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ABSTRACT

The Cross-Section and Time-Series of Stock and Bond Returns*

Value stocks have higher exposure to innovations in the nominal bond risk premium than growth stocks. Since the nominal bond risk premium measures cyclical variation in the market's assessment of future output growth, this results in a value risk premium provided that good news about future output lowers the marginal utility of wealth today. In support of this mechanism, we provide new historical evidence that low return realizations on value minus growth, typically at the start of recessions when nominal bond risk premia are low and declining, are associated with lower future dividend growth rates on value minus growth and with lower future output growth. Motivated by this connection between the time series of nominal bond returns and the cross-section of equity returns, we propose a parsimonious three-factor model that jointly prices the cross-section of returns on portfolios of stocks sorted on book-to-market dimension, the cross-section of government bonds sorted by maturity, and time series variation in expected bond returns. Finally, a structural dynamic asset pricing model with the business cycle as a central state variable is quantitatively consistent with the observed value, equity, and nominal bond risk premia.

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Value strategies buy stocks that have low prices relative to measures of fundamentals such as dividends or book assets, and sell stocks that have high prices relative to fundamentals. These strategies earn high returns that appear anomalous relative to standard models such as the CAPM (e.g., [Basu, 1977](#); [Fama and French, 1992](#)). The profession has hotly debated whether these superior returns reflect a compensation for systematic risk or a behavioral bias. Under the behavioral hypothesis, extrapolative investors push up the price of growth (“glamor”) stocks that performed well in the recent past, allowing contrarian investors to profit from their over-optimism by investing in out-of-favor value stocks and/or shorting the growth stocks ([De Bondt and Thaler, 1985](#)). In support of a risk-based explanation, [Cochrane \(2006\)](#) points out that “Our lives would be so much easier if we could trace price movements back to visible news about dividends or cash flows.” Early attempts to connect the cash flows of value and growth firms to macro-economic sources of risk came up empty handed ([Lakonishok, Schleifer, and Vishny, 1994](#), LSV).

In this paper we provide new evidence that links the excess return on high minus low book-to-market stock portfolios to business cycle risk. Our progress results from studying a much longer sample with more adverse macroeconomic events (1926-2011 compared to 1968-1989 in LSV, or 15 recessions compared to 4), and from using a more robust method for linking value spread returns to macroeconomic events. That link runs via a bond market variable, the [Cochrane and Piazzesi \(2005\)](#) factor (CP), which measures the bond risk premium. The nominal bond risk premium also measures cyclical variation in the market’s assessment of future output growth. The connection between value minus growth returns and the CP factor on the one hand, and between CP factor and the macro-economy on the other hand implies a previously unexplored link between the cross-section of stock returns and the time series of bond returns that imputes a central role to cyclical risks. We propose a unified and parsimonious three-factor asset pricing model that is able to account for most of the average return differences between book-to-market sorted equity portfolios, the aggregate stock market, and maturity-sorted government bond

portfolios.

The first contribution of the paper is to document that value portfolio returns have a higher covariance with innovations in the bond risk premium than growth portfolio returns. Figure 1 shows covariances between unexpected returns on each of the quintile book-to-market portfolios, ordered from growth (low B/M) to value (high B/M), with innovations to the CP factor. The monotonically increasing pattern in exposures will generate a value premium if the price of CP risk is positive. Standard ICAPM logic implies that the price of CP risk is positive provided that innovations to CP lower the marginal utility of wealth for the average investor. This is natural because CP innovations represent good news about future economic performance. Indeed, the CP factor is a strong predictor of the level of economic activity 12 to 24 months ahead.

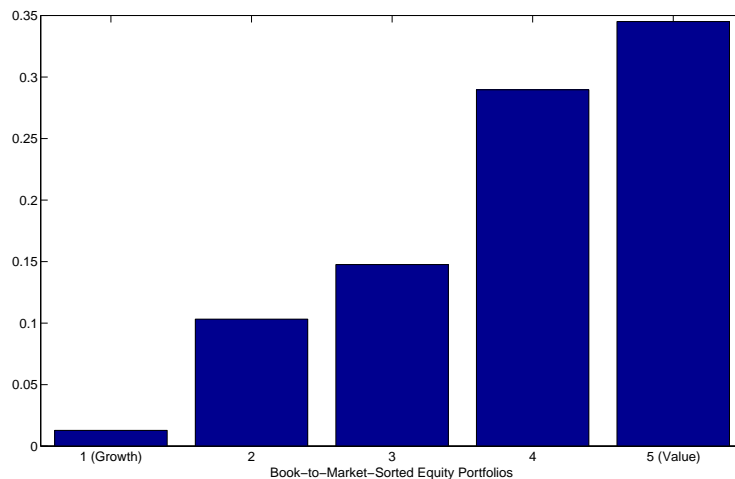


Figure 1. Exposure of value and growth portfolio returns to CP innovations.

The figure shows the covariance of innovations to the nominal bond risk premium (CP factor) and innovations to returns on five quintile portfolios sorted on the BM ratio. Portfolio 1 is the lowest book-to-market (growth) portfolio; portfolio 5 the highest book-to-market (value) portfolio. Innovations to CP and to returns are described in Section III. The covariances are multiplied by 10,000. The sample is monthly from June 1952 until December 2011.

The second contribution of the paper is to provide new evidence that value stocks experience negative cash-flow shocks in economic downturns. We find large differences in the behavior of cash-flow growth on value and growth over the macroeconomic cycle. For example, over the

course of the average NBER recession, dividends on value stocks fall 29% while dividends on growth stocks are flat. The 29% average gap hides interesting differences across recessions. It is much larger in really bad aggregate times than in shallower recessions. During the Great Recession of 2007-2009, the fall in value-minus-growth dividends was 37%. During the Great Depression the relative log change was -461%. The fall during the NBER recession months understates the fall during the broader bust period because the NBER dates may neither coincide with the peak nor the trough for real dividends, and because dividends may be sluggish to adjust to bad aggregate news. For the ten episodes in our sample that witness a protracted fall in real dividends on the market portfolio (28% decline on average), we find that real dividends on the highest book-to-market portfolio fall by 84% more than those on the lowest book-to-market portfolio. Finally, we show that periods where the CP factor is low are periods of significantly lower future dividend growth rates on value-minus-growth. The latter turn down with a lag of about 6-8 quarters, longer than the 3-4 quarter lag between the same low CP events and the level of macro-economic activity.

What results is a coherent picture of value-minus-growth returns, the CP factor, the level of macroeconomic activity, and dividend growth on value-minus-growth that is consistent with a risk-based resolution of the value premium puzzle. To drive this point home, we select periods (events) during which value stocks and the value-minus-growth strategy experience exceptionally low returns, “value crashes.” We find that value crashes are not only associated with low contemporaneous CP factor realizations, but also with low future economic activity and low future dividend growth on value-minus-growth, consistent with a risk-based explanation. This event-based approach provides a novel, robust method to detect the link between prices, cash-flows, and macroeconomic aggregates in high marginal utility states of the world that matter most for pricing. The traditional fixed lead-lag correlation approach applied to the full sample would turn up weaker and more unstable evidence. Our event-based approach could prove fruitful for investigating other return anomalies.

The evidence on the link between the value spread and the CP factor suggests a connection between stock and bond returns. Based on this connection, the third contribution of the paper is to provide a unified pricing model for the cross-section of book-to-market equity portfolios, the equity market portfolio, and the cross-section of maturity sorted bond portfolios. Naturally, our first pricing factor is CP : differential exposure of the five book-to-market portfolios accounts for the average value spread in the data. Second, differential exposure to shocks to the level of the term structure (LVL) accounts for the difference between the excess returns on five government bond portfolios, consistent with [Cochrane and Piazzesi \(2008\)](#). Third, exposure to the market return (MKT) accounts for the aggregate equity premium. This three-factor model reduces mean absolute pricing errors on our test assets from 4.8% per year in a risk-neutral benchmark economy to 0.5% per year. By having the price of LVL risk depend on the CP factor, the model also captures the predictability of bond returns by the CP factor. All estimated risk prices have the expected sign, and are collectively different from zero. We cannot reject the null hypothesis that the model's pricing errors are jointly zero. The three-factor model works for different sub-samples and for different sets of test assets. For example, it does a good job pricing corporate bond portfolios sorted by credit rating, jointly with equity and government bond portfolios. Finally, we present individual stock-level evidence that exposure to the CP shocks is priced and results in a higher equity risk premium.

The final contribution of the paper is to develop a model that clarifies the economic connections between value and growth stock returns, their dividends, the CP factor, and the macroeconomy. The business cycle plays a central role as the key state variable; its innovations carry a positive risk price. The CP factor in the model, which is constructed from endogenous bond prices of different maturity, is perfectly correlated with the business cycle state variable. Dividend growth of value stocks has a higher loading on the state of the business cycle than that of growth stocks; this is the source of the value premium in the model. The heterogeneous exposure is calibrated to match the observed fall in dividends on value and growth stocks over

the course of the average recession. More generally, the model replicates the exposure of nominal bond and stock portfolio returns to CP , LVL , and MKT shocks that we document in the data. The model is spelled out in an online appendix while the main text contains a verbal discussion of the results.

I. Related Literature

Surprisingly, in modern dynamic asset pricing theory, business cycle risk plays a secondary role.¹ Our work uncovers new evidence that business cycle risk in output and consumption growth is priced in stock markets. Value stock returns are more sensitive to bond risk premium innovations than growth stocks returns and hence are more exposed to cyclical news about the economy's future cash flow growth, because their subsequent cash flow growth is more sensitive to output growth. Value stocks earn a premium as a result. Relative to existing dynamic asset pricing models, our work uncovers the cyclical component in expected output growth as a new priced, state variable, distinct from the low frequency state variables in long-run risk and external habit models of [Bansal and Yaron \(2004\)](#) and [Campbell and Cochrane \(1999\)](#). The latter are designed to match the lower frequency variation in the market dividend yield.² Our state variable matches the differential cyclical variation in value and growth dividend yields that we uncover in the data.

¹For example, [Bansal, Kiku, and Yaron \(2010\)](#) extend the model of [Bansal and Yaron \(2004\)](#) by adding a cyclical component to consumption. In their model, good news about future output growth lowers the marginal utility of wealth today because investors have a preference for the early resolution of uncertainty so that the income effect dominates the substitution effect. However, in their preferred calibration, business cycle risk is not strong enough to generate large risk premia. A related literature studies the temporal composition of risk in asset prices, (e.g., [Cochrane and Hansen, 1992](#); [Kazemi, 1992](#); [Bansal and Lehman, 1997](#); [Hansen, Heaton, and Li, 2008](#)).

²These models are successful in accounting for many of the features of both stocks and bonds. For the external habit model, the implications for bonds were studied by [Wachter \(2006\)](#) and the implications for the cross-section of stocks were studied by [Menzly, Santos, and Veronesi \(2004\)](#) and [Santos and Veronesi \(2006\)](#). Likewise, the implications of the long-run risk model for the term structure of interest rates were studied by [Piazzesi and Schneider \(2006\)](#) and [Bansal and Shaliastovich \(2010\)](#), while [Bansal, Dittmar, and Lundblad \(2005\)](#); [Bansal, Dittmar, and Kiku \(2007\)](#); [Hansen, Heaton, and Li \(2008\)](#) study the implications for the cross-section of equity portfolios.

A small but growing literature models stock and bond returns jointly, most often in affine settings like ours, but confines its exploration to the relationship between aggregate stock and bond markets.³ [Lettau and Wachter \(2009\)](#) and [Gabaix \(2009\)](#) additionally study the cross-section of stock returns. The former is a model with common shocks to the risk premium in stock and bond markets, while the latter is a time-varying rare disasters model. Complementary work in production-based asset pricing has linked investment behavior of value and growth firms during a recession to the value premium [Zhang \(2005\)](#).

Our paper also advances an ICAPM literature, starting with [Chen, Roll, and Ross \(1986\)](#), where term structure factors are routinely used either as a predictor of the aggregate return on the stock market or as a conditioning variable in an estimation of a conditional beta model of the cross-section of stock returns. [Ferson and Harvey \(1991\)](#) study stock and bond returns' sensitivity to aggregate state variables, among which the slope of the yield curve. They conclude that time variation in equity risk premia is important for understanding the cross-sectional variation in size and industry equity portfolios, and that interest rate risk premia are important for understanding the cross-sectional variation in bond return portfolios. [Ang and Bekaert \(2007\)](#) find some predictability of nominal short rates for future aggregate stock returns. [Brennan, Wang, and Xia \(2004\)](#) write down an ICAPM model where the real rate, expected inflation, and the Sharpe ratio move around the investment opportunity set and show that this model prices the cross-section of stocks. Similarly, [Petkova \(2006\)](#) studies the connection between the Fama-French factors and innovations in state variables such as the default spread, the dividend-price ratio, the yield spread, and the short rate. Using a VAR model, [Campbell and Vuolteenaho \(2004\)](#) and [Campbell, Polk, and Vuolteenaho \(2010\)](#) argue that common variation in book-to-market portfolio returns can be attributed to news about future cash flow growth on the market. Cash flow innovations in this approach are largely permanent

³Examples are the affine models of [Bakshi and Chen \(1996, 1997\)](#) in a square-root diffusion setting, [Bakshi and Chen \(2005\)](#) and [Bekaert, Engstrom, and Grenadier \(2010\)](#); [Bekaert, Engstrom, and Xing \(2009\)](#) in a Gaussian setting, and the linear-quadratic model of [Campbell, Sunderam, and Viceira \(2008\)](#).

shocks. In contrast to this literature, our focus is on the joint pricing of stock and bond returns, business cycle shocks, and the link with dividend growth on stock portfolios. [Baker and Wurgler \(2012\)](#) show that government bonds comove most strongly with “bond-like stocks,” which are stocks of large, mature, low-volatility, profitable, dividend-paying firms that are neither high growth nor distressed. They propose a common sentiment indicator driving stock and bond returns. Finally, [Lustig, Van Nieuwerburgh, and Verdelhan \(2012\)](#) price both nominal bond yields and the aggregate stock market return in a no-arbitrage model in order to measure the wealth-consumption ratio in the data; they do not study the cross-section of bond nor stock returns.

II. Value Stocks are Risky

In this section, we provide new evidence that value stocks are risky. We start by documenting that value stocks suffer from bad cash-flow shocks during aggregate bad times, times of high marginal utility growth for the representative investor. Because dividends adjust to bad shocks with a lag, it is natural to look for early indicators of poor future economic performance. The literature has traditionally looked at bond markets for expectations about future economic activity. We follow in that tradition and document that a linear combination of bond yields, the Cochrane-Piazzesi (CP) factor, is a strong predictor of both future aggregate economic activity and future dividend growth on value minus growth stocks. This naturally suggests the CP factor as a pricing factor for the cross-section of book-to-market portfolios, an idea we pursue in detail in [Section III](#). In the last part of this section, we select periods where realizations on both the value and the value-minus-growth portfolio are exceptionally low and ask whether these are periods with bad news about future aggregate economic activity. This exercise links price movements to the CP factor and to news about future cash flows.

A. Value Stocks' Dividends Fall More in Recessions

We use monthly data on dividends and inflation from July 1926 until December 2011 (1026 observations). Inflation is measured as the change in the Consumer Price Index from the Bureau of Labor Statistics. Dividends on book-to-market-sorted quintile portfolios are calculated from cum-dividend and ex-dividend returns available from Kenneth French' data library. To eliminate seasonality in dividends, we construct annualized dividends by adding the current month's dividends to the dividends of the past 11 months.⁴ We form log real dividends by subtracting the log change in the CPI from the log of nominal dividends. Our focus is on cash dividends.⁵ We define recessions following the NBER's Business Cycle Dating committee. It is important to note that all quintile portfolios, including the growth portfolio 1, pay substantial amounts of dividends. The average annual dividend yield varies only modestly across book-to-market quintile portfolios: 3.1% (portfolio 1), 3.9% (2), 4.3% (3), 4.5% (4), and 3.9% (5).

The left panel of Figure 2 plots log real dividends on book-to-market quintile portfolios 1 (G), 5 (V), and the market portfolio (M). For consistency with the results below, the figure focuses on the post-1952 sample. The figure shows strong evidence that dividends on value stocks fall substantially more in recessions than in expansions. Value stocks show strong cyclical fluctuations whereas dividends on growth stocks are, if anything, slightly pro-cyclical. The picture for the pre-1952 period, reported in Online Appendix A, is consistent with this behavior. The two most obvious examples of the differential cash-flow behavior of value and growth are the Great Depression (September 1929 - March 1933) and the Great Recession (December 2007

⁴Investing dividends at the risk-free rate yields similar results. [Binsbergen and Koijen \(2010\)](#) show that reinvesting monthly dividends at the market return severely contaminates the properties of dividend growth.

⁵Cash dividends are the right measure in the context of a present-value model that follows a certain portfolio strategy, such as value or growth ([Hansen, Heaton, and Li, 2008](#)). An alternative is to include share repurchases to cash dividends, but this would correspond to a different dynamic strategy ([Larrain and Yogo, 2007](#)). However, in the recent recession, which is the largest downturn in cash dividends during the period in which repurchases become more popular, share repurchases also declined substantially. This suggests that during the episodes that we are most interested in, cash dividends and share repurchases comove positively and are exposed to the same aggregate risks.

- June 2009), but the same pattern holds during most post-war recessions (e.g., 1973, 1983, 1991, 2001). During the Great Depression, the log change in real dividends from the peak is -499% for value, -59% for the market, and -37% for Growth. In the Great Recession, dividends fell 27% for value, 12% for the market, while growth dividends rose 10%.

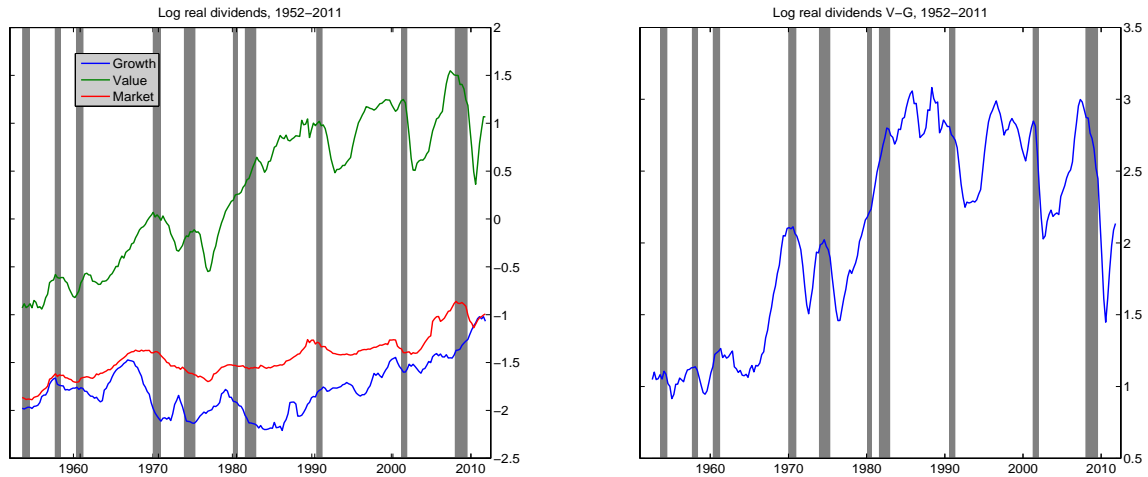


Figure 2. Dividends on value, growth, and market portfolios.

The left panel plots the log real dividend on book-to-market quintile portfolios 1 and 5 and on the CRSP value-weighted market portfolio. The right panel plots the log real dividend on book-to-market quintile portfolios 5 (value) minus the log real dividend on the boot-to-market portfolio 1 (growth), plotted against the right axis. The grey bars indicate official NBER recession dates. Dividends are constructed from cum- and ex-dividend returns on these portfolios. Monthly dividends are annualized by summing dividends received during the year. The data are monthly from July 1952 until December 2011 and are sampled every three months in the figure.

Strictly adhering to the NBER recession dates understates the change in dividends from the highest to their lowest point over the cycle. For example, annual dividends on value-minus-growth fall by 37% during the December 2007-June 2009 recession, but they fall 13% between April and December of 2007 and continue to fall longer after the official recession is over. The decline from June 2009 until June 2010 is a massive 115%. Thus, the total decline over the cycle measured from May 2007 until June 2010 is 165%, eclipsing the decline of 37% over the official NBER cycle. Similarly, value-minus-growth dividends fall by 89% (85%) in the period surrounding the 2001 (1991) recession compared to a 30% (13%) drop between the NBER peak and the last month of the recession. The right panel of Figure 2 plots the log difference

between value and growth portfolios (right axis) as well as NBER recessions (bars). The figure illustrates not only large declines in dividends on value-minus-growth around recessions, but also that declines usually lag the official recession, possibly capturing the downward stickiness in dividend adjustments that is well understood in the literature on firms' dividend payment behavior. The corresponding picture for the pre-1952 period, reported in Online Appendix A, is consistent with this behavior.

