DISCUSSION PAPER SERIES

No. 9373

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PUBLIC POLICY

Centre for Economic Policy Research

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Available online at: **www.cepr.org/pubs/dps/DP9373.asp**

ISSN 0265-8003

WINNING BY LOSING: INCENTIVE INCOMPATIBILITY IN MULTIPLE QUALIFIERS

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Discussion Paper No. 9373
March 2013

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CEPR Discussion Paper No. 9373

March 2013

ABSTRACT

Winning by Losing: Incentive Incompatibility in Multiple Qualifiers

In sport tournaments, the rules are presumably structured in a way that any team cannot be better off (e.g., to advance to the next round of competition) by losing instead of winning a game. Starting with a real-world example, we demonstrate that the existing national rules of awarding places for the UEFA Champions League and the UEFA Europa League, which are based on the results of the national championship, a round-robin tournament, and the national cup, a knock-out tournament, might produce a situation where a team will be strictly better off by losing a game. Competition rules of the European qualification tournament to the World Cup 2014 suffer from the same problem. We show formally that in qualifying systems consisting of several round-robin tournaments, monotonic aggregating rules always leave open such a possibility. Then we consider qualifying systems consisting of a round-robin tournament (championship) and a knock-out tournament (cup). We show that any redistribution rule that allows the cup's runner-up to advance in the case that the cup's winner advances based on its place in a championship, has the same drawback, and discuss possible fixes.

JEL Classification: D71
Keywords: football, organization, tournaments and UEFA Champions League

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Submitted 26 February 2013

1 Introduction

In any sports tournament the rules define a game, a strategic interaction between participants. In theory, these rules should be structured in such a way that a team cannot advance by losing instead of winning a game. In practice, those who design the rules might overlook adverse consequences for incentives that the rules create as in most real-world situations the corresponding game-theoretic analysis might be cumbersome. This is especially so when the situation where losing becomes strictly dominant is a low-probability event.

Consider the following set of rules that is common in European football (52 out of 53 UEFA, The Union of European Football Associations, countries use a variation of this system). Suppose that a country holds more than one tournament to qualify for international tournaments. Typically, teams that win top places[1] in the national championship (a round-robin tournament) qualify for the UEFA Champions League, the most important and profitable club tournament, while the next tier qualifies for the Europa League, the second tournament. The winner of the national cup (a knock-out tournament) qualifies for the UEFA Europa League. If the winner of the national cup qualifies for the Champions League, then the cup runner-up enters the Europa League. In this paper, we show that the described rule creates a possibility that, in certain circumstances, a team might benefit by deliberately losing a game in the championship. Furthermore, we show that a whole class of such redistribution rules is inherently flawed.

The intuition behind the misalignment of incentives is straightforward. A strategic loss by one team might help another team that otherwise goes to the Europa League as the cup winner, to advance to the Champions League, giving the cup runner-up a place in the Europa League. Trivially, the cup runner-up might prefer to lose to the cup winner in the national championship to help the latter to advance to the Champions League and free a place in the Europa League. Our results demonstrate that this is a general phenomenon.

The following very simple example illustrates the basic logic of our argument.

Example 1 *Let there be two domestic round-robin tournaments and 4 teams, namely A, B, C and D, participating in each of the tournaments, which we 'Tournament 1' and 'Tournament 2'. The best team in each tournament qualify for the Champions League; this qualification itself is the main prize. It could happen that one team wins both tournaments. In this case, there is one vacant place in the Champions League. Consider the following redistribution rule: if one team wins both tournaments, then the vacant place is allocated to the team that finished on the second place in Tournament 1. Now we construct the situation when team B is better off by losing*

[1] 1-4, depending on the country's ranking.

the game versus team A.Under any 'reasonable' ranking method (e.g., the standard football one

Tournament 1				
	A	B	C	D
A		Win	Win	Win
B	Loss		Win	Win
C	Loss	Loss		Win
D	Loss	Loss	Loss	

Tournament 2				
	A	B	C	D
A		?	Draw	Win
B	?		Draw	Loss
C	Draw	Draw		Win
D	Loss	Win	Loss	

Figure 3: Example of incentive incompatibility with two round-robin tournaments.

where each win gets the team 3 *points, a draw is* 1 *point, and loss is* 0), *in Tournament 1 team A will be ranked first and team B will be second. As for Tournament 2, teams A and C compete for the first position. If team B loses to A in the last match of the tournament, then team A wins both tournaments. In this case, according to the redistribution rule team B gets qualifies for the Champions League as the second team in Tournament 1. At the same time, if team B wins over A, team C is the first in Tournament 2 (instead of A). Consequently, team B has to lose the game against A in order to qualify.*

The same logic can be easily expanded to the general case with more than three teams, more winners qualifying for international competitions, and any 'reasonable' redistribution rules.[2]

There are a number of situations, in which a team might prefer losing a game, rather than winning.[3] First, some players may be bribed. Second, the teams that performed worse may

[2]Sport tournaments use various ranking methods based on match results. For example, the National Hockey League awards, during the regular season, two points for a win, one point for losing in overtime or a shootout, and zero points for a loss.

