DISCUSSION PAPER SERIES

No. 9327

DISCOUNT PRICING

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INDUSTRIAL ORGANIZATION

ISSN 0265-8003

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Discussion Paper No. 9327
February 2013

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CEPR Discussion Paper No. 9327

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ABSTRACT

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JEL Classification: D03, D18, D83 and M3
Keywords: consumer protection, consumer search, false advertising, price discrimination and reference dependence

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*We are very grateful to Kfir Eliaz, Glenn Ellison, David Gill, Salar Jahedi, Andrew Rhodes, Mike Riordan, Rani Spiegler, John Vickers and Jidong Zhou for helpful discussions.

Submitted 23 January 2013

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Abstract

We investigate the marketing practice of framing a price as a discount from an earlier price. We discuss two reasons why a discounted price—rather than a merely low price—can make a consumer more willing to purchase. First, a high initial price can indicate the product is high quality. Second, a high initial price can signal a bargain relative to other options, and there is less incentive to search. We also discuss a behavioral model where the propensity to buy increases when others pay more. A seller has an incentive to offer *false* discounts, where the initial price is exaggerated.

Keywords: Reference dependence, price discounts, price discrimination, consumer search, sales tactics, false advertising, consumer protection.

1 Introduction

In his account of sales practices, Cialdini (2001, page 12) writes about

> the Drubeck brothers, Sid and Harry, who owned a men's tailor shop [...] in the 1930s. Whenever Sid had a new customer trying on suits in front of the shop's three-sided mirror, he would admit to a hearing problem and repeatedly request that the man speak more loudly to him. Once the customer had found a suit he liked and asked for the price, Sid would call to his brother, the head

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> tailor, at the back of the room, 'Harry, how much for this suit?' Looking up from his work—and greatly exaggerating the suit's true price—Harry would call back, 'For that beautiful, all wool suit, forty-two dollars.' Pretending not to have heard and cupping his hand to his ear, Sid would ask again. Once more Harry would reply, 'Forty-two dollars.' At this point, Sid would turn to the customer and report, 'He says twenty-two dollars.' Many a man would hurry to buy the suit and scramble out of the shop with his [...] bargain before poor Sid discovered the 'mistake'.

As this anecdote suggests, consumers are more likely to buy an item if they perceive it to be a bargain. This is easily understood when the consumer is given an accidental discount, as occurs for instance if she sees that the product has been given the wrong price tag. If the product's genuine price—which reflects its cost, quality and competitive environment—is \$42, but by chance the consumer can get the product for \$22, this represents value-for-money and will make the consumer more inclined to buy. This rational response to a mistaken discount is exploited by the Drubecks' fraudulent sales tactic.

It is more of a challenge to explain why consumers might care about receiving a deliberate discount from a seller, as opposed simply to obtaining a low price. Why should a consumer be more likely to buy a jacket priced at \$100 accompanied by a sign which reads "50% of its previous price" than he would be if the price were merely stated as \$100? Why does a retailer advertise that its price is \$100 even though the "manufacturer's suggested price" was \$200? Despite its prevalence, the practice whereby a product's price is framed as a discount off a reference price—a practice we term *discount pricing*—has apparently received little economic analysis. In most of the literature on sales, consumers care only about the price level, and whether or not a low price is labelled as being discounted from some higher price plays no role. In this paper, we explore the phenomenon of discount pricing using three economic models.

In each model, a firm sells its product over two periods. Consumers are segmented over time and must buy in their own period. Consumers in the second period cannot directly observe the initial price, although the firm can report its previous price (for instance, with a sign reading "50% of its previous price"). The firm's environment can differ in two dimensions. First, the legal regime might permit the firm to make misleading claims about its initial price (we term this the "*laissez-faire*" regime); alternatively in the "honest" regime the firm is required to make only truthful claims about its previous price. Second, in the *laissez-faire* regime second-period consumers might be "savvy", and ignore unsubstantiated claims about previous prices—that is, they "discount the discount"—or they might be "trusting" and take the firm's price claims at face value. Thus, we consider three market environments: (i) an honest regulatory regime; (ii) a *laissez-faire* regime with savvy consumers, and (iii) a *laissez-faire* regime with trusting consumers. Policy may be able to move from environment (ii) to (i), or from (iii) to (i), with legal sanctions which act effectively to prohibit misleading price claims. Alternatively, policy may be able to move from (iii) to (ii) with an education campaign which informs consumers that firms are in fact often able to present misleading claims.

In our first model, presented in section 2, uninformed consumers take the firm's initial price as a signal of its choice of quality. The firm sells its product to two groups of consumers: keen buyers who can accurately determine the product's quality and wish to buy early, and casual buyers who arrive later and cannot directly observe quality. Since period-1 consumers will only buy the product at a high price if the quality is high, later buyers use the initial price offered as an indicator of quality. In this framework, it is more likely that the firm has an incentive to supply a high-quality product when later buyers can observe its initial price. Thus, the firm's ability to write "was \$200, now \$100", if credible, can induce it to supply a better product.

In the second model (section 3), the knowledge that a firm's previous price was high can induce consumers to buy immediately rather than investigate an alternative supplier. A firm sells a limited stock of its product over two periods. An alternative supplier supplies the same product at a price which is uncertain to both the firm and its potential customers, and customers must incur a search cost to discover this alternative price. If the firm's stock does not sell out in the first period, early customers must have purchased elsewhere, and later consumers infer that the alternative price was low in the first period (and hence likely to be low in the second period). As a result, the firm has a clearance sale at a new price. However, when the firm's initial price was higher, the estimate of the alternate price is revised downwards less drastically. Thus later consumers care about the initial price, and all else equal, they are less inclined to investigate the second supplier when the firm had a higher initial price.

Our third model, presented in section 4, incorporates consumer reference dependence. Consumers intrinsically care about the firm's earlier price, and all else equal their propensity to buy is higher when the product was first offered to others at a higher price. For instance, consumers simply like the feeling of "getting a bargain", or travellers enjoy knowing that others on the same plane paid more for their air tickets. (In the Appendix we analyze a related model in which the firm sells simultaneously to all consumers, which makes the fraction of consumers who obtain a bargain endogenous and which permits a more symmetric situation in which *all* consumers care about the prices offered to others.)

If, for any of these reasons, consumers care about getting a discount, a seller has an incentive to exploit this by making exaggerated claims about its previous price. The media regularly features stories in which a seller's claimed discounts are alleged to be fictitious, although the fraud is usually less intricate than our opening anecdote. For instance, a supermarket's heavily advertised 15% average price reduction may have been preceded by an unadvertised gradual price rise cancelling out the reduction, or a retailer may market virtually all of its stock at "70% off". If consumers are savvy and know that sellers are able to misrepresent their reference price without penalty, they will simply regard sale signs as puffery and pay them no attention. The result is that a potentially useful channel of information is absent. However, if instead consumers are trusting and believe a firm's false claims, the outcome is worse, as these consumers may be induced to pay more for the product than they would otherwise. In all three models, changing the regulatory regime from *laissez-faire* to honest will increase the firm's profit when consumers are savvy (as the change enables the firm to commit to an initial price), but will reduce its profit when consumers are trusting. While this regime change can sometimes improve aggregate consumer surplus and efficiency, a common theme across the three models is that the change will induce the firm to raise its initial price: early consumers are "exploited" to deliver a larger discount to later buyers.

There are a number of earlier contributions which discuss issues related to our models. Our first model, where an initial price of a product signals its quality, builds on a large literature which studies how (current) price can signal quality. For instance, Bagwell and

Riordan (1991) present a model where a firm has private information about the exogenous quality of its product. They find that high and declining prices signal high product quality: the firm distorts its price above the full-information level in order to signal high quality, and, as more consumers become informed, there is less price distortion in later periods. While their motivation is different from ours and their insights are derived mainly in a setting where the firm's current price signals quality, they also consider an extension where consumers can observe the firm's past price. In this case, the firm's prices may be more distorted in period 1 but less distorted in period 2, compared to when past price is not observed, and they find that the high-quality firm has an incentive to reveal past price information to uninformed consumers. Thus, when a firm makes sequential sales of a product, the exogenous quality of which is the firm's private information, a policy that bans false discounts could boost profit.

Our second model, where a firm's past price can indicate whether it is worthwhile for a consumer to buy immediately rather than investigate an outside option, essentially embeds Lazear's (1986) model of clearance sales into a simple search framework. Lazear's basic model supposes that a firm has a single item to sell over two periods, and there is a single consumer present in each period. The two consumers have the same valuation for the product, but the firm does not know this common valuation. If the item remains unsold after the first period, the firm infers that this valuation is below its initial price, and so offers a discount to the second consumer. In Lazear's model, the second consumer has no interest in the firm's initial price, and she will buy whenever the sale price is below her valuation. In our model, by contrast, the second consumer does care about the initial price since that is informative about the benefits of searching for another supplier.

Taylor (1999) also modifies (what is in effect) Lazear's model so that later consumers care about the initial price. He studies how best to sell a house when distinct pools of buyers arrive over two periods. The seller knows the quality of the house (about which buyers obtain a noisy signal), while buyers have idiosyncratic tastes for the house given its quality. If the house remains unsold after the first period, the seller adjusts his reservation price for the second period. The house may not sell in the first period if (a) no buyer was present, (b) all buyers had negative signals about the house's quality, or (c) all buyers had valuations below the asking price. Similar to the mechanism in our model, when the seller sets a high initial price which is observed by later buyers, this makes (c) a relatively more likely explanation, and a later buyer's belief about quality is less adversely affected.

Also somewhat related is Yang and Ye (2008), who propose a search model in which a consumer's incentive to search in the current period depends on previous prices. Their model has many firms, each of which has the same unit cost which varies over time in Markov fashion. A consumer buys the product repeatedly over time. Search is non-sequential, and in a given period a consumer either discovers all market prices or buys randomly. Consumers are less inclined to search if they believe the current cost is high, since the gap between high and low prices is then relatively small. Any consumer who searched in the previous period can infer that period's cost from observed prices, and high previous prices indicates previous cost was high, and hence that current cost is likely to be high. As such, consumers who observed high previous prices in the market will be less inclined to search in the current period.

