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**SOCIAL SECURITY, UNEMPLOYMENT
RISK AND EFFICIENT BARGAINING
BETWEEN UNIONS AND FIRMS**

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*INTERNATIONAL MACROECONOMICS
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ABSTRACT

Social Security, Unemployment Risk and Efficient Bargaining between Unions and Firms*

We construct an overlapping generations model with unemployment risk where wages, employment and severance payments are set through efficient bargaining between risk averse Unions and risk neutral firms. Assuming that a First Best cannot be achieved due to workers' shirking incentives, we characterize a Second Best allocation and show how this can be implemented in a market economy. We prove that the latter generates too little employment and consumption smoothing, an excessive young age consumption and too much saving with respect to the Second Best. This inefficiency can be reduced by increasing the intensity of a pay-as-you-go social security system even if the economy is dynamically efficient.

JEL Classification: A1, H2 and J5

Keywords: labor markets, risk, social security and unemployment

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Social Security, Unemployment Risk and Efficient Bargaining between Unions and Firms*

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February 8, 2013

Abstract

We construct an overlapping generations model with unemployment risk where wages, employment and severance payments are set through efficient bargaining between risk averse Unions and risk neutral firms. Assuming that a First Best cannot be achieved due to workers' shirking incentives, we characterize a Second Best allocation and show how this can be implemented in a market economy. We prove that the latter generates too little employment and consumption smoothing, an excessive young age consumption and too much saving with respect to the Second Best. This inefficiency can be reduced by increasing the intensity of a pay-as-you-go social security system even if the economy is dynamically efficient.

KEYWORDS. Social Security; Labor Markets; Unemployment.

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1 Introduction

Mandatory social security programs are often interacting with alternative insurance provisions defined by the bargaining process taking place in the labor market between Unions and firms, including severance payments, occupational pension funds (*i.e.*, collective insurance schemes for old age offered by employers to employees within a firm or an industry) and other contractual retirement benefits. For example, in the Italian private sector, about 7.5% of annual earnings is directly managed by the firm and returned to employees when the job is terminated. In Germany, many workers have been able to retire relatively early through unofficial schemes characterized by “a negotiable combination of unemployment compensation and a supplement or severance pay” (Borsh-Supan et al. (2004)). Occupational pension funds or age-related severance payments are widespread in many countries, such as the Netherlands, the UK and the US. These provisions have a strong impact on individuals' retirement decisions and make the effective labor participation rate of older workers highly variable (across individuals and countries). Gruber and

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Wise (2004) compute the total percentage of men not working at ages between 55 and 65 (*unused productive capacity*) across several advanced economies and find these numbers to range from 55% to 65% in the case of Belgium, France, Italy, the Netherlands and the UK, down to a minimum of about 20% for Japan. Although most observers (including Gruber and Wise (2004)) relate these patterns to the generosity of public pension schemes, other authors, such as Ebbinghaus (2001), consider other factors, such as employers' labor shedding policies in the face of restructuring or skill depreciation. Contractual agreements involving a wide spectrum of variables affecting workers' welfare (employment, severance pay, age of retirement) are particularly relevant in the "European (or corporatist) social model", *i.e.*, an institutional environment characterized by the Unions' strong bargaining power, labor protection laws and social rights based on employment entitlements¹. In these environments, older workers are typically over-represented among Union members (Lindbeck and Snower (1988)), so that laying them off for poor performance or skill depreciation is possible only through hard negotiations and conditional to generous compensations.

The basic motivation of this paper arises from the observation that public unfunded social security programs appear to be larger in countries where the bargaining process between Unions and firms includes the allocation of social insurance against age and/or employment related risks (corporatist countries). In other words, the workers' higher ability to mitigate the effects of age-related shocks in the labor market (through the Union's bargaining power) does not displace mandatory social insurance, but, to the contrary, seems to be reinforcing it. In fact, a positive correlation between Union density and public social security is well documented (see Esping-Andersen (1990), Brugiavini et al. (2001)). Figure 1 shows a cross-country relation between corporatist ranking (a scale ranging from 1 to 5) provided in Siaroff (1999) and the gross pension replacement rates of median earners in 2011 taken from the OECD database (where the slope of the regression line is 6.3). One explanation of this phenomenon is based on the political leverage of the Unions in countries with corporatist institutions (Esping-Andersen (1990)). An alternative explanation, which we explore in this paper, is that public social security programs may be complementary to contractual employment insurance when designing an optimal policy.

We consider an economy where the only source of uncertainty derives from old age labor productivity shocks and reconsider the welfare effects of social security with efficient bargaining between risk averse Unions and risk neutral firms over wages, severance pay and employment, and the absence of insurance markets against unemployment risks. We show that, although private bargaining provides workers with the maximum insurance against unemployment risk that is allowed for by a No-Shirking constraint, the market allocation is producing too much saving and too much consumption in young age with respect to the Second Best allocation. By enacting an unfunded social security program

¹According to Pryor (1988), "Corporatism is a type of organized or coordinated capitalism where power to make important economic policies is transferred from the parliament and government to semi-private organizations; these are based on economic function or industrial sector and include a strong representation of labor interests".

(which is effectively transferring resources from the young to the old and reducing the net present value of earnings), the government may increase old workers' employment and welfare at a market allocation, thereby increasing welfare. The basic intuition is that an unfunded social security program allows firms to offer less contractual insurance and this reduces the cost of employing older workers. These findings may help to explain why "union strength and early exit is not as clear cut as for welfare state expansion in general" (Ebbinghaus (2001)).

We set up an overlapping generations model where young and old workers face asymmetric conditions in the labor market. Output is produced using labor only and the young exchange their labor services in a competitive environment, whereas the old play the role of *insiders*, as they are able to bargain with firms (through a Union) and they are unable (or unwilling) to move across jobs, *i.e.*, they face the options of staying in the firm where they were hired originally or being fired. Following the available evidence (e.g., Lindbeck and Snower (1988)), we assume that unionization is acquired through seniority. An important feature of the model is that workers are risk averse and their individual productivity in old age is subject to random shocks. This last assumption reflects the idea that skills may depreciate randomly with age. Hence, firms may have an incentive to replace old workers with the young, but this action is limited by the Unions' bargaining power. When a worker turns old, her individual productivity level is revealed, and the firm decides whether to keep her or lay her off. Bargaining between firms and Unions is the solution of the Unions' expected utility maximization subject to non negative firms' profits and a No-Shirking condition. The latter requires that old workers' utility contingent on being employed to be above a reservation level and prevents the optimal labor contract from offering full insurance against unemployment risk. We follow Blanchard and Tirole (2008) by assuming that there is no aggregate uncertainty and that individual productivity is observed by the firm only (neither the worker nor the Union nor a third party). In particular, the firm's employment decision follows from setting a threshold such that all workers whose productivity falls below this threshold are fired and the remaining workers are kept in the payroll earning a productivity independent wage. In this case, some unemployment is optimal if and only if workers are risk averse.

