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ABSTRACT

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We analyse different forms of international debt mutualisation in a simple framework with a political distortion and (partial) default under adverse economic circumstances. One form is a debt repayment guarantee, which can be "unlimited" or "limited", i.e. only be invoked when the guarantee threshold is not exceeded. We also explore the "blue-red" bonds proposal, under which blue debt is guaranteed by the other countries in a union, while red debt is not guaranteed. Only a suitably chosen limited guarantee induces the government to reduce debt and raises social welfare. Making the guarantee also conditional on sufficient structural reform may stimulate reform effort. However, now a trade-off exists between extracting more reform and inducing the government to limit debt issuance.

JEL Classification: E60, E62, H60 and H63
Keywords: blue and red bonds, debt bias, debt guarantee, eurobonds, political distortions, social welfare and structural reform

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1 Introduction

According to many commentators, the current debt crisis has brought the eurozone to the verge of a break up. The situation in which some eurozone countries face high interest rates on public debt and other countries are forced to effectively guarantee those debts via the emergency funds is widely viewed as unviable in the long run.[1] While the ratio of aggregate public debt to GDP in the eurozone lies below that for the U.S., the latter country has faced no trouble so far in financing its debt. In view of all this, a number of experts as well as politicians, such as the current leaders France, Italy and Spain have pleaded for the introduction of "eurobonds", which take some form of collectively guaranteed public debt. Among the various proposals, there are the "blue and red bond" proposal by Delpla and Von Weizsäcker (2010), in which the EU countries pool their public debt up to at most 60% under joint and several liability as senior (blue) debt, while any debt above 60% of GDP would be issued as junior (red) debt. The proposal by Hellwig and Philippon (2011) foresees a maximum of 10% of GDP of mutually guaranteed short-term debt. De Grauwe and Moesen (2009) propose a collectively guaranteed eurobond, the interest paid on which is differentiated across the participating countries on the basis of their market interest rates. Further, the European Commission (2011) has issued a green paper on what it calls "stability bonds". Finally, Claessens et al. (2012) provide an in-depth discussion of the various proposals and the steps that need to be taken to arrive some form of common debt issuance.

While eurobonds have their proponents, of course they also have their adverseries such as Issing (2009), who points to the danger of moral hazard by countries that already have a weak record in terms of budgetary discipline. The perceived danger is that these countries, knowing that at least part of their debt is guaranteed by other countries, will increase their spending and start issuing more debt, because the interest rate on the guaranteed component of the debt is insensitive or largely insensitive to an individual debt increase. Although the European Commission (2011) and others acknowledge the potential problem of moral hazard, they believe that there are ways

[1] Lane (2012) reviews the European sovereign debt crisis, which has sparked interest in the analysis of sovereign default risk in Europe. For example, Corsetti and Dedola (2011) extend the model by Calvo (1988) and show how imposing an interest ceiling can break a self-fulfilling equilibrium in which default is forced despite sound fundamentals.

to work around this problem.

This paper provides a formal analysis of the budgetary and welfare consequences of different types of debt mutualisation in the eurozone giving some prominence to the alleged moral hazard problems associated with debt mutualisation.[2] One type takes the form of a guarantee provided by other countries for the repayment of public debt up to a certain maximum amount. In the case of a (partial) default, each debt holder gets the same fraction of his holdings repaid. Hence, the government issues a single type of debt. As a consequence, all the debt issued by the same country carries the same interest rate. The guarantee can be "unlimited" in the sense that the rest of the union provides financial support up to the guarantee level if necessary, even if debt exceeds the guarantee level. However, the guarantee can also be made "limited", meaning that any financial support from the rest of the union is lost when the government issues debt above the guarantee level. We also consider an alternative in which the government may issue two types of debt alongside each other. This would essentially come down to the "blue and red bond" proposal by Delpla and Von Weizsäcker (2010). Blue bonds are senior and collectively guaranteed, while red bonds are non-senior debt that is not collectively guaranteed. Its repayment depends solely on the fortune of the public finances of the issuing country. Since both types of debt carry different risks, they also feature different interest rates.

We set up a simple two-period model of a country featuring a political distortion along the lines of Alesina and Tabellini (1990) and Beetsma and Uhlig (1999) and also in the spirit of Cukierman et al. (1992). The present model allows for (partial) default on the debt when the country's resources become too small due to unfortunate economic circumstances. We do not need an explicit cost of default (such as in Calvo, 1988, and Beetsma, 1996) for the model to give meaningful results. Due to the political distortion the country issues an amount of debt that is excessive from society's perspective. While the possibility of default under adverse economic circumstances in itself induces the government to issue more debt, this effect is kept in check by the response of the interest rate required by risk-neutral investors to be prepared to hold the debt. Hence, debt is excessive, though to a limited extent.

The introduction of a debt guarantee eliminates the response of the inter-

[2] In the sequel we will refer to "debt mutualisation" rather than "eurobonds", as the former is more general.

est rate to an increase in debt as long as debt does not exceed the maximum guarantee level. This in itself gives the government an incentive to issue even more debt. The same is the case under the blue-red bonds proposal. However, depending on the precise choice for the guarantee level or the precise threshold for the amount of blue bonds, the equilibrium debt level may or may not exceed the debt level in the absence of the debt mutualisation. In fact, in the presence of a limited debt guarantee, setting the guarantee level sufficiently low, but not too low, induces the government to issue less debt than in the absence of the guarantee, thereby raising social welfare. Neither an unlimited guarantee, nor the blue-red bonds proposal can generate this beneficial outcome within our framework.

We also consider an extension in which we allow the government to select an optimal amount of structural reform in the first period, which makes the economy more efficient and yields additional resources in the second period. Making participation in a limited guarantee scheme also conditional on sufficient reform, may induce governments to reform more. Hence, the moral hazard in providing reform can be alleviated to some extent. However, there exists a trade-off between extracting more reform and inducing the government to issue less debt by reducing the guarantee level. Setting the reform threshold higher requires an increase in the guarantee level to pursuade the government to participate in the scheme.

Obviously, it is important to acknowledge the limitations of our analysis. The main one concerns the assumption that the bonds traded in our model are perfectly liquid, while enhancing liquidity is often cited as one of the main reasons for issuing eurobonds as it would lead to a large and homogeneous market for European debt, much like the market for U.S. government debt. However, this paper aims at scrutinising the perception that eurobonds provide a disincentive to limiting debt accumulation and undermine the willingness to undertake structural economic reforms that are claimed to be necessary in many European economies.

This paper proceeds as follows. Section 2 presents the model, while Section 3 solves the case in which there is no debt mutualisation. Section 4 analyses the case with a debt guarantee, while Section 5 explores the blue and red bonds proposal. In Section 6 we set up our social welfare criterion and analyse whether there exist forms of debt mutualisation that may raise social welfare. Section 7 expands the model by introducing the choice of structural reform. Finally, Section 8 concludes the main text of the paper. The appendix, which is not for publication, contains some technical details.

It will be posted on http://www1.feb.uva.nl/mint/beetsma.shtm.

2 The model

We focus on a small open country that is part of some large union that may allow for some degree of debt mutualisation. We may think of this union as the European Union (EU) or the eurozone. There are two periods, 1 and 2. The country's fiscal policy suffers from a political distortion leading to suboptimally-high spending on public goods and, hence, an overaccumulation of public debt in period 1. The model features the possibility of (partial) debt default under adverse economic circumstances.

2.1 Utility and political parties

Utilities and the political structure follow the specification in Alesina and Tabellini (1990) and Beetsma and Uhlig (1999). For the sake of clarity we use specifications that are as simple as possible, but still allow us to convey our main messages.

Society's expected utility is given by

$$U_S(f_1, g_1, f_2, g_2) = u(f_1 + g_1) + E[f_2 + g_2], \tag{1}$$

where $f_t \geq 0$ and $g_t \geq 0$ are public goods in period t and $E[.]$ denotes the expectations operator. This utility function can be thought of as a social welfare function that takes into account the preferences of all agents in society. Specifically, society is indifferent about the relative quantities of each public good and only cares about total public good provision and how this is allocated over time. The function $u(x)$ is twice continuously differentiable with $u'(x) > 0$ and $u''(x) < 0$. Further, $u'(x), u''(x) \rightarrow 0$ as $x \longrightarrow \infty$. For convenience, we also assume that $u(0) = 0$ and $u'(1) = 1$. Finally, we abstract from discounting.

The country features two political parties, F and G, which are selected to run the government by an election with a random outcome. The parties have specific ideological preferences over the public goods and party F cares only about public good f_t, while party G only cares about public good g_t. Hence, the utilities of parties F and G are given by, respectively:

$$\begin{aligned} U_F(f_1, g_1, f_2, g_2) &= u(f_1) + E[f_2], \\ U_G(f_1, g_1, f_2, g_2) &= u(g_1) + E[g_2]. \end{aligned}$$

Without loss of generality we assume that party F is in office in period 1. It will be re-elected at the end of the first period with an exogenous probability of p, where $0 < p \leq 1$. Except when we explicitly note otherwise, we assume that $p < 1$. This way we model a very simple political distortion that leads to suboptimally high spending on public goods in the first period and, hence, an overaccumulation of public debt. Debt will be excessive relative to what society at large would choose because the party in office in period 1 realises that it may not be re-elected for a new term, so that any resources available in the second period are spent on the public good that it attaches no utility to.

2.2 Debt and resource constraints

The resource constraints in the two periods are given by:

$$h_1 \equiv f_1 + g_1 = 1 + b, \tag{2}$$
$$h_2 \equiv f_2 + g_2 = 1 + \varepsilon - b(1 + r), \tag{3}$$

where b is the amount of debt that is issued, r the interest rate on this debt and ε a mean-zero shock distributed with density function $g(\varepsilon)$ and support $[\varepsilon_L, \varepsilon_H]$. In the following we confine ourselves to specifications and parameter values such that the outcome for debt is non-negative, i.e. we make:

Assumption 1: $b \geq 0$.

Further, to obtain analytically tractable solutions, we make:

Assumption 2: The shock ε is uniformly distributed on $[\varepsilon_L, \varepsilon_H]$ with mean zero.

For most of our results this is assumption is too restrictive. However, it will allow for simple analytical expressions for the response of the interest rate to a change in the public debt. Because shock ε is mean zero, $\varepsilon_L = -\varepsilon_H$. Hence, $g(\varepsilon) = 1/(\varepsilon_H - \varepsilon_L) = 1/(2\varepsilon_H)$. The term $b(1+r)$ in (3) is the sum of second-period debt repayment plus interest payment. We refer to this sum as the "debt-servicing costs". Debt is issued on the international capital market to risk-neutral investors.

We assume a lower bound on second-period resources of $0 < \rho_L < 1$. This minimum is motivated by the fact that the ability and willingness of a country to tax its population is only limited. As long as this minimum is not reached, debt and interest are paid off. However, if debt-servicing costs are so large that second-period resources would fall below ρ_L, then repayment is limited to the amount that leaves second-period resources exactly at ρ_L. In our view, these assumptions are quite well in line with what we observe in reality – see Panizza et al. (2009) for a recent overview. While in many historical instances of sovereign debt default countries would in principle have been able to repay their debt had they only raised tax rates by enough and cracked down hard enough on tax evasion, there are clearly limits to how far a government can go in burdening its citizen only to repay international investors. It is also unlikely to see governments defaulting when economic circumstances are favourable or only moderately bad. Moreover, the cases discussed by Panizza et al. (2009) show that defaults tend to be partial. Incidentally, to obtain our results we do not need to resort to the assumption of some explicit, but ad hoc, cost of default that may be hard to motivate.

