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GLOBALIZATION AND MULTIPRODUCT FIRMS

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ABSTRACT

Globalization and Multiproduct Firms

We present an international trade model of multiproduct firms where firms differ in their endowment of managerial resources and in how effectively these resources can be used in making production more efficient. The model gives rise to a trade-off between conglomerate and specialization strategies of firms, yielding testable predictions on the relationship between firm size, scope and productivity. More efficient firms become exporters, but not all exporters are large and not all large firms export. Following a trade liberalization, non-exporters experience a fall in their market-to-book ratio and consolidate the number of products they manage to lower their marginal costs while the opposite holds for exporters.

JEL Classification: F12 and F15
Keywords: diversification discount, firm heterogeneity, multiproduct firms, productivity and trade liberalization

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1 Introduction

Economists have documented huge heterogeneity across firms in many "performance measures" such as size, productivity, scope (the number of products managed), and export status. A number of systematic patterns have emerged. For instance, there is a tendency for large firms to be more productive, to produce more products and to have a greater likelihood of exporting. However, the correlation between these performance measures is far from perfect (Bartelsman, Haltiwanger and Scarpetta, 2011). While many large firms are highly profitable, there are many examples of large, unprofitable conglomerates. In fact, the corporate finance literature has documented a *diversification discount puzzle* according to which large, diversified firms trade at a discount (Lang and Stulz, 1994). Further, not all exporters are large and not all large firms export: over the entire spectrum of firm sizes, exporters and non-exporters co-exist (Hallak and Sivadasan, 2011).

In addition to the documented heterogeneity in the levels of performance measures and their co-variance, there is also heterogeneity in the response of firms to trade liberalization and other economic shocks. While non-exporting firms tend to consolidate the number of products they manage (Baldwin and Gu, 2009), the opposite has been documented for exporters (Iacovone and Javorcik, 2010). Further, Lileeva and Trefler (2010) show that only those firms that switch to become exporters following a trade liberalization experience an increase in their productivity.

In this paper, we develop a model with two-dimensional firm heterogeneity that allows us to span the observed variation in firm performance measures and in the response to trade shocks. In our model, a firm's size, scope and observed productivity are endogenous and reflect an interaction between underlying firm characteristics and the international trade environment. We study two sources of firm heterogeneity. First, firms differ in their endowment of managerial resources. This endowment can be used to expand the firm's product portfolio and to lower its marginal cost of production. Second, firms differ in how efficiently managerial resources can be applied to the reduction in marginal costs. The fact that managerial resources are scarce within the firm implies a trade-off between lowering marginal costs and expanding the firm's scope. Those firms that use managerial resources more efficiently than others will emphasize cost reduction at the expense of firm scope. Holding managerial efficiency fixed, however, the larger is a firm's endowment of managerial resources, the larger will be its product range.

The model gives rise to a number of subtle predictions on the relationship between firm size, scope and productivity. Controlling for product diversification, the model gives rise to a size premium: larger firms exhibit a larger market-to-book ratio (Tobin's Q). However, controlling for firm size, the model predicts a diversification discount: firms that choose to diversify more exhibit a lower market-to-book ratio. These results speak directly to the corporate finance literature on the diversification discount puzzle (Lang and Stulz, 1994).

In an international trade setting, the model provides a rich set of predictions on export status. In contrast to Melitz (2003) and other papers where firms differ only in their innate productivity, not all exporters in our model are large and not all large firms are exporters: small, highly efficient exporters focusing on a small number of products co-exist with large, inefficient conglomerates that sell a large range of products only domestically (which is consistent with the empirical findings of Hallak and Sivadasan, 2011). For a given endowment of managerial resources, more efficient firms sort into exporting while less efficient firms choose to sell only domestically (Bernard and Jensen, 1999).

Trade liberalization induces changes in productivity that reflect a change in the incentives facing each firm toward allocating its scarce managerial resources. Following a fall in trade costs, non-exporting firms drop products while exporting firms expand their product range (Baldwin and Gu, 2009; Iacovone and Javorcik, 2010). Firms that switch to become exporters experience the largest increase in productivity as they choose to be "leaner and meaner" so as to compete in the global market (Lileeva and Trefler, 2010).

The paper contributes to the international trade literature in several ways. While nesting Melitz (2003) as a special case, the paper relaxes the assumption of extreme diseconomies of scope typically found in the trade literature. As the marginal cost of each product is endogenous, unlike in standard selection-driven models, the model gives rise to a trade-off between conglomerate and specialization strategies of firms. Moreover, the model results in rich comparative statics. The paper also demonstrates how Tobin's Q can be used to assess productivity comparisons across large multiproduct firms and illustrates that the effects of globalization on firm-level productivity depend delicately on which measure of productivity is used.

Our paper is most closely related to the nascent literature that is concerned with multiproduct firms in international trade (Eckel and Neary, 2010; Bernard, Redding and

Schott, 2011; Mayer, Melitz and Ottaviano, 2012).[1] In these papers, firms typically draw a distribution of marginal costs for various products of different degrees of substitutability so that the marginal cost of any given product is exogenous. Moreover, these papers focus on the within-firm distribution of marginal costs. Only low marginal cost products are exported, and trade liberalization induces firms to shed weaker products to "focus on their core competencies." Instead, we abstract from within-firm heterogeneity in order to explore a rather different mechanism, namely one where a firm's marginal cost for any given product depend's on how the firm solves the trade-off between product proliferation and specialization, and firms differ in the extent of this trade-off. This allows us to explain additional features of the data, such as the diversification discount and the co-existence of large and small exporters.

Our model is also related to a number of papers in the trade literature in introducing multiple sources of firm heterogeneity such as Nocke and Yeaple (2008), Davis and Harrigan (2011), Harrigan and Reshef (2011), and Hallak and Sivadasan (2011). To the best of our knowledge, our paper is unique in considering managerial competencies that independently drive firm efficiency, exporting status, and the extent of firm scope.

The plan of the paper is as follows. In the next section, we set out the closed economy model. In Section 3, we derive the equilibrium in the closed economy and show how different firms allocate their managerial resources differently, resulting in rich patterns of firm scale and scope. We also demonstrate that the model gives rise to a diversification discount when controlling for firm size and to a size premium when controlling for firm scope. In Section 4, we embed the model in an international trade setting with two identical countries. We show that exporters will be more efficient than non-exporters but that not all exporters will be large firms and that not all large firms will become exporters. Further, we analyze the effects of globalization on firm scale and scope, and on productivity. We conclude in Section 5.

[1] Dingra (2011) analyzes R&D decisions of multiproduct firms. Firms' R&D efforts internalize the effect of cannibalization across products in their portfolio. Arkolakis and Muendler (2011) bring the model of Bernard, Redding and Schott (2011) to the data.

2 The Closed Economy Model

We consider a closed economy with a single (differentiated goods) sector and a single factor of production (labor). There is a mass L of identical consumers (workers) with a CES utility function:

$$U = \left[\int_{\Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}},$$

where $x(\omega)$ is consumption of product $\omega \in \Omega$, and $\sigma > 1$ the elasticity of substitution between products.

Each worker supplies a single unit of labor. The wage rate in the economy is denoted w, which we henceforth use as *numéraire* and set $w \equiv 1$. Aggregate income is thus equal to L. The resulting aggregate demand for product ω is given by

$$X(\omega) = Ap(\omega)^{-\sigma}, \tag{1}$$

where $p(\omega)$ is the price of product ω and

$$A \equiv \frac{L}{\int_{\Omega} p(\omega)^{-(\sigma-1)} d\omega}$$

the residual demand level.

There is a sufficiently large mass of atomless and *ex ante* identical potential entrants. If a potential entrant decides not to enter, it obtains a profit of zero. If it does decide to enter, the firm has to incur an irrecoverable entry cost F^e. A fraction $F/F^e \in (0, 1]$ of this entry cost is used to build firm-specific capital equipment for which there is no resale market. Upon entry, the firm receives a random draw of its type $(\widetilde{\theta}, T)$ from a continuous distribution function $\widetilde{G}$ with associated density $\widetilde{g}$ and support $(0, 1/(\sigma - 1)) \times [1, \infty)$. A firm's type $(\widetilde{\theta}, T)$ consists of two elements: its managerial efficiency, $\widetilde{\theta}$, and its endowment of managerial resources, T.