[3]A famous example of misaligned incentives in football tournaments is the Shell Caribbean Cup 1994 (see (Gardiner, 2005) for all details). In the last game of the preliminary group 1, Barbados had to win with the goal difference +2 or more, while for its competitor, Grenada, a loss with goal difference -1 was enough to advance to the next round. Barbados was leading 2-0, when Grenada scored on the 83rd minute. At 2-1, Grenada would qualify, so Barbados tried to score. However, due to the specific rules, there was another option. The rules were as follows. In the case of a draw after 90 minutes, the teams play extra 30 minutes. If a goal is scored during this extra time, the game ends. The unusual provision was that a goal scored in the extra time is counted as two goals. Thus, Barbados realized that they have a nonstandard option: instead of trying to score in Grenada's goal during the last minutes of the game, it is easier to score an own goal. Score 2-2 gives Barbados additional 30 minutes to score a goal and win with the goal difference +2. However, when Barbados scored an own goal, Grenada still

have legal advantages in the next season[4]. Third, being the second in qualification might result in having a preferred competitor in the knock-out stage[5]. However, in the first example reverse incentives are not generated by the tournament rules. In the second example prize distribution rules were deliberately designed to reward less fortunate teams. In the third case the focus is on the expected outcome (any team has a lower probability to win playing against *Barcelona* or *Chelsea* than against a weaker team). In this paper our focus is on the possibility that a team is strictly better off by losing.

In economics, the problem of the aggregation of results in sports tournaments is connected to the classic problem of the aggregation of voter preferences. It was initially noticed by Harary and Moser (1966), who discussed discrete properties of round-robin tournaments. Arrow (1963) in his seminal paper formulated several highly desired properties of aggregation rules of voter preferences and proved that there is only one aggregation rule (namely, dictatorship) that satisfies these properties. Ariel Rubinstein used a similar approach for the problem of ranking participants in a round-robin tournament (Rubinstein, 1980). There, he defined a tournament as a pair $(N, \to)$, where N is a set of all participants in the tournament and $\to$ is a binary complete asymmetric relation defined on set N. Relation $x \to y$, where $x, y \in N$, is interpreted as a win of team x over team y. The ranking rule is a function $\succcurlyeq$ that assigns to each possible tournament $T = (N, \cdot)$ a place for each participating team. Rubinstein defined the properties of anonymity, positive responsiveness and independence of irrelevant alternatives and proved that the only ranking rule that satisfies all 3 properties, is a ranking with respect to the number of wins. Several authors defined other desired sets of properties and found all ranking rules that satisfy those properties (see, for example, Bouyssou, 2004; van den Brink and Gilles, 2000; Herings, van der Laan and Talman, 2005; Slutzki and Volij, 2005, 2006).

Starting in 1970s, researchers began focusing on manipulability of voting systems. From the Gibbard–Satterthwaite (Gibbard, 1973; Satterthwaite, 1975) and Duggan–Schwartz (Duggan and Schwartz, 2000) theorems it follows that in the presence of "good enough" aggregation rules

had few minutes to change the things and escape the extra time. Grenada had to score one goal... no matter into which net! Barbados understood it as well and divided the players to defend both goals. Grenada's players unsuccessfully tried to score an own goal during the last moments of the second half. In the extra time Barbados successfully completed the plan and scored a legal goal which gave them a qualification to the next stage.

[4]In National Basketball Association draft lottery favours less successful teams in order to level off the teams chances next time.

[5]In London Summer Olympics 2012 four badminton pairs were disqualified for doing this. Badminton World Federation (BWF) charged them with "not using one's best efforts to win a match" and "conducting oneself in a manner that is clearly abusive or detrimental to the sport" (see BWF website, checked November 11, 2012: http://www.bwfbadminton.org/news_item.aspx?id=65297).

there is always a voter who can profitably deviate from his real preferences and vote strategically. A similar question arises in connection with aggregation of tournaments results: under the given ranking rule, is there a team that has a positive incentive to lose a game deliberately due to strategic issues? If only one tournament is being played, then under every reasonable ranking rule a team can not be better off by losing instead of winning. Some authors (see, e.g.,, Chen, Deng and Liu, 2011; Faliszewski, 2008; Russell and Walsh, 2009) consider the possibility of forming a coalition of several teams. In that case one team from the coalition may deliberately lose to another team from this coalition to enlarge the profits of the whole coalition.

The rest of the paper is organized as follows. In section 2 we provide a real-world example of incentive incompatibility with multiple qualifiers that illustrates the logic of the main results.. Section 3 contains the formal setup and proves the main theorem. Section 4 discusses implications of our formal results for European football competitions.

2 Real-world examples

In this section, we demonstrate, by means of the real-world examples, the logic of incompatibility of incentives to win a game in a system, consisting of multiple tournaments. Later, we shall prove that any rule that specifies that the runner up for the country's cup to qualify in the case that the cup's winner qualifies for based on its place in a round-robin tournament results in such a possibility (Theorem 1).

