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ABSTRACT

Fiscal Discoveries and Sudden Decouplings*

The recent Eurozone debt crisis has witnessed sharp decouplings in cross-country bond yields without commensurate shifts in relative fundamentals. We rationalize this phenomenon in a model wherein countries with different fundamentals are on different equilibrium paths all along, but which become discernable only during bad times. Key ingredients are cross-country differences in the volatility and persistence of fiscal revenue shocks combined with asymmetric information on country-specific fiscal shocks. Differences in the cyclicality of fiscal revenues affect the option value of borrowing and resulting default risk; unobservability of fiscal shocks makes bond pricing responsive to market actions. When tax revenues are hit by common positive shocks, no country increases net debt and interest spreads stay put. When a common negative revenue shock hits and is persistent, low volatility countries adjust spending while others resort to borrowing. This difference signals a relative deterioration of fiscal outlooks, interest spreads jump and decoupling takes place.

JEL Classification: E62, F34, G15 and H3
Keywords: default, eurozone debt crisis, fiscal gaps, information asymmetry, perfect bayesian equilibrium, pesistence, sovereign debt and volatility

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1 Introduction

From the times of Alexander Hamilton to Mario Draghi, debt crises have repeatedly dragged policy makers into taking a stand on a polemic question—namely, what triggers sudden decouplings of bond yields across national or sub-national borders following protracted spells of yield convergence? Such a reversal of fortunes has been at play in the Eurozone. As illustrated in Figure 1, from the eve of the monetary union in end-1998 to the onset of the global financial crisis ten years on, yield convergence was remarkable; this was so in terms of both magnitude and length, as well as in its defiance of widely known differences in institutions, fiscal performances, and productivity differentials across eurozone states. No less dramatic has been its post-2008 reversal: yield decoupling reached unprecedented heights, with the cross-country dispersion of bond yields surpassing even those during the severe market turbulences of the 1990s. These developments are all the more disconcerting since at the epicenter of the crisis lie countries not so long ago heralded as growth success stories of the advanced world, such as Ireland and Spain, as well as countries like Greece and Portugal which have been long declared as graduated from "debt intolerance" (Reinhart, Rogoff and Savastano 2003).

The aim of this paper is twofold. First, we provide a tractable model that endogenously generates yield coupling and sudden decoupling without a comensurate change in relative country fundamentals. In doing so we explain the pattern of interest spreads in Figure 1. Second, we examine how the distinct equilibria which underpin yield coupling and decoupling respond to fundamental parameters. Unlike earlier work on sovereign debt, we focus on two parameters – namely, the volatility and persistence of fiscal revenue shocks. We document substantial cross-country differences in those parameters and quantitatively examine how these differences affect equilibria and interest spreads during good and bad times.

At the core of our model is the interaction between two main ingredients highlighted in many accounts of the recent crisis. One is that countries have structurally distinct underlying volatility and persistence of fiscal revenues. The other ingredient is asymmetric information about country specific fiscal shocks. We combine these ingredients in a stripped-down three date, two-period model as follows. A country issues long-term (two-period) debt to finance investment that, upon maturity, is expected to yield sizeable revenue gains. Fiscal revenues follow a stochastic path with realizations at the intermediate and final periods, which may cause tax revenue collection to fall short of planned spending. In response to the middle period shock, the country can either adjust spending and refrain from borrowing, or it can tap capital markets to finance the emerging fiscal gap. In the

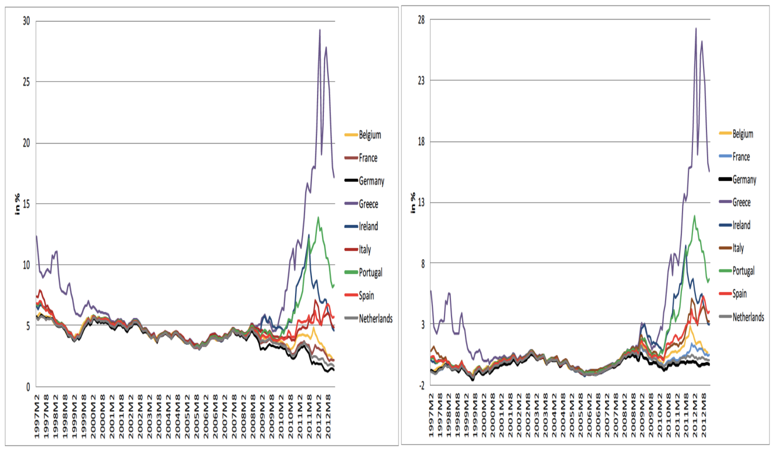

Figure 1: Left Panel: Long-Term Bond Yields of Selected Euroarea Sovereigns. Right Panel: Long-Term Bond Spreads Relative the U.S.

final period the country decides whether to repay or default. In the case of default, the country faces loss of fiscal revenues, as standard in the literature. In addition, the sovereign is also forced to repay part of the debt obligations.

All parameters are public knowledge except the middle shock realization which is directly observable by the sovereign but not by international lenders. Also importantly, the middle shock is known to be persistent. The central implication of investors being unable to observe shocks is that they make inferences about the country's fiscal outlook based on market actions by the sovereign. For tractability reasons and to focus on essentials, we restrict these market actions to two: either the country borrows in the middle period, or it does not, i.e., stays put. We model the interaction between the country and lenders as a game and numerically solve for Perfect Bayesian Equilibrium (PBE). Two types of equilibria exist: a separating equilibrium, in which the country's action depend on the shock realization, and a pooling equilibrium, in which the country's action is the same, regardless of the shock realization.

How can this framework generate the pattern of country spreads illustrated in Figure 1? Consider two countries, one at each end of the yield dispersion spectrum. Country-A, characterized by a set of weaker fundamentals (notably highly volatile tax revenues), sustains a separating equilibrium in which it stays put after a good fiscal shock and issues new debt after a bad fiscal shock. Country-B, characterized by a set of stronger fundamentals (notably lower tax revenue volatility), sustains a pooling equilibrium in which it never taps the market regardless of the fiscal realizations. Suppose countries are hit by a positive fiscal shock. In this case, investors observe both countries refraining from borrowing and this generates only a small spread between country yields. The gap in countnry spreads that should prevail due to the gap in country fundamentals is dampened by the presence of informational noise. This is how the model conceptualizes the 2000-2007 period of yield compression – when shocks to aggregate income and country-specific tax bases were either positive or only mildly negative across the eurozone (as we document in Section 2). The situation is quite different following a large negative shock. In this case, the informational noise will amplify the effects on spreads of whatever differences in fundamentals were there prior to the shock: investors learn that country-A is on a negative fiscal path relative to country-B. Hence the interest spread between the two countries widens, as observed from 2008. In this setting, fiscal discoveries can thus explain both yield convergence and sudden yield decoupling. It is important to be clear that we are not rationalizing convergence and sudden decoupling by a general cross-country shift from a pooling to a separating equilibrium. Instead, convergence and divergence obtain from the implications of pooling and separating equilibria simultaneously played by different countries.

Whether a country finds optimal to play a pooling or a separating strategy is non trivial: there are trade-offs in each strategy. What pins-down the dominant strategy is the set of common-knowledge fundamentals which are country specific. These fundamentals include the country's discount factor, initial debt levels, the income loss and hair-cut parameters that pin down the relative cost of default, and – mostly central to our story – the underlying shock volatility and persistence (but not actual shock realizations).

Attendant model calibrations yield the following results. First, countries with weaker fundamentals generally find it optimal to play a separating strategy, whereas those with stronger fundamentals opt for pooling. In particular, separating equilibrium is more prevalent in countries for which the underlying variance and persistence of the tax revenue shock are higher. Second, our model provides a calibratable tool to address related questions. One question is what happens to countries' strategies and spreads once short-term fiscal uncertainty (as measured by dispersion of shocks to tax revenues in the model's middle-period) is higher vs. lower, both in absolute terms and relative to long-term fiscal uncertainty (as measured by dispersion of shocks to tax revenues in the model's last period). Our simulations indicate that higher short-run volatility relative to long-run volatility increases the dominance of separating equilibrium and raises spreads. We also show that higher persistence of revenue shocks has an effect on interest spreads. If the equilibrium is separating and the (AR1) persistence of its tax revenue shocks is raised by 0.2 (a typical difference in fiscal revenue persistence across EU countries, as we document below), spreads can rise by more than 400 basis points. Interestingly, we find broad regions of relevant parameter values on which equilibrium abruptly changes from pooling to separating. Hence, the resulting equilibria turns out to be quite sensitive to small parameters changes. Finally, our simulations also show that default is more likely once a country plays a separating equilibrium game, despite being also plausible in a pooling equilibrium. Yet, our model could also deliver equilibrium regions where the country on a separating strategy will never default after a bad shock.

In modeling country risk, this paper relates to a large literature on sovereign borrowing and default risk, starting with Eaton and Gersovitz (1981) seminal contribution. As in the subsequent versions of the cannonical model by Aguiar and Gopinath (2006), Arellano (2008), our model builds on the volatility and persistence of output shocks (tax revenue shocks in our setting) as drivers of country risk. Asymmetric information in our model (absent in these previous papers) transforms market actions into signals with tangible implications for bond pricing. This can explain two documented facts on the recent Eurozone debt crisis: i) spreads can move sharply with market actions and before the pertinent output or fiscal revenue information is publicly available; ii) yields of some governments (those playing a pooling strategy) may not rise at all (relative to the risk-free rate), even when hit by a bad and persistent shock that raises debt-to-income ratios (cf. Figure 1). Indeed, as discussed further below, the stability of bond spreads among core Eurozone countries (and relative to the US treasury bill) following the common negative shock of 2008-09 has been a main development of the recent crisis. Another novelty of our analysis relative to that earlier work is the focus on the stochastic tax revenue (rather than aggregate income) as the central determinant of default risk. This difference may not be so subtle insofar as the elasticity of fiscal revenues to aggregate income varies widely across countries (and may be subject to structural breaks within countries), a fact that we document below and explore in our model calibrations. Finally, information asymmetry in our model also generates – relative to the full symmetric information game – a higher initial level of interest rates, even when preferences are linear.

In featuring asymmetric information and signaling through market tapping, our setting relates to Eaton (1996), Alfaro and Kanuzck (2005), Sandleris (2008), Catão, Fostel, and Kapur (2009), and D'Erasmo (2011) who also study how investors' uncertainty about either the country's type or shocks determine fluctuations in sovereign spreads. Yet, there are important departures. For one, in our model countries are not "types". Investors are not uncertain about whether the country is a "bad" or a "good" type, as they are defined by a perfectly observed set of fundamentals. That is, investors know that country A's fundamentals are weaker than country B's, but what they do not know is how bad (or good) country's A fiscal shock is. In short, it is the interaction between differences in known fundamentals (which pin-down the dominant country strategy) and unknown shocks that allow spreads to converge or diverge widely.

In highlighting the role of market tapping as a signal of fiscal prospects, our paper is also related to an early literature on the timing of fiscal consolidations as Alesina and Drazen (1991) and Drudi and Prati (2000). In this literature, governments may find optimal to deviate from optimal tax smoothing in order to signal their "types", defined as capacity to embark on a lower vs. higher default risk fiscal path. As in our model, Drudi and Pratti (2000) find that there can be a pooling or a separating equilibrium in this game. Yet, their separating equilibrium is characterized by a "weaker" government always defaulting and a "stronger" government signaling that it can abstain from borrowing during bad times and never defaulting. Unlike in our model, investors' uncertainty pertains to what "type" of government is in charge – there is no uncertainty regarding output or fiscal revenue realizations; indeed, when equilibrium is separating, default by the weak government occurs in the model's middle period as a myopic administration takes office, so all uncertainty is then resolved and no further borrowing ensues. Thus, their model does not rationalize yield decoupling amidst continuous borrowing; nor does it obtain a separating equilibrium in which default may not occur following a bad revenue outcome.

The remainder of the paper is structured as follows. Section 2 documents key facts on the 2008-2012 Eurozone debt crisis which motivate our modeling strategy. Section 3 lays out the model while Section 4 presents the simulation results. Section 5 concludes. Further specifics of model derivations are provided in the appendix.

2 Empirics of the Eurozone Crisis

In this section we describe the key empirical regularities that motivate our modeling strategy. In particular, we provide empirical justification for the model's two main building blocks—namely, the role of fiscal shocks and information asymmetries—in shifting country spreads. While the bond pricing dynamics that our model seeks to explain is featured in many past debt crises, other specifics of past crises may vary widely (for an overview and further references, see Rogoff and Reinhart, 2009). In what follows we keep the focus on Eurozone developments as the set of stylized facts that we seek to explain.

2.1 Heterogenous Fiscal Dynamics

The triggering event of the ongoing Eurozone crisis has well-known origins—a string of defaults on tranched U.S. mortgage bonds in the summer of 2007 that culminated with

Figure 2: Real GDP Growth (top) and Growth of General Government Revenues (bottom).

the closure of Lehmann Brothers.[1] From the Eurozone's viewpoint and for the purposes of our modeling exercise, this can basically be viewed as a common external shock that caused output and hence the tax base to contract, thus depressing real tax revenues. A comparison between top and bottom panels in Figure 2 shows, however, that while the output downturn was synchronized and of roughly similar magnitude across these countries with the exception of Ireland, there is greater dispersion in tax revenue performances. In particular, the four countries at the epicenter of the debt crisis in 2010 and 2011 were precisely the ones experiencing the sharpest falls in real government revenue collection. This is indicative of differences in the pro-cyclicality of national tax systems across the Eurozone. Top and bottom panels in Figure 3 plot the real GDP and real government revenues and show that tax revenues have been nearly four times as elastic to output in Spain, for instance, than in Germany since 1980. The same applies to other peripheral Eurozone countries.[2] Overall, Table 1 shows that crisis countries in the Eurozone have historically displayed significantly higher conditional volatility as well as persistence of government revenue shocks relative to non-crisis countries (Germany notably). That is, the exacerbated revenue volatility of recent years is not only a by-product of the crisis: some – if not much of it – seems to reflect structural factors. This has important implications

[1] For a broad overview of background developments and the role of gross international funding exposures in exacerbating the contraction of global output, see Gourinchas (2011).

[2] Estimates of the revenue-output elasticity for other countries are available from the authors upon request.

Figure 3: Real GDP and Real Government Revenues (HP-detrended): Spain (top), Germany (bottom).

for modeling and policy, as discussed in Sections 3 and 4. Similar considerations apply to shock persistence – it has been high in crisis-striken countries, even if one limits the estimation to the pre-2008 sample. This can be simply gleaned from a regression of (HP-filtered) de-trended real revenues on its first order lags, as reported in Table 1. It is also interesting to notice that volatility and persistence of fiscal revenue shocks is mostly higher in countries that are more closed to foreign trade. To the extent that the latter is a proxy for the cost of default, as discussed in Rose (2005) and Borensztein and Pannizza (2008), this also has important implications for explaining spread decoupling in Eurozone and for the way we calibrate our model, as discussed in Section 4.

Rather striking cross-country differences within the Eurozone lie in the response of public spending to the global financial crisis and the attendant output contraction of 2008-09. The top panel in Figure 4 shows that primary government spending ran well ahead of revenues in 2008 and 2009 in all four countries that subsequently went through the severe debt crises in 2010 and 2011.[3] In this regard, a comparison between Germany and Greece is particularly striking: while on the eve of the crisis in 2007, government primary spending was entirely financed with revenues in both countries (with an identical ratio of

[3]This is true even if we add debt servicing costs (i.e., interest spending) and use the general government expenditure metric. The respective chart is not plotted to save space but is available from the authors upon request.

Country	Tax Revenues Parameters Unconditional Volatility (sd)		Persistence (AR1)		Openess Exports/GDP
	1980-2007	1990-2007	1980-2007	1990-2007	2007
Austria	3.9	2.6	0.4	0.6	58%
Belgium	4.1	2.9	0.7	0.7	81%
France	2.5	1.6	0.6	0.6	27%
Germany	3.5	3.7	0.5	0.5	47%
Greece	6.1	6.6	0.6	0.7	21%
Italy	2.6	2.6	0.4	0.5	20%
Ireland	4.9	5.6	0.8	1.0	80%
Netherlands	2.8	3.5	0.5	0.6	71%
Portugal	7.1	3.4	0.7	0.3	32%
Spain	7.6	8.2	0.9	0.8	27%
Crisis Countries	*6.6*	*6.1*	*0.8*	*0.8*	*30%*
Non-Crisis Countries	*3.5*	*2.9*	*0.5*	*0.6*	*58%*

Table 1: Volatility and Persistence of (HP)-Detrended Real Tax Revenues.

94%), that gap rose by close to 15% by 2009. That is, Germany cut spending in tandem with its smaller fiscal revenue decline, whereas Greece did not. In the event, as country risk increased and the share of shorter-term in total debt rose, the share of interest spending in total government spending rose (bottom panel in Figure 4).

2.2 Public Debt Dynamics

The flip side of the fiscal revenue dynamics just discussed is that governments resorted to extensive market tapping. Hence government debt to GDP ratios skyrocketed, as shown in the top panel of Figure 5. In particular, countries that at the onset of the monetary union in 1999 had manageably low debt to GDP ratios (at around 50%) like Ireland and Portugal, saw their debt to GDP ratio approach 90% by 2010. Extensive government borrowing during 2008–10 implied that market access was not lost. Not only did total (public and private) national bond issuance in international capital markets remained high, but also the government share in total issuance rose (bottom panel in Figure 5). This stands in sharp contrast with Germany's government new issuance, which fell relative to total.[4] Overall, as these countries' relative fiscal positions sharply deteriorated, new issuance rose while their yields started to decouple from other Eurozone peers, most notably Germany. Figure

[4]Continuing issuance through 2010 and some of the 2011 (as of partial data at this point) suggests that the bond repurchase program by the ECB cannot account for all the financing that these countries received which allowed public debt to keep rising and preventing sharper current account reversals of the type observed in the emerging market crises of the 1990s.