Reference dependence, the context of our third model, came to prominence with the work of Kahneman and Tversky (1979) and Thaler (1985), and is now the subject of a vast literature in economics. Consumers can be loss-averse—that is, they care more about avoiding losses than they do about obtaining the same sized gains—or they might be bargain-loving, and experience pleasure from extracting a bargain from a seller. In our simple model in section 4, all that matters is that a consumer's propensity to buy increases if the reference price—in our context, the initial price—increases. (However, the model we present in the Appendix assumes bargain-loving preferences.) One implication of Thaler's theory is that firms might profit from a high "suggested retail price", which serves as a reference price, and a lower selling price may then provide consumers with a "transaction utility".[1] Evidence that consumers incorporate previous prices into their reference price, as we assume in our model, is surveyed in Kalyanaram and Winer (1995). They suggest that the plausibility of this method of reference price formation is called into question by the observation in surveys that many consumers cannot recall previous prices of a particular good; however, this problem is overcome if the current price were framed as a discount

[1] Bordalo, Gennaioli and Shleifer (2012) develop a model of "salience" in consumer decision making, which they use to explain a number of perplexing phenomena. Their analysis suggests that, by raising consumers' valuation of quality through salience, firms can benefit from "misleading sales"—artificially inflating the regular price and simultaneously offering a generous discount.

from a previous price, so that the firm itself recalls the earlier price.

A recent paper with reference dependence which addresses some similar themes to ours is Heidhues and Kőszegi (2012). They study a model with consumer loss-aversion, where a consumer's reference point is her rational expectations about the purchase. A monopolist commits to a probability distribution for its price from which each consumer is only offered a single price and cannot search for others. The authors find that it is often optimal for the firm to choose a price distribution which consists of an atomistic "regular" price together with a continuum of lower "sale" prices. (The firm uses a discounted price to "force" the consumer sometimes to buy, and loss-aversion then makes the consumer more willing to buy at the regular price too.) They derive equilibrium prices which are high, in the sense that consumers would be better off if the firm did not exist at all. The model we present in the Appendix differs in three ways from theirs: our consumers are bargain-lovers rather than loss-averse; their reference price is simply the average price offered by the firm, and we mainly study the situation in which the firm cannot commit to its price distribution. We find that the firm in equilibrium charges exactly two prices, neither of which is particularly high (no higher than the most profitable deterministic price).

2 Initial Price as Signal of Quality

Our first model, as do the models in sections 3 and 4, uses the following framework. A monopolist, which for simplicity does not discount the future, sells a product over two periods, with price p_1 in the first period and price p_2 in the second. Consumers in the two periods comprise distinct groups and can only buy in their own period, and second-period consumers do not directly observe the firm's initial price. (However, consumers do know which period they are in.) The firm's environment may differ along two dimensions:

Regulatory environment. There are two possible regulatory regimes: a *laissez-faire* regime, where the firm is free to make any claims about its initial price to period-2 consumers, and an *honest* regime, where any report it makes of its initial price is required to be truthful. In our models, in the honest regime the firm will always report its initial price to second-period consumers rather than remain silent.

Consumer sophistication. In the *laissez-faire* regime, second-period consumers might be *savvy*, so that they accurately forecast the firm's equilibrium choice of initial price and ignore any claims it makes about its initial price. Savvy consumers will recognize that the firm has an incentive to exaggerate its reported previous price. Alternatively, second-period consumers might be *trusting* and believe the firm's claim about its initial price. Our preferred interpretation of trusting consumers is that they mistakenly believe that regulation is already in place to ensure honest reporting from the firm. As such, we assume that a trusting consumer makes the same inferences from the firm's (possibly misleading) report as does a savvy consumer in the honest regime. In particular, in the honest regulatory regime there is no distinction between the two kinds of consumer.

In this section we modify a standard static model of quality choice so that the firm sells over time.[2] Specifically, a monopolist supplies a product over two periods, and chooses its quality *ex ante* which is then fixed for the two periods. The firm can choose one of two quality levels, L and H, and it has constant unit cost c_i if it chooses quality $i = L, H$. All consumers have unit demand.

An exogenous fraction σ of consumers buy in the first period and the remaining $1 - \sigma$ consumers buy in the second. Period-1 consumers are particularly interested in the product: they can discern directly the product's quality, and they are impatient to buy (which is why they buy only in the first period). Their valuation is v_i for the product when its quality is $i = L, H$. Period-2 consumers are casual buyers, and cannot directly observe quality. (Little of substance in the analysis would be affected if both informed and uninformed consumers were present in both periods; the crucial assumption is that there is some correlation between being informed and being impatient, so that a greater proportion of consumers in the first period can directly discern quality.) A period-2 consumer's valuation for the product is θv_i when quality is $i = L, H$, where the parameter $0 < \theta \leq 1$ reflects the likely situation where casual buyers have a lower willingness-to-pay for the item. To avoid discussing sub-cases involving non-supply, we assume that

$$\theta v_L > c_H \ , \tag{1}$$

so the high-quality product can profitably be sold even to casual buyers who believe quality

[2]This static model is taken from Tirole (1988, section 2.3.1.1), which itself incorporates elements from a number of earlier contributions.

is low. We also assume that supplying a high-quality product is socially efficient, so

$$[\sigma + (1-\sigma)\theta]\,\Delta_v > \Delta_c \ , \tag{2}$$

where $\Delta_v \equiv v_H - v_L$ and $\Delta_c \equiv c_H - c_L$.

A consumer buys the item if the price is no higher than her willingness-to-pay, which depends on observed (for a period-1 consumer) or anticipated (for a period-2 consumer) product quality. The firm's strategy consists of its choice of quality, the two prices, and the reporting of initial price. In equilibrium the firm's strategy is optimal given consumer buying behaviour and the regulatory constraint to be honest (if applicable), while the expectations of product quality by period-2 consumer, which might depend on observed or reported prices, are consistent with the firm's strategy when the period-2 buyers are savvy or when the regulatory regime is honest. (In the *laissez-faire* regime, if period-2 consumers are trusting, they react to p_2 and the reported p_1 the same way as in the honest regime. This reflects our preferred interpretation of trusting consumers, that they mistakenly believe the firm is forced to be honest in its report of its previous price.)

Consider first a *laissez-faire* regulatory regime where the firm can make any claim about its initial price to period-2 buyers. Since these buyers do not see the initial price, the firm will choose p_1 to maximize first-period profit, so that $p_1 = v_i$ if the firm offers quality $i = L, H$. Intuitively, if the fraction of period-1 buyers is large enough, the firm makes more profit by serving informed buyers with their preferred product than by supplying an inferior product to all consumers:

Lemma 1 *Suppose the regulatory regime is* laissez-faire. *Regardless of whether period-2 buyers are savvy or trusting, if*

$$\sigma > \frac{\Delta_c}{\Delta_v} \tag{3}$$

then any equilibrium involves the firm supplying a high-quality product, while if $\sigma < \Delta_c/\Delta_v$ any equilibrium involves low quality.

Proof. Note that in any equilibrium the firm must supply period-2 buyers. (The firm could charge $p_2 = \theta v_L$, which would induce period-2 consumers to buy regardless of their beliefs about quality, and from (1) this generates positive profit, while not serving these consumers yields zero profit in the second period.) Suppose in a candidate equilibrium that the firm supplies a low-quality product with initial price $p_1 = v_L$ and subsequent price p_2, and period-2 consumers are willing to buy at their price. In such an equilibrium we have $p_2 \geq \theta v_L$, since all period-2 consumers will surely buy at any price up to θv_L. (It is possible that $p_2 > \theta v_L$ if period-2 buyers are trusting and have been misled into thinking the product is high quality.) This strategy yields profit $\sigma(v_L - c_L) + (1-\sigma)(p_2 - c_L)$. The firm could deviate to offer high quality, in which case it makes profit $\sigma(v_H - c_H) + (1-\sigma)(p_2 - c_H)$. (With this deviation the firm can now charge $p_1 = v_H$ to the period-1 buyers, and it still wishes to serve period-2 buyers given (1).) The latter profit strictly exceeds the former if and only if (3) holds, in which case any equilibrium must involve high quality. A similar argument establishes that if $\sigma < \Delta_c/\Delta_v$ there can be no equilibrium with high quality. ■

Lemma 1 is silent about the second-period price. This price depends in part on whether period-2 consumers are savvy or trusting, but there is also the usual problem in signalling games of multiple equilibria. For now, suppose that period-2 buyers are savvy, so that they will disregard the firm's claims about its initial price. If $\sigma < \Delta_c/\Delta_v$, then any equilibrium involves low quality. Savvy buyers recognize that the firm will choose low quality, and so will pay no more than θv_L. Since they will always be willing to pay up to this threshold, it follows that $p_2 = \theta v_L$ is the unique equilibrium price. Suppose next that $\sigma > \Delta_c/\Delta_v$, so that any equilibrium will have high quality. Clearly, the most profitable such equilibrium extracts all surplus from period-2 buyers, so that $p_2 = \theta v_H$. However, other equilibria exist: if period-2 buyers for some reason believed that the firm chose high quality only if $p_2 = p_2^*$, where $\theta v_L \leq p_2^* < \theta v_H$, and otherwise they believed the product was low quality, then the firm's optimal price is to choose $p_2 = p_2^*$.

These lower-price equilibria can be eliminated by using the forward induction refinement. To see this, suppose (3) holds and $\theta v_L \leq p_2^* < \theta v_H$. Consider a deviation by the firm to $p_2 = \theta v_H$. Period-2 consumers will either buy or not buy at this price, depending on their beliefs about quality. If they do not buy, the firm enjoys profits $\sigma(v_H - c_H)$ with high quality and profits $\sigma(v_L - c_L)$ with low quality, and from (2) it is more profitable to have chosen high quality. If they do buy at this price, the firm enjoys profits $\sigma(v_H - c_H) + (1-\sigma)(\theta v_H - c_H)$ with high quality and profits $\sigma(v_L - c_L) + (1-\sigma)(\theta v_H - c_L)$

with low quality. The former is greater than the latter when (3) is satisfied. Therefore it is a dominant strategy for the firm offering $p_2 = \theta v_H$ to have supplied a high-quality product. Using the forward induction refinement, savvy consumers should infer that the firm must have chosen high quality, and they are thus willing to purchase the product at $p_2 = \theta v_H$. This eliminates any equilibrium with $\theta v_L \leq p_2^* < \theta v_H$.

We summarize this discussion as:

Lemma 2 *Suppose the regulatory regime is* laissez-faire *and period-2 buyers are savvy. If σ is large enough that (3) is satisfied, the unique equilibrium which satisfies forward induction is for the firm to supply a high-quality product, and to choose prices which fully extract consumer surplus (i.e., $p_1 = v_H, p_2 = \theta v_H$). If σ is small enough that (3) is strictly violated, the unique equilibrium is for the firm to supply a low-quality product, and to choose prices which fully extract consumer surplus (i.e., $p_1 = v_L, p_2 = \theta v_L$).*

Consider next the outcome in an honest regime, where the firm is forced to make only truthful claims about its initial price to period-2 buyers. For instance, these buyers see a price label which truthfully states "was \$200, now \$100", or "50% off, now \$100". We will show that the efficient level of quality can be supported in equilibrium for a wider range of market parameters than (3) when the initial price is revealed. This is because the initial price can serve as a signal of quality.