After learning its type, the entrant has to decide on the size of its product portfolio. For each of the N products it chooses to manage, the firm has to incur a fixed cost f to build product-specific capital equipment. In addition to the fixed cost per product, the firm has to incur a constant labor cost per unit of output. The marginal cost of product ω, denoted $c(\omega; t_\omega, \widetilde{\theta})$, is decreasing in the amount of managerial resources, t_ω, that the firm chooses to spend on the product:

$$c(\omega; t_\omega; \widetilde{\theta}) = z\,(t_\omega)^{-\widetilde{\theta}}$$

if $t_\omega \geq 1$, and $c(\omega; t_\omega; \widetilde{\theta}) = \infty$ otherwise, where $z > 0$ is a cost parameter that is common to all firms and products. The greater is the firm's managerial efficiency $\widetilde{\theta}$, the greater is the rate at which the firm's marginal cost decreases with managerial effort. However, the firm cannot use more than the managerial resources with which it is endowed. That is, the firm faces the following resource constraint:

$$\sum_{\omega \in \mathcal{I}} t_\omega \leq T,$$

where $\mathcal{I}$ is the set of products managed by the firm.

Remark 1 *Suppose all firms were restricted to manage a single product, i.e., $N \equiv 1$, so that each entrant would optimally set $t_\omega = T$ for the single product ω it manages. In that case, the model would boil down to the Melitz (2003) model where firm heterogeneity is only one-dimensional. To see this, note that if $N \equiv 1$ in our model, then any two firms $(\widetilde{\theta}', T')$ and $(\widetilde{\theta}'', T'')$ with $(T')^{-\widetilde{\theta}'} = (T'')^{-\widetilde{\theta}''}$ would face the same marginal cost, and thus behave identically, implying that the model with two-dimensional heterogeneity could be transformed into one with one-dimensional heterogeneity in productivity $\varphi \equiv zT^{-\widetilde{\theta}}$, exactly as in Melitz (2003).*

The sequence of moves is as follows. First, potential entrants decide whether or not to enter the market. Second, each entrant decides how many products to manage and how many managerial resources to spend on each of them. Third, each firm sets the prices of its various products so as to maximize its profit.

3 The Closed Economy: Analysis

In this section, we analyze equilibrium in the closed economy. We first derive the equilibrium and show how the firm's choice of scope varies with the firm's type. Next, we analyze the mapping from a firm's type to various measures of firm "performance," namely profit, sales, marginal cost, and Tobin's Q. Finally, we show that the model predicts a "size premium" (holding firm scope fixed) and a "diversification discount" (holding firm size fixed).

3.1 Derivation of Equilibrium

As the demand function (1) is iso-elastic, each firm will charge a constant markup over marginal cost, so that the profit-maximizing price is given by

$$p(\omega; t_\omega; \widetilde{\theta}) = \left(\frac{\sigma}{\sigma - 1}\right) c(\omega; t_\omega; \widetilde{\theta}).$$

The firm's post-entry profit over its $N = \#\mathcal{I}$ products is thus equal to

$$\frac{A}{\sigma}\left(\frac{\sigma - 1}{\sigma z}\right)^{\sigma-1} \sum_{\omega \in \mathcal{I}^+} (t_\omega)^\theta - Nf,$$

where $\mathcal{I}^+$ is the set of products for which $t_\omega \geq 1$, and $\theta \equiv \widetilde{\theta}(\sigma - 1) \in (0, 1)$. Note that this expression is increasing and concave in the t_ω's. For a given set $\mathcal{I}$ of products, it is thus optimal for the firm to exhaust all of its endowment of managerial resources (i.e., set $\sum_{\omega \in \mathcal{I}} t_\omega = T$) and, for each $\omega \in \mathcal{I}$, choose either $t_\omega = t \geq 1$ or $t_\omega = 0$. To ease notation, we will henceforth refer to $\theta \equiv \widetilde{\theta}(\sigma - 1)$ as to the firm's managerial efficiency, and to (θ, T) as to the firm's type, with associated cdf G and g, and support $\Theta \equiv (0, 1) \times [1, \infty)$.

Let $N(\theta, T) > 0$ denote the profit-maximizing number of products for a firm of type (θ, T). This choice of firm scope satisfies $N(\theta, T) \leq T$. (Otherwise, if $N(\theta, T) > T$, the firm would optimally allocate zero managerial resources to at least $T - N(\theta, T)$ goods and thus choose not to produce them; but then it would be optimal for the firm to set $N < N(\theta, T)$ so as to save on the fixed cost per product, a contradiction.) The marginal cost of a firm of type (θ, T) can thus be written as $c(\theta, T) = z\,(T/N(\theta, T))^{-\theta/(\sigma-1)}$, and the firm's profit-maximizing price as

$$p(\theta, T) = \left(\frac{\sigma}{\sigma - 1}\right) z \left(\frac{T}{N(\theta, T)}\right)^{-\frac{\theta}{\sigma-1}}. \tag{2}$$

The firm's post-entry profit is given by

$$\pi(\theta, T) = N(\theta, T) f \left[\zeta \left(\frac{T}{N(\theta, T)}\right)^\theta - 1\right], \tag{3}$$

where

$$\zeta \equiv \frac{A}{\sigma f}\left(\frac{\sigma - 1}{\sigma z}\right)^{\sigma-1}, \tag{4}$$

$$A = \frac{L}{M\left[\int_\Theta N(\theta, T) p(\theta, T)^{-(\sigma-1)} dG(\theta, T)\right]}, \tag{5}$$

and M is the mass of entrants. As ζ is proportional to A, we will henceforth (with a slight abuse of language) refer to ζ as to the markup-adjusted residual demand level.

For simplicity, we will in the following focus on the case where $\zeta > 1$. It is straightforward to show that, in a free entry equilibrium, $\zeta > 1$ if the entry cost F^e is sufficiently large. Moreover, we will be abstracting from the integer constraints on the number of products so that N can take the value of any nonnegative real number.

The following proposition characterizes the profit-maximizing choice of firm scope:

Proposition 1 *In equilibrium, a firm of type (θ, T) chooses to manage*

$$N(\theta, T) = \begin{cases} T & \text{if } \theta \in (0, \underline{\theta}] \\ T\left[(1-\theta)\zeta\right]^{1/\theta} & \text{if } \theta \in [\underline{\theta}, 1) \end{cases} \tag{6}$$

products, where $\underline{\theta} \equiv (\zeta - 1)/\zeta \in (0, 1)$. The profit-maximizing number of products, $N(\theta, T)$, is thus proportional to the firm's endowment of managerial resources, T, independent of its managerial efficiency θ for $\theta < \underline{\theta}$, and strictly decreasing in θ for $\theta > \underline{\theta}$.

Proof. See Appendix. ■

The proposition shows how the choice of scope varies across firms. Holding managerial efficiency θ fixed, firms with a greater endowment of managerial resources, T, choose to spread their resources over a proportionately larger number of products, implying that marginal cost $c(\theta, T)$ does not vary with T for a given value of θ. Holding the endowment T fixed, firms with greater managerial efficiency θ (above the threshold $\underline{\theta}$) choose a smaller number of products so as to invest more managerial effort per product. As a result, marginal cost $c(\theta, T)$ is decreasing in θ for $\theta \geq \underline{\theta}$ and a given value of T. The proposition also shows that $N(\theta, T) > 0$ for any type $(\theta, T) \in \Theta$. That is, all entrants choose to be active; there are no "selection effects" in our closed economy model.

Inserting (6) into (3), we can rewrite the post-entry profit of a firm of type (θ, T) as

$$\pi(\theta, T) = \begin{cases} Tf\left[\zeta - 1\right] & \text{if } \theta \in (0, \underline{\theta}], \\ Tf\left[(1-\theta)\zeta\right]^{1/\theta}\left(\frac{\theta}{1-\theta}\right) & \text{if } \theta \in [\underline{\theta}, 1). \end{cases}$$

That is, a firm's post-entry profit is proportional to its endowment of managerial resources, T. Differentiating $\pi(\theta, T)$ with respect to the firm's managerial efficiency θ further establishes that $\partial \pi(\theta, T)/\partial \theta = 0$ for $\theta < \underline{\theta}$, and $\partial \pi(\theta, T)/\partial \theta > 0$ for $\theta > \underline{\theta}$.

Since potential entrants are ex identical, free entry implies that they must be indifferent between entering and not:

$$\int_{\Theta} \pi(\theta, T) dG(\theta, T) - F^e = 0. \tag{7}$$

An equilibrium in the closed economy is given by the collection $\{M, N(\cdot,\cdot), p(\cdot,\cdot), \pi(\cdot,\cdot), \zeta\}$ satisfying equations (2)-(7).