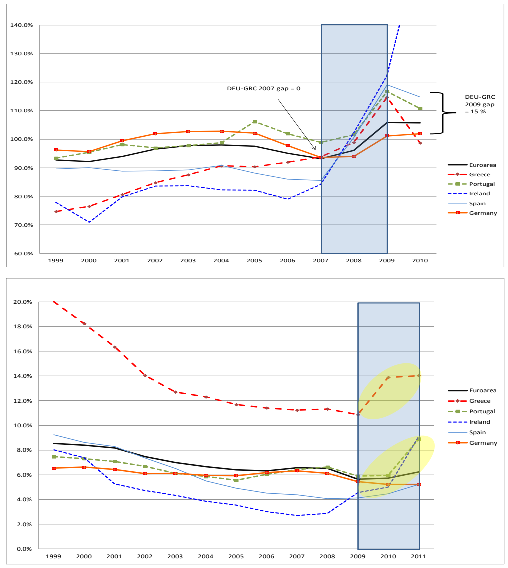

Figure 4: Ratio of General Government Primary Expenditures to Revenues (Top), Share of Interest Expenditure in Total Government Revenues (Bottom).

6 shows that current accounts remained highly negative in two of the four crisis countries, Greece and Portugal, while their respective country spreads relative to Germany rose.

2.3 Information Frictions

While fiscal deterioration played an important role, a common view is that the magnitude of fiscal shocks is not per se sufficient to explain the extend of yield decoupling. One hypothesis is that the interaction betweeen a negative fiscal shock and pervasive uncertainty about medium and long-term outlooks is key. This is consistent with the 5-year ahead IMF forecasts for the debt path in Eurozone crisis countries relative to others like the US and Germany.[5] While Figure 7 shows that all advanced countries—inside and outside the Eurozone—have seen their fiscal outlooks deteriorating, not only is the deterioration more dramatic in the Eurozone crisis countries but also the variance of such forecasts is much higher.[6]

Such cross-country differences in the uncertainty of fiscal outlooks can be ascribed to

[5]These forecasts are all based on those published in the IMF semi-annual "World Economic Outlook" report which is publicly available from the IMF website. They are deemed to contain the best information *publicly* available at the time of the report.

[6]Comparisons with other advanced countries tell a similar story and the respective data were omitted to save space but are available from the authors upon request.

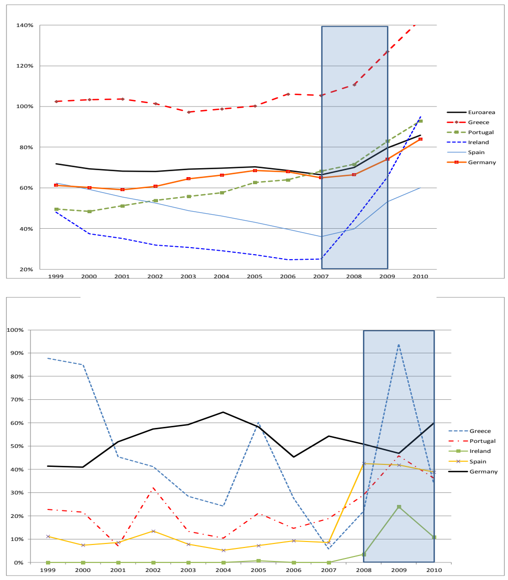

Figure 5: General Government Debt to GDP (Top), Government Share in Total Bond Issuance (Bottom).

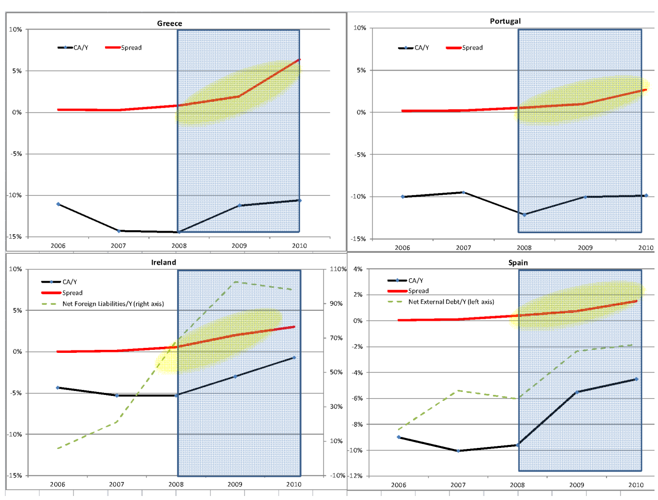

Figure 6: Net External Borrowing and Sovereign Spreads.

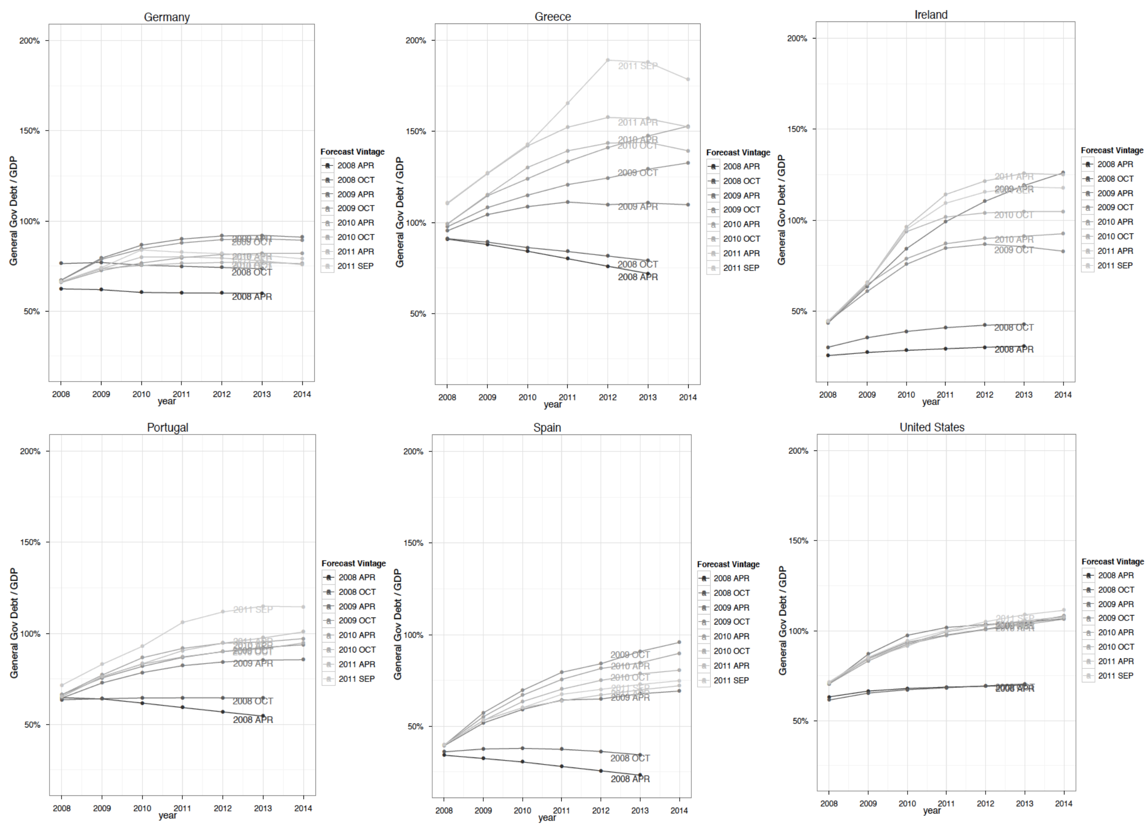

Figure 7: IMF Forecasts of Public Debt in the US and Euro Area Across Forecast Vintages.

three factors which have been highlighted by the press and which will also play a key role in the model presented below. The first is the uncertainty regarding the economic recovery and hence the recovery of the tax revenue base. The second is the uncertainty about the government's political capacity or resolve to undertake revenue enhancing measures and/or expenditure cuts which are necessary to stabilize debt along a sustainable (low default risk) path. In the model we combine these two factors together in the form of a higher variance of future fiscal positions.

The third factor is associated with uncertainty about the "true" fiscal position, both current and (sometimes) well past. The latter can also be gauged from Figure 7 by comparing the 2008 debt/GDP figures across the different forecast vintages. Specifically, for the 2009–2011 forecast vintages, the 2008 debt to GDP ratio is not a forecast but an outturn. For Germany and the U.S., all post-2008 forecast vintages start from around the same "starting point", indicating that revisions in the initial (2008) debt/GDP statistics have been minimal. This is clearly not the case for Greece or Portugal. Similarly, there has been greater uncertainty regarding the 2009 ratio for all Eurozone countries depicted (including Germany) than for the U.S., with such uncertainty being higher, once again, for crisis countries.

One may contend that such uncertainty may be equally shared by investors, governments, and general public, i.e., that information frictions are symmetric. Along these lines, it could be argued that statistical agencies in the crisis-striken countries are weak in terms of collecting and processing first round data, so substantial historical revisions can take place. Prima-facie, however, it is no less plausible that information about the true state of public finances is asymmetrically distributed, as current administrations have privileged access to tax records that most national laws shield away from public scrutiny. Indeed, when fiscal news are bad, the incumbent adminstration has clear incentives to either hide them or delay their release so as to smooth market reaction to the "true" shock. Recognition of this incentive to obfuscate or to only gradually reveal true information on fundamentals is a key centerpiece of a sizeable political economy literature on the pro-cyclicality of fiscal policy (see, e.g., Alesina et al, 2008). Sizeable ex-post revisions of actual outurns of the non-trivial magnitudes shown in Figure 7 are suggestive of this possibility. There are other indications that uncertainty on fiscal positions is asymmetrically distributed. One comes from candid official narratives. For instance, an official report by the European commission on the state of Greek government debt and deficit statistics dated January 2010, thus at a crucial turning point of the crisis, states:

> "On 2 and 21 October 2009, the Greek authorities transmitted two different sets of complete Excessive Deficit Procedure (EDP) notification tables to Eurostat, covering the government deficit and debt data for 2005–2008, and a forecast

for 2009. In the 21 October notification, the Greek government deficit for 2008 was revised from 5.0% of GDP (the ratio reported by Greece, and published and validated by Eurostat in April 2009) to 7.7% of GDP. At the same time, the Greek authorities also revised the planned deficit ratio for 2009 from 3.7% of GDP (the figure reported in spring) to 12.5% of GDP, reflecting a number of factors (the impact of the economic crisis, budgetary slippages in an electoral year and accounting decisions). According to the appropriate regulations and practices, this report deals with estimates of past data only."

Clues in a similar vein appear more tame elsewhere but are not preserve of Greece. Indeed, as shown in Figure 7, there have also been substantial revisions in fiscal out-turns elsewhere among crisis-striken countries. In this sense, the assumption of information asymmetry between governments, investors and general public seems again more widely applicable.[7] Actual market responses are also suggestive. If information on the state of public finances and related fundamentals were fully public and credible to market participants, one might not expect extra market tapping per se to have a substantial impact on bond yields – at least for small open economies that account for a minuscule share of global asset tradings, like those at the crisis epicenter. Yet, this has not been the case: as widely documented in the media and well-known to any engaged observer, country spreads have often reacted strongly – and sometimes in a matter of hours – to the increased frequency of market tapping, even in the absence of new official data or public announcements on fiscal difficulties meanwhile.[8] All in all, we take these various pieces of evidence as indicative of asymmetric information on fiscal shocks and that strategic market tapping can and have played a substantive role in sovereign bond pricing.

3 Model

3.1 Fiscal Revenue Shocks and Sovereign Debt.

We develop our argument in the simplest setting, which involves three periods, $t = 0, 1,$ and 2. A government issues bonds in international capital markets to finance investment in

[7] Related claims have also sometimes surfaced during public administration transitions within the Eurozone. Not uncommonly, the magnitude of the unfunded deficit is suddenly and massively revised upon such transitions and previous imcumbents are accused of successfully "hidding" the fiscal hole. For instance, on 31 January 2011, the Financial Times reported that: "Catalonia, one of the richest parts of Spain, needs to raise €10bn-€11bn in debt this year to cover deficits and repay earlier loans… Andreu Mas-Colell, finance minister in the newly elected Catalan nationalist government, conceded in an interview with the Financial Times that it was "not a negligible amount", as he added up the numbers and explained how he had inherited unfunded deficits from the previous, Socialist-led regional government. 'We're not yet guilty of anything,' he said, in an echo of the outraged complaints of Greek ministers in 2009 when they inherited a deficit from their predecessors in power that was much worse than previously announced."

[8] See, e.g., http://www.gardian.co.uk/business/2010/may/05/greece-debt-crisis-timeline.

a long-term project which can be related to physical infrastructure and/or human capital development. The project's investment requirement in period 0 (which we consider exogenous) generates expected fiscal revenues τ_0, τ_1 and τ_2 in periods $0, 1$ and 2 respectively.

In period 1 government's fiscal revenue is given by $F_1 = \tau_1 + \tilde{\epsilon_1}$, where $\tilde{\epsilon_1}$ is a shock which assumes two values: $\epsilon_1^H = \alpha\tau_1$ and $\epsilon_1^L = -\alpha\tau_1$, with probability p and $1-p$ respectively, and $\alpha < 1$. A key assumption throughout is that the shock is persistent, so that $\rho\epsilon_1$ still affects fiscal revenues in period 2, where $\rho < 1$ is the persistence parameter.

In period 2 the government's fiscal revenue is given by $F_2 = \tau_2 + \rho\epsilon_1 + \tilde{\epsilon_2}$, where the new shock $\tilde{\epsilon_2}$ can assume two values, ϵ_2^H or ϵ_2^L with probability q and $1-q$ respectively.

The government has access to debt markets in periods 0 and 1. In order to finance the initial investment requirement at time 0, the sovereign issues long-term debt to be paid in period 2. More precisely, it issues $D_0 = \tau_0$ at time $t = 0$, it pays interest $r_0\tau_0$ in $t = 1$ and it promises to pay $(1 + r_0)\tau_0$ at maturity in $t = 2$.

At period 1, upon receiving the fiscal shock $\tilde{\epsilon_1}$ in the middle period, the borrower has two options:

1. "No-Action" (N).

   In this case, the borrower just pays interest due at time 1. Notice that total outstanding debt at the end of the middle period is τ_0.

2. "Fresh Issuance" (I).

   In this case the borrower issues new fresh one-period debt D_1. It issues $D_1 = \alpha\tau_1$, and promise to pay $(1 + r_1)\alpha\tau_1$ at $t = 2$. Notice that in this case total outstanding debt at the end of the middle period is larger than the stock of debt at time 0. The stock of debt at time 1 is $\alpha\tau_1 + \tau_0$ compared to the stock of debt at time 0, τ_0.

In the final period, upon the realization of the shock $\tilde{\epsilon_2}$, the government decides whether to pay or default in all outstanding debt. We assume that all debt has the same seniority, so once a country defaults, it defaults in all its debt. We also assume that there is no default on interest payments in the middle period.[9]

[9] The first assumption is for simplification sake. Adding seniority would complicate the model without adding significant insight on the issue at hand. As discussed in Chatterjee and Eyigungor (2012), the expected effect would be to increase the cost of issuance in the middle period and discourage further debt. The second assumption is easily justifiable. For example, suppose $r = 5\%$ and $\tau_0 = 100\%$, this means that repayment of interest would amount only to 5% of revenues. Clearly, this payment would be easily met given that a very bad shock still leaves you with 40 or 50% of revenues, i.e. an amount 10 times higher than what you need to pay.

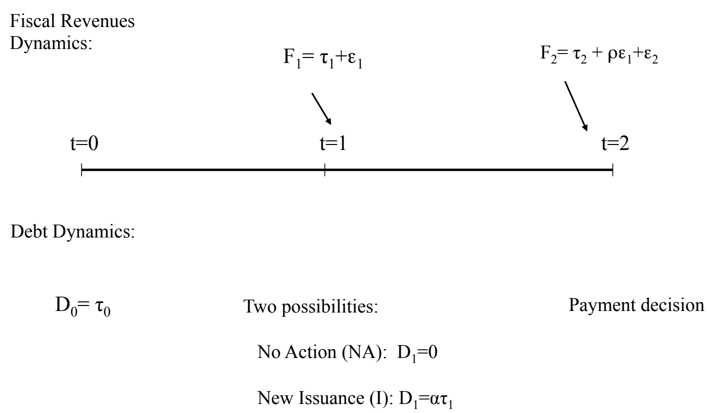

Figure 8: Fiscal Revenue and Debt Dynamics.

Figure 8 shows a timeline of fiscal revenue shocks and credit market access summarizing the previous discussion.

3.2 Lenders and Cost of Default.

The bond market is competitive, with risk-neutral lenders who are willing to subscribe to bonds at any price that, given their beliefs, allows them to break-even. For modeling simplicity we treat the mass of lenders at every period as a single lender.

Lenders have access to a risk-free technology in every period, which pays a risk-less interest rate r_f, which is taken as exogenous and constant across time. There are two debt markets, a long-term debt market at $t = 0$ and a short-term debt market at $t = 1$. We treat creditors at $t = 0$ as different from creditors at $t = 1$.

There is a punishment technology in the model that consists of recovery rates and fiscal confiscation. More precisely, in the case of default, creditors receive $c(1 + dr)D$, where D is the debt issued (τ_0 or $\alpha\tau_1$), and $1 - c$ represents haicuts.[10] This parametrization allows

[10] Recent work pins-down the hair-cut from an endogenous bargaining between sovereign and creditors over the surplus arising from default (Benjamin and Wright, 2008; Yue, 2010; D'Erasmo, 2011). One chief motivation is to generate endogenously debt to output ratios in a DSGE setting that resemble those observed in real data. In a finite horizon such as ours where default can take place in the last period, it is natural short-cut to take c as a exogenously given parameter as we do.

us to consider two extreme situations. In the case in which $d = 1$, the haircut is calculated over both, interest and principal. However, in the case in which $d = 0$, haircut is calculated over principal alone. Moreover, as in any finite-horizon framework, in the absence of other penalties in the final period the borrower would default with probability one. To avoid the trivialities associated with this case, we assume that default in the final period is punished with sanctions that cause the sovereign to lose a fraction η of its current fiscal revenues per unit of face value. A proportion f of this fix cost goes to creditors at time 0, whereas $1 - f$ goes to creditors at time 1. For example, if the sovereign decides to take no action in the middle period, $f = 1$ and $1 - f = 0$, i.e, the total proportion η of fiscal revenues goes to creditors at time 0. On the other hand, if the sovereign decides to issue new debt in the middle period, $f = \frac{1}{1+\alpha}$, which means that a proportion $f\eta = \frac{1}{1+\alpha}\eta$ of fiscal revenues goes to creditors at time 0, and $(1 - f)\eta = \frac{\alpha}{1+\alpha}\eta$ goes to creditors at time 1.[11]Hence, in this last scenario there is debt dilution in equilibrium.