A crucial feature of the model is that labor contracts are settled after workers have made their saving choices. This implies that the firms'-Union bargaining is efficient for given intertemporal consumption allocation. It turns out that the market allocation is characterized by too much consumption in young age with respect to old age consumption contingent on being unemployed (*bad-state consumption*). In other words, consumption smoothing falls short of a *Second-Best* level, where the latter is the one provided by a Planner selecting life-time consumption and employment subject to the No-Shirking condition. An unfunded social security system can improve efficiency in equilibrium by transferring resources from the young to the old, thereby reducing the gap between young age bad-state consumption and increasing old workers' employment rate. The positive effect on employment comes about because social security allows firms to provide less insurance than they would otherwise. Second-Best efficiency of the market allocation can be achieved by inducing individuals to reduce their desired saving below zero and

imposing a non-negative limit on it.

In a final section of the paper we investigate the consequences of two different generalizations. The first is to allow social security contributions to be contingent on the old workers' employment status. When the government transfers some of the revenue to unemployed individuals, firms can afford to save on severance payments and, then, they have an extra incentive to fire workers. This makes the implementation of a Second-Best even more problematic than it would be when contributions are state-independent, suggesting that an optimal social security program, in the present setting, should not be contingent on employment status. The second generalization allows for capital to be productive. In this case there is an extra-dimension in the analysis of the effects of social security. Although the latter improves efficiency by increasing consumption smoothing (for given investment), it may also decrease efficiency by reducing investment under the assumption that the equilibrium allocation is dynamically efficient.

The basic characteristics of the model are admittedly much too simple to be able to capture many features of a corporatist type of economy. The inability of old workers to move across firms is very strong, but it may be considered a natural consequence of Unions' bargaining power (*i.e.*, the relative attractiveness of the unemployment compensations) and human capital depreciation. In particular, one may argue that unemployment compensations, coupled with the difficulty of getting the new skills needed to be relocated to a different firm, may be enough to induce old workers' unwillingness (and inability) to change job. In general, the set of assumptions about workers' labor productivity and lack of labor mobility are made in order to represent, in a simple and manageable way, a situation in which workers' ability to adapt to evolving technologies and tasks are decreasing with age. Observe that we are only assuming that old workers' productivity is relatively risky, not lower on average. These assumptions reflect a view recently expressed by Van Ours and Stoeldraijer (2010), that older workers "have higher health care costs, lower flexibility in accepting new assignments and then may be less suitable for training" (Van Ours and Stoeldraijer (2010), p. 3), although they may be more experienced and productive on average. Our model is clearly unable to capture age-related productivity differences for more than two cohorts (young and old). This is a strong limitation, as most studies find the existence of a hump shaped relation between productivity and age² (workers being most productive between 35 and 55). Then, an old worker in this model must be considered as representing a set of somewhat heterogeneous individuals having access to various labor protections that are typically obtained with seniority. Regarding the way we model unemployment, one may think of an old workers deprived of his job as being in unemployment as well as in early retirement. An other limitation of the model is that capital does not enter the production function. Since, in our model, a welfare improvement is obtained through a fall in saving, this limitation may cast some doubts about the generality of our findings. As mentioned above, this point is addressed in a specific section.

There is a large literature on the potential benefits of social security as a risk sharing device. However, most of this literature studies the case in which returns to capital and

²Skirbekk (2003), Dostie (2006), Van Ours and Stoeldraijer (2010).

wages are imperfectly correlated and subject to aggregate shocks (Shiller (1999), Bohn (1998), Krueger and Kubler (2006)). In these models, social security is a way of forcing households to hold an asset with which capital income risk can be diversified. These arguments have little in common with our analysis, since we assume no aggregate risk and no stochastic returns on capital. An other strand of literature considers the interaction of private savings with optimal social insurance programs, unemployment benefits and job search efforts. Baily (1978) addresses the issue of optimal unemployment insurance (UI) in a model with private information regarding job search efforts. Assuming that UI is financed by proportional income tax and operating under balanced budget, he finds that, at the optimal level, the marginal benefits of consumption smoothing equal the welfare loss associated with the marginal increase in the average unemployment spell. He then shows that redundancy payment (*i.e.*, a lump-sum payment upon becoming unemployed) avoid the adverse incentive effects of a standard UI scheme. Bertola (2004) argues that some employment protection legislation, such as redundancy and severance payments may be beneficial when insurance markets are missing and workers are risk averse.

2 The Model

We consider an overlapping generations economy with a single consumption good that can be consumed, produced and stored (for one period only) at any time period. A unit of the consumption good stored at time t generates $R = 1/\beta$ units in $t+1$, with $\beta \in (0, 1)$. Production is carried out by a large number of identical firms operating under perfect competition and using a linear technology defined by a production function ωN , where N is the amount of labor in efficiency units and $\omega > 0$. Every period, $t \geq 0$, a continuum of mass 1 ex-ante identical individuals are born, each one living for two periods only and supplying one unit of labor inelastically every period. All individuals may be in one of two alternative states in old age: employment and unemployment. We may interpret the latter state as early retirement. The utility of an individual born at t is:

$$U(y_t, c_{t+1}^e, c_{t+1}^u, n_{t+1}) = u(y_t) + \beta (n_{t+1}u(c_{t+1}^e) + (1 - n_{t+1})u(c_{t+1}^u)), \quad (1)$$

where y denotes the young age consumption, c^e , c^u the old age consumptions conditional on the individual being, respectively, employed and unemployed, when old and n is the probability of being employed when old. The utility function, $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, is bounded, continuously differentiable, strictly increasing, strictly concave, such that $\lim_{c \rightarrow 0} u'(c) = \infty$, $\lim_{c \rightarrow \infty} u'(c) = 0$.

