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Yrjö Koskinen and Jøril Mæland

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Yrjö Koskinen, Boston University and CEPR
Jøril Mæland, Norwegian School of Economics

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Innovation, Competition, and Investment Timing*

In our model multiple innovators compete against each other by submitting investment proposals to an investor. The investor chooses the least expensive proposal and when to invest in it. Innovators have to provide costly effort and they learn privately the cost of investing. Multiple efforts have to be compensated for, but competition helps to erode innovators' informational rents, since innovators are more likely to lose the competition if they inflate investment costs. Consequently, competition leads to faster innovation, because the investor has less of a need to delay expensive investments. The investor's payoff sensitivity also increases with competition, thus enabling the investor to capture more of the upside of innovative activity.

JEL Classification: D44, D82, G24, G31, O31 and O32

Keywords: agency costs, auctions, innovation, investment timing and real options

Yrjö Koskinen
School of Management
Boston University
595 Commonwealth Avenue
Boston, MA 02215
USA

Jøril Mæland
Norwegian School of Economics
Helleveien 30
NO-5045 Bergen
NORWAY

Email: yrjo@bu.edu

Email: joril.maeland@nhh.no

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1. INTRODUCTION

Venture capital funds often invest in a portfolio of young firms engaged in similar projects, and subsequently VC funds grant further financing to only a very few of them deemed to have the highest potential. In internal capital markets, divisions submit competing budget requests to headquarters, who then engage in picking winners. Firms often organize innovation contests, where independent teams of engineers and scientists submit their solutions to technical problems. Then the firm chooses the best solution and the winning team receives a prize. All these situations are examples where the investor has to provide incentives for innovators to work hard and to relay their knowledge truthfully. In this paper we study how competition between innovators changes the compensation offered to them and the timing of investments. Our main results show that due to competition, innovators' compensation becomes less sensitive to the revenues and that investments occur earlier.

The scenario we have in mind is where innovators – portfolio firms, corporate divisions, teams of engineers and scientists – have to exert costly effort in order to come up with an investment proposal. While working on the project, innovators also learn privately how expensive it is to invest. The investor – a VC fund, corporate headquarters, R&D management – offers contracts that solve these moral hazard and adverse selection problems.

We employ a real options framework where investments are irreversible and the investor has to decide when to invest. An innovator has an incentive to inflate the costs if he thinks he will be awarded the contract: by declaring a high cost for the project the innovator can capture a difference between the declared and the true cost for himself. The investor can use two tools to solve this problem: provide higher compensation if the declared investment cost is low and delay the investment if the declared cost is high. By delaying the expensive investment the investor lowers the present value of the innovator's compensation, thus reducing the innovator's incentive to lie about the true cost of investing.

We formulate a principal-multiple agents model, in which the principal – the investor – can choose the number of agents – innovators – she can offer the contract to. The single agent case has been analyzed by Grenadier and Wang (2005). We add a feature from auction theory where agents compete to obtain an incentive contract, as described in Klemperer (1999) and Laffont and Tirole (1987). A key insight in our model is that when innovators have to compete for a contract, their

incentive to inflate costs is diminished. Each innovator would like to declare that the investment is expensive even if it is not the case and thus capture extra compensation for himself. This incentive to inflate the cost of investment is reduced by the presence of many innovators competing for the contract: by falsely declaring a high cost, an innovator would end up losing the contract to another agent that truthfully revealed his low cost.

Competition for contracts and the resulting erosion of informational rents has two implications: Firstly, the investment would not be delayed as much as it would have to be in the one agent case. As informational rents are decreasing in the number of innovators participating in the competition, it follows that investment delays are decreasing in the number of innovators, too. Secondly, the winning innovator's compensation becomes less sensitive to the cash flows from the investment. To induce the innovator not to inflate the cost the investor has to promise compensation that is increasing in the cash flows. The need for this also diminishes with competition.

We then proceed to show that when the investor can choose the number of agents she will contract with, there are no investment delays, i.e., first best investment policy is always achieved. Thus the informational rents have been completely eroded by competition. The agency problems are reduced to a pure moral hazard problem where the investor only needs to worry about providing incentives for the innovators to put in the high effort. As a consequence, the winning innovator's compensation becomes completely insensitive to the cash flows. No extra compensation is needed even for very valuable investments. The reason is that in a competition the winner receives the effort costs of all the participating innovators. Then winning the competition becomes so valuable that the expected value of inflating the investment cost is not enough to compensate for potentially losing the competition because of it.

We show that the optimal number of innovators is decreasing in effort costs. With very high effort costs, a single innovator is enough to achieve first best investment policy. Also, the harder the task, the more innovators should be invited to participate in the competition. The reason is that with an easy task (high *ex-ante* probability of a having a low cost investment), it is more likely that several innovators come up with low cost projects. Thus it is not worth the risk for any innovator to inflate the costs and likely lose the contract because of that. As a result, the investor can save some money by inviting fewer participants.

Interestingly, we also show that the optimal number of agents is decreasing in the volatility of the project. When volatility increases, the value of the investment option increases for both the low cost and high cost projects. However, the difference in value between these projects decreases. As a result the innovators would have less of an incentive to inflate the costs, leading to shorter investment delays. To accommodate this, the investor has an incentive to decrease the number of agents. Consequently, the outcome remains as a pure moral hazard problem.

This paper is related to the part of VC literature that deals with VC's portfolios and their optimal sizes. In our model the offer to participate and work on a project proposal can be thought of as a start-up investment. Then the subsequent competition and investment is like staged investment in VC financing: portfolio firms compete against each other and only the best one gets further financing. Kannianen and Keuschnigg (2003, 2004) were the first ones to introduce the concept of optimal portfolio size in VC financing. The VC would like to have a large portfolio, but having to advice too many companies would dilute the value of costly advice that the VC gives to its portfolio companies. The size of the portfolio increases in the profitability of portfolio companies, but declines in the size of initial investment and the effort cost of the entrepreneur¹.

Inderst, Mueller, and Münnich (2007) explore the incentive benefits of constrained VC financing. In their model with fixed portfolio size Inderst et al. (2007) let the VC limit the amount of financing that is available for its firms. The creation of shallow pockets forces the portfolio firms to work hard and compete against each other in order to receive scarce financing. The trade-off is that good firms might not get financing at all². Our paper provides a complimentary rationale for the shallow pockets argument: competition for scarce resources eliminates the informational rents portfolio firms enjoy.

To the extent that experienced VC firms invest in larger portfolios, Bengtsson and Sensoy (2011) provide evidence on entrepreneurial compensation that is consistent with our model. According to Bengtsson and Sensoy, experienced VCs are willing to settle for less downside protection in

¹Bernile, Cumming, and Lyandres (2007) extend the approach of Kannianen and Keuschnigg by endogenizing the sharing rule between the entrepreneur and the VC.

²In contrast, Fulghieri and Sevilir (2009) provide a model where small VC portfolios enhance the incentives of entrepreneurs to exert effort *ex-ante*. Large and focused portfolios improve the *ex-post* resource allocation. Large portfolios are optimal when firms are risky and their technologies are related, but small portfolios dominate when firms have high expected returns.

their financial contracts. We derive a similar results: when cash flows are low, investor's share is decreasing in number of competitors.

Our model is also connected to the literature on how firms allocate resources internally. Baiman, Rajan, and Saouma (2007) model the firm's internal resource allocation as an auction. Like in our model, the agents in Baiman et al. (2007) – divisional managers – have to exert costly effort to come up with a project. Then the divisional managers learn privately their costs of completing the projects. Baiman et al. (2007) don't consider the possibility that the firm could offer different contracts to divisional managers based on their realized costs, like we do. In contrast, Baiman et al. (2007) only allow for fixed completion bonuses. The firm has an incentive to choose a bonus that is too low to achieve optimal investments. Thus in Baiman et al. (2007) there are too few project completions, that would correspond to investment delays in our model.

In a related paper, Chen (2007) shows that auctioning off supply contracts can lead to optimal allocations. The firm commits to an auction where it specifies a price for each quantity that it is willing to buy. Privately informed suppliers submit bids for these contracts and the highest bid wins the contract. Chen shows that suppliers have an incentive to reveal their private information, just like in our model. In contrast to our model, there is no moral hazard and thus suppliers don't have to be compensated for their costly efforts.

In addition our paper is related to the literature on innovation contests. In an innovation contest the firm has an R&D problem and organizes a contest for outside agents to solve the technical problem. The agents submit their solutions and the agent that comes up with the best solution wins the pre-specified prize. Innovation contests may lead to underprovision of costly effort, but the upside is that the firms may receive an outstanding solution to its technical problem. The problem of a lower equilibrium effort can be mitigated by switching from fixed prize to performance contingent prize, as pointed out by Terwiesh and Xu (2008). Empirically, Boudreau, Lacetera, and Lakhani (2011) show that for less uncertain problems, the effort reducing effect of contests dominates, but for more uncertain problems the increased likelihood of an extreme solution makes organizing a contest worthwhile.

Bouvard (2010) too studies agency problems where the investment is a real option. He assumes that an entrepreneur also possesses private information about the quality of her project. In his signaling model, high quality projects are delayed, as opposed to our screening model, where lower

quality projects may be delayed. Bouvard doesn't consider moral hazard issues nor the effects of competition. Morellec and Schürhoff (2011) and Bustamante (2012) also develop real options models with signaling where firms have an incentive to speed up investments in order to convey positive information and thus gain access to financing with more lucrative terms. Grenadier and Malenko (2011) provide a more general real options model with signaling where firms have either an incentive to speed up investments or delay them. Firms will speed up investments if they benefit from highly valued projects, whereas they delay investments if they benefit from low valuations.

We are not the first ones to consider auctions in a real options framework. Board (2007) develops a model where a seller auctions off an asset – land, oil fields – among multiple agents and the winning agent chooses when to develop the asset. The agents have private information about the revenues that the asset can bring in. The revenue maximizing auction combines an up-front bid and a contingent fee paid when the agent starts using the asset. The contingent fee leads to delay of usage of the asset. The model of Board doesn't consider a moral hazard problem and also the number of agents participating in the auction is fixed.

While we consider the effects of competition in a model where a firm needs an agent to manage an investment, competition in product markets has also an effect on option exercise. Grenadier (2002) employs a standard real options model of investment, except that several firms hold these options and the value of these options depend on whether other firms exercise their options. Grenadier shows that competition erodes the value of waiting and firms invest at close to zero net present value threshold. However, Novy-Marx (2007) demonstrates that when firms' production technologies differ firms have an incentive to delay investments, even in the case that competition has eroded all the oligopoly profits.

Effects of competition on investment timing is also discussed in Lambrecht and Perraudin (2003). They assume that each competitor knows his own investment cost, but not the competitors' cost levels. A similarity to our model is that only the winner of the game can realize his investment project: the competitors lose the option to invest when the first investor has realized his investment project and thus captured the whole market. The focus in Lambrecht and Perraudin (2003) is different from ours as they discuss the trade-off between postponing the investment to maximize the option value and invest early to preempt competitors' from investing first. However, in both models

competition reduces each competitor's option value: in our model competition reduces informational rents, and in Lambrecht and Perraud's model it reduces monopoly rents of an investment option.

We organize the rest of the paper as follows. Section 2 outlines the model. Section 3 derives the investment triggers and expected compensations for innovators. Section 4 implements the optimal sharing rule between the investor and the innovator awarded the contract, whereas Section 5 optimizes the investor's value with respect to the number of innovators invited to participate. Section 6 discusses the comparative statics results, and Section 7 concludes.

2. SETUP OF THE MODEL

The optimization problem of the investor is formulated in a principal-multiple agents framework, in which agents obtain private information about the quality of their respective investment projects after they have exerted unobservable efforts. The term "agents" refers to innovators, and the "principal" is the investor. In this section we start with a description of the innovators' projects. We then go on to provide a benchmark of the investment problem: the value of an innovation when there are no problems with respect to private information and moral hazard. Finally, we present the full private information and hidden effort problem faced by the investor.

