (deviant) non-merging firm we only need to modify the payoff in (7) by properly taking into account that the decision to walk away from the last non-merging firm and

of-pricing-externalities effect in order to rule out the alternative equilibrium of Proposition 7. What is important is to weaken the power consumer beliefs have at dictating equilibrium prices and this happens for example when there is a sufficiently large number of consumers who have perfect information. The equilibrium in Proposition 1 as well as our merger paradox result in Proposition 3, by contrast, survive this modification.

[29] For details about this derivation, see our working paper Moraga-González and Petrikaitė (2013). When $\hat{p} > \hat{p}^*$ instead, the payoff function is slightly different but the FOC is exactly the same.

visit the merged entity depends on whether the best of the non-merging firms' deals gives a lower or higher utility than $\overline{\overline{x}} - \hat{p}^*$. The total payoff of a deviating non-merging firm equals:

$$\tilde{\pi}(\tilde{p};\tilde{p}^*,\hat{p}^*) = \tilde{p}\left\{\frac{1}{n-k}\left[\frac{1-\overline{x}^{n-k}}{1-\overline{x}}\left(1-\overline{x}+\tilde{p}^*-\tilde{p}\right)+\overline{x}^{n-k}-\left(\overline{\overline{x}}-\hat{p}^*+\tilde{p}^*\right)^{n-k}\right]\right\},$$
$$+\,\tilde{p}\int_0^{\overline{\overline{x}}-\hat{p}^*}\left(\varepsilon+\tilde{p}^*\right)^{n-k-1}\left(\varepsilon+\hat{p}^*\right)^k d\varepsilon.$$

Proposition 8 *Assume that $k<10$ firms merge and that the search cost is sufficiently high. Then in the long-run after the merged entity stocks all the products of the parent firms, an equilibrium where consumers prefer to search for a satisfactory product first at the non-merging firms and continue, if they wish so, at the merged entity does not exist.*[30]

The decision to sell all products under one roof

Next we study whether the merged entity wishes to sell all products in a single store over selling them in separate stores. We restrict ourselves to the extreme cases of high and low search costs because for those two cases there exists a unique symmetric equilibrium in each of the business organizations we compare.

For a given consumer search-order, a merged entity that puts on display all its products becomes more attractive for consumers since they probably find a product they like without incurring additional search costs. By this effect, we expect the merged entity to prefer to stock each of its shops with all the products of the parent firms. At the same time, stocking all products together increases competition with the non-merging firms, which tends to reduce (merging and non-merging) firm prices. By this second effect, we expect the merged entity to prefer to keep selling the products in separate stores. Finally, putting all products together on display may lead to a change in the order consumers visit the firms (cf. Propositions 1 and 4).

Consider the case in which search costs are high. In such a case, as shown in Propositions 4 and 8, if the merged entity sells its products together there exists a unique symmetric equilibrium where consumers search first the products of the merged entity. If the products of the merged entity are instead sold in separate shops, Propositions 1 and 7 show that there exists a unique symmetric equilibrium where consumers search first the products of the non-merging firms. By virtue of Propositions 2 and 5, it is obvious that stocking the merged entity with all the products of the parent firms is a more profitable business strategy than selling the products in separate shops. In fact, merging would be unprofitable in the latter case while it is beneficial in the former one.

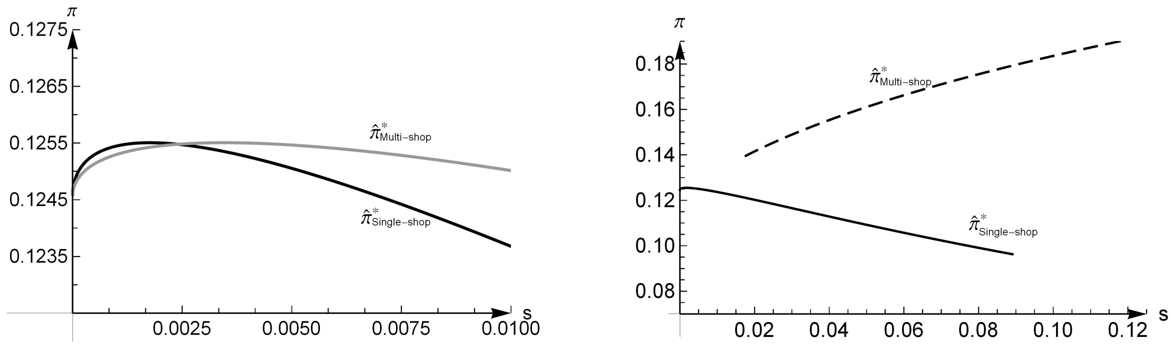
Consider now the case of low search costs. If the merged entity continues to sell its products in separate shops, then, by Propositions 1 and 7, we know the only equilibrium has consumers visiting

[30] The proof of this result is in our working paper Moraga-González and Petrikaitė (2013).

first the non-merging firms. Likewise, when the merged entity puts all the products under the same roof, the equilibrium of Proposition 4 does not exist and the only symmetric equilibrium that can exist has also consumers visiting first the non-merging firms. A comparison between these two equilibria leads to the result that selling the products in separate shops generates higher profits than selling them in a single shop provided that the search cost is sufficiently small.

Proposition 9 *(a) Assume that k firms merge and that s is high enough. Then, the merged entity prefers to sell all its products in a single shop over selling them in separate stores. By doing so, the merged entity gains market prominence and its profits soar. (b) Assume that $k = n - 1$ firms merge and that s is low enough so that consumers visit first the non-merging firms in equilibrium. Then the prices of the merged entity when it sells its products together are lower than when it sells them in separate shops. Moreover, if the search cost is sufficiently small, then the merged entity prefers to sell its products in separate shops.*

The graphs of Figure 8 illustrate this result. Figure 8a shows the case of low search costs. When s is very low, both the demand effect and the price effect are quite small but the price effect has a dominating influence. As a result, the profits of the single-shop merged entity are lower than the profits of the multi-shop one. For higher search costs, the demand effect is stronger than the price effect and profits when selling products together are higher than profits when selling them separately. Figure 8b shows the case of high search costs. Here, because the merged entity gains prominence in the market place, the merger is highly profitable.



(a) Low search cost (b) High search cost

Figure 8: Merged entity's profits when selling products jointly or separately ($n = 3, k = 2$).

6 Concluding remarks

This paper has studied the aggregate consequences of mergers in markets where consumers have to search in order to find satisfactory goods. We have used a model where firms compete in prices to sell differentiated products and consumers search sequentially to find product fit information. When the search cost is equal to zero, the model is similar to Perloff and Salop (1985) and merger analysis gives the same results as those in Deneckere and Davidson (1985). However, when search costs are sizable, the price divergence between merging and non-merging firms has implications for the order in which consumers visit firms when they search for good deals. Likewise, whether the merged entity continues to sell the products of the constituent firms in separate shops or else stocks them with a wider range of products has implications for the optimal order in which consumers search for satisfactory products.

We have distinguished between the short-run and the long-run effects of mergers. In the short-run, the merging firms coordinate their prices while everything else stays the same. We have shown that when search costs are relatively high, the unique post-merger symmetric equilibrium has the merging stores charging higher prices than the non-merging ones and consumers, correspondingly, starting their search at the non-merging stores. In such a situation, we have proven that merging is unprofitable when search costs are high. The paper has thus shown that a merger paradox may also arise when firms compete in prices to sell differentiated products. The paradox may arise because the insiders to a merger by actually merging put themselves at a disadvantage in the marketplace: consumers visit first the outsider firms because they charge lower prices and then, if unsatisfied with the products available there, proceed by visiting the insider and more expensive firms.

In the long-run, however, we have argued that the merged entity can choose to put on display all the products of the parent firms in a single shop. When search costs are significant, this business reorganization generates substantial *demand-side economies* because, everything else equal, consumers do not need to search as intensively as in the pre-merger situation to find satisfactory products. In contrast to a large literature on cost synergies and supply-side economies, this paper has emphasized the importance of these demand-side economies for the aggregate implications of merger activity. We have shown that firms that merge may gain a prominent position in the marketplace, in which case their incentives to raise prices are seriously dampened. In that case, consumers prefer to start searching for satisfactory products at the merged entity. In equilibrium, insider firms gain customers and increase their profits, while outsider firms lose out because they are placed all the way back in the optimal order consumers follow when they search for products. Importantly, we have shown that consolidation may create sufficiently large search economies so as to generate rents for consumers too.

We believe the arguments in this paper are novel and useful to further understand the effects of consolidation processes. Our merger paradox arises in a market where strategic variables are complements and our merger defence result is based on demand-side economies arising from sources other than complementarities (network externalities, complement products, one-stop shopping of an array of products, etc.) Moreover, because the main mechanisms at play are intuitive and powerful, they are expected to play a role in more general market settings provided search costs are significant. Ultimately, we hope this paper adds to a finer design of merger policy.

Efficiency gains arising from mergers may take a relatively long time to materialize. Our theory points out that after-merger business reorganization may lead to important search economies that in the long-run may even result in price decreases relative to the short-run. Whether supply- or demand-side economies are at the heart of after-merger potential welfare gains remains an empirical question. Developing methods to quantify the importance of economies of search and cost synergies seems a fascinating area for future empirical research.

Appendix

Proof of Proposition 1. The proof is organized as follows. Claim 1 shows that there is a pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies the FOCs (9) and (10). Claim 2 proves that such a pair of prices is unique. Claim 3 demonstrates that $\tilde{p}^* < \hat{p}^*$. Finally, we check that firms cannot gain by deviating from the equilibrium prices. In order to shorten the expressions, in this proof we drop the "$*$" super-indexes when they are not necessary.

Claim 1 *There is at least one pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies (9) and (10).*

Proof. We first rewrite the FOC (10) as $G(\hat{p}, \tilde{p}) = 0$, where

$$G(\hat{p}, \tilde{p}) \equiv \frac{1-\overline{x}^k}{k\overline{x}^{k-1}} - \hat{p} + g(\hat{p}, \tilde{p}) \tag{23}$$

and

$$g(\hat{p}, \tilde{p}) \equiv \frac{\int_0^{\overline{x}-\hat{p}} (\varepsilon+\hat{p})^{k-2} (\varepsilon+\tilde{p})^{n-k} (\varepsilon+k\hat{p})\, d\varepsilon}{(\overline{x}-\hat{p}+\tilde{p})^{n-k}\, \overline{x}^{k-1}}$$

Since G is continuously differentiable, the FOC $G(\hat{p}, \tilde{p}) = 0$ defines an implicit relationship between $\hat{p}$ and $\tilde{p}$. Let the function $\eta_1(\tilde{p})$ define this relationship. This function is represented in Figure 9 below. By the implicit function theorem we have

$$\frac{\partial \eta_1(\tilde{p})}{\partial \tilde{p}} = -\frac{\partial G/\partial \tilde{p}}{\partial G/\partial \hat{p}} = -\frac{\partial g/\partial \tilde{p}}{\partial g/\partial \hat{p} - 1}, \tag{24}$$

The numerator of (24) is positive. This is because

$$\frac{\partial g}{\partial \tilde{p}} = \frac{n-k}{\overline{x}^{k-1}(\overline{x}-\hat{p}+\tilde{p})^{n-k+1}} \int_0^{\overline{x}-\hat{p}} (\varepsilon+\hat{p})^{k-2} (\varepsilon+k\hat{p}) (\varepsilon+\tilde{p})^{n-k-1} (\overline{x}-\hat{p}-\varepsilon)\, d\varepsilon > 0$$

The denominator of (24) is however negative. To see this, we note first that

$$\begin{aligned}
\overline{x}^{k-1}(\overline{x}-\hat{p}+\tilde{p})^{n-k+1}\left(\frac{\partial g}{\partial \hat{p}} - 1\right) &= (n-k)\int_0^{\overline{x}-\hat{p}} (\varepsilon+\hat{p})^{k-2} (\varepsilon+\tilde{p})^{n-k} (\varepsilon+k\hat{p})\, d\varepsilon \\
&\quad + (\overline{x}-\hat{p}+\tilde{p})(k-1)\int_0^{\overline{x}-\hat{p}} (\varepsilon+\hat{p})^{k-3} (\varepsilon+\tilde{p})^{n-k} (2\varepsilon+k\hat{p})\, d\varepsilon \\
&\quad - (\overline{x}-\hat{p}+\tilde{p})^{n-k+1}\, \overline{x}^{k-2}\left[2\overline{x} + (k-1)\hat{p}\right].
\end{aligned} \tag{25}$$

Assuming $k > 2$, let us take the derivative of the RHS of (25) with respect to $\overline{x}$. After simplifying it, we obtain:

$$-\overline{x}^{k-2}(\overline{x}-\hat{p}+\tilde{p})^{n-k}\left[\overline{x}(n-k+2) + (k-1)\hat{p}\right] + (k-1)\int_0^{\overline{x}-\hat{p}} (\varepsilon+\hat{p})^{k-3} (\varepsilon+\tilde{p})^{n-k} (2\varepsilon+k\hat{p})\, d\varepsilon \tag{26}$$

If we now take the derivative of (26) with respect to $\overline{x}$ and simplify it we get

$$-(n-k)\,\overline{x}^{k-2}(\overline{x}-\hat{p}+\tilde{p})^{n-k-1}\left[\overline{x}(n+1) + \tilde{p}(k-1)\right] < 0.$$

This implies that the derivative of the RHS of (25) with respect to $\overline{x}$, given in equation (26), is decreasing in $\overline{x}$. Setting $\overline{x}$ equal to its lowest value, $\hat{p}$, in (26) gives

$$-\hat{p}^{k-2}\tilde{p}^{n-k}\left[\hat{p}(n-k+2) + (k-1)\hat{p}\right] < 0.$$

As a result, the RHS of (25) is also decreasing in $\overline{x}$. If we set now $\overline{x} = \hat{p}$ in the RHS of (25), we obtain $-\tilde{p}^{*n-k+1}\hat{p}^{k-1}(k+1) < 0$. From this we conclude that (25) is negative. As a result, since the numerator of $\partial\eta_1(\tilde{p})/\partial\tilde{p}$ is positive and the denominator is negative, we infer that the function $\eta_1(\tilde{p})$ increases in $\tilde{p}$.[31]

Now consider the other equilibrium condition. Let us denote the LHS of (9) as $H(\hat{p},\,\tilde{p})$. The condition $H(\hat{p},\,\tilde{p})=0$ also defines an implicit relationship between $\hat{p}$ and $\tilde{p}$. Let the function $\eta_2(\tilde{p})$ define such a relationship. This function is represented in Figure 9 below. By the implicit function theorem we have

$$\frac{\partial\eta_2(\tilde{p})}{\partial\tilde{p}}=-\frac{\partial H/\partial\tilde{p}}{\partial H/\partial\hat{p}}. \tag{28}$$

We note that H increases in $\hat{p}$. In fact,

$$\frac{\partial H}{\partial\hat{p}}=(n-k)(\overline{x}-\hat{p}+\tilde{p})^{n-k-1}(1-\overline{x}^k)+(n-k)k\int_0^{\overline{x}-\hat{p}}(\varepsilon+\tilde{p})^{n-k-1}(\varepsilon+\hat{p})^{k-1}d\varepsilon>0$$

Moreover, H decreases in $\tilde{p}$. In fact, for $k<n-1$ we have

$$\begin{aligned}\frac{\partial H}{\partial\tilde{p}}&=-\frac{1-\overline{x}^{n-k}}{1-\overline{x}}-(n-k)(\overline{x}-\hat{p}+\tilde{p})^{n-k-1}+(n-k)(n-k-1)\int_0^{\overline{x}-\hat{p}}(\varepsilon+\tilde{p})^{n-k-2}(\varepsilon+\hat{p})^k d\varepsilon\\&<-\frac{1-\overline{x}^{n-k}}{1-\overline{x}}-(n-k)(\bar{x}-\hat{p}+\tilde{p})^{n-k-1}(1-\overline{x}^k)-(n-k)\overline{x}^k\tilde{p}^{n-k-1}<0,\end{aligned}$$

while for $k=n-1$ we get $\partial H/\partial\tilde{p}=-2<0$. As a result, we conclude that the function η_2 is increasing in $\tilde{p}$.

Therefore both η_1 and η_2 increase in $\tilde{p}$. To show that at least one pair of prices $\{\hat{p}^*,\tilde{p}^*\}$ exists that satisfies the system of FOCs (10) and (9), we need to show that the functions η_1 and η_2 cross at least once in the space $[0;1/2]\times[0,\,p_k^m]$. As shown in Figure 9 we observe that $\eta_1(0)>0$. To demonstrate this, note that

$$G(\hat{p},\,0)=\frac{1-\overline{x}^k}{k\overline{x}^{k-1}}-\hat{p}+\frac{1}{\overline{x}^{k-1}(\overline{x}-\hat{p})^{n-k}}\int_0^{\overline{x}-\hat{p}}(\varepsilon+\hat{p})^{k-2}(\varepsilon+k\hat{p})\varepsilon^{n-k}d\varepsilon.$$

Since G decreases in $\hat{p}$ and since

$$G(0,0)=\frac{1-\overline{x}^k}{k\overline{x}^{k-1}}+\frac{1}{\overline{x}^{n-1}}\int_0^{\overline{x}}\varepsilon^{n-1}d\varepsilon>0,$$

[31] When $k=2$, equation (25) changes slightly. Therefore, we treat this case separately. If $k=2$ then

$$g(\hat{p},\tilde{p})=\frac{\int_0^{\overline{x}-\hat{p}^*}(\varepsilon+\tilde{p})^{n-2}(\varepsilon+2\hat{p})\,d\varepsilon}{(\overline{x}-\hat{p}+\tilde{p})^{n-2}\,\overline{x}}$$

and

$$\frac{\partial g(\hat{p},\tilde{p})}{\partial\hat{p}}=\frac{(n-2)\int_0^{\overline{x}-\hat{p}^*}(\varepsilon+\tilde{p})^{n-2}(\varepsilon+2\hat{p})\,d\varepsilon}{(\overline{x}-\hat{p}+\tilde{p})^{n-1}\,\overline{x}}+\frac{2\int_0^{\overline{x}-\hat{p}^*}(\varepsilon+\tilde{p})^{n-2}\,d\varepsilon}{(\overline{x}-\hat{p}+\tilde{p})^{n-2}\,\overline{x}}-\frac{\overline{x}+\hat{p}}{\overline{x}}.$$

Then equation (25) is

$$\begin{aligned}\overline{x}(\overline{x}-\hat{p}+\tilde{p})^{n-1}\left[\frac{\partial g}{\partial\hat{p}}-1\right]&=(n-2)\int_0^{\overline{x}-\hat{p}}(\varepsilon+\tilde{p})^{n-2}(\varepsilon+2\hat{p})\,d\varepsilon\\&\quad+\frac{2}{n-1}(\overline{x}-\hat{p}+\tilde{p})^n-\frac{2}{n-1}\tilde{p}^{n-1}(\overline{x}-\hat{p}+\tilde{p})-(2\overline{x}+\hat{p})(\overline{x}-\hat{p}+\tilde{p})^{n-1}\end{aligned} \tag{27}$$

The derivative of the RHS of (27) with respect to $\overline{x}$ is negative

$$\begin{aligned}&-(\overline{x}-\hat{p}+\tilde{p})^{n-2}\left[\overline{x}\frac{n^2-n-2}{n-1}+\hat{p}\frac{n+1}{n-1}-\frac{2}{n-1}\tilde{p}\right]-\frac{2}{n-1}\tilde{p}^{n-1}<\\&-(\overline{x}-\hat{p}+\tilde{p})^{n-2}\left[\overline{x}\frac{n^2-n-2}{n-1}+\hat{p}\frac{n+1}{n-1}-\frac{2}{n-1}\overline{x}\right]-\frac{2}{n-1}\tilde{p}^{n-1}=\\&-(\overline{x}-\hat{p}+\tilde{p})^{n-2}\left[\overline{x}\frac{n^2-n-4}{n-1}+\hat{p}\frac{n+1}{n-1}\right]-\frac{2}{n-1}\tilde{p}^{n-1}<0\end{aligned}$$

Since this expression is negative, the same arguments can be used to conclude that $\partial G/\partial\tilde{p}$ is positive also when $k=2$, which implies that $\eta_1(\tilde{p})$ increases in $\tilde{p}$.

we conclude that $\eta_1(0) > 0$.