We assume that young workers have identical labor productivity whereas old workers' individual productivity is equal to θ , where θ is a random variable, i.i.d. (across old workers) with mean equal to 1, c.d.f. $G(\theta)$ and density $g(\theta)$ on the support $[0, e]$. The realizations of θ of each worker are observable by the employer only, *i.e.*, neither by the worker nor by a third party such as a workers' Union or an insurance company. This implies that the individual's labor compensation cannot be made contingent on these realizations.

Denote with x_t the time t threshold of the random productivity shock, θ , below which a firm is planning to fire a worker. Hence, a firm fires a mass

$$G(x_t) = \int_0^{x_t} dG(\theta)$$

of workers at every period t , so that $n_t = (1 - G(x_t))$, and the size of the labor force contributing to production at any time t is

$$N_t = n_t^y + n_{t-1}^y H(x_t), \quad (2)$$

where n_t^y denotes the mass of young workers employed at any period t and

$$H(x) = 1 - \int_0^x \theta dG(\theta).$$

Following a simplified version of efficiency wage theories, we assume that workers may shirk in the workplace and firms engage in random monitoring. In particular, let $B > 0$ be the utility benefit of shirking and $q \in (0, 1)$ the probability of a worker being caught shirking. Then, the No-Shirking condition can be written as

$$u(c_t^e) \geq qu_t^r + (1 - q)(u(c_t^e) + B),$$

where the right hand side is the expected utility of a shirking worker and u_t^r is a reservation utility. Then, this condition can be written as $u(c_t^e) \geq \rho + u_t^r$, where $\rho = B(1 - q)/q$ is the utility premium (over the reservation value) for not shirking. Letting $\bar{c}_t \equiv u^{-1}(\rho + u_t^r)$, the No-Shirking condition will be simply written as

$$c_t^e \geq \bar{c}_t. \quad (\text{NS})$$

3 Efficient Allocations

In this section we provide a notion of constrained efficiency. The basic idea is that the Planner cannot remove the shirking incentives defined by condition (NS), but she can allocate investment and consumption at will. In a later section we will define a market economy and see what type of policies may implement the constrained efficient allocations.

Definition 1. *An allocation, $\{c_t, n_t, x_t, k_t\}_{t=0}^{\infty}$, is Constrained Efficient if, for given initial value, k_0 , and some arbitrary discount rate $\alpha \in (0, 1)$, it maximizes the social welfare function*

$$n_0 c_0^e + (1 - n_0) c_0^e + \sum_{t=0}^{\infty} \alpha^t U(y_t, c_{t+1}^e, c_{t+1}^u, n_{t+1}),$$

subject, at all $t \geq 0$, to the restrictions imposed by resource feasibility,

$$y_t + n_t c_t^e + (1 - n_t) c_t^u + k_{t+1} \leq \omega(1 + H(x_t)) + (1/\beta)k_t, \quad (3)$$

$$n_t = 1 - G(x_t), \quad (4)$$

non-negativity of consumption and capital and the No-Shirking condition, (NS).

Notice that, under the above definition, the Planner is able to provide individuals with the maximum feasible amount of insurance up to the one allowed by the no-shirking condition.

The constrained efficient allocation corresponds to the first best when (NS) is not binding at all $t \geq 0$. In this case, the (interior) solution to the above planning problem is characterized by full employment ($x = 0$, $n = 1$), full insurance ($c_t^e = c_t^u = c_t$) and consumption smoothing, *i.e.*, it is an allocation, $\{\hat{y}_t, \hat{c}_t, \hat{k}_{t+1}\}_{t=0}^{\infty}$, such that

$$\hat{y}_t = \hat{c}_{t+1}, \quad u'(\hat{y}_t) = \frac{\alpha}{\beta} u'(\hat{y}_{t+1}).$$

Under the assumption $\alpha = \beta$, the allocation is stationary, and such that

$$\hat{y} = \hat{c} = \omega + \frac{1 - \beta}{2\beta} k_0.$$

Notice that the first best is feasible only if it satisfies (NS), *i.e.*, $\hat{y}_t \geq \bar{c}_t$ for all $t \geq 0$.

If (NS) is binding, constrained efficiency cannot provide full insurance and full employment at all $t \geq 0$. The Planner's allocation is called, in this case a *Second-Best* (SB). Suppose, in particular, that the No-Shirking condition (NS) is binding at all $t \geq 0$. Then, by the first order conditions from the planning problem, a SB allocation,

$$\{c_t, n_t, x_t, k_t\}_{t=0}^{\infty},$$

is characterized, for all $t \geq 0$, by

$$y_t = c_{t+1}^u, \tag{5}$$

$$c_t^e = \bar{c}_t > c_t^u, \tag{6}$$

$$u'(c_t^u) ((c_t^e - c_t^u) - \omega x_t) = u(c_t^e) - u(c_t^u) \tag{7}$$

as well as the feasibility restrictions (3), (4). Observe that, when $\alpha = \beta$, c_t^u and y_t are time-invariant.

Equation (7) states that the Planner is implementing some sort of consumption smoothing across individuals' age. This follows from the assumption that utility is time additive and that the rate of return on storage equals both the Planner's and the individuals' rate of time preference. However, consumption smoothing is limited by the inability to provide full insurance, so that young age consumption equals bad-state consumption at old age. A further property of the Planner's allocation is that bad state-contingent consumptions and young age consumptions are all time-invariant. This follows, again, from the assumption that the rate of return on storage equals both the Planner's and the individuals' rate of time preference.

