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A THEORY OF COUNTERCYCLICAL GOVERNMENT-CONSUMPTION MULTIPLIER

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ABSTRACT

A Theory of Countercyclical Government-Consumption Multiplier*

This paper proposes a dynamic stochastic general equilibrium model in which the government-consumption multiplier doubles when unemployment rises from 5% to 8%. Theoretically, such countercyclicality arises because of a nonlinearity, namely, that labor supply is convex in a labor market tightness-employment diagram. In the model, as government consumption increases, public employment rises, stimulating labor demand. Equilibrium tightness increases, which reduces private employment and partially offsets the increase in public employment. Since labor supply is convex, the increase in tightness is small in recessions but large in expansions. Hence, government consumption reduces unemployment much more in recessions than in expansions.

JEL Classification: E24, E32 and E62
Keywords: business cycle, multiplier and unemployment

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1 Introduction

Conventional macroeconomic models are close to linear in the range of activity seen in the data. This property implies that the economy behaves similarly in recessions and expansions. A new class of models is nonlinear in the vicinity of the zero lower bound on nominal interest rates.[1] In these models, the economy behaves differently during recessions when the zero lower bound is reached. But in most recessions, the economy is away from the zero lower bound and it behaves as in an expansion. In fact in the last hundred years in the US, the economy has been at the zero lower bound only during the Great Depression and in the aftermath of the 2008 financial crisis.

This paper proposes a macroeconomic model that, in contrast to conventional models, is nonlinear in the range of activity seen in the data. The nonlinearity arises not from the zero lower bound but from the structure of the labor market, which is adapted from Michaillat [2012]. A consequence of the nonlinearity is that the economy behaves differently in recessions and expansions, and in particular, that an increase in government consumption reduces unemployment much more in recessions than in expansions. Simulations of the model calibrated to US data indicate that the government-consumption multiplier doubles when unemployment rises from 5% to 8%. The simulation results are consistent with existing empirical evidence about the cyclical fluctuations of government-consumption multipliers.

Why does government consumption reduce unemployment more effectively in recessions? In the model, government consumption expenditures arise entirely from the compensation of employees in the public sector.[2] Therefore, the government hires additional workers in the public sector when it increases its consumption. This is a simplification because in the US, broadly two-thirds of government consumption expenditures arise from the compensation of public employees and one-third arises from the purchase of intermediate goods and services from the private sector. But this simplification is useful because it allows me to focus on what seems to be the main lever with which the government is able to stimulate the economy. Indeed, Ramey [2012] finds that an

[1] See Eggertsson and Woodford [2003], Christiano, Eichenbaum and Rebelo [2011], or Eggertsson and Krugman [2012]. For a model with equilibrium unemployment accounting for the zero lower bound, see Rendhal [2012].

[2] Other researchers who modeled government consumption expenditures as compensation of public employees include Wynne [1992], Finn [1998], Cavallo [2005], or Pappa [2009].

increase in government consumption stimulates aggregate employment not because it stimulates employment in the private sector, but because the government hires workers in the public sector.

In expansions, an increase in government consumption does not reduce unemployment because public employment crowds out private employment almost one-for-one. Crowding out occurs because of matching frictions on the labor market, which require government and firms to post vacancies to hire workers. When unemployment is low, the competition to hire unemployed workers is intense. By posting vacancies, the government takes job applicants away from existing vacancies. Firms are required to post additional vacancies to attract applicants. Posting vacancies is costly so the marginal cost of labor rises and firms reduce employment significantly. Hence, the crowding out of private employment by public employment almost totally offsets the increase in public employment when unemployment is low.

In recessions, an increase in government consumption reduces unemployment effectively because public employment does not crowd out private employment much. When unemployment is high, there is no shortage of unemployed workers. By posting vacancies, the government brings job applicants out of unemployment, who would not have found a job otherwise. Since the government does not take applicants away from existing vacancies, firms do not need to post additional vacancies, the marginal cost of labor remains the same, firms maintain the same level of employment, and there is barely any crowding out.

The proposed model, described in Section 2, is a simple New Keynesian dynamic stochastic general equilibrium model that adopts the equilibrium unemployment framework of Pissarides [2000] to represent the labor market.[3] Exogenous technology shocks lead to business cycle fluctuations. There are seven endogenous variables: three prices (nominal interest rate, inflation, and labor market tightness), three quantities (consumption of final good, output of final good, and employment), and the real wage. Each price-quantity pair is associated with a market: bond market, final-good market, and labor market.

A particularity of the model is that the real wage can neither act as a price equalizing labor

[3] For other New Keynesian models with equilibrium unemployment, see for instance Walsh [2003], Moyen and Sahuc [2005], Trigari [2009], Blanchard and Galí [2010], and Monacelli, Perotti and Trigari [2010].

supply to labor demand, nor be set by monopolistic workers. The reason is that in presence of matching frictions, a surplus arises from worker-firm matches and the real wage is determined by a surplus-sharing rule, taken as given by workers and firms. In the model, I use the rule of Blanchard and Galí [2010], which imposes that the real wage be a function of current technology only. Instead of the wage, labor market tightness—the ratio of vacancies to unemployment—acts as a price that equalizes labor supply to labor demand.

In addition to the real wage, there are six other endogenous variables. The complete characterization of the equilibrium therefore requires six additional relationships. A large household, composed of employed and unemployed workers, allocates its income to consumption of a final good and savings using a one-period bond. The optimal choice of consumption and saving defines a first relationship: the New Keynesian IS curve. Monetary policy links the nominal interest rate to inflation according to a second relationship: the Taylor [1993] rule. The final good is produced by perfectly competitive firms from intermediate goods produced by monopolistic firms. The monopolistic firms face a quadratic price-adjustment cost, as in Rotemberg [1982]. They also face a cost to hire workers because of the matching frictions. The optimal pricing and hiring decisions of firms define a third relationship: the New Keynesian Phillips curve. The employment rate of the household depends on the search behavior of unemployed workers, and on the probability to find a job. The job-finding probability depends on labor market tightness. So employment and labor market tightness are related by a fourth relationship that I call labor supply. Employment and output are linked by a fifth relationship: the production technology. Output and consumption of final good are linked by a sixth relationship: the resource constraint.

To illustrate the nonlinearity of the model around the unemployment rates observed in the data, Section 3 represents the steady state with zero inflation using a labor supply-labor demand diagram plotted in a labor market tightness-employment plan, as in Figure 1(a).[4] Labor supply coincides with the Beveridge curve in steady state.[5] Labor supply is an increasing function of tightness and, when represented in the labor market tightness-employment plan, it is also convex. The convexity

[4] Landais, Michaillat and Saez [2010] introduced this representation to study optimal unemployment insurance over the business cycle.

[5] For more information on the Beveridge curve in the US, see Blanchard and Diamond [1989].

of labor supply is at the origin of the nonlinearity of the model. In steady state, the Phillips curve imposes that the marginal product of labor be a markup over the marginal cost of labor, the sum of the real wage plus hiring costs. This condition defines the labor demand of the private sector, a relationship between private employment and labor market tightness parameterized by the technology level. As in Michaillat [2012], I make two assumptions that shape private labor demand: (a) the production function of intermediate-good firms has diminishing marginal returns to labor; and (b) real wages are somewhat rigid, in the sense that they do not adjust one-for-one to technology. Because of (a), private labor demand decreases with tightness. Because of (b), private labor demand increases with technology. The aggregate labor demand—the sum of the private labor demand and an exogenous amount of public employment—shares these properties. In equilibrium, labor market tightness equalizes labor supply and labor demand. When technology falls, labor demand shifts inward, which raises unemployment and lowers tightness as in Figure 1(b).

An implication of the nonlinearity of the model is that the government-consumption multiplier—defined as the aggregate number of jobs created by employing one more worker in the public sector—is not only positive but also countercyclical. In the labor supply-labor demand diagram, increasing government consumption raises public employment and shifts aggregate labor demand outward, as depicted in Figure 1(c) for an expansion and in Figure 1(d) for a recession. Equilibrium tightness increases, which reduces private employment and partially offsets the increase in public employment. In other words, public employment crowds out private employment by making it more difficult for firms to hire workers. The increase in tightness is large in expansions but small in recessions. Hence the multiplier is small in expansions but large in recessions. Formally, I show that the multiplier depends positively on the ratio of the slopes of the labor demand and labor supply curves. I prove that the ratio decreases sharply with the technology level, mostly because of the convexity of labor supply. Hence the multiplier decreases with technology.

Section 4 calibrates the model with US data to show that the cyclical fluctuations of the multiplier are sizable. The model is simple enough to be solved exactly, without any linearization.[6]

[6]The choice of a quadratic price-adjustment cost, instead of the more common price-setting friction of Calvo [1983], allows me to study exactly the nonlinear dynamics of the model. Particularly, it allows to derive a closed-form expression for the Phillips curve. Braun, Körber and Waki [2012] also take advantage of the simplicity brought by a quadratic price-adjustment cost to compute the exact equilibrium of a nonlinear model of the zero lower bound. For

I compute the exact dynamics of the model in response to a collection of positive and negative technology shocks. Under the most positive shock, unemployment bottoms at 4.9%. Under the most negative shock, unemployment peaks at 8.2%. I repeat the simulations when the government hires a few public workers when the shock occurs. By comparing the dynamic responses with and without government intervention, I trace the marginal effect of public employment on aggregate employment in response to different shocks. An increase in government consumption is persistent because I assume that public workers cannot be dismissed: public jobs are closed only as a result of natural attrition. To account for this persistence, I compute a cumulative multiplier for each shock: it is the aggregate number of job$\times$weeks created by hiring a few workers in the public sector, divided by the number of job$\times$weeks added to the public sector. The cumulative multiplier doubles when the unemployment rate increases from 4.9% to 8.2%.[7]

To conclude, Section 5 discusses the empirical evidence presented by Auerbach and Gorodnichenko [2011] and Nakamura and Steinsson [2011] that the government-consumption multiplier is indeed higher when the unemployment rate is high than when the unemployment rate is low. Proofs and extensions are collected in the Appendix.

2 The Model

This section describes a simple model with four departures from the benchmark New Keynesian model. First, the labor market is subject to matching frictions instead of being perfectly competitive. Second, the frictional labor market combines real wage rigidity and diminishing marginal returns to labor as in Michaillat [2012]. Third, government consumption expenditures arise entirely from hiring costs and compensations of public employees, instead of arising entirely from the purchase of goods from the private sector. And fourth, monopolistic firms are subject to a quadratic price-adjustment cost, instead of the price-setting friction of Calvo [1983]. I present empirical support for these assumptions as I introduce them into the model.

other New Keynesian models with equilibrium unemployment using a quadratic price-adjustment cost, see Chéron and Langot [2000] and Krause, Lopez-Salido and Lubik [2008].

[7]The unemployment rate mentioned here is the extremum of the response of the unemployment rate to the underlying technology shock.

Source of fluctuations. Empirical evidence suggests that recessions in the labor market are driven by aggregate-activity shocks and not by reallocation shocks [for example, Blanchard and Diamond, 1989]. Thus, I assume that business cycles are driven by technology shocks and I model technology as a stochastic process $\{a_t\}_{t=0}^{+\infty}$.[8]

Labor market. A unit mass of workers participate in a labor market composed of two sectors. The public sector employs g_t workers.[9] The private sector, composed of a continuum of intermediate-good firms indexed by $i \in [0, 1]$ that employ $l_t(i)$ workers, employs l_t workers where

$$l_t = \int_0^1 l_t(i)di. \tag{1}$$

Aggregate employment is $n_t = l_t + g_t$. At the end of period $t-1$, a fraction s of the n_{t-1} existing worker-job matches is exogenously destroyed. Workers who lose their job apply for a new job immediately. At the beginning of period t, $u_t = 1 - (1-s) \cdot n_{t-1}$ unemployed workers search for a job. Jobseekers randomly apply to jobs, without directing their search toward private or public jobs. If they find a job, they start working in period t with the $(1-s) \cdot n_{t-1}$ incumbent workers.

By posting vacancies, firms hire workers in the private sector and the government hires workers in the public sector. The number h_t of matches made in a period is given by a Cobb-Douglas

[8] The assumption that technology shocks drive business cycles is prevalent in the equilibrium unemployment literature, and common in the business cycle literature. Of course it is not very realistic to assume that the only source of recessions are negative technology shocks. For instance, the recession that followed the 2008 financial crisis is not apparently caused by a collapse of productivity. To show that my theory of the countercyclical multiplier also operates in presence of aggregate demand shocks, Appendix E presents a simple variant of the model in which recessions are caused by the combination of low aggregate demand and nominal wage rigidity. Appendix E re-derives all the theoretical results of Section 3 in the model with aggregate demand shocks. The model, however, remains too simple to be calibrated and simulated. To quantify the cyclical fluctuations of the multiplier under aggregate demand shocks, one option would be to introduce aggregate demand shocks directly into the New Keynesian model. I could follow Christiano et al. [2011], who introduce a shock to the discount factor of households that makes them reluctant to consume, or Eggertsson and Krugman [2012], who introduce a shock to the borrowing constraint faced by households that forces them to deleverage by cutting consumption. Both shocks nicely capture an aggregate demand shock. These shocks, however, bring the economy against the zero lower bound on nominal interest rates (which is the particular issue studied by Christiano et al. [2011] and Eggertsson and Krugman [2012]). This is a problem for me, because I want to demonstrate that my theory of the countercyclical multiplier relies only on the structure of the labor market and not on the zero lower bound. As technology shocks do not bring the economy at the zero lower bound in reasonable circumstances, they are the focus of the simulations in Section 4.

[9] Quadrini and Trigari [2007] and Gomes [2010] also analyse models of equilibrium unemployment with a public sector.

matching function of unemployment u_t and vacancies o_t: $h_t = \omega_h \cdot u_t^{\eta} \cdot o_t^{1-\eta}$, with the restriction that $h_t \leq \min\{u_t, o_t\}$. $\omega_h > 0$ and $\eta \in (0,1)$ are parameters. Labor market conditions are summarized by labor market tightness $\theta_t \equiv o_t/u_t$. The matching technology prevents all unemployed workers from finding a job and all vacancies from being filled. Jobseekers find a job with probability $f(\theta_t) = \omega_h \cdot \theta_t^{1-\eta}$. Vacancies in the public and private sectors are filled with the same probability $q(\theta_t) = \omega_h \cdot \theta_t^{-\eta}$. In a tight market it is easy to find jobs—the job-finding probability $f(\theta_t)$ is high—but difficult to find workers—the vacancy-filling probability $q(\theta_t)$ is low. Integrating matching frictions in the New Keynesian model is necessary to capture the job destructions, job creations, and large flows of workers observed at all times on the labor market.[10]

Keeping a vacancy open has a per-period cost $r \cdot a_t$ in units of final good, where $r > 0$ captures resources spent recruiting workers.[11] A firm hires a worker with certainty by opening $1/q(\theta_t)$ vacancies and spending $r \cdot a_t/q(\theta_t)$.

Large household. All workers belong to a large household with expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot [\ln(c_t) + \omega_v \cdot \ln(z_t)], \tag{2}$$

where $\mathbb{E}_0$ is the mathematical expectation conditional on period-0 information, $\delta < 1$ is the discount factor, ω_v is a parameter, and c_t and z_t are consumption of final good (produced in the private sector) and consumption of public good (produced in the public sector).

I focus on changes in the employment rate of the household and abstract from changes in the number of hours worked by employed workers. This choice is consistent with the empirical evidence that most cyclical variations in total hours worked are due to variations in the number of employed workers and not to variations in hours per worker [Shimer, 2010]. The employment rate

[10] For empirical evidence on worker flows, job destruction, and job creation, see Blanchard and Diamond [1989] and Davis, Haltiwanger and Schuh [1996]. For an exhaustive treatment of the equilibrium unemployment framework, see Pissarides [2000] and Shimer [2010].

[11] As in Pissarides [2000] the cost of opening a vacancy is proportional to technology a_t. This is equivalent to assuming that the recruiting technology itself is independent of technology, but that it uses labor as unique input [Shimer, 2010]. This assumption is appealing since recruiting is a labor-intensive activity.

n_t of the household follows a law of motion governed by job destruction and job creation:

$$n_t = (1-s) \cdot n_{t-1} + [1-(1-s) \cdot n_{t-1}] \cdot f(\theta_t). \tag{3}$$

Workers have no control over their employment rate, which is determined by the separation rate s and the job-finding rate $f(\theta)$. The job-finding probability is determined endogenously in equilibrium, but it is exogenous to the worker. Yet in practice, people do have some control over their employment rate. They can adapt their search behavior to labor market conditions, which affects their chance of finding a job and their labor supply. I consider this possibility in Appendix C.

