## DISCUSSION PAPER SERIES

No. 9091
A FRAMEWORK FOR ANALYZING LANGUAGE AND WELFARE

Jacques Mélitz

INTERNATIONAL MACROECONOMICS and PUBLIC POLICY

# Centre for <br> www.cepr.org 

# A FRAMEWORK FOR ANALYZING LANGUAGE AND WELFARE 

Jacques Mélitz, Heriot-Watt University, CREST, CEPII and CEPR

Discussion Paper No. 9091
August 2012

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: $(44$ 20) 7183 8801, Fax: $(4420) 71838820$
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in INTERNATIONAL MACROECONOMICS and PUBLIC POLICY. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and nonpartisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.


#### Abstract

\section*{A framework for analyzing language and welfare}

The paper proposes a general model that will encompass trade and social benefits of a common language, a preference for a variety of languages, the fundamental role of translators, an emotional attachment to maternal language, and the threat that globalization poses to the vast majority of languages. With respect to people's emotional attachment, the model considers minorities to suffer losses from the subordinate status of their language. In addition, the model treats the threat to minority language as coming from the failure of the parents in the minority to transmit their maternal language (durably) to their children. Some familiar results occur. In particular, we encounter the usual social inefficiencies of decentralized solutions to language learning when the sole benefits of the learning are communicative benefits (though translation intervenes). However, these social inefficiencies assume a totally different air when the consumer gains of variety are brought in. One fundamental aim of the paper is to bring together contributions to the economics of language from labor economics, network externalities and international trade that are typically treated separately.


JEL Classification: D60, F10 and Z10
Keywords: language, trade and welfare
Jacques Mélitz
Department of Economics
Mary Burton Building
Heriot-Watt University
Edinburgh
EH14 4AS
Email: j.melitz@hw.ac.uk

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=100052

# A FRAMEWORK FOR ANALYZING LANGUAGE 

AND WELFARE

Jacques Mélitz

The subject of language arises prominently in a variety of contexts in economics, for example, the determination of wages, network externalities, and foreign trade (for an overview, see Grin (1996) and Ginsburgh and Weber (2011)). Yet despite a considerable literature on the subject, it has not yet entered the mainstream of economics. Perhaps the reason lies in the absence of a general framework. Such a framework, I propose, must meet four criteria. First, it must accommodate the fact that political and market integration and trade tend to diminish the number of languages in the world. Second, it must allow for the role of bilingualism and translation in tying the world's languages together. Third, it must admit diversity of languages as a benefit. Fourth, it must reflect the emotional attachment that people have to their maternal language. I will discuss the four criteria, and then try to sketch out an appropriate model.

Ethnologue records nearly 7,000 living languages in the world (some 450 of which are "nearly extinct"). "No one knows how many languages have already died since humans became able to speak, but it must be thousands," Crystal (1997, p.17). Of the roughly half with 10,000 or fewer speakers, only a small fraction can be expected to survive another century (see Krauss (1992), Dalby (2002), Lewis and Simons (2008) and Hagège (2011)). Languages disappear remarkably fast with the advance of trade and markets and regional political and economic integration within countries. Even among the 1,300 or so world languages that are still spoken by over 100,000 people, attrition is easy to imagine over the 21 st century. ${ }^{1}$ In addition, interpreters and translation enter in worldwide communication. To ignore this point necessarily exaggerates the required bilingualism for messages to get across. In theory, a trivial fraction of world population could carry messages to virtually all people on earth. Along

[^0]with numerous languages, there is also a "world language system" with "central languages" tying all languages together (see De Swann (2001) and Heilbron (1999)). Some approaches to multiple languages by economists evidently help to understand these facts. If communicating parties benefit from a common language, the globalization of commerce is likely to encourage concentration of learning of second languages on a limited number of languages. Yet the process through which the integration of markets within countries and internationally promotes the disappearance of the small languages does not draw particular attention of economists (Grin (1992) is a notable exception). Inter alia, the treatment of language as a medium of communication, or as a strict means to an end, suggests no reason why a single world language should not suffice. If languages are mere "communication technologies" (Church and King (1993)), there is no obvious reason why a person would value a variety of languages as such.

However, if we consider language as a source of stimulation and pleasures rather than a mere device, then explaining why a variety of languages would be better than one ceases to be a problem. ${ }^{2}$ In fact, language diversity as such, and not simply language in any one tongue, accounts for many pleasurable aspects of life - not only for people with broad linguistic skills. Monolinguals can read foreign-language works in translation; they can see and hear foreign films and television programs with sub-titles and dubbing. Over and above, much of the variety in world cultures from which we all benefit, going beyond the popular arts and entertainments but extending to cuisine, styles of manufactures, architecture, decors and dress, is probably related to language. Everyday observation suggests that a common language tends to narrow cultural differences.

To be sure, the variety coming from multiple languages can be carried too far for people's tastes. Heterogeneity of cultures, manners and speech, especially at home, can even pose a threat. Most people probably find cultural variety pleasurable only within limits. But the world dimension of the issue is important. There is a big difference between multiplicity of cultures and languages in the immediate neighborhood, in a different part of the country, and in the rest of the world. The local element can be intrusive and impossible to avoid. But as

[^1]regards diversity in distant lands, one can pick and choose. The foreign diversity can be tasted through travel or imported in selective packaging. To many minds, of course, this is insisting on the obvious. Thus, after evoking favorably the prospect of a future where every newborn on earth will learn English in infancy, the linguist Crystal adds: "If it [English] is by then the only language left to be learned, it will have been the greatest intellectual disaster that the planet has ever known" (1997, p. 140).

But neither questions of communication nor taste for variety come to grips with a fundamental aspect of the topic: the intensity of popular attachments to maternal languages. There are numerous movements in favor of minority languages in the world. Efforts to explain these movements with strict reference to issues of communication and diversity are futile. Indeed, we even encounter attempts to bring back languages from the grave, as in the case of Irish, Welsh, and Basque, and (for a successful example in Israel) Hebrew. People clearly associate their maternal language with their culture, and they often experience the rapid decline of this language with veritable dismay. As Bisin and Verdier (2011) emphasize, ethnic diversity within countries has been more resilient than it was predicted to be a few decades ago. Notwithstanding, even when younger generations continue to identify themselves with a minority culture, they often abandon the minority language.

Thus, the task, I believe, is to propose a general model that will encompass trade and social benefits of a common language, a preference for a variety of languages, the fundamental role of translators, an emotional attachment to maternal language, and the threat that globalization poses to the vast majority of languages. As regards the first 2 aims, the road is reasonably paved; on the last 3 , less so. With respect to the aim relating to emotional attachment, the model I propose will consider minorities to suffer losses from the subordinate status of their language, in opposition to Lazear (1999) who considers cultural assimilation as a possible benefit. I will also discuss this important conflict. In addition, as is common among ethnolinguists (see, in addition to the previous references to Krauss, Dalby, and Lewis and Simons, Fishman (1997)), I will treat the threat to minority language as coming from the failure of the parents in the minority to transmit their maternal language (durably) to their children. The tendency of minority parents to learn the majority language contributes heavily to the problem
(as Lazear agrees).
The next section will present the general framework; the subsequent one will discuss the decision choices; the third one will examine the welfare implications; the fourth will evoke the issue of the preservation of a variety of languages in a general discussion of language policy and the future; and the last will conclude.

## I. The model

I will begin with a world economy with a single language, and subsequently, modify the analysis to admit two languages, one of which dominates the other. At this next point, I will no longer assume a closed economy but an open one which includes numerous languages abroad. The open economy aspect will remain underdeveloped. But I prefer thinking of an open economy that can benefit from foreign languages through imports and travel once a second language enters at home. In general, the linguistic issues will not be related to choices of goods but instead the allocation of time. In addition, they will concern the sense of "being at home" or general comfort or discomfort in the country.

Let us assume a world where each household produces a single good but desires a large variety of goods in consumption, so trade between the households is very important. The trading also requires communication: talking, writing and reading. Each household divides its time between work and leisure and divides its leisure time between pleasurable and nonpleasurable activities. I will interpret the non-pleasurable leisure time as including trading, which is, of course, not fully correct. The households all have identical tastes. They also all dispose of a single production technology. There is only a single language for the moment, and I will assume that the transaction technology, like the production technology, is the same for everyone. Consequently, all households divide their time the same way between the three activities. However, as the single difference between them at this stage, they produce different goods. What they produce depends on who they are. Individuals specialize in producing different goods. They also divide into groups (industries) of equal size that produce individual goods and the members of which are essentially clones. For this reason, I index nothing except the individual household's output and output price.

The equations follow. The production function is

$$
\begin{equation*}
\mathrm{q}_{\mathrm{i}}=\alpha \mathrm{H} \tag{1}
\end{equation*}
$$

where H is the fraction of time spent on working, $\alpha$ is a production parameter, and q is the amount produced by household i. The individual household's budget restraint is:
(2) $\mathrm{p}_{\mathrm{i}} \mathrm{q}_{\mathrm{i}}=\sum_{\mathrm{k}=1}^{\mathrm{T}} \mathrm{p}_{\mathrm{ik}} \mathrm{c}_{\mathrm{k}}$
where T is the number of goods that the individual consumes in amounts $\mathrm{c}_{\mathrm{k}}$. T is also the total number of goods in the economy. There are $\mathrm{T}-1$ relative prices $\mathrm{p}_{\mathrm{i}}$ of the T goods in terms of the production good i , and $\mathrm{p}_{\mathrm{ii}}$ - written on the left hand side as $\mathrm{p}_{\mathrm{i}}-$ is identically equal to one. The utility function of the household is separable in goods and pleasurable time and takes the specific form:

$$
\begin{equation*}
\mathrm{U}=\mathrm{U}\left(\left\{\sum_{\mathrm{k}=1}^{\mathrm{T}} \mathrm{c}_{\mathrm{k}}^{\rho}\right\}^{1 / \mathrm{p}}+\beta(1-\mathrm{H}-\mathrm{S})\right) \quad \mathrm{U}^{\prime}(\bullet)>0 \quad \mathrm{U}^{\prime \prime}(\bullet)<0 \quad \mathrm{~T} \gg 1 \quad 0<\rho<1 \tag{3}
\end{equation*}
$$

where $\rho$ and $\beta$ are taste parameters, and $S$ is the percentage of time spent trading (the only non-pleasurable leisure activity thus far). $\rho$ is a parameter reflecting the individual's preference for variety. On the other hand, $\beta$ refers to valuation of the pleasurable part of leisure time $1-\mathrm{H}-\mathrm{S}$. In addition, the transactions technology is
(4) $\mathrm{S}=\delta(\mathrm{T}-1)$
where $\delta$ is the fraction of time required to convert good i into a different good k through trade. The idea is that widening consumption between different goods requires some extra search and time. $\delta$ is necessarily a tiny fraction of $S$ since $T$ is assumed to be large.

Since all production functions are the same and they all exhibit constant costs, the relative $p_{\mathrm{ik}}$ prices of all goods will obviously be one. Given the symmetric way in which all of the goods enter the utility function, the $\mathrm{c}_{\mathrm{k}}$ quantities of consumption will also all be equal. Thus, equations (2) and (3) reduce to
(2a) $\mathrm{q}_{\mathrm{i}}=\mathrm{Tc}$
and

$$
\begin{equation*}
\mathrm{U}=\mathrm{U}\left(\mathrm{~T}^{1 / \mathrm{p}} \mathrm{c}+\beta(1-\mathrm{H}-\mathrm{S})\right) \tag{3a}
\end{equation*}
$$

The solution to the consumer's problem yields $\mathrm{H}, \mathrm{S}, \mathrm{q}_{\mathrm{i}}$, and c .