The left hand side of equation (7) represents the cost of increasing (old workers') employment (as defined by the feasibility constraint) and the right hand side of the equation represents the expected utility-benefit of increasing employment. More specifically, $c^e - c^u$ is the amount of forgone output required to generate an extra unit of

employment, $\omega h x$ is the additional output generated by the extra unit of employment and $u'(c^u)$ is the shadow price of consumption. Now define

$$\Delta(c^e, c^u) = (c^e - c^u) - \frac{u(c^e) - u(c^u)}{u'(c^u)}.$$

This variable will play an extensive role in the sequel and it can be interpreted as the *net employment cost*, *i.e.*, the difference between the forgone output and the expected utility generated by an extra unit of employment. Then, equation (7) can be rewritten as

$$x_t = \frac{\Delta(c_t^e, c_t^u)}{\omega}. \quad (8)$$

Observe that $x > 0$ (some unemployment) occurs for $\Delta > 0$, *i.e.*, when forgone output exceeds the utility benefit. This requires some risk aversion (risk neutrality would make forgone output exactly equal to the utility benefit) or full insurance. In fact, $\Delta = 0$ for $c^e = c^u$ and, by strict concavity of the utility function, $\Delta > 0$ for $c^e \neq c^u$. Then, by equation (17) and the assumption $u'' < 0$, it is $x_t > 0$ for $c_t^e \neq c_t^u$. If the Planner is unable to provide full insurance, she will lay off some workers (and vice versa). Since $x_t = 0$ implies $c_t^e = c_t^u$, we can state that, when full insurance is not feasible, a constrained optimal allocation is characterized by equations (8). Notice that full insurance cannot be implemented if and only if the First Best is not feasible.

It is easily verified that Δ is increasing in c^e and decreasing in c^u . In other words, employment increases with c^u and decreases with c^e . This is not surprising, since Δ measures the net cost of employing an old individual. Furthermore, for given c^e , Δ is increasing in the consumption premium, $v = c^e - c^u$, as shown by

$$\frac{\partial \Delta}{\partial v} = -\frac{u(c^e) - u(c^u)}{u'(c^u)^2} u''(c^u) > 0.$$

The intuition is that, because Unions are risk averse, a higher consumption risk must be compensated by a higher good-state consumption for the firm to maintain a constant employment. In the next sections we will see how the market allocates resources in equilibrium and compare market allocations with the SB allocations that we have just defined.

4 Households' Choices and Labor Contracts

Households

We assume no bequests and no private insurance against unemployment. However, the government provides a public pay-as-you go social security system. Furthermore, we assume that individuals cannot borrow. In particular, let w_t^y be the young workers' wage rate, w_{t+1}^o their wage when old and μ_{t+1} a severance payment conditional on being

unemployed. Then, per-period budget constraints are

$$y_t + s_t = w_t^y(1 - \tau_t), \quad (9)$$

$$c_{t+1}^e = s_t/\beta + w_{t+1}^o + \gamma_t w_t^y, \quad (10)$$

$$c_{t+1}^u = s_t/\beta + \mu_{t+1} + \gamma_t w_t^y, \quad (11)$$

where $\tau_t \in [0, 1]$ is the rate at which young workers contribute to the social security system and $\gamma_t \geq 0$ is the rate of return of social security contributions. Then, letting c be the random variable with values in $\{c^e, c^u\}$, \mathbb{E} the expectation operator, the optimal consumption choice of a young born at time t is a solution to

$$\max_{(y_t, c_{t+1}^e, c_{t+1}^u) \geq 0} u(y_t) + \beta \mathbb{E}[u(c_{t+1})] \quad \text{s.t.: (9), (10), (11) and } y_t \leq w_t^y(1 - \tau). \quad (\text{CP})$$

A first order characterization of this solution is

$$u'(y_t) \geq \mathbb{E}[u(c_{t+1})]. \quad (12)$$

where the above holds with equality when $s_t = w_t^y(1 - \tau) - y_t > 0$.

Labor Contracts

The long-run profit at time t is

$$\Pi_t(n_{t-1}^y) = \sum_{j=0}^{\infty} \beta^j \pi_{t+j}, \quad (13)$$

where, at all $t \geq 0$,

$$\pi_t = \omega N_t - w_t^y n_t^y - w_t^o n_t n_{t-1}^y - \mu_t(1 - n_t)n_{t-1}^y$$

and N_t is defined in equation (2).

Recall that $n_t = 1 - G(x_t)$ and the No-Shirking condition defines a lower bound on old workers' utility when employed (as in (NS)). For any given (expected) sequence of young workers' wage, $\{w_{t+j}^y\}_{j=0}^{\infty}$ and previous period employment, n_{t-1}^y , an *optimal labor contract* is a non-negative sequence,

$$\{n_{t+j}^y, x_{t+j}, w_{t+j}^o, \mu_{t+j}\}_{j=0}^{\infty},$$

such that, for all $t \geq 0$, the expected utility of the old individuals born at time $t - 1$

$$n_t u(c_t^e) + (1 - n_t) u(c_t^u)$$

is maximized subject to (NS) and the non negative profit constraint $\Pi_t(n_{t-1}^y) \geq 0$. Notice that this setup implies efficient bargaining between firms and unions, with firms having positive and unions having zero discount factor. We now characterize the optimal labor contract in terms of first order conditions.

Profit maximization with respect to young workers' labor gives

$$\omega(1 + \beta H(x_{t+1})) = w_t^y + \beta (w_{t+1}^o n_{t+1} + \mu_{t+1}(1 - n_{t+1})). \quad (14)$$

This equation equalizes the expected discounted marginal products of a worker across his entire career, *i.e.*, as a young worker today and an old worker tomorrow, to his (gross of payroll tax) current wage rate plus the discounted expected old workers' wage and severance pay. Assuming that equation (14) is satisfied for all $s \geq t$ and plugging it into the time- t long-run zero profit condition, we get that, at the optimal contract

$$\Pi_t(n_{t-1}^y) = (\omega H(x_t) - n_t w_t^o - (1 - n_t)\mu_t) n_{t-1}^y = 0.$$

The latter delivers the following conditions

$$w_t^y = \omega, \quad (15)$$

$$n_t w_t^o + (1 - n_t)\mu_t = \omega H(x_t). \quad (16)$$

The second condition determines the threshold, x_t , below which workers are fired. By maximizing the firms' profit with respect to x_t , w_t^o and μ_t under the relevant constraints, we derive

$$g(x_t)(u(c_t^e) - u(c_t^u)) \geq \lambda_t g(x_t) [(w_t^o - \mu_t) - \omega x_t] n_{t-1}^y,$$

where λ is the Lagrange multiplier associated to the firms' non negative profit constraint and the inequality holds only if $x_t = 0$. The left hand side of the above is the benefit to the Union of a higher employment and the right hand side is the cost of a higher employment to the firms. Since the first order conditions give $\lambda_t = u'(c_t^u)/n_{t-1}^y$ and

$$w_t^o - \mu_t = (c_t^e - c_t^u),$$

the threshold x_t can be derived from the following condition

$$\omega x_t \geq (c^e - c^u) - \frac{u(c^e) - u(c^u)}{u'(c^u)} = \Delta(c_t^e, c_t^u). \quad (17)$$

The left hand side of equation (17) gives the opportunity cost to the firm of firing an extra worker (in terms of forgone output). The right hand side gives the net benefit of firing an extra worker. The latter is decomposed into two terms. The first term, $c^e - c^u$, is the firm's benefit from firing an extra worker. This term is a function of the "employment consumption premium", *i.e.*, the extra compensation received by an employed worker. The second term,

$$\frac{u(c^e) - u(c^u)}{u'(c^u)},$$

is the cost to the Union of an extra layoff. Observe that the net employment cost, $\Delta(c_t^e, c_t^u)$ is positive because firms are risk neutral and Unions are risk averse, so that the Union's evaluation of the "employment consumption premium" is smaller than the firm's evaluation.