3.2 Measures of Firm Performance

We now investigate how various measures of firm performance (sales, marginal cost and Tobin's Q) vary with firm type in equilibrium.

The standard measure of firm size is total firm sales. Total sales (over all products) of a firm of type (θ, T) are given by

$$\begin{aligned} S(\theta, T) &= N(\theta, T) A p(\theta, T)^{-(\sigma-1)} \\ &= \sigma f \zeta N(\theta, T) \left(\frac{T}{N(\theta, T)} \right)^{\theta}, \end{aligned}$$

or

$$S(\theta, T) = \begin{cases} \sigma f \zeta T & \text{if } \theta \in (0, \underline{\theta}], \\ \sigma f \zeta T \left[(1-\theta)\zeta \right]^{\frac{1-\theta}{\theta}} & \text{if } \theta \in [\underline{\theta}, 1). \end{cases} \tag{8}$$

So, the sales over all products is proportional to the firm's endowment of managerial resources T. For low values of θ, sales are independent of the firm's managerial efficiency, $\partial S(\theta, T)/\partial \theta = 0$ for $\theta < \underline{\theta}$, as these firms do not invest more than the minimum amount of managerial resources per product. For $\theta > \underline{\theta}$, sales are first decreasing and then increasing in the firm's managerial efficiency:

$$\frac{\partial \ln S(\theta, T)}{\partial \theta} = -\frac{1}{\theta^2} \left\{ \ln \left[(1-\theta)\zeta \right] + \theta \right\},$$

which is positive if and only if

$$\Lambda(\theta) \equiv \ln(1-\theta) + \ln \zeta + \theta < 0. \tag{9}$$

Note that $\Lambda(\underline{\theta}) = \underline{\theta} > 0$, and $\Lambda(\theta) < 0$ for θ sufficiently close to one (as $\zeta > 1$ by assumption). Moreover, $\Lambda'(\theta) = -1/(1-\theta) + 1 < 0$, implying that there exists a unique $\overline{\theta} \in (\underline{\theta}, 1)$ such that $\Lambda(\overline{\theta}) = 0$. Hence, $\partial S(\theta, T)/\partial \theta < 0$ for $\theta \in (\underline{\theta}, \overline{\theta})$, and $\partial S(\theta, T)/\partial \theta > 0$

for $\theta \in (\overline{\theta}, 1)$. That is, for intermediate values of managerial efficiency $\theta \in (\underline{\theta}, \overline{\theta})$, firms with greater managerial efficiency choose to be smaller (in terms of sales) because they prefer to focus their managerial effort on fewer products.

Let us now turn to measures of firm productivity. One such measure is the firm's marginal cost,

$$c(\theta, T) = \begin{cases} z & \text{if } \theta \in (0, \underline{\theta}], \\ z\left[(1-\theta)\zeta\right]^{\frac{1}{\sigma-1}} & \text{if } \theta \in [\underline{\theta}, 1), \end{cases}$$

which is independent of the firm's endowment of managerial resources T, and decreasing in the firm's managerial efficiency θ for $\theta > \underline{\theta}$ (and independent of θ for $\theta < \underline{\theta}$).[2]

Remark 2 *The empirical literature has shown that there is a tendency for large firms to have lower marginal costs than small firms (e.g., Bartelsman et al., 2011). If the distribution of firm types were such that it could be characterized as an ordered pair* $(\theta, T(\theta))$ *with*

$$\frac{T'(\theta)\theta}{T(\theta)} = 1 + \frac{\ln[(1-\theta)\zeta]}{\theta} > 1$$

for all $\theta \in (\underline{\theta}, 1)$*, then our model would be consistent with these facts. More generally, if there is a sufficiently strong positive correlation between* θ *and* T *in the population of firms, our model is able to match this fact.*

Variation in marginal cost across firms is the main object of interest in most of the literature on firm heterogeneity.[3] But there are many problems with measuring variation in marginal costs in practice. Moreover, and more fundamentally, measuring marginal costs leaves out other important dimensions of firm performance. As regards measuring marginal cost in practice, one approach consists in measuring TFP at the level of the plant or the firm. Such calculations, while common, suffer from a number of practical problems. First, factor inputs across firms may vary substantially in terms of their quality so that TFP measurements may not correspond to variation in marginal

[2] Note that marginal cost is negatively related to sales per product:

$$c(\theta, T) = \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{1}{A} \cdot \frac{S(\theta, T)}{N(\theta, T)}\right)^{-\frac{1}{\sigma-1}}.$$

[3] We have in mind marginal cost of producing a unit of utility which encompasses quality variation across firms.

costs. Second, most datasets lack information on output prices which is problematic for models that feature product differentiation and iso-elastic demand. Third, when firms (or plants) produce multiple products that use different technologies there is no mapping from firm-level input usages to product-level outputs.

The more fundamental problem with marginal cost as a performance measure is that it does not reflect the value that a firm creates by offering a wide range of products. In love-of-variety models consumers obtain value from the number of products a firm makes available as well as the quantity of each good. A proper assessment of firm performance should account for a firm's ability to provide a wide range of goods.

An alternative measure of firm performance that can readily be calculated from publicly available data is Tobin's Q or the market value of a firm relative to the book value of its assets. In standard firm heterogeneity models such as Melitz (2003), this measure provides a consistent measure of a firm's relative marginal costs. A firm that appears more productive because it uses more expensive inputs will be "penalized" by having a lower Tobin's Q unlike naïve measures of TFP. Further, Tobin's Q does not require product-level prices nor does it require information on the various technologies used to produce individual goods. Finally, Tobin's Q measures the value generated by a firm due to both its ability to produce a given set of products at low marginal cost and its ability to produce a wide range of products.

In our model, the book value of a firm is equal to its expenditure on capital equipment, $F + N(\theta, T)f$, whereas the firm's market value is equal to its profit gross of any capital cost, $\pi(\theta, T) + N(\theta, T)f$. (Recall that capital is specific to the firm, so the market value of the firm's capital equipment is zero.) That is, the market-to-book ratio of a firm of type (θ, T) is given by

$$Q(\theta,T) = \frac{N(\theta,T)f\zeta\left(\frac{T}{N(\theta,T)}\right)^{\theta}}{N(\theta,T)f+F} \tag{10}$$

$$= \begin{cases} \zeta\left[1+\frac{F}{Tf}\right]^{-1} & \text{if } \theta \in (0,\underline{\theta}], \\ \left[(1-\theta)\left(1+\frac{F}{Tf[(1-\theta)\zeta]^{1/\theta}}\right)\right]^{-1} & \text{if } \theta \in [\underline{\theta},1). \end{cases} \tag{11}$$

It can easily be verified that $Q(\theta, T)$ is strictly increasing in the firm's endowment of managerial resources, T. Holding T fixed, $Q(\theta, T)$ is independent of θ for $\theta < \underline{\theta}$, and strictly increasing in managerial efficiency θ for θ sufficiently close to one. However, for intermediate values of θ, the market-to-book ratio $Q(\theta, T)$ may be increasing or decreasing in θ, depending on parameter values.

As we will now show, even without imposing any assumptions on the joint distribution G of the endowment of managerial resources (T) and managerial efficiency (θ), the model makes empirically testable predictions on cross-sectional correlations when controlling for (endogenous) firm characteristics. Holding firm scope (the endogenous number of products) fixed, the model predicts a "size premium" in that larger firms have a larger market-to-book ratio; holding firm size (sales) fixed, the model also predicts a "diversification discount" in that more diversified firms have a smaller market-to-book ratio. These results speak to two empirical literatures: on the one hand, several studies have found a positive correlation between productivity and firm size (e.g., Bartelsman et al., 2011); on the other, the corporate finance literature has identified a "diversification discount puzzle" (Lang and Stulz, 1994), according to which more diversified firms have a smaller market-to-book ratio.[4]

In Figure 1, we depict "iso-size curves" and "iso-diversification" curves in firm type space $\Theta \equiv (0,1) \times [1,\infty)$. The iso-size curve corresponding to firm size $\overline{S}$ is defined as the graph of types (θ, T) such that $S(\theta, T) = \overline{S}$. Similarly, the iso-diversification curve corresponding to $\overline{N}$ products is defined as the graph of types (θ, T) such that $N(\theta, T) = \overline{N}$. We partition the type space into three regions, depending on the value of θ:

Region I Firm types (θ, T) such that $\theta \in (0, \underline{\theta})$;

Region II Firm types (θ, T) such that $\theta \in (\underline{\theta}, \overline{\theta})$;

Region III Firm types (θ, T) such that $\theta \in (\overline{\theta}, 1)$.