On the contrary, we now observe that $\eta_2(0) < 0$ (see Figure 9). This is because

$$H(\hat{p}, 0) = 1 - (\overline{x} - \hat{p})^{n-k} + (n-k) \int_0^{\overline{x}-\hat{p}} \varepsilon^{n-k-1} (\varepsilon + \hat{p})^k \, d\varepsilon$$

is increasing in $\hat{p}$ and $H(0,0) = 1 - \overline{x}^{n-k} + (n-k) \int_0^{\overline{x}} \varepsilon^{n-1} d\varepsilon > 0$.

Secondly, as depicted in Figure 9, we show that $\eta_1(1/2) < p_k^m < \eta_2(1/2)$, which ensures that the functions η_1 and η_2 cross at least once in the area $[0; 1/2] \times [0; p_k^m]$. To see that $\eta_2(1/2) > p_k^m$, we show that $H(p_k^m, 1/2) < 0$ where

$$H\left(p_k^m, \frac{1}{2}\right) = 1 - \frac{1 - \overline{x}^{n-k}}{1 - \overline{x}} \frac{1}{2} - \left(\overline{x} - p_k^m + \frac{1}{2}\right)^{n-k} + (n-k) \int_0^{\overline{x} - p_k^m} \left(\varepsilon + \frac{1}{2}\right)^{n-k-1} (\varepsilon + p_k^m)^k \, d\varepsilon.$$

Taking the derivative of $H(p_k^m, 1/2)$ with respect to $\overline{x}$ gives

$$-\frac{1 - (n-k)\,\overline{x}^{n-k-1} + (n-k-1)\,\overline{x}^{n-k}}{2(1-\overline{x})^2} - (n-k)\left(\overline{x} - p_k^m + \frac{1}{2}\right)^{n-k-1} \left(1 - \overline{x}^k\right) < 0,$$

so $H(p_k^m, 1/2)$ is decreasing in $\overline{x}$.[32] Setting $\overline{x}$ equal to its lowest possible value, p_k^m, we get

$$H\left(p_k^m, \frac{1}{2}\right)\Big|_{\overline{x}=p_k^m} = 1 - \frac{1}{2^{n-k}} - \frac{1 - (p_k^m)^{n-k}}{2(1 - p_k^m)}. \tag{29}$$

This expression is decreasing in n. In fact, its derivative with respect to n can be written as

$$\frac{2^{n-k-1}(p_k^m)^{n-k} \ln p_k^m + (1 - p_k^m) \ln 2}{2^{n-k}\left(1 - p_k^m\right)} < \frac{1}{2^{n-k}\left(1 - p_k^m\right)} \left[p_k^m \ln p_k^m + (1 - p_k^m) \ln 2\right] < 0.$$

The last inequality follows from the fact that $p_k^m \ln p_k^m + (1 - p_k^m) \ln 2 < 0$.[33] Since $H(p_k^m,\ 1/2)|_{\overline{x}=p_k^m}$ is decreasing in n, if we set n equal to its lowest possible value, $k+1$, in (29) we obtain

$$H\left(p_k^m, \frac{1}{2}\right)\Big|_{\overline{x}=p_k^m} \le H\left(p_k^m, \frac{1}{2}\right)\Big|_{\overline{x}=p_k^m; n=k+1} = 1 - \frac{1}{2^{k+1-k}} - \frac{1 - (p_k^m)^{k+1-k}}{2(1 - p_k^m)} = 1 - \frac{1}{2} - \frac{1}{2} = 0.$$

Therefore, since $H(p_k^m, 1/2)$ is decreasing in $\overline{x}$, we conclude that $H(p_k^m, 1/2)$ is always negative. And because H is increasing in $\hat{p}$, we obtain the result that $\eta_2(1/2) > p_k^m$.

We now show that $\eta_1(1/2) < p_k^m$. Since G is decreasing in $\hat{p}$, it suffices to demonstrate that

$$G\left(p_k^m, \frac{1}{2}\right) = \frac{1 - \overline{x}^k}{k\overline{x}^{k-1}} - p_k^m + \frac{\int_0^{\overline{x}-p_k^m} (\varepsilon + p_k^m)^{k-2} \left(\varepsilon + \frac{1}{2}\right)^{n-k} (\varepsilon + k p_k^m) \, d\varepsilon}{\left(\overline{x} - p_k^m + \frac{1}{2}\right)^{n-k} \overline{x}^{k-1}} < 0.$$

Taking the derivative of $G(p_k^m, 1/2)$ with respect to n gives

$$\left(\overline{x} - p_k^m + \frac{1}{2}\right)^{n-k} \overline{x}^{k-1} \frac{\partial G\left(p_k^m, \frac{1}{2}\right)}{\partial n} = \int_0^{\overline{x}-p_k^m} \left(\varepsilon + \frac{1}{2}\right)^{n-k} (\varepsilon + p_k^m)^{k-2} (\varepsilon + k p_k^m) \ln\left(\frac{\varepsilon + 1/2}{\overline{x} - p_k^m + 1/2}\right) d\varepsilon < 0.$$

[32] The inequality follows from noting that the expression $1 - (n-k)\,\overline{x}^{n-k-1} + (n-k-1)\,\overline{x}^{n-k}$ decreases in $\overline{x}$ and therefore it is higher than when we set $\overline{x} = 1$, that is, $1 - (n-k)\,\overline{x}^{n-k-1} + (n-k-1)\,\overline{x}^{n-k} \ge 1 - (n-k) + (n-k-1) = 0$.

[33] Taking the derivative of $p_k^m \ln p_k^m + (1 - p_k^m) \ln 2$ with respect to k gives $(\partial p_k^m / \partial k)\left(1 - \ln 2 + \ln p_k^m\right)$. The sign of this depends on the sign of $1 - \ln 2 + \ln p_k^m$, which is monotonically increasing in k, first negative and then positive. As a result, $p_k^m \ln p_k^m + \ln 2\,(1 - p_k^m)$ first decreases and then increases in k. At $k = 2$ it takes on a negative value while at $k \to \infty$ it is equal to zero. Therefore it is always negative.

Since $G(p_k^m, 1/2)$ decreases in n, we can set n equal to its lowest value and write

$$G\left(p_k^m, \frac{1}{2}\right) < G\left(p_k^m, \frac{1}{2}\right)\Bigg|_{n=k+1} = \frac{1}{k\overline{x}^{k-1}\left(\overline{x} - p_k^m + \frac{1}{2}\right)} T(\overline{x}),$$

where

$$T(\overline{x}) = \left(\overline{x} - p_k^m + \frac{1}{2}\right)\left(1 - \overline{x}^k - k\overline{x}^{k-1}p_k^m\right) + k\int_0^{\bar{x}-p_k^m} (\varepsilon + p_k^m)^{k-2}\left(\varepsilon + \frac{1}{2}\right)(\varepsilon + kp_k^m)\, d\varepsilon.$$

Note that $\left[k\overline{x}^{k-1}(\overline{x} - p_k^m + 1/2)\right]^{-1} > 0$. Thus, $G(p_k^m, 1/2)|_{n=k+1}$ is negative if $T(\overline{x}) < 0$. $T(\bar{x})$ decreases in $\overline{x}$ because $\partial T(\bar{x})/\partial\overline{x} = 1 - \overline{x}^k - k\overline{x}^{k-1}p_k^m$ and this expression decreases in $\overline{x}$. Therefore, using $\overline{x} = p_k^m$, we can write

$$\frac{\partial T(\overline{x})}{\partial\overline{x}} < \frac{\partial T(\overline{x})}{\partial\overline{x}}\Bigg|_{\overline{x}=p_k^m} = 1 - (p_k^m)^k - k(p_k^m)^{k-1}p_k^m = 0.$$

Since $T(\overline{x})$ decreases in $\overline{x}$, we conclude that $T(\overline{x}) < T(p_k^m) = 0$. As a result the functions η_1 and η_2 cross at least once in the region $[0; 1/2] \times [0; p_k^m]$.

Claim 2 *The pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies (9) and (10) is unique.*

Proof. To show this, it is enough to show that η_1 increases in $\tilde{p}$ at a rate less than 1, while η_2 does so at a rate greater than 1. From (24), since $\partial G/\partial\hat{p} < 0$, we know that η_1 increases in $\tilde{p}$ at a rate less than 1 if and only if $\partial G/\partial\hat{p} + \partial G/\partial\tilde{p} < 0$. For the case $k > 2$, we can then write

$$\overline{x}^{k-1}(\overline{x} - \hat{p} + \tilde{p})^{n-k}\left[\frac{\partial G(\hat{p},\tilde{p})}{\partial\tilde{p}} + \frac{\partial G(\hat{p},\tilde{p})}{\partial\hat{p}}\right] = (n-k)\int_0^{\overline{x}-\hat{p}} (\varepsilon + \hat{p})^{k-2}(\varepsilon + k\hat{p})(\varepsilon + \tilde{p})^{n-k-1}\, d\varepsilon$$
$$+ (k-1)\int_0^{\overline{x}-\hat{p}} (\varepsilon + \hat{p})^{k-3}(\varepsilon + \tilde{p})^{n-k}(2\varepsilon + k\hat{p})\, d\varepsilon - (\overline{x} - \hat{p} + \tilde{p})^{n-k}\overline{x}^{k-2}\left[2\overline{x} + (k-1)\hat{p}\right]. \tag{30}$$

We now notice that the RHS of (30) decreases in $\overline{x}$. In fact its derivative, after rearranging, is equal to $-(n-k)\overline{x}^{k-1}(\overline{x} - \hat{p} + \tilde{p})^{n-k-1} < 0$. Therefore, if (30) is negative when setting $\overline{x} = \hat{p}$, then it is always negative. Checking this, we obtain[34]

$$\overline{x}^{k-1}(\overline{x} - \hat{p} + \tilde{p})^{n-k}\left[\frac{\partial G(\hat{p},\tilde{p})}{\partial\tilde{p}} + \frac{\partial G(\hat{p},\tilde{p})}{\partial\hat{p}}\right] < -\tilde{p}^{n-k}\hat{p}^{k-1}(k+1) < 0.$$

Similarly, using (28), since $\partial H/\partial\hat{p} > 0$, we know that $\partial\eta_2/\partial\tilde{p} > 1$ if and only if $\partial H/\partial\hat{p} + \partial H/\partial\tilde{p} < 0$. For the case $k < n-1$, using the expressions above, we then compute

$$\frac{\partial H}{\partial\tilde{p}} + \frac{\partial H}{\partial\hat{p}} = -\frac{1 - \overline{x}^{n-k}}{1 - \overline{x}} + (n-k)(n-k-1)\int_0^{\overline{x}-\hat{p}} (\varepsilon + \tilde{p})^{n-k-2}(\varepsilon + \hat{p})^k\, d\varepsilon$$
$$- (n-k)(\overline{x} - \hat{p} + \tilde{p})^{n-k-1}\overline{x}^k + (n-k)k\int_0^{\overline{x}-\hat{p}} (\varepsilon + \tilde{p})^{n-k-1}(\varepsilon + \hat{p})^{k-1}\, d\varepsilon. \tag{32}$$

[34]The same holds for the case when $k = 2$. We have

$$\overline{x}(\overline{x} - \hat{p} + \tilde{p})^{n-2}\left[\frac{\partial G(\hat{p},\tilde{p})}{\partial\tilde{p}} + \frac{\partial G(\hat{p},\tilde{p})}{\partial\hat{p}}\right] = (n-2)\int_0^{\overline{x}-\hat{p}} (\varepsilon + \tilde{p})^{n-3}(\varepsilon + 2\hat{p})\, d\varepsilon$$
$$+ \frac{2}{n-1}(\overline{x} - \hat{p} + \tilde{p})^{n-1} - \frac{2}{n-1}\tilde{p}^{n-1} - (2\overline{x} + \hat{p})(\overline{x} - \hat{p} + \tilde{p})^{n-2} \tag{31}$$

After simplifying, the derivative of (31) with respect to $\overline{x}$ is $-\overline{x}(n-2)(\overline{x} - \hat{p} + \tilde{p})^{n-3}$, which is clearly negative. Then

$$\overline{x}(\overline{x} - \hat{p} + \tilde{p})^{n-2}\left[\frac{\partial G(\hat{p},\tilde{p})}{\partial\tilde{p}} + \frac{\partial G(\hat{p},\tilde{p})}{\partial\hat{p}}\right] < -3\hat{p}\tilde{p}^{n-2} < 0$$

This expression decreases in $\overline{x}$ because its partial derivative with respect to $\overline{x}$, after rearranging, is equal to

$$-\frac{1-(n-k)\,\overline{x}^{n-k-1}+(n-k-1)\,\overline{x}^{n-k}}{(1-\overline{x})^2}$$

and we have already shown above that the numerator of this expression is positive. Thus, using $\overline{x}=\hat{p}$ in (32) we can write[35]

$$\frac{\partial H}{\partial \tilde{p}}+\frac{\partial H}{\partial \hat{p}}<-\frac{1-\hat{p}^{n-k}}{1-\hat{p}}-(n-k)\,\tilde{p}^{n-k-1}\hat{p}^k<0.$$

The result then follows. ■

Claim 3 *The price of the merging stores is higher than the price of the non-merging ones, i.e.,* $\hat{p}^*>\tilde{p}^*$.

Proof. Let $\tilde{p}_1$ be the price at which the function η_1 crosses the 45 degrees line, i.e., $\eta_1(\tilde{p}_1)=\tilde{p}_1$; likewise, let $\tilde{p}_2$ be such that $\eta_2(\tilde{p}_2)=\tilde{p}_2$ ($\tilde{p}_1$ and $\tilde{p}_2$ are represented in Figure 9). Given the properties of η_1 and η_2, if we show that $\tilde{p}_1>\tilde{p}_2$ then we can conclude that $\hat{p}^*>\tilde{p}^*$.

If we set $\tilde{p}=\tilde{p}_1$ in the FOC $G(\hat{p},\tilde{p})=0$ we obtain:

$$\tilde{p}_1=\frac{1-\overline{x}^k}{k\overline{x}^{k-1}}+\frac{\int_0^{\overline{x}-\tilde{p}_1}(\varepsilon+\tilde{p}_1)^{n-2}(\varepsilon+k\tilde{p}_1)\,d\varepsilon}{\overline{x}^{n-1}} \tag{33}$$

Similarly, when $\tilde{p}=\tilde{p}_2$ the FOC $H(\hat{p},\tilde{p})=0$ gives:

$$\tilde{p}_2=1-\overline{x}+\frac{1-\overline{x}}{1-\overline{x}^{n-k}}(n-k)\int_0^{\overline{x}-\tilde{p}_2}(\varepsilon+\tilde{p}_2)^{n-1}\,d\varepsilon \tag{34}$$

For a contradiction, suppose that $\tilde{p}_2>\tilde{p}_1$. Then the difference between the RHS of (33) and the RHS of (34) must be negative. Let us denote this difference as V and note that

$$\begin{aligned} V\equiv\ &\frac{\int_0^{\overline{x}-\tilde{p}_1}(\varepsilon+\tilde{p}_1)^{n-2}(\varepsilon+k\tilde{p}_1)\,d\varepsilon}{\overline{x}^{n-1}}+\frac{1+(k-1)\,\overline{x}^k-k\overline{x}^{k-1}}{k\overline{x}^{k-1}}-\frac{1-\overline{x}}{1-\overline{x}^{n-k}}(n-k)\int_0^{\overline{x}-\tilde{p}_2}(\varepsilon+\tilde{p}_2)^{n-1}\,d\varepsilon\\ >\ &\frac{\int_0^{\overline{x}-\tilde{p}_1}(\varepsilon+\tilde{p}_1)^{n-1}\,d\varepsilon}{\overline{x}^{n-1}}+\frac{1+(k-1)\,\overline{x}^k-k\overline{x}^{k-1}}{k\overline{x}^{k-1}}-\frac{1-\overline{x}}{1-\overline{x}^{n-k}}(n-k)\int_0^{\overline{x}-\tilde{p}_2}(\varepsilon+\tilde{p}_2)^{n-1}\,d\varepsilon \end{aligned} \tag{35}$$

where the inequality follows from replacing $\varepsilon+k\tilde{p}_1$ by $\varepsilon+\tilde{p}_1$ in the first integral.

Since the second integral in (35) is equal to $[\overline{x}^n-(\tilde{p}_2)^n]/n$, the whole expression in (35) increases in $\tilde{p}_2$. Therefore, (35) must be higher than when we replace $\tilde{p}_2$ by $\tilde{p}_1$. That is, (35) is higher than

$$\begin{aligned} &\frac{1}{\overline{x}^{n-1}}\int_0^{\overline{x}-\tilde{p}_1}(\varepsilon+\tilde{p}_1)^{n-1}\,d\varepsilon+\frac{1-(k-1)\,\overline{x}^k-k\overline{x}^{k-1}}{k\overline{x}^{k-1}}-\frac{1-\overline{x}}{1-\overline{x}^{n-k}}(n-k)\int_0^{\overline{x}-\tilde{p}_1}(\varepsilon+\tilde{p}_1)^{n-1}\,d\varepsilon\\ &=\frac{\overline{x}^n-\tilde{p}_1^n}{n\left(1-\overline{x}^{n-k}\right)}\left[\frac{1-\overline{x}^{n-k}-(n-k)\,\overline{x}^{n-1}(1-\overline{x})}{\overline{x}^{n-1}}\right]+\frac{1+(k-1)\,\overline{x}^k-k\overline{x}^{k-1}}{k\overline{x}^{k-1}} \end{aligned} \tag{36}$$

This last expression is positive, which establishes a contradiction.[36] As a result, $\hat{p}^*>\tilde{p}^*$. ■

We finish the proof by making sure that firms do not gain by deviating from the equilibrium prices. Here we use again "$*$" in order to distinguish equilibrium prices from deviation prices. We start with the non-merging firms. We first note that the payoff in (7) is strictly concave, which implies that

[35] If $k=n-1$ then $\frac{\partial H}{\partial \tilde{p}}+\frac{\partial H}{\partial \hat{p}}=-2+\left(1-\overline{x}^{n-1}\right)+(n-1)\int_0^{\overline{x}-\hat{p}}(\varepsilon+\hat{p})^{n-2}\,d\varepsilon=-1-\hat{p}^{n-1}<0$.

[36] The term in squared brackets is positive. To see this, note that it is concave in k. Therefore, if it is positive for $k=2$ and $k=n-1$, then it is positive for all k. Setting $k=2$ gives $1-\overline{x}^{n-2}-(n-2)\overline{x}^{n-1}(1-\overline{x})$, which decreases in $\overline{x}$ since its derivative is $-(n-2)\overline{x}^{n-3}(1-\overline{x})[(1+\overline{x})+(n-1)\overline{x}]<0$. If we set $\overline{x}=1$ in the value for $k=2$ gives zero. Therefore it is positive for all $\overline{x}$ and $k=2$. Setting now $k=n-1$ gives $(1-\overline{x})(\overline{x}^{n-1})>0$.