Firms and government pay a real wage w_t to their employees, which is taxed at rate τ_t; thus, the real income of employed workers is $c_t^e = (1-\tau_t) \cdot w_t$. Unemployed workers receive a fraction ρ_t of the income of employed workers as unemployment benefits; thus, the real income of unemployed workers is $c_t^u = \rho_t \cdot (1-\tau_t) \cdot w_t$. The replacement rate $\rho_t = c_t^u / c_t^e$ measures the generosity of the unemployment insurance system. All the workers pool their income before choosing consumption and saving.[12] The household faces the budget constraint

$$p_t \cdot c_t + b_t = p_t \cdot n_t \cdot c_t^e + p_t \cdot (1-n_t) \cdot c_t^u + R_{t-1} \cdot b_{t-1} + T_t, \tag{4}$$

where p_t is the price level, b_t is the quantity of one-period bonds purchased by the household at time t, R_{t-1} is the one-period gross nominal rate of interest that pays off in period t, and T_t denotes the profits of firms, which are owned by the household.

The household chooses consumption $\{c_t\}$ to maximize utility (2) subject to the budget constraint (4) and to the no-Ponzi-game constraint

$$\mathbb{E}_0 \left[\lim_{t \to +\infty} b_t / \Pi_{i=0}^t R_{i-1} \right] \geq 0. \tag{5}$$

Let $\pi_t \equiv (p_t / p_{t-1}) - 1$ denote the rate of inflation at time t. The optimal consumption path is

[12] This formulation is standard since Merz [1995]. It avoids the complications that would result from having workers with heterogeneous wealth levels that depend on their employment history.

governed by the Euler equation

$$1 = \delta \cdot \mathbb{E}_t \left[\frac{R_t}{1+\pi_{t+1}} \cdot \frac{c_t}{c_{t+1}} \right]. \tag{6}$$

Wage. The wage paid by firms in the private sector is set once worker and firm have matched. Since search costs are sunk when workers and firms meet, there are always mutual gains from trade. There is no compelling theory of wage determination in such an environment [Hall, 2005]. Hence I assume that the wage follows the simple schedule from Blanchard and Galí [2010]:

$$w_t = \omega \cdot a_t^{\gamma}, \tag{7}$$

where $\omega > 0$ is a parameter. The parameter γ captures the flexibility of wages over the business cycle. If $\gamma = 0$, wages are independent of technology a_t and completely fixed over the cycle. If $\gamma = 1$, wages are proportional to technology and fully flexible over the cycle. As in the public finance literature, I assume that the incidence of the labor tax τ_t is entirely on the worker's side. Hence the wage does not respond to the labor tax rate.

Following Michaillat [2012], I make the following assumption on the wage schedule:

ASSUMPTION A1. The wage schedule is somewhat rigid: $\gamma < 1$.

In Section 4.1, I present empirical evidence from US micro-data that $\gamma < 1$. Assumption A1 is also motivated by historical and ethnographic studies that document and explain the sources of wage rigidity [for example, Bewley, 1999; Jacoby, 1984]. Under A1, unemployment increases when technology falls in recessions. In equilibrium, I impose that the wage be privately efficient. Private efficiency guarantees that worker-firm pairs exploit all opportunities for mutual improvement, such that the wage never causes the destruction of a match generating a positive bilateral surplus. Private efficiency is a reasonable equilibrium requirement when rational workers and firms engage in long-term interactions [Barro, 1977].

Representative final-good firm. The final good is produced in the private sector. There is perfect competition in the market for the final good. The representative final-good firm uses $y_t(i)$ units of each intermediate good i to produce y_t units of final good according to the following technology:

$$y_t = \left[\int_0^1 y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)}, \tag{8}$$

where $\epsilon > 1$ is the elasticity of substitution across intermediate goods.

Intermediate good i sells at the nominal price $p_t(i)$ while the final good sells at the nominal price p_t. The final-good firm chooses $y_t(i)$ for all $i \in [0, 1]$ to maximize its profits

$$p_t \cdot \left[\int_0^1 y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)} - \int_0^1 p_t(i) \cdot y_t(i) di.$$

The first-order condition with respect to $y_t(i)$ is

$$y_t(i) = y_t \cdot \left[\frac{p_t(i)}{p_t} \right]^{-\epsilon}. \tag{9}$$

This equation describes the demand for intermediate good i as a function of relative price $p_t(i)/p_t$.

Finally, perfect competition in the final-good market requires that the representative final-good firm sets its price p_t equals to its marginal cost:

$$p_t = \left[\int_0^1 p_t(i)^{1-\epsilon} di \right]^{1/(1-\epsilon)}. \tag{10}$$

Intermediate-good firms. There is no entry or exit into the production of intermediate good i.. Each intermediate good i is produced by a monopolist using the following technology:

$$y_t(i) = a_t \cdot x(l_t(i)), \tag{11}$$

where $y_t(i)$ is output of good i, a_t is technology level, $l_t(i)$ is employment by monopolist i, $x(l) = l^\alpha$, and $\alpha > 0$ is a parameter. The public good produced by public employment is consumed by workers but does not enter the production function of intermediate-good firms. Yet in practice, public employment may improve institutions, public service, or infrastructure, which in turn may improve the productivity of firms.[13] I consider this possibility in Appendix D.

Following Michaillat [2012], I make the following assumption on the production function:

ASSUMPTION A2. The production function has diminishing marginal returns to labor: $\alpha < 1$.

Assumption A2 is motivated by the slow adjustment of some production inputs over the business cycle. Crepon, Duflo, Gurgand, Rathelot and Zamora [2012] provide evidence, based on a large-scale field experiment, that the combination of Assumptions A1 and A2 provides a compelling description of the labor market over the business cycle.[14]

The monopolist faces a quadratic cost of adjusting its nominal price given by

$$\frac{\phi}{2} \cdot \left[\frac{p_t(i)}{p_{t-1}(i)} - 1\right]^2 \cdot c_t,$$

where the parameter $\phi > 0$ captures the resources that the firm devotes to adjusting prices. The cost of price adjustment is measured in units of final good and increases proportionally with the size of the economy, measured by aggregate consumption c_t. The adjustment cost makes the prices $p_t(i)$ of intermediate goods and the price p_t of the final good respond only gradually to nominal disturbances, allowing monetary policy to influence aggregate output in the short run.

Rotemberg [1982], who first formulated this price-setting friction, interprets the adjustment cost as a measure of the negative effects of price changes on customer-firm relationships, which increase in magnitude with the size of the price change and with the quantity purchased. Empirically, the negative effects of price changes on customer-firm relationships seem large. Zbaracki, Ritson, Levy, Dutta and Bergen [2004] study the pricing practices of a large industrial firm. They develop a detailed description of the pricing process to account for physical, managerial, and customer costs of changing prices. They find that customer costs are indeed more than twenty times the

[13] Evans and Karras [1994] provide evidence based on state-level data for the US that current government educational services are productive. Nadiri and Mamuneas [1994] provide evidence based on manufacturing industry data for the US that an increase in the stock of public R&D increases the labor demand of manufacturing firms.

[14] Section 5 in Landais et al. [2010] also discusses the empirical evidence provided by Crepon et al. [2012].

menu costs, and provide evidence of managers' fear of antagonizing customers by changing prices. They also find that customer costs are convex.

In addition to a price-adjustment cost, intermediate-good firm i also incurs a cost to hire new workers because of the matching frictions on the labor market. Firm i chooses employment $\{l_t(i)\}_{t=0}^{+\infty}$ and prices $\{p_t(i)\}_{t=0}^{+\infty}$ to maximize its expected discounted real profits

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \frac{\delta^t}{c_t} \cdot \left\{ \frac{p_t(i)}{p_t} \cdot y_t(i) - w_t \cdot l_t(i) - \frac{\phi}{2} \cdot \left[\frac{p_t(i)}{p_{t-1}(i)} - 1 \right]^2 \cdot c_t - \frac{r \cdot a_t}{q(\theta_t)} \cdot [l_t(i) - (1-s) \cdot l_{t-1}(i)] \right\},$$

where $l_t(i) - (1-s) \cdot l_{t-1}(i) \geq 0$ is the number of hires in period t. The multiplier $\delta^t / (c_t \cdot p_t)$ on the budget constraint in period t in the Lagrangian representation of the household's problem indicates the value of an additional dollar to the household in period t. The firm is subject to the production constraint (11) and the demand constraint (9). The Lagrangian of the firm's problem is

$$\begin{aligned} \mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{+\infty} \frac{\delta^t}{c_t} \cdot \Bigg\{ & \left[\frac{p_t(i)}{p_t} \right]^{1-\epsilon} \cdot y_t - w_t \cdot l_t(i) - \frac{\phi}{2} \cdot \left[\frac{p_t(i)}{p_{t-1}(i)} - 1 \right]^2 \cdot c_t \\ & - \frac{r \cdot a_t}{q(\theta_t)} \cdot [l_t(i) - (1-s) \cdot l_{t-1}(i)] + \Lambda_t(i) \cdot \left[a_t \cdot x(l_t(i)) - \left[\frac{p_t(i)}{p_t} \right]^{-\epsilon} \cdot y_t \right] \Bigg\}. \end{aligned}$$

The variables $\{\Lambda_t(i)\}_{t=0}^{+\infty}$ are Lagrange multipliers assigned to the production constraint. The first-order condition with respect to employment $l_t(i)$ is

$$\Lambda_t(i) = \frac{1}{x'(l_t(i))} \cdot \left\{ \frac{w_t}{a_t} + \frac{r}{q(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \cdot \frac{a_{t+1}}{a_t} \cdot \frac{r}{q(\theta_{t+1})} \right] \right\}$$

The multiplier $\Lambda_t(i)$ corresponds to the real marginal cost of producing one unit of intermediate good i in period t. The first-order condition with respect to price $p_t(i)$ is

$$\frac{p_t(i)}{p_t} = \frac{\epsilon}{\epsilon - 1} \cdot \Lambda_t(i) + \frac{\phi}{\epsilon - 1} \cdot \frac{c_t}{y_t} \cdot \left[\frac{p_t(i)}{p_t} \right]^{\epsilon} \cdot \left\{ \delta \cdot \mathbb{E}_t \left[\left[\frac{p_{t+1}(i)}{p_t(i)} - 1 \right] \cdot \frac{p_{t+1}(i)}{p_t(i)} \right] - \left[\frac{p_t(i)}{p_{t-1}(i)} - 1 \right] \cdot \frac{p_t(i)}{p_{t-1}(i)} \right\}.$$

In equilibrium the two first-order conditions yield the New Keynesian Phillips curve, which therefore captures the profit-maximizing choice of employment and price by intermediate-good firms.

Monetary policy. The monetary authority sets the gross nominal interest rate R_t according to a Taylor [1993] rule:

$$R_t = \frac{1}{\delta} \cdot (1+\pi_t)^{\mu_\pi \cdot (1-\mu_R)} \cdot (\delta \cdot R_{t-1})^{\mu_R}, \tag{12}$$

where π_t is the rate of inflation at time t, $\mu_R \in [0,1)$ measures interest-rate smoothing, and $\mu_\pi > 1$ measures the response of monetary policy to inflation. For convenience, I assume that steady-state inflation is zero. The steady-state gross nominal interest rate is $1/\delta$.

Government consumption. Before introducing government consumption into the model, I argue that the core of government consumption is public employment. First of all, two-thirds of government consumption in the data is compensation of government employees while only one-third is purchase of intermediate goods and services from the private sector. Let me explain. The National Income and Product Accounts (NIPA) represent the government as the consumer of government output. Government output is services produced by the government, such as education, health care, or national defense. Government consumption expenditures are the value of these services. But these services are provided at no cost to the population.[15] Thus, the services are valued at their cost of production in the NIPA. The NIPA tables compiled by the Bureau of Economic Analysis detail the production costs of government output.[16] On average over the 1947–2011 period, 62.5% of the variable production costs are compensation of government employees while only 37.5% are purchase of intermediate goods and services.[17]

Not only does compensation of public employees represent the majority of government consumption expenditures, but when the government increases its consumption to stimulate the economy in recessions, it usually increases public employment. During the Great Depression, the

[15] A small fraction (9.6%) of government output is sold to other sectors, mostly in the form of tuitions and health care by state and local governments. An even smaller fraction (1.2%) represents government own-account investment. I abstract from these complications here.

[16] The relevant table is Table 3.10.5, entitled "Government Consumption Expenditures and General Government Gross Output". Expenditures are measured in billions of dollars and seasonally adjusted.

[17] In fact, 54.8% of the production costs are compensation of government employees, 33.0% are purchase of intermediate goods and services, and 12.2% are consumption of government fixed capital—an inputed rent on government fixed capital. The inputed rent is a fixed cost of production, and I abstract from it here.

Roosevelt administration hired millions of unemployed workers to build dams, bridges, and roads [Neumann, Fishback and Kantor, 2010]. In 2011, the American Jobs Act presented to Congress by the Obama administration proposed to spend $130 billion to hire, or prevent the layoff of, teachers and other public-sector workers.[18]

Furthermore, Ramey [2012] provides empirical evidence based on vector autoregression methods that an exogenous increase in government consumption translates into an increase in public employment, not private employment. The Figure 14 in Ramey [2012] identifies exogenous increases in government consumption using the methodology of Ramey and Shapiro [1998]. The Figure 15 identifies exogenous increases using the methodology of Blanchard and Perotti [2002]. In Figure 14 in all samples (1939–2008, 1947–2008, and 1954–2008) and in Figure 15 in all but the 1947–2008 sample, an increase government consumption leads to a statistically significant increase in public employment. At the same time in Figure 14 in all samples, an increase government consumption has no effect on private employment; in Figure 15 in all but the 1954–2008 sample, an increase government consumption leads to statistically significant decrease in private employment.

In light of this argument, I assume in the model that government consumption expenditures arise entirely from the compensation of g_t employees in the public sector. Therefore, an increase in government consumption expenditures translates into an increase in public employment. Focusing on public employment allows me to highlight a new mechanism explaining countercylical government-consumption multipliers. I leave for future work the integration to the model of the purchase of intermediate goods and services by the government.

I assume that the government produces a public good according to the production function

$$z_t = \omega_z \cdot a_t \cdot x(g_t), \tag{13}$$

where z_t is the output of public good, g_t is the number of workers in the public sector, and ω_z is a parameter that scales the productivity of the public sector relative to that of the private sector.

The government is subject to a budget constraint each period. The government outlays have

[18]Details are provided at http://www.whitehouse.gov/the-press-office/2011/09/08/fact-sheet-american-jobs-act.

four components. First, the compensations $p_t \cdot w_t \cdot g_t$ of the workers in the public sector paid at the prevailing private-sector wage w_t. Second, the cost $p_t \cdot [r \cdot a_t / q(\theta_t)] \cdot [g_t - (1-s) \cdot g_{t-1}]$ incurred when hiring new public workers. Third, the service $R_{t-1} \cdot b_{t-1}$ of the debt inherited from the previous period. And fourth, the unemployment benefits $p_t \cdot \rho_t \cdot (1-\tau_t) \cdot w_t \cdot (1-n_t)$ transferred to unemployed workers. These outlays are financed by a labor tax τ_t paid by all employed workers, and by issuing new debt b_t. Therefore, the budget constraint is

$$n_t \cdot \tau_t \cdot w_t + \frac{b_t}{p_t} = \rho_t \cdot (1-\tau_t) \cdot w_t \cdot (1-n_t) + g_t \cdot w_t + \frac{r \cdot a_t}{q(\theta_t)} \cdot [g_t - (1-s) \cdot g_{t-1}] + \frac{R_{t-1}}{p_t} \cdot b_{t-1}.$$

Using the budget constraint (4) of the household and the definition of the real profits of firms, I rewrite the budget constraint of the government as a resource constraint:

$$y_t = c_t \cdot \left[1 + \frac{\phi}{2} \cdot \pi_t^2\right] + \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1-s) \cdot n_{t-1}]\,. \tag{14}$$

The final good is either consumed, allocated to hiring, or allocated to changing prices.

