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THE MYSTERY OF THE PRINTING PRESS: SELF-FULFILLING DEBT CRISES AND MONETARY SOVEREIGNTY

Giancarlo Corsetti and Luca Dedola

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Giancarlo Corsetti, Cambridge University, Rome III and CEPR
Luca Dedola, European Central Bank and CEPR

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

The Mystery of the Printing Press: Self-fulfilling debt crises and monetary sovereignty*

Building on Calvo (1988), we develop a stochastic monetary economy in which government default may be driven by either self-fulfilling expectations or weak fundamentals, and explore conditions under which central banks can rule out the former. We analyze monetary backstops resting on the ability of the central bank to swap government debt for its monetary liabilities, whose demand is not undermined by fears of default. To be effective, announced interventions must be credible, i.e., feasible and welfare improving. Absent fundamental default risk, a monetary backstop is always effective in preventing self-fulfilling crises. In the presence of fundamental default risk and institutional constraints on the balance sheet of the central bank, a credible monetary backstop is likely to fall short of covering government's financial needs in full. It is thus effective to the extent that it increases the level of debt below which the equilibrium is unique.

JEL Classification: E58, E63 and H63
Keywords: debt monetization, lender of last resort, seigniorage and sovereign risk and default

Giancarlo Corsetti
Department of Economics
Cambridge University
Sidgwick Avenue
Cambridge CB3 9DD

Email: gc422@cam.ac.uk

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Luca Dedola
Monetary Policy Research Division
European Central Bank
Postfach 16 03 19
D - 60066 Frankfurt am Main
GERMANY

Email: luca.dedola@ecb.int

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> "[T]he proposition [is] that countries without a printing press are subject to self-fulfilling crises in a way that nations that still have a currency of their own are not."

Paul Krugman, "The Printing Press Mystery", The conscience of a liberal, August 17, 2011.

> "Soaring rates in the European periphery had relatively little to do with solvency concerns, and were instead a case of market panic [...] [These countries] no longer had a lender of last resort, and were subject to potential liquidity crises."

Paul Krugman, "The Italian Miracle", The conscience of a liberal, April 29, 2013.

> "OMT has been probably the most successful monetary policy measure undertaken in recent times."

Mario Draghi, ECB Press Conference (Q&A), June 6, 2013.

1 Introduction

The recent sovereign debt crisis among some members of the euro area is commonly attributed to their loss of national sovereignty on monetary policy and the currency in which they issue government debt. As exemplified by the Krugman's quote above, a widely entertained hypothesis is that countries that issue debt in domestic currency and control the printing press can always finance deficits with money, and so eliminate the possibility of crises driven by self-validating expectations. Insofar as this option is precluded to countries without a currency of their own—the argument goes—their economies are inherently vulnerable to destabilizing speculation.

The historical record warns against overplaying the idea that inflationary financing be an easy way out of sovereign default. Outright default on public debt denominated in domestic currency is far from rare, also in countries where policymakers are in principle in control of the 'printing press.' In a long sample ending in 2005, Reinhart and Rogoff (2011) document 68 cases of overt domestic default (often coinciding with external debt default).[1] This

[1] According to the data, domestic default (usually but not necessarily in conjunction with default on external debt) tends to occur under extreme macroeconomic duress — in

evidence does not disprove the argument above: in the data, it is difficult to separate fundamental default from crises generated by self-fulfilling expectations. Eliminating the latter by no means implies that default cannot occur.[2] Rather, the evidence stresses the importance of identifying the conditions under which (and the concrete policy strategy by which) a central bank can stem disruptive speculation in the sovereign debt market, by providing an effective backstop to government debt. A striking example of monetary backstop has recently being provided by the OMT program launched by the ECB in 2012.

In this paper, we study a model in which debt crises may be driven by either self-fulfilling expectations, or weak fundamentals, and explore conditions under which central banks can rule out the former. Building on Calvo (1988), we analyze an economy where discretionary policymakers can choose to default (if only partially) on public debt by imposing losses on debt holders, either by outright repudiation ("haircuts") or by engineering surprise inflation.[3] In addition, we analyze a monetary backstop to public debt. Namely, we posit that, to dispel looming self-fulfilling debt crises, monetary authorities stand ready to purchase government paper at a pre-announced rate, financed by issuing their own monetary liabilities.[4]

The text builds up our model and our argument in two steps, each of interest on its own. In the first step, we generalize Calvo (1988) to an envi-

terms of high inflation and negative growth. Reinhart and Rogoff (2011) shows that, in the year in which a crisis erupts, on average, output declines by 4 percent if the country defaults on domestic debt, against a decline of 1.2 percent, if the country defaults on external debt only. The corresponding average inflation rates are 170 percent (in cases of domestic debt default) against 33 percent (external debt default).

[2] We should stress that the ability to prevent self-fulfilling crises does not rule out sovereign default altogether. Ex-post, defaults may be driven by weak fundamentals, and are typically associated with debt monetization and inflation — see the evidence in Reinhart and Rogoff (2009) and (2011) discussed above. As weak fundamentals may interact with self-fulfilling expectations of a crisis in determining sovereign risk, the case for central banks interventions remains strong.

[3] See also Cohen and Villemort (2011) and Cole and Kehoe (2000) among others. Our model differs from Calvo (1988) in several crucial dimensions. Namely, we model a stochastic economy where default can occur for fundamental reasons, and debt repudiation entails both fixed (output) and variable (budget) costs. In the monetary version of the model, default is not restricted to debasing debt via inflation as in Calvo (1988). Finally, we model the balance sheet of the central bank allowing for interest-bearing liabilities.

[4] The intervention rate should be sufficiently low as to rule out the bad equilibrium driven by self-fulfilling crises; as well as high enough to avoid ex ante losses. Namely, too low an interest rate would de facto translate into a transfer of resources covering the short-fall of fiscal revenues under weak fundamentals, effectively amounting to a bailout. As is well understood, anticipations of such a bailout would give rise to moral hazard.

ronment with fundamental fiscal stress and both proportional and fixed costs of default, abstracting from monetary backstop. In this step, we lay down in detail the main mechanism by which multiple equilibria emerge under lack of fiscal commitment. We show that the ability to generate seigniorage revenue and debase debt with inflation cannot prevent self-fulfilling debt crises altogether, even in economies with no fundamental default risk. Because of deadweight output costs of default, it will affect, however, the range of debt for which the equilibrium is not unique. The reason is straightforward: inflation is not costless. There are trade-offs between default, taxation and inflation that affect the degree to which the central bank is willing to ex-post inflate public debt away in response to financial distress, in line with the analysis by Aguiar et al. (2013).[5]

We then turn to the analysis of a monetary backstop to government debt, a distinctive feature of our contribution. First, we require a monetary backstop to be credible off equilibrium: announced interventions need to be both *feasible* and *welfare-improving* from the point of view of the monetary authorities. Indeed, a successful strategy does not require actual purchases of debt, as it works by coordinating market expectations on the fundamental equilibrium.

Second, we posit that monetary authorities' purchase of government paper are financed by issuing nominal liabilities ("reserves") remunerated at a default-free nominal market rate. In our analysis, here is where the 'printing press' argument comes into play: Independent modern central banks stand ready to honour their own liabilities (not necessarily government debt) by redeeming them for cash at their nominal value (see also Hall and Reis 2012). Hence, nominal liabilities issued by central banks (high powered money, not only in the form of cash but especially bank reserves, often interest-bearing) are exposed to the risk of inflation, but not to fears of outright default. However, in our model the interest rate on reserves will be increasing in expected inflation, reflecting any anticipation of discretionary attempts by the central bank to raise seigniorage revenues and debase nominal at via inflation in the future.[6]

[5] Our results are also relevant in relation to a key conclusion by Calvo (1988). In the monetary economy studied by this author, outright default is ruled out by assumption: multiple equilibria then obtain only by virtue of non-standard costs of inflation. That is, the equilibrium would be unique in the presence of standard convex costs of inflation. In a more general specification (such as ours), instead, multiplicity would still be possible, in the rates of outright default.

[6] See Gertler and Karadi (2012) for a similar idea applied to unconventional monetary policy. While our assumption is consistent with the idea that a monetary backstop to the government does not need to have immediate inflationary consequences, the interest rate

Our key results concern the characterization of the conditions for the above monetary backstop to sovereign debt to be successful in ruling out multiple equilibria. A monetary backstop is always credible and hence effective in preventing self-fulfilling crises in model economies in which default occurs exclusively per effect of self-fulfilling expectations. As long as markets coordinate on the fundamental equilibrium with no default, the central bank faces no risk of losses on its balance sheet, and can pick the desired rate of inflation ex post, satisfying the feasibility and welfare-improvement conditions.

Credibility is instead far from granted in economies in which, realistically, the probability of fundamental fiscal stress causing default, however small, is not nil. Even if the central bank need only threaten to intervene in the sovereign debt market, the monetary backstop exposes its balance sheet to fundamental risk of losses, which may undermine a central bank backstop strategy altogether. This will be so when losses would force policies, e.g. running extremely high inflation rates, that, from the perspective of the monetary authorities, are welfare-dominated by the alternative of facing non-fundamental default.

The link between interventions, losses and inflation is not mechanical, however, and reflects fiscal and monetary interactions. In our analysis, we focus on two polar cases. No inefficient inflation adjustment is required in the first case, in which benevolent monetary and fiscal authorities act under a consolidated budget constraint. Even when fundamental fiscal stress results in a default, positive transfers to the central bank ensure that its liabilities are honored without inefficient adjustment of inflation. The monetary authorities can then credibly stand ready to backstop the government financing needs *in full*.

The case of a consolidated budget constraint however downplays the complexity of actual policy interactions, that typically translate into institutional constraints on the central bank's balance sheet.[7] In our second case, we require benevolent fiscal and monetary authorities to operate under a separate budget constraint. We show that a monetary backstop strategy is not undermined, but its credibility is no longer granted for any level of debt. If the central bank is held responsible for servicing outstanding monetary liabilities without relying on fiscal transfers, the inflationary consequences of

on reserves is market determined.

[7] On the one hand, it is often the case that institutional arrangements or political constraints rule out or limit fiscal transfers to monetary authorities. On the other hand, central banks are typically wary of asking for fiscal support to guard their independence — see e.g. Goodfriend (2011) for a discussion.

losses in excess to the present discounted value of seigniorage at the desired rate of inflation, will bound the scale of credible interventions. Yet, due to fixed costs of default, there will generally be some threshold for the interest bill on debt below which the government does not default. The scale of credible interventions may be enough to reduce the interest burden below such threshold, even when markets charge high, non-fundamental rates.

While our analytical framework is close to Calvo (1988), our model and results also build on a vast and consolidated literature on self-fulfilling debt crises, most notably Cole and Kehoe (2002) and more recently Roch and Uhlig (2011), as well as sovereign default and sovereign risk, see e.g. Arellano (2008) and Uribe (2006) among others. A few recent papers and ours complement each other in the analysis of sovereign default and monetary policy. Jeanne (2011) addresses issues in lending of last resort using a finite-horizon model where, in case of default, the government repudiate its entire stock of debt, while Reis (2013) discusses debt crises by modelling the central bank balance sheet in a similar way as ours. In a continuous time framework, Aguiar, Amador, Farhi and Gopinath (2012) analyze a similar problem as the one we analyze in sections 2 and 3 (and appendix), but abstracting from fundamental risk. In their analysis, policymakers can rule out self-fulfilling equilibria by threatening to inflate away government liabilities that agents have to hold over a "grace period." Cooper (2012) and Tirole (2012) analyze debt guarantees and international bailouts in a currency union.

By the same token, while we encompass trade-offs across different distortions in a reduced-form fashion, in doing so we draw on a vast literature that has provided micro-foundations, ranging from the analysis of the macroeconomic costs of inflation, in the Kydland-Prescott but especially in the new-Keynesian tradition (see e.g. Woodford 2003), to the analysis of the trade-offs inherent in inflationary financing (e.g. Barro 1983), or the role of debt in shaping discretionary monetary and fiscal policy (e.g. Diaz et al. 2008 and Martin 2009), and, last but not least, the commitment versus discretion debate (e.g. Persson and Tabellini 1993).

The text is organized as follows. Sections 2 revisits the logic of self-fulfilling sovereign debt crises. Section 3 shows that the same mechanism survives under monetary sovereignty, when debt in national currency can also be inflated away. Section 4 discussed the preconditions for interventions in the debt market to stem self-fulfilling debt crises. Section 5 carries out a comparative analysis of backstop policies that can be pursued by monetary authorities. Section 6 concludes with a brief discussion of lessons for a currency union.

2 The logic of self-fulfilling debt crises

As in Calvo (1988), our main question concerns the determinants of the market price at which a government can borrow a given amount B from private investors at a point in time, and the consequences of agents expectations (determining this price) on the ex-post fiscal choices by the government.[8] The model is indeed solved under the maintained assumption that the government is unable to commit to a fiscal plan in a credible way, detailing how it will service public debt and finance public spending in future periods under different contingencies. Ex post, it may choose to default, partially or fully, on its liabilities.[9]

Since we are interested in the mechanism by which, for a given level of debt, default is precipitated by agents expectations (rather than, say, in the determinants of public debt accumulation), it is useful to carry out our analysis in a two-period economy— in an appendix we show how our two-period problem can be nested in an infinite horizon economy. Furthermore, to clarify the mechanism that may create instability in the debt market, in this section we abstract from the monetary dimension of the analysis altogether. A demand for monetary assets and the central bank will be introduced from the next section on.

The timeline of our model, summarized by Figure 1, is as follows. In the first period, private agents can invest a given stock of financial wealth W either in domestic public debt B, at the gross market interest rate $\widetilde{R}$, or in a real asset K, with an infinitely elastic supply, yielding an exogenously given "safe" interest rate R. Consumers' wealth in the first period is thus equal to the both assets, $W = B + K$.

In the second period, the output process is realized. The government sets taxes and may impose a haircut on the owners of government debt at the rate $\theta \in [0, 1]$, inducing distortions that affect net output and aggravate the budget — to be discussed below. All agents are risk neutral: domestic agents derive utility from consuming in period 2 only. Different from Calvo (1988), we explicitly allow for the possibility that default be driven by funda-

[8] In our two-period economy, the financial need of the government in the first period coincides with the stock of public debt. In multiperiod models, there would be a fundamental distinction between the stock of debt B, on which the government may impose haircuts, and the short-term financial needs of the public sector, which determine the exposure of the government to a self-fulfilling crisis — including the primary deficit, interest payments, as well as the rollover of outstanding bonds and bills coming to maturity during the period.

[9] In the model, under commitment there are no self-fulfilling default crises, — as shown by Calvo 1988. For an analysis of default under commitment, see Adam and Grill (2011).

