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ABSTRACT

Can we use seasonally adjusted indicators in dynamic factor models?*

We examine the short-term performance of two alternative approaches of forecasting from dynamic factor models. The first approach extracts the seasonal component of the individual indicators before estimating the dynamic factor model, while the alternative uses the non-seasonally adjusted data in a model that endogenously accounts for seasonal adjustment. Our Monte Carlo analysis reveals that the performance of the former is always comparable to or even better than that of the latter in all the simulated scenarios. Our results have important implications for the factor models literature because they show the that the common practice of using seasonally adjusted data in this type of models is very accurate in terms of forecasting ability. Using five coincident indicators, we illustrate this result for US data.

JEL Classification: C22, E27 and E32
Keywords: factor models, seasonal adjustment and short-term forecasting

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1. Introduction

The late-2000s recession, sometimes referred to as the Great Recession, magnified the interest of economic agents in having efficient short-term forecasting models that help monitor ongoing economic developments. This may be why the recent resurgence of dynamic factor models, first developed by Stock and Watson (1991), which have proven to be useful in growth and inflation forecasting. Among others, recent examples are Arouba, Diebold and Scotti (2009), Arouba and Diebold (2010) and Camacho and Perez-Quiros (2010).

To our knowledge, all of the forecasting analyses developed in this related literature used seasonally adjusted data, where the seasonal components were extracted individually from each variable either by the official statistical office that publishes the data or by the analyst (when seasonally adjusted data are not available) before estimating the models.[1] Therefore, only one common factor and several idiosyncratic components are estimated in these dynamic factor models. We will call this approach *traditional*, because it is the standard procedure in the literature.

This traditional approach has some limitations. First, behind the individual seasonal adjustments there exists the implicit assumption that the seasonal component for each variable is necessarily idiosyncratic (not common). Second, removing the seasonal component from the individual indicators before estimating the models may lead to losses of information about the seasonal components that could potentially be useful for forecasting.

As an alternative to this traditional approach, the *structural* dynamic factor models have the advantage of being formulated in terms of common components, such as trends, seasonal components and cycles that have a direct interpretation. Modelling these features could be of great benefit since they could be easily projected into the future, leading to potential forecasting improvements.

This paper aims to evaluate the performance of *traditional* versus *structural* factor models. We use a Monte Carlo exercise to show that when the data generating process exhibits idiosyncratic seasonal components the traditional dynamic factor model that uses seasonally adjusted data (the outcomes of TRAMO-SEATS) outperforms the structural dynamic factor model, especially when the idiosyncratic seasonal components are erroneously modelled as if they were common across series.[2] Interestingly, when the data are generated

[1] Note that the literature on large-scale dynamic factor models, which include a vast number of indicators, also uses seasonally adjusted data. Although our results can be extended to large-scale models, we focus on small-scale models for the sake of simplicity.

[2] We use the TRAMO-SEATS (Gomez and Maravall (1996)) version dated March 11, 2011, as downloaded from the Bank of Spain database. Alternative filters as X-11, X-12, and ARIMA models would lead to qualitatively similar results.

with common seasonal components, the performance of traditional factor model is still comparable to or even better than that of structural factor models, even in the case that the seasonal components are correctly modelled as common across the series. These results have important implications for the literature on factor models since they show the good forecasting performance of the standard models that use seasonally adjusted data with respect to alternative models that handle seasonally adjustments endogenously.

The results obtained in the Monte Carlo analysis are confirmed by using a set of five coincident US economic indicators. Our empirical results also suggest that the standard strategy of forecasting from dynamic factor models that use seasonally adjusted data is the most advisable way to compute the forecasts.

The paper is structured as follows. Section 2 describes the main features of structural and traditional dynamic factor models. Section 3 outlines the Monte Carlo simulation and discusses the results. Section 4 addresses the empirical analysis. Section 5 concludes.

2. Methodological framework

2.1. Structural factor decomposition

The trend stationary economic indicators are assumed to admit a structural factor decomposition. Therefore, each of the N trend stationary indicators, y_{it}, can be written as the sum of three stochastic components: a common component, f_t, which represents the overall business cycle conditions; an idiosyncratic component, u_{it}, which refers to the particular dynamics of the series; and a seasonal component, s_{it}, which refers to the periodic patterns and are allowed to be either idiosyncratic or common[3]. According to this structural factor decomposition, the structural dynamic factor model can be stated as

$$y_{it} = \alpha_i f_t + u_{it} + s_{it}, \quad (1)$$

where i=1,…,N, and the α_i are the loading factors.[4]

We assume the following dynamic specifications for the three components. The common component and the idiosyncratic components follow autoregressive processes of orders $p1$ and $p2$, respectively:

$$f_t = a_1 f_{t-1} + \ldots + a_{p1} f_{t-p1} + \varepsilon_{ft}, \quad (2)$$

[3] Note, that the series are trend stationary (or stationary at zero frequency), but allowed to be seasonally non-stationary (or integrated of order one at seasonal frequencies).

[4] To identify the model, we assume unit loading factor for the first variable ($\alpha_1 = 1$).

where $\varepsilon_{ft} \sim iN\left(0, \sigma_f^2\right)$, and

$$u_{it} = b_{i1}u_{it-1} + \ldots + b_{ip2}u_{it-p2} + \varepsilon_{it}, \tag{3}$$

where $\varepsilon_{it} \sim iN\left(0, \sigma_i^2\right)$, with i=1,…,N.

For the purposes of the paper, the treatment of the seasonal components deserves special comments. In standard applications that use factor decomposition analyses, which we called *traditional* models, the seasonal component of the series is extracted before estimating the model and, therefore, model selection, estimation and forecasting is carried on from seasonally adjusted series. The seasonal adjustment techniques are developed either by the researcher, usually with the help of automatic procedures, such as TRAMO-SEATS or X11, or by the statistical agencies, which in some cases publish official seasonally adjusted versions of the time series. In expression (1), this implies that $s_{it} = 0$, i=1,…,N.

Alternatively, the dynamic properties of the seasonal components could be accounted for within the structural dynamic factor model. In line with trigonometric seasonality model (see Harvey (1989) for details), we assume that the seasonal component can be viewed as the sum of its s/2 cyclical components

$$s_{it} = \sum_{j=1}^{s/2} s_{ijt}, \tag{4}$$

where s is the number of observations per year. In this expression, the cyclical components are modelled as trigonometric terms at the seasonal frequencies, $\lambda_j = 2\pi j/s$, through the model

$$\begin{pmatrix} s_{ijt} \\ s_{ijt}^* \end{pmatrix} = \begin{pmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{pmatrix} \begin{pmatrix} s_{ijt-1} \\ s_{ijt-1}^* \end{pmatrix} + \begin{pmatrix} \xi_{ijt} \\ \xi_{ijt}^* \end{pmatrix}, \tag{5}$$

where j=1,…,s/2, i=1,…,N, ξ_{ijt} and ξ_{ijt}^* are mutually uncorrelated noises with common variance $\sigma_{\xi_{ij}}^2$, and the term s_{ijt}^* appears by construction to form s_{ijt}. In addition, we use the standard assumption that the error terms exhibit the same variance across frequencies, i.e., $\sigma_{\xi_{ij}}^2 = \sigma_{\xi_i}^2$ for all j=1,…,s/2. To complete the statistical specification of the model, we assume that all the disturbances driving the three stochastic components are mutually and serially uncorrelated.

To facilitate simulations and estimations, we prove in the Appendix that this seasonal component can be alternatively expressed using a seasonal autoregressive integrated moving average specification. For quarterly data,[5] the seasonal components are

[5] For the sake of simplicity, we derive all the expressions for quarterly data. Although the expressions would be larger, all the results obtained in the paper could easily be obtained for monthly data.

$$\left(1+L+L^2+L^3\right)s_{it}=\left(1+0.3187L+0.1869L^2\right)\zeta_{it}, \quad (6)$$

where L is the backshift operator, $\zeta_{it}\sim iN\left(0,\sigma_{\zeta_i}^2\right)$ reflects that the seasonal effect is allowed to change over time, and i=1,…,N.[6] To derive this expression, we used s=4, since the seasonal behaviour of our quarterly indicators is often related to the time of a year.[7]

Regarding identification issues, it is crucial to distinguish between the cases of assuming common and idiosyncratic seasonal components. When the periodic patterns are common across the different indicators, the standard identification rules for multivariate unobserved components in factor models apply. Hence, to identify the two common components (the overall business conditions and the seasonal component) one needs at least five indicators in the model.[8] In addition, we assume that the first time series does not contain seasonal components ($s_{1t}=0$), and that the seasonal patterns are proportional across series ($s_{it}=\beta_i s_t$, with i=2,3,4,5 and $\beta_2=1$).