Symmetric equilibrium. I focus on a symmetric equilibrium in which all intermediate-good firms are identical. Equations (1), (8), and (10) imply that $l_t(i) = l_t$, $y_t(i) = y_t$, and $p_t(i) = p_t$ for all $i \in [0, 1]$. Given initial employment n_{-1}, initial bond holding b_{-1}, and stochastic processes $\{a_t, g_t\}_{t=0}^{+\infty}$ for technology and public employment, a symmetric equilibrium is a collection of nine stochastic processes

$$\{\theta_t, n_t, l_t, w_t, c_t, y_t, R_t, b_t, \pi_t\}_{t=0}^{+\infty}$$

that satisfy the following nine relationships: the wage schedule (7); the production function $y_t = a_t \cdot x(l_t)$; the budget constraint (4) of the household; the resource constraint (14); the New Keynesian IS curve (6); the Taylor rule (12); the New Keynesian Phillips curve

$$\pi_t \cdot (\pi_t + 1) = \frac{1}{\phi} \cdot \frac{y_t}{c_t} \cdot [\epsilon \cdot \Lambda_t - (\epsilon - 1)] + \delta \cdot \mathbb{E}_t \left[\pi_{t+1} \cdot (\pi_{t+1} + 1)\right], \tag{15}$$

where Λ_t is the marginal cost of producing one unit of final good

$$\Lambda_t \equiv \frac{1}{x'(l_t)} \cdot \left\{ \frac{w_t}{a_t} + \frac{r}{q(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \cdot \frac{a_{t+1}}{a_t} \cdot \frac{r}{q(\theta_{t+1})} \right] \right\};$$

the aggregate labor demand $n_t = l_t + g_t$; and the aggregate labor supply (3).

Efficient allocation. The efficient allocation solves the problem of a benevolent planner who faces the technological constraints and labor market frictions present in the economy. Proposition A1 in the Appendix proves that in the efficient allocation, labor market variables (labor market tightness, public and private employment) are independent of technology a_t. When technology falls, productivities in the private and public sectors as well as recruiting costs all fall in concert. Thus the trade-offs between productions in the private and public sector and between production and job search are unaffected by technology fluctuations. As a result , unemployment fluctuations over the business cycle are socially inefficient.[19]

3 Multipliers Across Steady States

This section focuses on a steady-state environment in which there are no technology shocks ($a_t = a$ for all t), no government-consumption shocks ($g_t = g$ for all t), inflation is zero ($\pi_t = 0$ for all t), and all other variables are constant over time. In this environment, I represent the equilibrium with a labor supply-labor demand diagram in a price θ-quantity n plan. The diagram illustrates why the economy is so nonlinear around the rates of unemployment seen in the data. In other words, the economy functions very differently in recessions and in expansions. One implication is that an increase in government consumption expenditures is especially effective to reduce unemployment when the labor market is depressed. To formalize this insight, I define the government-consumption multiplier as the reduction in unemployment achieved by hiring one additional worker in the public sector. I use the labor supply-labor demand representation to prove

[19]This result is not new to the literature. Blanchard and Galí [2010] show that in a New Keynesian model with equilibrium unemployment, the efficient allocation implies a constant unemployment rate over the business cycle.

that the multiplier is positive, and that it is high in steady-states parameterized by a low technology level and characterized by high unemployment. Section 4 complements these comparative statics by quantifying the dynamics of the multiplier in the model calibrated to US data.

I start by representing the equilibrium in a labor demand-labor supply diagram. Labor supply designates the employment rate of a worker given labor market conditions. In steady state, inflows to unemployment $s \cdot n$ balance outflows from unemployment $[1-(1-s)\cdot n]\cdot f(\theta)$ and labor supply satisfies

$$n^s(\theta) = \frac{f(\theta)}{s + (1-s)\cdot f(\theta)}. \tag{16}$$

Labor supply $n^s(\theta)$ increases with θ, because the job-finding probability $f(\theta)$ increases with θ. Furthermore, labor supply is convex in price θ-quantity n plan because $\partial^2 n^s/\partial\theta^2 < 0$. The labor supply curve represents the same locus of points as the Beveridge curve.

Private labor demand designates the employment level desired by intermediate-good firms given labor market conditions. The employment level satisfies the New Keynesian Phillips Curve (15) at zero inflation, namely,

$$x'(l) = \frac{\epsilon}{\epsilon-1}\cdot\left\{\frac{w}{a} + [1-\delta\cdot(1-s)]\cdot\frac{r}{q(\theta)}\right\}. \tag{17}$$

Equation (17) says that intermediate-good firms hire labor until marginal product of labor $a\cdot x'(l)$ equals a markup $\epsilon/(\epsilon-1) > 1$ over the marginal cost of labor, which is the sum of the real wage w plus the amortized hiring cost $[1-\delta\cdot(1-s)]\cdot r\cdot a/q(\theta)$. Private labor demand satisfies

$$l^d(\theta,a) = \left[\frac{1}{\alpha}\cdot\frac{\epsilon}{\epsilon-1}\cdot\left\{\omega\cdot a^{\gamma-1} + [1-\delta\cdot(1-s)]\cdot\frac{r}{q(\theta)}\right\}\right]^{\frac{-1}{1-\alpha}}. \tag{18}$$

Assumption A2 implies that $l^d(\theta,a)$ is a well-defined function of tightness θ. It also implies that $l^d(\theta,a)$ decreases with θ because the vacancy-filling probability $q(\theta)$ decreases with θ. When the labor market is tight, recruiting is expensive, which depresses hiring. Assumption A1 implies that $l^d(\theta,a)$ is a decreasing function of technology a. When technology is high, wages are low relative to the marginal product of labor, which stimulates hiring. Aggregate labor demand is the sum of public and private labor demands:

$$n^d(\theta, a, g) = g + l^d(\theta, a). \tag{19}$$

In presence of matching frictions, the wage itself cannot equalize labor supply and labor demand. Instead, labor market tightness acts as a price equilibrating labor supply and labor demand:[20]

$$n^s(\theta) = n^d(\theta, a, g). \tag{20}$$

Equation (20) implicitly defines equilibrium tightness θ. As technology a decreases, θ decreases because labor demand $n^d(\theta, a, g)$ is lower for all θ. Equilibrium employment n can be directly read off the labor demand curve:

$$n = n^d(\theta, a, g), \tag{21}$$

where equilibrium tightness θ satisfies (20). As technology a decreases, n decreases.

Figures 1(a) depicts the equilibrium in a steady state with high technology in a price θ-quantity n diagram. This equilibrium corresponds to an expansion. Equilibrium employment n and equilibrium tightness θ are given by the intersection of the downward-sloping labor demand curve with the upward-sloping labor supply curve. Figure 1(b) depicts the equilibrium in a steady state with low technology. This equilibrium corresponds to a recession. The labor demand shifts inward. The equilibrium has low aggregate and private employment, low tightness, and high unemployment.

Equipped with the labor supply-labor demand equilibrium representation, I can analyse the effect on unemployment of an increase in government consumption. An increase in government consumption translates into an increase in public employment. I define the government-consumption

[20] As I discuss the adjustments of labor market tightness necessary to reach the labor market equilibrium in the text, the reader must be aware that the equilibrium is actually reached through posting of vacancies. For instance if labor demand is above labor supply at the current tightness, the number of vacancies posted by firms is not sufficient to hire the desired number of workers. Consequently firms post more vacancies, which increases tightness. The job-finding probability rises so more jobseekers are able to find a job: labor supply increases. The vacancy-filling probability falls, which raises hiring costs and marginal cost of labor, so the employment level desired by firms fall: labor demand decreases. By posting more vacancies, firms increase labor market tightness and close the gap between labor supply and labor demand. The mechanism operates until the equilibrium is reached.

multiplier λ as the additional number of workers with a job when one additional worker is employed in the public sector:

$$\lambda \equiv \frac{\partial n}{\partial g}.$$

Before studying the fluctuations of the government-consumption multiplier when technology varies, I must specify the level of public employment associated with each technology level. I aim to characterize the effects of a marginal increase in public employment over the business cycle, leaving public employment policy unchanged. Thus, I assume that the share of public employment in total employment is unchanged over the business cycle.

ASSUMPTION A3. For any technology level a, the government sets $g = \zeta \cdot n$, where n is equilibrium employment and $\zeta \in (0, 1)$.

A justification for the assumption is that state and local governments in the US, which represent more than 85% of public employment, are required to balance their budget each year. The number of workers that state and local governments can employ is therefore directly tied to tax revenues, which are positively related to output and private employment. In fact in the aftermath of the 2008 financial crisis, state and local government drastically reduced their number of employees.

Proposition 1 establishes a few properties of the multiplier λ:

PROPOSITION 1.

(a) $\lambda < 1$. *Under Assumption A2,* $\lambda > 0$.

(b) Under Assumptions A1, A2, and A3, $d\lambda/da < 0$.

Consider the change dn in aggregate employment following a marginal increase $dg > 0$ in public employment. Part (a) shows that $dn < dg$. This result is illustrated in Figures 1(c) and 1(d). The labor demand curve shifts outward by $dn^d = dg > 0$. At the current tightness, labor supply falls short of labor demand. To reach the new equilibrium, tightness increases by $d\theta > 0$. With a higher tightness, firms face a lower vacancy-filling probability, a higher hiring cost, and a higher marginal cost of labor. It is more difficult and less desirable for firms to recruit workers because the government attracts job applicants away. Thus, firms reduce employment by $dl = \left(\partial l^d / \partial \theta\right) \cdot d\theta < 0$. This reduction is a movement inward along the demand curve. As a result, public employment necessarily crowds out private employment: $dn = dl + dg < dg$.

Part (a) also shows that $dn > 0$ under diminishing marginal returns to labor (Assumption A2). In Figures 1(c) and 1(d), diminishing returns translate into a downward-sloping labor demand, which clearly ensures that public jobs crowd out private jobs strictly less than one-for-one: $dn = dg + dl > 0$. More formally if public jobs crowded out private jobs one-for-one, the new equilibrium would have the same labor market tightness θ but lower private employment l. The marginal cost of labor $w + [1 - \delta \cdot (1 - s)] \cdot (r \cdot a)/q(\theta)$ would remain constant, but the marginal product of labor $a \cdot x'(l)$ would be higher by diminishing returns, thus violating condition (17).

Part (b) shows that under diminishing returns and wage rigidity (Assumption A1), dn is larger in a recession than in an expansion. In other words, government consumption is especially effective to reduce unemployment when the labor market is depressed. This property arises because the crowding out of private employment is weak in recessions. With wage rigidity, private labor demand falls when technology falls. Labor demand curve shifts inward from Figure 1(c) to Figure 1(d). Labor market tightness and employment fall. What is the effect of an increase in public employment in a recession? Since the matching process is congested by the high number of unemployed workers, vacancies are filled with high probability. The government only opens a few additional vacancies to hire dg additional workers. The equilibrium increase $d\theta > 0$ is small. In Figure 1(d), labor supply is fairly flat around the equilibrium point. Hence the reduction in private employment $dl = \left(\partial l^d / \partial \theta\right) \cdot d\theta < 0$ imposed by the increase in public employment is small. As a result, public employment does not crowd out private employment much in a recession. To the contrary in an expansion, the equilibrium increase $d\theta > 0$ is large. In Figure 1(c), labor supply is fairly steep around the equilibrium point. As a result, public employment crowds out private employment very much in an expansion.

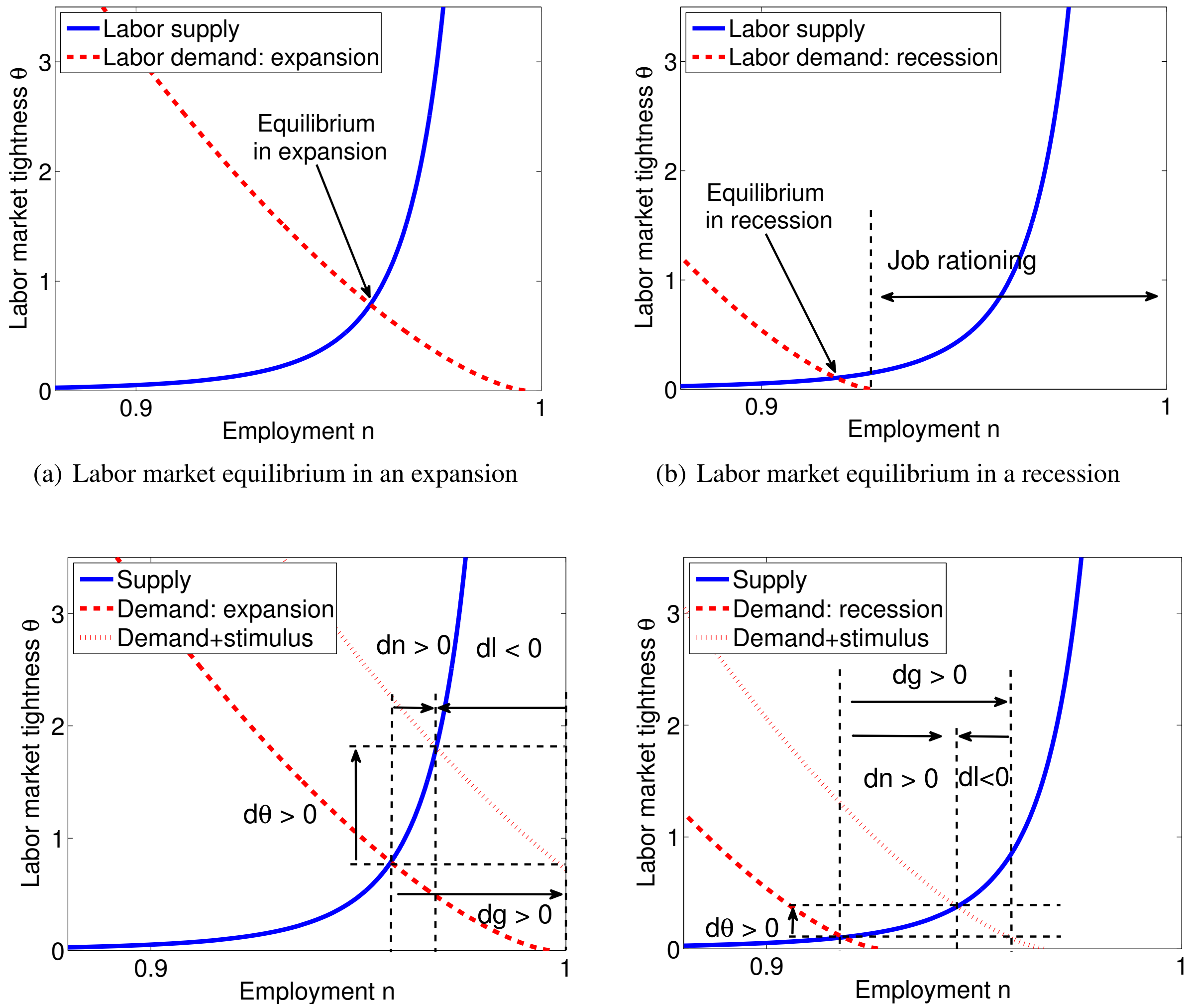
The proof of the proposition is relegated to Appendix A, but I provide a sketch here. Let $\epsilon^s \equiv (\theta / n^s) \cdot (\partial n^s / \partial \theta) > 0$ and $\epsilon^d \equiv -\left(\theta / n^d\right) \cdot \left(\partial n^d / \partial \theta\right) > 0$ be the elasticities of labor supply and labor demand with respect to tightness. Elasticity ϵ^d is normalized to be positive. I express the multiplier λ as a function of ϵ^s and ϵ^d by differentiating (20) and (21):

$$\lambda = \frac{\partial n^d}{\partial g} \cdot \left[1 - \frac{1}{1 + (\epsilon^s/\epsilon^d)}\right]. \tag{22}$$

The increase dn in aggregate employment following an increase dg in public employment equals the mechanical shift dn^d of labor demand attenuated by a factor $1/\left[1 + \left(\epsilon^s/\epsilon^d\right)\right] < 1$. This factor captures the crowding out of private employment. From (19), the shift of labor demand is simply $\partial n^d/\partial g = 1$. The magnitude of crowding out of depends on the elasticities ϵ^s and ϵ^d. The proof shows that $\lambda \in (0, 1)$ because $\epsilon^s \in (0, +\infty)$ and $\epsilon^d \in (0, +\infty)$. The proof also shows that (a) ϵ^s is countercyclical because it moves in proportion with unemployment u; and (b) ϵ^d is procyclical because it moves in proportion with the share of the hiring cost in the marginal cost of labor (in recessions, the share is small because hiring workers is easy). So ϵ^s/ϵ^d and λ are countercylical.