If the No-Shirking condition, (NS), is not binding, the optimal contract provides workers with full insurance against unemployment risk, *i.e.*, $w_t^o = \mu_t$. Hence, any equilibrium such that $x_t = 0$, so that the wage profile is flat, *i.e.*, $w_t^y = w_t^o = \omega$.

5 Equilibrium and Implementability

Given a *social security policy*, $\{\tau_t\}_{t=0}^\infty$, and an initial capital stock, k_0 , an equilibrium allocation is an array,

$$\{y_t, c_t^e, c_t^u, w_t^y, w_t^o, \mu_t, n_t^y, n_t, x_t, k_t, \gamma_t\}_{t=0}^\infty$$

of non negative variables such, for all $t \geq 0$,

- (a) workers chose consumption, $(y_t, c_{t+1}^e, c_{t+1}^u)$ by solving (CP),
- (b) Unions and firms choose w_t^o, x_t, n_t^y, n_t and μ_t by maximizing $U(y_{t-1}, c_t^e, c_t^u)$ for given y_{t-1} subject to non negative long-run profits, (13), and the No-Shirking condition, (NS),
- (c) young workers are fully employed (*i.e.*, $n^y = 1$),
- (d) the good markets and the capital markets clear, *i.e.*

$$y_t + \mathbb{E}[c_t] = \omega(1 + H(x_t)) + (1/\beta)k_t - k_{t+1}, \quad (18)$$

$$k_{t+1} = w_t^y(1 - \tau_t) - y_t, \quad (19)$$

- (e) and the social security policy satisfies the balanced budget condition

$$\tau_t w_t^y = \gamma_t w_{t-1}^y. \quad (20)$$

An equilibrium allocation is called *Under-Employment Equilibrium* (UE) allocation if $n_t < 1$ for all $t \geq 0$ and it is called *interior* if the credit constraint is not binding, *i.e.*, if equation (12) is satisfied with equality.

We now derive a reduced form characterization of SB and UE allocations in terms of the array

$$\{y_t, c_t^u, n_t, x_t\}.$$

First of all, an optimal labor contract with under-employment implies

$$c_t^e = \bar{c}_t \quad (21)$$

$$x_t = \frac{\Delta(c_t^e, c_t^u)}{\omega} > 0. \quad (22)$$

Secondly, solving forward equation (18) for k_t , we derive the present value representation

$$\sum_{t=0}^{\infty} \beta^t (y_t + \mathbb{E}[c_t] - \omega(1 + H(x_t))) = k_0/\beta. \quad (23)$$

Using (4) and (21), we get the additional restriction

$$n_t = 1 - G\left(\frac{\Delta(\bar{c}_t, c_t^u)}{\omega}\right). \quad (24)$$

By the solution to the firms-Unions bargaining problem, at the UE allocation,

$$w_t^y = \omega, \quad n_t w_t^o + (1 - n_t) \mu_t = \omega H(x_t), \quad c_t^e = \bar{c}_t.$$

Since young workers' wages are time invariant, we also derive $\gamma_t = \tau_t$ for all $t \geq 0$.

Both the SB and the UE allocations satisfy equations (23) and (24) at all $t \geq 0$, but the two type of allocations differ in the way consumption is allocated across ages, as can be seen from equations (5) and (12). Notice that the two restrictions, (5) and (12), are compatible only if (12) is satisfied with strict inequality, *i.e.*, the financial constraint, $y_t \leq \omega(1 - \tau)$, is binding. Hence, implementation of a SB allocation through markets is problematic because any interior equilibrium allocation implies

$$u'(y_t) = n_{t+1} u'(\bar{c}_t) + (1 - n_{t+1}) u(c_{t+1}^u),$$

and, then, $\bar{c}_t > y_t > c_{t+1}^u$ for all $n_{t+1} \in (0, 1)$, whereas the SB allocation implies $y_t = c_t^u < \bar{c}_t$. In other words, an interior equilibrium allocation generates “too much” consumption in young age or, equivalently, “too little” saving. Based on these premises, we will show now that the equilibrium allocation can implement the SB only if social security contributions are sufficiently high so as to reduce young age consumption to the SB level.

First of all we will establish some properties of the optimal consumption, y .

Proposition 1. *For any $\tau \in (0, 1)$ and for all utility functions satisfying the maintained assumptions, the optimal young age consumption, y_t , and saving $s_t = \omega(1 - \tau) - y_t$, solving (CP), are decreasing in τ . Furthermore, if the utility function, u , exhibits “prudence”, *i.e.*, $u'''(c) > 0$ for all $c > 0$, the optimal young age consumption is interior (*i.e.*, $y_t \leq \omega(1 - \tau)$ and condition (12) holds with equality) for all $\tau < (1 - H(x))/2$.*

The above proposition implies that the *laissez-fair* equilibrium at which $\tau = 0$ cannot implement the SB allocation and that non-interior solutions of the consumption-saving problem (CP) are possible only if τ is bigger than $(1 - H(x))/2$.

Proposition 2. *An equilibrium allocation may implement the SB allocations only if, at equilibrium, the financial constraint, $y_t \leq \omega(1 - \tau)$, is binding. If the latter is not binding, social welfare at stationary UE allocations is increasing in τ .*

Proof. The first part of the proposition follows by observing that (12) and (5) can be satisfied simultaneously only if the former holds with inequality.