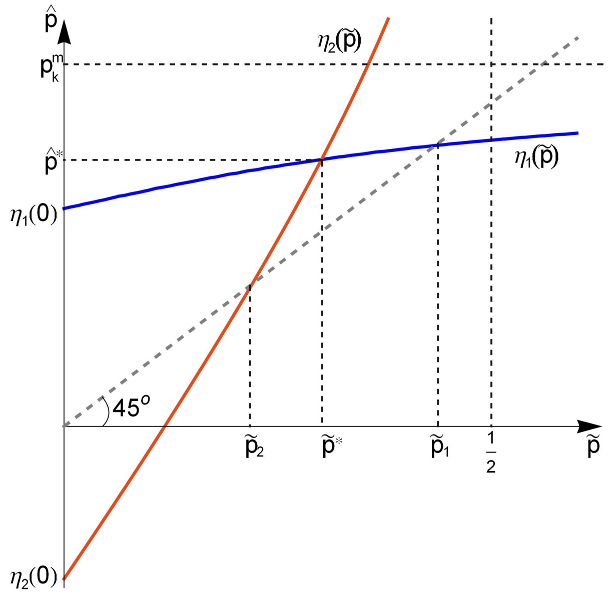


Figure 9: Existence and uniqueness of symmetric equilibrium

a non-merging firm does not gain by deviating to a price $\tilde{p}$ such that $\tilde{p} < 1 - \overline{x} + \tilde{p}^*$. Second, a non-merging firm may choose to deviate to a price $\tilde{p}$ such that $1 - \overline{x} + \tilde{p}^* \leq \tilde{p} < 1 - \overline{x} + \hat{p}^*$. Given this, all consumers walk away from the deviant non-merging store and therefore its demand is made of consumers who buy there either after visiting all non-merging stores or after visiting all stores in the market. Then, the payoff function of the deviant is

$$\widetilde{\pi}(\tilde{p}; \hat{p}^*, \tilde{p}^*) = \tilde{p}\left[\int_{\overline{x}-\hat{p}^*+\tilde{p}}^{1} (\varepsilon - \tilde{p} + \tilde{p}^*)^{n-k-1}\, d\varepsilon + \tilde{r}_a(\tilde{p}; \hat{p}^*, \tilde{p}^*)\right]$$
$$= \tilde{p}\left[\int_{\overline{x}-\hat{p}^*}^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1}\, d\varepsilon + \tilde{r}_a(\tilde{p}; \hat{p}^*, \tilde{p}^*)\right],$$

where $\tilde{r}_a(\tilde{p}; \hat{p}^*, \tilde{p}^*)$ is given in (6). Taking the FOC gives

$$\int_{\overline{x}-\hat{p}^*}^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1}\, d\varepsilon + \int_{0}^{\overline{x}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^{k}\, d\varepsilon - \tilde{p}\,(1 - \tilde{p} + \tilde{p}^*)^{n-k-1} = 0,$$

which can be rewritten as

$$(1 - \tilde{p} + \tilde{p}^*)^{n-k} - (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} + (n-k)\int_{0}^{\overline{x}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^{k}\, d\varepsilon - (n-k)\tilde{p}\,(1 - \tilde{p} + \tilde{p}^*)^{n-k-1} = 0. \tag{37}$$

From (9), we obtain the relationship

$$(n-k)\int_{0}^{\overline{x}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^{k}\, d\varepsilon = \frac{1 - \overline{x}^{n-k}}{1 - \overline{x}}\tilde{p}^* - 1 + (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k},$$

and use it in (37) to get

$$(1 - \tilde{p} + \tilde{p}^*)^{n-k} - 1 + \frac{1 - \overline{x}^{n-k}}{1 - \overline{x}}\tilde{p}^* - (n-k)\,\tilde{p}\,(1 - \tilde{p} + \tilde{p}^*)^{n-k-1} = 0,$$

or

$$(1 - \tilde{p} + \tilde{p}^*)^{n-k-1}\left[1 + \tilde{p}^* - (n-k+1)\,\tilde{p} - \frac{1}{(1 - \tilde{p} + \tilde{p}^*)^{n-k-1}}\left(1 - \frac{1 - \overline{x}^{n-k}}{1 - \overline{x}}\tilde{p}^*\right)\right] = 0. \tag{38}$$

We now argue that the LHS of (38) is always negative. Denote the term in squared brackets by $\phi(\tilde{p}, \tilde{p}^*)$. Taking its derivative with respecto to $\tilde{p}$ gives

$$\frac{\partial \phi}{\partial \tilde{p}} = -(n-k+1) - \frac{n-k-1}{(1-\tilde{p}+\tilde{p}^*)^{n-k}} \left(1 - \frac{1-\overline{x}^{n-k}}{1-\overline{x}} \tilde{p}^*\right),$$

which is negative because, using (9) again,

$$\begin{aligned} 1 - \frac{1-\overline{x}^{n-k}}{1-\overline{x}} \tilde{p}^* &= (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} - (n-k) \int_0^{\overline{x}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon \\ &> (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} - \overline{x}^k (n-k) \int_0^{\overline{x}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} \, d\varepsilon \\ &= (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} \left(1 - \overline{x}^k\right) + \overline{x}^k \tilde{p}^* > 0. \end{aligned}$$

Since $\phi(\tilde{p}, \tilde{p}^*)$ decreases in $\tilde{p}$ and in the deviation $\tilde{p} \geq 1 - \overline{x} + \tilde{p}^* > \tilde{p}^*$, if we prove that $\phi(\tilde{p}, \tilde{p}^*) < \phi(\tilde{p}^*, \tilde{p}^*) < 0$ then we can conclude that the LHS of (38) is always negative, which implies that the profit function is decreasing in $\tilde{p}$ and threfore the deviation is not profitable. But

$$\phi(\tilde{p}^*, \tilde{p}^*) = 1 + \tilde{p}^* - (n-k+1)\tilde{p}^* - \left(1 - \frac{1-\overline{x}^{n-k}}{1-\overline{x}} \tilde{p}^*\right) < 1 - (n-k)\tilde{p}^* - (1 - (n-k)\tilde{p}^*) = 0,$$

where the inequality follows from the fact that $1 - \overline{x}^{n-k} < (n-k)(1-\overline{x})$.

Finally, a non-merging firm may also deviate to a price $\tilde{p}$ such that $1 - \overline{x} + \hat{p}^* \leq \tilde{p} < 1/2$, in which case it would only sell to consumers who visit all firms in the market. Its payoff would be

$$\widetilde{\pi}(\tilde{p}; \hat{p}^*, \tilde{p}^*) = \tilde{p} \int_{\tilde{p}}^1 (\varepsilon - \tilde{p} + \tilde{p}^*)^{n-k-1} (\varepsilon - \tilde{p} + \hat{p}^*)^k \, d\varepsilon = \tilde{p} \int_0^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon.$$

Taking the FOC and rewriting it gives

$$\int_0^{1-\tilde{p}} \frac{(\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k}{(1 - \tilde{p} + \tilde{p}^*)^{n-k-1} (1 - \tilde{p} + \hat{p}^*)^k} d\varepsilon - \tilde{p} = 0 \tag{39}$$

The integrand of this expression increases in k. In fact its derivative with respect to k is equal to

$$\frac{(\varepsilon + \tilde{p}^*)^{n-k-1}}{(1 - \tilde{p} + \tilde{p}^*)^{n-k-1}} \frac{(\varepsilon + \hat{p}^*)^k}{(1 - \tilde{p} + \hat{p}^*)^k} \ln \frac{(\varepsilon + \hat{p}^*)(1 - \tilde{p} + \tilde{p}^*)}{(1 - \tilde{p} + \hat{p}^*)(\varepsilon + \tilde{p}^*)} > 0,$$

where the inequality follows from the observation that

$$\frac{(\varepsilon + \hat{p}^*)(1 - \tilde{p} + \tilde{p}^*)}{(1 - \tilde{p} + \hat{p}^*)(\varepsilon + \tilde{p}^*)} > \frac{(\varepsilon + \hat{p}^*)(1 - \tilde{p} + \hat{p}^*)}{(1 - \tilde{p} + \hat{p}^*)(\varepsilon + \hat{p}^*)} = 1.$$

Hence, setting $k = n-1$ in (39) we can write

$$\int_0^{1-\tilde{p}} \frac{(\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k}{(1 - \tilde{p} + \tilde{p}^*)^{n-k-1} (1 - \tilde{p} + \hat{p}^*)^k} d\varepsilon - \tilde{p} < \int_0^{1-\tilde{p}} \frac{(\varepsilon + \hat{p}^*)^{n-1}}{(1 - \tilde{p} + \hat{p}^*)^{n-1}} d\varepsilon - \tilde{p} \tag{40}$$

Now, because $\tilde{p} > \hat{p}^*$ and $\hat{p}^* > p^*$ (which is proven in Proposition 2) and the expression $1 - \tilde{p}^n - n\tilde{p}$ decreases in $\tilde{p}$ so $1 - \tilde{p}^n - n\tilde{p} < 1 - p^{*n} - np^* \leq 0$ (see the FOC (4) we can write that (40) is lower than

$$\int_0^{1-\tilde{p}} \frac{(\varepsilon + \tilde{p})^{n-1}}{(1 - \tilde{p} + \tilde{p})^{n-1}} d\varepsilon - \tilde{p} = \frac{1}{n}(1 - \tilde{p}^n - n\tilde{p}) < 0.$$

We then conclude that the LHS of (39) is always negative and therefore the deviation is not profitable either.

To complete the proof of existence and uniqueness of symmetric equilibrium we also need to make sure that the merged entity does not find it profitable to deviate to a price vector $(p_1, p_2, ..., p_k) \neq (\hat{p}^*, \hat{p}^*, ..., \hat{p}^*)$. Checking this in general is extremely difficult. In what follows we will show it for the case $k = 2$. We first prove analytically that the payoff of a deviant merged entity is strictly concave for all $(p_1, p_2) \in [0, 1 - \overline{x} + \hat{p}^*)^2$. (Note that when the search cost is large ($\hat{p}^* \to \overline{x}$), this analytical proof is sufficient because the set $[0, 1 - \overline{x} + \hat{p}^*)^2$ covers the entire strategy space.) For lower search costs we show numerically by plot that there do not exist incentives to deviate from the solution to the system of FOCs (9) and (10).[37]

For the $k = 2$ case, the payoff function of the deviant merged entity for $(p_1, p_2) \in [0, 1 - \overline{x} + \hat{p}^*)^2$ is

$$\hat{\pi}(p_i, p_j; \hat{p}^*, \tilde{p}^*) = \sum_i p_i d_i(p_i, p_j; \hat{p}^*, \tilde{p}^*) \quad i, j = \{1, 2\}, i \neq j$$

where

$$d_i = (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} \left(\frac{1}{2} + \frac{1}{2} (\overline{x} - \hat{p}^* + p_j) \right) (1 - \overline{x} + \hat{p}^* - p_i) + \int_0^{\overline{x} - \hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-2} (\varepsilon + p_j) \, d\varepsilon.$$

The second order derivatives are

$$\frac{\partial^2 \hat{\pi}}{\partial p_i^2} = -(\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} (1 + \overline{x} - \hat{p}^* + p_j), \quad i, j = \{1, 2\}, i \neq j$$
$$\frac{\partial^2 \hat{\pi}}{\partial p_i \partial p_j} = (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} (1 - \overline{x} + \hat{p}^* - p_i - p_j) + 2 \int_0^{\overline{x} - \hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-2} \, d\varepsilon$$

The first minor of the Hessian matrix is clearly negative. To show that the second minor is positive, we have to prove that the absolute value of the diagonal elements of the Hessian matrix are greater than the absolute value of the off-diagonal elements. Suppose that the off-diagonal elements are positive. Then we need to show that

$$\frac{\partial^2 \hat{\pi}}{\partial p_i^2} + \frac{\partial^2 \hat{\pi}}{\partial p_i \partial p_j} < 0.$$

Putting terms together, we obtain

$$\begin{aligned}
\frac{\partial^2 \hat{\pi}}{\partial p_i^2} + \frac{\partial^2 \hat{\pi}}{\partial p_i \partial p_j} &= (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} (-2\overline{x} + 2\hat{p}^* - p_i - 2p_j) + 2 \int_0^{\overline{x} - \hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-2} \, d\varepsilon \\
&< (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} (-2\overline{x} + 2\hat{p}^*) + 2 \int_0^{\overline{x} - \hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-2} \, d\varepsilon \\
&< -2 (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} (\overline{x} - \hat{p}^*) + 2 (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} \int_0^{\overline{x} - \hat{p}^*} d\varepsilon = 0,
\end{aligned}$$

where the first inequality follows from setting $p_i = p_j = 0$ and the second one from setting the integrand equal to its highest value, i.e., $\overline{x} - \hat{p}^* + \tilde{p}^*$.

If the off-diagonal elements are negative instead, we have to prove that

$$\frac{\partial^2 \hat{\pi}}{\partial p_i^2} - \frac{\partial^2 \hat{\pi}}{\partial p_i \partial p_j} < 0.$$

Putting terms together, we obtain

$$\frac{\partial^2 \hat{\pi}}{\partial p_i^2} - \frac{\partial^2 \hat{\pi}}{\partial p_i \partial p_j} = -(\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} (2 - p_i) - 2 \int_0^{\overline{x} - \hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-2} \, d\varepsilon < 0.$$

[37] For $k = 3$, we can also show analytically that the payoff function is strictly concave in a neighborhood of the symmetric equilibrium. In order to save space, we omit this proof.

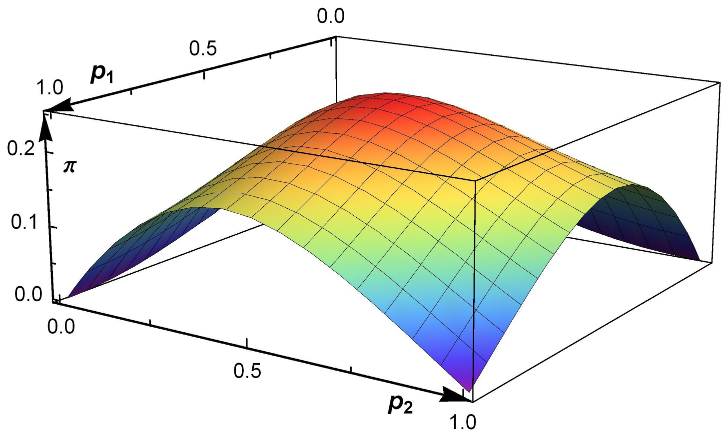


Figure 10: Payoff function of the deviant merged entity ($n = 3, k = 2$)

We conclude that the payoff of the deviant merged entity is strictly concave for $(p_1, p_2) \in [0, 1-\overline{x}+\hat{p}^*)^2$. And if search cost is high, as mentioned above, the payoff is globally strictly concave.

Consider now deviations to pairs $(p_1, p_2) \notin [0, 1-\overline{x}+\hat{p}^*)^2$. For this we proceed numerically. Let us start with deviations such that $0 \le p_i < 1-\overline{x}+\hat{p}^*$ and $1-\bar{x}-\hat{p}^* \le p_j \le 1$. In such deviations, the payoff of the merged entity is equal to

$$\pi(p_i, p_j; \hat{p}^*\tilde{p}^*) = \sum_i p_i d_i(p_i, p_j; \hat{p}^*, \tilde{p}^*) \quad i, j = \{1, 2\}, i \neq j$$

where

$$\begin{aligned}
d_i &= (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} (1 - \overline{x} + \hat{p}^* - p_i) + \int_{p_i}^{1+p_i-p_j} (\varepsilon + \tilde{p}^* - p_i)^{n-2} (\varepsilon + p_j - p_i)\, d\varepsilon \\
&+ \int_{1+p_i-p_j}^{\overline{x}-\hat{p}^*+p_i} (\varepsilon + \tilde{p}^* - p_i)^{n-2}\, d\varepsilon \\
&= (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-2} (1 - \overline{x} + \hat{p}^* - p_i) + \int_0^{1-p_j} (\varepsilon + \tilde{p}^*)^{n-2} (\varepsilon + p_j)\, d\varepsilon + \int_{1-p_j}^{\bar{x}-\hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-2}\, d\varepsilon \\
d_j &= \int_{p_j}^1 (\varepsilon + \tilde{p}^* - p_j)^{n-2} (\varepsilon + p_i - p_j)\, d\varepsilon = \int_0^{1-p_j} (\varepsilon + \tilde{p}^*)^{n-2} (\varepsilon + p_i)\, d\varepsilon
\end{aligned}$$

For deviations such that $1 - \overline{x} + \hat{p}^* \le p_i \le p_j \le 1$, we obtain a payoff equal to

$$\begin{aligned}
\pi &= p_i \left[\int_{p_i}^{1+p_i-p_j} (\varepsilon + \tilde{p}^* - p_i)^{n-2} (\varepsilon + p_j - p_i)\, d\varepsilon + \int_{1+p_i-p_j}^1 (\varepsilon + \tilde{p}^* - p_i)^{n-2}\, d\varepsilon \right] \\
&+ p_j \int_{p_j}^1 (\varepsilon + \tilde{p}^* - p_j)^{n-2} (\varepsilon + p_i - p_j)\, d\varepsilon \\
&= p_i \left[\int_{1-p_j}^{1-p_i} (\varepsilon + \tilde{p}^*)^{n-2}\, d\varepsilon + \int_0^{1-p_j} (\varepsilon + \tilde{p}^*)^{n-2} (\varepsilon + p_j)\, d\varepsilon \right] + p_j \int_0^{1-p_j} (\varepsilon + \tilde{p}^*)^{n-2} (\varepsilon + p_i)\, d\varepsilon
\end{aligned}$$

Figure 10 plots the payoff of the deviant merged entity for all possible pairs of deviation prices $(p_1, p_2) \in [0, 1]^2$ when the search cost is very small, namely, $s = 0.0002$. As it can be seen, the payoff function reaches its maximum at the symmetric equilibrium $(\hat{p}^*, \tilde{p}^*) \simeq (0.48, 0.39)$. We note that for other relatively low search cost values, the payoff function has a similar shape as in Figure 10. Moreover, as mentioned above, for high search cost values, the payoff function is strictly concave. These observations lead us to conclude that for $k = 2$, the solution to the FOCs (9) and (10) is the unique symmetric Nash equilibrium with $\hat{p}^* > \tilde{p}^*$. ■

Proof of Proposition 2. From Proposition 1, $\hat{p}^* > \tilde{p}^*$. Let us now show that $\tilde{p}^* > p^*$. In what follows we drop the "$*$" super-indexes.