In the Appendix, I show that the results of Proposition 1 are robust to two natural extensions. In Appendix C, I allow for a behavioral labor supply response: workers adjust their job-search effort to labor market conditions. In Appendix D, the output of the public sector contributes to a stock of public capital that enters the production function of firms. Proposition A2 and Proposition A3 show that the government-consumption multiplier remains positive and countercyclical in the two extensions. Furthermore, the proofs of the propositions are almost identical to that of Proposition 1. With a behavioral labor supply response, the elasticity ϵ^s also accounts for the dependence of search effort to tightness, so ϵ^s is no longer proportional to unemployment but it remains procyclical. With public capital, the direct effect of public employment on labor demand $\partial n^d/\partial g > 1$ because increasing public employment increases steady-state public capital, firm's marginal productivity, and firm's labor demand. But $\partial n^d/\partial g$ remains a constant.

Proposition 1 establishes that the government-consumption multiplier is positive and countercyclical with diminishing returns and wage rigidity. Models of equilibrium unemployment, however, typically assume fully flexible real wages ($\gamma = 1$) and constant marginal returns to labor ($\alpha = 1$). For example, if wages are determined by generalized Nash bargaining and the unemploy-



(a) Labor market equilibrium in an expansion

(b) Labor market equilibrium in a recession

(c) Increase in public employment in an expansion

(d) Increase in public employment in a recession

Figure 1: Labor market equilibria in a price θ-quantity n diagram

ment insurance replacement rate ρ is constant, then the wage w satisfies (7) with $\gamma = 1$.[21] Table 1

[21] Assume that $\alpha = 1$, that $c_t^u / c_t^e = \rho$ for all t, and that wages $\{w_t\}_{t=0}^{+\infty}$ are determined by generalized Nash bargaining for all t. Let $\chi \in (0,1)$ be worker's bargaining power. In steady state, w satisfies (7) with $\gamma = 1$ and

$$\omega = \frac{\chi}{1-\chi} \cdot \frac{r}{1-\rho} \cdot \left\{ [1 - \delta \cdot (1-s)] \cdot \frac{1}{q(\theta)} + \delta \cdot (1-s) \cdot \theta \right\}.$$

where θ is an implicit function of the parameters defined by

$$\frac{\epsilon - 1}{\epsilon} = \left[1 + \frac{\chi}{1-\chi} \cdot \frac{1}{1-\rho} \right] \cdot [1 - \delta \cdot (1-s)] \cdot \frac{r}{q(\theta)} + \frac{\chi}{1-\chi} \cdot \frac{1}{1-\rho} \cdot \delta \cdot (1-s) \cdot r \cdot \theta.$$

The wage moves in proportion to technology because the recruiting cost (which determines the outside option of the firm) is proportional to technology and unemployment benefits (which determine the outside option of workers) are a constant fraction ρ of the (post-tax) wage.

Table 1: Relationship between labor market structure and properties of the multiplier

Labor market structure		Multiplier		Labor market properties	
Production function $y = a \cdot n^{\alpha}$	Wage $w = \omega \cdot a^{\gamma}$	Sign	Cyclicality	Unemployment fluctuations	Job rationing
$\alpha = 1$	$\gamma = 1$	$\lambda = 0$	$d\lambda/da = 0$	no	no
$\alpha = 1$	$\gamma < 1$	$\lambda = 0$	$d\lambda/da = 0$	yes	no
$\alpha < 1$	$\gamma = 1$	$\lambda > 0$	$d\lambda/da = 0$	no	maybe
$\alpha < 1$	$\gamma < 1$	$\lambda > 0$	$d\lambda/da < 0$	yes	yes

Notes: The assumption that $\gamma < 1$ is Assumption A1. The assumption that $\alpha < 1$ is Assumption A2. The table records "yes" in the "Unemployment fluctuations" column if $du/da < 0$, where $u = 1 - (1 - s) \cdot n$ is steady-state unemployment; it records "no" if $du/da = 0$. The table records "yes" in the "Job rationing" column if $\lim_{r \to 0} n < 1$ for a low enough, where r is the recruiting cost; it records "no" if $\lim_{r \to 0} n = 1$ for all $a > 0$; it records "maybe" if the answer could be "yes" or "no" depending on other parameters than α and γ. The results in the "Sign" and "Cyclicality" columns are proved in Appendix A. The results in the "Unemployment fluctuations" and "Job rationing" columns are the subject of the work of Michaillat [2012].

shows how the results of Proposition 1 are modified under these common assumptions. The table also relates the results on the government-consumption multiplier to the work of Michaillat [2012].

Table 1 underlines four results. First, real wage rigidity has nothing to do with the result that the multiplier is positive; therefore, the fact that unemployment is high in recessions is not sufficient to explain why government consumption is more effective in recessions. Real wage rigidity is not sufficient to obtain a positive multiplier. Consider a model with real wage rigidity ($\gamma < 1$) and constant returns to labor ($\alpha = 1$). The multiplier is exactly zero in this model. And real wage rigidity is not necessary to obtain a positive multiplier. Consider a model with fully flexible real wages ($\gamma = 1$) and diminishing returns to labor ($\alpha < 1$). The multiplier is positive in this model.

Second, real wage rigidity ($\gamma < 1$) is necessary to obtain unemployment fluctuations. When wages are fully flexible ($\gamma = 1$), technology fluctuations do not lead to any labor market fluctuations. When technology falls, marginal product of labor, wage, and recruiting cost fall in proportion; thus private labor demand is invariant to technology. In fact technology a drops out of equation (18). In Figure 1, labor demand would not respond to technology.

Third, diminishing returns ($\alpha < 1$) are necessary to obtain a positive multiplier. With constant returns to labor ($\alpha = 1$), public employment crowds out private employment one-for-one so that government consumption has no effect on aggregate employment. Private and aggregate labor demands are perfectly elastic so $\epsilon^d = +\infty$, crowding out is $1/\left[1 + \left(\epsilon^s/\epsilon^d\right)\right] = 1$ and, using expression (22), $\lambda = 0$. In Figure 1, labor demand would be horizontal so it would not shift with an increase in public employment.

Fourth, there is a close connection between the results in Michaillat [2012] and the results in this paper. Proposition 4 in Michaillat [2012] shows that under Assumptions A1 and A2, jobs are rationed in recessions: some unemployment would remain even if matching frictions vanished. In recessions, the marginal product of the last workers in the labor force, who are least productive due to diminishing returns, falls below the wage. It becomes unprofitable for firms to hire these workers even if recruiting is costless at $\theta = 0$.[22] The property is illustrated in Figure 1(b), where labor demand intersects the x-axis on $(0, 1)$. The property is based on the behavior of the model at the limit when recruiting cost $r \to 0$. Here, Proposition 1 shows that the multiplier is countercyclical under Assumptions A1 and A2. This property, however, is based not on the limit behavior of labor demand when recruiting cost $r \to 0$ but on the relative slopes of labor demand and labor supply.

4 Multiplier Dynamics

A restriction of Section 3 is that it analyses the effect of government consumption in steady state. In this section, I simulate the dynamic effect of an increase in government consumption in response to a range of technology shocks. The simulations quantify the countercyclical fluctuations of the government-consumption multiplier.

4.1 Calibration

I calibrate all parameters at a weekly frequency as shown in Table 2. I calibrate as many parameters as possible directly from US micro- and macro-data. I start with the labor market parameters:

[22] The private efficiency of existing worker-firm matches, however, is always respected. This is because the wage is always below the marginal product of employed workers; otherwise the firm would not hire them. Of course the wage is above the marginal product of the least productive workers, but they are never matched with firms.

unemployment-elasticity η of the matching function, recruiting cost r, job destruction rate s, share ζ of public employment in total employment, and flexibility γ of the real wage. I set $\eta = 0.7$, in line with empirical evidence [Petrongolo and Pissarides, 2001]. I follow the strategy used in Michaillat [2012] to estimate r. Using micro-evidence gathered by Barron, Berger and Black [1997] and Silva and Toledo [2009], I set $r = 0.32 \cdot \omega$, where ω is the steady-state real wage.

I estimate s as the average of the seasonally-adjusted monthly series for the total separation rate in all nonfarm industries constructed by the Bureau of Labor Statistics (BLS) from the Job Openings and Labor Turnover Survey (JOLTS) for the January 2001–December 2011 period. The average separation rate is 0.036, so $s = 0.0090$ at weekly frequency. All nonfarm industries are composed of the nonfarm private sector and of the government sector (federal, state, and local government); therefore this separation rate is an average over jobs in the private and public sectors.

I estimate ζ as the average share of public employment in total employment in the seasonally-adjusted monthly data from the Current Employment Survey (CES) collected by the BLS. Public employment is the employment level in the government super sector, which includes federal, state, and local government. Total employment is the employment level in the total nonfarm super sector. I find $\zeta = 0.17$.

I calibrate γ based on estimates obtained in micro-data, less prone to composition effects than macro-data. It is mostly the flexibility of wages in newly created jobs, not in existing jobs, that drives job creation. The best estimate of this flexibility using US data is provided by Haefke, Sonntag and Van Rens [2008]. Using panel data following production and supervisory workers over the 1984–2006 period, they estimate an elasticity of total earnings of job movers with respect to productivity of 0.7. If the composition of jobs accepted by workers improves in expansions, 0.7 is an upper bound on the elasticity of wages in newly created jobs.[23] A lower bound on this elasticity is the elasticity of wages in existing jobs, estimated in the 0.1–0.45 range with US data

[23] If the composition of workers on each type of job remains the same over the business cycle, the flexibility of wages in newly created jobs is given by that of wages for newly hired workers. But the flexibility of wages for newly created jobs is different from that for newly hired workers if there is a change in the composition of jobs accepted by workers over the business cycle [Gertler and Trigari, 2009]. Firms could maintain rigid wages in each type of job, and workers taking new jobs could show procyclical wages if during expansions they face better opportunities to move to higher-paying industries, higher-paying firms within industry, or higher-paying jobs within firm.

[Pissarides, 2009]. Therefore, I set $\gamma = 0.5$, in the range of plausible values. Since the upper bound on wage flexibility is $0.7 < 1$, the assumption of real wage rigidity (Assumption A1) finds support in micro-data.

Next, I calibrate the monetary parameters: parameters μ_π and μ_R of the Taylor rule (12), and price-adjustment cost ϕ. μ_π is the elasticity of the gross nominal interest rate R to gross inflation $1 + \pi$ in steady state. μ_R is the elasticity of the current gross nominal interest rate R_t to the lagged gross nominal interest rate R_{t_1}. I set $\mu_\pi = 1.5$ and $\mu_R = 0.94$, which corresponds to 0.7 at quarterly frequency. These values are common in the literature.

Using time-and-motion methods, Zbaracki et al. [2004] find that in a one-billion-dollar industrial firm in a given year, the price-adjustment costs amounts to 1.22% of the revenue of the firm. The firm only changed the price of 25% of its 8,000 products that year, and the price changes were of the order of 4%. Therefore, $\phi/2 \cdot (0.04)^2 \cdot 0.25 = 0.0122$, which implies $\phi = 61$.[24]

I assume that log technology follows an AR(1) process: $\log(a_{t+1}) = \mu_a \cdot \log(a_t) + \nu_{t+1}$, where the error term ν_{t+1} is a centered normal random variable. To estimate autocorrelation μ_a, I construct log technology as a residual $\log(a_t) = \log(y_t) - \alpha \cdot \log(l_t)$. Output y_t and employment l_t are the quarterly, seasonally-adjusted, real output and employment in the nonfarm business sector constructed by the BLS Major Sector Productivity and Costs (MSPC) program. To isolate fluctuations at business cycle frequency, I take the difference between log technology and a low-frequency trend—a Hodrick-Prescott (HP) filter with smoothing parameter 10^5. I estimate an autocorrelation of 0.897, which yields $\mu_a = 0.992$ at weekly frequency.

So far, I have estimated parameters directly from the data, independently of the model. I calibrate a few additional parameters using values commonly found in the literature. I set the production function parameter to $\alpha = 0.66$; the elasticity of substitution across intermediate goods to $\epsilon = 11$, which corresponds to a monopolistic markup of 10%; and the discount factor to $\delta = 0.999$, which corresponds to an annual interest rate of 5%. And by matching the steady-state value of the variables in the model to the average of their empirical counterpart, I can calibrate the remain-

[24] This cost is similar to the cost estimated by maximum likelihood with a New Keynesian model by Ireland [2001]. In US data, he finds $\phi = 72$ over the 1959–1979 period and $\phi = 77$ over the 1979–1998 period.

Table 2: Steady-state targets and parameter values used in simulations (weekly frequency)

Steady-state target		Value	Source
$\overline{a}$	Technology	1	Normalization
$\overline{u}$	Unemployment	6.37%	JOLTS, 2001–2011
$\overline{l}$	Private employment	0.789	JOLTS, 2001–2011
$\overline{\theta}$	Labor market tightness	0.426	JOLTS, 2001–2011
Parameter		**Value**	**Source**
η	Elasticity of matching to unemployment	0.7	Petrongolo and Pissarides [2001]
r	Recruiting cost	0.206	Barron et al. [1997], Silva and Toledo [2009]
s	Separation rate	0.90%	JOLTS, 2001–2011
ζ	Share of public employment	0.165	CES, 2001–2011
γ	Real wage flexibility	0.5	Pissarides [2009], Haefke et al. [2008]
μ_π	Elasticity of Taylor rule to inflation	1.5	Convention
μ_R	Elasticity of Taylor rule to lag interest rate	0.973	Corresponds to 0.7 quarterly
ϕ	Price-adjustment cost	61	Zbaracki et al. [2004]
μ_a	Autocorrelation of technology	0.992	MSPC, 1947–2011
α	Marginal returns to labor	0.66	Convention
δ	Discount factor	0.999	Corresponds to an interest rate of 5% annually
ϵ	Elasticity of substitution	11	Corresponds to a markup of 10%
ω_h	Efficacy of matching	0.172	Matches steady-state targets
ω	Real wage level	0.644	Matches steady-state targets

ing parameters: matching efficacy ω_h, steady-state real wage. I normalize average technology to $\overline{a} = 1$. I compute average labor market tightness using seasonally-adjusted, monthly series for vacancy level (collected by the BLS in the JOLTS) and unemployment level (computed by the BLS from the CPS) over the January 2001– December 2011 period. I find $\overline{\theta} = \overline{v}/\overline{u} = 0.426$. I compute average unemployment using seasonally-adjusted monthly unemployment rate constructed by the BLS from the CPS over the January 2001– December 2011 period. I find $\overline{u} = 6.37\%$, which implies $\overline{l} = 0.789$. To calibrate ω_h, I exploit the equality of inflows to unemployment with outflows from unemployment in steady state: $u \cdot f(\theta) = s \cdot n = s \cdot (1-u)/(1-s)$. I find $\omega_h = s/(1-s) \cdot (1-\overline{u})/\overline{u} \cdot \overline{\theta}^{\eta-1} = 0.172$. I calibrate ω by exploiting (17), which implies $\omega = (\epsilon - 1)/\epsilon \cdot x'(\overline{l}) / \left\{1 + [1 - \delta \cdot (1-s)] \cdot 0.32/q(\overline{\theta})\right\} = 0.644$. Then, I recover $r = 0.32 \cdot \omega = 0.243$.

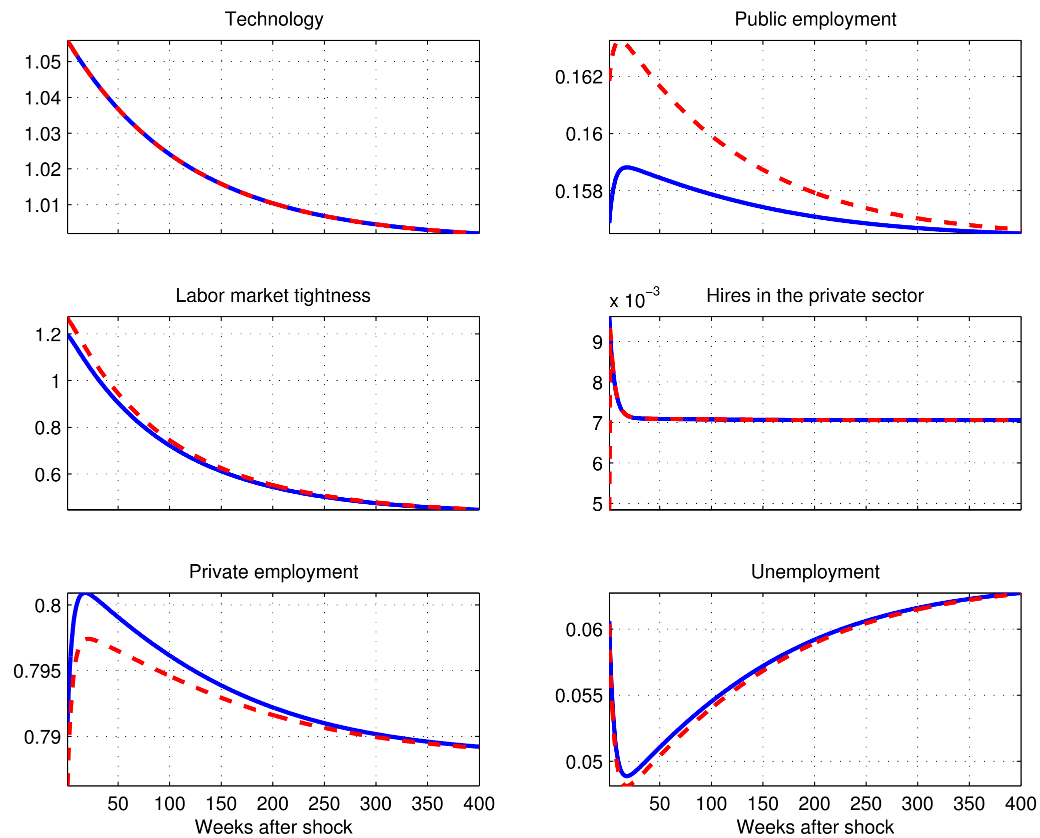


Figure 2: Impulse response functions to a positive technology shock

Notes: The solid blue lines are the dynamic responses to a positive technology shock of +5.4%. The dashed red lines are the dynamic responses to a positive technology shock of +5.4% accompanied by the hire of 0.5% of the labor force in the public sector. I solve the exact model using a shooting algorithm under perfect foresight. The model is calibrated in Table 2.

