# DISCUSSION PAPER SERIES

No. 9077

**CAPITAL REGULATION AND CREDIT FLUCTUATIONS**

Hans Gersbach and Jean-Charles Rochet

***FINANCIAL ECONOMICS and INTERNATIONAL MACROECONOMICS***

**Centre for Economic Policy Research**

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ISSN 0265-8003

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Discussion Paper No. 9077
August 2012

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# ABSTRACT

## Capital Regulation and Credit Fluctuations*

We provide a rationale for imposing counter-cyclical capital ratios on banks. In our simple model, bankers cannot pledge the entire future revenues to investors, which limits borrowing in good and bad times. Complete markets do not sufficiently stabilize credit fluctuations, as banks allocate too much borrowing capacity to good states and too little to bad states. As a consequence, bank credit, output, capital prices or wages are excessively volatile. Imposing a (stricter) capital ratio in good states corrects the misallocation of the borrowing capacity, increases expected output and can be beneficial to all agents in the economy. Although in our economy, all agents are risk-neutral, counter-cyclical capital ratios are an effective stabilization tool. To ensure this effectiveness, capital ratios have to be based on ex ante equity capital, as classical capital ratios can be bypassed.

JEL Classification: D86, G21 and G28
Keywords: complete markets, credit fluctuations, macroprudential regulation and misallocation of borrowing capacity

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*We would like to thank Felix Bierbrauer, Claudio Borio, Elena Carletti, Russ Cooper, Giancarlo Corsetti, Doug Diamond, Jordi Gali, Thomas Gehrig, Piero Gottardi, Zhiguo He, Ennisse Kharroubi, Arvind Krishnamurthy, Bob King, David Levine, Frederic Malherbe, Bruno Parigi, Richard Portes, Helene Rey, Lars Stole, Javier Suarez, and especially John Moore and Jean Tirole for helpful discussions. We are also grateful to seminar participants at the Bank of England, the Banque de France, the BIS, the European University Institute, the Federal Reserve Board, the IMF, the London Business School, the University of Chicago, Studienzentrum Gerzensee, the University of Cologne, the University of Tübingen, the Swiss National Bank, the University of Vienna and the University of Zürich for useful comments. The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) grant agreement 249415-RMAC and NCCR FinRisk (project Banking and Regulation)

Submitted 27 July 2012

# 1 Introduction

The implementation of counter-cyclical capital ratios, i.e. imposing (stricter) capital ratios in booms, is a central theme of macroprudential regulation.[1] Yet, the conceptional foundations for such a regulation are not entirely clear.

In this paper, we provide a conceptional foundation for counter-cyclical capital ratios by examining how banks allocate their borrowing capacity across good and bad states in a complete market setting. We find that banks allocate too much borrowing capacity to boom states and too little to bad states, creating excessive fluctuations of credit, output, asset prices and wages. This can be corrected by capital regulation in boom states. Imposing tighter capital ratios in booms increases expected output.

More specifically, we consider a simple two-sector economy in which a capital good can be used to produce a consumption good. A fraction of the capital good is owned by investors and the remaining part by bankers. All individuals are risk-neutral. In one sector, capital can be lent and borrowed frictionlessly. There are diminishing returns of capital in this sector. In the other sector, banks lend to entrepreneurs, who have access to a constant returns to scale technology. Banks alleviate the moral hazard problems of the entrepreneurs. In this sector, output and lending rates are affected by macroeconomic shocks. Good states and bad states refer to high or low capital productivity. Banks can only pledge a fraction of their future revenues to investors. The specific form of this financial friction - moral hazard of bankers, asset diversion or non-alienability of human capital - does not matter. The financial friction, however, limits borrowing of banks in good states and bad states. Before macroeconomic shocks occur, agents can trade in complete financial markets. This simple model yields the following insights.

First, complete financial markets allow bankers to reallocate borrowing capacity between good and bad states. This reallocation decisions are governed by the objective of bankers to maximize their rents. As a rule, the access to complete markets reduces the volatility of bank-lending and capital prices.

Second, with complete markets, however, banks allocate too much borrowing capacity to good states and too little to bad states. The reason is as follows. Bankers aim at maximizing their rents from lending, taking capital prices and prices of financial assets as given. At the competitive equilibrium, capital prices are such that bankers are indifferent between shifting

[1] The reasoning behind different notions and possible foundations of macroprudential regulation is outlined in Borio (2003, 2010).

one additional unit of borrowing capacity across states. A social planner facing the same borrowing constraints would recognize that reallocating borrowing capacity to the bad state would reduce capital prices in the good state and increase them in the bad state. This would increase the expected borrowing capacity and would allow to increase the expected lending by banks to entrepreneurs that have higher expected capital productivity than in the other sector. As a consequence, the expected output in the economy would increase.

Third, a regulatory capital ratio in the boom that is tighter than the market-imposed capital ratio can implement the expected output gain a social planner could achieve. Such capital regulation limits lending in the boom, and thus corrects the misallocation of borrowing capacity across good and bad states, thereby reducing fluctuations of lending and capital prices.

Fourth, regulatory equity capital ratio based on interim equity - and thus classical equity capital ratios - however, is not effective in implementing the social planner solution. Bankers could reallocate borrowing capacity ex ante and partially bypass this regulation. Effective capital regulation requires to relate lending to initial (or past) equity. Those ratios continue to be binding if bankers trade in complete financial markets.

Fifth, we provide three extensions of the model. The model can be extended easily to chained financial frictions when bankers themselves face the problem that entrepreneurs cannot fully pledge their output. In such circumstances, the misallocation of borrowing capacity of banks becomes more pronounced and the welfare gains that can be achieved by macroprudential regulation increase. In a second extension, we introduce labor as a further factor of production supplied by workers. We obtain a broader conclusion that tighter capital requirements for booms stabilize other factor prices such as wages. This may be particularly valuable socially when these workers are risk-averse and do not have access to financial markets to insure themselves against wage fluctuations. In a further extension, we explore various ways how output gains via macroprudential regulation can be distributed in the economy, so that all agents benefit and that such regulations engineer a Pareto improvement.

The paper is organized as follows. In section 2, we discuss the related literature. In section 3, we introduce the model. In section 4, we characterize the competitive equilibrium with and without contingent markets. In section 5, we provide the rationale for capital regulation. Section 6 contains the extension of the model with labor and capital. In section 7, we introduce several ways of distributing the gains from capital regulation within society. Section 8 concludes.

## 2 Relation to the literature

### 2.1 Empirical Evidence

Empirical work has identified several phenomena that motivate our analysis and for which it can provide an explanation. First, volatility of bank lending is typically a multiple of the volatility of GDP. In Figure 1 we provide evidence for a variety of countries since World War II. In almost all countries, bank lending is more volatile than GDP.[2]

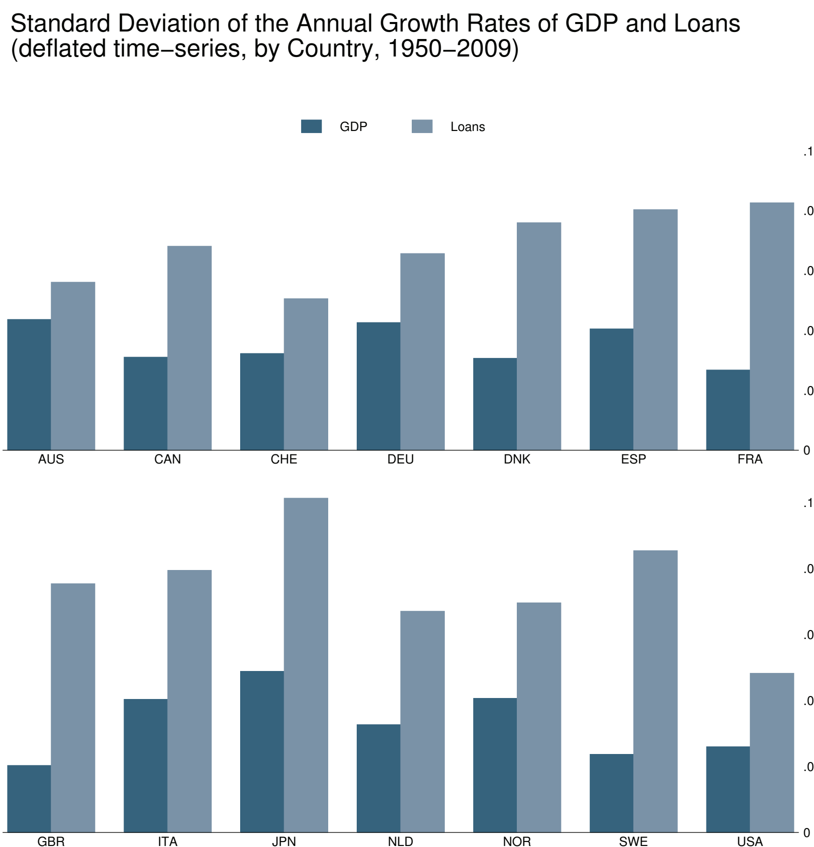


Figure 1: Evidence for Credit Cycles, Source: Calculations based on Schularick and Taylor (2012).

[2] We have used CPI data to deflate the series. Using the GDP deflator for the period 1961-2009 yields the same pattern with small variations of the multiples. We note that the multiple reported in Meh and Moran (2010) for the US in the last decades is over four.