However, when seasonality is different across indicators, it cannot be separately identified from the idiosyncratic component, u_{it}. Although we will estimate this model as an alternative way to model seasonal components, we acknowledge that in this case the noise can be transmitted from the seasonal component, s_{it}, to the idiosyncratic component, u_{it}, and vice versa, which may influence the in-sample fitting performance of the model negatively.

2.2. State-space representation

To estimate model's parameters and to infer unobserved components by using the Kalman filter, it is convenient to rewrite the equations that describe the model's dynamics in a state-space representation. In the case of N economic indicators which are collected in the vector Y_t, the appropriate state-space form of the model requires the specification of both the measurement equation, $Y_t=Hh_t+e_t$, with $e_t\sim iN(0,R)$, and the prediction equation $h_t=\Phi h_{t-1}+\eta_t$, with $\eta_t\sim iN(0,\Omega)$.

For this purpose, it is worth pointing out that the seasonal components

[6] In this case, the yearly sum of the seasonal effects is expected to be zero, since the disturbance term has zero expectation. A model of deterministic seasonality is easily obtained by imposing $\zeta_{it}=0$.

[7] Although we focus on trigonometric seasonality as in Harvey (1989), there are alternative ways of allowing seasonal variables to change over time, as in Hannan et al. (1970) or Harrison and Stevens (1976). However, the Hannan et al. (1970) seasonal model and the Harvey (1989) model with non-equal variances are the same models in the Gaussian case, or when innovations follow a mixture of normal distribution as in Bruce and Jurke (1992). The Harrison and Stevens (1976) seasonality with correlated disturbances model and the Hannan's model are the same models, which are also identical to the model that we use in the Gaussian case.

[8] An identified model with N variables and k common components should satisfy the inequality $N\geq 2k+1$.

$$s_{it}=\frac{\left(1+0.3187L+0.1869L^2\right)}{\left(1+L+L^2+L^3\right)}\zeta_{it} \tag{7}$$

can be written as

$$s_{it}=\left(1+0.3187L+0.1869L^2\right)\chi_{it} \tag{8}$$

where $\left(1+L+L^2+L^3\right)\chi_{it}=\zeta_{it}$, with $\mathrm{var}(\zeta_{it})=\sigma^2_{\zeta_i}$, i=2,…,5.

The specific forms of these two equations depend on the assumption about the seasonal component. Using the assumptions that N=5, $p1$=$p2$=1, when seasonal components are common across the different indicators, the state space representation of the model becomes:

$$\begin{pmatrix} y_{1t}\\ y_{2t}\\ y_{3t}\\ y_{4t}\\ y_{5t}\end{pmatrix}=\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \alpha_2 & 0 & 1 & 0 & 0 & 0 & 1 & 0.3187 & 0.1869\\ \alpha_3 & 0 & 0 & 1 & 0 & 0 & \beta_3 & 0.3187\beta_3 & 0.1869\beta_3\\ \alpha_4 & 0 & 0 & 0 & 1 & 0 & \beta_4 & 0.3187\beta_4 & 0.1869\beta_4\\ \alpha_5 & 0 & 0 & 0 & 0 & 1 & \beta_5 & 0.3187\beta_5 & 0.1869\beta_5 \end{pmatrix}\begin{pmatrix} f_t\\ u_{1t}\\ u_{2t}\\ u_{3t}\\ u_{4t}\\ u_{5t}\\ \chi_t\\ \chi_{t-1}\\ \chi_{t-2}\end{pmatrix}, \tag{9}$$

and

$$\begin{pmatrix} f_t\\ u_{1t}\\ u_{2t}\\ u_{3t}\\ u_{4t}\\ u_{5t}\\ \chi_t\\ \chi_{t-1}\\ \chi_{t-2}\end{pmatrix}=\begin{pmatrix} a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & b_2 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & b_3 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & b_4 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & b_5 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} f_{t-1}\\ u_{1t-1}\\ u_{2t-1}\\ u_{3t-1}\\ u_{4t-1}\\ u_{5t-1}\\ \chi_{t-1}\\ \chi_{t-2}\\ \chi_{t-3}\end{pmatrix}+\begin{pmatrix} \varepsilon_{ft}\\ \varepsilon_{1t}\\ \varepsilon_{2t}\\ \varepsilon_{3t}\\ \varepsilon_{4t}\\ \varepsilon_{5t}\\ \varsigma_t\\ 0\\ 0\end{pmatrix} \tag{10}$$

where R=0, and Ω is a diagonal matrix with main diagonal

$$diag(\Omega)=\left(\sigma_f^2,\sigma_1^2,\sigma_2^2,\sigma_3^2,\sigma_4^2,\sigma_5^2,\sigma_\zeta^2,0,0\right)'. \tag{11}$$

The state-space form of *traditional* dynamic factor models that use either the official seasonally adjusted data sets or the seasonally adjusted outcomes from TRAMO-Seats can easily be derived from these expressions. It is obtained by imposing $\chi_t=\chi_{t-1}=\chi_{t-2}=\chi_{t-3}=0$, and $\varsigma_t=0$.

However, when the seasonal component is assumed to be idiosyncratic for each economic indicator, the state-space representation of the model is

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} \\ \alpha_2 & 0 & 1 & 0 & 0 & 0 & A & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} \\ \alpha_3 & 0 & 0 & 1 & 0 & 0 & 0_{1\times3} & A & 0_{1\times3} & 0_{1\times3} \\ \alpha_4 & 0 & 0 & 0 & 1 & 0 & 0_{1\times3} & 0_{1\times3} & A & 0_{1\times3} \\ \alpha_5 & 0 & 0 & 0 & 0 & 1 & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & A \end{pmatrix} \begin{pmatrix} f_t \\ u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ u_{5t} \\ X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{pmatrix}, \tag{12}$$

and

$$\begin{pmatrix} f_t \\ u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ u_{5t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \\ X_{5t} \end{pmatrix} = \begin{pmatrix} a & 0 & 0 & 0 & 0 & 0 & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} \\ 0 & b_1 & 0 & 0 & 0 & 0 & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} \\ 0 & 0 & b_2 & 0 & 0 & 0 & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} \\ 0 & 0 & 0 & b_3 & 0 & 0 & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} \\ 0 & 0 & 0 & 0 & b_4 & 0 & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} \\ 0 & 0 & 0 & 0 & 0 & b_5 & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} \\ 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & B & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times3} & B & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times3} & 0_{3\times3} & B & 0_{3\times3} \\ 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & B \end{pmatrix} \begin{pmatrix} f_{t-1} \\ u_{1t-1} \\ u_{2t-1} \\ u_{3t-1} \\ u_{4t-1} \\ u_{5t-1} \\ X_{2t-1} \\ X_{3t-1} \\ X_{4t-1} \\ X_{5t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{ft} \\ \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \\ Z_{1t} \\ Z_{2t} \\ Z_{3t} \\ Z_{4t} \end{pmatrix}, \tag{13}$$

where $A = (1,0.3187,0.1869)'$, $X_{it} = (\chi_{it}, \chi_{it-1}, \chi_{it-2})'$, $Z_{it} = (\zeta_{it},0,0)'$, and i=2,..,5.

$$B = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \tag{14}$$

$R = 0$, and Ω is a diagonal matrix with main diagonal

$$diag(\Omega) = (\sigma_f^2, \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_{\varsigma_2}^2, 0,0, \sigma_{\varsigma_3}^2, 0,0, \sigma_{\varsigma_4}^2, 0,0, \sigma_{\varsigma_5}^2, 0,0)'. \tag{15}$$

3. Monte Carlo simulations

In this section, we design a Monte Carlo experiment to study some of the finite-sample properties of *structural* dynamic factor models that account for common or idiosyncratic seasonal components against *traditional* dynamic factor models that manage seasonally adjusted data. As shown in Table 1, the experiment is conducted with a comprehensive set of coefficients in order to capture a wide range of specifications, allowing for different degrees of common factor correlation, different persistence of idiosyncratic components, and idiosyncratic components that are heterogeneous.