To prove the second part of the proposition, let (y, c^u, n, x) be a stationary equilibrium for given $c^e = \bar{c}$ and $\tau \in [0, 1)$ and define the social welfare function at this stationary equilibrium as

$$\mathcal{V}(\tau) = \frac{1}{1 - \beta} (u(y) + n u(\bar{c}) + (1 - n) u(c^u)).$$

Then,

$$(1 - \beta) \mathcal{V}'(\tau) = u'(c^u) \left(\frac{u'(y)}{u'(c^u)} \frac{\partial y}{\partial \tau} + \mathbb{E} \left[\frac{\partial c}{\partial \tau} \right] + \frac{\partial n}{\partial \tau} \left(\frac{u(\bar{c}) - u(c^u)}{u'(c^u)} \right) \right).$$

By differentiation of the feasibility constraint (23) at stationarity,

$$\frac{\partial y}{\partial \tau} + \mathbb{E} \left[\frac{\partial c}{\partial \tau} \right] + \frac{\partial n}{\partial \tau} (\bar{c} - c^u) - \omega x \frac{\partial n}{\partial \tau} = 0.$$

Plugging the above into the expression for $\mathcal{V}'(\tau)$ gives

$$(1 - \beta)\mathcal{V}'(\tau) = u'(c^u) \left(\frac{u'(y)}{u'(c^u)} - 1 \right) \frac{\partial y}{\partial \tau}. \quad (25)$$

Since $\partial y / \partial \tau < 0$ and $y > c^u$ whenever the financial constraint is not binding, we derive the proposition. \square

We derive now a full characterization of stationary allocations with unemployment in terms of the variables y , c^u and n . First of all, we assume that a first best is not feasible, a condition equivalent to

$$\bar{c} > \omega + \frac{(1 - \beta)}{2\beta} k_0. \quad (\text{SB})$$

Then, imposing stationarity on the feasibility condition (23) and the optimal employment condition (24) we derive the pair of equations

$$y + n\bar{c} + (1 - n)c^u = (1 - \beta)k_0/\beta + \omega(1 + H(x(n))), \quad (26)$$

$$n = 1 - G(\Delta(\bar{c}, c^u)/\omega), \quad (27)$$

where $x(n) = G^{-1}(1 - n)$. Evidently, a triple $(n^o, y^o, c^{uo}) \in (0, 1) \times (0, \bar{c})^2$ such that $y^o = c^{uo}$ solving (26), (27) simultaneously is a SB allocation, whereas an interior UE allocation generated by some policy $\tau \in [0, 1]$ is a triple (n^*, y^*, c^{u*}) satisfying (26), (27) together with

$$u'(y) = nu'(\bar{c}) + (1 - n)u'(c^u). \quad (28)$$

Equations (26), (27) represent the (stationary) resource feasibility constraint and the labor market equilibrium condition, respectively, whereas equation (28) represents the optimal (private) consumption choice (*i.e.*, the first order condition for an interior optimal consumption choice for $c^c = \bar{c}$). We will show in a moment that the labor market equilibrium condition can be represented as a map relating bad-state consumption, c^u , to n , which we denote as $c^u = f_L(n)$. Plugging this relation into (26) and (28), these two additional restrictions define two more maps relating y to n , to be denoted as $y = f_R(n)$ and $y = f_C(n)$, respectively. This is a useful characterization since $f_L(n)$ is decreasing whereas $f_R(n)$ and $f_C(n)$ are both increasing in $[0, 1]$. Then, the unique intersection between $f_L(n)$ and $f_R(n)$ corresponds to the SB allocation (n^o, y^o, c^{uo}) and the unique intersection between $f_C(n)$ and $f_R(n)$ corresponds to the UE pair (n^*, y^*) . The remaining equilibrium value, c^{u*} , is equal to $f_L(n^*)$.

Proposition 3. *Under condition (SB), equations (26), (27), (28) have a unique solution $y = f_R(n)$, $c^u = f_L(n)$, $y = f_C(n)$, respectively, for all $n \in [0, 1]$, where $f_j : [0, 1] \rightarrow [0, \bar{c}]$ ($j = 1, 2, 3$) are differentiable functions such that $f'_R(n) < 0$, $f'_L(n) > 0$, $f'_C(n) > 0$ for all $n \in (0, 1)$ and $f_R(1) < f_L(1) = f_C(1)$, $f_L(n) < f_C(n)$ for all $n \in (0, 1)$.*

The situation is represented graphically in figure 2 for the case of a constant relative risk aversion (equal to 0.5) and a uniform distribution, $G(x) = x/2$. The intersection between the curves $f_R(n)$ and $f_L(n)$ defines a SB allocation, (n^o, y^o, c^{uo}) , such that

$$y^o = c^{uo} = f_R(n^o) = f_L(n^o),$$

whereas the intersection between $f_R(n)$ and $f_C(n)$ together with the value $f_L(n^*)$ represent an interior UE allocation, (n^*, y^*, c^{u*}) such that

$$y^* = f_R(n^*) = f_C(n^*) > c^{u*} = f_L(n^*).$$

Since $f_R(n)$ is decreasing and f_L, f_C are both decreasing with $f_C(n) > f_L(n)$ for all $n \in (0, 1)$, we derive the following proposition as a simple corollary of proposition 3.

Proposition 4. *Under condition (SB) and if $f_R(0) > f_L(0)$, there exist a unique SB allocation, (n^o, y^o, c^{uo}) and a unique UE allocation, (n^*, y^*, c^{u*}) , such that*

$$n^o < n^*, \quad y^* > c^{uo} > c^{u*}.$$

The intuition explaining the above proposition is based on the slopes of the three curves, f_L, f_R and f_C . The schedule f_L slopes upward because the gap between \bar{c} and c^u represents the net cost of employing an extra worker. The reason why f_R slopes downward is more involved. First, observe that a rise in n increases average old age consumption by more than aggregate output. In fact, a unit rise in n increases aggregate output by ωx and it increases average old age consumption by $(\bar{c} - c^u) + \partial c^u / \partial n$. Since the labor market equilibrium implies $\omega x < (\bar{c} - c^u)$ and $\partial c^u / \partial n > 0$, the former effect falls short of the latter. Then, young age consumption, y , must fall to restore an equilibrium. Finally, f_C slopes upward because a rise in n has a negative effect on the expected marginal utility of old age consumption. Given the characteristics of the model, a rise in social security contributions induces young individuals to increase bad-state consumption and, this, in turn, lowers the net cost of old workers' labor, allows firms to offer less insurance thereby generating more employment.