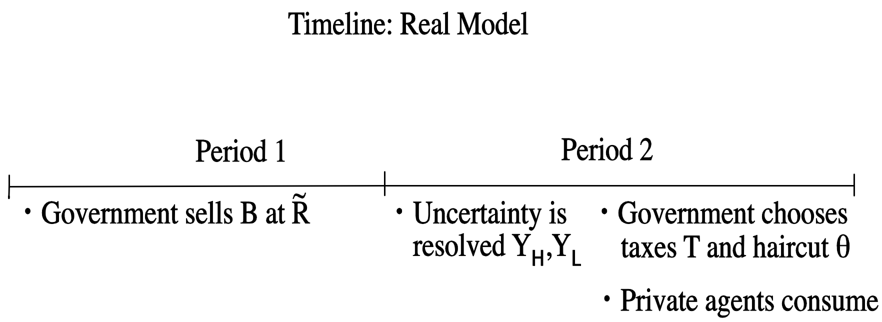

Figure 1

mental imbalances, in addition to self-fulfilling expectations. To this end, we posit that output varies across two states of the world, H and L, occurring with probability μ and $(1-\mu)$. As discussed below, for a sufficiently large stock of initial debt, variations in the state of the economy will enable us to contrast, if only in a stylized way, 'normal' and 'fiscal and macroeconomic stress' circumstances. This distinction will be crucial when contemplating the merits and limits of financial support to the fiscal authorities. Figure 1 underscores that fiscal policy cannot be pre-committed and is decided after agents have formed their expectations.

2.1 The optimal choice between taxes and haircuts under discretion

We start with the analysis of how the government choose the level of taxation and default in period 2 under discretion, i.e. taking the interest rate set by the market in the previous period as given. The policy trade-offs faced in this choice are of course rooted in the economic distortions implied by different policy options, i.e., defaulting versus running large primary surplus. In the spirit of Calvo, we proceed by specifying the relevant distortions in reduced-form, referring to the relevant literature which has provided microfoundations to them.

Following the disruption of domestic financial intermediaries and financial markets, sovereign default may entail different types of costs. These include both output and tax losses associated with a contraction of economic activity, as well as transaction costs in the repudiation of government liabilities. In the theoretical literature, some contributions (see e.g. Arellano 2008 and Cole and Kehoe 2000) posit that a default causes output to contract by a fixed amount; in other contributions (see e.g. Calvo 1988) the cost of default falls on the budget and is commensurate to the extent of the haircut imposed on investors. The relative weight of different costs is ultimately an empirical matter — see e.g. Cruces and Trebesch (2012). Yet, alternative assumptions on whether default costs are mainly lump-sum or proportional to the size of the haircut are bound to shape distinct policy trade-offs with far-reaching implications for policy analysis. For this reason, we find it appropriate not to restrict our model to one type of costs only.

Rather, we follow the literature in assuming, first, that default in period 2 entails a loss of ξ_θ units of output, regardless of the size of default and the state of the economy. This assumption squares well with the presumption that the decision to breach government contracts, even with a small haircut, marks a discontinuity in the effects of such policy on economic activity.[10] Second, we model variable costs of default falling on the budget, proportional to the size of the ex-post haircut $\theta B\widetilde{R}$ (which of course can be expected to vary across states of the economy).[11] We posit that, upon defaulting, the government incurs a financial outlay equal to a fraction $\alpha \in (0,1)$ of the size of default.[12]

Running a large primary surplus is also distortionary. Namely, in light of the literature on tax smoothing, we posit that taxation results in a dead-weight loss of output indexed by $z(T_i; Y_i)$. Given the level of gross output Y_i, the function $z(.)$ is convex function of T, satisfying standard regularity conditions. We realistically assume that, to raise a given level of tax revenue T, dead-weight losses are larger, and grow faster in T, if the economy is in

[10] A plausible alternative assumption could have the fixed costs paid only at a minimun threshold default rate.

[11] If $\theta_i = 0$, there is no default: the country repays the entire interest bill $B\widetilde{R}$ at market rates. If default occurs, repayment is reduced by θ_i.

[12] Calvo (1988) refers to legal and transaction fees associated to default. In a broader sense, one could include disruption of financial intermediaries (banks and pension funds) that may require government support. Note that our results would go through if the variable costs of default were in output, rather than in the budget.

a recessionary state, that is:

$$\begin{aligned} z\,(T;Y_L) &> z(T;Y_H), \\ z'\,(T;Y_L) &> z'(T;Y_H). \end{aligned}$$

Since what matters in our analysis is the size of the primary surplus, rather than the individual components of the budget, for simplicity we posit that government spending G is state invariant — an assumption that is not consequential for our main results.

Under these assumptions, in each state of nature (H or L) the budget constraint of the government in period 2 reads

$$T_i - G = (1-\theta_i)\,B\widetilde{R} + \alpha\theta_i B\widetilde{R}, \qquad \alpha, \theta_i \in [0,1] \qquad i = L, H \tag{1}$$

where $\widetilde{R}$ is the market interest rate on public debt, set in period 1. The primary surplus — defined as the difference between taxes T and government spending G — finances debt repayment net of the haircut $\theta_i B\widetilde{R}$, but gross of the transaction costs of defaulting $(\alpha\theta_i B\widetilde{R})$.[13]

In period 2, the budget constraint of the country's residents is

$$C_i = [Y_i - z\,(T_i;Y_i) - \xi_\theta] - T_i + KR + (1-\theta_i)\,B\widetilde{R} \tag{2}$$

Consumption is equal to output, Y, net of output losses from raising taxes and defaulting on liabilities, $z\,(T_i, Y_i) + \xi_\theta$, minus the tax bill, T, plus the revenue from portfolio investment. Consumers earns the safe (gross) interest rate R on their holdings of K, and the net (ex haircut) payoffs $(1-\theta)\,\widetilde{R}_i$ on their holding of public debt B.

Under discretion, the government decides its optimal policy plan (θ_i, T_i) by maximizing agents' utility (which coincides with consumption C_i), subject to its budget constraint and taking expectations (and thus $\widetilde{R}$) as given. The optimal discretionary plan is characterized by two notable features. First, fixed costs of default ξ_θ induce a non-linearity: a positive θ_i will be chosen if and only if the benefits of the haircut will be large enough compared to this cost. Second, there is a well-defined upper bound on the country's willingness to raise taxes, which vary depending on whether the country chooses to default or service its liabilities in full.

Conditional on default, let $\widehat{T}_i$ denote the level of taxes that maximizes private consumption under policy discretion (taking $\widetilde{R}$ as given). As long

[13] From an accounting perspective, the budget costs of default due to legal fees should be part of the the primary surplus. In what follows, we find it expositionally convenient to consider them as part of the debt service, hence we include them in the net interest bill of the government.

as an interior solution for θ_i exists, i.e., the constraint $0 < \theta_i \leq 1$ is not binding, the first order condition of the policy problem yields[14]

$$z'(\widehat{T}_i; Y_i) = \frac{\alpha}{1-\alpha}. \tag{3}$$

Conditional on default, the government chooses an optimal tax level $\widehat{T}_i$ trading-off the economic costs associated with raising revenue $z(T_i)$, with the variable budget cost of default, indexed by the parameter α. This trade-off is independent of the interest rate and the size of debt. Given G, the optimal taxation level $\widehat{T}_i$ determines the maximum primary surplus that the country finds it optimal to generate conditional on default, $\widehat{T}_i - G$, in turn nailing down net output $Y_i - z\left(\widehat{T}_i\right)$ as well as the optimal haircut rate. It may of course happen that the constraint $\theta_i \leq 1$ is binding in equilibrium. In this case, the government sets a tax level higher than $\widehat{T}_i$, to cover current non-interest expenditure including the variable budget costs of default evaluated at $\theta_i = 1$, namely:

$$\widehat{T}_i \leq \widehat{T}^+ = G + \alpha B\widetilde{R} \tag{4}$$

Conditional on the government choosing not to default, $\theta_i = 0$, tax revenue needs to rise enough to finance both current spending G and the debt service $B\widetilde{R}$ in full:

$$\widetilde{T}_i - G = B\widetilde{R} = \widetilde{T} - G \tag{5}$$

Note that with a state invariant G, the primary surplus required to service the outstanding liabilities is the same across states.

How far is the government willing to raise taxes before exercising (optimally) the option to default? The 'fiscal capacity' of the government is naturally defined as the maximum primary surplus that the country will find it optimal to generate to finance its interest bill in full. Comparing

[14] This is just the first order condition from choosing θ_i to maximize ex-post consumption C_i subject to 1:

$$-z'(T_i, Y_i)\frac{\partial T_i}{\partial \theta_i} - \frac{\partial T_i}{\partial \theta_i} - B\widetilde{R} = 0,$$
$$\frac{\partial T_i}{\partial \theta_i} = -(1-\alpha)\, B\widetilde{R}$$

consumption under full debt service and default,[15] such threshold primary surplus is identified by the right-hand-side of the following inequality:

$$\widetilde{T}_i \leq \widehat{T}_i + \xi_\theta + \frac{B\widetilde{R} + G - \widehat{T}_i}{1-\alpha} - \left[z\left(\widetilde{T}_i; Y_i\right) - z\left(\widehat{T}_i; Y_i\right)\right] \tag{6}$$

Relative to the optimal taxation conditional on default $\widehat{T}_i$, the fiscal capacity of a country is pinned down by the fixed output costs ξ_θ plus the variable budget costs of default $\frac{B\widetilde{R} + G - \widehat{T}_i}{1-\alpha}$ (the latter expressed in terms of forgone income), minus the incremental output loss due to tax distortions when the debt is repaid in full — the term in squared bracket. The larger this term, or the lower the overall default costs $\xi_\theta + \frac{B\widetilde{R} + G - \widehat{T}_i}{1-\alpha}$, the lower the fiscal capacity of the country.

Ultimately, the fiscal capacity is a function of the budget cost of default and the level of debt. Since $\widehat{T}_i$ is increasing in α, a high budget cost of default raises fiscal capacity. Conversely, a high stock of debt reduces it, via its effect on $\widetilde{T}_i$. To see this point most clearly, we rewrite the "no-default" condition as follows:

$$\xi_\theta \geq z(G + B\widetilde{R}; Y_i) - z(\widehat{T}_i(\alpha); Y_i) - \frac{\alpha}{1-\alpha}\left[B\widetilde{R} - \left(\widehat{T}_i(\alpha) - G\right)\right]$$

where for simplicity we have assumed that the constraint $\theta \leq 1$ is not binding in equilibrium. It should be clear by now that the term 'capacity' by no means indicates a technical limit, but is the outcome of a discretionary, welfare-maximizing decision by the government.

The above conditions are defined up to the size of the haircuts, to be determined jointly with equilibrium pricing by private markets.

[15] Namely:

$$\widetilde{T}_i \leq -\left[z\left(\widetilde{T}_i, Y_i\right) - z\left(\widehat{T}_i, Y_i\right)\right] + \widehat{T}_i + \xi_\theta + \theta_i B\widetilde{R}$$
$$\widetilde{T}_i - G = B\widetilde{R}$$
$$\widehat{T}_i - G = \left[1 - \theta_i(1-\alpha)\right] B\widetilde{R}$$
$$=>$$
$$\widetilde{T}_i \leq \xi_\theta - \left[z\left(\widetilde{T}_i, Y_i\right) - z\left(\widehat{T}_i, Y_i\right)\right] + \widehat{T}_i + \frac{B\widetilde{R} + G - \widehat{T}_i}{1-\alpha}$$

2.2 Debt pricing

The price of debt is pinned down by the interest parity condition, equating (under risk neutrality) the expected real returns on domestic bonds to the safe interest rate:

$$\widetilde{R}\left[\mu\left(1-\theta_H\right)+\left(1-\mu\right)\left(1-\theta_L\right)\right]=R \tag{7}$$

Under rational expectations, agents anticipate the optimal discretionary plan of the government conditional on the market interest rate $\widetilde{R}$. This condition, together with the conditionally optimal tax rates (3) or (4) and (5), the condition for choosing default (6), and the government budget constrain (1), define an equilibrium.

As already mentioned, we are interested in analyzing fundamental fiscal stress. For this reason, we now identify the range of debt levels that may cause fundamental default for low realizations of output. Namely, we assume B to be large enough that, in the low output state, the primary surplus under default will fall short of the interest bill of the government valued at the risk-free rate R:

$$\widehat{T}_L-G<BR. \tag{8}$$

This implies that, unless the fixed cost ξ_θ is prohibitively high, the government will default for fundamental reasons in the low-output state. Conversely, we posit that, given B, the primary surplus optimally chosen under default in the high-output state can comfortably finance the largest possible interest bill — corresponding to the case in which agents anticipate total repudiation in the low-output state:[16]

$$\widehat{T}_H-G>\frac{R}{\mu}B. \tag{9}$$

So, there is no fundamental reason for defaulting in the high-output state.

Moreover, we further restrict B and parameters such that, were agents to anticipate complete default in the low-output state and no default in the high one, the optimal primary surplus (including the variable budget costs of defaulting) in L is non-negative,

$$\widehat{T}_L-G-\alpha\frac{R}{\mu}B\geq 0. \tag{10}$$

[16] Note that a countercyclical G would increase 'fiscal stress' in the low output state, while raising fiscal surplus in the good output state. Generalizing our model in this direction would aggravate notation, without producing additional insight.

Hence, the haircut rate in this state is less than 100 percent. Note that, together with (8), the above condition restricts μ (the probability of the good output state) to be higher than α (the proportional budget cost of default). By the same token, we posit a sufficient condition for a default to occur (per effect of self-validating expectations) also in the high output state, with an haircut rate less than 100 percent, that is:[17]

$$BR > \frac{(\alpha + \mu)}{(1 - \alpha)} \left(\widehat{T}_H - \widehat{T}_L \right). \tag{11}$$

The overall purpose of these assumptions is straightforward: they ensure that (a) agents price the possibility that the government chooses to default on its liabilities, for purely fundamental reasons, and that (b), in response to non-fundamental debt crises, default occurs in both the low and the high-output state.[18]

2.3 Weak fundamentals and self-validating expectations as drivers of sovereign debt crises

We will now show that, depending on the relative weight of the costs of default and taxation, and the level of debt, different equilibrium outcomes are possible in the model, and the equilibrium is not necessarily unique. For expositional reasons, we find it convenient to extend at first the main result by Calvo (1988) — who posits no fixed costs of default — to our stochastic setting. As detailed in our first proposition below, we show that, if $\xi_\theta = 0$, there will be two equilibria. One is a fundamental equilibrium (denoted with the superscript F), in which the interest rate charged on debt reflects anticipations of default in the low-output state of nature in period 2, based on the correct probability that this state materializes. The other one is a non-fundamental equilibrium (denoted with N), in which market participants coordinate their expectations on default occurring in both the high and low output state — and thus charge a higher equilibrium interest rate than in F. The following proposition summarizes our results.

