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IN A HOMOGENEOUS PRODUCT
DUOPOLY**

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Discussion Paper No. 9106
August 2012

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ABSTRACT

On two-part tariff competition in a homogeneous product duopoly*

We explore the nature of two-part tariff competition between duopolists providing a homogeneous service when consumers differ with respect to their usage rates. Competition in only one price component (the fixed fee or the rate) may allow both firms to enjoy positive profits if the other price component has been set at levels different enough for each firm. Endogenous market segmentation emerges, with the heavier users choosing the lower rate firm and the lighter users choosing the lower fee firm. We therefore characterize how fixing one price component indirectly introduces an element of product differentiation to an otherwise homogeneous product market. We also examine the crucial role that non-negativity constraints play for the nature of market equilibrium.

JEL Classification: D43 and L13

Keywords: market segmentation, non-linear pricing and two-part tariffs

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*We are grateful to participants at the EARIE and ASSET conferences, as well as at seminars at various universities, for their helpful comments.

Submitted 17 August 2012

1 Introduction

One of the fixed points of oligopoly theory is that price competition in homogeneous goods markets leads to pricing at cost and zero profits, the “Bertrand outcome”. In this paper we set up a very simple homogeneous product duopoly model that allows us to explore in depth the nature of competition when pricing takes the form of a two-part tariff.¹ Indeed, in many markets pricing involves the use of a fixed fee and of a per unit payment rate. Several examples readily come to mind including credit cards, telephone services, car rentals, club memberships, equipment leasing, amusement parks, tv program subscriptions and many other.² While the relevant literature has been growing, the larger part of the contributions refer to pricing by a monopolist and to the role of selection under uncertainty. Here we explore the nature of two-part tariff competition under alternative assumptions about which price components can be chosen by the firms and when.

We set up a very simple duopoly model where all consumers view the products sold by the two firms as perfectly homogeneous. However, and importantly, consumers differ with respect to their usage levels.³ In this setting, if the two rival firms compete by choosing both their fixed fee and per-unit rate, the Bertrand logic prevails and competition leads to zero profit with pricing at cost. However, we characterize how if per-unit rates for the two firms are set at levels that are not too close to each other, competition via fixed fees leads to an equilibrium where both firms make positive profits. Moreover, both firms’ profits increase as the difference between the rates increases. Essentially, even though the products are homogeneous, fixing one of the firms’ pricing instruments (at levels that are not too close across firms) along with the assumption that usage levels vary in the population of consumers, serves to implicitly introduce some differentiation in the market since now consumers are no longer indifferent between the two products. This effect, in turn, allows firms to enjoy positive profits while the market becomes endogenously segmented. A similar result obtains if the fees are first fixed, and then firms compete via per-unit rates.

¹Modifications of the basic oligopoly model, apart from product differentiation, are known to allow firms to enjoy positive profits, including repeated interaction, capacity constraints or dynamic competition with switching costs.

²In the case of credit cards, the user often pays an annual fee and, in addition, interest on the amount he borrows using the card; telephone users may be paying a monthly fee and a per-minute rate; and car renters may be paying a daily rate and a charge per km.

³Examples of this usage level are the amount the user borrows in the case of a credit card, the mileage in case of a car rental and the number of photocopies in case of a copy machine.

A direct implication of our analysis is that if firms were allowed to set one of the two pricing instruments and to subsequently compete via the other instrument, they could assure themselves positive profits. In fact, when rates are given, both firms would prefer the difference in these rates to be as high as possible, since in that case their profits are maximized.⁴ It follows that price competition only in one dimension is not enough to lead to the perfectly competitive (Bertrand) outcome. This case, which is the core result in our analysis, is especially relevant when one of the price components has been set for the competing firms by a regulator, by some industry committee or by some other institutional procedure. Setting fees or rates at different levels across firms serves to introduce indirectly heterogeneity to an otherwise homogeneous product market. Regulators should be, therefore, aware that fixing one price component at levels different enough for each firm may allow these firms to make positive profits even though they compete in another component and even though the products are homogeneous. Likewise, a competition authority should not view competition in one price component as sufficient if the firms have been able to coordinate and set another price component at some level for each firm.

We also examine the case where the consumers could, in principle, purchase one product from more than one firms. We demonstrate that this case is formally identical to the case where firms can charge only non-negative prices. If we take the rates as given, the firm with the low rate captures a larger market share and makes higher profit than its rival. If we take the fees as given, the firm with the low fee captures the entire market and makes a positive profit.

Our analysis is related to the growing literature on two-part tariffs (see e.g. Oi, 1971, for a classic reference and Varian, 1989, Armstrong, 2006, Stole, 2007, and Vettas, 2011 for reviews and references). Much of the literature, though, has focused on the monopoly case.⁵ Despite its importance, the study of oligopoly competition with two-part tariffs has received relatively less attention, especially when also compared to the volume of work on oligopoly with linear pricing. Still there is a number of important papers that have studied various aspects of nonlinear competition in oligopolies. These include, among other, Spulber (1981), Oren *et al.* (1983), Hayes (1987), Mandy (1992), Stole (1995),

⁴We discuss an analogy of this aspect of our model with models such as Shaked and Sutton (1982) where firms prefer their products to be as differentiated as possible, to relax (linear) price competition.

⁵In addition, it has been examined how two-part tariffs can be used by upstream competitors when attempting to endow with commitment power their downstream counterparts. See e.g. Rey and Tirole (1986) for a classic analysis. Saggi and Vettas (2002) show how the use of two part tariffs overturns the result obtained under linear tariffs, that duopolist suppliers would like to establish relations with a large number of distributors.

Corts (1998), Armstrong and Vickers (2001), Harrison and Kline (2001), Rochet and Stole (2002), Yin (2004) and more recently, Calzolari and Denicolò (2011) on quantity discounts.⁶

The remainder of the paper is as follows. In Section 2 we set up the basic model. In Section 3 we describe the equilibrium prices when firms have available only one pricing instrument, when firms charge two-part tariffs simultaneously and when they set their prices sequentially. In Section 4, we consider one of the two pricing instruments exogenously given and analyze the equilibrium in fees (for given rates) and in rates (for given fees). In Section 5, we extend the basic model to study how prices are affected when firms cannot set negative prices. In Section 6 we assume that customers can purchase one product from more than one firms. Section 7 concludes. The proofs are relegated to the Appendix unless they are needed for the understanding of the core arguments.

2 The basic model

We consider competition between two firms, A and B, selling products viewed by consumers as perfect substitutes – we say that firm i sells product i , $i = A, B$. Pricing by each firm i has two parts, a fixed fee f_i , to be paid by each consumer that chooses product i , and a rate r_i , that is to be applied to the usage of the product. Demand is represented by a continuum of consumers with total mass normalized to 1. Consumers differ with respect to their usage levels. In particular, each consumer is represented by his level of usage, θ , where we further assume that θ is uniformly distributed on $[\theta_L, \theta_H]$.⁷ Without any loss of generality, we set $\theta_L = 0$ and $\theta_H = 1$.

The goal of each firm is profit maximization – as we assume, for simplicity, zero costs, this amounts to maximization of the revenue from each product.⁸ Each consumer, on the other hand, chooses the product that leads to the lowest total payment: a consumer with

⁶Our analysis is also related to studies where consumers may wish in principle to purchase from more than one firms. Hoernig and Valletti (2007) model competition between differentiated products à la Hotelling under linear pricing, two-part tariffs and fixed fee. Armstrong and Vickers (2010) present a two dimensional Hotelling model where consumers differ in their preferences for a supplier and also in the quantity they want to purchase. In our model of homogeneous products, we show that mixing (that is, a purchase from more than one firms) will never occur in equilibrium even if in principle possible.

⁷We assume for simplicity that the usage level, θ , for a given consumer is fixed and inelastic with respect to the rate. The logic of the analysis could be extended to apply to cases where the usage rates respond to price changes but only by a little relatively to how much they vary across the population.

⁸It is straightforward to introduce a constant unit cost, without altering the main results.

usage level θ will choose the product of firm A (“product A”) if $f_A + \theta r_A < f_B + \theta r_B$ and product B otherwise. For the main part of the analysis, and consistent with the “discrete choice” literature, we assume that consumers choose to purchase one of the two products.⁹ There is no uncertainty.¹⁰

We shall consider various aspects of competition between the firms, depending on which price components are the strategic variables. Independently of which choices are endogenous, we have to first describe the firms’ profits as functions of the fees and rates charged. We distinguish three cases. Given the lack of product differentiation, for both firms to have a positive market share it should be that either the two firms charge identical prices or, if they charge different prices, that one firm charges a higher fee and the other charges a higher rate. More precisely:

Case 1. If a firm has both a higher fee and a higher rate than its rival, no consumer uses its product and that firm ends up with zero profit. In that case, the firm with the lower fee and lower rate has demand equal to the entire market. The profit of such a firm, say firm i , that captures the entire market by charging f_i, r_i is $\int_0^1 (f_i + \theta r_i) d\theta = f_i + r_i \int_0^1 \theta d\theta = f_i + \frac{r_i}{2}$.

Case 2. If both firms have equal fees and equal rates (say f and r) then assume that they split the market equally and have equal profits, $\frac{(f+r)}{2}$. *Case 3.* Suppose now that one firm charges a higher fee and the other a higher rate. Without loss of generality, let firm A be the one that has the higher rate, that is, $r_A \geq r_B$. When $r_A > r_B$ and $f_A < f_B$, a consumer with usage level θ is indifferent between product A and product B if $f_A + \theta r_A = f_B + \theta r_B$. We denote this indifference usage level by

$$\tilde{\theta} \equiv \frac{f_B - f_A}{r_A - r_B}. \quad (1)$$

Both firms have positive market shares if $\tilde{\theta} \in (0, 1)$. In such a case, firm A sells to

⁹This formulation involves two implicit assumptions. First, that the utility of having a product is high enough that no consumer would choose not to have any of the two. Second, that no consumer purchases both products. As long as prices are positive, this assumption follows from the fact that the products are perfect substitutes. If some prices are negative, then, in principle, consumers may wish to obtain both products in case the total payment for them is negative. In some markets such behavior may be possible (e.g. in some segments of the credit/payment cards market), while in other cases it is not (e.g. a traveler may drive only one rental car at a time, or an office may have space only for one photocopy machine). For completeness, we extend later in the paper our analysis to cover two alternative assumptions. In Section 5 we constrain all prices to be non-negative (an assumption which implies that no consumer would ever obtain both products), while in Section 6 consumers could choose both products.