Suppose first that the firm in fact makes no report of its initial price (even though it could make a truthful report if it wished). Then, following the same argument as for Lemma 1, supplying a high-quality product is more profitable if and only if (3) holds. The outcome for the firm and consumers is then as described in Lemma 2.

Suppose next that the firm chooses and reports a low initial price such that $p_1 \leq v_L$. Then in any equilibrium the firm chooses low quality. To see this, we argue in a similar manner to the proof of Lemma 1. The firm will always sell to period-2 buyers in equilibrium, and at some price $p_2 \geq \theta v_L > c_H$. If the firm chooses high quality its profit is $\sigma(p_1 - c_H) + (1 - \sigma)(p_2 - c_H)$, while if it offers a low-quality product its profit is $\sigma(p_1 - c_L) + (1 - \sigma)(p_2 - c_L)$. Since the latter is always higher than the former, it is optimal for the firm to offer a low-quality product. Therefore, the unique equilibrium given an initial price $p_1 \leq v_L$ is for the firm to supply a low-quality product and to charge price $p_2 = \theta v_L$. Intuitively, since the price is low enough that the informed consumers buy regardless of quality choice, the firm's revenue is independent of quality and so it is more profitable to offer the less-costly, low-quality product. At the other extreme, if the firm chooses and reports a high initial price $p_1 > v_H$, then period-1 consumers will not buy regardless of quality. Again, it is a dominant strategy for the firm to have offered the low-quality product and to charge $p_2 = \theta v_L$.

Finally, suppose that the firm chooses and reports an intermediate initial price in the range $v_L < p_1 \leq v_H$. As before, in equilibrium the firm will sell to period-2 buyers, and say that an equilibrium second-period price is $p_2 \geq \theta v_L > c_H$. The firm's profit if it supplies a high-quality product is $\sigma(p_1 - c_H) + (1 - \sigma)(p_2 - c_H)$, while its profit with a low-quality product is $(1 - \sigma)(p_2 - c_L)$. (For this second expression, note that the firm cannot sell to the informed buyers since $p_1 > v_L$.) Thus, any equilibrium with initial price $v_L < p_1 \leq v_H$ involves supplying a high-quality product provided that

$$p_1 > c_L + \frac{\Delta_c}{\sigma} \ . \tag{4}$$

The reason is that truthfully reporting an initial price in this range makes a deviation to low quality more costly for the firm, since this involves foregoing profits from the informed buyers. As in Lemma 2, the unique second-period price which survives the forward induction refinement is $p_2 = \theta v_H$.

The best chance for high quality to be incentive compatible for the firm is to set $p_1 = v_H$ in (4). Doing this implies that first-best profit—where the firm supplies a high-quality product and chooses prices $p_1 = v_H$ and $p_2 = \theta v_H$— is feasible if $v_H \geq c_L + \frac{\Delta_c}{\sigma}$, i.e., if

$$\sigma > \frac{\Delta_c}{\Delta_v + [v_L - c_L]} \ . \tag{5}$$

If this condition does not hold, there is *no* initial price which could convince period-2 buyers that quality is high. In this case the firm supplies low quality and fully extracts the resulting consumer surplus. We summarize this discussion as:

Lemma 3 *Suppose the regulatory regime is honest, so the firm can only make truthful claims about its initial price. If σ is large enough that (5) is satisfied, the unique equilibrium which satisfies forward induction is for the firm to supply a high-quality product, and to* *choose prices which fully extract consumer surplus (i.e., $p_1 = v_H$, $p_2 = \theta v_H$). If σ is small enough that (5) is strictly violated, the unique equilibrium is for the firm to supply a low-quality product, and to choose prices which fully extract consumer surplus (i.e., $p_1 = v_L$, $p_2 = \theta v_L$).*

When σ lies in the intermediate range

$$\frac{\Delta_c}{\Delta_v + [v_L - c_L]} < \sigma < \frac{\Delta_c}{\Delta_v}\ , \tag{6}$$

and period-2 buyers are savvy, the firm is able to offer a high-quality product if and only if the regulatory regime is honest. The firm's profit rises in this case, so the firm *welcomes* a policy which requires it to be honest. Welfare—which equals profit in this setting with full extraction of consumer surplus—also rises in this case.

The third case we need to consider is when consumers in the *laissez-faire* regime are trusting, and believe the firm's report of its initial price. Recall that these consumers react to a firm's report exactly as a savvy consumer would in the honest regime, and so their demand is as described prior to Lemma 3. In this scenario, it is clear that the firm will supply a high-quality product if and only if (3) holds, but when the looser condition (5) holds it will claim to period-2 buyers that its initial price was high ($p_1 = v_H$) to induce the belief that its product is high quality. If σ lies in the range (6), trusting consumers will buy the product at a high price $p_2 = \theta v_H$ in the belief that the product is high quality, when in fact it is low quality. Such consumers will therefore suffer negative surplus. We summarize this outcome as the following:

Lemma 4 *Suppose the regulatory regime is* laissez-faire *and casual buyers are trusting. If σ is large enough that (3) is satisfied, the unique equilibrium which satisfies forward induction is for the firm to supply a high-quality product, and to choose prices which fully extract consumer surplus (i.e., $p_1 = v_H$, $p_2 = \theta v_H$). If σ is small enough that (3) is strictly violated, the firm supplies a low-quality product and chooses initial price which fully extracts period-1 consumer surplus (i.e., $p_1 = v_L$). If σ lies in the range (6), the firm reports that its initial price was $\bar{p}_1 = v_H$ which induces period-2 buyers to anticipate a high-quality product, but the true initial price is $p_1 = v_L$. Period-2 buyers are charged a price which extracts their anticipated surplus ($p_2 = \theta v_H$), but in fact the product is low quality.*

Thus, with trusting consumers a policy which prevents misleading claims about initial prices not only induces efficient quality choice for a wider set of market parameters (as was the case when consumers were savvy), but it now improves consumer welfare and reduces profit. We summarize the discussion as:

Proposition 1 *Consider a change in regime from* laissez-faire *to honest. If σ lies outside the range (6), the change has no impact. If (6) holds and casual buyers are savvy, the change boosts profit and welfare (and leaves consumers indifferent). If (6) holds and casual buyers are trusting, the change harms profit but boosts welfare and consumer surplus.*

3 Initial Price as Signal to Buy Immediately

A second reason why a consumer might like a discounted price is because this signals that the price is low relative to her other options, and she would do well to take advantage of it when search is costly. More precisely, the market phenomenon we wish to explore is when and how one firm's *past* price acts as a positive signal of another supplier's *current* price. Whenever this is the case, it follows that:

(a) consumers will be interested in a firm's previous price, even though they cannot pay that price, since that is informative about the payoff they obtain from investigating the alternative deal, and
(b) firms have an incentive to exaggerate their past prices if permitted, since that may deter consumers from searching elsewhere.

One framework to discuss this issue goes as follows. A firm sells its product over two periods, and for simplicity suppose it has no production costs and does not discount the future. The same product is available from an alternative supplier, whose price we take to be uncertain and exogenously determined. Consumers are short-lived and can buy only in their own period. A consumer needs to engage in costly search to discover the price from the alternative supplier. There are two ways to proceed with the analysis. One approach would be to assume the firm has private information about the alternate price when it chooses its own price, and so may attempt to signal market conditions in its choice of price(s). While simple versions of such models appear tractable, the analysis is very sensitive to assumptions about off-equilibrium consumer beliefs.[3]

In this section, though, we describe a model—a variant of Lazear's (1986) model of clearance sales—in which the firm has *no* private information about market conditions. A seller has some stock of a product to sell over two periods. The same product is available from an alternative source at a price which neither the firm nor consumers know *ex ante*.[4] If the product did not sell well in the first period, this indicates that the alternative price turned out to be relatively low, and early consumers decided to buy the product elsewhere. As a result, the firm typically offers a discount in the second period, since later consumers infer that when the product remains available it is likely that the alternative price is low, and this usually makes their demand more elastic than first-period demand.

In more detail, a firm has a single unit of its product to sell over two periods. The same product is available in unlimited quantities elsewhere in the market at a non-negative random price P. This alternative price might vary over the two periods, but with persistence.[5] Let the first-period's alternative price, P_1, be drawn from a distribution with c.d.f. $F_1(P_1)$, and let $F_2(P_2, P_1)$ denote the probability that the second-period's alternative price is no higher than P_2 given that the first-period alternative price was no higher than P_1. We assume there is positive correlation between P_1 and P_2, in the sense that F_2 strictly decreases with P_1. We also assume that the marginal distribution for the alternative price is the same in the two periods, i.e., $F_2(P, \infty) \equiv F_1(P)$. The firm sets price p_1 in the first period and, if the good remains unsold, sets price p_2 in the second period. The firm does not know the realization of either alternative price P_i, although, like consumers, it gains information about P_1—and hence about P_2—if its product did not sell in the first period.

There is a single consumer who buys only in the first period, and a second consumer

[3] Perhaps the simplest such model involves the price from the alternative supplier, P, taking binary values and remaining unchanged for the two periods. The firm knows the realization of P. One can investigate separating equilibrium in which the firm signals P by its choice of its own price(s), and analyze how the cost of achieving separation is affected by whether or not second-period consumers can observe the initial price. Such analysis would have some similarities to Bagwell and Riordan (1991)'s discussion of separating equilibria when consumers can observe both the firm's current price and its previous price.

[4] Clearly, the alternative source need not supply an identical product, and what matters for the consumer's search behaviour is the alternate price adjusted for the supplier's product quality.

[5] If the alternative price was independently distributed over time, there is no reason for the second-period consumer to care about the initial price when she decides on her search strategy. The firm will nevertheless offer a discount in the second period since that is its final opportunity to sell its stock.

who buys only in period 2.[6] Each consumer sees the firm's price for free, but needs to incur search cost s to discover the alternative price.[7] Consumers differ in their cost of search, and suppose that the search costs of the two consumers are independent draws from the distribution with c.d.f. $G(\cdot)$. If a consumer discovers that the alternative price is higher than the firm's price, she can return to buy from the firm without incurring a further search cost. If the second-period consumer arrives at the firm and discovers the product has sold out, she must still incur the search cost s to obtain the product from the alternative supplier. Suppose that a consumer's valuation for the product is high enough that she always buys from one source or the other.

