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PROFESSIONAL SERVICE OUTSOURCING, ASYMMETRIC INFORMATION AND WAGE INEQUALITY

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ABSTRACT

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The economy is experiencing a large shift towards professional services. Markets for these services are characterized by large information asymmetries: the difficulty in providing the necessary advice, the quality of the advice, and whether a problem is solved may all be unobservable. Our analysis considers these markets in a general equilibrium setting, which allows us to address the selection of talent into occupations and their efficiency and distributional implications. We first show that reductions in communications costs allow these markets to appear and increase wage inequality, as they favor the most skilled agents. However, under asymmetric information these markets are unable to exclude the least talented from posing as experts. If contingent contracts cannot be written, the market collapses and no services are bought or sold. If output contingent contracts are feasible, market exchanges weakly involve excessive trade. Despite the asymmetric information efficiency can be attainable when experts can solve many problems. Even when the allocation is efficient, the asymmetry of information has distributional consequences. It benefits moderately skilled agents at the expense of the least talented and most talented ones.

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Professional service outsourcing, asymmetric information and wage inequality*

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Abstract

The economy is experiencing a large shift towards professional services. Markets for these services are characterized by large information asymmetries: the difficulty in providing the necessary advice, the quality of the advice, and whether a problem is solved may all be unobservable. Our analysis considers these markets in a general equilibrium setting, which allows us to address the selection of talent into occupations and their efficiency and distributional implications. We first show that reductions in communications costs allow these markets to appear and increase wage inequality, as they favor the most skilled agents. However, under asymmetric information these markets are unable to exclude the least talented from posing as experts. If contingent contracts cannot be written, the market collapses and no services are bought or sold. If output contingent contracts are feasible, market exchanges weakly involve excessive trade. Despite the asymmetric information efficiency can be attainable when experts can solve many problems. Even when the allocation is efficient, the asymmetry of information has distributional consequences. It benefits moderately skilled agents at the expense of the least talented and most talented ones.

I Introduction

Over the last fifty years, the share of the service sector in the GDP of Western economies has grown substantially, e.g. in the USA by about 20 points. Approximately half of this increase is the result of the increase in the share of professional and business services such as computer services, consulting and legal services (Herrendorf, Rogerson and Valentinyi, 2009). An important aspect of this change, as Fuchs (1965) originally argued, is the increase in professional service outsourcing, i.e., the separation of the manufacturing activities undertaken by firms from some of the specialized

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knowledge needed to produce and sell goods.[1] When production and knowledge that originally occurred in the same location and under the same management are separated, the distribution of available information changes, and informational asymmetries may impede efficiency. In this paper, we study the efficiency and distributional consequences for the aggregate economy of the separation between problem solving and production that is characteristic of professional service outsourcing.

To conduct our analysis, we study a model in which there may exist, endogenously, two types of agents: production agents, who spend their time producing and who know how to deal only with routine problems; and specialized problem solvers or consultants, who assist production workers when they encounter unusual problems. An agent who encounters a problem while engaged in production may pay another agent to use his know-how to help him solve the particular problem. We argue that such trades are plagued by informational asymmetries. First, the difficulty of the problem is frequently difficult to assess. Second, the skill of the agent offering his services in the market (a 'consultant') is also unobservable. Finally, the output of the consultant (whether the problem is solved or not) is itself often uncertain or at least ambiguous in nature. As a result, these markets have tended to be replaced by 'knowledge-hierarchies' inside the firm that permit for the exchange of know-how within the firm (Garicano, 2000). However, as we indicated above, these exchanges have increasingly been outsourced and take place in the market. In this case, informational asymmetries make it difficult to match problems with the right consultants, as consultants may pretend to know more than they do, and those in need of advice may represent their problems as being simpler than they are.

Consider first trade and earnings when information asymmetries are absent. The first best in these markets, which we analyze in Section II, has the less knowledgeable agents specializing in production and more knowledgeable ones becoming specialized consultants or problem solvers. Furthermore, the first best involves positive sorting: the best consultants tackle the problems that are expected to be more difficult, which are those that the most skilled production agents could not solve by themselves. Intermediately skilled[2] agents do not enter the market, and thus neither buy nor sell services. They are not knowledgeable enough to offer to solve others' problems, but if they elect to purchase services, they pose questions that are, in expectation, difficult to solve, such that it is not worth the consultants' time to assist them. The existence of these agents 'in the middle' (too knowledgeable to ask questions but not intelligent enough to answer them), as we shall see, is a key obstacle for the development of a problem-solving market and is one of the features that distinguishes this type of market from a generic asymmetric information setting.

The impact of team production on agents' earnings is non-monotonic: it benefits the most skilled

[1] Fuchs (1965, p. 3) argued against demand shifts as the reason for the transformation and contended that 'growth in intermediate demand for services by goods producing industries' would account for 10% of the shift. Herrendorf, Rogerson and Valentinyi (2009), argue that outsourcing is unlikely to explain the entire trend, as business services only account for half of the increase in the expenditure share of services.

[2] We use skill and knowledge interchangeably here.

agents, who can leverage their know-how to solve a number of problems, and the least skilled agents, who on their own could extract very little value from the opportunities they originate. The intermediate agents, conversely, are not much, if at all, better off than in autarky. As buyers of services, they request advice on problems that are difficult to solve and thus require very talented (and expensive) consultants; when acting as consultants, intermediate agents are unable to solve a substantial share of the problems for which their assistance was requested. Thus reductions in communication costs benefit, under perfect information, the low- and high-skill agents at the expense of the 'middle class'. Contractual problems appear because, generally, agents' knowledge and hence the (expected) difficulty of the problems is unobservable.

As a first step, we show in Section III that if informational asymmetries are one-sided, the first best can still be attained, and earnings are as above, although the market structure is somewhat different. It is well known in bilateral trading relationships that if the party with private information can be made the residual claimant (and if agents are risk neutral) an efficient outcome can be attained. A similar logic extends to a two-sided market and allows us to predict the type of market institutions we will observe, depending on the distribution of information. Markets must be set such that prices are based on the observable type of actor: a referral market is needed when the producer's type is observable and a consulting market when the consultant's type is observable. Thus, equilibrium prices induce the side of the market with private information to self-select into the efficient match.

Efficiency is more difficult to attain when asymmetric information is double-sided, that is, neither the knowledge of the agents nor the expected difficulty of the problems they encounter in production can be observed. In this case, the market is characterized by double-sided adverse selection. In particular, those seeking assistance have incentives to pretend that their problems are simpler than they in fact are, in an effort to pay less for advice. However, consultants have an incentive to pretend that they are more intelligent than they are, in an effort to earn a higher fee. In other words, consultants want to play smart, whereas producers want to play dumb. Moreover, and further complicating the problem, whether a problem is actually solved is often unverifiable. Consider, for example, a firm that needs advice on its future strategy: how can the quality of the advice provided be evaluated? Not surprisingly, as we show in Section IV, a matching market where the quality of neither the sellers nor the buyers can be observed is inefficient. In fact, if output is unverifiable, the market completely breaks down because of the possibility of global deviations. Without contingent payments, the worst agents in the economy have strong incentives to pretend to be experts. Advice becomes worthless as the 'pretenders' cannot be identified or persuaded to stay out of the market via incentives. A matching market where neither the quality of sellers nor that of the buyers can be observed is inefficient.[3]

[3] A certification mechanism able to establish minimum standards for access to an advisory role provides a partial solution in this environment, which we studied in Fuchs and Garicano (2010). We also provided a characterization of the optimal certification level. In particular, we showed that entry regulation involves fewer experts than the first best, as those seeking advice are matched with the average advisor in the market, which makes advice less valuable than under optimal matching.

As we show in Section IV, if it is possible to verify whether the advice provided solved the problem, conditional payments (a fee paid only if the problem posed is solved) are possible, and contingent contracts can be used. We characterize the separating equilibrium in this case and compare it to the first best. Each producer pays or receives a different fixed fee and a contingent payment (or share in the venture) when the problem is solved. This, we show, disciplines both production agents and problem solvers. The first key result under double-sided informational asymmetries is that no agent can be excluded from the market; hence, even mediocre problem solvers and producers with extremely difficult problems wish to trade. As a result, market efficiency decreases to the extent that if communication is sufficiently expensive, the market completely breaks down and a (separating) equilibrium with trade is impossible to attain. When communication costs decrease, such that each consultant can work on a larger number of problems, an equilibrium with excessive trade exists. Finally, when communication costs are low enough, efficiency requires that all problems be solved; because all problems are solved in the market, the first best is attained. Thus, when communication costs are low enough, the first best allocation can be attained even in the presence of two sided-adverse selection.

We find that (as in, e.g., Leland and Pyle, 1978) output contingent payments increase with the difficulty of problems to the point that the most knowledgeable consultants (who tackle the most difficult problems) actually become full residual claimants to the output. More surprisingly, the fixed fee paid by producers is non-monotonic. Initially, it increases and then decreases to the point at which it becomes negative, i.e., consultants begin buying shares of the venture. The non-monotonicity arises because of the asymmetric implications with respect to matching that a local deviation produces for consultants relative to producers in different segments of the market. At the low end of the market, as there is high demand for consultants of low quality, pretending to be a slightly more knowledgeable producer leads to being matched with a much better consultant. However, for the consultants that are matched with these agents, a local deviation only leads to a slightly different match. The producers would then have a stronger incentive to exaggerate their quality, and thus the fixed fee they have to pay must increase to dissuade such behavior. However, high-producing actors do not demand much consultant time because they overcome most difficulties on their own. Thus, a local deviation will only lead to a slightly better match for them. In contrast, for the top consultants, a local deviation will produce a problem that is much simpler to solve. Thus, the consultants now have the strongest incentives to deviate. Therefore, the fixed fee must decrease to prevent these deviations. The non-monotonicity of the payments leads to two producers paying the same fixed fee but giving up different shares; separation is ensured because the matching is different, i.e., the producer that gives up a higher share is compensated by being matched with a more skilled consultant.

Even when the allocation coincides with the one attained under full information the lack of information has distributional consequences: it favors agents in the middle of the distribution.

To prevent them from either pretending to be better experts than they are or to have simpler problems than they do, they must be allowed to capture some rents. As a consequence, agents at the extremes of the distribution (who trade with them) become worse off. These effects are augmented when excessive entry of consultants occurs: the best problem solvers now also receive more difficult problems and the low producers are in turn matched with worse problem solvers. Thus, asymmetric information favors moderately skilled agents for two reasons. First, it allows them to enter the market as consultants, which would be impossible if their mediocrity were apparent. Second, it increases the rents they receive to ensure separation and second best matching.

No previous study has, to our knowledge, examined the double-sided adverse selection issue raised by matching consultants to problems under asymmetric information. The previous literature on consultant services emphasized moral hazard issues involved in the provision of consultant services, i.e., consultants have little incentive to supply the appropriate level of effort. Demski and Sappington (1987) examine the trade-off between productive effort and information gathering incentives faced by the consultant. In Wolinsky (1993), the issue is the incentives that must be provided for consultants to recommend the correct treatment, i.e., minor treatment for minor problems and major treatment for major problems. Wolinsky shows that specialization is optimal in this case. Similarly, Pesendorfer and Wolinsky (2003) study the provision of adequate diagnosis effort by consultants. Taylor (1995) studies how insurance can solve informational asymmetries in a context where only the consultant can determine the necessary treatment. Garicano and Santos (2004) study the matching of opportunities and consultants in a context with moral hazard and one-sided adverse selection. We depart from this literature in that we alone study the distributional consequences across the distribution and we focus on the double-sided nature of the informational asymmetries: the agent does not know the quality of the consultant, and the consultant does not know, a priori, the difficulty of the problem posed.