An investor seeks to invest in an innovation project and invites n innovators to come up with project proposals. Initially we analyze the situation in which n is fixed, but in Section 5 we endogenize n . At the time of the invitation, the investor announces that she will invest in one of the proposed projects, and offers a pre-specified contract to the innovator with the best proposal. We assume that the investor is able to commit to the terms of the contract offered. All parties are risk neutral. We also assume that innovators do not have any initial wealth and that they have limited liability, implying that innovators' compensation has to be non-negative.

The innovators' projects are developed through two phases. In the first phase each innovator has to provide effort to come up with a proposal. The higher the effort of an innovator, the higher is the probability that he is able to develop a good project. The quality of the project is privately revealed to the innovator after he has exerted effort.

In the second phase the winner of the contract is selected based on the submitted business proposals. If the investor chooses to invest in project i at time t , the payoff from the project is equal to $X_t - K_i$, where X_t is a stochastic variable that is observable to all parties, and K_i is

privately observed by innovator i . We interpret X_t as the time t value of future, uncertain cash flows, and K_i as the investment cost of innovator i 's project. The stochastic variable, X_t , is driven by the process,

$$(1) \quad dX_t = \mu X_t dt + \sigma X_t dW_t,$$

where μ is the expected change in X_t per period, σ is the volatility, or standard deviation, per unit of time, and dW_t is the increment of a standard Wiener process. Let X denote the asset value at time 0, i.e., $X \equiv X_0$, and assume there is no traded asset that is perfectly correlated with the cash flows from project.

As X_t changes stochastically over time, we maximize the project value by finding the optimal time to invest in the project. This means that we allow for the possibility that it may be optimal to postpone the investment. The investment options are assumed to be perpetual. We assume $r > \mu$ to ensure that it will be optimal to invest at some future time (if the growth rate μ is larger than the discount rate r it is always optimal to postpone the investment).

The investment cost for innovator i , K_i , can take one of two values, K^G or K^B , with $K^B - K^G > 0$. We interpret K^G as draw of a high quality (or a "good") project, i.e. a project with a low investment cost. Analogously, K^B refers to a high investment cost, which means that it is a low quality (or a "bad") project. By exerting effort innovator i can influence the probability of the level of the investment cost, K_i .

Initially, innovators can choose between two effort levels, high and low. Let q_H represent the probability of K^G when an innovator decides to exert high effort. If the innovator chooses to exert low effort, the innovator's probability of a good project is given by q_L . An innovator's cost of high effort is ξ_H , whereas the cost of low effort is equal to ξ_L . We assume $q_H > q_L$ and $\xi_H > \xi_L$. Effort cannot be observed by the investor, and is therefore not contractible.

A summary of the timing stages of the model is presented in Figure 1.

Before we move on to discuss contract schemes, we present the first-best case, i.e. the case when we have no agency costs. The first-best investment timing will serve as a benchmark for our mixed hidden effort and private information problem.

2.1. First best investment decisions: no hidden effort or private information (the benchmark case). Let $V(X, K_i)$ denote the value of a project with innovator i 's investment cost when there is no unobservable action and no asymmetric information. The investment project is formulated as a real option: the project value is maximized by finding the optimal time to invest. Let the function $X^*(K_i)$ represent the value of future cash flows that triggers investment. This means that it is optimal to invest immediately when $X > X^*(K_i)$. If $X < X^*(K_i)$ the project value is maximized by postponing the investment until X reaches the trigger $X^*(K_i)$. It is well known (shown in Brennan and Schwartz (1985), McDonald and Siegel (1985), and Dixit and Pindyck (1994), among others) that the project value then is given by the following proposition.

Proposition 2.1. *The value of an innovator i 's investment project when there are no agency problems:*

$$(2) \quad V(X, K_i) = \begin{cases} \left(\frac{X}{X^*(K_i)}\right)^\beta (X^*(K_i) - K_i) & \text{for } X < X^*(K_i) \\ X - K_i & \text{for } X \geq X^*(K_i), \end{cases}$$

where

$$(3) \quad X^*(K_i) = \frac{\beta}{\beta - 1} K_i,$$

and β is given by

$$(4) \quad \beta = \frac{1}{2} - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2} > 1.$$

A proof of Proposition 2.1 is given in Appendix A. Eq. (2) shows that if immediate investment is optimal, the value of the project is equal to $X - K_i$. The term $\left(\frac{X}{X^*(K_i)}\right)^\beta$ in Eq. (2) can be interpreted as a *discount factor* as it gives the present value of receiving one unit of money at the expected future time when X reaches $X^*(K_i)$. For the rest of the presentation we assume that X is below the investment trigger for all values of the investment cost, K_i . This simplifies the presentation of the model without loss of generalization.

The value of the project in Proposition 2.1 is based on *ex post* information, i.e. given that the parties know the investment cost of project i , and under the assumption that innovator i knows that his project will be financed by the investor. Initially, the investor and the innovators do not know whether their projects are of high or low quality. In order to increase the probability that at

least one of the innovators' projects is of high quality, the investor can invite multiple innovators to submit business proposals. However, this comes at a cost as the investor has to compensate the innovators for their effort costs of preparing proposals. In the first-best case, the investor's optimization problem with respect to how many innovators, n , to invite to the contest is a trade-off between these two considerations. We will assume that it is optimal for the innovators to exert high effort.³

In our model it is assumed that the investor has the bargaining power. Hence, in the first best case the investor obtains the entire value of the project the investor selects, and the innovators are compensated only for their effort costs. The investor optimally chooses to invest in one of the high quality projects. Let p_n^H represent the probability that there is at least one innovator with a K^G -type project, i.e., $p_n^H = 1 - (1 - q_H)^n$. The investor's ex ante value of the investment opportunity, as well as the optimal number of innovators to invite to submit investment proposals, are stated in the following proposition.

Proposition 2.2. *For a given n , the investor's first best value of the contract is equal to*

$$(5) \quad V_{FB}^P(X, n) = p_n^H \left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G) + (1 - p_n^H) \left(\frac{X}{X^*(K^B)} \right)^\beta (X^*(K^B) - K^B) - n\xi_H.$$

The optimal number of innovators in the first-best scenario can be expressed as

$$(6) \quad n_{FB}^* = \frac{\ln \left(\frac{\xi_H}{-\ln(1-q_H) \left[\left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G) - \left(\frac{X}{X^*(K^B)} \right)^\beta (X^*(K^B) - K^B) \right]} \right)}{\ln(1 - q_H)}.$$

The optimal number of innovators given by Eq. (6) is found by maximizing V_{FB}^P with respect to n , as shown in Appendix B. Eq. (6) illustrates that the optimal number of innovators is a trade-off between the effort costs of the innovators, ξ_H , and the probability q_H that an innovator has a high quality project.

2.2. Setting with hidden effort and private information. The investor offers a contract to n innovators. The contract is a function of the observable asset value, X , and the n innovators'

³The innovators are assumed to have to exert at least "low effort" to submit a project proposal. Therefore, if low effort were the optimal choice there would be no moral hazard problem to discuss, and the investor's optimization problem would be equal to a pure private information problem.

reports of their privately observed investment costs, $\hat{K} = [\hat{K}_1, \hat{K}_2, \dots, \hat{K}_n]$, where \hat{K}_i is innovator i 's report of his privately observed cost K_i , $i = 1, \dots, n$.

Using results of Laffont and Tirole (1987) we organize the competition as a "winner-takes-all" contract: The innovator who is awarded the contract shares the value of the project with the investor, whereas the competitors receive nothing. The profit sharing between the investor and the innovator who wins the contract takes place at the time of investment. If innovator i is awarded the contract the reported project value at the investment time, $X - \hat{K}_i$, is shared between innovator i and the investor. Let $s_i(X, \hat{K})$ be the compensation of innovator i , and $X - \hat{K}_i - s_i(X, \hat{K})$ be the investor's compensation. If innovator i 's report \hat{K}_i deviates from the true value K_i , innovator i 's value from the contract is equal to $s_i(X, \hat{K}) + \hat{K}_i - K_i$. The investment is made when X reaches the trigger $X^I(\hat{K}_i)$ if the winner, innovator i , reports \hat{K}_i . In short, the winner is offered the contract $\{X^I(\hat{K}_i), s_i(X, \hat{K})\}$.

In our model we have only two possible values of each innovator i 's investment cost K_i , K_i^G and K_i^B . This means that the innovator awarded the contract can choose between two contract schemes, depending on whether the innovator reports a good or a bad project. As all innovators with investment cost K^G have identical projects, and all innovators with investment cost K^B have identical projects, we drop the subscript i for innovator i in the notation below. We impose the condition that the investor has to offer the same menu of contracts for each agent. The two available contracts are then denoted $\{X^G, s^G\}$ and $\{X^B, s^B\}$, where $X^G \equiv X^I(K^G)$, $s^G \equiv s_i(X^I(K^G), \hat{K})$, $X^B \equiv X^I(K^B)$, $s^B \equiv s_i(X^I(K^B), \hat{K})$.

The project that receives the investment is selected randomly from the pool of projects with the highest value. Thus, at the stage when each innovator's private information is revealed, each innovator's probability of being awarded the contract when there is n competitors is represented by Y_n^G if the innovator announces a good project, or Y_n^B if the innovator reports a bad project. The probability Y_n^B is given by the probability that none of the other $n - 1$ innovators reports K^G ,

$$(7) \quad Y_n^B = \frac{1}{n}(1 - q_H)^{n-1}.$$

For innovators of K^G -type, the probability of winning is equal to,

$$(8) \quad Y_n^G = \sum_{j=0}^{n-1} \frac{1}{j+1} \binom{n-1}{j} q_H^j (1-q_H)^{n-1-j}.$$

To simplify notation we define each innovator's expected compensation, $S^G = Y_n^G s^G$ and $S^B = Y_n^B s^B$.

As each innovator ex ante has identical projects, the investor's portfolio of projects equals the investor's expected values from each project multiplied by the number of innovators competing for the contract, n ,

$$(9) \quad V^P(X, n) = n \left[q_H \left(\frac{X}{X^G} \right)^\beta (Y_n^G (X^G - K^G) - S^G) + (1 - q_H) \left(\frac{X}{X^B} \right)^\beta (Y_n^B (X^B - K^B) - S^B) \right].$$

For a fixed n the investor maximizes her value $V^P(X, n)$ by finding optimal investment strategies, X^G and X^B , and compensation functions, S^G and S^B . This optimization problem is solved in Section 3. In Section 5 we solve the investor's problem of finding the optimal number of innovators to offer the contract to.

3. OPTIMAL INVESTMENT TRIGGER AND EXPECTED COMPENSATION OF EACH INNOVATOR

To solve the optimization problem with respect to investment triggers and expected compensation we follow the approach of Grenadier and Wang (2005). They have a similar real options set-up to ours, except that in their paper there is only one agent who needs incentives to exert effort and reveal private information. Laffont and Martimort (2002) (pp. 294-298) also present an agency problem with one agent, in which moral hazard is followed by private information, but their model is in discrete time. We introduce competition in the Grenadier and Wang type framework using a similar approach to the private-value auctions described in Klemperer (1999) and Laffont and Tirole (1987). Laffont and Tirole assume that each firm has private information about its future cost at the contracting stage, whereas in our model there is no private information at that stage. They formulate the principal's maximization problem as a Vickrey auction, in which each bidder simultaneously submits a bid, without seeing others' bids. The contract is given to the bidder who makes the best bid, and is priced according to the second-best bidder. In this auction truth telling is a dominant strategy. Although we also apply a Vickrey auction in order to solve our agency

problem, it can be shown that the results do not depend on how the auction is organized (see Klemperer (1999) section 4 and references therein).

