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Ton van den Bremer and
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Ton van den Bremer, Oxford University
Frederick van der Ploeg, Oxford University

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Managing and Harnessing Volatile Oil Windfalls*

Three funds are necessary to manage an oil windfall: intergenerational, liquidity and investment funds. The optimal liquidity fund is bigger if the windfall lasts longer and oil price volatility, prudence and the GDP share of oil rents are high and productivity growth is low. We apply our theory to the windfalls of Norway, Iraq and Ghana. The optimal size of Ghana's liquidity fund is tiny even with high prudence. Norway's liquidity fund is bigger than Ghana's. Iraq's liquidity fund is colossal relative to its intergenerational fund. Only with capital scarcity, part of the windfall should be used for investing to invest. We illustrate how this can speed up the process of development in Ghana despite domestic absorption constraints.

JEL Classification: D91, E21, E22 and Q32
Keywords: economic development, Ghana, inefficiency, intergenerational fund, Iraq, liquidity fund, Norway, oil price volatility, precautionary buffers, public investment and sovereign wealth

Ton van den Bremer
OxCarre, Department of Economics
Oxford University
Manor Road Building
Manor Road
Oxford OX1 3UQ

Email:
ton.vanderbremer@eng.ox.ac.uk

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Frederick van der Ploeg
OxCarre, Department of Economics
Oxford University
Manor Road Building
Manor Road
Oxford OX1 3UQ

Email:
rick.vanderploeg@economics.ox.ac.uk

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1. Introduction

Many countries experience substantial oil revenue windfalls.[2] The consensus is that these should be put in a sovereign wealth fund to smooth the benefits across generations and that the countries should have buffers to cope with oil price volatility. This is sound advice for countries that are well integrated into world capital markets, but not for countries facing capital scarcity or constraints on external borrowing. Oil windfalls should then be used not only to accumulate sovereign wealth, but also to invest in the domestic economy and boost development (Collier, et al., 2010; van der Ploeg and Venables, 2011, 2012). Some countries harness these windfalls for growth and development, especially if institutions are good, but others suffer from poor growth despite large resource bonanzas (e.g., Sachs and Warner, 1997; Mehlum et al., 2006; Boschini et al., 2007; van der Ploeg, 2011). Given the political and institutional failures in many oil-rich developing economies, it is a challenge to transform subsoil wealth into productive growth-enhancing physical and human capital. An additional challenge facing oil-rich countries is the high volatility of oil prices given their adverse effects on growth, especially in countries that have poor financial systems, restrictions on international trade and unrestricted capital flows and that are landlocked and ethnically fractionalized (e.g., Blattman et al., 2007; van der Ploeg and Poelhekke, 2009; Aghion et al., 2009).

Our prime objective is thus to address the vexed question of how to put oil windfalls to good use and how to cope with the historically high volatility of oil prices.[3] In particular, we want to obtain an estimate of the optimal shares of the windfalls to be allocated to intergenerational saving when there is no uncertainty about future oil prices, on the one hand, and precautionary saving which is what is needed additionally when there is uncertainty about future oil prices, on the other hand, and thus assess how much of the windfall is left to boost consumption. Our second objective is to understand why capital scarcity requires that part of the windfall is spent on domestic investment and consumption (van der Ploeg and Venables, 2011, 2012). The empirical fact that fast growth often goes together with reductions in net foreign liabilities - the 'allocation puzzle' (e.g., Gourinchas and Jeanne, 2007; Aizenman et al., 2004; Pradad et al., 2006) – might occur if capital is not invested in high-debt economies due to the higher risk of expropriation.[4] Political economy frictions and the resulting debt dynamics may jointly explain the

[2] Whenever we refer to oil, it should be interpreted to refer more generally to natural resources (gas, diamonds, copper, bauxite, phosphate, etc.) and could also be interpreted as remittances (which decline as migrant workers return home or loose connection with the home country) or foreign aid. Oil is thus used as a short-hand for a windfall of foreign exchange and oil price as a short-hand for commodity prices.

[3] Earlier work uses a model of a small open economy that exports exhaustible resources to quantify optimal precautionary saving in response to volatile resource prices and demonstrates that current account balances of countries with a greater weight of resource revenue to future income are bigger (Bems and de Carvalho Filho, 2011).

[4] Limited commitment incentivizes the government to pay down external debt along the adjustment path (e.g., Cohen and Sachs, 1986; Aguiar and Amador, 2011). Debt overhang can exacerbate volatility (Aguiar et al., 2009).

empirically observed negative relationship between volatility and growth. Our model of capital scarcity is inspired by such considerations, but we abstract from micro foundations and simply postulate a relationship between sovereign debt and the risk premium and explore how this affects economic development. Our final objective is to allow for the inefficiency of public capital projects, since absorption constraints start to bite if the oil windfall is used to rapidly increase public investment.

The private sector cannot achieve the first-best outcome due to various market failures: it does not have good access to international capital markets and may be less able to smooth consumption than the government; derivatives and hedging may be too costly or politically infeasible; it does not internalize the interest-spread externality to do with sovereign risk and capital scarcity; public goods such as infrastructure, education or health are inadequately provided by the market; the economy may not be able to absorb a rapid buildup of public capital. Furthermore, even if rates of return on domestic capital and foreign assets are equalized, the marginal product of public capital may be higher and the supply of public capital thus lower due to various market and non-market distortions unless the government corrects for these distortions. We use the metaphor of capital scarcity to capture this. There are then two decisions to consider: the first is how much to save; the second is whether to invest at home or abroad.

Three funds are thus necessary to manage the revenues from an oil windfall: *intergenerational* funds to smooth benefits across generations; *liquidity* funds to cope with oil price volatility; *investment* funds to manage domestic investment in case of capital scarcity and absorption constraints.[5] We present an infinite-horizon, continuous-time framework for optimal management of stochastic oil windfalls and derive how much of the windfall should be allocated to each of these three funds. We show that the optimal size of the liquidity fund is larger if the windfall lasts longer and oil price volatility, prudence and the GDP share of oil rents are high. With capital scarcity and sluggish adjustment of public capital, part of the windfall must be spent on domestic investment. To illustrate how the importance of each fund depends on the particular features of the economy, we apply our framework to three very different oil-rich countries: Norway, Iraq and Ghana. This illustrates why the declining windfall of Norway implies a substantial intergenerational than liquidity fund, why the liquidity fund is the main concern for Iraq, and why the optimal liquidity fund for Ghana is tiny and why the real challenge for Ghana is how to use its windfall to invest in the domestic economy and cope with absorption constraints.

Section 2 discusses the principles of using sovereign wealth funds to manage and harness oil windfalls and points out some obstacles to using options and structured derivatives to hedge the risk of oil price volatility. Section 3 puts forward our theoretical framework to derive the optimal management of debt,

[5] Some share of oil windfall, typically, ends up in private hands, which may not end up in the funds administered by the government or independent authorities. We consider optimal size of the ‘funds’ for the economy as a whole.

assets and domestic investment in oil-rich economies faced with volatile oil prices and capital scarcity. Section 4 discusses calibration issues related to oil price volatility, the inefficiency of public investment and interest premium on foreign debt. Sections 5, 6 and 7 apply the theoretical framework to estimate the sizes of the intergenerational, liquidity funds and investment funds as well as the primary deficit increments for the windfalls of Norway, Iraq and Ghana, respectively. Finally, section 8 concludes and offers some policy suggestions.

2. Principles of managing volatile oil windfalls

2.1. Three types of funds for volatile managing windfalls

Oil windfalls last for a limited period, are often known some years in advance, and are highly volatile and unpredictable. Three types of funds can play a role in managing these features of oil windfalls:

1. An *intergenerational fund* to smooth the benefits of a temporary windfall over current and future generations abstracting from uncertainty about future oil prices. During the windfall when oil is produced and sold, the revenue is put into the intergenerational fund. Once the windfall has ceased, the returns on this fund are used to finance the general deficit.
2. A *liquidity fund* to collect additional precautionary buffers as prudent response to oil price volatility. This fund is designed to self insure against periods when the oil price is low.
3. An *investment fund* to temporarily park funds until domestic investment projects are ready to be undertaken and to collect any returns from these funds.

There are distinct reasons for having each of these funds (e.g., Collier et al., 2010; van der Ploeg and Venables, 2012). The *intergenerational* fund is used to smooth consumption in the face of foreseen changes in oil revenue. For example, it may be known that current oil reserves will last for another thirty years. The permanent income hypothesis then suggests that during the windfall the temporary component of the windfall is saved in foreign assets whilst the permanent component is used to fund the primary deficit. After thirty years, the interest on accumulated assets finances the same increase in the primary deficit as during the windfall.[6] Hence, through a judicious management of foreign assets accumulated in a sovereign wealth fund (i.e., the intergenerational fund) the temporary windfall leads to a permanent increase in the primary deficit, which can be split into a permanent increase in consumption or a permanent reduction in taxes depending on political preferences. If the oil windfall is expected to fade out in the future, the country must save to achieve the same consumption increment or tax cut in all time periods. If the windfall is anticipated some years ahead, the permanent income hypothesis suggests that

[6] This is related to the celebrated Hartwick rule which says that oil rents should be saved, so that exhaustible assets under the ground are fully transformed into assets above the ground (Hartwick, 1977).

the country borrows with the future windfall as security so that the permanent increase in the primary deficit extends to the announcement period. The so-called bird-in-hand rule rules out using oil windfalls as collateral and is thus a prudent variant of this permanent-income rule. For example, Norway puts all oil revenue in its intergenerational fund and takes out 4 percent each year to finance the general deficit.

A *liquidity* fund is used to cope with oil price volatility. This fund is larger if oil prices are more volatile (high standard deviation of oil price shocks as fraction of average consumption), oil price shocks are more permanent and policy makers more prudent, and the windfall lasts for a longer period. Prudence requires a positive third derivative of the utility function. There may also be precautionary saving in response to asset return uncertainty, but only if there is a motive for intergenerational saving or borrowing.

Finally, an *investment* fund is necessary if countries are not well integrated in global capital markets. In countries with perfect access to world capital markets no part of the windfall should be spent on domestic investment projects: the windfall should feed the intergenerational and liquidity fund and curb the general deficit, but not feed an investment fund. However, many developing countries suffer from capital scarcity and pay a premium when borrowing to fund investment projects. It is then optimal to spend part of the oil windfall on domestic investment to alleviate capital scarcity (van der Ploeg and Venables, 2011). Domestic investment projects may face all kinds of absorption, planning and legal constraints in which case it makes sense to temporarily park part of the oil windfall until it is feasible to undertake the project.

2.2. What assets should the intergenerational and liquidity funds invest in?

By careful choice of the sovereign wealth portfolio an oil rich country can hedge oil income risk. The key question is whether one should choose equity holdings in companies whose fortunes move inversely with the world price of oil or companies who are not affected by or benefit from increases in the oil price. Examples of the former are energy intensive users such as aluminum smelters, steel producers, oil companies, etc. whilst examples of the latter are companies that offer substitutes for fossil fuels, produce energy efficient cars, etc. Net asset holders that invest in companies whose share prices vary inversely with the price of oil need to hold less precautionary buffers. Net debtors need to hold bigger precautionary buffers. This suggests that net debtors should invest less in energy-intensive companies and more in companies whose fortunes do not vary inversely with the price of oil.

2.3. Hedging against volatile oil prices

An alternative way for an oil exporter to deal with the volatility of future oil prices is to hedge and transfer the risk to those who are better able to bear it (e.g., Daniel, 2001; Stulz, 2002). For example, Mexico bought a put option at a strike price of 140 US dollar per barrel in 2009 after oil prices reached heights of almost 140 US dollar per barrel in 2008. When the oil price went significantly below this strike price Mexico exercised and pocketed 8 billion US dollars. The costs of the option were 1.5 billion US dollar. The drop in oil revenue was compensated to a large extent by the profits on the option. The option thus offers an insurance policy against the risk of future oil price volatility. Ecuador, Columbia, Algeria, Texas and Louisiana have also used options to protect themselves against volatile oil and gas prices.