In Region I, all iso-size curves and iso-diversification curves are flat lines as firm sales and firm scope are independent of managerial efficiency θ. From equation (8), the iso-size curve corresponding to firm size $\overline{S}$ is given by $T = (\sigma f \zeta)^{-1} \overline{S}$, whereas equation (6) implies that the iso-diversification curve corresponding to $\overline{N}$ products is given by $T = \overline{N}$.

[4]Several explanations of the diversification discount puzzle have been proposed in the corporate finance literature. For instance, Rajan, Servaes and Zingales (2000) provide an explanation based on agency costs that result in the misallocation of resources across divisions. Maksimovic and Phillips (2002) argue that the diversification discount puzzle can better be explained by comparative advantage across sectors. There are also some who argue that the diversification discount may in fact be a statistical artifact of selection (Villalonga, 2004).

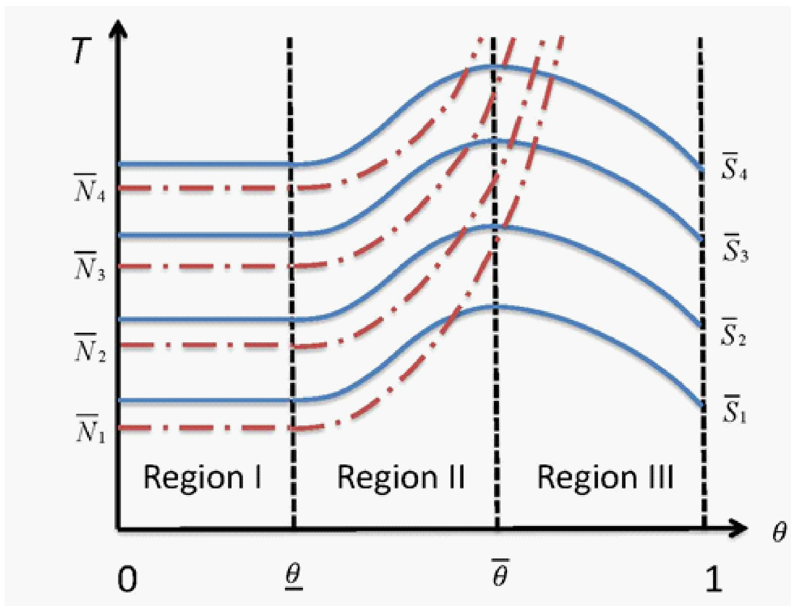

Figure 1: The iso-diversification curves (hatched curves with $\overline{N}_1 < \overline{N}_2 < \overline{N}_3 < \overline{N}_4$) are flat in Region I and upward-sloping in Regions II and III. The iso-size curves (solid curves with $\overline{S}_1 < \overline{S}_2 < \overline{S}_3 < \overline{S}_4$) are flat in Region I, upward-sloping in Region II, and downward-sloping in Region III.

Turning to Regions II and III, it follows from equations (6) and (8) that the iso-size and iso-diversification curves are given by

$$T = \frac{\overline{S}}{\sigma f \zeta \left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}}} \tag{12}$$

and

$$T = \frac{\overline{N}}{\left[(1-\theta)\zeta\right]^{\frac{1}{\theta}}}, \tag{13}$$

respectively. These equations can be written in logs as

$$\ln T = \ln \overline{S} - \ln (\sigma f \zeta) - \left(\frac{1-\theta}{\theta}\right) \ln \left[(1-\theta)\zeta\right]$$

and

$$\ln T = \ln \overline{N} - \left(\frac{1}{\theta}\right) \ln \left[(1-\theta)\zeta\right],$$

respectively. Differentiating with respect to θ, we obtain that

$$\begin{aligned} \left.\frac{d \ln T}{d\theta}\right|_{S(\theta,T)=\overline{S}} &= \frac{1}{\theta^2}\left\{\theta + \ln\left[(1-\theta)\zeta\right]\right\} \\ &< \frac{1}{\theta^2}\left\{\frac{\theta}{1-\theta} + \ln\left[(1-\theta)\zeta\right]\right\} \\ &= \left.\frac{d \ln T}{d\theta}\right|_{N(\theta,T)=\overline{N}}. \end{aligned}$$

That is, at any point of intersection, the iso-diversification curve cuts the iso-size curve from below. Now, recall from the discussion after equation (9) that $\Lambda(\theta) \equiv \theta + \ln\left[(1-\theta)\zeta\right] > 0$ if $\theta \in (\underline{\theta}, \overline{\theta})$, and $\Lambda(\theta) < 0$ if $\theta \in (\overline{\theta}, 1)$. Recall also from the discussion after equation (22) in the proof of Proposition 1 that $\Psi(\theta) \equiv \theta + (1-\theta)\ln\left[(1-\theta)\zeta\right] > 0$ for all $\theta \in (\underline{\theta}, 1)$. Hence, in Region II, all iso-size and iso-diversification curves are upward-sloping. In Region III, the iso-diversification curves continue to be upward-sloping, whereas the iso-size curves are downward-sloping.

We are now in the position to state the key result on the relationship between firm size and firm scope on the one hand and firm productivity, as measured by Tobin's Q, on the other:

Proposition 2 *Along an iso-diversification curve, there is a positive relationship between firm size $S(\theta, T)$ and the market-to-book ratio $Q(\theta, T)$. Along an iso-size curve,* *there is a negative relationship between firm scope $N(\theta, T)$ and the market-to-book ratio $Q(\theta, T)$. That is, the model predicts a* size premium *when controlling for firm scope, and a* diversification discount *when controlling for firm size.*

Proof. Consider first moving along an iso-diversification curve from the left to the right in type space. In Region I, the curve is flat and completely overlaps with one iso-size curve. From (11), the market-to-book ratio $Q(\theta, T)$ is independent of θ in Region I. Hence, all points on the same iso-diversification curve in Region I are associated with the same level of firm sales and the same value of Tobin's Q. In Regions II and III, the iso-diversification curve is upward-sloping and cuts the family of iso-size curves from below. That is, as we move along the iso-diversification curve to the right, firm size increases, as do the values of θ and T. From equation (10), the market-to-book ratio corresponding to firm scope $\overline{N}$ is given by

$$Q(\theta, T)|_{N(\theta,T)=\overline{N}} = \frac{\overline{N} f \zeta \left(\frac{T}{\overline{N}}\right)^{\theta}}{\overline{N} f + F},$$

which is strictly increasing in T and θ. Hence, along an iso-diversification curve, there is a positive relationship between firm size $S(\theta, T)$ and the market-to-book ratio $Q(\theta, T)$.

Consider now moving along an iso-size curve from the left to the right in type space. In Region I, the curve is flat and completely overlaps with one iso-diversification curve. From (11), the market-to-book ratio $Q(\theta, T)$ is independent of θ in Region I. Hence, all points on the same iso-size curve in Region I are associated with the same level of firm sales and the same value of Tobin's Q. In Regions II and III, the iso-size curve is first upward-sloping (in Region II) and then downward-sloping (in Region III), and cuts the family of iso-diversification curves from above. That is, as we move along the iso-size curve to the right, firm scope decreases with increasing θ. From equations (11) and (12), the market-to-book ratio corresponding to firm size $\overline{S}$ is given by

$$Q(\theta, T)|_{S(\theta,T)=\overline{S}} = \left[1 - \theta + \frac{\sigma F}{\overline{S}}\right]^{-1}, \tag{14}$$

which is strictly increasing in θ and independent of T. Hence, along an iso-size curve, there is a negative relationship between firm scope $N(\theta, T)$ and the market-to-book ratio $Q(\theta, T)$. ■

Equation (14) shows that the firm's endowment of managerial resources, T, affects Tobin's Q only through its effect on firm size, S. Once one controls for firm size, Tobin's Q depends only on the firm's managerial efficiency θ, and monotonically so.

4 The Open Economy

We now extend the model to incorporate a simple trading environment between two identical countries, home and foreign. Following Melitz (2003), we assume that for a firm to export any particular product, it must first incur a fixed capital cost of f^x in order to set up a distribution outlet in the foreign market that is specific to that product. Moreover, for each unit of a product shipped to a foreign market it must incur an iceberg-type trading cost $\tau > 1$. When firms are constrained to produce a single product, the open model thus simplifies to Melitz (2003).