Let us next allow for two separate cultures, each of which possesses a separate language. There are $\mathrm{N}_{1}$ households which belong to the majority culture, $\mathrm{N}_{2}$ which belong to the minority culture, with N equal $\mathrm{N}_{1}+\mathrm{N}_{2}$ and $\mathrm{N}_{1}$ much larger than $\mathrm{N}_{2}$. All households consist of two parents and two children. The adults of both cultures either already know the second language or decide whether to learn it. I shall use the notation $\mathrm{N}_{11}$ and $\mathrm{N}_{22}$ to designate the adult members of $\mathrm{N}_{1}$ and $\mathrm{N}_{2}$ who are monolingual, and $\mathrm{N}_{12}$ and $\mathrm{N}_{21}$ those of $\mathrm{N}_{1}$ and $\mathrm{N}_{2}$, respectively, who are bilingual. Henceforth all of the N notation will refer strictly to adults.

The revised model will introduce 8 basic differences, 7 of which depend on language alone and the 8th (the 5th in order of presentation) on the opening up of the economy. First, there will be an income penalty for ignorance of the majority language. The evidence on this point is overwhelming (See, for example, McManus, Gould and Welch (1983), Chiswick and Miller (1995, 2002, 2007), Dustmann and van Soest (2002), and Dustmann and Fabbri (2003)). Second, all monolinguals will suffer higher transaction costs. This is a market imperfection that will appear as a loss of time spent shopping. Third, non-shopping leisure time will be less satisfying to all monolinguals because they can no longer communicate with every one they meet. These first 3 differences yield a fourth one: the monolinguals may decide to devote time to learning the opposite language. This will be supposed to be another non-pleasurable leisure activity. Fifth, consumption will be more varied for all households because of production opportunities abroad and therefore imports. That is, the total number of available goods T will rise. Sixth, linguistic variety will contribute to T and do so partly through home production and partly through imports. This is the response to Crystal's alarm that a single language would be the "greatest intellectual disaster that the planet has ever known." The last two differences reflect people's emotional attachment to maternal language but are defined far more broadly. There is a general loss of welfare because of not feeling "at home" in the minority, which may afflict the majority too if there is a large enough minority presence, though it is always less important than for the minority. Last, one of the pleasurable activities to which members of the minority may decide to devote some leisure time as opposed to other pleasurable activities is to transmit their language and culture to their children.

As a basic simplification, I will assume that people limit themselves to learning a sin-
gle second language and this second one is the other one spoken at home even though there are very many languages abroad. As a further simplification, I will also assume that the required extra time spent on international transactions because of third languages in the rest of the world is exogenously given and does not affect the decision about learning the other home language. However, I will relax both of these assumptions in two separate appendices, where neither assumption will emerge as fundamental in the analysis.

The equations follow. The indices i and j now serve on the left hand side to distinguish between households that are members of $\mathrm{N}_{1}$ and $\mathrm{N}_{2}$, respectively. However, if the indices appear after the variables on the right hand side, they have the additional role of indicating differences between monolinguals and bilinguals within a given culture, that is, between $\mathrm{N}_{11}$ and $\mathrm{N}_{12}$ in the case of i and between $\mathrm{N}_{21}$ and $\mathrm{N}_{22}$ in the case of j . Still further, if the indices on the right hand side appear before the variables, they also signify differences within the relevant language group, $\mathrm{N}_{11}, \mathrm{~N}_{12}, \mathrm{~N}_{22}$ or $\mathrm{N}_{21}$, as the case may be. In the event of differences between households within a language group, I also always assume a continuous distribution in a specified range. The discussion that follows the equations shows that, contrary to possible impression, the model remains quite small.
(5) $\mathrm{q}_{\mathrm{i}}=\alpha \mathrm{H}$
(6) $\mathrm{q}_{\mathrm{j}}=\eta_{j} \alpha \mathrm{H}$
(8) $\mathrm{p}_{\mathrm{i}} \mathrm{q}_{\mathrm{i}}=\sum_{\mathrm{k}=1}^{\mathrm{T}_{\mathrm{i}}} \mathrm{p}_{\mathrm{ik}} \mathrm{c}_{\mathrm{ik}}$
(9) $\quad p_{j} q_{j}=\sum_{k=1}^{T_{j}} p_{j k} c_{j k}$

$$
\begin{equation*}
\mathrm{S}_{\mathrm{i}}=\left(1+\gamma_{\mathrm{i}}\right) \delta\left(\mathrm{T}_{\mathrm{i}}-1\right)+\mathrm{X}_{\mathrm{i}} \quad \text { where } \mathrm{X}_{\mathrm{i}}=0 \text { or } \overline{\mathrm{X}}_{\mathrm{i}} \quad \text { and } 0<\gamma_{\mathrm{i}} \ll 1 \tag{12}
\end{equation*}
$$

$$
\begin{align*}
& U_{i}=\theta_{\mathrm{i}} U\left(\left\{\sum_{\mathrm{k}=1}^{\mathrm{T}_{\mathrm{i}}} c_{\mathrm{ki}}^{\rho}\right\}^{1 / \rho}+\beta\left(1-\omega_{\mathrm{i}}\right)\left(1-\mathrm{H}-\mathrm{S}_{\mathrm{i}}\right)\right) \quad 0<\rho<1 \text { and } 0<\omega_{\mathrm{i}} \ll 1  \tag{10}\\
& \mathrm{U}_{\mathrm{j}}=\theta_{\mathrm{j}} \mathrm{U}\left(\left\{\sum_{\mathrm{k}=1}^{\mathrm{T}_{\mathrm{j}}} c_{\mathrm{kj}}^{\rho}\right\}^{1 / \rho}+\beta\left(1-\omega_{\mathrm{j}}\right)\left(1-\mathrm{H}-\mathrm{S}_{\mathrm{j}}-Y\right)+\Omega_{\mathrm{j}} \mathrm{Y}\right) \text { where } \mathrm{Y}=0 \text { or } \hat{Y} \text { and } 0<\omega_{\mathrm{j}} \ll 1 \tag{11}
\end{align*}
$$

$$
\begin{equation*}
\mathrm{S}_{\mathrm{j}}=\left(1+\gamma_{\mathrm{j}}\right) \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)+\mathrm{X}_{\mathrm{j}} \quad \text { where } \mathrm{X}_{\mathrm{j}}=0 \text { or } \overline{\mathrm{X}}_{\mathrm{j}} \quad \overline{\mathrm{X}}_{\mathrm{i}}>\overline{\mathrm{X}}_{\mathrm{j}} \text { and } 0<\gamma_{\mathrm{j}} \ll 1 \tag{13}
\end{equation*}
$$

(17) $\quad \omega_{j}=\left\{\begin{array}{cc}0 & \text { if } j \in N_{21} \\ { }_{j} \omega_{1} \frac{N_{11}}{N} & \text { if } j \in N_{22}\end{array}\right.$
(18) $\mathrm{T}_{\mathrm{i}}=\left(\mathrm{T}_{\mathrm{o}}+{ }_{\mathrm{i}} \tau_{1} \mathrm{~m} \frac{\mathrm{~N}_{2}}{\mathrm{~N}}+{ }_{\mathrm{i}} \tau_{2} \mathrm{~L}_{\mathrm{W}}\right)\left(1+{ }_{\mathrm{i}} \tau_{3}\left(\mathrm{~L}_{\mathrm{i}}-1\right)\right)$ where $\mathrm{L}_{\mathrm{i}}=1$ or 2 and $0<{ }_{\mathrm{i}} \tau_{3} \ll 1$
(19) $T_{j}=\left(T_{o}+{ }_{j} \tau_{1} \frac{N_{1}}{N}+{ }_{j} \tau_{2} L_{W}\right)\left(1+{ }_{j} \tau_{3}\left(L_{j}-1\right)\right)$ where $L_{j}=1$ or 2 and $0<{ }_{j} \tau_{3} \ll 1$ $\theta_{\mathrm{i}}=1-{ }_{\mathrm{i}} \theta_{1} \frac{\mathrm{~N}_{2}}{\mathrm{~N}}-{ }_{\mathrm{i}} \theta_{2} \frac{\mathrm{~N}_{22}}{\mathrm{~N}}$ and $0 \leq{ }_{\mathrm{i}} \theta_{1}+{ }_{\mathrm{i}} \theta_{2} \leq 0.5$
(21) $\quad \theta_{\mathrm{j}}=1-{ }_{\mathrm{j}} \bar{\theta}+{ }_{\mathrm{j}} \theta_{1} \frac{\mathrm{~N}_{2}}{\mathrm{~N}}$ where $0<{ }_{\mathrm{j}} \bar{\theta}-{ }_{\mathrm{j}} \theta_{1}<1$

$$
\begin{equation*}
\Omega_{\mathrm{j}}={ }_{\mathrm{j}} \chi(1+\overline{\mathrm{j}} \bar{\theta})\left(1+\frac{\mathrm{N}_{22}}{\mathrm{~N}_{2}}+\mathrm{n}\right) \mathrm{m} \tag{22}
\end{equation*}
$$

where n is the percentage of $\mathrm{N}_{2}$ households who choose $\mathrm{Y}=\hat{\mathrm{Y}}$
$\hat{Y}=$ the amount of time $S$ a minority household must set aside to inculcate its language in its children.
$\bar{X}_{i}, \bar{X}_{j}=$ the amount of time $S$ any household $i$ or $j$ requires in order to learn the language of the opposite culture at home.
$\mathrm{m}=$ the proportion of $\mathrm{N}_{2}$ households who speak the minority language and equal 1
$\mathrm{L}_{\mathrm{w}}=$ linguistic diversity in the world.
$L_{i}, L_{j}=$ the language repertoire of household $i$ or $j$.
With regard to the production function, or eqs. (5)-(7), the modeling says that all
households are equally productive except for the monolinguals in the minority, $\mathrm{N}_{22}$. The latter obtain a fraction $1-\bar{\eta}$ less output per hour of labor than the rest. Eqs. (8)-(13) correspond to the earlier eqs. (2)-(4) and all the differences from those earlier ones, except for the bulk of the rise in T, depend on language and are spelled out in the subsequent equations.

As regards the non-pleasurable time that people spend outside of work S, eqs. (14)(15) concern the part of this time they spend on transactions because of language. This extra time includes a $\gamma_{0}$ element for everyone and an additional element for the monolinguals. The $\gamma_{0}$ element, affecting everyone, comes from foreign trade. For the moment, this will be interpreted to mean either that neither home language is spoken abroad or else that any difference in $S$ for people resulting from foreign speakers of either language is negligible. However, I will lift this assumption in an appendix (the first one) and allow, for example, that if Russian is one of the two home languages, then households may profit from a larger reduction in S from learning Russian because of foreign Russian speakers. The change will be modest. The other element of $S$ in eqs. (14) and (15) results from ignorance of the second home language and differs for the two monolingual groups. As regards $\mathrm{N}_{11}$, it depends on the size of $\mathrm{N}_{22}$ relative to the total population N while in the case of $\mathrm{N}_{22}$, it depends on the size of $\mathrm{N}_{11}$ relative to N. Evidently, therefore, the problem is more severe for $\mathrm{N}_{22}$. However, this problem is mitigated for both monolingual groups by the parameter $\psi$ (since $\psi>1$ and $\mathrm{N}_{11} / \mathrm{N}$ and $\mathrm{N}_{22} / \mathrm{N}$ are both $<1$ ), which refers to the reduction in the needed time for transactions stemming from interpreters and translations. This parameter permits both monolingual groups to get along more easily with a single language. ${ }^{3}$ By indexing the parameter $\gamma_{1}$ in eqs. (14) and (15), I admit differences among individual monolinguals in overcoming language barriers.