Now we present the main optimization problem of the investor with respect to each innovator's investment trigger and expected compensation. For a given n we maximize the investor's value with respect to each innovator's project, which means that we optimize the part of the value in Eq. (9) that is inside the brackets,

$$(10) \quad \max_{X^G, X^B, S^G, S^B} q_H \left(\frac{X}{X^G} \right)^\beta (Y_n^G (X^G - K^G) - S^G) + (1 - q_H) \left(\frac{X}{X^B} \right)^\beta (Y_n^B (X^B - K^B) - S^B),$$

subject to *ex ante* incentive compatibility and participation constraints, and *ex post* incentive compatibility and participation constraints in Eqs. (11)-(15) below:

- Prior to exerting effort the innovators do not know their respective investment costs. Each innovator's probability of developing a high quality project depends on his level of effort. The *ex ante* incentive compatibility constraint (hidden effort/moral hazard) ensures that each innovator chooses to exert high effort instead of low effort,

$$q_H \left(\frac{X}{X^G} \right)^\beta S^G + (1 - q_H) \left(\frac{X}{X^B} \right)^\beta S^B - \xi_H \geq q_L \left(\frac{X}{X^G} \right)^\beta S^G + (1 - q_L) \left(\frac{X}{X^B} \right)^\beta S^B - \xi_L.$$

The left-hand side of the equation represents an innovator's value of the project if he exerts high effort, whereas the right-hand side states the value of low effort. We rearrange the expression as follows,

$$(11) \quad \left(\frac{X}{X^G} \right)^\beta S^G - \left(\frac{X}{X^B} \right)^\beta S^B \geq \frac{\Delta \xi}{\Delta q},$$

where $\Delta \xi \equiv \xi_H - \xi_L$ and $\Delta q \equiv q_H - q_L$.

- The *ex ante* participation constraint makes sure that the innovators participate in the competition, which means that the value of participating must be positive,

$$q_H \left(\frac{X}{X^G} \right)^\beta S^G + (1 - q_H) \left(\frac{X}{X^B} \right)^\beta S^B - \xi_H \geq 0.$$

We reorganize the *ex ante* participation constraint,

$$(12) \quad \left(\frac{X}{X^G} \right)^\beta S^G + \frac{1 - q_H}{q_H} \left(\frac{X}{X^B} \right)^\beta S^B \geq \frac{\xi_H}{q_H}.$$

- The ex post incentive constraints are necessary to ensure that the innovators report their true investment costs. If an innovator has a good project, his value of truthfully reporting a low investment cost must be higher than his value from reporting a high investment cost,

$$(13) \quad \left(\frac{X}{X^G}\right)^\beta S^G \geq \left(\frac{X}{X^B}\right)^\beta (S^B + Y_n^B \Delta K),$$

where $\Delta K \equiv K^B - K^G > 0$. The left-hand side of Eq. (13) represents the compensation of reporting a true investment cost. It must be at least as valuable as the compensation of lying, given by the right-hand side of Eq. (13), and therefore we define the left-hand side term as value of private information, or informational rents. Note that the investor has three tools at her disposal in order to reduce the value of an innovator's private information. She can increase X^B , thereby delaying the investment in the bad project. This reduces the K^G -type innovator's value of private information, as he would have to wait longer to realize his gain from a bad project. Secondly, she can increase the number of competitors. This reduces the value of private information through a reduced probability of being awarded the contract. Thus, the more competitors, the lower value of exploiting the private information. Thirdly, the investor can also reduce the compensation of an innovator reporting a bad project, S^B .

The private information constraint of an innovator with a bad project is given by,

$$(14) \quad \left(\frac{X}{X^B}\right)^\beta S^B \geq \left(\frac{X}{X^G}\right)^\beta (S^G - Y_n^G \Delta K).$$

- The ex post participation constraint requires that expected compensation for both types is positive,

$$(15) \quad S^G \geq 0, \quad S^B \geq 0.$$

Thus, contracts are bounded by limited liability.

The optimization problem in Eqs. (10)-(15) can be simplified. In Proposition 3.1 the simplifying results are summarized (and correspond to Propositions 2-5 in Grenadier and Wang (2005)).

Proposition 3.1. (i) *The expected compensation of an innovator with investment cost K^G , S^G , is strictly larger than zero.*

(ii) *The expected compensation of the K^B -type innovator, S^B , is equal to zero.*

- (iii) The ex post incentive constraint of a K^B -type in Eq. (14) never binds.
 (iv) At least one of the constraints in Eqs. (11), (12), and (13) always binds.

Proof. See Appendix C.1-C.2. The intuition of (i) is that to ensure truth-telling the compensation of the good type, S^G , must be strictly larger than the compensation of the bad type, S^B . In (ii) the expected compensation of an innovator with investment cost K^B equals zero since there is no reason to give an innovator with the highest investment cost any informational rents. With regard to (iii) the ex post incentive compatibility constraint of the bad type does not bind as long as $S^G \leq \left(\frac{X}{X^G}\right)^\beta Y_n^G \Delta K$, which we know from Eq. (13) must be true if the K^B -type innovator is to accept $S^B = 0$.

Proposition 3.1 leaves us with the following simplified optimization problem for the principal,

$$(16) \quad \max_{S^G, X^G, X^B} q_H \left(\frac{X}{X^G}\right)^\beta (Y_n^G (X^G - K^G) - S^G) + (1 - q_H) \left(\frac{X}{X^B}\right)^\beta Y_n^B (X^B - K^B),$$

subject to the private information constraint, the moral hazard constraint, and the participation constraint for high effort,

$$(17) \quad \left(\frac{X}{X^G}\right)^\beta S^G \geq \max \left\{ \left(\frac{X}{X^B}\right)^\beta Y_n^B \Delta K, \frac{\Delta \xi}{\Delta q}, \frac{\xi_H}{q_H} \right\}.$$

The constraint in Eq. (17) replaces the three constraints in Eqs. (11), (12), and (13), since we know from Proposition 3.1 (iv) that at least one of the constraints always binds. If the first term in the max-operator in Eq. (17) has the highest value of the three terms in the operator, the value of an innovator's compensation must be equal to his informational rents. If the second term is the largest, the binding constraint is the investor's cost of providing the innovators with incentives for exerting high instead of low effort. The third term is the investor's cost of guaranteeing that each innovator has a positive value from participating in the contest.

Below we present the solution to the optimization problem in Eqs. (16)-(17). First, we show that for type K^G projects, it is always optimal to follow the first best investment strategy:

Proposition 3.2. *If an innovator is of type K^G the optimal investment trigger X^G is equal to the first best trigger $X^*(K^G)$.*

Proof. See Appendix C.2. Without incentive constraints it is optimal to invest at the first-best trigger and share the profit between the investor and the winner of the contract. Hence, for a high

quality project agency problems do not imply a dead-weight loss in the contract. Agency problems only have an impact on how the value of the investment project is shared between the investor and the winner of the contract.

The properties of the optimal investment strategies for the K^B -type innovator depend on which of the constraints in Eq. (17) apply. Similarly to the model of Grenadier and Wang (2005), we identify three regions of possible combinations of the constraints: In the *private information region* only the private information constraint binds, and in the *hidden effort region* either the ex ante incentive constraint (the moral hazard constraint) or the ex ante participation constraint binds. In the *joint region* the private information constraint and one of the effort constraints bind simultaneously. The regions are in particular sensitive to effort costs, ξ_H , and number of competitors, n . The higher the number of innovators the investor invites to compete for the contract, the more expensive it is for the investor to give innovators incentives to provide effort. This diminishes the regions in which the private information problem apply.

3.1. The private information region. Let X^{PI} be the optimal investment trigger of an innovator with a low quality project when only the private information constraint binds. The first-order condition of Eq. (16) with respect to X^B results in the following optimal investment trigger for the private information region,

$$(18) \quad X^{PI} = \frac{\beta}{\beta - 1} \left(K^B + \frac{q_H}{1 - q_H} \Delta K \right).$$

The trigger X^{PI} is strictly higher than the first-best trigger $X^*(K^B)$. This means that the investment is delayed compared to first-best investment timing. The result is equivalent to the result in Grenadier and Wang (2005) that private information leads to under-investment. When private information is the binding constraint, Eq. (17) requires that the expected compensation for each K^G -type investor is given by

$$(19) \quad \left(\frac{X}{X^*(K^G)} \right)^\beta S^G = \left(\frac{X}{X^{PI}} \right)^\beta Y_n^B \Delta K.$$

The expected compensation in Eq. (19) represents each innovators's informational rents. The value of private information can be decomposed into the value of lying ΔK – the value of receiving K^B and paying the lower true investment cost K^G – adjusted by the discount factor, $\left(\frac{X}{X^{PI}} \right)^\beta$, and the

probability of winning the contract given that the innovator announces that he has a bad quality project, Y_n^B . Eq. (19) illustrates why it is optimal to delay the investment compared to a first best policy: A delayed investment reduces the value of the discount factor on the right-hand side of Eq. (19), which again reduces informational rents.

3.2. The hidden effort region. When one of the effort constraints binds, and the private information constraint does not, we let X^{HE} denote optimal investment trigger. Again, we find the optimal trigger by maximizing Eq. (16) with respect to X^B , which gives

$$(20) \quad X^{HE} = \frac{\beta}{\beta - 1} K^B.$$

In this scenario there is no investment delay compared to the first-best investment strategy as the optimal investment trigger is equal to the first best trigger, i.e., $X^{HE} = X^*(K^B)$. Let $C(\xi_H, q_H) \equiv \max \left\{ \frac{\Delta\xi}{\Delta q}, \frac{\xi_H}{q_H} \right\}$. The constraint in Eq. (17) requires that in the hidden effort region the expected compensation of an innovator with a good project is

$$\left(\frac{X}{X^*(K^G)} \right)^\beta S^G = C(\xi_H, q_H).$$

Hence, in the hidden effort region an innovator's expected compensation is equal to the maximum of

- the effort cost adjusted by the probability of being a high quality innovator, ξ_H/q_H , and
- the extra compensation an innovator requires in order to choose high effort instead of low effort divided by the increase in the probability of managing a high quality project from q_L to q_H , $\Delta\xi/\Delta q \equiv (\xi_H - \xi_L)/(q_H - q_L)$.

3.3. The joint region. When both the private information constraint and one of the effort constraints in Eq. (17) bind simultaneously, the optimal investment trigger requires that the investor's cost of private information and hidden effort must be equal to each other, i.e., we need $\left(\frac{X}{X^B}\right)^\beta Y_n^B \Delta K = C(\xi_H, q_H)$. In other words, the informational rent $\left(\frac{X}{X^J}\right)^\beta Y_n^B \Delta K$ is equal to the cost of providing an investor with incentives for high effort, $C(\xi_H, q_H)$. The optimal investment trigger, denoted X^J , is then given by

$$(21) \quad X^J = \left(\frac{Y_n^B \Delta K}{C(\xi_H, q_H)} \right)^{\frac{1}{\beta}} X.$$

Since both constraints bind simultaneously, we derive from Eq. (17) that the expected compensation of an innovator with a good project is equal to

$$\left(\frac{X}{X^*(K^G)}\right)^\beta S^G = \left(\frac{X}{X^J}\right)^\beta Y_n^B \Delta K = C(\xi_H, q_H).$$

3.4. Summary of findings with respect to investment timing and compensation. Our findings with respect to regions, investment triggers and innovators' values are summarized in Proposition 3.3:

Proposition 3.3. *The optimal investment trigger of a project with investment cost equal to K^B , is given by*

$$(22) \quad X^{B*} = \begin{cases} \frac{\beta}{\beta-1} \left(K^B + \frac{q_H}{1-q_H} \Delta K \right) & \text{for the private information region} \\ \left(\frac{Y_n^B \Delta K}{C(\xi_H, q_H)} \right)^{\frac{1}{\beta}} X & \text{for the joint region} \\ \frac{\beta}{\beta-1} K^B & \text{for hidden effort region} \end{cases}$$

The expected compensation for an innovator with low investment cost, K^G , is given by

$$(23) \quad \left(\frac{X}{X^*(K^G)}\right)^\beta S^{G*} = \begin{cases} \left(\frac{X}{X^{PI}}\right)^\beta Y_n^B \Delta K & \text{for the private information region} \\ \left(\frac{X}{X^J}\right)^\beta Y_n^B \Delta K = C(\xi_H, q_H) & \text{for the joint region} \\ C(\xi_H, q_H) & \text{for the hidden effort region} \end{cases}$$

The different regions can be identified through evaluation of Eq. (17), employing Eqs. (22) and (23):

(24)

$$\text{Private information region: } \max \left\{ \frac{\Delta \xi}{\Delta q}, \frac{\xi_H}{q_H} \right\} \leq \left(\frac{X}{X^{PI}}\right)^\beta Y_n^B \Delta K$$

$$(25) \quad \text{Joint region: } \left(\frac{X}{X^{PI}}\right)^\beta Y_n^B \Delta K \leq \max \left\{ \frac{\Delta \xi}{\Delta q}, \frac{\xi_H}{q_H} \right\} \leq \left(\frac{X}{X^*(K^B)}\right)^\beta Y_n^B \Delta K$$

$$(26) \quad \text{Hidden effort region: } \left(\frac{X}{X^*(K^B)}\right)^\beta Y_n^B \Delta K \leq \max \left\{ \frac{\Delta \xi}{\Delta q}, \frac{\xi_H}{q_H} \right\} \leq \left(\frac{X}{X^*(K^G)}\right)^\beta Y_n^G \Delta K$$

The expected compensation for each innovator in Eq. (23) is increasing and convex (recall that $\beta > 1$) in the underlying asset value X when private information is the dominating agency problem. When the moral hazard problem dominates, each innovator's value is independent of the X .

