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Massimo Motta and Helder Vasconcelos

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Massimo Motta, Universitat Pompeu Fabra and CEPR
Helder Vasconcelos, Faculdade de Economia do Porto and CEPR

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Exclusionary Pricing in a Two-Sided Market*

In this paper we provide a new way of modelling two-sided markets, and we then use this model to study anti-competitive conduct in an asymmetric two-sided market which captures the main features of some recent antitrust cases. We show that below-cost pricing on one market side can allow an incumbent firm to exclude a more efficient rival which does not have a customer base yet. This exclusionary behaviour is the more likely to occur the more mature the market and the stronger the established customer base of the incumbent.

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Keywords: demand externalities, predation and two-sided markets

Massimo Motta
Professor
Universitat Pompeu Fabra
C/ Ramon Trias Fargas 25-27
E-08005 Barcelona
SPAIN

Email: massimo.motta@upf.edu

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Helder Vasconcelos
Faculdade de Economia do Porto
Rua Dr. Roberto Frias
4200-464 Porto
PORTUGAL

Email: hvasconcelos@fep.up.pt

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1 Introduction

In recent years, there has been a large literature dealing with two-sided markets, that is, markets where a platform sells to two distinct groups of users which may affect each other's utility. Common examples of such markets are credit cards (card-holders and merchants), operating systems (computer users and software developers), malls (shoppers and shops), media (viewers or readers and advertisers).[1]

One important feature of such markets is that prices optimally take into account the externalities between the two sides of the market. Similarly to what happens for firms selling complementary products, it may be optimal to sell below cost to - or even subsidise - one group (the group whose demand is more price-sensitive) to increase demand on this side of the market, with the objective of increasing demand on the other side of the market.[2]

This has led several commentators to state that below-cost pricing in two-sided markets should not worry antitrust agencies, since - far from implying an exclusionary objective - they would reflect normal competitive behaviour in industries where there exist externalities between different sides of the market. For instance, Evans and Schmalensee (2007) claim that:

> "Price equals marginal cost (or average variable cost) on a particular side is not a relevant economic benchmark for two-sided platforms for evaluating either market power, predatory pricing, or excessive pricing under European Community law ... it is incorrect to conclude, as a matter of economics, that deviations between price and marginal cost on one side provide any indication of pricing to exploit market power or to drive out competition." (p. 27)[3]

Inspired by the (heterosexual) nightclubs example, Wright (2004) highlights that:

> "sometimes the cover charge women face is permanently set at zero, which is clearly below marginal cost. However, far from representing predatory pricing, below-cost prices may be used to generate greater surplus by attracting those users (women) that provide the greatest benefits to the network of other users (men). While such a price structure may represent an attempt by a firm to attract greater market share, since prices can be

[1] See, for instance, Evans (2002), Rochet and Tirole (2006) and Armstrong (2007) for additional examples of two-sided markets.

[2] As Caillaud and Jullien (2003) point out, "[d]ue to indirect network effects, the key pricing strategies are of a 'divide-and-conquer' nature, subsidizing the participation of one side (divide) and recovering the loss on the other side (conquer)." (p. 310)

[3] See also following pages, especially pp. 29-30.

profitably retained below cost, it would make no sense to think of this as predation." (pp. 48-49).

Undoubtedly, in most cases pricing below cost on one side of the market does not represent a threat to competition, and it may be the only way to get 'both sides on board', and to ensure that a product is viable. In this paper, however, we show that in certain circumstances - and in particular when the market is not at its initial stages and there is a strong asymmetry between a dominant firm and its rival(s) - pricing below cost on one side of the market may allow a dominant firm with an established and captive customer base to exclude a more efficient rival from both sides of the market, with detrimental consequences on welfare.

Intuitively, sacrificing profits on one side allows the dominant incumbent to enjoy monopoly profits on the other side, thus providing a rationale for predation. Obviously, the rival knows that getting consumers on each side is crucial for its overall existence, and this typically results in a price war on one side of the market (in our case, the side which is less affected by demand externalities).[4] There are two effects which determine which firm will win consumers on this market side. On the one hand, the rival is assumed to have lower production costs, and this allows it to make more aggressive price offers. On the other hand, if the incumbent excludes the rival from one side of the market, it will be *monopolist* on the other, whereas the rival would always have to compete with the incumbent which has already an installed base on both sides. In other words, other things being equal, the incumbent will set prices more aggressively on one side because it anticipates that, if it secures it, it will obtain monopoly (rather than duopoly) profits on the other market side. Only if the rival has a sufficiently strong cost advantage would it manage to overcome this latter effect.

In what follows, we briefly describe a few exclusionary pricing cases involving firms operating in markets which exhibit two-sided features.

In the early 1990s, the UK quality newspapers industry (newspapers being a prototypical two-sided market) - was the stage of a price war which prompted an intense public debate and several allegations of predatory pricing.[5] After many years in which prices and market shares had been stable, in September 1993 News International (Rupert Murdoch's company) which owned The Times, cut its price from 45 pence (same price as The Guardian and The Independent, and only

[4] Along these lines, and considering a situation where two groups (1 and 2) interact via one or more platforms, Armstrong (2007) highlights that "[i]f a member of group 1 exerts a large positive externality on each member of group 2, then it is natural to expect that group 1 will be targeted aggressively (i.e. offered a low price relative to the cost of supply) by platforms. In broad terms, especially in competitive markets, it is group 1's benefit to the other group that determines group 1's price, not how much group 1 benefits from the presence of group 2."

[5] We draw on Behringer and Filistrucchi (2010) for a description of this episode.

slightly lower than The Daily Telegraph) to 30 pence. Being probably the closest competitor to the Times, the Independent - which already had some financial difficulties - felt particularly threatened by this price cut. The Independent initially kept prices high (and even slightly increased them), but in Summer 1994 - after its sales had fallen by 20% - dropped its prices from 50p to 30p, while the Times decreased its price further to 20p. The Independent's complaints were joined by politicians who - estimating that the Times was making huge losses in order to force the Independent to exit the market - alleged predatory conduct. In October 1994, the Office of Fair Trading (OFT) decided not to intervene. In Summer 1995, also reflecting rising costs of news printing, newspapers' prices started to climb, and in January 1996 they recovered stability, with The Times selling at 30p and the rivals at prices around 40p. Between August 1993 and January 1996, The Times increased its market share from about 17% to 28%, whereas its rivals decreased it (The Independent from 16% to 12%, the Daily Telegraph from 49% to 43%, The Guardian from 18% to 17%).

We report this episode to show that in a two-sided market below-cost pricing (assuming that in this particular case this was the case) on one side of the market may be consistent with two situations, respectively procompetitive and anticompetitive. The procompetitive one, emphasised by the recent literature (see e.g. Rochet and Tirole (2003, 2006) and Armstrong (2006)) is that subsidising demand to one group of consumers (in this case, the readers) will increase sales to the other group of consumers (the advertisers).[6] The anticompetitive one is that below-cost pricing may be used to force the rival's exit, by depriving it of sales to one group of consumers, and thereby reducing also its demand from the other group. Our paper does not only provide a model which formally shows why the latter mechanism may take place, but it also tries to identify under which circumstances exclusionary pricing may take place. Far from claiming that below-cost pricing should always be interpreted as exclusionary conduct, we shall show that in cases like the UK newspaper market where the alleged predator does not have a dominant position and where it is unlikely that consumers are fully locked-in, exclusionary conduct is unlikely to arise. Our model then supports the idea that The Times did not breach competition laws, and that the OFT did well not to intervene.

Our model will suggest instead that there was anticompetitive exclusionary pricing in another recent UK case, *Napp*. In 2001 the OFT found that Napp, a pharmaceutical company, had infringed the UK Competition Act 1998 through its behaviour in the market for the supply and distribution of sustained release morphine in the United Kingdom. This infringement consisted of both predatory

[6] Indeed, think of the number of free newspapers in circulation everywhere today. Even in such situations predation may take place, but it would take the form of selling advertising space below cost, and would therefore not rely on two-sided market mechanisms (since readers pay zero price). Two such recent cases are *Aberdeen Journals* in the UK and *Media 24* in South Africa.

pricing in the hospital segment and excessive pricing in the community segment (Napp had a market share well in excess of 90% in both segments). Sustained release morphine is sold to two completely different groups of buyers. On one side there are hospitals, which have a high demand elasticity (pharmaceuticals have to be paid out of their budget) and can count on the advice of specialist doctors for an assessment of the competing products. On the other side, there is the so-called 'community segment', where general practitioners prescribe products for their patients (with the National Health System paying the bills), and who - not being experts - tend to choose those products which have already been chosen by hospitals (in some cases, this is automatic: for patients who have been in hospitals, the general practitioners limit themselves to follow the hospital's prescription).

Note that this can be seen as an asymmetric two-sided market, where hospitals care about prices only, not about demand by the other consumer group, while the demand of the community segment strongly depends on the choices made by hospitals. Formally, this is very much the same situation as the newspaper market, where readers largely do not care about advertising, while advertisers care about the number of readers that a newspaper sells.

An incumbent like Napp may want to sell below costs to the crucial side of the market (the hospital market in this case) to make sure the rival does not win it, thereby deterring buyers on the other side of the market (in this case, the community segment) - whose demand is characterised by a positive externality with that of the former side - from buying from the entrant. As a result, they will be obliged to buy from the incumbent, which can behave like a monopolist on this (community) side of the market, recouping any losses made to win the other (hospital) side.

In this case, the market was mature and it is excluded that Napp needed to sell below-cost to hospitals in order to have the community side of the market 'on board'. Furthermore, there clearly was an entrenched dominant position by Napp, which enjoyed strong incumbency advantages - taking the form of both it having previously won the vast majority of hospitals' orders, and its strong network in the community segment (promotion is very costly in such a segment). Its rivals were small and their market shares declined as an effect of its pricing strategy in the hospital market.