To cover a large variety of combinations, Table 1 reports that the loading factor of the first variable is set to unity in order to achieve identification, while other factor loadings are either positive for variables 2 and 3 (0.7 and 1.1, respectively) or negative for variables 4 and 5 (-0.8 and -0.5, respectively). We generate two alternative scenarios for the seasonal components.[9] The first scenario, called M1, tries to mimic the empirical forecasting exercise where seasonal components are idiosyncratic.[10] The second scenario, called M2, tries to mimic the case of common seasonal components, where the seasonal factor loadings are either positive (0.9 for variable 3) or negative (-0.8 and -0.7 for variables 4 and 5, respectively).

The common non-seasonal factor, f_t, and the individual components, u_{it}, are generated as first order autoregressive processes. Since these components are assumed to be non-seasonal, they are generated with positive autoregressive parameters to insure that they do not generate distortions in the variable at the high part of the spectrum. Also, if these components had been generated with relatively high negative autoregressive parameters, standard methods of univariate seasonal adjustments, such as TRAMO-SEATS or X11, would have considered the variance contribution of these components as seasonal and would have extracted it from the generated time series.

According to Table 1, in simulations S1, S2, and S3 we replicate situations where the economic indicators share a strong persistent non-seasonal common component (autoregressive parameter of 0.9). However, in simulations S4, S5 and S6 the persistence of the factor is weak (autoregressive parameter of 0.2) while it is moderate in simulations S7, S8 and S9 (autoregressive parameter of 0.5). In addition, these potential empirical cases are combined with weak idiosyncratic components (autoregressive parameters between 0.1 and 0.4) in simulations labelled S1, S4 and S7 and strong idiosyncratic components (autoregressive parameters between 0.6 and 0.9) in simulations labelled S2 S5 and S8. To cover all combinations, the idiosyncratic components are allowed to exhibit mixed persistence (autoregressive parameters between 0.2 and 0.9) in simulations S3, S6 and S9.

For each of these 18 cases, we generate a total of M=1000 sets of time series of length T=120 observations.[11] We use them to mimic three different empirical forecasting scenarios. The first scenario, called EId, mimics the case in which an analyst fits a *structural* dynamic

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[9] As mentioned before, for identification purposes, the seasonal component does not affect the first variable.

[10] In line with our empirical results, we set $\sigma^2_{\zeta_i} < \sigma^2_j \; \forall \, i, j$. In particular, we set $\sigma^2_{\zeta_i}$ around 0.1.

[11] The length of the generated time series is 120 since it would refer to 30 years of quarterly observations.

factor model to the non seasonally adjusted data, whose seasonal components are treated as idiosyncratic. The second scenario, called ECo, refers to a similar case but where the seasonal component is common to the last four time series. The third scenario, called EsaTS, mimics the case in which the analyst uses seasonally adjusted data before estimating the standard dynamic factor model, i.e., the *traditional* approach. In our analysis, the seasonal components are extracted from the generated time series using TRAMO-SEATS.

In each replication, *m*, we estimate the two structural factor models and the factor model that uses seasonally adjusted data, and we examine the performance of these models in Tables 2 and 3. In each of these tables, the figures in brackets analyze the ability of the models to infer the factor while the rest of the figures refer to the accuracy of the models to infer the time series. In Table 2, we examine the in-sample fit of the models by computing the averaged squared difference across the *T* observations between the generated and the estimated time series (Mean Squared Errors), which are also averaged across the *M* replications. In Table 3, we compare the out-of-sample forecasting accuracy by computing the errors in forecasting (one-step-ahead) the generated target series. For each *m*-th replication, the one-step-ahead forecasts are obtained by estimating the models with data from t=1 to $t=T-1$, and by computing the forecasts for *T*. Then, the Mean Squared Errors are computed as the averages of the squared errors across the *M* replications.

The main results of the Monte Carlo experiment are derived from the figures reported in Tables 2 and 3. First, there are two main potential sources of seasonal misspecification in structural dynamic factor models: when the data are generated with idiosyncratic seasons but the model incorrectly assumes common seasons and when the data are generated with common seasons but the model uses the erroneous assumption that the seasons are idiosyncratic. The former source of misspecification can be evaluated by using the figures reported in the second column of the panels labelled as M1. The MSEs achieved by the dynamic factor models that erroneously assume common seasons when they have been generated as idiosyncratic dramatically increase and become one or two orders of magnitude greater than the MSEs of dynamic factor models that correctly account for the idiosyncratic seasons. This result holds whether or not the idiosyncratic seasons are modelled within the model or extracted from the model before estimating. The latter source of misspecification, when data are generated with common seasons but the model erroneously assumes that the seasons are idiosyncratic, can be evaluated by using the figures reported in the first column of the panels labelled M2. According to the magnitude of the figures reported in these columns, the second source of misspecification seems to be much less damaging than the first. When the common seasons are erroneously fitted as idiosyncratic, the MSEs of the misspecified model are much closer to those of the correct specification.

Second, when the seasonal component is generated idiosyncratically across the time series, the *traditional* approach of dynamic factor models that use seasonally adjusted data unequivocally achieves the best performance. The figures reported in the third column of the tables show that this strategy outperforms the structural factor model that assumes idiosyncratic seasons. The potential explanation is that the structural factor model may suffer from an identification problem since it is hard to identify separately the variances of the individual components from those of the seasonal components when they are idiosyncratic. Another explanation would be that the greater number of parameters to be estimated within the structural approach generates larger uncertainty and noise in the estimation.

Third, according to the fifth columns of the tables, the structural model that correctly treats the seasonal components as common when they are actually generated as common usually exhibits the best performance. In the case of common seasons, the structural model exhibits a more parsimonious representation than in the case of idiosyncratic seasonal components, which makes it easier to identify the common seasonal factor. Interestingly, when comparing the fifth columns of the tables with the sixth columns of the tables, it is worth emphasizing that the accuracy of the structural factor model is similar to the accuracy of the standard factor model that uses seasonally adjusted data, which in many cases exhibits the lowest MSEs.

Fourth, the persistence of the idiosyncratic and the common components increases the size of the differences across specifications but does not alter the nature of the results. Regardless of whether the seasonal components are common or idiosyncratic, the *traditional* factor model achieves relatively better forecasting performance in the case of high persistence, which motivates the use of this approach in case of doubts about the nature of the seasonal components. This may happen because the potential misspecifications of the *structural* factor models last longer in the case of highly persistent dynamics than in the case of less persistent data generating processes.

Fifth, the conclusions obtained by analysing the MSEs achieved by the models on inferring the factor and those achieved by the models on fitting the indicators are of the same nature, i.e., good factor estimation implies good fitting of the data. In addition, the results of the out-of-sample analysis (Table 3) are qualitatively similar to those of the in-sample performance, although a little weaker. The intuition is that there is more noise in the out-of-sample analysis, which generates higher uncertainty across the models and makes it difficult to extract conclusions from the analysis.

Summing up, these results agree with the general strategy followed by analysts that apply dynamic factor models to time series that exhibit common, idiosyncratic, and seasonal components. This *traditional* approach consists, prior to fitting the factor model, of removing the seasonal components either by using the official seasonally adjusted version of the indicators or by using the outcomes of automatic procedures of seasonal adjustment, such as the TRAMO-SEATS. When seasonality is idiosyncratic, this strategy leads to the best results. When the seasonality is common across series, it leads to very good results, which are comparable to the results of estimating the structural factor model associated with the data generating process, but with the advantage that this strategy eliminates the potential damage of using structural factor models that assume common seasonality when it is actually idiosyncratic.

4. Empirical analysis

4.1. In-sample analysis

The five quarterly indicators used in the empirical analysis, which are plotted in quarterly frequency in Figure 1, are the University of Michigan consumer sentiment index, new passenger car and truck sales, median usual weekly earnings in constant dollars, Industrial Production, and employees on nonagricultural payrolls from 1978.1 to 2011.1.[12] According to our preliminary analysis of unit roots, we find that all of them contain unit roots; therefore all variables are used in growth rates.

The University of Michigan consumer sentiment index is a consumer confidence index published by the University of Michigan and Thomson Reuters. The index is normalized to have a value of 100 in December 1964 and it is based on at least 500 telephone interviews which are conducted each month in a United States sample to assess near-time consumer attitudes on the business climate, personal finance, and spending. The index does not contain seasonality. New passenger car and truck sales were obtained from the Department of Commerce's Bureau of Economic Analysis (BEA), the median usual weekly earnings and nonagricultural payrolls were obtained from the Bureau of Labor Statistics, and Industrial Production was downloaded from the Federal Reserve.