Our paper also fits within the literature that studies trade in markets with bilateral asymmetric information. Most of the literature stems from the original analysis by Myerson and Satherwaite (1983) of trade mechanisms under asymmetric information which addresses multiple buyers and sellers of a commodity with unknown valuation (e.g., Lu and Robert, 2001). In this literature, buyers and sellers do not care about each other's quality per se, as they only care about the value of the object at stake. Thus, matching is irrelevant. The only paper we are aware of that studies equilibria in matching markets with two-sided incomplete information is Gale (2001). There are several important differences between our models. First, Gale takes as exogenous the side of the market in which agents are, while in our model, agents select ex-into buyers or sellers of advice. This endogenous choice makes the analysis somewhat more difficult, but the indifference conditions of the cutoff types help us pin down the equilibrium. Additionally, in Gale's paper, all agents have equal outside option, while in our setting higher quality agents have a higher outside option, which increases the adverse selection problems because those agents exit the market first if it is unattractive. Finally, because our problem has more structure, we are able to go substantially further in characterizing the market equilibrium.

The paper is also related to the management-worker sorting literature and in particular Garicano and Rossi-Hansberg (2004 and 2006). As in those papers, the economy-wide problem studied is one of matching talent with problems. However, here we study market (rather than firm/hierarchy) problems, which naturally involve informational asymmetries absent from those papers. Formally, the setting (with heterogeneous agents whose knowledge is exogenous) is similar to that in the 2004 (short) paper, but that paper did not obtain and characterize the first best, nor did it set up a market, which we do here. The worker/manager sorting models have been studied and generalized by Eeckhout and Kircher (2011), who study the general conditions under which interactions in production between worker and managerial skills generate skill-scale effects and positive sorting effects.

II The Model and Full Information Benchmark: advice, matching and earnings

The economy is formed by a continuum of income maximizing agents who are indexed by their level of know-how $z \in [0,1]$. Without loss of generality, we choose the index z so that z is measured in percentiles of the know-how distribution – the distribution of z is thus uniform. Agents must first decide if they become producers or specialized problem solvers ('consultants'). We let P denote the set of agents that become producers and S the set of agents that become (problem) solvers. If an agent is a producer, then at the beginning of the period he draws a problem, with an associated difficulty level $q \in [0,1]$, where q is unobserved and i.i.d. across problems and distributed according to $F(q)$, a continuous function with density $f(q)$. If $z \geq q$, the agent can solve the problem by himself, produces a unit of output, and obtains a payoff of 1. If $z < q$ then he cannot solve the problem. However, he can choose to seek a consultant who can potentially solve the problem for a fee. Because not all producers choose to seek advice on unsolved problems, two subsets of P exist: A the set of agents that do seek advice or help and I the set of agents that remain independent.

The other option is for the agent to become a consultant. Consultants use their unit of time to help producers with their unsolved problems. It takes them $h < 1$ to help each particular producer: since they do not have to produce, it only costs a fraction h of time to help one other agent solve his problem. Consultants do not generate any problems of their own; that is, they specialize in solving problems for others. Like producers, consultants with know-how z can solve problems q as long as they are not 'too difficult' i.e. $z \geq q$. We will assume for simplicity that if neither the original agent nor the hired consultant (if one is hired) can solve the problem then the problem goes unsolved.

To represent the (potentially random) matching between producers and solvers, we use the CDF $M(s,z)$. That is, $M(s,z)$ determines the probability that agent $z \in A$ is matched with $s \in S$. We use

$\mu(s,z)$ to denote its density. In some cases the matching will not have any random component and $M(s,z)$ will hence be degenerate. In these cases we simply use the matching function $m(z): A \to S$

Information Structure.

Our objective is to characterize the optimal contracts and the division of labor that arise in this economy, which will depend critically on what is observable and what is verifiable/contractible. We begin by assuming that perfect information and contractibility: all agents types are observable and contingent contracts can be written. We then consider briefly the cases in which only one side of the market is observable. We then focus our study on the case in which all types are unobservable; for this last case we consider both verifiable and non-verifiable output.

Properties of the First Best Allocation. We begin by studying the first best. Suppose that a social planner can optimally sort agents into those who seek advice (A), those who produce but do not seek or provide advice (I for independent), and consultants or solvers (S), suppose he can also choose which type of consultants help which agents with their problems.

The planner's problem can then be written as:

$$\max_{A,I,M(s,z)} \int_{z\in A} \left(F(z) + (1-F(z)) \Pr\left(q < s | q > z, M(s,z)\right)\right) dz + \int_{z \in I} F(z)\, dz$$

With full information, the only constraint that the planner faces is the resource constraint that the demand for advice not be larger than the supply. Formally, for any subset $D \in S$:

$$\underbrace{\int_{z\in D} (1-F(z))\, dz}_{Demand} \leq \underbrace{\int_{z\in D} \frac{\int_{s\in S} \mu(s,z)}{h} dz}_{Supply} \tag{1}$$

Intuitively, it seems clear that, more skilled producers must ask questions of more skilled consultants, and that consultants should be those more skilled at problem solving. We show below that this is indeed the case. (Proofs are in the appendix)

Lemma 1 (Assortative Matching) *Let s be a solver or consultant who is solving problems posed by producer z and s' a solver who is solving problems posed by producer z'. If $z > z'$ optimality requires that $s > s'$.*

To illustrate why the planner would select assortative matching consider a producer of the highest quality. If he were unable to solve the problem, the problem would be expected to be difficult. In contrast, an unsolved problem from the lowest producer is expected to be fairly simple. Assigning the most knowledgeable consultant to the simple problem is inefficient. A less able consultant could most likely solve problem, and the most knowledgeable consultant's time would be better spent solving those problems that are expected to be difficulty and hence that less able consultants are unlikely to solve.

To show that some matching always takes place in equilibrium, consider a situation where all workers are unmatched, the least skilled agent $z = 0$ produces $F[0] = 0$ and the most skilled agent produces $F[1] = 1$. Now consider the value of the match between the best and worst workers. This value is $\frac{F[1]-F(0)}{h(1-F(0))} = \frac{1}{h} > 1$ as long as $h < 1$, and thus this match is welfare increasing.

Assortative matching combined with the fact that it is never optimal to have an under-utilized consultant implies that with full information $M(s,z)$ is degenerate and hence we can focus on characterizing the matching function $m(z)$. Before doing so, it is useful to establish the following two lemmas:

Lemma 2 (Independents are Smart) *Suppose there are producers with ability z that do not seek advice. Then, a producer $z' > z$ that seeks advice when he cannot solve a problem cannot exist. If $z \in I$ then if $z' > z$ $z' \notin O$.*

Intuitively, if some problems are going to be passed on, they must be (in expectation) simple, i.e. – those that have the highest likelihood of being solved by a problem solver. If a problem is too difficult to be passed on and is dropped, all the problems originated by more knowledgeable agents are even more difficult and hence are not worth passing on to the consultants.

Lemma 3 (Experts are Smarter) *Agents who become consultants are more knowledgeable than those who become producers. If $z \in S$ then for all $z' \in P$, $z > z'$.*

It is easy to see that if an agent is a consultant, someone more knowledgeable than him should not be a producer. If this was not the case, the roles could be reversed and output increased: the producer solves the problems that were formerly solved by the consultant, but with a higher probability. The gain is larger than the loss, as each consultant solves multiple problems.

The ordering implied by the two lemmas above combined with the fact that $M(s,z)$ is degenerate allows us to write the resource constraint on advising time (eq. 1) as follows:

$$\text{for all } z: \int_0^z (1 - F(q))\, dq = \int_{m(0)}^{m(z)} \frac{1}{h} dt$$

Equivalently, the integral equation above can be written as follows:

$$m(z) = m(0) + h \int_0^z (1 - F(q))\, dq \quad \forall z \tag{2}$$

and therefore:

$$m'(z) = h(1 - F(z))$$

We summarize our results in the following proposition:

Proposition 1 *The first best allocation can be characterized by a matching function* $m(z)$ *and 2 cutoff types* z_1 *and* z_2 *where* $z_1 \leq z_2$. *Types* $z \in [0, z_1] = A$ *are producers who, if necessary, seek advice, types* $z \in (z_1, z_2) = I$ *produce but never seek advice and* $z \in [z_2, 1] = S$ *are solvers or consultants. The matching function satisfies:* $m(0) = z_2$, $m(z_1) = 1$ *and* $m'(z) = h(1 - F(z))$.

This follows from the lemmas above and the need to guarantee the correct ratio of consultants to producers so that demand for consultants and their available time are equal across different quality levels.

$$0 \underbrace{------}_{A} z_1 \underbrace{---------}_{I} z_2 \underbrace{---------}_{S} 1$$

The first best allocation. Given the characterization provided in the proposition above, the only parameters that remain to be determined are the optimal cutoff types z_1 and z_2. In fact, the boundary conditions $m(z_1) = 1$ and $m(0) = z_2$ imply that:

$$\int_0^{z_1} (1 - F(z))\, dz = \frac{1 - z_2}{h}$$

Hence, the planner can only choose z_1. This in turn determines the cutoff type

$$z_2 \equiv Z(z_1) = 1 - h \int_0^{z_1} (1 - F(z))\, dz$$

and the matching function, which we now explicitly index by z_1 :

$$m(z; z_1) = 1 - h \int_z^{z_1} (1 - F(q))\, dq$$

Given the previous results, solving for the first best allocation is reduced to solving the following:

$$\max_{z_1} \int_0^{z_1} F(m(z; z_1)) dz + \int_{z_1}^{Z(z_1)} F(z)\, dz$$

Taking the derivative with respect to z_1 and grouping the terms to facilitate the interpretation, the FOC can be written as:

$$\begin{aligned} & -\left(\underbrace{\int_0^{z_1} f(m(z; z_1)) \frac{\partial m(z; z_1)}{\partial z_1} dz}_{\textit{loss from worse matches}} + \underbrace{F(Z(z_1)) \frac{\partial Z(z_1)}{\partial z_1}}_{\textit{loss from less originators}} \right) \\ = \; & \underbrace{F(m(z_1; z_1)) - F(z_1)}_{\textit{Extra output}} \end{aligned} \qquad (3)$$

The condition can be readily interpreted. As z_1 increases, the marginal gain (the second line of expression 3) comes from additional output being produced as more agents are able to seek help. They now produce with probability $F(m(z_1))$ rather than working on their own and producing with probability $F(z_1)$. There are two sources of losses. First, as more agents seek advice, the quality of advice each agent receives is reduced. The reason for this is that with positive sorting, as more agents become consultants, the worst consultant (the one who advises the worst producer), is of lower quality, and so on for all producers. The final loss is the output loss from those producers who were self-employed and now, rather than generating productive opportunities on their own, provide advice to other agents.

Proposition 2 *The number of independent agents is increasing in h. If $h > h'$, then $I' \sqsubseteq I$.*

Figure 1 illustrates the solution for the uniform case for each value of the parameter h. For a given level of h, we can trace a horizontal line at that level that divides the distribution of agents by skill into three (or two) categories: those originators who seek advice, those who do not and consultants. The middle category stops existing at $h = 0.75$, as for sufficiently low helping cost all originators seek advice.[4]

Thus the share of agents that participate in the consulting markets increases as communication costs decline. When communication costs are high, agents operate in autarky, i.e., if they confront a problem, they try to solve it on their own, and if they cannot, they drop it. As communication costs decrease, the share of agents who seek help with their problems increases monotonically. Similarly, the share of agents who produce on their own and do not seek advice (independents) decreases monotonically.

However, as can be seen from the figure above, the share of consultants is non-monotonic in the number of problems that a consultant can address. When consultants cannot leverage their expertise with many problems (h close to 1), it is not worth seeking advice and most agents are independents. As h decreases the share of consultants grows. For the uniform case, for $h \leq 0.75$ it is efficient to not have any independents, i.e. all agents who are engaged in production seek advice if they confront a problem that they cannot solve. From this point, if we continue to increase the number of problems that a consultant can address (decreasing communication costs h) the number of consultants begins to decline, simply because fewer consultants are needed to address all the unsolved problems.