Consider the consumer in the first period. If offered price p_1, she will buy immediately, without investigating the alternative price, if and only if

$$p_1 \leq \mathbb{E}[\min\{P_1, p_1\}] + s \ .$$

Here, $\mathbb{E}[\cdot]$ denotes taking expectations over the random variable P_1 so that the right-hand side is her expected outlay if she chooses to investigate the alternative supplier: she incurs the search cost, but then is able to buy the product at the lower of the two prices. This condition can be written as

$$\int_0^{p_1} F_1(P_1) dP_1 \leq s \ . \tag{7}$$

If her search cost s is below this threshold, the consumer will investigate the alternative price, but return to buy if $P_1 > p_1$. Putting these two sources of demand together implies that the probability of selling in the first period at price p_1 is

$$q_1(p_1) = 1 - G\left(\int_0^{p_1} F_1(P_1) dP_1\right) + G\left(\int_0^{p_1} F_1(P_1) dP_1\right)(1 - F_1(p_1))$$

$$= 1 - F_1(p_1) G\left(\int_0^{p_1} F_1(P_1) dP_1\right) \ . \tag{8}$$

[6] This starkly simplified framework corresponds to Lazear's most basic model (Lazear, 1986, section IB). As in Lazear (1986), this framework can be extended to allow the number of consumers in the two periods to be uncertain (so that the item might remain unsold simply because no one turned up rather than because the alternate price is low), or for early buyers to be able to wait and have a chance to buy later at the lower price.

[7] It would be possible to extend this simple framework so that some consumers first visit the alternative supplier before the firm in question. Such consumers will be fully informed about their options, and so will not care about the firm's previous price.

Importantly, if the product does not sell in the first period this implies that $P_1 \le p_1$. The only way that the initial consumer does not buy from the firm is if she decides to investigate the alternative price and discovers that that price is lower.

Next consider the second consumer, and suppose that the true initial price was p_1 while the consumer believes the initial price was $\bar{p}_1$. (When the second consumer is trusting, it is possible that $\bar{p}_1 \neq p_1$.) The consumer believes that the alternative price P_2 has distribution governed by c.d.f. $F_2(P_2, \bar{p}_1)$, whereas in fact P_2 has c.d.f. $F_2(P_2, p_1)$. When the second consumer is trusting, we assume that her belief about the initial price is generated entirely from the firm's report of its initial price, and does not depend on the firm's price p_2. (This is consistent with our preferred interpretation of a trusting consumer as one who mistakenly believes that the regulatory environment already acts to prevent false price claims.) As in expression (7), the consumer will buy immediately if her search cost satisfies

$$\int_0^{p_2} F_2(P_2, \bar{p}_1) dP_2 \le s \ . \tag{9}$$

Since the left-hand side of (9) decreases with $\bar{p}_1$, the consumer is more likely to buy from the firm without search when she thinks the initial price was higher. This is the key ingredient in this model: all else equal, a high initial price acts to deter search in the second period.

If the consumer's search cost does not satisfy (9), she will investigate the alternative price and then return to buy if $P_2 > p_2$, which occurs with (true) probability $1 - F_2(p_2, p_1)$. Putting this together shows that the firm's demand in the second period at price p_2, given that no sale occurred in the first period at true price p_1 and that the consumer believes the initial price was $\bar{p}_1$, is

$$q_2(p_2, p_1, \bar{p}_1) = 1 - F_2(p_2, p_1) G\left(\int_0^{p_2} F_2(P_2, \bar{p}_1) dP_1\right) \ . \tag{10}$$

This is increasing in both p_1 and $\bar{p}_1$. For convenience, write

$$\pi_2(p_1, \bar{p}_1) \equiv \max_{p_2} p_2 q_2(p_2, p_1, \bar{p}_1) \tag{11}$$

for the maximum available profit in the second period given that the product remained unsold at true initial price p_1 and the second consumer believes the initial price was $\bar{p}_1$. Clearly π_2 increases with both p_1 and $\bar{p}_1$. Let $\pi_1(p_1) \equiv p_1 q_1(p_1)$ denote first-period profit. The firm's profit over the two periods is

$$\Pi = \pi_1(p_1) + (1 - q_1(p_1))\pi_2(p_1, \bar{p}_1) \ . \tag{12}$$

In the honest regulatory regime (labelled environment (i) in the Introduction), we automatically have $\bar{p}_1 \equiv p_1$. Therefore, the firm's profit is

$$\Pi = \pi_1(p_1) + (1 - q_1(p_1))\pi_2(p_1, p_1) \tag{13}$$

and the firm chooses p_1 to maximize this expression. With initial price p_1, the initial consumer's prior for the alternative price has distribution $F_1(P)$, while second consumer's prior for the alternative price has distribution $F(P, p_1)$. The latter involves lower prices, in the sense of first-order stochastic dominance. Thus, for a *given* price p, the initial consumer is more likely than the second to buy the product without search.

In a *laissez-faire* regime where the second-period consumer is savvy (environment (ii)), the firm chooses p_1 to maximize profit in (12) given belief $\bar{p}_1$, and beliefs are required to be consistent so that $p_1 = \bar{p}_1$. This implies that an equilibrium initial price when the second consumer cannot observe the initial price, say p_1^*, satisfies the first-order condition

$$\pi_1'(p_1^*) + (1 - q_1(p_1^*))\frac{\partial \pi_2(p_1^*, p_1^*)}{\partial p_1} - q_1'(p_1^*)\pi_2(p_1^*, p_1^*) = 0 \ . \tag{14}$$

As with environment (i), the second consumer knows her alternative price comes from a distribution with lower prices than that faced by the initial consumer.

In a *laissez-faire* regulatory regime in which the second consumer is trusting (environment (iii)), the firm is free to choose both p_1 and $\bar{p}_1$ to maximize expression (12). Since Π increases with $\bar{p}_1$, the firm will claim its initial price was at the highest possible level.[8] As such, the second consumer does not believe the alternative price P_2 is shifted downwards at all, and she has the same prior for the alternative price as the initial consumer did.

As discussed in the Introduction, the firm's profits are highest in environment (iii) when it faces the fewest constraints on its price policy, and lowest in environment (ii). Without placing tight restrictions on the shape of demand, i.e., on the form of the functions F_1, F_2 and G, it appears hard to generate clear-cut results about price comparisons, either across regulatory regimes or over time. However, the following result can be derived:

Lemma 5 *If the regulatory regime changes from* laissez-faire *to honest, the initial price will rise when the second consumer is savvy.*

[8] If there is a choke price for q_1 in (8), then any claimed initial price $\bar{p}_1$ above this choke price induces the same second-period demand for the firm. We can take this choke price as the firm's announcement $\bar{p}_1$ when it faces a trusting consumer.

Proof. Note first that (13) can be written

$$\pi = \pi_1(p_1) + (1 - q_1(p_1)) [\pi_2(p_1, p_1^*) + \pi_2(p_1, p_1) - \pi_2(p_1, p_1^*)] \ .$$

Any price $p_1 < p_1^*$ must yield smaller profits in the above than setting $p_1 = p_1^*$, because

$$\begin{aligned} \pi_1(p_1) + (1 - q_1(p_1)) [\pi_2(p_1, p_1^*) + \pi_2(p_1, p_1) - \pi_2(p_1, p_1^*)] &\leq \pi_1(p_1) + (1 - q_1(p_1))\pi_2(p_1, p_1^*) \\ &\leq \pi_1(p_1^*) + (1 - q_1(p_1^*))\pi_2(p_1^*, p_1^*), \end{aligned}$$

where the first inequality follows from the fact that $\pi_2(p_1, \bar{p}_1)$ is an increasing function of $\bar{p}_1$, and the second follows because setting $p_1 = p_1^*$ maximizes the function of p_1 given by $\pi_1(p_1) + (1 - q_1(p_1))\pi_2(p_1, p_1^*)$. This implies that the optimal price in the honest regime cannot be lower than an equilibrium in the *laissez-faire* regime with savvy consumers. Second, note that profit in (13) is strictly increasing at price $p_1 = p_1^*$ given that $\pi_2(p_1, \bar{p}_1)$ is strictly increasing in $\bar{p}_1$. It follows that the price in the honest regime is strictly above any equilibrium price in the *laissez-faire* regime. ■

In situations in which second-period demand is more elastic than first-period demand, one can make further predictions. Consider for instance the situation with a savvy consumer (so that $\bar{p}_1 = p_1$). Suppose the own-price elasticity of second-period demand decreases with p_1, so that the most-profitable second-period price is an increasing function of p_1. Then Lemma 5 implies that a change in regime from *laissez-faire* to honest will increase the initial price, and hence also increase the second price. However, as we will see in an example below, even though both prices rise as we move from environment (ii) to (i), the second consumer can still be better off, since there is then a better chance that the product remains available to her.

If second-period demand is more elastic than initial demand, we can also deduce that the firm offers a discount in the second period (in either regulatory regime). To see this, note that the initial price p_1 which maximizes (12) or (13) is above the myopic price which maximizes first-period profit $\pi_1(\cdot)$.[9] However, the second-period price is below the price which maximizes $\pi_1(\cdot)$. (If the initial price was $p_1 = \infty$, the second-period price would

[9] This is because each function $(1 - q_1(p_1))\pi_2(p_1, p_1)$ or $(1 - q_1(p_1))\pi_2(p_1, \bar{p}_1)$ strictly increases with p_1, and so a maximizer of (13) or (12) is greater than a maximizer of $\pi_1(\cdot)$ alone.

maximize $\pi_1(\cdot)$ since the distribution of the alternative price would be the same as in the first period. But with a lower initial price, second-period demand is more elastic, and so the second-period price will be below this level.)

While in many cases second-period demand is indeed more elastic than initial demand, it is hard to find satisfying conditions which ensure this is so. Recall from (8) that demand from a consumer who believes that the alternative price has distribution $F(P)$ is

$$q(p) = \underbrace{1 - G\left(\int_0^p F(P)dP\right)}_{\text{buy without search}} + \underbrace{G\left(\int_0^p F(P)dP\right)(1 - F(p))}_{\text{buy after searching}} .$$

While the first component of demand, $1 - G(\int^p F)$, typically becomes more elastic as $F(\cdot)$ is shifted upwards, the second component $G(\int^p F)(1 - F(p))$ is less clear-cut. Indeed, even the scale of the "buy after search" demand, let alone its elasticity, could go up or down with F. For this reason, it is hard to be sure that second-period demand is more elastic.