(a) For a contradiction, assume that $\tilde{p} < p$ when $\overline{x} \to 1$. Denote the equilibrium quantity sold by a non-merged firm by $\tilde{q}$, that sold by all the merging firms together by $\hat{q}$ and the aggregate quantity sold in the market by all firms by Q. We note that $Q = 1 - \hat{p}^k \tilde{p}^{n-k}$.[38] Using the FOCs we can write that

$$Y(\tilde{p}, \hat{p}) \equiv Q - (n-k)\tilde{q} - \hat{q} = 0$$

where

$$(n-k)\tilde{q} = \frac{1-\overline{x}^{n-k}}{1-\overline{x}}\tilde{p}$$

$$\hat{q} = k\hat{p}\overline{x}^{k-1}(\overline{x} - \hat{p} + \tilde{p})^{n-k} - k(k-1)\hat{p}\int_0^{\overline{x}-\hat{p}} (\varepsilon + \hat{p})^{k-2}(\varepsilon + \tilde{p})^{n-k} d\varepsilon.$$

We now argue that $Y(\tilde{p}, \hat{p})$ is decreasing in $\tilde{p}$. This is because $\partial Q/\partial \tilde{p} < 0$, $\partial \tilde{q}/\partial \tilde{p} > 0$ and

$$\begin{aligned}
\frac{1}{k(n-k)\hat{p}}\frac{\partial \hat{q}}{\partial \tilde{p}} &= \overline{x}^{k-1}(\overline{x} - \hat{p} + \tilde{p})^{n-k-1} - (k-1)\int_0^{\overline{x}-\hat{p}} (\varepsilon + \hat{p})^{k-2}(\varepsilon + \tilde{p})^{n-k-1} d\varepsilon \\
&> \overline{x}^{k-1}(\overline{x} - \hat{p} + \tilde{p})^{n-k-1} - (k-1)(\overline{x} - \hat{p} + \tilde{p})^{n-k-1}\int_0^{\overline{x}-\hat{p}} (\varepsilon + \hat{p})^{k-2} d\varepsilon \\
&= (\overline{x} - \hat{p} + \tilde{p})^{n-k-1}\hat{p}^{k-1} > 0
\end{aligned}$$

Next, since Y is decreasing in $\tilde{p}$ and by assumption $\tilde{p} < p$ we must have $Y(p, \hat{p}) < 0$. In other words, using the notation $\lim_{\overline{x}\to 1} p = p_1$ and $\lim_{\overline{x}\to 1} \hat{p} = \hat{p}_1$, it must be the case that

$$\begin{aligned}
\lim_{\overline{x}\to 1} Y(p_1, \hat{p}_1) = 1 - \hat{p}_1^k p_1^{n-k} - (n-k)p_1 - k\hat{p}_1(1 - \hat{p}_1 + p_1)^{n-k} \\
+ k(k-1)\hat{p}_1\int_0^{1-\hat{p}_1} (\varepsilon + \hat{p}_1)^{k-2}(\varepsilon + p_1)^{n-k} d\varepsilon < 0
\end{aligned} \tag{41}$$

Now we invoke the FOC of the merged entity, denoted above by $G(\hat{p}, \tilde{p})$. The function $G(\hat{p}, \tilde{p})$ was shown to be increasing in $\tilde{p}$ so when $\tilde{p} < p$ we must have $G(\hat{p}, p) > G(\hat{p}, \tilde{p}) = 0$. Therefore:

$$\begin{aligned}
\lim_{\overline{x}\to 1} G(\hat{p}, p) = -\hat{p}_1 + \frac{(k-1)}{(1 - \hat{p}_1 + \tilde{p}_1)^{n-k}}\hat{p}_1\int_0^{1-\hat{p}_1} (\varepsilon + \hat{p}_1)^{k-2}(\varepsilon + p_1)^{n-k} d\varepsilon \\
+ \frac{1}{(1 - \hat{p}_1 + \tilde{p}_1)^{n-k}}\int_0^{1-\hat{p}_1} (\varepsilon + \hat{p}_1)^{k-1}(\varepsilon + p_1)^{n-k} d\varepsilon
\end{aligned}$$

must be positive, which implies that it must be the case that

$$-\hat{p}_1(1 - \hat{p}_1 + p_1)^{n-k} + (k-1)\hat{p}_1\int_0^{1-\hat{p}_1} (\varepsilon + \hat{p}_1)^{k-2}(\varepsilon + p_1)^{n-k} d\varepsilon > -\int_0^{1-\hat{p}_1} (\varepsilon + \hat{p}_1)^{k-1}(\varepsilon + p_1)^{n-k} d\varepsilon.$$

Using this inequality in (41), we get that

$$\lim_{\overline{x}\to 1} Y(p, \hat{p}) > 1 - \hat{p}_1^k p_1^{n-k} - (n-k)p_1 - k\int_0^{1-\hat{p}_1} (\varepsilon + \hat{p}_1)^{k-1}(\varepsilon + p_1)^{n-k} d\varepsilon. \tag{42}$$

[38] A consumer does not buy at all when the match value drawn at every firm is lower than its corresponding price.

This last expression is increasing in $\hat{p}_1$. This is because the sign of its derivative with respect to $\hat{p}_1$ is the same as the sign of the following expression

$$\begin{aligned}
&-\hat{p}_1^{k-1}p_1^{n-k}-(k-1)\int_0^{1-\hat{p}_1}(\varepsilon+\hat{p}_1)^{k-2}(\varepsilon+p_1)^{n-k}\,d\varepsilon+(1-\hat{p}_1+p_1)^{n-k}\\
&>-\hat{p}_1^{k-1}p_1^{n-k}-(k-1)(1-\hat{p}_1+p_1)^{n-k}\int_0^{1-\hat{p}_1}(\varepsilon+\hat{p}_1)^{k-2}\,d\varepsilon+(1-\hat{p}_1+p_1)^{n-k}\\
&=-\hat{p}_1^{k-1}p_1^{n-k}+(1-\hat{p}_1+p_1)^{n-k}\hat{p}_1^{k-1}>0.
\end{aligned}$$

We now argue that $\hat{p}_1 \geq p_1$. To show this, we first invoke the result in Proposition 3 of Zhou (2009) that the equilibrium price of firms visited last is higher than p. In our model, in addition to the search-order effect of Zhou, the firms visited last internalize the pricing externalities they confer on one another and this leads the firms to raise further their prices. As a result, here it must also be the case that $\hat{p}_1 \geq p_1$. Given this, (42) is greater than after setting $\hat{p}_1 = p_1$, that is, $\lim_{\overline{x}\to 1} Y(p,\hat{p})$ is larger than

$$1-(p_1)^n-(n-k)\,p_1-k\left[\frac{1}{n}-\frac{1}{n}(p_1)^n\right]=np_1-(n-k)\,p_1-kp_1=0,$$

where for the first equality we have used the FOC of a typical firm in the pre-merger market (when $\overline{x}\to 1$ the FOC of a firm in a pre-merger market becomes $1-np_1-(p_1)^n=0$). Consequently, if $\tilde{p}_1 > p_1$ then we have $\lim_{\overline{x}\to 1} Y(p,\hat{p}) > 0$, which establishes a contradiction.

(b) Let us take the limit of the LHS of (9) and (10) when $\overline{x}\to p_k^m$ and let $\tilde{p}_l \equiv \lim_{\overline{x}\to p_k^m}\tilde{p}$ and $p_k^m \equiv \lim_{\bar{x}\to p_k^m}\hat{p}$ (which is the monopoly price). Then we get the following expressions

$$\begin{aligned}
(\tilde{p}_l)^{n-k}\left[1-(k+1)\left(p_k^m\right)^k\right]&=0\\
\left(1-p_k^m\right)\left(1-(\tilde{p}_l)^{n-k}\right)-\tilde{p}_l\left[1-\left(p_k^m\right)^{n-k}\right]&=0.
\end{aligned}\tag{43}$$

The first equation is indeed zero given the definition of p_k^m and the second equation therefore gives the value of $\tilde{p}$ when $\bar{x}\to p_k^m$. We note that $\tilde{p}_l < p^m = 1/2$ because, as shown in the proof of proposition 1, $H\left(p_k^m, 1/2\right) \leq 0$.

Let $p_l \equiv \lim_{\bar{x}\to p_k^m} p$. We now argue that $\tilde{p}_l > p_l$. To show this, we take the limit when $\bar{x}\to p_k^m$ of the FOC that determines p_l. This gives $(1-p_k^m)(1-(p_l)^n)-p_l[1-(p_k^m)^n]=0$. The solution of this equation, p_l, decreases in n. Comparing this equation with (43), since $n-k<n$, it is immediately clear that $\tilde{p}_l > p_l$.

(c) If $n=3$ then the FOC of a merging firm may be rearranged as follows

$$\hat{p}^3-\hat{p}\overline{x}^2-\tilde{p}\left(3\hat{p}^2-1\right)=\frac{\overline{x}^3}{3}-\frac{\hat{p}^3}{3}-\overline{x}+\hat{p}.\tag{44}$$

The FOC of a non-merging firm gives us the relation $\overline{x}^3/3-\hat{p}^3/3-\overline{x}+\hat{p}=2\tilde{p}-1$. Using this expression in (44) we have $\hat{p}^3-\hat{p}\overline{x}^2-3\tilde{p}\hat{p}^2-\tilde{p}+1=0$, or

$$\tilde{p}=\frac{1+\hat{p}^3-\hat{p}\overline{x}^2}{1+3\hat{p}^2}.$$

From the FOC in the pre-merger market we know that

$$p=\frac{1-p^3}{1+\overline{x}+\overline{x}^2}.$$

Since, by strategic complementarity, $\tilde{p}$ increases in $\hat{p}$ and since $\hat{p} > p$, the difference $\tilde{p} - p$ is greater than when we replace $\hat{p}$ by p. Therefore

$$\tilde{p} - p = \frac{1 + \hat{p}^3 - \hat{p}\overline{x}^2}{1 + 3\hat{p}^2} - \frac{1 - p^3}{1 + \overline{x} + \overline{x}^2} > \frac{1 + p^3 - p\overline{x}^2}{1 + 3p^2} - \frac{1 - p^3}{1 + \overline{x} + \overline{x}^2} \tag{45}$$

The RHS of this expression is concave in $\overline{x}$ because its second derivative with respect to $\overline{x}$ is negative:

$$-\frac{2p}{1 + 3p^2} - \frac{6\left(1 - p^3\right)\overline{x}(1 + \overline{x})}{\left(1 + \overline{x} + \overline{x}^2\right)^3} < 0$$

Hence, if the RHS of (45) is positive with the highest and the lowest possible values of $\overline{x}$ then it is positive for all possible $\overline{x}$ values. Setting $\overline{x} = 1$ in the RHS of (45) gives

$$\frac{2 - 3p - 3p^2 + 4p^3 + 3p^5}{3\left(1 + 3p^2\right)}. \tag{46}$$

which, as shown in Figure 11, is always positive for all $p \in [0, 1/2]$. Setting $\overline{x} = p$ in (45) gives

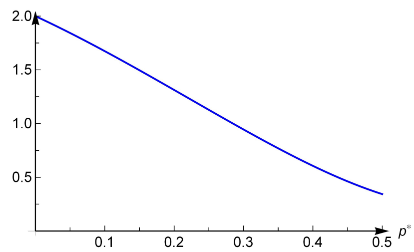


Figure 11: Plot of expression 46

$$\frac{p\left(1 - 3p + 3p^2\right)}{1 + 3p^2} > 0.$$

Thus, $\tilde{p} > p$. ■

Proof of Proposition 3. In symmetric equilibrium the payoff to the merged entity is equal to

$$\hat{\pi}(\tilde{p}^*, \hat{p}^*) = \hat{p}^* \left[(1 - \overline{x}^k)(\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k} + k \int_0^{\overline{x} - \hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k} (\varepsilon + \hat{p}^*)^{k-1} d\varepsilon\right],$$

while the payoff to a non-merging firm is equal to

$$\tilde{\pi}(\tilde{p}^*, \hat{p}^*) = \tilde{p}^* \left\{\frac{1}{n-k}\left[1 - (\overline{x} - \hat{p}^* + \tilde{p}^*)^{n-k}\right] + \int_0^{\overline{x} - \hat{p}^*} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k d\varepsilon\right\}.$$

(a) To prove this we set $k = 2$ in the profits difference $\hat{\pi}^*/k - \pi^*$ and study its sign when $\overline{x} \to p_2^m (= 1/\sqrt{3})$. For the profit of a merging firm we have

$$\lim_{\overline{x} \to 1/\sqrt{3}} \frac{\hat{\pi}^*}{2} = \frac{(\tilde{p}_l)^{n-2}}{3\sqrt{3}}$$

where, as in the proof of Proposition 2, $\tilde{p}_l \equiv \lim_{\overline{x} \to p_k^m} \tilde{p}^*$. We have shown above that $\tilde{p}_l < p^m = 1/2$. Therefore,

$$\lim_{\overline{x} \to 1/\sqrt{3}} \frac{\hat{\pi}^*}{2} < \frac{(p^m)^{n-2}}{3\sqrt{3}}$$

which implies that

$$\lim_{\overline{x}\to 1/\sqrt{3}}\left[\frac{\hat{\pi}^*}{2}-\pi^*\right]<\frac{(1/2)^{n-2}}{3\sqrt{3}}-\frac{(p_l)^2\left[1-(p_2^m)^n\right]}{n(1-p_2^m)} \tag{47}$$

where, again as in the proof of Proposition 2, $p_l \equiv \lim_{\overline{x}\to p_k^m} p^*$. If we demonstrate that (47) is negative, then the result follows. For this we need that

$$p_l>\sqrt{\frac{n(1-3^{-1/2})(1/2)^{n-2}}{3\sqrt{3}(1-3^{-n/2})}} \tag{48}$$

To show that (48) indeed holds, we now invoke the FOC in the pre-merger market; when $\overline{x}\to 1/\sqrt{3}$ the FOC writes

$$1-(p_l)^n-p_l\frac{1-3^{-n/2}}{1-3^{-1/2}}=0. \tag{49}$$

Now, using (49), if we replace p_l by $\left[\frac{n(1-3^{-1/2})}{2^{n-2}3\sqrt{3}(1-3^{-n/2})}\right]^{\frac{1}{2}}$ in this expression we get

$$1-\left[\frac{n(1-3^{-1/2})}{2^{n-2}3\sqrt{3}(1-3^{-n/2})}\right]^{\frac{n}{2}}-\sqrt{\frac{n(1-3^{-1/2})}{2^{n-2}3\sqrt{3}(1-3^{-n/2})}}\frac{\left(1-3^{-n/2}\right)}{1-3^{-1/2}}. \tag{50}$$

This last expression, as shown in Figure 12, is always positive for $n\geq 3$.

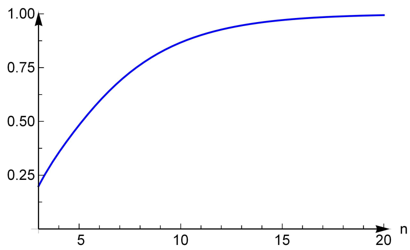


Figure 12: Plot of expression (50).

Since (49) is decreasing in p_l, then (48) must hold.

(b) Using the definition of p_k^m, we have that $1-(p_k^m)^k=k(p_k^m)^k$. Therefore we can write

$$\begin{aligned}\lim_{\overline{x}\to p_k^m;n\to\infty}\left[\frac{\hat{\pi}^*}{k}-\pi^*\right]&=\lim_{n\to\infty}\left[p_k^m(\tilde{p}_l)^{n-k}(1-(p_k^m)^k)-\frac{(p_l)^2\left(1-(p_k^m)^n\right)}{n\left(1-p_k^m\right)}\right]\\&<\lim_{n\to\infty}\left[\frac{\left(p_k^m\right)^{k+1}}{2^{n-k}}-\frac{(p_l)^2\left(1-(p_k^m)^n\right)}{n\left(1-p_k^m\right)}\right]\end{aligned}$$

where the inequality follows from the fact that $\tilde{p}_l<p^m=1/2$. Note that $1-(p_k^m)^n>1-(p_k^m)^k=k\left(p_k^m\right)^k$. Thus,

$$\lim_{n\to\infty}\left[\frac{\left(p_k^m\right)^{k+1}}{2^{n-k}}-\frac{(p_l)^2\left(1-(p_k^m)^n\right)}{n\left(1-p_k^m\right)}\right]<\frac{\left(p_k^m\right)^k}{1-p_k^m}\lim_{n\to\infty}\left[\frac{p_k^m\left(1-p_k^m\right)}{2^{n-k}}-\frac{(p_l)^2k}{n}\right]=0,$$

which shows that for any k, merging is not profitable whenever search costs and the number of competitors is sufficiently high.

(c) To prove this, we show that $\lim_{\overline{x}\to 1}\left[\hat{\pi}^*-k\pi^*\right]>0$. Notice that

$$\lim_{\overline{x}\to 1}\hat{\pi}^*=\hat{p}_1^*k\int_0^{1-\hat{p}_1^*}\left(\varepsilon+\tilde{p}_1^*\right)^{n-k}\left(\varepsilon+\hat{p}_1^*\right)^{k-1}d\varepsilon \tag{51}$$

where, as in Proposition 2 we use the notation $\hat{p}_1^* \equiv \lim_{\overline{x}\to 1} \hat{p}^*$ and $\tilde{p}_1^* \equiv \lim_{\overline{x}\to 1} \tilde{p}_1^*$. Since $\hat{p}_1^*$ solves the FOC (10), we can replace $\hat{p}_1^*$ in (51) and write

$$\lim_{\overline{x}\to 1} \hat{\pi}^* > \tilde{p}_1^* k \int_0^{1-\tilde{p}_1^*} (\varepsilon + \tilde{p}_1^*)^{n-k} (\varepsilon + \tilde{p}_1^*)^{k-1} d\varepsilon = \frac{k\tilde{p}_1^*}{n} (1 - \tilde{p}_1^{*n}).$$

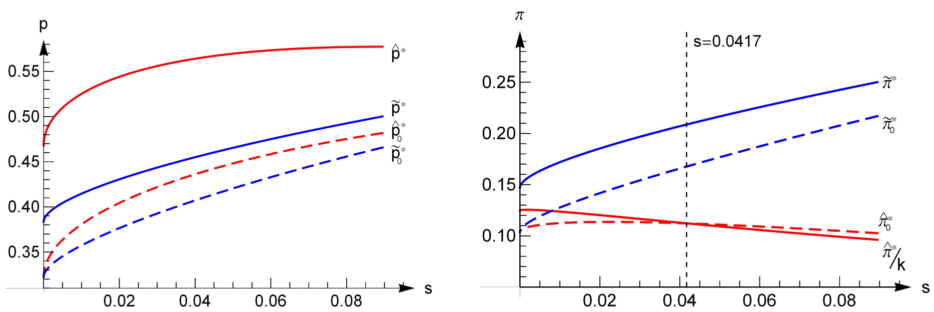
We note note that the polynomial $y(1-y^n)$ is increasing in y for all $y \le (n+1)^{-1/n}$. Therefore, because $\tilde{p}_1^* > p_1^*$ we can write that $\lim_{\overline{x}\to 1} \hat{\pi}^* > \frac{kp_1^*}{n} (1 - p_1^{*n}) = k \lim_{\overline{x}\to 1} \pi^*$, where $p_1^* \equiv \lim_{\overline{x}\to 1} p_1^*$. ■

Illustration that our merger paradox result of Proposition 3 also holds under a fixed order of search .

Here we argue that our paradoxical result in Proposition 3 that merging is not profitable in the short-run when search costs are sufficiently high is *not* just an artifact of the change from random search in the pre-merger market to directed search in the post-merger market. To illustrate this idea, we examine a model where in the pre-merger market the potentially merging firms are searched last, as in the post-merger equilibrium of Proposition 1. This signifies that a merger does not alter the search order and therefore the price and profits effects of a merger are just due to the internalization-of-pricing-externalities effect.

Post-merger the payoffs to merging and non-merging firms can easily be obtained from our derivations above. Let $\tilde{p}_0^*$ and $\hat{p}_0^*$ be the pre-merger equilibrium prices. Since in the pre-merger market consumer start searching at the non-merging firms, the payoff to a deviant non-merging firm is exactly equal to (7), where $\tilde{p}^*$ and $\hat{p}^*$ should be replaced by $\tilde{p}_0^*$ and $\hat{p}_0^*$. The payoff of a potentially merging firm in the pre-merger market is easily obtained by setting $d_m(\hat{p}; \tilde{p}^*, \hat{p}^*) = 0$ in the expression (8), where again $\tilde{p}^*$ and $\hat{p}^*$ should be replaced by $\tilde{p}_0^*$ and $\hat{p}_0^*$. Post-merger payoffs are given by (7) and (8).

The graphs of Figure 13 compare pre- and post-merger prices and profits. The dashed lines of Figure 13a represent the pre-merger prices. The firm that is visited first charges a lower price than the firms that are visited later. The solid lines represent the prices post-merger. The graph clearly shows that when the potentially merging firms do indeed merge, they raise their prices much more than the non-merging firms. When search costs are high, this reduces sharply their sales, which has consequences for merger profitability. Figure 13b shows the profits effects of a merger. Again, the dashed curves represent the pre-merger profits of the two types of firms, while the solid lines show the post-merger profits. The profits of the non-merging firms clearly increase after a merger takes place. However, the profits of the merging firms increase for low search costs but decrease for high search costs. In sum, this is not very different from our main analysis illustrated in Figure 2.



(a) Prices (b) Profits

Figure 13: Merger paradox in the short-run with fixed search-order $(n = 3, k = 2)$.