¹⁰Introducing uncertainty into our analysis would likely generate additional insights, as would the analysis of dynamics, see e.g. an approach as in Griva and Vettas (2003).

consumers with $\theta \in [0, \tilde{\theta}]$, firm B to those with $\theta \in [\tilde{\theta}, 1]$ and the profit functions become

$$\begin{aligned}\pi_A &= \int_0^{\tilde{\theta}} (f_A + \theta r_A) d\theta = f_A \tilde{\theta} + r_A \frac{\tilde{\theta}^2}{2} = \\ &= f_A \frac{f_B - f_A}{r_A - r_B} + r_A \frac{(\frac{f_B - f_A}{r_A - r_B})^2}{2}\end{aligned}\quad (2)$$

and

$$\begin{aligned}\pi_B &= \int_{\tilde{\theta}}^1 (f_B + \theta r_B) d\theta = f_B(1 - \tilde{\theta}) + r_B \frac{1 - \tilde{\theta}^2}{2} = \\ &= f_B \left(1 - \frac{f_B - f_A}{r_A - r_B}\right) + r_B \frac{1 - (\frac{f_B - f_A}{r_A - r_B})^2}{2}.\end{aligned}\quad (3)$$

If $\tilde{\theta} \geq 1$, firm A captures the entire market with profit equal to $\int_0^1 (f_A + \theta r_A) d\theta = f_A + r_A \int_0^1 \theta d\theta = f_A + \frac{r_A}{2}$, while firm B's market share and profit are zero. On the other hand, it is important to observe that $\tilde{\theta} < 0$ can only occur when one firm has both a higher fee and a higher interest rate. If a firm has a lower fee than its rival, it guarantees itself some positive market share – the question then is whether its rate is higher enough than its rival's for the rival to also have a positive share. The reason that a lower fee implies a positive market share, regardless of the rates, *is that there are always some consumers with usage level θ close to zero* (recall that θ is uniformly distributed on $[0, 1]$). Consumers with θ equal to zero (and, by continuity, also these near zero) make their selection only on the basis of which product has the lower fixed fee. Since their usage level is very limited, such consumers do not pay attention to the per unit rates. So, in summary, the profit functions when $r_A > r_B$ and $f_A < f_B$ are

$$\pi_A = \begin{cases} f_A \frac{f_B - f_A}{r_A - r_B} + r_A \frac{(\frac{f_B - f_A}{r_A - r_B})^2}{2} & \text{if } \tilde{\theta} \in (0, 1) \\ f_A + \frac{r_A}{2} & \text{if } \tilde{\theta} \geq 1 \end{cases}$$

and

$$\pi_B = \begin{cases} f_B \left(1 - \frac{f_B - f_A}{r_A - r_B}\right) + r_B \frac{1 - (\frac{f_B - f_A}{r_A - r_B})^2}{2} & \text{if } \tilde{\theta} \in (0, 1) \\ 0 & \text{if } \tilde{\theta} \geq 1. \end{cases}$$

3 Equilibrium: preliminaries

Let us now explore some aspects of equilibrium behavior in the model described above. We start with simultaneous choices of the firms. First, we observe that:

Proposition 1 *Suppose pricing is via rates only (that is, fees cannot be used). Then the only equilibrium of the game where firms choose their rates simultaneously is $r_A^* = r_B^* = 0$,*

with equilibrium profits zero. Similarly, suppose pricing is via fixed fees only (that is, rates cannot be used). Then the only equilibrium of the game where firms choose their fees simultaneously is $f_A^* = f_B^* = 0$ with equilibrium profits zero.

Proof: The proof is simple. Essentially, when only one pricing instrument exists, we are in a world of one-dimensional price competition with homogeneous goods. Standard Bertrand arguments then imply that the only equilibrium is with prices equal to cost and profits equal to zero. If a given firm charges a fee strictly above cost (that is, zero in our case) then the other firm would maximize its profit by slightly undercutting that fee – the same argument holds with respect to rates. ■

This is an important feature in our setting, that if price competition occurs via a single price instrument (either a fixed fee or a rate) competition drives profits to zero. What happens when both fees and rates can be used? When both instruments are chosen at the same time we have:

Proposition 2 *When firms are competing by setting fees and rates simultaneously, the only equilibrium is that firms set both prices equal to cost ($r_A^* = r_B^* = 0$, $f_A^* = f_B^* = 0$) and make zero profit.*

Proof: See the Appendix. ■

To obtain a better understanding of how the two firms compete when setting fees and rates simultaneously, Figures 1 and 2 are useful. Figure 1 presents the fees and profits for $r_A > r_B > 0$ and $f_A < f_B < 0$. Each of the two lines represents the total payment a consumer would make to obtain one of the two products. Thus, depending on their θ , consumers choose the product represented by the lower of the two lines. Indifference occurs where the two lines cross. From the viewpoint of the firms, for each customer they serve, the distance between the price line and the zero line represents the profit (or loss). As we can see, for $r_A > r_B > 0$ and $f_A < f_B < 0$, at least one firm makes necessarily non zero profit. Figure 2 presents the fees and profits for $r_A > 0 > r_B$ and $f_A < 0 < f_B$. As we can see, firm A can increase its profit by increasing its fee to $f_A = 0$.

Thus far, we have that competition leads to pricing at cost and to zero firms' profits when pricing is simultaneous and is expressed through fixed fees only, or through rates only, or through both fixed fees and rates. It is also natural to consider sequential moves. If firms make their choices sequentially rather than simultaneously, does the commitment implied allow them to obtain positive profit? Not surprisingly, we find that sequential moves is not a sufficient condition for both firms to enjoy positive profit. Like in the

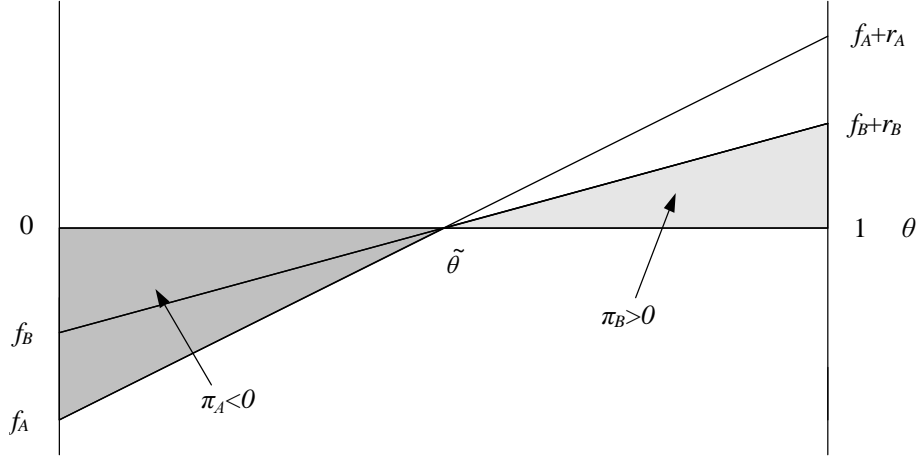


Figure 1: Profits for $r_A > r_B > 0$ and $f_A < f_B < 0$

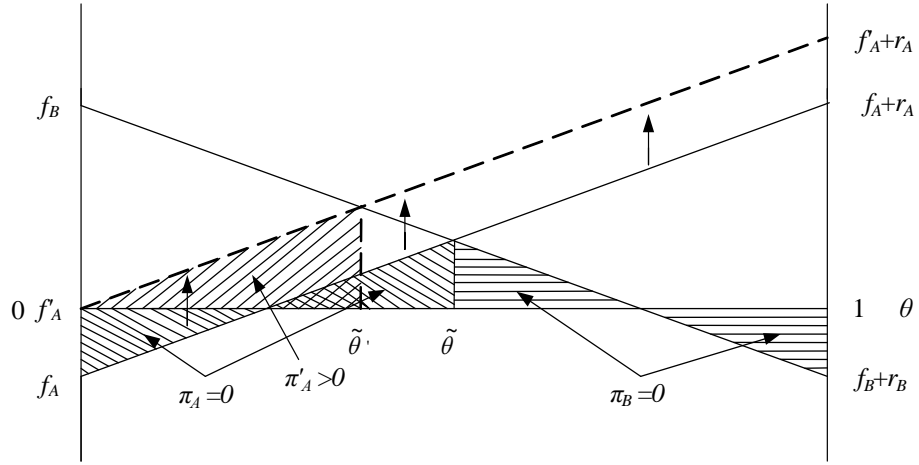


Figure 2: Profits for $r_A > 0 > r_B$ and $f_A < 0 < f_B$

case of simultaneous moves, a price undercutting incentive exists (although the arguments have to be appropriately modified). Formally, let us consider competition when firm A (the “leader”) chooses its prices before firm B (the “follower”). We are looking for a subgame perfect equilibrium. To simplify the statement of the following result let us assume that, when prices are equal, consumers choose to purchase from the follower.

Proposition 3 *Suppose that firms set their prices sequentially, so that (i) firms choose rates sequentially (and there are no fixed fees), (ii) firms choose fixed fees sequentially (and there are no rates) or (iii) firm A sets its fee and rate and then firm B sets its fee and rate. There is a continuum of equilibria at which the follower always matches the leader’s price (as long as that is non-negative; and prices at zero otherwise) and the leader sets some non-negative price. The follower always captures the entire demand and the leader makes zero profit.*

Proof: See the Appendix. ■

For a better understanding of Case (iii) see Figure 3. When $f_A + \theta r_A > 0$ for some

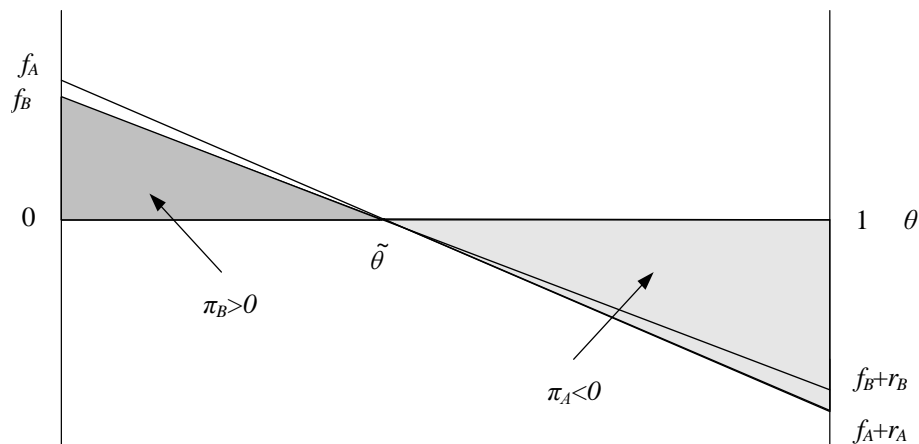


Figure 3: Profits when firms choose sequentially and $f_A + \theta r_A > 0$ for some $\theta \in (0, 1)$

$\theta \in (0, 1)$ and firm B wishes to capture the consumers that have $\theta \leq \tilde{\theta}$ (because these are the profitable ones), it will charge a fee slightly lower than f_A and a rate slightly higher than r_A and attract only the profitable consumers.

4 Equilibrium market segmentation with competition in one price component

Up to this point we have explored the equilibria of the game under alternative assumptions about the timing and when both price components are choice variables. We now explore the equilibrium of the game when one price component is set at some predetermined levels and competition takes place via the other component.

4.1 Competition in fixed fees

We proceed by analyzing the case with the fees as the strategic variables and taking the rates as exogenously given. We have:

Proposition 4 *Suppose that the rates are given as $r_A \geq r_B \geq 0$ and the firms choose their fees simultaneously. Then: (i) if $r_A = r_B = r$, the equilibrium fees are $f_A^* = f_B^* = -r/2$, total demand is divided equally between the two firms and both firms' profit is equal to zero, (ii) if $r_B < r_A < 2r_B$ there is no equilibrium in pure strategies and (iii) if $r_A \geq 2r_B$, total*

demand is divided equally between the two firms, the equilibrium fees are

$$f_A^* = -\frac{r_B}{2} \text{ and } f_B^* = \frac{1}{2}(r_A - 2r_B)$$

and firm B makes a higher profit than firm A.