To illustrate the operation of the model and gain further insights, we consider a numerical example. Suppose that s is uniformly distributed on $[0, \frac{1}{2}]$, P_1 is uniformly distributed on $[0, 1]$, and $P_2 \equiv P_1$ so there is perfect correlation between the alternative prices over time. Detailed calculations contained in the appendix reveal that the outcomes in the three market environments (i)–(iii) listed in the Introduction are as reported in Table 1.

	p_1	p_2	profit	total outlay	period-2 outlay	rival's revenue
environment (i)	0.779	0.533	0.600	1.371	0.684	0.531
environment (ii)	0.760	0.525	0.599	1.366	0.689	0.529
environment (iii)	0.776	0.579	0.616	1.387	0.702	0.534

Table 1: Outcomes with uniform example in the three market environments

If the second consumer was savvy, then a policy which forced the firm to make only truthful claims would have the effect of switching from environment (ii) to (i). As reported in this table, the impact of the policy would be to boost the firm's profit, harm consumers in aggregate, but help the second-period consumer who benefits from a high initial price, since that makes it more likely that the product remains available to her. However, at least in this example, the impact of the regime change does not seem significant for any party.

If instead the second consumer was trusting, then a policy which forced the firm to make only truthful claims would have the effect of switching from environment (iii) where the initial price was exaggerated to (i). As shown in the table, the impacts of the regime change are now more significant. This change harms the firm, and helps consumers in aggregate (and each consumer separately). When the consumer is trusting, the firm has an incentive to claim its initial price was so high that no sale would ever take place, i.e., $\bar{p}_1 = 1$. Because the second consumer's beliefs about the alternative price are then not revised downwards, the firm is able to charge a relatively high price to the second consumer, as she expects the alternative price to be relatively high. The outcome in this case is that the firm's true initial price is $p_1 \approx 0.776$ but it claims its initial price was 1. The second consumer believes that the alternative price is uniformly distributed on the interval [0,1], whereas in fact it is uniformly distributed on [0,0.776], and so this consumer is induced to search less often than she should.

Interestingly, the alternative supplier (who is a passive agent in this model) has expected revenue ranked in exactly the same way as the firm in the various market environments. In particular, the rival obtains highest revenue in the *laissez-faire* regime with trusting consumers. Intuitively, one might have expected that the rival would suffer from the firm's ability to stifle search via its misleading claims. However, the firm chooses a higher price in the second period when it can mislead the consumer, and here this price-raising effect outweighs the search-deterrent effect, and the rival also benefits from the firm's efforts to discourage price comparison.

4 A Model with Reference Dependence

Our final model is perhaps the simplest. As before, a monopolist, which for simplicity has no production costs and does not discount the future, sells a single product over two periods, with price p_1 in the first period and price p_2 in the second. Consumers in the two periods comprise distinct groups and can only buy in their own period.

Consumers in the first period are assumed to have no "reference price" with which to compare the price they are offered, and their demand, $Q_1(p_1)$, simply depends on the price they are offered. There is a second group of consumers who can buy only in the second period, and who take (their expectation of) the initial price as their reference price, and their demand takes the form $Q_2(\bar{p}_1, p_2)$ when they believe the initial price was $\bar{p}_1$. We suppose that $Q_2(\bar{p}_1, p_2)$ strictly increases with $\bar{p}_1$ (and has a strictly positive partial derivative) over the relevant range of prices, reflecting our focus on consumers who enjoy obtaining a discount.[10] We also suppose that $Q_2(p_1, p_1) \equiv Q_1(p_1)$, so that demand in the two periods is equal with a constant price over time. If own-price elasticity in the second period decreases with the initial price—i.e., if Q_2 is log-supermodular in $(\bar{p}_1, p_2)$—then a higher (expected) initial price induces the firm to raise its second-period price. Suppose that first-period profit $pQ_1(p)$ is strictly single peaked, is maximized at $p = p^*$, and the maximum value of $pQ_1(p)$ is $\pi^* = p^*Q_1(p^*)$. Note that if the firm were known to choose a uniform price over time, so $p_1 = \bar{p}_1 = p_2$, the most profitable uniform price is p^* which generates profit $2\pi^*$.

The firm will typically wish to offer a discount from its reported initial price in the second period:

Lemma 6 *If $\bar{p}_1 \geq p^*$ then a price p_2 which maximizes $p_2 Q_2(\bar{p}_1, p_2)$ satisfies $p_2 < \bar{p}_1$.*

Proof. Suppose to the contrary that $p_2 > \bar{p}_1$. Then second-period profit is $p_2 Q_2(\bar{p}_1, p_2) < p_2 Q_1(p_2) < \bar{p}_1 Q_1(\bar{p}_1)$, where the second inequality follows from $pQ_1(p)$ being single-peaked and $\bar{p}_1 \geq p^*$. But given $\bar{p}_1$ the firm can ensure itself second-period profit equal to $\bar{p}_1 Q_1(\bar{p}_1)$ by setting $p_2 = \bar{p}_1$, which is a contradiction. We deduce that $p_2 \leq \bar{p}_1$. However, profit $p_2 Q_2(\bar{p}_1, p_2)$ is strictly decreasing in p_2 at the point $p_2 = \bar{p}_1$, since

$$Q_2(\bar{p}_1, \bar{p}_1) + \bar{p}_1 \frac{\partial Q_2(\bar{p}_1, \bar{p}_1)}{\partial p_2} = Q_1(\bar{p}_1) + \bar{p}_1 \left[Q_1'(\bar{p}_1) - \frac{\partial Q_2(\bar{p}_1, \bar{p}_1)}{\partial \bar{p}_1} \right] < Q_1(\bar{p}_1) + \bar{p}_1 Q_1'(\bar{p}_1) \leq 0 \, .$$

(Here, the final inequality follows from $pQ_1(p)$ being single-peaked and $\bar{p}_1 \geq p^*$.) We deduce that the most profitable second-period price satisfies $p_2 < \bar{p}_1$ provided $\bar{p}_1 \geq p^*$. ■

[10] It is plausible that a very high initial price would be ignored by second-period consumers, if, for example, no consumer actually paid this high price. Therefore, it is implausible to suppose that Q_2 globally increasing in $\bar{p}_1$. While for convenience we assume that second-period demand is differentiable, it is straightforward to allow Q_2 to have a "kink" at the point $p_2 = \bar{p}_1$, as is typically assumed in models with consumer loss aversion.

Consider first an honest regime where any report by the firm of its initial price must be accurate (environment (i) in the Introduction's terminology). If the firm chooses and reports that its initial price was p_1, the firm chooses (p_1, p_2) to maximize total profit

$$\Pi = p_1 Q_1(p_1) + p_2 Q_2(p_1, p_2) \ . \tag{15}$$

Since Q_2 increases with p_1, the firm sets an initial price which is above the myopic price p^*. (If instead the firm makes *no* report of its initial price, consumers infer that the firm's dominant strategy is to choose $p_1 = p^*$, so that $\bar{p}_1 = p^*$. However, the firm's profit is higher if it chooses and reports a strictly higher initial price.) From Lemma 6 the firm offers a discounted price $p_2 < p_1$ in the second period.

Consider next a *laissez-faire* regime where the firm's pricing claims are not required to be truthful. If second-period consumers believe the initial price was $\bar{p}_1$, the firm chooses (p_1, p_2) to maximize

$$\Pi = p_1 Q_1(p_1) + p_2 Q_2(\bar{p}_1, p_2) \ . \tag{16}$$

Regardless of beliefs $\bar{p}_1$, since the actual initial price cannot affect second-period demand, it is a dominant strategy for the firm to choose the initial price to be the myopic price p^*. (Conceivably, beliefs $\bar{p}_1$ might depend on the second price p_2. Nevertheless, it remains true that the firm will choose $p_1 = p^*$.) We deduce that a change from the *laissez-faire* regime to an honest regime will induce the firm to raise its initial price. The regime change harms the first-period consumers, who are exploited in order to offer later consumers a perceived bargain.

In a *laissez-faire* regime in which the second-period consumers are savvy (environment (ii)), they infer the initial price will be p^* so that $\bar{p}_1 = p^*$. From Lemma 6, the second-period price is strictly below the initial price p^*. With savvy consumers, the firm will be better off with a change from a *laissez-faire* regime to an honest regime—it could set initial price p^* in the regime where consumers observe the initial price, but does better by setting a higher price—and so will welcome a policy which forbids misleading price claims. (The firm is also better off than in a situation where it must set a uniform price over time. The most profitable uniform price is p^*, but from Lemma 6 the firm does better in the second period by offering a discount.) When Q_2 is log-supermodular, the most profitable second-period price increases with $\bar{p}_1 = p_1$. In such cases the regime change will induce the firm to raise *both* prices. Second-period consumers may nevertheless benefit from the regime change, if their psychological pleasure from the discount outweighs the harm from rise in the price they pay.

Alternatively, in a *laissez-faire* regime where second-period consumers are trusting (environment (iii)), the firm can implement any belief $\bar{p}_1$ via its report of its initial price, and so it chooses the three parameters $(p_1, \bar{p}_1, p_2)$ to maximize (16). The firm has an incentive to claim its initial price was high, which induces a boost in demand from second-period consumers, but in fact sets the myopic price p^* which extracts maximum profit from the early consumers.[11] This scenario is particularly profitable for the firm—more profitable than the honest regime in (15) since the could could choose $\bar{p}_1 = p_1$ in (16) but generally its does better by choosing $\bar{p}_1 > p_1$—and so when consumers are trusting the firm is *harmed* by policy which forces it to make truthful claims about initial price. When Q_2 is log-supermodular, the change from a *laissez-faire* to an honest regulatory regime will induce the firm to raise its true initial price but reduce its subsequent price. When Q_2 is log-supermodular, in the *laissez-faire* regime trusting consumers are offered a "false" discount, in that the true initial price is below the price they are offered.