[4] In the uniform case:

$$\begin{aligned} m(z; z_1) &= 1 - h\left(z_1 - z - \frac{z_1^2 - z^2}{2}\right) \\ Z(z_1) &= 1 - h\left(z_1 - \frac{z_1^2}{2}\right) \end{aligned}$$

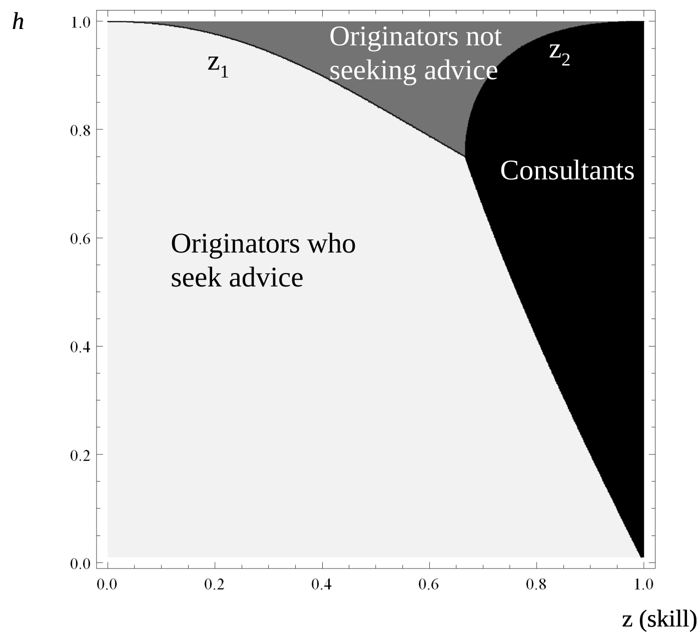

Figure 1: First best allocation of agents to occupations as a function of communication costs h

III Competitive Equilibrium and One-Sided Informational Asymmetries

With perfect information, the first best can be attained in a decentralized way as a competitive equilibrium. There are many different decentralizations that can implement the first best, and they are all equivalent in the allocation that they support, which is unique. In particular, we will specifically consider two decentralizations that are readily interpretable and will be useful subsequently. They differ in the agent who obtains the residual income from solving the problem. As a result, they address asymmetric information differently.

Letting the consultant of an agent z be $m(z)$, the joint output that a matched pair produces is given by the following expression:

$$F(z) + (1 - F(z))\frac{F(m(z)) - F(z)}{(1 - F(z))} = F(m(z))$$

That is, with probability $F(z)$ the producer produces on his own, and with probability $(1-F(z))$ he needs help. The (ex post) output of the match (conditional on advice being needed) is given by $\frac{F(m(z))-F(z)}{(1-F(z))}$ per worker or, as a problem solver may have $1/h$ producers, $y = \frac{F(m(z))-F(z)}{h(1-F(z))}$. For an arbitrary solver type z_s the output function displays increasing differences $\partial^2 y(z, z_s)/\partial z \partial z_s > 0$, hence the competitive equilibrium must be characterized by positive sorting, $m'(z) > 0$. The competitive equilibrium must result in occupational choices for all agents between originating or solving problems, in an earnings stream for producers and solvers, and in an allocation of producers to solvers (a matching function). The two decentralizations differ in who claims the residual income from the problem potentially being solved. We define them next.

Definition 1 *In a **consulting market** producers pay a fixed price for advice $w(z)$ and claim the residual income from the problem being solved.*

The earnings of producers z who hire solvers z_s are:

$$W_p^c(z, z_s) = F(z) + (1 - F(z)) \left(\frac{F(z_s) - F(z)}{(1 - F(z))} - w(z_s) \right),$$

while earnings of producers or consultants of skill z are:

$$W_s^c(z) = \frac{w(z)}{h}.$$

Definition 2 *A **referral market** has consultants claiming the residual income from the problem solved; they pay a fixed price $r(z)$ in exchange for the problem.*

The earnings of producers z are then

$$W_p^r(z) = F(z) + (1 - F(z))r(z);$$

and the earnings of consultants who buy problems from producers of skill z_p are

$$W_s^r(z, z_p) = \frac{1}{h} \left(\frac{F(z) - F(z_p))}{1 - F(z_p))} - r(z_p) \right).$$

We now characterize the equilibrium in each of these markets. We will show that the allocations and earnings are identical, and identical to the first best.

A Consulting Services Market

In a consulting services market producers hire consultants of skill z_s for a fixed fee $w(z_s)$. Producers are the residual claimants to output. The earnings of consultants do not depend on who they match with; their earnings are simply determined by the equilibrium consulting fee (they make no choices): $W(z) = \frac{w(z)}{h}$; however, producers earns the residual, so they cares directly about the choice of partners:

$$\begin{aligned} W_p^c(z; z_s) &= \max_{z_s} F(z) + (1 - F(z)) \left(\frac{F(z_s) - F(z)}{(1 - F(z))} - w(z_s) \right) \\ &= \max_{z_s} F(z_s) - (1 - F(z))\, w(z_s). \end{aligned}$$

With the first order condition for the optimal choice of consultant:

$$f(z_s) - (1 - F(z))w^{'}(z_s) = 0. \quad (4)$$

Before characterizing the competitive equilibrium, note that, because $w'(z_s)$ must be increasing in equilibrium, $\partial^2 W_p^c(z; z_s)/\partial z \partial z_s > 0$, and the matching function $z_s = m(z)$ must be increasing, $m'(z) > 0$. The competitive equilibrium in this case can be characterized as follows:

Definition 3 *A competitive equilibrium in a consulting service market consist of:*
(i) a fee schedule $w(z)$ paid for by problem producers to consultants,
(ii) a matching function $m(z) : A- > S$ allocating consultants to producers;
(iii) a pair of cutoffs $\{z_1, z_2\}$, such that $A = [0, z_1]$ is the set of producers who seek help when they necessary, $I = [z_1, z_2]$ is the set of producers who never seek advice (independent) and $S = [z_2, 1]$ is the set of problem solvers;
such that:
(1) Supply equals demand point by point;
(2) The matching is such that no producer can do better by choosing a different consultant;
(3) No agent can be made better off by an occupational (from producer to consultant) change or by deciding to seek or forgo advice.

To construct the equilibrium, start from the supply and demand conditions. Supply equaling demand pointwise implies: $m'(z) = (1 - F(z))h$. With $m(0) = z_2$, we can write the matching function as $m(z; z_2)$. Then for a given z_2, the matching function evaluated at the highest producer is:

$$m(z_1; z_2) = 1$$

which implies that the matching is entirely pinned down up to one constant z_1. Trivially, $m(z_1; z_2) = 1$ implies a function $z_2^{sd}(z_1)$ with $z_2^{sd'} < 0$ (intuitively, if the supply of problems requiring advice increases, i.e. z_1 goes up, more problem solvers are required, i.e., z_2 must decrease).

Notice also that the first order condition (4) must hold for all z. Thus given some matching $m(z, z_2)$), the first order condition determines a wage function for each z_2.

$$w^{'}(z; z_2) = \frac{f(z)}{(1 - F(m^{-1}(m(z; z_2)))} \quad (5)$$

This differential equation can be solved simply by integration, as there is no $w(.)$ on the right hand side, and generates a wage function $w(z, z_2)$. To solve for the constant of integration, use $\frac{1}{h}w(z_2; z_2) = F(z_2)$. Finally, optimal occupational choices also requires that the top producer in A be indifferent between seeking help or not: $W_p^c(z_1; 1) = 1 - (1 - F(z_1))\, w(1; z_2) = F(z_1)$, which implies $w(1; z_2) = 1$. This allows us to solve for z_2. The following proposition summarizes this analysis (see the Appendix for a detailed proof).

Proposition 3 *The competitive equilibrium in a consulting services market, is unique and achieves the first best.*

We show that the competitive equilibrium achieves the first best in the Appendix. Note that the argument above does not require that we observe the ability of the producers. The consultants do not make any choices, so they do not need to observe anything. Thus suppose that the producers' skills are unobservable, but the skills of consultants are not. This could be the case, for example, if consultants have developed a reputation that allows agents to know who is knowledgeable and who is not, whereas problem producers and their quality are unknown. In this case, the consulting market we have just described would work exactly in the way that we suggested. We state this in the following corollary.

Corollary 1 *Under one sided asymmetric information, where consultant skill can be observed but producer skill cannot, the consulting services market still attains the first best.*

A Referral Market

In a referral market producers transfer the entire residual ownership of the problem to problem solvers, in exchange for a fixed referral price $r(z)$. The earnings of producers, for a given price per problem, are given, i.e., producers now do not need to choose anything:

$$W_p^r(z) = F(z) + (1 - F(z))r(z) \tag{6}$$

Whereas problem solvers earnings are a function of whom they choose to buy problems from:

$$W_s^r(z; z_p) = \max_{z_p} \frac{1}{h} \left(\frac{F(z) - F(z_p)}{1 - F(z_p)} - r(z_p) \right) \tag{7}$$

The optimal choice of z_p by a solver with skill z requires:

$$-\frac{f(z_p)(1 - F(z))}{(1 - F(z_p))^2} = r'(z_p) \tag{8}$$

Again, note that as in the first best, and in the consulting services market, the competitive equilibrium must be characterized by assortative matching because $\frac{\partial^2 w(z_s)}{\partial z_s \partial z_p} > 0$. The competitive equilibrium is defined analogously to the consulting case:

Definition 4 *A competitive equilibrium in problem referrals consists of:*
(i) a price schedule $r(z)$ paid by consultants in exchange for an unsolved opportunity from type z,
(ii) a matching function $m(z) : A- > S$ allocating opportunities to consultants;
(iii) a pair of cutoffs $\{z_1, z_2\}$, such that $A = [0, z_1]$ is the set of producers who sell their unsolved opportunities, $I = [z_1, z_2]$ is the set of producers who do not sell their opportunities and $S = [z_2, 1]$ is the set of problem solvers;

Such that:
(1) Supply equals demand point by point;
(2) No consultant can do better by choosing to buy problems from a different producer;
(3) No agent can be made better off by an occupational change or by deciding to seek or forgo buying/selling unsolved opportunities.

The first part of the equilibrium construction, using the condition that supply equals demand, leads to the same function $m(z; z_2)$ and the supply and demand condition result in a downward sloping marginal manager function $z_2^{sd}(z_1)$. Substituting again in for the first order condition, we have:

$$-\frac{f(z)(1-F(m(z;z_2))}{(1-F(z))^2} = r'(z), \tag{9}$$

which we can integrate for each z_2 to obtain a function $r(z; z_2)$ and a constant. Again we can solve for the constant by using $W_s^r(z_2) = \frac{1}{h}\left(F(z_2) - r(z; z_2)\right) = F(z_2)$ so that $r(z, z_2) = F(z_2)(1-h)$. Finally, as above, we obtain a second condition for z_1 and z_2 by using the top producer's indifference condition between seeking help or not: $W_p^c(z_1; 1) = F(z_1) + (1 - F(z_1))\, r(z_1) = F(z_1)$ thus $r(z_1; z_2) = 0$. This generates a condition $z_2^r(z_1)$ with $z_2^{r'}(z_1) > 0$, as we show in the appendix. Moreover, occupational choice is optimal. The following proposition summarizes this analysis (see the Appendix for a detailed proof).

Proposition 4 *The competitive equilibrium allocation in the referrals market is unique and achieves the first best. Moreover, the equilibrium allocation and earnings in the referral market are the same as in the consulting market.*

Analogously to the consulting market, the equilibrium in this market does not require observing the skill of problem solvers or consultants. This means that a referral market can achieve the first best when the consultant's skill is unobservable. For example, suppose all agents can see the skill of agents less skilled than themselves. Then, one-sided asymmetric information follows, and establishing the informed side, i.e., the problem solvers , as the buying side, results in the first best.