There are other markets with similar features as in *Napp* where predation had been alleged. In 2006, following an initiative of the Economics Ministry, the French Autorité de la Concurrence investigated Aventis for giving away its product. The market at issue was that of Low Molecular Weight Heparine (LMWH), an anti-coagulant.[7] At first sight, the case is extraordinarily similar

[7] See the Autorité de la Concurrence's Decision n. 10-D-02 of 14 January 2010.

to *Napp*, except that: (1) Some of the rivals were large and well-established companies with non-negligible market shares (in 2001, when the alleged predation started, Aventis had 55-60% market share, Sanofi 20-25% and Pfizer 10-15% - later Sanofi and Aventis merged, and Glaxo distributed Sanofi's product); (2) Aventis did not itself start the strategy of distributing its products for free to hospitals, but limited itself to follow suit when its rivals undertook this strategy; and (3) the strategy did not have foreclosing effects: the main complainant, a firm called Léo Pharma, increased its market share in the community market from 10-15% in 2001 to 20-25% in 2006, while maintaining its very low share of the hospital market (thereby also questioning the hypothesis that hospital sales are a necessary condition for the community sales of LMWH).

Again, this is to stress that while our model provides a rationale for exclusionary pricing in two-sided markets, it does not imply that any below-cost pricing in such markets should be taken as anti-competitive conduct.

The reader will note that both the *UK newspapers* and the *Napp* cases concern industries where there is a clear asymmetry between the two groups of market participants: there is a demand externality from one group to the other, but not vice versa. Accordingly, after describing a general model of two-sided markets, we shall specialise it to fit the environment of the above-mentioned cases.

Our paper is related to two strands of the recent economic literature. The first strand of the literature is the one on two-sided markets, whose main references have already been mentioned. In terms of modelling assumptions, the closest paper to ours within this strand of the literature is that of Armstrong (2006). In particular, like us, he assumes that the fixed benefit a consumer enjoys from using a platform depends only on which side of the market the agent is on, platform charges are levied as a lump-sum fee and costs are incurred when agents join a platform.[8] There exist, however, a few differences between Armstrong's framework and ours. In particular, in our model the market is composed of a discrete number of buyers with inelastic demands. Further, we focus on a market that already exists at the moment the game starts, with an asymmetric position between an incumbent and an entrant. We shall also make some additional special assumptions that allow us to simplify the setting to fit the case of an "asymmetric" two-sided market like the one concerning the cases of *Napp* and the *UK newspapers* discussed above. In particular, in the baseline model we

[8] There exist a number of important differences in the modelling assumptions between Armstrong (2006) and the pioneering article by Rochet and Tirole (2003) that concern the characterisation of agent's gross utility, the structure of platforms' fees and the structure of platforms' costs (see Section 2 in Armstrong (2006) for a discussion). As Armstrong (2006, p. 671) highlights, "[w]hich assumptions concerning tariffs and costs best reflect reality depends on the context. Rochet and Tirole's model is well suited to the credit card context, for instance, whereas the assumptions here are intended to apply to markets such as nightclubs, shopping malls, and newspapers."

assume that while consumers on one market side do care about cross-group network externalities, the utility of consumers on the other market side is unaffected by the number of consumers on the other group using the same platform.[9]

The second strand of the literature deals with exclusionary conduct. In our paper, the incumbent firm exploits demand externalities across buyers to exclude a rival, a mechanism that is in the spirit of anticompetitive exclusion in presence of contracting externalities, as stressed by Bernheim and Whinston (1998), but whose main insight was probably first applied to exclusionary conduct by Aghion and Bolton (1987). In particular, in our model it is as if the incumbent and one consumer group (the one which does not care about externalities) made a coalition to the detriment of the other consumer group and the entrant. Apart from the literature on exclusive dealing (see e.g. Rasmusen *et al.* (1991), Segal and Whinston (2000)), a similar mechanism can be found in models of exclusionary pricing such as Karlinger and Motta (2012) and Fumagalli and Motta (2009). However, those papers do not deal with two-sided markets. Further, in the former paper exclusion takes place because of miscoordination among buyers (whereas our mechanism here does not rely on miscoordination), and in the latter consumers buy sequentially rather than simultaneously.

The remainder of the paper is structured as follows. In Section 2 we present the basic model, which is chosen as the simplest possible setting where the elements we are interested in could emerge. Section 3 analyses the benchmark case in which platforms have to set uniform prices, i.e. identical prices on both sides of the market. Section 4 investigates the more general case in which platforms can charge different prices across the two sides of the market. The main policy conclusions that can be drawn from our analysis are summarised in Section 5. Section 6 addresses what are the main implications of relaxing the baseline model assumption that cross-group network externalities are unidirectional. Finally, Section 7 concludes the paper.

2 The setup

In this Section, we first propose - in reasonably general terms - a model of two-sided markets which contains some new features: in particular, we model buyers as discrete. We shall then make some special assumptions which simplify the model.

Suppose there are two groups of agents, labelled $i = 1, 2$, which interact with each other via intermediaries or "platforms". At the moment the game starts, there exist two competing "platforms".

[9] Section 6 will extend the analysis by discussing possible equilibria when cross-group network externalities are bidirectional.

Platform I is the dominant incumbent, and has already an installed base of $\beta_i^I > 0$ consumers on side i, and platform E is a rival platform with an installed base of $\beta_i^E < \beta_i^I$ consumers. These installed bases can be interpreted in different ways, for instance the product at issue may be a durable good and old consumers are locked-in to the platform they have bought in the past and would not consider buying again; or the product may be non-durable but old consumers have arbitrarily large switching costs which make them captive to the platform they chose in the past. Throughout the paper, we choose the former interpretation[10] since it allows to disregard old consumers' purchasing decisions.

Clearly, we are focusing here on a situation similar to the one that antitrust agencies and courts are facing if there was a monopolisation (Section 2 of the US Sherman Act) or abuse of dominance (article 102 of the European Union Treaty) investigation. The market already exists and a firm enjoys an incumbency advantage.

On each side of the market, there exists a second group of ("new") consumers, of size N_i, that is on the market when the game starts. A consumer is assumed to either choose to deal with one platform or to deal with no platform (single-homing). In addition, a consumer in one group is assumed to care about the number of ("old" and "new") consumers of the *other group* who use the same platform. In particular, suppose the utility of an agent belonging to group i ($i = 1, 2$) if she joins platform k ($k = I, E$) is given by:

$$U_i^k = r_i^k + z_i v\left(\beta_j^k + N_j^k\right) - p_i^k, \tag{1}$$

where $r_i^k \geq 0$ is the fixed benefit the agent obtains from using platform k, $v(\cdot)$ is a function that represents the benefit the consumer has from interacting with agents on the other side of the platform, N_j^k denotes the number of "new" consumers on the other market side that join platform k, z_i is a parameter which measures the intensity of the 'cross-group externality', and p_i^k is the price charged by platform k to consumers on side i.[11] The externality function is taken to be twice-continuously differentiable, with $v_i(0) = 0$, $v'(\cdot) > 0$, $v''(\cdot) < 0$, and $\lim_{n\to\infty} v'(n) = 0$. Throughout

[10] Our motivating example, Napp, has features which resemble the assumption that the platform consists of a durable good: hospitals buy at a procurement auction and continue to use their stock until a new auction is made.

[11] Following Armstrong (2006), we assume that platforms charge for their services on a "lump-sum" basis. This approach is different from the one in the pioneering paper by Rochet and Tirole (2003), who focus on the case where charges are levied on a "per-transaction" basis. As Armstrong (2007) points out, "[t]he crucial difference between the two forms of tariff is that cross-group externalities are less important with per-transaction charges, since a fraction of the benefit of interacting with an extra agent on the other side is eroded by the extra charge incurred. For instance, when the charge for placing an advert in a newspaper is levied on a per-reader basis, an advertiser does not have to form a view about how many readers the newspaper will attract when it decides whether to place an advert."

the paper, we assume that the two platforms are incompatible.

For simplicity, we specialise the model of cross-group demand externalities described above. We assume that both generations of "new" consumers are of the same size, $N_1 = N_2 = N$. Likewise, let $\beta_1^I = \beta_2^I = \beta^I > 0 = \beta_1^E = \beta_2^E$ (the rival firm E is a new firm which does not have any customer base). We also assume that $N > \beta^I$. (As will be clearer later on, assuming $N \leq \beta^I$ would reinforce the exclusionary results.)

The most important assumption we make in the baseline model is that $z_1 > z_2 = 0$, implying that side-1 consumers care about the externality whereas side-2 consumers' utility is unaffected by the number of people on the other side of the market. This assumption is in line with the case of *Napp* described in the introduction (hospitals are not influenced by purchase decisions of General Practitioners (GPs), but GPs' demand increases with hospitals' adoption of the pharmaceutical product), and with the case of newspapers (readers' utility is not affected by the number and space of advertising messages included in the newspaper, but advertisers' demand increases with the number of readers of the newspaper). For simplicity, we also assume that firms' platforms are of the same quality: $r_i^I = r_i^E = r_i$ (the fixed benefit only depends on the side of the market the agent is on). Moreover, suppose $r_1 = 0 < r_2$ (if $r_2 = 0$, side-2 consumers would never be willing to pay for the product). Note, therefore, that - for equal number of users - platforms are perceived by buyers as homogenous.

Turning to the cost side, assume that costs are incurred when agents join a platform. Each platform k has a cost c_k, $k = I, E$, where $c_I > c_E \geq 0$. Note that, for simplicity, platform k's costs of serving side 1 or side 2 are assumed to be identical. In addition, the new platform E is assumed to face zero entry costs, to highlight that entry barriers come only from indirect network effects.

We also impose the following market viability condition.