Proposition 1 *In the economy described by (1), (2), and (7), where we posit $\xi_\theta = 0$, with the government optimally choosing taxes satisfying either (3) or (4) in case of default, depending on whether the constraint $\theta_i \leq 1$ is/is not binding, and (5) otherwise, under the maintained assumptions (8),*

[17] See the expression for the equilibrium haircut in (13) below.

[18] As in Calvo (1988), the initial stock of debt is not so high that there is no equilibrium price at which it can be sold to market participants.

(9), (10) and (11), namely, if $\mu > \alpha$ and the following restrictions on the initial debt level hold:

$$\begin{aligned}
\mu\left(\widehat{T}_H - G\right) &> BR > Max\left\{\widehat{T}_L - G, \frac{(\alpha+\mu)}{(1-\alpha)}\left(\widehat{T}_H - \widehat{T}_L\right)\right\} \\
\widehat{T}_L - G &\geq \frac{(\alpha+\mu)}{(1-\alpha)}\left(\widehat{T}_H - \widehat{T}_L\right) \Longleftrightarrow (1+\mu)\left(\widehat{T}_L - G\right) \geq (\alpha+\mu)\left(\widehat{T}_H - G\right)
\end{aligned}$$

an equilibrium will exist and will not be unique. There will be a fundamental equilibrium in which default will occur only in the low output state of the world, with the equilibrium haircuts given by $\theta_H^F = 0$ and

$$0 < \theta_L^F = \frac{RB - \left(\widehat{T}_L - G\right)}{(1-\mu)\left[RB - \left(\widehat{T}_L - G\right)\right] + (\mu - \alpha) RB} < 1. \tag{12}$$

There will be another equilibrium, driven by self-validating expectations, where $\theta_L^F < \theta_L^N$ and $0 < \theta_H^N \leq \theta_L^N$, with the rate of default in each state given by:

$$\begin{aligned}
\theta_H^N &= \frac{\left(\widehat{T}_H - G - BR\right) - \frac{(1-\mu)}{(1-\alpha)}\left(\widehat{T}_H - \widehat{T}_L\right)}{(1-\alpha)\left(\widehat{T}_H - G - BR\right) - (1-\mu)\left(\widehat{T}_H - \widehat{T}_L\right) + \alpha\left(\widehat{T}_H - G\right)} \qquad (13) \\
\theta_L^N &= \min\left\{\frac{\left(BR - \widehat{T}_L + G\right) - \frac{\mu}{(1-\alpha)}\left(\widehat{T}_H - \widehat{T}_L\right)}{(1-\alpha)\left(BR - \widehat{T}_L + G\right) - \mu\left(\widehat{T}_H - \widehat{T}_L\right) + \alpha\left(\widehat{T}_L - G\right)}, 1\right\}.
\end{aligned}$$

The equilibrium interest rate will generally be higher than the safe rate R. In the F-equilibrium, the difference is determined by expectations of default in the weak state. In the N-equilibrium, the difference is driven by self-confirming beliefs that the fiscal authority will default regardless of the level of output.[19] Note that, by virtue of the conditions stated at the end of the previous subsection, the haircut in the low-output state is strictly below 100 percent in the F-equilibrium. In the N-equilibrium, in contrast, no condition prevents self-validating expectations from pushing the government in this contingency to default on its entire stock of debt.

Haircuts and interest rates vary across equilibria. In the fundamental equilibrium, if the government defaults, it does so only in the low output

[19] The solution in Calvo (1988) is a special case of our analysis if, in addition to assuming $\xi_\theta \to 0$ (no fixed cost of default), we let $\mu \to 1$ (output is non stochastic). In the non-stochastic version of the model, the equilibrium may be unique for a special combinations of parameters' values.

state Y_L; in the non-fundamental equilibrium, the government imposes haircuts in both states of the world. In the model without fixed costs, non-fundamental equilibria are welfare-dominated because the level of taxation chosen by the government upon defaulting is higher in each state of nature, and so are the overall tax-related distortions reducing output.

The logic of multiplicity is illustrated by Figure 2a and 2b. The two graphs plot, against the market rate $\widetilde{R}$, the best-response default rate θ that satisfies the budget constraint and the optimality conditions of the government (light-colored line) and of the investors (dark-colored line), in the high-output and the low-output state, respectively. In each state of the world, the government best-response depends on the haircut in the other states only through the market interest rate $\widetilde{R}$. Specifically, for $0 < \theta_i < 1$ the government reaction function is increasing in $\widetilde{R}$ as follows:

$$\theta_i\text{-Government} = \frac{B\widetilde{R} - \left(\widehat{T}_i - G_i\right)}{(1-\alpha)\, B\widetilde{R}} > 0, \qquad i = L, H.$$

Conversely, from (7), the state-contingent haircut expected by investors depends not only on $\widetilde{R}$, but also the expected haircut in the other states. In Figure 2a, drawn for the high output state, the investors best response to $\widetilde{R}$,

$$\theta_H\text{-Investors} = 1 - \frac{1}{\mu}\left[\frac{R}{\widetilde{\widetilde{R}}} - (1-\mu)\,(1-\theta_L)\right],$$

is plotted under the assumptions that there is non-fundamental default in the low-output state (the curve in the center, with $\theta_L = \theta_L^N$). The curve crosses the government best response at a positive haircut rate: a default in the high-output state can only occur conditional on investors coordinating on self-validating expectations of fiscal stress also in the low-output state and accordingly bidding a sufficiently high interest rate $\widetilde{R}$. Moreover, because the government best response for some $\widetilde{R} < \widetilde{R}^N$ hits the non-negativity constraint on θ_H, it also crosses the investors best response for $\theta_H = 0$. At a sufficiently low sovereign rate, under our assumptions, in the H-state the government finds it optimal to repay its obligation in full, as it cannot set $\theta_H < 0$. Multiplicity arises *specifically* because of this non-negativity constraint.

The Figure 2b is drawn for the low-output state. Here, the government best response crosses *two* best responses for the investors, one conditional on no default in the high-output state (the fundamental equilibrium, with $\theta_H = 0$); the other conditional on default in this state (the non-fundamental equilibrium, with $\theta_H = \theta_H^N$).

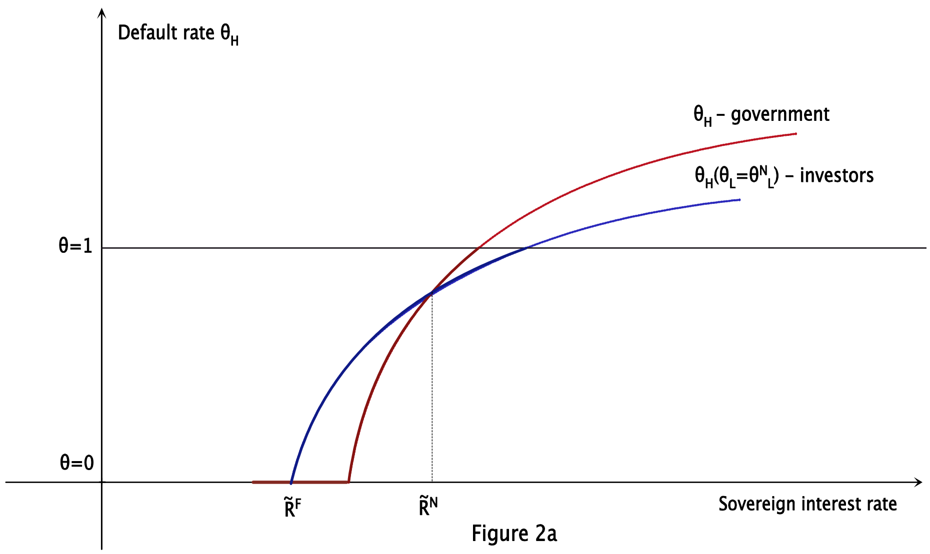

Figure 2a

An arguably unappealing feature of Calvo (1988) with $\xi_\theta = 0$, is that self-fulfilling debt crises are possible for any (even very low) level of debt. This is an implication of omitting fixed costs of default. We now further generalize Calvo's results, encompassing a specification of these costs that is standard in the literature on sovereign default. We will show that, with fixed costs, first, the equilibrium is characterized by a threshold value for debt, below which there is no multiplicity. Second, in the range of debt where equilibrium is unique, default may not occur at all, not even in the low-output state. These results are formally stated in the following proposition.

Proposition 2 *In the same economy described in proposition 1, for given fixed output costs of default ($\xi_\theta > 0$):*
(a) If the government debt B is sufficiently low so that (6) holds in the high-output state, namely B satisfies the following inequality:

$$\xi_\theta \geq z(G + B\widetilde{R}^N; Y_H) - z(\widehat{T}_H; Y_H) - \frac{\alpha}{1-\alpha}\left[B\widetilde{R}^N - \left(\widehat{T}_H - G\right)\right] \qquad (14)$$

where

$$\begin{aligned} B\widetilde{R}^N &= \frac{RB}{1 - \mu\theta_H^N - (1-\mu)\,\theta_L^N} = \\ &= \frac{(1-\mu)\left[RB - \left(\widehat{T}_L - G\right)\right] + (\mu - \alpha)\,RB - \mu\left(\widehat{T}_H - G\right)}{\alpha} \end{aligned}$$

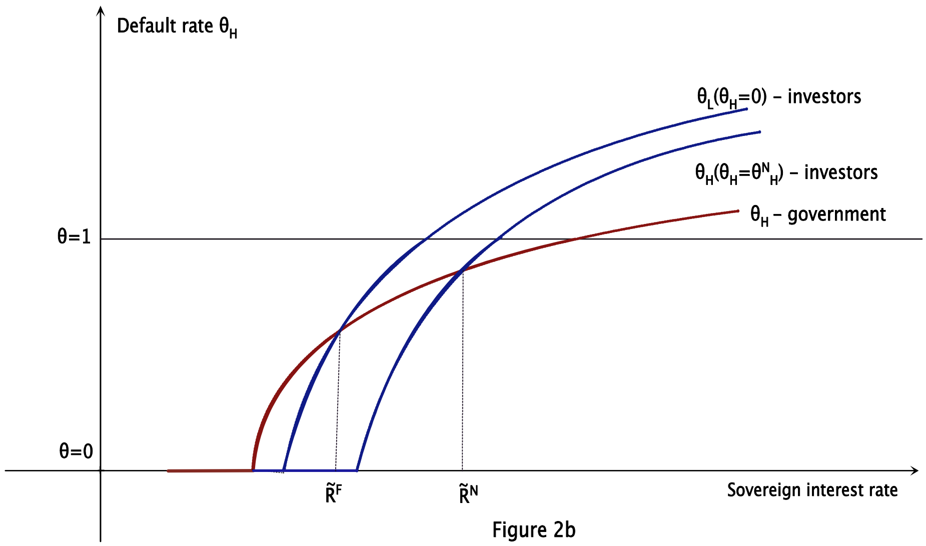

Figure 2b

there is a unique, fundamental equilibrium.
In this unique equilibrium, default will occur in the low output state only, provided (6) does not hold in this state, namely for a level of debt B satisfying the following inequality:

$$\xi_\theta < z(G + B\widetilde{R}^F; Y_L) - z(\widehat{T}_L; Y_L) - \frac{\alpha}{1-\alpha}\left[B\widetilde{R}^F - \left(\widehat{T}_L - G\right)\right] \qquad (15)$$

where

$$B\widetilde{R}^F = \frac{RB}{1-(1-\mu)\,\theta_L^F} = \frac{(1-\mu)\left[RB - \left(\widehat{T}_L - G\right)\right] + (\mu-\alpha)\,RB}{(\mu-\alpha)}$$

with the equilibrium rate of default being given by (12).[20] *For a lower level of debt, the fundamental equilibrium will display no default:* $\theta_H^F = \theta_L^F = 0$.

[20] Given the assumptions that $z(T;Y_L) > z(T;Y_H)$, $z\left(\widehat{T}_H;Y_H\right) - z(\widehat{T}_L;Y_L) \geq \frac{\alpha}{1-\alpha}\left(\widehat{T}_H - \widehat{T}_L\right) > 0$, a necessary and sufficient condition for a range of debt to exist so that such an equilibrium with fundamental default is possible, is that for $B \geq \underline{B}$:

$$z(\underline{B}\widetilde{R}^F + G; Y_L) - z\left(\widehat{T}_L; Y_L\right) - \frac{\alpha}{1-\alpha}\left(\underline{B}\widetilde{R}^F + G - \widehat{T}_L\right) \geq \xi_\theta,$$

where $\underline{B}$ is defined implicitly by the following expression:

$$\underline{B} : z(\underline{B}\widetilde{R}^N + G; Y_H) - z\left(\widehat{T}_H; Y_H\right) - \frac{\alpha}{1-\alpha}\left(\underline{B}\widetilde{R}^N + G - \widehat{T}_H\right) = \xi_\theta$$

(b) If the stock of debt is large enough that (6) is violated in the high output state, namely (14) does not hold, the equilibrium will generally be not unique. There will be two equilibria, characterized as the F- and the N-equilibrium in Proposition 1.

Our second proposition establishes that the equilibrium is unique for levels of debt that are low in relation to the non-variable costs of default. Specifically, a very low level of debt may discourage credit events even when the macroeconomic outcome turns out to produce fiscal stress — default costs would reduce welfare more than the distortions of running high fiscal surpluses in a downturn. Under these circumstances, haircuts become an attractive option only when the legacy debt of the government is sizeable enough. Still, the fixed cost ξ_θ may ensure uniqueness of equilibrium, insofar as they are large enough to rule out default in the high output case.

These different possibilities are illustrated by Figure 3, plotting the right-hand side of (14) and (15), together with the fixed costs of default, against a given initial stock of debt. The locus in the center of the figure is the relative net (variable) benefits from defaulting in the high output state in the non-fundamental equilibrium. The other locus is the corresponding net benefit from defaulting in the low state, in the fundamental equilibrium. Since the latter locus can lie above or below the former, the figure shows two of the same. It can be shown that, although all these loci may be non-linear over some regions of debt, they are always upward sloping around the point at which they cross the fixed-cost horizontal line, as depicted in the figure. This is due to the fact that the interest rate bills (i.e. $B\widetilde{R}^F$ and $B\widetilde{R}^N$) are increasing in the level of financing needs B, as shown in Proposition 2.[21]

Consider the region of debt to the left of the threshold T below which the equilibrium is unique (determined by the condition (14)). In this region, default in the low-output state may or may not occur. When the locus (15) lies above (14), the government will default in the low output state only if

and we have that:

$$
\begin{aligned}
B\widetilde{R}^N &= \frac{RB}{1-\mu\theta_H^N-(1-\mu)\,\theta_L^N} = \frac{(1-\mu)\left[RB-\left(\widehat{T}_L-G\right)\right]+(\mu-\alpha)\,RB-\mu\left(\widehat{T}_H-G\right)}{\alpha} \\
B\widetilde{R}^F &= \frac{RB}{1-(1-\mu)\,\theta_L^F} = \frac{(1-\mu)\left[RB-\left(\widehat{T}_L-G\right)\right]+(\mu-\alpha)\,RB}{(\mu-\alpha)}.
\end{aligned}
$$

[21] It is easy to show that the interest rate bill instead *falls* in B in the original, non-stochastic economy in Calvo (1988).