2.1 Russian season 2011/2012

The first example[6] is more complicated than the story described in introduction as teams strive to qualify for two, not one, international tournaments. Yet this does not affect the logic of the argument.

By May 8, 2012, each team in the Russian Premier League had one more game to play in the 2011-12 championship tournament. The final of the Russian Cup, the second major tournament, was to be held on the May 9. Conditional on the results of other games, Lokomotiv Moscow would have been better off losing its game against Spartak Moscow. This would let Spartak qualify for the UEFA Champions League, let Dynamo Moscow (if it wins over Rubin in the Cup final) qualify for the Europa League, leave Rubin out of the international competitions, and give Lokomotiv a place in the Europa League. If, instead, Lokomotiv beats Spartak, the other results being the same, Dynamo would qualify for the Champions League, thus getting

[6] This case was initially raised in a comment posted by Dr. Andrei Brichkin (nickname *quant*) on the http://www.eurocups.ru/guestbook/ website, message 170910.

Rubin, the Cup's runner-up, qualify for the Europa League, and leaving Lokomotiv out of the international competitions.

In the Russian Premier league, a win is awarded 3 points, and a draw is 1 point. In 2011-12, Russia switched from the Spring-Fall season to more conventional Fall-Spring season. In the Spring of 2012, the top eight teams after 2011 competed for places 1-8 while the bottom eight teams after 2011 competed for places 9-16. Both mini-tournaments were played in a double round-robin format and points were added to the points gained in 2011.

With one match to go, the top eight teams standings were as follows.

Place	Team	Pts
1	Zenit St.Petersburg	85
2	CSKA Moscow	73
3	Spartak Moscow	72
4	Dynamo Moscow	71
5	Anzhi Makhachkala	70
6	Lokomotiv Moscow	66
7	Rubin Kazan'	65
8	Kuban' Krasnodar	60

The remaining games were Kuban' – Dynamo, Lokomotiv – Spartak, Rubin – CSKA, and Anzhi – Zenit. The Cup final on May, 9 was scheduled to occur between Dynamo and Rubin.

The most valuable prize, save for the championship itself, is qualification for international tournaments, the UEFA Champions League and the UEFA Europa League. Participation in these tournaments brings substantial financial rewards for clubs and additional exposure for players, the Champions League being far more attractive in both respects. The number of slots for both tournaments is determined by UEFA using the past performance of the country's teams. For 2012-13, Russia was awarded 2 slots in the Champions League, and 4 slots in the Europa League.

Slots for participation in the UEFA Champions League and UEFA Europa League are distributed according to the following rules.

1. Teams that are ranked 1st and 2nd in the Russian national championship qualify for the Champions League.

2. Teams that are ranked 3rd to 5th in the national championship, qualify for the Europa League.

3. The Russian Cup winner qualifies for the Europa League.

4. If the Cup winner is ranked 1st or 2nd in the national championship, then it qualifies for the Champions League, and the Cup runner-up qualifies for Europa League.

5. If the Cup winner is ranked 3rd to 5th in the national championship, then the team ranked 6th qualifies for Europa League.

Now, let us consider the following scenario in some detail. First, suppose that Dynamo wins the Russian Cup and beats Kuban' in the championship. Second, suppose that Rubin vs. CSKA is a draw. With only two matches (Lokomotiv vs. Spartak, Anzhi vs. Zenit) unplayed, the teams' standings are as follows.

Place	Team	Pts
1	Zenit	85
2	Dynamo	74
3	CSKA	74
4	Spartak	72
5	Anzhi	70
6	Lokomotiv	66
7	Rubin	66
8	Kuban'	60

With equal number of points, ultimate relative standings are determined by the number of wins. Due to this rule Dynamo is above CSKA (both teams have 74 points) and Lokomotiv is above Rubin (both teams have 66 points). The outcome of Anzhi — Zenit match is irrelevant for further consideration as Zenit has clinched the championship in advance, and Anzhi has already earned the place in Europa League (regardless of the result of the last game, Anzhi cannot be ranked lower than 5th or higher than 4th).

Thus, the only game left is Lokomotiv–Spartak. There are three possible outcomes: Lokomotiv's win, draw and loss. Consider the final standing of teams in each of these cases. Teams that qualify for the Champions League are italicized; teams that qualify for the Europa League are in bold.