Such plain-vanilla put options are costly. Lu and Neftci (2008) thus argue for structured-reverse options which lower the cost of plain-vanilla options by selling simultaneously other options (zero-premium collar) but such products can lead to substantial losses if commodity prices rises above their cap. Barrier options (e.g., an up-and-out put option or a knockout option) are cheaper. Various developing countries use commodity derivatives markets to hedge against commodity price risk (e.g., Larson et al., 1998).

Options and other structured derivative products can help in managing oil price volatility, but are expensive and risky. For most commodities (including oil) maturities are too short and the financial markets too thin with insufficient depth to provide adequate protection. There are large political risks if a lot of money has been spent and options are not exercised. If the option is exercised with profit, it will be denounced as speculation. Big commodity exporters which hedge can influence the market price, especially if they have private information, and also stand to be accused of speculation rather than insurance. This is why liquidity funds are often more attractive.

2.4. General economic policy

Governments should also make sure that goods, labor and capital markets are flexible as this helps to deal better with oil price volatility. It also helps to avoid irreversible commitments which cannot be kept if oil prices fall by a large amount. Independent liquidity funds reduce the need for such politically difficult measures. Further, it helps to diversify into sectors whose fortunes are orthogonal to the commodity sector. The government can relate debt payments to the oil price to protect itself against oil price volatility. The idea is that, in the event of a crash in oil prices, the government's debt burden falls as well.

Governments may also help the private sector to hedge against oil price volatility and prevent changes in world oil prices being passed on fully to domestic consumers, especially if households are risk averse and face high adjustment costs, credit markets and self-insurance are imperfect and hedging opportunities for private individuals using futures contracts and options are limited (e.g., Federico et al., 2001).[7] The tradeoff between retail oil price stability and government fiscal stability poses important challenges, but is beyond the scope of the present paper.

[7] In many poor countries the share of petroleum consumption in household income is high, income and price elasticities for petroleum demand are low and households are relatively risk averse in which case the risk aversion effect dominates the effect of substituting away from petrol if its price is high and towards petrol if its price is low so that consumers benefit from petrol price stability (e.g., Turnovsky et al., 1980).

3. Theory of managing volatile oil windfalls

To make a quantitative assessment of the size of the intergenerational and liquidity funds and the optimal amount of domestic investment to undertake in response to a temporary and volatile oil windfall, we formulate a simple welfare-based, infinite-horizon, continuous-time model of an oil-exporting economy.[8] We also allow for capital scarcity and capital adjustment costs for investment.

The trend rate of growth equals the sum of the growth rates of population and labor augmenting technical progress, denoted by *n* and *g*, respectively. All variables are expressed in efficiency units (i.e., divided by $e^{(n+g)t}$). The instantaneous utility of per-capita consumption at time *t* is given by the CES utility function:

(1) $$U\left(C(t)e^{gt}\right) = \frac{\left(C(t)e^{gt}\right)^{1-1/\sigma} - 1}{1-1/\sigma}, \sigma \neq 1, \quad U\left(C(t)e^{gt}\right) = \ln\left(C(t)\right) + gt, \sigma = 1.$$

where $\sigma > 0$ denotes the elasticity of intertemporal substitution and *C* aggregate consumption in efficiency units. The coefficient of relative risk aversion equals $CRRA = -CU''/U' = 1/\sigma$ and also corresponds to the coefficient of relative intergenerational inequality aversion. It thus is crucial in determining the tradeoffs between present and future consumption. The coefficient of relative prudence is $CRP = -CU''/U' = 1+1/\sigma$. Public investment is denoted by *I* and is subject to internal adjustment costs, so the price of public investment goods equals $1+0.5\,\phi I/S$ with $\phi > 0$ the adjustment cost parameter and *S* the stock of public capital. One interpretation is that, when investment is increased rapidly, the price of investment goes up as a result of various absorption constraints. Denoting the depreciation rate of the public capital stock by $\delta^* > 0$, we write the accumulation of the public capital stock in efficiency units as:

(2) $$\frac{dS(t)}{dt} = I(t) - \delta S(t), \quad S(0) = S_0,$$

where $\delta \equiv \delta^* + n + g$ is the effective depreciation rate (including the trend rate of growth). Production by the representative firm (indicated by superscript *f*) operating under constant returns to scale is given by:

(3) $$Y^f(t) = F\left(K^f(t), S(t)/Y(t)\right) = E_0^{*1-\alpha} K^f(t)^{\alpha}[S(t)/Y(t)]^{\beta^*}, \quad F' > 0, F'' < 0, 0 < \alpha < 1, \beta^* > 1,$$

where $E_0^* > 0$ is the level of labor-augmenting technical progress, α the share of private capital in aggregate production and β^* the marginal effect of the share of public capital in aggregate production on firm-level output. Public capital thus boosts private production. Private capital is financed from abroad at a user cost of capital equal to the exogenous world interest rate r^* plus the depreciation rate of private

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[8] Bems and de Carvalho Filho (2011) offer a discrete-time approach for dealing with oil price uncertainty and precautionary buffers, but do not deal with capital scarcity.

capital δ^P, hence $\alpha\, Y^f/K^f = r^* + \delta^P$. The intensive-form production function with this private sector response substituted into (3) is in symmetric equilibrium with $Y = Y^f$ and $K = K^f$ equal to:

$$(4)\qquad Y(t) = E_0 S(t)^{\beta}, \quad E_0 \equiv \left[E_0^* \left(\frac{\alpha}{r^* + \delta^P} \right)^{\alpha} \right]^{\frac{1}{1+\beta^*-\alpha}}, \beta \equiv \frac{\beta^*}{1+\beta^* - \alpha} > 1.$$

Equation (4) says that total factor productivity and public investment act to increase private output.

The asset accumulation equation for the oil-exporting economy gives the increase in sovereign wealth as growth-corrected interest income $(r + \Pi)\, A$ plus oil revenue $(P - \Psi)\, O$ plus (non-oil) production income Y minus consumption and the cost of public investment (including the costs of absorption):

$$(5)\quad \frac{dA(t)}{dt} = \left[r + \Pi(D) \right] A(t) + \left[P(t) - \Psi \right] O(t) + Y(t) - C(t) - I(t) - 0.5\phi I(t)^2 / S(t), \quad A(0) = A_0,$$

where $r \equiv r^* - n - g$ denotes the growth-corrected world interest rate, Π the risk premium on borrowing from abroad, D national indebtedness, $A = -D$ the stock of foreign assets held by the country, P the exogenous world price of oil, Ψ the constant extraction cost per barrel of oil, and O the volume of oil production. Equation (5) thus gives the current account dynamics of the economy. Since variables are measured in efficiency units, the world interest rate r^* is corrected for the trend rate of economic growth. Without capital scarcity and accumulated sovereign wealth large enough, the risk premium Π is zero. If the economy is a substantial borrower from abroad, it has to pay an interest premium and this premium rises with indebtedness (in efficiency units), i.e., $\Pi > 0$ and $\Pi' > 0$. One could relate the size of the risk premium to the economy's ability to pay by specifying $\Pi(D/Y)$ or to the size of the anticipated windfall. Since there is no conclusive empirical support for oil windfalls alleviating the debt premium paid on international capital markets, we abstract from this.

Policy makers face various types of uncertainty: about oil prices, reserves, investment returns, asset returns, and general economic outcomes (notably growth prospects). We focus on the most important form of uncertainty for oil-rich economies, i.e., oil price volatility. We first describe the oil price by a geometric Brownian motion:

$$(6)\qquad dP(t) = \upsilon_P P(t)dt + \sigma_P P(t)dW(t) \quad \text{or} \quad d\ln(P(t)) = \left(\upsilon_P - 0.5\sigma_P^2 \right) dt + \sigma_P dW(t),$$

where $W(t)$ is a Wiener process satisfying $W(t) - W(s) \sim \mathrm{N}(0, t - s)$ for $t \geq s$. The constants υ_P and σ_P are the percentage drift and the percentage volatility, respectively. The solution to (6) is:

$$(6')\qquad P(t) = P_0 \exp\left[\left(\upsilon_P - 0.5\sigma_P^2 \right) t + \sigma_P W(t) \right]$$

with expectation and variance given by $E[P(t)] = P_0 e^{\upsilon_P t}$ and $\mathrm{var}[P(t)] = P_0^2 e^{2\upsilon_P t}(e^{\sigma_P^2 t} - 1)$, respectively. To reflect the strong effect of even a small degree of mean reversion on the propagation of shocks, we also consider the mean-reversion model of Schwartz (1997) for the oil price:[9]

(7) $$dP(t) = \eta_P \;\; m_P - \log(P(t)) \;\; P(t) + \upsilon_P \;\; dt + \sigma_P P(t) dW(t),$$

which can be rewritten as a homoskedastic AR(1) process for $\log(P(t))$:

(7′) $$d\log(P(t)) = \;\; \eta_P \;\; m_P^* + \upsilon_P t - \log(P(t)) \;\; + \upsilon_P \;\; dt + \sigma_P dW(t),$$

where $m_P^* = m_P - \sigma_P^2 / 2\eta_P$ (from Itō calculus).

The government thus maximizes the expected value of a utilitarian social welfare function, i.e., the sum of the utilities of per-capita consumption (1) over all members of the population,

(8) $$\mathrm{E}_0\left[\int_0^\infty e^{nt} U \;\; C(t)e^{gt} \;\; e^{-\rho^* t} dt\right] = \mathrm{E}_0\left[\int_0^\infty \frac{C(t)^{1-1/\sigma} - 1}{1 - 1/\sigma} e^{-\rho t} dt\right], \sigma \neq 1,$$
$$= \mathrm{E}_0\left[\int_0^\infty \left[\ln \;\; C(t) \;\; + gt\right] e^{-\rho t} dt\right], \sigma = 1,$$

subject to the public capital accumulation equation (2), the intensive-form production function (4), the asset accumulation equation (5) and the stochastic dynamics of the oil price (6) or (7). The social rate of discount is denoted by $\rho^* > 0$. Since social welfare (8) is in terms of consumption in efficiency units, the social discount rate is corrected for population growth and (depending on the income and substitution effect) for labor-augmenting technical progress: $\rho \equiv \rho^* - n - (1 - 1/\sigma) g$.

Using Itō calculus to solve this stochastic optimization problem, we obtain the following conditions:[10]

(9a) $$\frac{\mathrm{E}_t[dC]}{dt} = \sigma C \left\{ r + \Pi \;\; D \;\; + \Pi' \;\; D \;\; D - \rho + \frac{1}{2} CRP \left(\frac{\partial C}{\partial P}\right)^2 \left(\frac{\sigma_P P}{C}\right)^2 C \right\}, \quad C(0) \text{ free},$$

(9b) $$\frac{dS}{dt} = \left[\frac{1}{\phi}(q-1) - \delta\right] S, \quad S(0) = S_0,$$

(9c) $$\frac{\mathrm{E}_t[dq]}{dt} = \;\; r + \Pi(D) + \Pi'(D)D + \delta \;\; q - (1-\alpha)\beta E_0 S^{\beta-1} - \frac{1}{2\phi}(q-1)^2, \quad q(0) \text{ free},$$

[9] We thus avoid the heteroskedasticity problems which would arise when estimating a Geometric Ornstein-Uhlenbeck process (Dixit and Pindyck, 1994) and exploit that (7) can be written as a standard homoskedastic AR(1)-process after a logarithmic transformation.

[10] These optimality conditions extend the results of van der Ploeg (2012) to a stochastic setting.

(9d) $$\frac{dA}{dt} = E_0 S^{\beta} + (P - \Psi)O + \; r + \Pi(-A) \; A - C - \frac{1}{2\phi}(q^2 - 1)S, \quad A(0) = A_0,$$

where q is the social cost of capital, $\partial C/\partial P$ is the effect of an oil price shock on consumption ('marginal propensity to consume' out of the wealth generated by an oil price shock) and $CRP = 1 + 1/\sigma > 1$ is the coefficient of relative prudence. Oil production in efficiency units, $O(t)$, $\forall t \geq 0$, is exogenous.