Corollary 2 *Under one sided asymmetric information, where only the skill of producers can be observed (for example, all agents can observe the skill of those less skilled than themselves) the referrals market still attains the first best.*

Thus straightforward institutional arrangements can achieve efficiency if the informational problems are only one sided. In general, in bilateral relationships, letting the party with private information be the residual claimant allows to achieve efficiency. We have shown that a similar logic extends to this two sided market. As long as the market is set up so that prices are based on the

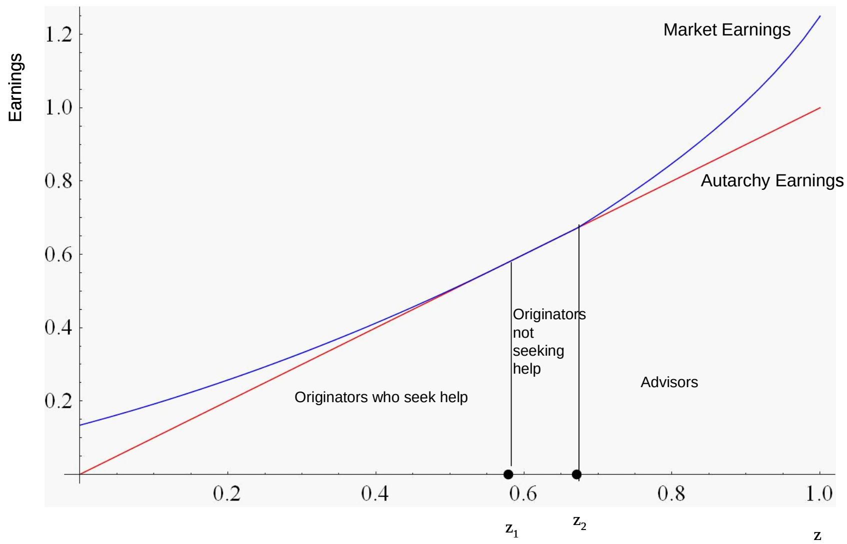

Figure 2: Market Allocation (the curved line) versus Autarchy (the straight, 45^oline)

observable type, i.e., a referral market when the producer's type is observable or a fee based market for advice when the consultant's type is observable, equilibrium prices will induce the side of the market with private information to self-select the efficient match.

Who gains most from the market for advice?

Inspection of figure 2 above provides important intuition regarding the value of skills in this market. The agents who gain the most from being able to provide and seek advice are those at the extremes: the best problem solvers and the worst producers. Thus, a better producer sells goods of lower quality, in the sense that he is seeking advice on more difficult problems. Thus 'quality' is here decreasing with regard to skill on the advice-seeking side of the market. At the margin, producer z_1 is indifferent with regard to seeking advice; to him, advice has no value, i.e., his earnings are the same with or without the advice. On the other side of the market, quality increases with skill, i.e., a better agent can provide better advice and thus benefits most from the market for skills.

IV Two-Sided Asymmetric Information

We now turn to the case in which the agents' quality is private information. This becomes a trading problem with two-sided adverse selection. Consultants may want to pretend they are more knowledgeable than they truly are and producers may want to pretend that their unresolved problems are simpler than they really are.

We first analyze the case in which output is unverifiable and ownership is non transferable. The only type of market that could be established in this case is one with uncontingent wages in exchange for consultant services. This market breaks down because at wages high enough to motivate individuals of high quality to become consultants, individuals of too low quality also want to enter the consultant sector.

Then we consider the case in which output is verifiable or an ownership stake can be sold, and hence fully contingent contracts can be written. In this case, for low values of h, the planner can achieve the first best allocation. This is possible when h is such that the first best demands that all producers seek advice on their unsolved problems. Keeping producers from bringing their problems to the market is not possible and leads to excessive trade relative to the first best when the first best would instead have some producers not seeking help with their unsolved problems.

A Unverifiable output and non-transferable ownership

When output contingent contracts cannot be written and ownership cannot be transferred, the producers are full residual claimants and consultants can only be paid an uncontingent fee. Because their payoff is uncontingent all consultants must receive the same payment. Under these circumstances, the market breaks down completely. No trade can take place, as the lowest-skilled agents in the economy can pretend to be more knowledgeable and become sellers of consulting services. Any fixed fee that is high enough to entice a highly skilled agent to become a consultant will induce the least-skilled agents to misrepresent their knowledge and offer their "services" for this fee.

Proposition 5 *When output contingent contracts cannot be written and ownership cannot be transferred there cannot be a competitive equilibrium with trade in consultant services.*

The intuition for this result is that the expected earnings from becoming a producer depend on the agent's type but the expected earnings of becoming a consultant (or pretending to be one) are independent of type. Therefore, if an individual of a particular type prefers to become a consultant all types of lesser quality will wish to follow the same path.

Note that in contrast to the classic lemons problem as in Akerlof (1970) the main reason for the market to break-down is not unravelling due to high types exiting, but rather from the excessive entry of individuals of low quality. To illustrate, consider the market for brain surgeons. That top brain surgeons are not differentially compensated from good brain surgeons is a second order consideration. The first order problem arises from the average Joe donning a white robe and offering to crack your head open. Therefore, to make the market operate, we must find a way to prevent the individuals of low quality from becoming false experts. In Fuchs and Garicano (2010) we show how a certification process achieves this and partially restores efficiency. As we show in the next section, if output is contractible full efficiency can be attained (for low values of h) without resorting to certification.

B Verifiable Output: Contingent Contracts

Suppose now that it is possible to contract on output. Agents can be paid on the condition that the solution is found. Without loss of generality, contracts can be characterized by two parameters w, α. w being the uncontingent payment to the problem solver and α the additional payment to the problem solver if he succeeds in solving the problem. Note that $w < 0$ would correspond to the consultant paying for giving advice, i.e., purchasing the problem in exchange for an output share.

We characterize next the incentive feasible allocations in which each type of producer offers a different contract and each type of solver works for a different producer, that is, the matching function is strictly monotonic. Let $\omega_z = \{w(z), \alpha(z)\}$ denote the contract offered by type z and $m(z)$ denote the solver type that attempts to solve a problem originated by type z.

Definition 5 *A set of contracts ω_z, a matching function $m(z)$ and a pair of cutoffs for occupational choice $\{z_1; z_2\} \in [0,1]^2$ are a separating incentive feasible allocation if: (i) the demand for help equals the supply of consultant services; (ii) only those agents $z > z_2$ choose to become consultants; (iii) only agents with $z < z_1$ choose to seek help with their unsolved problems; (iv) both types of agents truthfully reveal their types.*

We first show that the equilibrium must exhibit positive assortative matching.

Lemma 4 *Any separating incentive feasible allocation must exhibit positive assortative matching, $m'(z) > 0$.*

Because $m(z)$ is strictly increasing market clearing type by type essentially pins down $m'(z)$. The only degree of freedom left comes from z_1, the share of agents who become producers. In principle, there will be two possibilities, depending on whether there are independents: either types $z \in [0, z_1]$ will be producers, $[z_1, z_2]$ independents and types $z \in [z_2, 1]$ will be solvers; or $z_1 = z_2 = z^*$, where then $z \in [0, z^*]$ are producers and $z \in [z^*, 1]$ are solvers, i.e., no agents are independent. In the next Lemma we show that there does is no separating, incentive feasible allocation where some agents never seek advice, and hence we focus on the case where all producers seek advice.

Lemma 5 *In any separating equilibrium all producers must seek advice in equilibrium.*

Proof. For there to be an equilibrium where some agents do not seek advice there must exist a z_1 and $z_2 > z_1$ such that type z_2 is ex-ante indifferent between being a consultant or a producer who leaves his problem unsolved. If he remains a producer but does not seek advice, he earns $F(z_2)$. If he becomes a consultant, he is paired with the worst worker type (because he is the marginal consultant), and he earns $\frac{1}{h}\left(w(0) + \alpha(0)\frac{F(z_2)-F(0)}{1-F(0)}\right)$, where the last term is the conditional probability that he can solve a problem that worker 0 could not solve. Indifference requires:

$$F(z_2) = \frac{1}{h}\left(w(0) + \alpha(0) F(z_2)\right)$$

or

$$F(z_2)\left(1-\frac{\alpha(0)}{h}\right)=\frac{w(0)}{h}. \tag{10}$$

Furthermore, for an interval of types that prefer to be producers to exist, there must exist a type z_1 that must strictly prefer to be a producer over obtaining the contract offered to the worst consultant:

$$F(z_1)\geq\frac{1}{h}\left(w(0)+\alpha(0)F(z_1)\right)$$

or

$$F(z_1)\left(1-\frac{\alpha(0)}{h}\right)\geq\frac{w(0)}{h} \tag{11}$$

Now, note that $w_0 \geq 0$ is a necessary condition in an equilibrium where some problems remain unsolved. Thus the two conditions above only can hold if $z_1 \geq z_2$ which cannot be the case. ■

Intuitively, a separating incentive feasible allocation with some problems unsolved must keep some agents out of the market. This restricts the uncontingent payments to be positive $w \geq 0$ (because otherwise anyone not seeking advice could receive a positive payment by entering the market place) which excessively limits the space of available contracts. In particular, if a contract consisting of a share and a fixed payment is enough to leave an agent z_2 indifferent, the fixed payment must be high enough that it would be strictly preferred by any agent $z_1 < z_2$. The contract that is sufficient to keep the worst problem solver in the market makes it too attractive for the best producer to remain out of the market - he also wishes to be a problem solver.

Given the two lemmas above, we can conclude that there can be at most one separating equilibrium allocation.

Proposition 6 *There is at most one separating equilibrium allocation.*

The qualifier "at most" is used in the proposition above because for high values of h there is indeed no separating equilibrium. Before demonstrating this, we first characterize the equilibrium.

Producers and Problem Solvers Problem.

Let $V(z,\tilde{z})$ denote the value (ex-post) of a producer of type z who failed to solve his problem and is pretending to be type $\tilde{z}$:

$$\begin{aligned} V(z,\tilde{z}) &\equiv -w_{\tilde{z}}+(1-\alpha_{\tilde{z}})\Pr\left(q<m(\tilde{z};z_1)\,|q>z\right) \\ &\equiv -w_{\tilde{z}}+(1-\alpha_{\tilde{z}})\frac{F(m(\tilde{z};z_1))-F(z)}{(1-F(z))}. \end{aligned}$$

Hence, we can define the ex-ante expected value of becoming a producer and pretending to be type $\tilde{z}$ if trading with a consultant:

$$R(z,\tilde{z})\equiv F(z)+(1-F(z))\max\left\{V(z,\tilde{z}),0\right\}$$

Let $S(z,\tilde{z})$ denote the value of a problem solver of type z who pretends to be $\tilde{z}$ and thus buys problems from type $m^{-1}(\tilde{z})$:

$$\begin{aligned} S\left(z,\tilde{z}\right) &\equiv \frac{1}{h}\left(w_{m^{-1}(\tilde{z};z_1)} + \alpha_{m^{-1}(\tilde{z})}\Pr\left(q<z|q>m^{-1}\left(\tilde{z};z_1\right)\right)\right) \\ &\equiv \frac{1}{h}\left(w_{m^{-1}(\tilde{z};z_1)} + \alpha_{m^{-1}(\tilde{z})}\frac{F(z)-F(m^{-1}(\tilde{z};z_1))}{(1-F(m^{-1}(\tilde{z};z_1)))}\right) \end{aligned}$$

Equilibrium Contracts and Matching

Given that $z_1 = z_2 = z^*$ the conditions for equilibrium imply:

(i) **Supply and Demand.** Given that there is assortative matching, the matching function is as it is in the first best, i.e., it is given by (2) up to the parameter z^*,that is

$$m'\left(z\right) = h\left(1-F\left(z\right)\right) \tag{12}$$

with

$$m(0) = z^*; \quad m\left(z^*\right) = 1$$

(ii) **Occupational Choice.** Each type must voluntarily choose the occupation assigned to them in equilibrium. [5]

For consultants $(z > z^*)$ not to prefer to originate their own problems,

$$S\left(z,z\right) \geq R\left(z,\tilde{z}\right) \quad \textit{for all } \tilde{z} < z^*;$$

and for producers $(z < z^*)$ not to pretend to be consultants:

$$S\left(z,\tilde{z}\right) \leq R\left(z,z\right), \quad \textit{for all } \tilde{z} > z^*.$$

With equality for the boundary type:

$$S(z^*,z^*) = R\left(z^*,z^*\right)$$

(iii) **Ex post advice seeking.** Third, asking for help must be ex post optimal when prescribed by the equilibrium, which requires that those with $z < z^*$ must strictly prefer to seek advice, that is, $V(z,z) > 0$, for all $z < z^*$.