3 No debt mutualisation

First, consider the case when there is no form of debt mutualisation. The first-period government solves:

$$Max_b\, U_F = u(1+b) + p \left[\begin{array}{c} \int_{\varepsilon_L}^{\rho_L + b(1+r) - 1} \rho_L g(\varepsilon)\, \mathrm{d}\varepsilon + \\ \int_{\rho_L + b(1+r) - 1}^{\varepsilon_H} \left[1 + \varepsilon - b(1+r)\right] g(\varepsilon)\, \mathrm{d}\varepsilon \end{array} \right], \quad (4)$$

where r is determined by the requirement that the risk-neutral investors are repaid in expected terms. Hence, if the amount of debt issued is less than or equal to $1 + \varepsilon_L - \rho_L$, then all debt is repaid for sure and $r = 0$. If the debt exceeds $1 + \varepsilon_L - \rho_L$, then r is determined by the smallest non-negative solution to:

$$b = \int_{\varepsilon_L}^{\rho_L + b(1+r)-1} (1 + \varepsilon - \rho_L)\, g(\varepsilon)\, \mathrm{d}\varepsilon + \int_{\rho_L + b(1+r)-1}^{\varepsilon_H} b\,(1+r)\, g(\varepsilon)\, \mathrm{d}\varepsilon. \quad (5)$$

In the case of an adverse shock $\varepsilon_L \leq \varepsilon < \rho_L + b\,(1+r) - 1$, total debt-servicing costs are at most partly honoured, up to the amount $1 + \varepsilon - \rho_L$, while for a benign shock $\varepsilon \geq \rho_L + b\,(1+r) - 1$, debt-servicing costs are fully honoured. Note that a negative interest rate is excluded, because then the risk-neutral investor would under all states of the world receive a repayment less than his initial investment.

In the sequel, we assume that:

Assumption 3: $1 + \varepsilon_L = \rho_L$.

This assumption excludes the possibility that $1 + \varepsilon_L < \rho_L$, in which case, even with zero debt, there would be shock realisations for which second-period resources could fall below the minimum amount ρ_L. The assumption also excludes the possibility that $1 + \varepsilon_L > \rho_L$. This implies that, whenever debt is positive, it is never fully repaid for sure. This last assumption is simply made for convenience in order to simplify the algebra.

Differentiating the government's objective function, and applying Leibniz' integral rule, the first-order condition for an internal optimum is:

$$u'(1+b) = p\left[1 + r + b\frac{\mathrm{d}r}{\mathrm{d}b}\right]\left[1 - \frac{b\,(1+r)}{2\varepsilon_H}\right]. \quad (6)$$

where we have used Assumption 2 and $\varepsilon_L = -\varepsilon_H$. Appendix A shows that

$$\frac{\mathrm{d}r}{\mathrm{d}b} = \frac{1}{b}\frac{2\varepsilon_H - (1+r)\left[2\varepsilon_H - b\,(1+r)\right]}{2\varepsilon_H - b\,(1+r)}. \quad (7)$$

Substituting this into (6), the first-order condition of the government becomes

$$u'(1+b) = p. \quad (8)$$

Differentiating the first derivative $u'(1+b) - p$ of U_F with respect to b, we see that the second-order condition for an internal maximum is fulfilled.

Equation (8) has a unique solution for b, which, by the properties of function $u(.)$, is positive if $p < 1$. Define this solution as $b^{ND} > 0$ and the associated interest rate as r^{ND}, where the superscript ND indicates the case of "no debt mutualisation".

Appendix A also shows that the explicit solution for r is given by $r = 2\frac{\varepsilon_H - \sqrt[2]{\varepsilon_H(\varepsilon_H - b)}}{b} - 1$, hence a proper solution to the government's problem requires that $0 < b \leq \varepsilon_H$. However, it cannot a priori be excluded that b^{ND} exceeds ε_H. Hence, we make:

Assumption 4: The re-election probability is sufficiently high to ensure that $b^{ND} \leq \varepsilon_H$.

It is easy to see that if the debt burden ranges from 0 to ε_H, total debt servicing costs range from 0 to $2\varepsilon_H$. In fact, if $b \rightarrow \varepsilon_H$ the restriction that second-period resources cannot fall below ρ_L becomes binding except for the most favourable shock $\varepsilon = \varepsilon_H$. Only if $\varepsilon = \varepsilon_H$, debt-servicing costs can be honoured in full. In fact, as $b \rightarrow \varepsilon_H$, $r \rightarrow \infty$.

The response of the interest rate r to a change in the debt level deters the government from issuing more debt than b^{ND} when there is a possibility of (partial) default. We can see this as follows. Rewrite (5) as $\int_{\rho_L+b(1+r)-1}^{\varepsilon_H} b(1+r)\, g(\varepsilon)\mathrm{d}\varepsilon = b - \int_{\varepsilon_L}^{\rho_L+b(1+r)-1} (1+\varepsilon-\rho_L)\, g(\varepsilon)\mathrm{d}\varepsilon$ and substitute this into (4), which can now be written as:

$$Max_b\ U_F = u(1+b) + p\left[\int_{\varepsilon_L}^{\varepsilon_H} (1+\varepsilon-b)\, g(\varepsilon)\, \mathrm{d}\varepsilon\right].$$

This is the government's objective function when any default is excluded beforehand, because the restriction that second-period resources cannot fall below ρ_L no longer applies and, hence, the interest rate is zero. The first-order condition for this problem is again (8). By contrast, if we set $r = 0$ and $\mathrm{d}r/\mathrm{d}b = 0$ in (6), this condition reduces to:

$$u'(1+b) = p\left[1 - \frac{b}{2\varepsilon_H}\right]. \tag{9}$$

Because $u'(1+b^{ND}) = p$ and $b^{ND} > 0$, when evaluated at $b = b^{ND}$ the left-hand side of (9) exceeds its right-hand side. Hence, if the interest rate does not react to the amount of debt that is issued any internal solutions for debt implied by (9) must exceed b^{ND}. In other words, the response of the interest rate r is the counterveiling power that prevents additional debt issuance due to the possibility of (partial) default.

4 A debt repayment guarantee

Now we introduce the possibility of debt mutualisation. In this section, it takes the form of a repayment guarantee $\tilde{d} > 0$ on the debt-servicing costs. We distinguish two types of guarantees. Under an *unlimited* guarantee the rest of the union also guarantees repayment when debt-servicing costs exceed $\tilde{d}$, implying that debt-cum-interest payment will always be covered up to the minimum of $b(1+r)$ and $\tilde{d}$. Under a *limited* guarantee the rest of the union only guarantees repayment when debt-servicing costs do not exceed $\tilde{d}$. Hence, debt-servicing costs will be covered up to the minimum of $b(1+r)$ and $\tilde{d}$ as long as $b(1+r)$ does not exceed $\tilde{d}$, while when $b(1+r)$ exceeds $\tilde{d}$ the rest of the union does not provide any financial assistance and the country is solely responsible for servicing its own debt. Effectively, the system then returns to a situation in which there is no guarantee.[3] We assume that the rest of the union is so financially solid, that it will always be able to ensure that the guarantee it has given is honoured.

In solving for the outcomes, we now need to solve two restricted optimisation problems. The first is under the restriction that all debt-servicing costs fall under the guarantee, i.e. $b(1+r) \leq \tilde{d}$, while the other is under the restriction that this is not the case, i.e. $b(1+r) > \tilde{d}$.

Consider first the case $b(1+r) \leq \tilde{d}$. Because debt is fully serviced under any shock realisation, r is determined by the condition $b = \int_{\varepsilon_L}^{\varepsilon_H} b(1+r)\, g(\varepsilon) \mathrm{d}\varepsilon = b(1+r)$, hence $r = 0$. Hence, the government solves (4) with $r = 0$ imposed. Applying Leibniz' rule, the first-order condition for an internal solution is (9), implying that any internal solution when the debt guarantee applies must exceed b^{ND}. As explained above, breaking the relationship between debt and its interest rate by guaranteeing its repayment induces the government to issue more debt. Because the derivative of the government's utility with respect to debt when evaluated at $b = 0$ is positive for $p < 1$, an optimum under the

[3] One could also imagine the rest of the union issuing a guarantee in which debt-servicing costs are taken over only partially if the country issues more debt than $\tilde{d}$ and a bad shock materialises. We do not study this case.

restriction $b \leq \tilde{d}$ is either found for an internal maximum $0 < b < \tilde{d}$ or at $b = \tilde{d}$.

The second derivative of the objective function is $u''(1+b) + p/(2\varepsilon_H)$. We make:

Assumption 5: If it exists, for the smallest solution of (9), $u''(1+b) < -p/(2\varepsilon_H)$. For any other solutions of (9), $u''(1+b) > -p/(2\varepsilon_H)$.

This assumption is more natural than it may appear at first sight, because in the most commonly used case of constant relative risk aversion the third-order derivative u''' is positive and falling in its argument. Hence, for this utility specification the second derivative $u''(1+b)+p/(2\varepsilon_H)$ for the smallest solution to (9) is smaller than that for any other solutions. Assumption 5 ensures that, if it exists, the smallest solution to (9) corresponds to a maximum and any other solutions to a minimum. De facto, this assumption also excludes the possibility that (9) has more than two solutions, because with continuously-differentiable functions two minima cannot exist if there is no maximum in between them. Figure 1 illustrates the first-order conditions (8) and (9), by depicting the left- and right-hand sides of the two equations. The figure ignores the location of $\tilde{d}$. It shows the solution b^{ND} that prevails in the absence of a guarantee. In the presence of a guarantee an internal solution cannot exist if the left-hand side of (9) exceeds its right-hand side for any value $b \geq 0$. In that case the curved line $u'(1+b)$ lies uniformly above the downward-sloping straight line and the objective function is increasing over all $b \leq \tilde{d}$. In the following, if it exists, we denote by $b = b^G < 2\varepsilon_H$ the internal maximum implied by (9). Here, the superscript G indicates the case of a "guarantee". Obviously, this solution can only correspond to an internal global maximum if it does not exceed $\tilde{d}$. Note that (9) implies $\mathrm{d}b/\mathrm{d}p = (2\varepsilon_H - b)/[(2\varepsilon_H)\, u''(1+b)]$. Hence, an increase in the political distortion, i.e. a fall in the re-election probability, raises b^G.

The second case is when $b(1+r) > \tilde{d}$. Again, the government maximises (4), leading to the first-order condition (6). Under the *limited* guarantee, the rest of the union no longer guarantees the debt-servicing costs and, hence, the interest rate is determined by (5), implying that if an internal maximum exists under the restriction $b(1+r) > \tilde{d}$, it is again determined by (8), so that $b = b^{ND}$.

