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Luis Araujo and Bernardo Guimarães

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Luis Araujo, Michigan State University and Fundação Getulio Vargas
Bernardo Guimarães, Fundação Getulio Vargas and CEPR

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

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This paper studies how constraints on the timing of actions affect equilibrium in intertemporal coordination problems. We show that while the possibility of waiting longer for others'' actions helps agents to coordinate in the good equilibrium, the option of delaying one's' actions harms coordination and can induce severe coordination failures: if agents are very patient, they might get arbitrarily low expected payoffs even in cases where coordination would yield arbitrarily large returns. The risk-dominant equilibrium of the corresponding one-shot game is selected when the option to delay effort is commensurate with the option to wait longer for others'' actions. In an application to innovation processes, we show that protection of the domestic industry might hinder industrialization. We also argue that increased competition might have spurred the emergence of shadow banking in the last few decades.

JEL Classification: C72, C73 and D84
Keywords: coordination failures, delay, option and strategic complementarities

Luis Araujo
Department of Economics
Michigan State University
220A Marshall-Adams Hall
East Lansing, MI 48824
USA

Email: araujolu@msu.edu

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Bernardo Guimarães
São Paulo School of Economics
Fundacao Getulio Vargas
Rua Itapeva, 474
Sao Paulo
BRAZIL

Email: bernardo.guimaraes@fgv.br

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=160881

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The effect of options on coordination*

Luis Araujo† Bernardo Guimaraes‡

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Abstract

This paper studies how constraints on the timing of actions affect equilibrium in intertemporal coordination problems. We show that while the possibility of waiting longer for others' actions helps agents to coordinate in the good equilibrium, the option of delaying one's actions harms coordination and can induce severe coordination failures: if agents are very patient, they might get arbitrarily low expected payoffs even in cases where coordination would yield arbitrarily large returns. The risk-dominant equilibrium of the corresponding one-shot game is selected when the option to delay effort is commensurate with the option to wait longer for others' actions. In an application to innovation processes, we show that protection of the domestic industry might hinder industrialization. We also argue that increased competition might have spurred the emergence of shadow banking in the last few decades.

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1 Introduction

Coordination problems resulting from strategic complementarities are pervasive in economics. This paper focuses on intertemporal coordination problems, in which an agent's payoff depends on the future behavior of other agents. For instance, when deciding whether to innovate and create a new technology, a firm may consider that its eventual benefits critically depend on whether other firms will use this technology as a platform for future innovations. In intertemporal coordination problems,

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†Michigan State University and Sao Paulo School of Economics – FGV.

‡Sao Paulo School of Economics – FGV.

the possibility of delaying one's actions and the possibility of waiting longer for someone else's action may affect the equilibrium outcome. Building on the example above, if given the option, a firm may postpone the decision to innovate if it seems that it may take a long time for complementary future innovations to appear. In another direction, if the innovation does not depreciate over time, a firm may choose not to postpone the decision to innovate as it does not matter much when complementary future innovations will actually appear. Our objective is to provide a model which captures these effects, helping us to understand what affects coordination and the effects of policies.

The model works as follows. Consider an economy inhabited by two agents, an active agent and a passive agent. The passive agent is inactive, while the active agent has to choose between exerting effort in the current period and delaying effort for a later period, which includes the possibility of never exerting effort. If he chooses effort, he incurs a sunk cost and becomes a passive agent, while the passive agent receives a benefit and leaves the economy. Moreover, a newly born active agent enters the economy. If he chooses to delay effort, he faces an exogenous probability of being replaced by a newly born active agent, and the passive agent faces an exogenous probability of being replaced by a newly born passive agent. The former probability captures the "option" of delaying effort, while the latter probability captures the "option" of waiting for someone's action after exerting effort.[1] All variables in the model are common knowledge.

If costs and benefits are fixed, then as long as the present value of receiving the benefit in the next period is larger than the cost of exerting effort in the current period, this benchmark economy exhibits multiple equilibria: an equilibrium in which all active agents (i.e., the current active agent and all future active agents) always exert effort, and an equilibrium in which all active agents never exert effort. Thus equilibrium selection is completely driven by self-fulfilling beliefs. In particular, since the current active agent either believes that all future active agents will always exert effort or will never do so, options do not play any role. Indeed, options matter only if the current active agent believes that the behavior of active agents in the near future will be somewhat different from their behavior in the more distant future.

We then assume that the economy experiences different states, which evolve according to a random

[1] Even though we use the term option to characterize both possibilities, it is clear that, while the option of delaying an action is a choice of the agent, the option of waiting for someone's action is an opportunity, which depends on another agent's choice.

walk and are such that the cost of exerting effort is increasing in the current state. In a large region of states (the region of interest), the present value of receiving the benefit in the next period is larger than the cost of exerting effort in the current period. In this region, the decision of the current active agent depends on his belief about the behavior of future active agents. However, there exist faraway states in which it is strictly dominant to always exert effort and faraway states in which it is strictly dominant not to do so.

Our first main result is that there exists a unique symmetric equilibrium in cut-off strategies: the current active agent exerts effort if and only if the current state (thus, the current cost) is below a critical level. The equilibrium depends on the option to delay effort and the option to wait for someone's effort. It also depends on the properties of the random process which governs the evolution of the state. In particular, there exists a negative relation between the variance of the random process and the cut-off state. This negative relation captures the real-options effect (e.g., Bloom (2009)): an increase in uncertainty (variance) leads to a reduction in the likelihood that investment (effort) will be exerted because it increases the value of postponing effort. However, unlike the standard real-options effect, investment is not necessarily restored once the variance of the random process converges to zero. This is so because the decision not to exert effort is also the result of a coordination failure, i.e., an agent may opt to delay effort because he expects that future agents will also do so.

Our second main result provides an explicit characterization of the equilibrium when the variance of the random process converges to zero. The obtained cut-off can be seen as an upper bound to the cost of effort that active agents are willing to incur. The equilibrium shows a causal link between options and the equilibrium outcome. The cut-off state is decreasing in the option of delaying the effort decision and increasing in the option of waiting for someone's effort. A higher probability that the current active agent will be replaced if he chooses not to exert effort leads to a larger region of states in which active agents choose effort. In turn, a higher probability that the current passive agent is replaced if the current active agent chooses no effort shrinks the region of states in which active agents choose effort.

We then show that the option to delay effort can induce severe coordination failures. In particular, an agent might have a payoff arbitrarily close to 0 even in a region where the cost of exerting effort is zero and the benefit is positive (hence the return to effort is infinite). That will happen if agents are patient enough, the chances that an active agent will be replaced are small enough and the chances that a passive agent will be replaced are not so small. In this case, effort is only exerted in the region of states in which it is strictly dominant to do so. Intuitively, if the current active agent is patient and knows there is a high probability that he will have an opportunity to exert effort in a later period, he will only exert effort when he attributes a large probability to the future active agent exerting effort as well. The problem is that a similar reasoning will be made by all future active agents. Thus, beliefs that sustain a decision not to postpone effort cannot arise in equilibrium.

In the opposite direction, we obtain that if the probability with which a passive agent can wait for someone's effort is equal to one, then, as long as agents are patient enough, effort is always exerted whenever it is efficient to do so – as long as the probability with which an active agent can delay effort is not equal to one. Intuitively, if the option to wait as a passive agent is safer than the option to wait as an active agent, a patient agent has incentives to exert effort soon, knowing that eventually he will reap the benefits of this effort. Since all active agents make the same reasoning, effort is exerted whenever the cost of doing so is below the present value of receiving the benefit in the next period.

Our model is not build to match the specificities of any particular situation in which options affect coordination, but there are many instances to which it can be applied. We provide two applications at the end of the paper. First, we study how options affect industrialization in a setup where coordination matters.[2] In this context, an active agent has the option between choosing a new technology (say, industry) and keeping the old one (say, agriculture). We think of the option to delay choosing a new technology as capturing the absence of potential competition (competition means that a newly born active agent is likely to enter the economy in the following period and take the place of the current active agent). In turn, we think of the option to wait longer for someone' decision to choose a new technology as capturing either the non-depreciation of the technology, or the protection of property rights in the economy. One implication of our model is that the sheer ability of postponing innovation hinders coordination on an innovative path. This offers a novel, policy-based explanation as to why coordination on innovation might happen in some countries but not in others. Second, we apply our model to understand the emergence of shadow banking. While the link between the collapse of shadow banking and the onset of the great recession is relatively well understood, the emergence of the shadow banking system itself is much less well understood (Gorton (2010)). We argue that options

[2]For models of how industrialization depends on coordination, see e.g. Kiyotaki (1988), Murphy, Shleifer and Vishny (1989), Matsuyama (1991) and Ciccone (2002).

and coordination offer a promising route towards understanding this phenomenon.

Our model relates to the literature of equilibrium selection in coordination games, which comprises coordination games with incomplete information (global games)[3] and dynamic games with frictions that prevents agents from changing strategy at every moment.[4] A result which often emerges is that behavior in equilibrium follows a simple rule: agents choose the risk-dominant action of the one-shot coordination game.[5] In our economy, we do not have a standard one-shot game. However, in the particular case in which options play no role, i.e., effort cannot be delayed and a passive agent cannot wait for more than one period for the effort decision of the current active agent, our model can be interpreted as a sequence of one-shot coordination games between pairs of active agents: the active agent in the current period chooses between effort and no effort given his belief about the behavior of the future active agent he will be paired with. In this particular case, the optimal decision of an active agent coincides with the risk-dominant action of the underlying one-shot coordination game. Interestingly, when the variance of the random process goes to zero, this result can be generalized: the optimal decision of an active agent coincides with the risk dominant action of the underlying one-shot coordination game if and only if the probability that the current active agent is replaced by a newly born active agent is exactly equal to the probability that the current passive agent is replaced by a newly born passive agent. This result holds irrespective of the discount factor.[6]

The option of investing later provides an outside option for an active agent that chooses not to exert effort. In this sense, the paper is related to models that have explored the endogeneity of the outside option in coordination games. In Steiner (2008a), the expectation of successful coordination tomorrow undermines successful coordination today, which leads to coordination cycles. In Steiner (2008b), labor mobility increases workers' outside option and hence might hinder coordination between them. Chassang (2010) considers a dynamic global game with the possibility of exit, which restores

[3] See, e.g., Carlsson and van Damme (1993) and Morris and Shin (2003).

[4] See, e.g., Frankel and Pauzner (2000) and Burdzy, Frankel and Pauzner (2001).

[5] A general result in one-shot global games is that, if an action is part of a risk-dominant equilibrium, then for a sufficiently small amount of incomplete information, this action is the unique action that survives iterated elimination of strictly dominated strategies (Morris and Shin (2003)). Burdzy, Frankel and Pauzner (2001) consider a dynamic game with complete information and with frictions. They show that, as frictions vanish, each player ignores the behaviof of other players and switch to the risk-dominant action.

[6] A corollary to this result is that when the variance of the random process is bounded away from zero and the probabilities of being replaced are the same for the active and passive agents, coordinating on effort is more difficult than in a static game owing to the real option effect.

multiple equilibria to the model. Kovac and Steiner (forthcoming) study the strategic consequences of reversibility of actions and show that it might enhance or hamper efficient coordination. Araujo and Guimaraes (2012) analyze a fundamental model of money, and show that the option of accepting and spending money in future periods affects how agents coordinate in the use of money. Last, the paper is also related to work providing alternative explanations for why agents inneficiently delay exerting effort, either because of more hazard (Bonatti and Horner (2011)) or because their preferences are time inconsistent (O'Donoghue and Rabin (1999)).

Our paper proceeds as follows. In the next section, we present the environment and characterize the equilibrium. In section 3, we describe the causal link between options and coordination. In section 4, we discuss two applications of the model, and in section 5, we conclude. Proofs omitted in the main text are presented in the appendix.