(iv) **Truth telling.** For producers and solvers to be willing to report their type truthfully the

[5] Since we require thruthtelling to be optimal conditional on choosing the right occupation we assume without loss that agents would be truthful if they choose the right occupation.

following two conditions must hold:

$$\begin{aligned} V(z,z) &= \max_{\tilde{z}} V(z,\tilde{z}) \\ S(z,z) &= \max_{\tilde{z}} S(z,\tilde{z}) \end{aligned}$$

Equilibrium Construction To construct the equilibrium, we will first construct the marginal conditions for truth telling. We begin by considering the problem of the producer $z \in [0, z^*]$ who failed to solve his own problem. For him to report his type truthfully we require:

$$V(z,z) = \max_{\tilde{z}} V(z,\tilde{z}),$$

that is, $\forall z \in [0, z^*]$:

$$\left(\frac{\partial V(z,\tilde{z})}{\partial \tilde{z}} | \tilde{z} = z \right) = 0$$

In equilibrium ($\tilde{z} = z$) and since supply demand equality (12) mean that the slope of the matching function be given by $m'(z) = h(1 - F(z))$ we have:[6]

$$-w'(z) - \alpha'(z) \frac{F(m(z)) - F(z)}{(1 - F(z))} + (1 - \alpha(z)) f(m(z)) h = 0 \tag{13}$$

Where for a problem solver to report his type:

$$S(z,z) = \max_{\tilde{z}} S(z,\tilde{z})$$

that is, $\forall z \in [z_2, 1]$:

$$\left(\frac{\partial S(z,\tilde{z})}{\partial \tilde{z}} | \tilde{z} = z \right) = 0$$

thus, in equilibrium ($\tilde{z} = z$) in terms of producer skill, rather than consultant skill we have:

$$w'(z) + \alpha'(z) \frac{F(m(z)) - F(z)}{1 - F(z)} + \alpha(z) \frac{-f(z)(1 - F(m(z))}{(1 - F(z))^2} = 0 \tag{14}$$

By adding equations (14) and (13) we can solve for $\alpha(z)$:

$$\alpha(z) = \frac{h(1 - F(z))^2 f(m(z))}{h(1 - F(z))^2 f(m(z)) + f(z)(1 - F(m(z))}$$

This is a closed form solution for the $\alpha(z)$ function, i.e., everything is known and we can show:

Proposition 7 *(a)* $\alpha(z^*) = 1$

[6] The second order condition is:
$-w_{\tilde{z}}^{''} - \alpha_{\tilde{z}}^{''} \frac{m(\tilde{z}) - z}{1-z} - 2\alpha_{\tilde{z}}' \frac{m'(\tilde{z})}{1-z}$
$+ (1 - \alpha_{\tilde{z}}) \frac{m^{''}(\tilde{z})}{(1-z)} \leq 0.$

(b) $\alpha(0) = \frac{hf(z^*)}{hf(z^*)+f(0)(1-F(z^*))} > 0$
(c) $w(z^*) \leq 0$
(d) $w'(z^*) < 0$
(e) $w'(0) > 0$
(f) If $\frac{f(z)}{1-F(z)}$ *is weakly increasing then* $\alpha'(z) > 0$ *and* $h' > h$ *implies* $\alpha(z; h') > \alpha(z; h)$ *for* $z < z^*(h')$

(a) Means that the best consultant is the full residual claimant of the output, which is a natural result. In general to guarantee that $\alpha'(z) > 0$ we need (by f) that $\frac{f(z)}{1-F(z)}$ since the effect of matching must also be taken into account. (c) Follows from (a) which implies that $V(z^*, z^*) = -w(z^*)$ but since by occupational choice (ii) $V(z^*, z^*) > 0$ we must have that $w(z^*) \leq 0$: the best producer does not obtain any share, $1 - \alpha_{z^*} = 0$, and he is passing the problem up, so he cannot be paying for advice.

When $\alpha'(z) > 0$ one might be tempted to believe that fixed payments $w(z)$ would be strictly decreasing to compensate for the increasing variable share, but this is not the case. The non-monotonicity of the fixed fees arises from the fact that the matching function introduces an asymmetry in the local incentives to deviate. This asymmetry is observed in the difference in the last terms of 13 and 14.[7] In particular, consider the incentives for a type $z = 0$ to pretend to be of a slightly more knowledgeable. Because the slope of the matching function is high for z close to zero, as many originators seek help, pretending to be of slightly higher quality leads to a much better match for the originator. This incentive to exaggerate is partly offset by having $\alpha' > 0$ but note that the value of α' cannot be chosen arbitrarily because it must simultaneously provide incentives for z^* consultants to not desire to be of type $z^* + \varepsilon$. Because the cost to a consultant of exaggerating his quality is the result of receiving a more difficult match and the matching function is steep at zero, the α' needed to satisfy the consultants incentive compatibility is lower than the one necessary to guarantee incentive compatibility for producers. It is therefore necessary to have $w'(0) > 0$ to be able to satisfy both incentive compatibility constraints simultaneously. A similar logic implies that $w'(z^*) < 0$.

Finally (f) also shows that when a consultant can help a larger number of originators because of lower communication or helping cost h, the contigent share that the consultant receives *per problem* is reduced. Intuitively, the same amount of contingent pay can be attained with a lower per-problem share when the consultant deals with more problems and the aggregate is what matters for incentive compatibility.

To complete the equilibrium construction we must characterize the fixed payment schedule $w(z)$. For this we can use the fundamental theorem of calculus, to obtain the entire $w(z)$ schedule for a given $w(0)$,

$$w(z; w(0)) = w(0) + A(z) \tag{15}$$

[7] This also explains why the contingent schedule on its own is not sufficient to provide incentives to both type of agents.

Where:

$$A(z) = \int_0^z \left((1-\alpha(t))\, h - \alpha'(t) \frac{F(m(t)) - F(t)}{1-F(t)} \right) dt$$

The integral in $A(z)$ is involved and cannot be obtained in closed form. However, we can fully characterize this function for a given w_0. Because there are no independents for a given h supply and demand uniquely determine z^* (from $Z_2(z^*) = z^*$). Given z^*, the matching function is uniquely pinned down (see equation (12)), as is $\alpha(z)$. The only object left to solve for is $w(0)$ in (15). The condition that the marginal producer z^* is indifferent between being a producer and becoming a consultant determines $w(0)$:

$$\begin{aligned} S(z^*, z^*) &= R(z^*, z^*) \\ \frac{1}{h}(w(0) + \alpha(0) F(z^*)) &= F(z^*) - (1 - F(z^*)) w(z^*)(w(0), z^*) \end{aligned}$$

or:

$$w(0) = \frac{F(z^*)(1 - \frac{1}{h}\alpha(0)) - A(z^*)(1 - F(z^*))}{\frac{1}{h} + 1 - F(z^*)}$$

This completes the equilibrium construction.

Lastly, we verify that the candidate equilibrium above is indeed an equilibrium. Note first that conditions (i), and (iv) are met by construction. Thus we need to verify that ex post advice-seeking (condition iii) is optimal, that is $V(z,z) > 0$ for all $z < z^*$ and that occupational choice is optimal, that is condition (ii) holds. Unfortunately for a general distribution $F(z)$ it is difficult to verify these conditions analytically although it would be simple to check numerically for given parameter assumptions.

Assuming that $F(z) = z$ we can do this analytically. We show in the Appendix (see Lemma (11)) that condition (iii) is indeed satisfied. Finally, we verify that occupational choice is optimal, that is condition (ii) holds. Given that for $F(z) = z$ the expression in condition (ii) are continuous and monotonic in h one can establish a cutoff value h^* such that for $h < h^*$ the condition is satisfied whereas for $h > h^*$ some consultants prefer to exit the market.[8]

Proposition 8 *If $F(z) = z$ there exists a unique separating equilibrium iff $h < h^*$ for some $h^* \in (0,1)$.*

The equilibrium has the following properties:

1. *The share of the opportunity transferred $\alpha(0) = \frac{h(1-z)^2}{h(1-z)^2 + (1-m(z))}$, which is increasing in z and $\alpha(z^*) = 1$.*

2. *The fixed payment w_z is non monotonic, increasing at 0 and decreasing at z^* with $w(z^*) \leq 0$.*

3. *An increase in h increases the fixed price of advice (shifts $w(z)$ up) and reduces the share of the solution transferred to the consultants ($\alpha(z)$ goes down).*

[8] Furthermore, h^* can be numerically computed to be approximately 0.76.

4. The equilibrium is efficient for $h < 0.75$

Intuitively, truth-telling is attained through both the fixed and variable portions of the contract and the quality of the match. In fact, since $w(z)$ is non-monotonic, there exist $z < z'$ such that $w(z) = w(z')$ and yet truth-telling is attained even though $\alpha(z) < a(z')$. The high type z' would seem to prefer the contract for z, because it costs the same fixed payment to get advice and a lower share must be offered (and thus a higher share retained). However, at that price and share the advice received is worse, as $m(z) < m(z')$; this ensures truth-telling. Conversely, the low type z does not prefer the better advice which requires offering a higher share, and the problem is easy enough that it can be solved by less knowledgeable consultants with relatively high probability.

As to the last point, given that $h < 0.75$ is the condition for the equilibrium to not have independents in the first best, the equilibrium with double-sided asymmetric information is efficient in that case. This is important, as we have a market that can achieve, when communication costs h are low enough, first best efficiency even though both seller and buyer types are unobservable. The key inefficiency in this market is that it is impossible to exclude those in the middle; when communication costs are sufficiently low, there is no efficiency-related reason to exclude them, and the equilibrium is efficient.

Distributional Consequences:

We can compare the equilibrium payoffs relative to the first best. It is clear that, as long as the equilibrium exists, there is weakly more trade than with full information. This is quite straightforward when asymmetric information leads to strictly more trade (a more inefficient outcome). It is clear that those originators that would have remained out of the market with full information must be strictly better off when they take advantage of the asymmetry of information to bring their problems to the market. The losers are those types close to the extremes.

When the allocation under asymmetric information coincides with the first best and hence there are no aggregate efficiency implication, there still remain distributional consequences. The agents around z^* benefit at the expense of those in the extremes. The types around z^* are in some sense the worst types in the market, because they bring to consultants the most difficult problems in expectation (if they are originators) or because they are the least skilled experts offering their services (if they are consultants). When their type is not publicly observable they may have incentives to pretend to be a better expert or, as originators, to pretend to bring a simpler problem. As shown above, we can still construct contracts that will prevent this mimicry from taking place in equilibrium. However, a feature of those contracts is that the equilibrium payoffs to the experts do not increase as rapidly in their types as they do under full information. Lower expert types are capturing some informational rents. A similar effect occurs on the other side of the markets, i.e., easier problems (those generated by the least skilled originators) receive a lower reward in equilibrium than they would be with full information. Thus agents around z^* benefit at the expense of those in the extremes.

Proposition 9 *The agents around z^* benefit from the asymmetry of information at the expense of types close to the extremes* $\{0,1\}$

Proof. A. Lets consider first the case where the full information equilibrium has independents
The middle types (those who are independent) earn $F(z)$ in the full information case. In the asymmetric information case, they are in the market and hence earn strictly more than $F(z)$. The top agents, with skill $z=1$, pay 0 for the problems under full information. In the asymmetric information equilibrium they remain full residual claimants ($\alpha(z^*)=1$), but now they pay a nonnegative price and they must solve a harder problem, that is $z^* > z_1$. They are strictly worse off, as are types close to them by continuity. The bottom agents, with skill $z=0$, are matched with worse experts, as $z^* < z_2$. Note that type $z=0$ was making $(1-h)F(z_2)$ in the full information case, since type z_2 was making $hF(z_2)$ per match (and hence $F(z_2)$ on aggregate). Now type z^* is making at least $F(z^*)$ on aggregate which means that types $z=0$ now must make at most $(1-h)F(z^*) < (1-h)F(z_2)$ .Therefore $z=0$ must be strictly worse off, as are types close to them by continuity.
B. Now consider the case in which the first best has no independents.
Note first that because the matches remain the same, the surplus per match is the same. Therefore, if type z^* is better off then both types that deal with this type must be worse off (type 0 when z^* acts as an expert and type 1 when z^* acts as an originator) and vice versa. As shown in Lemma (10) (see Appendix) the slope of equilibrium expert payoffs is lower under asymmetric information than under full information. This immediately implies that in order for the surplus of the match to remain constant the types around z^*must be strictly better off and the types in the extremes strictly worse off. ■

Summarizing, the equilibrium with double sided asymmetric information has the following properties:
1. When the first best allocation has no independents (low values of h) efficiency can be attained even in the presence of two-sided asymmetric information. Redistributive are still observed.
2. For intermediate values of h the equilibrium exists but there exists excessive trade relative to the first best. Producers obtain weakly worse advice than in the first best. Consultants obtain weakly more difficult problems. The agents around z^* benefit from the asymmetric-information at the expense of types close to the extremes $\{0,1\}$
3. For high values of h there is no separating equilibrium.