Assumption 1 Let us assume that:

$$\min \left\{ z_1 v \left(\beta^I \right), r_2 \right\} > c_I. \tag{2}$$

In particular, note that $z_1 v \left(\beta^I \right) > c_I$ means that the market was viable when only the "old" cohort of buyers existed.

We consider the following two-stage game:

1. Active platforms set prices simultaneously on each side of the market.

2. "New" buyers (on each side) decide whether to join a platform, and which one if they have the choice, and pay the corresponding price. (Recall that old buyers have already purchased the product and do not buy any longer.)

In what follows, we characterize the equilibria of this game under the base model assumptions that prices are non-negative and that consumers purchase only once. Two alternative pricing regimes will be considered: (*i*) a uniform pricing regime where platforms charge the same price to consumers on sides 1 and 2; and (*ii*) an "asymmetric" pricing regime where we assume that platforms can charge different prices across the two sides of the market, but cannot price discriminate between subgroups of consumers on the same market side. Note that, in some situations, the case where the platforms have to set uniform prices, i.e. identical prices on both sides of the market, is unrealistic, but it would still represent a useful benchmark. In other cases, firms sell exactly the same good to two different groups of consumers (think of *Napp*, for instance) and therefore uniform pricing would be realistic and natural.

3 Uniform pricing

Assume that platforms charge the *same* price on both sides of the market, $p_1^k = p_2^k = p^k$ with $k = I, E$. To solve the game, we move backwards: we first look for the equilibrium solutions at the buyers' stage of the game, and then turn to the equilibrium of the whole game.

The following Lemma describes the buyers' equilibria:

Lemma 1 *Under uniform pricing:*

- *If* $p^E \leq \min\left\{p^I, z_1 v(N)\right\}$*, there is a buyers' equilibrium where all "new" buyers buy from* E*.*
- *If* $p^I < \min\left\{p^E, z_1 v(N+\beta^I)\right\}$*, there is a buyers' equilibrium where all "new" buyers buy from* I*.*
- *There is no buyers' equilibrium in which all "new" buyers from one side buy from* I *whereas all "new" buyers from the other side buy from* E*.*

Proof. See Appendix. ■

Armed with the equilibrium solutions at the buyers' stage, we can now move backwards to the first stage of the game.[12] Given that similar games with uncoordinated buyers and scale economies are typically characterised by multiple equilibria (see for instance Segal and Whinston, 2000), we investigate the existence of two types of pure-strategy equilibria in this game. The first type is an "entry" equilibrium, where the entrant is active and all ("new") consumers join its platform. The second type is an "exclusionary" equilibrium, where consumers buy from the incumbent despite the fact it is less cost-efficient. It turns out that, under uniform pricing, an entry equilibrium always exists whereas exclusionary equilibria do not exist in the present game, as the following propositions summarise.

Proposition 1 *Under uniform pricing, an* ***entry equilibrium*** *always exists where platforms I and E set, respectively, prices* $p^I = c_I$ *and* $p^E = c_I - \varepsilon$*, where* ε *is positive and arbitrarily small, and all "new" consumers (on both sides of the market) buy from E.*

Proof. See Appendix. ■

Proposition 2 *Under uniform pricing, there exists* ***no exclusionary equilibrium*** *where all "new" consumers (on each side of the market) join the incumbent's platform.*

Proof. See Appendix. ■

So, under uniform pricing, the unique equilibrium of the proposed game is the entry equilibrium where the Bertrand equilibrium solution applies: E enters, I sets a price c_I and E sets a price a shade below c_I, and all "new" consumers join the entrant's platform. The intuition behind this result is simple. In our model, consumers on market side 2 do not care about the number of consumers on market side 1 who use the same platform ($z_2 = 0$). In addition, platform E is more cost efficient, $c_I > c_E$. This then implies that firm E will always be able to undercut the incumbent and sell to side-2 buyers, in turn making it possible for it to sell to buyers on the other side as well. (These considerations hold only for uniform pricing; when asymmetric pricing is allowed, firm I could sell below cost on side 2, and recover losses on side 1: see below.) In our model, therefore, exclusion cannot take place because of buyers' miscoordination, unlike similar games where there are scale

[12] It is also possible that there are buyers' equilibria where some side-i ($i = 1, 2$) buyers buy from a firm and some from the other. For instance, if $p^I = p^E$, side-2 buyers are indifferent between I and E. Suppose m of them buy from E. Then, side-1 buyers will prefer to buy from I rather than E according to: $z_1 v(\beta^I + N - m) > z_1 v(m)$. The inequality $\beta^I + N - m \gtrless m$ will determine the platform side-1 buyers would sponsor, and in case $\beta^I + N - m = m$ side-1 buyers would also be perfectly indifferent.

economies and buyers are uncoordinated and move simultaneously (see e.g. Rasmusen et al. (1991) and Segal and Whinston (2000) for models of exclusive dealing, and Karlinger and Motta (2012) for a model where firms just compete in prices).

4 Asymmetric pricing

We now investigate the effect of asymmetric prices, namely we assume that platforms can charge different prices across the two sides of the market (where the price charged by platform k to consumers on side i, p_i^k, is non-negative), but cannot charge different prices to subgroups of consumers belonging to the same market side (there is no price discrimination on a given market side).

4.1 Equilibrium solutions under asymmetric pricing

Asymmetric pricing will change considerably the results of the game. Asymmetric pricing opens the possibility that the incumbent's platform sets a price below c_I for consumers on market side 2 (where consumers do not care about cross-group externalities) and recovers profits by attracting consumers on market side 1. In other words, if platform I manages to set a price $p_2^I < p_2^E$, it will induce every consumer on side 2 to strictly prefer platform I, in turn inducing consumers on side 1 to join platform I as well, since they would not enjoy any indirect network externality from joining platform E.[13] Of course, rival firm E will also try to lower prices on side 2 (if it lost side-2 consumers it would not sell to side-1 consumers). Aggressive price competition on side-2 will follow, and the result will depend among other things on the efficiency gap between the firms (the lower c_E relative to c_I the more likely that platform E will win side-1 buyers) and the proportion between "old" and "new" consumers (for given "new" consumers N, the higher the established base of the incumbent β^I the less likely that platform E will win side-1 consumers).

Proposition 3 ***(Entry equilibria under asymmetric pricing.)*** *If both platforms can set different (non-negative) prices across the two sides of the market, then:*

(i) If $c_I < z_1 v(\beta^I + N)/2$ and $c_E \leq \left(c_I + z_1\left[v(N) - v(\beta^I)\right]\right)/2$, all "new" buyers buy from the entrant at prices $p_1^E = c_I + z_1\left[v(N) - v(\beta^I)\right]$ and $p_2^E = 0$;

[13] Note that the incumbent platform would have no incentive to embark on a "reverse" deviation wherein it would charge a price below c_I to side-1 consumers. This is because this deviation would not induce side-2 consumers to join its platform (as they do not care about cross-group indirect network externalities).

(*ii*) *If* $c_I \geq z_1 v(\beta^I + N)/2$ *and* $c_E \leq \left(3c_I - z_1\left[v(\beta^I) + v(\beta^I + N) - v(N)\right]\right)/2$, *all "new" buyers buy from the entrant at prices* $p_1^E = c_I + z_1\left[v(N) - v(\beta^I)\right]$ *and* $p_2^E = 2c_I - z_1 v(\beta^I + N)$.

Proof. See Appendix. ■

The previous Proposition tells us that - contrary to the case of uniform pricing - under asymmetric pricing an 'entry' equilibrium does not exist if the incumbent firm does not have a too wide efficiency gap relative to the entrant. One can also note that the larger the incumbent's customer base the lower the price that the entrant would have to charge for an entry equilibrium to exist ($\partial p_i^E/\partial\beta^I \leq 0$). In turn, this implies that the larger the incumbent's base - for given N - the less likely that the entry equilibrium exists.

We saw in the previous section that under uniform prices, *exclusionary equilibria* did not exist. We now show that this is no longer true under asymmetric pricing.

Proposition 4 *(**Exclusionary equilibria under asymmetric pricing**.) If both platforms can set different (non-negative) prices across the two sides of the market, then:*

(*i*) *If* $c_I < z_1 v(\beta^I + N)/2$, *there exists an equilibrium in which all "new" buyers buy from the incumbent at prices* $p_1^I = z_1 v(\beta^I + N)$ *and* $p_2^I = 0$.

(*ii*) *If (a)* $c_I < z_1 v(\beta^I + N)/2$ *and* $c_E > z_1 v(N)/2$; *or (b)* $c_I \geq z_1 v(\beta^I + N)/2$ *and* $c_E > c_I - z_1\left[v(\beta^I + N) - v(N)\right]/2$, *there exists an equilibrium where all "new" buyers buy from the incumbent at prices* $p_1^I = z_1 v(\beta^I + N)$ *and* $p_2^I = 2c_E - z_1 v(N) < c_I$.

Proof. See Appendix. ■

This Proposition shows that when the efficiency gap between the two platforms is sufficiently small, then there exists an exclusionary equilibrium where *firm I sells at a below-cost price to side-2 consumers*, who do not care about indirect network externalities, and recovers losses on the other side of the market by charging the monopoly price to side-1 consumers. Intuitively, there cannot exist an exclusionary equilibrium where firm I charges above c_I on market side 2: firm E is more efficient and would slightly undercut firm I thereby getting side-2 consumers and attracting side-1 consumers as well.

Exclusion takes place because firm I can sacrifice profits on side-2 consumers to keep the more efficient firm E out of the market and getting in this way the full monopoly profits $z_1 v(\beta^I + N)$ on side-1 consumers.[14] One may wonder how it is possible that firm E, which is more cost-efficient, may not be able to make better offers to side-2 consumers. This is because, due to its established customer base, the incumbent constrains the price that firm E can charge on side-1 consumers, in turn making it less profitable for E to act more aggressively on side 2. In other words, firms compete very aggressively on side 2 but the losses they can make are limited by the profits they can extract on side-1 consumers if they get side-2 consumers. The higher profits that firm I can make on side 1 pushes it to be more aggressive than firm E when competing for consumers on market side-2.