4.2 Simulations

In the simulations the economy departs substantially from its steady state, so I cannot follow the standard procedure of simulating the log-linear approximation of the model. Instead I solve the exact model using a shooting algorithm under perfect foresight. I begin by simulating an expansion. At time 0, the economy is in steady state. At time 1, an unexpected positive technology shock $\nu_1 = +5.4\%$ occurs. Then, no further shock occur and technology converges back to its steady-state value. Let $\hat{x}_t$ denote the value of x at time t in this simulation. Public employment is set by the rule of Assumption A3 that $\hat{g}_t = \zeta/(1-\zeta)\cdot\hat{l}_t$ for all t.The solid blue lines in Figure 2 are the impulse response functions (IRFs). At time 1, labor market tightness increases because firms

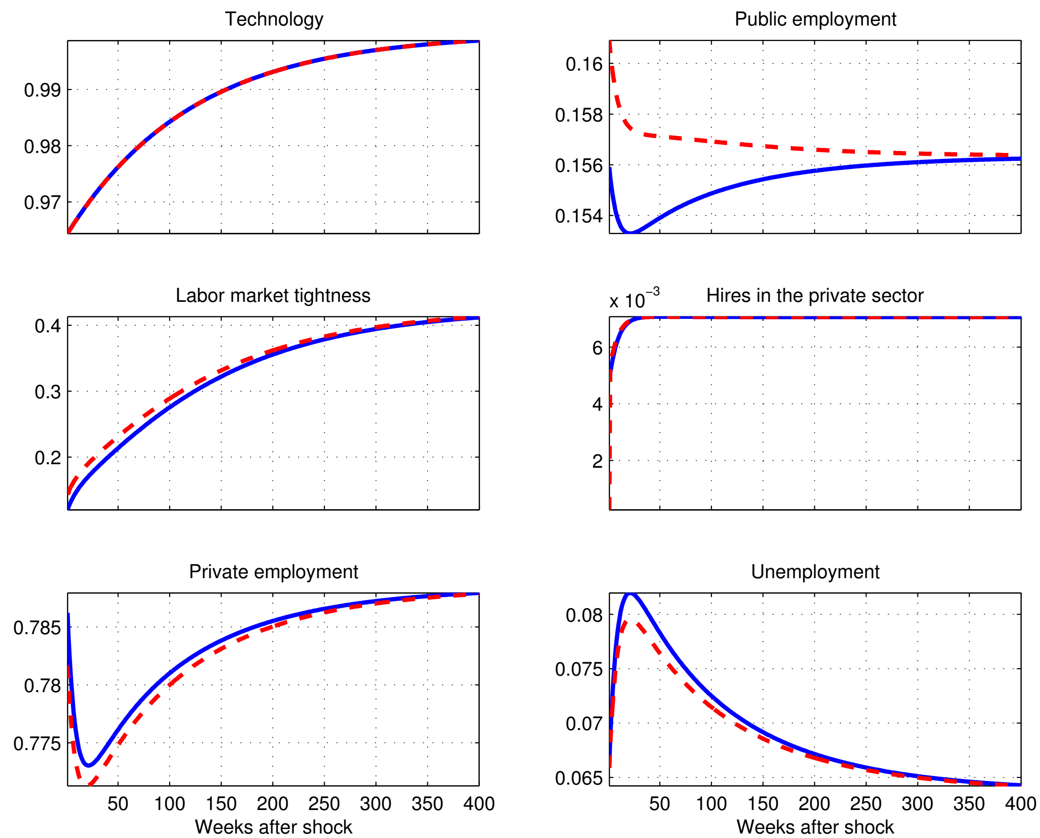


Figure 3: Impulse response functions to a negative technology shock

Notes: The solid blue lines are the dynamic responses to a negative technology shock of -3.6%. The dashed red lines are the dynamic responses to a negative technology shock of -3.6% accompanied by the hire of 0.5% of the labor force in the public sector. I solve the exact model using a shooting algorithm under perfect foresight. The model is calibrated in Table 2.

post more vacancies to hire more workers in response to the favorable shock. The number of new hires in the private sector increases sharply, which leads private employment to build up and peak after 25 weeks. Mechanically, public employment also increases. Since aggregate employment increases, unemployment drops, and bottoms at 4.8% after 25 weeks.

To illustrate the effect of an increase in government consumption during an expansion, I simulate the economy when the positive technology shock $\nu_1 = +5.4\%$ is accompanied by an increase in public employment by 0.5% of the labor force at time 1. Let x_t^* denote the value of x at a time t in this simulation. At time 1, $g_1^* = \hat{g}_1 + 0.005$. After that, the government hires just as many workers as in the previous experiment: $g_t^* - (1-s) \cdot g_{t-1}^* = \hat{g}_t - (1-s) \cdot \hat{g}_{t-1}$ for all $t \geq 2$.

The dashed red lines in Figure 2 are the IRFs. At time 1, the government post additional vacancies to hire additional workers in the public sector. As a result, public employment and labor market tightness rise above their previous level. Because labor market tightness is higher than in the absence of government intervention, it becomes more difficult for intermediate-good firms to recruit workers: firms in the private sector are subject to the competition from the government because they recruit from the same pool of unemployed workers. With a higher labor market tightness, it takes longer and it is more costly to fill a vacancy. Thus, the marginal cost of labor rises and firms reduce hiring accordingly. As a consequence, private employment falls below its previous level: public employment crowds out private employment. The net effect of an increase in public employment, however, is positive because unemployment falls below its previous level.

To illustrate the effect of an increase in government consumption during a recession, I repeat the two previous simulations but replace the positive technology shock by a negative technology shock $\nu_1 = -3.6\%$. Figure 3 displays the IRFs in these simulations. In response to the negative technology shock, labor market tightness, private employment, and public employment fall while unemployment increases. Note that the number of hires in the private sector falls sharply at time 1, when the negative technology shocks occurs. The number of new hires, however, remains positive, implying that the rigid wages remain privately efficient even in response to such a large negative shock. Qualitatively, an increase in government consumption has the same effects in an expansion and a recession; but quantitatively, the effects are very different. In an expansion, labor market tightness increases by 0.07 from 1.19 to 1.26 in response to hiring in the public sector; in a recession, it only increases by 0.02 from 0.12 to 0.14. As a consequence, the increase in hiring costs caused by hiring in the public sector is much larger in expansions than in recessions: in an expansion, the expected time to fill a vacancy increases by 0.26 week from 6.58 weeks to 6.84 weeks; in a recession, it only increases by 0.16 week from 1.35 week to 1.51 week. Hence, crowding out is larger in expansions than in recessions: in an expansion, private employment falls by 0.35% from 80.10% to 79.75% at its extremum; in a recession, it only falls by 0.18% from 77.30% to 77.12%. Thus, increasing government consumption reduces unemployment more effectively in recessions than in expansions: in an expansion, unemployment only falls by 0.07%

from 4.89% to 4.82% at its extremum; in a recession, it falls by 0.25% from 8.20% to 7.95%.

To measure the effects of an increase in government consumption period by period, I define the instantaneous multiplier

$$\lambda_t \equiv \frac{n_t^* - \hat{n}_t}{g_t^* - \hat{g}_t}. \tag{23}$$

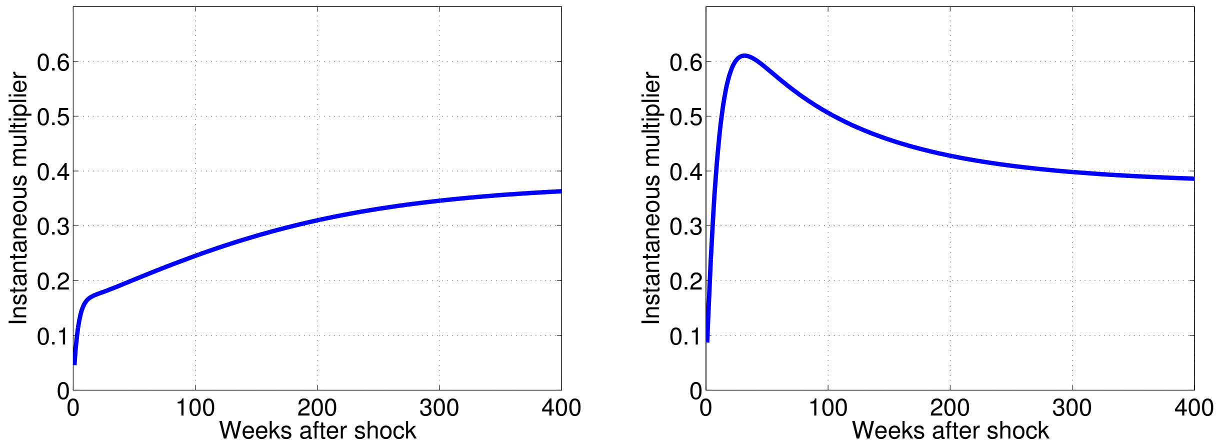
The instantaneous multiplier gives the aggregate number of jobs created for each additional public job in period t. Figure 4(a) shows the dynamics of λ_t after the positive technology shock $\nu_1 = +5.4\%$, and Figure 4(b) shows the dynamics after the negative technology shock $\nu_1 = -3.6\%$.[25] At time 1, the instantaneous multiplier is small (around 0.1) after both positive and negative shock. After the positive shock, the multiplier grows slowly over time to reach a steady-state value of 0.4. The increase in the multiplier concurs with the increase in unemployment as technology decreases to its steady-state level. After the negative shock, the multiplier grows quickly and peaks above 0.6 after 20 weeks. The peak of the multiplier concurs with peak of unemployment, which occurs broadly 20 weeks after the negative technology shock (see Figure 3). After the peak, the multiplier reverts to a steady-state value of 0.4 as unemployment reverts to its steady-state value.

To summarize the effect of an increase in government consumption for each technology shock, I define the cumulative multiplier

$$\lambda^C \equiv \frac{\sum_{t=0}^{T} [n_t^* - \hat{n}_t]}{\sum_{t=0}^{T} [g_t^* - \hat{g}_t]}, \tag{24}$$

where $T = 15,000$ is the horizon in the shooting algorithm, long enough for the economy to converge back to steady state. The cumulative multiplier measures the aggregate number of job$\times$weeks created by hiring a few workers in the public sector, divided by the number of job$\times$weeks added to the public sector. The cumulative multiplier takes into account the persistence of government consumption, which arises because I assume that public workers cannot be dismissed such that public jobs disappear only as a result of natural attrition. I repeat the simulation that allowed

[25] To isolate the marginal effect of an increase in government consumption, the series $\{n_t^*\}$ and $\{g_t^*\}$ are obtained when public employment only increases by 0.01% of the labor force at time 1. In the simulations displayed in Figures 2 and 3, I made the effect of an increase in government consumption more visible by increasing public employment by 0.5% at time 1.



(a) Instantaneous multiplier in an expansion

(b) Instantaneous multiplier in a recession

Figure 4: Multiplier dynamics in an expansion and a recession

Notes: The instantaneous multiplier is defined by equation (23). The multiplier applies to the hire of 0.01% of the labor force in the public sector when the technology shock occurs. The expansion follows a positive technology shock of +5.4%. The recession follows a negative technology shock of -3.6%. I solve the exact model using a shooting algorithm under perfect foresight. The model is calibrated in Table 2.

me to compute Figure 4 for a collection of 16 technology shocks, ranging from $\nu_1 = -3.6\%$ to $\nu_1 = +5.4\%$. I compute the IRFs to each technology shock, without and with an increase in government consumption. Then I associate to each technology shock the extremum of the IRF of the unemployment rate without government intervention. I also compute the cumulative multiplier associated with each technology shock. I link each cumulative multiplier to the associated unemployment rate and plots the 16 multiplier-unemployment rate pairs in Figure 5. The figure shows that the cumulative multiplier doubles from 0.24 to 0.49 when the unemployment rate increases from 4.9% to 8.2%. This means that on the one hand, an increase in government consumption is effective when it occurs in response to a negative shock; on the other hand, it is ineffective when it occurs in response to a positive shock. Figure A1 in the Appendix shows that these numerical results are robust to the presence of a behavioral labor supply response and public capital.

While the simulations are too simple to quantify the effects of actual policy interventions, they shed light on some practical issues. During the Great Depression, the Roosevelt administration was concerned that public jobs created as part the New Deal might make it more difficult for private firms to hire workers by taking away job applicants [Neumann et al., 2010]. Figure 5 addresses

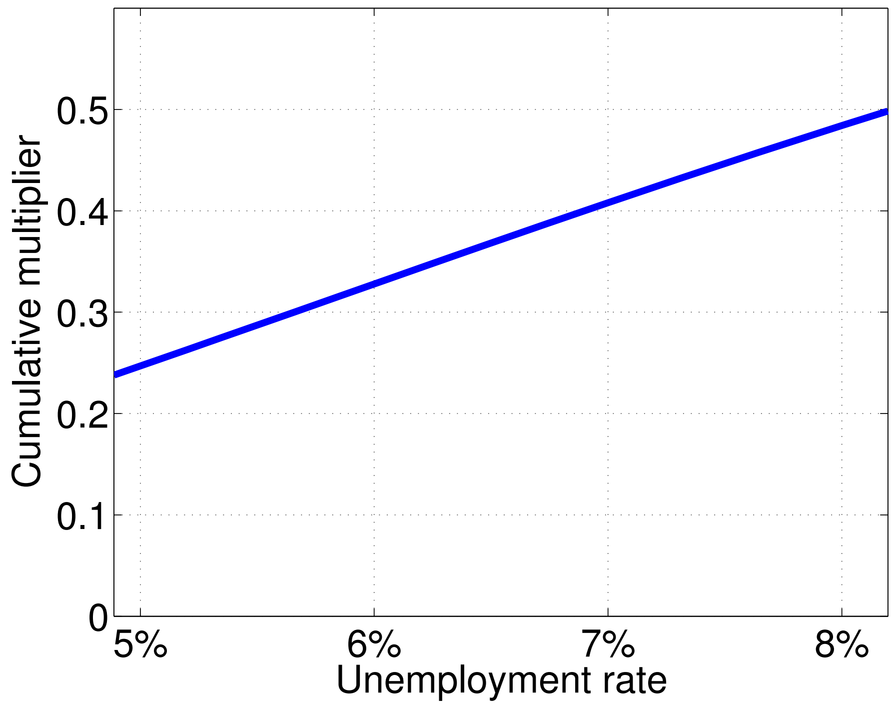


Figure 5: Cumulative multiplier over the business cycle

Notes: The cumulative multiplier is defined by equation (24). The multiplier applies to the hire of 0.01% of the labor force in the public sector when the technology shock occurs. The unemployment rate reported on the x-axis represents the extremum of the IRF of unemployment to the underlying technology shock, without government intervention. To obtain the graph, I impose a collection of 16 technology shocks to the model, ranging from -3.6% to +5.4%. I solve the exact model using a shooting algorithm under perfect foresight. The model is calibrated in Table 2.

this practical concern by showing that in recessions the crowding out of private jobs by public jobs is much weaker than in expansions. Parker [2011] argues that available estimates of government-consumption multipliers—obtained by averaging the effects of government consumption over the business cycle—should not be used by policymakers designing a stimulus in a recession, because the estimates are not valid in recessions. Figure 5 supports this argument. The multiplier obtained at an unemployment rate of 6.4%, the average unemployment rate in the US for the 2001–2011 period, is 0.35. But policymakers are more likely to design a stimulus in bad times, say when the unemployment rate reaches 8.2%. At that point the multiplier is 0.50, much higher than 0.35.