In the *joint region* in Eq. (22) the investment policy $X^{B*} = X^J$ depends explicitly on the number of innovators n : As n increases the trigger is pushed toward the first-best trigger for an innovator with a K^B -type project. To show this more explicitly, we rearrange the optimal investment trigger for the *joint region*. Rearranging the expression of the optimal investment trigger, $X^J = X^J(n)$, in (21) leads to

$$(27) \quad X^J(n) = \frac{\beta}{\beta - 1} \left(K^B + \lambda_1 \frac{q_H}{1 - q_H} \Delta K \right),$$

where

$$(28) \quad \lambda_1 = (X^J(n) - X^*(K^B)) \frac{\beta - 1}{\beta} \frac{1 - q_H}{q_H} \frac{1}{\Delta K},$$

for $0 \leq \lambda_1 \leq 1$. For decreases in n or ξ_H , λ_1 approaches 1 and this increases the optimal investment trigger, $X^J(n)$, until it reaches the investment trigger when private information is the only binding constraint, X^{PI} . Conversely, increases in n or ξ_H implies that λ_1 approaches 0, and the investment trigger approaches first-best. Thus the investment inefficiency caused by private information is mitigated by moral hazard, as in Grenadier and Wang (2005) and Laffont and Martimort (2002). Our main contribution is to study the effects of competition, and we find that an increase in competition is a first-order factor in overcoming investment inefficiencies due to informational problems.

Although Eq. (22) shows that only in the *joint region* the optimal investment trigger explicitly depends on n , the optimal investment trigger approaches the first-best trigger $X^*(K^B)$ as the number of n increases. The reason is that an increase in n reduces both the size of the *private information region* and the *joint region* and increases the size of *hidden effort region*. Hence, we conclude that increased competition speeds up innovation, as illustrated in Figure 2. Observe that as the number of innovators increases from one to four the optimal investment trigger approaches the first-best trigger⁴. We formalize the result in Proposition 3.4.

Proposition 3.4. *The size of private information region and the joint region are decreasing in n . This implies that as n increases the optimal investment trigger approaches the first-best policy $X^*(K^B)$.*

⁴Parameter values of numerical illustrations are given in Table 1.

Proof. From the regions given by Eqs. (24)-(26) we observe that if the probability that a low quality project is awarded the contract, Y_n^B , decreases, the *hidden effort region* increases, whereas the two other regions decreases. As Y_n^B is decreasing in n , we attain the result in Proposition 3.4.

The result in Proposition 3.4 predicts that when sufficiently many innovators compete for VC financing, or for winning a prize in an innovation contest, private information does not lead to serious inefficiency problems.

Most of the extant literature on VC financial contracts focuses on agency problems and risk sharing, and not on competition. Kaplan and Strömberg (2004) empirically study contracts in venture financing. They conclude that agency problems such as moral hazard and private information are more important to contract design than risk sharing concerns. Bengtsson and Sensoy (2011) draw similar conclusions for experienced VCs. In our paper we show the nature of agency problems changes in presence of competition among entrepreneurs in capital markets. Our model predicts that private information problems are reduced as competition for project financing is intensified, whereas costs of moral hazard become more important.

4. THE OPTIMAL SHARING RULE FOR FIXED n

Recall that the compensation function S^G represents the *expected* compensation of each innovator with a high quality project, $S^G = Y^G s^G$, where s^G is the compensation of the innovator awarded the contract. In Section 4.1 we evaluate the winner's compensation s^G and verify that this is indeed an optimal contract for a given n . Moreover, in Section 4.2 we discuss properties of the sharing rule between the innovator awarded the contract and the investor.

4.1. The compensation of the innovator awarded the contract. To maximize the investor's value the investor selects a winner from the pool of innovators with the lowest investment cost. As the innovators in this pool are identical, the winner is picked randomly within the pool. Only the winner of the contract obtains a compensation strictly larger than zero, s^G . The other innovators' compensations are equal to zero. Evaluation of the relationship $S^G = Y^G s^G$ and Eq. (23) leads to the following expression of the optimal compensation of the innovator awarded the contract.

Proposition 4.1. *The optimal compensation of the winner of the contract is given by*

$$(29) \quad s^{G*} = \begin{cases} \left(\frac{X^*(K^G)}{X^{PI}} \right)^\beta \frac{Y_n^B}{Y_n^G} \Delta K & \text{for the private information region} \\ \left(\frac{X^*(K^G)}{X^J} \right)^\beta \frac{Y_n^B}{Y_n^G} \Delta K = \left(\frac{X^*(K^G)}{X} \right)^\beta \frac{C(\xi_H, q_H)}{Y_n^G} & \text{for the joint region} \\ \left(\frac{X^*(K^G)}{X} \right)^\beta \frac{C(\xi_H, q_H)}{Y_n^G} & \text{for the hidden effort region} \end{cases}$$

In Appendix E we verify that Eq. (29) is an optimal compensation function. In the *private information region* the winner's compensation, s^{G*} , linearly depends on the fraction Y_n^B/Y_n^G , which decreases in the number of innovators n . This means that s^{G*} decreases in n . The intuition for the result is that the more competitors there are, the less incentive there is for an innovator to misreport his type. In the *joint region* and the *hidden effort region* the compensation function increases in n . The explanation is that the investor has to compensate each innovator for his effort costs adjusted for the probability of winning the contract. The effect of n on s^{G*} is illustrated in Figure 3. The curves represent the optimal compensation as a function of the cost of high effort, ξ_H , for the cases in which the number of competitors, n , is given by one to four, respectively. When the compensation is independent of ξ_H only the private information constraint is binding. The curves in Figure 3 illustrate that the value of private information decreases in n . The compensation increases linearly in ξ_H in the intervals where the hidden effort constraint binds. Note that the more competitors the winner has, the steeper is his compensation as a function of ξ_H . The reason is that the principal ex ante has to give incentives to all the innovators to exert costly effort. Thus, Figure 3 illustrates the trade-off the investor faces when she is optimizing over the optimal number of innovators: An increase in the number of competitors reduces the costs of private information, but increases the costs of compensating innovators for hidden effort.

4.2. Properties of profit sharing between the investor and the innovator awarded the contract.

We measure the contract winner's share of the project value as the compensation, $\left(\frac{X}{X^*(K^G)} \right)^\beta s^{G*}$ relative to the value of a high quality investment option, $V(X, K^G)$. The value for the innovator awarded the contract can be derived from Eqs. (23) and (29),

$$(30) \quad \left(\frac{X}{X^*(K^G)} \right)^\beta s^{G*} = \begin{cases} \left(\frac{X}{X^{PI}} \right)^\beta \frac{Y_n^B}{Y_n^G} \Delta K & \text{for the private information region} \\ \left(\frac{X}{X^J} \right)^\beta \frac{Y_n^B}{Y_n^G} X^J \Delta K = \frac{C(\xi_H, q_H)}{Y_n^G} & \text{for the joint region} \\ \frac{C(\xi_H, q_H)}{Y_n^G} & \text{for the hidden effort region} \end{cases}$$

Note that we have formulated the option to invest in a project, $V(X, \cdot)$, analogously to a financial call option, in which the value of future stochastic cash flows is replaced by the spot value of a financial asset, and the investment cost represents a contracted fixed strike price. It is well known that the value of a call option is increasing and convex in the underlying asset value, in our model denoted by X . Moreover, the value of the option increases as a function of volatility σ . Thus, instead we focus on the effects of X and σ on profit sharing between the contract winner and the investor.

We start by discussing effects of changes in the present value of future cash flows from the project. Since both the first and second derivatives of Eqs. (2) and (30) in the private information region and the joint region are positive for $\beta > 1$, we get the following result:

Proposition 4.2. *The contract winner's compensation, s^{G*} , is increasing and convex in X in the private information region and the joint region. In the hidden effort region the contract winner's compensation is independent of X .*

Figure 4 illustrates the contract winner's share relative to the total value of the investment project, $\left(\frac{X}{X^*(K^G)}\right)^\beta s^{G*}/V(X, K^G)$ as a function of X . For low values of X the contract winner's share is high as the investment option is far "out-of-the-money", and thus the value of the investment option is low compared to the effort cost. This result is consistent with Bengtsson and Sensoy (2011), who find that (experienced) investors require less downside protection. On the other hand, the investor receives most of the upside potential of the project value: As X increases in the hidden effort region, the contract winner's share decreases as a function of X . The reason is that the compensation for the contract winner is independent of X in the hidden effort region, whereas the value of the investment project, $V(X, K^G)$, increases in X .

For $X^G \leq X < X^{PI}$ the contract winner's share is increasing in X when $n = 1$, as shown in Figure 4. This reflects the fact that the informational rents the investor has to pay to the innovator in order to prevent him from lying increases more in X than the underlying investment project $V(X, K^G)$ does in this interval: The value of the total project is linear in X as $V(X, K^G) = X - K^G$ and the contract winner's value is convex in X for the private information region as shown in Eq. (30). When $X \geq X^{PI}$ the innovator's maximum value of private information, $\Delta K \frac{Y_n^B}{Y_n^G}$, is reached.

Hence, in this interval the investor obtains the full upside potential of the investment project. This explains the decreasing profit share in Figure 4 for $X \geq X^{PI}$.

Note also that, analogously to the values illustrated in Figure 3, the contract winner's share increases in n in the hidden effort region, and decreases in n in the private information region. Moreover, the private information region is decreasing in n .

An increase in the volatility of future cash flows, σ , has ambiguous effects on the values for the parties. On one hand, it enlarges the investment policy inefficiency through an increase in the investment trigger X^{PI} . On the other hand, we know that a higher volatility implies a higher value of the investment option. The curves in Figure 5 illustrate the contract winner's relative share of the investment values as a function of σ . In the private information region for $n = 1$ the curve is hump-shaped because of two opposing effects: For small values of σ the convexity in the value of private information dominates, which increases the innovator's share. For larger values of σ the under-investment effect dominates: a higher volatility leads to an increase in the investment trigger X^{PI} , which reduces the value of private information. In the hidden effort region, the contract winner's value of the compensation is relatively insensitive to σ , whereas the value of investment option increases. As we know that the value of an investment option increases in σ , this result too illustrates that the investor obtains almost all the upside value of the option. Also consistently with the previous result in Figure 3, Figure 5 illustrates that in the hidden effort region, the contract winner's share increases in n .

As the effort cost does not bind in the private information region, the winner's share is independent of ξ_H here. Figure 6 shows that as ξ_H becomes higher, the hidden effort cost starts to bind, the contract winner's share increases. The increase is larger the higher the number of competitors is. Moreover, as n increases the value of private information is reduced, and therefore both the contract winner's share and the private information region decreases in n .

5. OPTIMAL NUMBER OF INNOVATORS

So far we have optimized the investor's value with respect to investment triggers and compensation of the innovators. In this section, we let the investor optimize his value with respect to the number of innovators invited to submit project proposals.