Quite differently, the factor $\omega$ in eqs. (16)-(17) refers to the reduced ability of monolinguals to enjoy personal interactions with others in their leisure time. In this case, interpreters and translation are of no help. The role of the parameter $\omega_{1}$ in (16) and (17) $\left(\omega_{1}\right.$ small) is

[^2]not to exaggerate this last problem since the relevant reduction in welfare concerns only one of many possible allocations of pleasurable leisure time $1-\mathrm{H}-\mathrm{S}$. By indexing $\omega_{1}$ (like $\gamma_{1}$ ) the model admits differences in household decisions to learn the opposite language for reasons of personal interaction (as opposed to reasons of market efficiency).
$\bar{X}_{\mathrm{i}}$ and $\overline{\mathrm{X}}_{\mathrm{j}}$ in eqs. (12) and (13) distinguish between the required time to learn the two languages. Selten and Pool (1991) make the same distinction on the ground of the inherent differences in the difficulty of learning different languages (see also the interesting early paper by Marschak (1965)). My reason for the distinction differs. I wish to recognize the fact that learning a language is always easier if the language prevails in the environment. The prevalence of a language means continuous practice and reinforcement. As a result, I assume $\bar{X}_{i}>\bar{X}_{j}$, with some consequences later on.

There are important precedents for many of the previous aspects of the specification. Breton and Mieszkowski (1977) and Carr (1985) were the first to treat differences in language as impediments to trade. In their innovative contribution, Breton and Mieszkowski essentially drew an analogy between the role of language in trade and that of transportation costs, while Carr (in a French translation of a lost English original predating Breton and Mieszkowski) introduced the similar analogy with regard to multiple currencies. Breton and Mieszkowski implicitly recognize the external effects of the adoption of a language. However, a full development of the external effects of a common language awaited Church and King (1993). The latter also referred to communication in general rather than specifically to trade. Lazear (1999) took the subsequent (and arguable) step of assigning the externalities strictly to benefits in trade. He supposes random encounters between traders which consummate in trades when people with a common language meet but not otherwise. My proposed framework differs on this point. The issue of externalities occurs here in the same way as Church and King (1993) formulated it in the case of $1-\mathrm{H}-\mathrm{S}$, or with respect to social encounters, but only in an attenuated fashion as concerns S , with regard to which interpreters and translations intervene. Evidently trade occurs massively between people who could never speak to one another directly. However, it's hard to enjoy the company of others with whom one cannot speak. This point will recur.

Eqs. (18) and (19) mark a fundamental difference from the earlier model with a single language. Plural languages affect the number of goods T available for consumption and therefore the variety of consumption. The exogenous factor $\mathrm{T}_{\mathrm{o}}$ in these two equations is independent of linguistic variety (but still higher than T in the earlier single-language model because of imports). The other 3 terms introduce an endogenous component of T , associated with linguistic variety. The first of the 3 , which depends on ethnic composition at home, reflects the new consumption goods resulting from ethnic variety, for example, ethnic restaurants; the second, $\mathrm{L}_{\mathrm{W}}$, reflects the new goods available through import and travel; and the third, L , depends on the language repertoire of the individual household. Obviously the first 2 terms derive from cultural variety and not strictly language. However, they are positively correlated with language, and since the part that interests us is the one that depends on language, I have eliminated the rest. For the same reason, $m$ or the proportion of the minority $\mathrm{N}_{2}$ who speak the minority language, enters in eq. (18). m is currently 1 but will fall below 1 in the future as some children of the $\mathrm{N}_{2}$ households will not retain the language. L enters in the equation in a different way than the first 2 variables and in such a way that a move to bilingualism will raise the variety of consumption by a set percentage $\left(\tau_{3}\right)$ rather than a fixed amount. I will also take this percentage to be small. There is a fundamental reason for regarding all 3 terms as beneficial to the individual: namely, that they concern budget allocations that the household can choose at will (to visit abroad or to stay home, to read translated works or not to read them, etc.). Indexing the coefficients of all 3 terms in (18) and (19) allows for separate household responses to all 3 .

The precedent for the treatment of a variety of languages as a pleasure in the model is Grin and Vaillancourt (1997). ${ }^{4}$ The only deviation from them I would stress is the idea that the benefit extends to monolinguals as well. According to my proposal, the disappearance of all

[^3]the languages that a household does not understand would hurt every single household in some regard.
$\theta$ in the utility functions (10) and (11) refers to a sense of discomfort or estrangement in the society afflicting everyone in the minority and possibly afflicting people in the majority too. $\theta$ is independent of the allocation of income or time. As developed in eqs. (20) and (21), in the case of the majority (eq. (20)), there is not even necessarily any estrangement at all: $\theta_{1}+$ $\theta_{2}$ may be zero and thus $\theta$ equal to 1 . However, as $\mathrm{N}_{2} / \mathrm{N}$ rises, the heterogeneity of cultures and manners at home become less pleasurable, and as indicated in the introduction, beyond a certain point, even arouses discomfort. Thus, above a threshold of cultural diversity, low for some, high for others, the members of the majority will be invaded with the same feeling of not being "at home" as the one that prevails (more or less strongly) in the minority. In addition, according to the formulation, a rise in the proportion of minority members who do not speak the majority language, $\mathrm{N}_{22} / \mathrm{N}$, if anything, raises discomfort in the majority.

As regards the minority, eq. (21), $\theta$ is always less than one since $\bar{\theta}$ is positive. $\bar{\theta}$ depends essentially on the hospitality in the host country: the ease of obtaining citizenship, social attitudes toward foreigners, the ability to use a foreign language in pedestrian traffic and transport, the courts and the hospitals, etc. But, in any event, minority members suffer a utility loss. Quite significantly, however, this utility loss is tempered by a greater sense of solidarity within the sub-culture itself. Lazear (1999) offers corroborating evidence from the US. The solidarity in the minority has been modeled here as depending strictly on the size of $\mathrm{N}_{2} / \mathrm{N} .{ }^{5}$

The last equation, (22), is particularly important. The children of the minority learn the majority language automatically as a result of schooling and the linguistic environment where they grow up. This puts the minority language at a great disadvantage. Because of the impact of the majority culture on the children, if the parents wish to ingrain their culture and their native language so well in their children that the children will remain bilingual throughout their adult lives, the parents must make a special effort. To do so, they must allocate $\hat{\mathrm{Y}}$ of their leisure time to the endeavor (see eq. (11)). I interpret $\hat{\mathrm{Y}}$ as a set amount of time, just as I

[^4]interpret $\overline{\mathrm{X}}$ (the time needed to learn the opposite language). Those assumptions are obvious simplifications. Eq. (22) then models the pleasure resulting from this last allocation of time as a function of 3 variables. One is the percentage of the minority who do not speak the majority language $\mathrm{N}_{22} / \mathrm{N}_{2}$. Monolingualism in the minority promotes the parents' ability to transmit their native language to their children and makes the effort to do so more pleasurable for them. Another influence is $\bar{\theta}$, the previous measure of inhospitality. Low hospitality diminishes the integration of the minority households in the society and thereby increases their attachment to their culture and their desire to transmit their culture and their language to their children. This agrees with many observations (for example, the impact of the reduction of anti-Semitism in Europe since World War II on the decline of the Yiddish language in the Jewish population of Yiddish-speaking ancestry still living in Europe). The third influence is the percentage n of the $\mathrm{N}_{2}$ households who choose to allocate the time $\hat{\mathrm{Y}}$ to the effort. This percentage provides positive social reinforcement to the parental effort and increases both its pleasure and its effectiveness. m (currently $=1$ ) enters in a crucial way in eq. $(22)$ (where $\mathrm{m}=0$ leads to $\Omega=0$ ) on the idea that only parents who speak the minority language can transmit it to their children. Last, the indexing of the coefficient $\chi$ admits differences in the pleasure that individual minority parents get from allocating time $\hat{Y}$ to the effort. The dependent variable of eq. (22), $\Omega$, is perhaps the single most important reflection in the model of the minority households' attachment to their maternal language. While I assume that spending time $\overline{\mathrm{X}}$ means acquiring the opposite language, I will not insist below on the necessary success of the object of time $\hat{Y}$. There are some interesting affinities between eq. (22) and the general theoretical specification of the transmission of cultural traits to children in Bisin and Verdier (2001).

Two observations about this model are essential before moving on. First, labor time, H , still indicates a common number of hours (fraction of time) for everyone as it did earlier with a single language. However, whereas before this treatment of H had been consistent with viewing the variable as endogenous, this is no longer true since if given the choice, households would now make different labor/leisure choices based on linguistic considerations. Thus, the required associated assumption now is that H is exogenous. Next, it is no longer
possible to eliminate prices as simply as before. Marginal costs will now differ between industries depending upon whether monolinguals in the minority culture are present or not, and therefore the relative prices of all goods may no longer be the same. For this reason, I will add the special assumption that minority households are present in all industries. As a result, marginal costs will continue to be the same in all industries, all relative prices will also still be the same and identically one and they will still drop out from the household budget restraint, or eqs. (8) and (9).

As a result of these assumptions, eqs. (8)-(11) may be rewritten as follows, in accordance with the earlier passage from eqs. (2)-(3) to (2a)-(3a):
(8a) $\quad \mathrm{q}_{\mathrm{i}}=\mathrm{T}_{\mathrm{i}} \mathrm{c}_{\mathrm{i}}$
(9a) $\mathrm{q}_{\mathrm{j}}=\mathrm{T}_{\mathrm{j}} \mathrm{c}_{\mathrm{j}}$
(10a) $\quad U_{i}=\theta_{i} U\left(T_{i}^{1 / p} \mathrm{c}_{\mathrm{i}}+\beta\left(1-\omega_{\mathrm{i}}\right)\left(1-\mathrm{H}-\mathrm{S}_{\mathrm{i}}\right)\right)$

$$
\begin{equation*}
\mathrm{U}_{\mathrm{j}}=\theta_{\mathrm{j}} \mathrm{U}\left(\mathrm{~T}_{\mathrm{j}}^{1 / \mathrm{p}} \mathrm{c}_{\mathrm{j}}+\beta\left(1-\omega_{\mathrm{j}}\right)\left(1-\mathrm{H}-\mathrm{S}_{\mathrm{j}}-\mathrm{Y}\right)+\Omega_{\mathrm{j}} \mathrm{Y}\right) \tag{11a}
\end{equation*}
$$

## II. Individual decision problems and solutions

The decision problem differs by culture and by language group within each culture. For households $\mathrm{N}_{12}$, there is no decision problem at all. The model sets all endogenous values on the right hand side of the utility function (10a) and the equation simply yields utility U . Quite specifically, eqs. (5), (18) and (8a) determine q, T and c, and eqs. (12), (14), (16) and (20) determine $\mathrm{S}, \gamma, \omega$, and $\theta$. In the case of a household $\mathrm{N}_{11}$, there is a utility-maximization problem: whether to learn the minority language or not (the value of X). Eq. (10a) is needed to determine X . Otherwise, the system is the same as for the bilinguals in the majority. For any bilingual minority household $\mathrm{N}_{21}$, the decision problem instead is Y : whether to allot the necessary time to inculcating the minority language in its children. Eqs. (6), (7), (19) and (9a) determine $\eta, q, T$ and $c$, while eqs. (11a), (13), (15), (17), (21) and (22) determine $S, \gamma, \omega, \theta$ and Y . The monolingual minority households $\mathrm{N}_{22}$ have 2 problems: whether to learn the majority language and whether to inculcate the minority language in their children. Based on the model, however, the former decision, about X, does not depend on the latter and therefore can be discussed independently and first. In the case of this first decision, eqs. (6), (7), (9a), (11a),
(13), (15), (17), (19) and (21) determine $\eta, q, c, S, \gamma, \omega, T, \theta$, and $X$ simultaneously. As regards the second decision, about $Y$, the same equations yield the same solutions for $\eta, q, c, S$, $\gamma, \omega, \mathrm{T}$ and $\theta$ given X , but the system also solves for $\Omega$ and Y after adding eq. (22). The $\mathrm{N}_{22}$ households who opt to become bilingual simply have less leisure time available to choose $\mathrm{Y}=\hat{\mathrm{Y}}$ than the rest.