Second, credit cycles characterized by credit booms and credit busts have been recently characterized and thoroughly investigated in a series of papers (Igan et al. (2009), Elekdag and Wu (2011) and Claessens et al. (2011)). Schularick and Taylor (2012) present new evidence on the rapid rise of leverage in the financial sector in the second half of the twentieth century. They also demonstrate that credit booms predict busts.

Third, Jiménez et al. (2011) provide evidence how macroprudential policies in the form of counter-cyclical bank capital buffers have reduced fluctuations in total credit supply of banks in Spain. Our results can provide a conceptual foundation how tightened capital requirements in booms reduce supply of bank credit in good times and increase it in bad times.

## 2.2 Financial Intermediaries and Aggregate Economic Activity

There is a vast literature on financial contracting and aggregate economic activity. We highlight one important line of research that emphasizes how borrowers facing shocks to their net worth may become more or less credit constrained, causing shocks to amplify and persist. Two classical contributions have derived such mechanisms from first principles.

Bernanke and Gertler (1989) examine an overlapping generation model in which risk-neutral entrepreneurs, with private information about their project outcomes, borrow from lenders who have access to a costly auditing technology. Higher net worth of borrowers lowers financing costs. As a consequence, a positive technology shock not only increases current real capital investments, but it also propagates over time as it raises future net worth of borrowers. The opposite occurs in downturns.

Kiyotaki and Moore (1997) significantly extend these insights by considering shocks to net worth arising from changes in the value of a firm's asset. A small, temporary technology shock causes credit constrained firms to cut back on their investment expenditures in the current and in subsequent periods. As prices of assets reflect future revenue conditions, the shock may cause a significant decline in asset prices and thus in the net worth of credit constrained firms. As a consequence, those firms need to reduce further their investments. This intertemporal multiplier process can generate large, persistent changes of output and asset prices.

Recent work in the tradition of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) has focused on overborrowing coupled with insufficient insurance. In Caballero and Krishnamurthy (2003) and Lorenzoni (2008) entrepreneurs cannot protect themselves against the risk that they become financially constrained. In Korinek (2011) such insurance is available but costly. As a

result, in these papers, entrepreneurs invest too much. Bianchi (2011) assesses the aggregate and welfare consequences of such overborrowing phenomena quantitatively.

We focus on an environment with complete financial markets allowing banks to allocate their borrowing capacity across good and bad states flexibly. Atomistic banks allocate too much borrowing capacity to good times at the expense of borrowing capacity in bad times.[3]

## 3 The Model

Our model is closely related to the ones used in Lorenzoni (2008) and Gersbach and Rochet (2011). Like in these two models, we have three dates $t = 0, 1, 2$ and two goods, consumption and capital. We simplify Gersbach and Rochet (2011) by assuming that adjustment costs are zero. We simplify Lorenzoni (2008) by replacing collateral constraints by a simpler form of financial frictions. This allows us to have only one production period (namely between $t = 1$ and $t = 2$) instead of two. Date $t = 0$ is only there to allow ex-ante trade on contingent financial markets. On the other hand, we introduce new features in each of these models: banks into Lorenzoni's model and capital regulation into Gersbach and Rochet's. The other new crucial feature in our model is bankers' possibility to trade ex ante in complete contingent markets and to allocate their borrowing capacity across good and bad times. This last feature justifies macroprudential policies.

As in Lorenzoni (2008), there is a traditional sector characterized by a concave, twice continuously differentiable production function $F(\cdot)$, and another sector, which is interpreted as the banking sector in a broad sense that includes all financial intermediaries that contribute to providing credit to the economy.

There is a continuum of bankers with mass one, each endowed with some physical capital $e$ at date 0. They have access to a lending technology with constant returns to scale.

[3] As banks take capital prices as given and as the allocation decision of a subset of banks impacts on the borrowing capacity allocation of other banks, our paper is broadly related to the literature on pecuniary externalities. At least since Scitovsky (1954) it is well-known that pecuniary externalities may matter for welfare when we depart from the assumptions that guarantee the validity of the first welfare theorem in general equilibrium. Such pecuniary externalities have been the focus of classical contributions (see e.g. Geanakoplos and Polemarchakis (1986), Stiglitz (1982), Greenwald and Stiglitz (1986), and in more recent work, e.g., Caballero and Krishnamurthy (2001), Allen and Gale (2004), and Farhi et al. (2009) and in the literature discussed in subsection 2.1). In our model, pecuniary externalities operate through two relative prices: the relative price of capital in the good and bad state. To achieve welfare gains both prices need to adjust. Therefore, the inefficiency in our model can be called a two-dimensional welfare-reducing pecuniary externality.

The expected return on banks' assets is denoted by $R$.[4] Initially, capital $e$ is distributed according to some distribution with bounded support and aggregate amount $E < 1$. A crucial ingredient is the presence of a financial friction: Bankers can borrow from outside investors, but they cannot fully pledge their future income to these outside investors.

Like Holmström and Tirole (1997) we assume that the non-pledgeable income is a multiple $bk$ of the size $k$ of the investment.[5] The parameter $b$ measures the intensity of financial frictions. In the Appendix 1, we show that this non-pledgeability can be generated by several forms of financial frictions such as moral hazard, asset diversion, or non-alienability of human capital.

Like in Lorenzoni (2008) and Gersbach and Rochet (2011), there is an aggregate shock at $t = 1$. Conditionally on this shock, the expected return $R$ on the banking technology can be either $R_h$ (high return) or $R_l$ (low return, $R_l < R_h$) with probabilities $\pi_h$ and $\pi_l$ (with $\pi_h + \pi_l = 1$). The aggregate state $s = h, l$ that determines $R_s$ is called a "boom" (when $s = h$) or a "recession" (when $s = l$). By contrast, the traditional sector is not subject to aggregate shocks nor to financial frictions.

The total physical capital stock of the economy is normalized to 1. Bankers own (on aggregate) a fraction $E$ of this capital. $E$ can be interpreted as the aggregate capitalization of the banking sector. Investors own the remaining capital $1 - E$. They also own the firms in the traditional sector. All agents are risk-neutral and endeavor to maximize their expected consumption at date 2.

The timing of the model is as follows:

- at date 0, all agents can trade on contingent markets for capital. There is a market for each state, i.e. financial markets are complete.[6] Thanks to these markets, banks can obtain a state dependent capital endowment, denoted by $e_s$ in states $s = h, l$.

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[4]This is a summary of the lending activities of banks. At a more detailed level, we envision that there is a continuum of entrepreneurs operating a technology, with expected return $R$. Those entrepreneurs cannot raise funds directly from investors because of moral hazard. Each banker can alleviate the moral hazard of these entrepreneurs by monitoring them and enforcing contractual obligations. For simplicity, we assume that the costs of these activities are sufficiently small and can be neglected, or that $R$ is the net return after these costs have been taken into account. We assume that banks are efficient in eliminating moral hazard, and can secure $R$.

[5]Lorenzoni (2008) assumes instead that the non-pledgeable income is a constant fraction $(1-\theta)Rk$ of the asset's return.

[6]This is an important difference with Lorenzoni (2008) who, like in most of the literature on credit cycles, has to assume some form of market incompleteness.

- at date 1, all agents observe the aggregate state $s$. A typical banker borrows $k_s - e_s > 0$ units of capital from investors/depositors, and invests $k_s$ units of capital. The aggregate size of the banking sector (the integral of $k_s$ over all banks) is denoted by $K_s$. We interpret $K_s$ as the aggregate volume of credit to the economy in state $s$. The remaining amount of capital $1 - K_s$ is invested in the traditional sector.
- at date 2, output in the banking sector, $R_s K_s$, is shared between investors and bankers. More specifically, the investors who, at date 1, have lent $k_s - e_s$ units of capital to a typical bank receive $p_s(k_s - e_s)$ units of consumption at date 2. Since consumption is taken as a numeraire, the rate of return on banks' deposits, namely $p_s$, can also be interpreted as the price of capital in state $s$, as it corresponds to the number of units of consumption that are delivered at date 2 in exchange for one unit of capital at date 1. The output in the traditional sector $F(1 - K_s)$ accrues to investors.[7] The total output in the economy is $F(1 - K_s) + R_s K_s$.

An important remark regarding the concepts and the language is in order. At the beginning, a banker owns physical capital $e$. Through trading in complete financial markets, bankers end up with a state-dependent physical capital endowment $e_s$. By founding a bank and funding it with $e_s$ and deposits $k_s - e_s$ at the deposit rate $p_s$, the banker is the residual claimant, and $e_s$ turns into inside equity.[8] Therefore, we also call $e_s$ "bank equity" to simplify notation by renouncing additional notation for inside equity.

## 4 The Competitive Equilibrium

Our simple economy has a unique competitive equilibrium, which is easy to characterize. As a first benchmark, we start by the case where there are no contingent markets at date 0.

### 4.1 The case without contingent markets

Consider a banker who has equity $e$ (here it is the same in both states since we do not have any contingent markets) and assume that the macro state is $s$. Investors/depositors agree to provide additional capital $k_s - e$ if and only if they are promised an expected repayment of

[7] It accrues in two forms: return on invested capital $F'(1 - K_s)K_s$ and profits of firms operating the technology $F(1 - K_s) - F'(1 - K_s)K_s$. For details see Gersbach and Rochet (2011).

[8] In practice, banks use derivatives and other contingent securities to allocate equity across good and bad states.