Proof: (i) If $r_A = r_B = r$ and one firm had a higher fee than the other, then this firm would have no clients. Each firm has the incentive to slightly undercut its rival's fee in order to capture the entire market. This will lead both firms to set such fees that both profits are equal to zero and the market is equally divided. This case obeys the standard logic of a Bertrand competition model. In equilibrium, the fees are $f_A^* = f_B^* = -\frac{r}{2}$ and the profit for each firm is $\pi_i = \frac{1}{2}(f_i + \frac{r}{2}) = 0$. At this point, no firm will want to lower its fee because it will capture the entire market but will make losses, and either firm will be indifferent towards increasing its fee since it will lose all clientele and again make zero profit.

(ii) We check the second-order conditions and find that $\frac{\partial^2 \pi_A}{\partial f_A^2} = \frac{-r_A + 2r_B}{(r_A - r_B)^2} > 0$ for $r_B < r_A < 2r_B$, and therefore the profit function of firm A is strictly convex, while $\frac{\partial^2 \pi_B}{\partial f_B^2} = \frac{-2r_A + r_B}{(r_A - r_B)^2} < 0$ for $r_B < r_A < 2r_B$, and therefore the profit function of firm B is strictly concave. For $r_B < r_A < 2r_B$ the best response correspondence of firm A, given the fee of firm B, is:

$$f_A = R_A(f_B) = \begin{cases} f_B - r_A + r_B & \text{if } f_B \geq \frac{1}{2}(r_A - 2r_B) \\ \text{any fee} \geq f_B & \text{if } f_B < \frac{1}{2}(r_A - 2r_B) \end{cases}$$

and is derived as follows. The highest possible fee firm A can charge in order to attract all clientele is the one that, given f_B , will make $\theta = 1$, that is $\frac{f_B - f_A}{r_A - r_B} = 1$ and solving for f_A we obtain

$$f_A = f_B - r_A + r_B. \quad (4)$$

Since firm A has a convex profit function, it maximizes its profit when it serves the entire market, compared to when it shares it. Firm A has a positive profit when it attracts all clientele only if $f_B \geq \frac{1}{2}(r_A - 2r_B)$. If $f_B < \frac{1}{2}(r_A - 2r_B)$, firm A prefers to set $f_A \geq f_B$ and have no clients and zero profit, than to capture the entire market and make losses.

Next, we derive the best response correspondence of firm B, given firm A's fee:

$$f_B = R_B(f_A) = \begin{cases} f_A & \text{if } f_A > r_A - r_B \\ \frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B} & \text{if } -r_A \leq f_A \leq r_A - r_B \\ \text{any fee} \geq f_A + r_A - r_B & \text{if } f_A < -r_A. \end{cases}$$

This best response correspondence is derived as follows. If firm B wants to capture the entire market, the highest fee it can charge is the one that, given f_A , makes $\theta = 0$, that

is $\frac{f_B - f_A}{r_A - r_B} = 0$. Solving with respect to f_B , we obtain: $f_B = f_A$. When firm B shares the market with firm A, its reaction function, which is derived by setting $\frac{\partial \pi_B}{\partial f_B} = 0$ and solving with respect to f_B , is

$$f_B = R_B(f_A) = \frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B}. \quad (5)$$

This is the fee that maximizes firm B's profit when both firms operate in the market, that is when $\theta \in (0, 1)$, therefore $f_A \leq f_B \leq f_A + r_A - r_B \Rightarrow f_A \leq \frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B} \leq f_A + r_A - r_B$, which gives $-r_A \leq f_A \leq r_A - r_B$. If $f_A \geq r_A - r_B$, firm B maximizes its profit by charging $f_B = f_A$ and capturing the entire market.¹¹ If $f_A < -r_A$, firm B maximizes its profit ($\pi_B = 0$) by charging a fee that guarantees that no consumer would ever choose that firm, and this fee is any $f_B > (f_A + r_A - r_B)$.¹²

By combining these two best response correspondences we see that they never intersect. Firm A finds it profitable to either capture the entire market or have no clientele. It never aims at sharing the market with firm B. On the other hand, as long as firm A has some customers, firm B finds it more profitable to share the market with firm A. As a result, firm A can never capture all clientele.

(iii) If rates are positive and $r_A > 2r_B$, both profit functions are strictly concave since $\frac{\partial^2 \pi_A}{\partial f_A^2} = \frac{-r_A + 2r_B}{(r_A - r_B)^2} < 0$ and $\frac{\partial^2 \pi_B}{\partial f_B^2} = \frac{-2r_A + r_B}{(r_A - r_B)^2} < 0$.¹³ The best response correspondence for firm A, given firm B's fee, is

$$f_A = R_A(f_B) = \begin{cases} f_B - r_A + r_B & \text{if } f_B \geq r_A - 2r_B \\ \frac{-f_B r_B}{r_A - 2r_B} & \text{if } 0 < f_B < r_A - 2r_B \\ \text{any fee } \geq f_B & \text{if } f_B \leq 0 \end{cases}$$

and is derived as follows. The highest possible fee firm A can charge in order to capture all customers is $f_A = f_B - r_A + r_B$ (derived as in (4)). When firm A shares the market

¹¹If $f_A > r_A - r_B$, expression (5) is not the optimal reaction for firm B since $\theta = \frac{f_B - f_A}{r_A + r_B} = \frac{\frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B} - f_A}{r_A + r_B}$ becomes negative. If firm B charges $f_B = \frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B}$, it ends up capturing the entire market with a fee lower than the one that maximizes its profit.

¹²If $f_A < -r_A$, expression (5) is not the only optimal reaction for firm B because $\theta = \frac{f_B - f_A}{r_A + r_B} = \frac{\frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B} - f_A}{r_A + r_B}$ is greater than 1. If firm B charges $f_B = \frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B}$, it ends up having no clientele, as it would with any f_B greater than $(f_A + r_A - r_B)$. In order to have some clients firm B should charge such a low fee that it would make losses.

¹³If $r_A > r_B$ and $r_A \geq 0$, while $r_B \leq 0$, the same equilibrium values hold for any $r_A \geq 0$ and $r_B \leq 0$. Both profit functions are strictly concave since $\frac{\partial^2 \pi_A}{\partial f_A^2} = \frac{-r_A + 2r_B}{(r_A - r_B)^2} < 0$ and $\frac{\partial^2 \pi_B}{\partial f_B^2} = \frac{-2r_A + r_B}{(r_A - r_B)^2} < 0$ for any given $r_A > 0$ and $r_B < 0$. If $r_A > r_B$ and both r_A and r_B are negative, the same equilibrium values hold for $r_A > 2r_B$, since both profit functions are strictly concave for such negative rates.

with firm B, its reaction function, which is derived by setting $\frac{\partial \pi_A}{\partial f_A} = 0$ and solving with respect to f_A , is

$$f_A = R_A(f_B) = \frac{-f_B r_B}{r_A - 2r_B}. \quad (6)$$

This is the fee that maximizes firm A's profit when both firms operate in the market, that is when $\theta \in (0, 1)$, therefore $f_B - r_A + r_B < f_A < f_B \Rightarrow f_B - r_A + r_B < \frac{-f_B r_B}{r_A - 2r_B} < f_B$, which gives $0 < f_B < r_A - 2r_B$. If $f_B \geq r_A - 2r_B$, firm A makes a higher profit by charging $f_A = f_B - r_A + r_B$ and capturing the entire market.¹⁴ If $f_B < 0$ firm A maximizes its profit ($\pi_A = 0$) by charging a fee that guarantees that no consumer would ever choose that firm, and this fee is any $f_A \geq f_B$.¹⁵

Next, we derive the best response correspondence of firm B, given the fee of firm A:

$$f_B = R_B(f_A) = \begin{cases} f_A & \text{if } f_A \geq r_A - r_B \\ \frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B} & \text{if } -r_A < f_A < r_A - r_B \\ \text{any fee} \geq f_A + r_A - r_B & \text{if } f_A \leq -r_A \end{cases} \quad (7)$$

This best response correspondence is derived as in the previous case. The two best response correspondences intersect when $f_A^* = -\frac{r_B}{2}$ and $f_B^* = \frac{1}{2}(r_A - 2r_B)$.

Substituting the values of f_A^* and f_B^* into (1) we obtain $\theta_1^* = \frac{1}{2}$. Moreover, substituting f_A^* and f_B^* into (2) and (3), we obtain

$$\pi_A^* = \frac{1}{8}(r_A - 2r_B) \quad (8)$$

and

$$\pi_B^* = \frac{1}{8}(2r_A - r_B). \quad (9)$$

As we can see, $\pi_B^* > \pi_A^*$ if $r_A + r_B > 0$. ■

We have shown that when rates are positive and $r_A > 2r_B$ the two firms' best response correspondences intersect and there exist a unique equilibrium where $f_A^* = -\frac{r_B}{2}$ and $f_B^* = \frac{1}{2}(r_A - 2r_B)$. This is illustrated in Figure 4.

Some remarks on the properties of the equilibrium are in order here. First, for $r_A > 2r_B$, the firm with the higher rate (firm A) chooses in equilibrium a lower fee than its rival. Second, the equilibrium fee of firm B is non-negative. In contrast, the equilibrium fee of

¹⁴When $f_B \geq r_A - 2r_B$, expression (6) gives $\theta = \frac{f_B - f_A}{r_A + r_B} = \frac{f_B - (\frac{-f_B r_B}{r_A - 2r_B})}{r_A + r_B}$, which is greater than 1. As a result, if firm A sets $f_A = \frac{-f_B r_B}{r_A - 2r_B}$, it ends up capturing the entire market with a fee lower than the one that maximizes its profit. Firm B can make a higher profit by charging a fee that gives $\theta = 1$.

¹⁵If $f_B < 0$, then $f_A = \frac{-f_B r_B}{r_A - 2r_B}$ is not the only optimal reaction of firm B because θ becomes smaller than 0. If firm A charges $f_A = \frac{-f_B r_B}{r_A - 2r_B}$, it ends up having no clientele, as it would with any fee greater or equal to f_B . If firm A decides to have some clients, it makes losses.

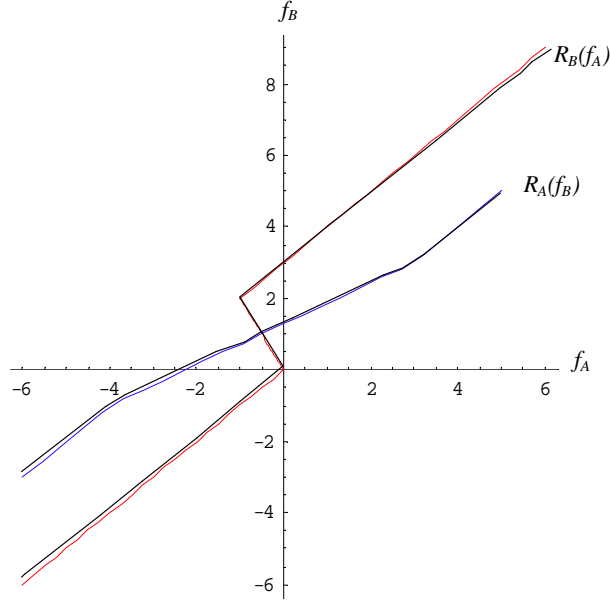


Figure 4: Both firms' best response correspondences when $r_A = 4$ and $r_B = 1$

firm A is negative. Note that if the firms have a per unit cost equal to c , the equilibrium fee for firm A would be $f_A = c - \frac{r_B}{2}$ and for $c > \frac{r_B}{2}$ the fee would be positive. Therefore, the fees in our formulation appear to be negative because we have assumed unit cost equal to zero.¹⁶ Third, the firm that obtains the higher equilibrium profit is the one that charges the relatively lower rate and higher fee, in other words, the one that attracts the consumers who tend to have a higher usage level.