Under the *unlimited* guarantee the interest rate r is now given by the smallest positive solution to:

$$
\begin{aligned}
b &= \int_{\varepsilon_L}^{\varepsilon_L+\tilde{d}} \tilde{d} g(\varepsilon)\, \mathrm{d}\varepsilon + \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+b(1+r)} (\varepsilon - \varepsilon_L) g(\varepsilon)\, \mathrm{d}\varepsilon \\
&\quad + \int_{\varepsilon_L+b(1+r)}^{\varepsilon_H} b(1+r) g(\varepsilon)\, \mathrm{d}\varepsilon,
\end{aligned} \tag{10}
$$

where we have used that $\varepsilon_L = \rho_L - 1$. Hence, for really bad shocks the guarantee is invoked and debt-servicing costs are honoured to a level of $\tilde{d}$ (the first term on the right-hand side of (10)). For intermediate shocks the full debt-servicing costs are only partially honoured, but to a level larger than $\tilde{d}$ (the second term on the right-hand side of (10)). Finally, for good shocks, debt-servicing costs are fully honoured – see the third term on the right-hand side of (10). Appendix B shows that $\mathrm{d}r/\mathrm{d}b$ is also now given by (7) and, hence, the first-order condition for an internal optimum is again (8), which implies that $b = b^{ND}$.[4]

By Assumption 5, and because $b^G > b^{ND}$, under both types of guarantees the full objective function of the government features at most one internal maximum. If $\tilde{d} < b^{ND}$, the only internal maximum can be at $b = b^{ND}$. If $\tilde{d} > b^{ND}$, the only internal maximum can be at $b = b^G$. However, the global maximum to the government's problem may or may not correspond to an internal maximum, as Figure 2 shows. The figure draws the government's objective function under both types of guarantees. The thick pieces are the parts over which the government optimises. The thick piece for $b \leq \tilde{d}$ is relevant under both types of guarantees. Here, for any given positive debt level, the objective function is higher than when the debt would not be guaranteed (the dashed line), because under the guarantee the government pays zero interest, while without the guarantee it would pays positive interest leading to a lower expected level of resources in the second period. As debt passes through the threshold $\tilde{d}$, under the limited guarantee interest payments jump from zero to a positive value, because none of the debt is any longer guaranteed, and the government's objective function jumps down to the lower thick line. Under the unlimited guarantee, it is easy to show that as $b \downarrow \tilde{d}$, then $r \downarrow 0$. Hence, interest payments gradually build up as debt increases further

[4]Appendix B also derives an explicit (positive) solution for the interest rate r. The admitted range of debt values for which this solution exists is wider than in the absence of the guarantee. Further, as expected, if $b \downarrow \tilde{d}$, hence the risk of non-repayment falls to zero, then $r \downarrow 0$.

beyond $\tilde{d}$, and government's objective function follows the upper thick line piece to the right of $\tilde{d}$. For the specific case drawn in Figure 2, under the limited guarantee the global maximum is achieved at $b = \tilde{d}$, while under the unlimited guarantee it is achieved at $b = b^{ND}$.

Summarising, we have:

Result 1: Consider the presence of a debt guarantee $\tilde{d} > 0$. (i) There can be at most one internal maximum to the government's optimisation problem. (ii) Under a limited guarantee, if $\tilde{d} \leq b^{ND}$, the global maximum is either achieved at $b = \tilde{d}$ or at $b = b^{ND}$, while if $\tilde{d} > b^{ND}$, the global maximum is either achieved at $b = b^G$ or at $b = \tilde{d}$. Hence, debt may be lower than, equal to, or higher than debt in the absence of a guarantee. (iii) Under an unlimited guarantee, if $\tilde{d} < b^{ND}$, the global maximum is achieved at $b = b^{ND}$, while if $\tilde{d} > b^{ND}$, the global maximum is either achieved at $b = b^G$ or at $b = \tilde{d}$. Hence, debt is equal to or higher than debt in the absence of a guarantee.

5 Blue and red bonds

In this section we assume that the government can issue two types of debt, in line with the "blue and red bond" proposal made by Delpla and Von Weizsäcker (2010). We will see that the outcomes for this arrangement largely coincide with those for the unlimited debt guarantee. The blue type of debt, also referred to as "senior" and indexed by superscript "s", is fully guaranteed and, therefore, carries an interest rate of zero. The amount of blue debt cannot exceed $\tilde{d}$. The red type of debt, also referred to as "non-senior" and indexed by superscript "n" is not guaranteed. Because it will carry a positive interest rate, it will only be issued when the amount of blue debt is set to its allowed maximum $\tilde{d}$.

Again, the government solves two optimisation problems and selects the debt level that yields its highest utility. The first case is when total debt does not exceed $\tilde{d}$ and the government only issues blue debt. Hence, the government maximises (4) over b^s, subject to the restrictions $b^s \leq \tilde{d}$ and $r = 0$. The first-order condition is again (9) and, hence, the analysis of this case is same as before under the repayment guarantee.

The second case is when the total debt exceeds $\tilde{d}$. In other words, $b^s = \tilde{d}$ and $b^n > 0$. In contrast to the case with the unlimited repayment guarantee, now only the excess of debt over $\tilde{d}$, i.e. $b^n > 0$, will carry a positive interest rate. Now, the government solves

$$Max_{b^n}\ U_F = u(1+\tilde{d}+b^n) + p\left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L+\tilde{d}+b^n(1+r^n)} \rho_L g(\varepsilon)\,\mathrm{d}\varepsilon + \\ \int_{\varepsilon_L+\tilde{d}+b^n(1+r^n)}^{\varepsilon_H} \left[1+\varepsilon-\tilde{d}-b^n(1+r^n)\right] g(\varepsilon)\,\mathrm{d}\varepsilon \end{array}\right], \tag{11}$$

subject to the restriction that $b^n > 0$ and where r^n is determined by the smallest positive solution to:

$$b^n = \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+\tilde{d}+b^n(1+r^n)} \left(\varepsilon-\varepsilon_L-\tilde{d}\right) g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L+\tilde{d}+b^n(1+r^n)}^{\varepsilon_H} b^n(1+r^n)\, g(\varepsilon)\,\mathrm{d}\varepsilon. \tag{12}$$

Note that this expression differs from (10). Because red debt does not carry any guarantee, its repayment kicks in only when $\varepsilon \geq \varepsilon_L + \tilde{d}$, i.e. when the shock is large enough to repay at least all the blue debt. Hence, in contrast to the case with the unlimited repayment guarantee there is a range of shocks $\varepsilon_L \leq \varepsilon < \varepsilon_L + \tilde{d}$, over which the holders of red debt are not repaid. Obviously, the interest rate r^n needs to adjust to compensate for the expected losses on the red debt when the shock falls in this range.

Now, the government's first-order condition for an internal optimum is:

$$u'(1+\tilde{d}+b^n) = p\left[(1+r^n) + b^n\frac{\mathrm{d}r^n}{\mathrm{d}b^n}\right]\left[\frac{2\varepsilon_H-\tilde{d}-b^n(1+r^n)}{2\varepsilon_H}\right].$$

Appendix C shows that

$$\frac{\mathrm{d}r^n}{\mathrm{d}b^n} = \frac{1}{b^n}\frac{2\varepsilon_H-(1+r^n)\left[2\varepsilon_H-b^n(1+r^n)-\tilde{d}\right]}{2\varepsilon_H-b^n(1+r^n)-\tilde{d}}. \tag{13}$$

Substitution into the first-order condition yields:

$$u'(1+\tilde{d}+b^n) = p. \tag{14}$$

Obviously, the second-order condition is fulfilled. Hence, an internal solution requires that $b = \tilde{d}+b^n = b^{ND}$.

For $\tilde{d} < b^{ND}$ the government's optimisation problem can be illustrated in a figure very similar to Figure 2 for the case of the unlimited guarantee. As b^n increases, interest payments gradually increase and the wedge between the function (4) with $r = 0$ and the function (11) with the interest rate determined by (12) gradually widens. We have the following result:

Result 2: Consider the red and blue bonds arrangement, with a threshold $\tilde{d} > 0$ on the amount of blue bonds. If $\tilde{d} \leq b^{ND}$, the global maximum is reached at $b = \tilde{d} + b^n = b^{ND}$. If $\tilde{d} > b^{ND}$, the global maximum is either reached at $b = b^G$ or at $b = \tilde{d}$. Hence, debt is equal to or larger than in the absence of any form of debt mutualisation.

6 Social welfare and optimal arrangements

To explore optimal arrangements we need to have a benchmark for evaluation. As the benchmark we will take the social welfare function (1), modified to take account of the financial assistance provided by the rest of union in servicing the public debt.

6.1 Social welfare

Since the public goods F and G are perfectly substitutable in the social welfare function (1), the precise division of each period's resources over the two goods is irrelevant from society's perspective. What is relevant, though, is the distribution of resources over time, hence the optimal amount of debt to be issued.

In the absence of any form of debt mutualisation we take as the relevant benchmark the social welfare function (1). The optimal debt level simply follows from maximising (4) with $p = 1$ imposed, subject to (5). Hence, the first-order condition is given by (8) for $p = 1$. Since, $u'(1) = 1$, the socially-optimal amount of debt is:

$$b = 0. \tag{15}$$

For $b > 0$ the social welfare function is monotonically decreasing in debt.

In the presence of a debt repayment guarantee, social welfare is "artificially" increased by the resources provided by the rest of the union when it needs to step in to ensure that the guarantee is honoured. Therefore, we take as our benchmark a "modified social welfare function", obtained by subtracting from the original social welfare function the resources supplied by the rest of the union under the various possible states of the world in period 2. Hence, the modified social welfare function takes account of the full consequences of our government's debt policy, because it incorporates the externality imposed on the rest of the union. For $b \leq \tilde{d}$ the modified social welfare function is:

$$u(1+b) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L+b} \left[\rho_L - (b + \varepsilon_L - \varepsilon)\right] g(\varepsilon)\, \mathrm{d}\varepsilon + \\ \int_{\varepsilon_L+b}^{\varepsilon_H} \left[1 + \varepsilon - b\right] g(\varepsilon)\, \mathrm{d}\varepsilon \end{array} \right], \tag{16}$$

where the term $(b + \varepsilon_L - \varepsilon)$ captures the resources provided by the rest of the union to guarantee the debt repayment. For the worst possible shock $\varepsilon = \varepsilon_L$, the need for resources from the rest of the union is at its maximum b. When ε approaches $b + \varepsilon_L$ the amount of resources needed from the rest of the union approaches zero. Appendix E shows that (16) can be rewritten as (4) for $p = 1$ subject to (5).

Under a *limited* debt guarantee, for $b(1+r) > \tilde{d}$ the modified social welfare function is directly given by the social welfare function, i.e. (4) for $p = 1$ and subject to (5). Hence, under the limited debt guarantee for all $b \geq 0$ the modified social welfare function coincides with the social welfare function in the absence of debt mutualisation. Appendix E shows that this is also the case for the unlimited debt guarantee and the red and blue bonds scheme.

In other words, for $b \geq 0$, under all debt mutualisation schemes the modified social welfare function coincides with the social welfare function in the absence of debt mutualisation. Hence, modified social welfare reaches a maximum at $b = 0$ and is monotonically decreasing for $b > 0$.