We first characterize the organization of production across firms of heterogeneous type (θ, T), focusing on endogenous firm scope, the number of countries served, and measures of firm productivity (including Tobin's Q). We then consider comparative static exercises in which trade costs between countries fall in symmetric fashion.

4.1 International Organization of Production

To avoid a taxonomy of cases, we impose (as in the closed economy case) an implicit restriction on parameters such that the markup-adjusted residual demand level ζ satisfies[5]

$$\frac{\ln(1 + f^x/f)}{\tau^{1-\sigma}} > \zeta > 1. \tag{15}$$

We first note that a firm that exports a good to the foreign country will always sell that good in the home market as there is no additional fixed cost to doing so. Let δ be the share of goods exported, t^d the managerial resources allocated to a product that is sold only domestically, and t^x the managerial resources allocated to an export product. The profit associated with a firm of type (θ, T) is then

$$\pi(\theta, T) = \max_{\delta, t^d, t^x, N} \left\{ N \left((1-\delta) f \left[\zeta \left(t^d\right)^{\theta} - 1 \right] + \delta \left[f \zeta (1+\rho) \left(t^x\right)^{\theta} - (f + f^x) \right] \right) \right\}, \tag{16}$$

where $\rho \equiv \tau^{1-\sigma}$ is the "freeness" of trade.

The following proposition shows how the choice of firm scope varies across firm types. It establishes that firms that choose to export are those with the greatest managerial efficiency and that such firms always choose to export all of their products.

[5] This restriction implies that, in equilibrium, any firm (θ, T) that chooses to export also chooses not to be maximally diversified in terms of its product range (i.e., $N(\theta, T) < T$).

Proposition 3 *In the equilibrium of the open economy, a firm of type (θ, T) chooses to manage*

$$N(\theta,T) = \begin{cases} T & \text{if } \theta \in (0, \underline{\theta}], \\ T\left((1-\theta)\,\zeta\right)^{\frac{1}{\theta}} & \text{if } \theta \in [\underline{\theta}, \theta^x), \\ T\left[\frac{1+\rho}{1+f^x/f}\,(1-\theta)\,\zeta\right]^{\frac{1}{\theta}} & \text{if } \theta \in (\theta^x, 1) \end{cases} \tag{17}$$

products, where

$$\theta^x \equiv 1 - \frac{\ln(1+\rho)}{\ln(1+f^x/f)} \in (\underline{\theta}, 1), \tag{18}$$

and exports share

$$\delta(\theta,T) = \begin{cases} 0 & \text{if } \theta \in (0, \theta^x), \\ 1 & \text{if } \theta \in (\theta^x, 1), \end{cases}$$

of its products,

Proof. See Appendix. ■

Proposition 3 demonstrates that a firm's export decision is independent of its endowment of managerial resources, T, depending only on its managerial efficiency, θ. A firm that decides to switch to exporting optimally chooses to lower the marginal cost of its production processes by focusing its endowment of managerial resources on fewer products. The opportunity cost of becoming productive enough to export is the reduction in domestic profits due to the reduced product range. As high-θ firms have an advantage at lowering marginal cost, the balance for such firms is tipped in favor of exporting. It can readily be verified that, holding fixed T, the number of products managed by a firm, $N(\theta,T)$, is strictly decreasing in θ for all $\theta > \underline{\theta}$ and drops discontinuously at θ^x. This discrete reduction in the number of products managed is due to the desire of a firm to become "leaner and meaner" once it has chosen to sell each of its products in both the domestic and foreign market. To see this, note that the endogenous marginal cost of production as a function of firm type is

$$c(\theta,T) = \begin{cases} z & \text{if } \theta \in (0, \underline{\theta}) \\ z\left((1-\theta)\zeta\right)^{\frac{1}{\sigma-1}} & \text{if } \theta \in (\underline{\theta}, \theta^x) \\ z\left[(1-\theta)\zeta\left(\frac{1+\rho}{1+f^x/f}\right)\right]^{\frac{1}{\sigma-1}} & \text{if } \theta \in (\theta^x, 1) \end{cases}.$$

To see that marginal cost drops discontinuously as θ increases from just below θ^x, recall that $\rho < f^x/f$. As in the closed economy case, the marginal cost of the firm is independent of T, and strictly decreasing in θ for all $\theta > \underline{\theta}$.

Remark 3 *Proposition 3 shows that exporters use their managerial resources more efficiently than non-exporters and, for a given endowment T, manage fewer products. In the data, as reported by Bernard, Redding and Schott (2011), it has been observed that firms that export tend to produce more products than firms that do not. Our framework can reproduce this result if there is a positive correlation between θ and T in the population of firms, as high-θ firms export and high-T firms choose to manage more products.*

A firm that has chosen to export tends to produce fewer goods but serves multiple markets at lower marginal cost than if it had chosen not to export. The post-entry profit of a firm of type (θ, T) is given by

$$\pi(\theta,T) = \begin{cases} Tf\left[\zeta - 1\right] & \text{if } \theta \in (0, \underline{\theta}], \\ Tf\left[(1-\theta)\zeta\right]^{1/\theta}\left(\frac{\theta}{1-\theta}\right) & \text{if } \theta \in [\underline{\theta}, \theta^x], \\ T(f+f^x)\left[\left(\frac{1+\rho}{1+f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}}\left(\frac{\theta}{1-\theta}\right) & \text{if } \theta \in [\theta^x, 1), \end{cases} \tag{19}$$

whereas its sales are given by

$$S(\theta,T) = \begin{cases} \sigma T f \zeta & \text{if } \theta \in (0, \underline{\theta}], \\ \sigma T f \zeta\left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}} & \text{if } \theta \in [\underline{\theta}, \theta^x), \\ \sigma T f \zeta (1+\rho)\left[\frac{1+\rho}{1+f^x/f}(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}} & \text{if } \theta \in (\theta^x, 1). \end{cases} \tag{20}$$

Equation (20) establishes that sales are discontinuous in θ at the export cutoff θ^x, jumping up as the number of products managed drops. To see this, consider the logarithm of the ratio of total sales of an exporter as θ goes to θ^x from above to total sales of a non-exporter as θ goes to θ^x from below:

$$\begin{aligned} \ln\left(\frac{\lim_{\theta\downarrow\theta^x} S(\theta,T)}{\lim_{\theta\uparrow\theta^x} S(\theta,T)}\right) &= \ln(1+\rho) + \left(\frac{1}{\theta^x} - 1\right)\ln\left[\frac{1+\rho}{1+f^x/f}\right] \\ &= \frac{1}{\theta^x}\ln\left(\frac{1+\rho}{1+f^x/f}\right) + \ln\left(1+\frac{f^x}{f}\right) \\ &= \frac{1-\theta^x}{\theta^x} + \ln\left(1+\frac{f^x}{f}\right) > 0, \end{aligned}$$

where we have used the definition of θ^x to establish the last equality. From (20) it is clear that our model is generally able to generate firms of equal size in terms of their aggregate sales in which some of these firms export and others do not. Specifically, one can always find a firm of type (T, θ) with $\theta > \theta^x$ and a firm of type (T', θ') with $T' > T$

and $\theta' < \theta^x$ such that $S(\theta, T) = S(\theta', T')$. Our model is therefore consistent with the empirical regularity noted by Hallak and Sivadasan (2011) that across the size spectrum exporters co-exist with non-exporters.

As previously noted, it is often difficult to measure the marginal costs even in the case of single product firms and that Tobin's Q sidesteps these issues by revealing the value of firms' intangible assets that give rise to heterogenous outcomes. If we treat f^x as being a tangible asset that is part of a firm's book value, then the value of Tobin's Q as a function of firm type is

$$Q(\theta, T) = \begin{cases} \frac{f\zeta}{f+\frac{F}{T}} & \text{if } \theta \in (0, \underline{\theta}), \\ \frac{f}{(1-\theta)\left(f+\frac{F}{N(\theta,T)}\right)} & \text{if } \theta \in (\underline{\theta}, \theta^x), \\ \frac{f+f^x}{(1-\theta)\left(f+f^x+\frac{F}{N(\theta,T)}\right)} & \text{if } \theta \in (\theta^x, 1), \end{cases} \tag{21}$$

where $N(\theta, T)$ is given by (17). As in the closed economy case, we note that a firm's type enters into the calculation of Tobin's Q in two places. First, for firms that are not maximally diversified (i.e., $N(\theta, T) < T$), an increase in θ raises profit per product, which tends to raise Tobin's Q, but also means that the firm spreads the fixed cost of capital expenditure at the entry stage, F, over fewer products. An increase in T induces a proportionate increase in the number of products managed and therefore does not affect the equilibrium level of marginal cost. However, firms that are endowed with a greater value of T have a higher market-to-book ratio because the book value of the entry cost, F, is is now spread over a larger number of products.