Lokomotiv's win

Place	Team	Pts
1	*Zenit*	*85*
2	*Dynamo*	*74*
3	**CSKA**	**74**
4	**Spartak**	**72**
5	**Anzhi**	**70**
6	Lokomotiv	69
7	**Rubin**	**66**
8	Kuban'	60

Draw

Place	Team	Pts
1	*Zenit*	*85*
2	*Dynamo*	*75*
3	**CSKA**	**74**
4	**Spartak**	**72**
5	**Anzhi**	**70**
6	Lokomotiv	67
7	**Rubin**	**66**
8	Kuban'	60

Lokomotiv's loss

Place	Team	Pts
1	*Zenit*	*85*
2	*Spartak*	*75*
3	**Dynamo**	**74**
4	**CSKA**	**74**
5	**Anzhi**	**70**
6	**Lokomotiv**	**66**
7	Rubin	66
8	Kuban'	60

If Lokomotiv beats Spartak (Table 3, left column) or there is a draw (Table 3, central column), then Lokomotiv is 6th in the national championship and does not qualify for the Europa

League while 7th-ranked Rubin qualifies as the runner up of the Cup. (Dynamo, the Cup's winner, is qualified for the Champions League as it is ranked 2nd in the national championship.)

Now, if Lokomotiv loses to Spartak, then Lokomotiv is still 6th in the national championship. However, Spartak is now 2nd and qualifies for the Champions League. This means that Dynamo gets its place in the Europa League as the Cup's winner, and Rubin, as a runner up, does not get anything. Lokomotiv, as the 6th-ranked team in the national championship, qualifies for the Europa League by point 5 of the allocation rule mentioned earlier.

In the scenario considered above, Lokomotiv has all the incentives to lose in the final game of the national championship. While the team would finish sixth in each case, losing would bring about qualification for the European tournament. This scenario wasn't realised, as Rubin won the Russian Cup, beating Dynamo.

2.2 World Cup 2014 European Qualification

The second example[7] deals with the qualification tournament in the UEFA zone for the FIFA World Cup 2014. There are 53 teams competing for 12 European places at the World Cup. These teams are split into 8 groups, each consisting of 6 teams, and 1 group, consisting of 5 teams. Teams from one group play each other two times on the home-away basis. Points are awarded as always: 3 points for a win, 1 point for a draw, 0 points for a loss. Each team finishing first in its group automatically qualifies for the final tournament. The worst of 9 second-placed teams is out. The other 8 second-placed teams are split into pairs and the winner from each pair also qualifies for the final tournament. Our subject of interest is how the best 8 second-placed teams are determined. According to the rules, for each second-placed team the number of points gained versus the 1st, 3rd, 4th and 5th teams is calculated, and all second-placed teams are ranked with respect to this number.

Now imagine the following. Take the group with 6 teams. Assume that team A has guaranteed 2nd place before the last matchday (it means that team A can neither get 1st place nor finish below 2nd). Let team A be scheduled to play on the last matchday with team B, which currently holds 6th position in the group, 1 point behind the 5th-placed team C. Finally, let team A collect 2 points from 2 games versus team C (this is possible in the case of two draws) and collect 3 points from 1 game versus team B (that is A won over B). In these circumstances team A has absolutely no reason to play for a win on the last matchday because if team B loses, B finishes in 6th place and points from matches played against 6th-placed teams are not counted when determining best 2nd-placed teams. Moreover, team A has positive incentives to lose to

[7]This issue was also discussed by user *MABP84* and Dr. Andrei Brichkin (nickname *quant*) on the http://www.eurocups.ru/guestbook/ website, messages 185008 and 185019.

team B. If team C loses on the last matchday and team B wins over team A, team B finishes in 5th place. Thus, for team A the number of points gained from matches played against 1st, 3rd, 4th and 5th team is greater by 1 point in the case of loss to team B than in the case of a win over B. It is easy to check that all described conditions are compatible and the tournament with the required properties exists.

3 Theory

In this section we formalize the problem of results aggregation in round-robin tournaments. Then, we demonstrate that incentives incompatibility necessarily arise under any monotonic ranking method or allocation rule when there are multiple round-robin qualifiers.

Definition 1 *A* tournament *is a pair* $(\mathcal{X}, v(x,y))$*, where* $\mathcal{X}$ *is a nonempty finite set of the teams and* $v(x,y)$ *is a function which satisfies the following three conditions:*

1) $v(x,y)$ *is defined on the set* $(\mathcal{X} \times \mathcal{X}) \setminus \{(x,y)|x=y\}$*;*

2) image of $v(x,y)$ *is a subset of the set* $\{-1,0,1\}$*;*

3) for each $x_0, y_0 \in \mathcal{X}$*,* $x_0 \neq y_0$*, the equality* $v(x_0,y_0) = -v(y_0,x_0)$ *holds.*

Function v *is called the* characteristic function *of the tournament* $(\mathcal{X}, v(x,y))$*..*

Let $x_0, y_0 \in \mathcal{X}$, $x_0 \neq y_0$. We say that the team x_0 wins over the team y_0 if and only if $v(x_0,y_0) = 1$; the team x_0 loses to the team y_0 if and only if $v(x_0,y_0) = -1$; the teams x_0 and y_0 tie if and only if $v(x_0,y_0) = 0$. This definition of the tournament corresponds to a round-robin tournament in which each two teams play versus each other once and function v defines the result of each match.

Fix the set $\mathcal{X}$ and consider different characteristic functions v. For each function S whose domain is the set of all characteristic functions v and that maps v into a partially ordered set $S(v)$ of elements of the set $\mathcal{X}$ we say that S is a ranking method. In other words, the ranking method is a rule that orders the participating teams in accordance with the results of all matches.