Since $f_1(0) + f_2(0) = \omega + (1 - \beta)k_0/\beta$, the assumption $f_R(0) > f_L(0)$ reduces to

$$f_L(0) < (\omega + (1 - \beta)k_0/\beta)/2,$$

where $f_L(0)$ is such that $\Delta(\bar{c}, f_L(0)) = \omega$. Recalling that $\Delta(\bar{c}, c^u) \geq \bar{c} - c^u$, with equality only if a individuals are risk averse, a sufficient condition for the existence of a SB allocation is

$$\frac{3}{2}\omega + \frac{(1 - \beta)}{2\beta}k_0 > \bar{c} > \omega + \frac{(1 - \beta)}{2\beta}k_0.$$

6 Some Extensions

Conditional Social Security Transfers

The above analysis was carried out under the assumption that employed and unemployed individuals receive the same social security contribution. Assume now that the government makes these contributions conditional on the occupational status of the recipient.

In particular, let $\alpha \in (0, 1)$ and define the old individuals budget constraints as

$$\begin{aligned} c_{t+1}^e &= s_t/\beta + w_{t+1}^o + \gamma_t \alpha w_t^y, \\ c_{t+1}^u &= s_t/\beta + \mu_{t+1} + \gamma_t (1 - \alpha) w_t^y. \end{aligned}$$

We will assume that $\alpha < 1/2$, so that the redistribution favors the unemployed (or the individuals who are retiring earlier). In this case the optimal labor contract represented by equation (17) modifies to

$$\omega x_t \geq \gamma \omega (1 - 2\alpha) + \Delta(c_t^e, c_t^u),$$

where γ satisfies the government budget balance condition

$$\tau = \gamma(\alpha n_t + (1 - \alpha)(1 - n_t)).$$

As $\alpha < 1/2$, an UE allocation implies $x_t > \Delta(\bar{c}, c_t^u)$, whereas the SB is such that $x_t = \Delta(\bar{c}, c_t^u)$. Remember that x equals the firm's benefit from firing an extra worker, which is a function of the "employment consumption premium", $\bar{c} - c^u$, *i.e.*, the extra compensation received by an employed worker. When the government sets $\alpha < 1/2$, some part of this premium, $\gamma \omega (1 - 2\alpha)$, represents the implicit benefit to the firm arising from the government subsidy: because unemployed individuals receive this transfer, firms can afford to save on severance payments and, then, they have an extra incentive to fire workers. This suggests that an optimal social security program, in the present setting, should not be redistributive.

Capital in the Production Function

We now briefly comment on the case in which capital enters the production function. Intuitively, this case should provide more ammunition to the theory according to which social security is harmful because it reduces saving and investment.

Assume that there is no storage technology and output is produced with a production function $F(K_t, N_t)$, where N_t is defined in equation (2). Then, assuming that the social and private time discount rates are both equal to $\beta \in (0, 1)$, one can easily derive the following characterization of a stationary SB allocation:

$$xF_N(K, N) = \Delta(\bar{c}, c^u), \tag{29}$$

$$u'(y) = \beta F_K(K, N) u'(c^u), \tag{30}$$

$$y + n\bar{c} + (1 - n)c^u = F(K, N) - K, \tag{31}$$

$$F_K(K, N) = 1/\beta. \tag{32}$$

with the usual additional constraint $1 - n = G(x)$.

An UE allocation satisfies equations (29), (31) and

$$u'(y) = \beta F_K(K, N) \mathbb{E}[u'(c)], \tag{33}$$

$$K = F_N(K, N)(1 - \tau) - y, \tag{34}$$

instead of (30) and (32).

Consider the welfare effect at equilibrium of a change in the social security contribution, τ . Letting $\mathcal{V}(\tau)$ be the individuals expected utility at equilibrium for a given τ , a replication of the procedure used in the proof of proposition 2 with the appropriate adjustments, allows to derive

$$(1 - \beta)\mathcal{V}'(\tau) = u'(c^u) \left[\left(\frac{u'(y)}{u'(c^u)} - 1 \right) \frac{\partial y}{\partial \tau} + (F_K - 1) \frac{\partial K}{\partial \tau} \right]. \quad (35)$$

Hence, assuming $F_K > 1$ (dynamic efficiency) and that a rising τ has a negative effect on both y and K , the sign of the above derivative is ambiguous. On the one hand, a fall in y induced by a higher τ narrows the gap between current and next period bad-state consumptions, thereby allowing for more consumption smoothing. On the other hand, the fall in K reduces net output.

7 Conclusions

We have shown that an unfunded social security program can be welfare improving in an economy with efficient bargaining between firms and Unions over wages, employment and severance pay, under the assumption that there is unemployment risk and lack of insurance markets. The reason is that private consumption-saving decisions are not part of the bargaining agreement. This creates an externality generating too little consumption smoothing and too little employment at equilibrium. Social security may increase employment and welfare because it increases old workers' consumption in unemployment allowing firms to provide less insurance. Implementation of the Second-Best allocation may be possible in a market economy only if individuals cannot borrow and social security contributions are sufficiently high to make the borrowing limit a binding constraint on consumption-saving decisions. The model also suggests that implementation of the Second-Best is even more problematic (in effect, impossible even in the case of a binding borrowing limit) if social security contributions are contingent on old individuals employment status.

The positive welfare effect of social security cannot be generalized to the case of economies with productive capital, although, in this case, the model suggests that the negative effects on savings of rising social security contributions may be less important than suggested in more standard models.

Our finding may provide a partial answer to the question why countries with corporatist institutions (*i.e.*, strong role of collective bargaining between Unions and firms) have "large" social security programs.

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9 Appendix

Proof of proposition 1.

Let $V(y, \tau) \equiv nu'(c^e) + (1 - n)u'(c^u)$, where

$$c^e = \frac{\omega(1 - \tau(1 - \beta)) - y + \beta w^o}{\beta}, \quad c^u = \frac{\omega(1 - \tau(1 - \beta)) - y + \beta \mu}{\beta}.$$

Notice that $V_1(y, \tau) > 0$, $V_2(y, \tau) > 0$ and $0 < V(0, \tau) < V(\omega(1 - \tau), \tau)$. These properties can be used to show that consumption y and saving $s = \omega(1 - \tau) - y$ solving problem (CP) are decreasing in τ .