To get at these broader boom-bust cycles in dividends more systematically, we alternatively define busts as periods where real dividends on the market portfolio drop by 5% or more over a protracted period (6 months or more). There are ten such periods in the 1926 to 2011 sample. They last an average of 38 months and real dividends on the market portfolio fall by 28% on average. Real dividends on the growth portfolio fall by 20% on average, while those on the value portfolio fall by 104%, a difference of 84%. In all but two of these periods (starting in 1941 and in 1951), dividends on value stocks fall by more than those on growth stocks. The average ratio of the fall in the V-G dividend to the fall in the market dividend is 1.5. In other words, the periods with large sustained decreases in real dividends on the market are associated with much larger declines in the dividends on value than on growth, fifty percent larger than the decline in the market dividend growth itself.

B. The CP Factor and the Economic Activity

Because dividends adjust to bad shocks with a lag, it is natural to look for early indicators of poor future economic performance. The literature has traditionally looked at bond markets for expectations about future economic activity. We follow in that tradition and document that a linear combination of bond yields, the Cochrane-Piazzesi (CP) factor, is a strong predictor of both future aggregate economic activity and future dividend growth on value minus growth stocks.

First, a substantial bond return predictability literature shows that bond risk premia vary over time. [Cochrane and Piazzesi \(2005\)](#) combine bond yields of maturities one to five years to form the *CP* factor and show that it does a good job predicting future excess bond returns. We follow [Cochrane and Piazzesi \(2005\)](#) in constructing the *CP* factor from the term structure of government bond yields.⁶

We consider the following predictive regression in which we forecast future economic activity, measured by the Chicago Fed National Activity Index (*CFNAI*),⁷ using the current *CP* factor:

$$CFNAI_{t+k} = c_k + \beta_k CP_t + \varepsilon_{t+k}, \quad (1)$$

where k is the forecast horizon expressed in months. The regressions are estimated by OLS and we calculate Newey-West standard errors with $k - 1$ lags. The sample runs from March 1967 until December 2011, dictated by data availability. [Figure 3](#) shows the coefficient β_k in the top panel, its t-statistic in the middle panel, and the regression R-squared in the bottom panel. The forecast horizon k is displayed on the horizontal axis and runs from 1 to 36 months. The figure shows the strong predictability of the *CP* factor for future economic activity. All three statistics display a hump-shaped pattern, gradually increasing until approximately 18 months and gradually declining afterwards. The maximum t-statistic is about 4.2, which corresponds to an R-squared value of 15%. The results suggest that a high *CP* factor precedes higher

⁶In particular, we use monthly Fama-Bliss yield data for nominal government bonds of maturities one-through five-years. These data are available from June 1952 until December 2011. We construct one- through five-year forward rates from the one- through five-year bond prices. We then regress the equally-weighted average of the one-year excess return on bonds of maturities of two, three, four, and five years on a constant, the one-year yield, and the two- through five-year forward rates. The yields are one-year lagged relative to the return on the left-hand side. The *CP* factor is the fitted value of this predictive regression. The R^2 of this regression in our sample of monthly data is 18.5%, roughly twice the 10.3% R^2 of the five-year minus one-year yield spread, another well-known bond return predictor.

⁷The CFNAI is a weighted average of 85 existing monthly indicators of national economic activity. It is constructed to have an average value of zero and a standard deviation of one. Since economic activity tends toward trend growth rate over time, a positive index reading corresponds to growth above trend and a negative index reading corresponds to growth below trend. CFNAI is normalized to have mean zero and standard deviation one.

economic activity about 12 to 24 months later. We obtain a similar result forecasting real GDP growth with the CP factor; the results are available in Online Appendix A. The CP factor has been linked to other macroeconomic series in recent work.⁸ We also note that CP predicts the business cycle much better than the level of the yield curve (see Online Appendix A).

While the CP factor clearly leads the cycle, the exact timing of the CP factor vis-a-vis the official NBER recession dating may be fragile because because the lead-lag pattern may fluctuate from one recession to the next. Thus, it may be more productive to isolate periods in which CP is low and then to ask how the level of economic activity behaves in *and around* such *events*. In each quarter since 1952:III, we compute quarterly CP as the CP factor value in the second month of that quarter, and we select the 30% of quarters with the lowest quarterly CP readings. Figure 4 shows how several series of interest behave six quarters before until eight quarters after the low- CP event, averaged across events. The quarter labeled ‘0’ is the event quarter with the low CP reading. The top RHS panel shows the dynamics of CP itself, which naturally falls from a positive value in the preceding quarters to a highly negative value in the event quarter, after which it recovers. The bottom RHS panel shows the economic activity index CFNAI over this CP cycle. There is a strong pattern in economic activity in the quarters surrounding the low CP event. When CP is at its lowest point, economic activity is about average (recall that CFNAI is mean-zero). CFNAI then turns negative for the next eight quarters, bottoming out four to five quarters after the CP event. This lead-lag pattern is consistent with the predictability evidence. The change in CFNAI from four quarters before until four quarters after is 1.2, an economically large change representing 1.2

⁸Brooks (2011) shows that the CP factor has a 35% contemporaneous correlation with news about unemployment, measured as deviations of realized unemployment from the consensus forecast. Gilchrist and Zakrajsek (2012) shows that a credit spread, and in particular a component related to the bond risk premium, forecasts economic activity. A related literature studies the predictability of macro-economic factors for future bond returns. Cooper and Priestly (2008) show that industrial production in deviation from its trend forecasts future bond returns; Joslin, Priebisch, and Singleton (2010) incorporate this finding in an affine term structure model. Ludvigson and Ng (2009) shows that a principal component extracted from many macroeconomic series also forecasts future bond returns. While macro-economic series do not fully soak up the variation in bond risk premia, there clearly is an economically meaningful link between them.

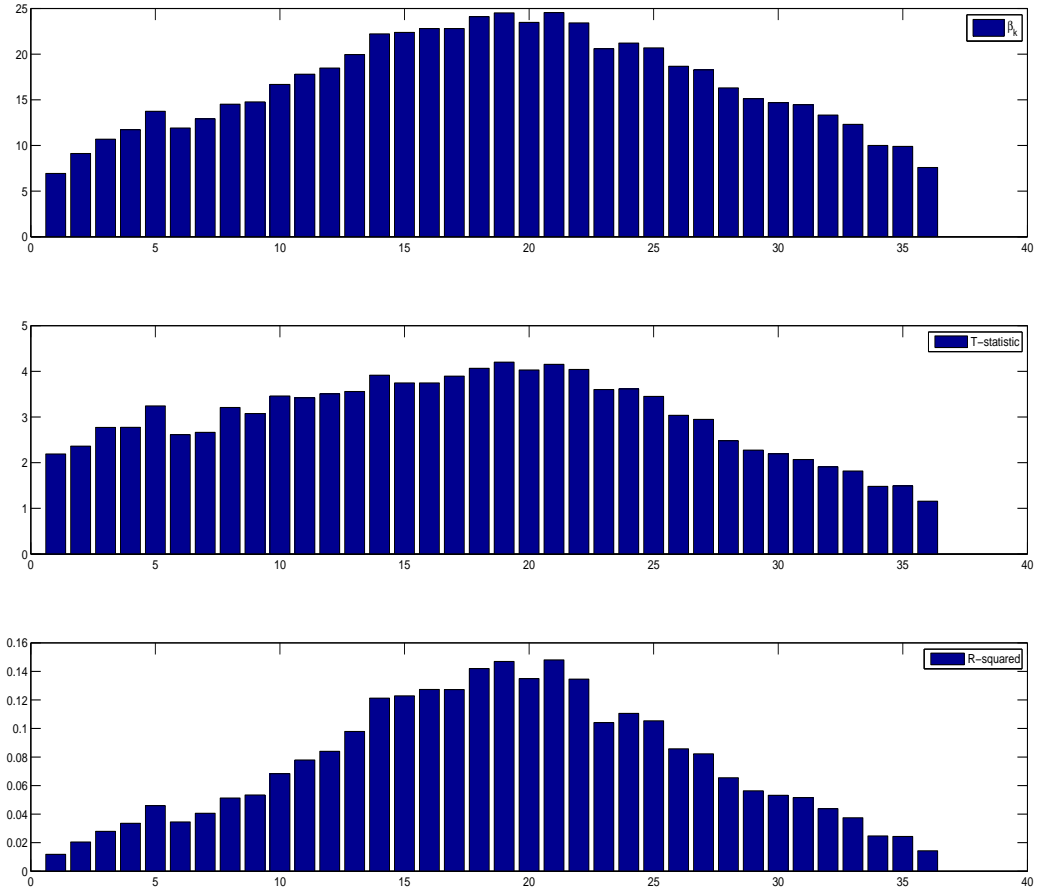


Figure 3. Economic activity predicted by the CP factor.

The top panel displays the predictive coefficient β_k in (1), the middle panel the t-statistic, and the bottom panel the corresponding R-squared value. We consider $k = 1, \dots, 36$ months of lags, displayed on the horizontal axis in each panel, and the t-statistics are computed using Newey-West standard errors with $k - 1$ lags. The sample is March 1967 until December 2009.

standard deviations of CFNAI. The Online Appendix shows similarly strong dynamics in real GDP growth in CP -event time.

The bottom left panel of Figure 4 shows annual dividend growth on value (fifth book-to-market) minus growth (first book-to-market portfolio) over the CP cycle. It connects the facts on CP and the business cycle to the facts on dividend growth of V-G over the business cycle, discussed above. Dividend growth on V-G is high when CP is at its nadir and starts falling

immediately afterwards. This decline in V-G dividend growth is persistent and economically very large. Over the eight quarters following the CP event, annual dividends on value stocks fall by 17.2% points more than on growth stocks, a 0.7 standard deviation decline. Comparing the bottom two panels, we see that dividend growth lags economic activity by several quarters. In sum, low CP realizations clearly predict low future dividend growth rates on V-G, but with a considerable lag.

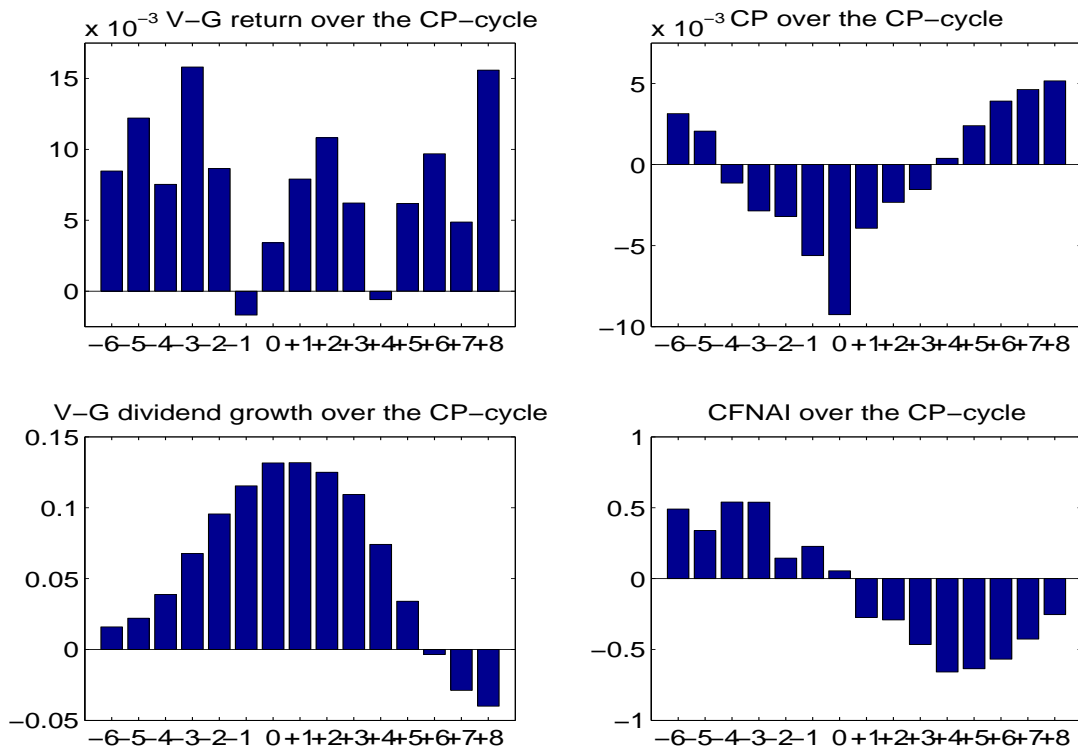


Figure 4. Low CP Events

The figure plots four quarterly series in event time. The event is defined as a quarter in which the quarterly CP factor is in its respective lowest 30% of observations. This selection leads to 71 events out of 238 quarters (30%). The sample runs from 1953.III until 2011.IV. In each panel, the period labeled '0' is the quarter in which the event takes place. The labels -1, -2, -3, etc refer to one, two, three, etc quarters before the event whereas the labels +1, +2, +3, etc. refer to one, two, three, etc quarters after the event. The top left panel plots the realization of the quarterly log return on value minus growth. The bottom left panel reports annual log dividend growth on value minus growth. The top right panel plots the CP factor. The bottom right panel plots the CFNAI index of economic activity. The latter is available only from 1967.II onwards.

Finally, the top left panel of Figure 4 shows quarterly returns on value minus growth. The evidence presented in the introduction suggests a link between *innovations* in CP and returns

on V-G. This panel is consistent with that evidence. Between quarter -1 and 0, the CP factor falls sharply while between quarter 0 and +1, CP rises sharply. The top LHS figure shows that realized returns on the V-G strategy are negative in quarter 0 and positive in quarter 1, consistent with the higher exposure of value stocks to CP innovations than the exposure of growth stocks. We return to the relationship between V-G returns and the CP factor in detail in the Section III.

C. Value Crashes

If value is risky then it should be useful to isolate periods in which value stocks do particularly poorly. In *or around* such periods, we should find evidence of poor performance of cash-flows and/or the macroeconomy. To investigate this possibility, we select quarters in which both the realized log real return on the fifth book-to-market portfolio (value) and the realized log return on value minus growth (first book-to-market portfolio) are in their respective lowest 30% of observations. These are periods in which value does poorly in absolute terms as well as relative to growth. The double criterion rules out periods in which value returns are average, but growth returns are very high. This intersection leads to 35 events out of 238 quarters (or about 15% of the sample). The top LHS panel plots the realization of the quarterly log return on value minus growth in the event quarter (labeled period 0), the six quarters preceding it (labeled with a minus sign), and the eight quarters following it (labeled with a plus sign). By construction, V-G returns are low in period 0, on average -7% per quarter. The value spread declines in the three quarters leading up to the event and rebounds in the three quarters following the event.

The first important result is that the CP factor, plotted in the top right panel of Figure 5 shows the same pattern as V-G returns when plotted in V-G event time. The level of CP falls in the quarters leading up to the low V-G return, becomes slightly negative in the quarter

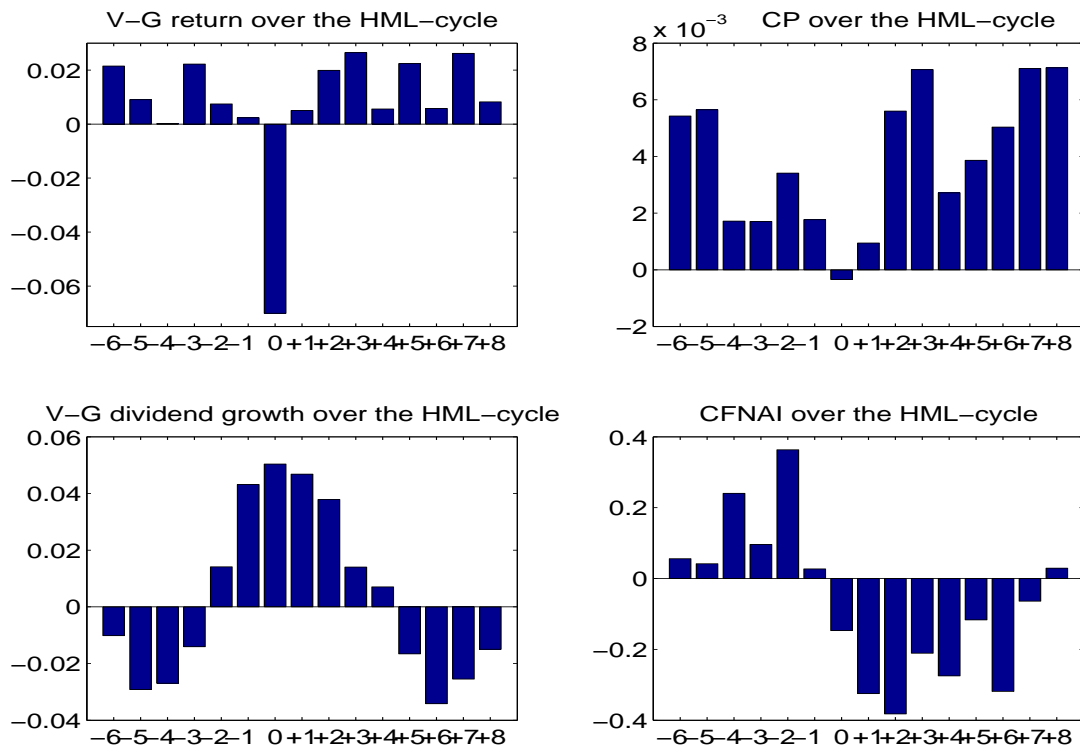


Figure 5. Value Crashes

The figure plots four quarterly series in event time. The event is defined as a quarter in which both the realized log real return on the fifth book-to-market portfolio (value) and the realized log return on value minus growth (first book-to-market portfolio) are in their respective lowest 30% of observations. This intersection leads to 35 events out of 238 quarters (15%). The sample runs from 1953.III until 2011.IV. In each panel, the period labeled '0' is the quarter in which the event takes place. The labels -1, -2, -3, etc refer to one, two, three, etc quarters before the event whereas the labels +1, +2, +3, etc. refer to one, two, three, etc quarters after the event. The top left panel plots the realization of the quarterly log return on value minus growth. The bottom left panel reports annual log dividend growth on value minus growth. The top right panel plots the CP factor. The bottom right panel plots the CFNAI index of economic activity. The latter is available only from 1967.II onwards. The annual dividend growth, CP, and CFNAI series are all available to monthly frequency. We take the average over the months in a given quarter to create a quarterly series.

of the V-G return, and increases in the following three quarters. There is a *contemporaneous* relationship between V-G returns and changes in the CP factor. This suggests that (innovations in) the CP factor captures the risk associated with low value-minus-growth returns.

The second important result, shown in the bottom left panel of Figure 5, is that dividend growth on value-minus-growth falls considerably in the aftermath of the return event. Annual dividend growth on V-G gradually falls from 5% in the quarter of the event to below -3% six quarters after the event. This 8% points fall is economically significant; it is fourfold

the unconditional mean and one-third of the unconditional standard deviation. This finding dovetails nicely with the fall in dividends on value-minus-growth over the course of recessions, shown above. Indeed, many of the value crashes occur just prior to the official start of NBER recessions.

Third, we see the same decline in macro-economic activity following the return event. The bottom right panel of Figure 5 shows the level of CFNAI, a continuous indicator of the level of economic activity. CFNAI peaks at the peak of the business cycle and bottoms out at the trough; it is normalized to have mean zero and standard deviation one. Since CFNAI is only available from 1967 onwards, the bottom RHS panel misses the first eight return events. As the figure shows, in the event quarter, the level of economic activity is only just below average. But in the ensuing quarters, it goes substantially below zero. The change in economic activity from two quarters before to two quarters after the event is three-quarters of a standard deviation of CFNAI. The Online Appendix shows an equally large effect on real GDP growth. The delayed adjustment in dividends is consistent with the previous evidence. The evidence in the bottom two panels suggests that firms only cut dividends (and those in the value more than those in the growth portfolio) after a prolonged period of below-average levels of economic activity. The dividend growth of V-G lags the economic activity cycle by about three quarters on average.

Methodologically, the advantage of our event-time approach is that it focuses on those periods where the investment strategy performs poorly. By looking at windows around these low value return events, the relationships between returns, cash flows, and macroeconomic activity become more transparent and therefore easier to detect. While the low V-G return events are clearly associated with recessions, the exact timing vis-a-vis the official NBER recession dates varies from recession to recession. This makes it hard to detect clear relationships between value returns and NBER recessions with traditional approaches that use fixed lead-lag timing. Our approach could prove fruitful to understand other return anomalies like size or momentum

(Daniel and Moskowitz, 2011).