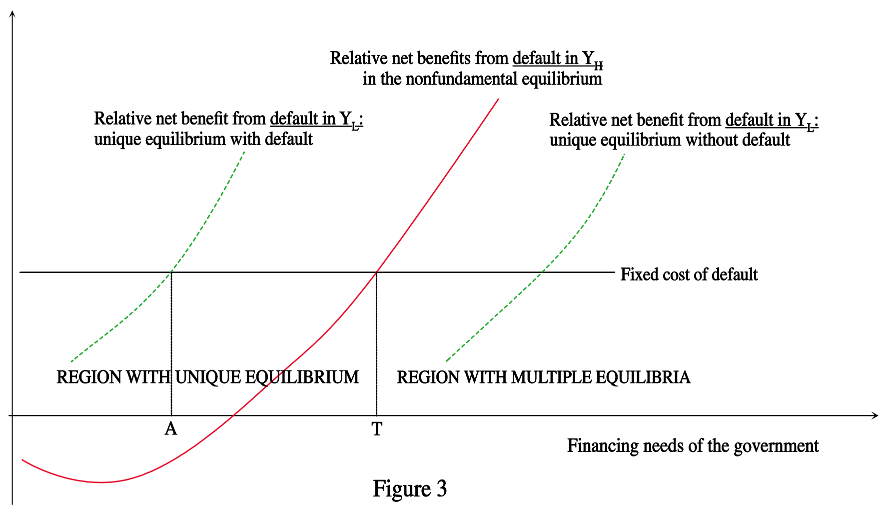

Figure 3

the initial debt is comprised between A and T — debt is sufficiently high to raise the benefits of haircuts above its fixed costs. Conversely, there will be no initial level of debt at which the government will default if the locus (15) happens to lie below (14).

According to our model, the self-fulfilling crises emphasized by Calvo (1988) emerge as a possibility only for a relatively high stock of government liabilities in relation to the fixed costs of default, similarly to Cole and Kehoe (2000). Fixed costs thus may explain while defaults are not frequent at relatively low debt level.

3 Sovereign default in a monetary economy with non-indexed debt

In the previous section, we have analyzed a mechanism that potentially makes a country vulnerable to self-fulfilling sovereign debt crises. In this section, the question we are interested in is whether the options to inflate away public debt ex post and raising revenue through the inflation tax dispose of equilibria in which the government ends up resorting to outright default per effect of self-fulfilling expectations. As stressed by Calvo (1988), some degree of repudiation is a natural outcome in a monetary economy, because unexpected changes in inflation rates affect the ex-post real returns on assets which are not indexed to the price level. Repudiation in period 2 can thus take the form of either outright default on debt holdings, or a reduction in the real value of debt through surprise in ex-post inflation, or both.[22]

To address this question, we focus on the benchmark policy scenario in which both the fiscal and the monetary authorities, while acting under discretion, are benevolent (maximize the utility of the representative agent) and act under their consolidated budget constraint. Moreover, as in the literature on discretionary policy and default, we stipulate that the (consolidated) budget constraint has to be satisfied for every policy strategy. We also intentionally abstract from issues in the determination of the value of nominal liabilities in the first, initial period, of the kind analyzed by the fiscal theory of the price level and related literature (see e.g. Uribe 2006 for a related approach). In what follows, we will show that the same non-uniqueness of equilibria analyzed in Section 2 also characterizes the monetary version of our economy where public debt is nominal.

3.1 The model setup

To minimize the use of new notation, we introduce the following modifications to our model specification. First, the stock of government liabilities B is now defined in nominal, rather than in real terms. Second, as in Calvo (1988), we restrict our attention to unit-velocity demand for (non-interest bearing) money M of the form:

$$M/P = \kappa, \tag{16}$$

where P is the price level. The seigniorage revenue — the amount of real resources the government can obtain by increasing the stock of high-powered money — in the second period will thus be:

$$\frac{M_i - M_1}{P_i} = \frac{\pi_i}{1+\pi_i}\kappa, \qquad i = L, H \tag{17}$$

where π_i ($\infty > \pi_i > -1$) is the inflation rate between period 1 and 2; as before, variables in the last period are indexed to the random realization of the high- and low-output states of the world. In addition to B, also M_1 and P_1 are exogenously given (we conveniently normalize $M_1 = P_1 = 1$).[23]

[22] This is different from the monetary model analyzed by Calvo (1988), where partial repudiation exclusively takes the form of inflation.

[23] As a simplification, the money demand (16) from Calvo implicitly bypasses the need to impose a transversality condition on M. Note that the setup can be easily generalized to encompass a Laffer curve, by positing that κ is a function of expected inflation.

Timeline: Monetary Model

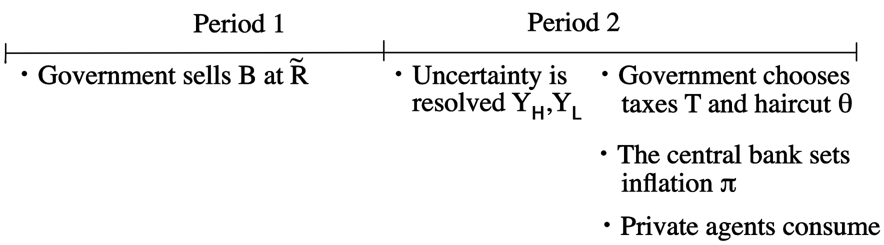

Figure 4

In period 2, the budget constraint of the government reads:

$$T_i - G = [1 - \theta_i (1 - \alpha)] \frac{B}{1 + \pi_i} \widetilde{R} - \frac{\pi_i}{1 + \pi_i} \kappa, \qquad \alpha, \theta_i \in [0, 1] \qquad i = L, H \tag{18}$$

A primary deficit — defined as the difference between government spending G and taxes T — can be financed at least in part through the inflation tax. The consumption/budget constraint of the country residents is

$$C_i = [Y_i - z (T_i; Y_i)] - \mathcal{C} (\pi_i) - \xi_\theta - T_i - \frac{\pi_i}{1 + \pi_i} \kappa + KR + (1 - \theta_i) \frac{B}{1 + \pi_i} \widetilde{R}, \tag{19}$$

where $C(\cdot)$ is the convex cost of inflation such that $C(0) = C'(0) = 0$ — a standard instance being given by $C(\pi) = \frac{\lambda}{2} \pi^2$. Consumption is equal to output Y_i net of losses from raising taxes and inflation, minus the costs of default (if any), minus the tax bill T_i including the inflation tax $\frac{\pi_i}{1 + \pi_i} \kappa$, plus the revenue from portfolio investment. The net real payoffs on public debt is $(1 - \theta_i) \frac{B}{1 + \pi_i} \widetilde{R}$.

The timeline is summarized by the Figure 4.

3.2 The optimal discretionary choice of inflation, taxation and default

The optimal policy plan under discretion (taking market expectations and thus $\widetilde{R}$ as given) is defined over T_i, θ_i, and π_i. These instruments could be controlled by different policymakers, raising issues in the specification of their objective functions and constraints, and the way they interact strategically. However, our goal is to verify whether the option to monetize the debt and the availability of seigniorage revenue reduces the vulnerability to self-fulfilling sovereign debt crisis. For this purpose, the natural benchmark is the case in which benevolent (discretionary) fiscal and the monetary authorities set their plans under coordination, hence subject to their consolidated budget constraint. Such a benchmark of course provides a reference allocation against which to assess the consequences of other policy scenarios, revolving around political economy considerations or institutional settings, which may differentiate the objectives and constraints of the monetary and fiscal authorities. We should note here that, even when the monetary and fiscal authorities are operationally independent, a common objective function and budget constraint fundamentally narrow the scope for opportunistic behavior. Under discretion, indeed, the policy plan below will be the same under Nash.[24]

According to the optimal discretionary plan, inflation and taxes are chosen by trading off the output benefits from reducing the need for distortionary income taxation and the costs of default (if any), with the output cost of inflation, according to the following condition

$$z'(T_i;Y_i)\left(B\widetilde{R}+\kappa\right)-\theta_i B\widetilde{R}\left[\alpha-z'(T_i;Y_i)\left(1-\alpha\right)\right]=\left(1+\pi_i\right)^2\mathcal{C}'\left(\pi_i\right) \quad (20)$$

where the tax level of course depends on whether the government defaults. Observe that the inflation rate would not be equal to zero even if printing money generated no seigniorage revenue ($\kappa = 0$). This is because a discretionary monetary authority will not resist the temptation to inflate the stock nominal debt, if only moderately so (according to the condition above).[25] On the other hand, positive costs of inflation prevent policymakers from wiping away the debt with infinite inflation.

Conditional on default, the optimal upper bound on the country's willingness to raise distortionary taxes is the same as before. If the constraint

[24] Fiscal and monetary policies could also be set sequentially, with one of the authorities acting as the leader — i.e., internalizing the reaction function of the other. It can be shown that, as long as both authorities have the same objective function and constraint, results tend to be either identical (when the fiscal authority leads), or quite close (when the monetary authority leads), to the one discussed in the main text.

[25] Under commitment the monetary authority would choose a lower inflation rate. However, it would not be able to undo the multiplicity due to the lack of commitment by the fiscal authority in choosing the size of the haircuts. We discuss committment later on in the paper.

$\theta_i \leq 1$ is not binding, taxes will satisfy

$$z'(\widehat{T}_i; Y_i) = \frac{\alpha}{1-\alpha}, \tag{21}$$

implying that the optimal inflation rate obeys the following trade-off:

$$\frac{\alpha}{1-\alpha}\left(B\widetilde{R} + \kappa\right) = (1+\widehat{\pi})^2\, \mathcal{C}'\left(\widehat{\pi}\right). \tag{22}$$

Note that, in this case, the optimal inflation rate is the same across states of the world, i.e., $\widehat{\pi}_H = \widehat{\pi}_L = \widehat{\pi}$.[26] If the constraint $\theta_i \leq 1$ is binding in equilibrium, instead, taxes and seigniorage will have to cover current non-interest expenditure. Specifically, taxes will have to be larger than $\widehat{T}_i$ (hence denoted by $\widehat{T}_i^+$), and will be state-contingent:

$$\widehat{T}_i \leq \widehat{T}_i^+ = G + \alpha B \frac{\widetilde{R}}{1+\pi_i} - \frac{\pi_i}{1+\pi_i}\kappa \tag{23}$$

while inflation will be correspondingly set according to

$$z'(\widehat{T}_i^+; Y_i)\left(B\widetilde{R} + \kappa\right) = (1+\pi_i)^2\, \mathcal{C}'\left(\pi_i\right) \tag{24}$$

Conditional on no default ($\theta_i = 0$), the revenue from taxation and seigniorage need to finance the government real expenditure and interest bill in full

$$\widetilde{T}_i + \frac{\widetilde{\pi}_i}{1+\widetilde{\pi}_i}\kappa - G = \frac{B\widetilde{R}}{1+\widetilde{\pi}_i} \tag{25}$$

where the tax and the inflation rates are set according to (20) with $\theta_i = 0$, that is

$$z'(\widetilde{T}_i; Y_i)\left(B\widetilde{R} + \kappa\right) = (1+\widetilde{\pi}_i)^2\, \mathcal{C}'\left(\widetilde{\pi}_i\right). \tag{26}$$

Both $\widetilde{T}_i$ and $\widetilde{\pi}_i$ are always state-contingent in this case.[27]

[26] This property of the optimal inflation rate depends on the simplifying assumption that the cost of inflation does not vary with the state of the world. It would be easy to relax this assumption, at the cost of cluttering the notation without much gain in terms of economic intuition.

[27] To see this, rewrite the implicit conditon for inflation replacing $\widetilde{T}_i$:

$$z'(B\widetilde{R} + G - \frac{\widetilde{\pi}_i}{1+\widetilde{\pi}_i}\kappa, Y_i)\left(B\widetilde{R} + \kappa\right) = (1+\widetilde{\pi}_i)^2\, \mathcal{C}'\left(\widetilde{\pi}_i\right).$$

Since the function $z'\left(\widetilde{T}_i, Y_i\right)$ is state contingent, also the left-hand-side has to be state contingent.

As in the previous section, the 'fiscal capacity' of the government is defined as the maximum taxation and seigniorage revenue the government is willing to raise to service its liabilities in full. In our monetary economy, it is identified by the right-hand-side of the following condition:

$$\begin{aligned}\widetilde{T}_i + \frac{\widetilde{\pi}_i}{1+\widetilde{\pi}_i}\kappa \quad &\leq \quad \xi_\theta + \widehat{T}_i + \frac{\widehat{\pi}}{1+\widehat{\pi}}\kappa - \left[z(\widetilde{T};Y_i) - z(\widehat{T}_i;Y_i)\right] \\ &\quad - \left[\mathcal{C}\left(\widetilde{\pi}_i\right) - \mathcal{C}\left(\widehat{\pi}_i\right)\right] + (1-\alpha)^{-1}\left[G + \frac{\widetilde{R}}{1+\widehat{\pi}_i}B - \widehat{T}_i - \frac{\widehat{\pi}_i}{1+\widehat{\pi}_i}\kappa\right]\end{aligned}$$

The 'fiscal capacity' of a country is now a function of the incremental costs of inflation, when the debt is repaid in full rather than partially.

To highlight the role of the inflation tax, we can rewrite the above condition as follows, again assuming that the constraint $\theta \leq 1$ is not binding:

$$\begin{aligned}\xi_\theta \quad \geq \quad & z(G + \frac{\widetilde{R}B - \widetilde{\pi}_i\kappa}{1+\widetilde{\pi}_i}; Y_i) - z(\widehat{T}_i;Y_i) + \left[\mathcal{C}\left(\widetilde{\pi}_i\right) - \mathcal{C}\left(\widehat{\pi}_i\right)\right] \qquad (27) \\ & - \frac{\alpha}{1-\alpha}\left[\frac{\widetilde{R}B - \widehat{\pi}_i\kappa}{1+\widehat{\pi}_i} - \left(\widehat{T}_i - G\right)\right]\end{aligned}$$

The above optimal conditions are defined up to the size of the haircut, to be determined jointly with equilibrium pricing by private markets.