Now observe that $\lim_{y \rightarrow 0} u'(y) = \infty$. Then, the optimal consumption choice is non-interior (*i.e.*, $s = \omega(1 - \tau) - y = 0$) if and only if

$$u'(\omega(1 - \tau)) \geq V(\omega(1 - \tau), \tau).$$

By prudence, $u'(c)$ is strictly convex and, then,

$$\begin{aligned} V(\omega(1 - \tau), \tau) &= nu'(\omega\tau + w^o) + (1 - n)u'(\omega\tau + \mu) \\ &> u'(nw^o + (1 - n)\mu + \omega\tau). \end{aligned}$$

Recalling that, at equilibrium, $nw^o + (1 - n)\mu = \omega H(x)$, we derive

$$V(\omega(1 - \tau), \tau) > u'(\omega(H(x) + \tau)).$$

Then, if the optimal consumption choice is non-interior, must be

$$u'(\omega(1 - \tau)) > u'(\omega(H(x) + \tau)),$$

i.e., $\tau > (1 - H(x))/2$.

Proof of proposition 3.

Since $\Delta(\bar{c}, c^u)$ is strictly decreasing in c^u and such that $\Delta(\bar{c}, \bar{c}) = 0$, it can be easily shown that, for all $n \in [0, 1]$, there exists a unique value $c^u = f_L(n) \in [0, \bar{c}]$ such that

$$n = 1 - G(\Delta(\bar{c}, f_L(n))/\omega),$$

with $f_L(1) = \bar{c}$, $f_L(0) \in (0, \bar{c})$. Furthermore, the implicit function theorem guarantees that $f_L(n)$ is differentiable with $f_L'(n) > 0$.

Now consider equation (26) and rewrite it as

$$y + n\bar{c} + (1 - n)f_2(n) = v_0 + \omega(1 + H(x(n))),$$

where $v_0 = (1 - \beta)k_0/\beta$. The above is verified for a unique value $y = f_R(n)$, where $f_R(n)$ is a differentiable function such that

$$f_R(0) = v + \omega - f_L(0), \quad f_R(1) = v + 2\omega - \bar{c},$$

and

$$f'_R(n) = \Delta(\bar{c}, f_L(n)) - (1 - n)f'_L(n) - (\bar{c} - f_L(n)) = -\frac{u(\bar{c}) - u(f_L(n))}{u'(f_L(n))} - (1 - n)f'_L(n) < 0.$$

Finally, notice that $f_R(1) \geq \bar{c}$ implies $v_0 + 2\omega \geq 2\bar{c}$, which is ruled out by (SB).

Now consider equation (28) and rewrite it as

$$u'(y) = nu'(\bar{c}) + (1 - n)u'(f_L(n)).$$

The above is verified for a unique value $y = f_C(n)$, where $f_C(n)$ is a differentiable function such that $f_C(0) = f_L(0)$, $f_C(1) = f_L(1) = \bar{c}$, $f_C(n) > f_L(n)$ for all $n \in (0, 1)$ and

$$f'_C(n) = \frac{u'(\bar{c}) - u'(c^u)}{u''(y)} + (1 - n)\frac{u''(c^u)}{u''(y)}f'_L(n) > 0$$

for all $n \in (0, 1)$.



Figure 1.

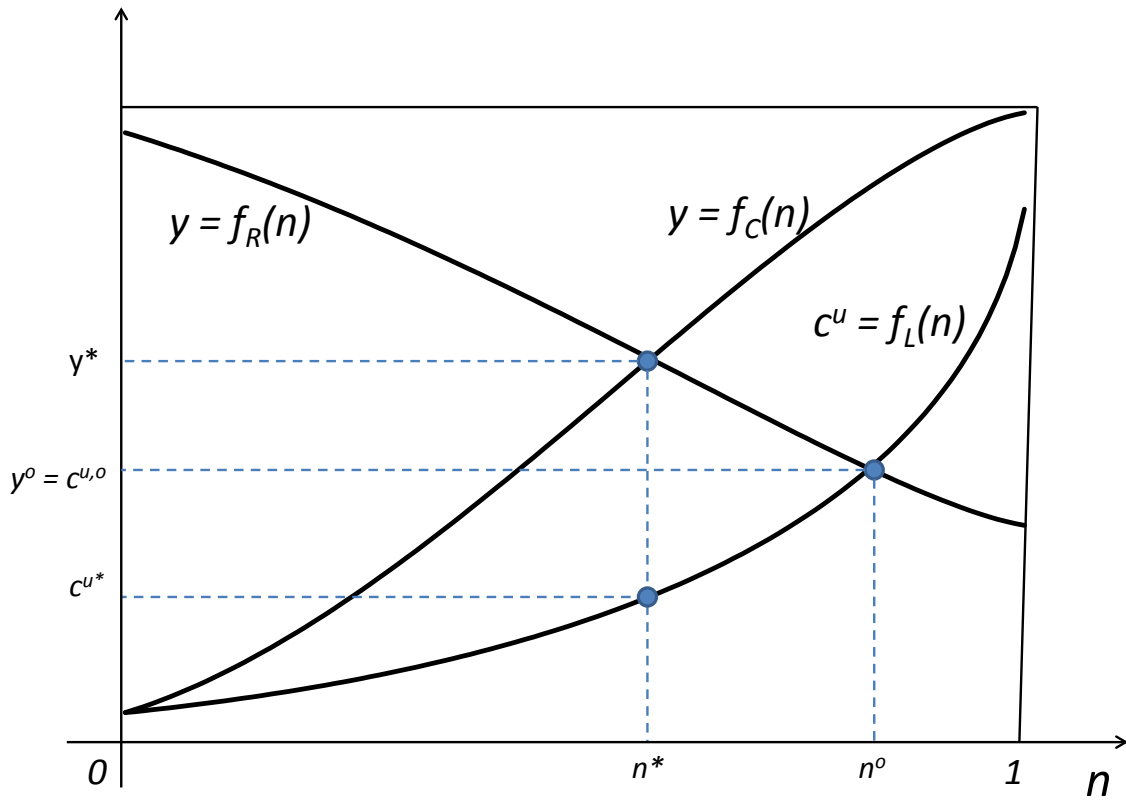


Figure 2.