Let us next consider the solutions themselves and begin with those about learning the second language for the two sets of monolinguals. In doing so, it is useful to proceed in stages: first to consider the solutions strictly on the basis of the communicative benefits; next to admit the benefits of higher variety of consumption; and last, as concerns the minority parents, to admit the income effect. We shall subsequently examine the separate problem for the minority parents whether to spend the required time transmitting the minority language to their children.

The respective solutions for the two cultures to the learning problem based on communicative benefits alone depend on the next 2 inequalities:

$$
\begin{aligned}
& \text { (23) } \bar{X}_{\mathrm{i}} \leq{ }_{\mathrm{i}} \gamma_{1}\left(\frac{\mathrm{~N}_{22}}{\mathrm{~N}}\right)^{\psi} \delta\left(\mathrm{T}_{\mathrm{i}}-1\right)+{ }_{\mathrm{i}} \omega_{1} \frac{\mathrm{~N}_{22}}{\mathrm{~N}}\left[1-\mathrm{H}-\left(1+\gamma_{\mathrm{o}}+{ }_{\mathrm{i}} \gamma_{1}\left(\frac{\mathrm{~N}_{22}}{\mathrm{~N}}\right)^{\psi}\right) \delta\left(\mathrm{T}_{\mathrm{i}}-1\right)\right] \\
& \text { (24) } \overline{\mathrm{X}}_{\mathrm{j}} \leq{ }_{\mathrm{j}} \gamma_{1}\left(\frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right)^{\psi} \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)+{ }_{\mathrm{j}} \omega_{1} \frac{\mathrm{~N}_{11}}{\mathrm{~N}}\left[1-\mathrm{H}-\left(1+\gamma_{\mathrm{o}}+{ }_{\mathrm{j}} \gamma_{1}\left(\frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right)^{\psi}\right) \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)\right]
\end{aligned}
$$

The first or $\gamma$ term on the right is the time that the household saves on transactions by learning the other language. The second or $\omega$ term is the leisure-time equivalent (based on the specification) of the benefit that the household gains from the ability to interact socially with the $\mathrm{N}_{22} / \mathrm{N}$ or $\mathrm{N}_{11} / \mathrm{N}$ part of the population during the pleasurable leisure time available at the start (the term in square brackets). $\mathrm{T}_{\mathrm{i}}$ and $\mathrm{T}_{\mathrm{j}}$ in these equations refer to the values prior to learning and will continue to be so in all the subsequent solutions.

If and only if $\bar{X}$ is lower than the sum of the 2 terms on the right hand side does learning the second language increase welfare. In case of a tie, I assume that the household learns the language.

It is interesting to compare eqs. (23)-(24) with the results of previous efforts to formalize the welfare improvements of language learning outside the labor market, beginning with

Selten and Pool (1991), continuing with Church and King (1993), and going on more recently with Ginsburgh et al. (2007) and Gabszewicz et al. (2011). In all these writings, the incentive to learn the second language is much higher for the minority than the majority precisely for the same reason as it is here: namely, that the monolingual majority population $\mathrm{N}_{11}$ is larger than the monolingual minority population $\mathrm{N}_{22}$. (The first $\gamma_{1}$ term on the right evidently dominates the second, $\omega_{1} \gamma_{1}$, one with the opposite sign.) The difference here is that the communicative benefits stemming from lower transactions costs can be driven down, possibly to insignificance, by translation (via the coefficient $\psi$ which enters exponentially). However, translation cannot similarly diminish the benefits of bilingualism stemming from social interactions. Notwithstanding, we remain in the realm where bilingualism in both cultures is sheer waste. The next step repairs this problem.

Once we admit the impact of a second language on widening the variety of goods that enter in the consumption basket, the condition for learning the second language (after substituting $\gamma_{i}(0)$ for $\gamma_{0}+{ }_{i} \gamma_{1}\left(\frac{N_{22}}{N}\right)^{\psi}$ and $\gamma_{j}(0)$ for $\left.\gamma_{o}+{ }_{j} \gamma_{1}\left(\frac{N_{11}}{N}\right)^{\psi}\right)$ becomes:

$$
\begin{array}{r}
\beta \overline{\mathrm{X}}_{\mathrm{i}} \leq \beta_{\mathrm{i}} \gamma_{1}\left(\frac{\mathrm{~N}_{22}}{\mathrm{~N}}\right)^{\psi} \delta\left(\mathrm{T}_{\mathrm{i}}-1\right)+\beta_{\mathrm{i}} \omega_{1} \frac{\mathrm{~N}_{22}}{\mathrm{~N}}\left[1-\mathrm{H}-\left(1+\gamma_{\mathrm{i}}(0)\right) \delta\left(\mathrm{T}_{\mathrm{i}}-1\right)\right] \\
+{ }_{\mathrm{i}} \tau_{3}\left(\frac{1}{\rho}\left(\mathrm{~T}_{\mathrm{i}}\right)^{\frac{1-\rho}{\rho}} \mathrm{c}_{\mathrm{i}}-\beta\left(1+\gamma_{\mathrm{o}}\right) \delta \mathrm{T}_{\mathrm{i}}\right) \\
\beta \overline{\mathrm{X}}_{\mathrm{j}} \leq \beta_{\mathrm{j}} \gamma_{1}\left(\frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right)^{\psi} \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)+\beta_{\mathrm{j}} \omega_{1} \frac{\mathrm{~N}_{11}}{\mathrm{~N}}\left[1-\mathrm{H}-\left(1+\gamma_{\mathrm{j}}(0)\right) \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)\right]  \tag{26}\\
+{ }_{\mathrm{j}} \tau_{3}\left(\frac{1}{\rho}\left(\mathrm{~T}_{\mathrm{j}}\right)^{\frac{1-\rho}{\rho}} \mathrm{c}_{\mathrm{j}}-\beta\left(1+\gamma_{\mathrm{o}}\right) \delta \mathrm{T}_{\mathrm{j}}\right)
\end{array}
$$

The negative component of the new $\tau_{3}$ term in these 2 equations reflects the extra time required to shop for the new goods that the second language makes available. However, the primary effect is in the first term and concerns the essential welfare gain of a wider variety of consumption. ( $\delta \mathrm{T}$ in the negative part is a small fraction while T in the positive part is a large number).

To complete the analysis, we must include the benefit of the rise in income that a minority household will obtain by learning the majority language. This leads to eq. (27):

$$
\begin{align*}
\beta \bar{X}_{\mathrm{j}} \leq \beta_{\mathrm{j}} \gamma_{1}\left(\frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right)^{\psi} \delta\left(\mathrm{T}_{\mathrm{j}}-1\right) & +\beta_{\mathrm{j}} \omega_{1} \frac{N_{11}}{\mathrm{~N}}\left[1-\mathrm{H}-\left(1+\gamma_{\mathrm{j}}(0)\right) \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)\right]  \tag{27}\\
& +{ }_{\mathrm{j}} \tau_{3}\left(\frac{1}{\rho}\left(\mathrm{~T}_{\mathrm{j}}\right)^{\frac{1-\rho}{\rho}} \mathrm{c}_{\mathrm{j}}-\beta\left(1+\gamma_{\mathrm{o}}\right) \delta \mathrm{T}_{\mathrm{j}}\right)+(1-\bar{\eta}) \mathrm{c}_{\mathrm{j}}\left(\left(1+{ }_{\mathrm{j}} \tau_{3}\right) \mathrm{T}_{\mathrm{j}}\right)^{\frac{1}{\rho}}
\end{align*}
$$

The last term in eq. (27) reflects the income gain from learning the majority language for the minority monolingual household. This term gives rise to a higher volume of consumption of each and every individual good $\left((1-\bar{\eta}) \mathrm{c}_{\mathrm{j}}\right) \cdot{ }^{6}$ Eqs.(25) and (27) represent the full conditions for learning the opposite language for the two sets of monolinguals.

There remains the minority decision whether or not to accord the required time to inculcating the native language in its children as opposed to devoting the time to other pleasurable pursuits, sports, socializing, reading, and other parental activities, etc. For the minority households who are already bilingual, $\mathrm{N}_{21}$, the formal decision rule is exactly the same as for the members of $\mathrm{N}_{22}$ who decide to become bilingual, even though the former households have more time left over for other pleasurable pursuits besides $\hat{Y}$. The solution is to adopt $\hat{Y}$ if

$$
\begin{equation*}
{ }_{\mathrm{j}} \chi\left(1+{ }_{\mathrm{j}} \bar{\theta}\right)\left(1+\frac{\mathrm{N}_{22}}{\mathrm{~N}}+\mathrm{n}\right) \mathrm{m} \geq \beta \tag{28}
\end{equation*}
$$

As for the minority households who decide to stay monolingual, the rule is to adopt $\hat{Y}$ if

$$
\begin{equation*}
{ }_{\mathrm{j}} \chi(1+\overline{\mathrm{\theta}})\left(1+\frac{\mathrm{N}_{22}}{\mathrm{~N}}+\mathrm{n}\right) \mathrm{m} \geq \beta\left(1-{ }_{\mathrm{j}} \omega_{1} \frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right) \tag{29}
\end{equation*}
$$

This last sub-group of $\mathrm{N}_{22}$ is therefore more prone to adopt $\hat{Y}$ than the former $\mathrm{N}_{22}$ sub-group. Their alternative uses of leisure time are less satisfying: they have fewer opportunities for satisfying encounters in everyday life.

## III. Social welfare implications

The decentralized solutions to both of the language problems in the model are inefficient. Four separate inefficiencies may be distinguished. There is a basic social dilemma, however, because repairing the fourth inefficiency, if not the third, runs contrary to the problem of repairing the first two.

[^5]First, any learning of the majority language by the minority to gain communicative benefits as such is socially inefficient. Learning of the majority language by the minority would yield a greater social advantage. Every unit addition to $\mathrm{N}_{21}$ produces communicative benefits to $\mathrm{N}_{11}$ individuals while every unit addition to $\mathrm{N}_{12}$ yields such benefits to only $\mathrm{N}_{22}$ individuals. Notwithstanding, some members of $\mathrm{N}_{11}$ will decide to learn the minority language for the sake of the communicative benefits alone. Two factors mitigate this problem while a third one can work either way. The first mitigating factor is translation. Translation cuts down the numbers in the majority who decide to learn the minority language for the sake of the trade benefits (that is, in connection with the $\gamma$ term, not the $\omega$ term in eq. (25)). The second attenuating factor is that the members of the majority who learn the minority language still obtain benefits of consumption diversity that they would otherwise not get (the last term in eq. (25)). Indeed, some of these people might have decided to learn the minority language regardless of any communicative benefits (if $\tau_{3}$ was high enough). The one factor that could go either way is the greater required time to learn the minority language than the majority one: $\bar{X}_{i}>\bar{X}_{j}$. So far as this greater difficulty cuts down the numbers who try to learn the minority language, the factor attenuates the social problem. So far as the majority members nevertheless decide to learn the minority language on account of the communicative benefits, the social waste is larger. (More resources are lost as compared with the opposite learning of the majority language by the minority.) Obviously the issue is one of an elasticity.

Quite independently, however, as a second inefficiency, there is too little learning of the majority language by the minority. That is, even if no $\mathrm{N}_{11}$ members decided to learn the minority language, there would still be too few minority households who decided to learn the majority language, and this is so for two separate reasons. First, the minority disregards the communicative benefits of a reduction in $\mathrm{N}_{22} / \mathrm{N}$ to the majority. ${ }^{7}$ Second, the minority also disregards the comfort in everyday life that the majority gets from lower $\mathrm{N}_{22} / \mathrm{N}$ (the $\theta_{2}$ term in eq. (20) for $\theta$ in the utility function eq. (10a)). Once again, two factors mitigate the problem. One is the income benefit of learning the majority language. This benefit clearly promotes

[^6]learning of the language by the minority. The second is the special ease of learning the majority language because of its currency in the environment (in this case, there is no issue of an elasticity: any learning of the majority language helps). On the other hand, translation increases the problem. Because of translation, fewer minority households will learn the majority language. While this does not reduce the majority's trade benefits, it still cuts down the group's communicative benefits in social interactions and the group's general sense of being "at home".