We summarize this discussion in the following:

Proposition 2 *In the honest regime or the* laissez-faire *regime with savvy consumers, the firm offers second-period consumers a discount from the initial price. (In the* laissez-faire *regime with trusting consumers, the firm offers second-period consumers a discount from its* reported *initial price.) If the regulatory regime changes from* laissez-faire *to honest, the firm's initial price will rise, which harms the initial consumers. If consumers are savvy [trusting], the regime change increases [decreases] the firm's profit.*

We present a variant of this model in the Appendix. In this alternative model, all consumers are *ex ante* identical, rather than being exogenously apportioned into period-1 and period-2 consumers, and the firm sells simultaneously to all consumers. This allows

[11] As mentioned above, in practice it is unlikely that Q_2 will be increasing in $\bar{p}_1$ for arbitrarily large $\bar{p}_1$, and there will be no impact on second-period demand for reports above some finite threshold. This threshold can be taken to be the equilibrium claim made to trusting consumers in the *laissez-faire* regime.

the fraction of consumers who receive a discount to be endogenous, and also makes the more symmetric assumption that *all* consumers care about prices paid by others (not just the period-2 consumers in the model presented in this section). In the model, consumers take their reference price to be the (anticipated) average price offered by the firm, and a consumer is more likely to buy if she is offered a price which is below this average price. The firm responds to this "demand for bargains" by artificially engaging in price dispersion, and identical consumers are offered distinct prices.

In a *laissez-faire* regime the firm can make claims about its average price which need not be accurate, but when consumers are savvy they anticipate the firm's equilibrium average price. In this environment, offering a uniform price is not credible: if a consumer believes that all other consumers pay the price p^*, the firm can obtain greater profit from that consumer by offering her a discounted price below p^*. Subject to regularity conditions, the only pricing policy which is incentive compatible when individual price offers are privately observed by consumers is a "high-low" policy where a fraction of consumers are offered the regular price p^*, while the rest are offered a fixed discount. Nevertheless, unlike Coasian price dynamics, here the firm's inability to refrain from undercutting its uniform price does not cause it to lose profits, and the discounted price boosts demand by enough to leave profits unchanged. However, the firm can do strictly better than this in an honest regime where any claim it makes about its average price is required to be accurate.

5 Conclusion

This paper has explored some economic effects of discount pricing. Although there are surely others, we suggested two reasons why a discounted price—as opposed to a merely low price—may make a rational consumer more willing to buy. First, the information that the product was initially sold at a high price may indicate the product is high quality. Second, a higher initial price can indicate that the product is an unusual bargain, and that there is little point searching for alternative prices. We also discussed discount pricing with behavioural consumers, who exhibit reference-dependent preferences in the sense that they are more likely to buy a product at a given price if they believe that other consumers paid a higher price.

Three environments were considered: (i) an honest regulatory regime in which a firm's report of its past price was required to be truthful; (ii) a *laissez-faire* regime with savvy consumers, and (iii) a *laissez-faire* regime with trusting consumers. Because it faces fewest constraints on its pricing policy in (iii), but the tightest constraints in (ii), in our models the firm obtained greatest profits with (iii) and lowest profits with (ii). We saw that a regime change which requires honest price claims will cause the firm to raise its initial price: early consumers are "exploited" to deliver a larger discount to later buyers. In the model where initial price signalled quality, a move from a *laissez-faire* to an honest regime enabled the firm to offer a more efficient level of quality, and if consumers were trusting this policy shift also benefited them. In the model where a high initial price signalled that search was less worthwhile, in a numerical example this shift in regime (slightly) harmed consumers if they were savvy but helped them if they were trusting.

This paper focused on one way a seller can deliver a bargain, which was to offer a discount on a previous price. There are other ways to give an impression of value-for-money, which could perhaps be analyzed along similar lines. For instance, a seller might offer a quantity discount, say of the form of "buy 3 books for the price of 2" in a bookstore. Such a tactic might reflect a more standard motive to discriminate between those who want a single book and those willing to buy more, alongside a desire to make a purchase appear a bargain. However, a common tactic is the more extreme "buy one, get one free", where it is harder to imagine a traditional price-discrimination motive. (A devious version of this would be first to double the regular price, and then to stick a "2 for the price of 1" label on, which keeps the unit price the same but adds the appearance of a bargain.)

Another motive to offer a bargain is if the seller "must sell". For instance, a product in a supermarket might be approaching its sell-by date, and the seller might offer a genuinely low price to get rid of its stock. A label which states "must sell today" or a shop with a "closing down" sale sign, if credible, may induce consumers to buy without search (or to buy in greater volume then they would do otherwise). Such a tactic could operate without any reference to a previous higher price.[12] As usual, though, there is much scope for deception. Ehrlich (1990, page 43), in his account of the market for pianos in the nineteenth century, wrote: "Another common practice was the advertisement of new [poor

[12] See Anderson and Simester (1998) for a model with this flavour.

quality] instruments as second hand for individual enforced sale by a 'recently bereaved widow' or 'gentleman about to emigrate'. Some were genuine bargains, others meretricious rubbish, deliberately assembled and falsely labelled to impress the gullible."

Policy may be able to move the market from a *laissez-faire* to an honest regulatory regime by means of legal sanctions which act effectively to prohibit misleading price claims. Several jurisdictions have rules in place to combat misleading pricing.[13] In the United States, the Federal Trade Commission's *Guides Against Deceptive Pricing* (para. 233.1) distinguishes between genuine and fictitious discounts. For instance, "where an artificial, inflated price was established for the purpose of enabling the subsequent offer of a large reduction - the 'bargain' being advertised is a false one; the purchaser is not receiving the unusual value he expects. In such a case, the 'reduced' price is, in reality, probably just the seller's regular price."

Nevertheless, as Rubin (2008) observes, in recent years there have been few attempts by the FTC to enforce its guidelines, although individual States sometimes do so. Rubin suggests this is in part because most consumers are "savvy" and disregard claims made by a firm about its prices at other times or in other outlets. However, even with savvy consumers there may be benefits in a move to an honest regime; for instance, in our model where initial price signalled quality such a move helped a firm offer high quality. Perhaps the principal reason why regulators are often reluctant to combat deceptive pricing is that it is hard to enforce, or perhaps even coherently to formulate, policy towards misleading pricing. As Rubin notes, a basic problem is how to determine how few sales need to occur at the full price, or for how short a time the full price is available, for a sales campaign stating "was \$200, now \$100" to be classified as misleading.[14] Sellers have a strong motive to make their customers feel they are getting a special deal, and myriad ways to achieve this. It is unrealistic and undesirable to suppose that regulation can address all forms of

[13] Some jurisdictions also have policies to prevent "permanent sales" by requiring all sales to occur on stipulated dates. Thus the winter sales in Paris in 2012 had to take place between 11 January and 14 February.

[14] At the time of writing, the UK regulator, the *Office of Fair Trading*, has agreed new principles governing price claims with a number of prominent supermarkets. A key principle is that a discount of the form "was £X, now £Y" should be displayed for no longer than the original price of £X was originally displayed. See their press release "Eight supermarkets sign up to OFT principles on special offers and promotions" dated 30 November 2012.

false discounting without unduly restricting a seller's marketing abilities.

In any event, the potential benefit from policy which shifts from *laissez-faire* to an honest regime can be realized only if it is effectively enforced. Indeed, as Rubin notes, and as with many kinds of consumer protection policy, weakly enforced policy may be worse than no policy. If consumers are under the impression that it is illegal for a firm to make misleading claims, they abandon their usual caution and act on these claims. If the policy is not in fact enforced, consumers are thereby converted into "trusting" types and environment (iii), arguably the worst case, is implemented.

An alternative approach might be for a regulator to mount a publicity campaign which informs consumers that firms are in fact often able to present misleading claims without penalty. Such a policy, if effective, could convert trusting consumers into savvy types and so move from environment (iii) to (ii). If it is simply too hard, except in the most flagrant cases, to combat deceptive pricing directly, the next best thing may be to alert consumers to the presence of the Drubecks of the world.

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APPENDIX

Details of Uniform Example presented in Section 3. Consider an example where s is uniformly distributed on $[0, \frac{1}{2}]$ and P_1 is uniformly distributed on $[0,1]$, and where $P_2 \equiv P_1$. Then (8) implies that

$$q_1(p_1) = 1 - p_1^3$$

if $p_1 \leq 1$. (If $p_1 > 1$, then the consumer always investigates the alternative price and never returns, so $q_1 = 0$.) The price which myopically maximizes first-period profit $\pi_1 = p_1 q_1$ is $p_1 = 1/\sqrt[3]{4} \approx 0.63$. From the discussion in the text, in the two-period setting we need only consider initial prices above 0.63.

We have $F_2(P_2, P_1) = P_2/P_1$ if $P_2 \le P_1 \le 1$, and otherwise $F_2 = 1$. It follows from (10) that when $p_2 \le p_1 \le 1$ and $p_2 \le \bar{p}_1 \le 1$ we have

$$q_2(p_2, p_1, \bar{p}_1) = 1 - \frac{p_2^3}{p_1\bar{p}_1} \ ,$$

and the firm's profit is

$$\Pi = p_1(1 - p_1^3) + p_1^3 p_2 \left(1 - \frac{p_2^3}{p_1\bar{p}_1}\right) \ . \tag{17}$$

Consider first the case of an honest regulatory regime, so $\bar{p}_1 \equiv p_1$. From (17), for given p_1, p_2 is chosen to maximize $p_2(p_1^2 - p_2^3)$, which entails $p_2 = (\frac{1}{2}p_1)^{\frac{2}{3}}$. As long as $p_1 \ge 0.63$, the second period price is below the initial price. Substituting this value for p_2 into (17), and setting $\bar{p}_1 = p_1$, shows that this profit is maximized by setting $p_1 \approx 0.779$, and hence $p_2 \approx 0.533$.

With a *laissez-faire* regime, profit is as given in (17). With a savvy second-period consumer, in equilibrium we will have $p_1 = \bar{p}_1$, in which case the most profitable choice for p_2 is $p_2 = (\frac{1}{2}\bar{p}_1)^{\frac{2}{3}}$. Substituting this value of p_2 into (17) shows profit to be

$$\Pi = p_1\left(1 - p_1^3\right) + p_1^3\left(1 - \tfrac{1}{4}\frac{\bar{p}_1}{p_1}\right)\left(\tfrac{1}{2}\bar{p}_1\right)^{\frac{2}{3}} \ .$$

Maximizing this expression with respect to p_1, and setting $\bar{p}_1 = p_1$, shows that the equilibrium initial price is $p_1 \approx 0.760$, which induces second-period price $p_2 \approx 0.525$ and total profit 0.599.

If the second consumer is trusting, and believes any report of the initial price, it is clear that the firm has an incentive to claim its initial price was so high that no sale would ever take place, so that it claims its initial price was $\bar{p}_1 = 1$. Profit in (17) then becomes

$$p_1(1 - p_1^3) + p_1^3 p_2\left(1 - \frac{p_2^3}{p_1}\right) \ .$$

For given p_1, the most profitable choice of p_2 is $p_2 = (\frac{1}{4}p_1)^{\frac{1}{3}}$. One can check that the price pair which maximizes this expression is $p_1 \approx 0.776$ and $p_2 \approx 0.579$, which yields total profit of about 0.616.