Equation (9a) is a modified version of the Keynes-Ramsey rule, which states that the growth rate in consumption is proportional to the social cost of borrowing minus the social discount rate. More intergenerational inequality aversion (lower σ) implies a smaller growth rate to avoid inequality between present and future generations. The social cost of borrowing corresponds to the world interest rate plus interest premium plus the term $\Pi'(D)D$ to correct for and internalize the interest spread externality. For an economy with capital scarcity, it is thus optimal to have a rising path of consumption; the economy consumes less upfront to pay of debt and lower the risk premium. Equation (9a) also contains a prudence term (see section 3.1 for discussion). Equation (9b) describes the public sector capital stock dynamics, where the rate of public investment is proportional to its social value, $I / S = (q-1)/\phi$. Equation (9c) gives the intertemporal efficiency condition for public sector investment: the marginal product of public capital plus the marginal reduction in adjustment cost must equal the social cost of borrowing (the market interest rate plus the interest premium on government debt, Π, plus the correction term to allow for the rising cost of public debt, $-\Pi' D$, plus the depreciation charge). Finally, equation (9d) gives the dynamics of sovereign wealth with the cost of public sector investment, transfers and output substituted. It supposes that the country has linear oil extraction costs and does not have monopoly power on the world market.

The five-dimensional system (6) or (7) and (9) has three predetermined state variables, P, A and S, and two non-predetermined state variables, C and q. Hence, $C(0)$ and $q(0)$ adjust instantaneously to ensure that the economy is on its stable manifold and thus satisfy a corresponding transversality condition on public debt and capital. We now consider two cases in turn: no capital scarcity ($\Pi = 0$), and no oil price volatility ($\sigma_P = 0$).

3.1. No capital scarcity: case for intergenerational and liquidity funds

Consider an economy with good access to world capital markets and no interest premium on national borrowing, $\Pi = 0$. The so-called separation theorem for public investment then holds: it is suboptimal to spend part of the oil windfall on public investment. How much public investment should be undertaken does not depend on the oil revenue coming in, but only on the costs and benefits of public investment itself. Any financing need is supplied by international capital markets. To see this, note that with $\Pi = 0$ (9b) and (9c) are decoupled from the rest of the economy (i.e., independent of (6) or (7), (9a), (9d) and

(9e)) and thus independent of the size of the oil windfall. This is a lesson often forgotten in highly developed oil- or gas-rich economies such as Norway and the Netherlands (even with the recent sovereign debt crisis). We suppose that the dynamics of (9b) and (9c) has played out, so the social value of public capital, the stock of public capital and aggregate production are independent of the size of the windfall:

$$(10) \qquad q = 1 + \phi\delta, \qquad S = \left[\frac{(1-\alpha)\beta E_0}{r+\delta+\phi\delta(r+0.5\delta)}\right]^{\frac{1}{1-\beta}}, \quad Y = E_0^{\frac{1}{1-\beta}}\left[\frac{(1-\alpha)\beta}{r+\delta+\phi\delta(r+0.5\delta)}\right]^{\frac{\beta}{1-\beta}}.$$

For purposes of optimally managing an oil windfall these variables can thus be treated as exogenous. Equations (9b), (9c) and (9d) then lead to the following asset accumulation equation:

$$(11) \qquad \frac{dA}{dt} = Y - (\delta + 0.5\phi\delta^2)S + (P-\Psi)O + rA - C, \quad A(0) = A_0,$$

where the first two terms indicate output net of costs (including absorption costs) of public investment. Further, oil price volatility induces an additional precautionary savings response as can be seen from:

$$(12) \qquad \frac{1}{dt}\mathrm{E}_t\ dC\ = \sigma\ r-\rho\ C + \frac{1}{2}CRP\left(\frac{\partial C}{\partial P}\right)^2\left(\frac{\sigma_P P}{C}\right)^2 C,$$

Since $r-\rho = -g/\sigma$ if $r^* = \rho^*$, the first term on the right-hand side of (12) sums up the *growth* effect. So in a growing economy, it is optimal to borrow if prospects are good. The second term in (12) is the *prudence* effect. With volatile oil prices the expected time path of consumption slopes upwards. Prudent saving implies consumption is initially low, especially if the coefficient of relative prudence and oil price uncertainty as share of consumption (i.e., $\sigma_P\, P/C$) are high. Further, under the permanent income hypothesis, countries with a temporary windfall save more of their windfall than those with a more permanent windfall, and thus have a smaller prudence effect as can be seen from:

$$(13) \qquad \frac{\partial C(t)}{\partial P(t)} = \ r-\sigma\ r-\rho\ \ \int_t^\infty \frac{\partial \mathrm{E}_t\ P(\tau)}{\partial P(t)} O(\tau) e^{-r(\tau-t)} d\tau,$$

where $r-\sigma\ r-\rho$ is the marginal propensity to consume out of wealth and (from (6) or (7))

$$(14) \qquad \frac{\partial \mathrm{E}_t\ P(\tau)}{\partial P(t)} = \begin{cases} 1 & \text{random walk} \\ \dfrac{\mathrm{E}\ P(\tau)}{P(t)} e^{-\eta_P\ \tau-t} = \dfrac{e^{m_P\left[1-e^{-\eta_P\ \tau-t}\right]+\ln(P(t))e^{-\eta_P\ \tau-t}}}{P(t)} e^{-\eta_P\ \tau-t} < 1 & \text{AR(1)} \end{cases}$$

for $\tau \geq t$. If $\eta_P = 0$, shocks are permanent and all expected future prices increase by the same amount as the initial shock. If $\eta_P = \infty$, shocks are purely transitory and have zero effect on future expected oil prices. In general, mean reversion implies that the effect of current price shocks on expected values of future price shocks is positive but less than one. Hence, the marginal propensity to consume future consumption out of a current shock to the oil price (13) is less with mean reversion and thus from the Euler equation (9a) or (12) the effect on precautionary saving is smaller too (cf., the discrete-time analysis of Bems and de Carvalho Filho (2011)). A higher trend growth rate cuts oil production in efficiency units and thus from (13) depresses the marginal propensity to consume and from (9a) or (12) also precautionary saving.

We solve the differential equations (11)-(12) and the partial differential equation (6) or (7) with a multiple-shooting algorithm.

3.2. Capital scarcity and the case for investing to invest

To capture the fact that developing economies experience capital scarcity and may have substantial sovereign debt before enjoying an oil bonanza, we suppose that countries pay a risk premium on their sovereign debt and take this as a metaphor for capital scarcity (cf., van der Ploeg and Venables, 2011, 2012). The separation theorem no longer holds as now the optimal level of domestic investment depends on the level of saving and the size of the windfall of foreign exchange. The relevant social cost of borrowing exceeds the private cost of borrowing, since the government (in contrast to the private sector) internalizes the higher cost of borrowing resulting from having debt. This results in a corresponding increase in the cost of public investment and explains why the separation theorem breaks down.

With capital scarcity both present and future consumption are lower even without windfall uncertainty. Further, present consumption is lower than future as capital scarcity prevents public debt from being raised to sufficiently high levels to fully smooth consumption. The higher social cost of borrowing holds back public investment and thus economic development. With volatile windfall income present and future consumption diverge even further, because for precautionary reasons public debt is cut and public investment raised. The precautionary buffer strikes the optimal balance between prudence and intergenerational equity. With capital scarcity the burden of smoothing consumption between generations is shared, so precaution demands borrowing and public investment are cut (viz. the arbitrage condition (9c) between public capital and debt).

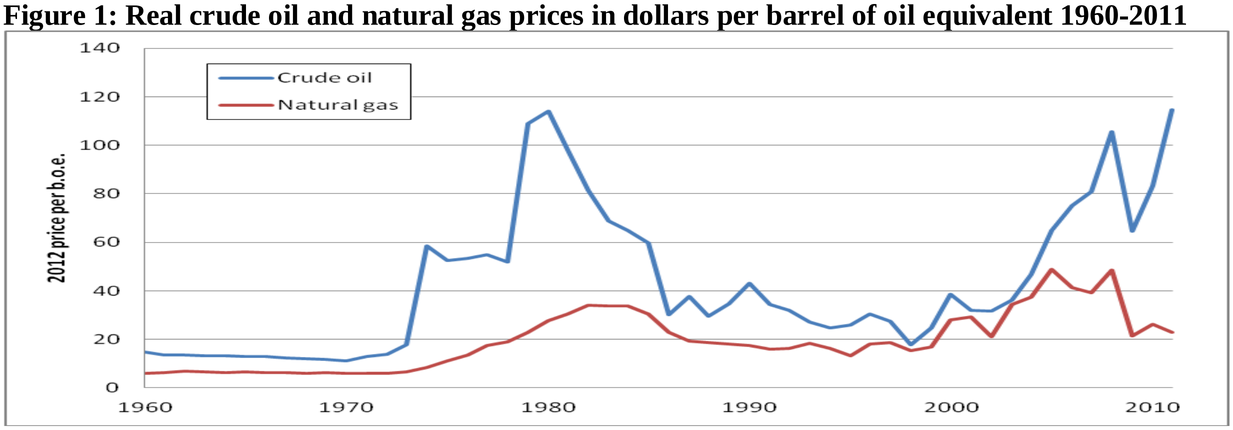
The effects of investment returns uncertainty on optimal management of oil windfalls obviously matters also. Without capital scarcity uncertain returns on public investment make it prudent to redeem more debt which depresses present consumption and increases future consumption. For high degrees of investment returns uncertainty future consumption becomes smaller again due to decreasing returns to public investment. Capital scarcity lowers public investment, debt and average consumption and forces a wedge between present and future consumption even if there is no investment returns uncertainty. With investment returns uncertainty, prudence acts to shift more income to the future. The drop in public investment is stronger if there is no capital scarcity, because the initial investment level is higher. With capital scarcity, saving is more powerful. It curbs both the debt and the interest to be paid on it, so the country needs to do less of it. Uncertainty about public investment returns thus cause countries to save more and invest less. The view that oil-rich countries facing capital scarcity tend to be big savers and small investors (Cherif and Hasanov, 2011) also accords well with an economy without capital scarcity and high degrees of uncertainty about returns on investment.

4. General calibration issues

4.1. Stochastic dynamics of the oil price

The time path of the world oil price is shown in fig. 1.[11] Evidence suggests that it is difficult to reject the hypothesis that the log of the real oil price follows a random walk without drift (Hamilton, 2009). We therefore first estimate a random-walk process. For the period 1960-2011, we obtain the ML estimates of the drift and volatility parameters in (6): $\hat{\upsilon}_P = 0.087$ and $\hat{\sigma}_P = 0.40$. Since $\hat{\upsilon}_P$ is statistically insignificant (t-ratio = 1.55), we set $\upsilon_P = 0$ and ignore long-run trends in the oil price.

Figure 1: Real crude oil and natural gas prices in dollars per barrel of oil equivalent 1960-2011



For Norway approximately half of resource revenues are from natural gas. Fig. 1 shows significant correlation between the oil and gas price and comparable volatility. We obtain the following ML

[11] We use a historical annual series for the price of crude oil (BP, 2012), expressed in 2012 prices using CPI data for the US (from OECD Economic Outlook No. 91). We use the UK natural gas price for the period of 1996-2011(BP, 2012), which seems the best available location. In the absence of generally available data for Europe for the period 1971-1995, we use the U.S. Natural Gas Wellhead Price for that period (USEIA, 2012). As all figures are in US dollars to compare between countries, we use US inflation data to obtain the real oil price. We abstract from purchase power parity conditions.

estimates of the drift and volatility parameters in (6) for the real price of natural gas: $\hat{\upsilon}_P = 0.048$ and $\hat{\sigma}_P = 0.21$ with the drift again insignificant (t-ratio = 1.67).