4.2 Properties and comparison to Shaked and Sutton (1982)

We can gain some further intuition for our analysis by drawing some analogies to the well-known work of Shaked and Sutton (1982). They offer a model of vertical product

¹⁶If we allow for a more general uniform distribution where θ is distributed on $[\theta_L, 1 + \theta_L]$, then $f_A^* = \frac{1}{2}(-r_B - 2r_A\theta_L)$. If θ_L can take negative values, then the equilibrium fee of firm A can be positive. Parameter θ_L can take negative values if we assume, for example, that the holders of a credit card who use their card only as a mean of payment and not in order to finance their purchases, benefit from the interest-free period between the day of purchase and the day they pay the bill. If they were to pay cash, their deposits in some bank account would have been reduced by the amount of the purchase, the day of the purchase. When they pay by card, they continue to earn the interest for this amount, although they have purchased the products or services. They continue to earn interest until the day they pay the bill for their card, which, according to present practice, can be up to 50 days after the purchase. As a result, the banks that have issued their cards pay to these “non-revolvers”, the interest on the amount of the purchases, every time they use their card only for payment.

differentiation (quality) in which consumers are heterogeneous with respect to their taste for quality, as in our model they are with respect to their usage levels. In their model, each product is priced linearly, that is, there is a given price for each unit of the product. Here, in contrast, a consumer takes into consideration both fees and rates. When firms take product quality as given and compete via linear prices, Shaked and Sutton (1982) show that the firm selling the higher quality charges a higher price than its rival, captures a larger market share (*twice* as large assuming preferences are uniformly distributed in $[0, 1]$), and enjoys higher profit. In our formulation, when rates are given (and are different enough so that $r_A > 2r_B$), in equilibrium, the firm that has been assigned the lower rate and therefore attracts the consumers with the higher usage levels, charges a higher fee than its rival, captures *equal* market share with its rival, and enjoys higher profit. This difference in the equilibrium of market shares is due to the different role that heterogeneity plays in the two models.

In Shaked and Sutton (1982), the heterogeneity of consumers with respect to quality makes each of them prefer a different firm but once a consumer has chosen a firm, no matter which one, he purchases one unit of the product by paying a fixed price. In our model, the heterogeneity of consumers with respect to their usage levels enters at two levels. Firstly, consumers, depending on their usage level, choose one of the two firms and obtain one product by paying the corresponding fixed fee f_i . Secondly, consumers, after obtaining the product they have chosen, use it at a different level $\theta \in [0, 1]$ by paying “price” r_i per unit of use. Formally, in Shaked and Sutton (1982) there is a distribution θ in the population of consumers, a given consumer chooses firm i to maximize $\theta q_i - p_i$, while the firm collects p_i . In our formulation, given the distribution θ , each consumer minimizes $f_i + \theta r_i$, which is also the amount that the firm collects.

It is important that in our analysis, like in Shaked and Sutton (1982), we obtain a result of maximal differentiation. In our model, the firm that attracts the consumers who tend to have a higher usage level gains from reducing its rate to the minimum, because this softens the competition through the fees. In Shaked and Sutton (1982), the low quality firm gains from reducing its quality, because this softens price competition.

Formally, since rates are taken as given, we can ask what happens when they change exogenously:

Remark 1 *For $r_A > 2r_B$, in equilibrium, both firms’ fees as well as both firms’ profits increase as the distance between the two rates increases.*

Proof: To see how the profits and the fees are affected by an increase in the rates we

simply differentiate with respect to these parameters of the problem:

$$\begin{aligned}\frac{\partial \pi_A^*}{\partial r_A} &= \frac{1}{8} \text{ and } \frac{\partial f_A^*}{\partial r_A} = 0, \\ \frac{\partial \pi_A^*}{\partial r_B} &= -\frac{1}{4} \text{ and } \frac{\partial f_A^*}{\partial r_B} = -\frac{1}{2}, \\ \frac{\partial \pi_B^*}{\partial r_A} &= \frac{1}{4} \text{ and } \frac{\partial f_B^*}{\partial r_A} = \frac{1}{2}\end{aligned}$$

and

$$\frac{\partial \pi_B^*}{\partial r_B} = -\frac{1}{8} \text{ and } \frac{\partial f_B^*}{\partial r_B} = -1. \quad (10)$$

We see that both firms' profits and fees increase as r_A increases and as r_B decreases. ■

4.3 Competition in rates

Now, we reverse the roles and examine the case where the fees are exogenously given and the rates are the strategic variables. We obtain:

Proposition 5 *Consider the case where the fees are taken as given and firms choose their rates simultaneously. (i) If $f_A = f_B = f$, then in equilibrium the rates are*

$$r_A^* = r_B^* = -2f$$

and total demand is divided equally between the two firms. (ii) If $f_A < f_B$ we distinguish three cases: (a) For $f_B > 0$ and $\frac{1}{2}f_B \leq f_A < f_B$, there is no equilibrium in pure strategies. (b) For $f_B > 0$ and $-f_B < f_A \leq \frac{1}{2}f_B$, the equilibrium rates are

$$r_A^* = \frac{-3f_A + f_B}{\sqrt{2}} \text{ and } r_B^* = -\frac{f_A + f_B}{\sqrt{2}}. \quad (11)$$

Firm A has a larger market share than firm B since $\theta^ = \frac{1}{\sqrt{2}}$. For $(-3 + 2\sqrt{2})f_B < f_A \leq \frac{1}{2}f_B$, firm A makes a higher profit than firm B, while for $-f_B < f_A < (-3 + 2\sqrt{2})f_B$, firm B makes a higher profit than firm A. (c) For $f_B > 0$ and $f_A \leq -f_B$, or $f_A, f_B < 0$, there is no equilibrium in pure strategies.*

Proof: See the Appendix. ■

Again, it is interesting to examine how an exogenous change in the fees affects profits. We obtain:

Remark 2 *For $f_B > 0$ and $-f_B \leq f_A \leq \frac{1}{2}f_B$, firm A's equilibrium profit increases the higher both fees become, while firm B's equilibrium profit increases as the distance between the fees increases.*

Proof: To see how the profits are affected by an increase in the fees, we differentiate the relevant equilibrium expressions (see the proof of Proposition 5 in the Appendix) with respect to f_A and f_B :

$$\begin{aligned}\frac{\partial \pi_A^*}{\partial f_A} &= \frac{1}{4\sqrt{2}} \text{ and } \frac{\partial \pi_A^*}{\partial f_B} = \frac{1}{4\sqrt{2}}, \\ \frac{\partial \pi_B^*}{\partial f_A} &= \frac{-1}{4\sqrt{2}} \text{ and } \frac{\partial \pi_B^*}{\partial f_B} = 1 - \frac{5}{4\sqrt{2}}.\end{aligned}$$

Clearly, firm A' profit increases as each of the fees increases. Since $f_B > 0$ and $-f_B \leq f_A \leq \frac{1}{2}f_B$, firm A's profit will be maximized when $f_A = \frac{1}{2}f_B$. Firm B's profit increases as f_A decreases and as f_B increases. As a result, firm B's profit will be maximized when $f_A = -f_B$. ■

5 Non-negative fees and rates

Now, we analyze the case where fees and rates can only take non-negative values. This is relevant in markets where for institutional or competition reasons no price component can be negative. We first examine the case where the rates are exogenously given and the fees are the strategic variables. We have:

Proposition 6 *When fees and rates can only take non-negative values, then for $r_A \geq r_B$ the equilibrium fees are*

$$f_A^* = 0 \text{ and } f_B^* = \frac{(r_A - r_B)^2}{2r_A - r_B}.$$

If $r_A > r_B$, firm B has a larger market share than firm A and the profits are

$$\pi_A^* = \frac{r_A(r_A - r_B)^2}{2(-2r_A + r_B)^2} \text{ and } \pi_B^* = \frac{r_A^2}{4r_A - 2r_B}.$$

If $r_A = r_B = r$, the two firms equally share the market by charging zero fees and both have customers from the entire range of θ . In this case, the profits are $\pi_A^ = \pi_B^* = r/4$.*

Proof: The best response correspondence of firm A is

$$f_A = R_A(f_B) = \begin{cases} f_B - r_A + r_B & \text{if } f_B \geq r_A - r_B \\ 0 & \text{if } 0 \leq f_B < r_A - r_B \end{cases} \quad (12)$$

and is derived as follows. The highest possible fee firm A can charge in order to capture the entire market is $f_A = f_B - r_A + r_B$. Firm A can only charge this price if it is non-negative, that is when $f_B \geq r_A - r_B$. If $f_B < r_A - r_B$, firm A makes a higher profit by sharing the market with firm B, since it can not charge the fee that maximizes its profit

(it is negative). Instead, it charges the highest possible fee it can in order to maximize its profit, which is $f_A = 0$.

The best response correspondence of firm B is

$$f_B = R_B(f_A) = \begin{cases} f_A & \text{if } f_A \geq r_A - r_B \\ \frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B} & \text{if } 0 \leq f_A < r_A - r_B. \end{cases} \quad (13)$$

This best response correspondence is derived the same way as in (7), without the possibility of a negative f_A . These two best response correspondences intersect when $f_A^* = 0$ and $f_B^* = \frac{(r_A - r_B)^2}{2r_A - r_B}$. Substituting the equilibrium fees into (1) we obtain that $\theta^* = \frac{r_A - r_B}{2r_A - r_B}$. For $r_A > r_B$, we have $\theta^* < \frac{1}{2}$ and therefore firm B has a larger market share than firm A. ■

Remark 3 For $r_A > r_B$, firm A's equilibrium profit increases as the distance between the given rates increases, while firm B's profit increases as the distance between the rates decreases. For $r_A \geq r_B$, firm A's equilibrium profit is maximized when $r_A = r_B$, while firm B prefers the two rates to be slightly differentiated.

Proof: See the Appendix. ■

If we compare each firm's profit when they can charge only non-negative fees (denoted as π_i^{*pf}), with when they can charge any fee (denoted as π_i^{*af}), we see that if firms can only charge non-negative fees, firm A makes a higher profit since $\pi_A^{*pf} - \pi_A^{*af} = \frac{4r_A^2 r_B - 5r_A r_B^2 + 2r_B^3}{8(2r_A - r_B)^2} > 0$ for $r_A \geq r_B$. When firms charge non-negative fees, firm A has a smaller market share compared to the case when firms can charge any fee. Firm B also makes a higher profit when it can charge only a non-negative fee since $\pi_B^{*pf} - \pi_B^{*af} = \frac{4r_A r_B - r_B^2}{8(2r_A - r_B)} > 0$ for $r_A \geq r_B$, and has a larger market share compared to the case when it can charge any fee.