Now we are ready to characterize lender's cash flows. Figure 9 describes the cash flow for a lender at $t = 0$. In period $t = 1$ the lender receives interest payments $r_0\tau$. With probability π_N the lender will face a "No-Action" situation. In this case with probability $\pi^{'}$ the creditor receives a total revenue of $(1 + r_f)r_0\tau_0 + (1 + r_0)\tau_0$, which consists of the revenues from investing the interest received in the middle period in the risk free technology and the interest plus principal. With probability $1 - \pi^{'}$ the borrower will default, and the creditor receives $(1 + r_f)r_0\tau_0 + c(1 + dr_0)\tau_0 + \eta F_2$. On the other hand, with probability π_I the lender will face a "New Issuance" situation in which case with probability $\pi^{''}$ the borrower will repay in the second period and with probability $1 - \pi^{''}$ it will default and the creditor will receive $(1 + r_f)r_0\tau_0 + (1 + r_0)\tau_0$ and $(1 + r_f)r_0\tau_0 + c(1 + dr_0)\tau_0 + f\eta F_2$ respectively.

Figure 10 shows the cash flows associated to lending at $t = 1$. With probability π' the creditor is paid back interest plus principal, $(1 + r_1)\alpha\tau_1$. With probability $1 - \pi'$ the creditor faces sovereign default, in which case she receives $c(1 + dr_1)\alpha\tau_1 + (1 - f)\eta F_2$.

[11]We are thus assuming that the fiscal cost in the last period is captured by the creditors. As made clearer in the technical appendix, this assumption simplifies the calculations. In standard models (like Cohen and Sachs 1985) a proportion η goes straight into the waste bin (deadweight losses). However, one can argue in favor of our modeling choice in several ways. For example, about 10% of Greek debt was issued in London so upon default, London courts could in principle get 10% of τ_0 back. If $\tau_0 = 2\tau_2$ under a bad shock, then we get an $\eta = 0.2$ (= 10% of $\tau_0 2$), which as we will see later is our approximated calibration of η. While the recovery record of vulture funds is far from exemplary, it is a zero return activity either; the November 2012 NYC court ruling on Argentine defaulted debt suggests that such a recovery assumption may not be too off-mark going forward. At any rate, if creditors are able to organize themselves better and extract fiscal surpluses later through all kinds of ways, this is realistic and all the more so under a common juridiction like the EU. In the model the present value of those would then be captured by $\eta\tau_2$. Further, the assumption of trivial deadweight losses may arguably be not so much of a stretch in the broader Eurozone context, as countries with stronger fundamentals indirectly benefit from the debt crisis via lower borrowing costs.

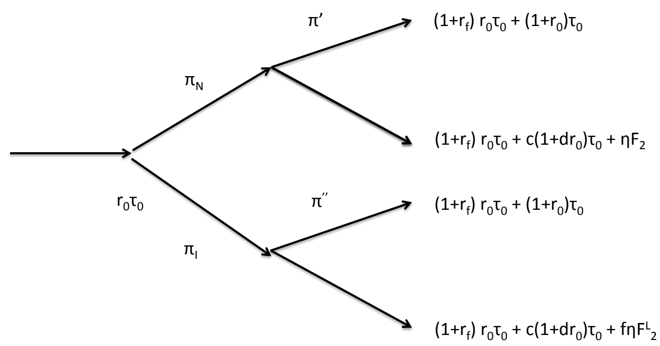

Figure 9: Lending at $t = 0$.

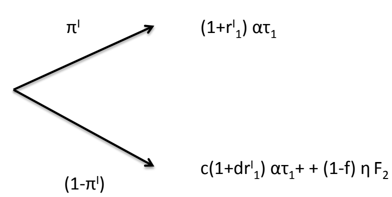

Figure 10: Lending at $t = 1$ to borrowers after Fresh Issuance.

3.3 Sovereign Payoffs.

The government is risk neutral, have a discount factor of β and maximizes expenditure $G = \sum_t \beta^t G_t$. The payoffs in each period are described below.

In period $t = 0$:

$$G_0 = \tau_0 \tag{1}$$

In period $t = 1$ there are two possibilities. If the borrower exerts No-Action, we have that

$$G_1 = F_1 - r_0\tau_0, \tag{2}$$

expenditures equals fiscal revenues at time 1, $F_1 = \tau_1 + \tilde{\epsilon_1}$, minus interest payment.

In the case the borrower issues fresh debt (I), then

$$G_1 = F_1 - r_0\tau_0 + \alpha\tau_1, \tag{3}$$

expenditures equals fiscal revenues at time 1 minus interest payments plus fresh debt issuance.

In the last period there are four possibilities. After No-Action the sovereign can repay or default. If it repays, we have that

$$G_2 = F_2 - (1 + r_0)\tau_0, \tag{4}$$

expenditure equals fiscal revenues at time 2, $F_2 = \tau_2 + \rho\epsilon_1 + \tilde{\epsilon_2}$, minus debt re-payments.

If it defaults

$$G_2 = F_2 - c(1 + dr_0)\tau_0 - \eta F_2, \tag{5}$$

expenditure equals fiscal revenues at time 2 minus punishment due to default, given by remaining debt obligations after haircuts and fiscal confiscations.

On the other hand, after new debt issuance (I), if the sovereign repays

$$G_2 = F_2 - (1 + r_0)\tau_0 - (1 + r_1)\alpha\tau_1, \tag{6}$$

expenditure equals fiscal revenues at time 2 minus debt re-payments of debt issued at $t = 0$ and $t = 1$.

If it defaults

$$G_2 = F_2 - c(1 + dr_0)\tau_0 - c(1 + dr_1)\alpha\tau_1 - \eta F_2, \tag{7}$$

expenditure equals fiscal revenues minus punishments due to default on all debt.[12]

3.4 Fiscal Discoveries and Sovereign Defaults

Motivated by the discussion on information asymmetries in Section 2, we assume that there is asymmetric information between the sovereign borrower and international lenders. While the borrower can perfectly observe the realization of the middle period fiscal shock $\tilde{\epsilon_1}$, lenders cannot.

The only way lenders can infer some information about the realization of the shock is through the borrower's action in the middle period: No-Action (N) or Fresh Issuance (I). Lenders at $t = 1$, after observing the borrower action will update (when possible) their beliefs of future default and re-price debt accordingly.

We model the borrower and lender interaction as a game. The borrower's strategy consists of an initial debt issuace D_0, an action after observing the shock realization in period 1, No-Action (N) or Fresh Issuance (I), and a repayment decision on all outstanding debt in period 2. The lender's strategy is to set break-even interest rates in each period. Given the information asymmetry described above, lenders will update beliefs about the shock realization in period 1 after observing the borrower's action. A Perfect Bayesian equilibrium (PBE) is an equilibrium in which everybody's response is optimal given everybody else's responses and beliefs, and beliefs are consistent with strategies and updated using Bayes' (whenever possible).

There are potentially two types of equilibria in pure strategies: Separating and Pooling. In a separating equilibrium actions following each shock realization will be different, and hence completely revealing. In this case, the equilibrium interest rate charged in period 1 will differ from the interest rate charged at time 0.[13]

On the other hand, in a pooling equilibrium actions following different shock realizations are the same. In this case, there is no information revelation and credit conditions remain unaffected.

In Section 4 we numerically solve for the PBE equilibria in this model for different sets of parameters and in the technical appendix we describe in detail all the equilibrium finding

[12] When calculating the PBE equilibrium for the game, one clearly needs to impose that parameter values are such that budget constraints at the end are satisfied. For details see Appendix.

[13] The separating equilibrium is fully revealing. The interest rate at 1 will reflect the low type's true probability of default. This is a consequence that in our model the space of signals, $\{N, I\}$ is as rich as the space of types $\{H, L\}$. If we were to add more types in our model, complete revelation would not happen anymore in a separating equilibrium, and hence asymmetric information would not only make pooling equilibrium possible but would also have an amplifying effect on pricing in the separating case. See Catão, Fostel and Kapur (2008).

procedures. It turns out that for some set of parameter values a separating equilibrium can be sustained. More precisely, in equilibrium the sovereign exerts no-action (N) after a good shock realization and issues fresh debt (I) after a bad shock realization. Hence, the sovereign action is completely revealing and has credit market repercussions producing a spike in spreads which could make future default more likely. Moreover, there is a set of parameter values such that a pooling equilibrium can be sustained. More precisely, in equilibrium the sovereign exerts no-action (N) regardless of the shock realization. In this case, there is no information revelation and hence pricing stays the same.

In our model, country finds convenient to borrow for two reasons. First, if the sovereign's discount factor β is higher from the investors' discount factor $1/(1+r_f)$, then the sovereign would find attractive to borrow in order to increase current consumption. Second, since default is possible, taking on more debt increases the value of the default option.[14] To see this, note that from equation (6) and (7) the expected benefit of borrowing at time 1 is given by $\alpha\tau_1 + \pi''(F_2 - R_0 D_0 - R_1 D_1 + (1-\pi'')((1-\eta)F_2 - c(R_0 D_0 + R_1 D_1))$, where $D_0 = \tau_0$ and $D_1 = \alpha\tau_1$ and $R_t = 1 + r_t, t = 0, 1$. It follows that for R_0, R_1 and D_0 fixed, the expected marginal benefit of borrowing in the intermediate period is $1 - \pi'' R_1 - (1-\pi'')cR_1 = 1 - R_1[\pi''(1-c) - c]$. Hence, holding initial debt and interest rates constant and $c < 1$, the marginal benefit of an extra unit of borrowing is declining on the probability of repayment π''. Following a bad shock, there is then an incentive to borrow in the middle period even with linear preferences. Importantly, this incentive rises on the volatility of tax revenues as the latter positively affects default probabilities. So, countries with higher underlying volatility of fiscal revenues will tend to value the mid-period option of extra-borrowing higher. It is not therefore surprising that, as we show in the next section, higher volatility has a bearing on the prevalence of separating relative to pooling equilibria.[15]

However, R_1 is clearly affected by D_1. In a standard setting with no information asymmetries, once extra borrowing takes place and the ratio of debt to revenues D_1/F_2 goes up, R_1 will go up to the point that investors' break-even condition is satisfied. Asymmetric information adds an extra channel through which D_1 affects R_1 given that borrowing becomes a signal. For one, information asymmetry generates the possibility of either a pooling or a separating equilibrium; and because there is debt dilution, the first investor

[14] Adding curvature to government preferences in the model would exacerbate this effect, rather than overturn it. So, it would not change qualitatively our results. It would change, however, the relative size of equilibrium regions: there would then be a consumption smoothing motive for debt, so separation equilibrium would be easier to sustain. Another incentive to increase debt in the middle period is the presence of tax lafer curve effects. When initial debt and tax rates are already sufficiently high that further hikes in rates are revenue-reducing, this can increase the incentive to borrow in t=1 upon a bad shock. For a discussion of tax lafer curve effects on sovereign risk, see Bi (2012).

[15] See Broner et.al. (2007) for a discussion and evidence on the incentives to issue short-term debt.

will internalize the possibility of middle period borrowing and R_0 will go up as well. Thus higher R_0 and R_1 will detract from the sovereign's incentive to play a separating strategy and borrow in the middle period aiming to default in the last period.

This trade-off is what determines whether a separating or pooling equilibrium can be sustained. For some parameter values, the country that receives a bad shock realization may find profitable to forego the option of borrowing so as to not face higher interest rates. When the benefit of issuing new debt is smaller than it cost, then only a pooling equilibrium can be sustained. Conversely, for some other parameter values, a country that received a bad shock may be willing to face higher interest rates. In this case, the benefits of issuing new debt far out-weight its costs. In this case a separating equilibrium can be sustained. We study how this trade-off plays out under distinct parametrizations in Section 4.

3.5 Fiscal Discoveries and Sudden Yield decoupling

As noted in the introduction, a key goal of this paper is to show how the existence of these two distinct equilibria can rationalize the behavior of spreads shown in Figure 1. That is, how can fiscal discoveries explain Eurozone spreads that were highly compressed for years prior to 2007 and then suddenly decouple?

Answering this question involves a clear distinction between the concept of "country" and "type" in this model. Let country i be defined by $\theta^i(\delta^i, \epsilon^i)$, where δ^i is a vector of country i's fundamentals (recovery functions, hair-cuts, discount factor, etc) and ϵ^i is the vector of country i's fiscal shocks.

In our model, for a given country i, ϵ^i_1 is private information and δ^i is common knowledge. Hence, a "type" in our model is defined by the shock realization, i.e., country i could be a high type (when shock realization is high) or a low type (when shock realization is low).[16] Hence, investors in our model can perfectly recognize the difference between different countries, but cannot directly observe specifics of fiscal outlooks within each country.

What is key is that the (common knowledge) vector of fundamentals δ^i is what determines whether separating or pooling equilibrium can be sustained. We explore quantitative aspects of such equilibria in Section 4, but the intuition is as follows. Consider for the sake of concreteness two countries: country-A, characterized by a set of weak fundamentals, who plays a separating strategy and country-B, characterized by a set of strong fundamentals, who plays a pooling strategy. Suppose countries are hit by a positive fiscal shock. In this

[16] In a more realistic situation the vector or fundamentals δ^i and fiscal shocks realizations ϵ^i, can be correlated and hence fundamentals would not be perfectly observable either. In this model we abstract from this matter. A way of understanding our model is about short-term information frictions.

case, investors will observe both countries refraining from borrowing and this will generate a very small difference in country spreads. The gap in country spreads that should prevail due to the gap in country fundamentals is dampened by the presence of informational noise. Though country-B received a good shock, investors do not learn in the pooling game, whereas they do learn in the separating game that country-A was on a good fiscal path. However, the situation is quite different following a negative shock. In this case, the informational noise works in the same direction as the gap in fundamentals: though investors do not learn from country-B's behavior, they do learn that country-A is on a negative fiscal path and hence spreads wildly diverge. Hence, fiscal discoveries can explain both, convergence and sudden decoupling.

It is important to be clear that we are not rationalizing convergence and sudden decoupling by a general cross-country shift from a pooling to a separating equilibrium. Instead, convergence and divergence obtain from the time series implications of pooling and separating equilibria played by different countries. In other words, our model-based interpretation is that yield decoupling takes place because distinct country groups were playing, at any given point of time (prior and post-2007), different games – some were playing a game with investors where the pooling strategy dominates, whereas others were playing (also all along) a game wherein a separating strategy dominates. As we discuss in section 4, within each of these pooling and separating games, there are distinct sub-cases in which default may or may not be optimal in the final period, depending on the sequencing of shocks and the relative costs of defaults. Likewise, in the group of countries playing a separating equilibrium, there will a sub-group facing higher (or lower) yields depending on whether the respective fundamentals are weaker (or stronger).

Finally, it is worth emphasizing the role of asymmetric information in this result. As discussed above, asymmetric information introduces a non-trivial trade off in the relationship between the option of taking extra borrowing and the interest rates; country-specific parameters will then determine whether countries find optimal to play a separating or a pooling strategy. Relative to the cannonical model, our model delivers the possibility of explaining two central features observed in the recent debt crisis. First, spread stability despite a bad shock that lowers income and hence raises debt-to-income (D/Y) ratios. In the cannonical model all country yields (relative to the risk free rate r_f) will go up. This is because the expected D/Y in the final period will go up even for those that do not borrow in the interim period. The possibility of pooling equilibrium in our model makes it possible that only a subset of countries spreads go up (see Figure 1 panel (b)). As a consequence, a model without information asymmetries will produce a smaller yield dispersion. Next section shows results for a Subgame Perfect Nash equilibrium that illustrate this points.

4 Calibration Results

In this section, we numerically solve for PBE in our theoretical model under alternative parametrizations. The goal is threefold. First, to identify sets of parameters for which either a pooling equilibrium or a separating equilibrium exist.[17] When either pooling or separating equilibria exist, we assess the corresponding impact on interest rates and default risk in good and bad times. Second, within these two possible types of equilibria, we distinguish various sub-cases regarding the relationship between the nature of the shock, the reaction of bond yields, and the optimality of default: in some cases, bad shocks will not be followed by either rising yields or a default; in others rising yields may or may not be followed by a default; and finally there is the possibility that default may not be preceded by higher yields. Third, we examine the sensitivity of such equilibria to parameter changes and demarcate regions between them where parametrization trade-offs arise. The fundamental parameters we are particularly interested in are: (i) parameters that characterize fiscal shocks–namely, the variance of the first and second period fiscal revenue shock and the persistence of the first period revenue shock; (ii) default costs, as gauged by the "haircut" on debt obligations and the confiscable share of fiscal revenues.

The simulations below show that for a given distribution of shocks, countries with "stronger" fundamentals generally find optimal to play a pooling strategy and those with weaker fundamentals find it optimal to play a separating strategy. We also explore the fact that there are trade-offs in the characterization of what is a country with stronger vs. weaker fundamentals: some countries may be stronger with regard to some fundamentals (e.g. deep capital markets and substantial worldwide linkages that make default extremely costly), but weaker with regard to others (e.g. being subject to larger shocks). We move the respective parameter values around, so as to see what difference does it make for the final equilibrium outcome and bond yields under good and bad shocks. In the Appendix we describe in detail the procedure to find Perfect Bayesian Equilibrium.