2 Model

2.1 Environment

Time is discrete and the discount factor is $\beta \in (0,1)$. The economy is populated by two agents, labeled active and passive. In every period, the active agent chooses between effort (e) and no effort (n), while the passive agent does not make any decision. If the active agent chooses effort, he incurs a cost c and becomes a passive agent. In turn, the passive agent receives a benefit b and is replaced by a newly born active agent. If the active agent chooses no effort, with probability $p_0 \in [0,1]$ he continues as an active agent, and with probability $1-p_0$ he is replaced by a newly born active agent. In turn, the passive agent receives no benefit and with probability $p_1 \in [0,1]$ he continues as a passive agent (with probability $1-p_1$ he is replaced by a newly born passive agent).

In every period, the economy is in some state $z \in \mathbb{R}$. States evolve according to a random process $z_t = z_{t-1} + \Delta z_t$, where Δz_t follows a continuous probability distribution that is independent of z and t with probability density $f(\Delta z)$. The process for Δz is symmetric around 0 and non-degenerate, hence $f(\Delta z) = f(-\Delta z)$, $E(\Delta z) = 0$ and $var(\Delta z) > 0$. Moreover, $f(\Delta z_1) \leq f(\Delta z_2)$ for all $\Delta z_1 > \Delta z_2 > 0$. This last condition is satisfied by the usual symmetric distributions, including normal and uniform. The benefit b is constant across states, but the effort cost c depends on the current state of the economy in the following way: there exists z_0 such that $c(z) = 0$ for $z \leq z_0$, and $c(z)$ is increasing and weakly convex for $z > z_0$, that is, $c' > 0$ and $c'' \geq 0$. This implies that there exists $z_H(\beta)$ such that $c(z_H(\beta)) = \beta b$. Finally, there exists $z_L < z_0$ such that, for all $z < z_L$, if the economy is in state z, the passive agent receives the benefit b regardless of the behavior of the active agent.

2.1.1 An interpretation of p_0 and p_1

This model deals with coordination problems of an intertemporal nature. When an agent chooses whether to exert effort or not, he is concerned about how the future behavior of other agents will impact his continuation payoff after he has chosen effort. In such a situation, the possibility of delaying one's action (as captured by p_0) and the possibility of waiting longer for someone's action (as captured by p_1) may have an important impact on the equilibrium outcome. We now discuss what the key parameters p_0 and p_1 might represent.

A quasi-rent is the expected return obtained by a sunk investment, which arises due to a temporary scarcity of the good or skill produced by the investment. In our environment, the sunk investment is the effort implemented by the active agent and the passive agent enjoys a quasi-rent, which is the expectation of receiving benefit b at some point in the future. As time evolves, there is a probability $1-p_1$ that the passive agent is replaced, in which case the quasi-rent is completely lost. One interpretation is that initially, the passive status is completely scarce, there is no passive agent in the economy but for the agent himself. The extent to which quasi rents may be enjoyed depend on the time span under which the skill produced by the sunk investment will continue to be scarce. In a real economy, this critically depends on the institutions in place, particularly patent and property rights. In the absence of such institutions, free-entry of agents with the ability to duplicate the scarce skill will no longer render it scarce, undermining its value. We think of p_1 as capturing the presence of such institutions in the economy. We can also think of $1 - p_1$ as the depreciation of the skills generated by the effort of the active agent. In this case, if p_1 is small, quasi rents are temporary not because the skills are no longer scarce, but because the skills are not permanent.

The quasi-rent is a result of a sunk investment from the active agent. One interpretation is that by exerting effort, the active agent precludes the establishment of potential competitors – which is termed preemptive competition. The threat of potential entrants is captured by p_0. A low value of p_0 implies that the active agent fears losing the possibility of moving first in the market, while a large p_0 can be seen as representing a low possibility of competitors entering the market, perhaps owing to some protection from the laws. In sum, p_0 might be seen as capturing the lack of competition from potential entrants. This idea is formalized in the Appendix A.1. Note that there is no contemporaneous competition between active agents in the model, there is only one active agent in the economy at a point in time.

The quasi-rent depends not only on p_1 but also on whether future active agents are expected to exert effort. If expectations regarding actions of future agents depend on the state of the economy, the benefit from exerting effort will depend on how the economy evolves. Exerting effort thus entails an opportunity cost, since it precludes the active agent from the option of exerting effort in the future. The value of the option of exerting effort in the future crucially depends on p_0.

2.2 Equilibrium

2.2.1 Benchmark case: $var(\Delta z) = 0$

Consider the case when $var(\Delta z) = 0$. If $z < z_L$, it is strictly dominant to exert effort, since effort entails no cost and the agent will obtain the benefit b irrespective of the behavior of the other agents. In turn, if $z > z_H(\beta)$, it is strictly dominant not to exert effort. Finally, if $z \in [z_L, z_H(\beta)]$, since $\beta b \geq c(z)$, there exists multiple equilibria. There exists a no-coordination equilibrium, in which active agents never exert effort and a coordination equilibrium in which active agents always exert effort. Importantly, both the coordination and the no-coordination equilibria do not depend on the values of p_0 and p_1.[7]

2.2.2 General case: $var(\Delta z) > 0$

Now consider the case when $var(\Delta z) > 0$. The main result in this section (Proposition 1) is that there is a unique symmetric equilibrium in cut-off strategies (generically): there exists a state z^* such that an active agent exerts effort if and only if $z < z^*$. By restricting attention to symmetric stationary equilibria in cut-off strategies, we are not imposing that an active agent has to follow a cut-off rule, we allow for deviations where an active agent chooses a strategy that is not of a cut-off type.

[7] In the region $z \in (z_0, z_H(\beta))$,there also exists a mixed strategy equilibrium in which active agents are indifferent and always exert effort with probability $\frac{c(z)}{\beta b} \frac{1-\beta p_1}{1-\frac{c(z)}{b} p_1}$. In this equilibrium, the probability of effort does not depend on p_0 but is decreasing in p_1.

The payoff of an active agent depends on the cost of exerting effort and on the action of future active agents. In a symmetric equilibrium in cut-off strategies, an active agent believes all future active agents will follow a cut-off rule. Hence the payoff of an agent does not depend on past states of the economy or on the time t.[8] Thus there is no further loss in generality in considering that agents' decisions depend only on the current state of the economy, so a strategy of an active agent is given by a mapping from the set of states z to the set of actions (effort and no effort).

The proof of Proposition 1 is divided in four Lemmas and is presented in the Appendix A.2. In what follows, we offer a brief discussion of the main steps involved in the proof.

First, we need to introduce some notation. Suppose the economy is at state z at time s. For $t > 0$ and $x > 0$, define $\phi(x,t)$ as the probability density that the economy will be at a state smaller than z at time $s+t$ but not before and that at time $s+t$ the economy will be at state $z-x$. Note that

$$\int_0^{\infty}\left(\sum_{t=1}^{\infty}\phi(x,t)\right)dx = 1$$

In words, since the process for z is symmetric, eventually the economy will be at a state smaller than z. We then define the function Γ_{0x} as the sum of probability densities that the economy will be at the state $z-x$ the first time it reaches a state smaller than z discounted by $(\beta p_0)^t$.

$$\Gamma_{0x} = \sum_{t=1}^{\infty}(\beta p_0)^t\,\phi(x,t).$$

Now, let $V_e(z_1, z_3)$ be the expected payoff from exerting effort in state z_1 if future active agents are following a cut-off rule at state z_3. We have

$$V_e(z_1, z_3) = -c(z_1) + \beta\int_{-\infty}^{\infty} f(w)V_1(z_1+w, z_3)dw, \tag{1}$$

where $V_1(z_1+w, z_3)$ is the expected payoff of being a passive agent in state z_1+w, given that future active agents are following a cut-off rule at state z_3. In turn, let $V_n(z_1, z_2, z_3)$ be the expected payoff from not exerting effort in state z_1, and exerting effort in the following periods if and only if $z < z_2$, given that future active agents are following a cut-off rule at state z_3. We obtain

$$V_n(z_1, z_2, z_3) = \int_{z_1-z_2}^{\infty}\Gamma_{0x}\left(-c(z_1-x) + \beta\int_{-\infty}^{\infty} f(w)V_1(z_1-x+w, z_3)\,dw\right)dx. \tag{2}$$

We are now ready to state Proposition 1.

[8] Here it is important that the process for Δz is independent of t and past realizations of Δz.

Proposition 1 *There exists a unique symmetric equilibrium in cut-off strategies. Agents exert effort if and only if $z < z^*$, where z^* solves*

$$V_e(z^*, z^*) = V_n(z^*, z^*, z^*).$$

Proof. *See Appendix A.2.* ■

Let $\Delta v(z) \equiv V_e(z, z) - V_n(z, z, z)$. In Lemma 3, we show that $\Delta v(z)$ is strictly decreasing in $z \leq z_L$, is constant in $z \in [z_L, z_0]$, and is strictly decreasing in $z \geq z_0$. Now, the assumption that the passive agent receives the benefit b in states $z < z_L$ regardless of what the active agent does, implies that $\Delta v(z)$ converges to a positive number when z is small. In turn, the assumption that $c(z)$ is increasing in $z > z_0$ implies that $\Delta v(z)$ is negative when z is large. Thus, with the exception of a set of parameters with measure zero in which $\Delta v(z) = 0$ for all $z \in [z_L, z_0]$, there exists a unique z^* such that $\Delta v(z^*) = 0$.

The state z^* is our candidate cut-off state. Indeed, there cannot be an equilibrium defined by a threshold $\widehat{z}$ such that $\Delta v(\widehat{z}) \equiv V_e(\widehat{z}, \widehat{z}) - V_n(\widehat{z}, \widehat{z}, \widehat{z}) \neq 0$. To see this, suppose the current and future active agents are playing a strategy defined by a threshold $\widehat{z}$ such that $\Delta v(\widehat{z}) > 0$. By continuity, in a neighborhood of $\widehat{z}$, $V_e(z, \widehat{z}) - V_n(z, \widehat{z}, \widehat{z}) > 0$. So the current active agent has an incentive to deviate and exert effort at the right of $\widehat{z}$. Likewise, if $\Delta v(\widehat{z}) < 0$, the current active agent has an incentive to deviate and not exert effort at the left of $\widehat{z}$.

Lemma 3 only provides a complete characterization of the equilibrium under the assumption that all active agents must choose the same cut-off rule. Since the current active agent can actually deviate and choose any strategy, including strategies which are not of a cut-off type, it remains to prove that the cut-off rule at state z^* is indeed a best reply to all future active agents choosing a cut-off rule at the same state z^*.

In every state $z \in \mathbb{R}$, the payoff of an active agent is given by $V^*(z) = \max\{V_e(z, z^*), V_n^*(z, z^*)\}$. $V_e(z, z^*)$ is the expected payoff of exerting effort in state z under the belief that all future active agents will choose the cut-off rule at state z^*. $V_n^*(z, z^*)$ is the expected payoff from not exerting effort in state z, and exerting effort in the following periods whenever it is optimal to do so, under the belief that all future active agents will choose the cut-off rule at state z^*.

Lemma 4 shows that $V_e(z, z^*) - V_n(z, z^*, z^*) < 0$ if $z > z^*$. Since, by definition, $V_n^*(z, z^*) \geq V_n(z, z^*, z^*)$, this implies $\Delta V^*(z, z^*) \equiv V_e(z, z^*) - V_n^*(z, z^*) < 0$ so that it is optimal not to exert effort to the right of z^*. Lemma 5 then shows that if $V_e(z, z^*) < V_n^*(z, z^*)$ for some $z < z^*$, then $\Delta V^*(z, z^*)$ is strictly decreasing in z. This implies that $\Delta V^*(z, z^*)$ crosses the zero line at most once. Moreover, since $\Delta V^*(z, z^*) > 0$ for z sufficiently small, together with Lemma 4, this result also implies that the best reply of the current active agent to all future active agents' following a cut-off rule at z^* is to also follow a cut-off rule at some state $z \leq z^*$. Finally, Lemma 6 proves that $z < z^*$ cannot be a cut-off, which implies that effort is exerted in all states $z < z^*$, and the cut-off is given by z^*.