6.2 Optimal arrangements

The debt guarantee $\tilde{d} \geq 0$ or the threshold $\tilde{d} \geq 0$ on the amount of blue bonds is an instrument that some system designer, for example the European

Council (the Heads of Government or State of the EU) or the European Commission, in principle could try to employ for improving social welfare by inducing the government to issue less debt. However, because under the unlimited debt guarantee and under the blue-red bonds proposal the government's utility is increasing in b for $b < b^{ND}$, for these arrangements it is not possible to set $\tilde{d}$ so as to induce less debt issuance than in the absence of any form of debt mutualisation.

By contrast in the case of a limited debt guarantee a suitable choice of $\tilde{d}$ is able to produce an increase in social welfare. Figure 3, which is based on Figure 2, shows how. The figure also draws the modified social welfare function. The government achieves its global maximum at $b = \tilde{d}^{opt}$.[5] The figure shows that by setting $\tilde{d}$ at the level $\tilde{d}^{opt}$ that makes the government's utility under the limited guarantee equal to the maximum utility in the absence of the guarantee, the government is induced to choose a debt level lower than b^{ND}. Setting $\tilde{d} < \tilde{d}^{opt}$ would lead the government to set $b = b^{ND}$, while setting $\tilde{d} > \tilde{d}^{opt}$ would yield $b = \tilde{d}$. In both cases, debt would be higher than $\tilde{d}^{opt}$. We have the following proposition:

Proposition 1: Suppose that the government is free to join a scheme involving a limited debt guarantee $\tilde{d}$. Then, a suitable choice of $\tilde{d}$ can reduce the debt bias due to the political distortion and raise social welfare. The maximum debt reduction and, hence, the maximum improvement in social welfare, is achieved for the guarantee $\tilde{d} < b^{ND}$, such that at $b = \tilde{d}$ the government's utility is equal to the government's utility at $b = b^{ND}$ in the absence of any debt mutualisation arrangement.

7 Structural reform

Delpla and Von Weizsäcker (2010) argue that eligibility for participation in a debt mutualisation scheme could be made conditional on being on an appropriate structural reform path. From an economic viewpoint such conditionality seems to make sense, since a properly reformed economy has more growth potential and is, therefore, less likely to run into trouble repaying its

[5] We break ties by assuming that if different debt levels yield the same utility for the government, then it selects the lowest of these debt levels.

debt and calling upon countries to provide financial assistance. In addition, if participation in some debt mutualisation scheme can be made sufficiently attractive, this in itself might induce countries to reform more. In this section we will scrutinise precisely this argument. We limit ourselves to the limited debt guarantee, as we found above that the other arrangements could not induce a reduction in debt, thereby raising social welfare.

7.1 Analysis of the modified model

We take the original model, modifying it in the vain of Beetsma and Jensen (2003). Utility of political party F is now given by

$$U_F(f_1, g_1, f_2, g_2) = -v(e) + u(f_1) + E[f_2],$$

where $e \geq 0$ stands for the amount of structural reform or "effort", while $v(.)$ is twice continuously differentiable with $v'(.) \geq 0$ and $v''(.) \geq 0$. For convenience, we assume that $v'(e) = 0$ for some value $e < 0$. Reforming the economy is costly in terms of the government's utility, because it reduces its political or public support. Further, through the convexity of $v(.)$ we implicitly assume that the disutility from a given increase in reform is bigger when the starting level of reform is higher. The benefit of more reform is indirect in the sense that it makes the economy more efficient, which is captured by an increase in the amount of resources in the second period such that:

$$h_2 \equiv f_2 + g_2 = 1 + \beta e + \varepsilon - b(1+r). \tag{17}$$

Parameter $\beta > 0$ measures the efficiency gains from reform. Obviously, more reform e reduces the chances of partial default for a given debt-servicing burden $b(1+r)$. The government now selects two instruments in the first period: the debt level and the amount of reform.

Consider first the case without the limited debt guarantee. Hence, the government solves:

$$Max_{b,e} \; -v(e) + u(1+b) + p \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L + b(1+r) - \beta e} \rho_L g(\varepsilon) \, \mathrm{d}\varepsilon + \\ \int_{\varepsilon_L + b(1+r) - \beta e}^{\varepsilon_H} [1 + \beta e + \varepsilon - b(1+r)] \, g(\varepsilon) \, \mathrm{d}\varepsilon \end{array} \right], \tag{18}$$

where r is determined by the smallest positive solution to:

$$b = \int_{\varepsilon_L}^{\varepsilon_L + b(1+r) - \beta e} (\beta e + \varepsilon - \varepsilon_L)\, g(\varepsilon)\, \mathrm{d}\varepsilon + \int_{\varepsilon_L + b(1+r) - \beta e}^{\varepsilon_H} b(1+r)\, g(\varepsilon)\, \mathrm{d}\varepsilon, \tag{19}$$

where we again repeatedly used Assumption 3. The first term on the right-hand side of (19) corresponds to partial repayment, because the shock realisation is unfortunate. In this case, second-period resources only equal ρ_L. The second term corresponds to the case of full repayment.

In the following we limit ourselves to situations in which $\varepsilon_L + b(1+r) - \beta e > \varepsilon_L$, hence $b(1+r) > \beta e$. In line with this, we assume that in the presence of a guarantee the guarantee exceeds βe and analogous to Assumption 1 we make:

Assumption 1′: $b(1+r) \geq \beta e$.

Appendix D shows that in the absence of a guarantee

$$\frac{\mathrm{d}r}{\mathrm{d}b} = \frac{1}{b} \frac{2\varepsilon_H - (1+r)\left[2\varepsilon_H + \beta e - b(1+r)\right]}{2\varepsilon_H + \beta e - b(1+r)}, \tag{20}$$

$$\frac{\mathrm{d}r}{\mathrm{d}e} = \frac{\beta}{b} \frac{\beta e - b(1+r)}{2\varepsilon_H + \beta e - b(1+r)}. \tag{21}$$

Hence, under Assumption 1′ and assuming that $b(1+r) < 2\varepsilon_H + \beta e$, the interest rate is decreasing in the effort level: more effort means more repayment capacity and, hence, a reduced chance of (partial) default in the second period. Therefore, investors demand a lower interest to be prepared to hold the debt. Substituting (20) and (21) into the first-order conditions for b and e yields:

$$u'(1+b) = p, \tag{22}$$

$$v'(e) = \beta p. \tag{23}$$

The second-order condition is fulfilled, because the second derivatives of the objective function with respect to e and b, respectively $-v''(e)$ and $u''(1+b)$, are both negative, while the second-order cross-derivative with respect to e and b is zero. Hence, the Hessian is negative definite. From the first-order conditions, we extract the following results:

Result 3: Consider the model with structural reform. In the absence of a debt guarantee, (i) the optimal debt choice is the same as in the model without structural reform and, hence, is given by b^{ND},[6] and (ii) an increase in the political distortion (a fall in p) raises debt and reduces reform.

It is not surprising that reform responds positively to an increase in the likelihood of re-election. A higher chance to be re-elected makes it more likely that the government can benefit from its reform efforts by spending more resources on its own preferred public good. We denote the optimal level of reform by e^*. For convenience, we normalise the function $v(.)$ such that $e^* = 0$.

The question is whether by making eligibility for the debt guarantee scheme conditional on meeting a certain level of reform, the government can be induced to increase its reform, thereby raising social welfare and reducing the likelihood that other countries need to step in to assist in servicing the debt. Suppose that participation in the scheme requires the government to at least implement an amount of reform $\tilde{e} \geq e^* = 0$. Hence, the government compares the indirect utilities under the following choices: staying out of the scheme, in which case it sets $b = b^{ND}$ and $e = e^* = 0$, as prescribed by (22) and (23). Alternatively, the government enters the scheme and sets reform at the optimal level allowed by the restriction $e \geq \tilde{e}$. Again, the government solves for the optimal amount of debt subject to $b \leq \tilde{d}$ and the optimal amount subject to $b(1+r) > \tilde{d}$. It selects the instrument combination that yields the highest utility and compares this to its utility from staying outside the scheme.

Under the debt guarantee, the first leg of the government's optimisation problem is

[6] We assume again that the re-election probability is sufficiently high to ensure the existence of a (positive) solution for the interest rate. With the normalisation for effort introduced below, the lower bound on the re-election probability is the same as implied by Assumption 4.

$$Max_b \; -v(e)+u(1+b)+p\left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L+b(1+r)-\beta e}\rho_L g(\varepsilon)\,\mathrm{d}\varepsilon + \\ \int_{\varepsilon_L+b(1+r)-\beta e}^{\varepsilon_H}\left[1+\beta e+\varepsilon-b(1+r)\right]g(\varepsilon)\,\mathrm{d}\varepsilon \end{array}\right], \tag{24}$$

subject to $b \leq \tilde{d}$, $e \geq \tilde{e}$ and $r = 0$. The first-order conditions for an internal maximum are:

$$u'(1+b) = p\left[1-\frac{b-\beta e}{2\varepsilon_H}\right], \tag{25}$$

$$v'(e) = \beta p\left[1-\frac{b-\beta e}{2\varepsilon_H}\right], \tag{26}$$

which differ from (22) and (23) only by the term in square brackets on the right-hand sides of both equations. Since this term is less than one under Assumption $1'$, and by the properties of the functions $u(.)$ and $v(.)$, any internal maximum must involve debt exceeding b^{ND} and effort lower than $e^* = 0$. Hence, by eliminating the advantage of more effort in terms of a lower interest rate, the debt guarantee creates a moral hazard problem with regard to the choice of effort. The second-period savings made by not paying interest on the debt reduce the need for reform. An internal maximum requires that $2\varepsilon_H u''(1+b)+p<0$ and $-2\varepsilon_H v''(e)+p\beta^2<0$. Hence, differentiating (25), at an internal maximum,

$$\frac{\mathrm{d}b}{\mathrm{d}e} = \frac{p\beta}{2\varepsilon_H u''(1+b)+p} < 0.$$

The intuition is that an increase in e widens the range of shocks, $1+\beta e+\varepsilon-b>\rho_L$, for which an increase in debt effectively leads to a reduction in second-period resources. This induces the government to issue less debt.

In the sequel we assume that the curvature of function $v(.)$ is so strong that the second derivative of the objective function with respect to effort, $-v''(e)+p\beta^2/(2\varepsilon_H)$, is negative for all e. Hence, because the first derivative of the objective function of the government with respect to e is decreasing in e for $e \geq e^* = 0$, for any given debt level it is optimal for the government to set $e = \tilde{e} > e^*$. Analogous to Assumption 5, we now make:

Assumption 5′: Set $e = \tilde{e}$. If it exists, for the smallest solution b of (25), $u''(1+b) < -p/(2\varepsilon_H)$. For any other solutions of (25), $u''(1+b) > -p/(2\varepsilon_H)$.

Hence, the government's optimisation problem under the restrictions $b \leq \tilde{d}$ and $e \geq \tilde{e}$ yields $(b, e) = \left(\min\left[\tilde{d}, \tilde{b}^G\right], \tilde{e}\right)$, where $\tilde{b}^G > b^{ND}$ is the solution of (25) for $e = \tilde{e}$, if it exists. Otherwise, $b = \tilde{d}$.