(at least) $p_s(k_s - e)$ units of consumption at date 2. This is only possible if this promised repayment does not exceed the maximum pledgeable income of the bank, namely $(R_s - b)k_s$: When $R_s < p_s + b$, which will always be satisfied at equilibrium, the participation constraint of investors implies that the bank is constrained in its investment choice by what we call a **market imposed solvency ratio**:

$$k_s \leq \frac{e}{1 - \frac{R_s - b}{p_s}}. \tag{1}$$

Note that the banks in our model can be interpreted as investment banks or hedge funds, that are financed by sophisticated investors. Thus there is no need, at this stage, for a regulation of capital ratios, imposed by a (micro-prudential) regulator. Investors themselves impose a limit to the banks' volume of lending. An alternative interpretation would be that these banks are traditional deposit-taking institutions and that (1) is a micro-prudential capital ratio, imposed by a traditional bank regulator. Both interpretations are possible, and our focus is different: Our objective is to find a conceptual foundation for a macro-prudential regulation of banks' capital. Thus we do not discuss possible motivations for micro-prudential regulation.

Since bankers take on as much leverage as they can, constraint (1) is binding for each bank. The aggregate size $K_s$ of the banking sector in state $s$ is obtained by integrating constraint (1) over all banks:

$$K_s = \frac{E}{1 - \frac{R_s - b}{p_s}} \equiv D_s(p_s). \tag{2}$$

The right hand-side of this equation is the demand for capital by the banking sector in state $s$. By absence of arbitrage opportunity, the price of capital in state $s$, namely $p_s$, must be equal to the marginal productivity of capital in the traditional sector:

$$p_s = F'(1 - K_s), \tag{3}$$

which can be rewritten as

$$K_s = S(p_s), \tag{4}$$

where $S(\cdot)$ is the supply of capital to the banking sector. The equilibrium price $p_s$ is determined by equaling (2) and (4). The demand for capital decreases from $+\infty$ (when $p_s = R_s - b$) to $\frac{R_s E}{b}$ (when $p_s = R_s$). On the same interval $(R_s - b, R_s)$ the supply of capital increases from $S(R_s - b)$

to $S(R_s)$. Thus when $E < b\frac{S(R_s)}{R_s}$, that is if bank capital is relatively scarce,[9] the intermediate value theorem implies that there is a unique value of $p_s$ in $(R_s - b, R_s)$ that equalizes supply and demand. We summarize these findings in the following proposition.

**Proposition 1**

*When there are no contingent markets, there is a unique competitive equilibrium denoted by* $K^0 = (K_h^0, K_l^0)$ *with prices* $(p_h^0, p_l^0)$. *When* $E < b\frac{S(R_s)}{R_s}$, *the price of capital in state* $s$ *is the unique value* $p_s^0$ *that equalizes supply and demand for bank capital in each state:*

$$S(p_s^0) = \frac{E}{1 - \frac{R_s - b}{p_s^0}}. \tag{5}$$

Figure 2 illustrates some properties of the competitive equilibrium in the absence of contingent markets.

Note that capital price $p_s^0$ and aggregate credit $K_s^0$ are pro-cyclical: $p_l^0 < p_h^0$ and $K_l^0 < K_h^0$. This is because the supply curve $K = S(p)$ is state independent, while the demand curve in state $h$ is above the demand curve in state $l$.

In fact, some procyclicality is not necessarily bad: the first-best allocation (that would prevail in our model if bank capital was abundant) is also pro-cyclical since $p_s^{FB} = R_s$ and $K_s^{FB} = S(R_s)$. However, the volatility of capital prices and credit volumes is typically higher in the competitive equilibrium than in the first-best allocation.

To illustrate this feature, consider the simple specification $S(p) = p$ (elasticity of credit supply identically equal to 1). In this case the competitive price can be computed explicitly:

$$p_s^0 = R_s - \max(b - E, 0). \tag{6}$$

To assess the variability of capital prices, we can use the coefficient of variation $\sigma_p = \frac{p_h - p_l}{\mathbb{E}[p_s]}$, which, by a slight abuse of language, we call the volatility of these prices. Formula (6) immediately shows that financial frictions exacerbate the volatility of capital prices (and credit volumes):

$$\sigma_p^0 = \frac{R_h - R_l}{R - \max(b - E, 0)} \geq \sigma_p^{FB} = \frac{R_h - R_l}{R},$$

where $R \equiv \mathbb{E}[R_s]$.

[9] When bank capital is abundant (a case we rule out as empirically irrelevant), $E > b\frac{S(R_s)}{R_s}$ and the constraint (1) does not matter anymore. The competitive equilibrium $(p_s = R_s, K_s = S(R_s))$ coincides with the first-best allocation.

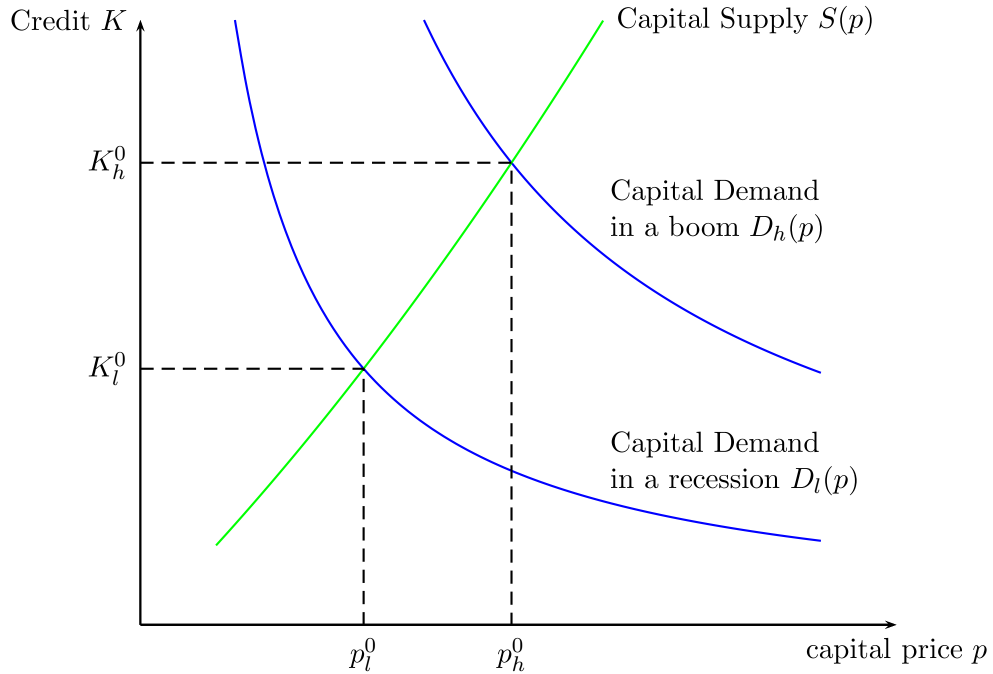


Figure 2: Equilibrium in the absence of contingent markets. Capital price and credit volume are pro-cyclical.

Moreover, we see that the equilibrium volatility of capital prices $\sigma_p^0$ increases with the severity of financial frictions (measured by $b$) and decreases with the capitalization of the banking sector (measured by $E$). Thus increasing bank capital has a stabilizing effect on capital prices and credit volumes.

However, the absence of contingent markets may not be a reasonable assumption, since modern banks have access to complex financial instruments that allow them to hedge against macro-shocks. This is why Section 4.2 below examines another benchmark, namely the case where financial markets are complete.

### 4.2 The Case of Complete Financial Markets

Suppose now that at date 0, banks and investors can trade on contingent capital markets. Denote by $e_h$ and $e_l$ the after-trade levels of capital of a typical bank with initial capital $e$. The simplest way to generate these trades is to consider that the bank will swap $e_h - e$ units of capital in state $h$ against $e - e_l$ units of capital in state $l$.[10] Since all agents are risk-neutral,

[10] In practice, such trades are often achieved through derivatives.

the swap rate must be equal to $\frac{\pi_h p_h}{\pi_l p_l}$. Thus the budget constraint of the bank writes

$$\pi_h p_h (e_h - e) = \pi_l p_l (e - e_l),$$

or more simply

$$\mathbb{E}[p_s(e_s - e)] = 0. \tag{7}$$

Rationally anticipating the values of equilibrium capital ratios that will prevail in each state at date 1, namely

$$e_s = k_s \left(1 - \frac{R_s - b}{p_s}\right), \tag{8}$$

the bank will select the contingent credit volumes $(k_h, k_l)$ that maximize expected profit $b\mathbb{E}[k_s]$ (which is equal to the expectation of non-pledgeable income) under the single constraint obtained by combining (7) and (8):

$$\mathbb{E}[k_s(p_s + b - R_s) - p_s e] = 0. \tag{9}$$

Note that both the objective function of the bank and constraint (9) are linear in $(k_h, k_l)$. Thus if we exclude corner solutions,[11] the only possible equilibrium is such that the coefficient of $k_s$ in constraint (9) is the same in both states:

$$p_h + b - R_h = p_l + b - R_l.$$

Denoting by $p \equiv \mathbb{E}[p_s]$ the expected price of capital (recall that $R \equiv \mathbb{E}[R_s]$ denotes the expected return on assets), this condition can be rewritten as

$$p_s = R_s - R + p. \tag{10}$$

Using (10), constraint (9) can be simplified:

$$\mathbb{E}[k_s](p + b - R) = pe.$$

By aggregating this condition over all banks, we obtain

$$\mathbb{E}[K_s] = \frac{E}{1 - \frac{R-b}{p}}. \tag{11}$$

[11] Such corner solutions are empirically irrelevant, since they imply that the banking sector is completely closed down in one of the states.