III. Empirical Link Between Stocks and Bonds

The evidence on the link between the value spread and the CP factor suggests a connection between stock and bond returns. Based on this connection, this section provides a unified asset pricing model for the cross-section of book-to-market equity portfolios, the equity market portfolio, and the cross-section of maturity sorted bond portfolios. The model is parsimonious in that only three pricing factors are needed to capture the bulk of the cross-sectional return differences. The model is a reduced-form stochastic discount factor model which imposes little more than unified pricing of risk (no arbitrage) between these equity and bond portfolios. Section IV presents a structural asset pricing model that provides an economic intuition for the empirical connection between stocks and bonds we document here.

A. Setup

Let P_t be the price of a risky asset and D_{t+1} its (stochastic) cash-flow, and R_{t+1} the cum-dividend return. Then the nominal stochastic discount factor (SDF) implies $E_t[M_{t+1}^{\$} R_{t+1}] = 1$. Lowercase letters denote natural logarithms: $m_t^{\$} = \log(M_t^{\$})$. We propose a reduced-form SDF, akin to that in the empirical term structure literature (Duffie and Kan, 1996; Duffee, 2002):

$$-m_{t+1}^{\$} = y_t^{\$} + \frac{1}{2}\Lambda_t'\Sigma\Lambda_t + \Lambda_t'\varepsilon_{t+1}, \quad (2)$$

where $y_t^{\$}$ is the nominal short-term interest rate, ε_{t+1} is a $N \times 1$ vector of shocks to the $N \times 1$ vector of demeaned state variables X_t , and where Λ_t is the $N \times 1$ vector of market prices of risk associated with these shocks at time t . The state vector in (3) follows a first-order

vector-autoregression with intercept γ_0 , companion matrix Γ , and conditionally normally, i.i.d. distributed innovations, $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$:

$$X_{t+1} = \Gamma X_t + \varepsilon_{t+1}, \quad (3)$$

$$\Lambda_t = \Lambda_0 + \Lambda_1 X_t. \quad (4)$$

The market prices of risk are affine in the state vector, where Λ_0 is an $N \times 1$ vector of constants and Λ_1 is an $N \times N$ matrix that governs the time variation in the prices of risk.

Log returns on an asset j can always be written as the sum of expected and unexpected returns: $r_{t+1}^j = E_t[r_{t+1}^j] + \eta_{t+1}^j$. Unexpected log returns η_{t+1}^j are assumed to be normally distributed and homoscedastic. We denote the covariance matrix between shocks to returns and shocks to the state variables by Σ_{Xj} . We define log excess returns to include a Jensen adjustment:

$$rx_{t+1}^j \equiv r_{t+1}^j - y_t^{\$}(1) + \frac{1}{2}V[\eta_{t+1}^j].$$

The no-arbitrage condition then implies:

$$E_t [rx_{t+1}^j] = Cov_t [rx_{t+1}^j, -m_{t+1}^{\$}] = Cov [\eta_{t+1}^j, \varepsilon'_{t+1}] \Lambda_t \equiv \Sigma_{Xj} (\Lambda_0 + \Lambda_1 X_t). \quad (5)$$

Unconditional expected excess returns are computed by taking the unconditional expectation of (5):

$$E [rx_{t+1}^j] = \Sigma_{Xj} \Lambda_0. \quad (6)$$

The main object of interest, Λ_0 , is estimated below. Equation (6) suggests an interpretation of our model as a simple three-factor model. In Section D, we estimate how the market prices of risk vary with X_t (Λ_1).

B. Data and Implementation

We aim to explain the average excess returns on the five value-weighted quintile portfolios sorted on their book-to-market ratio from Fama and French (1992), the value-weighted stock market return from CRSP (NYSE, Amex, and Nasdaq), and five zero-coupon nominal government bond portfolios with maturities 1, 2, 5, 7, and 10 years from CRSP. The return data are monthly from July 1952 until December 2011. Online Appendix Section E studies other sets of test assets for robustness.

We propose three asset pricing factors in X_t : the CP factor, the LVL factor, and the MKT factor. We construct the unexpected bond returns in η as the residuals from a regression of each bond portfolio's log excess return on the lagged CP factor. Similarly, we assume that stock returns are also predictable by the CP factor, and construct the unexpected stock returns in η as the residual from a regression of each stock portfolio's log excess return on the lagged CP factor.⁹ Second, we construct the level factor LVL as the first principal component of the one- through five-year Fama-Bliss forward rates. Third, the market factor (MKT) is the value-weighted stock market return from CRSP.

We estimate a single monthly VAR(1) with the CP , LVL , and MKT factors to extract factor innovations ε . Innovations to the state vector ε follow from equation-by-equation OLS estimation of the VAR model in (3). The innovation correlations between our three factors are close to zero: 0.05 between CP and LVL , 0.04 between CP and MKT , and -0.10 between LVL and MKT .¹⁰

⁹Cochrane and Piazzesi (2005) provide evidence of predictability of the aggregate market return by the lagged CP factor. In addition, we could include the aggregate dividend-price ratio (DP) as a predictor of the stock market. Given the low R^2 of these predictive regressions, the resulting unexpected returns are similar whether we assume predictability by CP , DP , both, or no predictability at all.

¹⁰In the context of an annual model, Cochrane and Piazzesi (2008) argue that the CP factor is not well described by an AR(1) process. In addition to the level of the term structure, they include the slope and the curvature (second and third principal components of the Fama-Bliss forward rates) as predictors in their VAR. The second difference is that they project forward rates on the CP factor before taking principal components of the forward rates. Our results (in a monthly VAR) are not sensitive to either including slope and curvature factors in our VAR to form innovations or to computing level, slope, and curvature in the alternative fashion,

The first column of Table I shows the average excess returns, expressed in percent per year, on our 11 test assets we wish to explain. They are the pricing errors resulting from a model where all prices of risk in Λ_0 are zero, that is, from a risk-neutral SDF model (*RN SDF*). Average excess returns on bonds are between 1.0 and 2.1% per year and generally increase in maturity. The aggregate excess stock market return is 6.4%, and the risk premia on the book-to-market portfolios range from 5.8% (BM1, growth stocks) to 9.8% (BM5, value stocks), implying a value premium of 4.0% per year.

C. Estimation Results

We estimate the three price of risk parameters in $\hat{\Lambda}_0$ by minimizing the root mean-squared pricing errors on our $J = 11$ test assets. This is equivalent to regressing the $J \times 1$ average excess returns on the $J \times 3$ covariances in Σ_{XJ} . The results from our model are in the second column of Table I (*Our SDF*). The top panel shows the pricing errors. Our model succeeds in reducing the mean absolute pricing errors (MAPE) on the 11 stock and bond portfolios to a mere 48 basis points per year. The model largely eliminates most of the value spread: The spread between the fifth and the first book-to-market quintile portfolios is 80 basis points per year. We also match the market equity risk premium and the average bond risk premium. Pricing errors on the stock and bond portfolios are an order of magnitude lower than in the first column and substantially below those in several benchmark models we discuss below. In sum, our three-factor pricing model is able to account for the bulk of the cross-sectional variation in stock and bond returns with a single set of market price of risk estimates.

The bottom panel of the table shows the point estimates for $\hat{\Lambda}_0$. We estimate a positive

or to making both changes at once. Results are available upon request. The only difference is that the VAR innovations for *CP*, *LVL*, and *MKT* are nearly uncorrelated in our procedure, whereas the correlation between *CP* shocks and *LVL* shocks is highly negative when forward rates are orthogonalized on *CP* before taking principal components. We focus on the three-factor structure because it is simpler and it maps more directly into the structural model of Section IV. The latter also implies a *MKT*, *LVL*, and *CP* factor structure whose innovations are nearly uncorrelated.

price of *CP* risk, while the price of *LVL* risk is negative and that of *MKT* risk is positive. The signs on these risk prices are as expected. As Section II explained at length, the positive price of *CP* risk arises because positive shocks to *CP* are good news for future economic activity, therefore indicating a negative innovation to the SDF or equivalently low marginal utility of wealth states for the representative investor. A positive shock to the level factor leads to a drop in bond prices and negative bond returns. A negative shock to bond returns increases the SDF and, hence, carries a negative risk price. A positive shock to the market factor increases stock returns and lowers the SDF, and should carry a positive risk price. We return to the *CP* factor and its positive risk price below. We also report asymptotic standard errors on the Λ_0 estimates using GMM with an identity weighting matrix. The standard errors are 34.02 for the *CP* factor price (point estimate of 90.82), 8.59 for the *LVL* factor price (-19.47), and 1.25 for the *MKT* factor price (2.20). Hence, the first two risk prices are statistically different from zero (with t-stats of 2.7 and -2.3), whereas the last one is only significant at the 10% level (t-stat of 1.8).

To help us understand the separate roles of each of the three risk factors in accounting for the risk premia on these stock and bond portfolios, we switch on only one risk factor and set the other risk prices to zero. Column 3 of Table I minimizes the pricing errors on the same 11 test assets but only allows for a non-zero price of level risk (Column *LVL*). This is the bond pricing model proposed by [Cochrane and Piazzesi \(2008\)](#). They show that the cross-section of average bond returns is well described by differences in exposure to the level factor. Long-horizon bonds have returns that are more sensitive to interest rate shocks than short-horizon bonds; a familiar duration argument. However, this bond SDF is unable to jointly explain the cross-section of stock and bond returns; the MAPE is 4.31%. All pricing errors on the stock portfolios are large and positive, there is a 4.14% value spread, and all pricing errors on the bond portfolios are large and negative. Clearly, exposure to the level factor alone does not help to understand the high equity risk premium nor the value risk premium. Value and growth

Table I
Unified SDF Model for Stocks and Bonds - Pricing Errors

Panel A of this table reports pricing errors on five book-to-market sorted quintile stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. They are expressed in percent per year (monthly numbers multiplied by 1200). Each column corresponds to a different stochastic discount factor (SDF) model. The first column contains the risk-neutral SDF and therefore reports the average pricing errors that are to be explained. The second column presents our SDF model with three priced factors (Our Model). The third column presents the results for a bond pricing model, where only the level factor is priced (*LVL*). In the fourth column, we only use the bond returns as moments to estimate the same SDF as in the third column (*LVL*-only bonds). The SDF model of the fifth column has the market return as the only factor, and therefore is the CAPM model (*MKT*). The sixth column allows for both the prices of *LVL* and *MKT* risk to be non-zero. The last column refers to the three factor model of Fama and French (1992). The last row of Panel A reports the mean absolute pricing error across all 11 securities (MAPE). Panel B reports the estimates of the prices of risk. The first six columns report market prices of risk Λ_0 for (a subset) of the following pricing factors : ε^{CP} (*CP*), ε^L (*Level*), and ε^M (*MKT*). In the last column, the pricing factors are the innovations in the excess market return (*MKT*), in the size factor (*SMB*), and in the value factor (*HML*), where innovations are computed as the residuals of a regression of these factors on the lagged dividend-price ratio on the market. Panel C reports asymptotic p-values of χ^2 tests of the null hypothesis that all market prices of risk in Λ_0 are jointly zero, and of the null hypothesis that all pricing errors are jointly zero. The data are monthly from June 1952 through December 2011.

Panel A: Pricing Errors (in % per year)								
	RN SDF	Our SDF	<i>LVL</i>	<i>LVL</i> -only bonds	<i>MKT</i>	<i>LVL</i> + <i>MKT</i>	<i>FF</i>	
1-yr	0.99	-0.55	-0.10	0.57	1.30	0.53	0.74	
2-yr	1.24	-0.75	-0.87	0.42	1.75	0.41	0.78	
5-yr	1.71	-0.25	-2.53	0.08	1.50	0.12	0.79	
7-yr	2.06	0.41	-2.95	0.13	1.09	0.13	0.84	
10-yr	1.73	0.18	-3.90	-0.44	0.88	-0.51	0.30	
Market	6.42	-0.76	5.13	5.92	-1.33	-1.26	0.05	
BM1	5.85	-0.29	4.58	5.36	-2.28	-2.18	0.43	
BM2	6.84	-0.04	5.34	6.26	-0.70	-0.71	-0.55	
BM3	7.78	0.70	6.26	7.19	0.71	0.66	-0.40	
BM4	8.44	-0.03	6.97	7.88	1.56	1.52	-0.73	
BM5	9.79	0.51	8.72	9.38	2.21	2.34	0.98	
MAPE	4.80	0.48	4.31	3.97	1.39	0.94	0.60	
Panel B: Prices of Risk Estimates Λ_0								
<i>MKT</i>	0	2.20	0	0	3.41	3.19	<i>MKT</i>	5.82
<i>LVL</i>	0	-19.47	-32.33	-12.46	0	-10.56	<i>SMB</i>	-10.45
<i>CP</i>	0	90.82	0	0	0	0	<i>HML</i>	6.38
Panel C: P-values of χ^2 Tests								
$\Lambda_0 = 0$	-	0.35%	0.01%	-	0.06%	0.02%		0.02%
Pr. err. = 0	-	8.83%	0.00%	-	0.00%	0.00%		0.02%

stocks have similar exposure to the level factor, i.e., a similar “bond duration.” The reason that this model does not do better pricing the bond portfolios is that the excess returns on stock portfolios are larger in magnitude and therefore receive most attention in the estimation. The estimation concentrates its efforts on reducing the pricing errors of stocks.

To illustrate that this bond SDF is able to price the cross-section of bonds, we estimate the same model by minimizing only the bond pricing errors (the first five moments in the table). The fourth column of Table I (*LVL - only bonds*) confirms that the bond pricing errors fall substantially: The mean absolute bond pricing error goes from 155bp with the risk-neutral SDF to 33bp with the “LVL-bonds only” kernel. However, the overall MAPE remains high at 3.97%. The canonical bond pricing model offers one important ingredient for the joint pricing of stocks and bonds, bonds’ heterogeneous exposure to the level factor, but this ingredient does not help to account for equity returns.

Another benchmark is the canonical Capital Asset Pricing Model. The only non-zero price of risk is the one corresponding to the *MKT* factor. The fifth column of Table I (*MKT*) reports pricing errors for the CAPM. Because past research has shown that the CAPM cannot price stock portfolios, it is not surprising that the CAPM is also unable to jointly price stock and bond returns. The MAPE is 1.39%. One valuable feature is that the aggregate market portfolio is priced well and the pricing errors of book-to-market portfolio returns go through zero. This means that the model gets the common level in all stock portfolio returns right. However, the pattern of pricing errors contains a 4.5% value spread. Pricing errors on bond portfolios are sizeable as well and are all positive. Neither book-to-market nor bond portfolios display interesting heterogeneity in their exposure to *MKT* shocks. So, while the *LVL* factor helps to explain the cross-sectional variation in average bond returns and the *MKT* factor helps to explain the level of equity risk premia, neither factor is able to explain why value stocks have much higher risk premia than growth stocks. The sixth column of Table I indeed shows that

having both the level and market factor priced does not materially improve the pricing errors and leaves the value premium puzzle in tact.

This is where the CP factor comes in. Figure 6 decomposes each asset's risk premium into its three components: risk compensation for exposure to the CP factor, the level factor, and the MKT factor. The top panel is for the five bond portfolios, organized from shortest maturity on the left (1-year) to longest maturity on the right (10-year). The bottom panel shows the decomposition for the book-to-market quintile portfolios, ordered from growth to value from left to right, as well as for the market portfolio (most right bar). This bottom panel shows that all book-to-market portfolios have about equal exposure to both MKT and LVL shocks. If anything, growth stocks (G) have slightly higher market (CAPM) betas than value stocks (V), but the difference is small. The spread between value and growth risk premia entirely reflects differential compensation for CP risk. Value stocks have a large and positive exposure to CP shocks while growth stocks have a low exposure (recall Figure 1). The differential exposure between the fifth and first book-to-market portfolio is statistically different from zero. Multiplying the spread in exposures by the market price of CP risk delivers a value premium of 0.30% per month or 3.6% per year. That is, the CP factor's contribution to the risk premia accounts for most of the 4% value premium. Given the monotonically increasing pattern in exposures of the book-to-market portfolios to CP shocks, a positive price of CP risk estimate is what allows the model to match the value premium.

The top panel of Figure 6 shows the risk premium decomposition for the five bond portfolios. Risk premia are positive and increasing in maturity due to their exposure to LVL risk. The exposure to level shocks is negative and the price of level risk is negative, resulting in a positive contribution to the risk premium. This is the duration effect mentioned above. But bonds also have a negative exposure to CP shocks. Being a measure of the risk premium in bond markets, positive shocks to CP lower bond prices and realized returns. This effect is larger the longer

the maturity of the bond. Given the positive price of CP risk, this exposure translates into an increasingly negative contribution to the risk premium. Because exposure of bond returns to the equity market shocks MKT is positive but near-zero, the sum of the level and CP contributions delivers the observed pattern of bond risk premia that increase in maturity.

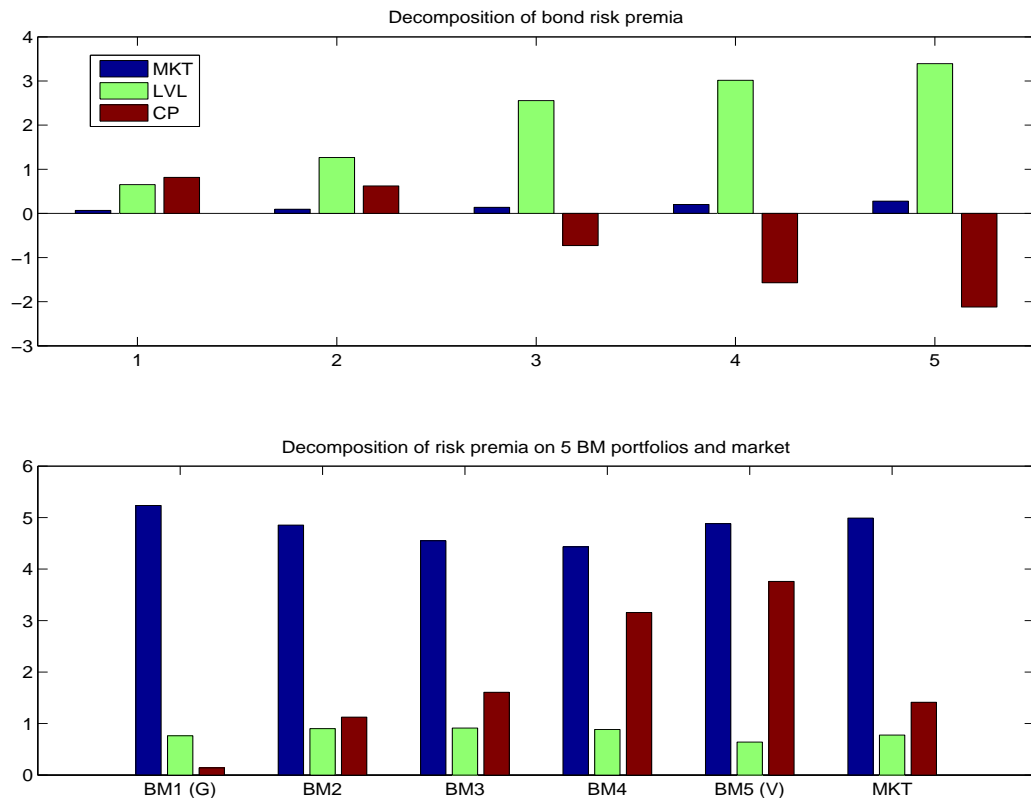


Figure 6. Decomposition of annualized excess returns in data.

The figure plots the risk premium (expected excess return) decomposition into risk compensation for exposure to the MKT , LVL , and CP factors. Risk premia, plotted against the left axis, are expressed in percent per year. The top panel is for the five bond portfolios: one-, two-, five-, seven-, and ten-year maturities from left to right, respectively. The bottom panel is for the book-to-market decile quintile portfolios, from growth (G) to value (V), and for the market portfolio (M). The three bars for each asset are computed as $\Sigma'_{XR}\Lambda_0$. The data are monthly from June 1952 until December 2009.

One might be tempted to conclude that any model with three priced risk factors can always account for the three salient patterns in our test assets. To highlight that such a conjecture is false and to highlight the challenge in jointly pricing stocks and bonds, online Appendix B develops a simple model where (1) the CP factor is a perfect univariate pricing factor for

the book-to-market portfolios (it absorbs all cross-sectional variation), (2) the *LVL* factor is a perfect univariate pricing factor for the bond portfolios, and (3) the *CP* and the *LVL* factor are uncorrelated. It shows that such a model generally fails to price the stock and bond portfolios jointly. This failure arises because the bond portfolios are exposed to the *CP* factor, while the stock portfolios are not exposed to the *LVL* factor. Consistent risk pricing across stocks and bonds only works if the exposures of maturity-sorted bond portfolios to *CP* are linear in maturity, with the same slope (in absolute value) as the level exposures. The data happen to approximately satisfy the three assumptions underlying the stark model, but this is not a foregone conclusion. Appendix B thus underscores the challenges in finding a model with consistent risk prices across stocks and bonds, or put differently, the challenge of going from univariate to multivariate pricing models.