The following proposition shows that our previous result on the diversification discount carries over to the open economy setting:

Proposition 4 *Consider two firms of different types, (θ, T) and (θ', T'), with the same level of sales, i.e., $S(\theta, T) = S(\theta', T')$. Then, the firm that produces the larger number of products will have a lower market-to-book ratio: $N(\theta', T') > N(\theta, T)$ implies $Q(\theta', T') < Q(\theta, T)$. That is, there is a diversification discount in the equilibrium of the open economy.*

Proof. See Appendix. ■

4.2 The Effects of Globalization

In this section, we explore the effects of a reduction in the iceberg-type trade cost τ, or equivalently an increase in trade freeness ρ, on firms' choice of scope and the resulting impact on firm performance measures. We confine attention to changes that are small enough to preserve the parameter restriction (15). (In the following, we will index post-liberalization variables by a prime.)

The following lemma shows how trade liberalization affects the effective market size for exporters and non-exporters.

Lemma 1 *Consider an increase in trade freeness from ρ to $\rho' > \rho$. This lowers the effective market size facing non-exporters, i.e., $\zeta' < \zeta$, and raises the effective market size facing exporters, i.e., $\zeta'(1+\rho') > \zeta(1+\rho)$.*

Proof. See Appendix. ■

An immediate implication of the lemma is that trade liberalization results in an increase in welfare by inducing a lower price index (or, equivalently, an increase in the markup-adjusted residual demand level). The lemma also makes clear that a fall in trade cost reduces the effective market size facing firms that do not export while raising the effective market size of exporting firms. As exporting becomes more attractive and the domestic market less attractive, the cutoffs for maximal diversification and exporting changes, as the next proposition shows.

Proposition 5 *Consider an increase in trade freeness from ρ to $\rho' > \rho$. This induces the thresholds for exporting and for maximal diversification to fall: $\theta^{x\prime} < \theta^x$ and $\underline{\theta}' < \underline{\theta}$.*

Proof. Equation (18) immediately implies that $\theta^{x\prime} < \theta^x$. Lemma 1, which establishes that $\zeta' < \zeta$, and the fact that $\underline{\theta} \equiv (\zeta - 1)/\zeta$ is increasing in ζ, imply that $\underline{\theta}' < \underline{\theta}$. ■

As in Melitz (2003), an increase in the freeness of trade lowers the efficiency threshold above which a firm selects into exporting: Following a trade liberalization, any firm (θ, T) with $\theta \in (\theta^{x\prime}, \theta^x)$ will switch from non-exporting to exporting.

In this setting, there is also another form of selection. As the effective size of the domestic market becomes smaller, due to a reduction in trade costs, the threshold above which firms opt to be less than maximally diversified falls as well. The following proposition formally considers how the choice of firm scope is affected by a trade liberalization.

Proposition 6 *Consider an increase in trade freeness from ρ to $\rho' > \rho$. This causes firms that initially sold only domestically to drop products, i.e., $N(\theta, T)' \leq N(\theta, T)$ for all $\theta \in (0, \theta^x)$, with a strict inequality if $\theta \in (\underline{\theta}', \theta^x)$, and all continuing exporters to increase the number of products they manage, i.e., $N(\theta, T)' > N(\theta, T)$ on $\theta \in (\theta^x, 1)$.*

Proof. See Appendix. ■

Trade liberalization causes firms that do not export prior to the trade shock to drop product lines. This effect is especially strong for firms that are induced by the trade liberalization to switch to exporting because an exporter optimally wants to be "leaner and meaner," as discussed before. On the other hand, for continuing exporters the trade shock results in a larger effective market size to which they respond by adding more products. This result is consistent with the empirical evidence for the response of Mexican firms to the NAFTA trade liberalization as shown by Iacovone and Javorcik (2010) and for Canadian firms in response to the Canadian-U.S. Free Trade Agreement as shown by Baldwin and Gu (2009). These results suggest systematic and asymmetric changes in firms' marginal costs, which the following corollary substantiates.

Corollary 1 *Consider an increase in trade freeness from ρ to $\rho' > \rho$. For firms that initially sold only domestically, this results in lower marginal costs, i.e., $c(\theta, T)' \leq c(\theta, T)$ for all $\theta \in (0, \theta^x)$, with a strict inequality if $\theta \in (\underline{\theta}', \theta^x)$. For continuing exporters, this results in higher marginal costs, i.e., $c(\theta, T)' > c(\theta, T)$ on $\theta \in (\theta^x, 1)$.*

Proof. This follows directly from the definition of marginal cost and Proposition 6. ■

The changes in the number of product lines managed by firms of different types implies a particular productivity effect that varies across firms. Those non-exporters that choose to drop products experience an increase in their productivity (as measured by marginal cost) as these firms become "leaner and meaner." Those firms that switch to become exporters after the reduction in trade costs also see their productivity rise: as they face the first-order effect associated with paying the additional fixed cost f^x per product, they choose to become "leaner and meaner," too. This last observation is consistent with Lileeva and Trefler (2008) who show that Canadian firms that were induced to export by the Canadian-U.S. Free Trade Agreement saw their productivity increase relative to those that were not induced to export. Finally, continuing exporters see their productivity fall as they adjust to an effectively larger market by expanding their product scope.

However, as discussed in the last section, marginal cost is a very imperfect measure of firm productivity. To complete our analysis, we therefore now investigate how a trade liberalization affects Tobin's Q across firms.

Proposition 7 *Consider an increase in trade freeness from* ρ *to* $\rho' > \rho$*. There exists a threshold value of managerial efficiency,* $\widehat{\theta} \in (\theta^{x\prime}, \theta^{x})$*, such that any firm* (θ, T) *with managerial efficiency below that threshold experiences a reduction in their market-to-book ratio, i.e.,* $Q(\theta, T)' < Q(\theta, T)$ *if* $\theta < \widehat{\theta}$*, while the opposite holds for any other firm, i.e.,* $Q(\theta, T)' > Q(\theta, T)$ *if* $\theta > \widehat{\theta}$*.*

Proof. See Appendix. ■

Tobin's Q falls for non-exporters because the trade liberalization induces a decrease in the home market's markup-adjusted residual demand level ζ, which directly reduces the profitability of such firms. For exporting firms the increase in access to the foreign market more than compensates for this fall in ζ as $(1+\rho')\zeta' > (1+\rho)\zeta$. Hence, we see that there is a negative relationship between the effect of trade liberalization on a firm's marginal cost and the effect on a firm's Tobin's Q! Once again, the model cautions analysts to carefully consider what drives firms' performance measures. Rising marginal costs can be associated with rising profitability if greater access to foreign markets induces firms to diversify their product mix. On the other hand, firms that have seen their market share contract due to trade liberalization need to become leaner, and so reduce their marginal cost, and this belt tightening is associated with falling profitability.

5 Conclusion

In this paper, we have introduced a model of firm heterogeneity in which firms' management teams vary in their endowment of managerial resources and the efficiency with which these resources can be used to lower the marginal cost of individual products. The more products a given firm chooses to manage, the higher are the firm's marginal cost as an increase in firm scope implies that managerial resources have to be spread more thinly. Firms also have to decide whether to export their products or to remain focused on the domestic market. For a given endowment of managerial resources, firms with greater managerial efficiency choose to focus on producing a smaller number of products at lower marginal cost. The return to being lean and mean by focusing on a small number of products is magnified for firms that choose to sell in a large global market. Because firms differ both in the endowment of managerial resources as well as in their managerial efficiency the model can span a wide range of observed behavior in the data: firms that are large and profitable but not geared toward foreign markets co-exist with firms that are small and profitable while also outward-oriented.

An important message of our paper is that there are multiple dimensions along which firm performance should be assessed. Large conglomerates that are not particularly productive and are not engaged in exporting may still provide an important role in the economy by providing a wide range of products to local customers. We have shown that standard measures of TFP likely would identify such firms as ill-performing but that Tobin's Q, which takes into account a wider range of intangible assets within the firm, accurately identifies the ability to manage a wide range of products as well as valuing firm efficiency properly. Further, we have shown that the efficiency advantage enjoyed by a firm is reflected in a diversification discount once the absolute size of a firm has been controlled.