Specifically, we consider four broad scenarios: (i) a *baseline* scenario in which the variance of the first and second period shock are equal; (ii) a *high short run risk* scenario in which the variance of the first period shock varies and is allowed to be much higher than in the final period. In the case of a negative shock, this scenario captures an immediate crisis situation in which the economy is subject to a sharp contraction in the first period while only facing business-cycle type fluctuations in the second period; (iii) a *high long run*

[17] We focus only on pure-strategies equilibria. We also found regions of non-existence. The possibility that no equilibrium exists for some regions of the parameters is a standard feature of models with asymmetric information. It may genuinely reflect either credit rationing in the first period or market freeze in the post-shock period. For the parameterizations that we considered, we also find that there is no multiple equilibria – equilibrium is either pooling or separating.

risk scenario in which the variance of the second period shock varies and is allowed to be much higher than the variance of the first period shock. This scenario is meant to describe an economy that is not subject to a major shock in the immediate future but faces large uncertainty in the medium-long run; (iv) a *high persistence* scenario in which the variances of the shocks are equal across periods but the persistence of the first period shock is higher than in the baseline scenario.

These alternative scenarios should be expected to change the dominance of one equilibrium vs. the other (separating vs. pooling) as well as the likelihood of default in period 2 following a good vs. a bad shock in period 1. For instance, consider a scenario where the variance of the first period shock is higher. When the shock is negative, this means a very large negative shock which requires a large welfare-reducing compression of government spending; so the country is more willing to borrow, and possibly exercise later the default option, to make up for such a revenue shortfall. At the same time, given persistence, this means that the act of borrowing in the middle period is extremely informative of the probability of default in the next period; so yields will rise sharply. This will lower the cost of default in the next period, raising the option value of borrowing. As corroborated by the numerical simulations below, this increases the prevalence of separating equilibrium. Likewise for an increase in shock persistence. Conversely, a higher variance of the final period shock should lead to more pooling and this is what we find below.

Distinct parameterizations also imply that, within each of the two equilibria, there can be distinct outcomes regarding the evolution of bond yields and whether defaults materialize or not. We index each of these distinct combinations of outcomes by a number, ranging from 1 to 6, mapping the shock sequence and the possibility of default in the second period.[18] The different equilibria are the following:

1. Equilibrium #1: Default never occurs.
2. Equilibrium #2: Default occurs only in case of two consecutive negative shocks.
3. Equilibrium #3: Default occurs only following a negative second period shock (for any first period shock).
4. Equilibrium #4: Default occurs only following a negative first period shock (for any second period shock).
5. Equilibrium #5: Default occurs following either a first or a second period negative shock.
6. Equilibrium #6: Default always occurs.

[18]See the on-line appendix for a full characterization of these different equilibrium cases.

4.1 Parametrization

The model contains eleven parameters: (i) the initial level of borrowing (τ_0) and the sequence of expected fiscal revenues in period 1 and period 2 (τ_1, τ_2); (ii) the sequence of shocks to fiscal revenues which are characterized by the variance of the first and second period shocks ($\sigma(\varepsilon_1), \sigma(\varepsilon_2)$), the persistence of the first period fiscal revenue shock (ρ). We assume that a positive and a negative shock are equiprobable ($p = q = 1/2$); (iii) the default costs that are captured by the recovery rate (c, d) –one minus the haircut– and the confiscated share of revenues (η) and finally, (iv) the discount factor (β) and the risk-free interest rate (r). Table 2 shows parameter values for the baseline and alternative cases we will consider.

In the baseline scenario the initial debt issuance in period 0 is normalized to 100. The mean fiscal revenues in period $1, \tau_1$, is set to 100. In many advanced countries – and particularly in the Eurozone, general government revenues are typically in the range of 40% to 50% of GDP, so this parametrization would thus imply an initial (pre-crisis) debt to GDP ratio in that range. This is not far-off the mark: the external debt to GDP ratios for Greece, Portugal, and Spain prior to the crisis (2005) were 55%, 45% and 36% respectively (see Catão and Milesi-Ferretti, 2013). We set the second period mean fiscal receipts, τ_2, to 135. This level is high enough so that a country hit by two negative shocks and defaulting is still be able to meet default payments, and low enough so that there is significant default risk. Since our model features only two periods, the second period fiscal revenues could be understood as the present value of future government revenues that can be used to pay off government liabilities, so should naturally be substantially higher than in the initial period. The discount factor β is set to 0.96 and the risk-free rate is set equal to $1 + r = 1/\beta + 0.001$, implying than international lenders are only infinitesimally more patient than domestic borrowers. This effectively mitigates one of the incentive for borrowing typically found in infinite horizons versions of the cannonical model, where the challenge is to engineer debt to GDP ratios in equilibria that are not unrealistically low, sometimes featuring much lower β values.[19] All the simulations are presented for a range of default costs varying between 0.1 and 0.3 for the confiscated share of revenues, following the rationale discussed earlier (see footnote 11). The other relevant default cost parameter is the debt recovery rate. We always consider $d = 1$ and we parameterize c to be between 0.5 and 0.9, consistent with the value range for the "haircut" (=1-recovery rate) between 10% and 50%, which is in line with extensive cross-country evidence (e.g. Cruces and Trebesh, 2011). The two i.i.d. shocks, ε_1 and $\varepsilon_{2,}$ have the same standard deviation ($\sigma(\varepsilon_1) = \sigma(\varepsilon_2) = 10$). The persistence of the tax revenue shock is 0.8. As illustrated in Table 1, these are of a similar order of magnitude of the actual cyclical volatility and

[19] See, e.g., Aguiar and Gopinath (2006), Table 2.

Parameter	Parameter Name	Baseline	Short-Run Risk	Long-Run Risk	Persistence
β	Discount Factor	0.96	–	–	–
τ_0	Initial Borrowing	100	–	–	–
τ_1	Fiscal Revenues at t=1	100	–	–	–
τ_2	Fiscal Revenues at t=2	135	–	–	–
c	Recovery (1-haircut)	[0.1,0.5]	0.25	0.25	0.25
η	Fiscal confiscation	[0.1,0.3]	–	–	–
p	Probability of good shock at t=1	0.5	–	–	–
q	Probability of good shock at t=2	0.5	–	–	–
ρ	Persistence	0.8	–	–	[0.5,1]
$\sigma(\epsilon_1)$	St Dev of shock at t=1	10	[0,0.3]	–	–
$\sigma(\epsilon_2)$	St Dev. of shock at t=2	10	–	[0,0.3]	–

Table 2: Parameter Values

persistence of real tax revenues shocks, notably in countries at the epicenter of the recent debt crisis (and even more so if we were to include the 2008-2012 period in the estimation of these parameters). In the three other scenarios, one of the default parameters (the fiscal confiscation) is set fixed under order to analyse the sensitivity to differences in short-run risk ($\sigma(\varepsilon_1)$), long-run risk ($\sigma(\varepsilon_2)$), and persistence (ρ).

4.2 Numerical Results

Each scenario is presented using two figures. In the first figure, we plot the nature of the equilibrium (separating or pooling) and no equilibrium regions, the type of separating equilibrium (ranging from 1 to 6), and the type of pooling (ranging from 1 to 6). In the second figure, we plot the interest rates following the realization of the interim period shock. Note that in the case of a pooling equilibrium, the interest rate is insensitive to the shock, while in the case of a separating equilibrium, the interest rate following a bad shock differs from the interest rate following a good shock.

Baseline Scenario

Figure 11 presents results for the baseline scenario. When default costs are large enough (i.e. the confiscation parameter η is high and hair cut is low) the optimal strategy for a country hit by a bad shock is not to issue new debt. This pooling equilibrium prevails for a wide range of parameters as long as a reduction in the recovery rate is compensated by an increase in the fiscal confiscation parameter in order to maintain default costs high enough. Further this pooling equilibrium is not followed by default for any shock realization in the second period (type-1 equilibrium). The reason is that by not issuing, the country hit by a bad shock maintains its debt burden at a low enough level that defaulting is not optimal.

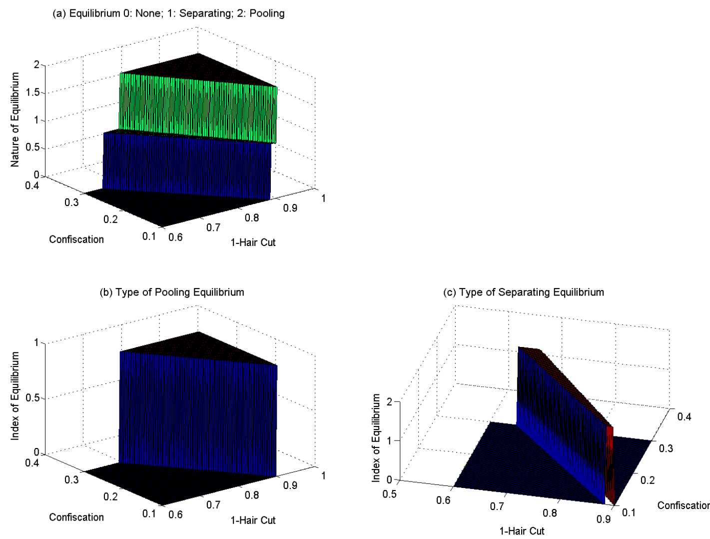

Figure 11: Baseline Scenario. Equilibrium

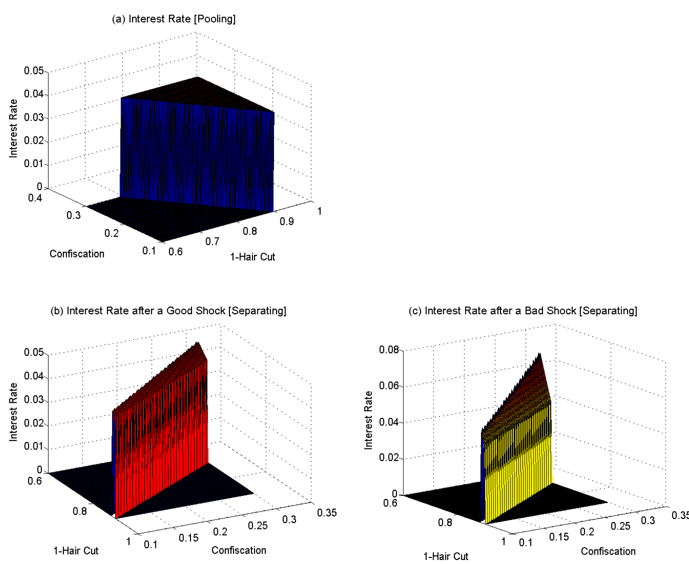

Figure 12: Baseline Scenario. Interest Rates.

If default costs are smaller however, incentives to issue following a bad shock increase and a separating equilibrium emerges. In this separating equilibrium (a type-2 equilibrium), a country hit by a bad shock chooses to re-issue and faces the prospect of a default if it experiences a second bad shock in the final period. Panel (a) in Figure 12 shows the interest rate in the case of a pooling equilibrium. Since this equilibrium is default-free, the interest rate is equal to the risk-free rate (4.27%). Panel (b) in the same figure shows the interest rate in a separating equilibrium following a good shock. Depending on the severity of the default costs, the interest in a separating equilibrium varies between 4.28% and 4.85%. In good times, the spread between a country playing a pooling and a country playing separating are therefore at most 0.58%, despite the fact that countries have different risks of defaulting in the final period. Notice that this feature will be true in all the sensitivity analyses presented below. As long as only good shocks occur, the pattern of interest rates remain very close accross countries that have very different fundamentals and are in a different equilibrium. Panel (c) shows the interest rate following a bad shock. In this case, the bond yield rate can be much higher (up to 7%) in proportion to the potential for the country to impose higher hair-cuts on investors.

These baseline results yield two important insights. First, countries experiencing the same negative shock but with different costs of defaulting can exhibit very different borrowing behavior and yields, and hence distinct default probabilities. In one case, countries will not issue debt and by doing so will remain riskless. In the other case, they will compensate a bad shock by issuing more debt at a higher interest rate therefore risking default in the second period. Second, the ranges of parameters for which a pooling equilibrium and a separating equilibrium exist are adjacent. This implies that small differences in default costs can generate large differences in equilibrium outcomes. Therefore, a switch from pooling to separating following a modest re-assessment of the default costs is plausible in the model, as arguably is in practice.

Distinct Short Run Risk Scenarios

Figures 13 and 14 present evidence on the sensitivity of the type of equilibria and of the attendant yields to changes in short-run uncertainty. Relative to the baseline calibration (which sets $\sigma(\varepsilon_1) = 10$), now the short-run volatility can be up to three times as large. Confiscation is fixed at $\eta = 0.25$ throughout these figures (see Table 2). One relevant question that these simulations allow us to address is how much higher does short-run volatility need to be in order to get a country with otherwise strong fundamentals (in terms of high η) into a separating equilibrium.

Given substantial persistence ($\rho = 0.8$), higher short-run volatility (relative to second period volatility) implies that a large part of the uncertainty can be resolved after the

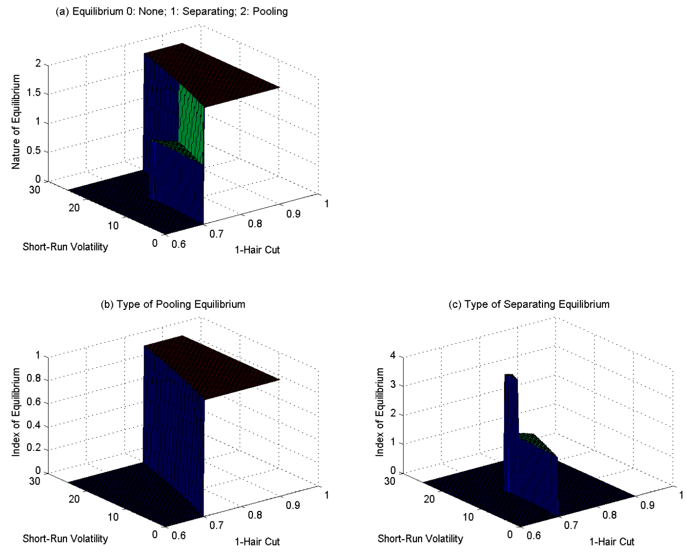

Figure 13: Short run risk Scenario. Equilibrium.

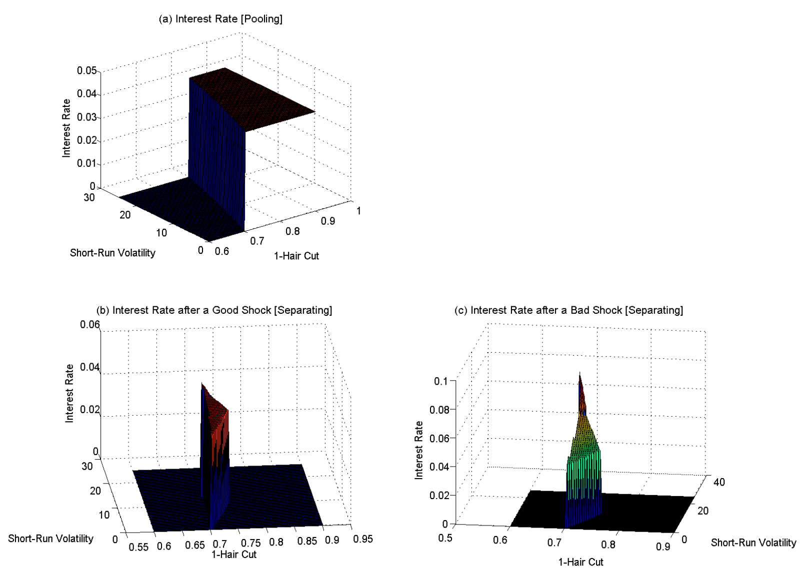

Figure 14: Short run risk Scneario. Interest Rates.

first period shock, strongly conditioning the default vs. repayment outcome in the second period. As a consequence, pooling is harder to sustain when short-run volatility increases. At baseline, for a short-run volatility equal to 10, pooling can be sustained for any haircut lower than 0.28. When short-run volatility is 20, the range of pooling is much smaller, with pooling sustainable only for haircut lower than 0.2. The flip side is that the range of parameters over which a separating equilibrium obtains is now larger. However, when short-run volatility is higher than 15, the separating equilibrium does not exists. Higher short-run volatility can lead countries to switch from a pooling to a separating equilibrium. Consider a country with a haircut equal to 0.25: when short-run volatility is set at the baseline, this country is in a pooling equilibrium. However as soon as short-run volatility increases beyond 12, it switches to a separating with associated default risk in the second period.

For some of this range, a separating equilibrium of type 4 prevails (see Figure 13 panel (c)) implying that the first period shock is a perfect predictor of second period's default. Countries hit by a bad shock face little prospect of recovery and default in the second period. Countries experiencing a positive shock remain solvent in the second period.[20] A consequence is that the bond yield is higher than in the baseline case, ranging from 500 to 950 bps (see Figure 14 panel (c)). As seen in Section 2, this range is in line with recent debt crisis experience. Note however that, once again, here "the Tarpeian Rock is not far from the Capitol" as the range of default costs for which following a bad shock, the economy remains risk-less (pooling equilibrium of type 1) or facing default for sure (separating equilibrium of type 4) are adjacent.

Distinct Long Run Risk Scenarios

Figures 15 and 16 present results for the case in which the standard deviation of the second period i.i.d shock varies between 0 and 0.3 (see Table 2). When the second period i.i.d shock is larger than in the baseline, there is a lot of future uncertainty on the ability to service debt. The timing of uncertainty resolution limits the informational value of revealing the nature of the shock to creditors' interim period. As a consequence, when the long-run volatility increases, the region of parameters for which a separating equilibrium exists tends to shrink. When long-run volatility is high enough, there is no separating equilibrium.

Interestingly, the timing of volatility matters a lot for the type of separating equilibrium that can occur. In the case of short-run volatility, an increase in short-run volatility shifts the type of separating equilibrium from type 2 (default occurs only in the eventuality of

[20] This illustrates a general feature of our model which is that default takes place during bad times. For empirical evidence on this regularity see Wright and Tomz (2007) and Yeyati and Panizza (2011).