Because the guarantee is conditional on debt not exceeding $\tilde{d}$, the second leg of the government's problem is to maximise (18) subject to $b(1+r) > \tilde{d}$ and $e \geq \tilde{e}$, where r is the smallest positive solution to (19). Hence, the new first-order conditions for an internal solution for b and e are again given by (22) and (23). In combination with the Hessian being negative definite, this implies that for any given debt level the government's utility is decreasing in effort for effort exceeding e^*. Hence, the government optimally sets $e = \tilde{e}$. The full solution of the government's optimisation problem under the restrictions $b(1+r) > \tilde{d}$ and $e \geq \tilde{e}$ is $(b, e) = \left(\max\left[\tilde{d}^+, b^{ND}\right], \tilde{e}\right)$, where $b = \tilde{d}^+$ is defined as the debt level larger than, but arbitrarily close to, $\tilde{d}$.

Taking the two partial optimisations under the guarantee together, analogous to Result 1 we have

Result $1'$: Consider participation in a scheme with a limited debt guarantee $\tilde{d} \geq \beta\tilde{e}$ conditional on $e \geq \tilde{e}$. (i) The government sets $e = \tilde{e}$. (ii) Given $e = \tilde{e}$, there can be at most one internal maximum for the government's debt choice. (iii) If $\tilde{d} < b^{ND}$, the global maximum is either achieved at $b = \tilde{d}$ or at $b = b^{ND}$, while if $\tilde{d} > b^{ND}$, the global maximum is either achieved at $b = \tilde{b}^G$ or at $b = \tilde{d}$. Hence, debt may be lower than, equal to, or higher than debt in the absence of a guarantee.

7.2 Social welfare

We assume that social welfare is still given by (1). Hence, while the cost of reform is born only by the government, society at large always benefits from more reform, because it raises the amount of resources in the second period. In the absence of the debt guarantee, the government would choose $e = e^* = 0$ and, given this effort level, the socially-optimal amount of debt follows from maximising (4) with $p = 1$ imposed, subject to (5). As before, the optimal debt level in this case is $b = 0$ and social welfare is decreasing in debt when debt is positive.

The above analysis has shown that with the debt guarantee, reform is given by $e = \tilde{e}$ both when $b \leq \tilde{d}$ and $b(1+r) > \tilde{d}$. Appendix E shows that, for given effort level $e = \tilde{e}$, modified social welfare, which subtracts the financial support provided by the rest of the union from the social welfare function when $b \leq \tilde{d}$, can be written as:

$$u(1+b) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L + b(1+r) - \beta\tilde{e}} \rho_L g(\varepsilon) \, \mathrm{d}\varepsilon + \\ \int_{\varepsilon_L + b(1+r) - \beta\tilde{e}}^{\varepsilon_H} \left[1 + \beta\tilde{e} + \varepsilon - b(1+r)\right] g(\varepsilon) \, \mathrm{d}\varepsilon \end{array} \right],$$

where r is determined by the smallest positive solution to (19) with $e = \tilde{e}$ imposed. This function is identical to social welfare in the absence of the debt guarantee scheme for $e = \tilde{e}$. The function is again maximised at $b = 0$ and decreasing in debt when debt is positive.

7.3 Optimal arrangements

The question is whether the introduction of a limited debt guarantee can again raise social welfare. The answer is yes. Figure 4 illustrates this. The figure depicts the government's utility when the country stays out of the scheme and optimally sets effort $e^* = 0$ and debt $b = b^{ND}$. It also depicts government utility when joining the scheme and effort is equal to $e = \tilde{e}$. The government optimises over the thick line pieces, the left one of which corresponds to the case in which debt does not exceed $\tilde{d}$ and the interest rate is zero, while the other corresponds to the case in which debt exceeds $\tilde{d}$ and the interest rate is positive. When debt crosses the threshold $\tilde{d}$, utility jumps down, because none of the debt is protected any longer by the guarantee. For $b > \tilde{d}$, under the guarantee scheme government utility lies below utility outside the limited guarantee system, because effort $\tilde{e}$ under the scheme exceeds optimal effort e^* outside the system. Obviously, if the government intends to issue more debt than $\tilde{d}$, it would never do this under a limited debt guarantee, because it would rather not join the scheme. We need to compare the government's global maximum under the limited guarantee with the global maximum outside the scheme achieved at $b = b^{ND}$. For the case shown in Figure 4, the government chooses to join the limited guarantee scheme and sets $b = \tilde{d}$.

Figure 4 also depicts (modified) social welfare as a function of debt both outside the guarantee system and under the guarantee system. For given debt level, modified social welfare under the guarantee lies above social welfare outside the guarantee system, because reform effort is higher. Now, the system designer has two instruments, the debt guarantee level $\tilde{d}$ and the threshold for reform effort $\tilde{e}$. Any alternatives under the guarantee scheme should at least yield government utility equal to the highest utility outside the guarantee scheme achieved for $b = b^{ND}$ and $e = e^*$. Figure 4 depicts a combination $\left(\tilde{d}, \tilde{e}\right)$ for which the maximised government utility levels in the two cases are identical. Under the guarantee scheme (modified) social welfare is higher than outside the scheme for two reasons: $b < b^{ND}$ and $e = \tilde{e} > e^*$. Trying to raise modified social welfare by lowering $\tilde{d}$ or raising $\tilde{e}$, which shifts (24) down for any given level of debt, leads the government to stay outside the guarantee system and, hence, these gains are foregone. In fact, the system designer faces a trade-off: by setting $\tilde{d}$ higher, it can also set $\tilde{e}$ higher, and vice versa. Obviously, the optimal combination $\left(\tilde{d}, \tilde{e}\right)$ depends on the marginal effect β of effort on second-period resources.

Summarising, we have the following proposition:

Proposition 2: A limited debt guarantee conditional on sufficient reform can raise (modified) social welfare, both by inducing more reform effort or by reducing public debt, or a combination of both. The optimal instrument setting $\left(\tilde{d}, \tilde{e}\right)$ involves a trade off. Setting the threshold $\tilde{e}$ for effort higher requires a higher repayment guarantee $\tilde{d}$ to induce the government to still join the scheme, while a reduction in the guarantee $\tilde{d}$ requires a lower effort threshold.

8 Conclusion

The call for some form of debt mutualisation in the eurozone is becoming louder. This paper has provided a formal analysis of different forms of debt mutualisation in the context of a model with a political distortion and the possibility of (partial) default if debt-servicing costs and adverse economic circumstances drive resources below some minimum. We explored a limited and an unlimited debt repayment guarantee and the blue and red bonds proposal. Under the latter two arrangements, public debt is at least equal to the (excessive) debt level in the absence of debt mutualisation, and potentially higher. Hence, social welfare cannot be raised under these two arrangements. By contrast, if the debt guarantee is made conditional such that all financial support from the rest of the union is lost if the guarantee threshold is exceeded, a suitable choice of this guarantee level can induce a reduction in the equilibrium debt level, thereby raising social welfare. Introducing structural reform into the model, we found that making participation in the guarantee scheme conditional on sufficient reform may induce governments to reform more. However, there is a trade-off between the amount of additional reform that can be extracted and the amount by which the government can be pursuaded to reduce their debt.

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APPENDIX

A Derivation of dr/db based on (5)

Working out condition (5) yields:

$$\begin{aligned} b = \; & \left[\tfrac{1-\rho_L}{\varepsilon_H-\varepsilon_L}\right]\left[\rho_L + b\left(1+r\right) - 1 - \varepsilon_L\right] + \\ & \left[\tfrac{1}{2(\varepsilon_H-\varepsilon_L)}\right]\left[\left(\rho_L + b\left(1+r\right) - 1\right)^2 - \varepsilon_L^2\right] + \\ & \left[\tfrac{1}{\varepsilon_H-\varepsilon_L}\right] b\left(1+r\right)\left[\varepsilon_H - \rho_L + 1 - b\left(1+r\right)\right]. \end{aligned} \tag{27}$$

Differentiating yields:

$$
\begin{aligned}
(\varepsilon_H - \varepsilon_L)\,\mathrm{d}b &= (1-\rho_L)\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] + \\
&\quad \left[\rho_L + b(1+r) - 1\right]\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] + \\
&\quad \left[\varepsilon_H - \rho_L + 1 - b(1+r)\right]\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] - \\
&\quad b(1+r)\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
(\varepsilon_H - \varepsilon_L)\,\mathrm{d}b &= \left[\varepsilon_H - \rho_L + 1 - b(1+r)\right]\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] \Leftrightarrow \\
\frac{\mathrm{d}r}{\mathrm{d}b} &= \frac{1}{b}\frac{(\varepsilon_H - \varepsilon_L) - (1+r)\left[\varepsilon_H - \rho_L + 1 - b(1+r)\right]}{\varepsilon_H - \rho_L + 1 - b(1+r)} \\
&= \frac{1}{b}\frac{2\varepsilon_H - (1+r)\left[2\varepsilon_H - b(1+r)\right]}{2\varepsilon_H - b(1+r)}.
\end{aligned}
$$

Actually, we can solve explicitly for the interest rate as a function of debt b. Write out (27):

$$
\begin{aligned}
(\varepsilon_H - \varepsilon_L)\, b &= -(1-\rho_L)^2 + (1-\rho_L)\, b(1+r) - (1-\rho_L)\,\varepsilon_L + \\
&\quad \tfrac{1}{2}(\rho_L - 1)^2 + \tfrac{1}{2}b^2(1+r)^2 + (\rho_L - 1)\, b(1+r) - \tfrac{1}{2}\varepsilon_L^2 + \\
&\quad b(1+r)\,\varepsilon_H - (\rho_L - 1)\, b(1+r) - b^2(1+r)^2
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & (\varepsilon_H - \varepsilon_L)\, b = -\tfrac{1}{2}(\rho_L - 1)^2 + (\rho_L - 1)\,\varepsilon_L - \tfrac{1}{2}\varepsilon_L^2 - \tfrac{1}{2}b^2(1+r)^2 + \\
& b(1+r)\,\varepsilon_H + (1-\rho_L)\, b(1+r)
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & 2(\varepsilon_H - \varepsilon_L)\, b + (\rho_L - 1)^2 + 2(1-\rho_L)\,\varepsilon_L + \varepsilon_L^2 + b^2(1+r)^2 \\
& -2b(1+r)\,\varepsilon_H - 2(1-\rho_L)\, b(1+r) = 0
\end{aligned}
$$

$$
\Leftrightarrow b^2(1+r)^2 - 2(1+\varepsilon_H - \rho_L)\, b(1+r) + 2(\varepsilon_H - \varepsilon_L)\, b + (1+\varepsilon_L - \rho_L)^2 = 0
$$

$$
\Leftrightarrow b^2(1+r)^2 - 2(1+\varepsilon_H - \rho_L)\, b(1+r) + 2(\varepsilon_H - \varepsilon_L)\, b = 0,
$$

where we have used Assumption 3. Further, using $\varepsilon_H = -\varepsilon_L = 1 - \rho_L$, this can be simplified further as:

$$\begin{aligned} b^2 (1+r)^2 - 4\varepsilon_H b (1+r) + 4\varepsilon_H b &= 0 \Leftrightarrow \\ b (1+r)^2 - 4\varepsilon_H (1+r) + 4\varepsilon_H &= 0. \end{aligned}$$

Hence,

$$\begin{aligned} r &= \frac{4\varepsilon_H \pm \sqrt[2]{16\varepsilon_H^2 - 16\varepsilon_H b}}{2b} - 1 \\ &= 2\frac{\varepsilon_H \pm \sqrt[2]{\varepsilon_H (\varepsilon_H - b)}}{b} - 1. \end{aligned}$$

By Assumption 1 we have already excluded negative values of b. Hence, a solution for r requires that

$$0 < b \leq \varepsilon_H = 1 - \rho_L.$$

The relevant solution for r is the smallest solution, given by:

$$r = 2\frac{\varepsilon_H - \sqrt[2]{\varepsilon_H (\varepsilon_H - b)}}{b} - 1.$$

It is easy to show that $r > 0$ for $b > 0$. Further, $b(1+r) = 2\left[\varepsilon_H - \sqrt[2]{\varepsilon_H (\varepsilon_H - b)}\right]$. Hence, when b ranges from 0 to ε_H, $b(1+r)$ ranges from 0 to $2\varepsilon_H$.