The third inefficiency relates to the failure of the decentralized decision-making to take any account of linguistic diversity in the world $\mathrm{L}_{\mathrm{w}}$. This world diversity increases everyone's welfare by increasing diversity of consumption (see eqs. (18)-(19) together with (10a)-(11a)). But while the presence of the minority language makes everyone better off in this respect, no household gives this a thought in its decision-making. True, depending on how we interpret $\mathrm{L}_{\mathrm{W}}$ (a point to which we will return), no resulting inefficiency will ensue as long as the minority language thrives elsewhere in the world. If French does not survive in Canada as a minority language, $\mathrm{L}_{\mathrm{W}}$ might remain the same (depending on how we define it). However, in the case of any language without majority status anywhere (or in the sort of circumstances that we currently suppose and from which we will only depart in the first appendix), the social inefficiency is evident.

The fourth inefficiency comes from the disregard of the future (which in the present case means the time after the current generation of adults is gone). This disregard is now total, but any element of it will do. The result is an inadequate concern with the benefit of encouraging bilingualism in the children of the minority. According to the model, this bilingualism (raising L to 2 in eq. (19) and thus maintaining m equal or close to 1 in eq. (18)) can do nothing but good to the next generation by enhancing consumption diversity for minority and majority alike. Yet this inefficiency stands in stark conflict with the first two, which say that more bilingualism among the adults in the minority is a benefit. The point is, of course, that any added bilingualism in the minority will damage the transmission of the minority language to the minority's children. We must pause here on a difference in viewpoint in Lazear (1999).

In a rich discussion covering much of the same terrain, but in some important regards,
especially immigration and the concentration of minorities in parts of urban areas ("ghettoes"), covering much more, Lazear treats the welfare benefit of the transmission of the minority language to the next generation as an open question. Some background is important. For most of his discussion, Lazear supposes a highly hospitable country tending to integrate minorities over time in conformity with the American ideal of the "melting pot." From this perspective, he reasonably considers that the factor of estrangement affecting all members of the minority culture in the model $\bar{\theta}$, if present, could disappear over time and instead of working on transmitting their culture to their children, which may be to the immigrant parents' own advantage (a point on which the model implicitly agrees), the parents would perhaps do their children more good by trying to rid them of their sense of estrangement in the country and eliminating $\bar{\theta}$ (see p. 122 in particular).

This part of Lazear's stand could be easily accommodated (surely as a possibility) without upsetting anything in the model by modeling $\bar{\theta}$ accordingly. However, some other differences cannot be reconciled. One of them concerns the learning of the majority language by the children of the minority. Whereas I assume that the children of the minority will learn the majority language automatically and the only question is whether they will become bilingual and retain the parents' native language, Lazear treats the children's entire language repertoire as essentially dependent on the parents. With regard to this difference, I believe that the general sociolinguistic literature favors my assumption, at least in a skeletal model about language. In addition, Lazear tends to minimize the advantage of the children's retention of the minority language both to themselves and to others, which I stress. On this next point, it bears note that ordinary humans are notoriously capable of learning several languages in childhood while studies of the impact of bilingualism on wages, and even more specifically on the income effect of possessing minority languages point, if anything, to a benefit of dual languages on wages. ${ }^{8}$

[^7]To return to the social inefficiencies in the model, there could be a fifth one relating to the minority's transmission of its language and culture to its children, and apart from any issue of the future. But the argument in this case is a bit precarious. Imagine that in deciding whether to choose $\hat{Y}$ minority households supposed that few others or no others in their own culture would make the same choice. Then too few of the minority households would adopt the choice from the standpoint of the collective welfare of the minority. However, if the minority is small enough and highly enough concentrated in urban areas (as Lazear reminds us), the assumption of an underestimate of n is questionable. Notwithstanding, there is clearly a coordination problem in the minority, as the model is formulated, stemming from the role of n in the transmission decision (eqs. (28) and (29)). Those minority households for which $\chi$ (eq. (22)) is not high enough would not wish to be drawn into any collective efforts to promote n that some others in their culture would wish, since those efforts would require excessive personal contributions to special schools, churches, political organizations, newspapers, for example. Yet these people's withdrawal from the collective effort would reduce the rewards that the rest can obtain. ${ }^{9}$ These observations are meant to reflect some aspects of reality. We do, in fact, encounter organized efforts of minorities in the world to bolster their culture and language by imposing restrictions within their own ranks. But those efforts are also the subject of tensions within the minority. ${ }^{10}$

## IV. General observations on policy, language survival and world linguistic diversity

The model harbors 4 parameters that could serve as instruments for language policy: $\bar{\theta}, \psi, \overline{\mathrm{X}}_{\mathrm{i}}$ and $\overline{\mathrm{X}}_{\mathrm{j}} \cdot \bar{\theta}$ depends on the facility of obtaining citizenship, permits for places of

[^8]worship, immigration policy, and attitudes toward the minority language in public places, including hospitals, transportation, and the courts. It is also affected by many factors relating to $\psi$, including status as an official language, school curricula and legal requirements about languages in broadcasting, labelling and advertising. Finally, $\bar{X}_{i}$ and $\bar{X}_{j}$ depend partly on public programs of adult education or at least may be said to depend so even though the programs may not affect the required time for learning since the programs do alter the cost of learning and thereby affect $\overline{\mathrm{X}}_{\mathrm{i}}$ and $\overline{\mathrm{X}}_{\mathrm{j}}$ (which represent non-pleasurable leisure time) in utilityequivalent terms.

In addition, the fundamental ambiguity in the model about the proper use of the tools of language policy to encourage or discourage the minority language resonates well with reality. Different countries make different choices. There are numerous instances of outright hostility to minority languages, such as Kurdish in Turkey and Iraq, Hungarian in Romania. There are also instances of countries that try to ignore the minority language(s) at home and implicitly hope they will disappear. Thus, France denies recognition of Breton, Corsican, Basque and Alsatian (the largest and essentially a dialect of German) as official languages, even regionally. Yet some countries go the opposite way and try to accommodate their main minority language(s). Witness the efforts of the Swiss to sustain Romansh, the Swedes to help the Tornelaan, the Dutch to preserve Frieden. The European Union generally takes a favorable attitude toward minority languages. Despite political debate in the US about according official status to Spanish, it is difficult to miss the basic accommodation of Spanish in everyday life in the country.

The manifest difficulty of achieving the objectives of language policy also finds clear expression in the model. In particular, hospitality to minority cultures, while possibly intended to keep the cultures alive, will tend to accelerate assimilation by the young and thereby undermine the minority language. If this is contrary to policy, the efforts to increase $\theta$ toward one and eliminate $\bar{\theta}$ completely will need to be offset by an effort to raise $\chi$ (eq. (22)) and make it easier and more satisfying for the minority to assure the durable transmission of their native language to their children. In the end, though, empirical evidence suggests that the language policy may not have the desired effect. The Irish have encountered problems in efforts
to keep the Irish language alive in isolated pockets of its existence. At the opposite end of the pole, outright hostility to minority languages, as in the case of Hungarian in Romania or Kurdish in Turkey, may increase solidarity in the minority and its will to hang on to language and culture. A general model of language should harbor such salient aspects of reality.

As regards the future, the model clearly reflects the threat to minority languages that was announced at the beginning. The minority language in our model is in danger. Unless steps are taken to avoid it, a portion of the minority members, $\mathrm{N}_{2} / \mathrm{N}$, will no longer possess the minority language in the next generation, and the proportion of those who do possess it, m (currently equal to one), may be expected to drop towards $n$. Short of immigration, which has not been modeled, there will be no monolinguals at home who possess strictly the minority language in the next generation. The only monolinguals will be those who speak the majority language alone. Moreover, those people will include members of the $\mathrm{N}_{2}$ population, around $(1-n) N_{2}$ of them. Since this last group of monolinguals in $N_{2}$ can hardly transmit the minority language to its children, the choice of allocating $\hat{Y}$ time to the effort befalls strictly the other $\mathrm{N}_{2}$ members and only will be made by those who find

$$
{ }_{\mathrm{j}} \chi\left(1+{ }_{\mathrm{j}} \bar{\theta}\right)(1+\mathrm{n}) \mathrm{m}-\beta \geq 0
$$

(as follows by setting $\mathrm{N}_{22}$ equal to zero in eq. (28)). Note also that in this last inequality m is now less than one while n is a maximum of m . On this reasoning, knowledge of the minority language in the $\mathrm{N}_{2}$ population could fall precipitously in the generation after the next. ${ }^{11}$ Clearly, according to this specification, the minority language could vanish from the home country within a few generations.

By extension, the only languages in the world that are safe from extinction are those with a national home base. With around 200 countries worldwide that makes around 200 safe languages at most. If we compare this view with reality, 200 is an exaggeration in one respect and an understatement in another. It is an exaggeration because some languages, like English

[^9]and Arabic, have a home base in several countries. It is an understatement because some languages are firmly rooted in regions of countries: Punjabi in Punjab, Catalan in Catalonia, Kurdish in Eastern Turkey and Western Iraq, etc. On a rough estimate, the understatement is probably more important. ${ }^{12}$ Indeed, a fundamental preserver of minority languages in the world may well be economic and political fractionalization (and warfare) within countries and domestic labor and capital immobility, all of which make the model less applicable. Yet if we expect future economic growth and advance of world trade to feed the disappearance of regional barriers to internal trade, the model will become progressively more applicable and it becomes difficult to envisage even as many as 400 languages in a couple of centuries.

From this perspective, the preservation of world linguistic diversity, $\mathrm{L}_{\mathrm{W}}$, enters as a fundamental consideration. On this issue, few people will see harm in the effort to preserve outside of their own country on earth, and some people will be strongly in favor. However, there is an issue of international coordination. Short of international agreement, there is no mechanism relating the number of surviving languages in the world to the desired number of languages. In the case of animals and plants, where a similar situation holds, some international actions have been taken to protect endangered species. But with the possible exception of the EU, no similar movement is afoot in the case of languages. What general guidelines could such a movement adopt?

In any attempt to safeguard the number of world languages, some system of priorities would be needed. Weinstein's (1992) analysis of an ordering on the grounds of diversity, which he essentially defined in connection with the evolution of species of animals and plants (though he mentions languages too), would suggest strictly philological considerations. The concern would be with keeping alive as many higher branches as possible in the evolution of language. Thus, a language like Basque, whose origins are clouded in mystery, would remain highly important, while Catalan, a close cousin of Castilian and French, all Latin derivatives, would be much less so. Similarly, preserving one of the Gaelic languages would matter a great

[^10]deal, but if Gaelic Welsh survived, keeping Breton alive as well would be of little concern, especially as opposed to preserving another genus. But literary specialists would be very much concerned with a different criterion: the presence of a literature in the language. In this event, the argument for Catalan would be much stronger than the one for Basque. In addition, if we consider the criterion as requiring a certain literary capital going beyond folk tales, only a fraction of the world's languages would matter.

Quite significantly, however, the issue may be broader. Do we really want a measure of $\mathrm{L}_{\mathrm{W}}$ that depends strictly on the surviving languages in the solution to a resource-constrained allocation problem? Suppose, for example, that a radical form of bilingual diglossia sets in on earth with a single language serving in speech and writing in government, business, trade and literature, while all other languages survive strictly in informal contact with kinsmen. Would there be no reduction in $\mathrm{L}_{\mathrm{w}}$ ? The worried accounts of the spread of English as a universal language envisage a certain tendency in this direction. In addition, the history of previous spreads of languages over broad territories since antiquity suggest grounds for such concern (Dalby (2002) and Da Landa (2011), chapter 3, are particularly interesting). ${ }^{13}$ My fundamental point, however, is simply that the vibrancy of languages is probably an issue as well as their survival in measuring $\mathrm{L}_{\mathrm{w}}$. Therefore we may want a measure of $\mathrm{L}_{\mathrm{w}}$ that incorporates welfare weights for surviving languages in a Weinstein type of analysis.