We recall that from the previous section when fees can take any value and $r_A \geq 2r_B$, both firms prefer maximal differentiation of the rates, while when fees can take only non-negative values and $r_A > r_B$, firm A prefers maximal differentiation of the rates and firm B prefers minimal differentiation. When fees can take any value, we have $\theta^* = \frac{1}{2}$, therefore the two firms share the market equally. Both profits are affected by the magnitude of the given rates only through prices, not through the market shares. In this case an increase in r_B by one unit (see relation (10)) decreases f_B by one unit, while the market share of firm B remains unchanged. Firm B collects the fee from all customers, while it collects, on average, $\frac{3r_B}{4}$ from each client. As a result firm B would prefer a minimum r_B in order to increase its profit from the revenues it gets from the fees.

When firms can charge only non-negative fees, we have $\theta^* = \frac{r_A - r_B}{2r_A - r_B}$. Consequently, a change in the given rates changes the profits of both firms not only through prices but

also through the market shares. In this case, an increase in r_B increases the market share of firm B (since now the two rates are less differentiated and fewer customers will choose firm A) and decreases f_B , since $\frac{df_B}{dr_B} = -\frac{(r_A-r_B)(3r_A-r_B)}{(-2r_A+r_B)^2} < 0$. In total, firm B's profit will increase as r_B increases, since the profit from the increase in firm B's market share outweighs the loss from the decrease in f_B . As a result, firm B prefers a maximum r_B (formally r_B to approach r_A) in order to maximize its profit.

Now, we consider the case where the rates are the strategic variables and the fees are exogenously given.

Proposition 7 *When fees and rates can only take non-negative values then, for $f_B \geq f_A$, the equilibrium rates are*

$$r_A^* = f_B - f_A \text{ and } r_B^* = 0,$$

and firm A captures the entire market.

Proof: See the Appendix. ■

For $f_B > f_A$, we substitute the equilibrium rates into (2) and (3) and find that the equilibrium profits are $\pi_A^* = (f_A + f_B)/2$ and $\pi_B^* = 0$. If $f_B = f_A = f$, then $\pi_A^* = \pi_B^* = f/2$ and the two firms share the market and have customers from the entire range of θ .

Remark 4 *If the given fees are set at different levels ($f_B > f_A$), firm A's profit increases as both fees increase, so firm A prefers minimum differentiation of fees while firm B is indifferent. If $f_B \geq f_A$ firm A prefers the two fees to be slightly differentiated while firm B prefers $f_B = f_A$.*

Proof: In order to see how firm A's profit is affected by an increase in the given fees, we differentiate the equilibrium profit with respect to each fee. For $f_B > f_A$ we obtain

$$\frac{d\pi_A^*}{df_A} = \frac{d\pi_A^*}{df_B} = \frac{1}{2}.$$

We see that firm A's profit increases as both fees increase. So firm A maximizes its profit when $f_A \rightarrow f_B$, so $\pi_A^* \rightarrow f_B$. Firm B never manages to have any clients when $f_B > f_A$. When $f_B = f_A = f$, we find that $\pi_A^* = \pi_B^* = \frac{f}{2}$, so we conclude that firm A's profit is smaller when the two fees are the same compared to when $f_B > f_A$, while firm B's profit is higher. Therefore, firm A maximizes its profit when the two fees are slightly differentiated, while firm B maximizes its profit when $f_B = f_A$. ■

By comparison to the case where firms can charge any rate, we observe that if firms can only charge non-negative fees and rates, firm A makes a higher profit and has a higher

market share, since it captures the entire market. In contrast, firm B makes a higher profit when it can charge a negative rate because, when it is limited to charging only non-negative rates, it loses all its clientele.

6 Consumers could purchase one product from each of the firms

In this section we analyze the case where consumers can purchase one unit of each product, when the rates are set at positive levels. Consumers here do not choose to purchase two products in order to use them both, they purchase both products only if this minimizes their total payment. Therefore, this can happen only if firms charge negative fees. We take the positive rates as given and let the fees be the strategic variables. We have:

Proposition 8 *When fees can take any value, rates are positive and consumers can purchase more than one products, then in equilibrium, no firm chooses to charge a negative fee. For $r_A \geq r_B$, the equilibrium fees are*

$$f_A^* = 0 \text{ and } f_B^* = \frac{(r_A - r_B)^2}{2r_A - r_B},$$

with profit levels

$$\pi_A^* = \frac{r_A(r_A - r_B)^2}{2(-2r_A + r_B)^2} \text{ and } \pi_B^* = \frac{r_A^2}{4r_A - 2r_B}.$$

If $r_A > r_B$, firm B has a larger market share than firm A, otherwise, if $r_A = r_B$, the two firms share the market equally and make positive profits $\pi_A^ = \pi_B^* = \frac{r}{4}$.*

Proof: See the Appendix. ■

We see that when consumers can in principle purchase more than one products, no firm will ever choose in equilibrium a negative fee. It follows that in this case, the analysis coincides with the analysis in the previous section when firms can charge only non-negative prices.

7 Conclusion

We have studied a simple duopoly model with the following features. Two firms offer homogeneous products and consumers differ with respect to how much they use the product they will purchase. Pricing takes the form of two-part tariffs and we explore various

possibilities. When firms choose both price components simultaneously, the Bertrand logic prevails and both firms make zero profit. When the two firms charge their prices sequentially, only the follower could make positive profit. Our main focus is on the case when one of the two pricing components is exogenously determined and firms compete via the other component. If and only if one price component has been fixed at levels that differ enough between the two firms, then both firms make positive profits in equilibrium. The market is segmented: consumers with a relatively low usage level choose the low fee firm, while consumers with a relatively high usage level choose the rival firm. So, although the products are homogeneous, in this equilibrium, fixing one of the price components leads to endogenous segmentation of the market with both firms making positive profit.

In the case where the rates are given, we show that, in equilibrium, the firm with the higher rate, charges a negative fee (or, more generally, one lower than its cost). Despite the fact that one price component is negative, this firm makes positive profit overall. The more differentiated the rates, the higher is the equilibrium profit for *both* firms; this is because competition via the fees becomes softer. Essentially, fixing one of the price components, when consumers differ with respect to their usage, makes competition operate as if the products were differentiated, when in fact they are not. Moreover, a Shaked and Sutton (1992) type logic emerges here since firms prefer to maximize the distance between them with respect to the price component that has been fixed.

We extend the basic model by examining the equilibrium when firms can set only non-negative prices. We characterize why this case is identical to the case where consumers could possibly purchase one product from more than one firms. Then, no firm charges a negative price and as a result, no consumer in fact chooses to purchase more than one products. When rates are given and are different enough, the firm with the high rate makes zero profit and has a smaller market share than its rival. When fees are given, the firm with the low fee captures the entire market.

We have kept our model as simple as possible though the analysis itself is not trivial. Our goal here is to examine in some depth the distinct role that each component plays in two-part tariff competition, as well as that of the differentiation of the consumers along their usage rates. We stress that fixing one price component indirectly introduces some elements of product differentiation, an important aspect of the problem and not yet exploited in the literature. We explore when this may happen and the implications for competition. These features of the problem should be kept in mind by competing firms as well as by market regulators when they are able to dictate one component of a two-part tariff. Hopefully our analysis contributes to an important and growing literature on

duopoly competition via two-part tariffs.

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Appendix

Proof of Proposition 2: Let us first observe that $r_A^* = r_B^* = 0$ and $f_A^* = f_B^* = 0$ is indeed an equilibrium. Assuming that one of the firms, say firm B, sets $r_B = f_B = 0$, it should be clear that there is no (f_A, r_A) combination that allows firm A to enjoy a strictly positive profit. The reason is that since the $r_B = f_B = 0$ combination is available, no

consumer would prefer instead the (f_A, r_A) combination unless it represented a negative net payment (i.e. a subsidy) for him (equal to $f_A + r_A\theta$). But this payment is equal to the profit for firm A. Thus, by integrating over all the consumers that would accept such a combination, we find that any (f_A, r_A) combination that is accepted by some consumers leads to a loss (or, at best to zero profit) for firm A. Thus we reach the conclusion that $r_A = f_A = 0$ is a best response to $r_B = f_B = 0$.

Now we turn to examining if any other equilibria exist. First, we cannot have an equilibrium with $r_A = r_B > 0$ and $f_A = f_B > 0$. In such a case the firms would share the market with positive profits and a profitable deviation exists for any firm that, by slightly lowering its fee or both of its prices, can capture the entire market and increase its profit.

Second, suppose that one firm, say A, sets both prices higher than its rival, firm B. That would imply that firm A makes zero profit. It is not possible to have such an equilibrium if firm B makes positive profit because then firm A could slightly undercut firm B's prices and increase its profit from zero to some positive level. But it is also not possible to have such an equilibrium if firm B makes zero profit or negative profit, because then firm B could increase its profit by raising its prices.

Finally, it remains to examine the case where one firm charges a higher fee and the other firm charges a higher rate, say, $r_A > r_B$ and $f_A < f_B$. Then we distinguish four possibilities. If one firm makes strictly positive profit and strictly higher than its rival, then the rival has a profitable deviation. It can increase its profit by setting both its prices at levels slightly lower than the firm that has the higher profit. By doing so, it will capture the entire market and obtain profit higher than its rival's and, *a fortiori*, higher than what the deviating firm itself would have at the candidate equilibrium. If the two firms make the same positive profit, a similar argument applies: each firm can slightly undercut the other and increase its profit. If one firm (or both) makes negative profit, that firm has a profitable deviation in setting both prices equal to zero. Thus, the only remaining possibility is to have $r_A > r_B$ and $f_A < f_B$ with $\pi_A = \pi_B = 0$. The first step is to observe that either $r_A > r_B > 0$ and $f_A < f_B < 0$, or $0 > r_A > r_B$ and $0 < f_A < f_B$ cannot constitute an equilibrium with $\pi_A = \pi_B = 0$, as in each of these cases at least one firm makes necessarily non zero profit. So we are left with $r_A > 0 > r_B$ and $f_A < 0 < f_B$. Suppose we have such a case and both firms make zero profit. Then firm A can increase its profit by increasing its fee to $f_A = 0$. As it then still has a lower fee than its rival, it captures part of the market (consumers with θ close to zero) and since $r_A > 0$ this deviation leads to strictly positive profit and is thus profitable. ■

Proof of Proposition 3: (i) and (ii) Assume firm A sets its price (fee or rate) first and firm B takes this price as given. The equilibrium behavior of firm A is to set any $p_A \geq 0$ and the best response correspondence of firm B is

$$p_B = R_B(p_A) = \begin{cases} p_A & \text{if } p_A \geq 0 \\ 0 & \text{if } p_A < 0. \end{cases}$$

Firm B's best response correspondence is derived as follows. Given firm A's price, firm B can capture the entire market by charging the same price with firm A or can charge a higher price and have no clientele. Firm B will never charge a price smaller than p_A , since it can capture the entire market by charging the same price with firm A. Firm B will choose to attract all clientele as long as it makes a non-negative profit and this happens for $p_A \geq 0$. If $p_A < 0$, firm B prefers to have no clientele than to attract all the customers and make losses.

Firm A's equilibrium behavior is derived as follows. Firm A, which sets its price first, takes the best response correspondence of firm B as given and tries to maximize its profit. Firm A knows that if it sets $p_A \geq 0$ firm B will set the same price and leave firm A without clientele and with zero profit. If it sets $p_A < 0$, firm B will charge a higher price in order to have no clientele and make zero profit. Firm A will then have captured the entire market but it will make losses equal to p_A . Therefore firm A prefers to charge any non-negative price and have zero profit than to charge a negative price and make losses.