Note also that we have assumed that $N > \beta^I$, that is, the number of "new" consumers is larger than the number of "old" ones: the market is growing. It is straightforward to see that, other things being equal, as β^I increases - for given N - the exclusionary equilibria become more likely. Hence, if we relaxed the assumption that the market is growing, it would be *more likely* for exclusionary equilibria to exist.

It is well-known in the literature (see, e.g. Evans (2002)) that indirect network effects imply that firms (platforms) rely on complex pricing strategies, often involving price discrimination across the two sides of the market, to try and get both sides on board and solve the so called 'chicken-and-egg' problem. The literature has completely neglected, however, that, as the previous proposition shows, the adoption of a pricing structure which is biased towards one side of the market may well aim at *excluding a more efficient entrant platform* from the market.

Figure 1 illustrates the results obtained in Propositions 3 and 4 (the figure is drawn for the case where N is not too large relative to β^I). The figure shows that if the efficiency gap between the incumbent and the entrant platforms is sufficiently large, then an entry equilibrium exists. The intuition is simple. As already mentioned, asymmetric pricing opens the possibility that the incumbent platform undercuts the entrant on market side 2, attracting all side-2 "new" buyers and thereby also inducing all "new" side-1 buyers to buy from I (as they would not enjoy any indirect network externality in case they joined the entrant's platform). However, the larger c_I with respect to c_E is, the more difficult for the incumbent platform to win side-1 buyers, which in turn makes it possible for the entrant to sustain higher prices which are immune from the incumbent's deviations.

[14] Our model, therefore, provides a rationale for predation in two-sided markets. It is interesting to note, however, that while in other models (e.g. Fumagalli and Motta (2009)) the exclusionary effect depends on future monopoly profits (sacrifice profits today to recoup later), in our model there is a distributional impact of the pricing policy: profits on market side 2 are sacrificed (consumption on this side is subsidised) so as to keep the entrant out and reap monopoly profits on market side 1.

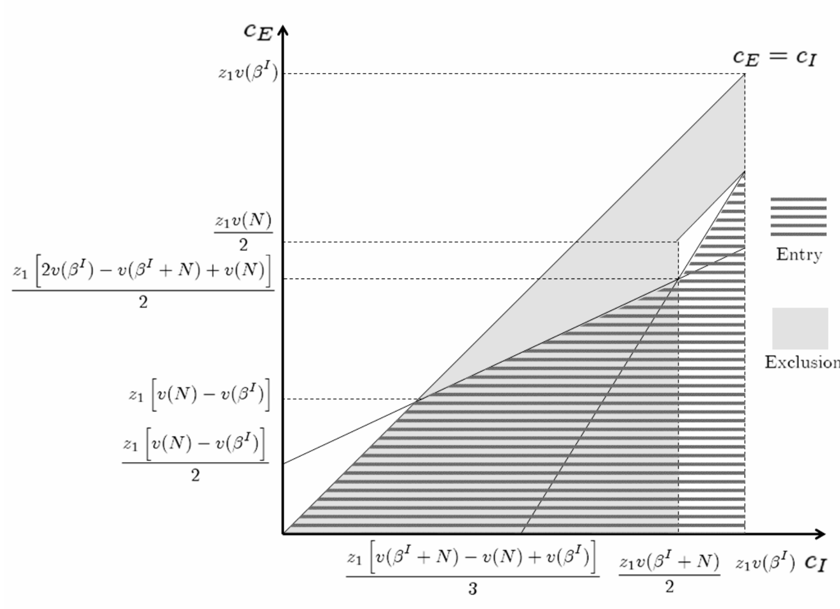

Figure 1: Entry and exclusionary equilibria (for N "not too big")

The figure also shows that there exists an area of parameter values in which, by discriminating across the two sides of the market, the incumbent firm might be able to exclude the more efficient rival from the market (while this was not possible under uniform pricing - see Proposition 2). This type of equilibrium exists if the incumbent platform is sufficiently efficient (relative to the entrant platform).

The figure further illustrates that under asymmetric pricing there is not only a region where multiple equilibria are possible, but also a region of parameter values where no equilibrium in pure strategies exists. It is beyond the scope of the paper to characterize the mixed strategy Nash equilibria that would arise in such region. It should be remarked, however, that at any mixed strategy equilibrium there would be a strictly positive probability that the entrant would *not* serve consumers, which can be interpreted as confirming the exclusionary potential of discriminatory pricing also in this region of parameter values.

4.2 Equilibrium solutions when below-cost pricing is prohibited

Another natural benchmark to study is one where there exists a policy which prohibits the incumbent (or both firms; since the entrant is more efficient this would be equivalent) from setting prices below cost on any side of the market. In this case, it is easy to see that the solution is the same as under uniform pricing: there is only the entry equilibrium. Intuitively, the incumbent could not set price below-cost on any side. So, the entrant, which is more efficient, would just need to set a price a shade below the cost of the incumbent to win the orders from both consumer groups.

Proposition 5 *If below-cost pricing were prohibited, then: (1) there always exists an entry equilibrium where* $p_1^I = p_2^I = c_I$, $p_1^E = c_I + z_1 \left[v(N) - v(\beta^I) \right]$, $p_2^E = c_I$, *and all buyers on both sides buy from the entrant; (2) there exists* ***no exclusionary equilibrium*** *where all "new" consumers join the incumbent's platform.*

Proof. See Appendix. ■

Hence, in our model, prohibiting below-cost pricing by the incumbent firm is an effective way of preventing exclusion of a more efficient entrant platform.

5 Policy discussion

5.1 Welfare analysis: asymmetric pricing versus uniform pricing and prohibition of below-cost pricing

As the previous analysis shows, price discrimination across market sides (which, interestingly, resembles a cross-subsidy) can lead to exclusion of more efficient rivals which do not have a customer base yet. Hence, an important question that should be raised at this point is whether this exclusionary pricing is welfare decreasing.

In this section, we address this question by focusing attention on the region of parameter values wherein there is exclusion under asymmetric pricing and by comparing the associated welfare to the one resulting from a situation where there is uniform pricing or prohibition of below-cost pricing.

Two preliminary remarks are in order at this point. First, in our model, side-1 consumers' utility increases continuously with the number of ("old" and "new") consumers on the other market side. Hence, the "old" generation of consumers on market side 1 cannot be ignored when studying welfare effects: even if they do not buy again, side-2 "new" consumers' decisions (also) have an impact on side-1 "old" consumers' utility. Second, note that since we assume inelastic demands, when computing total welfare, prices can be ignored as they reduce consumer surplus by the same amount as they increase profits. This implies that considering the case of uniform pricing or the case of prohibition of below-cost pricing as the benchmark case for welfare comparison is equivalent:

in both cases only an entry equilibrium exists (for all feasible parameter values) and even though equilibrium prices differ in both cases, prices play no welfare role.

Under entry, welfare will be:

$$W^{entry} = N\left[z_1 v(N) + r_2\right] + \beta^I \left[z_1 v(\beta^I) + r_2\right] - 2Nc_E. \tag{3}$$

Under an exclusionary equilibrium, welfare will be:

$$W^{exclusion} = \left(N + \beta^I\right)\left[z_1 v(\beta^I + N) + r_2\right] - 2Nc_I. \tag{4}$$

Therefore, exclusion will be welfare-detrimental if:

$$N\left[z_1 v(N) + r_2\right] + \beta^I \left[z_1 v(\beta^I) + r_2\right] - 2Nc_E > \left(N + \beta^I\right)\left[z_1 v(\beta^I + N) + r_2\right] - 2Nc_I \tag{5}$$

or, equivalently,

$$2N\left(c_I - c_E\right) > \left(N + \beta^I\right) z_1 v(\beta^I + N) - \left[N z_1 v(N) + \beta^I z_1 v(\beta^I)\right]. \tag{6}$$

There are two opposing forces at work. On the one hand, indirect network externalities benefitting side-1 consumers imply that society is better-off when consumers use the same platform rather than divide themselves across platforms: from the assumption that $v(\cdot)$ is an increasing function, it follows that $\left(N + \beta^I\right) z_1 v(\beta^I + N) > N z_1 v(N) + \beta^I z_1 v(\beta^I)$, *ceteris paribus*, making welfare higher when both "old" and "new" buyers are served by the incumbent platform. On the other hand, when it is the incumbent which serves all buyers, then there is a productive inefficiency, which is reflected in the l.h.s. of inequality (6). Whether the first or the second effect prevails, will depend on the particular parameter values assumed.

Consider for example the case where indirect network externalities enjoyed by side-1 consumers are weak, so that $z_1 \to 0$. In this case, it is the productive inefficiency effect that will be dominant, making welfare higher under entry. Likewise, if $\beta^I \to 0$ (or if $N \to \infty$), the "old" population of consumers becomes irrelevant, and again entry will lead to higher total surplus.