## V. Closing remarks

This has been an exercise in fitting pieces together. There exists an impressive range of contributions to the economics of language by economists working in different subdisciplines. Quite understandably, in their attempts to develop some aspect of this vast topic, authors have tended to abstract from much of the rest. My concern instead has been to unify

[^11]and bring together. ${ }^{14}$ As a result, my proposed framework is more complicated than some though I have tried to keep it as simple as possible. Two choices loom large in my efforts to simplify. First, I adopted conditions where language has no relative price effects at all. Second, while admitting the effect of language on both the variety of consumption and pleasurable leisure time, I assumed that variety of consumption and pleasurable leisure time enter separately in the utility function. Without both modeling choices, I could not cope.

[^12]
## APPENDIX 1

## Foreign speakers of the two home languages

Suppose we lift the assumption that neither home language is spoken abroad or the equivalent assumption that any difference in transactions time $S$ resulting from foreign speakers of either language is negligible. There is now less transaction time required for trade with foreigners. Accordingly, let us replace eqs. (14) and (15) with:

$$
\gamma_{\mathrm{i}}=\left\{\begin{array}{cc}
\gamma_{\mathrm{W}, \mathrm{o}} & \text { if } \mathrm{i} \in \mathrm{~N}_{12}  \tag{14a}\\
\gamma_{\mathrm{W}, \mathrm{o}}+{ }_{\mathrm{i}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 22}\right)^{-\psi}+{ }_{\mathrm{i}} \gamma_{1}\left(\frac{\mathrm{~N}_{22}}{\mathrm{~N}}\right)^{\psi} & \text { if } \mathrm{i} \in \mathrm{~N}_{11} \text { where } \quad \psi>1 \text { and } \\
{ }_{\mathrm{i}} \gamma_{\mathrm{W}, \mathrm{o}}+{ }_{\mathrm{i}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 22}\right)^{-\psi}={ }_{\mathrm{i}} \gamma_{\mathrm{W}, 1}<\gamma_{\mathrm{o}}
\end{array}\right.
$$

$$
\gamma_{\mathrm{j}}=\left\{\begin{array}{cc}
\gamma_{\mathrm{W}, \mathrm{o}} & \text { if } \mathrm{j} \in \mathrm{~N}_{21}  \tag{15a}\\
{ }_{\mathrm{j}} \gamma_{\mathrm{W}, \mathrm{o}}+{ }_{\mathrm{j}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 11}\right)^{-\psi}{ }_{\mathrm{j}} \gamma_{1}\left(\frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right)^{\psi} & \text { if } \mathrm{j} \in \mathrm{~N}_{22} \text { where } \psi>1 \text { and } \\
{ }_{\mathrm{j}} \gamma_{\mathrm{W}, \mathrm{o}}+{ }_{\mathrm{j}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 11}\right)^{-\psi}={ }_{\mathrm{j}} \gamma_{\mathrm{W}, 1}<\gamma_{\mathrm{o}}
\end{array}\right.
$$

$\gamma_{\mathrm{W}, \mathrm{o}}$ is associated with the transaction time in foreign trade that affects everyone because of third languages. $\gamma_{\mathrm{W}, \mathrm{o}}+{ }_{\mathrm{i}} \lambda\left(\mathrm{N}_{\mathrm{W}, 22}\right)^{-\psi}$ or $\gamma_{\mathrm{W}, \mathrm{o}}+{ }_{\mathrm{j}} \lambda\left(\mathrm{N}_{\mathrm{W}, 11}\right)^{-\psi}$ - alternatively, either ${ }_{\mathrm{i}} \gamma_{\mathrm{W}, 1}$ or ${ }_{\mathrm{j}} \gamma_{\mathrm{W}, 1}$ - is associated with the transactions time in foreign trade affecting the monolinguals only. ${ }_{i} \gamma_{\mathrm{W}, 1}$ and ${ }_{\mathrm{j}} \gamma_{\mathrm{W}, 1}$ are higher than $\gamma_{\mathrm{W}, \mathrm{o}}$ but still lower than $\gamma_{\mathrm{o}}$. The difference $\gamma_{\mathrm{W}, 1}-\gamma_{\mathrm{W}, \mathrm{o}}$ differs for the two monolingual populations. For $\mathrm{N}_{11}$, the difference depends positively on the total world number of foreign speakers of the minority language who do not understand the majority one, $\mathrm{N}_{\mathrm{W}, 22}$, while for $\mathrm{N}_{22}$, it depends positively on the total world number of foreign speakers of the majority language who do not understand the minority one, $\mathrm{N}_{\mathrm{W}, 11}$. For both groups, it depends negatively on translation (as indicated by the exponent $-\psi$ ). ${ }^{15}$

The new language learning solutions corresponding to eqs. (25) and (27) become:

[^13](25a) $\beta \bar{X}_{i} \leq \beta\left[{ }_{i} \lambda\left(N_{W, 22}\right)^{-\psi}+_{i} \gamma_{1}\left(\frac{N_{22}}{N}\right)^{\psi}\right] \delta\left(\mathrm{T}_{\mathrm{i}}-1\right)+\beta_{i} \omega_{1} \frac{N_{22}}{N}\left[1-H-\left(1+\hat{\gamma}_{\mathrm{i}}(0)\right) \delta\left(\mathrm{T}_{\mathrm{i}}-1\right)\right]$
$$
+{ }_{\mathrm{i}} \tau_{3}\left[\frac{1}{\rho}\left(\mathrm{~T}_{\mathrm{i}}\right)^{\frac{1-\rho}{\rho}} \mathrm{c}_{\mathrm{i}}-\beta\left(1+\gamma_{\mathrm{W}, \mathrm{o}}\right) \delta \mathrm{T}_{\mathrm{i}}\right]
$$
(27a) $\beta \overline{\mathrm{X}}_{\mathrm{j}} \leq \beta\left[{ }_{\mathrm{j}} \lambda\left(\mathrm{N}_{\mathrm{W}, 11}\right)^{-\psi}{ }_{\mathrm{j}} \gamma_{1}\left(\frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right)^{\psi}\right] \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)+\beta_{\mathrm{j}} \omega_{1} \frac{\mathrm{~N}_{11}}{\mathrm{~N}}\left[1-\mathrm{H}-\left(1+\hat{\gamma}_{\mathrm{j}}(0)\right) \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)\right]$
$$
+{ }_{j} \tau_{3}\left[\frac{1}{\rho}\left(T_{j}\right)^{\frac{1-\rho}{\rho}} c_{j}-\beta\left(1+\gamma_{W, o}\right) \delta T_{j}\right]+(1-\bar{\eta}) c_{j}\left(\left(1+{ }_{j} \tau_{3}\right) T_{j}\right)^{\frac{1}{\rho}}
$$
where $\hat{\gamma}_{\mathrm{i}}(0)={ }_{\mathrm{i}} \lambda\left(\mathrm{N}_{\mathrm{W}, 22}\right)^{-\psi}+{ }_{\mathrm{i}} \gamma_{1}\left(\frac{\mathrm{~N}_{22}}{\mathrm{~N}}\right)^{\psi}$ and $\hat{\gamma}_{\mathrm{j}}(0)={ }_{\mathrm{j}} \lambda\left(\mathrm{N}_{\mathrm{W}, 11}\right)^{-\psi}+{ }_{\mathrm{i}} \gamma_{1}\left(\frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right)^{\psi}$
There are two important changes. The first concerns the added economies in transactions costs from learning the second language and is in the first term on the right. The second concerns the greater leisure time available for pleasurable encounters and is in the second or $\omega$ term on the right (as reflected in the lower value of $\hat{\gamma}(0)$ than the previous $\gamma(0)$ ). In addition, but as a lesser consideration, the $\tau_{3}$ term on the right is somewhat higher than before because the rise in $T$ requires less transactions time than before ( $\gamma_{\mathrm{W}, \mathrm{o}}$ is lower than $\gamma_{\mathrm{o}}$ ).

According to the proposed modification, there will be more learning of the home languages by both sets of monolinguals because of foreign speakers of the languages, but translation will limit the part of this rise associated with trade. As regards the extra learning, the entire part resulting in more household pairs (worldwide) that are now capable of communicating in two separate languages will be a social waste from the standpoint of communication alone. However, some of this new social waste will no longer be reparable nationally and poses a problem of international coordination. Moreover, if the minority language at home is larger than the majority one on the world level (and this is reflected in trade, as implicitly the case), from an international perspective, the new social waste may mean too much learning of the majority language by the minority rather than the opposite. However, from the national perspective, which remains perhaps the more important one, so far as foreign trade yields any additional learning of the majority language by the minority it will obviously diminish the problem of inadequate learning of the majority language at home. For this reason, this last
problem of inadequate learning of the majority language becomes smaller, for example, for a country like England than one like Italy. By the same token, however, the problem of the insufficient transmission of the minority language to the children becomes larger in a country like England than one like Italy.

Of course, this view of the modifications of the earlier analysis abstracts from the possibility that people will wish to learn the second home language for the sake of communicative benefits of social interaction with tourists or during visits abroad (which is not in the $\omega$ term ${ }^{16}$ though it could be added) (compare Olivier et al (2008)). This view of the modifications also abstracts from the welfare advantages of greater consumption diversity that would stem from larger language repertoires as such (which is present in the $\tau_{3}$ term). Finally, the analysis is likely to remind us of the underlying assumption that there is no obstacle to trade except language. None of the other familiar obstacles to trade - distance, contiguity, etc. - are present. In this connection, we should keep in mind that any introduction of the other obstacles to trade would no longer permit us to drop all relative prices as equal and would therefore complicate everything from the start. (Compare the pioneering analysis of language in foreign trade in Breton and Mieszkowski (1977), who keep the other obstacles to trade in the analysis and center instead on the impact of language on the terms of trade.) The most obvious result of any move in this direction would be to limit foreign trade relative to domestic trade and thereby to limit the importance of the modifications in this appendix.

[^14]
## APPENDIX 2

## Learning of a foreign language

Suppose next we admit the possible decision to learn a foreign language for the sake of benefits of lower costs of foreign trade. Specifically, let the relevant foreign language be the most important one for the country in foreign trade. For bilinguals this means admitting the possible learning of a third language while for monolinguals it means a new choice of a second language. To simplify, I will now revert to the assumption (lifted in the previous appendix) that the two home languages are only spoken at home. It is obviously reasonable to think of the third language as English.