The last but one row of Table I tests the null hypothesis that the market price of risk parameters are jointly zero. This null hypothesis is strongly rejected for all models, including ours. The asymptotic p-value for the χ^2 test, computed by GMM using the identity weighting matrix, is less than 1% for our model. The last row reports the p-value for the χ^2 test that all pricing errors are jointly zero. Interestingly, ours is the only model for which the null hypothesis cannot be rejected; the p-value is 8.8%. Test of whether individual pricing errors are zero cannot be rejected for all but the aggregate market portfolio in our model (not reported). These tests lend statistical credibility to our results.

D. Time-varying Risk Prices

Having estimated the constant market prices of risk, Λ_0 , we turn to the estimation of the matrix Λ_1 , which governs the time variation in the prices of risk. We allow the price of level risk $\Lambda_{1(2)}$ and the price of market risk $\Lambda_{1(3)}$ to depend on the *CP* factor. We use two predictive regressions to pin down this variation in risk prices. We regress excess returns on a constant

and lagged CP :

$$rx_{t+1}^j = a_j + b_j CP_t + \eta_{t+1}^j,$$

where we use either excess returns on the stock market portfolio or an equally-weighted portfolio of all bond returns used in estimation. Using equation (5), it then follows:

$$\begin{pmatrix} \Lambda_{1(2)} \\ \Lambda_{1(3)} \end{pmatrix} = \begin{pmatrix} \Sigma_{X,Market(2:3)} \\ \Sigma_{X,Bonds(2:3)} \end{pmatrix}^{-1} \times \begin{pmatrix} b_{Market} \\ b_{Bonds} \end{pmatrix}.$$

Following this procedure, we find that $\hat{\Lambda}_{1(2)} = -847$ and $\hat{\Lambda}_{1(3)} = 49$. This implies that equity and bond risk premia are high when CP is high, consistent with the findings of [Cochrane and Piazzesi \(2005\)](#).

E. Robustness

Book-to-market Decile Portfolios We also study book-to-market *decile* instead of quintile portfolios, alongside the same bond portfolios and the stock market portfolio. Table II shows that the value spread between the tenth and first book-to-market portfolios is 4.7% per annum (Column 1), 0.7 percentage point higher than between the extreme quintile portfolios. The mean absolute pricing error among these 16 assets is 5.75% per year. Our model's residual pricing error is a mere 0.46% (Column 2). It eliminates the entire value premium. The market price of risk estimates are nearly identical to those obtained with the quintile portfolios. Again, the null hypothesis that all market prices of risk are jointly zero is strongly rejected, while the null that all pricing errors are jointly zero cannot be rejected (p-value is 16%).

Table II
Decile Book-to-Market Portfolios

The table is identical to Table I, except that it replaces the book-to-market quintile portfolios by book-to-market decile portfolios. The data are monthly from June 1952 through December 2011.

Panel A: Pricing Errors (in % per year)				
	RN SDF	Our SDF		FF
1-yr	0.99	-0.55		0.78
2-yr	1.24	-0.74		0.88
5-yr	1.71	-0.24		1.07
7-yr	2.06	0.43		1.22
10-yr	1.73	0.20		0.69
Market	6.42	-0.77		-0.09
BM1	5.73	0.06		0.65
BM2	6.16	-0.71		-0.05
BM3	7.03	-0.01		0.13
BM4	6.76	-0.15		-0.96
BM5	7.65	0.82		-0.23
BM6	7.72	0.22		-0.30
BM7	7.61	-0.89		-0.90
BM8	9.29	0.81		0.06
BM9	9.57	0.69		0.74
BM10	10.42	0.03		0.61
MAPE	5.75	0.46		0.59
Panel B: Prices of Risk Estimates Λ_0				
MKT	0	2.20	MKT	5.04
LVL	0	-19.38	SMB	-6.44
CP	0	91.02	HML	5.94
Panel C: P-values of χ^2 Tests				
$\Lambda_0 = 0$	-	0.51%	0.02%	
Pr. err. = 0	-	16.22%	0.12%	

Alternative Pricing Models In Section III, we compare our SDF model to the basic stock-pricing and bond-pricing models. It might be interesting to also compare the model to the three-factor model of Fama and French (1992), which offers a better-performing alternative to the CAPM for pricing the cross-section of stocks. We ask how well it prices the cross-section of book-to-market stocks and government bonds over our monthly sample from July 1952 until December 2011. We use the market return (MKT), the size (SMB), and the value factor (HML) as pricing factors and price the same 11 (16) test assets as in Tables I (II). The last column of each table contains the pricing errors for the Fama-French models. The MAPE is 60 (59) basis points per year with 11 test assets (16 test assets), which is somewhat higher than the 48 (46) basis points of our model in the second column. In both cases, the slightly worse fit in the last column is due to higher pricing errors on the bond portfolios. Tests of the null hypothesis that all pricing errors are jointly zero are rejected at conventional levels. We have verified that this rejection is due to the higher pricing errors on the bond moments. This finding is consistent with the findings in Fama and French (1993) who introduce additional pricing factors (beyond MKT, SMB, and HML) to price bonds. Our results suggest that three factors suffice. In unreported results, we find that the difference between the MAPE of our model and the Fama-French model increases when we weight the 11 Euler equation errors by the inverse of their variance as opposed to equally. In addition, there remains a statistical difference between the p-values of χ^2 tests of the null that all pricing errors are jointly zero between our model (5%) and the FF model (<1%) with the alternative weighting matrix. The reason is that our model fits the bond return moments better.

Subsamples and Other Test Assets Online Appendix C considers several exercises confirming the robustness of our empirical results. First, we use a different weighting matrix in the market price of risk estimation. Second, we do a subsample analysis. Third, we study the pricing of additional stock and bond portfolios. On the equity side, we consider ten size-sorted

portfolios, five earnings-to-price sorted portfolios, and twenty-five size and value double-sorted portfolios. Worthy of emphasis is our result that the same three factors also price corporate bonds portfolios, sorted by ratings class, alongside our eleven benchmark test assets. This result strengthens the connection between stock and government bond pricing by accounting for the return dynamics of an “intermediate” asset class. After all, stocks and corporate bonds are both risky claims on the firm’s cash flows albeit with different priority structure.

Twisted CP The *CP* factor is a linear combination of one- through five-year bond yields which predicts economic activity and whose innovations have a monotonic covariance pattern with returns on the book-to-market portfolios. There are other linear combinations of the same five yields which may be better predictors of economic activity. A natural question is whether the best linear predictor of economic activity continues to be a good pricing factor for the cross-section of test assets. To investigate this question, we regress $CFNAI_{t+k}$, with $k = 12$ ($k = 24$), on the five lagged bond yields dated t . We use the linear projections to form two *twisted CP* factors, and use each twisted *CP* factor alongside the *MKT* and *LVL* factors to price our 11 benchmark test assets. The best 12-month ahead predictor predicts economic activity considerably better than the standard *CP* factor: the R^2 is double the 8% for the standard factor. In contrast, the standard *CP* factor predicts economic activity 24 months ahead just about as well as the best linear predictor. The pricing exercises generate a MAPE of 57bp (57bp) for the 12- (24-)month factor, comparable to our benchmark 41bp pricing error. The 24-month factor explains pricing errors on the book-to-market portfolios much better than the 12-month factor and about as well as *CP*, while the 12-month factor makes lower pricing errors on the bond portfolios.

F. Individual Firm Returns

As a final robustness check, we investigate whether exposure to nominal bond risk premium (CP) shocks is associated with higher equity risk premia not only among stock and bond portfolios, but also among individual stocks. We look both at single-sorted portfolios as well as at equity portfolios that are double-sorted based on their CP exposure and their book-to-market ratio.

Our sample is the CRSP/Compustat universe between July 1963 and December 2010. For each stock-month pair, we estimate the covariance between monthly CP innovations and the stock's return based on 60-month rolling windows. If a shorter history is available of a certain stock, we require at least 12 observations to estimate the CP exposure. We start our first sort in July 1968. This ensures that we have 60 months of data for a substantial cross section of stocks to estimate the CP exposure more reliably. We sort stocks each year in June based on their CP -exposure and calculate the quintile portfolio returns over the next 12 months, value-weighting stocks within each portfolio.

We first study returns of portfolios sorted on CP exposure (see Table IA.V in the Online Appendix). We find a spread in average returns between the highest- CP exposure and the lowest- CP exposure of 2.4% per year. The standard CAPM cannot explain these portfolio returns. The spread in CAPM alphas is 2.4%, as high as the raw return spread. The MAPE of the CAPM for these CP-quintile portfolios is 82bp per year. In contrast, our three-factor model *can* explain the return spread in the CP portfolios. The MAPE falls to 39bp and the Q5-Q1 spread in the “KLN alphas,” the alphas with respect to our three factors, is only 0.5%. Encouragingly, the point estimates for the prices of risk are quite similar to those presented in our main estimation, even though we used no bond portfolios and different equity portfolios. The risk price on CP equals 102, the risk price on the LVL factor equals -45, and that of the MKT is close to zero. Because the exposure of the portfolios to LVL and MKT are about the

same, the risk prices on these factors are hard to estimate separately with these five portfolios. If we remove the level factor, we find that the risk price from *CP* hardly changes (from 102 to 97), but the price of market risk is now positive at 1. Finally, we compute the covariances of the five *CP* portfolios with the *CP* factor and find that the difference between the high- and low-*CP* beta portfolios is positive. The positive risk price and positive spread in covariances allows our model to explain most of the spread in average returns between the *CP* portfolios.

The second exercise double sorts stocks into five quintiles based on their *CP* exposure and then within *CP* quintile based on their book-to-market (*BM*) ratio. This results in a 5×5 sort (see Table IA.VI in the Online Appendix). For each *BM* group, we find a positive spread between high and low-*CP* exposure portfolios, with spreads ranging from 0.5% to 4.6% per year. We also find that the spread between high and low *BM* portfolios is positive in each *CP* group. This could imply that *CP* exposures and *BM* are related, yet not the same. Or it could reflect estimation error in *CP* exposures which prevents *CP* exposure from fully subsuming *BM* exposure. Turning to the pricing, we find that the CAPM model cannot explain the heterogeneity in average returns on the 25 portfolios along either dimension. The MAPE of the CAPM is 171bp per year. In contrast, our three-factor model eliminates a substantial fraction of the spread along *both CP* and *BM* dimensions. The MAPE reduces to 100bp. Ex-post *CP* exposures are higher for the portfolios with higher ex-ante *CP* exposures as well as for portfolios with higher *BM* ratios. In further support for our model, we find comparable market price of risk estimates to the benchmark ones, but from this double-sorted cross-section of equity portfolios (without bonds). For the market price of *CP* risk we find 71 (compared to 68 for the benchmark estimate), for *LVL* risk we estimate -24 (-20), and for *MKT* we have 0.8 (1.1); see Table IA.I of the Online Appendix for the post-1963 sample benchmark estimates. Taken together, these results suggest that there are separate spreads along the dimensions of ex-ante *CP* exposure and *BM* ratio. However, both spreads are to a large extent accounted for by our model with risk prices that are very similar to ones we estimated using other cross-sections of

assets.

IV. Structural Model with Business Cycle Risk

In the last part of the paper, we discuss a structural asset pricing model that connects our empirical findings in a transparent way. The model formalizes the relationships between the returns on value and growth stocks, the bond risk premium (CP), and the state of the macroeconomy. It does so in a unified pricing framework that can quantitatively account for the observed risk premia on stock and bond portfolios, while being consistent with the observed dynamics of dividend growth rates, inflation, and basic properties of the term structure of interest rates. Its role is largely pedagogical: to clarify the minimal structure necessary to account for the observed moments. In the interest of space, we focus here on a verbal discussion and relegate a full model description, solution, and calibration to Online Appendix D.

The model has one key state variable, s , which measures macroeconomic activity. One interpretation of s is as a leading business cycle indicator. This state variable follows an autoregressive process, with modest persistence, and its innovations ε_{t+1}^s are the first priced source of risk. Real dividend growth for value (V), growth (G), and market (M) equity portfolios is given by:

$$\Delta d_{t+1}^i = \gamma_{0i} + \gamma_{1i}s_t + \sigma_{di}\varepsilon_{t+1}^d + \sigma_i\varepsilon_{t+1}^i, \quad \forall i = \{V, G, M\}.$$

The shock ε_{t+1}^d is an aggregate dividend shock, the second priced source of risk, while ε_{t+1}^i is a non-priced idiosyncratic shock. The market portfolio has no idiosyncratic risk; $\sigma_M = 0$. The key parameter configuration is $\gamma_{1V} > \gamma_{1G}$ so that value stocks are more exposed to shocks in macroeconomic activity than growth stocks. As in the data (Section II.A), a low value for s is associated with lower future dividend growth on V minus G .

We construct recessions in the model in a procedure that mimics the NBER dating algorithm

and that matches the frequency and duration of recessions. Our calibration chooses γ_{1V} and γ_{1G} to match the decline in dividend growth value minus growth over the course of NBER recessions. Finally, inflation is the sum of a constant, a mean-zero autoregressive process which captures expected inflation, and an unexpected inflation shock. The innovation to expected inflation ε_{t+1}^x is the third and last priced source of risk.¹¹ Inflation and dividend growth parameters are chosen to match the unconditional mean and volatility of dividend growth and inflation, as well as the volatility and persistence of one- through five-year nominal bond yields.

To simplify our analysis, we assume that market participants' preferences are summarized by a real stochastic discount factor (SDF), whose log m evolves according to the process:

$$-m_{t+1} = y + \frac{1}{2}\Lambda_t'\Lambda_t + \Lambda_t'\varepsilon_{t+1}.$$

where the vector $\varepsilon_{t+1} \equiv (\varepsilon_{t+1}^d, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)'$ and y is the constant real rate of interest.¹² The market prices of risk are chosen to match the equity risk premium (the one associated with ε^d), slope of the yield curve (ε^x), and value risk premium (ε^s).

The model generates an affine nominal term structure of interest rates. It also generates a one-factor model for the nominal bond risk premium: All variation in bond risk premia comes from cyclical variation in the economy, s_t . Thus, the *CP* factor which measures the bond risk premium is perfectly positively correlated with s_t , the (leading) indicator of macroeconomic activity. Innovations to the *CP* factor are innovations to s (ε^s), lending a structural interpretation to *CP* shocks which is consistent with our empirical evidence. The constant component of the bond risk premium partly reflects compensation for cyclical risk and partly exposure to expected inflation risk. Exposure to the cyclical shock contributes negatively to excess bond

¹¹This inflation process is common in the literature (e.g., Wachter, 2006; Bansal and Shaliastovich, 2010).

¹²For similar approaches to the SDF, see Bekaert, Engstrom, and Xing (2009), Bekaert, Engstrom, and Grenadier (2010), Lettau and Wachter (2009), Campbell, Sunderam, and Viceira (2008), and David and Veronesi (2009).

returns: A positive ε^s shock lowers bond prices and returns, and more so for long than for short bonds. Exposure to expected inflation shocks contributes positively to excess bond returns: A positive ε^x shock lowers bond prices and returns but the price of expected inflation risk is negative. Since common variation in bond yields is predominantly driven by the inflation shock in the model, the latter acts like (and provides a structural interpretation for) a shock to the level of the term structure (*LVL*). Long bonds are more sensitive to level shocks, the traditional duration effect. Turning to stock pricing, the equity risk premium provides compensation for exposure to aggregate dividend growth risk (ε^d) and for cyclical risk (ε^s). Shocks to the market return (*MKT*) are a linear combination of ε^d and ε^s shocks. Like bond risk premia, equity risk premia vary over time with the state of the economy s_t . The model generates both an equity risk premium and a value premium. The reason for the value premium can be traced back to the fact that value stocks' dividends are more sensitive to cyclical shocks than those of growth stocks. Because the price of cyclical risk is naturally positive, the second term delivers the value premium. Put differently, in the model, as in the data, returns on value stocks are more exposed to bond risk premium shocks than returns on growth stocks.

For each asset, we can then compute covariances of unexpected returns with the *MKT*, *LVL*, and *CP* shocks, as defined inside the model. Interestingly, we are able to replicate the three-factor risk premium decomposition we uncovered in Section III. Figure 7 is the model's counterpart to Figure 6 in the data. It shows a good quantitative match for the relative contribution of each of the three sources of risk to the risk premia for growth, value, and market equity portfolios, as well as for maturity-sorted government bond portfolios. This fit is not a forgone conclusion, but results from the richness of the model and the choice of parameters.¹³

The model also generates interesting asset pricing dynamics over the business cycle as detailed

¹³For example, differential exposure to the market factor could have well been the source of the value risk premium in the model given that the market shocks are linear combinations of permanent dividend growth and transitory cyclical shocks. Or, bonds of different maturity could have differential exposure to the market factor shocks. The data show no heterogeneity in both types of exposures. The model has just enough richness to replicate these patterns.

in the Online Appendix.

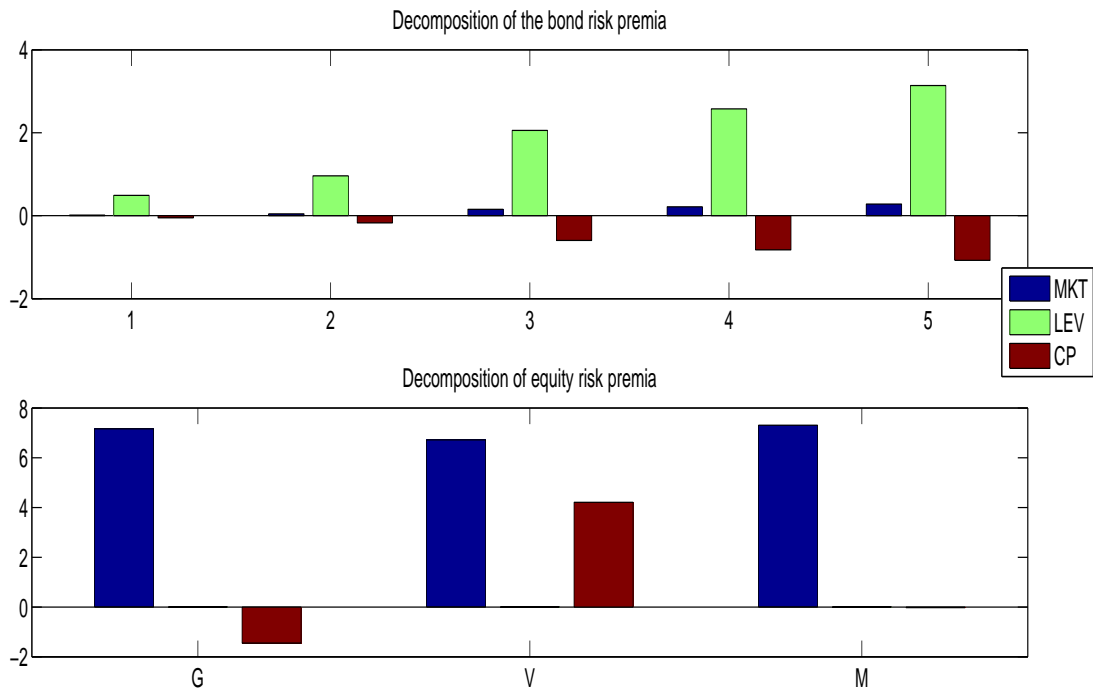


Figure 7. Decomposition of annualized excess returns in model.

The figure plots the risk premium (expected excess return) decomposition into risk compensation for exposure to the *CP* factor, the *LVL* factor, and the *MKT* factor. Risk premia, plotted against the left axis, are expressed in percent per year. The top panel is for the five bond portfolios (1-yr, 2-yr, 5-yr, 7-yr, and 10-yr) whereas the bottom panel is for growth (G), value (V), and market (M) stock portfolios. The results are computed from a 10,000 month model simulation under the calibration described in detail in Online Appendix D.3.

We conclude that the model delivers a structural interpretation for the *MKT*, *LVL*, and *CP* shocks. *CP* shocks reflect (transitory) cyclical shocks to the real economy, which naturally carry a positive price of risk. The *LVL* shock captures an expected inflation shock, and the *MKT* shock mostly captures a (permanent) dividend growth shock. The model quantitatively replicates the unconditional risk premium on growth, value, and market equity portfolios, and bond portfolios of various maturities, as well as the decomposition of these risk premia in terms of their *MKT*, *LVL*, and *CP* shock exposures. Furthermore, it matches some simple features of nominal term structure of interest rates and bond risk premia. It does so for plausibly calibrated dividend growth and inflation processes.