3.3 Debt pricing

The interest parity condition, pinning down that price of government debt, now includes expected inflation:

$$\widetilde{R}\left[\mu\left(1-\theta_H\right) + (1-\mu)\left(1-\theta_L\right)\right] = \left[\mu\left(1+\pi_H\right) + (1-\mu)\left(1+\pi_L\right)\right]R. \quad (28)$$

Under risk neutrality, expected real returns are the same on government bonds and on the real asset.

The rational expectations equilibrium is defined by these pricing conditions, together with the budget constraint (18), the two conditional optimal tax rates, either (21) or (23), or (25), optimal inflation, either (22) or (24), and the condition for choosing default (27).

Below we rewrite the conditions ensuring that our economy is under fundamental fiscal stress in the low output state, but not in the high output state. Similarly to the real economy, we posit that, in the low-output state, the government revenue under fundamental default will fall short of the interest bill of the government valued at the nominal risk-free rate

$\left[\mu\left(1+\widetilde{\pi}_H\right)+(1-\mu)\left(1+\widehat{\pi}^F\right)\right] R$. In the nominal version of the model, this must be true adding the revenue from the inflation tax, $\dfrac{\widehat{\pi}^F}{1+\widehat{\pi}^F}\kappa$, to that from taxation:

$$\widehat{T}_L - G + \frac{\widehat{\pi}_L^F}{1+\widehat{\pi}_L^F}\kappa < \left[\mu\frac{1+\widetilde{\pi}_H}{1+\widehat{\pi}_L^F} + (1-\mu)\right] RB < RB. \tag{29}$$

In the low-output state, unless the fixed cost ξ_θ is prohibitively high, the government will default for fundamental reasons.

Conversely, we assume that, in the high-output state, there will be no fundamental reason for defaulting. The primary surplus net of the inflation tax revenue will be above the largest possible interest bill, when agents anticipate total repudiation in the low-output state:

$$\widehat{T}_H + \frac{\widetilde{\pi}_H}{1+\widetilde{\pi}_H}\kappa - G > \left[\mu\frac{1+\widetilde{\pi}_H}{1+\widehat{\pi}_L^F} + (1-\mu)\right]\frac{R}{\mu}B \tag{30}$$

We also impose the analogs of (10) and (11): parameters are such that, when agents anticipate complete default in the low-output state and no default in the high one, the primary surplus (including the variable budget costs of defaulting) in L is non-negative

$$\widehat{T}_L + \frac{\widehat{\pi}_L^F}{1+\widehat{\pi}_L^F}\kappa - G - \left[\mu\frac{1+\widetilde{\pi}_H}{1+\widehat{\pi}_L^F} + (1-\mu)\right]\frac{\alpha}{\mu}RB \geq 0. \tag{31}$$

As above, the above conditions restrict μ (the probability of the good output state) to be higher than α (the proportional budget cost of default). By the same token, we posit

$$\left[\mu\frac{1+\widetilde{\pi}_H}{1+\widehat{\pi}_L^F} + (1-\mu)\right] BR > \frac{(\alpha+\mu)}{(1-\alpha)}\left(\widehat{T}_H + \frac{\widehat{\pi}_H^F}{1+\widehat{\pi}_H^F}\kappa - \widehat{T}_L - \frac{\widehat{\pi}_L^F}{1+\widehat{\pi}_L^F}\kappa\right) \tag{32}$$

to ensure that the government chooses to default, per effects of self-validating expectations of fiscal stress, also in the high output state.

3.4 Multiple equilibria and macroeconomic resilience

From the description of the economy and the optimal policy plans above, it is far from clear that a "printing press" *per se* alters the mechanism by which the economy is vulnerable to self-fulfilling debt crises. Indeed, the following two propositions, in analogy to propositions 1 and 2, states that the option to monetize debt — aiming at reducing the ex-post value of debt via inflation — and raise seigniorage revenues does not shield a country from confidence crises. Setting $\xi_\theta = 0$ (no fixed output costs of default) we so state the analog of proposition 1 for our monetary economy.

Proposition 3 *In the economy summarized by (18), (19), and (28), with the government optimally choosing taxes satisfying either (21) or (23) in case of default, or (25) otherwise, setting $\xi_\theta = 0$, under the maintained assumptions (29), (30), (31) and (32), the equilibrium will exist and will not be unique. There will be a fundamental equilibrium in which default will occur only the low output state of the world, with the equilibrium haircut given by $\theta_H^F = 0$ and*

$$0 < \widehat{\theta}_L^F = \frac{RB\frac{\mu(1+\widetilde{\pi}_H)+(1-\mu)\left(1+\widehat{\pi}^F\right)}{1+\widehat{\pi}^F} + G - \widehat{T}_L - \frac{\widehat{\pi}^F}{1+\widehat{\pi}^F}\kappa}{(1-\alpha)\,RB\frac{\mu(1+\widetilde{\pi}_H)+(1-\mu)\left(1+\widehat{\pi}^F\right)}{1+\widehat{\pi}^F} - (1-\mu)\left[\widehat{T}_L + \frac{\widehat{\pi}^F}{1+\widehat{\pi}^F}\kappa - G\right]} < 1, \tag{33}$$

the trade-off between taxation and inflation given by

$$\begin{aligned} z'(\widetilde{T}_H; Y_H)\left(B\widetilde{R}^F + \kappa\right) &= (1+\widetilde{\pi}_H)^2\,\mathcal{C}'\left(\widetilde{\pi}_H\right) \\ \frac{\alpha}{1-\alpha}\left(B\widetilde{R}^F + \kappa\right) &= \left(1+\widehat{\pi}^F\right)^2\mathcal{C}'\left(\widehat{\pi}^F\right) \end{aligned}$$

and the ex-ante interest rate determined as follows

$$\widetilde{R}^F = \frac{\mu\,(1+\widetilde{\pi}_H) + (1-\mu)\left(1+\widehat{\pi}^F\right)}{\mu + (1-\mu)\left(1-\widehat{\theta}_L^F\right)}R \tag{34}$$

There will be another equilibrium, driven by self-validating expectations, where the default rate, the tax rate and the inflation rate in each state are given by the solution to the following system

$$\begin{aligned} \widehat{T}_H - G - \left(1-\widehat{\theta}_H^N(1-\alpha)\right)\frac{\widetilde{R}^N}{1+\widehat{\pi}_H^N}B + \frac{\widehat{\pi}_H^N}{1+\widehat{\pi}_H^N}\kappa &= 0 \\ \widehat{T}_L - G - \left(1-\widehat{\theta}_L^N(1-\alpha)\right)\frac{\widetilde{R}^N}{1+\widehat{\pi}_L^N}B + \frac{\widehat{\pi}_L^N}{1+\widehat{\pi}_L^N}\kappa &= 0 \\ 0 &\leq \widehat{\theta}_L^N \leq 1 \end{aligned}$$

$$\widetilde{R}^N \left[\mu\left(1-\widehat{\theta}_H^N\right)+(1-\mu)\left(1-\widehat{\theta}_L^N\right)\right] = \left[\mu\left(1+\widehat{\pi}_H^N\right)+(1-\mu)\left(1+\widehat{\pi}_L^N\right)\right] R$$

and either

$$\frac{\alpha}{1-\alpha}\left(B\widetilde{R}^N+\kappa\right) = \left(1+\widehat{\pi}^N\right)^2 \mathcal{C}'\left(\widehat{\pi}^N\right)$$

if the constraint $\theta_L^N \leq 1$ *is not binding,*

$$z'(\overline{T}_L; Y_L)\left(B\widetilde{R}+\kappa\right) = (1+\pi_L)^2 \mathcal{C}'(\pi_L)$$

otherwise.

Including fixed output costs of default prevents multiplicity for a low stock of initial debt, as in proposition 2.

Proposition 4 *In the economy described by proposition 3, for given fixed output costs of default* ($\xi_\theta > 0$)*:*
(a) Equilibrium is unique if the government debt B *is sufficiently low so that (27) holds in the high-output state, namely* B *satisfies the following inequality:*

$$\begin{aligned}\xi_\theta \geq\ & z(G+\frac{\widetilde{R}^N B-\widetilde{\pi}_H\kappa}{1+\widetilde{\pi}_H}; Y_H) - z(\widehat{T}_H; Y_H) + \left[\mathcal{C}(\widetilde{\pi}_H) - \mathcal{C}\left(\widehat{\pi}^N\right)\right] \\ & -\frac{\alpha}{1-\alpha}\left[\frac{\widetilde{R}^N B-\widehat{\pi}^N\kappa}{1+\widehat{\pi}^N} - \left(\widehat{T}_H - G\right)\right]\end{aligned}$$

where

$$\begin{aligned}\widetilde{R}^N &= \frac{\left(1+\widehat{\pi}^N\right)}{\mu\left(1-\theta_H^N\right)+(1-\mu)\left(1-\theta_L^N\right)} R \\ \frac{\alpha}{1-\alpha}\left(B\widetilde{R}^N+\kappa\right) &= \left(1+\widehat{\pi}^N\right)^2 \mathcal{C}'\left(\widehat{\pi}^N\right)\end{aligned}$$

In this unique equilibrium, default may or may not be chosen by the government in the low-output state, depending on whether the level of cost satisfies the following inequality

$$\begin{aligned}\xi_\theta <\ & z(G+\frac{\widetilde{R}^F B-\widetilde{\pi}_L\kappa}{1+\widetilde{\pi}_L}; Y_L) - z(\widehat{T}_L; Y_L) + \left[\mathcal{C}(\widetilde{\pi}_L) - \mathcal{C}\left(\widehat{\pi}^F\right)\right] \\ & -\frac{\alpha}{1-\alpha}\left[\frac{\widetilde{R}^F B-\widehat{\pi}^F\kappa}{1+\widehat{\pi}^F} - \left(\widehat{T}_L - G\right)\right]\end{aligned}$$

(b) there are two equilibria, characterized as in proposition 3, if the government debt B is sufficiently large so that (27) is violated in the high-output state.

Together, these two propositions suggest that, relative to the real economy studied in the previous section, debt-monetization and seigniorage obviously affect the equilibrium policy trade-offs. But *per se* the option to print money does not rule out multiplicity.[28] The reason is straightforward: inflation is not costless from a macroeconomic perspective, and it will be set optimally in relation to the costs involved by raising taxes and/or defaulting.

Multiplicity is actually of exactly the same kind as in the real economy: partial repudiation via haircuts differs across equilibria. Conversely, for a given default rate θ, the inflation rate is uniquely determined — there is no multiplicity in debt monetization. This is because, while inflation can be negative, θ (as discussed in the previous section) can only be positive.

Our results differ from those in Calvo (1988), who also provides an example of monetary economy with multiple equilibria and self-fulfilling expectations of default. The difference depends on two crucial features of the model economy. First, in the monetary version of our model the government may still choose to impose haircuts on the holders of public debt — a possibility that is instead ruled out by assumption in the monetary economy studied by Calvo. Second, inflation costs are convex. In contrast, Calvo (1988) specifies non-convex costs $C(\pi)$, implying multiplicity in the rate of inflation itself.[29] Uniqueness of the inflation rate is a relevant result for the analysis in the rest of our paper, where we study conditions under which central banks can provide a backstop to government debt and rule out self-fulfilling debt crises.

An important question raised by a comparison of propositions 2 and 4 is whether, even if ineffective to rule out self-fulfilling crises, inflationary finance may nonetheless increase macro resilience to them. The question is whether the stock of debt for which the equilibrium is unique is higher in a monetary economy (everything else equal) than in an economy without inflation-related benefits (seigniorage) and distortions. Debt monetization

[28] Note that, as for the real economy in the previous section, by virtue of the regularity conditions we impose on the size of debt relative to the tax capacity of the country, default is always partial in the low-output state in the F-equilibrium, as well as in the high-output state in the N-equilibrium.

[29] Our model would also predict multiplicity in inflation rates, if we replaced our assumptions about $C(\pi)$ with the one in Calvo (1988). See also Obstfeld (1994) for a similar assumption.

has two opposing effects on the decision to default. Consider (27) evaluated at $\kappa = C(\pi) = \pi = 0$, hence determining the threshold stock of debt that marks the switch from equilibrium uniqueness to multiplicity in the real economy. At that point, some revenue from the inflation tax allows the government to reduce taxation and the associated loss of output. Through this channel, inflation raises the level of nominal debt at which the switch occurs. However, there are now output costs due to inflation. A large differential in the inflation costs without and with default tends to lower the switching threshold. If seigniorage revenue turns out to be low in equilibrium, it may be possible that equilibrium multiplicity becomes a problem for a lower stock of initial debt in a monetary economy, compared to the economy with indexed debt and no seigniorage studied in Section 2.[30]

4 Policy options to stem self-fulfilling debt crises

When multiple equilibria are possible, differences in welfare across equilibria are driven by differences in output losses caused by taxation, inflation and default. Specifically, the increase in the interest rate due to self-fulfilling expectations causes unnecessary output disruption not only in the low-output state, but also in the high output state.

The fact that equilibria with non-fundamental default are detrimental to social welfare raises the issue of what kind of policies can be deployed to prevent it. As emphasized by Calvo (1988), there is a straightforward policy that can improve welfare: self-fulfilling debt crises could be prevented by an institution that, in period 1, would credibly set a ceiling $\overline{R}$ on the interest rate, at which it stands ready to buy any amount of government debt. The ceiling $\overline{R}$ should be sufficiently low as to rule out the bad equilibrium driven in part by self-fulfilling expectations, and high enough to avoid

[30] Observe that the budget costs of debt default are independent of inflation, because the state-contingent monetization of the debt (relevant for their calculation) is indeed perfectly anticipated by agents.

ex-ante losses.[31] In our economy of section 2, this would imply:

$$\widetilde{R}^N > \overline{R} \geq \widetilde{R}^F = \frac{R}{1 - (1-\mu)\,\theta_L^F}.$$

Such ceiling would essentially coordinate market expectations on the fundamental equilibrium only. This is because knowing that interest rates cannot rise to the level $\widetilde{R}^N$, the only market equilibrium is one in which private agents find it optimal to bid for the government debt at the lower equilibrium rate $\widetilde{R}^F$. As a result, there is no need to actually purchase any amount of debt. The argument is summarized in the simple game depicted in Figure 5 below. If the lender of last resort is expected to play $\bar{R}$ at the node $\widetilde{R}^N$, the latter rate cannot be a market equilibrium (see De Grauwe 2011 for a recent application of this argument to interventions by the European Central Bank).[32]

[31] It is easy to verify that the ceiling cannot exceed the market rate at which the best response of the government is a strictly positive default rate in the high output state. Note that, while the ceiling on interest rates should be sufficiently low as to rule out the bad equilibrium driven in part by self-fulfilling crises, it should also be at least as high as the interest rate in the fundamental equilibrium in order to avoid a transfer of resources covering the short-fall of fiscal revenues under weak fundamentals – too low an interest rate would effectively amount to a bailout. This would occur if the cap rate were to be set *below* the rate in the fundamental equilibrium:

$$\widetilde{R}^N > \overline{R} \geq \widetilde{R}^F > R^{bail-out}$$

Of course, anticipations of a bailout of this kind is a distortion, creating all sort of destabilizing incentives ex-ante, giving rise to "moral hazard" (see e.g. Green 2010 and Prescott 2010 for a recent discussion).