Since $K_s = S(R_s - R + p)$, the expected price of capital $p$ can be determined by equaling expected supply and demand:

$$\mathbb{E}[S(R_s - R + p)] = \frac{E}{1 - \frac{R-b}{p}}. \tag{12}$$

We summarized these results in the following proposition.

**Proposition 2**

*When financial markets are complete, there is a unique competitive equilibrium denoted by $K^c = (K_h^c, K_l^c)$ with prices $(p_h^c, p_l^c)$. When bank capital is scarce (specifically $E < b\frac{\mathbb{E}[S(R_s)]}{R}$), this equilibrium is characterized by*

$$p_s^c = R_s - R + p^c, \tag{13}$$

*where $p^c$ solves (12).*

Note that banks use contingent markets at $t = 0$ to reallocate their equity across states in such a way that their returns on equity ($ROE_s$) in each state are proportional to their return on assets ($ROA_s$):

$$ROE_s = \frac{bk_s}{e_s} = \frac{bp_s^c}{p_s^c - R_s + b} = \frac{b}{p^c - R + b} p_s^c > p_s^c = ROA_s.$$

Thus our model predicts that, when financial markets are complete, banks will select their contingent plans so as to equalize the ratios $\frac{ROE_s}{ROA_s}$ in both states. Note that this ratio is larger than 1 when $E < b\frac{\mathbb{E}[S(R_s)]}{R}$: There is a wedge between the return on informed capital (equity) and the return on uninformed capital (deposits), due to the scarcity of informed capital. This wedge would disappear if $E$ was larger than $b\frac{\mathbb{E}[S(R_s)]}{R}$.

In Appendix 2, we provide a simple numerical calibration of our model with complete markets. In this example, capital prices fluctuate 40% more than returns and bank credit fluctuates 40% more than GDP.

### 4.3 Comparison

A natural question is whether the existence of contingent markets stabilizes the economy. In fact this is not necessarily the case, as illustrated by the simple, linear specification $S(p) = p$

that we already used. Equation (12) gives in this case

$$\mathbb{E}[S(R_s - R + p^c)] = p^c = \frac{E}{1 - \frac{R-b}{p^c}},$$

which gives an explicit solution:

$$p^c = R - \max(b - E, 0).$$

If we assume $E < b\frac{\mathbb{E}[S(R_s)]}{R} = b$, $p^c < R$. Moreover,

$$p^c_s = R_s - R + p^c,$$

thus

$$p^c_s = R_s - \max(b - E, 0).$$

This is the same expression as in equation (6) without contingent markets. With this particular specification of $S(\cdot)$, contingent capital markets do not make a difference: the two competitive equilibria $((p^c_h, p^c_l), K^c)$ and $((p^0_h, p^0_l), K^0)$, i.e. with and without contingent markets, are identical. However, this feature is not robust:

**Proposition 3**
*When the elasticity of capital supply is less than one and the volatility of aggregate shocks $\sigma_R = \frac{R_h - R_l}{R}$ is small, capital prices are less volatile with contingent markets than without:*

$$\sigma^c_p = \frac{p^c_h - p^c_l}{p^c} < \sigma^0_p = \frac{p^0_h - p^0_l}{p^0},$$

*where $p^c = \mathbb{E}[p^c_s]$ and $p^0 = \mathbb{E}[p^0_s]$.*

The proof of Proposition 3 is given in Appendix 3.

# 5 A Role for Macroprudential Regulation

## 5.1 Characterizing constrained efficiency

Because of financial frictions and scarcity of bank capital, the equilibrium volume of credit $K_s$ and capital price $p_s$ fluctuate more than the (first-best) optimal volume of credit $K^{FB}_s = S(R_s)$ and capital price $p^{FB}_s = R_s$. Of course the relevant comparison is with the (second-best)

optimum[12], which is subject to the same constraints as the competitive equilibrium:

$$p_s(K_s - E_s) = (R_s - b)K_s, \tag{14}$$

where $p_s = F'(1 - K_s)$,

and $$\mathbb{E}[p_s(E_s - E)] = 0. \tag{15}$$

Constraint (14) means that investors obtain, in each state $s$ the same return on bank deposits and on their direct investments in the traditional sector. Constraint (15) expresses the equilibrium in contingent commodity markets at date 1. By eliminating $E_s$, these two constraints can be combined into a single constraint that applies both to the competitive equilibrium and to the regulator. Indeed, the regulator can effectively control (through capital regulation) the volumes of lending $K_h$ and $K_l$ of the banks in each state, but he is subject to the same (ex-ante) financing constraint as the banks, namely the aggregate form of condition (9):

$$\mathbb{E}[K_s(p_s - R_s + b)] \leq \mathbb{E}[p_s]E, \tag{16}$$

where $$p_s = F'(1 - K_s),\ s = h, l. \tag{17}$$

**Definition 1**

*A constrained efficient capital allocation is a vector $(K_l, K_h)$ that maximizes the aggregate expected output of the economy under constraints (16) and (17).*

## 5.2 The Competitive Allocation is not constrained efficient

We now establish the main result of the paper.

**Proposition 4**

*The competitive allocation $(K_l^c, K_h^c)$ is generically constrained inefficient. In particular if capital supply is log-concave, aggregate expected output can be increased by reducing $K_h$ and increasing $K_l$.*

[12]If a central planner could impose lump-sum taxes on investors and distribute the proceeds to bankers (in such way that $E \geq b\frac{\mathbb{E}[S(R_s)]}{R}$) the competitive allocation would coincide with the first-best allocation. However such a forced redistribution seems hardly politically feasible.

**Proof of Proposition 4:**

Suppose by contradiction that $K^c = (K_l^c, K_h^c)$ maximizes aggregate expected output,

$$Y(K_l, K_h) = \mathbb{E}[F(1 - K_s) + R_s K_s]$$

under the participation constraint of investors

$$G(K_l, K_h) = \mathbb{E}[F'(1 - K_s)(K_s - E) + (b - R_s)K_s] \leq 0.$$

By the Kuhn-Tucker Theorem, the gradients of $Y$ and $G$ at $K^c$ must be colinear. Now

$$\begin{aligned}
\frac{\partial Y}{\partial K_s} &= \pi_s[-F'(1 - K_s^c) + R_s] = \pi_s[-p^c + R], \\
\frac{\partial G}{\partial K_s} &= \pi_s[F'(1 - K_s^c) + b - R_s - F''(1 - K_s^c)(K_s^c - E)] \\
&= \pi_s[p^c + b - R - F''(1 - K_s^c)(K_s^c - E)].
\end{aligned}$$

These gradients are colinear if and only if $F''(1-K_s^c)(K_s^c - E)$ is the same in both states. This is generically not true, since $K_h^c > K_l^c$. When $\log S$ is concave, $\frac{S'}{S}$ is a decreasing function and the direction of improvement can be determined. Indeed, $-F''(1 - K_s^c) = \frac{1}{S'(p_s^c)}$ and

$$-K_h^c F''(1 - K_h^c) = \frac{S(p_h^c)}{S'(p_h^c)} > \frac{S(p_l^c)}{S'(p_l^c)} = -K_l^c F''(1 - K_l^c).$$

This is because $p_h^c - p_l^c = R_h - R_l > 0$. Now we also have that $K_h^c > K_l^c$ and thus $\dfrac{K_h^c - E}{K_h^c} > \dfrac{K_l^c - E}{K_l^c} > 0$. Thus by multiplication and using that $F''(1 - K_s^c) < 0$, we obtain

$$-F''(1 - K_h^c)(K_h^c - E) > -F''(1 - K_l^c)(K_l^c - E),$$

and thus

$$\left(\frac{\partial G}{\partial K_h} / \frac{\partial G}{\partial K_l}\right)(K^c) > \left(\frac{\partial Y}{\partial K_h} / \frac{\partial Y}{\partial K_l}\right)(K^c).$$

As shown in Figure 3, aggregate expected output can be increased marginally around $K^c$ by reducing $K_h$ while increasing $K_l$ so that $G(K_l, K_h)$ remains constant.

■

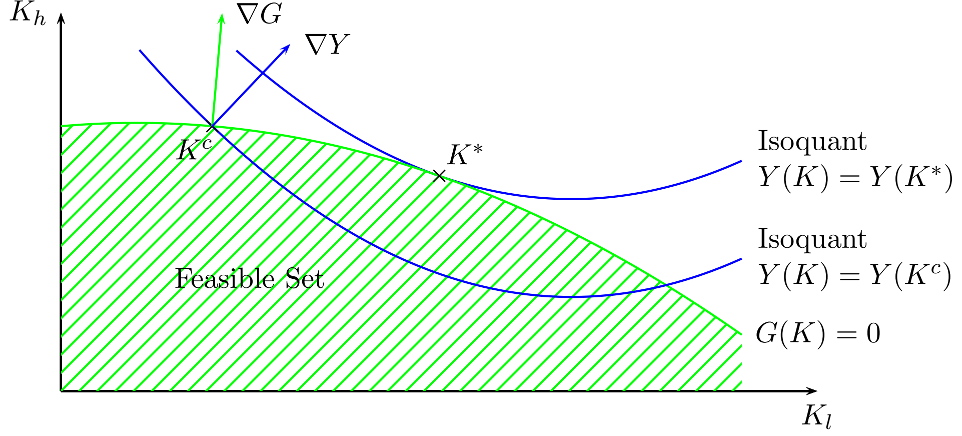


Figure 3: The competitive equilibrium $K^c$ is not constrained efficient. The constrained optimum is $K^*$.