3 Options and coordination

One implication of proposition 1 is that the unique symmetric equilibrium is determined by the point z^* where an agent is indifferent between effort and no effort, under the assumption that all agents (including himself) will exert effort in the future if and only if the economy is at the left of z^*.

An extreme case, which will prove interesting in what follows, occurs when an active agent does not have the option to delay his effort decision ($p_0 = 0$) and a passive agent must receive the benefit one period after exerting effort ($p_1 = 0$).

3.1 The no-options case

When $p_0 = p_1 = 0$, if an agent exerts effort in state z^*, his expected payoff is $V_e(z^*, z^*) = -c(z^*) + \frac{1}{2}\beta b$: since the process Δz is symmetric, there is a probability $\frac{1}{2}$ that the state of the economy in the following period will be to the left of z^*, in which case the agent receives the benefit b. If, instead, the agent does not exert effort, he is replaced by a newly born active active agent and leaves the economy with a payoff zero ($\Gamma_{0x} = 0$, hence $V_n(z^*, z^*, z^*) = 0$). This implies that z^* solves

$$c(z^*) = \frac{1}{2}\beta b.$$

This result has a natural interpretation. If $p_0 = p_1 = 0$, our economy can be thought of as a sequence of one-shot coordination games between pairs of active agents, where the active agent in the current period chooses between effort and no effort given his belief about the behavior of the active agent he will be paired with in the following period (this pairing actually only takes place if the current active agent chooses effort). In this game, effort is the risk dominant action if and only if $c(z^*) < \frac{1}{2}\beta b$, which is the same condition for effort to be the optimal decision in our economy.

		active agent	
		e	n
active agent	e	$\beta b - c(z^*), \beta b - c(z^*)$	$-c(z^*), 0$
	n	$0, -c(z^*)$	$0, 0$

3.2 The general case

For the general case in which options matter (i.e., $p_0 > 0$ or $p_1 > 0$), it is convenient to rewrite the expression in (1) so that the expected payoff of exerting effort at z^* in period s, conditional on all future active agents exerting effort at z^* is given by:

$$V_e(z^*, z^*) = -c(z^*) + \frac{1}{2}\beta b + \left[\int_0^\infty f(w)\Omega_{1w}dw\right]\beta b, \tag{3}$$

where Ω_{1w} is the sum of probabilities that the economy will be at a state smaller than z^* for the first time in period $s+t$ discounted by $(\beta p_1)^t$, given that the economy is currently in state $z^* + w$. The function Ω_{1w} can be recursively written as:

$$\Omega_{1w} = \int_w^\infty \Gamma_{1x} dy + \int_0^w \Gamma_{1x}\Omega_{1w-x}dx,$$

where

$$\Gamma_{1x} = \sum_{t=1}^{\infty} (\beta p_1)^t \phi(x, t).$$

The first integral considers all processes such that the economy is at a state smaller than z^* the first time it is at a state smaller than $z^* + w$. The second integral considers occurrences where the economy reaches a state between z^* and $z^* + w$ before eventually reaching a state smaller than z^* for the first time.

In words, an agent exerting effort at the state z^* incurs the cost $c(z^*)$ in period s. In period $s+1$, the economy will be at a state smaller than z^* with probability $\frac{1}{2}$, in which case the agent gets payoff βb and is replaced by a new active agent. Alternatively, the economy may be at a state larger than z^*. In this case, for each non-negative value of w, the economy will be at the state $z^* + w$ with probability density $f(w)$, and Ω_{1w} is the sum of probabilities that the economy will reach a state lower than z^* in period $s+t$ discounted by $(\beta p_1)^t$. The discount factor takes into account the time discount factor β and the probability the agent remains in the economy if the corresponding active agent makes no effort.

It is also convenient to rewrite the expression in (2) so that the expected payoff of not exerting effort at z^* in period s, conditional on all active agents (including the agent himself) following a cut-off rule at state z^* in all future periods is given by

$$V_n(z^*, z^*, z^*) = \int_0^\infty \Gamma_{0x} \left[-c(z^* - x) + \beta b \left(F(x) + \int_0^\infty f(x+w)\Omega_{1w} dw \right) \right] dx. \tag{4}$$

In words, the agent will exert effort when the economy reaches a state lower than z^*, and the term Γ_{0x} is the discounted sum of probability densities that the economy will be at the state $z^* - x$ the first time it reaches a state $z < z^*$. The discount factor takes into account both the time discount factor β and the probability p_0 that the agent remains in the economy conditional on having made no effort. At the state $z^* - x$, the agent exerts effort incurring a cost $c(z^* - x)$. In the following period, the economy will still be at a state lower than z^* with probability $F(x)$, in which case the agent will receive the benefit b. Alternatively, the economy may have jumped from the state $z^* - x$ to a state larger than z^*, say state $z^* + w$. In this case, for each non-negative value of w, the economy will be at the state $z^* + w$ with probability density $f(x + w)$, and Ω_{1w} is the sum of probabilities that the economy will reach a state lower than z^* in period $s + t$ discounted by $(\beta p_1)^t$. Again, the discount factor takes into account the time discount factor β and the probability the agent remains in the economy if the corresponding active agent makes no effort.

Intuitively, the advantage of exerting effort in state z^* is that it anticipates receiving the benefit b. The disadvantage is that the cost of effort is relatively high and there is a probability $\frac{1}{2}$ that the benefit may not be obtained in the following period. In turn, the disadvantage of not exerting effort in state z^* is that the benefit will only be accrued later in the future. The advantage is that the cost of effort will be relatively lower ($c(z - x) \leq c(z)$) and the interval between the cost and the benefit of effort will also be lower ($F(x) > \frac{1}{2}$).

This reasoning suggests that an increase in the variance of the process Δz would reduce an agent's willingness to exert effort as it increases the benefit of waiting for a lower cost. Proposition 2 formalizes this result.

Proposition 2 *Consider two situations: (i) Δz is described by cdf F; (ii) Δz is described by cdf G, such that that $F(kx) = G(x)$, for all x and some $k > 1$. The threshold z^* under F is strictly smaller than the threshold z^* under G as long as $p_0 > 0$ and $c(z^*) > 0$.*

Proof. See Appendix A.3. ■

The result in Proposition 2 is the real options effect (e.g., Bloom (2009)): an increase in uncertainty (captured here by an increase in the $var\left(\Delta z\right)$) leads to a drop in economic activity (captured here by the decision not to exert effort).

We now restrict attention to the scenario where $var(\Delta z) \to 0$. Proposition 2 thus implies that the threshold we characterize is an upper bound to the equilibrium obtained with the same parameters for any $var(\Delta z)$.

3.3 The general case with var$(\Delta z) \to 0$

The balance between the cost and the benefit of postponing effort depends on how the integral in the expression for $V_e(z^*, z^*)$ compares to the integrals in the expression for $V_n(z^*, z^*, z^*)$. The following lemma provides a key result which greatly simplifies the analysis.

Lemma 1

$$\int_0^\infty \Gamma_{1x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw \right] dx = \int_0^\infty f(w)\Omega_{1w}dw. \tag{5}$$

Proof. *See Appendix A.4.* ■

The LHS of (5) resembles the last term of the expression for V_n in (4). It means the following: suppose the economy is at z^* in period s and the agent will only exert effort the first period the economy is at the left of z^*. The LHS of (5) (multiplied by βb and excluding the cost $c\left(z\right)$) is the expected benefit the agent will receive considering only events where the economy goes to some state $z > z^*$ in period $s+1$ and the agent discounts the future at rate βp_1. The RHS of (5) (multiplied by βb and excluding the cost $c\left(z^*\right)$) is the expected benefit of an agent who exerts effort at z^* in period s and discounts the future at rate βp_1, again considering only events where the economy goes to some state $z > z^*$ in period $s+1$. Lemma 1 shows that both are equivalent. Proposition 3 then characterizes the equilibrium threshold in terms of the parameters of the economy in the case where $var(\Delta z) \to 0$.

Proposition 3 *When $var\left(\Delta z\right) \to 0$, the equilibrium is determined by λ, where*

$$\lambda = \frac{1}{2} + \frac{\int_0^\infty \left(\Gamma_{1x} - \Gamma_{0x}\right) \left(F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw\right) dx}{1 - \int_0^\infty \Gamma_{0x}dx}. \tag{6}$$

If $\lambda \geq 0$, $c(z^*) = \lambda\beta b$. Otherwise, $z^* < z_L$, which implies $c(z^*) = 0$.

Proof. *See Appendix A.5.* ■

The variable λ summarizes the equilibrium selection. If $\lambda \leq 0$, effort is never exerted at any z such that $c(z) > 0$, and if $\lambda = 1$, effort always is exerted as long as it is efficient to do so.

Corollary 1 *If $p_0 = p_1$, we have*

$$\lambda = \frac{1}{2}$$

Proposition 3 and Corollary 1 generalize the risk-dominance result obtained when $p_0 = p_1 = 0$ to any situation with $p_0 = p_1$, irrespective of the value of β. The equilibrium coincides with the risk dominant action of the corresponding one-shot game between the current and the future active agent. When $p_0 = p_1$, then $\Gamma_{1x} = \Gamma_{0x}$, and the expression for λ in (6) equals 1/2. The intuition is related to the result in Lemma 1: when $p_0 = p_1$, the gains from a smaller time between effort and rewards exactly cancel the gains from getting rewards sooner. Applying the result in Proposition 2, when var(Δz) is bounded away from 0, the threshold for effort z^* will correspond to some $c(z^*)$ smaller than $\frac{1}{2}\beta b$ because agents takes into account that it they wait and exert effort later, they will face a lower cost.

Proposition 4 examines how changes in the option values p_0 and p_1 affect the equilibrium outcome.

Proposition 4 *Let var$(\Delta z) \to 0$. Then*

1. *λ is decreasing in p_0.*

2. *λ is increasing in p_1.*

3. *If $p_0 = 1$ and $p_1 < 1$, $\lim_{\beta \to 1} \lambda \to -\infty$, and effort is never undertaken for any $z > z_L$.*

4. *If $p_0 < 1$ and $p_1 = 1$, $\lim_{\beta \to 1} \lambda = 1$, and effort is always undertaken for any $z < z_H(\beta)$.*[9]

Proof. *See Appendix A.6.* ■

[9] The assumption here is that

$$Var(\Delta z) \to 0 \quad , \quad \beta \to 1 \quad , \quad \frac{Var(\Delta z)}{1-\beta} \to 0$$

The first point of Proposition 4 states that the possibility of acting later represented by a larger p_0 increases the incentives for delays and might prevent agents from coordinating in the good equilibrium. Anticipating that future active agents will only act when it is "safe enough", the no-action equilibrium will be played. The result arises because the payoff of exerting effort is independent of p_0, but the payoff of not exerting effort is increasing in p_0. Intuitively the disadvantage of not exerting effort is that the agent defers benefits to the future (or make it less likely the agent will be able to reap any benefit). A high value of p_0 means that deferring benefits to the future is less costly.

The second statement of Proposition 4 tells us that a large p_1 helps agents to coordinate in the good equilibrium. Intuitively, the key advantage of deferring effort to the future is that the expected time between exerting effort and receiving the benefit b is smaller. A high value of p_1 implies that a larger time between exerting effort and receiving benefit is less costly to the agent.