5 Empirical Evidence

This paper proposes a dynamic stochastic general equilibrium model in which the government-consumption multiplier doubles when unemployment rises from 4.9% to 8.2%, as reported in Figure 5. This section concludes by discussing two notable studies that find empirical support for the result that government-consumption multipliers are larger in recessions than in expansions.[26]

Auerbach and Gorodnichenko [2011] find that the government-spending multiplier is countercyclical using data for a large number of OECD countries. They estimate the government-spending multiplier using a direct projection method that allows the multiplier to vary smoothly with the state of the economy. They define a recession as a period when the detrended unemployment rate is especially high, and an expansion as a period when the detrended unemployment rate is especially low. They compare the effect of an increase in government spending (which includes government consumption and government investment) on output in an expansion and in a recession. The results are reported on Line 5 in the Panel B of Table 3. The comparison of Column 1 to Column 2 shows that the multiplier is 0.50 (robust standard deviation: 0.22) when unemployment is high and -0.11 (robust standard deviation: 0.15) when unemployment is low. Therefore, the cumulative multiplier is quite large in recessions but it is not significantly different from zero in expansions.

Nakamura and Steinsson [2011] find that the government-spending multiplier is countercyclical using state-level data for the US. To estimate the government-spending multiplier, they focus on increases in government spending in US states during military build-ups. These increases are arguably unrelated to prevailing macroeconomic conditions. They are able to isolate the effect of government spending from a possible monetary policy response because US states are part of a monetary union. They sort periods into high-unemployment and low-unemployment peri-

[26] Evidence about the behavior of multipliers over the business cycle is scarce because studies estimating the effects of government consumption usually ignore the state of the economy when government consumption takes place [Parker, 2011]. While more empirical work is required to reach a consensus, a nascent literature estimate government-consumption multipliers over the business cycle and generally find that government-consumption multipliers are countercyclical [for example, Auerbach and Gorodnichenko, forthcoming; Bachmann and Sims, 2011]. An exception is Canova and Pappa [forthcoming], who find that multipliers are unlikely to be larger in recessions than in normal times. Their results may differ because they define recessions as periods when government deficit is large and the nominal interest rate cannot respond to shocks, whereas the other studies define recessions as period with either high unemployment or low output growth.

ods depending on whether the state unemployment rate at the start of the interval over which the government spending occurs is above or below the median unemployment rate. They measure the increase in state employment following an exogenous increase in government spending. They find that in high-unemployment periods the multiplier is roughly twice larger than in low-unemployment periods. As showed in Column 3 on Table IV, the multiplier at the state level is 1.10 in low-unemployment periods and 1.85 in high-unemployment.[27] Because the authors focus on state employment, one may worry that the findings result from migration of workers across states in response to government spending shocks. But migrations do not seem important: the authors find that government-spending shocks have small and statistically insignificant effects on state population at business cycle frequency.

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[27] The point estimates support the view that the government-consumption multiplier is countercylical. The multipliers, however, are not estimated precisely because the number of business cycles in their sample is limited.

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Appendix—For Online Publication

A Proof of Proposition 1

Part 1. In a steady state parameterized by technology a and public employment g, the key equilibrium condition is (20). Differentiating the condition with respect to g yields

$$\frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial g} = \frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial g} + \frac{\partial n^d}{\partial g}.$$

This equation allows me to express $\partial \theta / \partial g$ as a function of $\partial n^d / \partial \theta$, $\partial n^s / \partial \theta$, and $\partial n^d / \partial g$. In fact,

$$\frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial g} = -\frac{\partial n^d}{\partial g} \cdot \frac{1}{1 + (\epsilon^s / \epsilon^d)}. \tag{A1}$$

where I define the tightness-elasticities ϵ^s and ϵ^d of labor supply and labor demand:

$$\epsilon^s \equiv \frac{\theta}{n^s} \cdot \frac{\partial n^s}{\partial \theta} > 0, \tag{A2}$$

$$\epsilon^d \equiv -\frac{\theta}{n^d} \cdot \frac{\partial n^d}{\partial \theta} > 0. \tag{A3}$$

ϵ^d is normalized to be positive. The effect of public employment g on aggregate employment n appears by differentiating (21), and using the result from (A1):

$$\lambda \equiv \frac{\partial n}{\partial g} = \frac{\partial n^d}{\partial g} \cdot \left[1 - \frac{1}{1 + (\epsilon^s / \epsilon^d)} \right]. \tag{A4}$$

Since $\partial n^d / \partial g = 1$, $\epsilon^s > 0$, and $\epsilon^d > 0$, then $\lambda < 1$. Under Assumption A2, $\epsilon^d < +\infty$ so $\lambda > 0$. Note that if Assumption A2 does not hold and $\alpha = 1$, then $\epsilon^d = +\infty$ (as apparent in equation (A7)) and $\lambda = 0$.

Part 2. Lemma A1 establishes the cyclicality of the main variables in the model. The proof relies on this lemma to determine the cyclicality of the elasticities ϵ^s and ϵ^d.

LEMMA A1. *Under Assumptions A1, A2, and A3,* $d\theta/da > 0,\ dn/da > 0,\ dl/da > 0$, *and* $du/da < 0$.

Proof. Labor supply $n^s(\theta)$ satisfies (16). Therefore $\partial n^s / \partial \theta > 0$. Private labor demand $l^d(\theta, a)$ satisfies (18). Under Assumptions A2 and A1, $\partial l^d / \partial \theta < 0,\ \partial l^d / \partial a > 0$. Under Assumption A3, equilibrium condition (20) becomes $l^d(\theta, a) = (1 - \zeta) \cdot n^s(\theta)$. Differentiating this equilibrium condition with respect to a yields:

$$(1-\zeta)\cdot\frac{\partial n^s}{\partial\theta}\cdot\frac{d\theta}{da}=\frac{\partial l^d}{\partial a}+\frac{\partial l^d}{\partial\theta}\cdot\frac{d\theta}{da}$$

$$\frac{d\theta}{da}=\underbrace{\frac{\partial l^d}{\partial a}}_{+}\cdot(1-\zeta)\cdot\left[\underbrace{\frac{\partial n^s}{\partial\theta}}_{+}-\underbrace{\frac{\partial l^d}{\partial\theta}}_{-}\right]^{-1}.$$

Thus $d\theta/da>0$. I conclude by using $\partial n^s/\partial\theta>0$ and noting that in equilibrium $n=n^s(\theta)$, $u=1-(1-s)\cdot n$, and $l=(1-\zeta)\cdot n$. Note that if Assumption A1 does not hold and $\gamma=1$, then $\partial l^d/\partial a=0$ (as apparent in equation (18)) and $d\theta/da=0$. As in equilibrium $n=n^s(\theta)$, $u=1-(1-s)\cdot n$, and $l=(1-\zeta)\cdot n$, the assumption that $\gamma=1$ would also imply $dn/da=0,\ dl/da=0,\ du/da=0$.

To determine how the multiplier λ fluctuates over the business cycle, I study the tightness-elasticities ϵ^s and ϵ^d. Labor supply (16) implies an equilibrium relationship between elasticity ϵ^s and unemployment u:

$$\epsilon^s=(1-\eta)\cdot u. \tag{A5}$$

The elasticity ϵ^s is countercyclical because unemployment u is countercyclical.

Labor demand (18) implies that the elasticity ϵ^l_θ of private labor demand $l^d(\theta,a)$ with respect to tightness θ is

$$\epsilon^l_\theta\equiv\frac{\theta}{l}\cdot\frac{\partial l^d}{\partial\theta}=-\frac{\eta}{1-\alpha}\cdot\Omega,$$

where I define

$$\Omega\equiv\frac{[1-\delta\cdot(1-s)]\cdot r/q(\theta)}{[1-\delta\cdot(1-s)]\cdot r/q(\theta)+w/a}. \tag{A6}$$

Ω measure the share of the marginal recruiting costs in the marginal cost of labor. Using Assumption A3 and (19), I relate ϵ^d to Ω in equilibrium:

$$\epsilon^d=-(1-\zeta)\cdot\epsilon^l_\theta=\eta\cdot\frac{1-\zeta}{1-\alpha}\cdot\Omega. \tag{A7}$$

Using Lemma A1, the facts that $q'(\theta)<0$ and $d\left[w/a\right]/da<0$, and definition (A6): $d\Omega/da>0$. Using Lemma A1 and relations (A5) and (A7): $d\epsilon^d/da>0$ and $d\epsilon^s/da<0$. In addition, $\partial n^d/\partial g=1$. Hence (A4) implies that $d\lambda/da<0$.

B The Efficient Allocation

This Appendix derives the efficient allocation of the model presented in Section 2. Given the symmetry of the production function (8) for the final good, efficiency requires that identical quantities of each intermediate good be produced: $y_t(i) = y_t$ for all $i \in [0, 1]$. Accordingly, I focus on symmetric allocations in which all intermediate-good firms employ the same number of workers and produce the some about of good.

An *allocation* is a collection of stochastic processes $\{g_t, l_t, \theta_t, c_t, y_t, z_t\}_{t=0}^{+\infty}$ for public and private employment; labor market tightness; consumption; and output of final good and public good. A *feasible allocation* is an allocation that satisfies the following constraint: production constraint $y_t = a_t \cdot x(l_t)$ for the final good; production constraint (13) for the public good; resource constraint in the economy, which imposes that the private good be either consumed or allocated to recruiting:

$$y_t = c_t + \frac{r \cdot a}{q(\theta_t)} \cdot h_t;$$

and law of motion for aggregate employment $(1 - s) \cdot n_{t-1} + u_t \cdot f(\theta_t) = n_t$. I denote total employment $n_t = l_t + g_t$, unemployment $u_t = 1-(1-s)\cdot n_{t-1}$, and new hires $h_t = n_t-(1-s)\cdot n_{t-1}$. The *efficient allocation* is the feasible allocation that maximizes (2). Proposition A1 establishes that the efficient allocation is invariant to technology:

PROPOSITION A1. *In the efficient allocation, labor market variables $\{g_t, l_t, \theta_t\}_{t=0}^{+\infty}$ are deterministic, independent of the technology process $\{a_t\}_{t=0}^{+\infty}$. These variables remain constant over time for an appropriate choice of initial employments l_{-1} and g_{-1}. The ratios of consumption to technology $\{c_t/a_t\}_{t=0}^{+\infty}$, output of final good to technology $\{y_t/a_t\}_{t=0}^{+\infty}$, and output of public output to technology $\{z_t/a_t\}_{t=0}^{+\infty}$ share the same property.*

Proof. The Lagrangian of the planner's problem is

$$L = \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ \omega_v \cdot \ln(a_t \cdot \omega_z \cdot x(g_t)) + \ln(c_t) + \Lambda_t^1 \cdot \left[a_t \cdot x(l_t) - c_t - \frac{r \cdot a}{q(\theta_t)} \cdot \{n_t - (1 - s) \cdot n_{t-1}\} \right] \right.$$
$$\left. + \Lambda_t^2 \cdot [(1 - s) \cdot (1 - f(\theta_t)) \cdot n_{t-1} + f(\theta_t) - n_t] + \Lambda_t^3 \cdot [n_t - l_t + g_t] \right\}.$$

$\{\Lambda_t^1, \Lambda_t^2, \Lambda_t^3\}$ are Lagrange multipliers. The first-order conditions with respect to $\{c_t, \theta_t, n_t, l_t, g_t\}$

are

$$\frac{1}{c_t} = \Lambda_t^1 \tag{A8}$$

$$\Lambda_t^2 \cdot q(\theta_t) \cdot u_t = r \cdot a_t \cdot \frac{\eta}{1-\eta} \cdot \frac{1}{f(\theta_t)} \cdot \Lambda_t^1 \cdot h_t \tag{A9}$$

$$\Lambda_t^1 \cdot \frac{r \cdot a_t}{q(\theta_t)} + \Lambda_t^2 = \Lambda_t^3 + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\Lambda_{t+1}^1 \cdot \frac{r \cdot a_{t+1}}{q(\theta_{t+1})} + \Lambda_{t+1}^2 \cdot [1 - f(\theta_{t+1})]\right] \tag{A10}$$

$$\Lambda_t^3 = \Lambda_t^1 \cdot a_t \cdot x'(l_t) \tag{A11}$$

$$\Lambda_t^3 = \frac{\omega_v}{z_t} \cdot a_t \cdot \omega_z \cdot x'(g_t). \tag{A12}$$

Since hiring a worker has the same marginal cost in the private and public sectors, the marginal benefit of a worker must be equal in both sectors. Combining first-order conditions (A11) and (A12), I obtain

$$\omega_v \cdot \omega_z \cdot \frac{c_t}{z_t} = \frac{x'(l_t)}{x'(g_t)}$$

$$\omega_v \cdot \omega_z \cdot \frac{c_t/a_t}{z_t/a_t} = \left[\frac{g_t}{l_t}\right]^{1-\alpha}. \tag{A13}$$

In particular, it is always optimal to employ some workers in the public sector as long as the public good is valuable ($\omega_v > 0$) and the government is productive ($\omega_z > 0$).

Since $f(\theta_t) = h_t/u_t$, the first-order condition (A9) becomes

$$\frac{\Lambda_t^2}{\Lambda_t^1 \cdot a_t} = \frac{\eta}{1-\eta} \cdot \frac{r}{q(\theta_t)}.$$

I combine first-order conditions (A10) and (A11) and divide by $a_t \cdot \Lambda_t^1$:

$$x'(l_t) = \frac{r}{q(\theta_t)} + \frac{\Lambda_t^2}{\Lambda_t^1 \cdot a_t} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1 \cdot a_{t+1}}{\Lambda_t^1 \cdot a_t} \cdot \left\{\frac{r}{q(\theta_{t+1})} + \frac{\Lambda_{t+1}^2}{\Lambda_{t+1}^1 \cdot a_{t+1}} \cdot [1 - f(\theta_{t+1})]\right\}\right]$$

I combine these two relationships and multiply by $(1-\eta)$:

$$(1-\eta) \cdot x'(l_t) = \frac{r}{q(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{c_t/a_t}{c_{t+1}/a_{t+1}} \cdot \left\{\frac{r}{q(\theta_{t+1})} - \eta \cdot r \cdot \theta_{t+1}\right\}\right]. \tag{A14}$$

This relationship says that the marginal benefit from having workers search for jobs must equal the marginal benefit from having them produce goods.

The first-order conditions (A13) and (A14), together with the production constraints $y_t/a_t = x(l_t)$ and $z_t/a_t = \omega_z \cdot x(g_t)$, the resource constraint $c_t/a_t = x(l_t) - [r/q(\theta_t)] \cdot h_t$, and the law of motion for aggregate employment $(1-s) \cdot n_{t-1} + u_t \cdot f(\theta_t) = n_t$, constitute the system of 6 equations that characterize the efficient allocation. This system does not involve the technology stochastic process $\{a_t\}_{t=0}^{+\infty}$. If the efficient allocation is unique, then the efficient allocation is independent of the realizations of the stochastic process for technology: the efficient allocation is deterministic. If initial employment l_{-1} and g_{-1} are chosen adequately to be at their steady-state values, then the labor market variables in the efficient allocation remain constant over time.

C An Extension with Behavioral Labor Supply Response

This Appendix extends the baseline model of Section 2 to account for a behavioral labor supply response of the household to labor market conditions.

C.1 The model

Household members controls their employment rate by choosing a job-search effort e_t. As the choice of consumption, the choice of search effort is taken collectively by the household each period. As firms incur a cost for each vacancy advertised, the household incurs a cost $d(e_t)$ for each unemployed worker searching with effort e_t. The cost is measured in units of final good. I assume that $d(e_t) = \omega_d \cdot e_t^{1+\kappa} / (1+\kappa)$, where $\omega_d > 0$ and $\kappa > 0$ are parameters.

The number of matches is a Cobb-Douglas matching function of aggregate search effort $e_t \cdot u_t$ and vacancies o_t: $h_t = \omega_h \cdot (e_t \cdot u_t)^\eta \cdot o_t^{1-\eta}$. I redefine labor market tightness as $\theta_t \equiv o_t / (e_t \cdot u_t)$. The job-finding probability per unit of effort is $f(\theta_t)$. That is, a jobseeker searching with effort e_t finds a job with probability $e_t \cdot f(\theta_t)$. The vacancy-filling probability is $q(\theta_t)$.