Proof of Proposition 4. The proof is similar to the proof of Proposition 1. We first claim that there is a pair of prices $\{\hat{p}^*, \tilde{p}^*\}$ that satisfies the FOCs (18) and (19) and then argue that such a pair of prices is unique. After this, we show that the putative order of search is optimal when the search cost is sufficiently large. Finally, we check whether "large" deviations are profitable.

Let $G(\hat{p}^*, \tilde{p}^*)$ and $H(\hat{p}^*, \tilde{p}^*)$ denote the LHS of the FOCs (18) and (19), respectively. In what follows, we drop the "$*$" super-indexes to shorten the expressions.

Claim 4 *There is a pair of prices $\hat{p}$ and $\tilde{p}$ that satisfy the first-order conditions $G(\hat{p}, \tilde{p}) = 0$ and $H(\hat{p}, \tilde{p}) = 0$.*

Proof. The function G is differentiable and takes on real values for all $(\hat{p}, \tilde{p}) \in [0, p_k^m] \times [0, p^m]$. Therefore, the FOC $G(\tilde{p}, \hat{p}) = 0$ defines an implicit relation between $\hat{p}$ and $\tilde{p}$. Let us denote such a relationship by $\tilde{p} = \upsilon_1(\hat{p})$. We now argue that υ_1 is increasing. By the implicit function theorem

$$\frac{\partial \upsilon_1}{\partial \hat{p}} = \frac{-\partial G/\partial \hat{p}}{\partial G/\partial \tilde{p}} \tag{52}$$

We next note that G is decreasing in $\hat{p}$ and increasing in $\tilde{p}$. To see this, compute first

$$\begin{aligned}
\frac{\partial G}{\partial \hat{p}} &= \Big[-(k-1)(\overline{x} - \tilde{p} + \hat{p})^{k-2}(\overline{x} - \tilde{p} + (k+1)\hat{p}) - (k+1)(\overline{x} - \tilde{p} + \hat{p})^{k-1} \\
&+ k(k-1)\int_0^{\overline{x}-\tilde{p}} (\varepsilon + \tilde{p})^{n-k}(\varepsilon + \hat{p})^{k-3}(2\varepsilon + k\hat{p})\, d\varepsilon\Big] \\
&= -k(\overline{x} - \tilde{p} + \hat{p})^{k-2}(2\overline{x} - 2\tilde{p} + (k+1)\hat{p}) \\
&+ k(k-1)\int_0^{\overline{x}-\tilde{p}} (\varepsilon + \tilde{p})^{n-k}(\varepsilon + \hat{p})^{k-3}(2\varepsilon + k\hat{p})\, d\varepsilon
\end{aligned}$$

Note next that $\partial G/\partial \hat{p}$ decreases in $\overline{x}$. This is because

$$\begin{aligned}
\frac{1}{k}\frac{\partial^2 G}{\partial \overline{x} \partial \hat{p}} &= -(k-2)(\overline{x} - \tilde{p} + \hat{p})^{k-3}(2\overline{x} - 2\tilde{p} + (k+1)\hat{p}) - 2(\overline{x} - \tilde{p} + \hat{p})^{k-2} \\
&+ (k-1)\overline{x}^{n-k}(\overline{x} - \tilde{p} + \hat{p})^{k-3}(2\overline{x} - 2\tilde{p} + k\hat{p}) \\
&= -(k-1)(\overline{x} - \tilde{p} + \hat{p})^{k-3}(2\overline{x} - 2\tilde{p} + k\hat{p})\left(1 - \overline{x}^{n-k}\right) < 0
\end{aligned}$$

We know that $\overline{x} \geq \tilde{p}$. If we evaluate $\partial G/\partial \hat{p}$ at $\overline{x} = \tilde{p}$ we obtain $\partial G/\partial \hat{p} = -k(k+1)\hat{p}^{k-1} < 0$. Since $\partial G/\partial \hat{p}$ decreases in $\overline{x}$, then we conclude $\partial G/\partial \hat{p}$ is negative for all $\overline{x}$.

Compute now

$$\begin{aligned}
\frac{\partial G}{\partial \tilde{p}} &= (k-1)(\overline{x} - \tilde{p} + \hat{p})^{k-2}(\overline{x} - \tilde{p} + (k+1)\hat{p}) + (\overline{x} - \tilde{p} + \hat{p})^{k-1} \\
&+ k(n-k)\int_0^{\overline{x}-\tilde{p}} (\varepsilon + \tilde{p})^{n-k-1}(\varepsilon + \hat{p})^{k-2}(\varepsilon + k\hat{p})\, d\varepsilon \\
&- k\overline{x}^{n-k}(\overline{x} - \tilde{p} + \hat{p})^{k-2}(\overline{x} - \tilde{p} + k\hat{p}) \\
&= k(\overline{x} - \tilde{p} + \hat{p})^{k-2}(\overline{x} - \tilde{p} + k\hat{p})\left(1 - \overline{x}^{n-k}\right) \\
&+ k(n-k)\int_0^{\overline{x}-\tilde{p}} (\varepsilon + \tilde{p})^{n-k-1}(\varepsilon + \hat{p})^{k-2}(\varepsilon + k\hat{p})\, d\varepsilon > 0
\end{aligned}$$

As a consequence, υ_1 is increasing in $\hat{p}$.

We now observe that the solution of the equation $G(\hat{p}, \tilde{p}) = 0$ when $\hat{p} = 0$ is negative. We establish this by contradiction. Suppose that the solution to $G(0, \tilde{p}) = 0$ is some non-negative number. If this is so, since we know G increases in $\tilde{p}$, it should be the case that $G(0,0) < 0$. However,

$$G(0,0) = 1 - \overline{x}^k + k\int_0^{\overline{x}} \varepsilon^{n-1} > 0,$$

which leads to a contradiction. Summarizing, we have shown that the implicit function υ_1, defined on $[0, p_k^m]$, starts taking negative values and is increasing.

Consider now the second FOC $H(\hat{p},\tilde{p})=0$ and rewrite it as

$$\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}(1-\overline{x}-\tilde{p})+\frac{1}{(\overline{x}-\tilde{p}+\hat{p})^{k}}\int_{0}^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k}(\varepsilon+\tilde{p})^{n-k-1}\,d\varepsilon=0$$

Let us denote the LHS of this expression by $L(\hat{p},\tilde{p})$. The equation $L(\hat{p},\tilde{p})=0$ defines an implicit relationship between $\hat{p}$ and $\tilde{p}$, which we denote $\tilde{p}=\upsilon_2(\hat{p})$. We show next υ_2 is also increasing. By the implicit function theorem we have

$$\frac{\partial\upsilon_2}{\partial\hat{p}}=\frac{-\partial L/\partial\hat{p}}{\partial L/\partial\tilde{p}},$$

We note that L is increasing in $\hat{p}$ and decreasing in $\tilde{p}$. The first observation comes from

$$\frac{\partial L}{\partial\hat{p}}=\frac{k}{(\overline{x}-\tilde{p}+\hat{p})^{k+1}}\int_{0}^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k-1}(\varepsilon+\tilde{p})^{n-k-1}(\overline{x}-\tilde{p}-\varepsilon)\,d\varepsilon>0.$$

For the second, we compute

$$\begin{aligned}\frac{\partial L}{\partial\tilde{p}}&=-\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}+\frac{k}{(\overline{x}-\tilde{p}+\hat{p})^{k+1}}\int_{0}^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k}(\varepsilon+\tilde{p})^{n-k-1}\,d\varepsilon\\&\quad+\frac{n-k-1}{(\overline{x}-\tilde{p}+\hat{p})^{k}}\int_{0}^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k}(\varepsilon+\tilde{p})^{n-k-2}\,d\varepsilon-\overline{x}^{n-k-1}\end{aligned}\tag{53}$$

It is difficult to evaluate the sign of this derivative on inspection. To ease the evaluation, consider first the term in the second line of this derivative. We note that

$$\begin{aligned}&\frac{n-k-1}{(\overline{x}-\tilde{p}+\hat{p})^{k}}\int_{0}^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k}(\varepsilon+\tilde{p})^{n-k-2}\,d\varepsilon-\overline{x}^{n-k-1}\\&<(n-k-1)\int_{0}^{\overline{x}-\tilde{p}}(\varepsilon+\tilde{p})^{n-k-2}\,d\varepsilon-\overline{x}^{n-k-1}=-\tilde{p}^{n-k-1}<0\end{aligned}\tag{54}$$

Consider next the first term of (53) and note that

$$\begin{aligned}&\frac{k}{(\overline{x}-\tilde{p}+\hat{p})^{k+1}}\int_{0}^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k}(\varepsilon+\tilde{p})^{n-k-1}\,d\varepsilon-\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}\\&<\frac{k\overline{x}^{n-k-1}}{(\overline{x}-\tilde{p}+\hat{p})^{k+1}}\int_{0}^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k}\,d\varepsilon-\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}\\&=\frac{k\overline{x}^{n-k-1}}{k+1}-\frac{k\overline{x}^{n-k-1}\hat{p}^{k+1}}{(k+1)(\overline{x}-\tilde{p}+\hat{p})^{k+1}}-\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}\\&<\frac{k\overline{x}^{n-k-1}}{k+1}-\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}=\frac{1}{1-\overline{x}}\left[\frac{k\overline{x}^{n-k-1}(1-\overline{x})}{k+1}-\frac{1-\overline{x}^{n-k}}{n-k}\right]\end{aligned}\tag{55}$$

We now argue that the term in square brackets in the last line of (55) is negative for all $\overline{x}$. To see this, we first observe that it increases in $\overline{x}$. In fact, taking the derivative w.r.t. $\overline{x}$ we get

$$\begin{aligned}&\frac{k+1}{\overline{x}^{n-k-2}}\frac{\partial}{\partial\overline{x}}\left[\frac{k\overline{x}^{n-k-1}(1-\overline{x})}{k+1}-\frac{1-\overline{x}^{n-k}}{n-k}\right]=k(n-k-1)-k(n-k)\overline{x}\\&+(k+1)\overline{x}=-k(k+1)+(k+1)\overline{x}+k^{2}\overline{x}+nk(1-\overline{x})\\&\geq-k(k+1)+(k+1)\overline{x}+k^{2}\overline{x}+(k+1)k(1-\overline{x})=\overline{x}>0\end{aligned}$$

Since for the highest possible $\overline{x}$ we have

$$\lim_{\overline{x}\to1}\frac{k\overline{x}^{n-k-1}(1-\overline{x})}{k+1}-\frac{1-\overline{x}^{n-k}}{n-k}=0$$

we conclude that (55) is negative. This in turn implies that L decreases in $\tilde{p}$. Since L is increasing in $\hat{p}$ and decreasing in $\tilde{p}$, the function υ_2, defined implicitly by the first order condition $H(\hat{p}, \tilde{p}) = 0$, is also increasing in $\hat{p}$.

We finally observe that the solution to $L(\hat{p}, \tilde{p}) = 0$ when $\hat{p} = 0$ must be a positive number. By contradiction, suppose that the solution to $L(0, \tilde{p}) = 0$ is some negative number. If this is so, since we know L decreases in $\tilde{p}$, it should be the case that $L(0, 0) < 0$. However,

$$L(0,0) = \frac{1-\overline{x}^{n-k}}{n-k} + \frac{1}{\overline{x}^k}\int_0^{\overline{x}-\tilde{p}} \varepsilon^{n-1} d\varepsilon > 0,$$

which constitutes a contradiction. Summarizing, we have now shown that the implicit function υ_2 defined on $[0, p_k^m]$ starts taking positive values and is increasing.

To show that υ_1 and υ_2 cross at least once, we now prove that $\upsilon_1(p_k^m) = \overline{x} > \upsilon_2(p_k^m)$ (since both are increasing in $\tilde{p}$ and we know that $\upsilon_1(0) < 0 < \upsilon_2(0)$). Setting $\hat{p} = p_k^m$ in the FOC for the merged entity gives

$$\begin{aligned} G(p_k^m, \tilde{p}) &= 1 - (\overline{x} - \tilde{p} + p_k^m)^{k-1}(\overline{x} - \tilde{p} + (k+1) p_k^m) \\ &\quad + k\int_0^{\overline{x}-\tilde{p}} (\varepsilon + \tilde{p})^{n-k}(\varepsilon + p_k^m)^{k-2}(\varepsilon + kp_k^m) d\varepsilon = 0 \end{aligned}$$

which solution is $\tilde{p} = \overline{x}$ since $G(p_k^m, \overline{x}) = 1 - (k+1)(p_k^m)^k = 0$ by definition of p_k^m.

Likewise setting $\hat{p} = p_k^m$ in the FOC for the non-merging firm gives

$$\begin{aligned} L(p_k^m, \tilde{p}) &= \frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}(1 - \overline{x} - \tilde{p}) \\ &\quad + \frac{1}{(\overline{x} - \tilde{p} + p_k^m)^k}\int_0^{\overline{x}-\tilde{p}} (\varepsilon + p_k^m)^k (\varepsilon + \tilde{p})^{n-k-1} d\varepsilon = 0 \end{aligned}$$

Since $L(p_k^m, \overline{x}) = (1 - \overline{x}^{n-k})(1 - 2\overline{x}) / [(n-k)(1-\overline{x})] \leq 0$ and we know that L decreases in $\tilde{p}$, it is clear that the solution to $L(p_k^m, \tilde{p}) = 0$ must be some $\tilde{p} < \overline{x}$.

To complete the proof of existence, it remains to be shown that at the point(s) at which υ_1 and υ_2 cross we have $\tilde{p} \leq p^m = 1/2$. For this, it suffices to show that $L(p_k^m, 1/2) < 0$ because since L decreases in $\tilde{p}$, this means that the solution to $L(p_k^m, \tilde{p}) = 0$ must be some $\tilde{p} < 1/2$. In fact, setting $\tilde{p} = 1/2$, we get

$$\begin{aligned} L(p_k^m, 1/2) &= \frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}\left(\frac{1}{2} - \overline{x}\right) \\ &\quad + \frac{1}{\left(\overline{x} - \frac{1}{2} + p_k^m\right)^k}\int_0^{\overline{x}-\frac{1}{2}} (\varepsilon + p_k^m)^k \left(\varepsilon + \frac{1}{2}\right)^{n-k-1} d\varepsilon. \end{aligned} \tag{56}$$

We now note that $L(p_k^m, 1/2)$ decreases in $\overline{x}$. To see this, compute

$$\begin{aligned} \frac{\partial L\left(p_k^m, \frac{1}{2}\right)}{\partial \overline{x}} &= -\frac{1 + (n-k-1)\overline{x}^{n-k} - (n-k)\overline{x}^{n-k-1}}{2(n-k)(1-\overline{x})^2} \\ &\quad - \frac{k}{\left(\overline{x} - \frac{1}{2} + p_k^m\right)^{k+1}}\int_0^{\overline{x}-1/2} (\varepsilon + p_k^m)^k \left(\varepsilon + \frac{1}{2}\right)^{n-k-1} d\varepsilon \end{aligned} \tag{57}$$

and notice that $1 + (n-k-1)\overline{x}^{n-k} - (n-k)\overline{x}^{n-k-1} > 0$ for all $\overline{x}$ (since it decreases in $\overline{x}$ and equals zero when $\overline{x} = 1$). Therefore, if $L(p_k^m, 1/2) \leq 0$ for the lowest value of $\overline{x}$, then it is negative everywhere. In fact, setting $\overline{x} = 1/2$ in (56) yields $L(p_k^m, 1/2) = 0$. To summarize, we have now shown that υ_1 and υ_2 cross at least once on $[0, p_k^m] \times [0, p^m]$ so a candidate equilibrium exists. ■

Claim 5 *The pair of prices* $\{\hat{p}^*, \tilde{p}^*\}$ *that satisfies (18) and (19) is unique.*

Proof. We start by noting that υ_1 is increasing in $\hat{p}$ at a rate greater than 1. Using the derivations above, this follows from the following remarks. First, note that

$$\begin{aligned}\frac{1}{k}\left(-\frac{\partial G}{\partial \hat{p}}-\frac{\partial G}{\partial \tilde{p}}\right) &= (\overline{x}-\tilde{p}+\hat{p})^{k-2}\Big((\overline{x}-\tilde{p}+\hat{p})+\overline{x}^{n-k}(\overline{x}-\tilde{p}+k\hat{p})\Big)\\ &\quad -(k-1)\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\tilde{p})^{n-k}(\varepsilon+\hat{p})^{k-3}(2\varepsilon+k\hat{p})d\varepsilon\\ &\quad -(n-k)\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\tilde{p})^{n-k-1}(\varepsilon+\hat{p})^{k-2}(\varepsilon+k\hat{p})d\varepsilon. \qquad (58)\end{aligned}$$

Observe now that this expression is increasing in $\overline{x}$, as its derivative with respect to $\overline{x}$ equals $(k-1)(\overline{x}-\tilde{p}+\hat{p})^{k-2}\left(1-\overline{x}^{n-1}\right)\geq 0$. Therefore, if (58) is positive when $\overline{x}$ takes on its lowest value, then it is positive everywhere. Setting $\overline{x}=\tilde{p}$ in the RHS of (58) gives $\hat{p}^{k-2}\left(\hat{p}+k\tilde{p}^{n-k}\hat{p}\right)>0$, which proves that υ_1 increases with slope greater than 1.

We continue by noting that the rate at which υ_2 increases is lower than 1. Using the derivations above, since $\partial L/\partial\tilde{p}<0$, we need to show that

$$\begin{aligned}\frac{\partial L}{\partial \hat{p}}+\frac{\partial L}{\partial \tilde{p}} &= -\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}+\frac{k}{(\overline{x}-\tilde{p}+\hat{p})^k}\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k-1}(\varepsilon+\tilde{p})^{n-k-1}d\varepsilon\\ &\quad +\frac{n-k-1}{(\overline{x}-\tilde{p}+\hat{p})^k}\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k}(\varepsilon+\tilde{p})^{n-k-2}d\varepsilon-\overline{x}^{n-k-1} \qquad (59)\end{aligned}$$

is negative. Now notice that the last line of this expression is negative (from (54)). Moreover, regarding the first line of (59) we have

$$\begin{aligned}&-\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}+\frac{k}{(\overline{x}-\tilde{p}+\hat{p})^k}\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k-1}(\varepsilon+\tilde{p})^{n-k-1}d\varepsilon\\ &<-\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}+\frac{k\overline{x}^{n-k-1}}{(\overline{x}-\tilde{p}+\hat{p})^k}\int_0^{\overline{x}-\tilde{p}}(\varepsilon+\hat{p})^{k-1}d\varepsilon\\ &=-\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}+\frac{\overline{x}^{n-k-1}}{(\overline{x}-\tilde{p}+\hat{p})^k}\left[(\overline{x}-\tilde{p}+\hat{p})^k-\hat{p}^k\right]\\ &<-\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}+\overline{x}^{n-k-1}=-\frac{1+(n-k-1)\overline{x}^{n-k}-(n-k)\overline{x}^{n-k-1}}{(n-k)(1-\overline{x})}<0,\end{aligned}$$

where the last inequality follows from the remarks after equation (57). This implies that υ_2 increases at a rate less than 1. This, together with the arguments before shows that there exists a unique candidate equilibrium. ■

Claim 6 *If the search cost is sufficiently large, then* $\overline{\overline{x}}-\hat{p}>\overline{x}-\tilde{p}$ *and* $\hat{p}>\tilde{p}$.

