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ABSTRACT

The Wealth-Consumption Ratio

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Abstract

We derive new estimates of total wealth, the returns on total wealth, and the wealth effect on consumption. We estimate the prices of aggregate risk from bond yields and stock returns using a no-arbitrage model. Using these risk prices, we compute total wealth as the price of a claim to aggregate consumption. We find that US households have a surprising amount of total wealth, most of it human wealth. This wealth is much less risky than stock market wealth. Events in long-term bond markets, not stock markets, drive most total wealth fluctuations. The wealth effect on consumption is small and varies over time with real interest rates.

1 Introduction

The total wealth portfolio plays a central role in modern asset pricing theory and macroeconomics. Total wealth includes real estate, non-corporate businesses, other financial assets, durable consumption goods, and human wealth. The objective of this paper is to measure the amount of total wealth, the amount of human wealth, and the returns on each. The conventional approach to approximating the return on total wealth is to use the return on an equity index. Our approach is to measure total wealth as the present discounted value of a claim to aggregate consumption. The discount factor we use is consistent with observed stock and bond prices. Our preference-free estimation imposes only the household budget constraint and no-arbitrage conditions on traded assets. According to our estimates, stock market wealth is only one percent of total wealth and all non-human wealth only eight percent. Moreover, the returns on the vast majority of total wealth differ markedly from equity returns; they are much lower on average and have low correlation with equity returns. Thus, our results challenge the conventional approach.

Our main finding is that U.S. households have a surprising amount of total wealth, $3.5 million per person in 2011 (in 2005 dollars). Of this, 92 percent is human wealth, the discounted value of all future U.S. labor income. Our estimation imputes a value of $1 million to an average career spanning 35 years. The high value of total wealth is reflected in a high average wealth-consumption

ratio of 83, much higher than the average equity price-dividend ratio of 26. Equivalently, the total wealth portfolio earns a much lower risk premium of 2.38 percent per year, compared to an equity risk premium of 6.41 percent. Total wealth returns are only half as volatile as equity returns. The lower variability in the wealth-consumption ratio indicates less predictability in total wealth returns. Unlike stocks, most of the variation in future expected total wealth returns is variation in future expected risk-free rates, and not variation in future expected excess returns. The correlation between total wealth returns and stock returns is only 27 percent, while the correlation with 5-year government bond returns is 94 percent. Thus, the destruction and creation of wealth in the U.S. economy are largely disconnected from events in the stock market and are related to events in the bond markets instead. Between 1979 and 1981 when real interest rates rose, \$318,000 of per capita wealth was destroyed. Afterwards, as real yields fell, real per capita wealth increased steadily from \$860,000 in 1981 to \$3.5 million in 2011. Total U.S. household wealth was hardly affected by the spectacular declines in the stock market in 1973-74, 2000-2001, and 2007-09. The main message from these results is that equity is quite different from the total wealth portfolio.

A simple back-of-the-envelope Gordon growth model calculation helps explain the high wealth-consumption ratio. The discount rate on the consumption claim is 3.51 percent per year (a consumption risk premium of 2.38 percent plus a risk-free rate of 1.49 percent minus a Jensen term of 0.37 percent) and its cash-flow growth rate is 2.31 percent. The Gordon growth formula delivers the estimated mean wealth-consumption ratio: $83 = 1/(.0351 - .0231)$.

Our methodology also produces new estimates of the marginal propensity to consume out of wealth. We find that the U.S consumer spent only 0.76 cents out of the last dollar of wealth, on average over our sample period. The marginal propensity to consume tracks interest rates: It peaks in 1981 at 1.4 cents per dollar and bottoms out in 2010 at 0.6 cents per dollar. The 50 percent drop in the marginal propensity to consume out of wealth occurred because the newly created wealth between 1981 and 2010 reflected almost exclusively lower discount rates rather than higher future consumption growth. We estimate that all variation in the wealth-consumption ratio is due to variation in discount rates.

In addition to the low volatility of aggregate consumption growth innovations, the reason that total wealth resembles a real bond is that the value of a claim to aggregate risky consumption is similar to that of a claim whose cash flows grow deterministically at the average consumption growth rate. The latter occurs because innovations to current and future consumption growth carry a small market price of risk according to our calculations. This is not a foregone conclusion because the market prices of risk are estimated to be consistent with observed stock and bond prices. The finding that current consumption growth innovations are assigned a small price is not a complete surprise. That is the equity premium puzzle. But, we also know that traded asset prices predict future consumption growth. This opens up the possibility that shocks to future consumption demand a high risk compensation. A key finding of our work is that this channel

is not strong enough to generate a consumption risk premium that resembles anything like the equity risk premium. Discounting consumption at a low rate of return implies that the present discounted value of the stream (total wealth) is high, arguably higher than commonly believed.

A key assumption in the paper is that stock and bond returns span all priced sources of risk. We verify that our unspanned consumption growth innovations are essentially acyclical and serially uncorrelated. In addition, we check whether the pricing of consumption innovations that are not spanned by innovations to bond yields or stock returns can overturn our results. Even if we allow for unspanned priced risk that delivers Sharpe ratios equal to four times the observed Sharpe ratio on stocks, the consumption risk premium remains 2.5 percentage points below the equity risk premium. In the Online Appendix, we show that our valuation procedure is appropriate even in an economy with heterogeneous agents who face uninsurable labor income risk, borrowing constraints, and limited asset market participation.