For optimal choices of the decision variables, X^G, X^B, S^G, S^B , the investor's value in Eq. (9) is maximized with respect to n as shown in the following optimization problem,

$$(31) \quad \max_n n \left[q_H \left(\frac{X}{X^*(K^G)} \right)^\beta (Y_n^G(X^*(K^G) - K^G) - S^{G*}) + (1 - q_H) \left(\frac{X}{X^{B*}} \right)^\beta Y_n^B(X^{B*} - K^B) \right].$$

Recall that $p_n^H = 1 - (1 - q_H)^n$ is the probability that the winner of the contract is K^G -type. Alternatively, we can formulate the probability as $p_n^H = nq_H Y_n^G$, i.e. the probability that there is a good innovator in the pool of innovators is equal to the number of innovators times each innovator's probability that he has a good project, and times each innovator's probability of winning the contract given that he has a good project. Moreover, we have that $1 - p_n^H = n(1 - q_H)Y_n^B$. Thus, we can rewrite the optimization problem in Eq. (31) as

$$(32) \quad \max_n = p_n^H \left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G - s^{G*}) + (1 - p_n^H) \left(\frac{X}{X^{B*}} \right)^\beta (X^{B*} - K^B).$$

We analyze the optimization problem separately for each region.

In the *private information region*, evaluation of s^{G*} in (32) using Eqs. (23) and (29) leads to the following expression for the investor's value,

$$(33) \quad V_{PI}^P(n) = p_n^H \left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G) + (1 - p_n^H) \left(\frac{X}{X^{PI}} \right)^\beta \left(X^{PI} - K^B - \frac{q_H}{1 - q_H} \Delta K \right).$$

Observe that the probability that the winner is type K^G , p_n^H , increases in n . As the first term in Eq. (33) is larger than the second term, the value $V_{PI}^P(n)$ increases in n for all n in the region. Intuitively, as long as the hidden effort constraint does not bind, the investor's value increases in the number of competitors.

Evaluation of the investor's value in the *joint region* leads to

$$(34) \quad V_J^P(n) = p_n^H \left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G) + (1 - p_n^H) \left(\frac{X}{X^J(n)} \right)^\beta \left(X^J(n) - K^B - \frac{q_H}{1 - q_H} \Delta K \right).$$

In this region a higher number of innovators, n , increases the probability of the winner being an K^G -type and pushes the optimal investment trigger of a winner of K^B -type towards his first-best trigger. Both factors lead to a higher value for the investor. On the other hand, a higher n implies that the winner's compensation must be higher to motivate all innovators to exert high effort,

which decreases the investor's value. In sum, $\frac{\partial V_I^P(n)}{\partial n} > 0$ for all n in the region. A proof is given in Appendix F.

Our results so far are summed up in Proposition 5.1.

Proposition 5.1. *The investor chooses n so that first-best investment triggers are reached.*

This means that when the investor can freely choose the optimal number of innovators, the private information constraint will not be a binding constraint in the contract, and there is no investment inefficiency. Consequently, the *private information* and *joint regions* cease to exist, and only the *hidden effort region* remains relevant. Note also that the contract is renegotiation proof with respect to investment policy, as a contracted first best investment policy implies that the policy will be optimal after the investor has selected a contract winner. Thus when the investor is allowed to choose optimal n , there is no need to assume that the investor has to fully commit to the proposed contract.

As a result, the optimal number of competitors is found in the *hidden effort region*, where the value to the investor, V_{HE}^P , is equal to

$$(35) \quad \begin{aligned} V_{HE}^P(n) = & p_n^H \left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G) \\ & + (1 - p_n^H) \left(\frac{X}{X^*(K^B)} \right)^\beta (X^*(K^B) - K^B) - nq_H \max \left\{ \frac{\Delta\xi}{\Delta q}, \frac{\xi_H}{q_H} \right\}. \end{aligned}$$

In this region private information is not a binding constraint, and the optimal n is found based on the following trade-off: The probability that the winner of the contract is a K^G -type agent increases in n , which leads to a higher value for the investor. On the other hand, a larger n implies a higher compensation for the winner, as the investor has to give all the innovators incentives to exert high effort. This lowers the value to the investor.

Note that the trade-off is consistent with the empirical results in Boudreau et al. (2011). They find that for less uncertain problems, the effort effect is the largest, implying that for these problems fewer innovators are invited to compete for the prize of the contest. For more uncertain problems, the dominating effect is to invite many innovators to compete for the prize to increase the probability that one of the innovators come up with a good solution for the problem.

If $\frac{\xi_H}{q_H} \geq \frac{\Delta\xi}{\Delta q}$ in Eq. (35) the ex ante participation constraint binds, and not the ex ante incentive constraint, and consequently the investor's value is equal to her value in the first-best case in equation (5). Intuitively, both in the first-best case and in the situation in which effort is unobservable the investor needs to compensate the innovators for their costs of submitting business proposals. The investor's value is lower than in a first best situation only when the costs of providing each innovator with incentives for high effort is larger than the participation costs.

We simplify the optimization problem with respect to n by allowing n to be continuous. The following proposition gives a closed-form solution for the optimal number of innovators and is derived by maximizing the investor's value in Eq. (35) with respect to n .

Proposition 5.2. *For all n satisfied by the hidden effort region in equation (26), the optimal number of innovators, n^* , is equal to*

$$(36) \quad n^* = \frac{\ln \left(\frac{q_H C(\xi_H, q_H)}{-\ln(1-q_H) \left[\left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G) - \left(\frac{X}{X^*(K^B)} \right)^\beta (X^*(K^B) - K^B) \right]}{\ln(1-q_H)} \right)}{\ln(1-q_H)}.$$

Eq. (36) is well defined for parameter values such that the fraction in the logarithmic expression in the nominator gives values between 0 and 1.

Figure 7 illustrates the investor's project value as a function of the number of innovators, n . The upper curve represents the project value in the first-best case of no agency problems, whereas the lower curve is the value for the investor given a contract written under the assumption of full commitment. In this numerical example, it is optimal to choose approximately five innovators both in the first best case and when we have agency problems. In general, the optimal number of innovators is higher in the first best case than in the case in which we have a binding moral hazard constraint, as the effort cost per innovator is given by $\frac{\Delta\xi}{\Delta q} > \frac{\xi_H}{q_H}$.

Initially, we assumed that it is optimal for the investor to provide the innovators with incentives for exerting high effort. Thus, we need to verify numerically that high effort is the optimal choice, i.e., we need to make sure that the following inequality holds:

$$(37) \quad V_{HE}^P(n^*) \geq \max_n \left\{ p_n^L \left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G) + (1 - p_n^L) \left(\frac{X}{X^*(K^B)} \right)^\beta (X^*(K^B) - K^B) - n\xi_L \right\},$$

where $p_n^L \equiv 1 - (1 - q_L)^n$. The right-hand side of Eq. (37) is the value of the investor's contract if she had given all the innovators incentives for low effort. We can verify this inequality in our numerical example. Given the parameter values in Figure 1 we find that $n^* = 4.8$ and $V_{HE}^P(n^*) = 66.24$. If the investor instead gives the innovators incentives for low effort the value of the investor's project is a little lower, 65.91. Hence, in this example the difference in value between high effort and low effort is not large. However, the optimal number of innovators is much higher in the low effort case, 26.9. The reason is that each innovator's cost of exerting low effort, $\xi_L = 0.25$, is significantly smaller than the cost of exerting high effort, $\xi_H = 1.3$. Moreover, the probability that an innovator draws a high quality project is also smaller given low effort, $q_L = 0.1$, whereas $q_H = 0.45$. Note that if $\xi_L = 0$ there is no cost of inviting one more innovator to participate in the contest, and in that case it is therefore optimal to let the number of competitors go to infinity.

6. PROPERTIES OF THE CONTRACT

The optimal number of innovators will of course vary depending on types of projects, markets and industries. In this section we study how profitability, volatility, and effort costs change the optimal number of innovators, n^* . In addition we study the profit sharing characteristics of the optimal contract. When we study sharing properties we implicitly assume that the winning project is a high quality project with investment cost K^G . The situation in which the contract winner has a project of low quality is trivial when it comes to sharing properties, as the compensation of the contract winner, s^{B^*} , is equal to zero.

We start by discussing effects from changes in the value of future cash flows from the project, X .

Proposition 6.1. *An increase in X ,*

- (i) *leads to a higher n^* ,*
- (ii) *increases the value to the investor, $V_{HE}^P(n^*)$,*
- (iii) *increases the contract winner's compensation, $\left(\frac{X}{X^*(K^G)}\right)^\beta s^{G*} = \frac{C(\xi_H, q_H)}{Y_{n^*}^G}$,*
- (iv) *reduces the contract winner's profit share, $\frac{C(\xi_H, q_H)}{Y_{n^*}^G} \frac{1}{V(X, K^G)}$.*

Proof: See Appendix G.1

The intuition for the result in Proposition 6.1 (i) is as follows. A higher value of X implies that both a high quality and a low quality project increase in value. However, the increase is larger for the high quality project than low quality project, which means that the investor will have an incentive to invite more innovators to compete for the contract, as this increases the probability that a least of the projects will be of high quality.

Proposition 6.1 (i) and (ii) state that the value functions for both the investor and the innovator increase in X , as is the case for the total value of the investment option. However, the innovator's value is relatively insensitive to increases in X since his value of the compensation increases only through the denominator of his value function $C(\xi_H, q_H)/Y_{n^*}^G$. As n^* increases in X the probability that any good innovator wins the contract, $Y_{n^*}^G$, is reduced. This lower probability of winning must be compensated by increasing the compensation. The result in Proposition 6.1 (iv) implies that the investor's value increases more than the value for the innovator in X . The reason is that the innovators have no negotiation power in our model, and each innovator's expected value of participating in the contest therefore has an upper boundary equal to the cost of giving them incentives to choose high effort. In other words, the investor obtains most of the upside value of the investment option.

Proposition 6.2. *An increase in the volatility σ ,*

- (i) *reduces the optimal number of innovators, n^* , and*
- (ii) *increases the value to the investor, $V_{HE}^P(n^*)$.*
- (iii) *decreases the winning innovator's compensation, $\frac{C(\xi_H, q_H)}{Y_{n^*}^G}$.*

Proof: Appendix G.2. The result in Proposition 6.2 (i) is perhaps counterintuitive as we know that the value of the investment option in equation (2) increases in volatility. Thus, one could be led to believe that it is profitable to invite more innovators to compete for the contract, in order to ensure that there is at least one good project in the pool of potential investment projects. However, it turns out that it is actually optimal to decrease the number of innovators. The explanation is that the increase in the value of an investment option is concave in σ . This means that the difference between the value of a good project and the value of a bad project decreases in σ . Consequently, n^* is reduced for higher σ .

The value of the investor's investment option, $V_{HE}^P(n^*)$, is, as expected, increasing in σ . More surprisingly, in the hidden effort region the innovator is not compensated for higher volatility. On the contrary, Proposition 6.2 (ii) states that the contract winner's compensation decreases in σ . This result follows from in Proposition 6.2 (i): the number of competitors is reduced because of higher volatility. As each innovator's probability of winning the contest is higher, the winner's compensation is lower while still satisfying each innovator's ex ante participation and incentive constraints.

The two final parameters we present comparative statics for are each innovator's probability of drawing a high quality project, q_H , and each innovator's cost of exerting high effort, ξ_H . These two parameters correspond to the trade-off when it comes to the choice of n in innovation contest models, as described in Terwiesh and Xu (2008) and Boudreau et al. (2011): the trade-off between inviting many innovators to a contest as it increases the probability that one project is good, and the cost of effort if too many innovators are invited. The trade-off is also comparable to the results on VC portfolio size in Kannianen and Keuschnigg (2003, 2004) and Bernile et al. (2007): they show that the more entrepreneurs there are in a VC portfolio, the poorer is the quality of the venture capitalist's advice to each entrepreneur.

If q_H is high there is a high probability of coming up with a high quality project. A change in this parameter leads to the opposite effect on n^* compared to a change in σ , as stated in the following proposition.