B Derivation of (7) based on (10)

Working out condition (10) yields:

$$\begin{aligned} b = {} & \tfrac{\tilde{d}^2}{2\varepsilon_H} + \tfrac{1}{2}\left(b(1+r) - \tilde{d}\right) + \\ & \left[\tfrac{1}{4\varepsilon_H}\right]\left[(\varepsilon_L + b(1+r))^2 - \left(\varepsilon_L + \tilde{d}\right)^2\right] + \\ & \left[\tfrac{1}{2\varepsilon_H}\right] b(1+r)\left[2\varepsilon_H - b(1+r)\right] \end{aligned}$$

Differentiating:

$$
\begin{aligned}
2\varepsilon_H \mathrm{d}b &= \varepsilon_H \left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] + \\
&\quad \left[\varepsilon_L + b\,(1+r)\right]\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] + \\
&\quad \left[2\varepsilon_H - b\,(1+r)\right]\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] - \\
&\quad b\,(1+r)\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] \Leftrightarrow \\
2\varepsilon_H \mathrm{d}b &= \left[2\varepsilon_H - b\,(1+r)\right]\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] \Leftrightarrow \\
\frac{\mathrm{d}r}{\mathrm{d}b} &= \frac{1}{b}\frac{2\varepsilon_H - (1+r)\left[2\varepsilon_H - b\,(1+r)\right]}{2\varepsilon_H - b\,(1+r)}.
\end{aligned}
$$

As before, we can solve explicitly for the interest rate as a function of debt b. Write out (10):

$$
\begin{aligned}
2\varepsilon_H b &= \tilde{d}^2 + \tfrac{1}{2}\left(\varepsilon_L + b\,(1+r)\right)^2 - \tfrac{1}{2}\left(\varepsilon_L + \tilde{d}\right)^2 - \varepsilon_L\left(\varepsilon_L + b\,(1+r)\right) \\
&\quad + \varepsilon_L\left(\varepsilon_L + \tilde{d}\right) + b\,(1+r)\left(2\varepsilon_H - b\,(1+r)\right)
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad 2\varepsilon_H b = \tilde{d}^2 + \tfrac{1}{2}\left(\varepsilon_L + b\,(1+r)\right)^2 - \tfrac{1}{2}\left(\varepsilon_L + \tilde{d}\right)^2 \\
-\varepsilon_L\left(b\,(1+r) - \tilde{d}\right) + b\,(1+r)\left(2\varepsilon_H - b\,(1+r)\right)
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad 2\varepsilon_H b = \tilde{d}^2 + \tfrac{1}{2}\varepsilon_L^2 + b\,(1+r)\,\varepsilon_L + \tfrac{1}{2}\left(b\,(1+r)\right)^2 - \tfrac{1}{2}\left(\varepsilon_L + \tilde{d}\right)^2 \\
-\varepsilon_L\left(b\,(1+r) - \tilde{d}\right) + b\,(1+r)\left(2\varepsilon_H - b\,(1+r)\right)
\end{aligned}
$$

$$
\Leftrightarrow 2\varepsilon_H b = \tfrac{1}{2}\tilde{d}^2 + 2b\,(1+r)\,\varepsilon_H - \tfrac{1}{2}\left(b\,(1+r)\right)^2
$$

$$
\Leftrightarrow b^2\,(1+r)^2 - 4b\varepsilon_H\,(1+r) + 4\varepsilon_H b - \tilde{d}^2 = 0
$$

Hence,

$$
\begin{aligned}
r &= \frac{4\varepsilon_H \pm \sqrt[2]{16\varepsilon_H^2 - 16\varepsilon_H b + 4\tilde{d}^2}}{2b} - 1 \\
&= \frac{2\varepsilon_H \pm \sqrt[2]{4\varepsilon_H^2 - 4\varepsilon_H b + \tilde{d}^2}}{b} - 1.
\end{aligned}
$$

The relevant solution is

$$
r = \frac{2\varepsilon_H - \sqrt[2]{4\varepsilon_H^2 - 4\varepsilon_H b + \tilde{d}^2}}{b} - 1,
$$

which reduces to the one found earlier for $\tilde{d} = 0$. In addition we see that as $b \downarrow \tilde{d}$, then r converges to

$$
r = \frac{2\varepsilon_H - \left(2\varepsilon_H - \tilde{d}\right)}{\tilde{d}} - 1 = 0.
$$

C Derivation of (13)

Working out condition (12) yields:

$$
\begin{aligned}
b^n &= \left[\tfrac{-\varepsilon_L - \tilde{d}}{2\varepsilon_H}\right] b^n (1 + r^n) + \\
&\quad \left[\tfrac{1}{4\varepsilon_H}\right] \left[\left(\varepsilon_L + \tilde{d} + b^n (1 + r^n)\right)^2 - \left(\varepsilon_L + \tilde{d}\right)^2\right] + \\
&\quad \left[\tfrac{1}{2\varepsilon_H}\right] b^n (1 + r^n) \left[2\varepsilon_H - \tilde{d} - b^n (1 + r^n)\right] \\
&= \left[\tfrac{1}{2\varepsilon_H}\right] b^n (1 + r^n) \left[3\varepsilon_H - 2\tilde{d} - b^n (1 + r^n)\right] + \\
&\quad \left[\tfrac{1}{4\varepsilon_H}\right] \left[\left(\varepsilon_L + \tilde{d} + b^n (1 + r^n)\right)^2 - \left(\varepsilon_L + \tilde{d}\right)^2\right]
\end{aligned}
$$

Differentiating yields:

$$
\begin{aligned}
\mathrm{d}b^n &= \left[\tfrac{3\varepsilon_H - 2\tilde{d} - b^n(1+r^n)}{2\varepsilon_H}\right] \mathrm{d}\left[b^n\left(1+r^n\right)\right] - \\
&\quad \left[\tfrac{b^n(1+r^n)}{2\varepsilon_H}\right] \mathrm{d}\left[b^n\left(1+r^n\right)\right] + \left[\tfrac{\varepsilon_L + \tilde{d} + b^n(1+r^n)}{2\varepsilon_H}\right] \mathrm{d}\left[b^n\left(1+r^n\right)\right] \\
&= \left[\tfrac{2\varepsilon_H - b^n(1+r^n) - \tilde{d}}{2\varepsilon_H}\right] \mathrm{d}\left[b^n\left(1+r^n\right)\right].
\end{aligned}
$$

Hence,

$$
2\varepsilon_H \mathrm{d}b^n = \left[2\varepsilon_H - b^n\left(1+r^n\right) - \tilde{d}\right]\left[\left(1+r^n\right)\mathrm{d}b^n + b^n \mathrm{d}r^n\right].
$$

Hence,

$$
\begin{aligned}
&\left[2\varepsilon_H - \left(1+r^n\right)\left(2\varepsilon_H - b^n\left(1+r^n\right) - \tilde{d}\right)\right]\mathrm{d}b^n \\
= &\left[2\varepsilon_H - b^n\left(1+r^n\right) - \tilde{d}\right] b^n \mathrm{d}r^n.
\end{aligned}
$$

Rewriting yields (13).

D Derivation of (20) and (21) based on (19)

Working out condition (19) yields:

$$
\begin{aligned}
b &= \left[\tfrac{\varepsilon_H + \beta e}{2\varepsilon_H}\right]\left(b\left(1+r\right) - \beta e\right) + \\
&\quad \left[\tfrac{1}{4\varepsilon_H}\right]\left[\left(\varepsilon_L + b\left(1+r\right) - \beta e\right)^2 - \varepsilon_L^2\right] + \\
&\quad \left[\tfrac{1}{2\varepsilon_H}\right] b\left(1+r\right)\left[2\varepsilon_H - b\left(1+r\right) + \beta e\right]
\end{aligned}
\tag{28}
$$

To find $\mathrm{d}r/\mathrm{d}b$, we differentiate (28) holding e constant:

$$
\begin{aligned}
2\varepsilon_H \mathrm{d}b &= (\varepsilon_H + \beta e)\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] + \\
&\quad \left[\varepsilon_L + b\,(1+r) - \beta e\right]\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] + \\
&\quad \left[2\varepsilon_H + \beta e - b\,(1+r)\right]\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] - \\
&\quad b\,(1+r)\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
2\varepsilon_H \mathrm{d}b &= \left[2\varepsilon_H + \beta e - b\,(1+r)\right]\left[(1+r)\,\mathrm{d}b + b\mathrm{d}r\right] \Leftrightarrow \\
\frac{\mathrm{d}r}{\mathrm{d}b} &= \frac{1}{b}\frac{2\varepsilon_H - (1+r)\left[2\varepsilon_H + \beta e - b\,(1+r)\right]}{2\varepsilon_H + \beta e - b\,(1+r)}.
\end{aligned}
$$

To find $\mathrm{d}r/\mathrm{d}e$, we differentiate (28) holding b constant:

$$
\begin{aligned}
0 &= \beta\left[b\,(1+r) - \beta e\right]\mathrm{d}e + (\varepsilon_H + \beta e)\left[b\mathrm{d}r - \beta\mathrm{d}e\right] + \\
&\quad \left[\varepsilon_L + b\,(1+r) - \beta e\right]\left[b\mathrm{d}r - \beta\mathrm{d}e\right] + \\
&\quad \left[2\varepsilon_H + \beta e - b\,(1+r)\right] b\mathrm{d}r + b\,(1+r)\left[\beta\mathrm{d}e - b\mathrm{d}r\right] \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
0 &= \beta\left[b\,(1+r) - \beta e\right]\mathrm{d}e + b\,(1+r)\left[b\mathrm{d}r - \beta\mathrm{d}e\right] + \\
&\quad \left[2\varepsilon_H + \beta e - b\,(1+r)\right] b\mathrm{d}r - b\,(1+r)\left[b\mathrm{d}r - \beta\mathrm{d}e\right] \Leftrightarrow
\end{aligned}
$$

$$
\beta\left[\beta e - b\,(1+r)\right]\mathrm{d}e = \left[2\varepsilon_H + \beta e - b\,(1+r)\right] b\mathrm{d}r \Leftrightarrow
$$

$$
\frac{\mathrm{d}r}{\mathrm{d}e} = \frac{\beta}{b}\frac{\beta e - b\,(1+r)}{2\varepsilon_H + \beta e - b\,(1+r)}.
$$

E The modified social welfare function

We show that the modified social welfare function under a debt mutualisation scheme, which takes account of the resources supplied by the rest of the union to fulfill the scheme, is equivalent to the social welfare function in the absence of the scheme. We demonstrate this first for the case of a limited guarantee with effort. The case of the limited guarantee in the benchmark model without effort follows as a special case.