V. Conclusion

This paper makes three contributions. First, we argue that the value premium reflects compensation for macroeconomic risk. Times of low returns on value stocks versus growth stocks are times when future economic activity is low and future cash-flows on value stocks are low relative to those on growth stocks. A bond market variable, the bond risk premium as measured by the [Cochrane and Piazzesi \(2005\)](#) (*CP*) factor, is an early warning signal of such poor future economic performance. Innovations to the *CP* factor are contemporaneously highly positively correlated with returns on value stocks, but uncorrelated with returns on growth stocks.

Based on this connection, our second contribution is to estimate a parsimonious three-factor pricing model that explains return differences between average excess returns on book-to-market sorted stock portfolios, the aggregate stock market portfolio, government bond portfolios sorted by maturity, and corporate bond portfolios. The first factor is the traditional market return factor, the second one is the level of the term structure, and the third factor is the *CP* factor. We estimate a positive market price of risk of bond risk premium shocks, consistent with the notion that positive *CP* innovations represent good news about future economic activity.

Third, we show how to quantitatively account for the empirical links between nominal bond risk premium, the returns and dividend growth rates on value and growth stocks, and macroeconomic activity in a simple model that starts from fundamental shocks. Transitory shocks to the real economy that operate at business cycle frequency play a key role in connecting the bond risk premium to the value premium. Taken together, this paper provides a new mechanism linking the properties of stock and bond prices, obviating the need for a behavioral explanation of the value anomaly. Rather, it resuscitates a central role for business cycle risk in asset pricing.

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A. Additional Results for Section II

Dividends Around NBER Recessions pre-1952 The main text shows the behavior of log annual real dividends on value (fifth book-to-market), growth (first book-to-market), and market portfolios in Figure 2 as well as the difference in dividend growth between value and growth portfolios in Figure 2. Figures IA.1 and IA.2 show the corresponding evidence for the period 1926 until 1952. The message of these figures is very much consistent with the discussion in the main text.

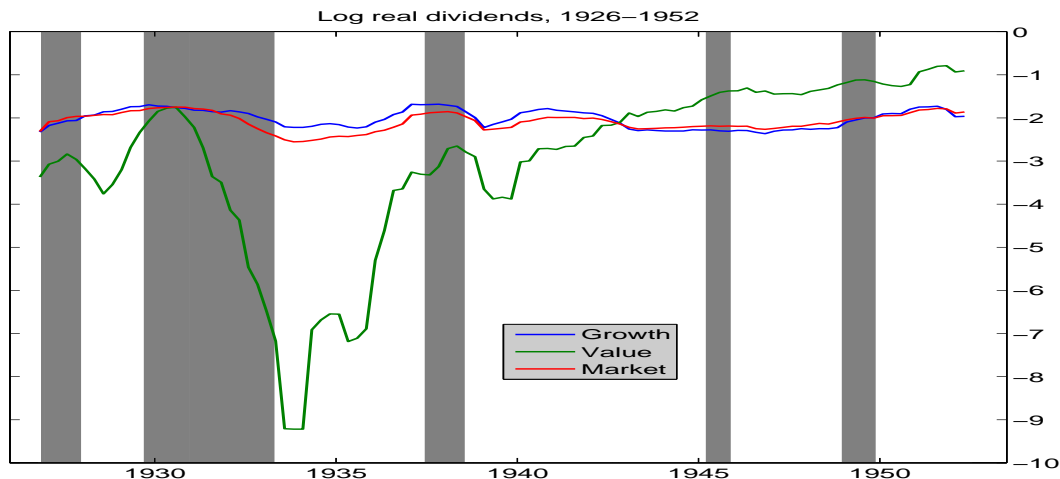


Figure IA.1. Dividends on value, growth, and market portfolios.

The figure plots the log real dividend on book-to-market quintile portfolios 1 and 5 and on the CRSP value-weighted market portfolio. Dividends are constructed from cum- and ex-dividend returns on these portfolios. Monthly dividends are annualized by summing dividends received during the year. The data are monthly from October 1926 until June 1952 and are sampled every three months in the figure.

Predicting GDP growth with CP In the main text we show that CP forecasts future economic activity, as measured by the $CFNAI$ index. As an alternative to $CFNAI$, we consider real gross domestic product (GDP) growth (seasonally adjusted annual rates) from the National Income and Product Accounts. This updates a regression that appears in the working paper version of [Cochrane and Piazzesi \(2005\)](#). The GDP data are available only at quarterly frequency, but go back to 1952 when the CP series starts. The CP factor in a given quarter is set equal to the value in the middle month of the quarter. We estimate

$$\Delta GDP_{t+k} = c_k + \beta_k CP_t + \varepsilon_{t+k}, \quad (\text{IA.1})$$

where k is the forecast horizon expressed in quarters. Figure IA.3 shows the coefficient β_k in the top panel, its t-statistic in the middle panel, and the regression R-squared in the bottom panel. The forecast horizon k is displayed on the horizontal axis and runs from 1 to 15 quarters. The figure shows the strong predictability of the CP factor for future real GDP growth. All three statistics

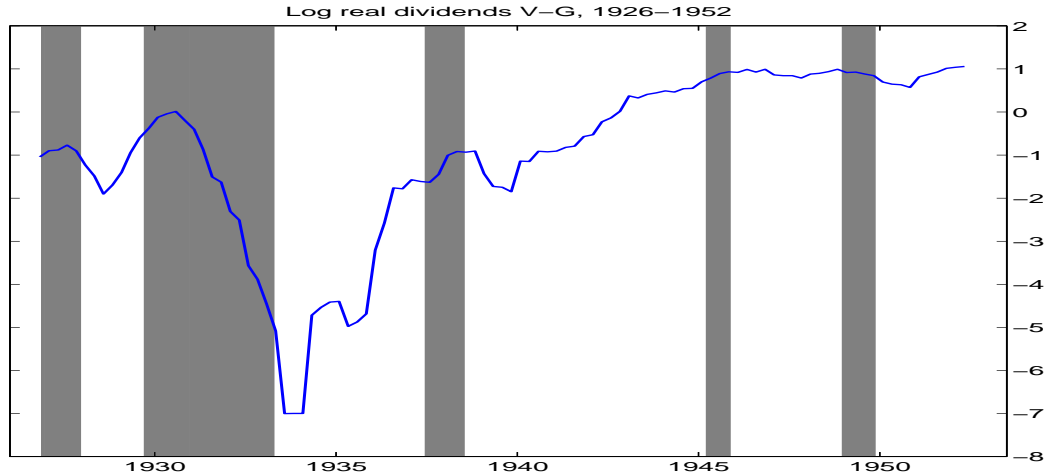


Figure IA.2. Dividends on value-minus-growth and NBER recessions.

The figure plots the log real dividend on book-to-market quintile portfolios 5 (value) minus the log real dividend on the boot-to-market portfolio 1 (growth), plotted against the right axis. The grey bars indicate official NBER recession dates. Dividends are constructed from cum- and ex-dividend returns on these portfolios. Monthly dividends are annualized by summing dividends received during the year. The data are monthly from October 1926 until June 1952 and are sampled every three months in the figure.

display a hump-shaped pattern, gradually increasing until five quarters ahead, and gradually declining afterwards. The maximum t-statistic is 3.9 and coincides with an R-squared value just below 12%. The point estimate at that horizon implies that a one standard deviation change in CP is associated with 0.55% points higher annualized GDP growth. The effect becomes statistically indistinguishable from zero after two years (quarter 9 and beyond). In sum, CP predicts future real GDP growth two to eight quarters ahead in economically and statistically significant ways.

Predicting CFNAI with the LVL factor Figure IA.4 predicts aggregate economic activity index CFNAI using the level factor of the term structure. The level factor is constructed as the first principal component of one- through five-year Fama-Bliss bond yields. Comparing the results to those in Figure 3, we see that the highest predictability is reached three years later rather than between one and two years, and that the predictability is much weaker than that of CP . The R-squared peaks at 4% compared to 15% for CP .

CP and NBER Recessions Figure IA.5 plots the CP factor over time (right axis) while drawing in NBER recessions (shaded areas). Consistent with the economic forecasting regressions, the CP factor is low before the start of most recessions in the post-1952 sample. It subsequently increases over the course of a recession, especially towards the end of the recession when better times are around the corner. In nearly every recession, the CP factor is substantially higher at the end than at the beginning of the recession. In the three deepest post-war recessions, the 1974, 1982, and 2008 recessions, CP dips in the middle of the recession -suggesting that bond markets fear a future deterioration of future economic prospects- before recovering.

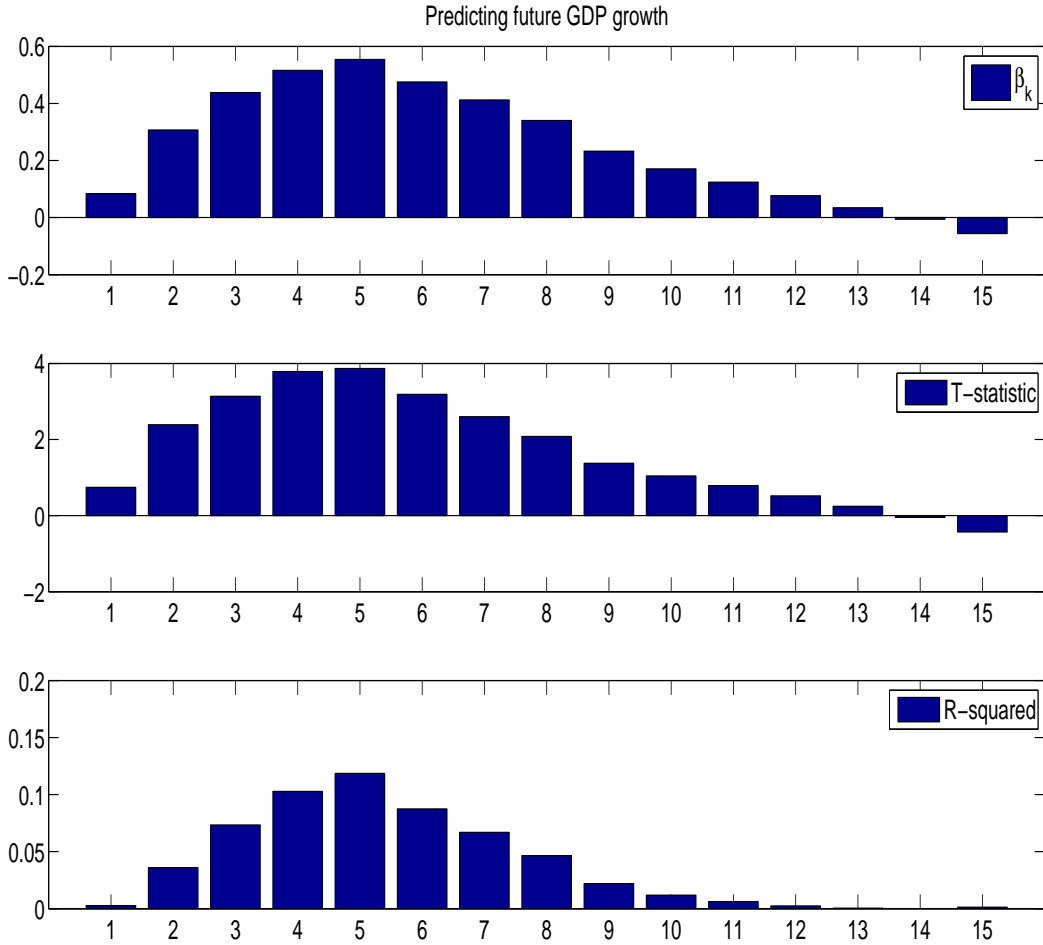


Figure IA.3. GDP growth predicted by the CP factor.

The top panel displays the predictive coefficient β_k in (IA.1), the middle panel the t-statistic, and the bottom panel the corresponding R-squared value. We consider $k = 1, \dots, 15$ quarters of lags, displayed on the horizontal axis in each panel, and the t-statistics are computed using Newey-West standard errors with $k - 1$ lags. The sample is 1952.III until 2011.IV (238 quarters).

Real GDP in CP -event Time We also study the behavior of real annual GDP growth in CP -event time. GDP growth rates are available over the entire post-war sample, whereas CFNAI only starts in 1967. Figure IA.6 is the same as Figure 4 in the main text, except that real GDP growth is plotted in the bottom right-hand side panel instead of CFNAI. Like CFNAI, GDP growth also shows a clean cycle around low- CP events. GDP grows at a rate that is 1% point above average four quarters before the event, the growth rate slows down to to the average in the event quarter, and growth further falls to rates 1.4% points below average four quarters after the event. The amplitude of this cycle (2.4% points) is economically large, representing one standard deviation of GDP growth.

Real GDP around Value Crashes Figure IA.7 shows the analogous figure to Figure 5 in the main text, except that real GDP growth is plotted in the bottom right-hand side panel instead of

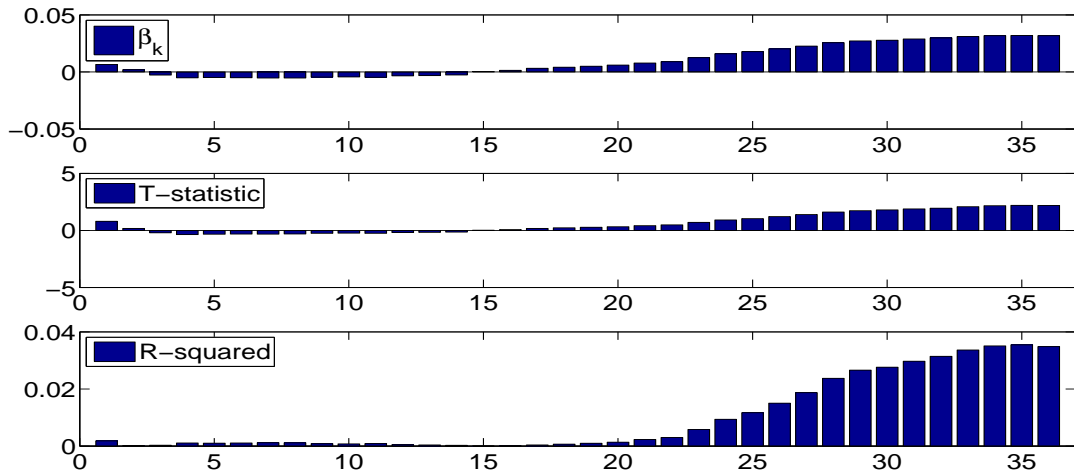


Figure IA.4. Predictive power of *LVL* to forecast future economic activity as measured by CFNAI.

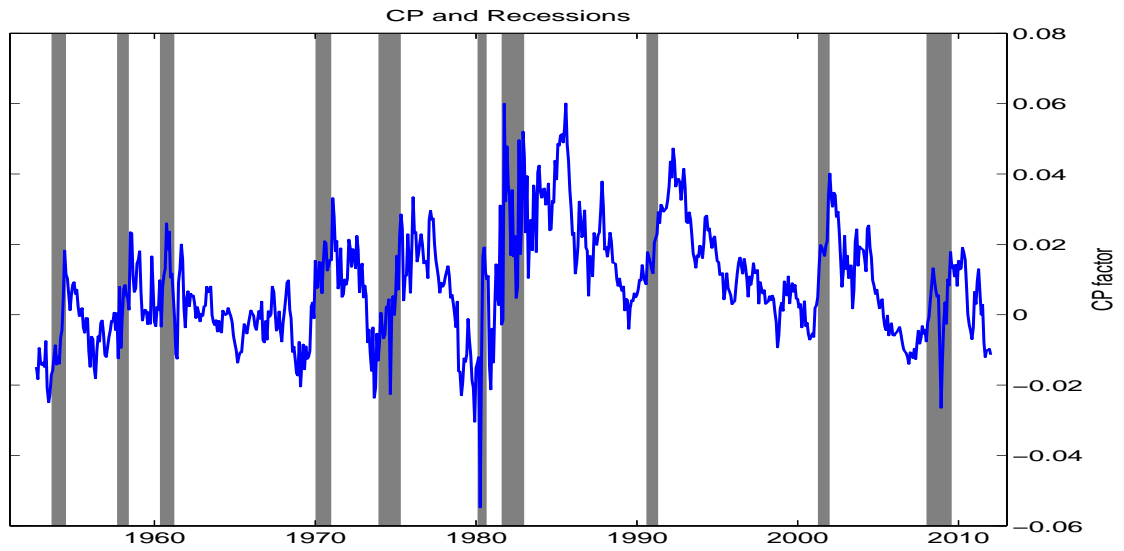


Figure IA.5. *CP* factor and NBER recessions.

The figure plots the *CP* factor (solid line, against the right axis) and the NBER recessions (shaded areas). The sample is July 1952 until December 2011.

CFNAI. GDP growth is only modestly below average in period 0 (-0.3% points), but falls to -1.2% points below average two-to-three quarters after the event. The change from 3 quarters before to 3 quarters after is 1.8% points, which is three-quarters of a standard deviation of real GDP growth.

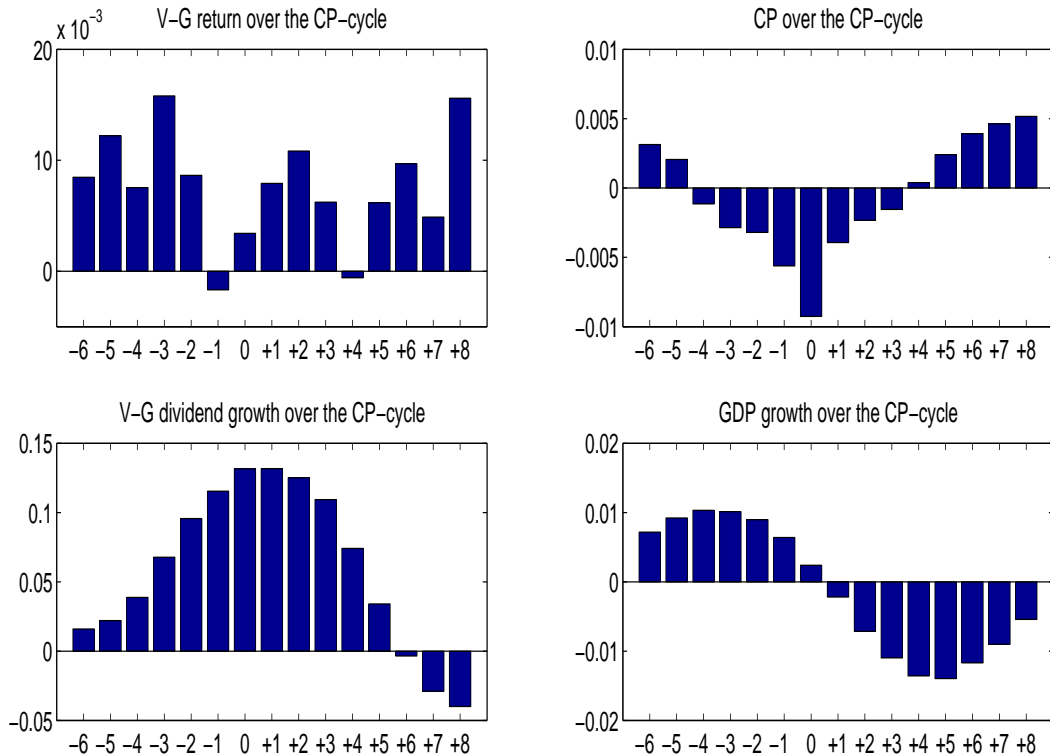


Figure IA.6. Low CP Events

The figure plots four quarterly series in event time. The event is defined as a quarter in which the quarterly CP factor is in its respective lowest 30% of observations. This selection leads to 71 events out of 238 quarters (30%). The sample runs from 1953.III until 2011.IV. In each panel, the period labeled '0' is the quarter in which the event takes place. The labels -1, -2, -3, etc refer to one, two, three, etc quarters before the event whereas the labels +1, +2, +3, etc. refer to one, two, three, etc quarters after the event. The top left panel plots the realization of the quarterly log return on value minus growth. The bottom left panel reports annual log dividend growth on value minus growth. The top right panel plots the CP factor. The bottom right panel plots real GDP growth.

B. How Pricing Stocks and Bonds Jointly Can Go Wrong

Consider two factors F_t^i , $i = 1, 2$, with innovations η_{t+1}^i . We normalize $\sigma(\eta_{t+1}^i) = 1$. Let $cov(\eta_{t+1}^1, \eta_{t+1}^2) = \rho = corr(\eta_{t+1}^1, \eta_{t+1}^2)$. We also have two cross-sections of test assets with excess, geometric returns r_{t+1}^{ki} , $i = 1, 2$ and $k = 1, \dots, K_i$, with innovations ε_{t+1}^{ki} . We assume that these returns include the Jensen's correction term. Suppose that both cross-sections exhibit a one-factor pricing structure:

$$E(r_{t+1}^{ki}) = cov(\varepsilon_{t+1}^{ki}, \eta_{t+1}^i) \lambda_i, \quad i = 1, 2.$$

The first factor perfectly prices the first set of test assets, whereas the second factor prices the second set of test assets. We show below that this does *not* imply that there exists a single SDF that prices both sets of assets.