Example 2 *Consider a tournament* $T = (\mathcal{X}, v_0)$*, where* $\mathcal{X} = \{A,B,C,D\}$ *and characteristic function* v_0 *is given by the following table:*

	A	*B*	*C*	*D*
A	-	*1*	*-1*	*-1*
B	*-1*	-	*1*	*0*
C	*1*	*-1*	-	*-1*
D	*1*	*0*	*1*	-

Let S *be the following ranking method:*

1) a team earns 3 points for each victory, 1 point for each draw, 0 points for each loss;

2) if one team gets more points than another, then the former team is ranked higher than the latter;

3) if two or more teams get the same number of points, then the team that gets more points in matches between these teams will be ranked higher;

4) if after applying rules 2) and 3) a total order is not achieved, then the teams with the same number of overall points and the same number of points in matches between themselves are ordered according to the following initial seeding: $A \succ B \succ C \succ D$.

Note that for any characteristic function v ranking method S defines a totally ordered set $S(v)$ of the teams from $\mathcal{X}$. In particular, for given function v_0 we obtain $S(v_0) = D \succ B \succ A \succ C$, that is D gets the 1st place, B — 2nd, A — 3rd, and C — 4th.

When $S(v)$ is a totally ordered set, we put it into one-to-one correspondence with a collection of teams' places $(s_1(v), ..., s_K(v))$, where $s_i(v)$ is the rank assigned to the team number i by the ranking method S in the tournament with characteristic function v, $i = 1, ..., K$. If for any two teams i and j either $s_i(v) < s_j(v)$ or $s_j(v) < s_i(v)$ holds, we say that $S(v)$ is a strict totally ordered set.

Definition 2 *We say that the ranking method S is* well-defined *if for any characteristic function v, the set $S(v)$ is strictly totally ordered. For the tournament with characteristic function v and the team x denote by $N_v^1(x)$, $N_v^0(x)$ and $N_v^{-1}(x)$ numbers of wins, draws, and losses respectively.*

Definition 3 *We say that the ranking method S satisfies* the monotonicity property *if and only if for any characteristic function v and for any two teams $x, y \in \mathcal{X}$ such that*

$$N_v^1(x) \geqslant N_v^1(y), \quad N_v^1(x) + N_v^0(x) \geqslant N_v^1(y) + N_v^0(y),$$

where at least one of these two inequalities in is strict, $s_x(v) < s_y(v)$ holds.

It is easy to see that if ranking method S satisfies the monotonicity property, then in a single tournament with the ranking method S a win is not worse than a draw and a draw is not worse than a loss. However, the incentive to play for a win may disappear in the case when the same teams participate in several tournaments and the results of one tournament affect distribution of prizes in other tournaments.

Let one international tournament and N domestic tournaments take place, $N \geqslant 1$. A team can proceed to the international tournament only after a successful performance in one of domestic competitions. An opportunity to play in the international tournament is the only prize in the domestic tournaments. Denote $G_K = \{1, 2, ..., K\}$. Let the set of teams competing domestically be $\mathcal{X} = G_K$, $K \geqslant 1$. Let b_i be the number of available places for the international tournament in tournament i, $i = 1, ..., K$.

It might happen that after all the domestic tournaments are completed, one team gets earns more than one place in the international tournament, i.e. this team finishes in the prize zone in several tournaments. In this case, there are vacant places in the international tournament. For example, in the extreme case, when all the teams are ranked the same in each tournament, there will be only $\max\limits_i b_i$ contested places instead of $\sum\limits_i b_i$. Then all the vacant places must be distributed among the other teams. It is easy to see that there can not be more than $\sum\limits_i b_i - \max\limits_i b_i$ vacant places.

Allocating the vacant places to the remaining teams might be done in many different ways. It is natural to allow only such distributions of vacant places that a team can win a place only if all teams that finished above it in this tournament also got placed. Below we give a formal definition.

Definition 4 *The redistribution rule is a labeled tree that can be obtained by applying the following algorithm:*

1) v_0 is a root; vertex v_0 is labeled with the collection $(b_1, ..., b_N)$ — number of places to international tournament that can be won in each of domesctic tournaments, $b_i \geqslant 1$ for each $i = 1, ..., N$.

2) if e is a vertex labeled with the collection $(a_1, ..., a_N)$ and $\max\limits_i a_i < \sum\limits_i b_i$, then there are $\sum\limits_i b_i - \max\limits_i a_i$ edges oriented away from the vertex e. These edges are labeled with numbers $1, ..., \sum\limits_i b_i - \max\limits_i a_i$. The vertex different from e that is incident to the edge, labeled with number l, $1 \leqslant l \leqslant \sum\limits_i b_i - \max\limits_i a_i$, is labeled with the collection $(c_1^l, ..., c_N^l)$, where $c_i^l \geqslant a_i$ for each $i = 1, ..., N$ and at least one of the latter inequalities is strict. Collection $(c_1^l, ..., c_N^l)$ is the new number of places to the international tournament that remain available in each of the domestic tournaments. Here "new" means the situation when in the state, corresponding to vertex e, exactly l places to the international tournament were not distributed. A path from the root to the leaf describes the order of redistributing places to the international tournament.