The labor supply (3) becomes

$$n_t = (1-s) \cdot n_{t-1} + [1 - (1-s) \cdot n_{t-1}] \cdot e_t \cdot f(\theta_t). \tag{A15}$$

It accounts for the influence of search efforts on the probability to find a job. The budget constraint (4) becomes

$$p_t \cdot c_t + p_t \cdot u_t \cdot d(e_t) + b_t = p_t \cdot n_t \cdot c_t^e + p_t \cdot (1-n_t) \cdot c_t^u + R_{t-1} \cdot b_{t-1} + T_t. \tag{A16}$$

It accounts for the cost of searching for a job. The household chooses consumption and effort $\{c_t, e_t\}$ to maximize utility (2) subject to the budget constraint (A16), the labor supply(A15), and the no-Ponzi-game constraint (5). The optimal consumption path is governed by the Euler equation (6). The optimal path of search effort satisfies

$$\frac{d'(e_t)}{f(\theta_t)} = [c_t^e - c_t^u] + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{c_{t+1}}{c_t} \cdot \left(\frac{d'(e_{t+1})}{f(\theta_{t+1})} - \kappa \cdot d(e_{t+1}) \right) \right], \tag{A17}$$

which is obtained by taking combining first-order conditions with respect to e_t and n_t in the Lagrangian representation of the household's problem.

Since some resources are devoted by the household to searching for a job, the resource constraint (14) becomes

$$y_t = c_t \cdot \left[1 + \frac{\phi}{2} \cdot \pi_t^2\right] + \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1-s) \cdot n_{t-1}] + d(e_t) \cdot [1 - (1-s) \cdot n_{t-1}]. \tag{A18}$$

Compared to the symmetric equilibrium described in Section 2, the symmetric equilibrium of the model with a behavioral labor supply response has one additional variable: search effort e_t. Accordingly, it has one additional relationship: the optimal search condition (A17). Furthermore, three equilibrium relationships are modified by the presence of a behavioral labor supply: the budget constraint of the household becomes (A16); the resource constraint becomes (A18); and the aggregate labor supply becomes (A15).

C.2 Multipliers across steady states

The optimal search condition (A17) indicates that the consumption gain from work

$$c_t^e - c_t^u = (1 - \tau_t) \cdot w_t \cdot (1 - \rho_t) \tag{A19}$$

is a key determinant of labor supply. The consumption gain is determined by the government through the choice of the labor tax τ_t and of the replacement rate ρ_t of unemployment insurance. Note that the government can levy any amount of revenue without affecting search effort and labor supply by increasing labor taxes τ_t, while keeping $c_t^e - c_t^u$ constant by adjusting ρ_t. To study the multiplier across steady states parameterized by different technology levels, I need to specify the consumption gain from work associated with each technology level. I consider a policy experiment in which the consumption gain is constant across steady states.

ASSUMPTION A4. For any technology a, the government sets $c^e - c^u = \Delta$, where $\Delta \in (0, 1)$.

In steady state, the household's optimal choice of effort satisfies

$$[1 - \delta \cdot (1-s)] \cdot \frac{d'(e)}{f(\theta)} + \kappa \cdot \delta \cdot (1-s) \cdot d(e) = \Delta. \tag{A20}$$

Under Assumption A4, this equation implicitly defines the supply of search effort $e^s(\theta)$ as an increasing function of labor market tightness θ. Since search effort responds to labor market tightness, the labor supply (16) becomes

$$n^s(\theta) = \frac{e^s(\theta) \cdot f(\theta)}{s + (1-s) \cdot e^s(\theta) \cdot f(\theta)}. \tag{A21}$$

Labor supply $n^s(\theta)$ is the employment rate chosen by jobseekers through their choice of search effort, given the matching frictions on the labor market. $n^s(\theta)$ increases with θ because both the supply of effort $e^s(\theta)$ and the per-unit job-finding probability $f(\theta)$ increase with θ. In Figures 1(c) and 1(d), the labor supply curve would be flatter.

Even though the slope of the labor supply curve is modified by the behavioral labor supply response, Proposition A2 establishes that the results of Proposition 1 remain valid:

PROPOSITION A2 (Multiplier with behavioral labor supply response)**.**

(a) $\lambda < 1$. *Under Assumption A2,* $\lambda > 0$.

(b) Under Assumptions A1, A2, A3, and A4, $d\lambda/da < 0$.

Proof. The proof of Proposition 1 remains mostly valid. Part 1 is unaltered. There are slight differences in Part 2. First, there is an additional equilibrium variable: search effort e.

LEMMA A2. *Under Assumptions A1, A2, A3, and A4,* $d\theta/da > 0,\ de/da > 0,\ dn/da > 0,\ dl/da > 0$, *and* $du/da < 0$.

Proof. Effort supply $e^s(\theta)$, given by (A20), and labor supply, given by (A21), satisfy: $\partial e^s/\partial\theta > 0,\ \partial n^s/\partial\theta > 0,\ \partial n^s/\partial e > 0$. Under Assumptions A2 and A1, private labor demand $l^d(\theta, a)$, given by (17), satisfies $\partial l^d/\partial\theta < 0,\ \partial l^d/\partial a > 0$. Proceeding as in the proof of Lemma A1, I obtain:

$$\frac{d\theta}{da} = \underbrace{\frac{\partial l^d}{\partial a}}_{+} \cdot \left[(1-\zeta)\cdot\left(\underbrace{\frac{\partial n^s}{\partial e}}_{+}\cdot\underbrace{\frac{\partial e^s}{\partial\theta}}_{+} + \underbrace{\frac{\partial n^s}{\partial\theta}}_{+}\right) - \underbrace{\frac{\partial l^d}{\partial\theta}}_{-}\right]^{-1}.$$

Thus $d\theta/da > 0$. I conclude by using $\partial e^s/\partial\theta > 0,\ \partial n^s/\partial\theta > 0,\ \partial n^s/\partial e > 0$, and noting that in equilibrium $e = e^s(\theta)$, $n = n^s(e,\theta)$, $u = 1-(1-s)\cdot n$, and $l = (1-\zeta)\cdot n$.

The equilibrium variables behave as in the baseline model. In addition, effort e is procyclical.

A second difference is that the elasticity ϵ^s of labor supply is affected by the presence of behavioral labor supply: search effort e responds to a change in tightness θ. I log-linearize the worker's optimality condition (A20) to obtain the effect of a marginal change in dg in public employment. Using the iso-elasticity of the disutility $d(e)$ I obtain:

$$\begin{aligned}
\ln(1/\rho) &= [1-\delta\cdot(1-s)]\cdot\frac{d'(e)}{f(\theta)} + \kappa\cdot\delta\cdot(1-s)\cdot d(e)\\
0 &= (1-\delta\cdot(1-s))\cdot\left[\frac{d'(e)}{f(\theta)}\right]\left[\kappa\check{e} - (1-\eta)\cdot\check{\theta}\right] + \kappa\cdot\delta\cdot(1-s)\cdot d(e)\cdot[1+\kappa]\,\check{e}\\
0 &= \left[\kappa\check{e} - (1-\eta)\cdot\check{\theta}\right] + \check{e}\cdot\kappa\cdot\frac{\delta\cdot(1-s)}{1-\delta\cdot(1-s)}\cdot f(\theta)\frac{(1+\kappa)d(e)}{d'(e)}\\
(1-\eta)\cdot\check{\theta} &= \kappa\cdot\check{e}\cdot\left[1+\frac{\delta\cdot(1-s)}{1-\delta\cdot(1-s)}\cdot f(\theta)\cdot e\right]\\
\epsilon^e_\theta &= \frac{\check{e}}{\check{\theta}} = (1-\eta)\cdot\frac{1}{\Delta},
\end{aligned}$$

where

$$\Delta = \kappa \cdot \left[1 + s \cdot \frac{\delta \cdot (1-s)}{1 - \delta \cdot (1-s)} \cdot f(\theta) \cdot e\right]. \tag{A22}$$

Labor supply (A21) implies an equilibrium relationship between ϵ^s, u, and Δ:

$$\epsilon^s = u \cdot [\epsilon^e_\theta + (1-\eta)] = (1-\eta) \cdot \left[1 + \frac{1}{\Delta}\right] \cdot u.$$

This elasticity is similar to that in the baseline model: there is only an additional $1/\Delta$ term. Note that since Δ depends critically on κ, the parameters κ (the elasticity of the cost of search) and η (the elasticity of the matching function with respect to unemployment) control the elasticity ϵ^s of labor supply. Using definition (A22) and Lemma A2: $d\Delta/da > 0$. Therefore it remains that $d\epsilon^s/da < 0$. The rest of the proof proceeds as the proof of Proposition 1.

C.3 Multiplier dynamics

I first calibrate the parameters of the cost from job search $d(e) = \omega_k \cdot e^{1+\kappa}/(1+\kappa)$. I normalize average effort $\overline{e} = 1$. In the US, weekly unemployment benefits replace between 50% and 70% of the last weekly pre-tax earnings of a worker [Pavoni and Violante, 2007]. I set the benefit rate to 60%, the midpoint of the range of plausible values. Since earnings are subject to a 7.65% payroll tax, the replacement rate is $\rho = 0.6/(1 - 0.0765) = 0.650$. In the US, both firm and workers are subject to the 7.65% payroll tax, so the labor tax is $\tau = 0.153$. Using (A19), I set the consumption gain from work to $\Delta c = (1 - 0.153) \cdot 0.644 \cdot (1 - 0.650) = 0.191$.

The convexity κ is related to a statistic ϵ_1 that has been estimated empirically. The statistic $\epsilon_1 \equiv (c^u/\xi) \cdot (\partial\xi/\partial c^u)$ captures the reduction in the hazard rate $\xi \equiv e \cdot f(\theta)$ out of unemployment when an unemployed worker receives an increase $dc^u > 0$ in benefits, keeping labor market tightness θ constant. Assume that the worker receives an increase $dc^u > 0$ in benefits, and reduces search effort by $de < 0$, which leads to a reduction $d\xi = f(\theta) \cdot de < 0$ in the hazard rate (we consider a change in benefits for one worker only, so labor market tightness θ is not affected by the policy experiment). As established by Landais et al. [2010], the elasticity ϵ_1 relates to κ so that we use an empirical estimate of ϵ_1 to calibrate κ. They prove that if $\delta \approx 1$,

$$\frac{\Delta c}{e} \cdot \frac{\partial e}{\partial \Delta c} \approx \frac{1}{\kappa} \cdot \frac{u + \kappa}{1 + \kappa}.$$

Since $\Delta c/c^u = 1/\rho - 1$ and $\partial \Delta c = -\partial c^u$, κ is related to ϵ_1 by

$$-\epsilon_1 \cdot \left(\frac{1}{\rho} - 1\right) \approx \frac{1}{\kappa} \cdot \frac{u + \kappa}{1 + \kappa}.$$

There are numerous estimates of ϵ_1 in the literature. Chetty [2008] reports that the estimates fall in the 0.4–0.8 range. I pick $\epsilon_1 = 0.6$, the midpoint of that range. With $\rho = 0.650$ and $\overline{u} = 6.37\%$, I obtain $\kappa = 2.10$. Last, I set $\omega_k = 0.256$ to match $\bar{e} = 1$ with the optimal choice of effort (A20).

Figure A1 plots the cumulative multiplier in the model with behavioral labor supply. The multiplier is nearly identical to the multiplier obtained in the baseline model, plotted in Figure 5. The multiplier is slightly larger because in presence of a behavioral labor supply, unemployment workers respond to an increase in public employment by searching more, which stimulates slightly private employment.

D An Extension with Public Capital

This Appendix extends the baseline model of Section 2 to account for public capital. Public capital is created by workers in the public sector. It contributes to the productivity of the private sector.

D.1 The model

I follow Baxter and King [1993] to model public capital. I assume that in period t, public-sector workers produce a public good z_t that contributes to public capital k_{t+1}. The production function of public capital is

$$k_{t+1} = (1 - \beta) \cdot k_t + z_t. \tag{A23}$$

β is the depreciation rate of public capital. Public capital k_t enters the production function (11) of intermediate-good firm i, which becomes

$$y_t(i) = a_t \cdot k_t^{\xi} \cdot x(l_t(i)).$$

The parameter $\xi > 0$ is the elasticity of output with respect to public capital, which indicates the productiveness of public capital. I assume that public capital does not enter the production function (13) of public good.

Compared to the symmetric equilibrium described in Section 2, the symmetric equilibrium of the model with public capital has two additional variables: the output of public good z_t, and the stock of public capital k_t. Accordingly, it has two additional relationships: the production function of public good (13), and the production function of public capital (A23). Furthermore, two equilibrium relationships are modified by the presence of public capital: the production function becomes $y_t = a_t \cdot k_t^{\xi} \cdot x(l_t)$, and the marginal cost of producing one unit of final good becomes

$$\Lambda_t \equiv \frac{1}{k_t^{\xi} \cdot x'(l_t)} \cdot \left\{ \frac{w_t}{a_t} + \frac{r}{q(\theta_t)} - \delta \cdot (1 - s) \cdot \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \cdot \frac{a_{t+1}}{a_t} \cdot \frac{r}{q(\theta_{t+1})} \right] \right\}.$$

D.2 Multipliers across steady states

In steady state, public capital k remains constant over time so $\beta \cdot k = z$. Therefore, the production function (13) of public good implies the following relationship between public capital and public employment:

$$k = \frac{\omega_z}{\beta} \cdot a \cdot x(g), \tag{A24}$$

As the production function of firms depends on public capital, private labor demand (18) becomes

$$l^d(\theta, a, g) = \left[\frac{1}{\alpha} \cdot \left\{\frac{\beta}{\omega_z \cdot a \cdot x(g)}\right\}^{\xi} \cdot \frac{\epsilon}{\epsilon - 1} \cdot \left\{\omega \cdot a^{\gamma-1} + [1 - \delta \cdot (1-s)] \cdot \frac{r}{q(\theta)}\right\}\right]^{\frac{-1}{1-\alpha}}. \tag{A25}$$

Private labor demand $l^d(\theta, a, g)$ increases with public employment g because the production function $x(g)$ increases with g. In Figures 1(c) and 1(d), the shift of the labor demand curve after an increase in public employment would be larger.

Even though the labor demand shift after an increase in public employment is larger, Proposition A3 establishes that the results of Proposition 1 remain valid:

PROPOSITION A3 (Multiplier with public capital)**.**

(a) $\lambda < 1 + \xi \cdot [\alpha/(1-\alpha)] \cdot [(1-\zeta)/\zeta]$. *Under Assumption A2,* $\lambda > 0$.

(b) Under Assumptions A1, A2, and A3, $d\lambda/da < 0$.

Proof. Compared to the proof of Proposition 1, Part 1 is unaltered. There are slight differences in Part 2. First, equilibrium labor demand is affected by public capital. Under Assumption A3, I rewrite private labor demand (A25):

$$l^d(\theta, a) = a^{\xi/[(1-\alpha)\cdot\alpha\cdot\xi]} \cdot \left\{\left[\frac{\beta}{\omega_z}\right]^{\xi} \cdot \left[\frac{1-\zeta}{\zeta}\right]^{\alpha\cdot\xi} \cdot \frac{1}{\alpha} \cdot \left[\frac{w}{a} + \frac{r}{q(\theta)}\right]\right\}^{-1/[(1-\alpha)\cdot\alpha\cdot\xi]}.$$

While the expression for private labor demand l^d is different, its properties remain the same as in the baseline model: $l^d(\theta, a)$ is increasing in a, decreasing in θ. Thus the results from Lemma A1 remain valid.

A second difference is that public employment g now influences private labor demand l^d; thus I need to re-calculate $\partial n^d/\partial g = 1 + \partial l^d/\partial g \neq 1$. Using the expression (A25) of private labor demand l^d, I consider the effect dl^d of a marginal change dg in public employment keeping tightness θ constant:

$$\begin{aligned} 0 &= \xi \cdot \alpha \cdot \check{g} + (\alpha - 1) \cdot \check{l}^d \\ \frac{\partial n^d}{\partial g} &= 1 + \frac{\partial l^d}{\partial g} = 1 + \frac{l}{g} \cdot \frac{\check{l}^d}{\check{g}} \\ \frac{\partial n^d}{\partial g} &= 1 + \xi \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1-\zeta}{\zeta}. \end{aligned} \tag{A26}$$

Since $\partial n^d/\partial g$ is a function of parameters only, the proof of Proposition 1 carries through.