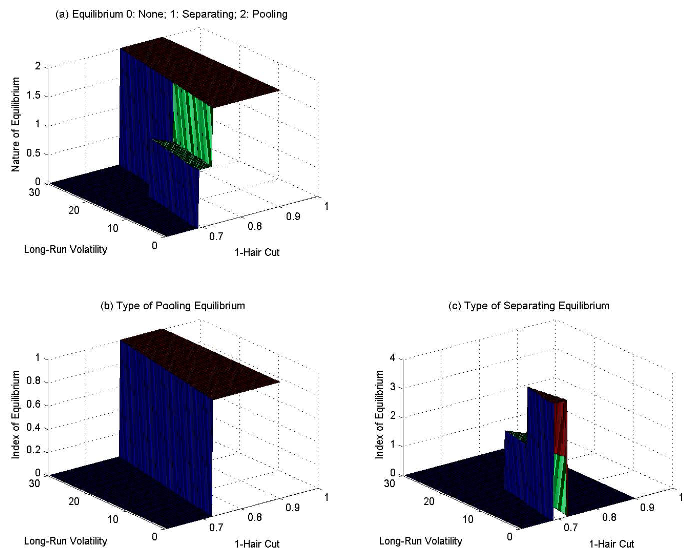

Figure 15: Long run risk Scenario. Equilibrium.

two subsequent negative shocks) to type 4 (defaults occurs for sure in the final period if a bad shock occurs in the interim period). The exact opposite occurs with long-run volatility. The reason for this contrast is due to the fact that under high short-run volatility, default risk is very dertermined by the first period shock with little chance of avoiding default after a negative interim period shock. Under high long run volatility, uncertainty about the future means more chances to escape default.

Pooling dominates, specially when default costs (lower hair-cut) are high enough. Note that despite the high variance of the final period shock, the economy remains riskless, as shown in Figure 15 panel (b) by the sizeable zone where the equilibrium type is 1 conditional on a pooling equilibrium. In anticipating that borrowers will not find optimal to issue new debt following a bad shock, creditors do not face any dilution risk and hold claims that are securely collateralized. Symmetrically, the combination of low debt level and low interest rate (see Figure 16 panel (a)) makes it optimal for debtors to choose not to default even in the situation where they suffer two bad shocks in a row. In the tiny region where equilibrium is separating, interest rate can be much higher than the risk free rate. However, as long-run volatility increase, the possibility of remaining default-free in all circumstances is somewhat reduced, implying that the combination of parameters for which pooling is an equilibrium shrinks slightly.

Distinct Persistence Scenarios

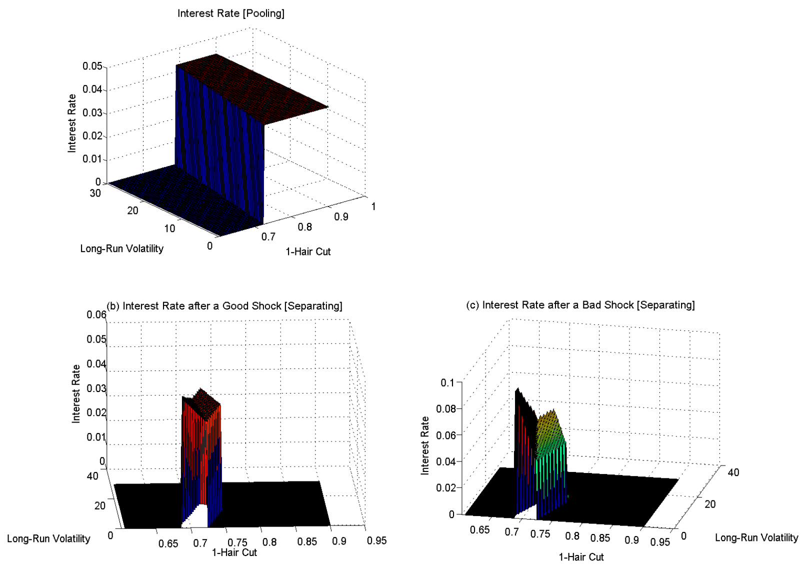

Figure 16: Long run risk Scenario. Interest Rates.

Figures 17 and 18 presents the results when we let persistence to vary from 0.5 to 1 (see Table 2). Higher persistence improves the value of the information provided by the first period shock to assess default risk in the second period. The parameter range for pooling (separating) is somewhat smaller (larger) than in the baseline case. When equilibrium is pooling default never occurs (Figure 17 panel (b)) and hence the interest rate is very insensitive to persistence (Figure 18 panel (a)). When persistence increases, the value of information revelation increase. As a consequence, the region of parameters for which an equilibrium is separating expands, allowing countries with weaker fundamentals (i.e. higher hair-cut) to sustain a separating equilibrium. This illustrates an important difference between higher volatility and higher persistence. As shown in Figure 13 and Figure 14, a separating equilibrim that exists with low volatility would dissapear when volatility is high enough. This is not the case with persistence. As Figure 15 shows, a separating equilibrium that exists with low persistence, will continue to exist with higher persistence.

Two types of separating equilibria exist: type-2 equilibrium when persistence is low enough, and type 4 equilibrium when persistence is very high, and so a default will happen for sure after a bad shock in t=1. When the equilibrium switches between type-2 to type-4, interest rate increase sharply as the economy evolves from a situation in which default next period will occur only in case of a repeated bad shock to a situation in which default is unavoidable and the bond yield rate sky rockets (Figure 18 panel (d)). This means that

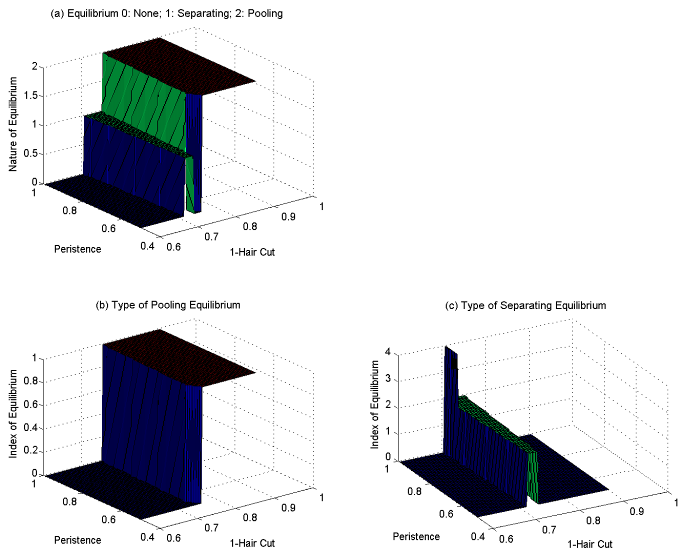

Figure 17: Persistence Scenario. Equilibrium.

when fundamentals are not strong enough to ensure pooling, default risk and hence bond yields are extremely sensitive to shock persistence.

Cross-Country Time Series Patterns

Given that a main empirical motivation of this paper is to explain the time-series evolution of cross-country bond yields, we can illustrate how our model can roughly reproduce the time series pattern of actual bond yield depicted in Figure 1. Using our baseline calibrations, we characterize two countries: a "strong-fundamentals" country, country B, (defined as having an expected hair-cut of only 0.15) vs. a "weak-fundamentals" one, country A, (defined as having a hair-cut twice as large). The two countries have the same value for all the other fundamentals including a common confiscation parameter of 0.25. In particular, they are facing the same shock structure. Under this parametrization, country B is in a pooling equilibrium and country A in an type-2 separating equilibrium (meaning that it will default only if it experiences two negative shocks in a raw). Figure 19 plots a simulation of the interest rate of the two countries for the following sequence of shocks: 9 successive positive shocks followed by one negative shock.[21] In this case, during good times (i.e., before the negative shock hits), the bond of the weak fundamental country A yields 4.7% return, whereas that of the strong fundamental country B yields 4.3%. As in

[21]We can interpret this sequence of shocks as reflecting the post euro-adoption period followed by the crisis of 2008.

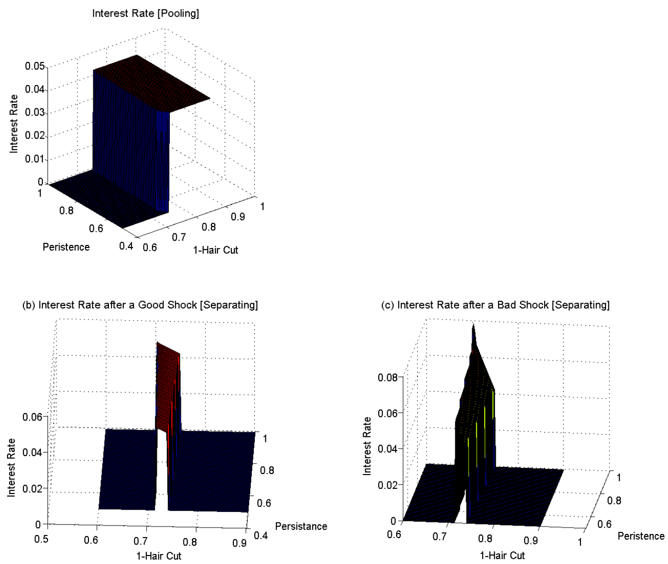

Figure 18: Persistence Scenario. Interest Rates.

the data, the good times spread between a weak and strong fundamentals country is very narrow despite the fact that the strong fundamental country is default-free while the weak fundamental country face a 25% change of default in two periods.[22] Post-shock the yield of the country with weak fundamentals rise to 6.5%. So a significant yield decoupling takes place.

Keeping the parameterization of default costs at the same level ($c = 0.3$ for A and $c = 0.15$ for B), Figure 20 shows what happens if the weaker country is faced with higher volatility (both short and long term) as well as with higher persistence. When short-run volatility increases from 10% to 15%, the interest rate in good times increases very modestly, by 10 bps, but the interest rate following a bad shock, increases sharply form 6.5% to 9.1%. Raising persistence from baseline 0.8 to 0.9 also implies a rise in interest rate following a bad shock but more modest (to 7.2%). Halving long-run volatility from 10% to 5% reduces sligthly the good times interest rate (by 5 bps) but actually increases the interest rate following a bad shock (from 6.46% to 6.87%) reflecting the lower probability to offset the consequence of a negative shock in the interim period by a good shock in the final one. These results indicate that, conditional on some countries playing a separating equilibrium game, a bad shock can generate yield dispersion of considerable degree depending on the size of the shock and its persistence. This is broadly consistent with the evidence discussed

[22]The default probability correspond to the probability of experiencing two successive negative shocks.

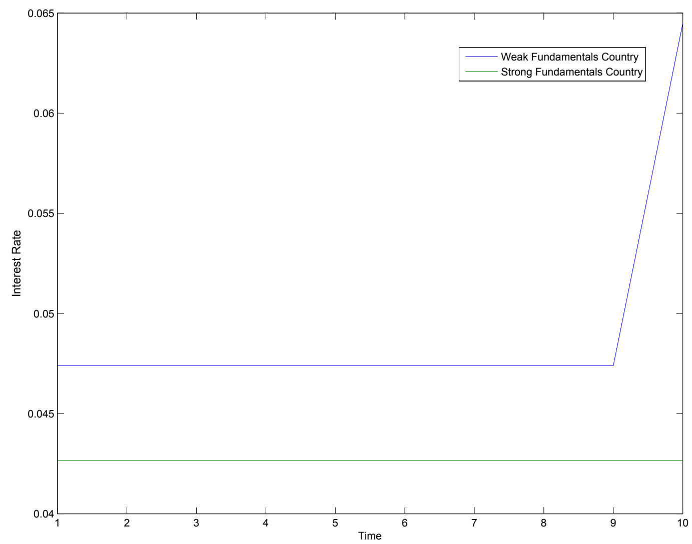

Figure 19: Yield Decoupling in the Model. Baseline Case.

in Section 2, where it is shown that the magnitude of tax revenue shocks – even if negative for all countries – displayed considerable cross-country differences.

Finally, Table 3 reports the effects of information asymmetry on the magnitude of interest rates. As discussed, in a simplified set-up where shocks can be either good or bad, middle-period borrowing is fully revealing on the magnitude of the (bad) shock. So, conditional on a separating strategy, our model delivers no amplification and so no higher R_1 than in the full information game. However, this does not mean that R_0 will be the same in the full vs. the asymmetric information equilibria. Table 3 shows that information asymmetry generally increases the cost of first period debt. The table also shows that the difference between R_0 in both cases is higher on persistence.

5 Conclusion

Through the lens of our model, sovereign yield dispersion across the Eurozone can be rationalized by the co-existence of two country groups on distinct equilibrium paths all along: for one group, fundamentals are strong enough for a pooling strategy to be optimal; but not for all others. As the two groups were subject to smaller and mostly positive shocks between 1999 and 2007, yields converged except for residual gaps due to differences in common-knowledge fundamentals. But when the large negative common shock of 2008-09 hit and tax revenues dropped sharply, it was still optimal for the first group to adjust

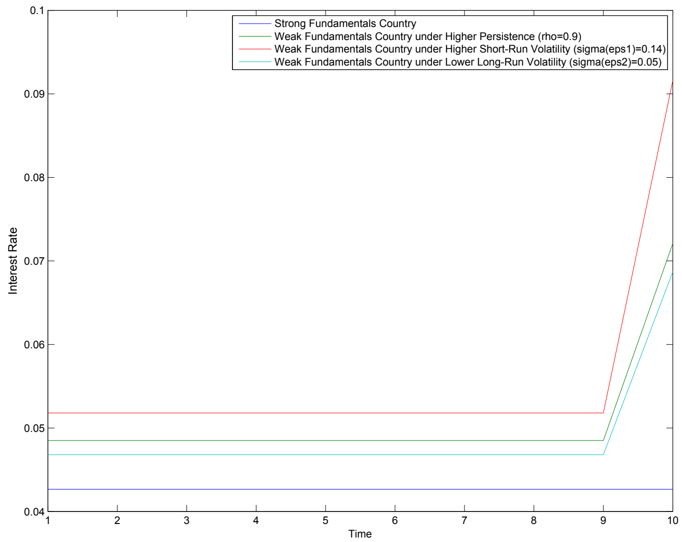

Figure 20: Yield Decoupling in the Model. Alternative Parametrizations.

Full Information Case					
	Haircut	Confiscation	Persistence	Volatility	R_0
Baseline	0.3	0.25	0.8	0.1	4.66%
High Persistence	0.3	0.25	0.9	0.1	4.66%
High Volatility	0.3	0.25	0.8	0.14	5.17%
Asymmetric Information Case					
	Haircut	Confiscation	Persistence	Volatility	R_0
Baseline	0.29	0.25	0.8	0.1	4.73%
High Persistence	0.29	0.25	0.9	0.1	4.85%
High Volatility	0.29	0.25	0.8	0.14	5.18%

Table 3: Interest Rate Comparisons between Full and Asymmetric Information Cases.

spending and refrain from borrowing, thus signaling a brighter fiscal outlook; in contrast, for those on a separating equilibrium it was optimal to resort to extensive borrowing. Higher debt ratios and expected deterioration of the fiscal outlook in the latter group translate into higher country risk.

Two key fundamentals that distinguish country groups in the model are the underlying volatility and persistence of fiscal revenue shocks. They give rise to distinct valuations for the option of extra borrowing: more volatile countries tend to benefit more from the option of borrowing to either default or gamble for resurrection. These differences in the underlying stochastics of revenue shocks have been non-trivial across the Eurozone. As we have documented, countries at the epicenter of the recent crisis have also been the ones where fiscal revenues have been historically more volatile and subject to more persistent shocks.

The model also fleshes out how asymmetric information on country-specific public finances amplifies yield decoupling. It does so while also explaining the stability of yields (relative to the US benchmark) for countries on a pooling equilibrium. This is a salient feature of the data (cf. second panel of Figure 1) that cannot be explained by the cannonical sovereign debt model with fully observable shocks. In addition, asymmetric information allows for the extra realism of generating sharp responses of country spreads to changing patterns of market tapping. Again, this has been a noted pattern in the recent Eurozone crisis.

Four results of our model calibration stand out. First, the model re-assuringly delivers spreads that roughly match real world counterparts for sensible parameterizations. Second, it posits that high short-term uncertainty regarding fiscal revenues increases the dominance of separating equilibrium. Conversely, higher long-run uncertainty increases pooling. Third, there are sizeable regions of continuity between the two equilibria around some (empirically) relevant parameter ranges; so some equilibria can be quite fragile. This can have far-reaching implications if (and when) parameter uncertainty is substantial. Fourth, we never obtain equilibrium regions where default occurs when equilibrium is pooling. If, instead, the equilibrium is separating, default may or may not ultimately materialize depending on the size and sequencing of shocks as well as the relative cost of default.

We draw the following policy implications from these results. One is that structural reforms that bring the volatility of tax bases and of revenue collection closer to those of pooling countries seem important to mitigate yield dispersion during bad times. At the same time, as we document, intra-Eurozone differences in volatility and persistence of fiscal revenues have been long lasting; so, their seemingly structural nature suggests that reforms may take time in narrowing such differences. Thus, a well-functioning system of fiscal transfers or a full-fledge fiscal union increase their appeal if mitigating yield dispersion and default risk during bad times is a central goal. Until such a system is fully functional, central banking policies may also have a role to play in mitigating default risk during bad times, for instance along the lines discussed in Reis (2013). While an analysis of the welfare implications of the various policy options clearly requires a more encompassing setting, we see our model as providing a tractable and readily calibratable tool that can highlight some of the main crisis transmission mechanisms and equilibria at play.

6 References

1. Aguiar, Mark, and Gita Gopinath, G., 2006, "Defaultable debt, interest rates and the current account", Journal of International Economics 69(1), 61-89.

2. Alesina A., Cammpante F., and Tabellini G. 2008, "Why is Fiscal Policy So Often Procyclical?", Journal of the European Economic Association, September 2008,Vol. 6 No. 5, 1006-1036

3. Alfaro, Laura, and Fabio Kanczuk, 2005, "Sovereign debt as a contingent claim: a quantitative approach", Journal of International Economics 65, 297-314.