We note that the log-concavity of capital supply can be related to the hazard rate. Suppose that there is a continuum of entrepreneurs, and each entrepreneur has access to an indivisible project of size one. The productivity of these projects is distributed according to some differentiable distribution function that generates the production function $F(\cdot)$.[13] In such a set-up, log-concavity of capital supply is identical to the condition that the hazard rate of entrepreneurs' productivity is declining.

### 5.3 A counter-cyclical capital ratio can improve social welfare

Suppose that the regulator imposes to all banks, but only during booms, a capital ratio based on their **initial** capital $e$:

$$e \geq \rho^* k_h. \tag{18}$$

By appropriately selecting $\rho^*$, and letting banks compete to attract investors, the regulator can implement the constrained efficient allocation of capital $K^*$ as we show next:

**Proposition 5**
*We assume that $S$ is log-concave. If the regulator sets $\rho^* = \dfrac{E}{K_h^*}$, the regulated competitive equilibrium allocation coincides with the constrained efficient allocation $K^*$.*

[13]Formally, if $G(x)$ is the distribution function, then $F(1-K_s) = \int_{G^{-1}(K_s)}^{\infty} x dG(x)$.

**Proof of Proposition 5:**

Consider first the behavior of banks. A typical bank, with capital $e$, will select contingent plans for lending $k_s$ and capital $e_s$ so as to maximize $b\mathbb{E}[k_s]$ under two constraints: the regulatory constraint (18) and the participation constraint of investors:

$$H(k_l, k_h) \equiv \mathbb{E}[k_s(p_s - R_s + b)] \leq e\mathbb{E}[p_s]. \tag{19}$$

The feasible set of the bank now has a kink $\hat{k}$. It is represented in Figure 4, together with the iso-profit line that passes through the kink $\hat{k}$.

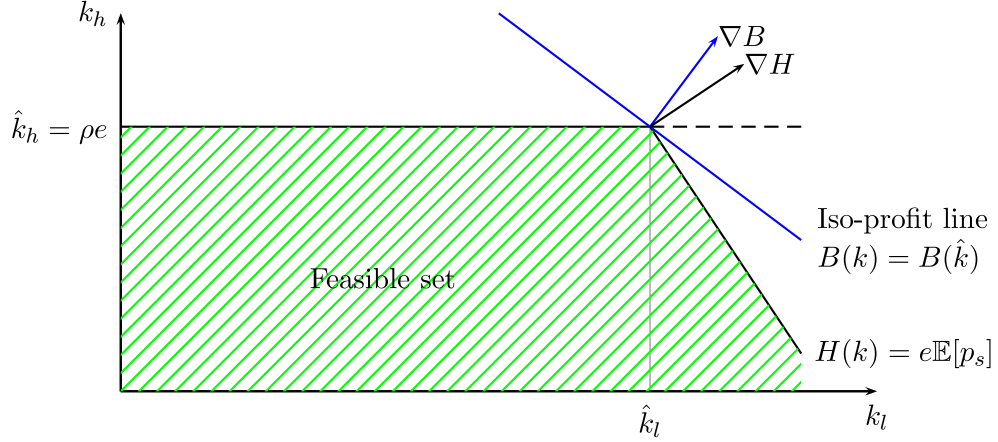


Figure 4: Feasible plans for a regulated bank

Let the minimal capital ratio $\rho^* = \frac{E}{K_h^*}$ be imposed in state $h$. Since $\hat{p}_s = F'(1 - K_s^*)$, where $(K_l^*, K_h^*)$ are the aggregate amounts invested in the banking sector, the kink $\hat{k}$ coincides with the contingent plans of the bank in the constrained efficient allocation:

$$\hat{k}_s = \frac{e}{E} K_s^*.$$

This property is tautological for $s = h$, and results from equation (19) for $s = l$. If we prove that $\hat{k}$ indeed maximizes the bank's profit on the feasible set, Proposition 5 will be established. Now recall that, by definition, $K^*$ maximizes expected output $Y(K)$ under the aggregate feasibility constraint $G(K) \leq 0$. Thus $\nabla Y$ and $\nabla G$ are colinear at $K^*$, implying the existence of a multiplier $\lambda$ such that:

$$\frac{\partial Y}{\partial K_s} = \pi_s(-p_s + R_s) = \lambda \frac{\partial G}{\partial K_s} = \lambda \pi_s \left[ p_s + b - R_s + \frac{K_s^* - E}{S'(p_s)} \right]$$

where we have used the fact that $-F''(1-K_s) = \dfrac{1}{S'(p_s)}$. Solving for $p_s - R_s$, we get

$$p_s - R_s = \frac{\lambda\left[-b - \frac{K_s^*-E}{S'(p_s)}\right]}{1+\lambda}.$$

We need to establish that the gradient of the profit function $B(k) = b\mathbb{E}[k_s]$ is above the gradient of the constraint $H(k) = e\mathbb{E}[p_s]$ (see Figure 4). Now

$$\frac{\partial B}{\partial k_s} = b\pi_s \quad \text{and} \quad \frac{\partial H}{\partial k_s} = \pi_s(p_s - R_s + b).$$

Since $S$ is log-concave and $p_h > p_l$, we know that

$$\frac{S(p_h)}{S'(p_h)} > \frac{S(p_l)}{S'(p_l)} \quad \text{and} \quad \frac{K_h^* - E}{K_h^*} > \frac{K_l^* - E}{K_l^*}.$$

Thus by multiplying these inequalities we obtain

$$\frac{K_h^* - E}{S'(p_h)} > \frac{K_l^* - E}{S'(p_l)},$$

which establishes the desired result, namely

$$\frac{\partial B/\partial k_h}{\partial B/\partial k_l} > \frac{\partial H/\partial k_h}{\partial H/\partial k_l}.$$

■

An important comment is in order at this stage. The regulation that we propose in Proposition 5 (and which allows to correct the constrained inefficiency of the competitive equilibrium) is **in the spirit** of counter-cyclical capital regulations that are currently considered by the Basel committee and several domestic regulators: It amounts to impose a cap on bank lending during booms. However these proposed regulations are **classical** capital ratios, based on **interim** levels of bank capital, i.e.

$$e_h \geq \rho k_h. \tag{20}$$

As we show below, such a regulation is inefficient in our model: Through contingent contracts signed at $t = 0$, banks can guarantee themselves a high level of capital $e_h$ in the boom (the counterpart being a low level of capital $e_l$ in the recession), which still leads to a highly

pro-cyclical lending policy, in spite of the regulator's attempt to stabilize credit fluctuations, through the counter-cyclical capital requirement.

In contrast with constraint (18), which directly limits the volume of lending in the boom, constraint (20) can largely be by-passed by banks through contingent financing contracts. To effectively limit the volume of lending in the boom, with a "classical" capital ratio, the regulator must choose a very high value for this ratio $\rho$ which distorts banks' capital allocation and indirectly reduces the volume of lending in the recession.

To illustrate the inefficiency of "classical" capital regulation, we now characterize the set of contingent credit allocations that can be implemented by such a regulation, allowing $\rho$ to be chosen arbitrarily by the regulator. Suppose $K = (K_l, K_h)$ is such an allocation. The corresponding capital prices are $p_s = F'(1 - K_s)$, $s = h, l$. We have to check that, confronted with $p$, $p_h$ and $p_l$, an individual bank with initial capital $e$ selects the individual allocation $\frac{e}{E}K$.

By linearity, we can also study the "representative" bank that owns the entire capital of the banking sector ($e = E$) but behaves competitively. In other words we only have to check that $K$ maximizes the representative bank's expected profit under the regulatory constraint (20) and the usual financing constraints:

$$\begin{aligned} & \max b\mathbb{E}[K_s] \\ \text{s.t.} \quad & K_h \leq \frac{E_h}{\rho}, \\ & K_l \leq \frac{E_l}{1 - \frac{R_l - b}{p_l}}, \\ & \mathbb{E}[p_s(E_s - E)] \leq 0. \end{aligned}$$

It is easy to see that all the constraints are binding at the solution. By eliminating $E_h$ and $E_l$, one obtains a unique constraint:

$$\pi_h p_h \rho K_h + \pi_l (p_l + b - R_l) K_l \leq \mathbb{E}[p_s] E. \tag{21}$$

The solution can only be interior if

$$p_h \rho = p_l + b - R_l, \tag{22}$$

which determines $\rho$. In this case, constraint (21) becomes

$$(p_l + b - R_l)(\pi_h K_h + \pi_l K_l) \leq \mathbb{E}[p_s]E. \tag{23}$$

Conversely, if $K$ satisfies (23) (with $p_s = F'(1 - K_s)$, $s = h, l$), it can be implemented by regulation (20) by choosing

$$\rho = \frac{p_l + b - R_l}{p_h} = \frac{F'(1 - K_l) + b - R_l}{F'(1 - K_h)}.$$

Thus we have established

**Proposition 6**

*A contingent credit allocation $K = (K_l, K_h)$ can be implemented by a "classical" capital ratio such as (20) if and only if two conditions are satisfied:*

$$J(K) \equiv (F'(1 - K_l) + b - R_l)\mathbb{E}[K_s] - \mathbb{E}[F'(1 - K_s)]E \leq 0, \tag{24}$$