The limiting results (statements 3 and 4) show that the effect is very strong when agents are patient and p_0 or p_1 are very high. If $p_1 < 1$, $p_0 = 1$ and agents are extremely patient, effort will only be exerted when the economy reaches the region where effort is the dominant strategy. An active agent is willing to wait a large amount of time to become a passive agents so that he is sure he won't lose the opportunity of getting b. Such an agent at a state z^* will then not choose to exert effort (even if the cost c is zero and there is a large benefit associated with becoming a passive agent!). He knows that all future active agents will behave in the same way, hence effort is never incurred in the region (z_L, z_H).

A corollary to this limiting result is that if $p_1 < p_0 = 1$ and $\beta \in (\bar{\beta}, 1)$, for some $\bar{\beta} < 1$, an agent at $z \in (z_L, z_0)$ will choose no effort although $c(z) = 0$. That is a case where returns could be infinite if they could coordinate (cost is zero, benefit is b), but the expected payoff of an active agent is arbitrarily close to 0. An active agent does not exert effort knowing that following agents will not exert effort as well until a point in a very distant future when z is around z_L. Severe coordination failures arise when p_0 and β are large enough if p_1 is smaller than p_0.[10]

Conversely, if $p_0 < p_1 = 1$ and agents are arbitrarily patient, effort is always undertaken whenever it is efficient to do so as long as the variance of Δz is arbitrarily small. A low value of p_0 helps agents to coordinate by reducing the option value of waiting.

[10] Applying the result in Proposition 2, when $var(\Delta z)$ bounded away from zero, there are even less incentives for effort, so incentives to delay until the cost of effort is zero can only increase for a larger $var(\Delta z)$.

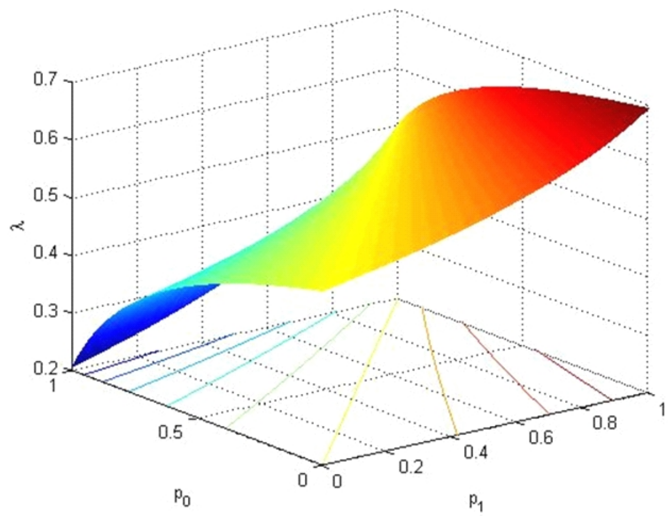

Figure 1: Values of λ for $\beta = 0.8$

Numerical examples In general, the value of λ depends on the process for z. Here we assume Δz is normally distributed. Using numerical techniques, the values of $\phi(x,t)$ can be found and equation (6) yields the value of λ. Figures 1 and 2 show the value of λ as a function of p_0 and p_1 for $\beta = 0.8$ and $\beta = 1$, respectively (negative values of λ are shown as $\lambda = 0$). The lines below the graph are iso-lambda curves. As one would expect from the analytical results, p_0 and p_1 have a larger influence on the equilibrium when β is higher.

The numerical results highlight the possibility of severe coordination failures in the model. With $\beta = 1$, the slope of the function $\lambda(p_0, p_1)$ close to $p_1 = 1$ is very large, which means that λ only gets close to 1 for extreme parameters. In contrast, $\lambda < 0$ for a much larger set of parameters. For instance, for $\beta = 0.99$, $p_0 = 1$ and $p_1 = 0.95$, the value of λ is negative: the expected payoff of an agent at some z away from z_L is arbitrarily close to 0 even though the cost of exerting effort is zero.

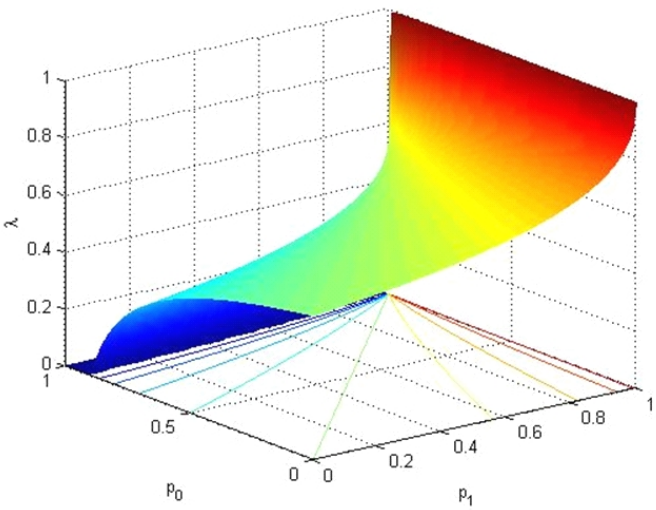

Figure 2: Values of λ for $\beta = 1$

4 Applications

4.1 Industrialization

Several models relate industrialization and coordination. Murphy, Shleifer and Vishny (1989) consider an economy in which the profit of a firm who is capable to choose a superior technology and industrialize depends on the demand for its product which, in turn, is an increasing function of the number of firms who industrialize. Industrializing requires incurring a fixed cost, hence it is only profitable if a sufficiently large number of firms industrialize. Coordination problems also arise in an economy with linkages in the production chain (e.g., Ciccone (2002) and Jones (2011)). Intuitively, if an intermediate good has a high productivity, this high productivity may increase the productivity of the next good in the chain. Strategic complementarities along the input chain are potentially important in this context, i.e., a firm's decision on whether to choose a new technology may depend on her expectation that the next firm in the chain will also eventually do the same. The list of papers in this literature also includes Kiyotaki (1988), Matsuyama (1991) and Durlauf (1993).

Industries in which innovation plays a central role are natural candidates to exhibit coordination problems.[11] Glaeser (2011) shows how agglomeration fosters the development of a new industry using

[11] As an illustration, the internet would probably be less present in our lives weren't for the possibility of transmitting

Detroit in the dawn of the 20^{th} century and the sylicon valley in the early 21^{st} century as examples – which highlights the importance of coordination for innovation. The ability to coordinate may also determine the fate of industrialization in developing countries as long as the demand externalities highlighted by Kiyotaki (1988) and Murphy, Shleifer and Vishny (1989) or the production externalities analyzed in Ciccone (2002) and Jones (2011) are important and the costs associated with international trading reduce the externalities from industrialization in other countries.

In this section we build on our model to examine how the option to delay an investment and the possibility of waiting longer for others' investments affect the process of industrialization in economies in which coordination matters. One difference between our model and most of this literature is that the coordination problem considered here is of an intertemporal (dynamic) nature: the decision of the current entrant firm depends on her belief about the behavior of future entrant firms. It is natural to assume that firms will not act at the same time for technological reasons (computers came before the internet, then came routers and tablets), and the issue we are analyzing requires a dynamic model.

There are two streams of production technologies, dynamic (industry) and sluggish (agriculture). The economy is initially populated by a *mature* firm positioned in the dynamic technological stream and an *entrant* firm, initially positioned in the sluggish technological stream. Each firm discounts the future with a factor β. The flow output of a firm in the sluggish stream is equal to y_0. In any period, the entrant firm can choose between remaining in the sluggish stream and innovating, i.e., moving to the dynamic stream. The latter choice involves a sunk cost C. The flow output of a mature firm in the dynamic stream is given by $y_1(z)$, where z behaves as in the environment laid out in section 2.1. Whenever an entrant firm moves to the dynamic stream (i.e., becomes mature), a new entrant firm in the sluggish stream enters the economy, and the previous mature firm in the dynamic stream becomes an *established* firm. The flow output of an established firm is given by $y_1(z) + \Delta y$, where Δy captures the positive externalities associated with industrialization.

Production technologies within the dynamic stream may "depreciate": at the end of every period there is a probability δ that the technology will become obsolete, in which case the flow output of established firms will revert to $y_1(z)$ in all future periods. Lastly, at the beginning of every period,

large amounts of information in little time (broadband technology), and the technology that allows wireless transmission of data (routers). In turn, those technologies would probably not have been so profitable if tablets and smart phones had not been invented. Finally, the popularity of tablets and smart phones depends to a large extent on the availability of the so-called apps, which will become really profitable once consumers start buying things through their apps.

there is a probability ρ that an entrant firm loses the ability to incur the cost C and innovate. In this case, the firm remains stuck in the sluggish stream obtaining an output flow y_0 from that period on, and a new entrant firm enters the economy.

This setup captures the positive impacts of new developments on previous efforts to industrialize in a stylized way that fits our model. The overall cost of becoming a mature firm within the dynamic technological stream is given by the fixed cost plus the opportunity cost, i.e.,

$$C + \frac{y_0}{1-\beta}.$$

Moreover, if the entrant firm never incurs the cost C, the expected profit of the mature firm within the dynamic technological stream is given by

$$\frac{y_1(z)}{1-\beta}.$$

In turn, if the entrant firm incurs the cost C, from that period on the mature firm becomes an established one and her expected profit is given by

$$\frac{y_1(z)}{1-\beta} + \frac{\Delta y}{1-\beta(1-\delta)}.$$

If we define

$$c(z) \equiv C + \frac{y_0 - y_1(z)}{1-\beta},$$

and

$$b \equiv \frac{\Delta y}{1-\beta(1-\delta)},$$

and if we let $p_1 = 1-\delta$, $p_0 = 1-\rho$, and make appropriate assumptions on $y_1(z)$ so that the cost $c(z)$ fits the assumptions laid down in Section 2.1, this setup collapses into our model. If $var(\Delta z) = 0$, there are multiple equilibria as long as $\beta b \geq c$. There is an equilibrium (agriculture) in which firms are stuck in the sluggish technological stream, and an equilibrium (industry) in which all firms move into the dynamic technological stream. Whether the equilibrium with agriculture or industry is selected does not depend on the probability ρ that an entrant firm loses her ability to move to the dynamic stream. Since b decreases with δ, the region of parameters in which agriculture is the unique equilibrium increases in the probability that the dynamic technology becomes obsolete.

The key contribution of our analysis is in the introduction of the option values into the coordination problem, and in providing a framework in which we can execute a meaningful comparative statics on the effects of options on coordination. Indeed, as shown in Proposition 1, once we have $\mathrm{var}(\Delta z) > 0$, there exists a unique equilibrium in cut-off strategies. A straightforward application of the proof of Propositions 1 and 3 then imply that an entrant firm chooses to incur the fixed cost and innovate if

$$\frac{1-\beta(1-\delta)}{1-\beta}\frac{(1-\beta)\,C+y_0-y_1(z)}{\beta\Delta y} < \lambda(\rho,\delta)$$

where $\lambda(\rho,\delta)$ is given by (6).

An interesting implication of our model is that a reduction in the probability δ that the dynamic technology becomes obsolete has two effects. It has the standard effect of increasing the payoff of investing in the dynamic technology (b is decreasing in δ) but it also helps agents to coordinate on the superior equilibrium ($\lambda(\rho,\delta)$ increases when δ decreases). Moreover, an increase in ρ, which can be thought of as an increase in the preemptive competition for the dynamic technology, helps agents in their effort to industrialize ($\lambda(\rho,\delta)$ increases when ρ increases), even though changes in ρ have no impact on the values of $c(z)$ and βb.

The fact that ρ affects the equilibrium outcome offers a well grounded rationale to the notion that protection of the domestic industry may be a counterproductive policy. In principle, it is not clear that bestowing a monopoly position to the domestic industry harms innovation – a monopolist also has incentives to choose a superior technology. But if coordination is important, protection harms the economy for its negative impact on the ability of firms to coordinate on the effort to industrialize, even though industrialization is more profitable than agriculture.