The economic indicators, which are plotted in Figure 1, exhibit a key advantage for our study: they are available as both non seasonally adjusted and seasonally adjusted. In addition, the selection of these indicators follows the line suggested by the influential paper of Stock and Watson (1991). We start the analysis with a set of indicators that includes an indicator from the supply side of the economy (Industrial Production), an indicator from the demand side (car and truck sales), an indicator from the income side (weekly earnings), and

[12] The analysis with monthly data would have achieved qualitatively similar results.

an indicator of the labor market (employees on nonagricultural payrolls), with the restriction that they had to be released seasonally and non-seasonally adjusted.[13] Then, we enlarge initial set of indicators with the University of Michigan consumer sentiment in order to incorporate a non-seasonal series which agrees with the evolution of the business cycle.

Table 4 displays the maximum likelihood estimates of structural dynamic factor models that account for seasonal adjustments and factor models that use seasonally adjusted data where the seasonal components are extracted before estimation. Among the *structural* dynamic factor models, we distinguish the case of models that assume idiosyncratic seasonal patterns (results labeled EId) from the case in which the models assume that season is common (results labeled ECo). Among the *traditional* dynamic factor models, we distinguish the case of models that use the officially published seasonally adjusted series (results labeled Esa) from the case of models that use indicators whose seasonal component is extracted from the series by using TRAMO-SEATS (results labeled EsaTS). The choice of model specifications is always based on the Schwarz criterion.

There are several noteworthy features from the estimates reported in Table 4. First, the estimated parameters obtained from traditional dynamic factor models based on official seasonally adjusted series are similar to those obtained from dynamic factor models that use the seasonally adjusted outcomes of TRAMO-SEATS. Accordingly, the results from these two specifications are expected to be very similar.

Second, the estimated common factor is very persistent since the estimates for its first order autocorrelation range from 0.73 to 0.89. Accordingly, when we compare the empirical results with those obtained in the Monte Carlo experiment, we focus on cases S1, S2 and S3 of Tables 2 and 3. In addition, Figure 2 shows that independently of the model used to extract the factor, the common factor can be interpreted as an indicator of the broad business cycle conditions. According to the figure, all the factors cohere strongly with the NBER chronology, plunging during NBER recessions. However, the common factor extracted from the structural factor model that assumes common seasons is too smooth, reflecting that part of the common business dynamics component remains embedded in the seasonal common component.

Third, the persistence of the individual components of the first four variables is very low since the autoregressive parameters are small and negative. Hence, the individual components contain only a limited amount of information about the behavior of the corresponding series at the business cycle frequencies apart from the information already

[13] This is why we substitute manufacturing and trade sales, originally used in Stock and Watson (1991) for car and truck sales. The same applies to the substitution of real personal income less transfers by weekly earnings.

contained in the common factor. In this sense, it seems that model S1 is the most relevant theoretical framework with which the empirical results are comparable.

Fourth, Figure 3, which displays the official non seasonally adjusted series and their respective seasonally adjusted versions, shows that when new passenger car and truck sales and median weekly earnings are seasonally adjusted with the structural model that assumes a common season, the seasonally adjusted versions still contain seasonal components. This agrees with the view that the seasonal component is idiosyncratic. Therefore, M1 (idiosyncratic seasons) seems to be the theoretical framework which is closest to the empirical analysis and the results from the Monte Carlo experiment labeled S1 are the most appropriate.

4.2. Out-of-sample analysis

In this section, we develop an out-of-sample forecasting analysis. For this purpose, we assume that the time series of interest which we want to forecast with *structural* and *traditional* dynamic factor models are the official seasonally adjusted versions of the series.[14] The first forecast is obtained by estimating the models with data from t=1 to $t=\tau$, and by computing the forecast τ+1. Then, the models are re-estimated with data from t=1 to $t=\tau+1$, and the forecast is obtained for τ+2. This process is repeated until $\tau=T$-1, which implies a total of T-τ forecasts. Therefore, the one-period-ahead forecasts were computed recursively and the analysis was conducted to simulate real-time forecasting.

The first simulated out-of-sample forecast was made in 2003.4. To construct these forecasts, the parameters and the unobserved components for all the models were estimated using only data available from 1978.1 through 2003.4. Thus, all parameters and unobserved components were re-estimated with data from 1978.1 through 2003.4, and forecasts from these models were then computed for 2004.1. Following this recursive forecasting procedure, the latest simulated out-of-sample forecasts were made in 2010.4 for 2011.1.[15] The analysis ends up with 25 quarterly out-of-sample forecasts for each variable.

For each time series, the averaged differences between the one-step-ahead forecast and the targeted variables are computed. To compare the results across time series easily, the figures reported in the top panel of Table 5 show these MSEs divided by the in-sample variance of each time series. EId and ECo show the results of forecasting with *structural*

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[14] The official seasonally adjusted time series are quite similar to the seasonally adjusted outcomes of TRAMO-SEATS. Accordingly, if the latter were the series of interest, the results would be qualitatively similar to those presented in the paper.

[15] Each quarter, we updated the database as if all the variables had been observed in that quarter. Therefore, we did not develop a real-time analysis since data revisions or publication delays are not treated. For a careful analysis of these real-time forecasting problems, see Camacho, Perez-Quiros and Poncela (2012).

factor models that assume idiosyncratic and common seasonal components, respectively. Finally, Esa shows the results of forecasting with the *traditional* factor model, where the forecast are computed from the official seasonally adjusted versions of the business cycle indicators. The bottom panel of Table 5 shows the p-values of the Diebold and Mariano (1995) tests of the null hypothesis of the same accuracy between the pairwise comparisons.

The figures reported in the table suggest some conclusions that are in line with the findings obtained in the Monte Carlo analysis: regardless of the data generating process, the forecasts computed from the standard dynamic factor model that uses seasonally adjusted series are at least of comparable performance with any other forecasts. Therefore, cleaning up the economic indicators from seasonality either by using the seasonally adjusted series or by using automatic univariate procedures before using the variables in the dynamic factor model seems to be a reasonably strategy to follow.

5. Conclusion

Despite the efforts of recent studies to evaluate the empirical short-term forecasting performance of dynamic factor models, it still remains an open question whether it is better to use seasonally adjusted indicators before estimating a standard dynamic factor model or to account for the seasonal components of raw data within a structural factor model. The first strategy implicitly assumes that the seasonal components are idiosyncratic and the latter strategy could lead to unnecessary complexity, especially for practitioners that are not familiar with seasonal processes.

We use Monte Carlo experiments to analyze the extent to which these two alternatives exhibit relative forecasting performance gains. Our simulation results suggest that when the data are generated under the assumption that the seasonal components are idiosyncratic the standard dynamic factor model that uses seasonally adjusted indicators exhibits the best forecasting performance. Interestingly, when the seasonal components are common to all the time series, the forecasting deterioration of a standard dynamic factor model that uses seasonally adjusted indicators with respect to a structural dynamic factor model is usually negligible in our experiment. Notably, the former improves on the latter in some cases.

In empirical applications, it is difficult to decide a priori if the seasonality is common or idiosyncratic across series. Given that the deterioration of the in-sample fitting and out-of-sample forecasting performance of the standard dynamic factor model that uses seasonally adjusted indicators is very small, while the performance of the structural common seasonal component model is very poor in the case of idiosyncratic seasonal factors, we strongly recommend the use of seasonally adjusted series in factor models.

We illustrate these results by using US data from 1978.1 to 2011.1 of the University of Michigan consumer sentiment index, new passenger car and truck sales, median usual weekly earnings, Industrial Production, and employees on nonagricultural payrolls share a common seasonal component. The forecasting performance of a standard dynamic factor model that uses the seasonally adjusted versions of these series is comparable to or even better than that of structural dynamic factor models that assume common or idiosyncratic seasonal components.