[32] In Corsetti and Dedola (2011), we show that, in contrast to a transfer implicit in an intervention rate below the fundamental rate, liquidity support does not discourage costly reforms that improve government budget (see also Corsetti et al 2005 and Morris and Shin 2006).

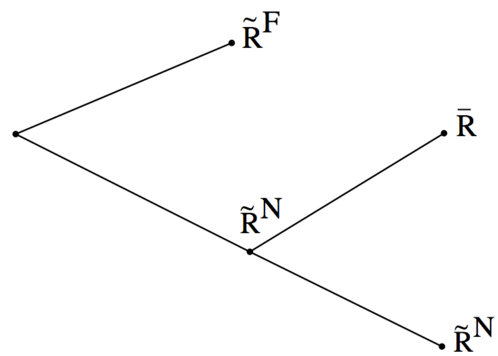

Figure 5

A key feature of the intervention policy is that, to be effective, the rate cap should be fully credible. In other words, private agents must believe that, if they coordinated on the bad equilibrium, the intervening institution would have no incentive to deviate from the announced policy of buying debt at $\overline{R}$, effectively playing the role of lender of last resort. In principle, one could just assume that the intervening institution is able to commit to the pre-announced policy. It is however more realistic and interesting to explore the determinants of its behavior. For interventions to be a sustainable belief, they need to be (i) feasible (the lender of last resort must have sufficient resources) and (ii) welfare improving from the perspective of the intervening institution. Assuming that such an institution is benevolent, this means that domestic welfare must be higher than at $\widetilde{R}^N$. So, a benevolent lender of last resort implies that $\bar{R}$ must be sufficiently lower than $\widetilde{R}^N$, as not to induce the government to default in the high state.

Clearly, a government unable to commit to future policies (as we have assumed in our analysis so far) would not be able to coordinate expectations on its own offering to pay an interest rate not higher than $\bar{R}$. If investors believe there will be default, they will simply refuse to buy debt at a price inconsistent with their expectations, independently of any government announcement. A natural candidate would rather be a deep-pocket external public institution, such as the International Monetary Fund.[33]

[33] A full analysis would require the specification of this institution's objectives and budget constraint (see e.g. Corsetti, Guimaraes, Roubini 2005 and Morris and Shin 2006, and Zwart 2007 among others)

5 Debt default and central bank interventions in the debt market

The question we want to address in the rest of the paper is whether the central bank can rule out self-fulfilling sovereign debt crises under any circumstances. What makes this question particularly intriguing is that, from an aggregate perspective, any purchase of government debt by the monetary authorities is at best backed by their consolidated budget with the fiscal authorities — i.e. there are no additional resources to complement tax and seigniorage revenues.

5.1 The extended model

In modern economies, central bank liabilities (high powered money) include cash and especially bank reserves, often interest-bearing, which are clearly exposed to inflation risk, but not to outright default risk — central banks stand ready to exchange their nominal liabilities with cash at the par value. The reason why this is so may be a consequence of the very high costs that default on assets at the core of the financial system possibly entails, or simply reflects central bank 'commitment'.[34] In our analysis, we do not explore possible explanations, but simply posit by assumption that, while government debt is exposed to the risk of default via both outright haircuts and inflation, central bank liabilities such as high powered money are subject only to ex-post inflation risk.[35] Based on this assumption, we work out conditions under which the central bank can carry out successful interventions in the debt market, without compromising its own budget constraint and welfare objectives. We will thus show that it is by the specific attribute of monetary sovereignty just described, that central banks may be able to redress the problem of equilibrium multiplicity in the debt market.

[34] Another dimension of commitment concerns the ability of a central bank to keep promises on future policies. This raises the question of whether the central bank would be able to eliminate self-fulfilling debt crises by committing to an optimal (state-contingent) inflation plan. In an appendix, we show that this is the case under strict (and arguably unrealistic) conditions on the size of seigniorage relative to public debt.

[35] See Gertler and Karadi (2011) for an analysis of 'unconventional monetary policy' by which central banks exploit their advantage in issuing riskless liabilities to act as financial intermediaries during financial crises, providing funds to private firms. Virtually all monetary model assumes that the central bank does not tamper with the face value of 'money,' although there are historical examples to the contrary (see e.g. Velde 2007).

5.1.1 Budget constraints

Reconsider our model in Section 3, encompassing the possibility that the central bank purchases the amount ωB of the outstanding stock of debt at some pre-announced rate $\overline{R}$. The central bank finances its debt purchases by issuing, in addition to M, monetary liabilities in the form of "reserves" $\mathcal{H}$, remunerated at the default-free nominal rate $(1+\boldsymbol{i})$.[36] As discussed above, while ex-post inflation surprises affect the real value of the outstanding stock of all nominal liabilities (at the price of distortions induced by inflation), outright haircuts θ are applied to B only (at the price of output and budget costs, as discussed in the previous sections).

There are two key motivations for distinguishing between M and $\mathcal{H}$ in our framework. Firstly, from a modelling perspective, in our two-period framework, assuming interest-bearing reserves $\mathcal{H}$ allows us to introduce a demand for central bank liabilities for a given price level in the first period, consistent with the discretionary choice of inflation (the monetary policy "instrument") in the second period.[37] Of course, a backstop to the government may impact expectations of future inflation (in our framework, in the second period), to the extent that the central bank is anticipated to make good on its eventual losses via seigniorage revenue and/or the 'printing press'. The interest rate $1 + i$ paid on reserves will generally reflect inflation expectations: the larger the anticipated monetary expansions, the higher the market-determined nominal interest rate on reserves at the time the central bank issues them. Secondly, from a policy perspective, our treatment of $\mathcal{H}$ reflects a key institutional feature of modern central bank liabilities. In practice, central banks are able to expand their balance sheet without feeding inflationary pressures and expectations, by paying an interest rate on reserves anchored by short-term rates.

Denoting by $\mathcal{T}_i$ the transfers from the central bank to the fiscal authority,

[36] One way to think about this approach is as sterilized interventions that do not change the amount of "liquidity" (M in our model) in the economy, with no consequences for current inflation.

[37] In dynamic monetary models, buying government debt by increasing the money stock does not necessarily result in higher current inflation, as the latter mainly reflects future money growth (see e.g. Diaz et al. (2008) and Martin (2009), placing this consideration at the heart of their analysis of time inconsistency in monetary policy).

the budget constraint of the central bank in the second period reads:

$$\begin{aligned}\mathcal{T}_i &= \frac{\pi_i}{1+\pi_i}\kappa + \frac{(1-\theta_i)}{1+\pi_i}\omega B\overline{R} - \frac{(1+\boldsymbol{i})}{1+\pi_i}\mathcal{H} = \\ &= \frac{\pi_i}{1+\pi_i}\kappa + \left(\frac{(1-\theta_i)}{1+\pi_i}\overline{R} - \frac{(1+\boldsymbol{i})}{1+\pi_i}\right)\omega B.\end{aligned}$$

In writing the budget constraint of the fiscal authority, we find it analytically convenient to proceed under the following two assumptions. First, the government cannot discriminate the central bank's from agents' holding of debt when applying the haircut θ_i. Second, the cost of defaulting on the central bank, α_{CB}, may be different from the cost of defaulting on the private sector, α. The government budget constraint in either state of nature in period 2 then reads:

$$T_i - G = [1-\theta_i(1-\alpha)]\frac{\widetilde{R}}{1+\pi_i}(1-\omega)B + [1-\theta_i(1-\alpha_{CB})]\frac{\overline{R}}{1+\pi_i}\omega B - \mathcal{T}_i \tag{35}$$

where $\alpha, \alpha_{CB}, \theta_i \in [0,1]$, and $\widetilde{R}$ is again the equilibrium interest rate at which agents are willing to buy the outstanding government debt $(1-\omega)B$.

Consolidating the budget of the fiscal and the monetary authorities yields the following key expression:

$$T_i + \frac{\pi_i}{1+\pi_i}\kappa - G = [1-\theta_i(1-\alpha)]\frac{\widetilde{R}}{1+\pi_i}(1-\omega)B + \left[\theta_i\alpha_{CB}\frac{\overline{R}}{1+\pi_i} + \frac{(1+\boldsymbol{i})}{1+\pi_i}\right]\omega B \tag{36}$$

Ultimately, the primary surplus cum seigniorage (on the left-hand-side) finances both the interest payments by the government to private investors (net of default but gross of the transaction costs associated to it); *and* the interest bill of the central bank — always paid in full under our assumptions. The consolidated budget constraint so clarifies our earlier point, that the purchase of government debt financed by issuing reserves today does not mechanically translate into higher inflation in the future. It raises inflation only to the extent that, after repaying the bonds in the hands of private investors (net of default but gross of transaction costs), the primary surplus falls short of the interest bill on reserves at the desired level of inflation (and thus at the desired seigniorage level).

The budget constraint of the representative agent is

$$C_i = [Y_i - z(T_i; Y_i) - \xi_\theta] + KR - T_i + (1-\theta_i)(1-\omega)\frac{B}{1+\pi_i}\widetilde{R} + \frac{(1+\boldsymbol{i})}{1+\pi_i}\mathcal{H} - \frac{\pi_i}{1+\pi_i}\kappa - \mathcal{C}(\pi_i),$$

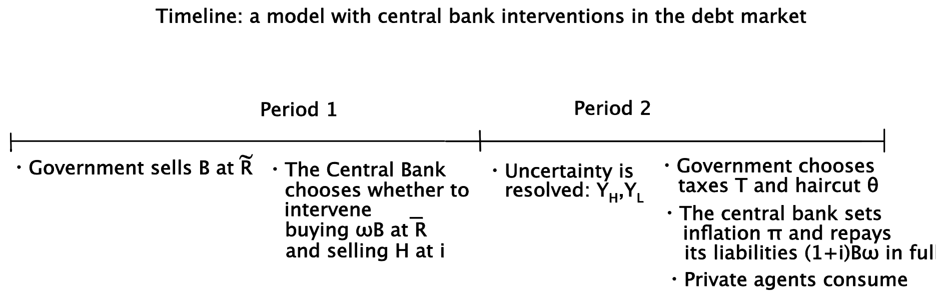

Figure 6

Combining the three constraints above we can write the objective function of benevolent policymakers:

$$\begin{aligned} C_i &= [Y_i - z(T_i; Y_i) - \xi_\theta] + KR + \\ &\quad -T_i + (1-\theta_i)(1-\omega)\frac{B}{1+\pi_i}\widetilde{R} - \mathcal{T}_i + \frac{\pi_i}{1+\pi_i}\kappa + \frac{(1-\theta_i)}{1+\pi_i}\omega B\overline{R} - \frac{\pi_i}{1+\pi_i}\kappa - \mathcal{C}(\pi_i) \\ &= [Y_i - z(T_i; Y_i) - \xi_\theta] - G - \theta_i\left[(1-\omega)\alpha\widetilde{R} + \omega\alpha_{CB}\overline{R}\right]\frac{B}{1+\pi_i} - \mathcal{C}(\pi_i). \end{aligned} \tag{37}$$

The timeline is summarized by Figure 6.

5.1.2 Fiscal and monetary policy reaction functions under discretion

Below we characterize optimal policies under discretion by benevolent policymakers under the consolidated budget constraint (36), taking the central bank intervention strategy (ω and $\overline{R}$) and the interest rate $\boldsymbol{i}$ set in period 1 as given. Hereafter, a bar above a variable refers to an allocation where $\omega > 0$, i.e. conditional on positive purchases by the central bank. As discussed in section 4, with a succesful backstop strategy the policies derived below characterize the off-equilibrium path.

Conditional on default, the upper bound on the country's willingness to raise distortionary taxes is:

$$z'(\widehat{\overline{T}}_i; Y_i) = \frac{\alpha(1-\omega) + \omega\alpha_{CB}}{1 - [\alpha(1-\omega) + \omega\alpha_{CB}]}, \tag{38}$$

where a bar above variables now indicates outcomes for an intervention policy $\{0 < \omega < 1 \text{ and } \overline{R}\}$, under either no default (e.g., $\widetilde{\overline{T}}$) or default (e.g., $\widehat{\overline{T}}$). The maximum primary surplus (net of the inflation tax) that the country finds it optimal to generate in the second period, $\widehat{\overline{T}}_i - G$, and the associated net output $Y_i - z\left(\widehat{\overline{T}}_i\right)$, are now a function of the product between ω and the difference between α and α_{CB}. Insofar as the variable costs of default falls with debt purchases by the central bank, i.e. $\alpha > \alpha_{CB}$, so does the optimal taxation $\widehat{\overline{T}}$. However, while central bank purchases do affect the incentives to increase taxes relative to imposing a higher haircut θ_i, when $\theta_i = 1$ taxes will still have to adjust to satisfy the budget constraint (at the equilibrium level of transfers from the central bank $\widehat{\overline{\mathcal{T}}}$, and inflation $\widehat{\overline{\pi}}_i$):

$$\widehat{\overline{T}}_i \leq \widehat{\overline{T}}_i^{+} = G + \alpha \frac{\widetilde{\overline{R}}}{1+\widehat{\overline{\pi}}_i}(1-\omega)B + \alpha_{CB}\frac{\overline{R}}{1+\widehat{\overline{\pi}}_i}\omega B, \quad i = L, H \tag{39}$$

Conditional on no default, given seigniorage revenues, taxes must be raised to cover total public spending.

The optimal inflation rate (associated with the optimal tax rates) is given implicitly by the following two equations, one conditional on default:

$$\frac{\alpha(1-\omega)+\omega\alpha_{CB}}{1-[\alpha(1-\omega)+\omega\alpha_{CB}]}\left[B\widetilde{\overline{R}} + \kappa - \left(\widetilde{\overline{R}} - (1+\boldsymbol{i})\right)\omega B\right] = \left(1+\widehat{\overline{\pi}}\right)^2 \mathcal{C}'\left(\widehat{\overline{\pi}}\right), \tag{40}$$

the other conditional on no default ($\theta_i = 0$):

$$z'(\widetilde{\overline{T}}_i; Y_i)\left[B\widetilde{\overline{R}} + \kappa - \left(\widetilde{\overline{R}} - (1+\boldsymbol{i})\right)\omega B\right] = \left(1+\widetilde{\overline{\pi}}_i\right)^2 \mathcal{C}'\left(\widetilde{\overline{\pi}}_i\right). \tag{41}$$

It is easy to verify that, for $\omega = 0$, these optimality conditions are the same as in Section 3.