At the other extreme, if the efficiency gap between the entrant and the incumbent shrinks, then welfare will tend to be higher under an equilibrium where all consumers are served by the

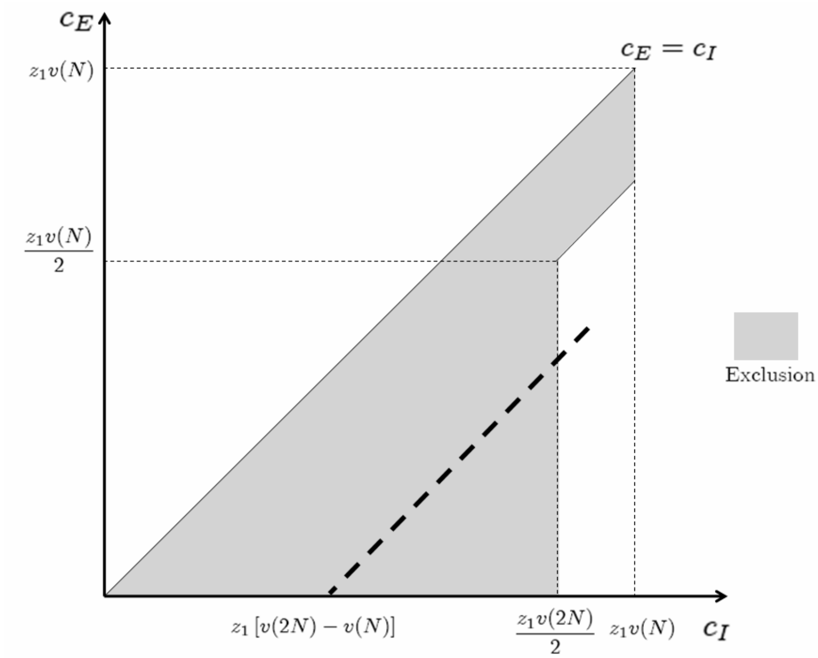

Figure 2: Anticompetitive and pro-competitive exclusionary equilibria (when $\beta^I \to N$)

incumbent. In the limit, when $c_I \to c_E$, it is clear that $W^{exclusion} > W^{entry}$.

To illustrate this trade-off in a simple way, consider the case where $\beta^I \to N$. By substitution into inequality (6), one obtains that $W^{entry} > W^{exclusion}$ if and only if:

$$c_I - c_E > z_1 [v(2N) - v(N)] . \tag{7}$$

Under this simplifying assumption, we know from Proposition 4 that an exclusionary equilibrium exists if: (i) $c_I < z_1 v(2N)/2$; or (ii) $c_I \geq z_1 v(2N)/2$ and $c_I - c_E < z_1 [v(2N) - v(N)]/2$. This region of parameter values where an exclusionary equilibrium exists when $\beta^I \to N$ is represented by the shaded area in Figure 2.

Now, the upward sloping dashed line in Figure 2 represents condition (7) when it is binding. As the Figure illustrates, this dashed line divides the region of parameter values where an exclusionary equilibrium exists into two different subregions. In the subregion above the dashed line, exclusion may arise, but it would be welfare beneficial (in this area, if entry occurs, it will be welfare detrimental). However, in the subregion below the dashed line, exclusion may arise and it would be welfare detrimental (if entry occurs, it will be beneficial). In this subregion, the efficiency gap between the two platforms is so high that when all "new" consumers are served by the incumbent platform, the resulting productive inefficiency outweighs the gain for society in terms of (higher) indirect network externalities benefiting side-1 consumers.

5.2 How to distinguish predatory versus competitive pricing?

It is often argued that pricing below cost in a two-sided market should not be presumed to be an anticompetitive practice since even a monopolist platform may find it optimal to embark on such a pricing strategy on a particular market side so as to exploit two-sided indirect network externalities and get 'both sides on board'. As the following Lemma shows, however, in our model, a monopolist in a mature two-sided market would not embark on below-cost pricing.

Lemma 2 *If the market already existed but there was no entry threat, the incumbent platform would maximize profits by setting prices* $p_1^M = z_1 v\left(\beta^I + N\right)$ *and* $p_2^M = r_2$*.*

Proof. See Appendix. ■

It is certainly true that, in some circumstances, below-cost pricing would allow a (monopolist) two-sided platform to avoid market failures and obtain the critical mass of consumers on both sides of the market.[15] However, as the previous Lemma shows, this is not true in our context, where the market already exists and a firm enjoys an incumbency advantage.

In the *Napp* case we discussed in the introduction, the firm's argument was that below-cost pricing in the hospital market was not an anticompetitive strategy because any unit sold in hospitals would have led to more units sold in the community market and this conduct was therefore not due to exclusionary reasons. The judges of the Competition Appeals Tribunal rejected this defence since they noticed that in all hospital auctions in which Napp was behaving as a monopolist it did not offer any discounts to hospitals. In other words, the comparison of the conduct adopted by the same firm in markets where it is contested and markets where it is not, can tell us something about the motivations of below-cost pricing. Like in *Napp*, in our model below-cost pricing is not necessary in order to have 'both sides on board', and such an efficiency defence cannot therefore be accepted.

Our results therefore suggest that it is very important to distinguish between a new market and a mature market such as the one we consider in this paper and that will typically be investigated in

[15] See, for instance, Section 3 in Armstrong (2006) for a monopoly platform model where the profit-maximizing outcome may involve one group of consumers being offered a subsidized (below-cost) service.

antitrust cases. Antitrust Authorities should stay away from cases where there is an infant market for two main reasons. First, because in such cases it is indeed possible that below-cost pricing is beneficial, and necessary to get 'both sides on board'. Second, because our model shows that exclusionary conduct is less likely to occur when the established customer base is small compared with the number of "new" consumers (see Proposition 4).

6 Extension: two-sided network externalities

So far, we have assumed that network externalities are unidirectional: consumers on market side 2 were assumed to be indifferent as to the number of consumers on market side 1. In this section, we address what are the main implications of relaxing this assumption. In particular, we consider the case where: (*i*) firms may set asymmetric prices to consumers on different sides of the market; and (*ii*) $z_1 > z_2 > 0$, implying that side-2 consumers care less about cross-group demand externalities than side-1 consumers, but are not indifferent as to the number of consumers on the other market side. This being the case, the market viability condition is now:

Assumption 1' Let us assume that:

$$\min\left\{z_1 v\left(\beta^I\right), r_2 + z_2 v\left(\beta^I\right)\right\} > c_I. \tag{8}$$

As before, this condition ensures that the market was viable (on both sides) when only the "old" cohort of buyers existed. We also impose the following assumption:

Assumption 2 Let us assume that:

(*i*) $r_2 > z_2\left[v\left(N\right) - v\left(\beta^I\right)\right]$;

(*ii*) $\frac{z_1 - z_2}{z_2} > \frac{v(N) - v\left(\beta^I\right)}{v(N+\beta^I)}$.

Assumption 2 amounts to require that z_2 is (positive but) sufficiently small, and/or that N is not much larger than β_I: the market is growing at a sufficiently low rate.

First, it is important to highlight that, as the next proposition shows, considering the more general case in which there are two-way network externalities does *not* remove the possibility that exclusion takes place in equilibrium.

Proposition 6 ***(Exclusionary equilibrium under asymmetric pricing and two-way network externalities)*** *If both platforms can set different (non-negative) prices across the two sides of the market and $z_1 > z_2 > 0$, then if $c_I < (z_1 + z_2)\, v(\beta^I + N)/2$, there exists an equilibrium in which all "new" buyers buy from the incumbent at prices $p_i^I = z_i v(\beta^I + N)$, with $i = 1, 2$.*

Proof. See Appendix. ■

Hence, similarly to what happened with regards to the baseline model, also in a more general context wherein there are two-way network externalities, the adoption of a biased pricing structure favouring the group of consumers that exert higher externalities on each consumer belonging to the other market side can still be aimed at excluding a more efficient rival from the market. However, the exclusionary equilibria of Proposition 6 arise only because of miscoordination among buyers (in the same spirit as in Rasmusen et al. (1991)) whereas the mechanism described in Proposition 4 does not rely on miscoordination. In addition, contrary to what happened in the baseline model, this biased pricing structure does not necessarily involve charging a price below cost to those consumers who are targeted more aggressively: prices will only be set below c_I for consumers on market side 2 if z_2 is sufficiently low.[16]

Now, as for entry equilibria, there is a key difference between the analysis regarding this more general case with two-way network externalities and the case of unidirectional network externalities addressed in the previous sections. In the "asymmetric" two-sided market studied in the previous sections, a deviation by the incumbent platform on market side 1 would never induce consumers on market side 2 to also buy from the incumbent platform (as they were assumed not to care about cross-group network externalities). This being the case, it suffices for the entrant to ensure that its price for consumers on market side 1 is such that the incumbent platform cannot offer more surplus than the entrant to side 1 consumers by charging its best price (c_I). If, however, one considers the existence of two-way network externalities, then there are two types of deviations by the incumbent platform that should be considered. In particular, if the price set by the incumbent platform on market side i, $i = 1, 2$, is sufficiently low to induce every side-i consumer to strictly prefer to join the incumbent platform, then consumers on market side j, where $j \neq i$, can be easily induced to join the incumbent platform as well since they would not enjoy any indirect network externality from joining the entrant's platform instead. As a result, as the next proposition shows, an entry

[16] Clearly, if z_2 is sufficiently close to zero, then $p_2^I = z_2 v(\beta^I + N)$ will be smaller than c_I. However, in order for this below-cost pricing to be possible, the fixed benefit that side-2 consumers obtain from using the incumbent's platform, r_2, must be sufficiently high, $r_2 > z_2 \left[v(N) - v(\beta^I)\right]$.

equilibrium will only exist if it is immune to deviations of the type just described, i.e., if the prices charged by the entrant's platform to consumers *on each side of the market* are so low that the incumbent platform cannot profitably set a sufficiently low price on one side of the market while extracting more surplus from consumers on the other side of the market.

Proposition 7 ***(Entry equilibria under asymmetric pricing and two-way network externalities)*** *If both platforms can set different (non-negative) prices across the two sides of the market and $z_1 > z_2 > 0$, then there exists a $c_E^*(c_I, \beta_I, N)$ such that entry equilibria (i.e. equilibria in which all "new" buyers buy from the entrant) exist if and only if $c_E \leq c_E^*(c_I, \beta_I, N)$. (See proof for the equilibrium prices.)*

Proof. See Appendix. ■

Hence, the fact that the entrant's platform must simultaneously offer to "new" buyers on both sides of the market at least as much as the incumbent platform's best offer implies that, for an entry equilibrium to exist, the efficiency gap between the entrant and the incumbent should be sufficiently large, similarly to what happened in the baseline model.