Appendix

Since the empirical data are quarterly, the seasonal component of each time series, s_i, is the sum of two cyclical components, $s_{it} = s_{i1t} + s_{i2t}$, which are evaluated at the seasonal frequencies, $\lambda_1 = \pi/2$, and $\lambda_2 = \pi$. According to (4), the dynamics of the first cyclical component is

$$\begin{pmatrix} s_{i1t} \\ s_{i1t}^* \end{pmatrix} = \begin{pmatrix} \cos\pi/2 & \sin\pi/2 \\ -\sin\pi/2 & \cos\pi/2 \end{pmatrix} \begin{pmatrix} s_{i1t-1} \\ s_{i1t-1}^* \end{pmatrix} + \begin{pmatrix} \xi_{i1t} \\ \xi_{i1t}^* \end{pmatrix}. \tag{A1}$$

Using $\cos\pi/2 = 0$ and $\sin\pi/2 = 1$ and rearranging terms, one can obtains

$$\begin{pmatrix} s_{i1t} \\ s_{i1t}^* \end{pmatrix} = \frac{1}{1+L^2} \begin{pmatrix} 1 & L \\ -L & 1 \end{pmatrix} \begin{pmatrix} \xi_{i1t} \\ \xi_{i1t}^* \end{pmatrix}, \tag{A2}$$

which implies that $\left(1+L^2\right)s_{i1t} = \zeta_{i1t}$, where $\zeta_{i1t} = \xi_{i1t} + \xi_{i1t-1}^*$. If $\mathrm{var}\left(\xi_{i1t}\right) = \mathrm{var}\left(\xi_{i1\tau}^*\right) = \sigma_{\xi_{i1}}^2, \forall t, \tau$, then $\mathrm{var}\left(\zeta_{i1t}\right) = 2\sigma_{\xi_{i1}}^2$.

Similarly, the dynamics of the second cyclical component can be obtained from

$$\begin{pmatrix} s_{i2t} \\ s_{i2t}^* \end{pmatrix} = \begin{pmatrix} \cos\pi & \sin\pi \\ -\sin\pi & \cos\pi \end{pmatrix} \begin{pmatrix} s_{i2t-1} \\ s_{i2t-1}^* \end{pmatrix} + \begin{pmatrix} \xi_{i2t} \\ \xi_{i2t}^* \end{pmatrix}, \tag{A3}$$

which, using $\cos\pi = -1$ and $\sin\pi = 0$, leads to

$$\begin{pmatrix} s_{i2t} \\ s_{i2t}^* \end{pmatrix} = \frac{1}{(1+L)^2} \begin{pmatrix} 1+L & 0 \\ 0 & 1+L \end{pmatrix} \begin{pmatrix} \xi_{i2t} \\ \xi_{i2t}^* \end{pmatrix}. \tag{A4}$$

This expression implies that $(1+L)s_{i2t} = \zeta_{i2t}$, where $\zeta_{i2t} = \xi_{i2t}$ and $\mathrm{var}\left(\zeta_{i2t}\right) = \sigma_{\xi_{i2}}^2$. Let us additionally assume that $\sigma_{\xi_{i1}}^2 = \sigma_{\xi_{i2}}^2 = \sigma_{\xi_i}^2$.

Accordingly, the seasonal component of each time series can be expressed as

$$s_{it} = \frac{\zeta_{i1t}(1+L) + \zeta_{i2t}\left(1+L^2\right)}{\left(1+L^2\right)(1+L)}, \tag{A5}$$

or

$$\left(1+L+L^2+L^3\right)s_{it} = \zeta_{i1t}(1+L) + \zeta_{i2t}\left(1+L^2\right). \tag{A6}$$

Since the greatest polynomial of the two terms from the right-hand side is of power two, the resulting polynomial (the result of summation) is of power two as well

$$\left(1+L+L^2+L^3\right)s_{it}=\left(1+\alpha L+\beta L^2\right)\zeta_{it}\,. \tag{A7}$$

To find the unknown coefficients α and β, we derive the spectra of right-hand sides of both expressions. The spectrum of (A6) is

$$\left(2+2\cos\omega\right)\sigma^2_{\zeta_{1i}}+\left(2+2\cos(2\omega)\right)\sigma^2_{\zeta_{2i}}=\left(6+4\cos\omega+2\cos 2\omega\right)\sigma^2_{\xi_i}\,, \tag{A8}$$

and the spectrum of (A7) is

$$\left(\left(1+\alpha^2+\beta^2\right)+2\left(\alpha+\alpha\beta\right)\cos\omega+2\beta\cos 2\omega\right)\sigma^2_{\zeta_i}\,. \tag{A9}$$

Since the two spectra must represent the same dynamics, one can use the system of three equations with three unknowns $6\sigma^2_{\xi_i}=\left(1+\alpha^2+\beta^2\right)\sigma^2_{\zeta_i}$, $4\sigma^2_{\xi_i}=2\left(\alpha+\alpha\beta\right)\sigma^2_{\zeta_i}$ and $2\sigma^2_{\xi_i}=2\beta\sigma^2_{\zeta_i}$ to derive an equation for the first parameter $\alpha^4-4\alpha^3+12\alpha^2-16\alpha+4=0$. The real solutions of this equation are $\alpha_1=0.3187$ and $\alpha_2=1.6813$, and using again the system of equations, it is easy to obtain that they correspond to values $\beta_1=0.1869$ and $\beta_2=5.2745$. Using the first pair of real solutions to ensure invertibility, $\sigma^2_{\zeta_i}=5.3505\sigma^2_{\xi_i}$.

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Table 1. Parameters used in Monte Carlo simulations

Fixed parameters for all simulations
$\alpha_1 = 1, \alpha_2 = 0.7, \alpha_3 = 1.1, \alpha_4 = -0.8, \alpha_5 = -0.5$
$\sigma_f^2 = 1, \sigma_1^2 = 0.7, \sigma_2^2 = 0.8, \sigma_3^2 = 0.9, \sigma_4^2 = 1, \sigma_5^2 = 0.9$

Parameters that control idiosyncratic versus common seasons		
	M1 (idiosyncratic seasons)	M2 (common seasons)
	$\sigma_{\zeta_2}^2 = 0.1, \sigma_{\zeta_3}^2 = 0.09, \sigma_{\zeta_4}^2 = 0.08, \sigma_{\zeta_5}^2 = 0.1$	$\sigma_{\zeta_2}^2 = 0.1, \beta_3 = 0.9, \beta_4 = -0.8, \beta_5 = -0.7$

Parameters that control non-seasonal factor and individual components									
Not seasonal component	S1	S2	S3	S4	S5	S6	S7	S8	S9
Common	strong	strong	strong	weak	weak	weak	mod	mod.	mod
Idiosynchr.	weak	strong	mixed	weak	strong	mixed	weak	strong	mixed
	a=0.9	a=0.9	a=0.9	a=0.2	a=0.2	a=0.2	a=0.5	a=0.5	a=0.5
	b_1=0.3	b_1=0.7	b_1=0.9	b_1=0.3	b_1=0.7	b_1=0.9	b_1=0.3	b_1=0.7	b_1=0.9
	b_2=0.2	b_2=0.8	b_2=0.2	b_2=0.2	b_2=0.8	b_2=0.2	b_2=0.2	b_2=0.8	b_2=0.2
	b_3=0.4	b_3=0.9	b_3=0.5	b_3=0.4	b_3=0.9	b_3=0.5	b_3=0.4	b_3=0.9	b_3=0.5
	b_4=0.1	b_4=0.7	b_4=0.9	b_4=0.1	b_4=0.7	b_4=0.9	b_4=0.1	b_4=0.7	b_4=0.9
	b_5=0.3	b_5=0.6	b_5=0.3	b_5=0.3	b_5=0.6	b_5=0.3	b_5=0.3	b_5=0.6	b_5=0.3

Notes. Parameters α_i refer to the loading factors. Parameters σ_f^2 and σ_i^2 refer to the variance of noises of the common non-seasonal factor and the idiosyncratic components, respectively. Parameters $\sigma_{\zeta_i}^2$ refer to the variances of the noises of the cyclical components. Parameters a and b_i refer to the autoregressive parameters of the common factor and the idiosyncratic components, respectively.