C Contracts without fixed transfers: Pooling equilibria

Some instances of service outsourcing do not involve a fixed transfer or payment for participation, most likely because of problematic contract enforcement. This is the case for instance in an increasing number of internet sites that simply pay the problem solver a contingent success payment, and no fixed transfer or payment for participation. As can be easily verified by setting $w(z)=0$ in (13) and (14) and attempting to solve the two differential equations in one function $\alpha(z)$, no separation of types exists in this case.

Corollary 3 *No separation of types can exist when using only contingent payments, $w(z) = 0$.*

Pooling equilibria do exist, however, in this case. The simplest case, which we examine briefly here, involves announcing a single contingent fee and having all problem solvers match with all problem producers randomly.[9] No problem remains unmatched, as it is costless for a producer to offer a contingent fee to a problem solver for assistance in solving his problem. Thus the only equilibrium object in this case are the contingent fees (α) (because no type separation exists, the price cannot depend on the difficulty of the problem or on the ability of the producer; it only depends on whether output was produced) and the cutoff z^*.

Market clearing conditions imply a unique cutoff type z^* that must satisfy the following:

$$\underbrace{\frac{1-z^*}{h}}_{Demand} = \underbrace{\int_0^{z^*} (1 - F(q))\, dq}_{Supply} \tag{16}$$

Note that demand is decreasing in h so z^* must also be decreasing in h.[10]

Given z^* the contingent price α must ensure that type z^* be indifferent between becoming a producer or a consultant.

The expected earnings for the marginal producer z^* who is matched with a random problem solver are given in this pooling equilibrium p by:

$$R^p(z^*, \alpha) = F(z^*) + (1 - F(z^*)) \left(\frac{EF[z|z > z^*] - F(z^*)}{1 - F(z^*)} \right) (1 - \alpha) \tag{17}$$

$$= \alpha F(z^*) + (1 - \alpha) EF[z|z > z^*] \tag{18}$$

which is strictly decreasing in α, since the second term is larger than the first.

$$EF[z|z > z^*] = \int_{z^*}^{1} F dx$$

The expected earnings of problem solvers are given by:

$$S^p(z^*, \alpha) = \frac{\alpha}{h} \left(E_x \Pr\left[q < z^* | q > x\right] \right).$$

The expectation is given by

[9] Of course as we showed above, if the fee is non-contingent no equilibrium exists.

[10] Note that demand is decreasing in z^*, and supply is increasing in z^*. Moreover, if $z^* = 0$, there is excess demand, whereas if $z^* = 1$ there is excess supply.

$$E_x \Pr\left[q < z^* | q > x\right] = \frac{\int_0^{z^*}\left(\int_x^{z^*} f(q)\,dq\right)dx}{\int_0^{z^*}\left(\int_x^{1} f(q)\,dq\right)dx} = 1 - \frac{z^*\left(1 - F(z^*)\right)}{z^* - \int_0^{z^*} F(x)\,dx}$$

and

$$S^p(z^*, \alpha) = \frac{\alpha}{h}\left(1 - \frac{z^*\left(1 - F(z^*)\right)}{z^* - \int_0^{z^*} F(x)\,dx}\right)$$

Because $R^p(z, \alpha)$ is strictly decreasing in α and $S^p(z^*, \alpha)$ is strictly increasing in α we can find at most a unique α^* that guarantees that $R^p(z, \alpha) = S^p(z^*, \alpha)$,

$$\alpha = \frac{EF[z|z > z^*]}{\alpha EF[z|z > z^*] + \frac{1}{h}\left(1 - \frac{z^*(1-F(z^*))}{z^* - \int_0^{z^*} F(x)dx}\right) - F(z^*)}$$

Proposition 10 (Pooling) *When only a contingent payment can be offered, only pooling equilibria can exist. Moreover, if a pooling equilibrium exists, there is a unique referral share $\alpha(h)$, common to all problem solvers and originators, such that:*
i) Agents, $z \in [0, z^*(h)]$ are problem producers and agents $z \in [z^*(h), 1]$ are problem solvers,

ii) There is a common contingent fee $\alpha^*(h)$ per problem referred. The cutoff $z^*(h)$ is strictly decreasing in h and is larger than the first level cutoff z_1.
ii) For any $h > h^*$ a continuum of equilibria exists with $\alpha = 1$. The equilibria differ in the value of h and the distribution of problems that are passed on to consultants. Both the lower bound upper bounds of the equilibrium values for z are increasing in h and as $h \to 1$ there is a unique equilibrium with $z = 1$.

Thus, although the pooling market with a single contingent payment does not break down, it suffers from three types of efficiency losses with respect to the full information problem: first, problems that are too difficult to be referred are in the pool of problems passed on; second, there is excessive entry into consulting; and third, there is inefficient matching, i.e., conditional on a problem being passed on, the probability that it is solved is much lower, as the matching is now random rather than assortative.

In the uniform case,

$$E_x \Pr\left[q < z^* | q > x\right] == \frac{z^*}{2 - z^*}$$

and

$$S^p(z^*, \alpha) = \frac{\alpha}{h} \frac{z^*}{2 - z^*}$$

$$\alpha = \frac{2 + z^*(1 - z)}{\frac{2}{h} z^* + 2 - (3 - z^*) z^*} \tag{19}$$

In the expression above, α is strictly increasing in h and for $h = \frac{3}{4}$, $\alpha = 1$. Therefore, for the uniform case, an equilibrium with $\alpha \leq 1$ only exists for $h \leq \frac{3}{4}$. Note that an equilibrium with $\alpha > 1$ is not incentive compatible, as producers would have to pay for asking for help with their problem.

With $\alpha = 1$ there could still be equilibria in which some agents decide to become consultants. In fact, there is a continuum of such equilibria. In all of them, there is an excess supply of problems. Essentially, in all of these equilibria a producer abandons his claim to the problem, and entirely transfers the problem to the problem solver. We can find the upper and lower bounds for entry into the consultants market by assuming that the problems from the least knowledgeable and most knowledgeable producers are passed on to the consultants. Letting the interval of consultants be $[z_2, 1]$, the easiest case corresponds to the case where the problems are in the interval $[0, \bar{z}]$ with $\bar{z} < z_2$; the hardest case occurs when the problems are in $[\underline{z}, z_2]$.

For example, for the uniform case and in the intermediate case in which problems are drawn randomly from the pool of unsolved problems:

$$\begin{aligned} S^p(z^*, 1) &= R^p(z^*, 1) \\ &\Updownarrow \\ z^* &= 2 - \frac{1}{h}; \quad h > \frac{3}{4} \end{aligned}$$

Figure 3 illustrates the properties of the equilibria for the uniform case, with the bound on the worst consultant $(z^*(h))$ for each h depending on whether the problems selected are the worst problems, a random selection of problems, or the best problems drawn by producers.

V Implications

As we discussed at the beginning of this paper, the economy is undergoing an important structural change, as increasing expenditure and employment shares are allocated to the professional services sector. Much is unusual about this sector, but one thing in particular stands out: advice, the object of the exchange in many cases, is completely intangible and its quality is difficult to measure; additionally, the quality of those providing and receiving advice is unverifiable. In this paper, we

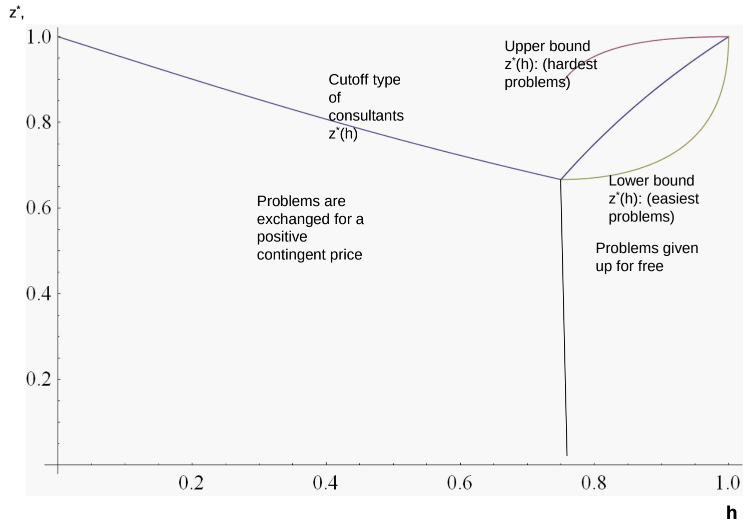

Figure 3: Pooling Equilibrium, Uniform case. For $h > .75$ the contingent fee is $\alpha = 1$:the entire claim is transferred. In this case there always exists an excess supply of problems, and the decision to enter into consulting depends on the selection of problems that do get transferred. Maximum entry (lower bound) occurs when the easiest problems get transferred; minimum entry (upper bound) when only the hardest problems get transferred.

studied the implications of trading advice for efficiency and distribution in a general equilibrium setting, where the entry of agents into this sector is endogenous and results from reductions in communication costs. We summarize these implications here.

Efficiency. As we show in Section III, when information is one-sided, the market achieves efficient matches between problems and solutions. If output is not verifiable, two-sided informational asymmetries completely destroy the market, irrespective of the parameters or functional form (in contrast to the situation in generic adverse selection problems) and no trade takes place. The reason is that the lowest-skilled agents, who should be originating problems, can always pretend to have more skill and become consultants to others.

If output is verifiable, we find that the optimal contract involves a fixed payment and an equity stake for problem solvers and producers; the equity share increases with the quality of the problem solving required. The market price for these opportunities will be such that, again, consultants sort themselves such that the best consultant ends up with the more difficult (in expectation) problems, those generated by the most talented originators. Depending on communication costs, this contract may attain full efficiency. When communication costs are low, the first best requires that all producers seek help; in this case, spot markets can achieve the first best, as the key potential efficiency loss in these advice markets is the impossibility of excluding some agents from trade. As communication costs increase, the market exhibits excessive trade relative to the first best, as no agent can be excluded from it.

If only equity shares are used, possibly because of enforcement or liquidity constraints, then separation is unattainable. As we show, only pooling equilibria exist in this case, where positive sorting cannot be sustained and the matching between problems and problem solvers is random. This random matching implies an additional efficiency loss, as good agents talents are wasted on simple problems and hard problems are allocated to agents who are insufficiently skilled.

Distribution. The distributional consequences of these markets are unusual. Informational asymmetries tend to favor the less well off, as rents must be left to them. In advice markets, as we show, informational asymmetries benefit the 'middle class', i.e., those who are either the producers of the hardest problems (the most skilled among producers) or as worst problem solvers (the least skilled among problem solvers) and who, absent informational asymmetries, would not be trading in the market.

This finding has interesting political economy implications. The median voter benefits from the muddied waters of contracting under asymmetric information. Thus we expect to see little political pressure to improve transparency. It has been well understood, since Friedman's dissertation, that professional service providers lobby to restrict access to their professions.[11] Here we observe a countervailing force from the median voter: less talented people may lobby not just for access, but against efforts to increase transparency, as they benefit from informational asymmetries. [12]

Direction of the informational asymmetry and efficient institutions. As we show in Section III, when information about the quality of those providing advice can be easily obtained (potentially through reputation or well functioning certification mechanisms) contracts should take the form of consulting contracts: consultant expertise is provided in exchange for a fee. Therefore those purchasing advice can easily internalize the difficulty of their problems and trade-off the probability of solving them against the cost of the advice. This results in efficient matches between problems and solutions and is consistent with the use of consulting by firms in many contexts, where essentially the consultant names a price and a quality pair and the client sorts among firms.[13]

Conversely, when opportunities are transferable and the quality of consultants who would be appropriate for a given opportunity is more difficult to observe, we expect referral contracts to be preferred. In this context, originators post their opportunities in exchange for a fee and consultants bid for them. The market price for these opportunities will be such that, again, consultants will

[11] As he puts it "It is hard to regard altruistic concern for their customers as the primary motive behind their determined efforts to get legal power to decide who may be a plumber." (Friedman, 1962).