(iii) We distinguish three cases. Case (1) If $f_A + \theta r_A \geq 0$ for all $\theta \in [0, 1]$, firm B wishes to attract the entire market and does so by charging $(f_B, r_B) = (f_A, r_A)$. Case (2) If $f_A + \theta r_A > 0$ for some $\theta \in (0, 1)$, firm B wishes to attract the customers that are profitable, that is, only those with θ such that $f_A + \theta r_A > 0$. Given that $f_A + \theta r_A$ is linear with respect to θ , there is a threshold $\tilde{\theta} \in (0, 1)$ such that either $f_A + \theta r_A \geq 0$ for all $\theta \leq \tilde{\theta}$, or $f_A + \theta r_A \geq 0$ for all $\theta \geq \tilde{\theta}$. For the case where firm B wants to capture the consumers with $\theta \leq \tilde{\theta}$ (respectively the consumers with $\theta \geq \tilde{\theta}$), it will charge a fee slightly lower than f_A (respectively, higher) and a rate slightly higher than r_A (respectively, lower) and attract only the profitable consumers. Case (3) If $f_A + \theta r_A \leq 0$ for all $\theta \in [0, 1]$, firm B does not wish to have any customers since this would imply a strictly negative profit. In this case, firm B's best response is to charge $(f_B, r_B) > (f_A, r_A)$.

Firm A's equilibrium behavior is derived as follows. Firm A, which sets its rate and fee first, takes the best response correspondence of firm B as given and tries to maximize its profit. Firm A knows that if it sets a pair of prices such that $f_A + \theta r_A \geq 0$ for every $\theta \in [0, 1]$, firm B sets the same prices and leaves firm A with no clientele and zero profit. If it sets a pair of prices such that $f_A + \theta r_A \geq 0$ for some $\theta \in (0, 1)$, firm B will attract

all the profitable consumers and leave firm A with the consumers that have θ such that $f_A + \theta r_A < 0$. Finally, if firm A sets a pair of prices such that $f_A + \theta r_A \leq 0$ for all $\theta \in [0, 1]$, firm B leaves firm A to attract all clientele by charging both a higher fee and rate. In this case, firm A makes losses. As a result, in equilibrium, firm A charges any pair (f_A, r_A) such that $f_A + \theta r_A \geq 0$ for every $\theta \in [0, 1]$ and has no customers, and firm B charges $(f_B, r_B) = (f_A, r_A)$, attracts the entire market and makes $\pi_B \geq 0$. ■

Proof of Proposition 5:

(i) Suppose $f_A = f_B = f$. This case follows partially the standard Bertrand case since, in equilibrium, profits are zero but the rate is not equal to cost (which is zero in our case). Each firm has an incentive to slightly undercut its rival's rate in order to capture the entire market. This leads both firms to set rates that make their profits equal to zero and the market is equally split between the two products. No firm can capture the entire market and have positive profit since the rival firm can set the same rate and gain half of the market. The equilibrium profit for each firm is $\pi_i = \frac{1}{2}(f + \frac{r_i}{2}) = 0$ and therefore, the equilibrium rates are $r_A^* = r_B^* = -2f$. For $r_A^* = r_B^* = -2f$, no firm wants to deviate. If a firm decreases its rate, it will capture the entire market but will make losses. A firm is indifferent towards increasing its rate since it will lose all clientele and again make zero profit.

(ii) Suppose $f_A < f_B$. First note that, in this case, firm A always attracts the customers with usage level equal to zero, no matter what the rates are. Let us investigate how the two profit functions behave. Firm A's second-order condition with respect to its own rate is

$$\frac{\partial^2 \pi_A}{\partial r_A^2} = -\frac{(f_A - f_B)(f_A(r_A - 4r_B) + f_B(r_A + 2r_B))}{(r_A - r_B)^4} \quad (A.1)$$

and firm B's is

$$\frac{\partial^2 \pi_B}{\partial r_B^2} = -\frac{(f_A - f_B)(f_B(-4r_A + r_B) + f_A(2r_A + r_B))}{(r_A - r_B)^4}. \quad (A.2)$$

Let us first note that $(f_A - f_B)$ is always negative for $f_A < f_B$. In order for equation (A.1) to be negative, and therefore firm A's profit function to be concave, the following must hold: if $f_B, f_A > 0$ and $\frac{1}{2}f_B < f_A < f_B$, then

$$r_B > \frac{r_A(f_A + f_B)}{2(2f_A - f_B)}$$

and if $f_B > 0$ and $f_A < \frac{1}{2}f_B$ or $f_B, f_A < 0$, then

$$r_B < \frac{r_A(f_A + f_B)}{2(2f_A - f_B)}.$$

In order for equation (A.2) to be negative, and therefore firm B's profit function to be concave, the following must hold: if $f_B, f_A > 0$, or $f_B > 0$ and $-f_B < f_A < f_B$, then

$$r_B < \frac{2r_A(2f_B - f_A)}{(f_A + f_B)}$$

and if $f_B > 0$ and $f_A < -f_B$ or $f_B, f_A < 0$, then

$$r_B > \frac{2r_A(2f_B - f_A)}{(f_A + f_B)}.$$

As we can see, when we are looking for an equilibrium, we have to investigate the responses of the two firms in three different cases: for $f_B > 0$ and $\frac{1}{2}f_B \leq f_A < f_B$, for $f_B > 0$ and $-f_B < f_A \leq \frac{1}{2}f_B$ and for $f_B > 0$ and $f_A \leq -f_B$, or $f_A, f_B < 0$.

In order to grasp a basic understanding of how the profit functions behave, we take the first-order condition of each profit function, set it equal to zero and solve it with respect to its own rate. For firm A we have that

$$r_A = R_A(r_B) = \frac{(3f_A - f_B)r_B}{f_A + f_B}. \quad (A.3)$$

Since we have only one solution we conclude that firm A's profit function is strictly concave or strictly convex depending on the given fees. For firm B we have that

$$r_B = R_B(r_A) = \begin{cases} r_A + \frac{-3(f_A^2 - f_B^2) + (-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{2/3}}{3(-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{1/3}} \\ r_A + \frac{(3+3i\sqrt{3})(f_A^2 - f_B^2) + i(i+\sqrt{3})(-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{2/3}}{6(-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{1/3}} \\ r_A + \frac{(3-3i\sqrt{3})(f_A^2 - f_B^2) - (1+i\sqrt{3})(-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{2/3}}{6(-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{1/3}} \end{cases} \quad (A.4)$$

where $i \equiv \sqrt{-1}$. As we can see, two of the roots are complex numbers, but depending on the values of f_A and f_B , the imaginary part may be edited out and we may end up with two or three real numbers. Since we have three solutions we conclude that when all solutions are acceptable, so as $\tilde{\theta} \in (0, 1]$, firm B's profit function becomes from concave convex or the opposite, as r_B increases.

We now explore the existence of equilibrium of rates, when the fees are given, in the three specific ranges deduced from the second-order conditions above. Analytically, for $f_A < f_B$ we have three distinct cases:

(a) If $f_B > 0$ and $\frac{1}{2}f_B \leq f_A < f_B$, the best response correspondence of firm A is

$$r_A = R_A(r_B) = \begin{cases} r_B + f_B - f_A & \text{if } r_B \geq -\frac{f_A + f_B}{2} \\ \frac{(3f_A - f_B)r_B}{f_A + f_B} & \text{if } r_B \leq -\frac{f_A + f_B}{2} \end{cases}$$

and is derived as follows. When firm A wants to capture the entire market, the highest possible rate it can charge is the one that makes $\tilde{\theta} = 1$, that is $\frac{f_B - f_A}{r_A - r_B} = 1$ and solving for r_A we obtain

$$r_A = r_B + f_B - f_A. \quad (A.5)$$

If firm A shares the market with firm B, its reaction function which is derived as in (A.3), is

$$r_A = R_A(r_B) = \frac{(3f_A - f_B)r_B}{f_A + f_B}.$$

In order for both firms to operate in the market, $\tilde{\theta} \in (0, 1)$. Since $\tilde{\theta} = \frac{f_B - f_A}{r_A - r_B}$, we find that in order for $0 < \tilde{\theta} < 1$, the following must hold: $f_B > f_A$ (which is exogenously given) and $r_A > f_B - f_A + r_B \Rightarrow \frac{(3f_A - f_B)r_B}{f_A + f_B} > f_B - f_A + r_B$ which gives $r_B < -\frac{(f_A + f_B)}{2}$. If $r_B > -\frac{(f_A + f_B)}{2}$, firm A prefers to capture the entire market than to share it. If $r_B < -\frac{(f_A + f_B)}{2}$ firm A makes a higher profit by sharing the market with firm B than by capturing all clientele. Firm A never leaves the entire market to firm B, since it has the lower fee and, at least, the consumers with zero usage rate choose product A, independently of the rates.

Next, we derive the best response correspondence of firm B, given the rate of firm A:

$$r_B = R_B(r_A) = \begin{cases} W & \text{if } r_A \geq f_A - 3f_B + 2\sqrt{2}\sqrt{f_B(-f_A + f_B)} \\ \text{any rate} \geq r_A - f_B + f_A & \text{if } r_A \leq f_A - 3f_B + 2\sqrt{2}\sqrt{f_B(-f_A + f_B)}. \end{cases}$$

where $W \equiv r_A + \frac{(3+3i\sqrt{3})(f_A^2 - f_B^2) + i(i+\sqrt{3})(-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{2/3}}{6(-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{1/3}}$. As we have already stated, firm B can never capture the entire market ($\tilde{\theta}$ can never become 0). For $f_B > 0$ and $\frac{1}{2}f_B \leq f_A < f_B$ and for certain values of r_A , the profit function of firm B crosses the axes of r_A more than two times (see, for example, Figure 5). In this case, to

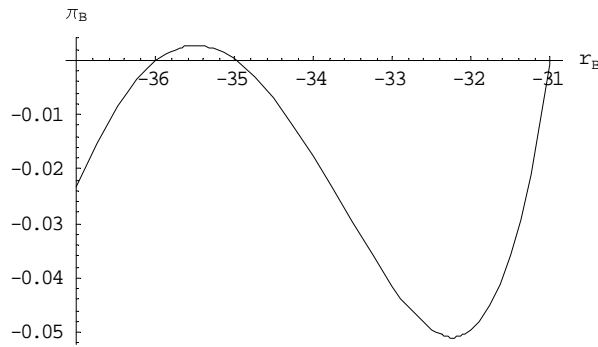


Figure 5: Firm B's profit function when $f_A = 20$, $f_B = 30$ and $r_A = -21$

find the values of r_B , we set the profit function equal to zero, solve it with respect to its own rate and obtain

$$r_B = r_A - f_B + f_A, \quad (A.6)$$

$$r_B = \frac{1}{2}(-f_A - f_B + r_A + \sqrt{(f_A + f_B - r_A)^2 + 8f_B r_A}) \quad (A.7)$$

and

$$r_B = \frac{1}{2}(-f_A - f_B + r_A - \sqrt{(f_A + f_B - r_A)^2 + 8f_B r_A}). \quad (A.8)$$

For $r_B \geq r_A - f_B + f_A$, firm B has no clients and is left with zero profit. As long as expression (A.7) is higher than (A.8), that is for $r_A > f_A - 3f_B + 2\sqrt{2}\sqrt{f_B(-f_A + f_B)}$, firm B makes a higher profit when it charges W and shares the market with firm A than when it has no clientele.¹⁷ When (A.7) is equal to (A.8), firm B makes zero profit both when it shares the market with firm A and when it has no clientele (see Figure 6). When

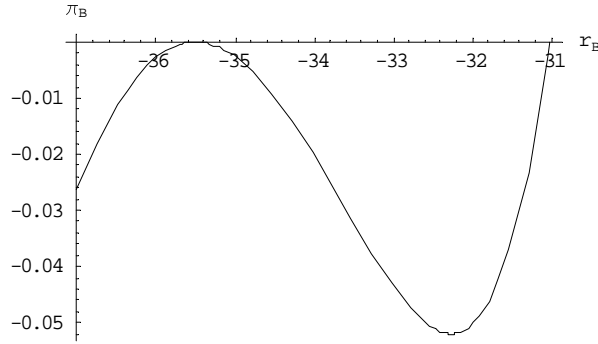


Figure 6: Firm B's profit function when $f_A = 20$, $f_B = 30$ and $r_A = -21.01$

expression (A.7) is smaller than expression (A.8) firm B prefers to have no clients because if it shares the market with firm A it makes losses.