What is consumer outlay in the three regimes? If the initial price is p_1, the first-period consumer will buy immediately at price p_1 if $s \ge \frac{1}{2}p_1^2$, she will return to buy later at total cost $p_1 + s$ if $s \le \frac{1}{2}p_1^2$ and $P_1 > p_1$, and she will buy from the alternative supplier at total cost $P_1 + s$ if $s \le \frac{1}{2}p_1^2$ and $P_1 < p_1$. Integrating over these three regions in (P_1, s)-space shows that expected outlay of the first-period consumer is

$$p_1(1 - p_1^2) + (1 - p_1)\int_0^{\frac{1}{2}p_1^2} 2(p_1 + s)ds + \int_0^{p_1}\left(\int_0^{\frac{1}{2}p_1^2} 2(P_1 + s)ds\right)dP_1 = p_1 - \tfrac{1}{4}p_1^4 \ . \tag{18}$$

(This differentiates to give $q_1(p_1)$, as we would expect.)

The calculation of the second consumer's outlay is more complex, as we need to separate the cases where the item remains unsold after the first period and where the item has already been sold. The item remains unsold when the first consumer's search cost satisfies $s \leq \frac{1}{2}p_1^2$ and when $P_1 < p_1$, which occurs with probability p_1^3. Conditional on the item being unsold at the end of the first period, suppose the consumer believes the initial price was $\bar{p}_1$ while the true initial price was p_1. The consumer therefore will buy immediately if $s \geq p_2^2/(2\bar{p}_1)$. In a similar manner to expression (18), one can show that the consumer's expected outlay is then

$$p_2(1-\frac{p_2^2}{\bar{p}_1}) + \frac{p_1 - p_2}{p_1}\int_0^{\frac{p_2^2}{2\bar{p}_1}} 2(p_2+s)ds + \frac{1}{p_1}\int_0^{p_2}\left(\int_0^{\frac{P_2^2}{2\bar{p}_1}} 2(P_2+s)ds\right)dP_2$$

$$= p_2 - p_2^4\left(\frac{1}{2p_1\bar{p}_1} - \frac{1}{4\bar{p}_1^2}\right) . \tag{19}$$

Notice that for given (p_1, p_2), this outlay is minimized by setting $\bar{p}_1 = p_1$. As one would expect, the consumer's outlay is minimized if her beliefs about the distribution of the alternative price coincide with the true distribution. Thus, the portion of second-period outlay generated by the event that the item remains unsold in the first period is p_1^3 multiplied by the expression (19). Consider next the event in which the item is sold in the first period. The consumer then has no search decision to make (although she still incurs the search cost). The portion of second-period outlay corresponding to the event in which the item is sold in the first period is

$$(1-p_1^2)(\frac{1}{2}+\frac{1}{4}) + p_1^2(1-p_1)(\frac{1+p_1}{2}+\frac{1}{4}) . \tag{20}$$

(Here, the first term represents the portion generated by the event in which the first consumer buys immediately, which occurs if that consumer's search cost satisfies $s \geq \frac{1}{2}p_1^2$, i.e., with probability $1 - p_1^2$. In this event, the second consumer pays the unconditional expected alternative price, which is $\frac{1}{2}$, plus the average search cost, which is $\frac{1}{4}$. The second term represents the event in which the first consumer does search, but returns to buy from the firm when $P_1 > p_1$. This occurs with probability $p_1^2(1-p_1)$, and then the expected alternative price is $\frac{1}{2}(1+p_1)$.) Putting these various portions together shows that the second consumer's outlay is

$$\frac{1}{4}(1-p_1)(3p_1+3p_1^2+2p_1^3+3) + p_1^3\left[p_2 - p_2^4\left(\frac{1}{2p_1\bar{p}_1} - \frac{1}{4\bar{p}_1^2}\right)\right] . \tag{21}$$

In the case where the consumer is not misled, so that $\bar{p}_1 = p_1$, this simplifies to

$$\tfrac{3}{4} - \tfrac{1}{2}p_1^4 + p_1^3 p_2 - \tfrac{1}{4}p_1^3 - \tfrac{1}{4}p_1 p_2^4 . \tag{22}$$

In the region $p_2 \leq p_1 \leq 1$, this outlay (22) is increasing in the second-period price, as one would expect, but is *decreasing* in the initial price. This reflects the fact that a high initial price implies that the item is more likely to be available to buy in the second period, which gives the second consumer more options.

Finally, consider the alternative supplier's revenue in the three regimes. Suppose that $p_2 < p_1 < 1$, and consider a particular realization of the alternative price P. If $P \geq p_1$, the alternative supplier will never sell in the first period, and so always sell in the second, and so will always sell exactly one unit. If $p_2 \leq P \leq p_1$, the supplier will sell its product in the first period if $s \leq \frac{1}{2}p_1^2$, i.e., with probability p_1^2. If it does sell in the first period, it will certainly not sell in the second (since $p_2 \leq P$ and the item remains available at the firm). If the supplier does not sell in the first period (i.e., the firm does sell), the item is no longer available at the firm and so the supplier certainly sells. In sum, if $p_2 \leq P \leq p_1$ the supplier sells exactly one unit. Finally, consider the case $P \leq p_2$. Again, the supplier will sell its product in the first period with probability p_1^2. If it does sell in the first period, it will sell again in the second if $s \leq p_2^2/(2\bar{p}_1)$, which occurs with probability $p_2^2/\bar{p}_1$. If the supplier does not sell in the first period (i.e., the firm does sell), the item is no longer available at the firm and so the supplier certainly sells. In sum, if $P \leq p_2$, the number of units supplied by the alternative source is

$$p_1^2(1 + \frac{p_2^2}{\bar{p}_1}) + 1 - p_1^2 = 1 + \frac{p_1^2 p_2^2}{\bar{p}_1} \ .$$

Integrating over P shows that the expected revenue of the alternative supplier is

$$\left[1 + \frac{p_1^2 p_2^2}{\bar{p}_1}\right] \int_0^{p_2} PdP + \int_{p_2}^1 PdP = \frac{1}{2} + \frac{p_1^2 p_2^4}{2\bar{p}_1} \tag{23}$$

Substituting the prices p_1 and p_2 (and, where applicable, the firm's claim about its initial price $\bar{p}_1$) into expressions (18)-(23) in the three regimes yields the figures for consumer outlay and alternative supplier revenue reported in Table 1 in the text.

Selling to bargain-loving consumers: In this appendix we describe a variant of the model of reference dependence presented in section 4. Here, the firm sells to a single group of consumers in a static interaction. Unlike most of the recent papers in industrial organization which focus on loss-aversion, we suppose consumers enjoy a benefit if they pay a price below the reference price but no loss if they encounter a non-bargain price.[15]

[15] Jahedi (2011) experimentally investigates a kind of bargain which we do not study in this paper, where a seller offers two units of its product for little more than the price of one unit. He shows how consumers are *less* likely to buy two units when faced with the choice from {buy nothing, buy two units for \$1} than they are when faced with the larger choice set {buy nothing, buy one unit for \$0.97, buy two units for

In our model, a consumer's reference price is simply her anticipated average price offered by the firm.

A monopolist, which has costless production, sells to a unit mass of consumers. The firm chooses its price according to a mixed strategy with c.d.f. $G(p)$ which has expected value denoted P. (The firm can offer a deterministic, or uniform, price as a special case.) As in Heidhues and Kőszegi (2012), we assume that an individual consumer is offered a single price, and cannot search for additional prices. For instance, the firm makes its price contingent on some arbitrary aspect of the consumer (e.g., location) which cannot easily be altered. If the consumer is offered a price above the expected offered price, so $p \geq P$, suppose that she buys with probability $Q(p)$. However, if she is offered a bargain price $p \leq P$, suppose her probability of purchase is $q(p, P)$. The demand function $q(p, P)$ is defined in the region $p \leq P$, and suppose that $q(p, p) \equiv Q(p)$ so that demand is continuous in the anticipated average price P. Suppose that $Q(p)$ and $q(p, P)$ are strictly decreasing in p in the usual way, while $q(p, P)$ strictly increases with P. Since $q(p, p) \equiv Q(p)$ and q is strictly increasing in its second argument, it follows that q is "more decreasing" in its first argument than Q; formally, we assume that

$$\left.\frac{\partial q(p, P)}{\partial p}\right|_{P=p} < Q'(p) \ . \tag{24}$$

In other words, demand has an "inward kink" at the point $p = P$. Suppose that profit $pQ(p)$ is single peaked in price, and maximized at $p = p^*$. Let $\pi^* = p^* Q(p^*)$ denote the maximum profit available with uniform pricing.

We first investigate the outcome when prices are secret, in the sense that a consumer observes only the price offered to her but not the prices offered to others, and she holds equilibrium beliefs about the average price offered to the population as a whole. This situation is analogous to a *laissez-faire* regime with savvy consumers.[16] In this case we have the following result.[17]

Proposition 3 *Suppose consumers observe only their own price, but form rational expectations about the average price. Then (i) offering a uniform price is not an equilibrium;*

$1}. Jahedi designs the experiments so that subjects know that prices have no signaling role (such as the signaling roles we analyze in our models), and deduces that some of his subjects have an intrinsic "taste for bargains", as we assume in this model. Spiegler (2011, section 9.4.2) briefly outlines a related model to the one presented here, although his construction perhaps uses implausibly high prices (higher than any consumer's raw valuation for the product).

[16] Here, we assume consumers have "passive beliefs" about the average price, and the price p a consumer is offered does not alter her beleifs about the average price P.

[17] In formal terms, this result resembles the analysis in Zhou (2011). Like us, he finds that a seller faces demand with an inward kink and chooses prices according to a mixed strategy with exactly two prices; in his case, the prominent seller uses "sales" to influence a loss-averse consumer's reference point when she evaluates the rival offer, while our firm uses "sales" to satisfy a consumer's demand for bargains.

(ii) in any equilibrium the firm obtains profit π^*, the same profit it obtains with uniform pricing; (iii) an equilibrium exists in which the firm offers two prices, a regular price p^* (the most profitable uniform price) and a discounted price p_B^*, and (iv) if the demand function $q(p.P)$ is log-concave in p, this "high-low" pricing equilibrium is unique.