*and investors participation constraint:*

$$G(K) = \mathbb{E}[(F'(1 - K_s) + b - R_s)K_s - EF'(1 - K_s)] \leq 0. \tag{25}$$

An immediate corollary of Proposition 6 is that a "classical" capital ratio such as (20) is inefficient. In the relevant region (south-east of the competitive equilibrium, i.e. ($K_h < K_h^c$, $K_l > K_l^c$), the curve defined by (24) is strictly below the curve defined by binding the participation constraint of investors:

$$G(K) = \mathbb{E}[(F'(1 - K_s) + b - R_s)K_s - EF'(1 - K_s)] = 0.$$

Indeed

$$J(K) - G(K) = \pi_h K_h [F'(1 - K_l) - R_l - F'(1 - K_h) + R_h].$$

In the relevant region ($K_h < K_h^c$, $K_l > K_l^c$) we have

$$F'(1 - K_l) - F'(1 - K_h) > F'(1 - K_l^c) - F'(1 - K_h^c) = R_l - R_h.$$

Thus in that region $J(K) > G(K)$ as was to be established. By contrast, in the north east quadrant of the competitive equilibrium, the market imposed ratio is more restrictive than the regulatory ratio, which means that $J(K) < G(K)$. Thus the feasible set, which is characterized by $\max(J(K), G(K)) \leq 0$, has a kink at the competitive equilibrium.

Figure 5 illustrates the inefficiency of "classical" capital ratios: the second best optimum K*, which can be implemented by a capital regulation based on the bank's **initial equity**, as established in Proposition 5, cannot be implemented by a "classical" capital ratio, based on the bank's **interim equity**.[14]

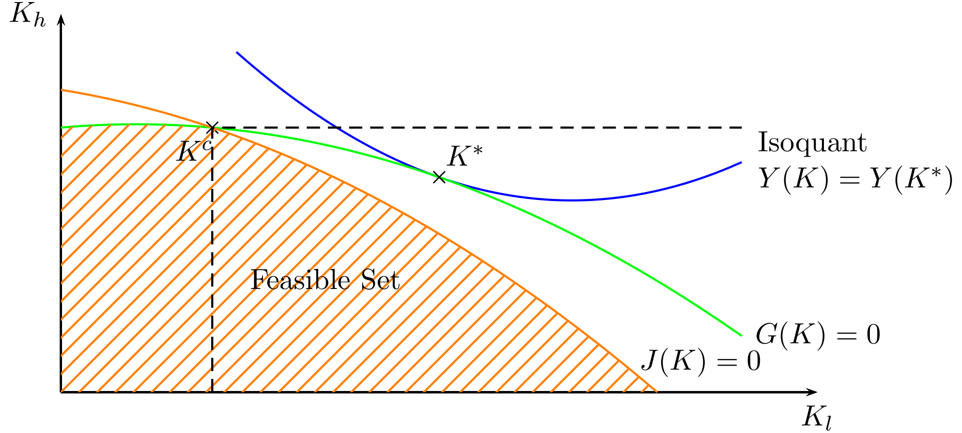


Figure 5: The feasible set with a "classical" capital ratio. The constrained efficient allocation $K^*$ does not belong to this set.

## 5.4 Robustness checks

This section relaxes two assumptions, namely the binary support of the macro-shock $s$ and the constant friction parameter $b$. Suppose that $s$ is a real number and can take an arbitrary number of values (with the convention that a higher $s$ corresponds to a higher return $R_s$) and the friction parameter $b_s$ varies with $s$. If markets are complete, a bank having equity $e$ will choose its contingent lending plans $k_s$ so as to maximize $\mathbb{E}[b_s k_s]$ under the participation constraint of investors:

$$\mathbb{E}[k_s(p_s + b_s - R_s) - p_s e] \leq 0. \tag{26}$$

An interior solution is only possible when $b_s$ is proportional to $p_s + b_s - R_s$. Denoting by $b$ and $p$ the expectations of $b_s$ and $p_s$, this implies that

$$p_s = R_s - (R - p)\frac{b_s}{b}. \tag{27}$$

Thus the equilibrium price in state $s$ is a convex combination of the return on banks' assets $R_s$ and of the pledgeable income $R_s - b_s$. If $b_s$ is lower in good states and higher in bad states

[14] In practice, such an approach requires that bank equity capital ratios for trend growth of GDP are determined and regulatory capital requirements are based on these data.

(thus $b_s$ and $R_s$ move in opposite directions, i.e. $b_s$ decreases in $s$) then instability is reinforced: even in absolute terms (i.e. not dividing by expectations), capital prices fluctuate more than fundamentals. However the opposite is true if $b_s$ and $R_s$ move together, for example if they are positively proportional, an assumption often made in the literature on financial frictions.

We can establish the existence and uniqueness of a competitive equilibrium like in Proposition 2. The equilibrium value of $p$, which we denote $p^c$ is the unique value of $p$ that equalizes demand and average supply:

$$\mathbb{E}[\frac{b_s}{b}S(R_s - (R-p)\frac{b_s}{b})] = \frac{E}{1-\frac{R-b}{p}}. \tag{28}$$

The variance of $b_s$ and its covariance with $R_s$ have an impact on the equilibrium value of $p$. As an illustration, consider the case where $S(p) = p$ and $b_s$ proportional to $R_s$. Easy computations show that

$$p^c = R - b + \frac{R^2 E}{\mathbb{E}[R_s^2]}. \tag{29}$$

Since $R^2 < \mathbb{E}[R_s^2]$, we see that in this case, more banking capital is needed to stabilize the economy than in the case where frictions are constant.

Proposition 4 can be extended to this more general set-up. The competitive equilibrium is generically constrained inefficient: the equilibrium allocation of capital does not maximize expected welfare $Y(K)$ under the participation constraint of investors to the financing of the banks, i.e.

$$G(K) = \mathbb{E}[(F'(1-K_s) + b_s - R_s)K_s - EF'(1-K_s)] \leq 0. \tag{30}$$

Of course, in this more complex set-up, it is not possible anymore to implement the second best optimum by a simple (uniform) capital ratio. But when $S$ is log concave and $b_s$ is decreasing in $s$ (more frictions in bad times), it can be established that, starting from the competitive allocation, social welfare can be improved by forcing banks to reduce lending in good states (i.e. when $s$ is greater than some threshold $s^*$) and increase lending in the other states, in such a way that the participation constraint of investors remains binding.

### 5.5 Chained Financial Frictions

In the basic version, we have assumed that bankers cannot pledge all future revenues to investors, but they can lend frictionlessly to entrepreneurs. The model, however, can easily be

extended to circumstances when bankers themselves face the problem that entrepreneurs cannot fully pledge their output. Suppose, e.g., that entrepreneurs can secretly divert part of the output by an (inefficient) technology. If the entrepreneur chooses to divert, he obtains $b^{en}$ ($b^{en} > 0$) consumption goods per unit of assets diverted. The banker, therefore, has to set the lending rate in such a way that the entrepreneur obtains at least $b^{en}$ per unit of loan granted to him in order to avoid diversion. Hence, bankers receive $R_s - b^{en}$. Bankers themselves could divert the assets, thereby earning $b^{ba}$ ($b^{ba} > 0$) per unit of assets diverted. Hence, in total, the maximal pledgeable income the bankers can offer to investors is $R^s - b^{en} - b^{ba} = R^s - b$, where $b := b^{en} + b^{ba}$ reflects the total intensity of financial frictions.

All of our preceding results can be applied to this chain of financial frictions, captured by the total intensity of financial frictions $b$. Counter-cyclical capital ratios of the form $e \geq \rho^* K_h$ with $\rho^* = \frac{E}{K_h^*}$ can improve welfare, as outlined in Proposition 5. Entrepreneurs and bankers will obtain the expected rents $b^{en}\mathbb{E}[K_s^*]$ and $b^{ba}\mathbb{E}[K_s^*]$, respectively.

## 6 Stabilization of Wage Fluctuations

The preceding analysis has focused on the case of a single input for production. However, the model is easily reformulated with more standard macroeconomic production functions where capital and labor are inputs. More specifically, suppose that there is an additional continuum of workers endowed with an aggregate amount of labor $\overline{L}$, which is normalized to $\overline{L} = 1$.[15] $\overline{L}$ is supplied inelastically in the labor market. The traditional sector is characterized by a production function with constant returns to scale

$$\tilde{F}(1-K, L)$$

where $1-K$ and $L$ are the inputs of capital and labor respectively. $\tilde{F}(1-K, L)$ is assumed to fulfill the standard assumptions of positive but decreasing marginal products of capital and labor $\left(\frac{\partial F}{\partial(1-K)} > 0, \frac{\partial^2 F}{\partial(1-K)^2} < 0, \frac{\partial F}{\partial L} > 0, \frac{\partial^2 F}{\partial L^2} < 0\right)$. The textbook example is the Cobb-Douglas production function

$$\tilde{F}(1-K, L) = A(1-K)^{\alpha} L^{1-\alpha}$$

[15] The equilibrium would be the same if investors are endowed with labor and no separate group of workers is present.

with $0 < \alpha < 1$, $A > 0$. The technology in the traditional sector is operated by a representative firm that acts competitively in the markets for capital, labor and consumption goods. We observe that profit maximization of the representative firm and market clearing in the labor and capital market in state $s$ imply

$$\frac{\partial \tilde{F}(1-K_s, 1)}{\partial(1-K_s)} = p_s, \tag{31}$$

$$\frac{\partial \tilde{F}(1-K_s)}{\partial L} = w_s. \tag{32}$$

In the case of Cobb-Douglas production functions, we obtain

$$\begin{aligned} \alpha A(1-K_s)^{\alpha-1} &= p_s, \\ (1-\alpha)A(1-K_s)^{\alpha} &= w_s. \end{aligned}$$

In this case, workers and investors share the output in the traditional sector according to

$$\begin{aligned} p_s(1-K_s) &= \alpha A(1-K_s)^{\alpha}, \\ w_s\overline{L} &= (1-\alpha)A(1-K_s)^{\alpha}. \end{aligned}$$

We also note that all preceding considerations continue to hold when we define

$$F(1-K_s) = \tilde{F}(1-K_s, 1).$$

Hence, we obtain

**Proposition 7**
*If capital supply is log-concave, the capital requirement $k_h \leq \rho^* e$ where $\rho^* = \dfrac{E}{K_h^*}$ reduces the fluctuations of wages in the economy.*

Proposition 7 follows from Proposition 5 and the equilibrium condition (32) for wages together with the properties of $\tilde{F}(\cdot,\cdot)$.