Proposition 6.3. *An increase in the probability of a high quality project, q_H , leads to*

- (i) *a reduction in n^* ,*
- (ii) *an increase in the investor's value, $V_{HE}^P(n^*)$,*
- (iii) *a reduction in the contract winner's compensation, $\frac{C(\xi_H, q_H)}{Y_{n^*}^G}$.*

Proof: Appendix G.3. When the probability that each innovator draws a high quality project increases, the probability that at least one of n innovators submits a business proposal for a high quality project increases as well. Therefore the investor will have incentives to invite fewer innovators to compete for the contract, as stated in Proposition 6.3 (i). The reason is that the investor can save costs by inviting fewer innovators, which also leads to the result in Proposition 6.3 (ii).

The interpretation of Proposition 6.3 (iii) is that when q_H increases, it is sufficient to pay the innovator awarded the contract less.

A higher effort cost ξ_H increases the region in which hidden effort binds, as can be seen from Eq. (17). Since ξ_H is a cost incurred by the innovators participating in the contest, the expected value from participating must cover these costs, as formulated in (11) and (12). Thus, we expect n^* to decrease in ξ_H . Furthermore, we expect the investor's value to decrease in ξ_H as the payment to the contract winner would have to increase to cover his higher effort cost, and the contract winner's compensation to increase correspondingly. These conjunctures are indeed confirmed in the following proposition, and proved in Appendix G.4.

Proposition 6.4. *An increase in the cost of high effort, ξ_H , leads to*

- (i) *a reduction in n^* ,*
- (ii) *a decrease in the investor's value, $V_{HE}^P(n^*)$,*
- (iii) *an increase in the winning innovator's compensation, $\frac{C(\xi_H, q_H)}{Y_{n^*}^G}$.*

Proposition 6.4 (ii) and (iii) tell us that ξ_H is important for profit sharing between the investor and the innovator awarded the contract: the higher the cost of effort, the larger is the winning innovator's share of the value of the investment option.

7. CONCLUSION

We have presented an investment problem involving both moral hazard and private information in a real options framework where one of the choice variables is when to invest. In our screening model the investor designs the contract so that the innovators have an incentive to truthfully reveal their information and provide high effort. In order to elicit information revelation expensive investments are delayed when the investor contracts only with one innovator. Competition among innovators alters this result dramatically: we show that when the investor can choose the number of innovators freely, investment options are exercised so that first best investment trigger is always achieved. Also as a result of competition among innovators, all the informational rents are dissipated and the winner of the competition is only compensated for the effort costs. Thus the investor is able to capture the upside potential of the investments. While we achieve these results in a simple model where innovators have only two levels of effort and there are only two kinds of projects, the

effects of competition carry over to more complex models. Competition will erode the value to wait and the exercise decision will be closer to the first best, even if it can't be exactly achieved.

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APPENDIX A. VALUE OF INVESTMENT PROJECT IN FIRST BEST CASE

As shown by Dixit and Pindyck (1994), among many others, the ordinary differential equation of the project value $V(X, \cdot)$ can be formulated as

$$(38) \quad \frac{1}{2}\sigma^2 X^2 V_{XX} + \mu X V_X - rV = 0,$$

where V_X and V_{XX} are the first and second derivatives of the value function $V(X, K_i)$ with respect to X . Boundary conditions of the value are

$$(39) \quad V(X^*(K_i), K_i) = X^*(K_i) - K_i,$$

$$(40) \quad V_X(X^*(K_i), K_i) = 1,$$

$$(41) \quad V(0, K_i) = 0.$$

The boundary condition in Eq. (39) tells us that at the investment trigger, $X^*(K_i)$, the project value must be equal to the payoff from the project when investment takes place. Eq. (40) is an optimality condition, ensuring that the investment trigger $X^*(K_i)$ is determined so as to maximize the value of the investment option. We assume that zero is an absorbing barrier for X , as reflected by the boundary condition in Eq. (41). The solution of the differential equation in Eq. (38), subject to the boundary conditions in Eqs. (39)-(41), is given in Proposition 2.1.

APPENDIX B. FIRST BEST SOLUTION OF n

First-order differentiation of Eq. (5) with respect to n leads to

$$(42) \quad \frac{dV(X)}{dn} = -\ln(1-q_H)(1-q_H)^n \left[\left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G) - \left(\frac{X}{X^*(K^B)} \right)^\beta (X^*(K^B) - K^B) \right] - \xi_H.$$

We find the expression of n_{FB}^* in Eq. (6) by setting the first-order differentiation above equal to 0 and solve for n .

APPENDIX C. DERIVING THE OPTIMAL CONTRACT

C.1. Proof of Proposition 3.1 (i). The following inequalities show that the expected compensation of innovators with good projects must be strictly positive:

$$(43) \quad \left(\frac{X}{X^G} \right)^\beta S^G \geq \left(\frac{X}{X^B} \right)^\beta (S^B + Y_n^B \Delta K) \geq \left(\frac{X}{X^B} \right)^\beta Y_n^B \Delta K > 0,$$

for $X > 0$. The first inequality in Eq. (43) follows from the ex post incentive constraint of an innovator with a high quality project in Eq. (13). The second inequality follows from the limited liability condition in Eq. (15).

C.2. Optimal investment triggers and expected compensation. We define a Lagrangian function in order to solve the optimization problem in Eqs. (10)-(15),

$$\begin{aligned}
(44) \quad L = & \left(\frac{X}{X^G}\right)^\beta (Y_n^G(X^G - K^G) - S^G) + \frac{1-q_H}{q_H} \left(\frac{X}{X^B}\right)^\beta (Y_n^B(X^B - K^B) - S^B) \\
& + \lambda_1 \left[\left(\frac{X}{X^G}\right)^\beta S^G - \left(\frac{X}{X^B}\right)^\beta (S^B + Y_n^B \Delta K) \right] \\
& + \lambda_2 \left[\left(\frac{X}{X^B}\right)^\beta S^B - \left(\frac{X}{X^G}\right)^\beta (S^G - Y_n^G \Delta K) \right] \\
& + \lambda_3 \left[\left(\frac{X}{X^G}\right)^\beta S^G - \left(\frac{X}{X^B}\right)^\beta S^B - \frac{\Delta \xi}{\Delta q} \right] \\
& + \lambda_4 \left[\left(\frac{X}{X^G}\right)^\beta S^G + \frac{1-q_H}{q_H} \left(\frac{X}{X^B}\right)^\beta S^B - \frac{\xi_H}{q_H} \right] \\
& + \lambda_5 S^B.
\end{aligned}$$

The first-order condition with respect to S^G gives

$$(45) \quad \lambda_1 - \lambda_2 + \lambda_3 + \lambda_4 = 1.$$

The first-order condition with respect to S^B is equal to

$$(46) \quad \left(\frac{X}{X^B}\right)^\beta \left[-\lambda_1 + \lambda_2 - \lambda_3 - \frac{1-q_H}{q_H}(\lambda_4 + 1) \right] + \lambda_5 = 0.$$

We use the relationship in Eq. (45) to simplify the condition in Eq. (46), which gives $\left(\frac{X}{X^B}\right)^\beta \left(\lambda_4 \frac{1}{q_H} + \frac{1}{q_H} \right) + \lambda_5 = 0$. We conjecture for now that $S^B = 0$.

The first-order conditions with respect to the investment triggers X^G and X^B are found to be equal to

$$(47) \quad X^G = \frac{\beta}{\beta - 1} (K^G - \lambda_2 \Delta K),$$

and

$$(48) \quad X^B = \frac{\beta}{\beta - 1} \left(K^B + \lambda_1 \frac{q_H}{1 - q_H} \Delta K \right).$$

It can be shown that the ex post incentive compatibility constraint for innovators of type K^B never binds (to be verified later), i.e., that $\lambda_2 = 0$ (which for the ex post incentive compatibility

constraint to hold means that we need to have $S^G \leq Y_n^G \Delta K$). Thus, the investment trigger for good projects, X^G , is equal to the first best trigger.

The investment trigger for bad projects will deviate from first best trigger. Note that with $\lambda_2 = 0$, the relationship in Eq. (45) will be equal to $\lambda_1 + \lambda_3 + \lambda_4 = 1$. This means that at least of the incentive compatibility constraints, private information or hidden effort, or the ex ante participation constraint, will always bind.

C.3. Private information. If the ex ante incentive and participation constraints do not bind we have $\lambda_1 = 1$. The optimal investment trigger for the agent of type B will be equal to

$$X^{PI} = \frac{\beta}{\beta - 1} \left(K^B + \frac{q_H}{1 - q_H} \Delta K \right).$$

C.4. Hidden effort. When the private information constraint does not bind, i.e., when one of the effort constraints binds, we have $\lambda_1 = 0$. Thus, the investment trigger X^B is in this region equal to the first-best investment trigger,

$$X^{HE} = X^*(K^B) = \frac{\beta}{\beta - 1} K^B.$$

C.5. Both the private information and on of the hidden effort restrictions bind. The combined first-order conditions of the Lagrangian function with respect to λ_1 and $\max\{\lambda_3, \lambda_4\}$ yield

$$X^J = \left(\frac{1}{\max\{\frac{\Delta \xi}{\Delta q}, \frac{\xi_H}{q_H}\}} Y_n^B \Delta K \right)^{\frac{1}{\beta}} X.$$

Consistency with the expression of X^B in Eq. (48) requires that

$$(49) \quad \lambda_1 = (X^J - X^*(K^B)) \frac{\beta - 1}{\beta} \frac{1 - q_H}{q_H} \frac{1}{\Delta K}.$$

APPENDIX D. PARAMETER VALUES FOR NUMERICAL ILLUSTRATIONS

The parameter values used in the base case of the numerical illustrations are presented in Table 1.

APPENDIX E. VERIFYING THAT s^{G*} AND s^{B*} ARE OPTIMAL COMPENSATIONS

Define the value of an innovator's participation in the competition as V^G and V^B , depending on whether the innovator has developed a high or a low quality project. Using this more general function for the value of each innovator, the Lagrange formulation of the investor's optimization problem is given by

$$\begin{aligned}
(50) \quad \bar{L} = & \left(\frac{X}{X^G}\right)^\beta Y_n^G (X^G - K^G) - V^G + \frac{1-q_H}{q_H} \left[\left(\frac{X}{X^B}\right)^\beta Y_n^B (X^B - K^B) - V^B\right] \\
& + \lambda_1 \left[V^G - \left(V^B + \left(\frac{X}{X^B}\right)^\beta Y_n^B \Delta K \right) \right] \\
& + \lambda_2 \left[V^B - \left(V^G - \left(\frac{X}{X^G}\right)^\beta Y_n^G \Delta K \right) \right] \\
& + \lambda_3 \left[V^G - V^B - \frac{\Delta \xi}{\Delta q} \right] \\
& + \lambda_4 \left[V^G + \frac{1-q_H}{q_H} \left(V^B - \frac{\xi_H}{q_H} \right) \right] \\
& + \lambda_5 V^B.
\end{aligned}$$

Maximizing the Lagrange function with respect to X^G , X^B , V^G , and V^B leads to identical investment triggers in Eq. (22). This implies that $V^G = Y_n^G s^G$, and we conclude that s^{G*} in Eq. (29) and $s^{B*} = 0$ are optimal compensation functions.