Table 2. In-sample Monte Carlo results

Specification	M1 (idiosyncratic seasons)			M2 (common seasons)		
	EId	ECo	EsaTS	EId	ECo	EsaTS
	(0.2296)	(0.3604)	**(0.2087)**	(0.2540)	(0.2283)	**(0.2095)**
	0.1658	1.2640	**0.0755**	0.1972	0.1078	**0.0783**
S1	0.1862	1.1408	**0.0687**	0.2745	0.0877	**0.0742**
	0.2085	1.9162	**0.1023**	0.1651	**0.0737**	0.0837
	0.1857	2.5292	**0.0606**	0.1895	**0.0561**	0.0610
	(0.5366)	(1.3527)	**(0.5346)**	(0.5477)	(0.5098)	**(0.5008)**
	0.1325	0.8465	**0.0582**	0.1638	0.0874	**0.0572**
S2	0.1643	1.2420	**0.0610**	0.2257	0.0709	**0.0654**
	0.1566	1.0715	**0.0610**	0.1199	**0.0572**	0.0679
	0.1644	2.3105	**0.0522**	0.1480	0.0441	**0.0404**
	(0.3797)	(0.8903)	**(0.3574)**	(0.4329)	(0.3810)	**(0.3595)**
	0.1606	1.9786	**0.0768**	0.1995	0.1022	**0.0786**
S3	0.1565	0.9464	**0.0728**	0.2707	0.0819	**0.0723**
	0.1395	0.8384	**0.0541**	0.1202	0.0655	**0.0550**
	0.1757	2.4502	**0.0715**	0.1906	**0.0530**	0.0608
	(0.2206)	(0.3728)	**(0.2063)**	(0.2538)	(0.2165)	**(0.2125)**
	0.1651	1.3549	**0.0987**	0.2250	0.1140	**0.0785**
S4	0.1798	0.9566	**0.0935**	0.3020	**0.0954**	0.1010
	0.1872	1.3925	**0.0843**	0.1791	**0.0770**	**0.0763**
	0.1756	1.7659	**0.0792**	0.2132	**0.0593**	0.0606
	(**0.3195**)	(0.5480)	(0.3217)	(0.3402)	(**0.3116**)	(0.3188)
	0.1323	0.9188	**0.0724**	0.1732	0.0919	**0.0632**
S5	0.1533	1.0317	**0.0888**	0.2401	**0.0735**	0.1162
	0.1479	1.3982	**0.0737**	0.1353	**0.0613**	0.0720
	0.1549	1.9859	**0.0679**	0.1578	**0.0476**	0.0493
	(0.2638)	(0.4559)	(**0.2585**)	(0.3081)	(**0.2688**)	(0.2743)
	0.1698	2.7783	**0.0870**	0.2077	0.1033	**0.0850**
S6	0.1795	0.9881	**0.0915**	0.3428	**0.0803**	0.1034
	0.1452	1.0195	**0.0681**	0.1246	**0.0638**	0.0726
	0.1739	2.1565	**0.0763**	0.1928	**0.0508**	0.0536
	(0.2234)	(0.3855)	(**0.2082**)	(0.2569)	(0.2237)	(**0.2117**)
	0.1557	1.1952	**0.0847**	0.2100	0.1113	**0.0780**
S7	0.1769	0.9532	**0.0856**	0.2989	0.0892	**0.0863**
	0.2020	1.3583	**0.0939**	0.1739	**0.0761**	0.0868
	0.1676	2.2314	**0.0646**	0.2000	0.0580	**0.0572**
	(**0.3871**)	(0.7016)	(0.3890)	(0.4052)	(0.3752)	(**0.3728**)
	0.1268	1.0032	**0.0646**	0.1787	0.0957	**0.0602**
S8	0.1540	0.8299	**0.0829**	0.2796	**0.0791**	0.0841
	0.1553	1.0842	**0.0613**	0.1384	**0.0648**	0.0649
	0.1484	2.2465	**0.0619**	0.1792	**0.0499**	0.0544
	(0.3034)	(0.5282)	(**0.2967**)	(0.3385)	(0.2981)	(**0.2919**)
	0.1574	1.5334	**0.0846**	0.1995	0.0984	**0.0840**
S9	0.1631	1.0800	**0.0835**	0.2605	**0.0804**	0.1031
	0.1434	0.7315	**0.0606**	0.1347	0.0618	**0.0602**
	0.1649	2.4018	**0.0651**	0.1871	0.0527	**0.0494**

Notes. For each specification, figures in parentheses refer to the MSE of the common factor while other figures refer to the MSE of series 2 to 5. Columns labelled as M1 and M2 refer to data generated processes with idiosyncratic and common seasonal components, respectively. Expressions S1 to S9 refer to data generated processes of different inertia of common non-seasonal and idiosyncratic components (see Table 1). EId, Eco, and EsaTS refer to models with idiosyncratic seasons, common season, and models whose indicators are seasonally adjusted before estimation, respectively. Lowest MSE are highlighted in bold.

Table 3. One-period-ahead Monte Carlo results

Specification	M1 (idiosyncratic seasons)			M2 (common seasons)		
	EId	ECo	EsaTS	EId	Eco	EsaTS
S1	(1.0285)	(1.0503)	**(0.9642)**	(1.1727)	(1.1613)	**(1.0922)**
	1.5254	1.4827	**1.4808**	1.5445	1.5255	**1.4798**
	1.2167	1.7344	**1.1598**	1.4283	1.4134	**1.3639**
	1.8004	2.1443	**1.6510**	2.1070	2.0468	**1.9764**
	1.8063	3.3232	**1.7004**	1.7021	**1.6783**	1.6890
	0.8723	3.0359	**0.8076**	1.1927	1.1757	**1.1542**
S2	**(1.2248)**	(1.5648)	(1.2389)	(1.1352)	(1.1701)	(**1.1272**)
	1.5306	1.5540	1.5369	**1.4860**	1.5084	1.4912
	1.2562	1.6171	**1.1302**	1.3734	1.3964	**1.2922**
	1.8043	2.6477	**1.6586**	2.0683	2.0672	**1.9311**
	1.6693	2.3276	**1.5508**	1.5971	1.5932	**1.4974**
	0.9098	2.5761	**0.8107**	0.9647	0.9978	**0.9166**
S3	**(1.2080)**	(1.3256)	(1.2284)	(1.1558)	(1.1470)	**(1.0478)**
	1.3881	**1.3936**	**1.3891**	1.3771	1.3769	**1.3642**
	1.3166	2.2820	**1.3198**	1.4314	1.4161	**1.3549**
	1.7298	2.4210	**1.6846**	1.9949	1.9485	**1.8915**
	1.6600	2.0486	**1.5618**	1.5561	1.4850	**1.4651**
	1.2535	2.9884	**1.2181**	1.1653	1.1540	**1.1352**
S4	(1.2884)	(**1.2876**)	(**1.2759**)	(0.9254)	(**0.9111**)	(0.9245)
	1.6961	**1.6875**	1.7138	1.6563	**1.6504**	1.6702
	1.7101	2.2406	1.6815	1.5435	**1.5193**	1.5253
	2.0945	2.5689	**2.0391**	1.8608	**1.8252**	1.8511
	2.2071	2.8995	**2.1785**	1.2118	**1.1892**	1.2022
	1.1072	2.2254	**1.0603**	0.9766	**0.9563**	0.9945
S5	(1.2997)	(1.3141)	**(1.2848)**	**(1.2145)**	(1.2072)	**(1.1946)**
	1.7973	1.8342	1.8561	**1.9969**	1.9997	2.0275
	1.6701	2.0850	**1.5649**	1.4003	1.3980	**1.3467**
	2.0773	2.6325	**1.8878**	2.2085	2.2895	**2.1773**
	2.2389	2.5183	**2.1263**	1.8022	1.7638	**1.7103**
	1.1109	2.3670	**1.0210**	0.9992	0.9936	**0.9863**
S6	(0.9422)	**(0.9261)**	(0.9373)	(1.3150)	(1.3121)	**(1.3100)**
	1.6985	1.8446	1.7299	1.9390	**1.9317**	1.9398
	1.4558	3.8554	**1.4418**	1.1852	1.1689	**1.1408**
	2.2430	2.3240	**2.1780**	2.2602	2.1738	**2.1207**
	1.5354	1.8670	**1.4567**	2.1693	2.1397	**1.9854**
	0.9894	2.3433	**0.9595**	1.2148	1.2166	**1.1832**
S7	(1.1757)	**(1.1415)**	(1.1543)	(0.9638)	(0.9707)	**(0.9451)**
	1.7445	**1.7174**	1.7223	1.4785	1.4723	**1.4678**
	1.4815	1.7508	**1.4684**	1.3954	1.3932	**1.3522**
	1.9694	2.2988	**1.9355**	1.8642	1.8119	**1.7568**
	1.8813	2.3477	**1.8673**	1.5606	1.5399	**1.5147**
	1.1827	3.4174	**1.1476**	1.1355	1.1150	**1.0858**
S8	**(1.1216)**	(1.2348)	**(1.0847)**	**(1.0076)**	(1.0292)	(1.0194)
	1.6801	1.7609	1.6934	**1.3680**	1.3734	1.3883
	1.4992	1.9186	**1.4574**	1.4477	1.4198	**1.3177**
	2.2177	2.7736	**2.0637**	1.9445	1.8643	**1.8388**
	1.8517	2.6887	**1.7671**	1.4601	**1.4248**	1.4279
	1.1678	3.6573	**1.0929**	1.1591	1.1255	**1.1115**
S9	**(1.1583)**	(1.3051)	**(1.1521)**	**(0.9554)**	(0.9874)	(0.9770)
	1.6988	1.7243	1.7017	1.5994	**1.5965**	1.6121
	1.4258	1.8413	**1.4141**	1.1158	1.1262	**1.0907**
	1.9802	2.6016	**1.9365**	1.5288	1.5639	**1.4619**
	1.9018	2.4207	**1.7362**	1.7502	1.7330	**1.6686**
	1.1540	3.3826	**1.1225**	0.8697	0.8510	**0.8492**

Notes. See notes of Table 2.