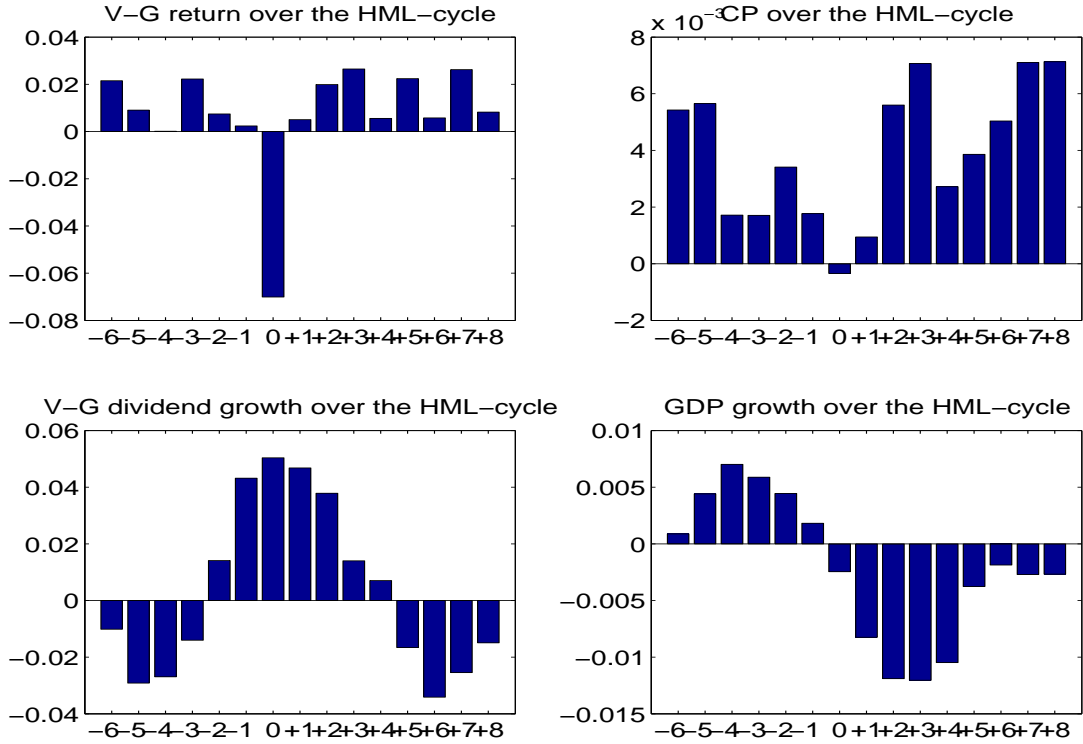


Figure IA.7. Value Crashes

The figure plots four quarterly series in event time. The event is defined as a quarter in which both the realized log real return on the fifth book-to-market portfolio (value) and the realized log return on value minus growth (first book-to-market portfolio) are in their respective lowest 30% of observations. This intersection leads to 35 events out of 238 quarters (15%). The sample runs from 1953.III until 2011.IV. In each panel, the period labeled ‘0’ is the quarter in which the event takes place. The labels -1, -2, -3, etc refer to one, two, three, etc quarters before the event whereas the labels +1, +2, +3, etc. refer to one, two, three, etc quarters after the event. The top left panel plots the realization of the quarterly log return on value minus growth. The bottom left panel reports annual log dividend growth on value minus growth. The top right panel plots the CP factor. The bottom right panel plots real GDP growth.

Consider the following model of unexpected returns for both sets of test assets:

$$\begin{aligned}\varepsilon_{t+1}^{k1} &= E\left(r_{t+1}^{k1}\right)\eta_{t+1}^1, \\ \varepsilon_{t+1}^{k2} &= E\left(r_{t+1}^{k2}\right)\eta_{t+1}^2 + \alpha_{2k}\eta_{t+1}^3,\end{aligned}$$

with $cov\left(\eta_{t+1}^2, \eta_{t+1}^3\right) = 0$. Unexpected returns on the first set of test assets are completely governed by innovations to the first factor, whereas unexpected returns on the second set of test assets contain a component $\alpha_{2k}\eta_{t+1}^3$ that is orthogonal to the component governed by innovations to the second factor. These η^3 shocks are not priced (by assumption). We assume that they are correlated with the η^1 shocks: $cov\left(\eta_{t+1}^1, \eta_{t+1}^3\right) \neq 0$.

This structure implies:

$$cov\left(\varepsilon_{t+1}^{ki}, \eta_{t+1}^i\right) = E\left(r_{t+1}^{ki}\right)var\left(\eta_{t+1}^i\right) = E\left(r_{t+1}^{ki}\right),$$

and hence $\lambda_i = 1$, $i = 1, 2$. Then we have:

$$\begin{aligned} \text{cov}(\varepsilon_{t+1}^{k1}, \eta_{t+1}^1) &= E(r_{t+1}^{k1}), & \text{cov}(\varepsilon_{t+1}^{k1}, \eta_{t+1}^2) &= E(r_{t+1}^{k1})\rho, \\ \text{cov}(\varepsilon_{t+1}^{k2}, \eta_{t+1}^1) &= (r_{t+1}^{k2})\rho + \alpha_{2k}\text{cov}(\eta_{t+1}^1, \eta_{t+1}^3), & \text{cov}(\varepsilon_{t+1}^{k2}, \eta_{t+1}^2) &= E(r_{t+1}^{k2}). \end{aligned}$$

The main point is that, if α_{2k} is not proportional to $E(r_{t+1}^{k2})$, then there exist no Λ_1 and Λ_2 such that:

$$E(r_{t+1}^{ki}) = \text{cov}(\varepsilon_{t+1}^{ki}, \eta_{t+1}^1)\Lambda_1 + \text{cov}(\varepsilon_{t+1}^{ki}, \eta_{t+1}^2)\Lambda_2.$$

On the other hand, if there is proportionality and $\alpha_{2k} = \alpha E(r_{t+1}^{k2})$, then we have:

$$\text{cov}(\varepsilon_{t+1}^{k2}, \eta_{t+1}^1) = E(r_{t+1}^{k2})(\rho + \alpha\text{cov}(\eta_{t+1}^1, \eta_{t+1}^3)) = E(r_{t+1}^{k2})\xi,$$

and Λ_1 and Λ_2 are given by:

$$\Lambda_1 = \frac{1 - \rho}{1 - \xi\rho}, \text{ and } \Lambda_2 = \frac{1 - \xi}{1 - \xi\rho}.$$

This setup is satisfied approximately in our model, where the first set of test assets are the book-to-market portfolios, η^1 are *CP* innovations, the second set of test assets are the bond portfolios, and η^2 are level innovations. Unexpected bond returns contain a component η^3 that is uncorrelated with level innovations, but that is correlated with *CP* innovations. Unexpected book-to-market portfolio returns, in contrast, are largely uncorrelated with level innovations. The result above illustrates that consistent risk pricing is possible because unexpected bond returns' exposure to *CP* shocks has a proportionality structure. This can also be seen in the top panel of Figure 6.

C. Additional Results for Section III

This section considers several exercises investigating the robustness of our empirical results in Section III. First, we use a different weighting matrix in the market price of risk estimation. Second, we do a subsample analysis. Third, we study additional stock and bond portfolios.

C.1. Weighted Least-Squares

Our cross-sectional estimation results in Table I assume a GMM weighting matrix equal to the identity matrix. This is equivalent to minimizing the root mean-squared pricing error across the 11 test assets. The estimation devotes equal attention to each pricing error and leads to a RMSE of 48bp per year. A natural alternative to the identity weighting matrix is to give more weight to the assets that are more precisely measured. We use the inverse covariance matrix of excess returns, as in Hansen and Jagannathan (1997). This amounts to weighting the bond pricing errors more heavily than the stock portfolio pricing errors in our context. When using the Hansen-Jagannathan distance matrix, we find a MAPE of 53bp per year compared to 41bp per year. However, the weighted RMSE drops from 48bp to 25bp per year. The reason for the improvement in RMSE is that the pricing errors on the bonds decrease substantially, from a MAPE of 43bp to 12bp per year. Finally, the price of risk

estimates in $\hat{\Lambda}_0$ are comparable to those in the benchmark case. The price of *CP* risk remains positive and is estimated to be somewhat lower than in the benchmark case, at 48.3 (with a standard error of 12.2). The market price of *LVL* risk remains statistically negative (-14.7 with standard error of 6.3), and the price of *MKT* risk remains positive (2.67 with a standard error of 1.1). The null hypothesis that all pricing error parameters are jointly zero continues to be strongly rejected. We conclude that our results are similar when we use a different weighting matrix.

C.2. Subsample Analysis

Table [IA.I](#) shows that our main empirical results are robust for various subsamples. These subsample results use *LVL* and *CP* factors which are estimated over the subsample in question. We have also investigated the same subsamples without re-estimating *CP* with similar results.

When we start the analysis in 1963, an often-used starting point for cross-sectional equity analysis (e.g., [Fama and French, 1993](#)), we find very similar results. The left columns in Panel A of Table [IA.I](#) shows a MAPE of 40 basis points per year, somewhat lower than the 48 basis points MAPE in the full sample. Our model improves relative to the Fama-French three-factor model, which has a pricing error of 71 basis points in this subsample (and 60 basis points in the full sample). The value premium is 4.7% in this sample, higher than the 4% in the full sample. The model accounts for all but 0.9% of this spread. In the right columns, we investigate the results in the second half of our sample, 1980-2011. Mean absolute pricing errors fall further to 36 basis points, while the MAPE under the Fama-French model rises to 82 basis points.

Panel B of Table [IA.I](#) shows that the price of risk estimates are similar to the ones from the benchmark estimation. The price of *CP* risk is lower in both subsamples than in the full sample, but statistical tests (not reported) confirm that they are not statistically different from the full sample point estimate. Finally, Panel C shows that in both subsamples, we reject the null that the prices of risk are zero at the 5% level. Also, we fail to reject the null that all pricing errors are jointly zero with p-values of 8.3% and 30.6%, respectively. In contrast, for the Fama-French model, we can reject the null that the pricing errors are zero at the 5% level in both subsamples.

C.3. Other Test Assets

Given that we found a unified SDF that does a good job pricing the cross-section and time-series of book-to-market sorted stock and maturity-sorted bond returns, a natural question that arises is whether the same SDF model also prices other stock or bond portfolios. In addition, studying more test assets allows us to address the [Lewellen, Shanken, and Nagel \(2009\)](#) critique. They argue that explanatory power of many risk-based models for the cross-section of (size and) value stocks may be poorly summarized by the cross-sectional R^2 .

One of their proposed remedies is to use more test assets in the evaluation of asset pricing models. Our benchmark results address this criticism already by adding maturity-sorted government bond portfolios to the cross-section of book-to-market stock portfolios. In addition, we now study several other sets of test assets. We start by adding corporate bond portfolios. Then we study replacing ten decile book-to-market portfolios by ten size decile portfolios, 25 size and book-to-market portfolios, and ten earnings-price portfolios.

Table IA.I
Other Sample periods - Pricing Errors

This table reports robustness with respect to different sample periods. It is otherwise identical to Table I. The data are monthly from January 1963 through December 2011 in the left columns and from February 1980 until December 2011 in the right columns.

Panel A: Pricing Errors (in % per year)						
	1963-2011			1980-2011		
	RN SDF	Our SDF	FF	RN SDF	Our SDF	FF
1-yr	1.03	-0.53	0.76	1.33	-0.02	0.86
2-yr	1.26	-0.89	0.74	1.94	-0.26	0.90
5-yr	1.92	-0.11	0.85	3.18	-0.11	0.82
7-yr	2.35	0.46	0.94	3.94	0.14	0.70
10-yr	2.11	0.11	0.41	4.07	0.08	0.11
Market	5.32	-0.83	-0.02	6.48	-0.86	0.71
BM1	4.76	-0.12	0.70	6.37	-0.20	0.36
BM2	5.91	0.11	-0.57	7.79	0.82	-0.70
BM3	6.30	0.28	-1.11	7.25	0.26	-1.19
BM4	7.75	-0.17	-0.55	7.76	-0.52	-0.99
BM5	9.36	0.77	1.25	9.26	0.61	1.64
MAPE	4.37	0.40	0.71	5.40	0.36	0.82
Panel B: Market Prices of Risk						
MKT		1.15	5.14		1.98	5.99
LVL/SMB		-19.72	-8.36		-23.38	-19.77
CP/HML		68.14	6.00		44.33	2.14
Panel C: P-values of χ^2 Tests						
$\Lambda_0 = 0$	-	1.74%	0.18%	-	1.08%	0.73%
Pr. err. = 0	-	8.29%	0.04%	-	30.58%	2.56%

C.3.1. Adding Corporate Bond Portfolios

One asset class that deserves particular attention is corporate bonds. After all, at the firm level, stocks and corporate bonds are both claims on the firm’s cash flows albeit with different priority structure. We ask whether, at the portfolio level, our SDF model is able to price portfolios of corporate bonds sorted by ratings class. Fama and French (1993) also include a set of corporate bond portfolios in their analysis but end up concluding that a separate credit risk factor is needed to price these portfolios. Instead, we find that the same three factors we used so far also do a good job pricing the cross-section of corporate bond portfolios, better than, for example, the standard Fama-French three-factor model.

We use data from Citibank’s Yield Book for four investment-grade portfolios: AAA, AA, A, and BBB. Return data for these portfolios are available monthly from February 1980 until December 2011, which restricts our estimation to this sample. Their annualized excess returns are listed in the first column of Table IA.II. There is a 68 basis point annual excess return spread between the BBB and the AAA portfolio in this period.

In a first exercise, we calculate Euler equations errors for these four portfolios, using our SDF model presented in Section III. That is, we do not re-estimate the market price of risk parameters $\hat{\Lambda}_0$, but simply calculate the pricing errors for the corporate bond portfolios. The market prices of risk are the same as those reported in Table IA.I for the post-1980 sample. The resulting annualized pricing errors are listed in the second column of Table IA.II. The model does a good job pricing the corporate bonds: mean absolute pricing errors on the credit portfolios shrink to 0.6% per year, compared to excess returns of more than 4% per year under risk-neutral pricing. None of the pricing errors is statistically different from zero. The mean absolute pricing error among all fifteen test assets is a low 42 basis points per year; only 6 basis points are added by the corporate bond portfolios.

Equally interesting is to re-estimate the market price of risk parameters of the SDF model when the corporate bond portfolios are included in the set of test assets. We do not allow for additional priced factors; the *CP*, *LVL*, and *MKT* factors remain the only three priced risk factors. The third column of Table IA.II shows that the corporate bond pricing errors are now below 50 basis points per year and they go through zero. the MAPE on the credit portfolios is 33 basis points per year. The overall MAPE on all fifteen assets is 39 basis points per year, 3 basis points above the MAPE when corporate bonds are not considered, and 3 basis points less than when the corporate bonds were not included in the estimation. Finally, comparing Columns 2 and 3, the point estimates for the market prices of risk Λ_0 in Panel B are similar for the models with or without corporate bonds. The last column reports results for the Fama-French three-factor model. Its pricing errors are higher than in our three-factor model; the MAPE is 89 basis points. Average pricing errors on the corporate bond portfolios are around 1% per year. We reject the null that all pricing errors are zero at the 5% level. We fail to reject the null that all pricing errors are jointly zero (p-value of 30.3%), while the FF model marginally fails to reject it (p-value of 5.7%). The low pricing errors for these corporate bond

C.3.2. Other Equity Portfolios

Table IA.III shows three exercises where we replace the book-to-market sorted equity portfolios by other equity portfolios. In the first three columns we use ten market capitalization-sorted portfolios alongside the bond portfolios and the market. The first column shows the risk premia to be explained (risk neutral SDF). Small firms (S1) have 3.6% higher risk premia than large stocks (S10). Our model in the second column manages to bring the overall mean absolute pricing error down from 6.17%

Table IA.II
Unified SDF Model for Stocks, Treasuries, and Corporate Bonds

Panel A of this table reports pricing errors on five book-to-market-sorted stock portfolios, the value-weighted market portfolio, five Treasury bond portfolios of maturities 1, 2, 5, 7, and 10 years, and four corporate bond portfolios sorted by S&P credit rating (AAA, AA, A, and BBB). They are expressed in percent per year. The sample is February 1980 until December 2011.

Panel A: Pricing Errors (% per year)				
	RN SDF	Our SDF not re-estimated	Our SDF re-estimated	FF
1-yr	1.33	-0.02	0.09	0.80
2-yr	1.94	-0.26	-0.08	0.74
5-yr	3.18	-0.10	0.16	0.44
7-yr	3.94	0.14	0.44	0.16
10-yr	4.07	0.08	0.40	-0.53
Market	6.48	-0.95	-0.97	0.88
BM1	5.80	-0.20	-0.31	0.41
BM2	7.31	0.82	0.78	-0.97
BM3	6.71	0.26	0.25	-1.42
BM4	7.63	-0.52	-0.37	-1.29
BM5	9.93	0.61	0.75	1.67
Credit1	3.86	-0.83	-0.48	0.64
Credit2	3.88	-0.74	-0.41	0.55
Credit3	4.12	-0.64	-0.31	0.89
Credit4	4.54	-0.19	0.12	1.92
MAPE	5.05	0.42	0.39	0.89
Panel B: Prices of Risk Estimates				
MKT		1.98	2.07	6.49
LVL/SMB		-23.38	-21.39	-23.20
CP/HML		44.33	40.64	2.12
Panel C: P-values of χ^2 Tests				
$\Lambda_0 = 0$	-	-	1.37%	0.95%
Pr. err. = 0	-	-	30.28%	5.87%

per year to 0.42% per year, comparable to the 0.48% (0.46%) we obtained with the book-to-market quintile (decile) portfolios. The market prices of risk are not statistically different from those estimated on book-to-market portfolios instead of size portfolios. Moreover, we fail to reject the null that all pricing errors are zero at the 5% level. Our model’s MAPE is somewhat lower than the 0.57% in the Fama-French model in the third column. The Fama-French model does better eliminating the spread between small and large stocks, whereas our model does better pricing the bond portfolios.

The next three columns use earnings-price-sorted quintile stock portfolios. The highest earnings-price portfolio has an average risk premium that is 6.3% higher per year than the lowest earnings-price portfolio. Our model reduces this spread in risk premia to 2.5% per year, while continuing to price the bonds reasonably well. The MAPE is 98 basis points per year compared to 72 in the Fama-French model.

The last three columns use the five-by-five market capitalization and book-to-market double sorted portfolios. Our three-factor model manages to bring the overall mean absolute pricing error down from 7.64% per year to 1.27% per year. This is again comparable to the three-factor Fama-French model’s MAPE of 1.17%. Relative to the FF model, ours reduces the pricing errors on the hard-to-explain S1B1 portfolio, but makes a larger error on the S1B4 and S1B5 portfolios.

The market price of risk estimates Λ_0 in Panel B of Table IA.III are comparable to those we found for the book-to-market portfolios in Table I. Panel C shows that we reject the null hypothesis that all market prices of risk are zero for all three sets of test assets. We fail to reject the null hypothesis that all pricing errors are zero on the size and earnings-price portfolios, and marginally reject the null (at the 1% level) for the twenty-five portfolios. We conclude that these results are in line with our benchmark results and that they further strengthen the usefulness of our empirical three-factor model.

C.4. Twisted CP

In Table IA.IV, we explore the pricing implications of two different yield-based pricing factors, which we refer to as “twisted CP” factors. They are the fitted values of a regression of macroeconomic activity $CFNAI_{t+k}$ on the one- through five-year yields at time t , where k is 12 and 24, respectively. We discuss these results in the main text.

C.5. Individual Firm Returns

In Table IA.V, we sort individual stocks into five portfolios based on their exposure to CP shocks in the previous 60 months. The table reports the average excess return per portfolio, the CAPM alphas, the alphas for the KLN model, the CP exposures of the five portfolios, the risk prices, and MAPE for the different models. We also consider a version of the KLN model where we omit the LVL factor as the exposures of the five portfolios to LVL and MKT are very similar. We provide further details on the sample selection and sorting procedure in the main text where we also discuss the results in detail.

In Table IA.VI, we first sort individual stocks into five portfolios based on their exposure to CP shocks in the previous 60 months. We then sort the stocks in each of the groups into five groups based on their B/M ratio. This results in 25 portfolios that differ by their CP exposure and B/M ratio. We report the same statistics as in Table IA.V for the single sorts based on CP exposures only. We discuss the sample selection, sorting procedure, and results in the main text.