Eqs. (12) and (13) regarding time $S$ remain the same except that X must now cover the cost of the time of learning the third language, $\bar{X}_{3}$. I will assume that the time required to learn this language is the same as that of learning the minority one for everyone at home, while the time required to learn the majority language is still smaller than that of learning either of the other two for the same reasons as before, or in other words, $\bar{X}_{3}=\bar{X}_{i}>\bar{X}_{j}$. Evidently the language repertoire L that affects the total number of goods in the consumer basket in eqs. (18) and (19) may now equal 3. The fundamental changes come in eqs. (14) and (15) and concern the coefficient $\gamma$. Suppose now:

$$
\begin{align*}
& \text { (14b) } \quad \gamma_{i}=\left\{\begin{array}{lr}
{ }_{i} \gamma_{W, 3}+{ }_{i} \lambda\left(N_{W, 3}\right)^{-\psi} & \text { if } i \in N_{12} \text { and } X_{i}=0 \\
{ }_{i} \gamma_{W, 3} & \text { if } i \in N_{12} \text { and } X_{i}=\bar{X}_{3}
\end{array}\right.  \tag{14b}\\
& \text { (14c) } \quad \gamma_{i}=\left\{\begin{array}{lr}
{ }_{i} \gamma_{W, 3}+{ }_{i} \lambda\left(N_{W, 3}\right)^{-\psi}+{ }_{i} \gamma_{1}\left(\frac{N_{22}}{N}\right)^{\psi} & \text { if } i \in N_{11} \text { and } X_{i}=0 \\
{ }_{i} \gamma_{W, 3}+{ }_{i} \lambda\left(N_{W, 3}\right)^{-\psi} & \text { if } i \in N_{11} \text { and } X_{i}=\bar{X}_{i} \\
{ }_{i} \gamma_{W, 3}+{ }_{i} \gamma_{1}\left(\frac{N_{22}}{N}\right)^{\psi} & \text { if } i \in N_{11} \text { and } X_{i}=\bar{X}_{3}
\end{array}\right. \\
& \text { (15b) } \gamma_{j}= \begin{cases}{ }_{j} \gamma_{W, 3}+{ }_{j} \lambda\left(N_{W, 3}\right)^{-\psi} & \text { if } j \in N_{21} \text { and } X_{j}=0 \\
{ }_{j} \gamma_{W, 3} & \text { if } j \in N_{21} \text { and } X_{j}=\bar{X}_{3}\end{cases}
\end{align*}
$$

(15c) $\quad \gamma_{j}=\left\{\begin{array}{lr}{ }_{j} \gamma_{W, 3}+{ }_{j} \lambda\left(N_{W, 3}\right)^{-\psi}+{ }_{j} \gamma_{1}\left(\frac{N_{11}}{N}\right)^{\psi} & \text { if } j \in N_{22} \text { and } X_{j}=0 \\ { }_{j} \gamma_{W, 3}+{ }_{j} \lambda\left(N_{W, 3}\right)^{-\psi} & \text { if } j \in N_{22} \text { and } X_{j}=\bar{X}_{j} \\ { }_{j} \gamma_{W, 3}+{ }_{j} \gamma_{1}\left(\frac{N_{11}}{N}\right)^{\psi} & \text { if } j \in N_{22} \text { and } X_{j}=\bar{X}_{3}\end{array}\right.$

$$
\begin{align*}
& { }_{i} \gamma_{\mathrm{W}, 3}+{ }_{\mathrm{i}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 3}\right)^{-\psi}=\gamma_{\mathrm{o}}  \tag{30}\\
& { }_{\mathrm{j}} \gamma_{\mathrm{W}, 3}+{ }_{\mathrm{j}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 3}\right)^{-\psi}=\gamma_{\mathrm{o}} \tag{31}
\end{align*}
$$

Since all foreign trade still takes place in foreign languages, the time spent on foreign transactions because of language is reflected in $\gamma_{0}$ as before. However, $\gamma_{0}$ breaks up in two parts, one associated with foreign languages other than the third one, $\gamma_{\mathrm{w}, 3}$, and the rest, associated with the third language $\gamma_{0}-\gamma_{\mathrm{W}, 3}$ (see eqs. (30) and (31)). This last part is a positive function of the total world number of speakers of the third language $\mathrm{N}_{\mathrm{W}, 3}$ and a negative function of translation or $\psi(\psi>1)$. In addition, the monolinguals $\mathrm{N}_{11}$ and $\mathrm{N}_{22}$ still bear the same linguistic cost as before of trading in home goods unless they learn the second home language, which is modeled precisely as before as a positive function of $\mathrm{N}_{22} / \mathrm{N}$ or $\mathrm{N}_{11} / \mathrm{N}$, as the case may be, and a negative function of translation (see the $\gamma_{1}$ term in eqs. (14c) and (15c)).

The solutions to the learning problems for the two home languages are still eqs. (25) and (27), which are repeated for convenience:
(25) $\beta \overline{\mathrm{X}}_{\mathrm{i}} \leq \beta_{\mathrm{i}} \gamma_{1}\left(\frac{\mathrm{~N}_{22}}{\mathrm{~N}}\right)^{\psi} \delta\left(\mathrm{T}_{\mathrm{i}}-1\right)+\beta_{\mathrm{i}} \omega_{1} \frac{\mathrm{~N}_{22}}{\mathrm{~N}}\left[1-\mathrm{H}-\left(1+\gamma_{\mathrm{i}}(0)\right) \delta\left(\mathrm{T}_{\mathrm{i}}-1\right)\right]$

$$
\left.+{ }_{i} \tau_{3}\left(\frac{1}{\rho}\left(\mathrm{~T}_{\mathrm{i}}\right)^{\frac{1-\rho}{\rho}} \mathrm{c}_{\mathrm{i}}-\beta\left(1+\gamma_{\mathrm{o}}\right) \delta \mathrm{T}_{\mathrm{i}}\right)\right)
$$

(27) $\beta \overline{\mathrm{X}}_{\mathrm{j}} \leq \beta_{\mathrm{j}} \gamma_{1}\left(\frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right)^{\psi} \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)+\beta_{\mathrm{j}} \omega_{1} \frac{\mathrm{~N}_{11}}{\mathrm{~N}}\left[1-\mathrm{H}-\left(1+\gamma_{\mathrm{j}}(0)\right) \delta\left(\mathrm{T}_{\mathrm{j}}-1\right)\right]$

$$
+{ }_{j} \tau_{3}\left(\frac{1}{\rho}\left(T_{j}\right)^{\frac{1-\rho}{\rho}} c_{j}-\beta\left(1+\gamma_{o}\right) \delta T_{j}\right)+(1-\bar{\eta}) c_{j}\left(\left(1+{ }_{j} \tau_{3}\right) T_{j}\right)^{\frac{1}{\rho}}
$$

The solutions to the new problem of learning the foreign language differ some for monolinguals and bilinguals. These solutions are:

$$
\begin{array}{ll}
\beta \overline{\mathrm{X}}_{3} \leq \beta\left({ }_{\mathrm{i}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 3}\right)^{-\psi}\right)+{ }_{\mathrm{i}} \tau_{3}\left(\frac{1}{\rho}\left(\mathrm{~T}_{\mathrm{i}}\right)^{\frac{1-\rho}{\rho}} \mathrm{c}_{\mathrm{i}}-\beta\left(1+{ }_{\mathrm{i}} \gamma_{\mathrm{W}, 3}\right) \delta \mathrm{T}_{\mathrm{i}}\right) & \text { if } \mathrm{i} \in \mathrm{~N}_{12} \\
\beta \overline{\mathrm{X}}_{3} \leq \beta\left({ }_{\mathrm{i}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 3}\right)^{-\psi}\right)+{ }_{\mathrm{i}} \tau_{3}\left(\frac{1}{\rho}\left(\mathrm{~T}_{\mathrm{i}}\right)^{\frac{1-\rho}{\rho}} \mathrm{c}_{\mathrm{j}}-\beta\left[1+{ }_{\mathrm{i}} \gamma_{\mathrm{W}, 3}+{ }_{\mathrm{i}} \gamma_{1}\left(\frac{\mathrm{~N}_{22}}{\mathrm{~N}}\right)^{\psi}\right] \delta \mathrm{T}_{\mathrm{i}}\right) & \text { if } \mathrm{j} \in \mathrm{~N}_{11} \\
\beta \overline{\mathrm{X}}_{3} \leq \beta\left({ }_{\mathrm{j}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 3}\right)^{-\psi}\right)+{ }_{\mathrm{j}} \tau_{3}\left(\frac{1}{\rho}\left(\mathrm{~T}_{\mathrm{j}}\right)^{\frac{1-\rho}{\rho}} \mathrm{c}_{\mathrm{j}}-\beta\left(1+{ }_{\mathrm{j}} \gamma_{\mathrm{W}, 3}\right) \delta \mathrm{T}_{\mathrm{j}}\right) & \text { if } \mathrm{j} \in \mathrm{~N}_{21} \\
\beta \overline{\mathrm{X}}_{3} \leq \beta\left({ }_{\mathrm{j}} \lambda\left(\mathrm{~N}_{\mathrm{W}, 3}\right)^{-\psi}\right)+{ }_{\mathrm{j}} \tau_{3}\left(\frac{1}{\rho}\left(\mathrm{~T}_{\mathrm{j}}\right)^{\frac{1-\rho}{\rho}} \mathrm{c}_{\mathrm{j}}-\beta\left[1+{ }_{\mathrm{j}} \gamma_{\mathrm{W}, 3}+{ }_{\mathrm{j}} \gamma_{1}\left(\frac{\mathrm{~N}_{11}}{\mathrm{~N}}\right)^{\psi}\right] \delta \mathrm{T}_{\mathrm{j}}\right) & \text { if } \mathrm{j} \in \mathrm{~N}_{22}
\end{array}
$$

Bilinguals $\mathrm{N}_{12}$ rest their decision about learning the foreign language strictly on eq. (25a) while bilinguals $\mathrm{N}_{21}$ rest theirs on eq. (27a). However, both sets of monolinguals will have a choice to make between learning one language or the other in case learning either one would be an advantage. To make this choice, monolinguals $\mathrm{N}_{11}$ will need to choose based on the higher of the two right hand sides of eqs. (25) and (25b) (since $\bar{X}_{i}=\bar{X}_{3}$ on the left), while monolinguals $\mathrm{N}_{22}$ will need to make the same choice on the basis of the higher of the two differences between the right hand and the left hand sides of eqs. (27) and (27b) (since the two left hand sides differ).

According to this formulation, if the number of third-language speakers, $\mathrm{N}_{\mathrm{W}, 3}$, is important, there could be learning of the third language by the majority despite the fact that it does not yield any social benefits of communication outside of trade (except via tourism, which I have ignored though this can be repaired). The same holds true for the minority. However, in the minority's case, both the special facility of learning the majority home language ( $\overline{\mathrm{X}}_{\mathrm{j}}<\overline{\mathrm{X}}_{3}$ ) and the income bonus of learning it, load the dice in favor of learning the home language. Any tendency of the minority to learn the foreign language nevertheless would make the majority less happy. On the other hand, this choice by the minority would promote the transmission of the minority language to the next generation since the danger to this transmission comes strictly from the majority language, as the model is formulated. On this reasoning, the spread of English (and foreign languages in general) for the sake of the benefits in foreign trade, helps to preserve minority languages. Yet the reasoning may be questioned from the broader perspective of Section IV where the worldwide applicability of the model
was put in doubt and the possibility was raised that the spread of common languages, whether they be native or foreign, would contribute to the breakdown of internal barriers to domestic trade and make the model more applicable. On this alternative view, by promoting market integration at home, the adoption of a foreign language reduces the protection of regional languages against the corrosive winds of the competition with the majority language. The issue is open.

## References

Bisin, Alberto and Thierry Verdier, 2001. The economics of cultural transmission and the dynamics of preferences. Journal of Economic Theory 97, 298-319.

Bisin, Alberto and Thierry Verdier, 2011. The economics of cultural transmission and socialization. In Jess Benhabib, Alberto Bisin and Matthew Jackson, eds., Handbook of Social Economics, volume 1A, chapter 9. North Holland, Amsterdam.

Breton, Albert and Peter Mieszkowski, 1977. The economics of bilingualism. In Wallace Oates, ed., The Political Economy of Fiscal Federalism. D. C. Heath and Co., Lexington, MA.

Carr, Jack, 1985. Le bilinguisme au Canada : l'usage consacre-t-il l'anglais monopole naturel ? In François Vaillancourt, ed., Économie et langue : recueil de textes. Conseil de la langue française, Université de Montréal.

Chiswick, Barry and Paul Miller, 1995. The endogeneity between language and earnings: International analyses. Journal of Labor Economics 13, 246-248.

Chiswick, Barry and Paul Miller, 2002. Immigrant earnings: Language skills, linguistic concentration and the business cycle. Journal of Population Economics 15 (1), 312-57.