Table 4. Maximum likelihood estimaes

	Estimated models			
	EId	Eco	Esa	EsaTS
α_2	1.70*	4.04	1.71*	1.76*
α_3	-0.12	-0.12	-0.12	-0.11
α_4	1.26**	4.12	1.28**	1.33*
α_5	0.28**	2.09	0.26*	0.27*
a	0.73**	0.89**	0.79**	0.78**
b_1	-0.06	-0.03	-0.06	-0.06
b_2	-0.36**	-0.14	-0.39**	-0.33**
b_3	-0.29**	-0.78**	-0.21**	-0.21**
b_4	-0.14	0.34	-0.03	-0.16
b_5	0.89**	0.35*	0.91**	0.91**
σ_f	0.78**	0.13	0.63*	0.62*
σ_1	6.15**	6.25**	6.15**	6.16**
σ_2	5.73**	10.80**	5.93**	5.54**
σ_3	0.84**	1.47**	0.80**	0.78**
σ_4	0.37**	0.92**	0.40**	0.54**
σ_5	0.18**	0.01	0.16	0.17
σ_{ζ_2}	1.90**	0.01	-	-
σ_{ζ_3}	0.13**	-	-	-
σ_{ζ_4}	0.28**	-	-	-
σ_{ζ_5}	0.01	-	-	-
β_3	-	-25.48**	-	-
β_4	-	-3.55**	-	-
β_5	-	16.51**	-	-

Notes. See notes of Table 2. Esa refers to a dynamic factor model that uses the official seasonally adjusted series. The estimated model in the case of Eid is displayed in (10) and (11) and in the case of Eco in (8) and (9).

** - statistically significant at 5% significance level

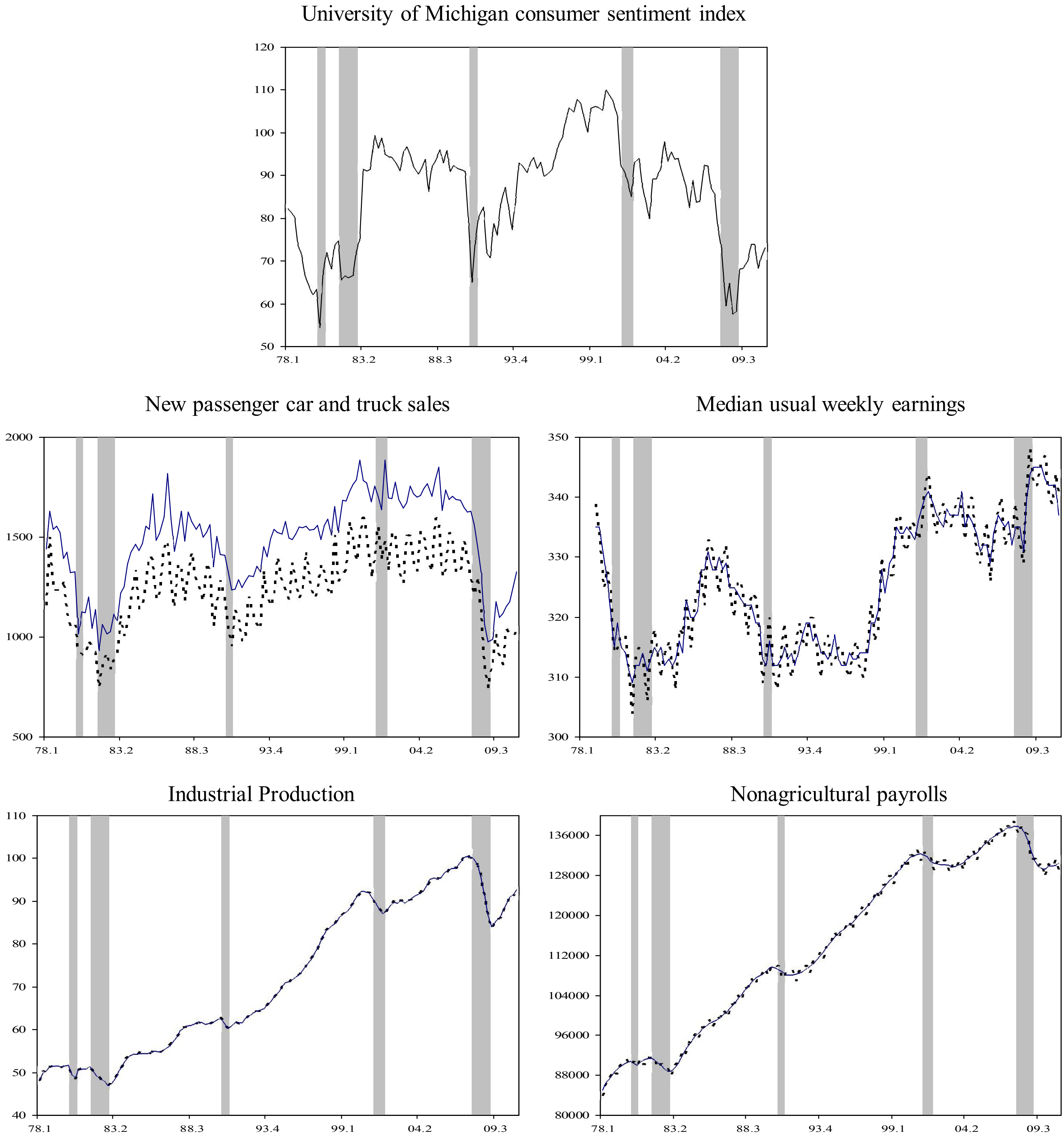
* - statistically significant at 10% significance level

Table 5. Empirical forecasting analysis

	Cars	Wages	IPI	Employ
		MSE		
EId	1.17	0.84	0.99	0.41
Eco	1.15	2.77	1.09	0.37
Esa	1.09	0.83	1.04	0.44
		p-values of DM test		
EId vs ECo	0.87	0.002	0.77	0.08
EId vs Esa	0.15	0.489	0.38	0.43
ECo vs Esa	0.51	0.002	0.85	0.12

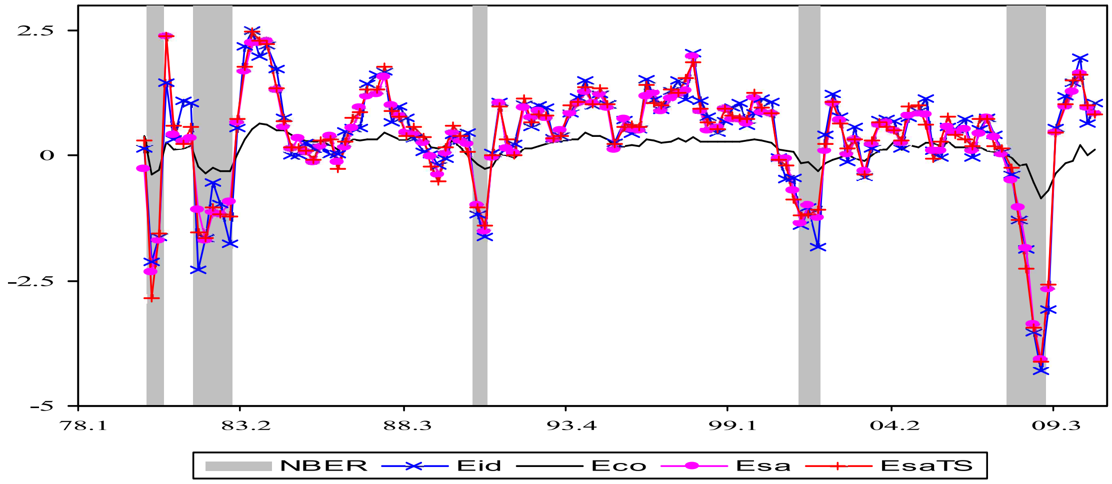
Notes. In the top panel, EId, Eco, and Esa refer to models with idiosyncratic seasons, common season, and models whose indicators are the official seasonally adjusted, respectively. To compare the results across time series easily, the figures show the MSE divided by the in-sample standard deviations of each time series. In the bottom panel, the table reports the *p*-values of the Diebold-Mariano test of the null of no different accuracy.