Our analysis of trade liberalization has revealed how productivity endogenously responds across firms. Increased import competition forces non-exporting firms to become leaner and meaner by shedding products and by so doing reducing their marginal cost. This effect is particularly strong for firms that switch to become exporters in response to a trade liberalization and so is consistent with a number of facts unearthed by a growing empirical literature. Finally, changes in Tobin's Q induced by a reduction in trade barriers reflect the changes in the relative value of different kinds of firm capabilities: the ability to manage a sprawling conglomerate due to a large endowment of managerial resources loses value relative to the managerial ability to focus on a few products in a more competitive global market. We believe that these observations offer a wide range of predictions that could be tested on publicly available data.

An interesting avenue of future research consists in extending the model by introducing heterogeneity at the product level: after deciding how many products to manage, the firm gets an independent random draw of the cost parameter z for each one of its products. This extension would allow the model to match at least two additional empirical facts: First, the distribution of sales across products within the same firm is skewed and, second, exporters typically choose not to export all of their products.

6 Appendix: Proofs

Proof of Proposition 1. Let $\widetilde{N}(\theta, T)$ denote the solution to the first-order condition of profit maximization with respect to the number N of products, i.e.,

$$f\left[\zeta\left(\frac{T}{\widetilde{N}(\theta,T)}\right)^{\theta}-1\right]-\theta f\zeta\left(\frac{T}{\widetilde{N}(\theta,T)}\right)^{\theta}=0,$$

or

$$\widetilde{N}(\theta,T)=T\left[(1-\theta)\zeta\right]^{\frac{1}{\theta}}.$$

Note that $\widetilde{N}(\theta,T)>0$ for all $(\theta,T)\in\Theta$; that is, each entrant chooses to be active. (In fact, by choosing $N=T$, a firm can ensure itself a strictly positive post-entry profit, independently of its type.) But note that the solution to the first-order condition, $\widetilde{N}(\theta,T)$, is the solution to the problem of profit maximization only if $\widetilde{N}(\theta,T)\geq T$, as otherwise $c(\theta,T)=\infty$, implying that the firm's profit is negative, a contradiction. Hence, the profit-maximizing number of products is given by

$$N(\theta,T)=\min\left\{T,\widetilde{N}(\theta,T)\right\}.$$

Next, we show that $\widetilde{N}(\theta,T)$ is strictly decreasing in θ. Taking the partial derivative of $\widetilde{N}(\theta,T)$ with respect to θ, and dividing by T, we obtain

$$\begin{aligned}\frac{\partial\widetilde{N}(\theta,T)}{\partial\theta}\frac{1}{T} &= -\frac{\zeta}{\theta}\left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}}-\frac{1}{\theta^{2}}\left[(1-\theta)\zeta\right]^{\frac{1}{\theta}}\ln\left((1-\theta)\zeta\right)\\ &= -\frac{\zeta}{\theta^{2}}\left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}}\Psi(\theta),\end{aligned}$$

where

$$\Psi(\theta)\equiv\theta+(1-\theta)\ln\left((1-\theta)\zeta\right). \tag{22}$$

Hence, $\partial\widetilde{N}(\theta,T)/\partial\theta<0$ if and only if $\Psi(\theta)>0$. Now, we have $\Psi(0)=\ln(\zeta)>0$ as $\zeta>1$ by assumption. Further,

$$\Psi'(\theta)=-\ln\left((1-\theta)\zeta\right),$$

$$\Psi''(\theta)=\frac{1}{1-\theta}>0,$$

so that $\Psi(\theta)$ achieves its unique minimum at $\theta^m \equiv (\zeta-1)/\zeta$, which is the unique solution on $(0,1)$ to $\Psi'(\theta^m) = 0$. But note that $\Psi(\theta^m) = \theta^m > 0$, implying that $\Psi(\theta) > 0$ for all θ. Hence, $\partial\widetilde{N}(\theta,T)/\partial\theta < 0$, and thus

$$N(\theta,T) = \begin{cases} T & \text{if } \theta \in (0,\underline{\theta}], \\ \widetilde{N}(\theta,T) & \text{if } \theta \in [\underline{\theta},1), \end{cases}$$

where $\underline{\theta} \equiv (\zeta-1)/\zeta$ is such that $\widetilde{N}(\underline{\theta},T) = T$. ■

Proof of Proposition 3. Consider first a firm that chooses to be maximally diversified, $N(\theta,T) = T$, implying that $t^d = t^x = 1$ in equation (16). But then the firm optimally chooses not to export, i.e., $\delta = 0$, as otherwise (by (15)) the second term in (16), the gross profit from selling a product in both markets, would be negative. But from the analysis of the closed economy, we know that a firm of type (θ,T) that sells only domestically chooses to be maximally diversified if and only if $\theta \in (0,\underline{\theta})$. Hence, $N(\theta,T) = T$ if $\theta \in (0,\underline{\theta})$.

Consider now a firm (θ,T) that chooses not to be maximally diversified, $N(\theta,T) < T$, i.e., $\theta \geq \underline{\theta}$. Given N and δ, the Lagrangean associated with the firm's managerial resource allocation problem can be written as

$$L = f\zeta N\left[(1-\delta)\left(t^d\right)^\theta + \delta(1+\rho)\left(t^x\right)^\theta\right] - \lambda N\left[(1-\delta)\left(t^d\right)^\theta + \delta\left(t^x\right)^\theta - \frac{T}{N}\right],$$

where λ is the Lagrange multiplier on the firm's managerial resource constraint. From the first-order conditions, we obtain that $t^x = (1+\rho)^{\frac{1}{1-\theta}}t^d$. Inserting this expression into the managerial resource constraint, yields

$$t^d = \frac{T}{N\left[(1-\delta) + \delta(1+\rho)^{\frac{1}{1-\theta}}\right]}.$$

The post-entry profit of a firm is then given by

$$\pi(N,\delta;\theta,T) = \max_{N,\delta} N\left\{f\zeta\left(\frac{T}{N}\right)^\theta\left[(1-\delta) + \delta(1+\rho)^{\frac{1}{1-\theta}}\right]^{1-\theta} - (f+\delta f^x)\right\}.$$

The first-order condition with respect to the number of products is

$$(1-\theta)f\zeta\left(\frac{T}{N}\right)^\theta\left[(1-\delta) + \delta(1+\rho)^{\frac{1}{1-\theta}}\right]^{1-\theta} - (f+\delta f^x) = 0,$$

which can be rewritten as

$$\left(\frac{T}{N}\right)^{\theta} = \frac{(f + \delta f^x)}{(1-\theta) f \zeta \left[1 + \delta \Delta\right]^{1-\theta}}, \tag{23}$$

where $\Delta \equiv (1+\rho)^{\frac{1}{1-\theta}} - 1$. The first-order condition with respect to the share of products produced for both markets is

$$f\zeta \left(\frac{T}{N}\right)^{\theta} (1-\theta)\Delta \left[1+\delta\Delta\right]^{-\theta} - f^x = 0.$$

Substituting for $(T/N)^{\theta}$ yields

$$f\zeta \frac{(f + \delta f^x)}{(1-\theta) f \zeta \left[1 + \delta \Delta\right]^{1-\theta}} (1-\theta)\Delta \left[1+\delta\Delta\right]^{-\theta} - f^x = 0,$$

or

$$\Delta = \frac{f^x}{f}.$$

Hence, an interior solution for δ is only optimal in the non-generic case that $\Delta = f^x/f$. The optimal choice of δ thus satisfies:

$$\delta = \begin{cases} 1 & \text{if } \Delta > \frac{f^x}{f}, \\ 0 & \text{if } \Delta < \frac{f^x}{f}. \end{cases} \tag{24}$$

That is, if $\Delta > f^x/f$, the firm chooses to export all of its products, whereas if $\Delta < f^x/f$ it will sell all of its products only domestically. As Δ is increasing in θ, the cutoff to become an exporter is given by

$$\theta^x(f^x, \rho) \equiv 1 - \frac{\ln(1+\rho)}{\ln(1 + f^x/f)}.$$

If $\theta > \theta^x$, the firm chooses to be an exporter; otherwise, it is a domestic firm only. Note that $\theta^x < 1$, and that, by (15), $\theta^x > \underline{\theta}$. Inserting (24) into equation (23), we obtain that $N(\theta, T) = T\left((1-\theta)\zeta\right)^{\frac{1}{\theta}}$ if $\theta \in (\underline{\theta}, \theta^x)$, and $N(\theta, T) = T\left[\frac{1+\rho}{1+f^x/f}(1-\theta)\zeta\right]^{\frac{1}{\theta}}$ if $\theta \in (\theta^x, 1)$. ■