Since the size of precautionary saving may be quite sensitive to mean reversion (cf., Bems and Carvalho Filho, 2011), we also estimate the AR(1) process (7′), but without trend, and obtain for crude oil $\hat{m}_P^* \hat{\eta}_P = 0.27$ (t-ratio = 1.33) and $\hat{\eta}_P = 0.066$ (t-ratio = 1.16) and $\hat{\sigma}_P^* = 0.29$, which corresponds to a mean price of $\exp(\hat{m}_P) = 110$ US$ per barrel and a volatility of $\hat{\sigma}_P = 0.26$. For the gas price over the same period 1960-2011, we obtain $\hat{m}_P^* \hat{\eta}_P^* = 0.21$ (t-ratio = 1.70) and $\hat{\eta}_P^* = 0.066$ (t-ratio = 1.54), which corresponds to a mean price of $\exp(\hat{m}_P) = 32$ US$ per barrel of oil equivalent, $\hat{\eta}_P = 0.064$ and a volatility of $\hat{\sigma}_P = 0.20$. A random walk process has drawbacks (e.g., uncertainty that grows monotonically with time and the unlimited persistent effect of current shocks into the future) and the AR(1)-process as well (e.g., the estimated coefficients being very sensitive to the sample period and the inclusion of a trend). Hence, we use both the estimated random walk and AR(1)-process for the oil and natural gas price and conduct a thorough sensitivity analysis.

4.2. Population growth and technical progress

For Norway we use a population growth rate of n = 0.5% [12] and a rate of technical progress of g = 1% per year, which implies a trend growth rate of 1.5% per year. For Iraq we use a population growth rate of n = 2.3%. Iraq's growth rate has been quite volatile. We use Iraq's average growth rate during 2005-2011 for the trend rate of real GDP growth of 4.1% per year. We thus set the rate of technical progress to g = 1.8% per year. Ghana's population growth has fluctuated between 2.3% and 2.4% per year during 2001-2012 (from World Bank Development Indicators), so we set n = 2.3% per year. Ghana's productivity growth rate has averaged 3% during 1991-2001 and 4.4% per year during 2001-11 (from World Bank Development Indicators), but we set a more modest g = 2.5% for the long horizon under consideration. Real GDP growth for Ghana has risen sharply from 4.1% in 2009 to 13.6% in 2011 reflecting the start of oil production in 2010. We take a more modest but still large annual trend rate of growth of $n + g$ = 4.8%.

4.3. Interest rates and interest premium on national debt

Our model assumes that countries with a high debt or capital scarcity pay an interest premium on their foreign debt. We assume this is the case for Ghana, but not for Norway or Iraq. From cross-country regressions (van der Ploeg and Venables, 2011) we obtain the interest spread schedule $\Pi(D) = 10^{-4} \exp(6.294)\left[\exp\left(1.9D/31.3\right) - 1\right]$ where 6.294 is the mean log of the spread. This implies

[12] This comes from Norway's official long-term forecast http://www.ssb.no/folkfram_en/main.html .

that a 10%-point increase in the debt-GDP ratio raises the interest differential by 6.9%-points if the economy has a debt-GDP ratio of 100 percent (or 1.3%-points if it has zero foreign debt). There is no empirical support for an effect of the size of the windfall on the interest premium.[13]

Although the real rate of return on the Norwegian SWF has been 2.4% on average during the period of its existence, 1998-2011 (NBIM, 2011), we use the larger historical average of the real interest rate on short-term Norwegian government bonds during 1970-2011 of 3.4% as proxy for the real risk-free rate r^*.[14] This yields a growth-corrected interest rate r of 1.9%. For the benchmark we ensure that growth in consumption in efficiency units is zero, so $\rho = r$ = 1.9%. For Iraq we assume r^* = 6% per year, so the growth-corrected interest rate r is 1.9%. We set ρ equal to 1.9% also. For Ghana we set r^* = 7%, so the growth-corrected interest rate r is 2.2%. We set for Ghana ρ = 2.2% also.[15]

4.4. Public investment: productivity and inefficiency

Recent survey evidence suggests that only 40 to 60 percent of spending on public investment realizes effective accumulation of public sector capital (Dabla-Norris et al., 2011; Gupta et al., 2011). As public investment is increased, its efficiency deteriorates (Berg, et al., 2011; van der Ploeg, 2012). To capture that absorption problems frustrate rapid economic development and to allow a more realistic calibration of the model, we have internal costs of adjustment. The ratio of investment that delivers public capital to total investment spending is the 'public investment measure of inefficiency' or *PIMI* for short. It is given by $PIMI = 1/(1+0.5\phi I/S)$, so that ramping up public investment lowers the *PIMI*. A high value of ϕ reflects absorption constraints so higher marginal returns on public capital are required. Investment is more inefficient in the early stages of economic development when public investment rates are high. Supposing that only 40% of spending on public investment is effective, the steady-state *PIMI* equals $1/(1+0.5\phi\delta) = 0.4$, but in the early stages of development and during the windfall less of investment outlays is delivered (the *PIMI* falls) as public investment rates (I/K) are higher.

A ballpark estimate for the output elasticity with respect to the stock of public capital is 0.15 (Bom and Ligthart, 2009). In line with this evidence, we set $\beta = 0.15/(1-\alpha)$. If we set the share of private capital in aggregate production equal to α = 0.1 (not unreasonable for a country like Ghana), we obtain a reduced-form output elasticity of β = 0.167. Hence, doubling the stock of public capital boosts output by roughly

[13] The coefficients for the effects of the public and publicly guaranteed debt to GNI, the ratio of reserves to GDP and the probability of default on the log of the interest rate spread are, respectively, 1.89, − 4.14 and 0.296.

[14] Excluding recent crisis years when rates in some safe countries have plummeted to below zero, we would have obtained 3.7%. We do not take the 2.4% of the Norwegian SWF as this may be low due to the dominant effect of the downturn in recent years. Bems and Carvalho Filho (2011) also use a high rate of return of 4%.

[15] Hence, $\rho^* = r^* - g/\sigma$ = 1.4%, 2.4% and 2.0% for Norway, Iraq and Ghana, respectively if σ = 0.5.

17 percent. Since we use Ghana to illustrate how capital scarcity affects how the windfall is used for investing to invest, we calibrate our model very roughly to Ghanaian data. We set the depreciation rate of public capital to $\delta^* = 0.025$, which corresponds to an expected lifetime of 40 years, so that $\delta = \delta^* + n + g = 0.087$ for Ghana. The adjustment cost parameter for public investment follows from the steady-state expression for the PIMI: $\phi = 3/\delta = 34.48$. We set the initial stock of public capital to half its steady-state value, $S(0) = 0.5\left[\frac{(1-\alpha)\beta E_0}{r+\delta+\phi\delta(r+0.5\delta)}\right]^{\frac{1}{1-\beta}}$ (using (10)), and calibrate E_0 to match Ghana's non-resource GDP in 2012, $\beta E_0 S^\beta = 36.65$ billion US\$. This gives $S(0) = 10.1$ billion US\$ and $E_0 = 24.9$ for Ghana.

5. Norway: Declining oil windfall, no capital scarcity

Norway discovered its first oil field Ekofisk (one of the world's largest offshore oil basins) in 1969 and started production in 1971. Today there are 57 oil and gas fields in production. Norway is the 5th largest exporter and 11th largest producer of oil in the world and was the third largest exporter and sixth largest producer of natural gas in 2006. Production is shifting from oil and other liquids to gas. The oil and gas industry constitutes about a quarter of GDP and half of exports. The peak of oil production was around the turn of the millennium. Government revenue comes from various sources: ordinary and special tax rates on value added, which have been quite volatile with special taxes taking over from ordinary taxes in importance since the early 1990s (together almost 35 percent of value added); net cash flow from the State's Direct Financial Interest in the gas/oil industry (after initial investment outlays of up to 20 percent of value added in the mid 1980s, net return is now more than one fifth of value added); production fees (no longer an important source); dividends from Statoil (about 3 percent of value added).

Norway puts this revenue in a fund called the Government Pension Fund Global, which started receiving funds in 1996 and has since grown rapidly in size.[16] Its aim is to counter the fall of expected petroleum income and smooth the disrupting effects of oil price volatility, so it has elements of both an intergenerational and a liquidity fund. Roughly 4 percent per annum from the fund is used to finance public spending or tax cuts. This 4-percent rule was implemented in 2001 and allows Norway to spread

[16] This fund comprises two separately managed funds. The main one is the Government Pension Fund Global renamed 1 January 2006 and part of the Norwegian Central Bank (formerly The Government Petroleum Fund established in 1990 and receiving money since 1996). It manages the surplus wealth produced by Norwegian petroleum income (taxes and licenses) and is second largest pension fund in the world. Since 1998 the fund was allowed to invest up to 40 percent of its assets in the international stock market (60 percent from 2007). The other fund is the Government Pension Fund Norway renamed 1 January 2006 (formerly The National Insurance Scheme Fund established in 1967) is much smaller.

oil and gas revenues to future generations.[17] Since future oil revenue cannot be used as collateral for borrowing, Norway's fiscal rule resembles a bird-in-hand rule. Since Norway's budgetary policies take account of declining oil revenue, it also has features of a permanent income rule. Given that Norway's economy is well integrated into world capital markets, capital scarcity is not an issue.

The 1983 Tempo Committee recommended to convert assets under the ground into a Fund and to decouple oil and gas income from spending. The 1988 Steigum Committee advised that public income should depend on the permanent income of total oil and gas wealth (i.e., the value of in-situ oil and gas plus the Fund). Norway wanted a pragmatic, operational and easy-to-understand policy rule, which requires credible, robust estimates of future, unproduced oil and gas revenues and the need to avoid political manipulation of say forecasts of future oil prices. Since smoothing of consumption, public spending and taxes ex ante may require large variations in the net liabilities or asset position in response to changes in the relevant present values, actual policy was more driven by bird-in-hand than permanent-income considerations. This might also have been an appropriate response to deal with oil price volatility.

5.1. The Norwegian oil and gas windfall

Production from proven oil and gas reserves is expected to fall substantially during the next twenty five years. Even allowing for improved recovery, discoveries of new fields and undiscovered resources, forecasts show a decline in oil production levels. Projected net oil and gas cash flows to the government decline up to 2060 and are sensitive to the projected oil price. We take the annual investment cost of the entire Norwegian oil and gas sector including exploration costs as a measure of extraction costs. Using historical data for 1970-2010 from Norwegian Petroleum Directorate (2011) we find that, apart from the initial years 1970-75 when extraction costs were still very high as the very first exploratory and extraction activity took place, average extraction costs were 9 US$/b.o.e. in the period 1990-2000, 6 US$/b.o.e. for 2000-2005 and 14 US$ for 2005-2010. For the future, we adopt constant extraction costs of 15 US$/b.o.e.

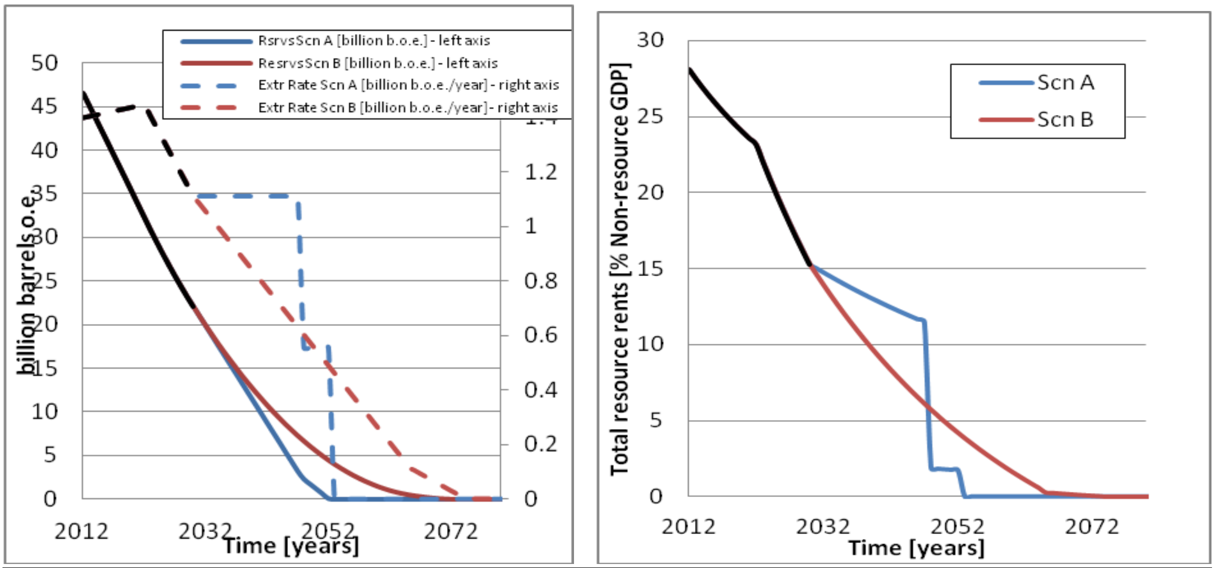
The Norwegian Petroleum Directorate (2011) provides a range of total reserve estimates between 31.5 and 66.7 billion b.o.e. with an average of 46.6 billion b.o.e. at the end of 2011. Because of their different prices and price behavior, we distinguish between oil[18] and gas reserves: 24.5 billion b.o.e. of gas and 22.0 billion b.o.e. of liquids (mainly oil and henceforth denominated as oil). For the extraction scenario we use official forecasts by the Norwegian Petroleum Directorate (2011), which are available until 2030 and are shown in the left panel of fig. 2.