APPENDIX F. EFFECT ON THE VALUE OF THE INVESTOR WHEN n INCREASES IN THE PRIVATE INFORMATION AND THE JOINT REGIONS

The probability that the investor will contract with a good innovator, $p_n^H = 1 - (1 - q_H)^n$, increases in n , as

$$(51) \quad \frac{\partial p_n^H}{\partial n} = -(1 - q_H)^n \ln(1 - q_H) \geq 0.$$

In the *private information region* the effect on the investor's value in Eq. (33) of a small increase in n is given by

$$(52) \quad \frac{\partial V_{PI}^P(n)}{\partial n} = \frac{\partial p_n^H}{\partial n} \left[\left(\frac{X}{X^*(K^G)} \right)^\beta (X^*(K^G) - K^G) - \left(\frac{X}{X^{PI}} \right)^\beta \left(X^{PI} - K^B - \frac{q_H}{1 - q_H} \Delta K \right) \right] \geq 0.$$

In the *joint region* the investment trigger of a low quality project, $X^J(n)$, depends on n . However, note that the investor's value function, $V_J^P(n)$, is monotonic in n , which means that it is sufficient to check the derivatives of V_J^P at the lower and upper boundaries of the region. At the lower

boundary of the *joint region* given by (25) we have $X^J(n) = X^{PI}$, and Eq. (52) applies here, too. At the upper boundary we find that $X^J(n) = X^*(K^B)$. The derivative is given by

$$(53) \quad \begin{aligned} & \left. \frac{\partial V_J^P(n)}{\partial n} \right|_{C=\left(\frac{X}{X^*(K^B)}\right)^\beta Y_n^G \Delta K} \\ &= \frac{\partial p_n^H}{\partial n} \left[\left(\frac{X}{X^*(K^G)}\right)^\beta (X^*(K^G) - K^G) - \left(\frac{X}{X^*(K^B)}\right)^\beta \left(X^*(K^B) - K^B - \frac{q_H}{1-q_H} \Delta K\right) \right] \geq 0. \end{aligned}$$

As $V_J^P(n)$ is monotonic in n the investor's value increases for all possible n in the *joint region*.

APPENDIX G. PROOFS OF COMPARATIVE STATICS ANALYZES IN SECTION 6

G.1. Proof of Proposition 6.1: Properties of the optimal contract as a function of X .

The derivative of n^* with respect to X is given by

$$(54) \quad \frac{dn^*}{dX} = -\frac{\beta}{\ln(1-q_H)X} > 0.$$

Evaluation of the first derivative of $V_{HE}^P(n^*)$ with respect to X leads to the expression,

$$(55) \quad \frac{dV_{HE}^P(n^*)}{dX} = \frac{\beta}{X} \left[\left(\frac{X}{X^*(K^G)}\right)^\beta (X^*(K^G) - K) - \frac{q_H}{-\ln(1-q_H)} C(\xi_H, q_H) \right],$$

which is positive as $C(\xi_H, q_H) \leq \left(\frac{X}{X^*(K^G)}\right)^\beta \Delta K$ by Eq. (26) and $\frac{q_H}{-\ln(1-q_H)} < 1$ for $0 < q_H < 1$.

The second derivative $V_{HE}^P(n^*)$ with respect to X is then given by

$$(56) \quad \frac{d^2 V_{HE}^P(n^*)}{dX^2} = \beta(\beta-1)X^{\beta-2} \left[\left(\frac{1}{X^*(K^G)}\right)^\beta (X^*(K^G) - K) - (-1)X^{-2}\beta \frac{q_H}{-\ln(1-q_H)} C(\xi_H, q_H) \right] \geq 0.$$

The contract winner's value of the contract, $\frac{C(\xi_H, q_H)}{Y_{n^*}^G}$, increases in X as $Y_{n^*}^G$ is negative in X ,

$$(57) \quad \begin{aligned} \frac{dY_{n^*}^G}{dX} &= \frac{\partial Y_{n^*}^G}{\partial n^*} \frac{dn^*}{dX} \\ &= \frac{(1-q_H)^{n^*} \ln(1-q_H)^{n^*+1} - (1-q_H)^{n^*}}{(n^*)^2 q_H} \frac{\beta}{\ln(1-q_H)X} < 0, \end{aligned}$$

as $(1-q_H)^{n^*} \ln(1-q_H)^{n^*+1} - (1-q_H)^{n^*} > 0$ for all $n^* \geq 1$ and $0 < q_H < 1$.

The derivative of the contract winner's value relative to value of a high quality investment project, $\frac{d\left(\frac{C(\xi_H, q_H)/Y_{n^*}^G}{V(X, K^G)}\right)}{dX} \leq 0$, as

$$(58) \quad \frac{d(Y_{n^*}^G V(X, K^G))}{dX} = \frac{1}{n^* q_H} \frac{\beta}{X} \left(\frac{X}{X^*(K^G)}\right)^\beta (X^*(K^G) - K^G) \left[\frac{1 - (1 - q_H)^{n^*}}{n^* \ln(1 - q_H)} + 1\right] \geq 0.$$

Eq. (58) is positive since $-1 < \frac{1 - (1 - q_H)^{n^*}}{n^* \ln(1 - q_H)} < 0$ for $n^* > 0$ and $0 < q_H < 1$.

G.2. Proof of Proposition 6.2. Differentiation of n^* with respect to β leads to

$$(59) \quad \frac{\partial n^*}{\partial \beta} = A \left[(K^G)^{1-\beta} \ln\left(\frac{X}{X^*(K^G)}\right) - (K^B)^{1-\beta} \ln\left(\frac{X}{X^*(K^B)}\right) \right],$$

where

$$A = \frac{\left(X \frac{\beta-1}{\beta}\right)^\beta}{-\ln(1 - q_H) \left[\left(\frac{X}{X^*(K^G)}\right)^\beta K^G - \left(\frac{X}{X^*(K^B)}\right)^\beta K^B \right]} > 0.$$

The first derivative in Eq. (59) is positive, as the exponential of the right-hand side of Eq. (59) is larger than 1,

$$\exp \left\{ A \left[(K^G)^{1-\beta} \ln\left(\frac{X}{X^*(K^G)}\right) - (K^B)^{1-\beta} \ln\left(\frac{X}{X^*(K^B)}\right) \right] \right\} = \frac{\left(\frac{X}{X^*(K^G)}\right)^{A(K^G)^{1-\beta}}}{\left(\frac{X}{X^*(K^B)}\right)^{A(K^B)^{1-\beta}}} > 1,$$

which means that the right-hand side of equation (59) is positive. Hence we find that

$$(60) \quad \frac{dn^*}{d\sigma} = \frac{\partial n^*}{\partial \beta} \frac{d\beta}{d\sigma} < 0,$$

since $\frac{\partial n^*}{\partial \beta} > 0$ and $\frac{d\beta}{d\sigma} < 0$.

The first derivative of $V_{HE}^P(n^*)$ with respect to σ is given by

$$\frac{dV_{HE}^P(n^*)}{d\sigma} = \frac{dV_{HE}^P}{d\beta} \frac{d\beta}{d\sigma},$$

where

$$(61) \quad \frac{dV_{HE}^P}{d\beta} = \left[\frac{\partial V_{HE}^P}{\partial p_{n^*}^H} \frac{\partial p_{n^*}^H}{\partial n^*} + \frac{\partial V_{HE}^P}{\partial n^*} \right] \frac{dn^*}{d\beta} + \frac{\partial V_{HE}^P}{\partial \beta}.$$

Evaluation of Eq. (61) leads to

$$\frac{dV_{HE}^P}{d\beta} = \frac{\left(\frac{X}{X^*(K^G)}\right)^\beta K^G \ln\left(\frac{X}{X^*(K^G)}\right)}{\beta - 1} - q_H C \frac{dn^*}{d\beta} > 0.$$

As $\frac{d\beta}{d\sigma} < 0$, we obtain $\frac{dV_{HE}^P}{d\sigma} < 0$.

G.3. Proof of Proposition 6.3. The first derivative of n^* with respect to q_H can be written as

$$(62) \quad \frac{dn^*}{dq_H} = \frac{n^* + \frac{1-q_H}{q_H} + \ln(1-q_H)^{-1}}{\ln(1-q_H)(1-q_H)} < 0$$

since $n^* + \frac{1-q_H}{q_H} + \ln(1-q_H)^{-1} > 0$ for $q_H \in (0, 1)$ and $n^* \geq 1$.

The total derivative of $V_{HE}^P(n^*)$ with respect to q_H is given by

$$\begin{aligned} \frac{dV_{HE}^P(n^*)}{dq_H} &= \frac{\partial V_{HE}^P(n^*)}{\partial n^*} dn^* dq_H \\ &= -(1-q_H)^{n-1} [V(X, K^G) - V(X, K^B)] \left(\frac{1-q_H}{q_H} + \ln(1-q_H)^{-1} \right) \\ &\quad + n^* \left\{ \frac{\Delta q_H}{\Delta q_H^2}, 0 \right\} - q_H C \frac{dn^*}{dq_H} > 0, \end{aligned}$$

since $\frac{1-q_H}{q_H} + \ln(1-q_H)^{-1} < 0$.

The derivative of $C(\xi_H, q_H)/Y_{n^*}^G$ with respect to q_H is equal to

$$\frac{d(C(\xi_H, q_H)/Y_{n^*}^G)}{dq_H} = \frac{\frac{\partial C}{\partial q_H} Y_{n^*}^G - C(\xi_H, q_H) \frac{\partial Y_{n^*}^G}{\partial q_H}}{(Y_{n^*}^G)^2} < 0,$$

since $\frac{\partial(\Delta\xi)/(q_H - q_L)}{\partial q_H} < 0$, $\frac{\partial(\xi_H)/q_H}{\partial q_H} < 0$, and

$$\begin{aligned} \frac{dY_{n^*}^G}{dq_H} &= \frac{(1-q_H)^{n^*-1} q_H n^* - 1 + (1-q_H)^{n^*}}{n^* q_H^2} \\ &\quad - \frac{(1-q_H)^{n^*} \ln(1-q_H) n^* + 1 - (1-q_H)^{n^*}}{(n^*)^2 q_H} - \frac{n^* \frac{1-q_H}{q_H} + \ln(1-q_H)^{-1}}{\ln(1-q_H)(1-q_H)} > 0. \end{aligned}$$

G.4. Proof of Proposition 6.4. First-order differentiation of n^* with respect to ξ_H equals

$$\frac{dn^*}{d\xi_H} = \frac{1}{C(\xi_H, q_H) \ln(1-q_H)} \frac{dC(\xi_H, q_H)}{d\xi_H} < 0.$$

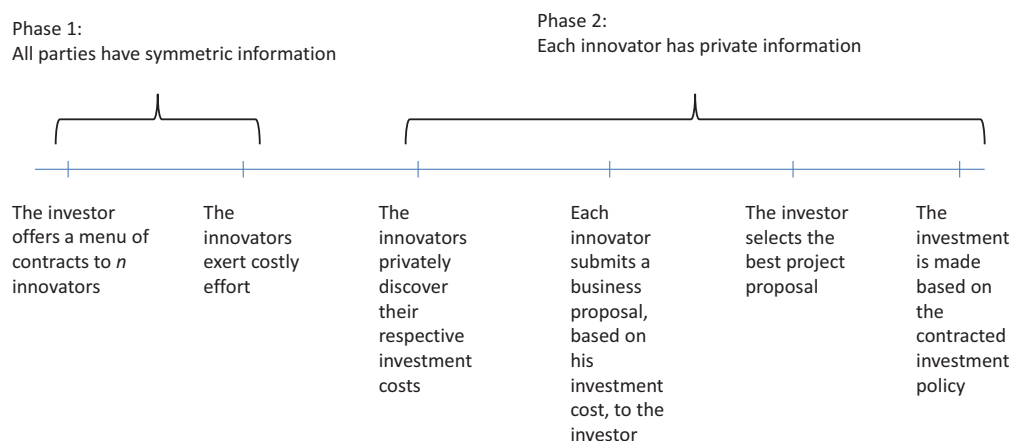


FIGURE 1. An overview of the stages of the model.

Value of observable asset	X	150
Risk-adjusted drift	μ	0.00
Volatility	σ	0.15
Risk-free interest rate	r	0.05
Privately observed investment cost if high quality project	K^G	75
Privately observed investment cost if low quality project	K^B	125
Probability of a high quality project if high effort	q_H	0.45
Probability of a high quality project if low effort	q_L	0.10
Cost of high effort	ξ_H	1.30
Cost of low effort	ξ_L	0.25
Resulting values:		
First-best investment trigger of high quality project	$X^*(K^G)$	120
First-best investment trigger of low quality project	$X^*(K^B)$	200
Investment trigger of low quality project when only private information constraint binds	X^{PI}	265.45

TABLE 1. Base case parameter values.