This application can be used to evaluate the impact of patents on coordination. Good enforcement of patents could be interpreted as a low value for δ: patents allow agents to profit longer from their inventions, and prevent others from investing and taking their places. Patents do increase the fundamental value of an invention, but once we consider that the value of inventions might also depend on coordination, patents have other important effects. The model also makes it clear that patents ought to be granted only after the corresponding sunk investment has been undertaken: a patent that protects an idea and so provides firms with an option to delay investment can be seen as a larger value of ρ, and thus harms coordination.

4.2 The Emergence of Shadow Banking

Shadow banking is the network of financial institutions and financial instruments which channels resources from investors in capital markets to ultimate borrowers in the real sector of the economy. The investigation into the collapse of the shadow banking system and its connections to the great recession has been the object of much research (see, for example, Adrian and Shin (2010), Brunnermeier (2009), and Gorton (2010, 2012). Less effort though has been directed towards understanding the origins and the development of shadow banking.[12] In what follows, we argue that the interplay between options and coordination may help explain the emergence of shadow banking and the development of securitization.

Coordination seems to be important in shadow banking. According to Pozsar et al (2012), "like the traditional banking system, the shadow banking system conducts credit intermediation. However, unlike the traditional banking system, where credit intermediation is performed under one roof – that of a bank – in the shadow banking system, it is performed through a daisy-chain of non-bank financial intermediaries in a multi step process" (page 10). In fact, this intermediation chain often involves seven or more steps. Hence the coordination problem in shadow banking looks like that of an economy with linkages in the production chain.

Gorton (2010) argues that the transformation of banking in the last 30 years is the result of three forces: (i) increased competition from non-banks, (ii) increase in capital requirements and changes in regulation, and (iii) innovation in financial products. The argument runs as follows. Up to the 1980's traditional banks did not engage in risky activities because they feared the loss of their charter value and the benefits that came with it (e.g., discount window at the Fed, deposit insurance). Starting in the 1980's, the increase in the competition from non-banks on both the asset side (e.g., junk bonds competing with bank loans) and the liability side (e.g., money market mutual funds competing for depositors), combined with a strengthening of capital requirements on banks, led to a decrease on the benefits of having a charter. This decrease made banks more prone to engage in riskier activities and with a desire to innovate. This desire was enhanced by the lifting of some restrictions on banking activities (e.g., interest rate ceilings on deposits where phased out), and found a concrete channel with securitization, a financial innovation that allowed for the sale of portfolios of loans into the capital

[12] Gorton (2010) argues that while the role played by securitization in the great recession is relatively well-understood, the origins of securitization are much less clear.

markets.

The reasoning put forth by Gorton raises an important question. If securitization turned out to be a profitable innovation, why did traditional banks waited for a decrease in charter values in order to undertake it?[13] One possibility is that the first factor pointed by Gorton (2010), the increased competition from non-banks, reduced the option value of waiting and investing later. Competition from non-banks corresponds to a lower p_0 in our model. In a situation where financial firms are willing to invest on securitization as long as they anticipate that others will provide the other inputs in the intermediation chain, fear of competition from potential entrants spurs investment and facilitates coordination. In terms of our basic environment, this corresponds to an increase in λ caused by a reduction in p_0.

The second factor pointed by Gorton (2010) can be seen as a reduction in the opportunity cost of engaging in securitization, and thus can be seen as a decrease in the parameter c. Increases in capital requirements and increases in restrictions imposed by regulators reduced the profitability of the traditional banking industry and hence the opportunity cost of investing in new activities. Proposition 3 shows that a reduction in c may facilitate securitization by mitigating the underlying coordination problems that it involves. The additional element that increased the profitability of securitization was the emergence of the repo market and the increase in the demand for collateral that it ensued. In terms of our basic environment, this can be thought of as an increase in b, which would again helps agents to coordinate.

5 Conclusion

In many contexts, coordination problems are of an intertemporal nature, where an agent's behavior does not depend on the current behavior of other agents but it depends on their future behavior. We showed that, in environments in which the disutility of effort changes over time, there is a unique symmetric equilibrium in cut-off strategies, and that the option to delay effort can have an important impact on the equilibrium outcome. This impact is twofold. First, an agent may exercise the option to delay effort because he expects that in the near future the disutility of effort may fall bellow its current

[13] Gorton (2010) points out that "we do not know for certain if [the decrease in charter value] in fact caused capital to exit regulated banking for the shadow banking system. But, in any case, the shadow banking system developed coincidently with the disappearance of charter value".

value (real-options effect). More interestingly, an agent may choose to delay effort because he expects that agents in the near future will also do the same (coordination effect). We showed that, while the real-options effect vanishes when the variance of the disutility of effort goes to zero, the coordination effect persists. Indeed, the cut-off cost an agent is willing to incur increases in the probability that the agent may have another option to choose effort in the future, and decreases in the probability that an agent will be able to wait longer for the effort decision of someone else. We also showed that the option to delay effort can induce severe coordination failures. In particular, coordination on effort may not happen even if the cost of effort is zero. Intuitively, if an agent is patient and knows there is a high chance he will have an opportunity to exert effort in a later period, he will only exert effort when he attributes a large probability to other agents exerting effort in the near future. The problem is that a similar reasoning is made by all agents. Thus beliefs that sustain a decision not to postpone effort cannot arise in equilibrium.

Our model captures key features of the impact of options on the equilibrium outcome in intertemporal coordination problems. It does not capture all possible features though. For example, when deciding whether to innovate, an agent may choose to delay this decision because he wants to mimic the innovation of another agent. Our model does not capture this type of incentive due to our assumption that there is only one active agent (i.e., only one agent who is able to innovate) in the economy at a point in time.[14] This assumption simplified our analysis and allowed us to focus on the incentives to delay innovation which stem from the lack of potential competition. A natural way in which the possibility of imitation can be introduced in our setting is by assuming that more than one agent is able to innovate in any given period, and that innovations can be replicated at some cost. The incorporation of such contemporaneous competition between active agents is a natural direction for future research.

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A Appendix

A.1 Interpreting p_0 as lack of competition

The parameter p_0 can be interpreted as reflecting the (lack of) competition that threatens the position of a active agent. This section formalizes this claim and clarifies which aspects of competition can be precisely represented by a lower p_0.

As in the basic model, the economy is initially populated by two types of agents, active and passive, and the active agent decides between effort (e) and no effort (n). The difference here is that if the active agent chooses no effort, he continues in the following period as an active agent with probability 1, but is joined by N newly born active agents. There exists a pecking order among the active agents. The one that tops the list is labeled the "best" agent. Each newly born active agent is better is better than the previous "best one" with probability γ.

At the beginning of every period, active agents have opportunities to plan investment. That has a small cost $\varepsilon > 0$ and allows an agent to invest later in that period. All active agents observe whether others have planned to invest, and there are further rounds of opportunities to plan investment until no active agent incurs the cost. Then all agents that planned investment have the opportunity to pay a cost $c - \varepsilon$, which transforms their plans to invest on real investment. Among those who chose to make effort, only the highest in the pecking order will become a passive agent, all the other active agents exit the economy. If investment was planned but the effort cost is not incurred, the agents exits the economy as well.

All other aspects of the game are unchanged.

Lemma 2 *In equilibrium:*

1. *Only the "best" active agent invests; whenever he plans investment, he also invests.*

2. *His problem is the same as that of an agent in the basic model with $p_0 = (1-\gamma)^N$, which is decreasing in N.*

Proof. Consider the (hypothetical) problem of an agent with only 2 possibilities: invest at the current period and become a passive agent next period with probability 1; or leave the game. Say agents are expected to invest if and only if $z > z^*$ and the economy is at state x.

If at the current period exerting effort is not the best option for the agents, noone will plan investment because paying the ε cost is worse than leaving the game with nothing.[15] So suppose investing at the current period is the best option for this agent. Now note that this is exactly the problem faced by the "best" active agent once someone else has planned investment: the active agent can either plan investment, invest and become a passive or leave the game. Hence if any other active

[15] Since all active agents have the same continuation payoff upon becoming passive agents, if it is optimal for an active agent to incur the overall cost c and become a passive agent, it must be optimal for all other active agents to do the same.

agent plans investment, the "best one" will follow suit and invest. But that implies that no other agent has an incentive to plan investment (it costs ε and yields 0).

Therefore active agents below the top of the pecking order play no role in this game, which is the first statement.

A direct implication of this result is that the problem of the best active agent is identical to the problem of the active agent in the basic environment. The best active agent knows he is the best in the current period but he also knows that the may not be the best in the following period. He will remain the best active agent with probability $p_0 \equiv (1-\gamma)^n$, which is the probability of the event that all newly born active agents who enter the economy next period are worse than him. That proves the second statement. ■

The first statement in the lemma shows that competition from "worse" active agents does not play a role in this model. Owing to the structure of the model, there is no way a low-ranked active agent can threaten the position of the top ranked agent. Thus there is an important aspect of competition that doesn't fit precisely in our framework, as this simple example shows. Nevertheless, competition from potential entrants plays an important role. Effort exerted by the active agent can be interpreted as preemptive competition as it prevents him being superseded by a future active agent. That happens with probability $1-(1-\gamma)^N$, so a larger N implies a lower p_0.

A.2 Proof of Proposition 1

In what follows, we let $\psi(z_1, z_2, z_3, t)$ denote the probability density that the state of the economy is z_1 in period $s+t$, conditional on z_2 being the state of the economy in period s, and conditional on the economy not being in any state $z \leq z_3$ in periods $\{s+1, ..., s+t-1\}$. Note that $\psi(z-x, z, z, t) = \phi(x, t)$.

Lemma 3 $\Delta v(z) \equiv V_e(z,z) - V_n(z,z,z)$ *is strictly decreasing in* $z \leq z_L$*, is constant in* $z \in [z_L, z_0]$*, and is strictly decreasing in* $z \geq z_0$*.*

Proof. *We start with* $z \geq z_L$*.* $\Delta v(z)$ *can be written as*

$$-c(z) + \beta \int_{-\infty}^{\infty} f(w) V_1(z+w, z) dw - \int_0^{\infty} \Gamma_{0x} \left\{ -c(z-x) + \beta \int_{-\infty}^{\infty} f(w) V_1 (z-x+w, z) \, dw \right\} dx,$$

where

$$\Gamma_{0x} \equiv \sum_{t=1}^{\infty} (\beta p_0)^t \psi(z-x, z, z, t).$$

Since $V_1(z+w,z)=V_1(z_L+w,z_L)$ *and* $V_1\left(z-x+w,z\right)=V_1\left(z_L-x+w,z_L\right)$*, the benefit part of the payoff does not depend on* z*. This implies that*

$$\frac{\partial \Delta v(z)}{\partial z}=-c'(z)+\int_0^{\infty}\Gamma_{0x}c'(z-x)dx\leq -c'(z)\left(1-\int_0^{\infty}\Gamma_{0x}dx\right),$$

where the inequality comes from the convexity of $c(z)$*. If* $z\leq z_0$*,* $c'(z)=0$ *and* $\Delta v(z)$ *is constant. If* $z>z_0$*, the strict convexity of* $c(z)$ *implies that the inequality is strict and* $\Delta v(z)$ *is strictly decreasing in* z*.*

Consider now $z<z_L$*. The payoff from exerting effort at* z *is*

$$V_e(z,z)=\beta\int_{-\infty}^{\infty}f(w)V_1(z+w,z_L)dw,$$

where we used the fact that $V_1(z,z)=V_1(z,z_L)$ *when* $z<z_L$*. Hence*

$$\frac{\partial V_e(z,z)}{\partial z}=\beta\int_{-\infty}^{\infty}f(w)\frac{\partial V_1(z+w,z_L)}{\partial z}dw.$$