Proof. (i) We first show that a uniform price cannot be an equilibrium. If to the contrary P is an equilibrium uniform price, anticipated by consumers, the firm cannot make greater profit by offering a consumer a bargain price $p < P$, i.e.,

$$Q(P) + P\frac{\partial q(P,P)}{\partial p} \geq 0 \ ,$$

and neither can it make greater profit by offering her a price $p > P$, i.e.,

$$Q(P) + PQ'(P) \leq 0 \ .$$

However, these inequalities are inconsistent given (24).

(iii) We construct a "high-low" pricing equilibrium as follows. Suppose consumers anticipate average price P. If the firm chooses a price strictly above P, this price p must locally maximize $pQ(p)$, and since this profit is single-peaked there is at most one such price, which is $p = p^*$. Since the firm must be indifferent between any price it charges in equilibrium, we deduce immediately that in any equilibrium the firm obtains profit π^*, the same profit it obtains with uniform pricing. (This proves part (ii).)

Let

$$\pi_B(P) = \max_{p \leq P} : pq(p,P) \tag{25}$$

be the maximum profit available from a consumer when she anticipates the average offered price is P and is offered a bargain price $p \leq P$, and let $p_B(P) \leq P$ be the solution to problem (25). If the firm follows a price policy such that

(a) it offers a fraction α of consumers the "regular" price p^*;
(b) it offers the remaining $1-\alpha$ consumers the discounted or bargain price $p_B^* < p^*$, so that the average offered price is $P^* = \alpha p^* + (1-\alpha)p_B^*$;
(c) $p_B^* = p_B(P^*)$, and
(d) $\pi_B(P^*) = \pi^*$,

then this comprises an equilibrium. To understand this, note that (a) implies the firm offers the price p^* which maximizes $pQ(p)$, i.e., the profit subject to offering an above-average price, (c) implies the firm's low price maximizes profit $pq(p,P^*)$ subject to offering a below-average price, where average price P^* is given in (b), and finally (d) implies that the firm is indifferent between offering a consumer the regular price p^* or the discounted price p_B^*, and so is willing to play a mixed strategy which offers a price p^* to some consumers and p_B^* to others.

We next demonstrate that such an equilibrium exists. Note that $\pi_B(0) = 0$ and $\pi_B(P)$ strictly increases with P. Note that the function of p given by $pq(p, p^*)$ is strictly decreasing at $p = p^*$, due to (24) and the fact the p^* maximizes $pQ(p)$. It follows that $\pi_B(p^*) > \pi^*$. Note that if $\pi_B(P) \geq \pi^*$, we must have $p_B(P) < P$. (This is because if $p_B(P) = P$ then $\pi_B(P) = PQ(P)$, and so if $P \neq p^*$ this contradicts the requirement that $\pi_B(P) \geq \pi^*$. And we have already established that if $P = p^*$ then $p_B(P) < P$.) Putting these results together implies that there is a unique $P^* < p^*$ which satisfies $\pi_B(P^*) = \pi^*$, and this price P^* also satisfies $p_B(P^*) < P^*$. If we write $p_B^* = p_B(P^*)$ and choose α so that $P^* = \alpha p^* + (1-\alpha)p_B^*$, then all requirements (a)-(d) are satisfied.

(iv) To establish uniqueness, we argue as follows. We know from part (i) that any equilibrium involves at least two prices being offered. Therefore at least one price is strictly above the average price and one is strictly below. The above argument shows that an above-average price must equal p^*. If P is the average offered price in some equilibrium, then any below-average price must maximize $pq(p, P)$, thus generating profit $\pi_B(P)$. To make the firm indifferent between choosing a below-average price and an above-average price, we must have $\pi_B(P) = \pi^*$ which uniquely determines the average price. Finally, if $q(p, P)$ is log-concave in p, then profit $pq(p, P)$ is single-peaked in p, and hence there can be only one price that solves problem (25). This completes the proof. ■

Part (i) of this result reveals that when prices are secret, the firm is unable to implement a uniform price. If a consumer believes that all other consumers pay the price p^*, the firm obtains more profit from that consumer by offering her a discounted price. Subject to regularity conditions, the only pricing policy which is incentive compatible when prices are secret is a "high-low" policy where a fraction of consumers are offered the regular price p^*, while the rest are offered a discount.

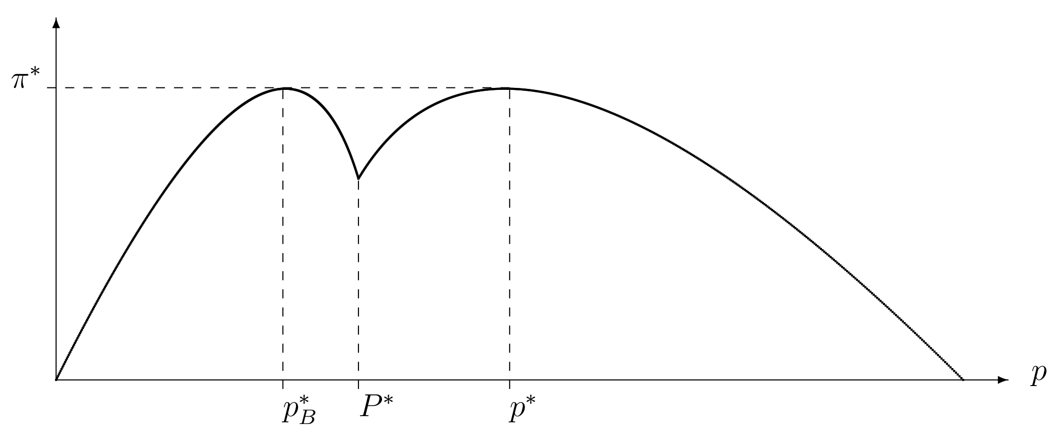

Figure 1: Expected profit when firm offers price p to a consumer

The construction of the "high-low" pricing equilibrium is depicted on Figure 1. The firm's average offered price is P^*, and if the firm offers a non-bargain price $p \geq P^*$, the firm's profit is $pQ(p)$, while if the firm offers a bargain price $p \leq P^*$ its profit is $pq(p, P^*)$. The respective peaks of these two profit functions are found at $p = p^*$ and $p = p_B^*$. The equilibrium is constructed so that the heights of the two peaks are equal (and equal to π^*), so that the firm is indifferent between choosing these two prices (which are strictly preferred to any other prices).

Consider an example with linear demand $Q(p) = 1 - p$ and $q(p, P) = 1 - p + \lambda(P - p)$, where $\lambda \geq 0$ is a parameter which reflects the strength of "preference for bargains". Here, the most profitable uniform price is $p^* = \frac{1}{2}$, which is therefore the equilibrium regular price. Over the relevant range, the function (25) is $\pi_B(P) = \frac{1}{4}\frac{(P\lambda+1)^2}{\lambda+1}$, and $p_B(P) = \frac{1+\lambda P}{2(1+\lambda)}$. Therefore, the condition $\pi_B(P^*) = \pi^* = \frac{1}{4}$ implies

$$P^* = \alpha = \frac{1}{1 + \sqrt{1 + \lambda}} \; ; \; p_B^* = \frac{1}{2\sqrt{1 + \lambda}} \; .$$

The firm's profit is $\pi^* = \frac{1}{4}$, regardless of λ, and this is also the profit achievable if the firm could commit to set a uniform price. Average price P^* falls when λ is larger, but the demand boost from bargain-loving consumers when λ is larger exactly off-sets this. When $\lambda = 0$ there is no price dispersion, and all consumers are offered the same price $p^* = \frac{1}{2}$. When $\lambda = 3$, though, two-thirds of consumers are offered the bargain price $p_B^* = \frac{1}{4}$, which is a 50% discount from the regular price.

We can compare this outcome to the situation with public prices, where the firm accurately reveals its entire price distribution to each consumer. In particular, consumers know the average price P, which, together with their own price, is what they care about. This situation is akin to an honest regulatory regime. As was the case with secret prices, it cannot be an equilibrium to offer a uniform price:

Lemma 7 *When consumers can observe the firm's price policy, the firm prefers to offer dispersed prices than to offer a uniform price*

Proof. Let $p > 0$ be any uniform price. Suppose the firm deviates from this uniform price by offering two prices, $p - \varepsilon$ and $p + \varepsilon$, where each price is offered to half the consumer population. (This policy leaves the average offered price unchanged at p.) The firm's profit with this new policy is

$$\pi(\varepsilon) \equiv \tfrac{1}{2}(p + \varepsilon)Q(p + \varepsilon) + \tfrac{1}{2}(p - \varepsilon)q(p - \varepsilon, p) \; .$$

Differentiating this expression with respect to ε shows that

$$\pi'(0) = \tfrac{1}{2}p\left\{Q'(p) - \frac{\partial q(p,p)}{\partial p}\right\} > 0 \; ,$$

where the inequality follows from (24). Thus, starting from any uniform price, the firm's profit is increased by implementing a small mean-preserving spread in its prices. ■

Thus, the presence of bargain-loving consumers gives the firm an incentive to offer distinct prices to otherwise identical consumers: in order to satisfy a "demand for bargains", the firm creates bargains by artificially dispersing its prices. We deduce that the firm is better off when it can accurately publicize its entire price policy relative to the case where it makes secret deals to individual consumers. Regulatory policy which forbids false price claims enables the firm to credibly reveal its pricing policy, and so helps the firm.

The optimal pricing policy with public prices is potentially complex, and we do not pursue it in detail here. Formally, the firm chooses its distribution for prices, $G(p)$, in order to maximize its profit

$$\int_{p\leq P} pq(p,P)dG(p) + \int_{p\geq P} pQ(p)dG(p)$$

subject to the requirement that

$$P = \int pdG(p) \ . \tag{26}$$

If one is prepared to assume that profit $pQ(p)$ was strictly concave and profit $pq(p,P)$ was strictly concave in p for relevant P, then the optimal policy consists of exactly two prices, just as in Proposition 3. For instance, if there were more than one price which was above the average, then Jensen's Inequality implies that profits increase if these above-average prices were replaced by their average. Doing this does not change the overall average price P, and so does not affect demand from those consumers given a bargain. With the strong assumption of concave profit, then, it is a straightforward matter to calculate the optimal price policy.

Finally, in a *laissez-faire* regime where consumers are trusting, the firm's profits are increased when it is able to make misleading claims. It can then obtain the benefit of boosting demand from perceived "bargains" without the cost of sometimes having to set inefficiently high prices. It would like to claim average price P was high, so that it could then set high actual prices without cutting demand. That is to say, constraint (26) need not be imposed in this situation. With trusting consumers, the welfare impact of a policy banning false discounts is complicated, and depends on how one views a consumer's utility from getting a "false bargain".