[17] The Fund also allows Norway to stabilize the economy across the business cycle, since the 4 percent is meant to be an average over the business cycle. Since value of the Fund varies with world asset markets, the government has the discretion to deviate from the 4 percent rule when it deems this necessary.

[18] We group oil and other liquids only excluding gas as liquids, henceforth referred to as oil, which follows the Norwegian Petroleum Directorate (2011).

Thereafter we consider two scenario's for the extraction rate: (A) continue at 2030 rate until resources are fully exhausted so that gas reserves are exhausted in 2052 and oil reserves in 2048 (base case); (B) linear decline from 2030 until resources are fully exhausted so that gas reserves are exhausted in 2075 and oil reserves in 2064. The oil rents are plotted in the right panel of fig. 2. Annual oil rents are thus estimated to be 92 billion US$ in 2012 or 28% of GDP and taper off to 15% of non-resource GDP in 2030 under the official production forecast and our estimated process for the oil price.

Figure 2: Reserves, extraction rates and oil/gas rents for Norway



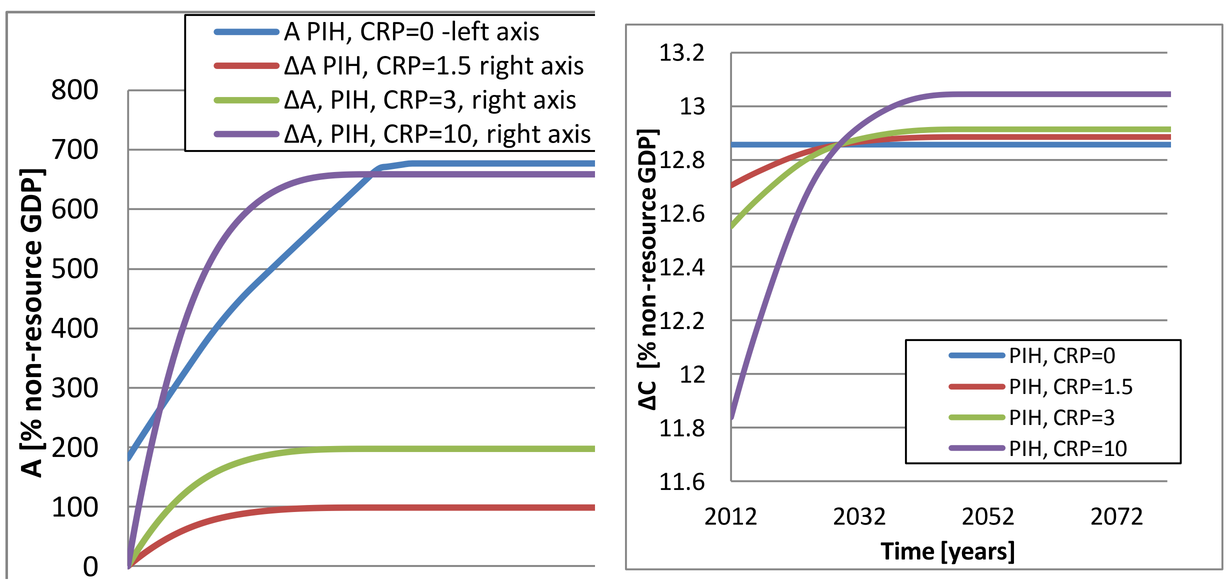
The size of the Norwegian Government Pension Fund Global (A_0) was a staggering 3,312 billion NOK at the end of 2011 (NBIM, 2012) corresponding to 592 billion US$ or 122% of GDP and 181% of non-resource GDP. Norwegian GDP in 2011 was 2,720 billion NOK or 484 billion US$ and non-resource GDP was 326 billion US$.

5.2. Estimates of optimal intergenerational and liquidity buffers for Norway

To determine the size of the optimal intergenerational buffer, we calculate the optimal savings responses when oil price volatility is set to zero or prudence is absent (CRP = 0). The blue line in the left panel of fig. 3 shows that this eventually results in a gradual accumulation from 1.8 to 6.8 of non-resource GDP of additional sovereign wealth (see left axis). This saving response permits a permanent increase in consumption of 12.9% of non-resource GDP. This amounts to an annual annuity of 8,537 US$ for each Norwegian citizen in 2012 (and then grows at the trend rate of g = 1.5% per year). This buffer shows the optimal development of the intergenerational fund.

To get an idea of the optimal size of the liquidity fund, we also calculate the optimal savings response with oil price volatility described by the AR(1)-estimate of the stochastic dynamics of the world oil price (7) given in section 4.1.Taking a ballpark figure for the coefficient of relative prudence of 3, the green line in the left panel (right axis) indicates the prudent saving response. Norway should thus accumulate an additional 3% of non-resource GDP in its liquidity fund, which does not appear large relative to the size of the intergenerational fund. Still, it corresponds to an additional fund of 1966 US$ per Norwegian citizen in 2012 and grows at the rate of 1.5% per year. If the coefficient of relative prudence is 10, the purple line in the left panel (right axis) indicates that it is optimal to have a liquidity fund of 10% of non-resource GDP which still amounts to only about one seventieth of the intergenerational fund. With a prudence coefficient at the lower end of the plausible range of 1.5, the red line shows that the optimal liquidity fund is only 1.5% of non-resource GDP or 0.2% of the intergenerational fund.

Figure 3: Optimal savings and consumption responses to Norwegian windfall (scenario A)



The right panel of fig. 3 shows the corresponding consumption increments. Taking account of oil price volatility and using the ballpark coefficient of relative prudence of 3, consumption increases initially by only 12% and then rises to 12.9% of non-resource GDP. The prudent saving response thus implies less consumption today (−0.3% of non-resource GDP with $CRP = 3$) and more consumption in the long run (0.06% with $CRP = 3$). These prudential tilts in the consumption profile are bigger if relative prudence is bigger, begging the question what the socially optimal degree of prudence is. For $CRP = 10$ (purple line) precautionary savings are 1% of non-resource GDP in 2012, resulting in a sustained increase in consumption of 0.19% of non-resource GDP after the windfall. Finally, the temporary nature of Norway's windfall implies that the marginal propensity to consume with respect to the price of a barrel of oil (14) falls monotonically and thus the prudence effect also falls with time and the upward tilt of the path for the consumption increment reduces with time.

5.3. Sensitivity analysis

The benchmark presented in fig. 3 had initial reserves of 46.6 billion b.b.o. with extraction scenario A, σ = 0.5, n = 0.5%, g = 1%, r^* = 3.4%, for crude oil $\hat{m}_P = \log(110), \hat{\eta}_P = 0.06, \sigma_P = 0.26$ and zero trend, and for natural gas with $m_P = \log(32)$, $\eta_P = 0.06$, $\sigma_P = 0.20$ and zero trend in (7). Initial price levels in 2011 were 110 US$/b.o.e. for oil and 51 US$/b.o.e. for gas. Table 1 gives some sensitivity results.

If linearly declining extraction rates are used from 2030 (scenario B), the windfall is spread out over a longer period which results in a somewhat smaller intergenerational fund and thus a smaller permanent increase in consumption, but thanks to discounting the effect is small. On the other hand, a less rapidly declining oil depletion rate induces a bigger liquidity fund and thus consumption rises initially by less and rises in the long by a little more (compared to the no-windfall outcome). Since initial oil and gas reserves are highly uncertain, table 1 also report what happens if reserve levels are at the higher end of the estimated range. Evidently, both the intergenerational and the liquidity funds and the resulting consumption increments are larger but due to discounting less than proportional.

Table 1: Sensitivity results for Norway (percent of non-resource GDP)

	ΔC^I	$-\Delta C^L(2012)$	$\Delta C^L(\infty)$	$A^I(\infty)$	$A^L(\infty)$
Base case	12.86	0.30	0.06	676.7	2.96
Extraction scenario B	12.64	0.36	0.07	665.3	3.92
Initial reserves +50%	14.41	0.39	0.11	758.2	5.70
r^* = 2.4%	6.74	0.09	0.01	749.0	0.87
g = 0.5%	16.24	0.49	0.12	676.7	5.13
Random walk	13.69	1.26	0.20	720.5	10.5
AR(1) with η_P=0.09	12.70	0.19	0.04	668.3	2.00
Lower oil/gas price	10.65	0.19	0.03	560.4	1.63

A lower real return on sovereign wealth has dramatic effects. Although it boosts the intergenerational fund by more than 70% of non-resource GDP, the resulting permanent increase in consumption is only 6.74% instead of 12.86% of non-resource GDP. As is clear from equation (14), the marginal propensity to consume out of an oil price shock is much less with a smaller real return and thus from (8a) or (11) we see that the prudence effect and the liquidity fund that is accumulated are much smaller. The prudential tilt in the consumption function is consequently smaller too. A lower economic growth rate boosts oil production in efficiency units and thus from (14) boosts the marginal propensity to consume out of an oil price shock and from (11) induces a much bigger liquidity buffer, i.e., 5.13% instead of 2.96% of non-resource GDP. Although the size of the intergenerational fund is unaffected, the increase in the growth-corrected real return induces a much bigger permanent increase in consumption.

If our estimates of a random walk for the oil price are used instead of an AR(1) process, we find a bigger prudence effect as shocks now persist forever (see (13) and (14)) and thus the optimal size of the liquidity fund is a factor 3.5 bigger. The precautionary consumption tilt is thus larger as well. With a bigger mean-reversion parameter of 0.09, the results go the opposite way: the prudence effect is small and thus the optimal size of the liquidity fund is smaller. It is a pity that it is so tough to pin down statistically the degree of mean reversion in oil prices, since the resulting optimal size of the liquidity fund depends on it.

Finally, a lower mean price for oil and gas ($\exp(m_P) = 70$ US$ for crude oil and $\exp(m_P) = 20.4$ for natural gas, i.e., a little more than a third less for both of them) scales down the optimal size of the intergenerational fund less than proportionally and scales down the liquidity fund more than proportionally (as initial prices are unchanged). Hence, the sustained increase in consumption rises less than proportionally and the precautionary tilt in the consumption path increases more than proportionally.

5.4. Comparison of optimal fiscal rule with historical fiscal rule

Norway's fiscal rule for the non-oil/gas primary public sector deficit B_t has been estimated for 1954-2007 using official forecasts for the expectations on future oil/gas revenue to calculate permanent values (Harding and van der Ploeg, 2012). The permanent income hypothesis implies values of 0 and 1 for the coefficients on current and permanent oil/gas revenue and is rejected by the data: the estimated rule suggests that a third not zero of each Krone of oil/gas revenue is used to raise the primary public sector deficit and for permanent oil/gas revenue the effect is 0.3, not 1. The bird-in-hand hypothesis implies a zero coefficient on permanent oil and gas income as only already accumulated assets should affect spending decisions. Hence, the estimated rule has features of both the permanent-income and bird-in-hand rule. Continuation of the estimated historical fiscal rule gives rise to an inverse hump-shaped time path of accumulation of sovereign wealth, i.e., a gradual rise to additional wealth of 76% of GDP in 2023, then falling to 29% of GDP in 2060 and in the very long run vanishing altogether. This inverse hump shape does not occur in our calculations of the wealth flowing into the intergenerational and liquidity funds that are based on the permanent income hypothesis.