Now, the reason why entry equilibria might not exist (for some parameter values) is that the incumbent platform may attract consumers through very attractive offers on one market side, very possibly using below cost prices, thus depriving the rival platform (the entrant) of its customers. If, however, below cost price was forbidden, then entry equilibria would continue to arise as in the uniform pricing benchmark.

7 Conclusions

We have presented in this paper a very simple model which captures some of the key features of some recent antitrust cases. We have showed that under two-sided markets, an incumbent firm may resort to below-cost pricing on one side of the market in order to exclude a (more efficient) rival from the industry.

The model we have used is admittedly very streamlined, but the qualitative conclusions could also arise in more general settings. In particular, we show that relaxing the assumption that the externality flows from one side to the other only (but not vice versa: one side is indifferent as to the number of buyers on the other side) does not remove the possibility that exclusion takes place. However, in a simultaneous move game exclusion would take place only because of miscoordination: the model is one where indirect network effects exist, and demand-side scale economies imply that unless a sufficient number of customers buy on one side of the market, consumers on the other side would not buy from the new (and more efficient) entrant either. Therefore, buyer's miscoordination may lead to exclusion, in the same spirit as in Rasmusen et al. (1991).[17]

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8 Appendix

This Appendix contains the proofs of the Lemmata and Propositions stated in the text.

Proof. (Lemma 1) Depending on the values of p^I and p^E, there are multiple equilibria which can arise at this stage.

Consider first a situation where *all buyers, on both sides, buy from the incumbent.* This is an equilibrium if and only if (i) a consumer on side i extracts more surplus when buying from I rather than by unilaterally deviating and buying from E (i.e., $CS_i^I\left(p^I,\beta^I+N\right) > CS_i^E\left(p^E,0\right)$), and (ii) a consumer on side i obtains a higher surplus when buying from I than if she does not buy (i.e., $CS_i^I\left(p^I,\beta^I+N\right) > 0$). It is easy to check that $p^I < p^E$ and $p^I < z_1 v(\beta^I + N)$ make sure that buyers from side 2 and side 1 respectively buy from the incumbent.

There can also exist an equilibrium where *all buyers buy from the entrant.* This occurs if $CS_i^E\left(p^E,N\right) > CS_i^I\left(p^I,\beta^I\right)$ and $CS_i^E\left(p^E,N\right) > 0$, i.e., if $p^E < \min\left\{p^I, z_1 v(N)\right\}$. Indeed, $p^E < p^I$ implies that side-2 buyers will buy from the entrant; for side-1 buyers to buy from the entrant as well, it must be $z_1 v(N) - p^E > z_1 v(\beta^I) - p^I$, which holds good since $p^E < p^I$, and $z_1 v(N) - p^E > 0$.

Finally,[18] we show that there is no equilibrium where *side*-2 *"new" buyers buy from E whereas side*-1 *"new" buyers buy from I*. In order for this equilibrium to occur, it must be that $p^E < p^I$ so that side-2 buyers prefer to buy from E. For side-1 buyers to buy from the incumbent when all new side-2 buyers buy from E, it must be $p^I < p^E - z_1\left[v(N) - v(\beta^I)\right]$, but this contradicts $p^I > p^E$ since we have assumed that $N > \beta^I$ (the market is growing).

Of course, there exist no equilibria where side-1 buyers buy from E and side-2 buyers buy from I: if no side-2 buyers buys from E, there is no positive price p^E at which side-1 buyers would want to buy from E. ■

Proof. (Proposition 1) First of all, note from Lemma 1 that "new" buyers have no incentive to deviate, as at the equilibrium $p^E < p^I$. Let us now turn to the firms. Firm I has no incentive to decrease its price, as it would have to sell at a loss. Moreover, it has no incentive to increase its price either, because when $p^I > p^E$ all side-2 buyers will continue to buy from E, which rules out miscoordination equilibria. As for firm E, at the candidate equilibrium its profits are given by $\pi^E = (c_I - c_E)(2N) > 0$. A price decrease would not increase its demand but only reduce its unit margins. A price increase would make it lose all its customers to firm I, as for $p^E > p^I = c_I$ all

[18] Obviously, there may also be situations where either on one side or on both sides buyers do not purchase from either firm. However, we do not write them down since they will never occur in equilibria of the whole game.

buyers would buy from the incumbent (see Lemma 1). ■

Proof. **(Proposition 2)** We know from Lemma 1 that in order for an equilibrium to exist at the buyers' stage where all "new" buyers buy from I, it must be that $p^I < p^E$. Moreover, this equilibrium will be profitable for firm I if and only if $p^I \geq c_I$. Now, at such a candidate equilibrium, firm E makes zero profits (all consumers would be served by the incumbent platform). So, even if firm I sets its lowest possible price $p^I = c_I$, firm E can deviate by setting a price a shade below c_I. By so doing, it will be able to attract all "new" consumers on both sides of the market and to make a deviation profit of $(c_I - c_E)(2N) > 0$. This is for two reasons. First, all side-2 buyers will buy from the entrant if $p^E < p^I = c_I$. Second, anticipating this, "new" buyers on side 1 will also prefer to buy from firm E if $z_1 v(N) - c_I > z_1 v(\beta^I) - c_I$ which is verified given that we have assumed that the externality function $v(\cdot)$ is increasing and that the market is growing ($N > \beta^I$). Hence, an exclusionary candidate equilibrium cannot exist under uniform pricing. ■

Proof. **(Proposition 3)** First of all, consider buyers' decisions. On side 2 there is standard Bertrand competition, and consumers will buy at the lowest price. On side 1, consumers will buy from E if: (i) $p_2^E < p_2^I$ (else, no side-2 buyer is buying from E); and (ii) $z_1 v(N) - p_1^E > z_1 v(\beta^I) - p_1^I$. They will buy from I if: (i) $p_2^I < p_2^E$ and (ii) $z_1 v(N + \beta^I) - p_1^I \geq 0$.[19] Let us now turn to the firms' decisions.

Consider a candidate equilibrium where E sets prices (p_1^E, p_2^E) and all "new" consumers buy from E. Consider platform I's possible deviations. First, note that I will never set a price below c_I on market side 1 because, by so doing, it would not attract any additional consumers on market side 2 (their utility is not affected by market side 1 number of users). Therefore, the maximum surplus that platform I can offer to side-1 consumers is $CS_1^I(c_I, \beta^I) = z_1 v(\beta^I) - c_I$ whereas, at the candidate equilibrium prices, $CS_1^E(p_1^E, N) = z_1 v(N) - p_1^E$. Thus, at any entry equilibrium where all "new" consumers buy from the entrant, it must be that $p_1^E < c_I + z_1 [v(N) - v(\beta^I)] \equiv \widetilde{p}_1^E$.

Second, the incumbent platform may, however, set a price $p_2^I < p_2^E$ which *attracts all side-*2 *"new" buyers*, thereby also inducing all side-1 consumers to buy from I, whatever the (non-negative) price p_1^E, as long as $p_1^I \leq z_1 v(\beta^I + N) \equiv \overline{p}_1^I$.

Therefore, the optimal incumbent's deviation will consist of the pair of prices $(\overline{p}_1^I, p_2^E - \varepsilon)$, where ε is positive and arbitrarily small. However, this deviation will be profitable only if $\pi^I(\overline{p}_1^I, p_2^E - \varepsilon) =$

[19] It is also possible that side-2 buyers will buy from E and side-1 buyers buy from I. This would occur if $p_2^E < p_2^I$ and $p_1^E > p_1^I + z_1 [v(N) - v(\beta^I)]$, or that there are equilibria where one or both sides do not buy because firms are charging too high prices, but of course these situations will never emerge as equibria of the whole game.

$N\left(\overline{p}_1^I + p_2^E - 2c_I\right) > 0$. Therefore, in order for the candidate equilibrium to be immune from a deviation by the incumbent platform attracting all side-1 consumers, it must be that $p_2^E \leq 2c_I - z_1 v(\beta^I + N)$. Define $\widetilde{p}_2^E \equiv \max\left\{0, 2c_I - z_1 v(\beta^I + N)\right\}$, where Assumption 1 ensures that $\widetilde{p}_2^E < r_2$. By setting $(\widetilde{p}_1^E, \widetilde{p}_2^E)$ firm E will therefore get all "new" buyers from each side. However, this is possible only insofar as $\pi_E(\widetilde{p}_1^E, \widetilde{p}_2^E) \geq 0$, which corresponds to:

1. If $c_I < z_1 v(\beta^I + N)/2$, then $\widetilde{p}_2^E = 0$, and $\pi^E(\widetilde{p}_1^E, \widetilde{p}_2^E) \geq 0$ becomes:

$$c_E \leq \frac{c_I + z_1\left(v(N) - v(\beta^I)\right)}{2}. \tag{9}$$

2. If instead $c_I \geq z_1 v(\beta^I + N)/2$, then $\widetilde{p}_2^E = 2c_I - z_1 v(\beta^I + N)$ and $\pi^E(\widetilde{p}_1^E, \widetilde{p}_2^E) \geq 0$ amounts to:

$$c_E \leq \frac{3c_I + z_1\left[v(N) - v(\beta^I) - v\left(\beta^I + N\right)\right]}{2}. \tag{10}$$

This completes the proof. ■

Proof. (Proposition 4) We have already analysed buyers' equilibria in the beginning of the proof of the previous proposition. Let us turn to firms' decisions and look for deviations which may disrupt an equilibrium where the incumbent sets prices (p_1^I, p_2^I) and all buyers buying from I. First of all, note that there is no room for side-1 deviations only. Indeed, if it does not get any buyer on side 2, the entrant will not be able to attract any buyer on side 1 either, no matter how low p_1^E is. This implies that the incumbent is free to set $\widetilde{p}_1^I = z_1 v(\beta^I + N)$ as long as it gets also side 2 buyers. (Recall that this was not true in case of entry equilibria: since the incumbent has already a customer base β^I, firm E had to reduce its price below $z_1 v(N)$ to be immune from side 1 deviations from the incumbent.)