[12] Friedman and Kuznets (1946) argued that licencing far from helping consumers, resulted in higher prices and lower quantity and quality of service. Empirical support for this view of licensing as an inefficeint regulation has been found in accounting (Young, 1988), dentistry (Kleiner and Kudrle, 2000) and optometry (Haas Wilson, 1986). After Arrow (1963) first advanced the hypothesis that entry regulation was a way to protect consumers under asymmetric information on expert quality, Stigler (1971) countered that entry regulations were captured by insiders. Shapiro (1986) argues that licensing provides incentives for human capital accumulation by experts under moral hazard. Leland (1979) discussed entry requirements in a market with asymmetric information regarding quality, and showed that if insiders were in charge, minimum standards would be set too high.

[13] Of course, there is an element of risk sharing in the hourly fee structure, but the total price of the project is actually basically known in advance with a high degree of certainty in this market.

sort themselves to ensure that the best consultant will end up with the a priori more difficult opportunities. Such markets are observed in biotech, for example, where firms that have discovered molecules and want to take them to market attempt to find the right company to do so by selling their IP, i.e., the profitable opportunity they generated; they post the opportunity and idea, and the pharma companies sort themselves among opportunities.

Rent sharing and referrals in the law As in our model, lawyers generally pass on clients to one another in exchange for a referral fee. This is particularly the case in litigation, where these payments take the form, as in the contracts we describe, of referral shares. While such compensation arrangements clearly involve team production and moral hazard issues (see Garicano and Santos, 2004), there is also a sorting element along the lines of our analysis. Thus, we expect better lawyers to receive larger output shares, and to be matched with harder problems. Empirically we should find referral shares increasing along with the quality of the claim or the lawyer originating it.[14]

Venture capital: sorting and contracting Venture capital markets have features that are similar to those in our second best contracts. Individuals who originated a business idea must find consultants to aid them in taking the idea to market. Venture capital contracts generally involve cash transfers and equity stakes, i.e., a share in the profits if the idea is successful.

Our model has clear implications for this type of markets. First, as noted above, there should be positive sorting between the quality of the deal and the quality of the venture capitalist; only a good venture capitalist is able to add sufficient value to a good entrepreneur. A direct test of this is Sorensen (2007), who finds that more experienced venture capitalists make more successful investments and invest in 'better' companies, i.e., late stage and biotechnology companies. Second, this positive sorting should covary with increasing revenue shares for the venture capitalist, i.e. an entrepreneur (and a venture capitalist) with higher unobserved quality should leave a higher revenue share to the venture capitalist, as the better entrepreneur signals his high quality by offering a large residual share. Kaplan and Stronberg (2002) come closest to being able to test this, as they have contracting data on VC contracts; however, their regressions do not test for these sorting and share effects.

VI Conclusions

In this paper we have made some progress towards understanding the role that markets may play in intermediating the supply and demand for advice, implications for the efficiency of these markets, and their distributional implications. Our starting point is the observation that these markets are

[14] A test along these lines was conducted in small sample by Stephen Spurr (1988). However, rather than presenting the regression of share on either claim value or quality of lawyer he includes both in the only specification he studies and finds them insignificant.

likely to be subject to large, two sided, informational asymmetries: it is difficult to truly know how talented a consultant is, and it is also difficult to know how difficult a problem is before solving it. We have proceeded by first studying the market absent information constraints, then introducing one-sided informational asymmetries, and finally introducing double sided information asymmetries. Our analysis shows that, even in the worst case of informational asymmetry, if contingent contracts are possible, the market may attain the first best.

Moreover, we have shown that information asymmetries limit the gains obtained in these markets by the most and the least skilled to the benefit of those in the middle, who are, unusually, capturing the informational rents here. In that sense, our key distributional result is that reductions in asymmetric information in outsourcing service markets increase wage inequality in two unusual ways: first, by allowing the worst consultants and the most difficult problems to be excluded from the market and second, by reducing the informational rents captured by those in the middle of the distribution at the expense of those at top and the bottom.

Our paper raises a number of questions for future study. One of current relevance involve the use of the internet to set up tournament-like structures in which a prize is announced and many potential solvers may choose to participate. Such ideas have been attempted by a range of companies that have emerged to match companies with problem solvers online.[15] The market has two sides, those who post problems for which no solution is yet known (called the 'seekers' by the pioneer in this field, Innocentive) and those who attempt to provide a solution (who are called 'solvers' by that site). As in our case, there is asymmetric information regarding how difficult the challenge will eventually prove to be and about the skill of those attempting a solution. [16]

This generation of sites operates along the lines of a tournament model; a prize is posted, and it is awarded to those who solved the challenge. The system is inefficient in that the effort of those who do not win the challenge is wasted. Moreover, this inefficiency is compounded strategically, as participants attempt to determine which challenges will attract just the right number of solvers to ensure an adequate probability of winning.[17] Of course, it is difficult to know a priori how difficult and how attractive a challenge will be, but the system is inherently unstable: if it becomes too popular, the probability of being the chosen solution collapses, and those with a substantially higher opportunity cost of time, presumably the best solvers, drop out of it. Clearly, given these inefficiency, researchers must study the optimal design in this setting. Our analysis above suggests that a design that reduces the risk fo this outcome may involve the use of a fee and an output share.

[15] See innocentive.com, fellowforce.com, ninesigma.com, yet2.com yourencore.com.

[16] According to InnoCentive.com (A) HBS case 9-6098-170, by Karim R. Lakharni, dated June 10, 2008, it had 145000 solvers registered, who had submitted to date 7,011 solutions; 620 challenges have been posted, with a total award of $16m, of which 215 had been solved with $3m paid out.

[17] A top solver in Innocentive, David H. Tracy declared "I'm good enough mathematician (barely) to know better than to play the lottery. ... If I thought that a given Challenge would attract say 100 strong solutions –solutions likely to be roughly as wonderful as mine- then I might choose not to invest the time needed to create and submit a solution with just 1% probability of winning." (http://www.innocentive.com/blog/2008/04/29/tracy/)

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VII Appendix A: Ommited Proofs

Lemma (1) **Assortative Matching.** We show that $s < s'$ for $z > z'$ cannot be optimal. Conditional on being consulted on one problem each, the expected number of solved problems is:

$$\frac{F(s)}{1-F(z)} + \frac{F(s')}{1-F(z')}$$

If instead we reverted the matching so that type $m(z)$ is tries to solve type z' problems and vice-versa the number of solved problems would be:

$$\frac{F(s')}{1-F(z)}+\frac{F(s)}{1-F(z')}$$

We show that the second arrangement is more productive if $s < s'$ for $z > z'$:

$$\begin{aligned}
s' &> s \\
F(s') &> F(s) \\
F(s')\left(F(z')-F(z)\right) &< F(s)\left(F(z')-F(z)\right) \\
F(s')F(z')-F(z)F(s') &< F(s)F(z')-F(z)F(s) \\
F(s')F(z')+F(z)F(s) &< F(s)F(z')+F(z)F(s') \\
F(s')\left(1-F(z')\right)+(1-F(z))F(s) &> F(s)\left(1-F(z')\right)+(1-F(z))F(s') \\
\frac{F(s')}{1-F(z)}+\frac{F(s)}{1-F(z')} &> \frac{F(s)}{1-F(z)}+\frac{F(s')}{1-F(z')}
\end{aligned}$$

■

Lemma (2) Independents are Smart. If $z' > z$ could not solve a problem it means that the problem is harder to solve than the unsolved problem by type z. Hence, it is more likely that type $m(z')$ will solve problem z than problem z' therefore the planner would be better off by leaving z' unmatched and matching z. This implies that no producer can be smarter than an independent. ■

Lemma (3)Experts are Smarter. Consider two agents and independent $z_I \in I$ and a consultant with type $m(z)$. Assume in search of a contradiction that $z_I > m(z)$ is optimal. The joint output of these two types is:

$$F(z_I)+\frac{F(m(z))}{hF(z)}$$

Since $\frac{1}{h} > 1$, if $z_I > m(z)$ then:

$$F(m(z))+\frac{F(z_I)}{hF(z)} > F(z_I)+\frac{F(m(z))}{hF(z)}$$

Therefore the planner could improve by having z_I and $m(z)$ switch their roles. Hence consultants must always be smarter than independents. ■

Proof of Proposition 2. Follows from taking the derivative of the FOC with respect to h. ■

Rather than providing a separate proof for Propositions (3) and (4) separately we will prove them jointly with the Proposition below.

Proposition 11 *The competitive equilibrium allocation exists and is unique. It may be implemented equivalently through a referral or a consulting market. It attains the first best.*

We shall show that the competitive equilibrium is unique and attains the first best. To do this we abstract first from the actual implementation for now proceed and proceed through a series of three lemmas which follow below. We then show that the earnings and allocation in the referral and consulting formulations match the ones in the general derivation we follow.

Lemma 6 *The competitive equilibrium must display positive assortative matching.*

Proof. To see this consider the production function of a firm that hires solvers and producers of skill z_s and z_p. This firm's production function will be:

$$\pi(z_s, z_p) = F(z_s)n - w_p n - w_s$$

Subject to the time constraint of the problem solver, $h(1-F(z_p)n = 1$. That is the profit function of this firm is:

$$\pi(z_s, z_p) = \frac{F(z_s) - w_p}{h(1 - F(z_p)} - w_s$$

It is clear this production function displays increasing differences (since $\frac{\partial^2 y}{\partial z_p z_s} > 0$ where $y = F(z_s)n$) and thus positive sorting must hold in equilibrium. ■

Lemma 7 (Market Clearing) *Equality of supply and demand means that the competitive equilibrium is pinned down up to the two cutoffs z_1 and z_2. Moreover, for each z_1 there exists a unique z_2, $z_2^{sd}(z_1)$ such that supply equals demand. Finally, $z_2^{sd'} < 0$.*

Proof. Suppose first that some agents are unmatched– there are independent producers. Supply equals demand pointwise implies: $m'(z) = (1-F(z))h$. With $m(0) = z_2$, we can write the matching function as $m(z; z_2)$. Then for a given z_2 and

$$m(z_1; z_2) = 1$$

implies that the match is entirely pinned down up to one constant z_1. Trivially, $m(z_1; , z_2) = 1$ implies a function $z_2^{sd}(z_1)$ with $z_2^{sd'} < 0$ (intuitively, if there are more supply of problems, you need a larger supply of problem solvers). ■

Lemma 8 (Uniqueness) *There always exists a competitive equilibrium for $h < 1$. This equilibrium is unique.*

Proof. The proof is by construction. We move along the $z_2^{sd}(z_1)$ curve until either $z_2 = z_1$ or $z_2 = w_c(z_2)$.

Consider a profit maximizing firm that hires teams of producers z_p and problem solvers z_s. Given that each problem solver can solve $1/h$ problems per unit of time, and that a producer only needs help with probability $(1 - F(z_p))$, the firm will need a measure $n = 1/(h(1 - F(z_p))$ of producers per problem solver, so that earnings are given by:

$$\pi(z_s, z_p) = F(z_s)n - w_p n - w_s$$

by the 0 profit condition these can be through of equivalently as the rents or wages of consultants who hire the producers:

$$w_s(z_s, z_p) = F(z_s)n - w_p n = \frac{F(z_s) - w_p}{h(1 - F(z_p)}$$

For the choice of producers of quality z_p to be an optimum, it must be the case that the wages are such that the choice of z_p is optimum:

$$w_s(z_s, z_p) = \max_{z_p} \frac{F(z_s) - w_p(z_p)}{h(1 - F(z_p)}$$

From here, using the first order condition and then the envelope we can obtain the slope of the earnings curve along the equilibrium:

$$\begin{aligned} \frac{\delta w_p(z)}{\delta z} &= \frac{f(z)}{(1 - F(z))}(F(m(z)) - w_p(z)) < f(z)\frac{(F(m(z)) - F(z)}{(1 - F(z))} < f(z) \\ \frac{\delta w_s(z)}{\delta z} &= \frac{f(z)}{h(1 - F(m^{-1}(z))} > f(z) \end{aligned}$$

Where we are using the matching schedule definition $z_s = m(z)$. The inequality in the first line uses the fact that, for producers who actually choose to be producers, earnings as producers are higher than earnings as self employed.