Proposition 7 shows that imposing a cap on lending in booms by capital requirements has indirect effects on other factor prices in the economy. This property has implications beyond the results derived so far, if workers are risk-averse and lack access to contingent markets (or asset markets which allow the same type of trades). In such cases, counter-cyclical capital regulations partially insure workers against wage fluctuations and thus partially substitute their lack of access to advanced financial markets.

## 7 Distributing the Gains from Macroprudential Policies

### 7.1 Forms of Redistribution

So far, we have focused on how bank capital requirements can increase aggregate output. From a utilitarian perspective, therefore, imposing stricter capital ratios in booms is welfare-improving, as all individuals are risk-neutral. Besides this traditional macroeconomic focus, one might also be interested in the distributional consequences of macroprudential policies and in how such consequences might be altered. This is in line with recurrent monetary policy debates of the welfare costs of inflation and their distribution within society. Like inflation - or its absence -, macroprudential policy generates winners, but it may also hurt some segments of society, depending on the dispersion of factors of production and the organization of the economy. In our most simple set-up with only investors and bankers, we face a particular form of distribution of welfare gains. The gains accrue solely to bankers, and the expected consumption of investors declines.[16]

There are at least three ways how gains of bankers from macroprudential policies are or can be distributed to the other agents in the economy, thereby ensuring a Pareto improvement. For this purpose, the simple model in the preceding sections has to be embedded into a broader context. We briefly outline the first two channels and then provide a detailed analysis of the third channel.

The first way are transfers that occur in multi-member households. For instance, let us consider the economy in the last section and suppose that a typical household is composed of two individuals: a banker (a male with endowment $e_1$ of the capital good) and a worker (a female with endowment $e_2$ of the capital good and labor endowment $l$). Household members stay together, as they benefit from group externalities or they may use household formation to pool resources. Total endowment of the household that the banker can use as capital in his bank is $e = e_1 + e_2$. Then, the expected amount of consumption goods of the household increases when capital requirements are imposed in booms. Moreover, both household members gain if they allocate income according to their contributions and $e_2$ is sufficiently high, or if they Nash bargain over consumption using exit and income shares determined by law as threat point, or simply allocate consumption in the household according to some sharing rule.

[16] At the competitive equilibrium a social welfare function $\beta\{\mathbb{E}[(F(1-K_s)] + \mathbb{E}[(R_s - b)K_s]\} + (1-\beta)b\mathbb{E}[K_s]$ is maximized for some value of $\beta \in (0,1)$. Imposing capital requirements in the boom increases $\mathbb{E}[K_s]$ at the expense of the expected consumption of investors.

A second type of redistribution is possible in a monetary version of our model, when cash has to be used to buy consumption goods and financial frictions are caused by cash diversion. In such circumstances, bankers who divert cash have to go to the marketplace to buy consumption goods. Then, it is possible to tax such purchases of consumption goods and tax revenues can be distributed to investors. The important point is that bankers cannot avoid such taxation - whether they divert cash or receive rents from their lending activities. Hence, the intensity of financial frictions does not increase when consumption goods purchases are taxed. As a consequence, when capital regulation is introduced, the gains of bankers can be collected by introducing (or increasing) sale taxes and the tax revenues can be redistributed to investors.

In the next subsection, we outline in detail how taxation can engineer a Pareto improvement in the context of multiple consumption goods, which represents a third type of redistribution of gains from capital regulation.

### 7.2 Taxation of Complementary Consumption Goods

If we embed the economy in a broader context with multiple consumption goods, the gains of bankers can be distributed by taxing consumption goods that cannot be completely substituted by the non-pledgeable output in the banking sector. In such circumstances, taxation does not, or only moderately increases the intensity of financial frictions, i.e. the non-pledgeable share of the output. Tax revenues are distributed to investors (and workers if present).[17] We illustrate the working of such a redistribution with a simple example deliberately designed to preserve the structure and the results of our model in sections 1 to 5.[18] We focus on financial frictions generated by asset diversion or inalienability of human capital as those forms are the easiest to illustrate.

We assume that all agents in the model variant in sections 1 to 5 or section 6 derive utility from two goods in $t = 2$. Their preferences are expressed by Bernoulli utility functions

$$u(c_1^s, c_2^s) = c_1^s + \mu \ln c_2^s$$

[17] This is related to the classical observation of e.g. Arnott and Stiglitz (1990) that models with moral hazard have to be put into a broader context with multiple consumption goods, to be able to derive robust conclusions. In our context, however, the purpose of taxation is not to alleviate moral hazard, but to redistribute the gains from macroprudential policies.

[18] The same procedure can be applied to the variant of the model in section 6.

where $c_1^s$ (resp. $c_2^s$) is the first (resp. second) good consumed in state $s$ and agents maximize their expected utilities $\mathbb{E}[u(c_1^s, c_2^s)]$. The first good is the output in the traditional and in the banking sector and we call it car. The second good called bread can be produced in $t = 2$ according to a linear production function

$$Z = \gamma X \tag{33}$$

where $\gamma > 0$ and $X$ is the amount of the first good that serves as input to this production. Typically, transportation equipments can be used for both purposes, consumption and production of other consumption goods.

The bread sector is operated by a continuum of (passive) entrepreneurs which will make zero profit in equilibrium. The price of bread in terms of cars is denoted by $p_Z$. Due to the linear technology the equilibrium price $p_Z^c$ will be equal to $\gamma$.

We first derive the competitive equilibrium without regulation. With asset diversion or inalienability of human capital, in $t = 2$, a banker receives $bk_s$ of the first consumption good in state $s$. Hence, he solves the following problem

$$\max_{\{c_1^s, c_2^s\}} \{c_1^s + \mu \ln c_2^s\}$$
$$\text{s.t.} \quad c_1^s + p_Z c_2^s = bk_s$$

which yields in equilibrium[19]

$$\begin{aligned} c_1^s &= bk_s - \mu, \\ c_2^s &= \frac{\mu}{p_Z} = \frac{\mu}{\gamma}. \end{aligned}$$

We note that $c_2^s$ is independent of the realization of the macroeconomic shock $s$.

Anticipating their behavior in $t = 2$ and the state independent equilibrium price $p_Z = \gamma$, the expected utility of the banker in $t = 0$ can be rewritten as

$$\mathbb{E}[c_1^s] + \mathbb{E}[\mu \ln c_2^s] = \mathbb{E}[bk_s - \mu] + \mu \ln(\frac{\mu}{p_Z}) = \mathbb{E}[bk_s] - \mu + \mu \ln(\frac{\mu}{\gamma}). \tag{34}$$

[19]Throughout this section, we assume interior solutions ($c_1^s > 0$, $c_2^s > 0$).

Hence, the problem of the banker in $t = 0$ is

$$\max_{\{k_s\}} \{\mathbb{E}[bk_s] - \mu + \mu \ln(\frac{\mu}{\gamma})\}$$
$$\text{s.t.} \quad \mathbb{E}[k_s(p_s + b - R_s) - p_s e] \leq 0.$$

As $\mu$ and $\gamma$ are constants, we obtain

**Proposition 8**

*The competitive equilibrium $(K_h^c, K_l^c)$ in the car-bread economy coincides with the competitive equilibrium in the one-consumption good economy.*

We next observe that we can apply the preceding results on capital regulation to the first consumption good.

**Corollary 1**

*If capital supply is log-concave, the aggregate amount of the first consumption good can be increased by imposing the capital ratio $k_h \leq \rho^* e$ with $\rho^* = \dfrac{E}{K_h^*}$ in the boom.*

We next describe how the increase of the expected aggregate consumption good can be redistributed to investors in order to engineer a Pareto improvement. For this purpose, we impose a sales tax $\tau$ ($\tau > 0$) on buying bread. As the equilibrium producer price stays at $\gamma$, agents demanding bread have to pay $\gamma(1 + \tau)$ per unit of bread. We observe

**Lemma 1**

*For asset diversion or non-alienability of human capital, the payment to the banker is $b$ per unit of assets and is independent of the tax rate $\tau$.*

We show the Lemma for the case of asset diversion. Suppose that bread is taxed at tax rate $\tau$, and thus $p_Z = (1 + \tau)\gamma$. If the banker has received capital goods $k_s$, he can divert secretly $bk_s$ units of cars. He will buy bread in $t = 2$, facing the price $p_Z = (1 + \tau)\gamma$. Using the expression derived in (34), the banker's expected utility upon diversion is

$$\mathbb{E}[bk_s] - \mu + \mu \ln\left(\frac{\mu}{(1 + \tau)\gamma}\right).$$

Hence, by paying at least $b$ per unit of capital invested, asset diversion is avoided, independently of the tax rate $\tau$.