E.1 Limited guarantee with effort

Effort is $e = \tilde{e}$. Social welfare outside the guarantee scheme is:

$$u(1+b) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L + b(1+r) - \beta\tilde{e}} \rho_L g(\varepsilon)\,\mathrm{d}\varepsilon + \\ \int_{\varepsilon_L + b(1+r) - \beta\tilde{e}}^{\varepsilon_H} \left[1 + \beta\tilde{e} + \varepsilon - b(1+r)\right] g(\varepsilon)\,\mathrm{d}\varepsilon \end{array} \right],$$

where r is determined by

$$b = \int_{\varepsilon_L}^{\varepsilon_L + b(1+r) - \beta\tilde{e}} (\beta\tilde{e} + \varepsilon - \varepsilon_L)\, g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L + b(1+r) - \beta e}^{\varepsilon_H} b(1+r)\, g(\varepsilon)\,\mathrm{d}\varepsilon.$$

We can rewrite this last expression as:

$$\begin{aligned} & \int_{\varepsilon_L}^{\varepsilon_L + b(1+r) - \beta\tilde{e}} \left[b - (\beta\tilde{e} + \varepsilon - \varepsilon_L)\right] g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L + b(1+r) - \beta e}^{\varepsilon_H} bg(\varepsilon)\,\mathrm{d}\varepsilon \\ = & \int_{\varepsilon_L + b(1+r) - \beta e}^{\varepsilon_H} b(1+r)\, g(\varepsilon)\,\mathrm{d}\varepsilon. \end{aligned}$$

We can substitute the left-hand side of this expression into the expression for social welfare to give:

$$\begin{aligned} & u(1+b) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L + b(1+r) - \beta\tilde{e}} \rho_L g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L + b(1+r) - \beta\tilde{e}}^{\varepsilon_H} (1 + \beta\tilde{e} + \varepsilon)\, g(\varepsilon)\,\mathrm{d}\varepsilon - \\ \int_{\varepsilon_L}^{\varepsilon_L + b(1+r) - \beta\tilde{e}} \left[b - (\beta\tilde{e} + \varepsilon - \varepsilon_L)\right] g(\varepsilon)\,\mathrm{d}\varepsilon - \int_{\varepsilon_L + b(1+r) - \beta e}^{\varepsilon_H} bg(\varepsilon)\,\mathrm{d}\varepsilon \end{array} \right] \\ = \; & u(1+b) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L + b(1+r) - \beta\tilde{e}} \left[\rho_L - (b - \beta\tilde{e} + \varepsilon_L - \varepsilon)\right] g(\varepsilon)\,\mathrm{d}\varepsilon + \\ \int_{\varepsilon_L + b(1+r) - \beta\tilde{e}}^{\varepsilon_H} (1 + \beta\tilde{e} + \varepsilon - b)\, g(\varepsilon)\,\mathrm{d}\varepsilon \end{array} \right] \\ = \; & u(1+b) + \left[\int_{\varepsilon_L}^{\varepsilon_H} (1 + \beta\tilde{e} + \varepsilon - b)\, g(\varepsilon)\,\mathrm{d}\varepsilon \right] \qquad (29) \\ = \; & u(1+b) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L + b - \beta\tilde{e}} \left[\rho_L - (b - \beta\tilde{e} + \varepsilon_L - \varepsilon)\right] g(\varepsilon)\,\mathrm{d}\varepsilon + \\ \int_{\varepsilon_L + b - \beta\tilde{e}}^{\varepsilon_H} \left[1 + \beta\tilde{e} + \varepsilon - b\right] g(\varepsilon)\,\mathrm{d}\varepsilon \end{array} \right], \end{aligned}$$

where we have used that $\varepsilon_L = \rho_L - 1$. The term $(b - \beta\tilde{e} + \varepsilon_L - \varepsilon)$ in the last expression captures the resources supplied by the rest of the union to fulfill the guarantee. Hence, for given effort level $e = \tilde{e}$ modified social welfare under the limited guarantee scheme coincides with social welfare in the absence of a guarantee. We have shown the equivalence for any value of b, hence the equivalence applies in particular for $b \leq \tilde{d}$. Since the guarantee is limited, it cannot be invoked for $b > \tilde{d}$. Hence, for those values of debt modified social welfare is directly given by social welfare in the absence of a guarantee for effort $e = \tilde{e}$.

E.2 Limited guarantee in model without effort

This is simply a special case of the previous one with $\tilde{e} = 0$.

E.3 Unlimited guarantee in model without effort

For $b \leq \tilde{d}$, this case corresponds to the previous one, while for $b > \tilde{d}$, the modified social welfare function can be written as:

$$u(1+b) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L+\tilde{d}} \left[\rho_L - \left(\tilde{d} + \varepsilon_L - \varepsilon\right)\right] g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+b(1+r)} \rho_L g(\varepsilon)\,\mathrm{d}\varepsilon + \\ \int_{\varepsilon_L+b(1+r)}^{\varepsilon_H} \left[1 + \varepsilon - b(1+r)\right] g(\varepsilon)\,\mathrm{d}\varepsilon \end{array} \right], \tag{30}$$

subject to (10). The term $\left(\tilde{d} + \varepsilon_L - \varepsilon\right)$ captures the resources provided by the rest of the union to guarantee debt repayment. We can rewrite (10) as:

$$\begin{aligned} & \int_{\varepsilon_L}^{\varepsilon_L+\tilde{d}} \tilde{d} g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L+b(1+r)}^{\varepsilon_H} b(1+r)\, g(\varepsilon)\,\mathrm{d}\varepsilon \\ = \;& b - \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+b(1+r)} (\varepsilon - \varepsilon_L)\, g(\varepsilon)\,\mathrm{d}\varepsilon. \end{aligned}$$

Substitute this into (30).

$$\begin{aligned} & u(1+b) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L+\tilde{d}} (\rho_L + \varepsilon - \varepsilon_L)\, g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+b(1+r)} \rho_L g(\varepsilon)\,\mathrm{d}\varepsilon + \\ \int_{\varepsilon_L+b(1+r)}^{\varepsilon_H} (1+\varepsilon)\, g(\varepsilon)\,\mathrm{d}\varepsilon - b + \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+b(1+r)} (\varepsilon - \varepsilon_L)\, g(\varepsilon)\,\mathrm{d}\varepsilon \end{array} \right] \\ = \;& u(1+b) + \int_{\varepsilon_L}^{\varepsilon_H} (1 + \varepsilon - b)\, g(\varepsilon)\,\mathrm{d}\varepsilon, \end{aligned}$$

where we have used that $\varepsilon_L = \rho_L - 1$. This expression coincides with (29) for $\tilde{e} = 0$ and, hence, it coincides with social welfare in the absence of debt mutualisation when effort is $\tilde{e} = 0$.

E.4 Blue and red bonds in model without effort

For $b \leq \tilde{d}$, this case corresponds to the previous ones, while for $b > \tilde{d}$, the modified social welfare function can be written as:

$$u(1+\tilde{d}+b^n) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L+\tilde{d}} \left[\rho_L - \left(\tilde{d}+\varepsilon_L-\varepsilon\right)\right] g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+\tilde{d}+b^n(1+r^n)} \rho_L g(\varepsilon)\,\mathrm{d}\varepsilon \\ \int_{\varepsilon_L+\tilde{d}+b^n(1+r^n)}^{\varepsilon_H} \left[1+\varepsilon-\tilde{d}-b^n(1+r^n)\right] g(\varepsilon)\,\mathrm{d}\varepsilon \end{array}\right] \tag{31}$$

subject to (12). The term $\tilde{d}+\varepsilon_L-\varepsilon$ captures the resources needed from the rest of the union. We can rewrite (31) as:

$$u(1+\tilde{d}+b^n) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_L+\tilde{d}} \left[\rho_L - \left(\tilde{d}+\varepsilon_L-\varepsilon\right)\right] g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+\tilde{d}+b^n(1+r^n)} \rho_L g(\varepsilon)\,\mathrm{d}\varepsilon \\ \int_{\varepsilon_L+\tilde{d}+b^n(1+r^n)}^{\varepsilon_H} \left[\rho_L - \left(\tilde{d}+\varepsilon_L-\varepsilon\right) - b^n(1+r^n)\right] g(\varepsilon)\,\mathrm{d}\varepsilon \end{array}\right]$$

$$= u(1+\tilde{d}+b^n) + \left[\begin{array}{c} \int_{\varepsilon_L}^{\varepsilon_H} \left[\rho_L - \left(\tilde{d}+\varepsilon_L-\varepsilon\right)\right] g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+\tilde{d}+b^n(1+r^n)} \left(\tilde{d}+\varepsilon_L-\varepsilon\right) g(\varepsilon)\,\mathrm{d}\varepsilon \\ - \int_{\varepsilon_L+\tilde{d}+b^n(1+r^n)}^{\varepsilon_H} \left[b^n(1+r^n)\right] g(\varepsilon)\,\mathrm{d}\varepsilon. \end{array}\right]$$

We can rewrite (12) as

$$\int_{\varepsilon_L+\tilde{d}+b^n(1+r^n)}^{\varepsilon_H} b^n(1+r^n)\, g(\varepsilon)\,\mathrm{d}\varepsilon = b^n + \int_{\varepsilon_L+\tilde{d}}^{\varepsilon_L+\tilde{d}+b^n(1+r^n)} \left(\tilde{d}+\varepsilon_L-\varepsilon\right) g(\varepsilon)\,\mathrm{d}\varepsilon.$$

Substitute this into the preceding expression to give:

$$
\begin{aligned}
& u(1+\tilde{d}+b^n)+\int_{\varepsilon_L}^{\varepsilon_H}\left[\rho_L-\left(\tilde{d}+b^n+\varepsilon_L-\varepsilon\right)\right]g\left(\varepsilon\right)\mathrm{d}\varepsilon \\
= \;\; & u(1+\tilde{d}+b^n)+\int_{\varepsilon_L}^{\varepsilon_H}\left[1+\varepsilon-\left(\tilde{d}+b^n\right)\right]g\left(\varepsilon\right)\mathrm{d}\varepsilon,
\end{aligned}
$$

which for $b=\tilde{d}+b^n$ coincides with (29) for $\tilde{e}=0$ and, hence, coincides with social welfare in the absence of debt mutualisation when effort is $\tilde{e}=0$.