Since we assume that $k\leq 10$, proving the statement for $s\to 1/8$ ($\overline{x}\to 1/2$) suffices. It takes a few steps to check that the solution to the FOCs (18) and (19) is $\tilde{p}=p^m=1/2=\overline{x}$ and $\hat{p}=p_m^k=(1+k)^{-1/k}$ when $s\to 1/8$; therefore $\hat{p}>\tilde{p}$. Given that $\overline{x}-\tilde{p}=0$ when $s\to 1/8$, proving that $\overline{\overline{x}}-\hat{p}>\overline{x}-\tilde{p}$ boils down to showing that $\overline{\overline{x}}-\hat{p}>0$. We know $\overline{\overline{x}}$ satisfies $\int_x^1 k(\varepsilon-x)\varepsilon^{k-1}d\varepsilon-s=0$, or

$$\frac{k(1-\overline{\overline{x}})-\overline{\overline{x}}(1-\overline{\overline{x}}^k)}{k+1}-s=0. \qquad (60)$$

Equation (60) can be rewritten as

$$\overline{\overline{x}}=\frac{k+\overline{\overline{x}}^{k+1}}{k+1}-s$$

Deducting $\hat{p}$ on both sides of this equality gives

$$\overline{\overline{x}} - \hat{p} = \frac{k + \overline{\overline{x}}^{k+1}}{k+1} - \hat{p} - s. \tag{61}$$

When $s \to 1/8$, $\hat{p} = (1+k)^{-1/k}$. As a result, when $s \to 1/8$, equation (61) can be rewritten

$$\overline{\overline{x}} - p_m^k = \frac{k + \overline{\overline{x}}^{k+1}}{k+1} - \frac{1}{(1+k)^{\frac{1}{k}}} - \frac{1}{8} \tag{62}$$

Note now that the RHS of (62) increases in $\overline{\overline{x}}$. Therefore using the lowest admissible value for $\overline{\overline{x}}$, we can write

$$\overline{\overline{x}} - p_m^k > \frac{k + (\frac{1}{2})^{k+1}}{k+1} - \frac{1}{(1+k)^{\frac{1}{k}}} - \frac{1}{8} > 0$$

for all $k \leq n-1$. ■

Finally, we check that firms cannot profitably deviate from the pair of prices given by the solution to the FOCs (18) and (19). We use again "$*$" to distinguish equilibrium prices from deviation prices. We start with deviations by a non-merging firm. We first note that the payoff function (17) is strictly concave, which implies that a non-merging firm does not gain by deviating to a price $\tilde{p} \in [0, 1-\overline{x}+\tilde{p}^*]$. Consider now 'large' deviations to prices $\tilde{p} > 1 - \overline{x} + \tilde{p}^*$. In that case, the payoff to the deviant would be

$$\widetilde{\pi} = \tilde{p} \int_0^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon.$$

Taking the FOC gives

$$\int_0^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon - \tilde{p} (1 - \tilde{p} + \tilde{p}^*)^{n-k-1} (1 - \tilde{p} + \hat{p}^*)^k = 0,$$

which we can rewrite as

$$\int_0^{1-\tilde{p}} \frac{(\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k}{(1 - \tilde{p} + \tilde{p}^*)^{n-k-1} (1 - \tilde{p} + \hat{p}^*)^k} d\varepsilon - \tilde{p} = 0 \tag{63}$$

The LHS of (63) decreases in $\tilde{p}$. In fact, its derivative with respect to $\tilde{p}$ is

$$\frac{(n-k-1)(1 - \tilde{p} + \hat{p}^*) + k(1 - \tilde{p} + \tilde{p}^*)}{(1 - \tilde{p} + \tilde{p}^*)^{n-k} (1 - \tilde{p} + \hat{p}^*)^{k+1}} \int_0^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon - 2 < 0,$$

where the inequality follows from

$$\begin{aligned}
&\frac{[(n-k-1)(1 - \tilde{p} + \hat{p}^*) + k(1 - \tilde{p} + \tilde{p}^*)] \int_0^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon}{(1 - \tilde{p} + \tilde{p}^*)^{n-k} (1 - \tilde{p} + \hat{p}^*)^{k+1}} = \\
&\frac{(n-k-1) \int_0^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon}{(1 - \tilde{p} + \tilde{p}^*)^{n-k} (1 - \tilde{p} + \hat{p}^*)^k} + \frac{k \int_0^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k \, d\varepsilon}{(1 - \tilde{p} + \tilde{p}^*)^{n-k-1} (1 - \tilde{p} + \hat{p}^*)^{k+1}} < \\
&\frac{(n-k-1) \int_0^{1-\tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1} \, d\varepsilon}{(1 - \tilde{p} + \tilde{p}^*)^{n-k}} + \frac{k \int_0^{1-\tilde{p}} (\varepsilon + \hat{p}^*)^k \, d\varepsilon}{(1 - \tilde{p} + \hat{p}^*)^{k+1}} = \\
&\frac{n-k-1}{n-k} \left[1 - \frac{(\tilde{p}^*)^{n-k}}{(1 - \tilde{p} + \tilde{p}^*)^{n-k}} \right] + \frac{k}{k+1} \left[1 - \frac{(\hat{p}^*)^{k+1}}{(1 - \tilde{p} + \hat{p}^*)^{k+1}} \right] < 2
\end{aligned}$$

Then, (63) is lower than when we set $\tilde{p} = 1 - \bar{x} + \tilde{p}^*$, that is

$$\int_0^{1-\tilde{p}} \frac{(\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k}{(1 - \tilde{p} + \tilde{p}^*)^{n-k-1} (1 - \tilde{p} + \hat{p}^*)^k} d\varepsilon - \tilde{p} < \int_0^{\overline{x}-\tilde{p}^*} \frac{(\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \hat{p}^*)^k}{\overline{x}^{n-k-1} (\overline{x} - \tilde{p}^* + \hat{p}^*)^k} d\varepsilon - (1 - \bar{x} + \tilde{p}^*).$$

Using the FOC (19), this last expression is equal to

$$-\frac{1}{\overline{x}^{n-k-1}}\frac{1-\overline{x}^{n-k}}{(n-k)(1-\overline{x})}(1-\overline{x}-\tilde{p}^{*})-(1-\bar{x}+\tilde{p}^{*})=-\frac{1-\overline{x}-\tilde{p}^{*}}{1-\overline{x}}\int_{\overline{x}}^{1}\frac{\varepsilon^{n-k-1}}{\overline{x}^{n-k-1}}d\varepsilon-(1-\overline{x}+\tilde{p}^{*}),$$

which increases in k. Therefore, it is lower than when we set $k=n-1$, which gives

$$-\frac{1-\overline{x}-\tilde{p}^{*}}{1-\overline{x}}\int_{\overline{x}}^{1}d\varepsilon-(1-\overline{x}+\tilde{p}^{*})=-2(1-\overline{x})\leq 0$$

Hence, there the deviation is not profitable.

We now move to study deviations by the merged entity. Checking that there does not exist any profitable deviation for arbitrary k is extremely difficult so we proceed as in Proposition 1 and study in detail the case of $k=2$. Consider first that the deviant firm deviates to a vector of prices $(p_1,p_2)\in[0,1-\overline{x}+\tilde{p}^{*})^2$ and assume without loss of generality that $p_1\leq p_2$. In this case the payoff to the deviant merged entity is given

$$\hat{\pi}=\sum_{i=1}^{2}p_i d_i \tag{64}$$

where

$$d_1=p_2-p_1+\int_{\overline{x}-\tilde{p}^{*}}^{1-p_2}(\varepsilon+p_2)\,d\varepsilon+\int_{0}^{\overline{x}-\tilde{p}^{*}}(\varepsilon+\tilde{p}^{*})^{n-2}(\varepsilon+p_2)\,d\varepsilon,\ \text{and}$$

$$d_2=\int_{\overline{x}-\tilde{p}^{*}+p_2}^{1}(\varepsilon-p_2+p_1)\,d\varepsilon+\int_{0}^{\overline{x}-\tilde{p}^{*}}(\varepsilon+\tilde{p}^{*})^{n-2}(\varepsilon+p_1)\,d\varepsilon.$$

The FOCs are

$$\frac{\partial\hat{\pi}}{\partial p_1}=d_1-p_1+p_2\left(1-\overline{x}+\tilde{p}^{*}-p_2+\int_{0}^{\overline{x}-\tilde{p}^{*}}(\varepsilon+\tilde{p}^{*})^{n-2}\,d\varepsilon\right)=0$$

$$\frac{\partial\hat{\pi}}{\partial p_2}=p_1\left(1-\overline{x}+\tilde{p}^{*}-p_2+\int_{0}^{\overline{x}-\tilde{p}^{*}}(\varepsilon+\tilde{p}^{*})^{n-2}\,d\varepsilon\right)+d_2-p_2(1-p_2+p_1)=0.$$

The second order derivatives are

$$\frac{\partial^2\hat{\pi}}{\partial p_1^2}=-2<0$$

$$\frac{\partial^2\hat{\pi}}{\partial p_2^2}=-2+3p_2-3p_1<0$$

$$\frac{\partial^2\hat{\pi}}{\partial p_1\partial p_2}=2-3p_2-2\overline{x}+2\tilde{p}^{*}+2\int_{0}^{\overline{x}-\tilde{p}^{*}}(\varepsilon+\tilde{p}^{*})^{n-2}\,d\varepsilon$$

We now note that the payoff in (64) is not necessarily strictly concave because the second derivative $\partial^2\hat{\pi}/\partial p_2^2$ might be positive. In particular this occurs when the search cost is large ($\bar{x}-\tilde{p}^{*}\simeq 0$), p_1 is small ($p_1\simeq 0$) and p_2 is large ($p_2\simeq 1$). This observation undermines standard attempts to prove existence and uniqueness and we, consequently, proceed numerically.

Notice that for deviations such that $0\leq p_i<1-\overline{x}+\tilde{p}^{*}$ and $1-\overline{x}+\tilde{p}^{*}\leq p_j\leq 1$ the payoff of the deviant merged entity is

$$\pi=p_i\left(1-(\overline{x}-\tilde{p}^{*}+p_i)+\int_{1-p_j}^{\overline{x}-\tilde{p}^{*}}(\varepsilon+\tilde{p}^{*})^{n-2}\,d\varepsilon+\int_{0}^{1-p_j}(\varepsilon+\tilde{p}^{*})^{n-2}(\varepsilon+p_j)\,d\varepsilon\right)$$
$$+p_j\int_{0}^{1-p_j}(\varepsilon+\tilde{p}^{*})^{n-2}(\varepsilon+p_i)\,d\varepsilon$$

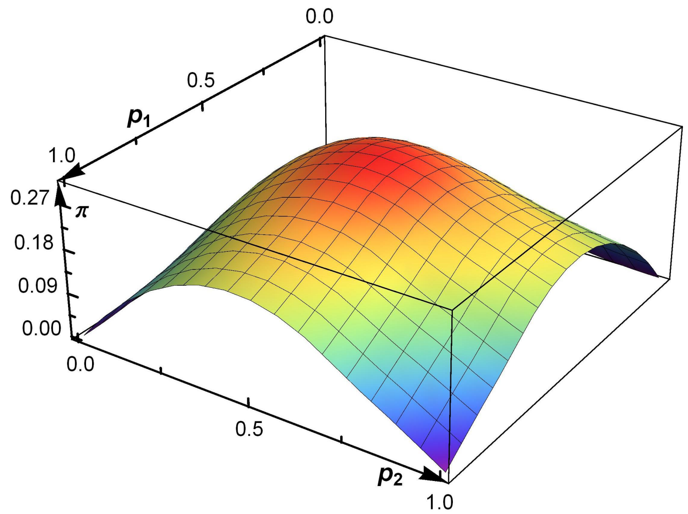


Figure 14: Payoff function of the deviant merged entity $(n = 3, k = 2)$

and for deviations such that $1 - \overline{x} + \tilde{p}^* \leq p_i \leq p_j \leq 1$, the payoff is

$$\pi = p_i \left(\int_{1-p_j}^{1-p_i} (\varepsilon + \tilde{p}^*)^{n-2} \, d\varepsilon + \int_0^{1-p_j} (\varepsilon + \tilde{p}^*)^{n-2} (\varepsilon + p_j) \, d\varepsilon \right) + p_j \int_0^{1-p_j} (\varepsilon + \tilde{p}^*)^{n-2} (\varepsilon + p_i) \, d\varepsilon.$$

Figure 14 plots the payoff of the deviant merged entity for all possible deviation prices when the search cost $s = 0.04$. Observation of the graph shows that for $k = 2$, the solution to the FOCs (9) and (10), $(\hat{p}^*, \tilde{p}^*) \simeq (0.50, 0.46)$, is the unique symmetric Nash equilibrium with $\overline{\overline{x}} - \hat{p}^* > \overline{x} - \tilde{p}^*$. ■

Proof of Proposition 5. (a) We first show that the merging stores increase their profits after the merger. The difference between the profit per product of the merged entity, $\hat{\pi}^*/k$, and the typical pre-merger profit of a firm, π^*, equals:

$$\frac{\hat{\pi}^*}{k} - \pi^* = \frac{\hat{p}^*}{k} \left[1 - (\overline{x} - \tilde{p}^* + \hat{p}^*)^k + k \int_0^{\overline{x} - \tilde{p}^*} (\varepsilon + \hat{p}^*)^{k-1} (\varepsilon + \tilde{p}^*)^{n-k} d\varepsilon \right] - \frac{p^*}{n} (1 - p^{*n})$$

Since $\hat{p}^*$ is an equilibrium price, then, given the non-merging firm's price, $\hat{\pi}^* (\hat{p}^*)$ is greater than $\hat{\pi} (\hat{p})$ for any $\hat{p} \neq \hat{p}^*$. Therefore, replacing $\hat{p}^*$ by $\tilde{p}^*$ gives

$$\frac{\hat{\pi}^*}{k} - \pi^* > \tilde{p}^* \left[\frac{1 - \overline{x}^k}{k} + \frac{1}{n} (\overline{x}^n - \tilde{p}^{*n}) \right] - \frac{p^*}{n} (1 - p^{*n}) \tag{65}$$

We now note that $\frac{1 - \overline{x}^k}{k} = \int_{\overline{x}}^1 \varepsilon^{k-1} d\varepsilon$ and is decreasing in k. Therefore, the RHS of (65) is greater than when we set $k = n - 1$, which gives

$$\begin{aligned} &\tilde{p}^* \left[\frac{1 - \overline{x}^{n-1}}{n-1} + \frac{1}{n} (\overline{x}^n - \tilde{p}^{*n}) \right] - \frac{p^*}{n} (1 - p^{*n}) \\ &= \frac{\tilde{p}^*}{n(n-1)} \left[n(1 - \overline{x}^{n-1}) + (n-1) (\overline{x}^n - \tilde{p}^{*n}) \right] - \frac{p^*}{n} (1 - p^{*n}) . \end{aligned} \tag{66}$$

Observe next that this expression is decreasing in $\overline{x}$ since its derivative with respect to $\overline{x}$ is proportional to $-\overline{x}^{n-2} n(n-1)(1 - \overline{x}) < 0$. Therefore, (66) is larger than when we set $\overline{x} = 1$, which gives

$$\frac{\tilde{p}^*}{n(n-1)} (n - 1 - (n-1) \tilde{p}^{*n}) - \frac{p^*}{n} (1 - p^{*n}) = \frac{\tilde{p}^*}{n} (1 - \tilde{p}^{*n}) - \frac{p^*}{n} (1 - p^{*n}) .$$

Finally, note that the expression $\tilde{p}^*(1-\tilde{p}^{*n})$ is increasing in $\tilde{p}^*$ because its derivative with respect to $\tilde{p}^*$ is equal to $1-(n+1)\tilde{p}^{*n}>0$. We then conclude that $\hat{\pi}/k-\pi^*>0$ if $\tilde{p}^*>p^*$. But, as argued in the proof of Proposition 2, the last inequality is true because $\tilde{p}^*$ is the price of the firms that are visited last.

(b) We showed in the proof of Proposition 4 that when the search cost is large $\hat{p}^*\to p_k^m$ and $\tilde{p}^*\to\frac{1}{2}$. Therefore, we have:

$$\begin{aligned}\lim_{\overline{x}\to 1/2}\left[\frac{\hat{\pi}^*}{k}-\tilde{\pi}^*\right]&=\lim_{\overline{x}\to 1/2}\left[\frac{\hat{p}^*}{k}\left(1-(\overline{x}-\tilde{p}^*+\hat{p}^*)^k+k\int_0^{\overline{x}-\tilde{p}^*}(\varepsilon+\tilde{p}^*)^{n-k}(\varepsilon+\hat{p}^*)^{k-1}d\varepsilon\right)\right.\\&\left.-\tilde{p}^*\left(\frac{(\overline{x}-\tilde{p}^*+\hat{p}^*)^k}{n-k}\left(1-\overline{x}^{n-k}\right)+\int_0^{\overline{x}-\tilde{p}^*}(\varepsilon+\tilde{p}^*)^{n-k-1}(\varepsilon+\hat{p}^*)^k d\varepsilon\right)\right]\\&=\frac{1}{k}p_k^m\left(1-(p_k^m)^k\right)-\frac{1}{2(n-k)}(p_k^m)^k\left(1-\frac{1}{2^{n-k}}\right)\\&=\frac{1}{k}p_k^m\left(1-(p_k^m)^k\right)-\frac{(p_k^m)^k}{2}\int_{1/2}^1\varepsilon^{n-k-1}d\varepsilon.\end{aligned}$$

This expression is increasing in n because its derivative with respect to n equals

$$-\frac{(p_k^m)^k}{2}\int_{1/2}^1\varepsilon^{n-k-1}\ln\varepsilon d\varepsilon>0$$

Then

$$\begin{aligned}\lim_{\overline{x}\to 1/2}\left[\frac{\hat{\pi}^*}{k}-\tilde{\pi}^*\right]\geq\frac{p_k^m}{k}\left(1-(p_k^m)^k\right)-\frac{1}{2}\frac{(p_k^m)^k}{k+1-k}\left(1-2^{k-k-1}\right)=\\\frac{p_k^m}{k+1}-\frac{(p_k^m)^k}{2}\left(1-\frac{1}{2}\right)=\frac{p_k^m}{k+1}-\frac{1}{4(k+1)}=\frac{1}{4(k+1)}\left[4p_k^m-1\right]>0\end{aligned}$$

where the first inequality follows form replacing n by $k+1$. ■

Proof of Proposition 6. (a) We first note that the equilibrium of Proposition 4 has $\hat{p}^*>\tilde{p}^*$ when the search cost is sufficiently high. The difference between post- and pre-merger total industry profits is $\Delta\Pi\equiv\hat{\pi}^*+(n-k)\tilde{\pi}^*-n\pi^*$. Using the expressions for profits above, we have

$$\begin{aligned}\Delta\Pi=\hat{p}^*\left(1-(\overline{x}-\tilde{p}^*+\hat{p}^*)^k+k\int_0^{\overline{x}-\tilde{p}^*}(\varepsilon+\tilde{p}^*)^{n-k}(\varepsilon+\hat{p}^*)^{k-1}d\varepsilon\right)+\\\tilde{p}^*\left((\overline{x}-\tilde{p}^*+\hat{p}^*)^k\left(1-\overline{x}^{n-k}\right)+(n-k)\int_0^{\overline{x}-\tilde{p}^*}(\varepsilon+\tilde{p}^*)^{n-k-1}(\varepsilon+\hat{p}^*)^k d\varepsilon\right)-p^*(1-p^{*n})\end{aligned}$$

Note now that this expression is clearly increasing in $\hat{p}^*$ (the derivative of the first line, by the FOC, is zero and that of the second line is positive). Hence,

$$\Delta\Pi>\Delta\Pi|_{\hat{p}^*=\tilde{p}^*}=\tilde{p}^*(1-\tilde{p}^{*n})-p^*(1-p^{*n})>0, \tag{67}$$

as shown in the proof of Proposition 5.

(b) In the pre-merger market, consumer surplus is given by

$$\begin{aligned}CS^*&=\frac{1-\overline{x}^n}{1-\overline{x}}\int_{\overline{x}}^1(\varepsilon-p^*)d\varepsilon+n\int_{p^*}^{\overline{x}}\varepsilon^{n-1}(\varepsilon-p^*)d\varepsilon-\frac{1-\overline{x}^n}{1-\overline{x}}s\\&=\frac{1-\overline{x}^n}{1-\overline{x}}\int_{\overline{x}}^1(\varepsilon-p^*)d\varepsilon+n\int_{p^*}^{\overline{x}}\varepsilon^{n-1}(\varepsilon-p^*)d\varepsilon-\frac{(1-\overline{x}^n)(1-\overline{x})}{2}.\end{aligned}$$

In the post-merger market, consumer surplus is given by $CS = \widehat{CS} + \widetilde{CS} - Sc$ where $\widehat{CS}$, $\widetilde{CS}$ and Sc are given by (20), (21) and (22), respectively.