4. Arellano, C., 2008, "Default risk and income fluctuations in emerging economies", American Economic Review Vol. 98, pp. 690-712

5. Bi, Huixin, 2012, "Sovereign Default Risk Premia, Fiscal Limits, and Fiscal Policy", European Economic Review, 56, pp. 2012.

6. Benjamin, D. and M. Wright, 2008, "Recovery Before Redemption? A Theory of Delay in Sovereign Default", unpublished manuscript.

7. Broner, Fernando, Guido Lorenzoni, and Sergio Smuckler, 2007, "Why do emerging markets borrow short-term?" NBER working paper 13076

8. Borensztein, Eduardo and Ugo Panizza, 2008, "The Costs of Sovereign Defaults", IMF working paper 08/238.

9. Catão, Luis, and Gian Maria Milesi-Ferretti, 2013, "External Liabilities and Crises", IMF Working Paper forthcoming.

10. Catão, Luis, Ana Fostel, and Sandeep Kapur, 2009, "Persistent Gaps and Default Traps", Journal of Development Economics.

11. Chatterjee, S. and B. Eyigungor, 2012, Debt Dilution and Seniority in a Model of Defaultable Sovereign Debt, Working paper 12-14, Federal Reserve Bank of Philadelphia.

12. Cruces J., and C.Trebesch, 2011, Sovereign Defaults: The Price of Haircuts. Unpublished Manuscript.

13. D'Erasmo, Pablo, 2011, "Government Reputation and Debt Repayment in Emerging Economies", University of Maryland.

14. Drudi, Francesco and Alessandro Prati, 2000, "Signaling Fiscal Regime Sustainability", European Economic Review 44, pp. 1897-1930.

15. Eaton, J., 1996, "Sovereign debt, reputation, and credit terms," International Journal of Finance and Economics, 1, 25-36.

16. Gourinchas, P.O, 2011, "Global Imbalances and Global Liquidity," unpublished manuscript, Berkeley.

17. Gourinchas, P.O, and Obstfeld M. 2011, Stories of the Twentieth Century for the Twenty-First. American Economic Journal: Macroeconomics.

18. Lindert, P.H., Morton, P.J., 1989, "How sovereign debt has worked", In: Sachs, Jeffrey (Ed.). Developing country debt and the world economy. Chicago.

19. Reinhart, C., Rogoff, K., Savastano, M., 2003, "Debt intolerance" NBER Working Paper 9908. National Bureau of Economic Research: Cambridge, Mass.

20. Reinhart, C., Rogoff, K., 2009, This Time Is Different: Eight Centuries of Financial Folly. Princeton University Press.

21. Reis, Ricardo, 2013, "The Mystique Surrounding the Central Bank's Balance Sheet Applied to the European Debt Crisis", NBER WP 18730.

22. Rose, Andrew K., 2005, "One Reason Countries Pay Their Debts: Renegotiation and International Trade," Journal of Development Economics, Vol. 77, No. 1, pp. 189–206.

23. Sachs, J., Cohen, D., 1985. "LDC borrowing with default risk" Kredit und Kapital, Special Issue, 211-235.

24. Sandleris, G. 2008, "Sovereign defaults: information, investment and credit", Journal of International Economics, 76, 267-275.

25. Sturzenegger F., and Jeromim Zettelmeyer, 2006. Debt defaults and lessons from a decade of crises. MIT Press: Cambridge, Mass.

26. Wright, M, Tomz, M, 2007, “Do Countries Default in Bad Times?” Journal of the European Economic Association, Volume 5, Numbers 2-3, pp. 352-60.

27. Yeyati Eduardo and Ugo Panizza, 2011, “The elusive costs of sovereign defaults,” Journal of Development Economics, Elsevier, vol. 94(1), pages 95-105.

28. Yue, Vivian, 2010, “Sovereign Default and Debt Renegotiation”, Journal of International Economics, 80, pp.176-187

Fiscal Discoveries and Sudden Decouplings: Appendix

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January 30, 2013.

Appendix 1: Asymmetric Information Case.

Separating Equilibrium

In this section we describe the way we find a separating equilibrium. Step 1 begins by assuming that the borrowers exerts no-action after a good shock and issues fresh debt after a bad shock and establishes the optimality of all other choices and beliefs. Step 2 confirms the optimality of the period 1 borrowers strategy assumed before.

Step 1:

We assume that the borrower after receiving $\epsilon_1^H = \alpha\tau_1$ decides to follow strategy (N) and after receiving $\epsilon_1^L = -\alpha\tau_1$ decides to follow strategy (I). Total confiscation losses are given by η. In the case of re-issuance, I, a proportion of total confiscation $f = \frac{1}{1+\alpha}$ goes to creditors at time $t = 0$ and a proportion $1 - f = \frac{\alpha}{1+\alpha}$ goes to creditors at time $t = 1$.

1. Lender's beliefs at $t = 1$.

   Lender's beliefs are given by $\mu(H/N) = 1$ and $\mu(L/I) = 1$. The equilibrium is completely revealing.

2. Borrower's strategy at $t = 2$.

   Let us consider first the borrower that received a good shock in the middle period, an H-borrower. His revenue after repayment is $\tau_2 + \rho\epsilon_1^H + \tilde{\epsilon_2} - (1+r_0)\tau_0$.

On the other hand, if he defaults his revenue is $\tau_2 + \rho\epsilon_1^H + \tilde{\epsilon_2} - c(1 + dr_0)\tau_0 - \eta(\tau_2 + \rho\epsilon_1^H + \tilde{\epsilon_2})$. Hence an H borrower repays at the end if and only if

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2 + \rho\alpha\tau_1)\eta + (1 + r_0)\tau_0 - c(1 + dr_0)\tau_0) = H_2 \tag{1}$$

Now, let us consider an L- borrower. His revenue after repayment is $\tau_2 + \rho\epsilon_1^L + \tilde{\epsilon_2} - (1 + r_0)\tau_0 - (1 + r_1)\alpha\tau_1$. On the other hand, if he defaults his revenue is $\tau_2 + \rho\epsilon_1^L + \tilde{\epsilon_2} - c(1 + dr_0)\tau_0 - c(1 + dr_1)\alpha\tau_1 - \eta(\tau_2 + \rho\epsilon_1^L + \tilde{\epsilon_2})$. Hence an L borrower repays at the end if and only if

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2 - \rho\alpha\tau_1)\eta + (1+r_0)\tau_0 + (1+r_1)\alpha\tau_1 - c(1+dr_0)\tau_0 - c(1+dr_1)\alpha\tau_1) = L_2 \tag{2}$$

Before moving on to determine the pricing, notice that when we consider the last period shock, there are six cases:

- Case 1: $H_2 < L_2 < \epsilon_2^L < \epsilon_2^H$. Nobody defaults.
- Case 2: $H_2 < \epsilon_2^L < L_2 < \epsilon_2^H$. H never defaults, L only for a bad shock.
- Case 3: $\epsilon_2^L < H_2 < L_2 < \epsilon_2^H$. Both default for a bad shock.
- Case 4: $H_2 < \epsilon_2^L < \epsilon_2^H < L_2$. H never defaults, L always defaults.
- Case 5: $\epsilon_2^L < H_2 < \epsilon_2^H < L_2$. H defaults only for a bad shock. L always defaults.
- Case 6: $\epsilon_2^L < \epsilon_2^H < H_2 < L_2$. Both always default.

3. Lender's pricing at $t = 1$.

- Case 1: $H_2 < L_2 < \epsilon_2^L < \epsilon_2^H$. In this case

$$r_1 = r_f \tag{3}$$

- Case 2: $H_2 < \epsilon_2^L < L_2 < \epsilon_2^H$. Break-even condition implies that $q(1 + r_1)\alpha\tau_1 + (1-q)(c(1+dr_1)\alpha\tau_1 + (1-f)\eta F_2^{LL}) = (1+r_f)\alpha\tau_1$, where $F_2^{LL} = \tau_2 - \rho\alpha\tau_1 + \epsilon_2^L$. This gives

$$r_1 = \frac{1 + r_f}{q + (1-q)cd} - \frac{(q + (1-q)c)\alpha\tau_1 + (1-q)(1-f)\eta F_2^{LL}}{(q + (1-q)cd)\alpha\tau_1} \tag{4}$$

- Case 3: $\epsilon_2^L < H_2 < L_2 < \epsilon_2^H$. In this case r_1 is given by equation (11).

- Case 4: $H_2 < \epsilon_2^L < \epsilon_2^H < L_2$. In this case r_1 is given by

$$r_1 = \frac{1+r_f}{cd} - \frac{c\alpha\tau_1 + (1-f)\eta EF_2^L}{cd\alpha\tau_1} \tag{5}$$

  where $EF_2^L = qF_2^{LH} + (1-q)F_2^{LL}$, and $F_2^{LH} = \tau_2 - \rho\alpha\tau_1 + \epsilon_2^H$.

- Case 5: $\epsilon_2^L < H_2 < \epsilon_2^H < L_2$. In this case r_1 by equation (12).
- Case 6: $\epsilon_2^L < \epsilon_2^H < H_2 < L_2$. In this case r_1 is given by equation (12).

4. Lender's pricing at $t = 0$.

- Case 1: $H_2 < L_2 < \epsilon_2^L < \epsilon_2^H$. Break-even condition implies that $r_0\tau_0(1 + r_f) + (1 + r_0)\tau_0 = (1 + r_f)^2\tau_0$. Which gives

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0}{(1+r_f)\tau_0 + \tau_0} \tag{6}$$

- Case 2: $H_2 < \epsilon_2^L < L_2 < \epsilon_2^H$. By the same break-even logic we have that

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0(p + (1-p)(q + (1-q)c)) - (1-p)(1-q)f\eta F_2^{LL}}{(1+r_f)\tau_0 + \tau_0(p + (1-p)(q + (1-q)cd))} \tag{7}$$

- Case 3: $\epsilon_2^L < H_2 < L_2 < \epsilon_2^H$.

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0(q + (1-q)c) - (1-q)(\eta pF_2^{HL} + f\eta(1-p)F_2^{LL})}{(1+r_f)\tau_0 + \tau_0(q + (1-q)cd)} \tag{8}$$

- Case 4: $H_2 < \epsilon_2^L < \epsilon_2^H < L_2$.

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0(p + (1-p)c) - (1-p)f\eta EF_2^L}{(1+r_f)\tau_0 + \tau_0(p + (1-p)cd)} \tag{9}$$

- Case 5: $\epsilon_2^L < H_2 < \epsilon_2^H < L_2$.

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0(p(q + (1-q)c) + (1-p)c) - p(1-q)\eta F_2^{HL} - (1-p)f\eta EF_2^L}{(1+r_f)\tau_0 + \tau_0(p(q + (1-q)cd) + (1-p)cd)} \tag{10}$$

- Case 6: $\epsilon_2^L < \epsilon_2^H < H_2 < L_2$.

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0 c - (\eta p E F_2^H + f\eta(1-p)EF_2^L)}{(1+r_f)\tau_0 + \tau_0 cd} \tag{11}$$

Step 2:

We first describe the payoffs of each borrower. Let us start with the L-borrower. His payoffs under no deviations, i.e. when playing the strategy assumed, I, are given by, first, in the case in which the borrower always repays:

$$\tau_1 - r_0\tau_0 + \beta(\tau_2 - \rho\alpha\tau_1 - (1+r_0)\tau_0 - (1+r_1)\alpha\tau_1) + E\tilde{\epsilon_2}) \tag{12}$$

when it repays only for a good shock:

$$\tau_1 - r_0\tau_0 + \beta(qP^R + (1-q)P^D) \tag{13}$$

where $P^R = \tau_2 - \tau_1\rho\alpha + \epsilon_2^H - (1+r_0)\tau_0 - (1+r_1)\alpha\tau_1$ and $P^D = \tau_2 - \tau_1\rho\alpha + \epsilon_2^L - c(1+dr_0)\tau_0 - c(1+dr_1)\alpha\tau_1 - \eta F_2^{LL}$. Finally, when he always defaults

$$\tau_1 - r_0\tau_0 + \beta(\tau_2 - \tau_1\rho\alpha - c(1+dr_0)\tau_0 - c(1+dr_1)\alpha\tau_1 + E\tilde{\epsilon_2} - \eta E F_2^L)) \tag{14}$$

There are two things that change when an L-borrower decides to deviate and play the B strategy after receiving a bad shock in the middle period: the second period repayment threshold and his payoffs. Let us first describe the deviation threshold. By the same logic as before, he repays if and only if

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2 - \rho\alpha\tau_1)\eta + (1+r_0)\tau_0 - c(1+dr_0)\tau_0) = L_2^d \tag{15}$$

His payoffs under deviations, i.e. when playing B, are given by, first, in the case in which the borrower always repays:

$$\tau_1(1-\alpha) - r_0\tau_0 + \beta(\tau_2 - \rho\alpha\tau_1 - (1+r_0)\tau_0 + E\tilde{\epsilon_2}) \tag{16}$$

when it repays only for a good shock:

$$\tau_1(1-\alpha) - r_0\tau_0 + \beta(qP^R + (1-q)P^D) \tag{17}$$

where $P^R = \tau_2 - \tau_1\rho\alpha + \epsilon_2^H - (1+r_0)\tau_0$ and $P^D = \tau_2 - \tau_1\rho\alpha + \epsilon_2^L - c(1+dr_0)\tau_0 - \eta F_2^{LL}$.

Finally, when he always defaults:

$$\tau_1(1-\alpha) - r_0\tau_0 + \beta(\tau_2 - \tau_1\rho\alpha - c(1+dr_0)\tau_0 + E\tilde{\epsilon_2} - \eta EF_2^L)). \tag{18}$$

Next we describe the payoffs of the H-borrower. His payoffs under no deviations, i.e. when playing the strategy assumed, N, are given by, in the case in which the borrower always repays:

$$\tau_1(1+\alpha) - r_0\tau_0 + \beta(\tau_2 + \rho\alpha\tau_1 - (1+r_0)\tau_0 + E\tilde{\epsilon_2}) \tag{19}$$

when it repays only for a good shock:

$$\tau_1(1+\alpha) - r_0\tau_0 + \beta(qP^R + (1-q)P^D) \tag{20}$$

where $P^R = \tau_2+\tau_1\rho\alpha+\epsilon_2^H-(1+r_0)\tau_0$ and $P^D = \tau_2+\tau_1\rho\alpha+\epsilon_2^L-c(1+dr_0)\tau_0-\eta F_2^{HL}$. Finally, when he always defaults:

$$\tau_1(1+\alpha) - r_0\tau_0 + \beta(\tau_2 + \tau_1\rho\alpha - c(1+dr_0)\tau_0 + E\tilde{\epsilon_2} - \eta EF_2^H)). \tag{21}$$

There are two things that change when an H-borrower decides to deviate and play the I strategy after receiving a good shock in the middle period: the second period repayment threshold and his payoffs. Let us first describe the deviation threshold. By the same logic as before, he repays if and only if

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2+\rho\alpha\tau_1)\eta+(1+r_0)\tau_0+(1+r_1)\alpha\tau_1-c(1+dr_0)\tau_0-c(1+dr_1)\alpha\tau_1) = H_2^d \tag{22}$$

His payoffs under deviation, i.e. when playing I, are given by, first, in the case in which the borrower always repays:

$$\tau_1(1+2\alpha) - r_0\tau_0 + \beta(\tau_2 + \rho\alpha\tau_1 - (1+r_0)\tau_0 - (1+r_1)\alpha\tau_1) + E\tilde{\epsilon_2}) \tag{23}$$

when it repays only for a good shock:

$$\tau_1(1+2\alpha) - r_0\tau_0 + \beta(qP^R + (1-q)P^D) \tag{24}$$

where $P^R = \tau_2 + \tau_1\rho\alpha + \epsilon_2^H - (1+r_0)\tau_0 - (1+r_1)\alpha\tau_1$ and $P^D = \tau_2 + \tau_1\rho\alpha + \epsilon_2^L -$

$c(1+dr_0)\tau_0 - c(1+dr_1)\alpha\tau_1 - \eta F_2^{HL}$. Finally, when he always defaults:

$$\tau_1(1+2\alpha) - r_0\tau_0 + \beta(\tau_2 + \tau_1\rho\alpha - c(1+dr_0)\tau_0 - c(1+dr_1)\alpha\tau_1 + E\tilde{\epsilon_2} - \eta EF_2^H)) \quad (25)$$

In order to check for the existence of the separating equilibrium, we need to check for possible deviations for each type. We have two cases in terms of thresholds: I) $H_2 < H_2^d < L_2^d < L_2$ and II) $H_2 < L_2^d < H_2^d < L_2$. Note, however, that for pricing we still just need to consider the six original cases since investors cannot observe deviations. However, in order to check for deviations some of these six cases may get subdivided in sub-cases, when we consider also the deviation thresholds. Tables 1 and 2 show all the possible cases that we need to check. Clearly, there will be parameter values that can sustain a separating equilibrium. This is ultimately a numerical question, which we discuss extensively in section 4. Finally, we need to check that parameter values are such that budget constraint holds at the final period. Repayments and after default payment obligations need to be feasible.

Pooling Equilibrium

Step 1:

We assume that the borrower after receiving $\epsilon_1^H = \alpha\tau_1$ and $\epsilon_1^L = -\alpha\tau_1$ the borrower decides to follow strategy N.

1. Lender's beliefs at $t = 1$.

   Lender's beliefs are given by the prior distribution, so $\mu(H) = p$ and $\mu(L) = 1 - p$.

2. Borrower's strategy at $t = 2$

   Let us consider first the H-borrower. By the same logic as before we have that

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2 + \rho\alpha\tau_1)\eta + (1+r_0)\tau_0 - c(1+dr_0)\tau_0) = H_2 \quad (26)$$

   Now, let us consider an L-borrower. L repays if and only if

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2 - \rho\alpha\tau_1)\eta + (1+r_0)\tau_0 - c(1+dr_0)\tau_0) = L_2 \quad (27)$$

3. Lender's pricing at $t = 1$.

   There is no credit market at 1.