By combining these two best response correspondences we see that they never intersect (see Figure 7). As long as firm A tries to capture the entire market, firm B never chooses to have no clientele, and as long as firm B tries to share the market with firm A, firm A makes a higher profit by capturing all clientele.¹⁸

(b) If $f_B > 0$ and $-f_B \leq f_A \leq \frac{1}{2}f_B$, the response correspondence of firm A, given the rate of firm B, is:

$$r_A = R_A(r_B) = \begin{cases} r_B + f_B - f_A & \text{if } r_B \geq -\frac{f_A + f_B}{2} \\ \frac{(3f_A - f_B)r_B}{f_A + f_B} & \text{if } r_B \leq -\frac{f_A + f_B}{2}. \end{cases}$$

¹⁷From (A.4) we find that the only root that satisfies $r_B < r_A - f_B + f_A$ for the specific range of given fees is W .

¹⁸It is easier to see why the two best response correspondences never intersect by giving specific values to the parameters. For $f_A = 2$ and $f_B = 3$, we find that firm A will try to capture the entire market as long as $r_B \geq -2.5$. In this case, firm A will charge $r_A = -1.5$, but then $r_A = -1.5 > f_A - 3f_B + 2\sqrt{2}\sqrt{f_B(-f_A + f_B)} = -2.10102$, so firm B will want to share the market by charging $r_B = W = -3.33424$. If firm B charges such a rate firm A will try to capture the entire market.

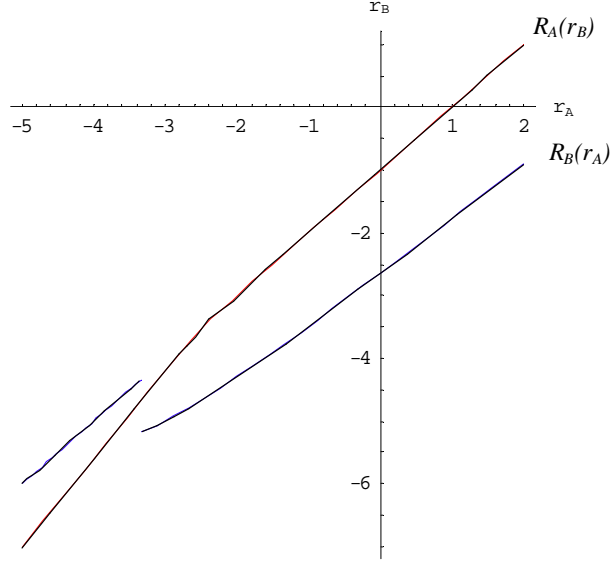


Figure 7: The best response correspondences when $f_A = 3, f_B = 4$

The best response correspondence of firm A is derived as in the previous case.

Next, we derive the best response correspondence of firm B, given the rate of firm A:

$$r_B = R_B(r_A) = \begin{cases} W & \text{if } r_A \geq -f_A \\ \text{anything} \geq r_A - f_B + f_A & \text{if } r_A \leq -f_A. \end{cases}$$

The best response correspondence of firm B is derived as in the previous case with one major difference: for $f_B > 0$ and $-f_B < f_A < \frac{1}{2}f_B$, we have that expression (A.6) is lower than expression (A.7) and therefore the profit function of firm B crosses the axes or r_B , at most, twice. In particular, for $r_A > -f_A$, firm B's profit function becomes zero when r_B takes the value of expression (A.6) or of (A.8) and is concave with a maximum at $r_B = W$. For $r_A < -f_A$ we have that expression (A.6) is even lower than expression (A.8) and the profit function of firm B crosses the axes of r_B only once and it crosses it from below.¹⁹ The maximum profit firm B can have in this case is zero by charging any $r_B \geq r_A - f_B + f_A$ (this rate makes $\tilde{\theta} \geq 1$), so no customer chooses product B. To summarize, when $r_A > -f_A$, firm B makes a positive profit by sharing the market with firm A, while, when $r_A < -f_A$, firm B prefers to have no clients and to make zero profit than to have some clients and make losses.

The two best response correspondences intersect when both firms want to share the market and the equilibrium rates are given by (11). As we can see, for $f_B > 0$ and $-f_B <$

¹⁹Another way to deduct the threshold $r_A > -f_A$ is by analyzing $\tilde{\theta}$. We know that firm B has some clients when $\tilde{\theta} \in (0, 1)$ and solving for r_B , we find that $r_B < r_A - f_B + f_A \Rightarrow W < r_A - f_B + f_A$ which gives $r_A > -f_A$.

$f_A \leq \frac{1}{3}f_B$ we have that $r_A^* \geq 0$ and $r_B^* < 0$, while for $f_B > 0$ and $\frac{1}{3}f_B < f_A < \frac{1}{2}f_B$, both rates are negative. Substituting (11) into (1) we find that, in equilibrium, the indifference usage level is at $\theta^* = \frac{1}{\sqrt{2}} \approx 0.7071$, therefore firm A has a larger market share than firm B. Substituting (11) into (2) and (3) we obtain the equilibrium profits:

$$\pi_A^* = \frac{f_A + f_B}{4\sqrt{2}} \text{ and } \pi_B^* = \frac{-f_A + (4\sqrt{2} - 5)f_B}{4\sqrt{2}}. \quad (\text{A.9})$$

For $(-3 + 2\sqrt{2})f_B \leq f_A \leq \frac{1}{2}f_B$, we have that $\pi_A^* \geq \pi_B^*$, while for $-f_B < f_A < (-3 + 2\sqrt{2})f_B$, we observe that $\pi_A^* < \pi_B^*$.

(c) If $f_B > 0$ and $f_A \leq -f_B$, or if $f_A, f_B < 0$, then the reaction correspondence of firm A, given the rate of firm B, is:

$$r_A = R_A(r_B) = \begin{cases} f_B - f_A + r_B & \text{if } r_B \geq -(f_A + f_B) \\ \infty & \text{if } r_B < -(f_A + f_B). \end{cases}$$

The best response correspondence of firm A is derived as follows. The highest possible rate firm A can charge in order to attract all clientele is $r_A = f_B - f_A + r_B$, derived as in expression (A.5). In this case firm B's profit is $\pi_B = f_A + \frac{f_B - f_A + r_B}{2} = \frac{f_B + f_A + r_B}{2}$ and since both fees are negative, condition $r_B \geq -(f_A + f_B)$ must hold in order for firm B's profit to be non-negative. If $r_B < -(f_A + f_B)$ firm A makes losses when it tries to capture the entire market, so it prefers to share it. When firm A shares the market by charging $r_A > f_B - f_A + r_B$, it attracts the consumers with a low usage level, and depending on the fee it charges, it can also attract some consumers with a higher usage level.²⁰ Since firm A has a negative fee, it needs to cover the losses from the fee with the payments from the per-usage rate. The highest profit that firm A can make is zero by charging $r_A \rightarrow \infty$ (see Figure 8).

Next, we derive the best response correspondence of firm B, given the rate of firm A:

$$r_B = R_B(r_A) = \begin{cases} Z & \text{if } r_A \geq -f_A \\ \text{anything} \geq r_A - f_B + f_A & \text{if } r_A \leq -f_A, \end{cases}$$

where $Z = r_A + \frac{-3(f_A^2 - f_B^2) + (-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{2/3}}{3(-27(f_A - f_B)^2 r_A + 3\sqrt{3}\sqrt{(f_A^2 - f_B^2)^3 + 27(f_A - f_B)^4 r_A^2})^{1/3}}$. Firm B has no clientele when $\tilde{\theta} \geq 1 \Rightarrow r_B \geq r_A - f_B + f_A$. When firm B shares the market with firm A it charges $r_B < r_A - f_B + f_A$ and from (A.4) we find that the only root that satisfies this condition, and is a real number for the specific range of given fees, is Z . To find when firm B is indifferent between sharing the market with firm A or having no clientele, we set

²⁰Since $f_A < f_B$ there are always some customers that choose firm A, no matter what rate it charges.

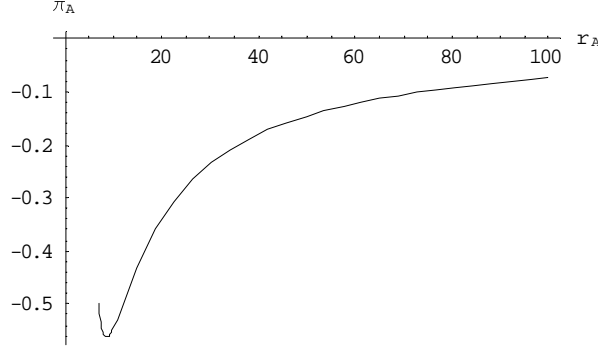


Figure 8: Firm A's profit function when $f_A = -4$, $f_B = 1$ and $r_B = 2$

$Z = r_A - f_B + f_A$ and solve it with respect to r_A . We find that when $r_A \geq -f_A$, firm B makes a higher profit when sharing the market. On the other hand, when $r_A \leq -f_A$, firm B makes losses when it attracts some clients and therefore maximizes its profit ($\pi_B = 0$) when it leaves all customers to firm A.