Due to the finite number of tournaments and competing teams each redistribution rule is a finite tree. It is easy to see that there are only a finite number of redistribution rules.

Example 3 *Let there be 2 domestic tournaments and 3 participating teams. Also there are 2 places to international tournament, these places will be giben to the winners of each domestic tournament, that is $b_1 = b_2 = 1$. Consider the following redistribution rule: if one team wins* *both tournaments (it means that there is one vacant place), then the second place is given to the team that finished on the second place in the first tournament. The corresponding tree is shown on Figure 4.*

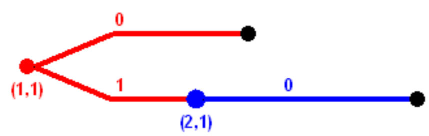

Figure 4: An example of a redistribution rule.

Definition 5 *A tournament is called* the simplest *if and only if for any* $i = 0, 1, ..., K-1$ *exactly one team won* i *matches.*

The following theorem is the main result of the paper. It says that if there are more than two tournaments, each providing at least one winner with the prize (e.g., a place in Champions League), and at least three teams, than any (monotonic) ranking method and any redistribution rule allow for a situation, in which a team is better off by losing, rather than winning, a game. The idea of the proof is straightforward: given a ranking method and a redistribution rule, we provide a collection of characteristic functions ("tournaments' outcomes") such that there is a team and a game that this team is better off losing than winning.

Theorem 1 *Suppose that* $N \geqslant 2$*;* $b_i \geqslant 1$ *for each* $i = 1, ..., N$*; and* $K > \max\left(\sum_i b_i, 3\right)$*. Then for any well-defined monotonic ranking methods* $S_1(\cdot), ..., S_N(\cdot)$ *and for any redistribution rule* R*, there exist characteristic functions* $v_1, ..., v_N, w$ *and* i*,* $1 \leqslant i \leqslant N$*, such that*

(i) there exists collection (x_0, y_0) *such that* $v_i(x_0, y_0) = 1$ *and* $w(x_0, y_0) = -1$*;*

(ii) for any collection (x, y)*, different from* (x_0, y_0)*, the equality* $w(x, y) = v_i(x, y)$ *holds;*

(iii) according to the standings $S_1(v_1), ..., S_{i-1}(v_{i-1}), S_i(v_i), S_{i+1}(v+1), ..., S_N(v_N)$ *team* x *qualifies to the international tournament;*

(iv) according to the standings $S_1(v_1), ..., S_{i-1}(v_{i-1}), S_i(w), S_{i+1}(v+1), ..., S_N(v_N)$ *team* x *does not qualify to the international tournament.*

Proof. Assign number 1 to the domestic tournament in which additional place to international tournament will be awarded in the case if exactly 1 place is vacant, according to redistribution rule R. From the condition 3) it follows that $K \geqslant 4$. Fix three arbitrary teams and call them X, Y and Z.

Define characteristic functions $v_3, ..., v_N$ as characteristic functions of $N-2$ simplest tournaments that jointly satisfy two following conditions:

1) if there are no redistributions of places concerning tournaments $3, ..., N$, neither team wins a place in more than one of these tournaments;

2) if there are no redistributions of places concerning tournaments $3, ..., N$, teams X, Y and Z do not win places.

To ensure compliance of these conditions, it is sufficient to replace arbitrarily the teams in the prize zone of tournaments, leaving teams X, Y and Z with 0,1 and 2 victories respectively in each tournament at the same time. From the monotonicity property of ranking methods S_i, $i = 3, ..., N$, it follows that teams X, Y and Z would be ranked last 3 in each of the domestic tournaments with numbers $3, ..., N$. If there are no redistributions of places concerning tournaments $3, ..., N$, neither of the teams X, Y and Z can win a place in neither of tournaments $3, ..., N$ because there are no more than $K-3$ places in the total prize pool of tournaments 3,...,N:

$$\sum_{i=3}^{N} b_i \leqslant \sum_{i=3}^{N} b_i + (b_1 - 1) + (b_2 - 1) \leqslant \sum_{i=1}^{N} b_i - 2 \leqslant K - 3,$$

where the first inequality is true due to condition 2) of the theorem and last inequality is true due to condition 3) of the theorem.

We define the characteristic function of the first tournament v_1 as a characteristic function of the simplest tournament with the following properties:

1) team X won $K - b_1 - 1$ matches and, consequently, due to monotonicity property was ranked on the $(b_1 + 1)$-th place;

2) team Y won $K - b_1$ matches and, consequently, due to monotonicity property was ranked on the b_1-th place;

3) team Z won 0 matches and, consequently, due to monotonicity property was ranked on the K-th place;

4) neither of the teams ranked from 1-st to $(b_1 - 1)$-th place won places in tournaments $3, ..., N$ in the absence of redistributions concerning those tournaments.