Figure 1: first-order conditions without and with guarantee

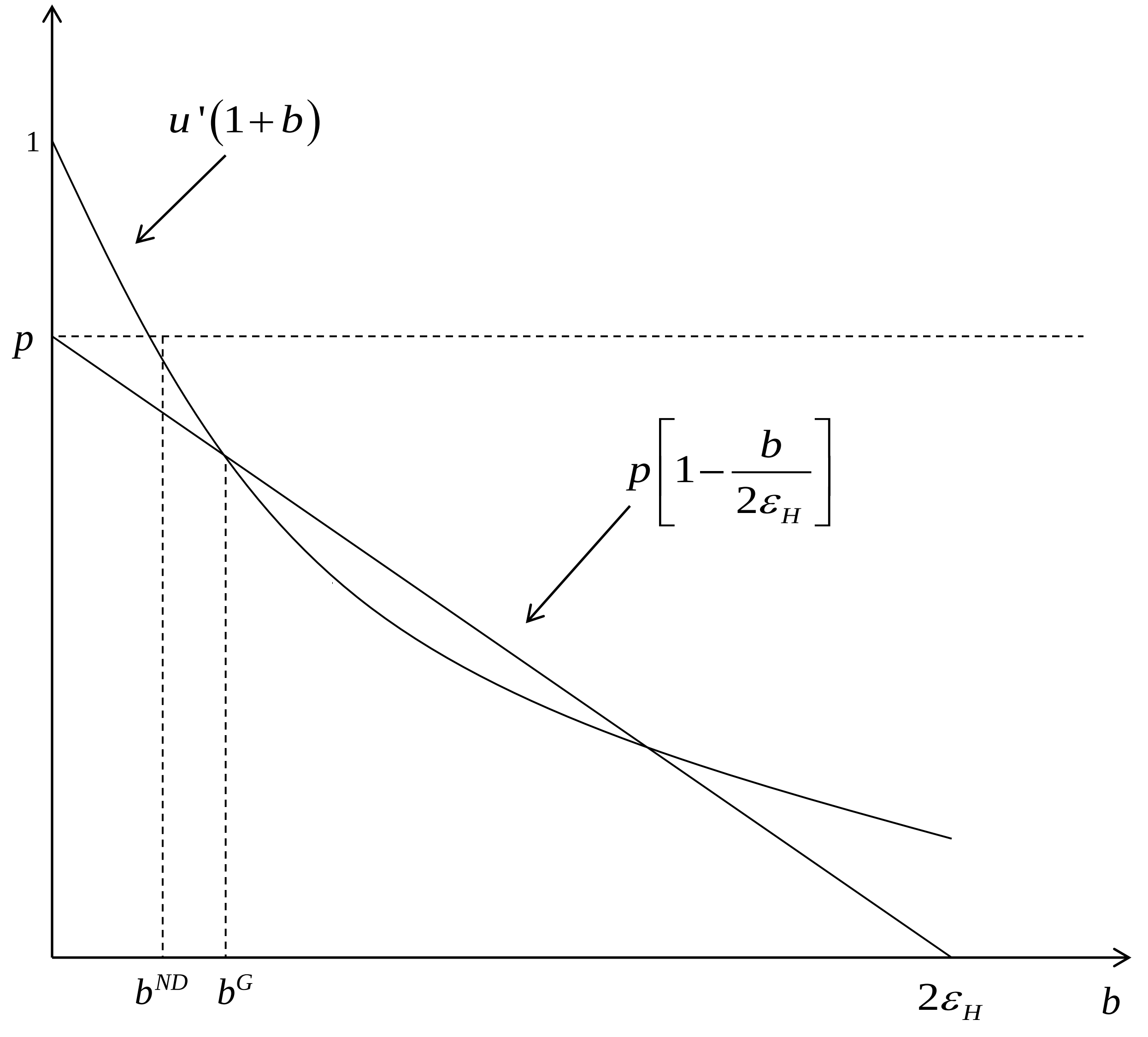

Figure 2: Government objective functions

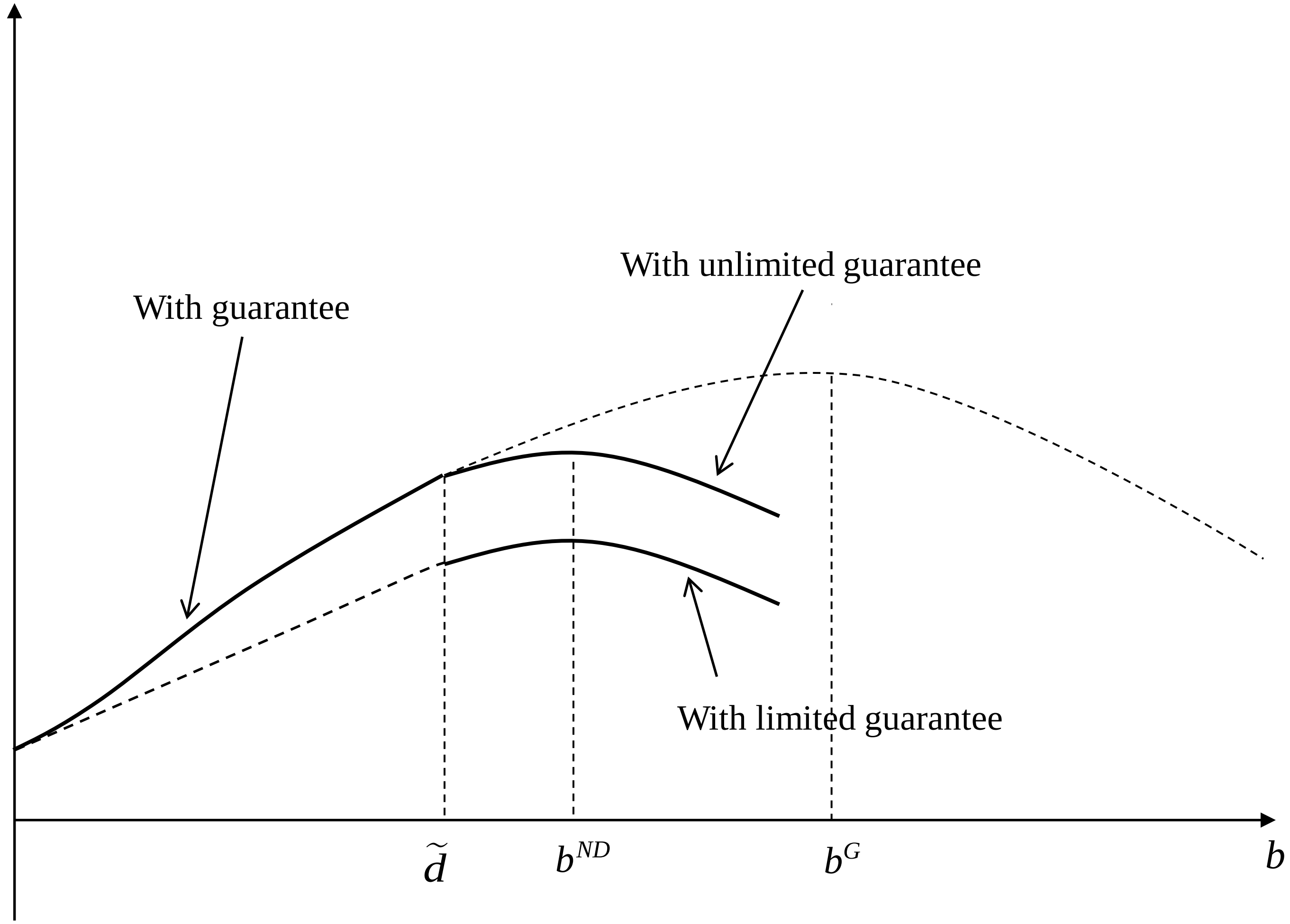

Figure 3: Choosing the optimal guarantee level

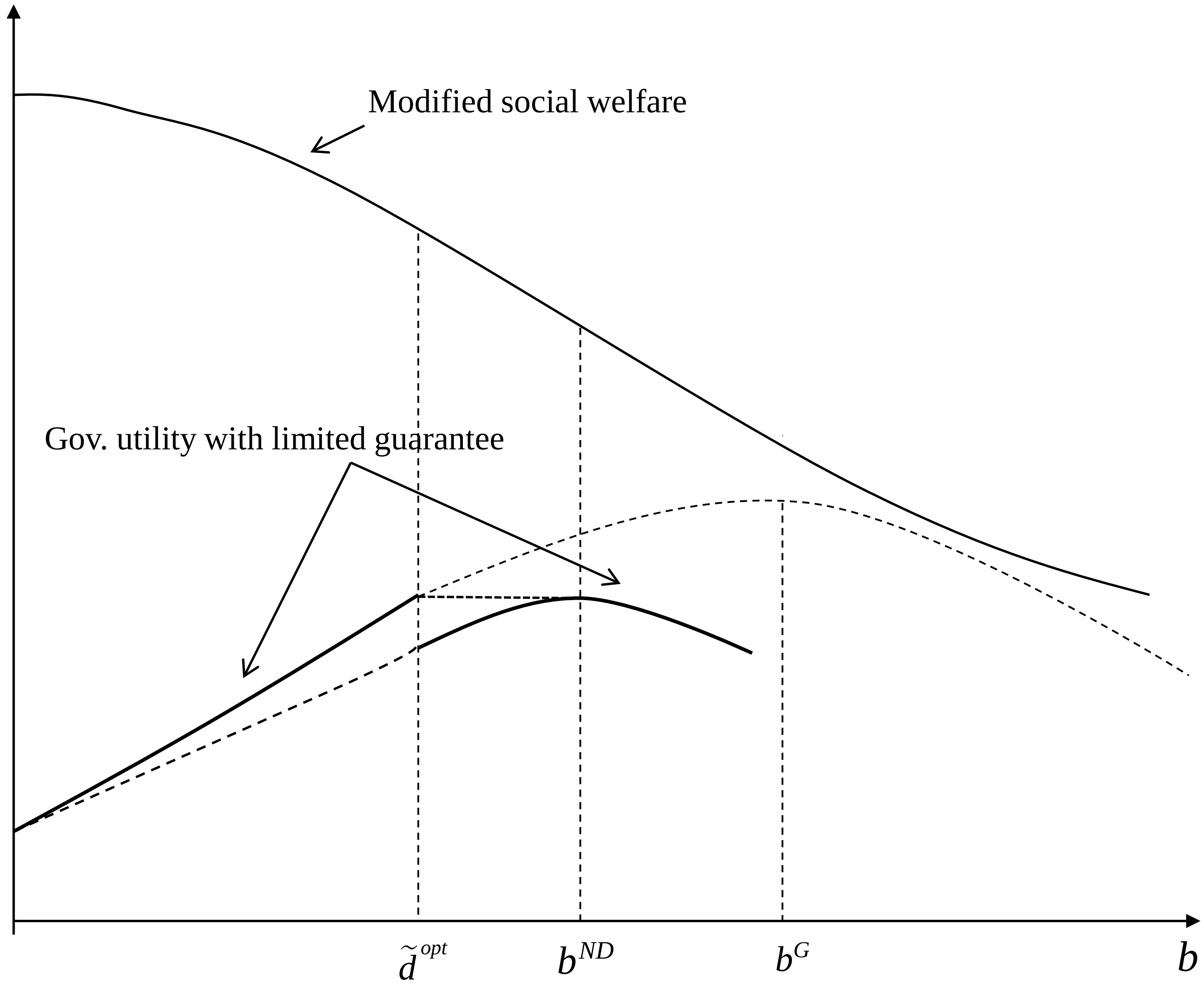

Figure 4: Government objective functions with effort