4. Lender's pricing at $t = 0$.

   - Case 1: $H_2 < L_2 < \epsilon_2^L < \epsilon_2^H$. Break-even condition implies that $r_0\tau_0(1 + r_f) + (1 + r_0)\tau_0 = (1 + r_f)^2\tau_0$. Which gives

$$r_0 = \frac{(1 + r_f)^2\tau_0 - \tau_0}{(1 + r_f)\tau_0 + \tau_0} \tag{28}$$

   - Case 2: $H_2 < \epsilon_2^L < L_2 < \epsilon_2^H$. By the same break-even logic we have that

$$r_0 = \frac{(1 + r_f)^2\tau_0 - \tau_0(p + (1 - p)(q + (1 - q)c)) - (1 - p)(1 - q)\eta F_2^{LL}}{(1 + r_f)\tau_0 + \tau_0(p + (1 - p)(q + (1 - q)cd))} \tag{29}$$

   - Case 3: $\epsilon_2^L < H_2 < L_2 < \epsilon_2^H$. In this case, r_0 is given by

$$r_0 = \frac{(1 + r_f)^2\tau_0 - \tau_0(q + (1 - q)c) - (1 - q)\eta(pF_2^{HL} + (1 - p)F_2^{LL})}{(1 + r_f)\tau_0 + \tau_0(q + (1 - q)cd)} \tag{30}$$

   - Case 4: $H_2 < \epsilon_2^L < \epsilon_2^H < L_2$. In this case, by the same logic we have that

$$r_0 = \frac{(1 + r_f)^2\tau_0 - \tau_0(p + (1 - p)c) - (1 - p)\eta EF_2^L}{(1 + r_f)\tau_0 + \tau_0(p + (1 - p)cd)} \tag{31}$$

   - Case 5: $\epsilon_2^L < H_2 < \epsilon_2^H < L_2$. In this case r_0 is given by

$$r_0 = \frac{(1 + r_f)^2\tau_0 - \tau_0(p(q + (1 - q)c) + (1 - p)c) - p(1 - q)\eta F_2^{HL} - (1 - p)f^I\eta EF_2^L}{(1 + r_f)\tau_0 + \tau_0(p(q + (1 - q)cd) + (1 - p)cd)} \tag{32}$$

   - Case 6: $\epsilon_2^L < \epsilon_2^H < H_2 < L_2$. In this case r_0 is given by

$$r_0 = \frac{(1 + r_f)^2\tau_0 - \tau_0 c - \eta(pEF_2^H + (1 - p)EF_2^L)}{(1 + r_f)\tau_0 + \tau_0 cd} \tag{33}$$

Step 2:

We first describe the payoffs of each borrower. Let us start with the L-borrower. His payoffs under no deviations, i.e. when playing the strategy assumed, N, are given by, first, in the case in which the borrower always repays:

$$\tau_1(1 - \alpha) - r_0\tau_0 + \beta(\tau_2 - \rho\alpha\tau_1 - (1 + r_0)\tau_0) + E\tilde{\epsilon}_2) \tag{34}$$

when it repays only for a good shock:

$$\tau_1(1-\alpha) - r_0\tau_0 + \beta(qP^R + (1-q)P^D) \quad (35)$$

where $P^R = \tau_2 - \tau_1\rho\alpha + \epsilon_2^H - (1+r_0)\tau_0$ and $P^D = \tau_2 - \tau_1\rho\alpha + \epsilon_2^L - c(1+dr_0)\tau_0 - \eta F_2^{LL}$. Finally, when he always defaults

$$\tau_1(1-\alpha) - r_0\tau_0 + \beta(\tau_2 - \tau_1\rho\alpha - c(1+dr_0)\tau_0 + E\tilde{\epsilon_2} - \eta E F_2^L)) \quad (36)$$

There are two things that change when an L-borrower decides to deviate and play the I strategy after receiving a bad shock in the middle period: the second period repayment threshold and his payoffs. Let us first describe the deviation threshold. Hence, he repays if and only if

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2 - \rho\alpha\tau_1)\eta + (1+r_0)\tau_0 + (1+r_1)\alpha\tau_1 - c(1+dr_0)\tau_0 - c(1+dr_1)\alpha\tau_1) = L_2^d \quad (37)$$

where r_1 is given by the value in the separating proof.

His payoffs under deviations, i.e. when playing I, are given by, first, in the case in which the borrower always repays:

$$\tau_1 - r_0\tau_0 + \beta(\tau_2 - \rho\alpha\tau_1 - (1+r_0)\tau_0 - (1+r_1)\alpha\tau_1 + E\tilde{\epsilon_2}) \quad (38)$$

when it repays only for a good shock:

$$\tau_1 - r_0\tau_0 + \beta(qP^R + (1-q)P^D) \quad (39)$$

where $P^R = \tau_2 - \tau_1\rho\alpha + \epsilon_2^H - (1+r_0)\tau_0 - (1+r_1)\tau_1\alpha$ and $P^D = \tau_2 - \tau_1\rho\alpha + \epsilon_2^L - c(1+dr_0)\tau_0 - c(1+dr_1)\tau_1\alpha - \eta F_2^{LL}$. Finally, when he always defaults:

$$\tau_1 - r_0\tau_0 + \beta(\tau_2 - \tau_1\rho\alpha - c(1+dr_0)\tau_0 - c(1+dr_1)\tau_1\alpha + E\tilde{\epsilon_2} - \eta E F_2^L)). \quad (40)$$

Next we describe the payoffs of the H-borrower. His payoffs under no deviations, i.e. when playing the strategy assumed, N, are given by, in the case in which the borrower always repays:

$$\tau_1(1+\alpha) - r_0\tau_0 + \beta(\tau_2 + \rho\alpha\tau_1 - (1+r_0)\tau_0 + E\tilde{\epsilon_2}) \quad (41)$$

when it repays only for a good shock:

$$\tau_1(1+\alpha) - r_0\tau_0 + \beta(qP^R + (1-q)P^D) \tag{42}$$

where $P^R = \tau_2 + \tau_1\rho\alpha + \epsilon_2^H - (1+r_0)\tau_0$ and $P^D = \tau_2 + \tau_1\rho\alpha + \epsilon_2^L - c(1+dr_0)\tau_0 - \eta F_2^{HL}$. Finally, when he always defaults:

$${}_1(1+\alpha) - r_0\tau_0 + \beta(\tau_2 + \tau_1\rho\alpha - c(1+dr_0)\tau_0 + E\tilde{\epsilon_2} - \eta EF_2^H) \tag{43}$$

There are two things that change when an H-borrower decides to deviate and play the I strategy after receiving a good shock in the middle period: the second period repayment threshold and his payoffs. Let us first describe the deviation threshold. Hence, he repays if and only if

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2+\rho\alpha\tau_1)\eta+(1+r_0)\tau_0+(1+r_1)\alpha\tau_1-c(1+dr_0)\tau_0-c(1+dr_1)\alpha\tau_1) = H_2^d \tag{44}$$

His payoffs under deviation, i.e. when playing I, are given by, first, in the case in which the borrower always repays:

$$\tau_1(1+2\alpha) - r_0\tau_0 + \beta(\tau_2 + \rho\alpha\tau_1 - (1+r_0)\tau_0 - (1+r_1)\alpha\tau_1 + E\tilde{\epsilon_2}) \tag{45}$$

when it repays only for a good shock:

$$\tau_1(1+2\alpha) - r_0\tau_0 + \beta(q + (1-q)P^D) \tag{46}$$

where $P^R = \tau_2 + \tau_1\rho\alpha + \epsilon_2^H - (1+r_0)\tau_0 - (1+r_1)\tau_1\alpha$ and $P^D = \tau_2 + \tau_1\rho\alpha + \epsilon_2^L - c(1+dr_0)\tau_0 - c(1+dr_1)\tau_1\alpha - \eta F_2^{HL}$. Finally, when he always defaults:

$$\tau_1(1+2\alpha) - r_0\tau_0 + \beta(\tau_2 + \tau_1\rho\alpha - c(1+dr_0)\tau_0 - c(1+dr_1)\tau_1\alpha + E\tilde{\epsilon_2} - \eta EF_2^H)). \tag{47}$$

In order to check for the existence of a N-pooling equilibrium, we need to check for possible deviations for each type.This is analogous to procedure in the first part of the theorem. All cases are described in table 3. Finally, the same check regarding end of the game budget constraint need to be checked.

Appendix 2: Symmetric Information Case.

In this section we describe the way we find a SPNE (subgame perfect Nash Edquilibrium). The key thing is that now lenders do not form beliefs and the interest rates are a function of the shock realization and not of the borrower action as before. Step 1 begins by assuming that the borrower issues fresh debt always and establishes the optimality of all other choices and beliefs. Step 2 confirms the optimality of the period 1 borrowers strategy assumed before.

Step 1:

We assume that the borrower after receiving $\epsilon_1^H = \alpha\tau_1$ decides to follow strategy (I) and after receiving $\epsilon_1^L = -\alpha\tau_1$ decides to follow strategy (I). Total confiscation losses are given by η. In the case of re-issuance, I, a proportion of total confiscation $f = \frac{1}{1+\alpha}$ goes to creditors at time $t = 0$ and a proportion $1 - f = \frac{\alpha}{1+\alpha}$ goes to creditors at time $t = 1$.

1. Borrower's strategy at $t = 2$.

   Let us consider first the borrower that received a good shock in the middle period, an H-borrower. His revenue after repayment is $\tau_2 + \rho\epsilon_1^H + \tilde{\epsilon_2} - (1 + r_0)\tau_0 - (1 + r_1^H)\alpha\tau_1$. On the other hand, if he defaults his revenue is $\tau_2 + \rho\epsilon_1^H + \tilde{\epsilon_2} - c(1 + dr_0)\tau_0 - c(1 + dr_1^H)\alpha\tau_1 - \eta(\tau_2 + \rho\epsilon_1^H + \tilde{\epsilon_2})$. Hence an H borrower repays at the end if and only if

   $$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2+\rho\alpha\tau_1)\eta+(1+r_0)\tau_0+(1+r_1^H)\alpha\tau_1-c(1+dr_0)\tau_0-c(1+dr_1^H)\alpha\tau_1) = H_2 \tag{48}$$

   Now, let us consider an L- borrower. By the same logic we have that

   $$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2-\rho\alpha\tau_1)\eta+(1+r_0)\tau_0+(1+r_1^L)\alpha\tau_1-c(1+dr_0)\tau_0-c(1+dr_1^L)\alpha\tau_1) = L_2 \tag{49}$$

2. Lender's pricing at $t = 1$.

   - Case 1: $H_2 < L_2 < \epsilon_2^L < \epsilon_2^H$. In this case

     $$r_1^H = r_1^L = r_f \tag{50}$$

   - Case 2: $H_2 < \epsilon_2^L < L_2 < \epsilon_2^H$. r_1^H is given by equation (50). Break-even condition implies that $q(1+r_1^L)\alpha\tau_1+(1-q)(c(1+dr_1^L)\alpha\tau_1+(1-f)\eta F_2^{LL}) =$

$(1+r_f)\alpha\tau_1$, where $F_2^{LL} = \tau_2 - \rho\alpha\tau_1 + \epsilon_2^L$. This gives

$$r_1^L = \frac{1+r_f}{q+(1-q)cd} - \frac{(q+(1-q)c)\alpha\tau_1 + (1-q)(1-f)\eta F_2^{LL}}{(q+(1-q)cd)\alpha\tau_1} \tag{51}$$

- Case 3: $\epsilon_2^L < H_2 < L_2 < \epsilon_2^H$. In this case r_1^L is given by equation (51). And

$$r_1^H = \frac{1+r_f}{q+(1-q)cd} - \frac{(q+(1-q)c)\alpha\tau_1 + (1-q)(1-f)\eta F_2^{HL}}{(q+(1-q)cd)\alpha\tau_1} \tag{52}$$

  where $F_2^{HL} = \tau_2 + \rho\alpha\tau_1 + \epsilon_2^L$.

- Case 4: $H_2 < \epsilon_2^L < \epsilon_2^H < L_2$. In this case r_1^H is given by equation (50), and r_1^L is given by

$$r_1^L = \frac{1+r_f}{cd} - \frac{c\alpha\tau_1 + (1-f)\eta EF_2^L}{cd\alpha\tau_1} \tag{53}$$

  where $EF_2^L = qF_2^{LH} + (1-q)F_2^{LL}$, and $F_2^{LH} = \tau_2 - \rho\alpha\tau_1 + \epsilon_2^H$.

- Case 5: $\epsilon_2^L < H_2 < \epsilon_2^H < L_2$. In this case $r_1^H$is given by equation (52) and r_1^L is given by equation (53).

- Case 6: $\epsilon_2^L < \epsilon_2^H < H_2 < L_2$. In this case r_1^L is given by equation (53) and $r_1^H$is given by

$$r_1^H = \frac{1+r_f}{cd} - \frac{c\alpha\tau_1 + (1-f)\eta EF_2^H}{cd\alpha\tau_1} \tag{54}$$

  where $EF_2^H = qF_2^{HH} + (1-q)F_2^{HL}$, and $F_2^{HH} = \tau_2 + \rho\alpha\tau_1 + \epsilon_2^H$

3. Lender's pricing at $t = 0$.

- Case 1: $H_2 < L_2 < \epsilon_2^L < \epsilon_2^H$. Break-even condition implies that $r_0\tau_0(1 + r_f) + (1+r_0)\tau_0 = (1+r_f)^2\tau_0$. Which gives

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0}{(1+r_f)\tau_0 + \tau_0} \tag{55}$$

- Case 2: $H_2 < \epsilon_2^L < L_2 < \epsilon_2^H$. By the same break-even logic we have that

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0(p+(1-p)(q+(1-q)c)) - (1-p)(1-q)f\eta F_2^{LL}}{(1+r_f)\tau_0 + \tau_0(p+(1-p)(q+(1-q)cd))} \tag{56}$$

- Case 3: $\epsilon_2^L < H_2 < L_2 < \epsilon_2^H$.

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0(q+(1-q)c) - (1-q)(f\eta p F_2^{HL} + f\eta(1-p)F_2^{LL})}{(1+r_f)\tau_0 + \tau_0(q+(1-q)cd)} \tag{57}$$

- Case 4: $H_2 < \epsilon_2^L < \epsilon_2^H < L_2$.

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0(p+(1-p)c) - (1-p)f\eta EF_2^L}{(1+r_f)\tau_0 + \tau_0(p+(1-p)cd)} \tag{58}$$

- Case 5: $\epsilon_2^L < H_2 < \epsilon_2^H < L_2$.

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0(p(q+(1-q)c)+(1-p)c) - p(1-q)f\eta F_2^{HL} - (1-p)f\eta EF_2^L}{(1+r_f)\tau_0 + \tau_0(p(q+(1-q)cd)+(1-p)cd)} \tag{59}$$

- Case 6: $\epsilon_2^L < \epsilon_2^H < H_2 < L_2$.

$$r_0 = \frac{(1+r_f)^2\tau_0 - \tau_0 c - (f\eta p EF_2^H + f\eta(1-p)EF_2^L)}{(1+r_f)\tau_0 + \tau_0 cd} \tag{60}$$

Step 2:

We first describe the payoffs of each borrower. Let us start with the L-borrower. His payoffs under no deviations, i.e. when playing the strategy assumed, I, are given by, first, in the case in which the borrower always repays:

$$\tau_1 - r_0\tau_0 + \beta(\tau_2 - \rho\alpha\tau_1 - (1+r_0)\tau_0 - (1+r_1^L)\alpha\tau_1) + E\tilde{\epsilon_2}) \tag{61}$$

when it repays only for a good shock:

$$\tau_1 - r_0\tau_0 + \beta(qP^R + (1-q)P^D) \tag{62}$$

where $P^R = \tau_2 - \tau_1\rho\alpha + \epsilon_2^H - (1+r_0)\tau_0 - (1+r_1^L)\alpha\tau_1$ and $P^D = \tau_2 - \tau_1\rho\alpha + \epsilon_2^L - c(1+dr_0)\tau_0 - c(1+dr_1^L)\alpha\tau_1 - \eta F_2^{LL}$. Finally, when he always defaults

$$\tau_1 - r_0\tau_0 + \beta(\tau_2 - \tau_1\rho\alpha - c(1+dr_0)\tau_0 - c(1+dr_1^L)\alpha\tau_1 + E\tilde{\epsilon}_2 - \eta EF_2^L)) \tag{63}$$

There are two things that change when an L-borrower decides to deviate and play the N strategy: the second period repayment threshold and his payoffs. Let us first describe the deviation threshold. By the same logic as before, he repays if and only if

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2 - \rho\alpha\tau_1)\eta + (1 + r_0)\tau_0 - c(1 + dr_0)\tau_0) = L_2^d \tag{64}$$

His payoffs under deviations, i.e. when playing N, are given by, first, in the case in which the borrower always repays:

$$\tau_1(1 - \alpha) - r_0\tau_0 + \beta(\tau_2 - \rho\alpha\tau_1 - (1 + r_0)\tau_0 + E\tilde{\epsilon_2}) \tag{65}$$

when it repays only for a good shock:

$$\tau_1(1 - \alpha) - r_0\tau_0 + \beta(qP^R + (1 - q)P^D) \tag{66}$$

where $P^R = \tau_2 - \tau_1\rho\alpha + \epsilon_2^H - (1+r_0)\tau_0$ and $P^D = \tau_2 - \tau_1\rho\alpha + \epsilon_2^L - c(1+dr_0)\tau_0 - \eta F_2^{LL}$. Finally, when he always defaults:

$$\tau_1(1 - \alpha) - r_0\tau_0 + \beta(\tau_2 - \tau_1\rho\alpha - c(1 + dr_0)\tau_0 + E\tilde{\epsilon_2} - \eta EF_2^L)). \tag{67}$$

Next we describe the payoffs of the H-borrower. His payoffs under no deviations, i.e. when playing the strategy assumed, I, are given by, in the case in which the borrower always repays:

$$\tau_1(1 + 2\alpha) - r_0\tau_0 + \beta(\tau_2 + \rho\alpha\tau_1 - (1 + r_0)\tau_0 - (1 + r_1^H)\alpha\tau_1) + E\tilde{\epsilon_2}) \tag{68}$$

when it repays only for a good shock:

$$\tau_1(1 + 2\alpha) - r_0\tau_0 + \beta(qP^R + (1 - q)P^D) \tag{69}$$

where $P^R = \tau_2 + \tau_1\rho\alpha + \epsilon_2^H - (1 + r_0)\tau_0 - (1 + r_1^H)\alpha\tau_1$ and $P^D = \tau_2 + \tau_1\rho\alpha + \epsilon_2^L - c(1 + dr_0)\tau_0 - c(1 + dr_1^H)\alpha\tau_1 - \eta F_2^{HL}$.