Table IA.III
Other Stock Portfolios - Pricing Errors

This table reports robustness with respect to different stock market portfolios, listed in the first row. Panel A of this table reports pricing errors (in % per year) on various stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. Each column corresponds to a different stochastic discount factor (SDF) model. The first column contains the risk-neutral SDF. The second column presents our SDF model with three priced factors (Our). The third column refers to the three factor model of Fama and French (FF). The last row of Panel A reports the mean absolute pricing error across all securities (MAPE). Panel B reports the estimates of the prices of risk. The data are monthly from June 1952 through December 2011.

Panel A: Pricing Errors (in % per year)											
10 Size Portfolios				5 Earnings-Price Portfolios				25 Size and Value Portfolios			
Assets	RN SDF	Our SDF	FF	Assets	RN SDF	Our SDF	FF	Assets	RN SDF	Our SDF	FF
1-yr	0.99	-0.12	0.70		0.99	-1.33	0.74		0.99	-1.39	0.86
2-yr	1.24	-0.17	0.85		1.24	-1.59	0.83		1.24	-1.69	1.08
5-yr	1.71	0.07	1.37		1.71	-0.58	1.00		1.71	-0.73	1.61
7-yr	2.06	0.51	1.86		2.06	0.54	1.17		2.06	0.39	2.01
10-yr	1.73	0.10	1.10		1.73	0.58	0.58		1.73	0.44	1.47
Market	6.42	-0.62	-0.21		6.42	-1.13	0.03		6.42	0.14	0.09
S1	9.39	1.10	-0.24	EP1	5.13	-0.71	0.71	S1B1	3.52	-3.76	-5.11
S2	9.07	-0.08	0.06	EP2	6.06	-1.01	-0.87	S1B2	9.52	-0.52	0.39
S3	9.61	0.69	0.27	EP3	7.62	-0.13	-1.07	S1B3	9.77	-0.63	0.49
S4	9.04	0.30	0.28	EP4	9.45	1.37	-0.03	S1B4	11.97	3.13	2.18
S5	9.19	-0.02	0.43	EP5	11.43	1.77	0.89	S1B5	13.37	2.97	1.72
S6	8.66	-0.05	0.08					S2B1	5.65	-1.58	-1.60
S7	8.52	-0.33	0.23					S2B2	8.90	-0.15	0.17
S8	8.03	-0.66	-0.50					S2B3	11.04	1.22	1.70
S9	7.34	-0.18	-0.82					S2B4	11.11	1.24	0.90
S10	5.79	-0.34	0.15					S2B5	12.38	2.47	0.04
								S3B1	6.68	-1.74	0.44
								S3B2	9.30	0.09	0.98
								S3B3	9.59	-0.14	0.44
								S3B4	10.67	0.18	0.65
								S3B5	12.01	1.48	0.55
								S4B1	7.18	0.16	1.75
								S4B2	7.43	-1.13	-0.63
								S4B3	9.33	0.11	0.18
								S4B4	9.87	-1.69	0.27
								S4B5	9.81	-1.67	-1.71
								S5B1	5.92	2.23	1.99
								S5B2	6.56	1.73	0.31
								S5B3	6.97	1.31	-0.09
								S5B4	6.84	-1.50	-1.99
								S5B5	7.41	-1.66	-2.94
MAPE	6.17	0.42	0.57		4.89	0.98	0.72		7.64	1.27	1.17

Panel B: Market Prices of Risk					
MKT		2.31	6.39		1.80 5.65
Level/SMB		-16.69	0.58		-26.01 -6.96
CP/HML		71.34	21.38		154.70 9.25

Panel C: P-values of χ^2 Tests					
$\Lambda_0 = 0$	-	1.18%	1.42%	-	0.36% 0.00%
Pr. err. = 0	-	6.92%	1.11%	-	4.38% 0.02%

Table IA.IV
Twisted CP Factors

This table reports pricing errors on five book-to-market sorted quintile stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. They are expressed in percent per year (monthly numbers multiplied by 1200). Each column corresponds to a different stochastic discount factor (SDF) model, where the *CP* factors differ. We also report the mean absolute pricing error across all 11 securities (MAPE) and the estimates of the prices of risk. The first column repeats our benchmark estimates. The second and third columns replace the standard *CP* factor by a “twisted CP” factor in the pricing model. They are the fitted values of a regression of macro-economic activity $CFNAI_{t+k}$ on the one- through five-year yields at time t , where k is 12 and 24, respectively. The data are monthly from June 1952 through December 2011.

	<i>CP</i>	<i>Twisted CP</i> ($k = 12$)	<i>Twisted CP</i> ($k = 24$)
Pricing errors:			
Market	-0.76	-0.73	-0.99
Bond portfolios	0.18	0.19	0.07
	0.41	0.26	0.55
	-0.25	-0.17	-0.19
	-0.75	-0.51	-1.09
	-0.55	-0.69	-0.39
B/M portfolios	-0.29	-0.14	-0.90
	-0.04	-1.26	0.28
	0.70	0.50	0.89
	-0.03	1.15	0.18
	0.51	0.63	0.70
Risk prices:			
<i>CP</i>	90.82	106.97	117.25
<i>LVL</i>	-19.47	-5.21	-24.35
<i>MKT</i>	2.20	1.11	2.33
MAPE	0.41	0.57	0.57
RMSE	0.48	0.67	0.66
Correlation with <i>CP</i>	100%	59%	95%
Correlation with <i>CP</i> shocks	100%	17%	87%

Table IA.V
Individual Firm Returns: Single Sorts

This table reports the results of sorting individual firms into five portfolios based on their exposure to *CP* shocks. We use 60-month rolling window estimates of *CP* betas, where we require at least 12 months of data for a stock to be included in one of the five portfolios. The table reports the average excess return per portfolio, the CAPM alphas, the alphas for our three-factor model (“KLN alphas”), the *CP* exposures of the five portfolios, the risk prices, and the mean absolute pricing error (MAPE) for the different models. The last row reports results for a version of our model where we omit the *LVL* factor; we do this because the exposures of the five portfolios to *LVL* and *MKT* are very similar. The data are monthly from July 1963 through December 2010.

	low <i>CP</i> Exposure					High <i>CP</i> Exposure	H-L <i>CP</i> beta	Risk prices			
								<i>CP</i>	<i>LVL</i>	<i>MKT</i>	MAPE
Avg. excess ret.	4.3%	4.9%	5.9%	5.8%	6.8%	2.4%					
CAPM alphas	-1.8%	-0.2%	1.0%	0.5%	0.6%	2.4%			2.07	82bp	
KLN alphas	-0.2%	-0.3%	0.6%	-0.5%	0.4%	0.5%	102.22	-45.28	-0.15	39bp	
<i>CP</i> covariances ($\times 10^5$)	2.30	2.13	1.96	2.95	3.60	1.30					
KLN alphas w/o <i>LVL</i>	-1.1%	0.1%	1.4%	0.0%	-0.2%	0.9%	96.6		0.93	56bp	

Table IA.VI
Individual Firm Returns: Double Sorts

This table reports the results of sorting individual firms into 25 portfolios based on their exposure to *CP* shocks and B/M ratio. We use 60-month rolling window estimates of *CP* betas, where we require at least 12 months of data for a stock to be included in one of the portfolios. We first sort stocks on *CP* betas into five portfolios, and then sort each of these groups into 5 portfolios based on their B/M ratio. The table reports the average excess return per portfolio, the CAPM alphas, the alphas for the KLN model, the *CP* exposures of the five portfolios, the risk prices, and MAPE for the different models. The data are monthly from July 1963 through December 2010.

Average excess returns	Low B/M				High B/M	H-L B/M				
Low <i>CP</i> exposure	2.1%	3.2%	7.7%	7.0%	8.0%	5.9%				
	4.6%	5.1%	4.7%	7.3%	7.4%	2.8%				
	5.2%	5.1%	6.0%	6.8%	8.8%	3.6%				
	5.4%	5.4%	7.1%	10.9%	8.7%	3.3%				
High <i>CP</i> exposure	4.7%	7.8%	8.2%	9.2%	10.1%	5.4%				
High-low <i>CP</i> exposure	2.6%	4.6%	0.5%	2.2%	2.1%					
							Risk prices			
CAPM alphas	Low B/M				High B/M	H-L B/M	<i>CP</i>	<i>LVL</i>	<i>MKT</i>	MAPE
Low <i>CP</i> exposure	-5.8%	-3.8%	0.9%	0.1%	0.5%	6.3%			2.50	171bp
	-1.8%	-1.1%	-0.8%	1.5%	0.7%	2.5%				
	-1.0%	-0.9%	0.8%	1.2%	2.7%	3.7%				
	-1.3%	-0.9%	0.8%	4.8%	2.2%	3.5%				
High <i>CP</i> exposure	-3.1%	0.6%	1.2%	2.0%	2.5%	5.6%				
High-low <i>CP</i> exposure	2.7%	4.5%	0.3%	1.9%	2.0%					
							Risk prices			
KLN alphas	Low B/M				High B/M	H-L B/M	<i>CP</i>	<i>LVL</i>	<i>MKT</i>	MAPE
Low <i>CP</i> exposure	-1.3%	-2.3%	0.8%	-0.8%	-1.0%	0.3%	71.34	-24.44	0.78	100bp
	1.0%	0.1%	-1.6%	-0.9%	-0.1%	-1.1%				
	1.2%	0.0%	-1.2%	-1.0%	1.8%	0.6%				
	-0.6%	-0.4%	0.6%	2.5%	1.7%	2.3%				
High <i>CP</i> exposure	-0.6%	2.2%	-0.8%	0.5%	0.2%	0.8%				
High-low <i>CP</i> exposure	0.7%	4.5%	-1.6%	1.3%	1.2%					
							Risk prices			
<i>CP</i> covariances ($\times 10^5$)	Low B/M				High B/M	H-L B/M				
Low <i>CP</i> exposure	-0.27	2.34	3.61	5.37	6.91	7.18				
	-0.02	1.55	3.41	5.36	5.07	5.09				
	-0.05	1.76	4.43	4.89	4.60	4.64				
	2.12	2.18	3.46	5.60	4.54	2.42				
High <i>CP</i> exposure	2.02	2.51	5.60	5.86	7.05	5.03				
High-low <i>CP</i> exposure	2.29	0.16	1.99	0.49	0.14					

D. Structural Model

This appendix provides the details of the structural asset pricing model that is sketched in the main text. It is the simplest structural model that provides the link between the state of the economy, the nominal bond risk premium, and value/growth stocks. We start by describing the setup and provide the derivations of the asset pricing expressions. We also discuss the parameters used in the numerical example, and how they were chosen.

D.1. Setup

The main driving force in the model is the mean-reverting process for s_t , which describes the state of the business cycle:

$$s_{t+1} = \rho_s s_t + \sigma_s \varepsilon_{t+1}^s.$$

Higher values of s denote higher economic activity. Note that the model permits an interpretation of s as a signal about *future* economic activity. Since this variable moves at business cycle frequency, the persistence ρ_s is moderate.

Real dividend growth on asset $i = \{G, V, M\}$ (Value, Growth, and the Market) is given by:

$$\Delta d_{t+1}^i = \gamma_{0i} + \gamma_{1i} s_t + \sigma_{di} \varepsilon_{t+1}^d + \sigma_i \varepsilon_{t+1}^i. \quad (\text{IA.2})$$

If $\gamma_{1i} > 0$, dividend growth is pro-cyclical. The shock ε_{t+1}^d is an aggregate dividend shock, while ε_{t+1}^i is an (non-priced) idiosyncratic shock; the market portfolio has no idiosyncratic risk; $\sigma_M = 0$. The key parameter configuration is $\gamma_{1V} > \gamma_{1G}$ so that value stocks are more exposed to cyclical risk than growth stocks. As is the data, a low value for s is associated with lower future dividend growth on V minus G . In fact, in the calibration below we will calibrate γ_{1V} and γ_{1G} to capture the decline in dividend growth value minus growth over the course of recessions.

Inflation is the sum of a constant, a mean-zero autoregressive process which captures expected inflation, and an unexpected inflation term:

$$\begin{aligned} \pi_{t+1} &= \bar{\pi} + x_t + \sigma_\pi \varepsilon_{t+1}^\pi, \\ x_{t+1} &= \rho_x x_t + \sigma_x \varepsilon_{t+1}^x. \end{aligned}$$

All shocks are cross-sectionally and serially independent and standard normally distributed. It would be straightforward to add a correlation between inflation shocks and shocks to the business cycle variable.

To simplify our analysis, we assume that market participants' preferences are summarized by a real stochastic discount factor (SDF), whose log evolves according to the process:

$$-m_{t+1} = y + \frac{1}{2} \Lambda_t' \Lambda_t + \Lambda_t' \varepsilon_{t+1}.$$

where the vector $\varepsilon_{t+1} \equiv (\varepsilon_{t+1}^d, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)'$ and y is the real interest rate. The risk price dynamics are affine in the state of the economy s_t :

$$\Lambda_t = \Lambda_0 + \Lambda_1 s_t$$

As in the reduced form model, the structural model features three priced sources of risk: aggregate dividend growth risk, which carries a positive price of risk ($\Lambda_0(1) > 0$), inflation risk ($\Lambda_0(2) < 0$), and

cyclical risk ($\Lambda_0(3) > 0$). Choosing $\Lambda_1(2) < 0$ makes the price of inflation risk counter-cyclical. As we show below, this makes bond risk premia increase pro-cyclical. We also set $\Lambda_1(1) > 0$ resulting in a pro-cyclical price of aggregate dividend risk. The log nominal SDF is given by $m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$.

D.2. Asset Prices

We now study the equilibrium bond and stock prices in this model.

D.2.1. Bond Prices and Risk Premia

It follows immediately from the specification of the real SDF that the real term structure of interest rates is flat at y . Nominal bond prices are exponentially affine in the state of the economy and in expected inflation:

$$P_t^{\$(n)} = \exp\left(A_n^{\$} + B_n^{\$}s_t + C_n^{\$}x_t\right),$$

with coefficients that follow recursions described in the proof below. As usual, nominal bond yields are $y_t^{\$(n)} = -\log(P_t^{\$(n)})/n$.

Proof. The nominal SDF is given by:

$$\begin{aligned} m_{t+1}^{\$} &= m_{t+1} - \pi_{t+1} \\ &= -y - \bar{\pi} - x_t - \frac{1}{2}\Lambda'_t\Lambda_t - \Lambda'_t\varepsilon_{t+1} - \sigma_{\pi}\varepsilon_{t+1}^{\pi} \end{aligned}$$

The price of an n -period bond is given by:

$$P_t^n = \exp\left(A_n^{\$} + B_n^{\$}s_t + C_n^{\$}x_t\right).$$

The recursion of nominal bond prices is given by:

$$\begin{aligned} P_t^n &= E_t\left(P_{t+1}^{n-1}M_{t+1}^{\$}\right) \\ &= E_t\left(\exp\left(A_{n-1}^{\$} + B_{n-1}^{\$}s_{t+1} + C_{n-1}^{\$}x_{t+1} - y - \bar{\pi} - x_t - \frac{1}{2}\Lambda'_t\Lambda_t - \Lambda'_t\varepsilon_{t+1} - \sigma_{\pi}\varepsilon_{t+1}^{\pi}\right)\right) \\ &= \exp\left(A_{n-1}^{\$} - y - \bar{\pi} - x_t - \frac{1}{2}\Lambda'_t\Lambda_t + B_{n-1}^{\$}\rho_s s_t + C_{n-1}^{\$}\rho_x x_t\right) \times \\ &\quad E_t\left(\exp\left(B_{n-1}^{\$}\sigma_s\varepsilon_{t+1}^s + C_{n-1}^{\$}\sigma_x\varepsilon_{t+1}^x - \Lambda'_t\varepsilon_{t+1} - \sigma_{\pi}\varepsilon_{t+1}^{\pi} + 1\right)\right) \\ &= \exp\left(A_{n-1}^{\$} - y - \bar{\pi} - x_t + B_{n-1}^{\$}\rho_s s_t + C_{n-1}^{\$}\rho_x x_t\right) \times \\ &\quad \exp\left(\frac{1}{2}[B_{n-1}^{\$}]^2\sigma_s^2 + \frac{1}{2}[C_{n-1}^{\$}]^2\sigma_x^2 - B_{n-1}^{\$}\sigma_s\Lambda_t(3) - C_{n-1}^{\$}\sigma_x\Lambda_t(2) + \frac{1}{2}\sigma_{\pi}^2\right), \end{aligned}$$

which implies:

$$\begin{aligned} A_n^\$ &= A_{n-1}^\$ - y - \bar{\pi} + \frac{1}{2}[B_{n-1}^\$ \sigma_s]^2 + \frac{1}{2}[C_{n-1}^\$ \sigma_x]^2 + \frac{1}{2}\sigma_\pi^2 - B_{n-1}^\$ \sigma_s \Lambda_0(3) - C_{n-1}^\$ \sigma_x \Lambda_0(2), \\ B_n^\$ &= B_{n-1}^\$ \rho_s - C_{n-1}^\$ \sigma_x \Lambda_1(2), \\ C_n^\$ &= -1 + C_{n-1}^\$ \rho_x. \end{aligned}$$

The starting values for the recursion are $A_0^\$ = 0$, $B_0^\$ = 0$, and $C_0^\$ = 0$. □

The expression for $C_n^\$$ can be written more compactly as:

$$C_n^\$ = -\frac{1 - \rho_x^n}{1 - \rho_x} \rho_x < 0,$$

implying that bond prices drop -and nominal interest rates increase- when inflation increases: $C_n^\$ < 0$. Consistent with the data, we assume that $\Lambda_1(2) < 0$. It follows that $B_n^\$ < 0$, implying that nominal bond prices fall -and nominal interest rates rise- with the state of the economy (s_t). Both signs seem consistent with intuition.

The nominal bond risk premium, the expected excess log return on buying an n -period nominal bond and selling it one period later (as a $n - 1$ -period bond), is given by:

$$\begin{aligned} E_t \left[rx_{t+1}^\$(n) \right] &= -cov_t \left(m_{t+1}^\$, B_{n-1}^\$ s_{t+1} + C_{n-1}^\$ \pi_{t+1} \right) \\ &= cov_t \left(\Lambda_t' \varepsilon_{t+1}, B_{n-1}^\$ s_{t+1} + C_{n-1}^\$ x_{t+1} \right) \\ &= \Lambda_t(2) C_{n-1}^\$ \sigma_x + \Lambda_t(3) B_{n-1}^\$ \sigma_s \\ &= \underbrace{\Lambda_0(2) C_{n-1}^\$ \sigma_x + \Lambda_0(3) B_{n-1}^\$ \sigma_s}_{\text{Constant component bond risk premium}} + \underbrace{\Lambda_1(2) C_{n-1}^\$ \sigma_x s_t}_{\text{Time-varying component bond risk premium}}, \end{aligned}$$

In this model, all of the variation in bond risk premia comes from cyclical variation in the economy, s_t . This lends the interpretation of CP factor to s_t . Because $C_{n-1}^\$ < 0$, $\Lambda_1(2) < 0$ generates lower bond risk premia when economic activity is low ($s_t < 0$). The constant bond risk premium contains compensation for inflation shocks and cyclical shocks. Inflation exposure results in a positive risk compensation (first term); it increases in maturity. Since most of the common variation in bond yields is driven by the inflation shock, we can interpret it as a shock to the level of the term structure (*LVL*). Long bonds are more sensitive to level shocks, the traditional duration effect. Exposure to the cyclical shock subtracts from the risk premium (second term). Indeed, a positive shock to the bond risk premium lowers bond prices and returns, and more so for long than for short bonds.

D.2.2. Stock Prices, Equity Risk Premium, Value Premium

The log price-dividend (pd) ratio on stock (portfolio) i is affine in s_t :

$$pd_t^i = A_i + B_i s_t,$$

where

$$B_i = \frac{\gamma_{1i} - \Lambda_1(1) \sigma_{di}}{1 - \kappa_{1i} \rho_s},$$

and the expression for A_i is given in the proof below.