By combining the two response correspondences, we see that they never intersect. Firm B never sets $r_B \geq -(f_A + f_B)$, since firm A will capture the entire market and firm B will be left with zero profit. On the other hand, if firm B sets $r_B < -(f_A + f_B)$, firm A is forced to charge $r_A \rightarrow \infty$, in order not to make losses. Once firm B knows that firm A will charge an infinite rate, it will also want to increase its rate towards infinity in order to maximize its profit, but once it increases its rate beyond $-(f_A + f_B)$, firm A enters the market and attracts all clientele. Firm A never captures the entire market because, in order to force firm B to have no clientele, it has to set $r_A \leq -f_A$, and firm A ends up with losses equal to $\pi_A = \frac{f_A}{2}$. As a result, there is no equilibrium in pure strategies for $f_B > 0$ and $f_A \leq -f_B$, or for $f_A, f_B < 0$. ■

Proof of Remark 3: In order to see how the equilibrium profits are affected by an increase in the given rates we use the first-order conditions

$$\frac{\partial \pi_A^*}{\partial r_A} = \frac{(r_A - r_B)(2r_A^2 - r_A r_B + r_B^2)}{2(2r_A - r_B)^3} > 0 \text{ and } \frac{\partial \pi_A^*}{\partial r_B} = \frac{r_A^2(-r_A + r_B)}{(2r_A - r_B)^3} < 0$$

$$\frac{\partial \pi_B^*}{\partial r_A} = \frac{r_A(r_A - r_B)}{(-2r_A + r_B)^2} > 0 \text{ and } \frac{\partial \pi_B^*}{\partial r_B} = \frac{2r_A^2}{(4r_A - 2r_B)^2} > 0.$$

Firm A's profit increases as $r_A \rightarrow \infty$ and as $r_B \rightarrow 0$; in this case $\pi_A^* \rightarrow \frac{r_A}{8}$. Firm B's profit increases as $r_A \rightarrow \infty$ and as $r_B \rightarrow r_A$; in this case $\pi_B^* \rightarrow \frac{r_A}{2}$. As we can see, firm A wants a maximal differentiation of rates while firm B wants a minimal differentiation, as long as $r_A > r_B$. When $r_A = r_B$, the two firms share the market and attract half of the customers

from the entire range of θ . In this case, $\pi_A^* = \pi_B^* = \frac{r_A}{4}$ and firm A makes a higher profit than when the rates are differentiated. So, firm A maximizes its profit when the rates are equal, while firm B prefers the two rates to be slightly differentiated. ■

Proof of Proposition 7: If $f_B \geq f_A > 0$ and $r_B \geq 0$ the reaction function of firm A, given the rate of firm B is:

$$r_A = R_A(r_B) = r_B + f_B - f_A$$

Firm A would want to share the market with firm B only if $r_B < -\frac{f_A+f_B}{2}$ which is negative and therefore can never occur. For a positive r_B , firm A maximizes its profit by capturing the entire market.

The best response correspondence of firm B, given the rate of firm A, is:

$$r_B = R_B(r_A) = \begin{cases} W & \text{if } r_A \geq \frac{\sqrt{(f_A-3f_B)(f_A-f_B)^4(f_A+f_B)^3}}{\sqrt{(f_A^2-f_B^2)^3}} \\ 0 & \text{if } f_B - f_A \leq r_A \leq \frac{\sqrt{(f_A-3f_B)(f_A-f_B)^4(f_A+f_B)^3}}{\sqrt{(f_A^2-f_B^2)^3}} \\ \text{any rate } > 0 & \text{if } 0 \leq r_A \leq f_B - f_A. \end{cases}$$

where $W \equiv r_A + \frac{(3+3i\sqrt{3})(f_A^2-f_B^2)+i(i+\sqrt{3})(-27(f_A-f_B)^2r_A+3\sqrt{3}\sqrt{(f_A^2-f_B^2)^3+27(f_A-f_B)^4r_A^2})^{2/3}}{6(-27(f_A-f_B)^2r_A+3\sqrt{3}\sqrt{(f_A^2-f_B^2)^3+27(f_A-f_B)^4r_A^2})^{1/3}}$. The best response correspondence of firm B is derived as follows. In order for firm B to have some clientele, $\tilde{\theta} < 1$ which gives that $r_A - f_B + f_A > r_B$. But when $r_A \leq f_B - f_A$, firm B can not charge a negative rate in order to attract some clientele and as a result it is left without any clients, whatever positive rate it charges. Firm B maximizes its profit when it charges W and shares the market with firm A. But for $f_B - f_A \leq r_A \leq \frac{\sqrt{(f_A-3f_B)(f_A-f_B)^4(f_A+f_B)^3}}{\sqrt{(f_A^2-f_B^2)^3}}$, $W \leq 0$ so firm B can not charge W . As a result firm B maximizes its profit by charging the closest fee to W which is allowed to, which is zero. For $r_A \geq \frac{\sqrt{(f_A-3f_B)(f_A-f_B)^4(f_A+f_B)^3}}{\sqrt{(f_A^2-f_B^2)^3}}$, firm B can charge the rate which maximizes its profit which is W , since $W \geq 0$.

The reaction function of firm A and the best response correspondence of firm B intersect when $r_A^* = f_B - f_A$ and $r_B^* = 0$. Substituting the equilibrium rates into (1) we find that $\theta^* = 1$, therefore firm A captures the entire market and firm B is left without any clients. ■

Proof of Proposition 8: For $f_A < f_B$ we distinguish three cases. *Case 1:* When $f_A, f_B > 0$, some consumers purchase product A because it charges a lower fee and some consumers purchase product B because it charges a lower rate. Every consumer will use only one product (products are perfect substitutes), so, when the fees are positive, no consumer will purchase both products since he will have to pay a positive fee for a second product that he would not use. A consumer is indifferent between purchasing and using

product A or product B, when $f_A + \theta_1 r_A = f_B + \theta_1 r_B$, (where now, the index indicates the specific case we are examining) so the indifference usage level is

$$\tilde{\theta}_1 = \frac{f_B - f_A}{r_A - r_B}. \quad (A.10)$$

In this case, $\tilde{\theta}_1$ consumers purchase and use product A, $(1 - \tilde{\theta}_1)$ consumers purchase and use product B and the profit functions are

$$\pi_{A1} = f_A \tilde{\theta}_1 + \frac{r_A}{2} \tilde{\theta}_1^2 \text{ and } \pi_{B1} = f_B (1 - \tilde{\theta}_1) + \frac{r_B}{2} (1 - \tilde{\theta}_1)^2. \quad (A.11)$$

Case 2: When $f_A < 0$ and $f_B > 0$, all consumers certainly purchase product A in order to earn $|f_A|$. From these consumers, some also choose to purchase product B, because it has a lower rate. So all the consumers purchase product A and some consumers purchase both products. Now, let's examine which consumers use product A and which use product B. A consumer is indifferent between using product A or product B when $f_A + \theta_2 r_A = f_A + f_B + \theta_2 r_B$, so the indifference usage level is

$$\tilde{\theta}_2 = \frac{f_B}{r_A - r_B}.$$

All consumers purchase product A but $\tilde{\theta}_2$ consumers use product A and $(1 - \tilde{\theta}_2)$ consumers purchase and use product B. In this case, the profit functions are

$$\pi_{A2} = f_A + \frac{r_A}{2} \tilde{\theta}_2^2 \text{ and } \pi_{B2} = f_B (1 - \tilde{\theta}_2) + \frac{r_B}{2} (1 - \tilde{\theta}_2)^2.$$

Case 3: When both $f_A, f_B < 0$, all consumers clearly purchase both products in order to make money from each purchase. A consumer is indifferent between using product A or product B if $f_B + f_A + \theta_3 r_A = f_A + f_B + \theta_3 r_B$, so the indifference usage level is $\tilde{\theta}_3 = 0$. In this case, all consumers purchase both products but use only product B, since it has the lower rate. The profit functions are

$$\pi_{A3} = f_A + \frac{r_A}{2} \tilde{\theta}_3^2 = f_A \text{ and } \pi_{B3} = f_B + \frac{r_B}{2} (1 - \tilde{\theta}_3)^2 = f_B + \frac{r_B}{2}.$$

Based on the above three cases, the best response correspondence of firm A is

$$f_A = R_A(f_B) = \begin{cases} f_B - r_A + r_B & \text{if } f_B \geq r_A - r_B \\ 0 & \text{if } 0 < f_B \leq r_A - r_B \\ \text{any fee} \geq 0 & \text{if } f_B \leq 0 \end{cases}$$

and is derived as follows. If $f_B > 0$, firm A will never charge a negative fee, because all consumers will purchase product A, in order to collect f_A , and firm A will make less profit

than if it charged a zero fee (since, from case 2, we know that $\pi_{A2} = f_A + \frac{r_A}{2}\tilde{\theta}_2^2 < \frac{r_A}{2}\tilde{\theta}_2^2$, if $f_A < 0$). When both fees are non-negative, we are in case 1, which is identical to the analysis when fees and rates can only take non-negative values (expression (12)). In this case, firm A charges $f_A = 0$ and shares the market with firm B when $f_B < r_A - r_B$. If $f_B \geq r_A - r_B$, firm A maximizes its profit by charging $f_A = f_B - r_A + r_B$ and capturing the entire market. If $f_B < 0$, firm A will never charge a negative fee because all consumers will choose product A in order to collect f_A , but no consumer will use it because it has the higher rate. As a result, firm A will make losses. In this case, firm A maximizes its profit ($\pi_B = 0$) by charging $f_A \geq 0$, in order to make sure that no customer will purchase its product. To summarize, firm A will never charge a negative fee, regardless of what firm B charges.

The best response correspondence of firm B is

$$f_B = R_B(f_A) = \begin{cases} f_A & \text{if } f_A \geq r_A - r_B \\ \frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B} & \text{if } 0 \leq f_A \leq r_A - r_B \\ \frac{(r_A - r_B)^2}{2r_A - r_B} & f_A \leq 0 \end{cases}$$

and is derived as follows. If $f_A \leq 0$, we are either in case 2 or in case 3. In either case, the indifferent consumer does not take into consideration the fee of firm A in order to decide which product to use. Firm B will never charge a negative fee because it could make a higher profit by charging a zero fee (in this case, all consumers would prefer to use product B, since it offers the lower rate). So we need to examine the case where firm B charges a non-negative fee. From the analysis of case 2 we know that when $f_A \leq 0$, while $f_B \geq 0$, the indifferent consumer is located at point $\tilde{\theta}_2 = \frac{f_B}{r_A - r_B}$. In this case, firm B's reaction function (which is derived by calculating the first-order condition, setting it equal to zero and solving it with respect to f_B) is:

$$f_B = R_B(f_A) = \frac{(r_A - r_B)^2}{2r_A - r_B}.$$

When $f_A > 0$, we are in case 1, which is identical to the analysis when fees and rates can only take non-negative values (expression 13). In this case, firm B charges $f_B = f_A$ and captures the entire market when $f_A \geq r_A - r_B$. If $0 \leq f_A \leq r_A - r_B$, firm B maximizes its profit by sharing the market with firm A and charging $\frac{f_A r_A + (r_A - r_B)^2}{2r_A - r_B}$.

The two best response correspondences intersect when

$$f_A^* = 0 \text{ and } f_B^* = \frac{(r_A - r_B)^2}{2r_A - r_B}. \quad (\text{A.12})$$

Substituting (A.12) into (A.10), we obtain that $\theta^* = \frac{r_A - r_B}{2r_A - r_B} < \frac{1}{2}$, therefore, for $r_A > r_B$, firm B has a larger market share than firm A.

When $r_A = r_B$, our analysis is similar to that of Proposition 5, case (i). Since rates are equal, each firm can capture the entire market by undercutting the fee of its rival. This will lead both firms to charge $f_A = f_B = 0$. When $r_A = r_B = r \geq 0$ and $f_A = f_B = 0$, the two firms share the market equally by attracting customers from the entire range of θ and enjoy profits equal to $\pi_A = \pi_B = \frac{r}{2}$. No firm will deviate from these fees because no firm can lower its fee and if a firm increases its fee, it will lose all clientele and make zero profit. Substituting (A.12) into (A.11), we obtain

$$\pi_A^* = \frac{r_A(r_A - r_B)^2}{2(-2r_A + r_B)^2} \text{ and } \pi_B^* = \frac{r_A^2}{4r_A - 2r_B}.$$

If $r_A = r_B = r$, the two firms equally share the market and both have customers from the entire range of θ . In this case the equilibrium profits are $\pi_A^* = \pi_B^* = \frac{r}{4}$. ■