The payoff from not exerting effort at z *is*

$$V_n(z,z,z)=\int_0^{\infty}\Gamma_{0x}\beta\int_{-\infty}^{\infty}f(w)V_1\left(z-x+w,z_L\right)dwdx.$$

Since x *is defined as the distance between the current state* $z-x$ *and the state* z*, a change in* z *does not affect* Γ_{0x}*. This implies that*

$$\frac{\partial V_n(z,z,z)}{\partial z}=\int_0^{\infty}\Gamma_{0x}\beta\int_{-\infty}^{\infty}f(w)\frac{\partial V_1\left(z-x+w,z_L\right)}{\partial z}dwdx.$$

Therefore

$$\frac{\partial \Delta v(z)}{\partial z}\frac{1}{\beta}=\int_{-\infty}^{\infty}f(w)\frac{\partial V_1(z+w,z_L)}{\partial z}dw-\int_0^{\infty}\Gamma_{0x}\int_{-\infty}^{\infty}f(w)\frac{\partial V_1\left(z-x+w,z_L\right)}{\partial z}dwdx.$$

Since $\int_0^{\infty}\Gamma_{0x}dx<1$*, we have*

$$\frac{\partial \Delta v(z)}{\partial z}\frac{1}{\beta}<\int_0^{\infty}\Gamma_{0x}\int_{-\infty}^{\infty}f(w)\frac{\partial V_1(z+w,z_L)}{\partial z}dwdx-\int_0^{\infty}\Gamma_{0x}\int_{-\infty}^{\infty}f(w)\frac{\partial V_1\left(z-x+w,z_L\right)}{\partial z}dwdx.$$

After a change in variables, we can rewrite the above inequality as

$$\frac{\partial \Delta v(z)}{\partial z}\frac{1}{\beta}<\int_0^{\infty}\Gamma_{0x}\int_{-\infty}^{\infty}f(w)\frac{\partial V_1(z+w,z_L)}{\partial z}dwdx-\int_0^{\infty}\Gamma_{0x}\int_{-\infty}^{\infty}f(\widetilde{w}+x)\frac{\partial V_1\left(z+\widetilde{w},z_L\right)}{\partial z}d\widetilde{w}dx.$$

Now since $\frac{\partial V_1(z,z_L)}{\partial z} = 0$ *for* $z < z_L$, $\frac{\partial V_1(z,z_L)}{\partial z} < 0$ *for* $z \geq z_L$, *and* $f(w) > f(w+x)$ *for all* $x > 0$ *and* $w > 0$, *we obtain*

$$\frac{\partial \Delta v(z)}{\partial z} \frac{1}{\beta} < \int_0^{\infty} \Gamma_{0x} \int_{z_L - z}^{\infty} [f(w) - f(w+x)] \frac{\partial V_1(z+w, z_L)}{\partial z} dwdx < 0.$$

This concludes our proof. ■

Note that $\Delta v(z)$ converges to a positive number when z goes to $-\infty$,and it converges to a negative number when z goes to ∞. Thus, with the exception of a measure zero set of parameters implying $\Delta v(z) = 0$ for all $z \in [z_L, z_0]$, there exists a unique $z \in \mathbb{R}$ such that $\Delta v(z) = 0$. This completes the first step of the proof. Henceforth, we let z^* denote the unique value of z such that $\Delta v(z) \equiv V_e(z,z) - V_n(z,z,z) = 0$.

Lemma 4 $\Delta V^*(z,z^*) \equiv V_e(z,z^*) - V_n^*(z,z^*) < 0$ *if* $z > z^*$.

Proof. Since $V_n^*(z,z^*) \geq V_n(z,z^*,z^*)$, it is enough to show that $V_e(z,z^*) - V_n(z,z^*,z^*) < 0$. We start with the case where $p_1 \geq p_0$.

Fix some period s and consider the effects on $V_e(z,z^*)$ of an arbitrarily small increase in z. First, an arbitrarily small increase in z implies an increase $c'(z)$ in the cost of exerting effort. Second, an arbitrarily small increase in z implies a decrease in the continuation payoff of exerting effort. This decrease is given by the discounted probability density associated with the event that z^* is the first state $z \leq z^*$ reached after period s, multiplied by the difference between obtaining the benefit b at z^* and not obtaining the benefit b at z^*. Indeed, it is only when z^* is the first state $z \leq z^*$ reached that the continuation payoff of exerting effort at z is different from the continuation payoff of exerting effort at an state $z + dz$, where dz is arbitrarily close to zero. Formally, we have

$$\frac{\partial V_e(z,z^*)}{\partial z} = -c'(z) - \beta \sum_{t=1}^{\infty} (\beta p_1)^{t-1} \psi(z^*, z, z^*, t) \left[b - \widehat{V}_e(z^*) \right],$$

where $\widehat{V}_e(z^*)$ is the payoff of being a passive agent at z^* under the assumption that effort is only exerted by active agents in states to the left of z^*.

Now fix some period s and consider the effects on $V_n(z,z^*,z^*)$ of an arbitrarily small increase in z. First, if dz is arbitrarily small and the agent is currently at state $z > z^*$, the expected payoff of exerting effort only at states $z' < z^*$ coincides with the expected payoff of exerting effort only at states

$z' < z^* + dz$. This implies that, for dz arbitrarily small, $V_n(z, z^*, z^*) = V_n(z + dz, z^* + dz, z^*)$. Now, since the process for Δz does not depend on z, this also implies that the first period in which effort is exerted is the same for an agent who is currently in state z and follows a cut-off rule at z^* and an agent who is currently in state $z + dz$ and follows a cut-off rule at $z^* + dz$. Thus, as in the case of $V_e(z, z^*)$, the only difference in payoffs associated with an arbitrarily small increase in z comes from differences in the marginal cost of effort and differences in continuation payoffs which arise if the state z^* is the first state $z' \leq z^*$ that is reached after the agent has made effort. Formally, we have that $\frac{\partial V_n(z,z^*,z^*)}{\partial z}$ equals

$$\int_0^\infty \sum_{t=1}^\infty (\beta p_0)^t \psi(z^* - x, z, z^*, t) \left\{ -c'(z^* - x) - \beta \sum_{k=1}^\infty (\beta p_1)^{k-1} \psi(z^*, z^* - x, z^*, k) \left[b - \widehat{V}_e(z^*) \right] \right\} dx.$$

Now, note that

$$\beta \sum_{k=1}^\infty (\beta p_1)^{k-1} \psi(z^*, z^* - x, z^*, k) \leq 1,$$

so that a lower bound for $\frac{\partial V_n(z,z^*,z^*)}{\partial z}$ is given by

$$-\int_0^\infty \sum_{t=1}^\infty (\beta p_0)^t \psi(z^* - x, z, z^*, t) c'(z^* - x) dx - \int_0^\infty \sum_{t=1}^\infty (\beta p_0)^t \psi(z^* - x, z, z^*, t) \left[b - \widehat{V}_e(z^*) \right] dx.$$

Moreover, the assumption that the density of Δz is weakly decreasing in Δz for $\Delta z > 0$ and weakly increasing in Δz for $\Delta z < 0$ implies that, for all $x > 0$,

$$\sum_{t=1}^\infty (\beta p_0)^t \psi(z^*, z, z^*, t) \geq \sum_{t=1}^\infty (\beta p_0)^t \psi(z^* - x, z, z^*, t).$$

Combining this fact with the convexity of $c(z)$, we obtain that a lower bound for $\frac{\partial V_n(z,z^*,z^*)}{\partial z}$ is given by

$$-c'(z) \int_0^\infty \sum_{t=1}^\infty (\beta p_0)^t \psi(z^* - x, z, z^*, t) dx - \sum_{t=1}^\infty (\beta p_0)^t \psi(z^*, z, z^*, t) \left[b - \widehat{V}_e(z^*) \right].$$

Thus, to prove that $V_e(z, z^*) - V_n(z, z^*, z^*) < 0$, it suffices to show that

$$-c'(z) \left[1 - \int_0^\infty \sum_{t=1}^\infty (\beta p_0)^t \psi(z^* - x, z, z^*, t) dx \right] - \sum_{t=1}^\infty \beta^t \left(p_1^{t-1} - p_0^t \right) \psi(z^*, z, z^*, t) \left[b - \widehat{V}_e(z^*) \right] < 0.$$

This last expression is verified whenever $p_1 \geq p_0$. Before we consider the case in which $p_1 < p_0$, note that in the analysis above we implicitly assumed that $z^* \geq z_L$. The proof in the case where $z^* < z_L$ is essentially the same. The only difference is that, when considering the impact of an arbitrarily small increase in z on the benefit of exerting effort, we need to replace the state z^* by the state z_L. Indeed, when $z^* < z_L$, the only difference in payoffs associated with an arbitrarily small increase in z comes from differences in the marginal cost of effort and differences in continuation payoffs which arise if the state z_L is the first state $z \leq z_L$ that is reached after the agent has made effort.

We now consider the case where $p_1 < p_0$. Fix some period s and look at the behavior of an active agent in state $z > z^*$. If he exerts effort, he obtains

$$V_e(z, z^*) = -c(z) + \beta b \int_0^\infty \sum_{t=1}^\infty (\beta p_1)^{t-1} \psi(z^* - x, z, z^*, t) dx.$$

In turn, if he does not exert effort, he obtains

$$V_n(z, z^*, z^*) = \int_0^\infty \sum_{t=1}^\infty (\beta p_0)^t \psi(z^* - x, z, z^*, t) V_e(z^* - x, z^*) dx.$$

Thus, $V_e(z, z^*) - V_n(z, z^*, z^*)$ is given by

$$-c(z) + \int_0^\infty \sum_{t=1}^\infty \beta^t \psi(z^* - x, z, z^*, t) \left[p_1^{t-1} b - p_0^t V_e(z^* - x, z^*)\right] dx,$$

while $V_e(z^*, z^*) - V_n(z^*, z^*, z^*)$ satisfies

$$-c(z^*) + \int_0^\infty \sum_{t=1}^\infty \beta^t \psi(z^* - x, z^*, z^*, t) \left[p_1^{t-1} b - p_0^t V_e(z^* - x, z^*)\right] dx = 0.$$

Since $c(z^*) \leq c(z)$, $V_e(z, z^*) - V_n(z, z^*, z^*) \geq 0$ is only possible if

$$\int_0^\infty \sum_{t=1}^\infty \beta^t \left[\psi(z^* - x, z, z^*, t) - \psi(z^* - x, z^*, z^*, t)\right] \left[p_1^{t-1} b - p_0^t V_e(z^* - x, z^*)\right] dx \geq 0$$

If we consider the most favorable case in which $\psi(z^* - x, z, t) \geq \psi(z^* - x, z^*, t)$ for all x and $t > 1$, and if we let $p_0 = p_1$, the inequality above can only be satisfied if

$$\int_0^\infty \sum_{t=1}^\infty (\beta p)^{t-1} \left[\psi(z^* - x, z, t) - \psi(z^* - x, z^*, t)\right] \left[b - p_0 V_e(z^* - x, z^*)\right] dx > 0,$$

which cannot be true when $z > z^*$. Thus, it must be the case that $V_e(z, z^*) - V_n(z, z^*, z^*) < 0$. Lastly, we need to consider the case where $p_1 < p_0$ and $z^* < z_L$. In this case, the payoff from exerting effort at $z > z^*$ is

$$V_e(z, z_L) = -c(z) + \beta b \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_1)^{t-1} \psi(z_L - x, z, z_L, t) dx.$$

In turn, if the agent does not exert effort, he obtains

$$V_n(z, z^*, z^*) = \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_0)^t \psi(z^* - x, z, z^*, t) V_e(z^* - x, z_L) dx.$$

Thus, $V_e(z, z^*) - V_n(z, z^*, z^*)$ is given by

$$-c(z) + \beta b \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_1)^{t-1} \psi(z_L - x, z, z_L, t) dx - \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_0)^t \psi(z^* - x, z, z^*, t) V_e(z^* - x, z_L) dx,$$

while $V_e(z^*, z_L) - V_n(z^*, z^*, z_L)$ satisfies

$$\beta b \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_1)^{t-1} \psi(z_L - x, z^*, z_L, t) dx - \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_0)^t \psi(z^* - x, z^*, z^*, t) V_e(z^* - x, z_L) dx = 0.$$

Since $c(z) \geq 0$, $V_e(z, z^*) - V_n(z, z^*, z^*) \geq 0$ is only possible if

$$\begin{aligned} & \beta b \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_1)^{t-1} \psi(z_L - x, z, z_L, t) dx - \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_0)^t \psi(z^* - x, z, z^*, t) V_e(z^* - x, z_L) dx \\ \geq \quad & \beta b \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_1)^{t-1} \psi(z_L - x, z^*, z_L, t) dx - \int_0^{\infty} \sum_{t=1}^{\infty} (\beta p_0)^t \psi(z^* - x, z^*, z^*, t) V_e(z^* - x, z_L) dx, \end{aligned}$$

which cannot be true when $z > z^*$. Thus, it must be the case that $V_e(z, z_L) - V_n(z, z^*, z_L) < 0$. This completes our proof. ■

We now show that, if $\Delta V^*(z, z^*) \equiv V_e(z, z^*) - V_n^*(z, z^*) < 0$, then $\Delta V^*(z, z^*)$ is strictly decreasing in z. This implies that $\Delta V^*(z, z^*)$ crosses the zero line at most once. Moreover, since $\Delta V^*(z, z^*) > 0$ for z sufficiently small, together with Lemma 2, this result also implies that the best reply to other agents' following a cut-off rule at z^* is to also follow a cut-off rule at some state $z \leq z^*$.