Consider first the candidate equilibrium where $p_1^I = z_1 v(\beta^I + N)$, $p_2^I = 0$ and all N "new" buyers (on either side) buy from the incumbent platform. At this candidate equilibrium, $CS_1^I\left(p_1^I, \beta^I + N\right) = 0$ whereas $CS_2^I\left(p_2^I, \beta^I + N\right) = r_2$. This candidate equilibrium cannot be disrupted by the entrant (whatever the prices $p_1^E \geq 0$, $p_2^E \geq 0$ are). This is because $CS_1^E(p_1^E, 0) = -p_1^E \leq 0$ and $CS_2^E(p_2^E, 0) = r_2 - p_1^E \leq r_2$. Hence, consumers on either market side have no incentives to deviate (buying from the entrant would not improve their payoff). Now, in order for this equilibrium to exist, it must be that $\pi^I(p_1^I, p_2^I) = N\left(z_1 v(\beta^I + N) - 2c_I\right) > 0$, which in turn implies that $c_I < z_1 v(\beta^I + N)/2$.

Let us now investigate the case wherein at the candidate equilibrium $p_2^I > 0$ and all "new" buyers on both market sides buy from the incumbent platform. In this case, the entrant might deviate by setting $p_2^E = p_2^I - \varepsilon$ and get all N side-2 "new" buyers. In this case, it will get as well all N "new" side-1 buyers as long as $p_1^E \leq \min\left\{p_1^I + z_1\left[v(N) - v(\beta^I)\right], z_1 v(N)\right\}$. Since $p_1^I = z_1 v(\beta^I + N)$, and because of $v(\cdot)$ being an increasing function, this condition amounts to saying that the highest side-1 price that firm E could obtain in a side-2 deviation would be: $\overline{p}_1^E = z_1 v(N)$. Such a side-2 deviation would therefore be profitable if and only if: $\pi^E = N\left(p_2^I + z_1 v(N) - 2c_E\right) \geq 0$. In other words, to prevent a profitable deviation by the entrant, it must be $p_2^I < 2c_E - z_1 v(N)$. (Let us define $\widetilde{p}_2^I \equiv 2c_E - z_1 v(N) - \varepsilon$.) This will lead to two different cases, as follows.

1. If $c_E \leq z_1 v(N)/2$, then an exclusionary equilibrium where $p_2^I > 0$ will never exist. (Recall that we are assuming that prices are non-negative)

2. If $c_E > z_1 v(N)/2$, the exclusionary equilibrium where $p_2^I > 0$ will exist as long as

$$\pi^I(\widetilde{p}_1^I, \widetilde{p}_2^I) = N\left(z_1 v(\beta^I + N) + 2c_E - z_1 v(N) - 2c_I\right) \geq 0 \tag{11}$$

   or, equivalently,

$$c_E \geq c_I - \frac{z_1\left[v(\beta^I + N) - v(N)\right]}{2}. \tag{12}$$

   Therefore, this exclusionary equilibrium will exist iff $c_E > \max\left\{c_I - \frac{z_1\left[v(\beta^I+N) - v(N)\right]}{2}, \frac{z_1 v(N)}{2}\right\}$. This condition can also be rewritten as: (a) if $c_I < z_1 v(\beta^I + N)/2$, an exclusionary equilibrium with $p_2^I > 0$ exists if $c_E > z_1 v(N)/2$; (b) if $c_I \geq z_1 v(\beta^I + N)/2$, an exclusionary equilibrium with $p_2^I > 0$ exists if $c_E > c_I - \frac{z_1\left[v(\beta^I+N) - v(N)\right]}{2}$. At this equilibrium, all consumers will buy from firm I at prices $\left(z_1 v(\beta^I + N), 2c_E - z_1 v(N)\right)$. In concluding, note that $\widetilde{p}_2^I \equiv 2c_E - z_1 v(N) < c_I$. This is because $c_E < (c_I + z_1 v(N))/2$ is always verified for $c_E < c_I$.

This completes the proof. ■

Proof. (Proposition 5) (1) At the candidate equilibrium buyers have no incentive to deviate since buying from the incumbent would not improve their payoff. Notice that, on the one hand,

$$CS_1^E\left(c_I + z_1\left[v(N) - v(\beta^I)\right], N\right) = CS_1^I\left(c_I, \beta^I\right) = z_1 v(\beta^I) - c_I > 0 \tag{13}$$

and $CS_2^E\left(c_I, N\right) = CS_2^I\left(c_I, \beta^I\right) = r_2 - c_I > 0$ (see Assumption 1), on the other.[20] The incumbent cannot decrease its price by the policy assumption; increasing its price would not win any order on either side. The entrant could decrease its price because $c_E < c_I$ but it has no incentive to do so because it would decrease profits. If it increased prices on either side it would lose customers to the incumbent. (2) Suppose there is a candidate exclusionary equilibrium at which $p_1^I \geq c_I$ and $p_2^I \geq c_I$ and both groups of new buyers buy from I. The entrant could slightly undercut the incumbent on side 2, and set $p_1^E = c_I + z_1\left[v(N) - v(\beta^I)\right]$, thereby making positive profits on both sides. Therefore, no such equilibrium would exist. ■

Proof. (Lemma 2) To sell to all "new" buyers, the monopolist platform needs to make sure that side-2 consumers buy. Hence, it will set $p_2^M = r_2$, extracting all possible surplus from side-2 consumers. Now, anticipating this, side-1 consumers will also buy from the incumbent monopolist if and only if $CS_1^I\left(p_1^I, \beta^I + N\right) = z_1 v\left(\beta^I + N\right) - p_1^I \geq 0$. Thus, the highest price p_1^I that the incumbent monopolist can charge to side-1 consumers is the one that extracts all ensuing externality benefits on side-1 consumers, $p_1^I = p_1^M = z_1 v\left(\beta^I + N\right)$, which leaves side-1 consumers (as well) indifferent between buying or not. The corresponding monopoly profit is then $\pi^M = N\left(z_1 v\left(\beta^I + N\right) + r_2 - 2c_I\right)$ which is always positive (see Assumption 1). ■

Proof. (Proposition 6) At the candidate equilibrium, $p_i^I = z_i v(\beta^I + N)$, for $i = 1, 2$, and all "new" consumers on each market side join the incumbent's platform. This being the case, $CS_1^I\left(p_1^I, \beta^I + N\right) = 0$ whereas $CS_2^I\left(p_2^I, \beta^I + N\right) = r_2$.

Clearly, this candidate equilibrium cannot be disrupted by the entrant whatever the prices $p_1^E \geq 0$ and $p_2^E \geq 0$ are. This is because $CS_1^E\left(p_1^E, 0\right) = -p_1^E \leq 0$ and $CS_1^E\left(p_2^E, 0\right) = r_2 - p_2^E \leq r_2$. Hence, consumers on either market side would not be able to improve their payoff by deviating and buying from the entrant's platform.

Now, in order for this equilibrium to exist, it must be that $\pi^I\left(p_1^I, p_2^I\right) = N\left((z_1 + z_2)\, v(\beta^I + N) - 2c_I\right) \geq 0$, which in turn implies that $c_I < (z_1 + z_2)\, v(\beta^I + N)/2$. ■

Proof. (Proposition 7) For an equilibrium where all "new" consumers buy from E to exist we need to find a pair of prices (p_1^E, p_2^E) such that neither firms nor buyers have an incentive to deviate.

There exist two possible deviations by platform I which we discuss in turn. First, platform I

[20] To be more precise, the entrant will set prices a shade below $c_I + z_1\left[v(N) - v(\beta^I)\right]$ and c_I to market sides 1 and 2, respectively.

may set a price p_1^I which *attracts all side* 1 *"new" buyers* even when only the "old" β^I buyers use platform I on side 2, that is a price at which $z_1 v(\beta^I) - p_1^I > z_1 v(N) - p_1^E$, or:[21]

$$p_1^I < p_1^E - z_1 \left[v(N) - v(\beta^I) \right] \equiv \underline{p}_1^I, \tag{14}$$

Now, a deviation which induces all side 1 buyers to buy from I, will induce all side 2 consumers to (also) buy from I as long as $r_2 + z_2 v(\beta^I + N) - p_2^I > r_2 + z_2 v(0) - p_2^E$, or:

$$p_2^I \leq z_2 v(\beta^I + N) + \min \left\{ p_2^E, r_2 \right\} \equiv \overline{p}_2^I. \tag{15}$$

Optimally, this first possible deviation will consist of the pair $(\underline{p}_1^I, \overline{p}_2^I)$. However, this deviation would will only take place insofar as: (1) $\underline{p}_1^I \geq 0$, i.e., $p_1^E \geq z_1 \left[v(N) - v(\beta^I) \right]$ (recall here we consider only non-negative prices); and (2) $\pi^I(\underline{p}_1^I, \overline{p}_2^I) = N \left(\underline{p}_1^I + \overline{p}_2^I - 2c_I \right) > 0$. Therefore, in order for a price pair (p_1^E, p_2^E) to be immune from both a deviation attracting all side 1 consumers, it must be that either condition (1) or condition (2) above are not satisfied, i.e.,

$$\text{either: } p_1^E \leq z_1 \left[v(\beta^I) - v(N) \right],$$

$$\text{or: } p_1^E \leq z_1 \left[v(N) - v(\beta^I) \right] - z_2 v(\beta^I + N) - \min \left\{ p_2^E, r_2 \right\} + 2c_I \equiv \widehat{p}_1^E \left(p_2^E \right)$$