Finally, we can determine the maximal tax rate, denoted by $\tau^{max}$, that can be levied on bankers. In the aggregate, bankers are equally well off in the unregulated economy as in the economy with capital regulation and taxation of bread if

$$\mathbb{E}[bK_s^c] - \mu + \mu \ln(\frac{\mu}{\gamma}) = \mathbb{E}[bK_s^*] - \mu + \mu \ln \left( \frac{\mu}{(1+\tau^{max})\gamma} \right)$$

which yields

$$\ln(1+\tau^{max}) = \frac{b}{\mu} \left\{ \mathbb{E}[K_s^*] - \mathbb{E}[K_s^c] \right\}.$$

Tax revenues in this case from bankers amount to $\tau^{max} c_2^s = \dfrac{\mu}{1+\tau^{max}} \tau^{max}$, which are independent of the state of the world. If $\mu$ is not too small, those tax revenues are sufficiently large to engineer a Pareto improvement.

# 8 Discussion and Conclusion

Our paper develops a very simple model where financial frictions generate excessive credit fluctuations that can be dampened by counter-cyclical regulation of banks' capital. The source of this inefficiency is a distortion of the allocation of borrowing capacity between good and bad times. Interestingly, in this model, regulation does not impose any cost on the banking industry, but instead, it works as a coordination device: banks collectively gain from the imposition of a counter-cyclical capital ratio.

Numerous extensions deserve further scrutiny. Two avenues for future research are particularly valuable. First, an alternative rationale for counter-cyclical capital regulation may be that such regulation reduces the likelihood of banking crises. The conceptional foundation of such a rationale and the interplay with the rationale developed in this paper are pressing issues. Second, dynamic extensions of the current framework may reveal that capital regulation may have persistent welfare effects, as it helps to stabilize the future equity levels of the banks.[20] It is of utmost importance for policy, as well as for macroeconomic theory, to develop such types of models.

[20]Details on such dynamic extension are available upon request.

# Appendix 1

### How different forms of financial frictions generate similar leverage constraints

### Asset Diversion

Suppose that managers can secretly divert assets like in Gertler-Karadi (2009) and Gertler-Kiyotaki (2011). The diversion technology is inefficient and only gives to the managers a return $b < R$ per unit of assets diverted. Inefficient asset diversion is avoided if and only if managers get (at least) an expected payment of $b$ per unit of capital invested.

### Moral Hazard

Suppose, like Holmström and Tirole (1997) that the banks' assets are risky (they default with probability $\pi$) and that bankers can secretely select an inferior technology, characterized by a higher probability of default $\pi + \Delta\pi$, but that provides bankers with a private benefit $B$ per unit of investment. In order to avoid the choice of this inferior technology, bankers must be promised a bonus of $\frac{B}{\Delta\pi}k$ in case of success, which means that the non pledgeable income of investors is at least $b \equiv \pi\frac{B}{\Delta\pi}$ per unit of capital.

### Inalienability of Human Capital

Suppose, like Hart and Moore (1994) or Diamond and Rajan (2001), that bank managers can threaten to walk away from their jobs, in which case investors have to replace them by new, less efficient managers that generate lower returns $(1 - \theta)R_s$. To keep the initial managers, investors must promise them a payment of at least $b_s = \theta R_s$ per unit of capital.

### Haircuts and limits to arbitrage

Suppose, like Gromb and Vayanos (2010), that investors are only ready to lend a fraction of the value of the assets they finance (haircut). A simple case arises when these investors are infinitely risk-averse. Then bankers will only offer them debt that is completely riskless. If by contrast bank loans are risky, the maximum borrowing capacity of the bank in state $s$ (per unit of capital) is the (minimum) amount that can be recovered in case of default $R_s^{min}$, which is strictly less than $R_s$.

# Appendix 2

## Calibration of the Model

We propose here a very simple calibration of our model with complete markets based on a capital ratio $CR = 1 - \frac{R-b}{p}$ of 2%, a ratio of financial frictions $FF = \frac{b}{R}$ of 30% and an elasticity of capital supply $\epsilon^s = \frac{pS'(p)}{S(p)}$ of 1. We renounce the notation that indicates complete markets in this Appendix.

We measure "volatilities" of random variables by their dispersion coefficients:

$$\sigma_R = \frac{R_h - R_l}{R},\ \sigma_p = \frac{p_h - p_l}{p},\ \sigma_Y = \frac{Y_h - Y_l}{\mathbb{E}[Y]} \quad \text{and} \quad \sigma_K = \frac{K_h - K_l}{\mathbb{E}[K]}.$$

We assume that these "volatilities" are small and use first-order Taylor expansions:

$$\sigma_K \sim \frac{S(p_h) - S(p_l)}{S(p)} \sim \frac{pS'(p)}{S(p)} \left( \frac{p_h - p_l}{p} \right) = \epsilon^s \sigma_p.$$

Similarly $Y_s = F(1 - K_s) + R_s K_s$. Thus

$$\begin{aligned} Y_h - Y_l &\sim -F'(1-K)(K_h - K_l) + (R_h - R_l)K + R(K_h - K_l) \\ &\sim (R - p)(K_h - K_l) + K(R_h - R_l), \end{aligned}$$

so that

$$\begin{aligned} Y_h - Y_l &\sim (R_h - R_l)[(R-p)S'(p) + K], \\ \text{and} \quad \sigma_Y = \frac{Y_h - Y_l}{Y} &\sim \frac{RK}{Y}[(\frac{R}{p} - 1)\epsilon^s + 1]\sigma_R. \end{aligned}$$

Finally, we can compute the volatility of (the value of) bank credit $pK$:

$$\sigma_{pK} = \sigma_p + \sigma_K = (1 + \epsilon^s)\sigma_p.$$

Recall that $p_s = R_s - R + p$ where $p = \mathbb{E}[p_s]$. Thus $\sigma_p = \frac{p_h - p_l}{p} = \frac{R_h - R_l}{p}$.

### Volatility of capital prices

$$\frac{\sigma_p}{\sigma_R} = \frac{R}{p} = \frac{1 - CR}{1 - FF} = \frac{98}{70} = 1.4.$$

Thus capital prices fluctuate 40% more than returns.

### Volatility of credit to GDP ratio

$$\frac{\sigma_{pK}}{\sigma_Y} = \frac{\sigma_p + \sigma_K}{\sigma_Y} \sim \frac{(1+\epsilon^s)R/p}{1 + (\frac{R}{p} - 1)\epsilon^s} \frac{Y}{RK} = \frac{(1+\epsilon^s)}{1 + (\frac{R}{p} - 1)\epsilon^s} \frac{Y}{pK}.$$

Thus if we take an average credit to GDP ratio $\frac{pK}{Y}$ of 1, the volatility of this ratio is

$$\frac{\sigma_{pK}}{\sigma_Y} = \frac{2}{1.4} \approx 1.4.$$

Again, credit fluctuates 40% more than GDP.

# Appendix 3

**Proof of Proposition 3:**

It results from a Taylor expansion around $p^0 \equiv \mathbb{E}[p_s^0]$ and $R \equiv \mathbb{E}[R_s]$ of the following equation, which is equivalent to (5):

$$S(p_s^0)[p_s^0 + b - R_s] = p_s^0 E.$$

We obtain for $s = h, l$

$$(p_s^0 - p^0)[S'(p^0)(p^0 + b - R) + S(p^0)] - (R_s - R)S(p^0) \sim (p_s^0 - p^0)E.$$

The approximation is accurate when the macro shock is small. Now take the difference between these two equations:

$$(p_h^0 - p_l^0)[S'(p^0)(p^0 + b - R) + S(p^0) - E] \sim (R_h - R_l)S(p^0).$$

Now $p_h^c - p_l^c = R_h - R_l$ and $p^c \sim p^0$ when $\sigma_R$ is small. Dividing the above relation by $p^0 S(p^0)$ we obtain

$$\sigma_p^c = \frac{p_h^c - p_l^c}{p^c} \sim \frac{R_h - R_l}{p^0} \sim \sigma_p^0 \left[ \frac{S'(p^0)}{S(p^0)}(p^0 + b - R) + 1 - \frac{E}{S(p^0)} \right].$$

As the macro shock is small, $p^c + b - R = p^c \frac{E}{S(p^c)}$ holds in equilibrium with contingent markets and $p^c \sim p^0$, we obtain $p^0 + b - R \sim p^0 \frac{E}{S(p^0)}$. Therefore, we have finally established that

$$\sigma_p^c \sim \sigma_p^0 \left[ 1 - \frac{E}{S(p^0)} \left( 1 - \frac{p^0 S'(p^0)}{S(p^0)} \right) \right].$$

Thus when $\frac{p^0 S'(p^0)}{S(p^0)} < 1$, and $\sigma_R$ is small, $\sigma_p^c < \sigma_p^0$.

■

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