Proof. The return definition implies:

$$\begin{aligned} r_{t+1} &= \ln(\exp(pd_{t+1}) + 1) + \Delta d_{t+1} - pd_t \\ &\simeq \ln(\exp(\overline{pd}) + 1) + \frac{\exp(\overline{pd})}{\exp(\overline{pd}) + 1} (pd_{t+1} - \overline{pd}) + \Delta d_{t+1} - pd_t \\ &= \kappa_0 + \kappa_1 pd_{t+1} + \Delta d_{t+1} - pd_t, \end{aligned}$$

where:

$$\begin{aligned} \kappa_0 &= \ln(\exp(\overline{pd}) + 1) - \kappa_1 \overline{pd}, \\ \kappa_1 &= \frac{\exp(\overline{pd})}{\exp(\overline{pd}) + 1}. \end{aligned}$$

We conjecture that the log price-dividend ratio is of the form:

$$pd_t = A + Bs_t,$$

The price-dividend ratio coefficients are obtained by solving the Euler equation:

$$E_t \left(M_{t+1}^{\$} R_{t+1}^{\$} \right) = 1.$$

We suppress the dependence on i in the following derivation:

$$\begin{aligned} 1 &= E_t(\exp(m_{t+1} - \pi_{t+1} + \kappa_0 + \kappa_1 pd_{t+1} + \Delta d_{t+1} - pd_t + \pi_{t+1})) \\ 0 &= E_t(m_{t+1}) + \frac{1}{2} V_t(m_{t+1}) + E_t(\kappa_0 + \Delta d_{t+1} + \kappa_1 pd_{t+1} - pd_t) \\ &\quad + \frac{1}{2} V_t(\Delta d_{t+1} + \kappa_1 pd_{t+1}) + Cov_t(-\Lambda'_t \varepsilon_{t+1}, \Delta d_{t+1} + \kappa_1 pd_{t+1}) \\ &= -y + \kappa_0 + \gamma_0 + \gamma_1 s_t + (\kappa_1 - 1)A + (\kappa_1 \rho_s - 1)Bs_t \\ &\quad + \frac{1}{2} \sigma_d^2 + \frac{1}{2} \sigma^2 + \frac{1}{2} \kappa_1^2 B^2 \sigma_s^2 - \Lambda_t(1) \sigma_d - \Lambda_t(3) \kappa_1 B \sigma_s. \end{aligned}$$

This results in the system:

$$\begin{aligned} 0 &= -y + \kappa_0 + \gamma_0 + (\kappa_1 - 1)A + \frac{1}{2} \sigma_d^2 + \frac{1}{2} \sigma^2 + \frac{1}{2} \kappa_1^2 B^2 \sigma_s^2 - \Lambda_0(1) \sigma_d - \Lambda_0(3) \kappa_1 B \sigma_s, \\ 0 &= (\kappa_1 \rho_s - 1)B - \Lambda_1(1) \sigma_d + \gamma_1, \end{aligned}$$

Rearranging terms, we get the following expressions for the pd ratio coefficients, where we make the dependence on i explicit:

$$\begin{aligned} A_i &= \frac{\frac{1}{2} \sigma_{di}^2 + \frac{1}{2} \sigma_i^2 + \frac{1}{2} \kappa_{1i}^2 B_i^2 \sigma_s^2 - \Lambda_0(1) \sigma_{di} - \Lambda_0(3) \kappa_{1i} B_i \sigma_s - y + \kappa_{0i} + \gamma_{0i}}{1 - \kappa_{1i}}, \\ B_i &= \frac{\gamma_{1i} - \Lambda_1(1) \sigma_{di}}{1 - \kappa_{1i} \rho_s}. \end{aligned}$$

□

We note that B_i can be positive or negative depending on the importance of dividend growth predictability (γ_{1i}) and fluctuations in risk premia ($\Lambda_1(1)\sigma_{di}$). Stock i 's price-dividend ratios is pro-cyclical ($B_i > 0$) when dividend growth is more pro-cyclical than the risk premium for the aggregate dividend risk of asset i : $\gamma_{1i} > \sigma_{di}\Lambda_1(1)$.

The equity risk premium on portfolio i can be computed to be:

$$\begin{aligned} E_t[r_{t+1}^i] &= \text{cov}_t\left(-m_{t+1}^{\$}, r_{t+1}^i + \pi_{t+1}\right) \\ &= \text{cov}\left(\Lambda_t' \varepsilon_{t+1}, \kappa_{1i} B_i \sigma_s \varepsilon_{t+1}^s + \sigma_{di} \varepsilon_{t+1}^d\right) \\ &= \underbrace{\Lambda_0(1)\sigma_{di} + \Lambda_0(3)\kappa_{1i} B_i \sigma_s}_{\text{Constant component equity risk premium}} + \underbrace{\Lambda_1(1)\sigma_{di} s_t}_{\text{Time-varying component equity risk premium}}. \end{aligned}$$

The equity risk premium provides compensation for aggregate dividend growth risk (first term) and for cyclical risk (second term). Risk premia vary over time with the state of the economy (third term). As we showed above, the data suggest that value stocks' dividends fall more in recessions than those of growth stocks ($\gamma_{1V} > \gamma_{1G}$). With $\sigma_{dV} \approx \sigma_{dG}$, this implies that $B_V > B_G$. Because the price of business cycle risk $\Lambda_0(3)$ is naturally positive, the second term delivers the value premium.

D.2.3. Link with Reduced-form Model

To make the link with the reduced-form model of Section III clear, we study the link between the structural shocks and the reduced form shocks. In the model, shocks to the market return (MKT) are given a linear combination of ε^d and ε^s shocks:

$$\varepsilon_{t+1}^{MKT} \equiv r_{t+1}^M - E_t[r_{t+1}^M] = \sigma_{dM} \varepsilon_{t+1}^d + \kappa_{1M} B_M \sigma_s \varepsilon_{t+1}^s$$

We construct the CP factor in the same way as in the data, from yields on 1- through 5-year yields and average excess bond returns. See footnote 6. Since the model has a two-factor structure for bond yields and forward rates, we use only the two- and the five-year forward rates as independent variables in the CP regression of average excess returns on forward rates. The model's CP factor is perfectly correlated with the process s , and has a innovations that differs by a factor σ^{CP} : $\varepsilon_{t+1}^{CP} = \varepsilon_{t+1}^s \sigma^{CP}$. Finally, since expected inflation drives most of the variation in bond yields in the model, LVL shocks in the model are proportional to expected inflation shocks: $\varepsilon_{t+1}^{LVL} = \varepsilon_{t+1}^x \sigma^L$. Denote $\tilde{\varepsilon} = [\varepsilon^{MKT}, \varepsilon^{LVL}, \varepsilon^{CP}]'$. Associated with $\tilde{\varepsilon}$, we can define market prices of risk $\tilde{\Lambda}$, such that SDF innovations remain unaltered: $\Lambda_t' \varepsilon_{t+1} = \tilde{\Lambda}_t' \tilde{\varepsilon}_{t+1}$. It is easy to verify that $\tilde{\Lambda}_0(1) = \Lambda_0(1)/\sigma_{dM}$, $\tilde{\Lambda}_0(2) = \Lambda_0(2)/\sigma^L$, and $\tilde{\Lambda}_0(3) = \Lambda_0(3)/\sigma^{CP} - \kappa_{1M} B_M \sigma_s \Lambda_0(1)/(\sigma_{dM} \sigma^{CP})$.

For each asset, we can compute covariances of unexpected returns with the MKT , LVL , and CP shocks inside the model. In the model that first covariance is given by:

$$\text{cov}(r_{t+1}^i - E_t[r_{t+1}^i], \varepsilon_{t+1}^{MKT}) = \sigma_{dM} \sigma_{di} + \kappa_{1M} B_M \kappa_{1i} B_i \sigma_s^2.$$

A calibration where $B_M \approx 0$ and $\sigma_{dV} \approx \sigma_{dG}$ will replicate the observed pattern (the linearization constant κ_{1i} will be close to 1 for all portfolios). Second, the covariance of stock portfolio returns with CP shocks is given by:

$$\text{cov}(r_{t+1}^i - E_t[r_{t+1}^i], \varepsilon_{t+1}^{CP}) = \kappa_{1i} B_i \sigma_s \sigma^{CP}.$$

The model generates a value premium because of differential exposure to CP shocks when $B_V > B_G$. When $\sigma_{dV} \approx \sigma_{dG}$, the stronger loading of expected dividend growth of value stocks to s_t ($\gamma_{1V} > \gamma_{1G}$)

makes $B_V > B_G$. Put differently, in the model -as in the data- returns on value stocks are more exposed to bond risk premium shocks than returns on growth stocks. Third, stock return innovations have a zero covariance with *LVL* shocks in the model by construction, similar to the small exposures in the data.

Likewise, we can compute covariances of bond return innovations with the *MKT*, *LVL*, and *CP* shocks. In that order, they are:

$$B_n^{\$} \kappa_{1M} B_M \sigma_s, \quad C_n^{\$} \sigma_x \sigma^L, \quad B_n^{\$} \sigma_s \sigma^{CP}.$$

When $B_M \approx 0$, exposure of bond returns to the market factor shocks is close to zero. Exposure to level shocks is negative: an increase in the level of interest rates reduces bond prices and returns. Exposure to CP shocks is also negative: an increase in the bond risk premium reduces bond prices and returns. Both exposures become more negative with the horizon because $B_n^{\$}$ and $C_n^{\$}$ increase in absolute value with maturity n .

D.3. Calibration

In this appendix, we provide the details of our calibration. We start by describing how we define recessions in the model. Second, we describe the calibration of dividends and inflation processes. Third, we describe the choice of market price of risk parameters. A summary of this discussion is found in the main text.

Recessions in the Model In order to measure how dividends change over the recession, we have to define recessions in the model. Our algorithm mimics several of the features of the NBER dating procedure: (i) The recession is determined by looking back in time at past real economic activity (s_t in the model) and its start is not known in real time, (ii) there is a minimum recession length, and (iii) it captures the notion that the economy went through a sequence of negative shocks and that economic activity is at a low level. We split each recession into three equal periods and refer to the last month of each period as the first, second, and third stage of the recession. The s process is negative at the start of the recession, falls considerably in the first stage of a recession, continues to fall in the second stage, and partially recovers in the last stage. Our recession dating procedure is novel, matches the empirical distribution of recession duration, and generates interesting asset pricing dynamics during recessions, to which we return to below. We now describe the recession dating procedure in detail.

Recessions in the model are determined by the dynamics of the state process s_t . Define the cumulative shock process $\chi_t \equiv \sum_{k=0}^K \varepsilon_{t-k}^s$, where the parameter K governs the length of the backward-looking window. Let $\underline{\chi}$ and $\bar{\chi}$ be the p_1^{th} and p_2^{th} percentiles of the distribution of χ_t , respectively, and let \underline{s} be the p_3^{th} percentile of the distribution of the s process. Whenever $\chi_t < \underline{\chi}$, we find the first negative shock between $t - K$ and t ; say it occurs in month $t - j$. If, in addition, $s_{t-j} < \underline{s}$, we say that the recession started in month $t - j$. We say that the recession ends the first month that $\chi_{t+i} > \bar{\chi}$, for $i \geq 1$. We assume that a new recession cannot start before the previous one has ended.

We find the recession parameters (K, p_1, p_2, p_3) by matching features of the fifteen recessions in the 1926-2009 data. In particular, we consider the fraction of recession months (19.86% in the data), the average length of a recession (13.3 months), the minimum length of a recession (6 months), the 25th percentile (8 months), the median (11 months), the 75th percentile (14.5 months), and the maximum length (43 months). We simulate the process for s_t for 10,000 months, determine recession months as

described above, and calculate the weighted distance between the seven moments in the simulation and in the data. We iterate on the procedure to find the four parameters that minimize the distance between model and data.¹⁴ The best fit has 19.70% of months in recession, an average length of 12.0 months, a minimum of 6, 25th percentile of 8, median of 11, 75th percentile of 14, and maximum of 43 months. The corresponding parameters are $K = 7$ months, $p_1 = 17$, $p_2 = 37$, and $p_3 = 29$.

To describe how the variables of interest behave over the course of a recession, it is convenient to divide each recession into three equal stages, and to keep track of the value in the last month of each stage. More precisely, we express the variable in percentage difference from the peak, which is the month before the recession starts. For example, if a recession lasts 9 (10) months, we calculate how much lower dividends are in months 3, 6, and 9 (10) of the recession, in percentage terms relative to peak. Averaging these numbers over recessions indicates the typical change of the variable of interest in three stages of a recession. The third-stage number summarizes the behavior of the variable over the entire course of the recession. We apply this procedure equally to the data and the model simulation.

We set $\rho_s = .9355$ to exactly match the 12-month autocorrelation of the CP factor of .435. This low annual autocorrelation is consistent with the interpretation of s as a business-cycle frequency variable. We set $\sigma_s = 1$; this is an innocuous normalization. The s process is negative at the start of the recession (1.6 standard deviations below the mean), falls considerably in the first stage of a recession (to 3.2 standard deviations below the mean), continues to fall in the second stage (to -3.9 standard deviations), and partially recovers in the last stage (to -2.9 standard deviations).

Dividend and Inflation Parameters We calibrate parameters to match moments of real dividend growth on the market portfolio, value portfolio (fifth book-to-market quintile), and growth portfolio (first quintile) for 1927-2009 (997 months). Since nominal bond yields are unavailable before 1952, we compare our model’s output for nominal bond yields and associated returns to the average for 1952-2009. In our model simulation, we reinvest monthly dividends at the risk-free rate to compute an annual real dividend series, replicating the procedure in the data. We calculate annual inflation as the twelve-month sum of log monthly inflation, as in the data.

The most important parameter is γ_{1i} , which measures how sensitive dividend growth is to changes in real economic activity. In light of the empirical evidence presented in Section II.A of the main paper, we choose γ_{1i} to match the log change in annual real dividends between the peak of the cycle and the last month of the recession. In the data, the corresponding change is -21.0% for value stocks (the fifth BTM portfolio), + 2.2% for growth stocks (first BTM portfolio), and -5.2% for the market portfolio (CRSP value-weighted portfolio). Given the parameters governing the s dynamics and the recession determination described above, the model matches these changes exactly for $\gamma_{1G} = -.4e - 4$, $\gamma_{1V} = .976e - 4$, and $\gamma_{1M} = .248e - 4$. Note that $\gamma_{1V} > \gamma_{1G}$ delivers the differential fall of dividends on value and growth stocks. This is the central mechanism behind the value premium.

The rest of the dividend growth parameters are chosen to match the observed mean and volatility. We choose $\gamma_{0G} = .0010$, $\gamma_{0V} = .0044$, and $\gamma_{0M} = .0010$ to exactly match the unconditional mean annual log real dividend growth of 1.23% on growth, 5.26% on value, and 1.23% on the market portfolio. We choose $\sigma_{dM} = 2.09\%$ to exactly match the unconditional volatility of annual log real dividend growth of 10.48%. We set $\sigma_{dG} = 1.94\%$ and $\sigma_{dV} = 2.23\%$ in order to match the fact that the covariance of growth stocks with market return innovations is slightly higher than that of value stocks. However,

¹⁴The weighting matrix is diagonal and takes on the following values: .9, .9, .7, .5, .7, .5, and .5, where the weights are described in the same order as the moments in the text. We use an extensive grid search and limit ourselves to integer values for the parameters.

the difference needs to be small to prevent the value premium from being due to differential exposure to market return shocks. To be precise, this difference makes the contribution of the market factor to the value premium equal to 0.44% per year, the same as in the data. We set the idiosyncratic volatility parameter for growth $\sigma_G = 3.48\%$ to match exactly the 13.75% volatility of dividend growth on growth stocks, given the other parameters. We set $\sigma_V = 10.94\%$ because the volatility of dividend growth on value stocks of 48.93%. The 12-month autocorrelation of annual log real dividend growth in the model results from these parameter choices and is -.01 for G, .21 for V, and .29 for M, close to the observed values of .11, .16, and .29, respectively.

Inflation parameters are chosen to match mean inflation, and the volatility and persistence of nominal bond yields. We choose $\bar{\pi} = .0026$ to exactly match average annual inflation of 3.06%. We choose $\rho_x = .989$ and $\sigma_x = .03894\%$ to match the unconditional volatility and 12-month autocorrelation of nominal bond yields of maturities 1- through 5-years (1952-2009 Fama-Bliss data). In the model, volatilities decline from 3.13% for 1-year to 2.58% for 5-year bonds. In the data, volatilities decline from 2.93% to 2.72%. The 12-month autocorrelations of nominal yields range from .88 to .84 in the model, and from .84 to .90 in the data. Our parameters match the averages of the autocorrelations and volatilities across these maturities. We choose the volatility of unexpected inflation $\sigma_\pi = .7044\%$ to match the volatility of inflation of 4.08% in the data. The 12-month autocorrelation of annual inflation is implied by these parameter choices and is .59 in the model, close to the .61 in the data. We set the real short rate $y = .0018$, or 2.1% per year, to match the mean 1-year nominal bond yield of 5.37% exactly, given all other parameters.

Market Prices of Risk We set $\Lambda_0(1) = .2913$ to match the unconditional equity risk premium on the market portfolio of 7.28% per year (in the 1927-2009 data). The market price of expected inflation risk $\Lambda_0(1) = -.0986$ is set to match the 5-1-year slope of the nominal yield curve of 0.60%. The term structure behaves nicely at longer horizons with 10-year yields equal to 6.27% per year, and 30-year yields equal to 6.49% per year. The average of the annual bond risk premium on 2-year, 3-year, 4-year, and 5-year bond returns, which is the left-hand side variable of the *CP* regression, is 0.75% in the model compared to 0.87% in the data. The mean *CP* factor is .0075 in model and .0075 in the data. We set the market price of cyclical risk $\Lambda_0(3) = .0249$ in order to match the 5.22% annual value premium (in the 1927-2009 data).

We set $\Lambda_1(1) = .1208$ in order to generate a slightly negative $B_M = -0.000624$. As argued above, the near-zero B_M prevents the value premium from arising from exposure to market return shocks, and it prevents bond returns from being heavily exposed to market risk. The slight negative sign delivers a slightly positive contribution of exposure to market return shocks to bond excess returns, as in the data (recall Figures 6 and 7). In particular, it generates a 15 basis point spread between ten-year and 1-year bond risk premia coming from market exposure, close to the 30 basis points in the post-1952 data. Finally, we set $\Lambda_1(1) = -0.0702$ in order to exactly match the volatility of the *CP* factor of 1.55%. The volatility of the average annual bond risk premium on 2-year, 3-year, 4-year, and 5-year bonds is 3.93% in the model and 3.72% in the data. As mentioned above, ρ_s is chosen to match the persistence of *CP*. Thus the model replicates the mean, volatility, and persistence of the *CP* factor and the nominal bond risk premium. The maximum annualized log Sharpe ratio implied by the model, $E[\sqrt{\Lambda_t^i \Lambda_t}] \sqrt{12}$ is 1.44. Unfortunately, there is no easy comparison with the numbers in the empirical section (bottom panel of Table I).

D.4. Asset Pricing Dynamics over the Cycle

Finally, our model implies interesting asset pricing dynamics over the cycle. The CP factor, or nominal bond risk premium, starts out negative at the start of the recession, falls substantially in the first stage of the recession, falls slightly more in the second stage, before increasing substantially in the third stage of the recession. This pattern for bond risk premia is reflected in realized bond returns. In particular, the negative risk premium shocks at the start of a recession increase bond prices and returns, and more so on long-term than short-term bonds. An investment of \$100 made at the peak in a portfolio that goes long the 30-year and short the 3-month nominal bond gains \$8.0 in the first stage of the recession. The gain further increases to \$11.7 in the second stage, before falling back to a \$7.4 gain by the last month of the recession. The latter increase occurs as consequence of the rising bond risk premium. Taken over the entire recession, long bonds gain in value so that they are recession hedges [Campbell, Sunderam, and Viceira \(2008\)](#). The same is true in the data, where the gain on long-short bond position is \$6.1 by the last month of the recession. Value stocks are risky in the model. Their price-dividend ratio falls by 21% in the first stage compared to peak, continues to fall to -34%, before recovering to -29% by the end of the recession. In the data, the pd ratio on value stocks similarly falls by 16% in the first stage, falls further to -26%, before recovering to +4%. Value stocks perform poorly, losing more during the recession than growth stocks, both in the model and in the data.

One important feature the model (deliberately) abstracts from are discount rate shocks to the stock market. As a result, the price-dividend ratio and stock return are insufficiently volatile and reflect mostly cash-flow risk. While obviously counter-factual, this assumption is made to keep the exposition focussed on the main, new channel: time variation in the bond risk premium, the exposure to cyclical risk, and its relationship to the value risk premium. One could write down a richer model to address this issues, but only at the cost of making the model more complicated. Such a model would feature a market price of aggregate dividend risk which varies with some state variable z . The latter would follow an AR(1) process with high persistence, as in [Lettau and Wachter \(2009\)](#). All price-dividend ratios and expected stock returns would become more volatile and more persistent, generating a difference between the business-cycle frequency behavior of the bond risk premium and the generational-frequency behavior of the pd ratio. This state variable could differentially affect value and growth stocks, potentially lead to a stronger increase in the pd ratio of value than that of growth in the last stage of a recession. This would shrink the cumulative return gap between value and growth stocks during recessions, which the model now overstates.