When $s \to 1/8$, $\overline{x} \to 1/2$, $p^* \to 1/2$ and $\hat{p}^* \to p_k^m$. Then, we can establish the comparison

$$\lim_{s \to 1/8} \left[\widehat{CS} + \widetilde{CS} - Sc - CS^* \right] = \int_{p_k^m}^{1} k \left(\varepsilon - p_k^m \right) \varepsilon^{k-1} d\varepsilon - \frac{1}{8} > 0$$

The proof is now complete. ■

Proof of Proposition 7. Before presenting the proof of this proposition, let us derive the payoff to a deviant merged entity that deviates by charging $\hat{p} \neq \hat{p}^*$, $\hat{p} < 1 - \overline{x} + \hat{p}^*$ for its product i. The deviation has an impact on the demand for all the products of the merged entity. Consider first the demand for product i. Take a consumer who visits firm i in her h^{th} search, $h = 1, 2, ..., k-1$. Conditional on the deviant being in h^{th} position, the probability the consumer buys product i right away is $\Pr[\varepsilon_i - \hat{p} > \overline{x} - \hat{p}^* > z_{h-1} - \hat{p}^*]$. This gives a demand equal to $\overline{x}^{h-1} \left(1 - \overline{x} + \hat{p}^* - \hat{p}\right)$. The consumer may walk away from product i, search the rest of the products of the merged entity and decide to buy i without searching at the non-merging firms. Conditional on product i being searched in h^{th} position, this happens with probability $\Pr\left[\max\left\{\overline{x} - \tilde{p}^*, z_{k-1} - \hat{p}^*\right\} < \varepsilon_i - \hat{p} < \overline{x} - \hat{p}^*\right]$, which is equal to

$$\int_{\overline{x} - \tilde{p}^* + \hat{p}}^{\overline{x} - \hat{p}^* + \hat{p}} \left(\varepsilon + \hat{p}^* - \hat{p}\right)^{k-1} d\varepsilon = \int_{\overline{x} - \tilde{p}^*}^{\overline{x} - \hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon.$$

Suppose now that product i is inspected after having searched the other $k-1$ products of the merged entity. In this case, the consumer stops searching and buys product i with probability $\Pr[\varepsilon_i - \hat{p} \geq \max\left\{\overline{x} - \tilde{p}^*, z_{k-1} - \hat{p}^*\right\}]$ which gives a demand equal to

$$\overline{x}^k \left(1 - \overline{x} + \hat{p}^* - \hat{p}\right) + \int_{\overline{x} - \tilde{p}^*}^{\overline{x} - \hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon.$$

No matter the position in which firm i is visited, it also obtains demand from consumers who walk away from it, visit the rest of the merging firms (if any), visit all the non-merging firms and decide to return to firm i to buy there. This occurs with probability $\Pr[\max\left\{0, z_{k-1} - \hat{p}^*, z_{n-k} - \tilde{p}^*\right\} < \varepsilon_i - \hat{p} < \overline{x} - \tilde{p}^*]$, which gives a returning demand equal to

$$\int_{\hat{p}}^{\overline{x} - \tilde{p}^* + \hat{p}} \left(\varepsilon + \tilde{p}^* - \hat{p}\right)^{n-k} \left(\varepsilon + \hat{p}^* - \hat{p}\right)^{k-1} d\varepsilon = \int_{0}^{\overline{x} - \tilde{p}^*} \left(\varepsilon + \tilde{p}^*\right)^{n-k} \left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon.$$

Summing the previous demands for all h, and taking into account that each position occurs with probability $1/k$, we obtain the expression

$$\begin{aligned} d_i(\hat{p}; \hat{p}^*, \tilde{p}^*) &= \frac{1 - \bar{x}^k}{k(1 - \bar{x})} \left(1 - \bar{x} + \hat{p}^* - \hat{p}\right) + \int_{\bar{x} - \tilde{p}^*}^{\bar{x} - \hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon + \int_{0}^{\bar{x} - \tilde{p}^*} \left(\varepsilon + \tilde{p}^*\right)^{n-k} \left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon \\ &= \frac{1 - \overline{x}^k}{k(1 - \overline{x})} \left(\hat{p}^* - \hat{p}\right) + \int_{\overline{x} - \tilde{p}^*}^{1 - \hat{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon + \int_{0}^{\overline{x} - \tilde{p}^*} \left(\varepsilon + \tilde{p}^*\right)^{n-k} \left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon. \end{aligned}$$

The deviation also affects the demand for the other products of the merged entity. Let us now calculate how the demand for, say, product m is affected by the deviation. Suppose a consumer visits firm m in her h^{th} search, $h = 1, 2, ..., k-1$. With probability $\frac{h-1}{k-1}$, the consumer has already inspected product i. In this case, the consumer will buy product m with probability $Pr[\varepsilon_m > \overline{x} > \max\{\varepsilon_i - \hat{p} + \hat{p}^*, z_{h-2}\}]$. With probability $\frac{k-h}{k-1}$, the consumer has not yet inspected firm i's product, in which case she will buy product m with probability $Pr[\varepsilon_m > \overline{x} > z_{h-1}]$. Putting these terms together, we obtain a demand equal to

$$\begin{aligned} &\sum_{h=1}^{k} \frac{h-1}{k(k-1)} \left(\overline{x} - \hat{p}^* + \hat{p}\right) \overline{x}^{h-2} \left(1 - \bar{x}\right) + \sum_{h=1}^{k-1} \frac{k-h}{k(k-1)} \overline{x}^{h-1} \left(1 - \overline{x}\right) \\ &= \frac{1 - k\overline{x}^{k-1} + (k-1)\overline{x}^k}{k(k-1)(1 - \overline{x})} \left(\overline{x} - \hat{p}^* + \hat{p}\right) + \frac{k - 1 - k\overline{x} + \overline{x}^k}{k(k-1)(1 - \overline{x})} \end{aligned}$$

With probability $\Pr[\max\{\overline{x}-\tilde{p}^*, z_{k-2}-\hat{p}^*, \varepsilon_i-\hat{p}\} \leq \varepsilon_m - \hat{p}^* < \overline{x}-\hat{p}^*]$, the consumer will return to buy product m without searching at the non-merging firms. This gives a demand

$$\int_{\overline{x}-\tilde{p}^*+\hat{p}^*}^{\overline{x}} \varepsilon^{k-2}\left(\varepsilon+\hat{p}-\hat{p}^*\right) d\varepsilon = \int_{\overline{x}-\tilde{p}^*}^{\overline{x}-\hat{p}^*} \left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+\hat{p}\right) d\varepsilon$$

Finally, a consumer will buy product m also when she walks away from it, visit the rest of the stores of the merged entity as well as the non-merging firms and returns to m. This happens with probability $\Pr[\max\{0, \varepsilon_i-\hat{p}, z_{k-2}-\hat{p}^*, z_{n-k}-\tilde{p}^*\} < \varepsilon_m-\hat{p}^* < \overline{x}-\tilde{p}^*]$, which gives a returning demand for product m equal to

$$\int_0^{\overline{x}-\tilde{p}^*} \left(\varepsilon+\tilde{p}^*\right)^{n-k}\left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+\hat{p}\right) d\varepsilon.$$

Putting terms together and simplifying we obtain the total demand for product m is

$$\begin{aligned}
d_m(\hat{p};\hat{p}^*,\tilde{p}^*) &= \frac{1-k\bar{x}^{k-1}+(k-1)\,\bar{x}^k}{k\,(k-1)\,(1-\bar{x})}\left(\bar{x}-\hat{p}^*+\hat{p}\right)+\frac{k-1-k\bar{x}+\bar{x}^k}{k\,(k-1)\,(1-\bar{x})} \\
&+\int_{\bar{x}-\tilde{p}^*}^{\bar{x}-\hat{p}^*}\left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+\hat{p}\right) d\varepsilon+\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon+\tilde{p}^*\right)^{n-k}\left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+\hat{p}\right) d\varepsilon \\
&= \frac{1-k+k\bar{x}-\bar{x}^k}{k\,(k-1)\,(1-\bar{x})}\left(\hat{p}-\hat{p}^*\right) \\
&+\int_{\bar{x}-\tilde{p}^*}^{1-\hat{p}^*}\left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+\hat{p}\right) d\varepsilon+\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon+\tilde{p}^*\right)^{n-k}\left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+\hat{p}\right) d\varepsilon
\end{aligned}$$

The total payoff to the deviant merged entity is

$$\hat{\pi}(\hat{p};\hat{p}^*,\tilde{p}^*) = p_i d_i(\hat{p};\hat{p}^*,\tilde{p}^*)+(k-1)\,\hat{p}^* d_m(\hat{p};\hat{p}^*,\tilde{p}^*).$$

The FOC of the merged entity is then

$$1-\left(\overline{x}-\tilde{p}^*+\hat{p}^*\right)^{k-1}\left(\overline{x}-\tilde{p}^*+(k+1)\,\hat{p}^*\right)+k\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon+\tilde{p}^*\right)^{n-k}\left(\varepsilon+\hat{p}^*\right)^{k-2}\left(\varepsilon+k\hat{p}^*\right) d\varepsilon = 0, \quad (68)$$

while the FOC of the non-merging firm is

$$\frac{\left(\overline{x}-\tilde{p}^*+\hat{p}^*\right)^k}{n-k}\frac{1-\overline{x}^{n-k}}{1-\overline{x}}\left(1-\overline{x}-\tilde{p}^*\right)+\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon+\tilde{p}^*\right)^{n-k-1}\left(\varepsilon+\hat{p}^*\right)^k d\varepsilon = 0. \quad (69)$$

(a) We now prove that when search cost is sufficiently high then $\hat{p}^* > \tilde{p}^*$ and therefore consumer expectations are violated. We start by noting that, because the price of the non-merging firms is less than or equal to $1/2 < p_k^m$ for all $\overline{x} \in [p_k^m; 1]$, the integral in (68) is positive. As a result, for an equilibrium to exist, the rest of the LHS of (68), $1-\left(\overline{x}-\tilde{p}^*+\hat{p}^*\right)^k - k\left(\overline{x}-\tilde{p}^*+\hat{p}^*\right)^k \hat{p}^*$, must be negative. Note that this expression decreases in $\hat{p}^*$. Then, it must be higher than when we set $\hat{p}^* = \tilde{p}^*$ because $\hat{p}^* < \tilde{p}^*$ by assumption. That is, it must be the case that

$$1-\left(\overline{x}-\tilde{p}^*+\hat{p}^*\right)^k - k\left(\overline{x}-\tilde{p}^*+\hat{p}^*\right)^k \hat{p}^* > 1-\bar{x}^k-k\overline{x}^{k-1}\tilde{p}^* \quad (70)$$

We now note that when $\overline{x} \to p_k^m$ the expression $1-\bar{x}^k-k\overline{x}^{k-1}p_k^m$ is equal to zero. Since $\tilde{p}^* \leq 1/2 < p_k^m$, it is clear that $1-\overline{x}^k-k\overline{x}^{k-1}p_k^m > 0$ when $\overline{x} \to p_k^m$. But this constitutes a contradiction because then the LHS of (68) cannot be negative. As a result, there is no such a pair of prices $\hat{p}^*$ and $\tilde{p}^*$ that satisfy (68) and (69) when $\overline{x} \to p_k^m$ and $\hat{p}^* < \tilde{p}^*$.

(b) We prove now that when search cost goes to zero again we obtain $\hat{p}^* > \tilde{p}^*$, which violates consumer expectations. To show this we use again the equality

$$Y(\tilde{p}^*,\hat{p}^*) \equiv Q-\hat{q}^*-(n-k)\,\tilde{q}^* = 0$$

where $Q = 1 - \hat{p}^{*k}\tilde{p}^{*n-k}$ denotes the aggregate quantity sold in the market and $\hat{q}^*$ and $\tilde{q}^*$ denote the equilibrium quantities of the merged entity and the non-merging firms. From the FOCs, these quantities are given by

$$\hat{q}^* = k\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k-1}\hat{p}^* - k\left(k-1\right)\hat{p}^* \int_0^{\bar{x}-\tilde{p}^*} \left(\varepsilon + \hat{p}^*\right)^{k-2}\left(\varepsilon + \tilde{p}^*\right)^{n-k} d\varepsilon$$
$$\tilde{q}^* = \frac{\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k-1}}{n-k}\frac{1-\overline{x}^{n-k}}{1-\overline{x}}\tilde{p}^*$$

The partial derivative of Y with respect to $\hat{p}^*$ is negative because $\tilde{q}^*$ increases with $\hat{p}^*$ and the derivative of $\hat{q}^*$ with respect to $\hat{p}^*$ is positive:

$$\begin{aligned}
\frac{\partial \hat{q}^*}{\partial \hat{p}^*} &= k\left(\bar{x} - \tilde{p}^* + \hat{p}^*\right)^{k-1} + k\left(k-1\right)\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k-2}\hat{p}^* - k\left(k-1\right)\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon + \hat{p}^*\right)^{k-2}\left(\varepsilon + \tilde{p}^*\right)^{n-k} d\varepsilon \\
&\quad - k\left(k-1\right)\left(k-2\right)\hat{p}^*\int_0^{\bar{x}-\tilde{p}^*}\left(\varepsilon + \hat{p}^*\right)^{k-3}\left(\varepsilon + \tilde{p}^*\right)^{n-k} d\varepsilon \\
&> k\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k-1} + k\left(k-1\right)\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k-2}\hat{p}^* - k\left(k-1\right)\overline{x}^{n-k}\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon + \hat{p}^*\right)^{k-2} d\varepsilon \\
&\quad - k\left(k-1\right)\left(k-2\right)\hat{p}^*\overline{x}^{n-k}\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon + \hat{p}^*\right)^{k-3} d\varepsilon \\
&= k\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k-2}\left(\overline{x} - \tilde{p}^* + k\hat{p}^*\right)\left(1 - \overline{x}^{n-k}\right) > 0.
\end{aligned}$$

Therefore, given that $\hat{p}^* < \tilde{p}^*$, if we set $\hat{p}^* = \tilde{p}^*$ then Y must be negative when $\bar{x} \to 1$. That is, it must be the case that

$$\begin{aligned}
\lim_{\overline{x}\to 1} Y\big|_{\hat{p}^*=\tilde{p}^*} &= \lim_{\overline{x}\to 1}\left[1 - \tilde{p}^{*n} - k\tilde{p}^* + k\left(k-1\right)\tilde{p}^*\int_0^{1-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-2} d\varepsilon - \left(n-k\right)\tilde{p}^*\right] \\
&= \lim_{\overline{x}\to 1}\left[1 - \tilde{p}^{*n} - n\tilde{p}^* + \frac{k\left(k-1\right)\tilde{p}^*}{n-1}\left(1 - \tilde{p}^{*n-1}\right)\right] < 0. \qquad (71)
\end{aligned}$$

The FOC (69) may be rearranged as

$$1 - \overline{x} - \tilde{p}^* + \frac{\left(n-k\right)\left(1-\overline{x}\right)}{1-\overline{x}^{n-k}}\frac{1}{\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^k}\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^k d\varepsilon = 0 \qquad (72)$$

The LHS of (72) increases in $\hat{p}^*$ because

$$\begin{aligned}
&\frac{k\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^k \int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon - k\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k-1}\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^k d\varepsilon}{\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{2k}} \\
&= \frac{k\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^{k-1} d\varepsilon - k\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^k d\varepsilon}{\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k+1}} \\
&= \frac{k\int_0^{\overline{x}-\tilde{p}^*}\left(\varepsilon + \tilde{p}^*\right)^{n-k-1}\left(\varepsilon + \hat{p}^*\right)^{k-1}\left(\overline{x} - \tilde{p}^* - \varepsilon\right) d\varepsilon}{\left(\overline{x} - \tilde{p}^* + \hat{p}^*\right)^{k+1}} > 0
\end{aligned}$$

Therefore, given that $\hat{p}^* < \tilde{p}^*$, if we set $\hat{p}^* = \tilde{p}^*$ then the LHS of (72) must be positive, that is,

$$1 - \overline{x} - \tilde{p}^* + \frac{\left(n-k\right)\left(1-\overline{x}\right)}{1-\overline{x}^{n-k}}\frac{1}{\overline{x}^k}\frac{1}{n}\left(\overline{x}^n - \tilde{p}^{*n}\right) > 0 \qquad (73)$$

If we take the limit of the LHS of (73) when $\bar{x} \to 1$, then we get the following inequality:

$$\lim_{\overline{x}\to 1}\left[-\tilde{p}^* + \frac{1}{n}\left(1 - \tilde{p}^{*n}\right)\right] > 0$$

This inequality implies that $1-\tilde{p}^{*n}-n\tilde{p}^{*}>0$ in the limit when $\overline{x}\to 1$. As a result, (71) is positive. But this constitutes a contradiction and therefore it cannot be the case that $\hat{p}^{*}>\tilde{p}^{*}$ when $\overline{x}\to 1$.

(c) Now we prove that $\hat{p}^{*}>\tilde{p}^{*}$ if $n=3$. We will use the results from the proof of part (b) of this proposition. If $n=3$,

$$Y(\cdot)_{\hat{p}^{*}=\tilde{p}^{*}}=1-\tilde{p}^{*3}-2\overline{x}\tilde{p}^{*}+\tilde{p}^{*}\left(\overline{x}^{2}-\tilde{p}^{*2}\right)-\overline{x}\tilde{p}^{*}=1-2\tilde{p}^{*3}+\left(\overline{x}^{2}-3\overline{x}\right)\tilde{p}^{*}$$

while condition (73) reduces to $\tilde{p}^{*}<1-\bar{x}+\frac{1}{3\overline{x}^{2}}\left(\overline{x}^{3}-\tilde{p}^{*3}\right)$.

Then,

$$\begin{aligned}Y(\cdot)_{\hat{p}^{*}=\tilde{p}^{*}}&>1-2\tilde{p}^{*3}+\left(\overline{x}^{2}-3\overline{x}\right)\left(1-\overline{x}+\frac{\overline{x}^{3}-\tilde{p}^{*3}}{3\overline{x}^{2}}\right)=1+\tilde{p}^{*3}\frac{3-7\overline{x}}{3\overline{x}}-\frac{\overline{x}}{3}\left(3-\overline{x}\right)\left(3-2\overline{x}\right)\\&>1+\left(\frac{1}{2}\right)^{3}\frac{3-7\overline{x}}{3\overline{x}}-\frac{\overline{x}}{3}\left(3-\overline{x}\right)\left(3-2\overline{x}\right)=\frac{1}{24\overline{x}}\left(3+17\overline{x}-72\overline{x}^{2}+72\overline{x}^{3}-16\overline{x}^{4}\right)\end{aligned}\qquad(74)$$

which is always positive. Therefore, $Y_{\hat{p}^{*}=\tilde{p}^{*}}>0$ and $\hat{p}^{*}>\tilde{p}^{*}$ if $n=3$.

(d) Finally, in the limit when $n\to\infty$ the FOC of the merged entity becomes

$$1-\left(\overline{x}-\tilde{p}^{*}+\hat{p}^{*}\right)^{k}-k\left(\overline{x}-\tilde{p}^{*}+\hat{p}^{*}\right)^{k-1}\hat{p}^{*}=0\qquad(75)$$

while that of a non-merging firm becomes

$$\frac{1}{1-\overline{x}}\left(\overline{x}-\hat{p}^{*}+\tilde{p}^{*}\right)^{k}\left(1-\overline{x}-\tilde{p}^{*}\right)=0.$$

This implies that $\lim_{n\to\infty}\tilde{p}^{*}=1-\overline{x}$.