When both B and $\mathcal{H}$ are held by the private sector, there are two equilibrium interest parity conditions. First, the interest rate on reserves $1+\boldsymbol{i}$, free from the outright default risk, must differ from the real rate R by private agents' expectations of inflation:

$$(1+\boldsymbol{i}) = [\mu(1+\pi_H) + (1-\mu)(1+\pi_L)]\,R. \tag{42}$$

Second, as long as the central bank does not buy up the whole stock of outstanding debt, the interest rate on government debt required by the private sector must exceed the interest paid on central bank's liabilities by the expected rate of default:

$$(1+\boldsymbol{i}) = \widetilde{\widetilde{R}}\left[\mu\left(1-\theta_H\right)+\left(1-\mu\right)\left(1-\theta_L\right)\right] \qquad \text{if } 0<\omega<1. \tag{43}$$

These expressions underscore that interest rates in period 1 are rising in expectations of both inflation and default.

5.2 The credibility of the monetary backstop

In characterizing the optimal plans above, the market interest rates $\widetilde{R}$ and $\boldsymbol{i}$, and the central bank purchases ω and intervention rate $\overline{R}$ are treated as predetermined. We now focus on the question of characterizing intervention policies $\{\omega$ and $\overline{R}\}$ that are feasible and welfare-improving over the N-equilibrium. As argued above, these conditions are necessary for central bank interventions to rule out multiplicity.

Through the lens of our default model, we can identify the precise conditions under which the central bank can be an effective 'lender of last resort to the government.' The problem at hand is compelling because in most circumstances it is difficult to exclude a positive probability of a fundamental default, causing a fiscal shortfall and thus potential losses on the debt owned by the central bank ex-post. In this case, either taxes or seigniorage, or possibly both, must adjust, in line with the classical analysis by Sargent and Wallace (1981).

The question once again pertains to the interactions between fiscal and monetary authorities. Consistent with the approach taken so far in the paper, we assume that authorities pursue the same objective — maximizing the residents' welfare ex post. However, we will now contrast the case when the monetary and fiscal authorities face a consolidated budget constraint (analyzed above) with the other polar case, of separate budget constraints. In the case of budget separation (likely to arise from institutional or political frictions among policymakers), we will focus on the consequences of imposing that the transfers from the central bank to the government can only be positive, de facto making the central bank responsible for fully absorbing its losses.

A specific issue of interest is whether, in order to rule out self-fulling debt crises, the central bank will have to credibly threaten to intervene up to satisfying the entire financial needs of the government; or threaten just to carry out possibly large but bounded purchases of government paper.

5.2.1 Consolidated budget constraint

Under a consolidated budget constraint (the case assumed in characterizing discretionary policy reaction functions in the previous subsections), it is easy to show that a strategy by which the central bank stands ready to underwrite the government debt issuance in full ($\omega = 1$) is feasible and welfare-improving relative to the non-fundamental equilibrium allocation. Omitting the proof, we state the main result: At $\omega = 1$, there exists a $\overline{R}$ such that all the budget constraints and the first-order conditions for the optimal discretionary policy plan spelled out in the previous subsection are satisfied, and welfare is higher than in the N-equilibrium.

What makes the threat to implement interventions up to the scale $\omega = 1$ credible is the understanding that, if fundamental fiscal stress in the low output state causes default, the fiscal authorities are willing to make positive contingent transfers to the central bank, as to ensure that monetary liabilities are fully honored at the desired level of taxation and inflation (that is, without the need of raising inflation above the optimal discretionary plan).

The intuition for this result is straightforward: by buying public debt, the central bank is able to swap risky government liabilities with monetary liabilities on which no discrete default is expected ex post — in practice redressing the government lack of commitment to service its debt, and allowing it to borrow at a rate even lower than $\widetilde{R}^F$.

However, this result heavily relies on very benign assumptions regarding the interactions between fiscal and monetary authorities. While these interactions can be expected to affect many dimensions of the policy problem, for our purpose their key implication is that they generally translate into constraints that prevent budget consolidation. Indeed, in the actual conduct of monetary policy, central banks are held responsible for backing their own liabilities, possibly using the printing press, in all states of the world.

Under budget separation then the threat to fully backstop all outstanding debt ($\omega = 1$) would hardly be credible. First, seigniorage revenue is bounded (in our model $\lim_{\pi\to\infty} \frac{\pi_i}{1+\pi_i}\kappa = \kappa$) and inflation is anticipated by rational agents: unless κ is implausibly large, or B too small to create a situation of fiscal stress, seigniorage and debt monetization may be insufficient for the central bank to repay $(1+\boldsymbol{i})\,\mathcal{H}$ in full under all circumstances. Second, with convex costs of inflation, using only the printing press, even when feasible, may not improve over the N-equilibrium allocation. As discussed above, it is optimal to use all instruments, taxes, default and inflation, to minimize their combined distortions. It follows that a break even constraint on the central bank, requiring positive fiscal transfer $\mathcal{T}_i \geq 0$ in $i = H, L$, can easily undermine the effectiveness of interventions with scale $\omega = 1$.

Under budget separation, central bank solvency might not be jeopardized if the backstop only targets a fraction of government debt, $\omega < 1$. The question is then whether central bank interventions of limited scale may actually succeed in coordinating market expectations on a good equilibrium. To answer this question, we now build on the analysis in sections 2 and 3, stressing a key implication of fixed costs of default, namely, there is a threshold value for the stock of government liabilities, below which the equilibrium is unique, and self-fulfilling debt crises do not occur. In the next subsection, we will build on this result to explain why a monetary backstop to the government can be effective, even under strict constraint on fiscal and monetary interactions.

5.2.2 Separate budget constraint

Can interventions be effective, when their size is necessarily bounded by the fact that the central bank operates a separate budget constraint? In the presence of fixed and variable costs of default, the size of the prospective interventions can be chosen by striking a balance between two competing forces. On the one hand, we have seen above that the larger the central bank holdings of government paper, the larger the risk of adverse inflationary consequences, in case of fundamental default. On the other hand, by buying a sufficiently large amount of public debt, the central bank can substantially lower the interest bill of the government, reducing the taxation cum seigniorage (and the associated costs) required to service public debt at market rates.

Because of fixed costs of default, large enough interventions will at some point make the alternative of not defaulting in the high state Y_H more attractive for the fiscal authority. Depending on the properties of the cost functions $z(\cdot)$ and $C(\cdot)$, there will be some value of $\omega < 1$, i.e., an intervention consisting of limited purchases of debt, that will ensure a unique equilibrium without default. By the logic of liquidity support reviewed in Section 2, threatening to buy this amount debt (instead of going for $\omega = 1$) would enable a central bank to coordinate the market away from a nonfundamental equilibrium, while satisfying its budget constraint.

Formally, for a strategy of limited purchases to be effective, it must be the case that for $\omega < 1$ default is suboptimal even if markets charge $\overline{\widetilde{R}}^N$, that is:

$$F(\omega) \equiv z\left(\tilde{\tilde{T}}_H; Y_H\right) + \mathcal{C}\left(\tilde{\tilde{\pi}}_H\right) - \left[z\left(\hat{\hat{T}}_H; Y_H\right) + \mathcal{C}\left(\hat{\hat{\pi}}\right)\right] \tag{44}$$
$$-\frac{\alpha}{(1-\alpha)}\left\{\left[1-\left(1-\frac{(1+\boldsymbol{i})}{\tilde{\tilde{R}}^N}\right)\omega\right]\frac{\tilde{\tilde{R}}^N B}{1+\hat{\hat{\pi}}} - \frac{\hat{\hat{\pi}}}{1+\hat{\hat{\pi}}}\kappa - \left(\hat{\hat{T}}_H - G\right)\right\} \leq \xi_\theta,$$

whereas, at $\tilde{\tilde{R}}^N$ (priced according to (43) for non-fundamental haircut rates) default would be unavoidable if the central bank were unwilling to intervene, that is, $F(\omega = 0) > \xi_\theta$. The condition (44) emphasizes that central bank purchases have to be large enough to decrease the distortions due to taxation and inflation required to avoid default in the high state, namely $z\left(\tilde{\tilde{T}}_H, Y_H\right) + \mathcal{C}\left(\tilde{\tilde{\pi}}_H\right)$, by more than they reduce distortions conditional on default, associated to $\hat{\hat{\pi}}$ and $\hat{\hat{T}}_H$. Intuitively, by decreasing the effective (cost of) debt, the effects of limited interventions would be akin to moving the economy from the region of multiple equilibria to the region of uniqueness region depicted in Figure 3, ruling out $\tilde{\tilde{R}}^N$ as an equilibrium interest rate.

In the above expression, $\tilde{\tilde{T}}_H$ and $\tilde{\tilde{\pi}}_H$ must satisfy the marginal conditions under no default evaluated at $\tilde{\tilde{R}}^N$:

$$\tilde{\tilde{T}}_H - G = \frac{\tilde{\tilde{R}}^N}{1+\tilde{\tilde{\pi}}_H}(1-\omega)B - \frac{\tilde{\tilde{\pi}}_H}{1+\tilde{\tilde{\pi}}_H}\kappa + \frac{(1+\boldsymbol{i})}{1+\tilde{\tilde{\pi}}_H}\omega B \tag{45}$$
$$z'(\tilde{\tilde{T}}_H; Y_i)\left[B\tilde{\tilde{R}}^N + \kappa - \left(\tilde{\tilde{R}}^N - (1+\boldsymbol{i})\right)\omega B\right] = \left(1+\tilde{\tilde{\pi}}_H\right)^2 \mathcal{C}'\left(\tilde{\tilde{\pi}}_i\right).$$

Note that these conditions are the same as under budget consolidation, since without default the break-even constraint does not bind. Conversely, $\hat{\hat{T}}_H$ and $\hat{\hat{\pi}}_H$ are defined, respectively, by (38), and, if the break-even constraint does not bind, by the counterpart of **(40)** evaluated at $\tilde{\tilde{R}}^N$

$$\frac{\alpha(1-\omega)+\omega\alpha_{CB}}{1-[\alpha(1-\omega)+\omega\alpha_{CB}]}\left[B\tilde{\tilde{R}}^N + \kappa - \left(\tilde{\tilde{R}}^N - (1+\boldsymbol{i})\right)\omega B\right] = \left(1+\hat{\hat{\pi}}\right)^2 \mathcal{C}'\left(\hat{\hat{\pi}}\right). \tag{46}$$

However, with non-fundamental default possible also in the high state, we should now allow for the possibility that the break-even constraint binds. With a binding constraint, the inflation rate (denoted $\hat{\hat{\pi}}_H^+$) must increase, as to satisfy

$$\mathcal{T}_H = \frac{\kappa \widehat{\overline{\pi}}_H^{+} + \left(\left(1 - \widehat{\overline{\theta}}_H^{N} \right) \overline{R} - (1 + \boldsymbol{i}) \right) \omega B}{1 + \widehat{\overline{\pi}}_H^{+}} \geq 0. \tag{47}$$

It is apparent that, as far as $\widetilde{\overline{R}}^{N} > (1 + \boldsymbol{i})$ (a condition always met in equilibrium), $\widetilde{\overline{T}}_H$ and $\widetilde{\overline{\pi}}_H$ and thus the associated distortions are both decreasing in ω: larger interventions make servicing the debt without default a more attractive option. On the other hand, while $\widehat{\overline{\pi}}_H$, $\widehat{\overline{T}}_H$ (with $\alpha_{CB} < \alpha$) are also decreasing in ω (thus reducing the variable cost of default), large enough interventions will make the break even constraint bind, leading to inefficiently high inflation $\widehat{\overline{\pi}}_H^{+}$—decreasing the attractiveness of default in (44).

In determining the maximum size of credible interventions, however, constraints on the central bank's balance sheet are more relevant in the low state, since, off the equilibrium path, fundamental fiscal stress is compounded by the high non-fundamental interest rates required by investors on their holdings of government debt. In the low state the inflation rate is the highest between the above efficient rate under default, $\widehat{\overline{\pi}}_L$ (defined by the counterpart of (46)) or the rate $\widehat{\overline{\pi}}_L^{+}$ determined residually to meet the break-even constraint

$$\mathcal{T}_L = \frac{\kappa \widehat{\overline{\pi}}_L^{+} + \left(\left(1 - \widehat{\overline{\theta}}_L^{N} \right) \overline{R} - (1 + \boldsymbol{i}) \right) \omega B}{1 + \widehat{\overline{\pi}}_L^{+}} \geq 0. \tag{48}$$

This condition effectively bounds the size of interventions ωB — defining the worst case scenario for losses that impinge on the credibility of backstop announcement.

We should note here that, were the probability of fundamental fiscal stress zero ($\mu \to 1$), a monetary backstop strategy would always be effective. Sufficiently large interventions that decrease the interest rate burden for the government to satisfy (44) would always be credible. An instance of such strategies is provided by the limiting case $\omega \to 1$, so that the government would not even face $\widetilde{\overline{R}}^{N}$, implying that (44) trivially holds.

Deriving specific conditions under which credible interventions in the above class exist would require further specialization of the model's primitives, regarding the output and budget costs of taxation, inflation and default. Ultimately, the merits of specific assumptions on these costs are to be grounded on an empirical assessment, which is beyond the scope of this paper. Nonetheless, establishing the logical possibility that limited interventions can be effective in stemming self-fulfilling debt crises is a novel result, shedding some light on why the announcement of actual backstop policies tend to have a significant impact on debt prices, even when their scale is not unbounded.

6 Conclusions

This paper has reconsidered in detail the question of whether and why the central bank can provide a backstop to the government, as to rule out self-fulfilling debt crises. We have shown that successful intervention strategies build on the ability of the central bank to swap (default-) risky government debt with nominal liabilities which can always be redeemed against currency — exposed to inflation risk but free of the fear of default. We have then shown that interventions not covering the government's financial needs in full may be as effective as the large scale interventions envisioned by drawing on the conventional wisdom on the lender of last resort. By leaving a substantial share of debt issuance in the hands of investors, the central bank can actually take advantage of the potentially large costs of default on investors. It can credibly stand ready to buy debt up to an amount which would make a decision to default suboptimal (against these costs) even at the high market rates associated with a non-fundamental expectations of a crisis. The threat of such intervention would rule out these rates as an equilibrium outcome.

Our main conclusions resonate with the widespread policy view, that a central bank has indeed the power to backstop the fiscal authority, although for different reasons that many observers invoke. This power is quite strong when either there is no fundamental risk of default, or the monetary and fiscal authorities operate under the mutual understanding that possible contingent losses by the central bank due its interventions in the debt market are covered by the treasury. With fundamental fiscal risk, absent this understanding, the size of monetary interventions is constrained by (the present discounted value of) seigniorage and the gains from inflating debt, at the rate of inflation the central bank is willing to purse according to its own plans. Indeed, the monetary backstop we study is effective because it actually prevents scenarios of runaway inflation, enabling the central bank to intervene even extensively without compromising on its objectives. This is a different argument from the one stressing the ability of countries with monetary sovereignty to run the printing press at will and engineer an inflationary debt debasement, essentially affecting the range of debt over which self-fulfilling crises are possible.