So, in order to avoid a deviation by the incumbent platform attracting all side 1 "new" buyers, the entrant's pricing strategy should be such that:

$$p_1^E = \max \left\{ \widehat{p}_1^E \left(p_2^E \right), z_1 \left[v(N) - v(\beta^I) \right] \right\} \equiv \widetilde{p}_1^E, \tag{16}$$

where $\widetilde{p}_1^E = \widehat{p}_1^E \left(p_2^E \right)$ if:

$$c_I \geq \frac{z_2 v(\beta^I + N) + \min \left\{ p_2^E, r_2 \right\}}{2}. \tag{17}$$

Consider now the following alternative deviation by the incumbent platform. Platform I may set a price p_2^I which *attracts all side* 2 *"new" buyers* even when only the "old" β^I buyers use platform

[21] In the baseline model, since $z_2 = 0$, a deviation by the incumbent platform attracting all "new" buyers on side 1 would never induce side 2 "new" buyers to also buy from the incumbent platform (side 2 buyers do not care about cross group network externalities). This being the case, by setting a price $p_1^E = c_I + z_1 \left[v(N) - v(\beta^I) \right]$, the entrant platform ensures that the incumbent will never be able to make a better offer to consumers on market side 1 (condition (14) is violated for all $p_1^I \geq c_I$).

I on side 1, that is a price at which $r_2 + z_2 v(\beta^I) - p_2^I > r_2 + z_2 v(N) - p_2^E$, or:

$$p_2^I < p_2^E - z_2 \left[v(N) - v(\beta^I) \right] \equiv \underline{p}_2^I, \tag{18}$$

Now, a deviation which induces all side 2 buyers to buy from I, will induce all side 1 consumers to (also) buy from I, whatever the price p_1^E is, as long as $z_1 v(\beta^I + N) - p_1^I \geq 0$ (recall here that $r_1 = 0$), or:

$$p_1^I \leq z_1 v(\beta^I + N) \equiv \overline{p}_1^I. \tag{19}$$

Optimally, this second possible deviation will consist of the pair $(\overline{p}_1^I, \underline{p}_2^I)$. However, this deviation would will only take place insofar as: (i) $\underline{p}_2^I \geq 0$, i.e., $p_2^E \geq z_2 \left[v(N) - v(\beta^I) \right]$; and (ii) $\pi^I(\overline{p}_1^I, \underline{p}_2^I) = N \left(\overline{p}_1^I + \underline{p}_2^I - 2c_I \right) > 0$. Therefore, in order for a price pair (p_1^E, p_2^E) to be immune from both a deviation attracting all side 1 consumers, it must be that either condition (i) or condition (ii) above are not satisfied, i.e.

$$\text{either: } p_2^E \leq z_2 \left[v(N) - v(\beta^I) \right],$$

$$\text{or: } p_2^E \leq z_2 \left[v(N) - v(\beta^I) \right] - z_1 v(\beta^I + N) + 2c_I \equiv \widehat{p}_2^E$$

Therefore, in order to avoid a deviation by the incumbent platform attracting all side 2 "new" buyers, the entrant's pricing strategy should be such that:

$$p_2^E = \max \left\{ \widehat{p}_2^E, z_2 \left[v(N) - v(\beta^I) \right] \right\} \equiv \widetilde{p}_2^E, \tag{20}$$

where $\widetilde{p}_2^E = \widehat{p}_2^E$ if:[22]

$$c_I \geq \frac{z_1 v(\beta^I + N)}{2}. \tag{21}$$

The problem for firm E will therefore be:

[22] In the baseline model (i.e. when $z_2 = 0$), condition (20) boils down to $\widetilde{p}_2^E = \max \left\{ 2c_I - z_1 v(\beta^I + N) +, 0 \right\}$, implying that $\widetilde{p}_2^E = 2c_I - z_1 v(\beta^I + N)$ if $c_I > z_1 v(\beta^I + N)/2$ (see condition (21) and the proof of Proposition 3).

$$\max_{p_1^E, p_2^E} \pi^E(p_1^E, p_2^E) = N(p_1^E + p_2^E - 2c_E),$$

$$s.to: \; p_1^E \geq 0 \text{ and } p_2^E \geq 0.$$

Four different combinations of prices should be studied, which we discuss in turn.

1. If $c_I \geq \left(z_1 v(\beta^I + N)\right)/2$, then $\widetilde{p}_2^E = \widehat{p}_2^E$ (see eqs. (20) and (21)) and $\widehat{p}_2^E > 0$ in this region of parameter values. Now, two subcases should be considered.

   (a) If $c_I \in \left[z_1 v(\beta^I + N)/2, \left[r_2 + z_1 v(\beta^I + N) - z_2\left(v(N) - v(\beta^I)\right)\right]/2\right]$,[23] then we know that $\widetilde{p}_2^E = \min\left\{\widehat{p}_2^E, r_2\right\} = \widehat{p}_2^E$. Now, with regards to $\widetilde{p}_1^E$, Assumption 2 ensures that, in this region of parameter values, condition (17) holds, implying that

   $$\widetilde{p}_1^E = \widehat{p}_1^E\left(\widehat{p}_2^E\right) = (z_1 - z_2)\left[v(N) + v(\beta^I + N) - v(\beta^I)\right]$$

   and, therefore, in order for an entry equilibrium to exist, one must have that $\pi^E(\widehat{p}_1^E\left(\widehat{p}_2^E\right), \widehat{p}_2^E) \geq 0$, i.e.,

   $$c_E \leq c_I + \frac{z_1\left(v(N) - v(\beta^I)\right) - z_2 v(\beta^I + N)}{2}.$$

   (b) If instead $c_I \geq \left[r_2 + z_1 v(\beta^I + N) - z_2\left(v(N) - v(\beta^I)\right)\right]/2$, then we have that $\widetilde{p}_2^E = \widehat{p}_2^E$ and $\min\left\{\widehat{p}_2^E, r_2\right\} = r_2$ (see eqs. (20) and (21)). As for $\widetilde{p}_1^E$, note that, in this region of the parameters space, $c_I > \left[z_2 v(\beta^I + N) + r_2\right]/2$ (eq. (17)) and, hence,[24]

   $$\widetilde{p}_1^E = \widehat{p}_1^E\left(\widehat{p}_2^E\right) = z_1\left[v(N) - v(\beta^I)\right] - z_2 v(\beta^I + N) - r_2 + 2c_I.$$

   Thus, in order for an entry equilibrium to exist, one must have that

[23] Assumption 2 (part (i)) ensures that this interval is non-empty.

[24] Note that $\widetilde{p}_1^E \geq 0$ if $c_I \geq \left[r_2 + z_2 v(\beta^I + N) - z_1\left(v(N) - v(\beta^I)\right)\right]/2$, which is always true in this region of parameter values (recall that $z_1 > z_2 > 0$).

$\pi^E(\widehat{p}_1^E\left(\widehat{p}_2^E\right), r_2) \geq 0$, or equivalently:

$$c_E \leq c_I + \frac{z_1\left[v(N) - v(\beta^I)\right] - z_2 v(\beta^I + N)}{2}.$$

2. If $c_I \in \left[0, \left(z_1 v(\beta^I + N)\right)/2\right]$, then $\widetilde{p}_2^E = z_2\left[v(N) - v(\beta^I)\right]$ (see eqs. (20) and (21)). Moreover, Assumption 2 guarantees that $\min\left\{\widetilde{p}_2^E, r_2\right\} = \widetilde{p}_2^E$. Now two subcases should be studied:

   (a) If $c_I \in \left[z_2\left(v(\beta^I + N) + v(N) - v(\beta^I)\right)/2, \left(z_1 v(\beta^I + N)\right)/2\right]$,[25] then condition (17) holds, implying that:

   $$\widetilde{p}_1^E = \widehat{p}_1^E\left(z_2\left[v(N) - v(\beta^I)\right]\right) =$$

   $$= (z_1 - z_2)\left[v(N) - v(\beta^I)\right] - z_2 v(\beta^I + N) + 2c_I,$$

   which is always non-negative in the region of parameter values under consideration.[26] Therefore, in order for an entry equilibrium to exist, one must have that $\pi^E(\widehat{p}_1^E\left(z_2\left[v(\beta^I) - v(N)\right]\right), z_2\left[v(\beta^I) - v(N)\right]) \geq 0$, i.e.,

   $$c_E \leq c_I + \frac{z_1\left[v(N) - v(\beta^I)\right] - z_2 v(\beta^I + N)}{2}.$$

   (b) If instead $c_I \in \left[0, z_2\left(v(\beta^I + N) + v(N) - v(\beta^I)\right)/2\right]$, then condition (17) does not hold, implying that:

   $$\widetilde{p}_1^E = z_1\left[v(N) - v(\beta^I)\right].$$

   Thus, in order for an entry equilibrium to exist, one must have that $\pi^E(z_1\left[v(N) - v(\beta^I)\right], z_2\left[v(N) - v(\beta^I)\right]) \geq 0$, i.e.

   $$c_E \leq \frac{(z_1 + z_2)\left[v(N) - v(\beta^I)\right]}{2}.$$

This completes the proof. ∎

[25] Assumption 2 ensures that this interval is non-empty.

[26] It is straightforward to show that in order for $\widetilde{p}_1^E \geq 0$, one must have that the following condition holds: $c_I \geq \left[z_2 v(\beta^I + N) - (z_1 - z_2)\left(v(N) - v(\beta^I)\right)\right]/2$.