The LHS of (75) decreases in $\hat{p}^{*}$. Then, if $\hat{p}^{*}<\tilde{p}^{*}$, the LHS of (75) must be negative if we replace $\hat{p}^{*}$ by $\tilde{p}^{*}=1-\overline{x}$. However,

$$1-\overline{x}^{k}-k\overline{x}^{k-1}\left(1-\overline{x}\right)=1+\left(k-1\right)\overline{x}^{k}-k\overline{x}^{k-1}\geq 0$$

where the inequality follows from setting $\overline{x}=1$. This establishes a contradiction so $\hat{p}^{*}<\tilde{p}^{*}$ cannot hold in the limit when $n\to\infty$. ■

Proof of Proposition 8. Before presenting the proof of this proposition, let us derive the payoff to a deviant merged entity that deviates by charging $\hat{p}<\hat{p}^{*}$ for its product i. The deviation has an impact not only on the demand for product i but also on the demand for products other than i. Consider first the demand for product i. Take a consumer who walks away from all non-merging firms and visits the merger entity. The consumer will buy product i with probability $\Pr\left[z_{n-k}-\tilde{p}^{*}<\overline{\overline{x}}-\hat{p}^{*}\text{ and }\varepsilon_{i}-\hat{p}>\max\left\{z_{n-k}-\tilde{p}^{*},z_{k-1}-\hat{p}^{*},0\right\}\right]$. This gives a demand for product i equal to

$$\begin{aligned}d_{i}(\hat{p};\hat{p}^{*},\tilde{p}^{*})&=\left(\overline{\overline{x}}-\hat{p}^{*}+\tilde{p}^{*}\right)^{n-k}\left(\hat{p}^{*}-\hat{p}\right)+\left(\overline{\overline{x}}-\hat{p}^{*}+\tilde{p}^{*}\right)^{n-k}\int_{\overline{\overline{x}}-\hat{p}^{*}+\hat{p}}^{1-\hat{p}^{*}+\hat{p}}\left(\varepsilon+\hat{p}^{*}-\hat{p}\right)^{k-1}d\varepsilon\\&\quad+\int_{\hat{p}}^{\overline{\overline{x}}-\hat{p}^{*}+\hat{p}}\left(\varepsilon+\hat{p}^{*}-\hat{p}\right)^{k-1}\left(\varepsilon+\tilde{p}^{*}-\hat{p}\right)^{n-k}d\varepsilon\\&=\frac{\left(\overline{\overline{x}}-\hat{p}^{*}+\tilde{p}^{*}\right)^{n-k}}{k}\left(k\hat{p}^{*}-k\hat{p}+1-\overline{\overline{x}}^{k}\right)+\int_{0}^{\overline{\overline{x}}-\hat{p}^{*}}\left(\varepsilon+\hat{p}^{*}\right)^{k-1}\left(\varepsilon+\tilde{p}^{*}\right)^{n-k}d\varepsilon\end{aligned}$$

Likewise, the consumer will buy product m with probability

$$\Pr\left[z_{n-k}-\tilde{p}^{*}<\overline{\overline{x}}-\hat{p}^{*}\text{ and }\varepsilon_{m}-\hat{p}^{*}>\max\left\{\varepsilon_{i}-\hat{p},z_{n-k}-\tilde{p}^{*},z_{k-2}-\hat{p}^{*},0\right\}\right],$$

which gives a demand for product m equal to

$$d_m(\hat{p};\hat{p}^*,\tilde{p}^*) = \int_{\hat{p}^*}^{\overline{\overline{x}}} (\varepsilon+\hat{p}^*-\hat{p})(\varepsilon-\tilde{p}^*+\hat{p}^*)^{n-k}\varepsilon^{k-2}d\varepsilon + (\overline{\overline{x}}-\hat{p}^*+\tilde{p}^*)^{n-k}\int_{\overline{\overline{x}}}^{1}(\varepsilon-\hat{p}^*+\hat{p})\,\varepsilon^{k-2}d\varepsilon$$

$$= \left(\overline{\overline{x}}-\hat{p}^*+\tilde{p}^*\right)^{n-k}\int_{\overline{\overline{x}}}^{1}(\varepsilon-\hat{p}^*+\hat{p})\,\varepsilon^{k-2}d\varepsilon + \int_0^{\overline{\overline{x}}-\hat{p}^*}(\varepsilon+\hat{p}^*)^{k-2}(\varepsilon+\hat{p})(\varepsilon+\tilde{p}^*)^{n-k}d\varepsilon.$$

The total payoff to a deviant merged entity is then

$$\hat{\pi}(\hat{p};\hat{p}^*,\tilde{p}^*) = p_i d_i(\hat{p};\hat{p}^*,\tilde{p}^*) + (k-1)\,\hat{p}^* d_m(\hat{p};\hat{p}^*,\tilde{p}^*).$$

Taking the first-order derivatives of the payoffs to non-merging and merging firms, imposing the condition that and $\tilde{p}=\tilde{p}^*$ and $\hat{p}=\hat{p}^*$ and simplifying gives the FOCs for the two types of firms:

$$\frac{1-\overline{x}^{n-k}}{1-\overline{x}}(1-\overline{x}-\tilde{p}^*)+\overline{x}^{n-k}-\left(\overline{\overline{x}}-\hat{p}^*+\tilde{p}^*\right)^{n-k}+(n-k)\int_0^{\overline{\overline{x}}-\hat{p}^*}(\varepsilon+\tilde{p}^*)^{n-k-1}(\varepsilon+\hat{p}^*)^k d\varepsilon = 0 \quad (76)$$

$$\left(\overline{\overline{x}}-\hat{p}^*+\tilde{p}^*\right)^{n-k}\left(1-\overline{\overline{x}}^k-k\overline{\overline{x}}^{k-1}\hat{p}^*\right)+k\int_0^{\overline{\overline{x}}-\hat{p}^*}(\varepsilon+\tilde{p}^*)^{n-k}(\varepsilon+\hat{p}^*)^{k-2}(\varepsilon+k\hat{p}^*)\,d\varepsilon = 0 \quad (77)$$

Note the similarity between these FOCs and the FOCs (9) and (10).

Assume that there is a pair of non-negative prices $\hat{p}^*$ and $\tilde{p}^*$ that satisfy the system of equations (76) and (77). For these prices to be consistent with equilibrium, first, they must be lower than or equal to the monopoly prices p_k^m and p^m, respectively; moreover, the reservation utility at the merged entity must be lower than the reservation utility at a non-merging firm.

Take the LHS of the FOC of a non-merging firm, equation (76). Note that the integral in this equation is positive. Observe now that the second summand is also positive because the assumption $\overline{x}-\tilde{p}^* > \overline{\overline{x}}-\hat{p}^*$ implies that $\overline{x} > \overline{\overline{x}}-\hat{p}^*+\tilde{p}^*$. As a consequence, if an equilibrium exists, the first term of the FOC (76) must be negative. This implies that in equilibrium, it must be the case that $\tilde{p}^* > 1-\overline{x}$.

Take now the limiting case where search cost is high so that $\bar{x}\to 1/2$. If this is so, for an equilibrium to exist, it must be the case that $\tilde{p}^* > 1/2$. But this is a contradiction because $\tilde{p}^* \le p^m = 1/2$. ■

Proof of Proposition 9. (a) This part follows straightforwardly from Propositions 2 and 5.

(b) We start by noting that the FOCs when the merged entity sells all products together, given above in (76) and (77) have exactly the same shape as the FOCs when the merged entity sells its products separately, given in (9) and (10). The only difference between these pairs of FOCs is that $\overline{x}$ is replaced by $\overline{\overline{x}}$ where necessary in order to account for the fact that the merged entity sells the products together.

Let us take the FOCs of the merged entity (10) and (77). Following the arguments in the proof of Proposition 1, both FOCs implicitly define the functions $\eta_1(\tilde{p})$ and $\tilde{\eta}_1(\tilde{p})$. We now show that $\tilde{\eta}_1(\tilde{p}) < \eta_1(\tilde{p})$ for all $\tilde{p}$. Upon observing the two FOCs, and noting that $\overline{\overline{x}} > \overline{x}$, it is obvious that we just need to show that $\partial\eta_1/\partial\overline{x} < 0$. Building on the proof of Proposition 1 and using the same notation, since we already now that $\partial G/\partial\hat{p} < 0$, we just need to show that $\partial G/\partial\overline{x} < 0$.

$$\frac{\partial G}{\partial \overline{x}} = -\frac{(k-1)\left(1-\overline{x}^k-k\hat{p}\overline{x}^{k-1}\right)}{k\overline{x}^k} - \frac{(k-1)(\overline{x}-\hat{p}+\tilde{p})+(n-k)\,\overline{x}}{\overline{x}^k(\overline{x}-\hat{p}+\tilde{p})^{n-k+1}}\int_0^{\overline{x}-\hat{p}}(\varepsilon+\hat{p})^{k-2}(\varepsilon+\tilde{p})^{n-k}(\varepsilon+k\hat{p})\,d\varepsilon.$$

Using the FOC (10) we can rewrite $\partial G/\partial \overline{x}$ as follows:

$$\begin{aligned}\frac{\partial G}{\partial \overline{x}} &= -\frac{(k-1)\left(1-\overline{x}^k-k\hat{p}\overline{x}^{k-1}\right)}{k\overline{x}^k}+\left[\frac{(k-1)\left(\overline{x}-\hat{p}+\tilde{p}\right)+(n-k)\,\overline{x}}{\overline{x}^k\left(\overline{x}-\hat{p}+\tilde{p}\right)^{n-k+1}}\right]\frac{\left(\overline{x}-\hat{p}+\tilde{p}\right)^{n-k}\left(1-\overline{x}^k-k\overline{x}^{k-1}\hat{p}\right)}{k}\\
&= -\frac{\left(1-\overline{x}^k-k\hat{p}\overline{x}^{k-1}\right)}{k\overline{x}^k}\left[k-1-\frac{(k-1)\left(\overline{x}-\hat{p}+\tilde{p}\right)+(n-k)\,\overline{x}}{\left(\overline{x}-\hat{p}+\tilde{p}\right)}\right]\\
&= \frac{\left(1-\overline{x}^k-k\hat{p}\overline{x}^{k-1}\right)(n-k)}{k\overline{x}^{k-1}\left(\overline{x}-\hat{p}+\tilde{p}\right)}<0\end{aligned}$$

where the last inequality follows from the fact that for the FOC (10) to be satisfied it must be the case that $1-\overline{x}^k-k\bar{x}^{k-1}\hat{p}<0$.

Let us now take the FOCs of the non-merging firms (9) and (76). Following the same arguments, both FOCs implicitly define the functions $\eta_2(\tilde{p})$ and $\tilde{\eta}_2(\tilde{p})$. We now show that $\tilde{\eta}_2(\tilde{p})>\eta_2(\tilde{p})$ for all $\tilde{p}$.

Upon observing the two FOCs, and noting that $\overline{\overline{x}}>\overline{x}$, it is obvious that we just need to show that $\partial\tilde{\eta}_2/\partial\overline{\overline{x}}>0$. Building on the proof of Proposition 1 and using the same notation, since we already now that $\partial H/\partial\hat{p}>0$, we just need to show that the LHS of (76) decreases in $\overline{\overline{x}}$. This derivative is equal to $-(\overline{\overline{x}}-\hat{p}+\tilde{p})^{n-k-1}(1-\overline{\overline{x}}^k)<0$.

Since $\tilde{\eta}_1(\tilde{p})<\eta_1(\tilde{p})$ and $\tilde{\eta}_2(\tilde{p})>\eta_2(\tilde{p})$, we conclude that both prices decrease when the merged entity stocks all the products together.

Regarding the profits result, we start by noting that the profits of the merged entity are independent of the decision to sell products together when the search cost is equal to zero. We now study the difference in profits of the merged entity across the two business strategies in a neighborhood of $s=0$ for the case $k=n-1$.[39]

When the merged entity sells its products in different stores, it gets a payoff given in (8). Using the FOC (9) we can isolate

$$\tilde{p}^*=\frac{1}{2}\left(1-\overline{x}+\hat{p}^*+\frac{1}{n}\overline{x}^n-\frac{1}{n}\left(\hat{p}^*\right)^n\right)$$

and rewrite (8) as follows

$$\hat{\pi}_S^*=\hat{p}^*\left[1-\tilde{p}^*\left(\hat{p}^*\right)^{n-1}\right]=\hat{p}^*\left[1-\frac{1}{2}\left(1-\overline{x}+\hat{p}^*+\frac{\overline{x}^n-\hat{p}^{*n}}{n}\right)\left(1+\hat{p}^{*n-1}\right)\right].$$

When the merged entity sells all its products in a single store, the payoff is given in proposition 8.

We are interested in the behavior of the difference in profits in a neighborhood of $s=0$. For this we study the expression

$$\lim_{s\to 0}\left(\frac{\partial\hat{\pi}_S^*}{\partial s}-\frac{\partial\hat{\pi}_J^*}{\partial s}\right)$$

where $\hat{\pi}_S^*$ denotes the profits when the products are sold in separate shops and $\hat{\pi}_J^*$ denotes the payoff when all products are sold together.

Since the functional forms for $\hat{\pi}_S^*$ and $\hat{\pi}_J^*$ are exactly the same and since the derivative of the equilibrium prices with respect to $\overline{x}$ are also exactly the same we can write

$$\lim_{s\to 0}\left(\frac{\partial\hat{\pi}_S^*}{\partial s}-\frac{\partial\hat{\pi}_J^*}{\partial s}\right)=\lim_{s\to 0}\left(\frac{\partial\hat{\pi}_S^*}{\partial\hat{p}^*}\frac{\partial\hat{p}^*}{\partial\overline{x}}+\frac{\partial\hat{\pi}_S^*}{\partial\overline{x}}\right)\left(\frac{\partial\overline{x}}{\partial s}-\frac{\partial\overline{\overline{x}}}{\partial s}\right)$$

Proceeding along the lines above, we can now that $\frac{\partial\hat{p}^*}{\partial\overline{x}}<0$ (for details see Moraga-González and Petrikaitė (2011)). Note next that

$$\lim_{s\to 0}\frac{\partial\hat{\pi}_S^*}{\partial\overline{x}}=\lim_{s\to 0}\frac{\hat{p}^*(1+\hat{p}^{*n-1})(1-\overline{x}^{n-1})}{2}=0$$

[39] Other cases are more complicated to check analytically because the FOCs (9) and (10) are not linear in $\tilde{p}^*$ and therefore we cannot obtain a closed form expression for $\tilde{p}^*$. Nevertheless, we have checked numerically that our result holds for other values of k. For example, when $n=5$ and $k=2$, the result is true for all s such that $0.001458<s<2*10^(-6)$.

Denoting $\lim_{s\to 0} \hat{p}^*$ by p_1^*, it takes a couple of steps to show that

$$\lim_{s\to 0} \frac{\partial \hat{\pi}_S^*}{\partial \hat{p}^*} = \frac{1}{2n}\left(2n - 1 - \left(n^2-1\right)\left(p_1^*\right)^n - n\left(p_1^*\right)^{n-1} + 2n\left(p_1^*\right)^{2n-1} - 2p_1^* n\right) \tag{78}$$

To study the sign of this expression, we use the FOC of the merged entity (10) to obtain

$$\tilde{p^*} = \frac{n\hat{p}^* - \left(\hat{p}^*\right)^n (n+1) - n\overline{x} + n\hat{p}^*\overline{x}^{n-1} + \overline{x}^n}{n\left(1 - n\left(\hat{p}^*\right)^{n-1}\right)}$$

and equate it to (6) to obtain

$$\left(n(1-\overline{x}+\hat{p}^*) + \overline{x}^n - \left(\hat{p}^*\right)^n\right)\left(1 - n\left(\hat{p}^*\right)^{n-1}\right) - 2n\hat{p}^* + 2\left(\hat{p}^*\right)^n (n+1) + 2n\overline{x} - 2n\hat{p}^*\overline{x}^{n-1} - 2\overline{x}^n = 0$$

Taking the limit when $s \to 0$ (for $\overline{x} = 1$) and using the notation $\lim_{s\to 0} \hat{p}^* = \hat{p}_1^*$ above this becomes

$$\left(\hat{p}_1^*\right)^n\left(2n - n^2 + 1\right) + 2n - 1 + n\left(\left(\hat{p}_1^*\right)^{2n-1} - 3\hat{p}_1^* - \left(\hat{p}_1^*\right)^{n-1}\right) = 0 \tag{79}$$

Combining (79) and (78) we can write the following series of inequalities

$$\begin{aligned}
&2n - 1 - \left(n^2-1\right)\left(\hat{p}_1^*\right)^n - n\left(\hat{p}_1^*\right)^{n-1} + 2n\left(\hat{p}_1^*\right)^{2n-1} - 2\hat{p}_1^* n \\
&= -\left(\hat{p}_1^*\right)^n\left(2n - n^2 + 1\right) - n\left(\left(\hat{p}_1^*\right)^{2n-1} - 3\hat{p}_1^* - \left(\hat{p}_1^*\right)^{n-1}\right) - \left(n^2-1\right)\left(\hat{p}_1^*\right)^n - n\left(\hat{p}_1^*\right)^{n-1} + 2n\left(\hat{p}_1^*\right)^{2n-1} - 2\hat{p}_1^* n \\
&= -2n\left(\hat{p}_1^*\right)^n + n\left(\hat{p}_1^*\right)^{2n-1} + n\hat{p}_1^* = n\hat{p}_1^*\left(1 - \left(\hat{p}_1^*\right)^{n-1}\right)^2 > 0.
\end{aligned}$$

Therefore we conclude that

$$\lim_{s\to 0}\left(\frac{\partial \hat{\pi}_S^*}{\partial \hat{p}^*}\frac{\partial \hat{p}^*}{\partial \overline{x}} + \frac{\partial \hat{\pi}_{SR}^*}{\partial \overline{x}}\right) < 0.$$

We now study the sign of

$$\lim_{s\to 0}\left(\frac{\partial \overline{x}}{\partial s} - \frac{\partial \overline{\overline{x}}}{\partial s}\right).$$

Note the following relationships

$$s = (1-\overline{x})^2/2 \text{ and } s = \frac{n-1}{n} - \overline{\overline{x}} + \frac{1}{n}\overline{\overline{x}}^n \text{ and } \overline{x} = 1 - \sqrt{2\left(\frac{n-1}{n} - \overline{\overline{x}} + \frac{1}{n}\overline{\overline{x}}^n\right)}$$

Therefore,

$$\frac{\partial \overline{x}}{\partial s} = \frac{1}{\partial s/\partial \overline{x}} = -\frac{1}{1-\overline{x}}, \quad \frac{\partial \overline{\overline{x}}}{\partial s} = \frac{1}{\partial s/\partial \overline{\overline{x}}} = -\frac{1}{1-\overline{\overline{x}}^{n-1}}$$

Then

$$\lim_{s\to 0}\left(\frac{\partial \overline{x}}{\partial s} - \frac{\partial \overline{\overline{x}}}{\partial s}\right) = \lim_{\overline{x}\to 1, \overline{\overline{x}}\to 1} \frac{\left(\overline{\overline{x}}^{n-1} - \overline{x}\right)}{(1-\overline{x})\left(1 - \overline{\overline{x}}^{n-1}\right)} = \lim_{\overline{\overline{x}}\to 1} \frac{\overline{\overline{x}}^{n-1} - 1 + \sqrt{2\left(\frac{n-1}{n} - \overline{\overline{x}} + \frac{1}{n}\overline{\overline{x}}^n\right)}}{\sqrt{2\left(\frac{n-1}{n} - \overline{\overline{x}} + \frac{1}{n}\overline{\overline{x}}^n\right)}\left(1 - \overline{\overline{x}}^{n-1}\right)} = -\infty$$

The result now follows. ■

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