Lemma 5 *Let $z < z^*$ and assume that $\Delta V^*(z, z^*) \equiv V_e(z, z^*) - V_n^*(z, z^*) < 0$. Then, $\Delta V^*(z, z^*)$ is strictly decreasing in z.*

Proof. Consider an agent at state $z < z^*$ in some period s. If he exerts effort his payoff is

$$V_e(z, z^*) = -c(z) + \beta \int_{-\infty}^{\infty} f(w) V_1(z + w, z^*) dw,$$

which implies

$$\frac{\partial V_e(z,z^*)}{\partial z}=-c'(z)+\beta\int\limits_{-\infty}^{\infty}f(w)\frac{\partial V_1(z+w,z^*)}{\partial z}dw.$$

We now look at the agent's optimal payoff if he chooses not to exert effort at z. Let ϑ be the set of states in which it is optimal to exert effort. Let $\vartheta_1=\vartheta\cap(-\infty,z)$ and $\vartheta_2=\vartheta\cap(z,\infty)$. Define $\phi_1(z_1,z_2,t)$ as the probability density associated with reaching the state z_1 in period $s+t$, conditional on z_2 being the state of the economy in period s, and conditional on the economy not reaching any state in the set ϑ_1 in periods $\{s+1,...,s+t-1\}$. Also define $\phi_2(z_1,z_2,z_3,t)$ as the probability density associated with reaching state $z_3\in\vartheta_2$ at least once in periods $\{s+1,...,s+t-1\}$, conditional on z_2 being the state of the economy in period s and z_1 being the state of the economy in period $s+t$. Finally, let

$$\Phi_1(z_1,z_2)=\sum_{t=1}^{\infty}(\beta p_0)^t\,\phi_1(z_1,z_2,t)$$

and

$$\Phi_2(z_1,z_2,z_3)=\sum_{\tau=1}^{t-1}(\beta p_0)^t\,\phi_2(z_1,z_2,z_3,t).$$

The payoff $V_n^*(z,z^*)$ of not exerting effort at z is then given by (where we used the fact that, for every $z'\in\vartheta_1$, there exists $x\geq 0$ such that $z'=z-x$)

$$\int\limits_{\vartheta_1}\Phi_1(z',z)\left\{-c(z-x)+\beta\int\limits_{-\infty}^{\infty}f(w)V_1(z-x+w,z^*)dw+\int\limits_{\vartheta_2}\Phi_2(z',z,y)\left[\widehat{V}_e(.)-\widehat{V}_n^*(.)\right]dy\right\}dz'.$$

To understand the expression above, consider the hypothetical case in which $\vartheta_2=\varnothing$, i.e., effort is never optimal to the right of state z. In this case, the second integral in the term inside brackets is zero, and the expression is similar to the one in the case where the agent is following a cut-off rule at z. Thus, the second integral in the term inside brackets should be interpreted as the additional payoff an agent receives when it is also optimal to choose effort in states $z\in\vartheta_2$. Even though the exact expression for $\widehat{V}_e(.)-\widehat{V}_n^*(.)$ is complex, we know that it is positive by the definition of ϑ_2.

Consider now an arbitrarily small increase in z. If we add the same increase dz to each state in the set ϑ_1, we obtain that, whenever an agent who is currently in state z chooses effort upon reaching some state $z'\in\vartheta_1$, then an agent who is currently at state $z+dz$ will also choose effort. Note though that, while the behavior of the former agent is optimal (by the definition of ϑ_1), the same is not true of the latter agent. This implies that a lower bound on the increase in the payoff $V_n^*(z,z^*)$ associated with an arbitrarily small increase dz in z is given by

$$\int_{\vartheta_1} \Phi_1(z',z)\left\{-c'(z-x)+\beta\int_{-\infty}^{\infty} f(w)\frac{\partial V_1(z-x+w,z^*)}{\partial z}dw+\int_{\vartheta_2}\frac{\partial \Phi_2(z',z,y)}{\partial z}\left[\widehat{V}_e(.)-\widehat{V}_n^*(.)\right]dy\right\}dz'.$$

Now, since an agent who is currently at state $z+dz$ is more likely to eventually reach a state in $\vartheta_2=\vartheta\cap(z,\infty)$ than an agent who is currently at state z, we know that $\frac{\partial \Phi_2(z',z,y)}{\partial z}>0$. This implies that a lower bound on the expression above is given by

$$\int_{\vartheta_1} \Phi_1(z',z)\left\{-c'(z-x)+\beta\int_{-\infty}^{\infty} f(w)\frac{\partial V_1(z-x+w,z^*)}{\partial z}dw\right\}dz'.$$

Thus, an upper bound on $\frac{\partial V_e(z,z^*)}{\partial z}-\frac{\partial V_n^*(z,z^*)}{\partial z}$ is

$$-c'(z)+\beta\int_{-\infty}^{\infty} f(w)\frac{\partial V_1(z+w,z^*)}{\partial z}dw-\int_{\vartheta_1} \Phi_1(z',z)\left\{-c'(z-x)+\beta\int_{-\infty}^{\infty} f(w)\frac{\partial V_1(z-x+w,z^*)}{\partial z}dw\right\}dz'.$$

Since $c(z)$ is convex and $\int_{\vartheta_1}\Phi_1(z',z)dz'<1$, an upper bound on the expression above is

$$\beta\int_{-\infty}^{\infty} f(w)\frac{\partial V_1(z+w,z^*)}{\partial z}dw-\int_{\vartheta_1} \Phi_1(z',z)\beta\int_{-\infty}^{\infty} f(w)\frac{\partial V_1(z-x+w,z^*)}{\partial z}dwdz'.$$

Moreover, since $\frac{\partial V_1(z+w,z^*)}{\partial z}\leq 0$ (and using $\int_{\vartheta_1}\Phi_1(z',z)dz'<1$), an upper bound on the expression above is

$$\beta\int_{\vartheta_1} \Phi_1(z',z)\left[\int_{-\infty}^{\infty} f(w)\frac{\partial V_1(z+w,z^*)}{\partial z}dw-\int_{-\infty}^{\infty} f(w)\frac{\partial V_1(z-x+w,z^*)}{\partial z}dw\right]dz'.$$

Finally, a change in variables allows us to rewrite the above expression as

$$\beta\int_{\vartheta_1} \Phi_1(z',z)\left[\int_{-\infty}^{\infty} f(w)\frac{\partial V_1(z+w,z^*)}{\partial z}dw-\int_{-\infty}^{\infty} f(w+x)\frac{\partial V_1(z+w,z^*)}{\partial z}dw\right]dz'.$$

We know that $\frac{\partial V_1(z+w,z^*)}{\partial z}=0$ whenever $z+w<z^*$, i.e., $w<z^*-z$. Thus, we can rewrite the above expression as

$$\beta\int_{\vartheta_1} \Phi_1(z',z)\int_{z^*-z}^{\infty}\left[f(w)-f(w+x)\right]\frac{\partial V_1(z+w,z^*)}{\partial z}dwdz'.$$

Finally, since $w > 0$ and $x \geq 0$, we have $f(w) > f(w + x)$ and the above expression is negative. ■

We have thus shown that the best reply to other agents following a cut-off rule at z^* is to also follow a cut-off rule at some state $z \leq z^*$. Lemma 4 proves that $z < z^*$ cannot be a cut-off, which implies that effort is exerted in all states $z < z^*$.

Lemma 6 *For all $z < z^*$, $\Delta V^*(z, z^*) \equiv V_e(z, z^*) - V_n^*(z, z^*) > 0$.*

Proof. *Consider the decision of an agent at $z < z^*$. The payoff from exerting effort is given by*

$$V_e(z, z^*) = -c(z) + \beta \int\limits_{-\infty}^{\infty} f(w) V_1(z + w, z^*) dw,$$

while the payoff from not exerting effort and following a cut-off rule at z is

$$V_n(z, z, z^*) = \int_0^{\infty} \Gamma_{0x} \left[-c(z - x) + \beta \int_{-\infty}^{\infty} f(w) V_1(z - x + w, z^*) dw dx \right],$$

where

$$\Gamma_{0x} \equiv \sum_{t=1}^{\infty} (\beta p_0)^t \psi(z - x, z, z, t).$$

Now, note that $V_e(z, z^*) - V_n(z, z, z^*) - \Delta v(z^*)$ is given by

$$\begin{aligned}
& c(z^*) - c(z) - \int_0^{\infty} \Gamma_{0x} \left[c\left(z^* - x\right) - c(z - x) \right] dx \\
& + \beta \int_{-\infty}^{\infty} f(w) \left[V_1(z + w, z^*) - V_1(z^* + w, z^*) \right] dw \\
& - \beta \int_0^{\infty} \Gamma_{0x} \int_{-\infty}^{\infty} f(w) \left[V_1(z - x + w, z^*) - V_1\left(z^* - x + w, z^*\right) \right] dw dx.
\end{aligned}$$

We will show that this expression is positive. Since $\Delta v(z^*) = 0$, this implies that $V_e(z, z^*) - V_n(z, z, z^*) > 0$. Thus, z is not an optimal response to all other agents choosing a cut-off at z^*.