Chiswick, Barry and Paul Miller, 2007. Computer usage, destination language proficiency and the earnings of natives and immigrants. Review of the Economics of the Household 5 (2), 129-157.

Chiswick, Barry, Harry Patrinos and Michael Hurst, 2000. Indigenous language skills and the labor market in a developing country: Bolivia. Economic development and cultural change 48, 349-367.

Christofides, Louis and Robert Swidinsky, 2010. The economic returns to the knowledge and use of a second official language: English in Quebec and French in the Rest-of-Canada. Canadian Public Policy, 36 (2), 137-158.

Church, Jeffrey and Ian King, 1993. Bilingualism and network externalities. Canadian Journal of Economics, 337-345.

Crystal, David, 1997. English as a Global Language. Cambridge University Press, Cambridge.

Dalby, Andrew, 2002. Language in Danger. Allen Lane: The Penguin Press, London.
De Swaan, Abram, 2001. Words of the World. Polity Press, Cambridge, UK.
Dustmann, Christian and Francesca Fabbri, 2003. Language proficiency and labour market performance of immigrants in the UK. Economic Journal, 113 (489), 695-717.

Dustmann, Christian and Arthur van Soest, 2002. Language and the earnings of immigrants. Industrial and Labor Relations Review 55 (3), 473-492.

Ethnologue: Languages of the World, 2009. 16th ed. Summer Institute of Linguistics, International Academic Bookstore, Dallas, TX, available online.

Fishman, Joshua 1991. Reversing Language Shift. Multilingual Matters Ltd., Clevedon, UK.
Fry, Richard and Lindsay Lowell, 2003. The value of bilingualism in the U.S. labor market. Industrial \& Labor Relations Review, 57 (1), 128-140.

Gabszewicz, Jean, Victor Ginsburgh and Shlomo Weber, 2011. Bilingualism and communicative benefits. Annals of Economics and Statistics 101-102. 271-286

Ginsburgh, Victor, Ignacio Ortuno-Ortin and Shlomo Weber, 2007. Learning foreign languages. Theoretical and empirical implications of the Selten and Pool model. Journal of Economic Behavior and Organization 64, 337-347.

Ginsburgh, Victor and Juan Prieto-Rodriguez, 2007. Returns to foreign languages of native workers in the EU. Industrial \& Labor Relations Review 64, 599-618.

Ginsburgh, Victor, Shlomo Weber and Sheila Weyers, 2011. The economics of literary translation: Some theory and evidence. Poetics 39, 228-246.

Ginsburgh, Victor and Shlomo Weber, 2011. How many languages do we need? The economics of linguistic diversity. Princeton University Press.

Gould, David, 1994. Immigrant links to the home country: Empirical implications for US bilateral trade flows. Review of Economics and Statistics 69, 301-316.

Grin, François, 1992. Towards a threshold theory of minority language survival. Kyklos 45, 69-97.

Grin, François, 1996. The economics of language: Survey, assessment, and prospects. International Journal of the Sociology of Language 121, 17-44.

Grin, François, 1999. Compétences et récompenses: La valeur des langues en Suisse. Fribourg, Éditions Universitaires de Fribourg.

Grin, François and François Vaillancourt, 1997. The economics of multilingualism: Overview and analytical framework, Annual Review of Applied Linguistics 17, 43-65.

Hagège, Claude, 2009. On the death and life of languages. New Haven, Yale University Press.

Heilbron, Johan, 1999. Towards a sociology of translation: Book translations as a cultural world-system. European Journal of Social Theory 2 (4), 429-444.

John, Andrew and Kei-Mu Yi, 1997. Language, learning, and location. Mimeo, available online.

Krauss, Michael, 1992. The world's languages in crisis. Language 69, 4-10.
Laitin, David, 1993. The game theory of language regimes. International Political Science Review 14 (3), 227-239.

Lang, Kevin 1986. A language theory of discrimination. Quarterly Journal of Economics 101 (2), 363-382.

Lang, Kevin 1993. Language and economists' theories of discrimination. International Journal of the Sociology of Language 103, 165-183.

Lazear, Edward, 1999. Culture and language. Journal of Political Economy 107(S6), S95S126.

Lewis, Paul and Gary Simons, 2010. Assessing endangerment: Expanding Fishman's GIDS. Revue Roumaine de Linguistique 2, 103-119.

Marschak, Jacob 1965. Economics of language. Behavioral Science 10, 135-140.
McManus, Walter, William Gould and Finis Welch, 1983. Earnings of Hispanic men: The role of English language proficiency. Journal of Labor Economics 1, 101-130.

Olivier, Jacques, Mathias Thoenig and Thierry Verdier, 2008. Globalization and the dynamics of cultural identity. Journal of International Economics 76, 356-370.

Pool, Jonathan, 1991. The official language problem. American Political Science Review 85, 495-514.

Sabourin, Conrad, 1985. La théorie des environnements linguistiques. In François Vaillancourt, ed., Économie et langue : recueil de textes, Conseil de la langue française, Université de Montréal.

Selten, Reinhard and Jonathan Pool, 1991. The distribution of foreign language skills as a game equilibrium. In Reinhard Selten, ed., Game Equilibrium Models IV: Social and Political Interaction. Springer-Verlag, Berlin, 64-87.

Tamura, Robert, 2001. Translators: Market makers in merging markets. Journal of Economic Dynamics \& Control 25, 1775-1800.

Vaillancourt, François, 1996. Language and socioeconomic status in Quebec: Measurements, findings, determinants and policy costs. International Journal of the Sociology of Language 121, 69-92.

Van Parijs, Philippe (2011). Linguistic Justice for Europe and the World. Oxford University Press.

Weitzman, Martin, 1992. On diversity. Quarterly Journal of Economics 107 (2), 363-405.


[^0]:    1 "Remember, though, the case of Breton, with perhaps a million speakers in living memory but now with very few children speakers, or Navajo, with well over 100,000 speakers a generation ago but now also with an uncertain future" (Krauss (1992), p. 7).

[^1]:    ${ }^{2}$ Compare van Parijs (2011) who relies instead on a normative issue of justice.

[^2]:    ${ }^{3}$ Note that I do not distinguish between $\psi$ depending on which of the two languages is the source and which is the target in a translation. In addition, I disregard the fact that any monolingual party to a trade might have a preference about translation one way or the other. I also overlook a fundamental complication surrounding cultural goods: the goods themselves typically differ to one degree or another between the original-language and the translated version (as evident in the continuous flow of new translations of the classics of literature into the same target languages as before). There is a limited economic literature on these aspects. See, for example, Ginsburgh et al (2011). I ignore all these matters.

[^3]:    ${ }^{4}$ Apart from Church and King, there were a few notable theoretical contributions in the main tradition of treating multiple languages essentially as a problem. See, in particular, Sabourin (1985) (who sees a matching problem in the labor market depending on one's favored linguistic environment), Lang (1986) (who poses the issue of Beckerian discrimination by employers between groups of workers who simply speak the same language differently), John and Ki (1997) (who are clearly sympathetic to the introduction of consumption benefits of multiple languages and are also concerned with world issues) and Tamura (2001) (who focuses on the role of translation). Lang (1993) provides a nice general review and extension of his basic argument that problems of communication help to understand Beckerian discrimination in the labor market.

[^4]:    ${ }^{5}$ Note the associated restriction in eqs. (20) and (21) that $\theta_{\mathrm{i}}$ can never be below 0.5 for any individual but $\theta_{\mathrm{j}}$ can be close to zero.

[^5]:    ${ }^{6} \mathrm{c}$ goes up to the common level in the majority and the continuing differences between the household that has chosen to learn the majority language and the majority concern T , the allocation of time and the sense of comfort in the society.

[^6]:    ${ }^{7}$ Of course, the majority similarly disregards the welfare benefits to the minority of its learning of the minority language. But this last disregard only helps at this stage of the analysis since it reduces the social misallocation resulting from the first inefficiency.

[^7]:    ${ }^{8}$ See Ginsburgh and Prieto-Rodriguez (2011) in a major study of 9 European countries and Grin (1999) regarding Switzerland. True, U.S. evidence shows no wage advantage of possessing the principal minority language at home. However, it does not show any wage penalty from it either. See Fry and Lowell (2003). Likewise, in Quebec bilingual native speakers of the minority language, English in this case, earn no more than the monolingual French (though the bilingual French do earn more from possessing English). See Vaillancourt (1996). (Compare also more recent results by Christofides and Swidinsky (2010).) The only study I know that may mean that retention of a minority language, as such, is a wage liability is Chiswick et al (2000), relating to Bolivia, where know-

[^8]:    ledge of an Amerindian language together with Spanish damages earnings. According to the authors, this result could come from social discrimination against indigenous language speakers, which would indeed make a case against the language. However, the authors also consider that it could come from a poor command of Spanish or lower schooling or a combination of all three; the evidence will not permit saying.
    ${ }^{9}$ In such a context, one could also easily imagine a "free rider" problem. To elaborate, suppose, in an extension of the model, that the required time $\hat{Y}$ to obtain the benefit of bilingualism for one's own children would drop for the individual as $n$ rose as the collective effort paid off, and yet the collective engagement calls for a minimal amount of time $\hat{\mathrm{Y}}$. Then there might be households who would make the engagement but following the commitment would prefer to "free ride" and choose lower $\hat{Y}$ than agreed. Compare Laitin (1993).
    ${ }^{10}$ In this regard, Lazear makes the important point that the presence of large urban centers in which a minority language flourishes will affect the kind of immigrants a country attracts.

[^9]:    ${ }^{11}$ This is not to deny that the remaining monolingual $\mathrm{N}_{2}$ population might retain a strong sense of separate cultural identity. This qualification is important in connection with Olivier et al (2008) who effectively model the interesting possibility that following the vanishing of $\mathrm{N}_{22}$ ethnic diversity would survive and remain steady. It would be an impoverished ethnic diversity all the same. (I italicize "effectively" because the authors do not refer to language as such.)

[^10]:    ${ }^{12}$ In addition, a few countries, like Switzerland, provide adequate public support at the national level for several regional languages. There may also be a few lingua franca in the world like Swahili in parts of East Africa or Malay in parts of Southeast Asia whose safety does not depend on status as a majority language in any particular country or region.

[^11]:    ${ }^{13}$ Many optimistic views of a universal language envisage a stable situation with multiple languages and vibrant bilingualism most everywhere on earth. The nineteenth-century inventor of Esperanto, Zamenhof, a native of Bialystok, Poland who was inspired by the hostile relations he encountered between Polish-speakers, Germanspeakers, Russian-speakers and Yiddish-speakers, certainly belongs to this mould. However, Dalby notably suggests the instability of such situations in the past. He also shows clearly that this instability stemmed from internal forces even though the spread of a common language in the relevant examples, including ancient Greek and Latin, never covered more than a modest fraction of the world by any relevant measure and preceded the internet and mass communications.

[^12]:    14 The one important area of contributions to the study of language by economists that I am aware of having ignored entirely relates to the costs of multiple languages within international organizations like the United Nations and the European Union. The outstanding contribution with clear application on this subject though of broader application as well, is Pool (1991), who demonstrated the existence of a theoretical solution to the problem of reconciling fairness and efficiency in language management in any multi-group population operating under a single set of political rules and therefore in these organizations.

[^13]:    ${ }^{15}$ I evidently assume that the $\lambda$ coefficient is properly adapted to make it possible to write the relationship of $\gamma_{\mathrm{w}, 1}$ $-\gamma_{\mathrm{W}, \mathrm{o}}$ to $\mathrm{N}_{\mathrm{W}, 22}$ or $\mathrm{N}_{\mathrm{W}, 11}$ and $\psi$ as simply as I do.

[^14]:    ${ }^{16}$ Since the term remains associated strictly with $\mathrm{N}_{22} / \mathrm{N}$ or $\mathrm{N}_{11} / \mathrm{N}$.