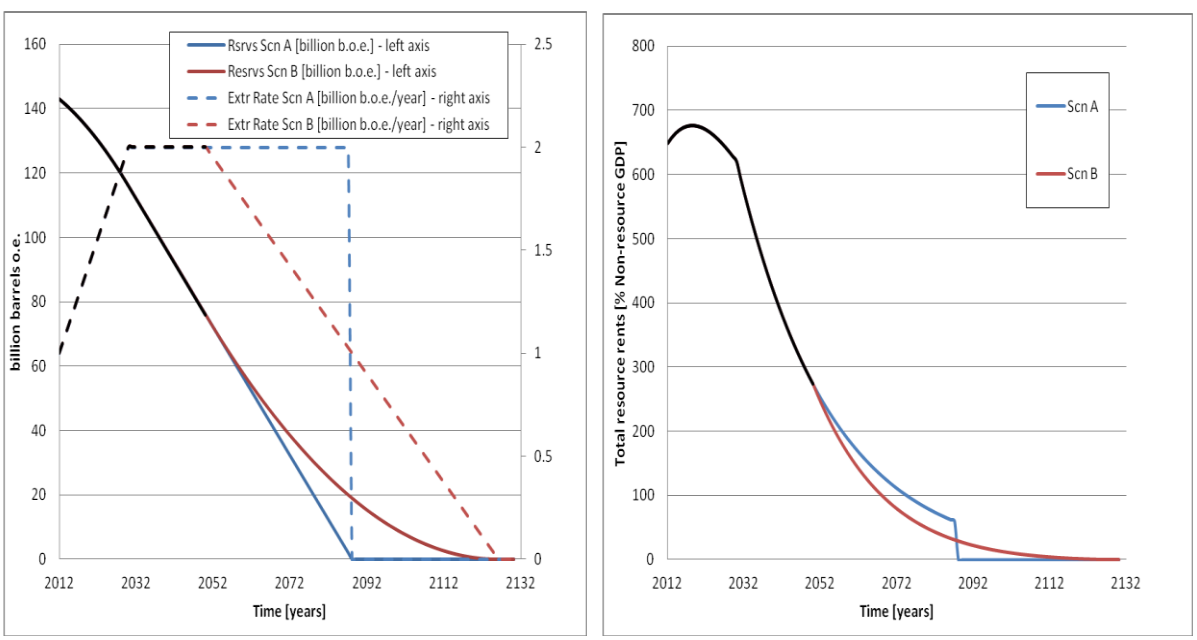
6. Iraq: Huge and long-lasting oil windfalls

From 1980 to1988 Iraq was embroiled in the war with Iran. In 1990-1991 the invasion of Kuwait led to the first Gulf War. After a decade of relative peace, 2003 saw the beginning of the second Gulf War with many casualties and about five million refugees. In late 2011 the U.S. withdrew the last of its troops, but there are still sectarian violence and homicides in Baghdad. Still, Iraq has made significant economic progress since the Transitional Government was established in 2005. Annual headline inflation has fallen from over 60% to single digits and the dinar has remained stable against the US dollar. Debt sustainability has improved with the 80% reduction of the Paris Club in 2004 and negotiations are underway with non-Paris Club creditors. Domestic fuel subsidies have been removed. The reduction in conflict and ensuing economic progress has led to a large increase in oil extraction.

6.1. The Iraqi oil windfall

Oil reserves at the end of 2011 are 143 billion barrels of oil, which has been strongly revised upwards from the estimate of 115 billion barrels of oil at the end of 2010. Production is projected to reach 2 billion barrels per year in 2030 from 1 billion barrels in 2011 (BP Statistical Review, 2012). We follow this forecast and let extraction grow linearly until 2030. Thereafter, we consider two scenarios: (A) continued extraction at the 2030 rate until exhaustion, which will take place in 2088 (base scenario); (B) continued extraction at the 2030 rate until 2050 followed by linear decline until exhaustion, which will take place in 2126. Based on the estimated extraction costs for Norway of 4-14 US$/b.o.e. (part of which is offshore), we guess extraction costs for Iraq, onshore and generally cheap to extract, to be 10 US$/barrel. The resulting reserves, extraction rates and oil rents are plotted in fig. 4. Resource rents start at 650% of non-resource GDP in 2012 and then taper off in both scenario A and B.

Figure 4: Reserves, extraction rates and oil rents for Iraq

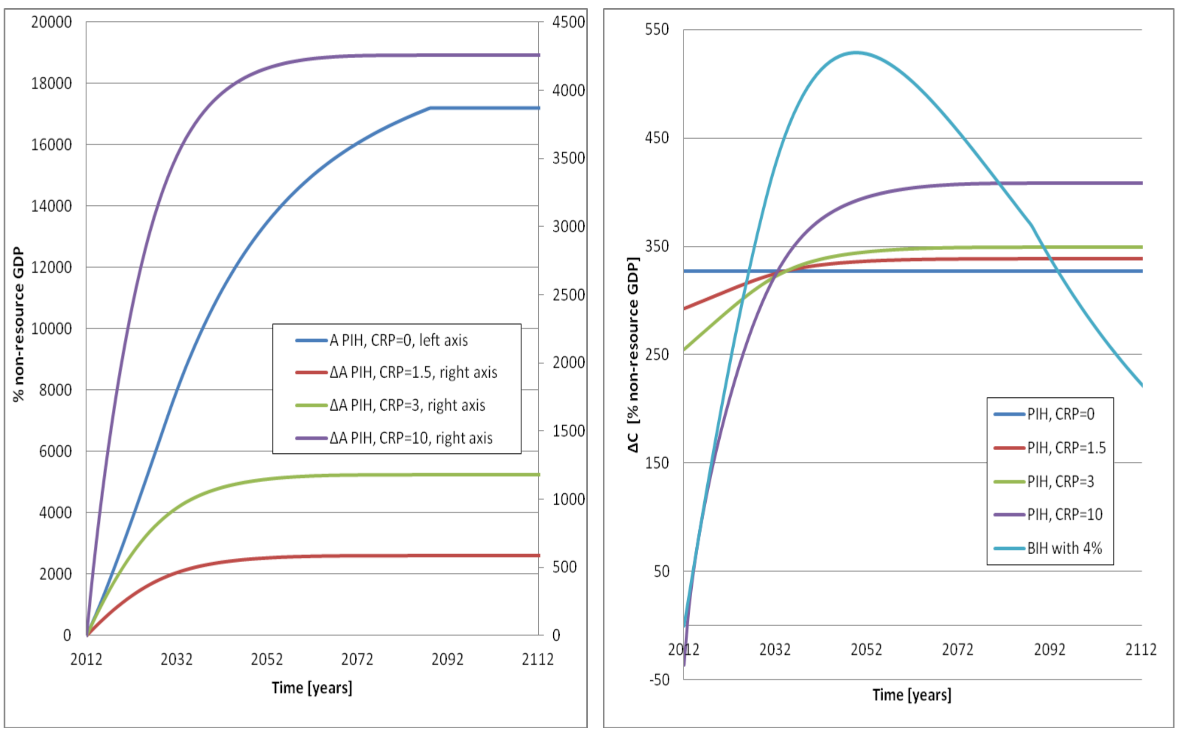


In 2010 oil production accounts for over half of GDP and 83% of government income (IMF, 2010) and these rates are even higher at current high oil prices. With estimated extraction costs of 10US$ per barrel and oil prices at 110 US$/barrel in 2011, oil rents reach a staggering 87% of total GDP (115.4 billion US$, 2011 GDP) or 650% of non-resource GDP. Fig. 4 indicates that oil rents are projected, at an oil price of 110 US$/barrel, to rise to 200 billion US$ in 2030.

6.2. Dominance of the liquidity buffer

Not taking account of oil price volatility, Iraq should accumulate sovereign wealth amounting to a colossal 172 times non-resource GDP reflecting both the huge size of its windfall and its low level of non-resource GDP. This ensures a sustained annual consumption increment of 3.3 times non-resource GDP. This amounts to a final fund of 80,429 US$ and ever-lasting annual annuity of 1,528 US$ for each Iraqi citizen in 2012 which will grow from then on at 4.1% per year. Although the Norwegian windfall is much smaller than that of Iraq, relative to its population it is larger and therefore Iraq's annuity for each citizen is smaller than for Norway (8537 US$).

Fig. 5: Optimal savings and consumption responses to Iraqi windfall (scenario A)



Since the Iraqi windfall is so large and lasts so long, the marginal propensity to consume out of oil wealth (13) is relatively large and thus the prudence effect shown in (12) is very large. Iraq is thus very vulnerable to oil price volatility and needs to build up a relatively large volatility buffer or stabilization fund compared to its generational fund. In fact, for *CRP* = 10 (see purple line in fig. 5) the consumption increment in 2012 is negative. This does not happen for lower degrees of prudence such as *CRP* equal to 1.5 or 3 (red and green lines, respectively). The prudent gradual accumulation of financial assets over eight decades leads in the base case with *CRP* = 3 to a volatility buffer of 11.8 times non-resource GDP (5,510 US$ per Iraqi), which amounts to 6.9% of the intergenerational fund. This brings the total sovereign wealth fund up to 184 times non-resource GDP or 85,839 US$ per Iraqi citizen, both growing at 4.1% per year. To achieve this amount of prudent saving, consumption has to fall compared with the no-prudence outcome, so that it rises by 255% instead of 327% of non-resource GDP initially (i.e., by 338 US$ per citizen less) and then rises to 349% of non-resource GDP in the long run. Interestingly, the cumulated assets path is for very high degrees of prudence not that different from the BIH path.

6.3. Sensitivity analysis

The base case reported in fig. 5 used n = 2.3%, g =1.8%, r^*=6%, extraction scenario A, and the same estimated oil price process (7) used for Norway. Table 2 offers some sensitivity results. Whether extraction scenario A or B is used does not make much difference for the size of the intergenerational and liquidity funds and for the consumption increments. Due to the high discount rate (r^* = 6.2%) and long time horizons involved for the Iraqi windfall, additional oil reserves do not have much effect either. More disappointing growth prospects (g = 0.8%) does not affect the final size of the intergenerational fund (as consumption in efficiency units is smoothed across generations), but does lead to a bigger permanent consumption increment. It also leads to a bigger liquidity fund and precautionary tilt of the consumption profile.

Table 2: Sensitivity results for Iraq (percent of non-resource GDP)

	ΔC^I	$-\Delta C^L(2012)$	$\Delta C^L(\infty)$	$A^I(\infty)$	$A^L(\infty)$
Base case	327.1	72	22.4	17,214	1,179
Extraction scenario B	322.2	73	22.4	16,960	1,177
Initial reserves +50%	330.9	73.0	23.0	17,414	1,209
r^* = 5%(-1%)	188.7	32.2	4.7	20,969	520
g = 0.8% (-1%)	499.2	140.6	81.7	17,214	2,817
Random walk	327.1	427.1	146.7	17,214	9,043
AR(1) with η_P=0.09	327.1	45.1	14.4	17,214	757
Lower oil price	249.2	49.1	13.5	13,115	710

A random walk for the world oil price (or more mean reversion in the oil price) does not affect the size of the optimal intergenerational fund, but leads to a factor 8 bigger (about a third smaller) liquidity fund.

Finally, a lower oil price ($\exp(m_P) = 70$ US$ instead of 110 US$ per barrel) leads to a less than proportional fall in the intergenerational fund and to a more than proportional fall in the liquidity fund.