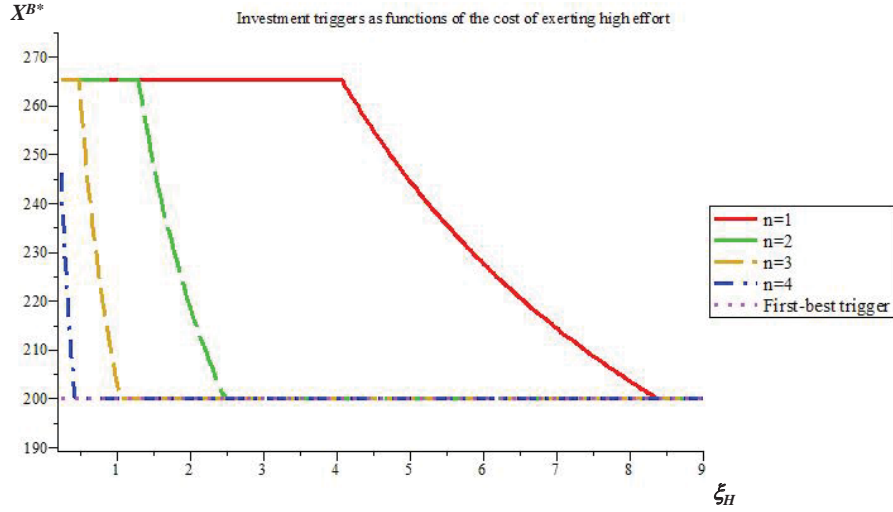


FIGURE 2. The first-best trigger $X^*(K^B)$ corresponds to the optimal trigger when we have no private information or hidden effort problems, and is given by the lower horizontal line in the figure. The four other curves represent optimal investment triggers under private information and hidden effort. The upper horizontal parts of these curves correspond to regions where only the private information constraint binds. In this case there is a large investment deviation from first-best trigger, which results in inefficient investment triggers and contracts that are not renegotiation-proof. In the region where the investment triggers decline towards the first-best trigger, both the private information and the effort constraints bind. When only one of the effort constraint binds, the optimal investment trigger equals the first-best trigger. Note that as the investor increases the number of competing innovators, the intervals of effort cost levels in which private information constraints bind are reduced. Thus, as the number of competitors increase, the optimal investment policy is pushed toward first-best investment.

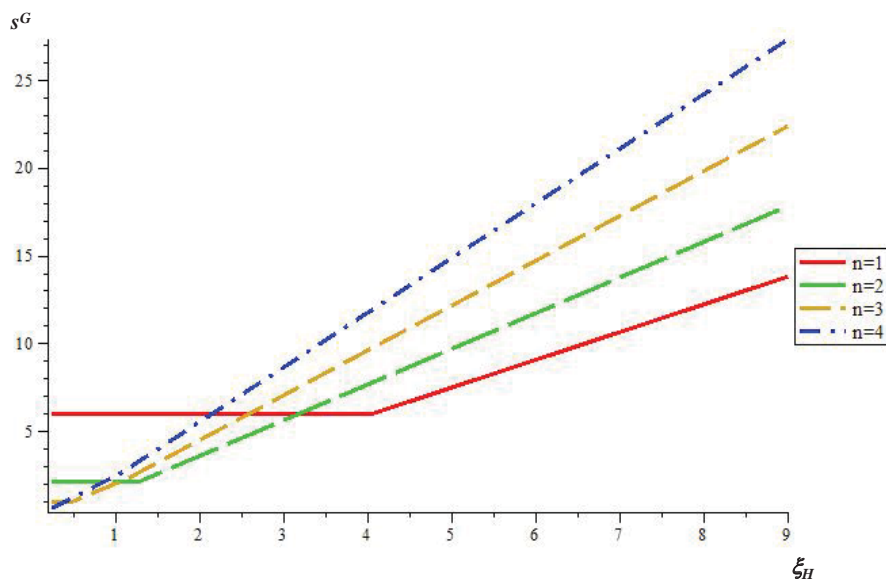


FIGURE 3. In the region of ξ_H -values where the compensation s^G is independent of ξ_H , only the private information constraint binds. The graphs illustrate that as n increases the informational rents decrease, but also that the region of ξ_H -values where private information is a binding constraint decreases. In the region where effort is a binding constraint, s^G increases linearly in ξ_H , and also increases in n . Note also that the hidden effort region increases in n , and that the compensation increases more steeply in ξ_H for a higher value of n .

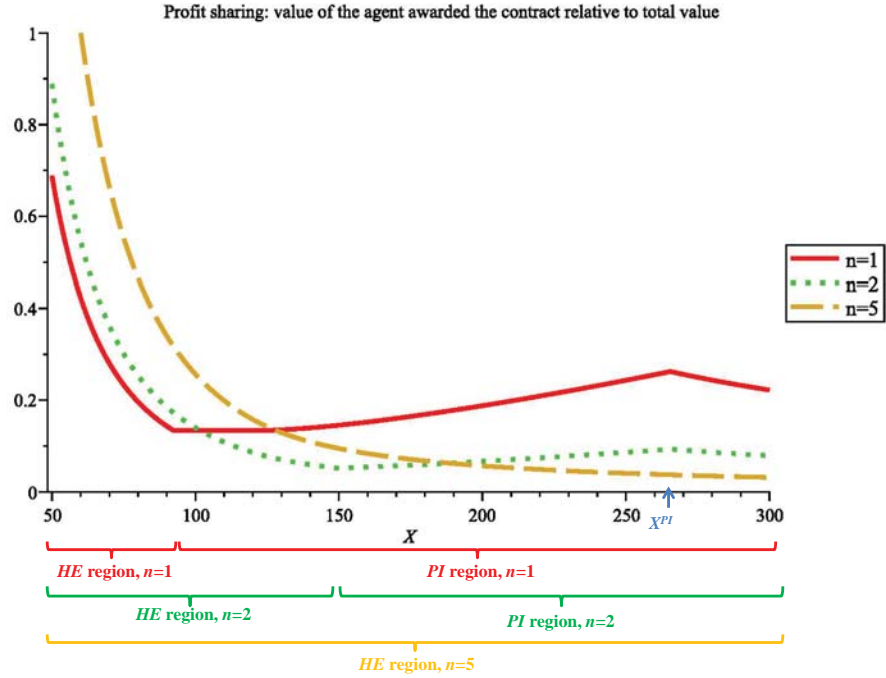


FIGURE 4. The contract winner’s relative share as a function of the observable part of the asset value, X .

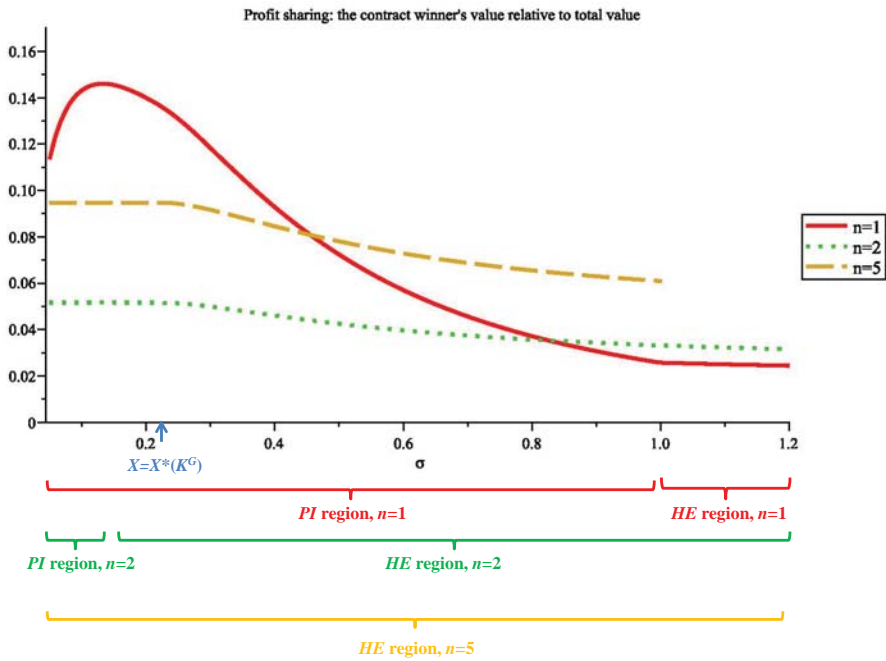


FIGURE 5. The contract winner’s relative share as a function of volatility, σ .

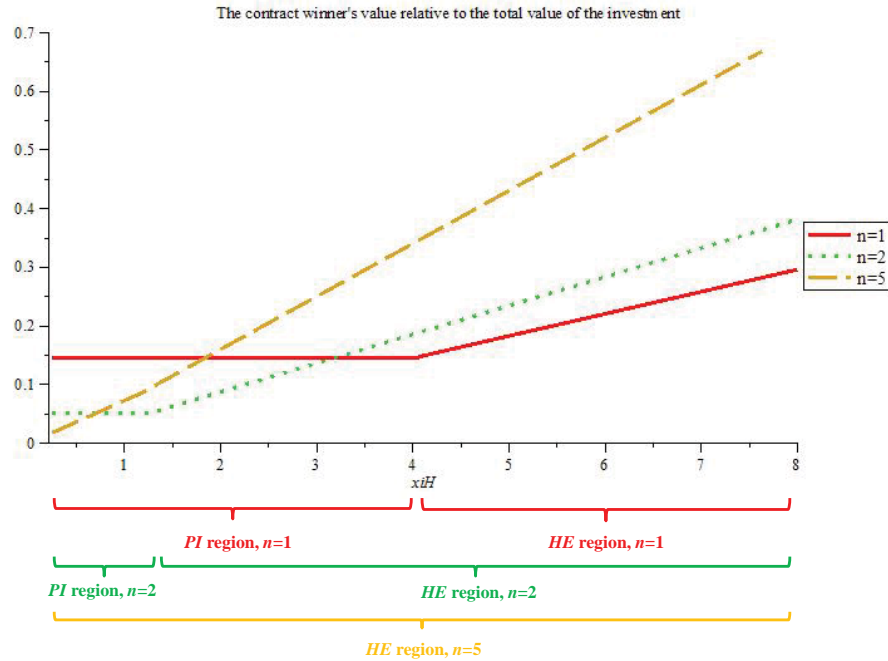


FIGURE 6. The contract winner's relative share as a function of each agent's cost of high effort, ξ_H .

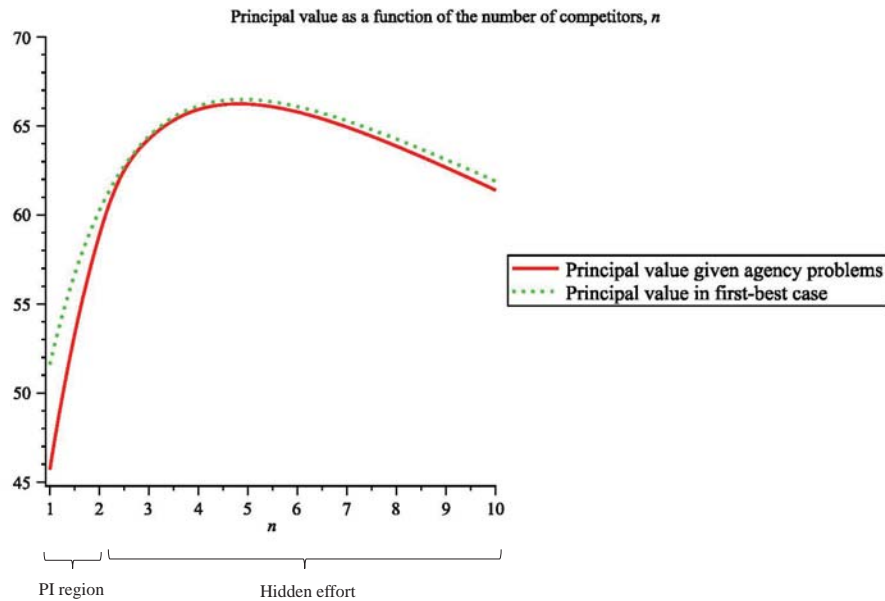


FIGURE 7. The upper curve represents the principal's project value as a function of the number of agents, n , in the first-best case. The lower curve gives optimal project values given agency problems.