Finally, when he always defaults:

$$\tau_1(1 + 2\alpha) - r_0\tau_0 + \beta(\tau_2 + \tau_1\rho\alpha - c(1 + dr_0)\tau_0 - c(1 + dr_1^H)\alpha\tau_1 + E\tilde{\epsilon_2} - \eta EF_2^H)) \tag{70}$$

There are two things that change when an H-borrower decides to deviate and play the N strategy after receiving a good shock in the middle period: the second period repayment threshold and his payoffs. Let us first describe the deviation threshold.

By the same logic as before, he repays if and only if

$$\tilde{\epsilon_2} \geq (1/\eta)(-(\tau_2 + \rho\alpha\tau_1)\eta + (1 + r_0)\tau_0 - c(1 + dr_0)\tau_0) = H_2^d \quad (71)$$

His payoffs under deviation, i.e. when playing I, are given by, first, in the case in which the borrower always repays:

$$\tau_1(1 + \alpha) - r_0\tau_0 + \beta(\tau_2 + \rho\alpha\tau_1 - (1 + r_0)\tau_0 + E\tilde{\epsilon_2}) \quad (72)$$

when it repays only for a good shock:

$$\tau_1(1 + \alpha) - r_0\tau_0 + \beta(qP^R + (1 - q)P^D) \quad (73)$$

where $P^R = \tau_2+\tau_1\rho\alpha+\epsilon_2^H-(1+r_0)\tau_0$ and $P^D = \tau_2+\tau_1\rho\alpha+\epsilon_2^L-c(1+dr_0)\tau_0-\eta F_2^{HL}$. Finally, when he always defaults:

$$\tau_1(1 + \alpha) - r_0\tau_0 + \beta(\tau_2 + \tau_1\rho\alpha - c(1 + dr_0)\tau_0 + E\tilde{\epsilon_2} - \eta E F_2^H)). \quad (74)$$

Finally, in order to check for the existence of a separating equilibrium, we need to check for possible deviations for each type. We have two cases in terms of thresholds: I) $H_2^d < H_2 < L_2^d < L_2$ and II) $H_2^d < L_2^d < H_2 < L_2$. Note, however, that for pricing we still just need to consider the six original cases since investors cannot observe deviations. However, in order to check for deviations some of these six cases may get subdivided in sub-cases, when we consider also the deviation thresholds. Tables 4 and 5 show all the possible cases that we need to check. Feasible constraints should be checked.

Table 1: Separating Equilibrium Deviation Conditions.

Case I) $H_2 < H^d_2 < L^d_2 < L_2$

REGION	H: CONDITION FOR NO DEVIATION	L: CONDITION FOR NO DEVIATION
1.1) $H_2 < H^d_2 < L^d_2 < L_2 < \varepsilon^L_2 < \varepsilon^H_2$	(19) ≥ (23)	(12) ≥ (16)
2.1) $H_2 < H^d_2 < L^d_2 < \varepsilon^L_2 < L_2 < \varepsilon^H_2$	(19) ≥ (23)	(13) ≥ (16)
2.2) $H_2 < H^d_2 < \varepsilon^L_2 < L^d_2 < L_2 < \varepsilon^H_2$	(19) ≥ (23)	(13) ≥ (17)
2.3) $H_2 < \varepsilon^L_2 < H^d_2 < L^d_2 < L_2 < \varepsilon^H_2$	(19) ≥ (24)	(13) ≥ (17)
3.1) $\varepsilon^L_2 < H_2 < H^d_2 < L^d_2 < L_2 < \varepsilon^H_2$	(20) ≥ (24)	(13) ≥ (17)
4.1) $H_2 < H^d_2 < L^d_2 < \varepsilon^L_2 < \varepsilon^H_2 < L_2$	(19) ≥ (23)	(14) ≥ (16)
4.2) $H_2 < H^d_2 < \varepsilon^L_2 < L^d_2 < \varepsilon^H_2 < L_2$	(19) ≥ (23)	(14) ≥ (17)
4.3) $H_2 < \varepsilon^L_2 < H^d_2 < L^d_2 < \varepsilon^H_2 < L_2$	(19) ≥ (24)	(14) ≥ (17)
4.4) $H_2 < H^d_2 < \varepsilon^L_2 < \varepsilon^H_2 < L^d_2 < L_2$	(19) ≥ (23)	(14) ≥ (18)
4.5) $H_2 < \varepsilon^L_2 < H^d_2 < \varepsilon^H_2 < L^d_2 < L_2$	(19) ≥ (24)	(14) ≥ (18)
4.6) $H_2 < \varepsilon^L_2 < \varepsilon^H_2 < H^d_2 < L^d_2 < L_2$	(19) ≥ (25)	(14) ≥ (18)
5.1) $\varepsilon^L_2 < H_2 < H^d_2 < L^d_2 < \varepsilon^H_2 < L_2$	(20) ≥ (24)	(14) ≥ (17)
5.2) $\varepsilon^L_2 < H_2 < H^d_2 < \varepsilon^H_2 < L^d_2 < L_2$	(20) ≥ (24)	(14) ≥ (18)
5.3) $\varepsilon^L_2 < H_2 < \varepsilon^H_2 < H^d_2 < L^d_2 < L_2$	(20) ≥ (25)	(14) ≥ (18)
6.1) $\varepsilon^L_2 < \varepsilon^H_2 < H_2 < H^d_2 < L^d_2 < L_2$	(21) ≥ (25)	(14) ≥ (18)

Table 2: Separating Equilibrium Deviation Conditions.

Case II) $H_2 < L^d_2 < H^d_2 < L_2$

REGION	H: CONDITION FOR NO DEVIATION	L: CONDITION FOR NO DEVIATION
1.1) $H_2 < L^d_2 < H^d_2 < L_2 < \varepsilon^L_2 < \varepsilon^H_2$	(19) ≥ (23)	(12) ≥ (16)
2.1) $H_2 < L^d_2 < H^d_2 < \varepsilon^L_2 < L_2 < \varepsilon^H_2$	(19) ≥ (23)	(13) ≥ (16)
2.2) $H_2 < L^d_2 < \varepsilon^L_2 < H^d_2 < L_2 < \varepsilon^H_2$	(19) ≥ (24)	(13) ≥ (16)
2.3) $H_2 < \varepsilon^L_2 < L^d_2 < H^d_2 < L_2 < \varepsilon^H_2$	(19) ≥ (24)	(13) ≥ (17)
3.1) $\varepsilon^L_2 < H_2 < L^d_2 < H^d_2 < L_2 < \varepsilon^H_2$	(20) ≥ (24)	(13) ≥ (17)
4.1) $H_2 < L^d_2 < H^d_2 < \varepsilon^L_2 < \varepsilon^H_2 < L_2$	(19) ≥ (23)	(14) ≥ (18)
4.2) $H_2 < L^d_2 < \varepsilon^L_2 < H^d_2 < \varepsilon^H_2 < L_2$	(19) ≥ (24)	(14) ≥ (16)
4.3) $H_2 < \varepsilon^L_2 < L^d_2 < H^d_2 < \varepsilon^H_2 < L_2$	(19) ≥ (24)	(14) ≥ (17)
4.4) $H_2 < L^d_2 < \varepsilon^L_2 < \varepsilon^H_2 < H^d_2 < L_2$	(19) ≥ (25)	(14) ≥ (16)
4.5) $H_2 < \varepsilon^L_2 < L^d_2 < \varepsilon^H_2 < H^d_2 < L_2$	(19) ≥ (25)	(14) ≥ (17)
4.6) $H_2 < \varepsilon^L_2 < \varepsilon^H_2 < L^d_2 < H^d_2 < L_2$	(19) ≥ (25)	(14) ≥ (18)
5.1) $\varepsilon^L_2 < H_2 < L^d_2 < H^d_2 < \varepsilon^H_2 < L_2$	(20) ≥ (24)	(14) ≥ (17)
5.2) $\varepsilon^L_2 < H_2 < L^d_2 < \varepsilon^H_2 < H^d_2 < L_2$	(20) ≥ (25)	(14) ≥ (17)
5.3) $\varepsilon^L_2 < H_2 < \varepsilon^H_2 < L^d_2 < H^d_2 < L_2$	(20) ≥ (25)	(14) ≥ (18)
6.1) $\varepsilon^L_2 < \varepsilon^H_2 < H_2 < L^d_2 < H^d_2 < L_2$	(21) ≥ (25)	(14) ≥ (18)

Table 3: Pooling Equilibrium Deviation Conditions.

Case II) $H_2 < H^d_2 < L_2 < L^d_2$

REGION	H: CONDITION FOR NO DEVIATION	L: CONDITION FOR NO DEVIATION
1.1) $H_2 < H^d_2 < L_2 < L^d_2 < \varepsilon^L_2 < \varepsilon^H_2$	(41) ≥ (45)	(34) ≥ (38)
1.2) $H_2 < H^d_2 < L_2 < \varepsilon^L_2 < L^d_2 < \varepsilon^H_2$	(41) ≥ (45)	(34) ≥ (39)
1.3) $H_2 < H^d_2 < L_2 < \varepsilon^L_2 < \varepsilon^H_2 < L^d_2$	(41) ≥ (45)	(34) ≥ (40)
2.1) $H_2 < H^d_2 < \varepsilon^L_2 < L_2 < L^d_2 < \varepsilon^H_2$	(41) ≥ (45)	(35) ≥ (39)
2.2) $H_2 < \varepsilon^L_2 < H^d_2 < L_2 < L^d_2 < \varepsilon^H_2$	(41) ≥ (46)	(35) ≥ (39)
2.3) $H_2 < H^d_2 < \varepsilon^L_2 < L_2 < \varepsilon^H_2 < L^d_2$	(41) ≥ (45)	(35) ≥ (40)
2.4) $H_2 < \varepsilon^L_2 < H^d_2 < L_2 < \varepsilon^H_2 < L^d_2$	(41) ≥ (46)	(35) ≥ (40)
3.1) $\varepsilon^L_2 < H_2 < H^d_2 < L_2 < L^d_2 < \varepsilon^H_2$	(42) ≥ (46)	(35) ≥ (39)
3.2) $\varepsilon^L_2 < H_2 < H^d_2 < L_2 < \varepsilon^H_2 < L^d_2$	(42) ≥ (46)	(35) ≥ (40)
4.1) $H_2 < H^d_2 < \varepsilon^L_2 < \varepsilon^H_2 < L_2 < L^d_2$	(41) ≥ (45)	(36) ≥ (40)
4.2) $H_2 < \varepsilon^L_2 < H^d_2 < \varepsilon^H_2 < L_2 < L^d_2$	(41) ≥ (46)	(36) ≥ (40)
4.3) $H_2 < \varepsilon^L_2 < \varepsilon^H_2 < H^d_2 < L_2 < L^d_2$	(41) ≥ (47)	(36) ≥ (40)
5.1) $\varepsilon^L_2 < H_2 < H^d_2 < \varepsilon^H_2 < L_2 < L^d_2$	(42) ≥ (46)	(36) ≥ (40)
5.2) $\varepsilon^L_2 < H_2 < \varepsilon^H_2 < H^d_2 < L_2 < L^d_2$	(42) ≥ (47)	(36) ≥ (40)
6.1) $\varepsilon^L_2 < \varepsilon^H_2 < H_2 < H^d_2 < L_2 < L^d_2$	(43) ≥ (47)	(36) ≥ (40)

Table 4: SPNE Equilibrium.

Case I) $H^d_2 < H_2 < L^d_2 < L_2$

REGION	H: CONDITION FOR NO DEVIATION	L: CONDITION FOR NO DEVIATION
1.1) $H^d_2 < H_2 < L^d_2 < L_2 < \varepsilon^L_2 < \varepsilon^H_2$	(68) ≥ (72)	(61) ≥ (65)
2.1) $H^d_2 < H_2 < L^d_2 < \varepsilon^L_2 < L_2 < \varepsilon^H_2$	(68) ≥ (72)	(62) ≥ (65)
2.2) $H^d_2 < H_2 < \varepsilon^L_2 < L^d_2 < L_2 < \varepsilon^H_2$	(68) ≥ (72)	(62) ≥ (66)
3.1) $H^d_2 < \varepsilon^L_2 < H_2 < L^d_2 < L_2 < \varepsilon^H_2$	(69) ≥ (72)	(62) ≥ (66)
3.2) $\varepsilon^L_2 < H^d_2 < H_2 < L^d_2 < L_2 < \varepsilon^H_2$	(69) ≥ (73)	(62) ≥ (66)
4.1) $H^d_2 < H_2 < L^d_2 < \varepsilon^L_2 < \varepsilon^H_2 < L_2$	(68) ≥ (72)	(63) ≥ (65)
4.2) $H^d_2 < H_2 < \varepsilon^L_2 < L^d_2 < \varepsilon^H_2 < L_2$	(68) ≥ (72)	(63) ≥ (66)
4.3) $H^d_2 < H_2 < \varepsilon^L_2 < \varepsilon^H_2 < L^d_2 < L_2$	(68) ≥ (72)	(63) ≥ (67)
5.1) $H^d_2 < \varepsilon^L_2 < H_2 < L^d_2 < \varepsilon^H_2 < L_2$	(69) ≥ (72)	(63) ≥ (66)
5.2) $\varepsilon^L_2 < H^d_2 < H_2 < L^d_2 < \varepsilon^H_2 < L_2$	(69) ≥ (73)	(63) ≥ (66)
5.3) $H^d_2 < \varepsilon^L_2 < H_2 < \varepsilon^H_2 < L^d_2 < L_2$	(69) ≥ (72)	(63) ≥ (67)
5.4) $\varepsilon^L_2 < H^d_2 < H_2 < \varepsilon^H_2 < L^d_2 < L_2$	(69) ≥ (73)	(63) ≥ (67)
6.1) $H^d_2 < \varepsilon^L_2 < \varepsilon^H_2 < H_2 < L^d_2 < L_2$	(70) ≥ (72)	(63) ≥ (67)
6.2) $\varepsilon^L_2 < H^d_2 < \varepsilon^H_2 < H_2 < L^d_2 < L_2$	(70) ≥ (73)	(63) ≥ (67)
6.3) $\varepsilon^L_2 < \varepsilon^H_2 < H^d_2 < H_2 < L^d_2 < L_2$	(70) ≥ (74)	(63) ≥ (67)

Table 5: SPNE Equilibrium.

Case II) $H^d_2 < L^d_2 < H_2 < L_2$

REGION	H: CONDITION FOR NO DEVIATION	L: CONDITION FOR NO DEVIATION
1.1) $H^d_2 < L^d_2 < H_2 < L_2 < \varepsilon^L_2 < \varepsilon^H_2$	(68) ≥ (72)	(61) ≥ (65)
2.1) $H^d_2 < L^d_2 < H_2 < \varepsilon^L_2 < L_2 < \varepsilon^H_2$	(68) ≥ (72)	(62) ≥ (65)
3.1) $H^d_2 < L^d_2 < \varepsilon^L_2 < H_2 < L_2 < \varepsilon^H_2$	(69) ≥ (72)	(62) ≥ (65)
3.2) $H^d_2 < \varepsilon^L_2 < L^d_2 < H_2 < L_2 < \varepsilon^H_2$	(69) ≥ (72)	(62) ≥ (66)
3.3) $\varepsilon^L_2 < H^d_2 < L^d_2 < H_2 < L_2 < \varepsilon^H_2$	(69) ≥ (73)	(62) ≥ (66)
4.1) $H^d_2 < L^d_2 < H_2 < \varepsilon^L_2 < \varepsilon^H_2 < L_2$	(68) ≥ (72)	(63) ≥ (65)
5.1) $H^d_2 < L^d_2 < \varepsilon^L_2 < H_2 < \varepsilon^H_2 < L_2$	(69) ≥ (72)	(63) ≥ (65)
5.2) $H^d_2 < \varepsilon^L_2 < L^d_2 < H_2 < \varepsilon^H_2 < L_2$	(69) ≥ (72)	(63) ≥ (66)
5.3) $\varepsilon^L_2 < H^d_2 < L^d_2 < H_2 < \varepsilon^H_2 < L_2$	(69) ≥ (73)	(63) ≥ (66)
6.1) $H^d_2 < L^d_2 < \varepsilon^L_2 < \varepsilon^H_2 < H_2 < L_2$	(70) ≥ (72)	(63) ≥ (65)
6.2) $H^d_2 < \varepsilon^L_2 < L^d_2 < \varepsilon^H_2 < H_2 < L_2$	(70) ≥ (72)	(63) ≥ (66)
6.3) $\varepsilon^L_2 < H^d_2 < L^d_2 < \varepsilon^H_2 < H_2 < L_2$	(70) ≥ (73)	(63) ≥ (66)
6.4) $H^d_2 < \varepsilon^L_2 < \varepsilon^H_2 < L^d_2 < H_2 < L_2$	(70) ≥ (72)	(63) ≥ (67)
6.5) $\varepsilon^L_2 < H^d_2 < \varepsilon^H_2 < L^d_2 < H_2 < L_2$	(70) ≥ (73)	(63) ≥ (67)
6.5) $\varepsilon^L_2 < \varepsilon^H_2 < H^d_2 < L^d_2 < H_2 < L_2$	(70) ≥ (74)	(63) ≥ (67)