First, consider the first line in the expression above. For any $x \geq 0$,

$$c(z^*) - c(z) \geq c(z^* - x) - c(z - x) \geq 0,$$

which comes from $z^* > z$ and c being convex. Since the integral of Γ_{0x} in x is smaller than one, the first line in the expression above must be positive. Now consider the second and third lines. We have

$$\begin{aligned}
& \beta \int_{-\infty}^{\infty} f(w) \left[V_1(z + w, z^*) - V_1(z^* + w, z^*) \right] dw \\
& - \beta \int_0^{\infty} \Gamma_{0x} \int_{-\infty}^{\infty} f(w) \left(V_1(z - x + w, z^*) - V_1\left(z^* - x + w, z^*\right) \right) dw dx
\end{aligned}$$

Since the integral of Γ_{0x} in x is smaller than one and the first integral is positive ($V_1(z,z^*)$ is decreasing in z), a lower bound on the expression above is

$$\begin{aligned}
&\beta \int_0^\infty \Gamma_{0x} \int_{-\infty}^\infty f(w) \left\{ \begin{array}{c} [V_1(z+w,z^*) - V_1(z^*+w,z^*)] \\ -[V_1(z-x+w,z^*) - V_1(z^*-x+w,z^*)] \end{array} \right\} dwdx \\
= \; &\beta \int_0^\infty \Gamma_{0x} \int_{0}^\infty f(w) \left\{ \begin{array}{c} [V_1(z+w,z^*) - V_1(z^*+w,z^*)] \\ -[V_1(z-x+w,z^*) - V_1(z^*-x+w,z^*)] \end{array} \right\} dwdx
\end{aligned}$$

where the lower limit of the inner integrals where changed from $-\infty$ to 0. This can be done because $V_1(z) = b$ for $z \leq z^*$.A change in variables allows is to rewrite the above expression as

$$\beta \int_0^\infty \Gamma_{0x} \int_0^\infty [f(w) - f(w+x)] [V_1(z+w,z^*) - V_1(z^*+w,z^*)] \, dwdx.$$

This last expression is non-negative since $f(w) \geq f(w+x)$ for w and x nonnegative. ■

We have thus shown that there exists a unique equilibrium in symmetric strategies. In this equilibrium, agents exert effort if and only if $z < z^*$, where z^* solves

$$-c(z^*) + \beta \int\limits_{-\infty}^{\infty} f(w) V_1(z^*+w,z^*) dw = \int_0^\infty \Gamma_{0x} \left[-c(z^*-x) + \beta \int_{-\infty}^\infty f(w) V_1(z^*-x+w,z^*) dwdx \right].$$

A.3 Proof of Proposition 2

We need to show that $\Delta v(z^*)|_F - \Delta v(z^*)|_G < 0$. Note that

$$\Delta v(z^*)|_F - \Delta v(z^*)|_G = \int_0^\infty (\Gamma_{0x}|F) \, c(z^*-x) dx - \int_0^\infty (\Gamma_{0x}|G) \, c(z^*-x) dx$$

can be written as

$$\Delta v(z^*)|_F - \Delta v(z^*)|_G = \int_0^\infty (\Gamma_{0x}|G) \, c(z^*-kx) dx - \int_0^\infty (\Gamma_{0x}|G) \, c(z^*-x) dx < 0.$$

Since $\Delta v(z^*)$ is decreasing in z^*, $\Delta v(z^*)|_F < \Delta v(z^*)|_G$ implies that z^* under F is smaller than z^* under G.

A.4 Proof of Lemma 1

First, using the definition of Ω_{1w}, we can write the right-hand side of (5) as

$$\int_0^\infty f(w) \Omega_{1w} dw \equiv \int_0^\infty f(x) \Omega_{1x} dx = \int_0^\infty \int_x^\infty f(x) \Gamma_{1w} dwdx + \int_0^\infty \int_0^x f(x) \Gamma_{1w} \Omega_{1x-w} dwdx. \quad (7)$$

Manipulating the first term on the right-hand side of (7), we get:

$$\int_0^\infty \int_x^\infty f(x)\Gamma_{1w} dw dx = \int_0^\infty \int_0^w \Gamma_{1w} f(x) dx dw = \int_0^\infty \Gamma_{1w} \left[F(w) - \frac{1}{2}\right] dw, \tag{8}$$

where the first equality comes from changing the order of variables in the double integral. In turn, manipulating the second term on the right-hand side of (7), we get

$$\int_0^\infty \int_x^\infty f(x)\Gamma_{1w}\Omega_{1x-w} dw dx = \int_0^\infty \int_w^\infty \Gamma_{1w} f(x)\Omega_{1x-w} dx dw = \int_0^\infty \int_0^\infty \Gamma_{1w} f(y+w)\Omega_{1y} dy dw, \tag{9}$$

where the first equality comes from changing the order of variables in the double integral and the second line comes from making $y = x - w$. We can thus rewrite the left-hand side of (7) as

$$\int_0^\infty f(x)\Omega_{1x} dx = \int_0^\infty \Gamma_{1x} \left[F(x) - \frac{1}{2}\right] dx + \int_0^\infty \int_0^\infty \Gamma_{1x} f(w+x)\Omega_{1w} dw dx,$$

which yields the claim.

A.5 Proof of Proposition 3

In case $var(\Delta z) \to 0$, the expression for $V_e(z^*, z^*)$ is the same as in (11), but the expression for $V_n(z^*, z^*, z^*)$ is given by

$$V_n(z^*, z^*, z^*) = \int_0^\infty \Gamma_{0x} \left[-c(z^*) + \beta b \left(F(x) + \int_0^\infty f(x+w)\Omega_{1w} dw\right)\right] dx \tag{10}$$

Using Lemma 1, we can rewrite (3) as

$$V_e(z^*, z^*) = -c(z^*) + \frac{1}{2}\beta b + \left(\int_0^\infty \Gamma_{1x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w} dw\right] dx\right) \beta b. \tag{11}$$

Equating (11) and (10), we obtain that the cut-off state z^* is the solution to

$$\left[\frac{c(z^*)}{\beta b} - \frac{1}{2}\right] \left(1 - \int_0^\infty \Gamma_{0x}\right) dx = \int_0^\infty (\Gamma_{1x} - \Gamma_{0x}) \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w} dw\right] dx.$$

Rearranging yields:

$$\frac{c(z^*)}{\beta b} = \frac{1}{2} + \frac{\int_0^\infty (\Gamma_{1x} - \Gamma_{0x}) \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w} dw\right] dx}{1 - \int_0^\infty \Gamma_{0x} dx} \equiv \lambda.$$

It is easy to show that relative incentives for effort are decreasing in the cost $c(z^*)$, hence $V_e(z^*, z^*) > V_n(z^*, z^*, z^*)$ whenever $\frac{c(z^*)}{\beta b}$ is smaller than λ. If $\lambda > 0$, that is the equilibrium threshold. If $\lambda < 0$, then $V_e(z^*, z^*) - V_n(z^*, z^*, z^*) < 0$ for all $z > z_L$, which is the region described by the value functions in (11) and (10), and hence the equilibrium threshold occurs at some $z^* < z_L$.

A.6 Proof of Proposition 4

1. The expression for $V_e(z^*, z^*)$ if an agent is indifferent between effort and no effort is given by

$$\frac{V_e(z^*, z^*)}{\beta b} = -\lambda + \frac{1}{2} + \int_0^\infty f(x)\Omega_{1x}dx, \tag{12}$$

since $\lambda = \frac{c}{\beta b}$ if the agent is indifferent. This expression is a function of λ, p_1, β, b, and the parameters of the stochastic process for Δz. This expression implies that λ is decreasing in p_0 if $V_e(z^*, z^*)$ is increasing in p_0 (when expressed as a function of p_0, p_1, β and the parameters of the stochastic process for Δz). So we need to prove that $V_e(z^*, z^*)$ is increasing in p_0.

Combining equation (12) with the expression for λ given by 6, we get that the agent is indifferent when

$$\frac{V_e(z^*, z^*)}{\beta b} = -\frac{\int_0^\infty (\Gamma_{1x} - \Gamma_{0x}) \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw\right] dx}{1 - \int_0^\infty \Gamma_{0x}dx} + \int_0^\infty f(x)\Omega_{1x}dx.$$

Using equation (5) and rearranging,

$$\frac{V_e(z^*)}{\beta b} = \frac{\int_0^\infty \Gamma_{0x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw\right] dx - \int_0^\infty \Gamma_{0x}dx \int_0^\infty f(w)\Omega_{1w}dw}{1 - \int_0^\infty \Gamma_{0x}dx},$$

which can be further rearranged to

$$\frac{V_e(z^*)}{\beta b} = \frac{\int_0^\infty \Gamma_{0x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw - \int_0^\infty f(w)\Omega_{1w}dw\right] dx}{1 - \int_0^\infty \Gamma_{0x}dx}.$$

The denominator is decreasing in p_0. The term inside brackets in the numerator can be written as

$$\int_0^x f(w)dw + \int_x^\infty f(y)\Omega_{1y-x}dy - \int_0^\infty f(w)\Omega_{1w}dw = \int_0^x f(w)(1-\Omega_{1w})dw + \int_x^\infty f(y)(\Omega_{1y-x} - \Omega_{1y})dy,$$

which is positive for all x since $\Omega_w < 1$ and Ω_y is decreasing in y. Hence the numerator is increasing in p_0, which completes the proof.

2. The expressions for $V_e(z^*, z^*)$ and the expression for $V_n(z^*, z^*, z^*)$ if an agent is indifferent between effort and no effort are given by

$$\frac{V_e(z^*, z^*)}{\beta b} = -\lambda + \frac{1}{2} + \int_0^\infty f(x)\Omega_{1x}dx,$$

and

$$\frac{V_n(z^*, z^*, z^*)}{\beta b} = \int_0^\infty \Gamma_{0x}dx \left(-\lambda + \frac{1}{2}\right) + \int_0^\infty \Gamma_{0x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw\right] dx.$$

Since $V_e(z^*, z^*) = V_n(z^*, z^*, z^*)$, we have

$$\left(\frac{1}{2} - \lambda\right)\left(1 - \int_0^\infty \Gamma_{0x} dx\right) = \int_0^\infty \Gamma_{0x}\left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w} dw\right] dx - \int_0^\infty f(w)\Omega_{1w} dw.$$

Since the left-hand side of the above equation is decreasing in λ, by the implicit function theorem, λ is increasing in p_1 if the right-hand side of the equation above is decreasing in p_1 (under the conditions on f). We thus need to prove that derivative of

$$\int_0^\infty \Gamma_{0x}\left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w} dw\right] dx - \int_0^\infty f(w)\Omega_{1w} dw$$

with respect to p_1 is negative. It is given by

$$\int_0^\infty \Gamma_{0x}\left(\int_0^\infty f(w+x)\frac{\partial \Omega_w}{\partial p_1} dw\right) dx - \int_0^\infty f(w)\frac{\partial \Omega_w}{\partial p_1} dw.$$

The condition $f(w) > f(w')$ for all $w' > w > 0$, implies that this expression is smaller than or equal to

$$\int_0^\infty \Gamma_{0x}\left(\int_0^\infty f(w)\frac{\partial \Omega_{1w}}{\partial p_1} dw\right) dx - \int_0^\infty f(w)\frac{\partial \Omega_{1w}}{\partial p_1} dw,$$

which is equal to

$$\left(\int_0^\infty f(w)\frac{\partial \Omega_{1w}}{\partial p_1} dw\right)\left(\int_0^\infty \Gamma_{0x} dx\right) - \int_0^\infty f(w)\frac{\partial \Omega_{1w}}{\partial p_1} dw.$$

The last expression is negative since $\int_0^\infty \Gamma_{0x} dx < 1$ as long as $\beta p_0 < 1$. This completes the proof.

3. If $p_0 = 1$, $\lim_{\beta \to 1} \int_0^\infty \Gamma_{0x} dx \to 1$. Hence as β approaches 1, the denominator of equation (6) approaches $+\infty$. Since $p_1 < 1$, $\Gamma_{1x} - \Gamma_{0x} < 0$, hence the numerator is negative, which yields the result.

4. If $p_1 = 1$, $\lim_{\beta \to 1} \Omega_{1x} \to 1$. Thus $\int_0^\infty f(w+x)\Omega_{1w} dw$ becomes $1 - F(x)$. Then λ can be written as

$$\lambda = \frac{1}{2} + \frac{1}{2}\frac{\int_0^\infty \Gamma_{1x} dx - \int_0^\infty \Gamma_{0x} dx}{1 - \int_0^\infty \Gamma_{0x} dx}$$

If $p_1 = 1$, $\lim_{\beta \to 1} \int_0^\infty \Gamma_{1x} dx \to 1$, which yields the claim.