Now we move along $z_1^{sd}(z_2)$. Since the top consultant matches with producer z_1 and $w(z_1) = F(z_1)$ for optimal occupational choice, top consultants earnings are fixed at $\frac{1-F(\varepsilon)}{h(1-F(\varepsilon))} = \frac{1}{h} > 1$ as long as $h < 1$, and as long as the equilibrium is interior (there are producers). The earnings schedule of consultants thus starts at $w_c(1) = 1/h$ and decreases with slope $\frac{f(z)}{1-F(m^{-1}(z;z_2))}$. Specifically, since increasing z_1 raises the value of the match of every problem solver, this means that the rate of decrease of earnings as we reduce z is larger the higher $z_1 : \frac{d}{dz_1}\left(\frac{f(z)}{1-F(m^{-1}(z;z_2))}\right) > 0$. Start from $z_2 = 1$, $z_1 = 0$ We know this is a market clearing pair (that is $z_1^{sd}(1) = 0$), since there is no supply or demand of problems. The worst workers earn $F[0] = 0$ and the best ones earn $F[1] = 1$. Now consider a deviation along the market clearing condition so that $z_1 = \varepsilon$ and $1 = z_2^{sd}(z_1)$. Now the value of the match is $\frac{1-F(\varepsilon)}{h(1-F(\varepsilon))} = \frac{1}{h} > 1$ as long as $h < 1$. Managers clearly will chose to hire workers ε, pay them $z_1 = \varepsilon$ and earn themselves $1/h > 1$. However, this is not an equilibrium, as the agents at $z_2 = 1$ strictly prefer being problem solvers than independents(the earnings function is discontinuous at $z_2 = 1$). Raise now z_1 to $z_1^* = 2\varepsilon$. Now earnings of top solvers $z = 1$ are still

$1/h$. Construct the earnings function of consultants by using $\frac{f(1)}{1-F(2\varepsilon))}$.The earnings of $z_2^* = z_2(2\varepsilon)$ are either still $w(z_2^*) > z_2$ or $w(z_2^*) = z_2$. In the second case, we have a competitive equilibrium and stop. In the first case, we go back and increase z_1 again by ε. Now the slope of the earnings function at 1 is stepper at every point, $\frac{f(1)}{1-F(3\varepsilon))} > \frac{f(1)}{1-F(2\varepsilon))}$ etc. Since $\frac{f(z)}{1-F(m^{-1}(z_1;z_2))} > f(z)$, and $z_2^{**} < z_2^*$ the distance $w(z_2) - z_2$ is unambiguously reduced with each step. We can continue taking these steps till $z_1 = z_2$. If at any point $w(z_2) = z_2$, we have an equilibrium, since $w(z_1) = z_1$, market clears, and matches are optimal (agents cannot gain by deviating since, by construction, the slope is always equal to the marginal contribution. Moreover, since $\frac{\delta w_p(z)}{\delta z} < f(z)$, if worker z_1 is indifferent between being a worker or a producer, all workers with $z < z_1$ strictly prefer to be workers. If instead at this point it is still the case that $w_c(z_2) > z_2$, then we have no independents, and we can obtain the cutoff simply from the market clearing condition, $z_1 = z_2 = z^*$, where $m(z^*; z^*) = 1$, ■

Lemma 9 *The unique competitive equilibrium is Pareto Optimal.*

Proof. All we need to show is that the cutoff types coincide. We do this by showing the that the FOC of the planner's problems is satisfied with the CE cutoff z_1^{CE}

$$\begin{aligned} & -\left(\underbrace{\int_0^{z_1^{CE}} f(m\left(z;z_1^{CE}\right))\frac{\partial m\left(z;z_1^{CE}\right)}{\partial z_1}dz}_{loss\ from\ worse\ matches} + \underbrace{F\left(Z\left(z_1^{CE}\right)\right)\frac{\partial Z\left(z_1^{CE}\right)}{\partial z_1}}_{loss\ from\ less\ producers}\right) \\ = & \underbrace{F\left(m\left(z_1;z_1^{CE}\right)\right) - F\left(z_1^{CE}\right)}_{Extra\ output} \end{aligned} \tag{20}$$

$$\begin{aligned} & -\left(\int_0^{z_1^{CE}} \left(f\left(m\left(z;z_1^{CE}\right)\right)\left(-h\left(1-F\left(z_1^{CE}\right)\right)\right)\right)dz + F\left(z_2\right)\left(-h\left(1-F\left(z_1^{CE}\right)\right)\right)\right) \\ = & \ F\left(1\right) - F\left(z_1^{CE}\right) \end{aligned} \tag{21}$$

$$\left(\int_0^{z_1^{CE}} f\left(m\left(z;z_1^{CE}\right)\right)dz + F\left(z_2\right)\right) = \underbrace{\frac{F\left(1\right)-F\left(z_1^{CE}\right)}{h\left(1-F\left(z_1^{CE}\right)\right)}}_{Earnings\ of\ z=1}$$

Making a change of variables in the integral, the LHS can be written as:

$$\left(\int_{z_2^{CE}}^1 \underbrace{\frac{f\left(z\right)}{h\left(1-F\left(z\right)\right)}}_{\frac{\partial w_S(z)}{\partial z}}dz + \underbrace{F\left(z_2\right)}_{Earnings\ of\ z=z_2}\right) = Earnings\ of\ z=1$$

■

Proof of Proposition (5). If consultants are paid a fixed fee ϕ and a given type z chooses to become a consultant then all types $z' < z$ will choose to become consultants as well. This follows from noting that for type z :

$$\frac{\phi}{h} \geq F(z) + (1 - F(z))(\max\{0, \Pr(sol|z) - \phi\})$$

where $\Pr(sol|z)$ is the probability that the problem gets solved conditional on hiring a consultant and the difficulty of the problem being above z.

Furthermore, since type z could choose to not solve a problem of difficulty $q < z$ it must also follow that he can pretend his ability level is $\tilde{z} < z$ and therefore:

$$\frac{\phi}{h} > F(\tilde{z}) + (1 - F(\tilde{z}))(\max\{0, \Pr(sol|\tilde{z}) - \phi\}) \quad \forall \tilde{z} < z$$

but the RHS of the equation is exactly what any type $\tilde{z} < z$ would get. Note that these agents can also get $\frac{\phi}{h}$ since their type is not verifiable. Hence, they would all choose to become consultants. As a result, there would be nobody interested in hiring a consultant because consultants would not be able to solve the problem. ■

Proof of Lemma 4. For the equilibrium to be separating $m(z)$ must be strictly monotonic. In search of a contradiction, suppose that $m'(z)$ were negative. Suppose first that there are no independents. This would imply that the lowest problem solver z^* is meant to solve problems for the best producer, z^*. Clearly, no problem posed is solved, $\frac{F(z^*)-F(z^*)}{1-F(z^*)} = 0$ and hence there cannot be any trade between them. Second, suppose there are independents. Then type z_1, the highest producer must turn to z_2, the lowest problem solver, for help. The gains to z_1 from hiring z_2 must be 0, since he must be indifferent between getting help or not:

$$0 = -w_{z_1} + (1 - \alpha_{z_1})\frac{F(z_2) - F(z_1)}{1 - F(z_1)}$$

while z_2 must be indifferent between becoming a problem solver or an independent:

$$F(z_2) = \frac{1}{h}\left(w_{z_1} + \alpha_{z_1}\frac{F(z_2) - F(z_1)}{1 - F(z_1)}\right)$$

For a given z_1, the z_2 that would result from this system of equations is generically different that the one required to satisfy the market clearing condition: $\int_0^{z_1}(1 - F(q))\,dq = \frac{1-z_2}{h}$. ■

Lemma 10 *In the assymetric information case the equilibrium payoff of consultants increases in type at a slower rate than it does in the full information case:*

Proof. Let w_{FI} and w_{AI} denote the equilibrium payoffs per match of consultants that match with an originator of type z with Full Information (FI) and Asymmetric Information (AI). *In the full* *information case*, in equilibrium $w'_{FI}(z) = \frac{f(m(z))}{1-F(z)} > 0 \quad \forall z \in [0, z*]$ (eq. 5). *In the asymmetric information case*, in equilibrium $w_{AI}(z) = w(z) + \alpha(z)\frac{F(m(z))-F(z)}{1-F(z)}$. Computing the derivative wrt z:

$$w'_{AI}(z) = w'(z) + \alpha'(z)\frac{F(m(z)) - F(z)}{1 - F(z)} + \alpha(z)\left\{\frac{[f(m(z))m'(z) - f(z)](1 - F(z)) + f(z)[F(m(z)) - F(z)]}{(1 - F(z))^2}\right\}$$

$$w'_{AI}(z) = w'(z) + \alpha'(z)\frac{F(m(z)) - F(z)}{1 - F(z)} + \alpha(z)\left\{\frac{-f(z)(1 - F(z)) + f(z)(F(m(z)) - F(z))}{(1 - F(z))^2}\right\} + \alpha(z)\frac{f(m(z))}{(1}$$

$$w'_{AI}(z) = w'(z) + \alpha'(z)\frac{F(m(z)) - F(z)}{1 - F(z)} + \alpha(z)\frac{-f(z)(1 - F(m(z)))}{(1 - F(z))^2} + \alpha(z)\frac{f(m(z))m'(z)(1 - F(z))}{(1 - F(z))^2}$$

$$w'_{AI}(z) = \alpha(z)\frac{f(m(z))m'(z)(1 - F(z))}{(1 - F(z))^2}$$

since the first three terms sum up to zero, due to the truthtelling constraint for consultants (eq. 13). Some algebraic manipulation delivers:

$$w'_{AI}(z) = \frac{f(m(z))}{1 - F(z)}\alpha(z)m'(z)$$

Since $m'(z) = h(1 - F(z))$, we get:

$$w'_{AI}(z) = w'_{FI}(z)\alpha(z)h(1 - F(z)) < w'_{FI}(z) \quad \forall z \in [0, z*]$$

since $\alpha(z) \leq 1, h < 1, 1 - F(z) \leq 1$.

■

Lemma 11 *For $F(z) = z$ the candidate equilibrium satisfies ex-post advice seeking (condition iii). That is $V(z,z) > 0$ for all $z < z^*$*

Proof.

$$V(z,z) = -(w_0 + A(z,z^*)) + \left(\frac{\left(1 - (hz - \frac{1}{2}hz^2 + z^*\right)\left(hz - \frac{1}{2}hz^2 + z^* - z\right)}{\left(h(1-z)^2 + \left(1 - (hz - \frac{1}{2}hz^2 + z^*)\right)(1-z)\right)}\right)$$

$$V(z,z) = -w_z + (1 - \alpha_z)\frac{m(z;z^*) - z}{1 - z}$$

$$\frac{\partial V(z,z)}{\partial z} = -w'_z - \alpha'_z\frac{m(z;z^*) - z}{1 - z} + (1 - \alpha_z)\left(\frac{(1 - z)(m'(z;z^*) - 1) - (m(z;z^*) - z)}{(1 - z)^2}\right)$$

and using the first order condition for truthtelling:

$$\begin{aligned}\frac{\partial V(z,z)}{\partial z} &= -\left(1-\alpha_z\right)h + \left(1-\alpha_z\right)\left(\frac{\left(1-z\right)\left(h(1-z)-1\right)-\left(m(z;z^*)-z\right)}{\left(1-z\right)^2}\right) \\ &= \frac{\left(1-\alpha_z\right)}{\left(1-z\right)^2}\left(\left(1-z\right)\left(h(1-z)-1\right)-\left(m-z\right)-\left(1-z\right)^2 h\right)\end{aligned}$$

which is negative if $h - m + 4z - 2hz - z^2 + hz^2 - 2 < 0$. Replacing m in and simplifying:

$$h + 4z - 3hz - z^2 + \frac{3}{2}hz^2 + \frac{1}{h}\left(\sqrt{h^2+1}-1\right) - 3 < 0$$

which is easy to verify to be true. Now we need to show that it is indeed always positive. Since $\alpha_{z^*} = 1$, we require that $w_{z^*} < 0$. One can verify this is indeed the case. ■