Here we construct the characteristic functions of the second tournament v_2 and w. These functions have the same values except for exactly one collection. Firstly, consider the simplest tournament with the following properties:

1) team X won 0 matches and, consequently, due to monotonicity property was ranked on the K-th place;

2) team Y won $K - b_2$ matches and, consequently, due to monotonicity property was ranked on the b_2-th place;

3) team Z won $K - b_2 - 1$ matches and, consequently, due to monotonicity property was ranked on the $(b_2 + 1)$-th place;

4) neither of the teams ranked from 1-st to $(b_2 - 1)$-th place won places in tournaments $3, ..., N$ in the absence of redistributions concerning those tournaments.

5) neither of the teams ranked from 1-st to $(b_2 - 1)$-th place finished among top b_1 teams in the first tournament.

Denote the characteristic function of this tournament $\widehat{v}_2$. Secondly, construct function v_2. Let $v_2(X, Z) = v_2(Y, Z) = 0$ (i.e. Z drawn the matches versus X and Y) and $v_2(\widetilde{\alpha}, \widetilde{\beta}) = \widehat{v}_2(\widetilde{\alpha}, \widetilde{\beta})$ for any collection $(\widetilde{\alpha}, \widetilde{\beta}) \in \mathcal{X} \times \mathcal{X} \setminus \{(X, Z), (Y, Z)\}$. Now, consider team Y and its place $s_Y(v_2)$. Team Y's record is worse than that of $b_2 - 1$ teams which won at least $K - b_2 + 1$ matches, so $s_Y(v_2) \geqslant s_Y(\widehat{v}_2) = b_2$. At the same time team Y's record is better than that of all other teams, including teams X and Z (we again use monotonicity property), so $s_Y(v_2) \leqslant b_2$. Thus, $s_Y(v_2) = b_2$. From the results of all tournaments $S_1(v_1), S_2(v_2), S_2(v_3), ..., S_N(v_N)$ it follows that team Y won a place to international tournament both in the first and in the second domestic tournaments. According to the redistribution rule R and our definition of the first tournament, in this case the team that finished on the (b_1+1)-th place in the first tournament gets the vacant place. Since $s_X(v_1) = b_1 + 1$, it is team X that qualifies.

Finally, we define the characteristic function w. Let $w(X, Y) = 1$ and $w(\widetilde{\alpha}, \widetilde{\beta}) = v_2(\widetilde{\alpha}, \widetilde{\beta})$ for any collection $(\widetilde{\alpha}, \widetilde{\beta}) \in \mathcal{X} \times \mathcal{X} \setminus \{(X, Y)\}$. Because of monotonicity of the ranking method S_2 and inequality $K \geqslant 4$ the following relations hold:

$$\begin{aligned} s_X(w) &> b_2, \\ s_Y(w) &> b_2, \\ s_Z(w) &= b_2. \end{aligned}$$

But in this case team X does not get a place to the international tournament if results $S_1(v_1), S_2(w), S_2(v_3), ..., S_N(v_N)$ are obtained. ■

4 Discussion

Theorem 1 proves that there is no acceptable qualification system consisting of several round-robin tournaments in which the possibility of profitable deliberate losing is excluded. This result may be generalized in different ways.

Most European football national championships are run in two rounds on a home-away basis, i.e. each two participating teams play against each other two times. To describe this kind of competition formally, the notion of a generalized tournament is defined (see, for example, Slutzki and Volij, 2005). We call k-rounds tournament the collection $(\mathcal{X}, v_1(x,y), ..., v_k(x,y))$, where $\mathcal{X}$ is the set of all participating teams and v_i is a characteristic function of i-th round, satisfying the same conditions as in the definition of a round-robin tournament, $i = 1, ..., k$, $k \geqslant 1$. By repeating the argument in the proof of theorem 1, it is easy to prove that the statement of Theorem 1 remains true after the substitution of "tournament" by "k-rounds tournament", $k \geqslant 2$.

Sometimes teams compete for places in several international tournaments. For example, national football federations from the UEFA zone delegate their teams for two international tournaments — the Champions League and the Europa League. A general formal analysis is cumbersome as the number of types of "joint wins" of domestic tournaments increases dramatically. Thus, it is harder to define general redistribution rules. Below we consider in detail one important special case, which is particularly relevant for the real world.

So far, our analysis was focused on round-robin tournaments. In most UEFA countries, qualification for the Champions League and the Europa League is decided after two tournaments: the national round-robin championship and the national cup which is held according to a knock-out system. There are several exceptions: for example, in Liechtenstein only national cup is held, whereas in England it is possible to get a place to international tournaments from three competitions: the Premier League, the FA Cup, and the League Cup.