Figure 1. Economic indicators: 1978.1-2011.1



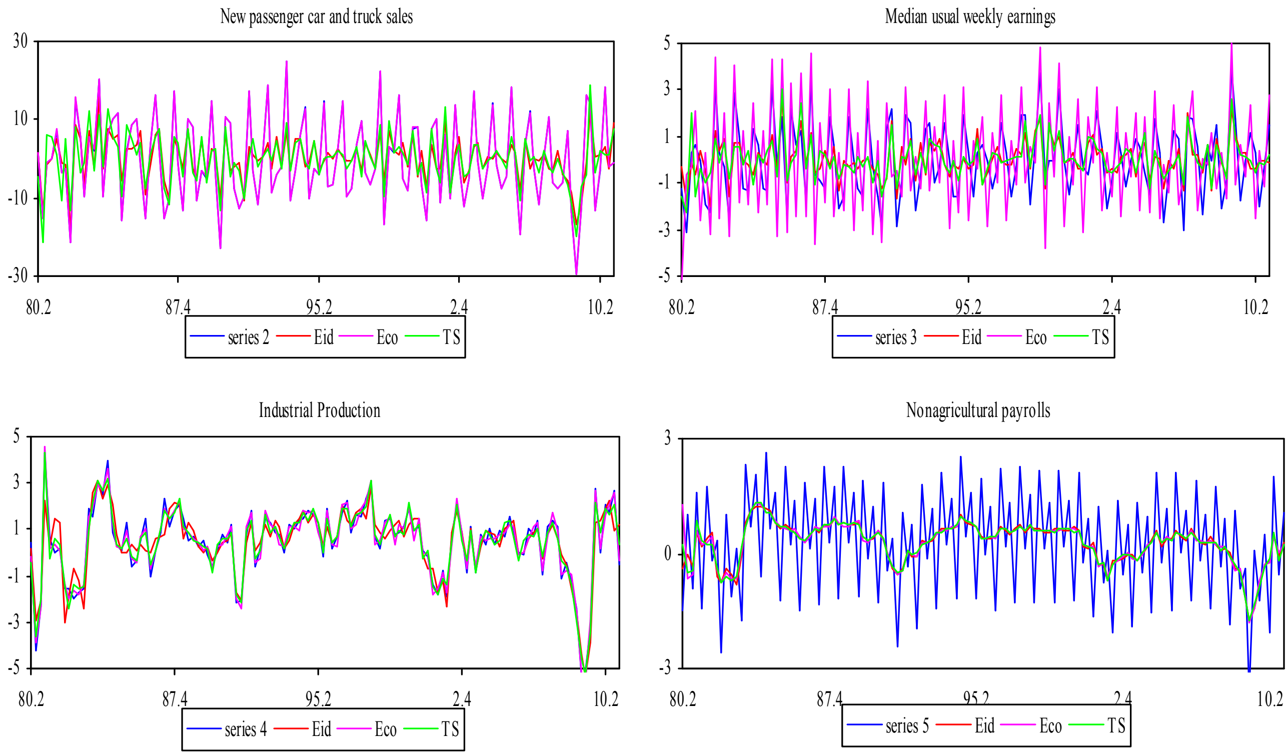
Notes: Straight lines refer to raw data while dotted lines refer to seasonally adjusted data. Shaded areas correspond to the NBER recessions.

Figure 2. Not seasonal common factor



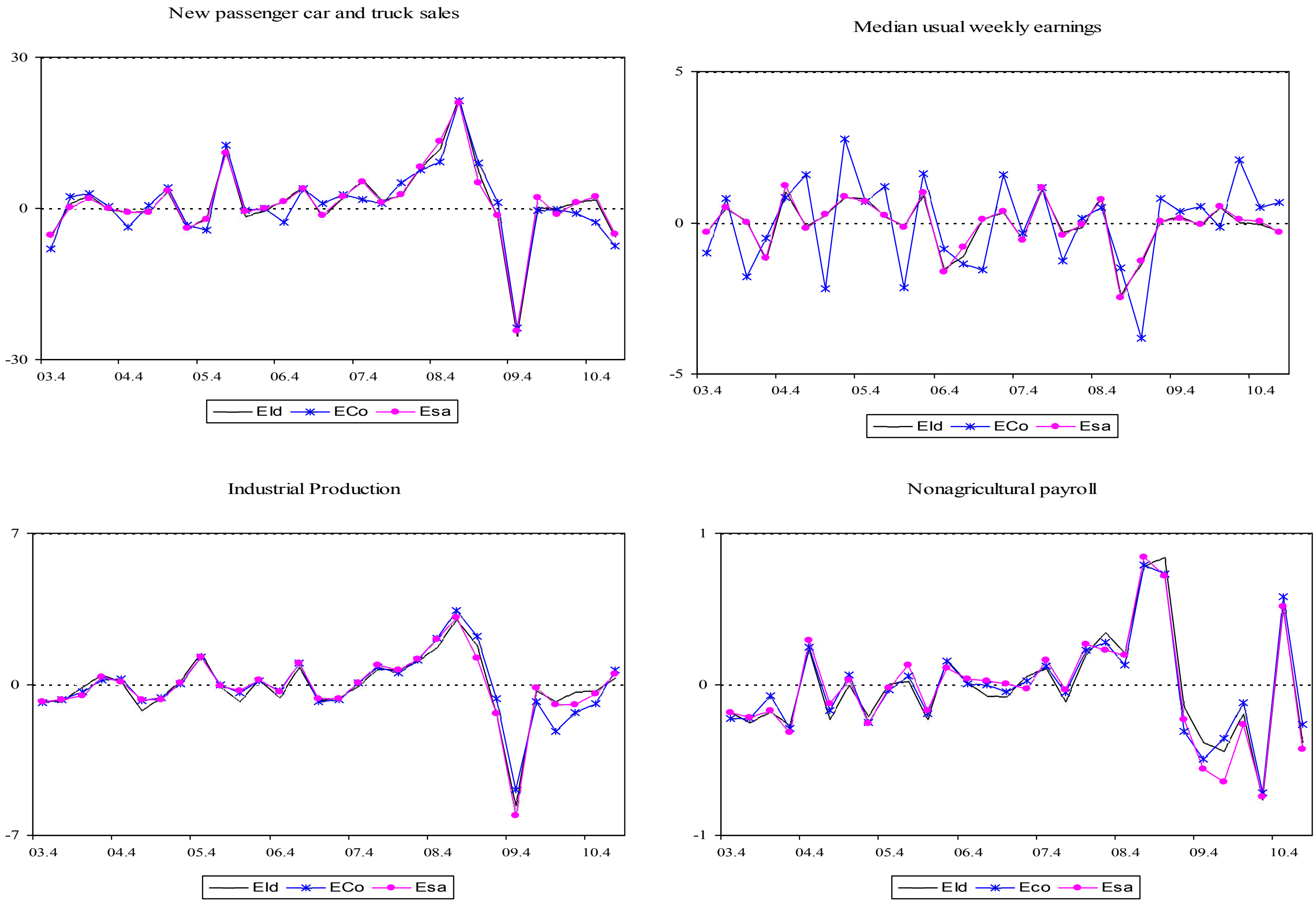
Notes: EId, Eco, refer to structural factor models with idiosyncratic and common seasons. Esa and EsaTS refer to factor models whose indicators the official seasonally adjusted series and whose seasonal component is removed by using TRAMO-Seats. Shaded areas correspond to the NBER recessions.

Figure 3. Seasonally adjusted series



Notes: See notes of Figure 2. In each graph, the original series (series 2 to 5) are not seasonally adjusted. TS is the TRAMO-Seats seasonally adjusted series

Figure 4. Out-of-sample forecasting errors



Notes: See notes of Figures 2 and 3. The closer to zero are the series the better performance of the forecasting model.