Although our analysis is carried out in closed economy, the lessons for a currency area are apparent. Arguably, the extent of fiscal support to the central bank is the key condition most critics of 'incomplete' monetary unions, e.g. lacking political union, have in mind, when praising the advantage of retaining a national currency. Fiscal support to the central bank, if only limited to financial stress situations, is clearly much more difficult in a currency union among essentially independent nations. Countries in a monetary union could thus be more vulnerable to debt crises, to the extent that the common central bank cannot count on the joint support of all national fiscal authorities. But as our analysis has shown, the common central bank still has the ability to engineer successful interventions. Namely, it will have to weigh the benefits of providing a backstop to countries exposed to debt crises, against the cost of drawing on seigniorage accruing to all countries in the union.

References

[1] Adam K. and M. Grill, "Optimal sovereign debt default", mimeo

[2] Aguiar M., M. Amador, E. Farhi and G. Gopinath (2012) Crisis and Commitment: Inflation Credibility and the Vulnerability to Sovereign Debt Crises, mimeo

[3] Barro R.J. (1983). Inflationary finance under discretion and rules, *Canadian Journal of Economics* 16: 1-16

[4] Calvo G. (1988). "Servicing the Public Debt: The Role of Expectations". *American Economic Review* 78(4): 647-661

[5] Cole, H. L. and Kehoe, T. (2000). "Self-Fulfilling Debt Crises". *Review of Economic Studies*, 67: 91–116

[6] Cohen, D. and Villemot, S. (2011). "Endogenous debt crises". CEPR Discussion Paper no. 8270. London, Centre for Economic Policy Research.

[7] Cooper R. (2012). "Fragile Debt and the Credible Sharing of Strategic Uncertainty", mimeo

[8] Corsetti G. and L. Dedola (2011). "Fiscal Crises, Confidence and Default: A Bare-bones Model with Lessons for the Euro Area". mimeo, Cambridge University.

[9] Corsetti G., Guimaraes B. and Roubini N. (2005). "International lending of last resort and moral hazard: A model of IMF's catalytic finance." *Journal of Monetary Economics*, 53(3): 441-471.

[10] Cruces, J. and C. Trebesch (2012). "Sovereign Defaults: The Price of Haircuts", mimeo.

[11] De Grauwe P. (2011), "The governance of a fragile eurozone", Economic Policy CEPS Working Documents.

[12] Diaz J., G. Giovannetti, R. Marimon and P. Teles (2008). "Nominal Debt as a Burden on Monetary Policy", *Review of Economic Dynamics*, 11(3): 493-514.

[13] Gertler M. and P. Karadi (2011). "A Model of Unconventional Monetary Policy", Journal of Monetary Economics 58: 17-34.

[14] Goodfriend M. (2011). "Central Banking in the Credit Turmoil: An Assessment of Federal Reserve Practice," Journal of Monetary Economics, 58(1): 1-12.

[15] Green E. (2010), "Bailouts." *Richmond Federal Reserve Bank Economic Quarterly*, 96(1).

[16] Jeanne O. (2012). "Fiscal Challenges to Monetary Dominance in the Euro Area: A Theoretical Perspective". mimeo.

[17] Martin F. (2009). "A Positive Theory of Government Debt", *Review of Economic Dynamics*, 12:608-631

[18] Morris S. and H.S. Shin (2006). "Catalytic Finance: When Does it Work?". *Journal of International Economics*, 70(1): 161-177.

[19] Prescott E. (2010). "Introduction to the Special Issue on the Diamond and Dybvig Model". *Richmond Federal Reserve Bank Economic Quarterly*, 96(1): 1-9

[20] Reinhart C. and K. Rogoff (2009). *This Time Is Different: Eight Centuries of Financial Folly*, Princeton University Press

[21] Reinhart, Carmen M. and Rogoff, Kenneth S. (2011), “The Forgotten History of Domestic Debt.” *The Economic Journal*, 121(552): 319-350.

[22] Reis. R. (2013). “The Mystique Surrounding the Central Bank’s Balance Sheet, Applied to the European Crisis”. *National Bureau of Economic Research*, Working Paper No. 18730

[23] Roch F. and H. Uhlig (2011). “The Dynamics of Sovereign Debt Crises and Bailouts”, mimeo

[24] Sargent T. and N. Wallace (1981). “Some unpleasant Monetarist Arithmetic”, *Quarterly Review,* Federal Reserve Bank of Minneapolis, 5(3): 1-17.

[25] Persson T. and G. Tabellini (1993). “Designing institutions for monetary stability” *Carnegie-Rochester Conference Series on Public Policy* 39: 53–84

[26] Uribe M. (2006). “A Fiscal Theory of Sovereign Risk”. *Journal of Monetary Economics* 53: 1857-1875.

[27] Velde F. (2007), “John Law’s System”, The American Economic Review 97(2): 276-279

[28] Tirole J. (2012). “Country Solidarity, Private Sector Involvement and the Contagion of Sovereign Crises,” mimeo

[29] Zwart S. (2007). “The mixed blessing of IMF intervention: Signalling versus liquidity support”. *Journal of Financial Stability*, 3(2): 149-174.

7 Appendix

7.1 Infinite horizon

Consider the following dynamic version of the Calvo model with real debt, whereas, without loss of generality, we assume deterministic output (as in the original paper by Calvo). The fiscal authority under discretion solves the following program:

$$\begin{aligned}
V(B;Y) &= \max_{T,\theta,B'} C + \beta V\left(B';Y'\right) \\
C &= \left(Y - z\left(G - B' + [1-\theta(1-\alpha)]\, B\widetilde{R}\right)\right) + (1-\theta)\, B\widetilde{R} - T - B' \\
T &= G + [1-\theta(1-\alpha)]\, B\widetilde{R} - B' \\
\widetilde{R}' &= \frac{R}{1-\theta(B')}, \theta \in [0,1] \\
R\beta &= 1
\end{aligned}$$

With linear utility and a discount factor β, the competitive equilibrium is characterized by the last condition — equating the risk free rate to the agents' discount factor. Agents are thus indifferent between present and future consumption. Rewrite the above substituting the budget constraints, we obtain

$$\begin{aligned}
V(B;Y) &= \max_{\theta,B'} \left\{Y - G - z\left(G - B' + [1-\theta(1-\alpha)]\, B\widetilde{R}\right) - \theta\alpha B\widetilde{R}\right\} + \beta V\left(B';Y'\right) \\
\beta\widetilde{R}' &= \frac{1}{1-\theta(B')}, \theta \in [0,1],
\end{aligned}$$

where the function $z(\cdot)$ satisfies the conditions in Calvo (1988), and we are using the standard dynamic notation with a $'$ denoting variables in the next period.

Focusing on Markov perfect equilibria, specifically assuming the fiscal authority next period will decide on debt issuance B'' and default rate θ' as a function of B', the first order conditions and are as follows:

$$\theta : \theta\left(z_T\left(G - B' + [1-\theta(1-\alpha)]\, B\widetilde{R}\right) - \frac{\alpha}{1-\alpha}\right) = 0$$

$$B' : z_T\left(G - B' + [1-\theta(1-\alpha)]\, B\widetilde{R}\right) = -\beta V_{B'}\left(B';Y'\right)$$

where because of the envelope condition:

$$-V_{B'}\left(B';Y'\right)=$$
$$\left[\alpha\theta'+z_T\left(G-B''+\left[1-\theta'\left(1-\alpha\right)\right]B'\widetilde{R}'\right)\left(1-\theta'\left(1-\alpha\right)\right)\right]\widetilde{R}'\left[1+\frac{B'}{1-\theta\left(B'\right)}\theta_{B'}\left(B'\right)\right].$$

We thus obtain the following second order difference equation in B :

$$z_T\left(G-B'+\left[1-\theta\left(1-\alpha\right)\right]B\widetilde{R}\right)=$$
$$\beta\widetilde{R}'\left[\alpha\theta'+z_T\left(G-B''+\left[1-\theta'\left(1-\alpha\right)\right]B'\widetilde{R}'\right)\left(1-\theta'\left(1-\alpha\right)\right)\right]\cdot\left[1+\frac{B'}{1-\theta\left(B'\right)}\theta_{B'}\left(B'\right)\right]$$

Observe that the optimality condition wrt θ implicitly defines it as a function of B and B' :

$$\theta=\frac{G+B\widetilde{R}-B'-z_T^{-1}\left(\frac{\alpha}{1-\alpha}\right)}{\left(1-\alpha\right)B\widetilde{R}},$$

also allowing to compute its partial derivative relative to B:

$$\theta_{B'}=\begin{cases}0, & \theta=0,1\\ \dfrac{1-\theta'\left(1-\alpha\right)}{\left(1-\alpha\right)B'}, & otherwise\end{cases}.$$

While a full characterization of this model would be of interest on its own, for our purpose we note that the Calvo two-period model is nested in a special version of the above, whereas current government act under discretion and can choose default ex post, while future governments cannot default. This implies $\theta'=0$, and $\beta\widetilde{R}'=1$. In this case, the equilibrium conditions simplify as follows

$$\begin{aligned}\theta\left(z_T\left(G-B'+\left[1-\theta\left(1-\alpha\right)\right]B\widetilde{R}\right)-\frac{\alpha}{1-\alpha}\right)&=0\\ z_T\left(G-B'+\left[1-\theta\left(1-\alpha\right)\right]B\widetilde{R}\right)&=z_T\left(G-B''+B'R\right)\\ z_T\left(G-B''+B'R\right)&=z_T\left(G-B'''+B''R\right).\end{aligned}$$

From the last equation, it is clear that any level of debt B' inherited by the future governments, it will be rolled over forever:

$$-B''+B'R=-B'''+B''R<=>B'''=B''=B'.$$

Future taxes will also be constant and equal to $T'=T''=G+B'\left(R-1\right).$ By the first two order conditions above, it must be the case that this level of taxation does not exceed the level associated with the maximum amount of distortions, $T' \leq \widehat{T} = z_T^{-1}\left(\frac{\alpha}{1-\alpha}\right)$. This level effectively provides an upper bound on the debt that can be issued by the discretionary government in the first period, namely:

$$B' \leq \frac{\widehat{T} - G}{(R-1)}.$$

Conditional on this upper bound on the level of debt that can be rolled over, the optimal policy for the government in the first period consists of minimizing distortions, according to the following choice of default rate as a function of initial debt and market interest rates:

$$\begin{aligned} \widehat{T} - G + B' &= [1 - \theta(1-\alpha)]\, B\widetilde{R} \\ \theta &= \frac{B\widetilde{R} - \left(\widehat{T} - G\right)\left(1 + \frac{1}{R-1}\right)}{(1-\alpha)\, B\widetilde{R}}. \end{aligned}$$

The optimal default is identical to the one derived in the two-period Calvo model,

$$\theta = \frac{B\widetilde{R} - \left(\widehat{T} - G\right)}{(1-\alpha)\, B\widetilde{R}}$$

up to a term capturing the present discounted value of primary surpluses.

Given the private sector reaction function $\widetilde{R} = \frac{R}{1-\theta}$, by the logic of the model, two equilibria will always exist in the infinite horizon model for the range of initial debt satisfying $0 < B < \left(\frac{\widehat{T} - G}{R-1}\right)$. Note that this range is wider than the corresponding range in the two-period model, that is, $0 < B < \left(\frac{\widehat{T} - G}{R}\right)$. This is of course implied by the cumulation of primary surpluses in the future.

In the infinite horizon model, one equilibrium will be characterized by no default in the first period. The initial level of debt B is rolled over forever, associated with the (low) level of taxes $T = G + B(R-1) < \widehat{T}$. The other equilibrium will feature default in the first period, and higher tax distortions associated with a higher level of debt $G + B'(R-1) = \widehat{T}$.

Note that, in this model, the only connection between the current and future period is the possibility that the incumbent discretionary government issues debt even when defaulting. Without this possibility the model would be exactly the same as the one studied in the text.

7.2 Central bank with commitment

Using the model specification of Section 3, we now assume a policy scenario in which the fiscal authority still acts under discretion, but the benevolent central bank can credibly commit to a state-contingent inflation policy. In our setting, this implies that inflation will be chosen before agents form expectations and bid for the government debt at the interest rate $\widetilde{R}$ (see Persson and Tabellini 1993). For simplicity of exposition, we posit $\mu \to 1$, so that there is no fundamental fiscal stress in the economy. Under policy discretion, multiplicity of course still obtain with $\mu \to 1$, as a special case of propositions 3 and 4.

Under the hypothesis that the central bank can commit, the marginal conditions governing the choice of inflation become, respectively:

$$\left(1 + \pi_H^F\right)^2 \mathcal{C}'\left(\pi_H^F\right) = z'\left(T_H, Y_H\right)\kappa \tag{49}$$

in the fundamental equilibrium without default, and

$$\left(1 + \pi_H^N\right)^2 \mathcal{C}'\left(\pi_H^N\right) = -\kappa, \tag{50}$$

conditional on (non-fundamental) default, if any. Note that, contrary to the analysis in Section 3, in the fundamental equilibrium inflation is positive only to the extent that seigniorage revenue is optimally traded-off against distortionary taxation — there is no systematic attempt by the central bank to resort to surprise inflation (compare the above expressions with equation (26)). Indeed, conditional on non-fundamental default, the optimal inflation rate could even be negative to support consumption by increasing the real value of money. Moreover, as apparent from the above expressions, the optimal inflation is decreasing in the demand for real balances κ — capping the seigniorage revenue (seigniorage and thus inflation is zero for $\kappa \to 0$).

By virtue of commitment, however, under certain conditions the central bank may be able to rule out multiplicity. Namely, the central bank may find it optimal to threaten to raise inflation and seigniorage in response to speculative pressure in the debt market driving the interest rate away from the fundamental value — with the result of undermining $\widetilde{R}^N$ as an equilibrium outcome. It can be shown that such a threat can indeed be part of the optimal inflation policy of the central bank under two strict conditions. First, debt cannot be too high relative to seigniorage revenue, so that the budget constraint would still be satisfied under inflationary financing (note that if $\kappa = 0$, seigniorage and thus optimal inflation would always be zero independently of default); second, the fixed output costs of default are large enough relative to the costs of inflation, so that inflationary financing would be welfare-improving over the N-equilibrium. Holding these conditions, indeed, the benefits from increased inflation and seigniorage (mitigating the need for raising taxes) in terms of avoiding the output losses due to default, would largely exceed the costs of inflation. Yet, the range of applicability of this result is rather narrow.