D.3 Multiplier dynamics

In the extension with public capital, I calibrate the depreciation rate of public capital at 2.5% at quarterly frequency as Baxter and King [1993], implying $\beta = 0.19\%$ at weekly frequency. The productivity of public capital ξ is critical to determine the size of the multiplier. A large literature studies the impact of public capital on productivity, but no consensus emerges; therefore, I follow Baxter and King [1993] and choose $\xi = 0.05$. I normalize average public capital to $\overline{k} = 1$. I use (A24) to set the relative productivity of the public sector at $\omega_z = \beta \cdot \overline{k}/\overline{g}^{\alpha} = 0.66\%$.

Figure A1 plots the cumulative multiplier with public capital. As in the baseline model in the text, the cumulative multiplier is quite countercyclical: it nearly doubles from 0.45 to 0.78 when the unemployment rate increases from 4.8% to 8.3%. The cumulative multiplier is also much higher than in the baseline model because in this case, public employment contributes to public capital, which in turn contributes to the productivity of private firms. As private firms are more productive with more public capital, they hire more workers.

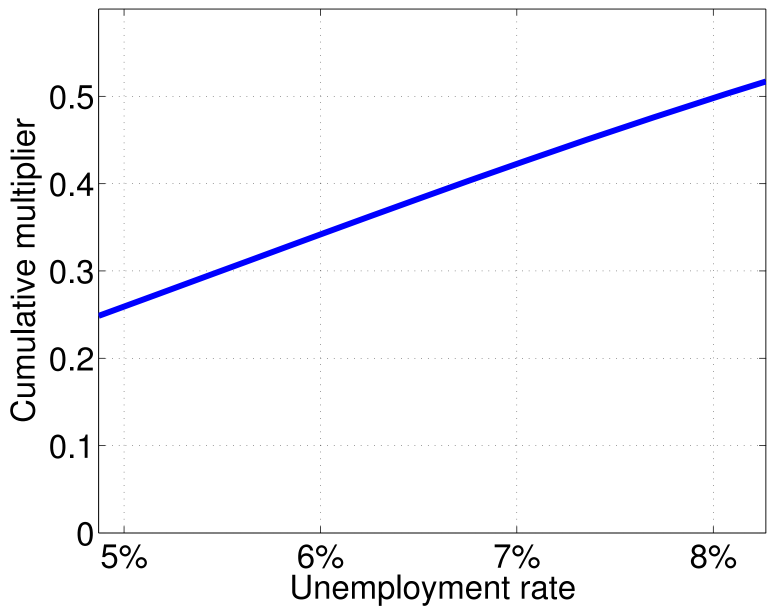
E A Variant with Aggregate Demand Shocks

In the model presented in Section 2, technology shocks combined with real wage rigidity cause recessions. The mechanism is as follows. After a negative technology shock the real wage, which is somewhat rigid, does not fall as much as the marginal product of labor. Therefore the marginal cost of labor is high relative to the marginal product of labor, which depresses hiring and raises unemployment. This appendix proposes a model in which the combination of aggregate demand shocks and nominal wage rigidity cause recessions. The mechanism is as follows. After a negative aggregate demand shock, prices fall. Nominal wage rigidity, combined with a lower price level, leads to a higher real wage and a higher marginal cost of labor, which depresses hiring and raises unemployment. The mechanism loosely captures one possible story for the Great Depression in the US: contractionary monetary policy lead to a deflation, which raised real wages because of nominal wage rigidity, which in turn depressed employment.[28] The model is too simplistic to be calibrated and simulated; therefore I limit the analysis to the characterization of the sign and cyclicality of the government-consumption multiplier using the comparative statics of Section 3.

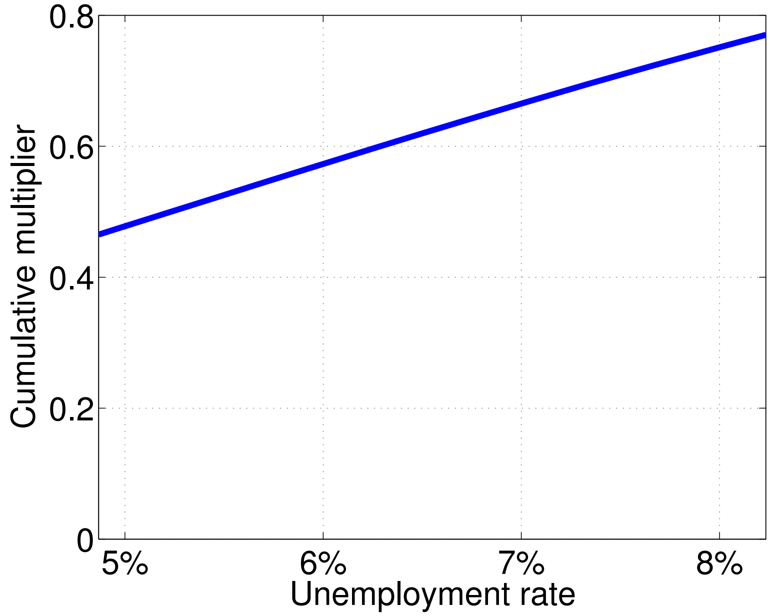
E.1 The model

Aggregate demand shocks $\{m_t\}_{t=0}^{+\infty}$ drive fluctuations. Technology remains constant and I normalize it to $a = 1$. The labor market remains the same as in the text. I simplify very much the structures of the household and of the private sector. The household neither borrows nor saves, and simply consumes its income each period. The labor supply of the household is its employment rate

[28] Romer [1993] singles out the contractionary monetary policy started in 1928 as one of the main trigger of the Great Depression. Jacoby [1984] explains nominal wage rigidity at the onset of the Great Depression by the prevalence of internal labor markets in corporations, and Temin [1990] by government intervention in the private sector.



(a) Cumulative multiplier in presence of behavioral labor supply



(b) Cumulative multiplier in presence of public capital

Figure A1: Cumulative multiplier over the business cycle in two extensions

Notes: The cumulative multiplier is defined by equation (24). The multiplier applies to the hire of 0.01% of the labor force in the public sector when the technology shock occurs. The unemployment rate reported on the x-axis represents the extremum of the IRF of unemployment to the underlying technology shock, without government intervention. To obtain the graph, I impose a collection of 16 technology shocks to the models, ranging from -3.6% to +5.4%. I solve the exact models using a shooting algorithm under perfect foresight. The models are calibrated in Table 2. The model with behavioral labor supply is described in Appendix C. The model with public capital is described in Appendix D.

n_t as a function of labor market conditions:

$$n_t = (1-s)\cdot n_{t-1} + [1-(1-s)\cdot n_{t-1}]\cdot f(\theta_t).. \tag{A27}$$

The private sector is composed of a representative firm that produces a final good, taking the price p_t of the good as given. A risk-neutral entrepreneur, with the same discount factor $\delta < 1$ as workers, owns the firm. As the intermediate-good firms in the text, the firm hires workers by posting vacancies. The firm chooses employment $\{l_t\}$ to maximize expected discounted profits

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left[x(l_t) - w_t \cdot l_t - \frac{r}{q(\theta_t)} \cdot [l_t - (1-s)\cdot l_{t-1}]\right],$$

where w_t is the real wage, $x(l_t) = l_t^\alpha$ is the production function, $r/q(\theta_t)$ is the hiring cost, and $l_t - (1-s)\cdot l_{t-1}$ is the number of hires in period t. The first-order condition with respect to l_t is

$$x'(l_t) = w_t + \frac{r}{q(\theta_t)} - \delta\cdot(1-s)\cdot \mathbb{E}_t\left[\frac{r}{q(\theta_{t+1})}\right]. \tag{A28}$$

The firm hires labor until the marginal product of labor $x'(l_t)$ equals the marginal cost of labor, which is the sum of real wage w_t, plus hiring cost $r/q(\theta_t)$, less discounted cost of hiring next period $\delta\cdot(1-s)\cdot\mathbb{E}_t\left[r/q(\theta_{t+1})\right]$.

I assume that the nominal wage $p_t \cdot w_t$ follows a simple wage schedule:

$$p_t \cdot w_t = \omega \cdot p_t^\gamma, \tag{A29}$$

where p_t is the price level, ω is a parameter, and $\gamma \in [0,1]$ indicates the flexibility of nominal wages. If $\gamma = 1$, the nominal wage $w_t \cdot p_t = \omega \cdot p_t$ is proportional to the price level such that the real wage $w_t = \omega$ does not depend on the price level. In that case, aggregate demand is neutral: it does not influence the equilibrium. If $\gamma = 0$, the nominal wage $w_t \cdot p_t = \omega$ is completely rigid such that the real wage $w_t = \omega/p_t$ increases when the price level falls. In that case, fluctuations in aggregate demand lead to fluctuations in unemployment. To obtain fluctuations in unemployment, I assume some nominal wage rigidity:

ASSUMPTION A5. The nominal wage schedule is somewhat rigid: $\gamma < 1$.

The production of the firm is sold in a perfectly competitive final-good market. As in Mankiw and Weinzierl [2011] the aggregate demand for the final good takes the form m_t/p_t, borrowed from the quantity theory of money. The output $x(l_t)$ produced at a price p_t is the aggregate supply of final good. When the final-good market is in equilibrium, the price clears the market:

$$\frac{m_t}{p_t} = x(l_t). \tag{A30}$$

Given the price level p_t determined by (A30), the equilibrium real wage is a function of employment

$$w_t = \omega \cdot \left[\frac{l_t^\alpha}{m_t}\right]^{1-\gamma}. \tag{A31}$$

When aggregate demand m_t is lower, the real wage w_t tends to be higher.

There is no monetary policy, and government consumption expenditures take the same form as in the text: the government employ g_t workers in the public sector in period t. The final good is either consumed or allocated to hiring such as the resource constraint is

$$x(l_t) = c_t + \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1-s) \cdot n_{t-1}]. \tag{A32}$$

I can now define the equilibrium of this model with aggregate demand shocks. Given initial employment n_{-1} and stochastic processes $\{m_t, g_t\}_{t=0}^{+\infty}$ for aggregate demand and public employment, an equilibrium is a collection of six stochastic processes

$$\{\theta_t, n_t, l_t, w_t, c_t, p_t\}_{t=0}^{+\infty}$$

that satisfy the following six relationships: the wage schedule (7); the resource constraint (14); the profit-maximization condition (A28) of the representative firm; the final-good market clearing condition (A30); the aggregate labor demand $n_t = l_t + g_t$; and the aggregate labor supply (A27).

E.2 Multipliers across steady states

The equilibrium can be represented with a labor supply-labor demand diagram in a price θ-quantity n plan. This diagram shares the same properties as the diagram presented in Figure 1(a). The diagram illustrates why the economy is so nonlinear around the rates of unemployment seen in the data. I also use the labor supply-labor demand representation to prove that the government-consumption multiplier is positive and, and that it is higher in steady-states parameterized by lower aggregate demand and therefore higher unemployment rates.

I start by representing the equilibrium in a labor demand-labor supply diagram. Labor supply $n^s(\theta)$ satisfies (16). Inserting the equilibrium real wage (A31) into the firm's profit-maximization condition (A28) defines implicitly the private labor demand curve $l^d(\theta, m)$ as the solution to

$$l^{\alpha-1} \cdot \left[\alpha - \omega \cdot m^{\gamma-1} \cdot l^{1-\alpha\cdot\gamma}\right] = [1 - \delta \cdot (1-s)] \cdot \frac{r}{q(\theta)}. \tag{A33}$$

The private labor demand $l^d(\theta, m)$ is the employment rate in the private sector such that in equilibrium, both the firm's profit-maximization condition (A28) and the final-good market-clearing condition (A30) hold. Under Assumption A5, private labor demand $l^d(\theta, m)$ is a well-defined function of aggregate demand m and labor market tightness θ, increasing in m and decreasing in θ. When aggregate demand is low, the price level is low and real wages are relatively high, depressing hiring. Unlike in the text, private labor demand does not require diminishing marginal returns to be decreasing with labor market tightness: nominal wage rigidity suffices.

Aggregate labor demand is the sum of public and private labor demands:

$$n^d(\theta, m, g) = g + l^d(\theta, m). \tag{A34}$$

Tightness θ acts as a price equilibrating labor supply and labor demand:

$$n^s(\theta) = n^d(\theta, m, g). \tag{A35}$$

Equation (A35) implicitly defines equilibrium tightness θ. As aggregate demand m decreases, θ decreases because labor demand $n^d(\theta, m, g)$ is lower for all θ. Equilibrium employment n can be directly read off the labor demand curve:

$$n = n^d(\theta, m, g), \tag{A36}$$

where equilibrium tightness θ satisfies (A35). As technology m decreases, n decreases.

Even though fluctuations are driven by aggregate demand shocks, the labor market equilibrium shares the same structure as the equilibrium in Figures 1(a) and 1(b). Under Assumption A5, the labor supply is upward and convex and the labor demand is downward-sloping in the $n - \theta$ plan. When aggregate demand falls in recessions, the labor demand curve shifts inwards and unemployment increases. Jobs are also rationed in recessions, as some unemployment would remain even if matching frictions vanished when demand is low enough. Higher employment implies more production, lower prices in the goods market, higher real wages because of nominal wage rigidity, and requires a lower tightness for firms to be willing to hire: the aggregate labor demand curve is downward sloping in a price θ-quantity n plan. If demand is low enough ($m < \omega^{1/(1-\gamma)}$), private labor demand falls below zero for $l < 1$: private jobs are rationed.

The structure of the labor market equilibrium in this model is similar to that in the model with technology shocks, suggesting that the properties of the government-consumption multiplier should be similar in the two models. Proposition A4 establishes that indeed, the results from Proposition 1 remain valid in the model with aggregate demand shocks. But unlike Proposition 1, Proposition A4 does not require diminishing marginal returns to labor (Assumption A2). The reason is that nominal wage rigidity (Assumption A5) yields both a downward-sloping labor demand and shifts of labor demand when aggregate demand fluctuates.

PROPOSITION A4 (Multiplier with aggregate demand shocks)**.**

(a) $\lambda < 1$. *Under Assumption A5,* $\lambda > 0$.

(b) Under Assumptions A5 and A3, $d\lambda/dm < 0$.

Proof. Compared to the proof of Proposition 1, Part 1 is unaltered. There are slight differences in Part 2. In the model with aggregate demand shocks, the equilibrium private labor demand is given by (A33). While the expression for equilibrium private labor demand $l^d(\theta, m)$ is different, its properties remain the same as in the baseline model: $l^d(\theta, m)$ is increasing in m and decreasing in θ. Therefore I can use a proof similar to that of Lemma A1 to characterize the qualitative behavior of equilibrium variables in response to an adverse aggregate demand shock.

LEMMA A3. *Under Assumptions A5 and A3,* $d\theta/dm > 0$, $dn/dm > 0$, $dl/dm > 0$, *and* $du/dm < 0$.

Given that equilibrium conditions (A35) and (A36), which determine equilibrium employment and tightness, are similar to conditions (20) and (21) in the text, the relationship (22) between the multiplier $\lambda \equiv \partial n/\partial g$ and the tightness-elasticities $\epsilon^s \equiv (\theta/n^s) \cdot (\partial n^s/\partial\theta)$ and $\epsilon^d \equiv -\left(\theta/n^d\right) \cdot \left(\partial n^d/\partial\theta\right)$ of labor supply and labor demand remains valid. As in the text, $\partial n^d/\partial g = 1$.

The labor supply is the same as in the model in the text so ϵ^s is given by (A5). Since the labor demand is different, I derive the properties of the elasticity ϵ^d. The tightness-elasticity of private labor demand ϵ^l_θ becomes

$$\epsilon^l_\theta \equiv \frac{\theta}{l} \cdot \frac{\partial l^d}{\partial\theta} = -\frac{\eta}{1-\gamma} \cdot \Omega,$$

where I define

$$\Omega \equiv \frac{[1-\delta\cdot(1-s)]\cdot r/q(\theta)}{[(1-\alpha\cdot\gamma)\,/\,(1-\gamma)]\cdot x'(l) - \alpha\cdot[1-\delta\cdot(1-s)]\cdot r/q(\theta)}. \tag{A37}$$

Using Assumption A3 and (A34), I relate ϵ^d to Ω in equilibrium:

$$\epsilon^d = \eta\cdot\frac{1-\zeta}{1-\gamma}\cdot\Omega. \tag{A38}$$

Suppose that Assumptions A5 and A3 hold. Lemma A3 and definition (A37) imply that $d\Omega/dm > 0$. Lemma A3 and the relations (A5) and (A38) imply $d\epsilon^d/dm > 0$ and $d\epsilon^s/dm < 0$. Therefore, relationship (22) implies that $d\lambda/dm < 0$.