7. Ghana: a temporary, small oil windfall and for investing to invest

Despite a long history of exploration, oil production in Ghana has been negligible until it started producing oil from the offshore Jubilee field in 2010. Based on current proven reserves, production from the Jubilee field is expected to peak from 2013-2015 at 120,000 barrels of oil per day, and last for 20 years (Oil, March 2011). This has the potential to generate up to US$ 1.8 billion per annum at peak production. Ghana is approximately 50th in the world in terms of proven oil reserves. Ghana's potential of possibly 4 billion barrels is much lower than those of Saudi Arabia (265 billion), Canada (175 billion), Venezuela (98 billion) and Nigeria (38 billion). At 160 barrels of oil per person Ghana's deposits are far less than those of Kuwait (40,000), Saudi Arabia (10,000), Venezuela (3,500) and Nigeria (240). Ghana's reserves per dollar of GDP are approximately 15th in the world, on par with Angola and Nigeria (CIA, 2011). Production from the Jubilee field at its peak will generate up to 30% of the government's income, if oil is at 75US$ per barrel. Ghana has GDP per capita of 1,470 US$.

Ghana's small and temporary oil windfall must be harnessed to promote sustainable economic growth and to provide for future generations. This differs from Nigeria, which has a large and temporary windfall and needs to worry more about alleviating absorption constraints and preventing inequality and corruption. Iraq has a much more permanent windfall and it has a real need to manage oil price volatility to safeguard recurrent spending on things like government jobs. In contrast, a small oil-rich country like Kuwait has no serious absorption constraints as labor and capital can be imported.

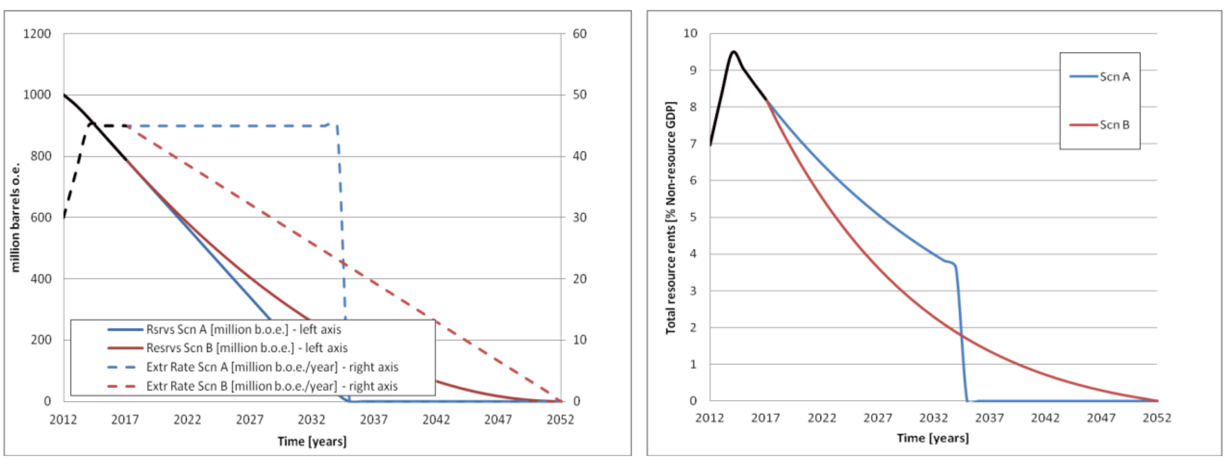
Ghana has adopted the Petroleum Revenue Management Act (PRMA) in 2011. It specifies how oil revenues are to be allocated between the annual budget and sovereign wealth funds as the income is received. The resulting spending profile is brought forward significantly in comparison to the permanent income benchmark. This may be sub-optimal given Ghana's stock of outstanding foreign debt, and will only be appropriate if the spending stimulates growth through domestic investment and cutting distortive taxes. The government's oil revenue from the Jubilee field has four components: (1) a royalty of 5 percent of gross oil revenues; (2) 13.75 percent of the field's commercial net profits go to the Ghana National Petroleum Corporation; (3) an "additional oil entitlement" of 10-25 percent of petroleum revenue (net of royalties and the GNPC interest) accrues if the project rate of return is between 18 and 33 percent; (4) a company income tax on all net profits of 35 percent.

The PRMA allocates government oil revenues between the annual budget and sovereign wealth funds. The allocation is based on “benchmark revenue”, using a seven year moving average of oil prices (including three projected years), and a three year average of output (including projections for the next year). From the benchmark revenue 50-70% is allocated to the annual budget, of which a minimum of 70% must go to investment in eleven priority areas and the remainder is consumed. The 30-50% of benchmark revenue not allocated to the annual budget is put in sovereign wealth funds which invest in foreign assets; a minimum of 30% must go to a heritage or intergenerational fund and the rest to a stabilization or liquidity fund. The spending profile outlined by the PRMA is brought forward significantly from the permanent-income benchmark. It may be too heavily weighted towards short-term spending, particularly given Ghana’s outstanding stock of public debt, which may be increasing the cost of borrowing and restricting private investment. If funds are to be spent in the short term, they should be used to boost public capital and cut distorting taxes and thus boost private investment and growth.

7.1. Ghana’s offshore oil windfall

In contrast to Norway, Ghana has only recently enjoyed an oil windfall. The base case extraction scenario used in a recent comprehensive report by the World Bank is based on proven recoverable oil reserves of 490 million barrels (Dessus et al., 2009) and probably underestimates Ghana’s future production and projects an oil production profile until 2029.

Figure 6: Reserves, extraction rates and oil rents for Ghana



We adopt a more optimistic estimate of 1 billion barrels of oil reserves at the end of 2011 and assume a linear increase from 30 in 2012 to 45 billion barrels of oil in 2014 followed by constant production at that level until 2017. Hence, we consider two scenarios in fig. 6: (A) continue at 2017 level until exhaustion in 2034; (B) linear decline until exhaustion in 2052.

Based on the estimated extraction costs for Norway of 4-14 US$/b.o.e., part of which are offshore, we guess extraction costs to be rather higher at 25 US$/barrel reflecting the fact that the reserves are offshore and in deep water. At peak production in 2014, annual resource rents are approximately 3.8 billion US$, 140 US$ per citizen (not in efficiency units) or 0.5% of non-resource GDP and declining rapidly thereafter.

7.2. Intergenerational and especially liquidity buffers relatively small

Our estimates of the optimal size of the intergenerational and liquidity funds apply to the economy as a whole as we do not distinguish between private and public oil rents. For the base case with n = 2.3%, g = 2.5%, r^* =7% and extraction scenario A, we find that the optimal size of the intergenerational fund in the long run is 115% of non-resource GDP which permits a sustained increase in consumption of 2.5% of non-resource GDP. This amounts to an annuity of a mere 37 US$ per citizen, albeit growing in real terms at 4.8% per year. The optimal size of the liquidity fund for Ghana and the resulting precautionary tilt of the consumption time path are hardly noticeable. Qualitatively the same insight is obtained for extraction scenario B, higher oil reserves and a random walk process for the world oil price or more mean reversion in the process for the world price. A lower growth rate of the economy leads to a less rapid decline of oil rents in efficiency units and to a larger, albeit still hardly significant liquidity fund. Since the windfall is relatively small and temporary, equations (12) and (13) indicate that the prudence effect is much smaller than for Norway (and a fortiori than for Iraq).

Table 3: Sensitivity results for Ghana [all results as % of non-resource GDP]

	ΔC^I	$-\Delta C^L$(t=2012)	ΔC^L(t=∞)	A^I(t=∞)	A^L(t=∞)
Base case	2.53	0.026	0.0036	115.0	0.16
Extraction scenario B	2.35	0.031	0.0045	106.7	0.21
Initial reserves +50%	2.90	0.035	0.0064	131.6	0.29
g = 1.5% (-1%)	3.68	0.058	0.012	115.0	0.39
Random walk	2.53	0.064	0.0078	115.0	0.35
AR(1) with η_P=0.09	2.53	0.019	0.0027	115.0	0.12
Lower oil price ($\exp(m_P) = 70$)	2.04	0.018	0.0022	92.8	0.10

7.3. Investing to invest

Given that Ghana is likely to suffer from capital scarcity, it should allocate (in contrast to Norway) part of its windfall not to sovereign wealth but to investment in the domestic economy. Present consumption should then be below that of future consumption for two reasons. First, if there is capital scarcity, it is optimal to pay off debt and reduce the interest burden. This effect is especially strong if intergenerational inequality aversion is not so large ($1/\sigma$ small) and capital scarcity is substantial. Second, the need to build a precautionary saving buffer tilts consumption towards the present but we have seen that given the small and temporary nature of Ghana's windfall this effect is tiny. Hence, we abstract from oil price volatility and focus at the tradeoff between capital scarcity and the need to invest versus the desire to spread wealth towards future generations.

The calibration is discussed in sections 4.3 and 4.4. [19] It implies the following steady state for Ghana: $S(\infty) = 20.2$, $Y(\infty) = 41.1$ and $C(\infty) = 23.6$ billion US$ (2012 prices). We set Ghana's initial external stock of public and publicly guaranteed external debt for 2012 equal to $D_0 = 6$ billion US$ (14% of GDP). The consumption increment that can be sustained under the permanent income rule is 0.93 billion US$ (2012 prices) in 2012 and grows at 4.8% per year from then on. Fig. 7 portrays, in contrast, the optimal development paths for Ghana without windfall (dashed lines) and with windfall (solid lines) for variables of endogenous variables measured in efficiency units. [20]

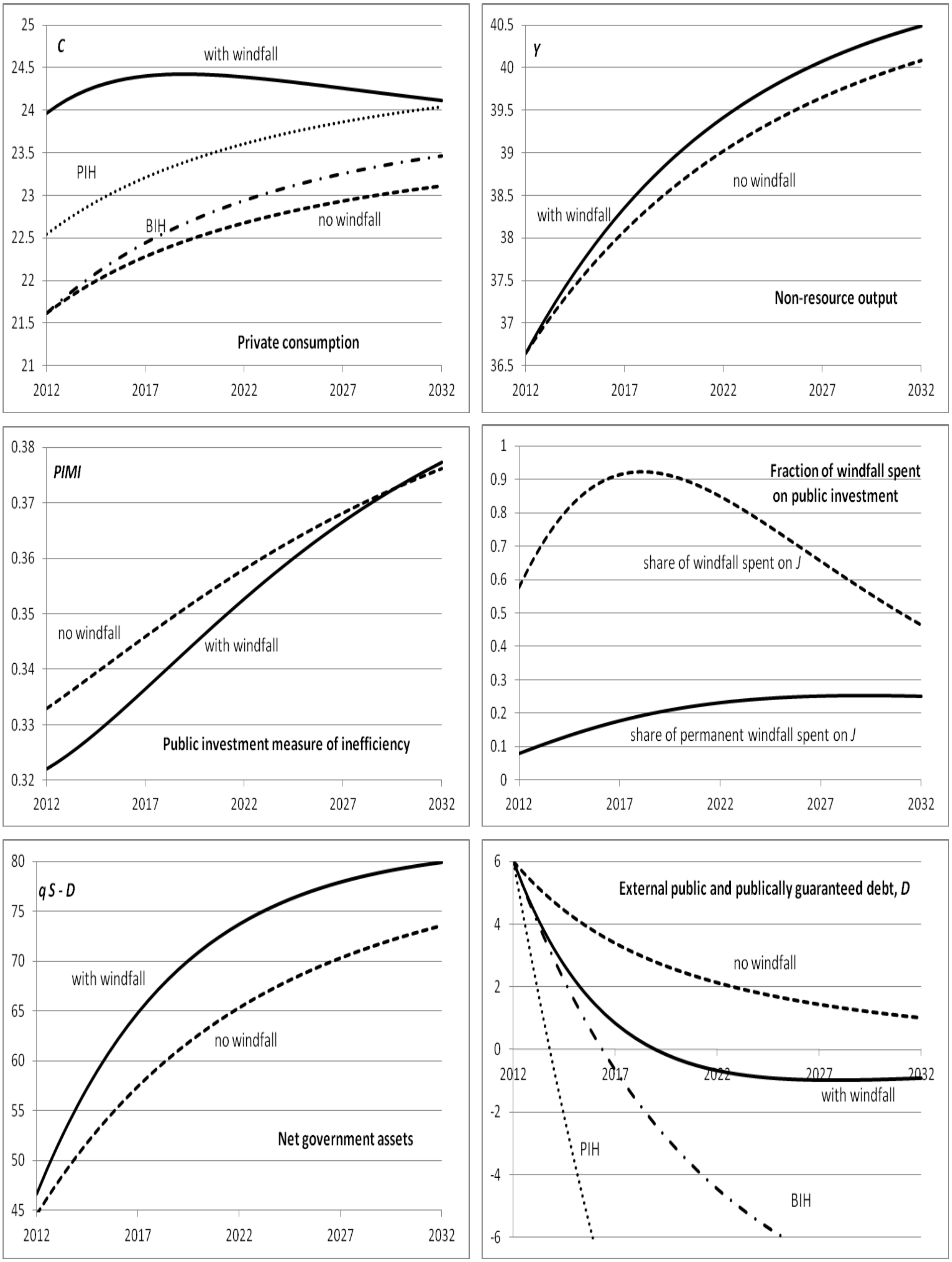
Without the windfall, Ghana will grow from a sub-optimally low level of its public sector capital stock along its development path. The gradual rise in public capital and output is associated with a rising path for public investment. Because of the absorption constraints the economy faces in the early periods of development when investment is high, the efficiency of public investment gradually improves with time (witness the rise in the *PIMI*). In the very long run output grows from 36.6 to 41.1 billion US$ whilst the public and publically guaranteed debt vanishes.