Proof of Proposition 4. For two firms that are either both exporters ($\min\{\theta, \theta'\} > \theta^x$) or both non-exporters ($\max\{\theta, \theta'\} < \theta^x$), the argument in the proof of Proposition 2 carries over to the open economy case. What still needs to be shown is that the result obtains for an exporting firm with $\theta > \theta^x$ when compared to a non-exporting firm for which $\theta' < \theta^x$. From the definition of Tobin's Q in (21), we have

$$Q(\theta, T) = Q(\theta', T')\left(\frac{fN(\theta', T') + F}{(f + f^x)N(\theta, T) + F}\right).$$

As the two firms have the same sales level, $S(\theta, T) = S(\theta', T')$, by assumption, equation (20) implies

$$\begin{aligned} N(\theta, T) &= N(\theta', T')\left(\frac{1-\theta}{1-\theta'}\right)\left(\frac{f}{f + f^x}\right) \\ &< N(\theta', T'), \end{aligned}$$

where the inequality follows as $\theta > \theta^x > \theta'$ by hypothesis. Substituting this expression into the first yields

$$\begin{aligned} Q(\theta, T) &= Q(\theta', T')\left[\frac{fN(\theta', T') + F}{fN(\theta', T')\left(\frac{1-\theta}{1-\theta'}\right) + F}\right] \\ &> Q(\theta', T'), \end{aligned}$$

where the inequality follows again as $\theta > \theta^x > \theta'$ by hypothesis. ■

Proof of Lemma 1. Inserting (19) into the free entry condition (7) we obtain

$$\begin{aligned} & f\int_1^\infty T\left\{(\zeta - 1)\int_0^{\underline{\theta}} g(\theta, T)d\theta + \int_{\underline{\theta}}^{\theta^x}\left(\frac{\theta}{1-\theta}\right)[(1-\theta)\zeta]^{\frac{1}{\theta}} g(\theta, T)d\theta \right. \\ & \left. + \int_{\theta^x}^1\left(\frac{\theta(1 + f^x/f)}{1-\theta}\right)\left[\left(\frac{1+\rho}{1 + f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}} g(\theta, T)d\theta\right\} dT - F^e \\ = {} & 0. \end{aligned} \tag{25}$$

Totally differentiating this expression yields

$$\begin{aligned} & \frac{d\zeta}{\zeta} f\int_1^\infty T\left\{\int_0^{\underline{\theta}} \zeta g(T, \theta)d\theta + \int_{\underline{\theta}}^{\theta^x}\left(\frac{1}{1-\theta}\right)[(1-\theta)\zeta]^{\frac{1}{\theta}} g(\theta, T)d\theta \right. \\ & \left. + \int_{\theta^x}^1\left(\frac{1 + f^x/f}{1-\theta}\right)\left[\left(\frac{1+\rho}{1 + f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}} g(\theta, T)d\theta\right\} dT \\ & + \frac{d\rho}{(1+\rho)} f\int_1^\infty T\int_{\theta^x}^1\left(\frac{1 + f^x/f}{1-\theta}\right)\left[\left(\frac{1+\rho}{1 + f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}} g(\theta, T)d\theta dT \\ = {} & 0, \end{aligned}$$

which establishes that $d\zeta d\rho < 0$,and thus $\zeta' < \zeta$. Now suppose that $\zeta(1+\rho)$ were to fall as well so that $\zeta'(1+\rho') \leq \zeta(1+\rho)$. Then, the LHS of (25) would be negative, a contradiction. Hence, $\zeta'(1+\rho') > \zeta(1+\rho)$. ■

Proof of Proposition 6. Consider first a firm (θ, T) with $\theta \in (0, \underline{\theta}']$. By Proposition 5 and the definition of the threshold $\underline{\theta}$, we have $N(\theta,T)' = N(\theta,T) = T$ if $\theta \in (0, \underline{\theta}']$ and $N(\theta,T)' < N(\theta,T) = T$ if $\theta \in (\underline{\theta}', \underline{\theta}]$. Consider now a firm (θ, T) with $\theta \in (\underline{\theta}, \theta^{x\prime}) \cup (\theta^{x}, 1)$. Differentiating equation (17) in conjunction with Lemma 1 implies that $N(\theta,T)' < N(\theta,T)$. Finally, consider a firm (θ, T) with $\theta \in (\theta^{x\prime}, \theta^{x})$. From (17), we obtain the ratio between the number of products post-liberalization and pre-liberalization:

$$\frac{N(\theta,T)'}{N(\theta,T)} = \left(\frac{\zeta'}{\zeta}\right)^{\frac{1}{\theta}} \left(\frac{1+\rho'}{1+f^{x}/f}\right)^{\frac{1}{\theta}} < 1,$$

where the inequality follows from Lemma 1 and the parameter restriction (15). ■

Proof of Proposition 7. Consider first a firm (θ, T) with $\theta \in (0, \underline{\theta}')$. From equation (21) and the fact that $\zeta' < \zeta$ by Lemma 1, it follows immediately that $Q(\theta,T)' < Q(\theta,T)$ for such a firm. Consider now a firm (θ, T) with $\theta \in (\underline{\theta}', \underline{\theta})$. From (21) and (17), the ratio between Tobin's Q post-liberalization and pre-liberalization equals

$$\frac{Q(\theta,T)'}{Q(\theta,T)} = \frac{f + \frac{F}{T}}{(1-\theta)\,\zeta\left(f + \frac{F}{T[(1-\theta)\zeta']^{1/\theta}}\right)}.$$

By definition of the thresholds $\underline{\theta}$ and $\underline{\theta}'$, we have for any $\theta \in (\underline{\theta}', \underline{\theta})$ that $(1-\theta)\,\zeta > 1 > (1-\theta)\,\zeta'$. Hence, $Q(\theta,T)' < Q(\theta,T)$. Next, consider a firm (θ, T) with $\theta \in (\underline{\theta}, \theta^{x\prime}) \cup (\theta^{x}, 1)$. Proposition 6 combined with (21) immediately implies that $Q(\theta,T)' < Q(\theta,T)$ if $\theta \in (\underline{\theta}, \theta^{x\prime})$ and $Q(\theta,T)' > Q(\theta,T)$ if $\theta \in (\theta^{x}, 1)$. Finally, consider a firm (θ, T) with $\theta \in (\theta^{x\prime}, \theta^{x})$. It is useful to note that for those firms that are not maximally diversified Tobin's Q can be written as

$$Q(\theta,T) = \left[1 - \theta + \frac{\theta F}{\pi(\theta,T)}\right]^{-1}.$$

Hence, the ratio between Tobin's Q post-liberalization and pre-liberalization equals

$$\frac{Q(\theta,T)'}{Q(\theta,T)} = \frac{\frac{1-\theta}{\theta} + \frac{F}{\pi(\theta,T)}}{\frac{1-\theta}{\theta} + \frac{F}{\pi(\theta,T)'}}.$$

To sign the effect of an increase in trade freeness on Tobin's Q it thus suffices to establish whether a firm's profits have risen or fallen. Given Lemma 1, it follows immediately that

$\pi(\theta^{x'}, T)' < \pi(\theta^{x'}, T)$ and $\pi(\theta^{x}, T)' > \pi(\theta^{x}, T)$. To complete the proof, we merely need to show that $\pi(\theta, T)'/\pi(\theta, T)$ is monotonically increasing in θ. From (19), and noting that any firm with $\theta \in (\theta^{x\prime}, \theta^{x})$ is switching from non-exporting to exporting, we have

$$\begin{aligned} \ln\left(\frac{\pi(\theta,T)'}{\pi(\theta,T)}\right) &= \ln\left(1+\frac{f^x}{f}\right)+\frac{1}{\theta}\ln\left[\left(\frac{1+\rho'}{1+f^x/f}\right)\frac{\zeta'}{\zeta}\right] \\ &= \log\left(1+\frac{f^x}{f}\right)\left(1-\frac{\theta^{x'}}{\theta}\right)-\frac{1}{\theta}\log\left(\frac{\zeta}{\zeta'}\right), \end{aligned}$$

where the second line follows from (18). As $\theta > \theta^{x'}$ and $\zeta > \zeta'$, this expression is strictly increasing in θ and is equal to zero at

$$\theta = \widehat{\theta} \equiv \theta^{x\prime} + \frac{\log(\zeta/\zeta')}{\log(1+f^x/f)}.$$

■

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