Now, we call a cup tournament (or, simply, a cup) a pair $(\mathcal{X}, T)$, where $\mathcal{X}$ is the set of all participating teams and T is a binary tree which is rooted from leaves to root and satisfies the following properties:

1) There are 2 edges arriving at each vertex except for leaves, where there are no edges arriving at leaves;

2) There is 1 edge leaving from each vertex except for the root, where there are no edges leaving from the root;

3) All vertices are labelled with one team from the set $\mathcal{X}$;

4) If parent vertices are labelled with teams X and Y, then the child vertex is labelled with either X or Y;

5) There is a one-to-one correspondence between the set $\mathcal{X}$ and the set of all leaf labels.

Tree T can be considered as a protocol of the cup tournament. A label of the child vertex corresponds to the winner of the match between the teams assigned to parent vertices. The label of the root corresponds to the winner of the cup.

Contrary to a case with several round-robin tournaments, there are no incentives to lose a match in the cup deliberately. Thus, the remaining interesting case is the case with one round-robin championship and one cup. The question is whether a team can extract profit from losing a game in the round-robin competition in the presence of the cup. Remember that this is the case of qualification to UEFA international tournaments. In the example of Russia-2012, described in the beginning of this paper, there were two tournaments – one round-robin championship and one cup. It appears that the key point in this case is the redistribution rule. If vacant places are always redistributed in favour of the championship, there are no incentives to lose a game in the championship due to the monotonicity of the ranking rule in the championship and the impossibility of awarding any extra places to the cup.

Thus, there is an important practical implication: *if one wants to avoid deliberate losses, define the redistribution rule in such a way that all vacant places are awarded to the teams away from the round-robin tournament.*

However, in many European countries the regulation of awarding places to UEFA international tournaments leave the chance for incentives incompatibility. Most often, if the cup winner qualifies to the Champions League, the vacant Europa League place goes to the cup runner-up. In Example 1 Lokomotiv had to lose in order to push Spartak to the Champions League at the expense of Dynamo, forcing the redistribution of the vacant Europa League place to the 6$^{\text{th}}$ place in the round-robin championship.

We can look more closely at one particular case. Consider two domestic tournaments: a round-robin championship $(\mathcal{X}, v(x,y))$ and a cup $(\mathcal{X}, T)$, as well as two international tournaments: the Champions League (the most prestigious tournament) and the Europa League (the second prestigious tournament). Let the best a_1 of the championship teams get places in the Champions League and the next b_1 teams get places to the Europa League along with cup winner, $a_1, b_1 \geqslant 1$. Redistribution rules must describe what would happen if the cup winner qualifies for the Champions League or the Europa League through the championship. Thus, there exist 4 possible redistribution rules. Denote them R_1, R_2, R_3, R_4 and define how they redistribute the vacant place in the following table:

	R_1	R_2	R_3	R_4
$CL + EL$ intersection	championship	championship	cup	cup
$EL + EL$ intersection	championship	cup	championship	cup

The following formal result holds.

Theorem 2 *Suppose that* $a_1, b_1 \geqslant 1$ *and* $K > \max(a_1 + b_1, 3)$*. Then*
(I) for any well-defined monotonic ranking method $S(\cdot)$ *and for each redistribution rule from the rules* R_2, R_3 *and* R_4*, there exist characteristic functions* v, w *such that*

(i) there exists a collection (x_0, y_0) *such that* $v(x_0, y_0) = 1$ *and* $w(x_0, y_0) = -1$*;*

(ii) for any collection $(x, y) \neq (x_0, y_0)$*, holds the equality* $w(x, y) = v(x, y)$*;*

(iii) according to standings $T, S(v)$ *team* x *gets a place to international tournament;*

(iv) according to standings $T, S(w)$ *team* x *does not get a place to international tournament.*

(II) If redistribution rule R_1 *is used, characteristic functions* v, w *that satisfy (i)-(iv) do not exist.*

The proof of this theorem for redistribution rules R_2, R_3 and R_4 is similar to the proof of theorem 1. In the case of redistribution rule R_1 a deliberate loss is useless because the team will be ranked in the round-robin tournament worse than in the case of a win, while an additional place will be awarded to the best of the teams which finish outside the prize zone in the round-robin tournament. As we already mentioned, most of UEFA national federations exploit redistribution rule R_3.

5 Conclusion

Optimal design of the rules of aggregation for tournaments is an important theoretical problem. Neglecting the analysis of incentive compatibility, the organizers of a tournament may suddenly face a situation, where one of (or even several) the teams would prefer to lose a game. The fact that this is a low-probability event, the potential costs of the rational misbehavior of the teams are too high. In this paper, we demonstrated that the existing regulations that determine who qualifies for the major football tournaments allow for a situation in which a team would need to lose in order to qualify. We showed that the existence of incentives compatible ranking methods and redistribution rules depends on the structure of qualifiers. In a single round-robin tournament any monotonic ranking method prevents the deliberate losses. If there are at least 2 round-robin qualifiers, then it is impossible to implement an appropriate ranking method (Theorem 1). Finally, if we have 1 round-robin and several knock-out qualifiers, one can solve the problem by redistributing the vacant places according to the teams performance in the round-robin tournament.

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