The effects of the oil windfall are to allow a more rapid buildup of public investment (signaled by a temporary higher social value of public investment, q) and a speeding up of the process of economic development. This inevitably leads to a temporary deterioration of the efficiency of public investment (lower *PIMI*). For a considerable amount of time the stock of public capital in efficiency units is higher than without the windfall which leads to long-lasting higher levels of output in efficiency units. Given that Ghana's windfall is fairly short and small it is a modest but significant increase. Still, the windfall allows for an increase in consumption from 21.6 to 24.0 billion US$ at the start of the windfall and from 23.0 to 24.2 billion US$ in efficiency units (49.6 to 52.2 billion US$ unadjusted) in 2028. The windfall allows external public and publically guaranteed debt to be paid off more rapidly, so that the social cost of

[19] We approximate oil rents in efficiency units by $N(t) = 3.8\exp[-0.068(t-2012)]$, so $N_0 = 3.8$ billion US$ and oil rents decline in 2012 dollars at 2% per year (as $n + g = 4.8\%$). This matches 2012 in situ oil wealth of 42 billion US$ and the annuity value of oil wealth in efficiency units as 2.2 percent of that, i.e., $N^P(0) = 0.93$ billion US$/year.

[20] The optimal policy simulations are obtained from a linearization of the state-space model.

Figure 7: Harnessing Ghana's windfall for domestic investment



Key: *Y*, *C*, *D* and *q S – D* are in billions US$ of 2012 (in efficiency units). *I /S* = 2(1 – *PIMI*)/*PIMI* and *q* = 1 + ϕ*I*/*S*.

borrowing falls more rapidly and public investment is stimulated. Net government assets (value of public capital minus public and publically guaranteed debt) jump up from 44.6 to 46.6 billion US$ on impact due to the jump increase in the social value of public capital, q. Afterwards, net assets continue to grow initially more rapidly with the windfall than without the windfall but eventually less rapidly and the difference in net government assets tapers off.

There are two key insights. First, much of the consumption increment comes in the early periods of the windfall. This reflects that future generations will be richer so more of the windfall is spent on current generations to avoid too much intergenerational inequality (more so if σ is smaller). Second, a substantial part of the windfall is spent on public investment which reflects that capital scarcity means that the economy has a sub-optimally low level of public investment. The windfall must thus be used to fund an investing-to-invest strategy, too boost consumption and to pay off foreign debt in a balanced way.

Partial-equilibrium application of permanent-income (PIH) and bird in hand (BIH) rules do not affect capital formation and output. Compared with the no-windfall paths, the PIH rule leads to a permanent increase in consumption of 927 million US$ and a long-run size of the intergenerational fund of 42.15 billion US$ (both in efficiency units). The BIH rule leads to a temporary buildup of more than 9 billion US$. These rules do little to stimulate the economy and thus lead to much lower consumption in the next three or four decades than the optimal 'investing to invest' trajectories.

Although our calibration is ad hoc with data on many variables hard to come by, the qualitative insight from fig. 7 that the 'investing to invest' strategy is better for developing economies with capital scarcity than the naive PIH and BIH rules remains valid.

8. Concluding remarks

We have highlighted the different roles that intergenerational, liquidity and investment funds can play in managing and harnessing oil windfalls. An intergenerational fund is needed to smooth the benefits of time-varying windfalls across generations. A liquidity fund is required to protect the economy against stochastic oil price volatility in addition to the usual arguments in favor of more political stability and flexibility of the economy. A liquidity fund offers an important alternative for hedging against oil price volatility, since hedging and related structured products have too many economic costs and political risks. Furthermore, markets are too thin to deal with size and duration of a windfall of a country like Iraq. The size of the liquidity fund should be larger if oil income volatility is higher, it makes up a larger share of income, and governments are more prudent. More notably, its size also depends on the marginal propensity to consume out of a windfall and thus strongly on the interest rate: only if oil price shocks lead to consumption shocks, do they necessitate precautionary buffers.

If the windfall is temporary, the oil rents are largely saved and little precautionary saving is needed (Ghana). If the windfall is permanent combined with the random walk behavior of the oil price, shocks in the oil price lead directly to shocks in consumption and large precautionary buffers are required (Iraq).The relative size of oil rents to GDP also matters for prudence and precautionary saving.

Table 4 sums up our base case estimates of the optimal sizes of the intergenerational and liquidity funds and the resulting consumption increments. Iraq's windfall is the largest and lasts longest, so its liquidity fund is relatively much larger: it is 6.85% of its intergenerational fund whilst for Norway it is only 0.44% and for Ghana 0.17%. This reflects that Iraq's windfall lasts longer than that of Norway and longer still than that of Ghana and that the Iraqi windfall makes up an enormous share of consumption for the foreseeable future. Ghana and Iraq thus have relatively a larger intergenerational fund. Since Iraq has a much larger population (33 million) than Norway (5 million), its oil annuity per citizen is less despite having a bigger windfall in absolute terms. Ghana's windfall does not really necessitate a liquidity fund.

Table 4: Optimal sizes of intergenerational and liquidity funds (*CRP* =3)

Country			Norway	Iraq	Ghana
Intergenerational fund	Final fund size	[% non-resource GDP]	677	17,214	115
		[USD per citizens]*exp(-*gt*)	449,292	80,429	1,688
	Permanent consumption annuity	[% non-resource GDP]	12.9	327	2.5
		[USD per citizens]*exp(-*gt*)	8,537	1,528	37
Liquidity fund	Additional final fund size	[% non-resource GDP]	3	1179	0.2
		[USD per citizens]*exp(-*gt*)	1966	5510	2
	Precautionary saving in 2012	[% non-resource GDP]	0.3	72	0.03
		[USD per citizens]*exp(-*gt*)	202	338	0.4
	Additional permanent consumption annuity	[% non-resource GDP]	0.06	22	0.004
		[USD per citizens]*exp(-*gt*)	37	105	0.05

By setting $r = \rho$, we have effectively assumed that future growth cannot be used as collateral for borrowing on the international capital market and we thus smooth consumption in efficiency units, not consumption itself. Productivity growth of non-resource income then acts as a natural mechanism that curbs the need for precautionary saving. An alternative is to allow future growth to act as collateral in which case we smooth per-capita consumption across generations (by setting $r^* = \rho^*$). Assets per capita will then vanish asymptotically as non-resource income grows forever. If the windfall is big enough, the majority of the windfall is consumed and a small part is still saved in an intergenerational fund.[21] A larger part is accumulated in a liquidity fund. [22]

Out of a sample of 31 oil producers 21 have funds of which 10 focus on stabilization and 8 on stabilization and saving (IMF, 2005). In practice, the size of the liquidity fund should depend on the features highlighted in our analysis, but also on the costs of volatility to the domestic economy or the opportunities for borrowing in the downturn. Stabilization or liquidity funds are typically contingent on the oil price or oil revenue. The political risk of such funds being looted also matters. If this is a serious risk, government will have a bias towards partisan, illiquid investment projects at the expense of saving in liquid sovereign assets and/or growth-enhancing (neutral) investment projects.

For developed countries with good access to world capital markets, not even part of the windfall should be spent on domestic investment. However, for countries with capital scarcity it makes sense to channel their windfalls of foreign exchange into a domestic investment fund if the expected return on domestic investment and the cost of borrowing are more than the return on sovereign wealth. Not all developing economies can absorb the extra windfall-induced demand for non-traded consumption and investment goods, especially if absorptive capacity is limited. In that case, there is a rationale for a parking fund in addition to an intergenerational and a liquidity fund.

Our illustrative calculations indicate that the optimal liquidity buffer for Ghana is very small relative to its intergenerational fund even for very high degrees of prudence, Norway's optimal liquidity fund is small and Iraq should a much more significant liquidity fund as well as an intergenerational fund. Especially in the early years precautionary saving constitutes a large share of Iraq's oil rents. Given capital scarcity and inefficient adjustment of public capital, we argue that resource-rich, developing countries like Ghana should aim to use part of their windfall for public investment rather than hedging against commodity price volatility. This gives a boost to their economy and delivers more consumption than a permanent-income or bird-in-hand rule. Iraq does suffer much less from capital scarcity, but might have a real problem absorbing its large and growing windfall. In that case Iraq should have a relatively large parking fund. Iraq's main challenge is to deal with a very volatile and growing stream of oil revenues which will continue to make up more than half of GDP for years to come.

[21] For Iraq the windfall is so large that assets per capita do not start to decline until 2050 in the base case. The consumption increment in 2012 is now 5.8 times non-resource GDP whilst the size of the windfall in 2012 is 6.5 times non-resource GDP. Given the greater need for precautionary saving as future generations are no longer much richer, consumption in 2012 is reduced by 1.2 times non-resource GDP in the base case (so the total consumption increment is 5.8 − 1.2 = 4.6 times non-resource GDP), thus leaving approximately 70% available for consumption.

[22] Normally future growth cannot be used as collateral for substantial borrowing, but oil-rich countries such as Iraq have sufficient oil income readily available. Whether we smooth consumption in efficiency units or in per capita terms is thus ultimately a normative question about how large the social discount rate should be.

Finally, we make four caveats. First, we focused at oil price uncertainty but uncertainty about growth prospects and oil reserves also matter both for the degree of precautionary saving and the rate of oil extraction (e.g., Pindyck, 1980, 1981). Typically, oil extraction is more aggressive either to probe to establish whether there are more, hitherto unknown fields or to have a smaller stock of oil reserves susceptible to oil price fluctuations. Secondly, Dutch disease effects of oil windfalls may matter, especially strong if the windfall is temporary and not smoothed (e.g., Corden, 1984). The optimal policy should therefore strike a balance between smoothing real exchange rate fluctuations and consumption, investing to invest and mitigating Dutch disease, which is tougher if a greater part of consumption and public investment has to be produced at home since adjustment is then more sluggish. Thirdly, we have abstracted from the fact that countries such as Iraq or Ghana may use part of their oil windfalls to expand their extraction capacity. Given that most of this is undertaken by foreign multinationals, capital scarcity should not be an issue if fields are profitable in which case revenue from earlier field should not be used to finance investments in extraction capacity. Fourthly, optimal policies need to take account of uncertainty about future asset returns within an integrated framework that includes a CAPM model of portfolio investments. To explain the puzzle that oil-rich countries often have large net financial asset positions yet their trade balances exceed their current accounts, one may need to allow for remittances or the political economy of siphoning off of oil revenue.

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