Connection to the Literature To derive our wealth estimates, we use a vector auto-regression (VAR) model for the state variables as in Campbell (1991, 1993, 1996). We combine the estimated state dynamics with a no-arbitrage model for the stochastic discount factor (SDF). As in Duffie and Kan (1996), Dai and Singleton (2000), and Ang and Piazzesi (2003), the log SDF is affine in innovations to the state vector while market prices of aggregate risk are affine in the same state vector. We estimate the market prices of risk by matching salient features of nominal bond yields, equity returns and price-dividend ratios, and expected returns on factor mimicking portfolios, linear combinations of stock portfolios that have the highest correlations with consumption and labor income growth. This approach is similar to that in Bekaert, Engstrom, and Xing (2009), Bekaert, Engstrom, and Grenadier (2010), and Lettau and Wachter (2011) who use affine models to match features of stocks and bonds. By using precisely-measured stock and bond price data, our approach avoids using data on housing, durable, and private business wealth from the Flow of Funds. These wealth variables are often measured at book values and with substantial error.

Our approach also avoids making arbitrary assumptions on the expected rate of return (discount rate) of human wealth, which is unobserved. In earlier work, Campbell (1993), Jagannathan and Wang (1996), Shiller (1995), and Lettau and Ludvigson (2001a, 2001b) all make particular, and very different, assumptions on the expected rate of return on human wealth. In a precursor paper, Lustig and Van Nieuwerburgh (2008) back out human wealth returns to match properties of consumption data. Bansal, Kiku, Shaliastovich, and Yaron (2012) emphasize the role of macro-economic volatility in a related exercise. Using market prices of risk inferred from traded assets, we obtain a new estimate of expected human wealth returns that fits none of the previously proposed models. We estimate human wealth to be 92 percent of total wealth. This estimate is consistent with Mayers (1972) who first pointed out that human capital forms a major part of the aggregate capital stock in advanced economies, and with Jorgenson and Fraumeni (1989) who also

calculate a 90 percent human wealth share. Our result is also consistent with the share of human wealth obtained by Palacios (2011) in a calibrated version of his dynamic general equilibrium production model.

Our results differ from earlier attempts to measure the wealth-consumption ratio and the return to total wealth. Lettau and Ludvigson (2001a, 2001b) estimate *cay*, a measure of the inverse wealth-consumption ratio. Their wealth-consumption ratio has a correlation of 24 percent with our series. Alvarez and Jermann (2004) do not allow for time-varying risk premia and measure total wealth returns as a linear combination of equity portfolio returns. They estimate a smaller consumption risk premium of 0.2 percent, and hence a much higher average wealth-consumption ratio.

Our paper connects to the literature that studies the valuation of an asset for which one only observes the dividend growth and not the price. The retirement and social security literature studies related questions when it values claims to future labor income (e.g., De Jong (2008), Geanakoplos and Zeldes (2010), and Novy-Marx and Rauh (2011)).

Our paper also contributes to the large literature on measuring the propensity to consume out of wealth. The seminal work of Modigliani (1971) suggests that a one dollar increase in wealth leads to five cents increase in consumption. Similar estimates appear in textbooks, models used by central banks, and in monetary and fiscal policy debates (see Poterba (2000) for a survey). A wealth effect of five cents on the dollar implies a wealth-consumption ratio that is four times lower than our estimates, or equivalently, a consumption risk premium as high as the equity risk premium. Our first contribution to this literature is to propose a wealth effect on consumption that is much smaller than previously thought. Second, we are the first to provide an estimate consistent with the budget constraint and no-arbitrage restrictions.[1] Third, we find that the dynamics of this wealth effect relate to bond market rather than stock market dynamics. This would explain the modest contraction in total wealth and aggregate consumption in response to the large stock market wealth destruction of 1973-74 (e.g., Hall (2001)). Our results are consistent with Bernanke and Gertler's (2001) suggestion that inflation-targeting central banks should ignore movements in asset values that do not influence aggregate demand. We find that traded assets amount to a relatively small share of total wealth. As a result, their price fluctuations do not affect much consumer spending, the largest component of aggregate demand.

Finally, our work contributes to the consumption-based asset pricing literature. It offers a new set of moments to evaluate their empirical performance. Too often, such models are evaluated on their implications for equity returns. But the models' primitives are the preferences and the dynamics of aggregate consumption growth. Moments of returns on the consumption claim are the most *primitive asset pricing moments* and should be the most informative for testing these

[1]Ludvigson and Steindel (1999) and Lettau and Ludvigson (2004) start from the household budget constraint but do not impose the absence of arbitrage, and assume a constant price-dividend ratio on human wealth.

models. In contrast, the dividend growth dynamics of stocks can be altered without affecting equilibrium allocations or prices of traded assets other than stocks; modeling them entails more degrees of freedom. The NBER working paper version of this paper carries out a comparison of two leading endowment economy models: the external habit model of Campbell and Cochrane (1999) and the long-run risk model of Bansal and Yaron (2004). Our work also has implications for production-based asset pricing models. As Kaltenbrunner and Lochstoer (2010) point out, such models usually generate the prediction that the claim to dividends is less risky than the claim to consumption. Our results indicate that this is counterfactual and suggest that stocks are special. Modeling realistic dividend dynamics (by introducing labor income frictions, operational leverage, or financial leverage) is necessary to reconcile the low consumption risk premium with the high equity risk premium.

The rest of the paper is organized as follows. Section 2 describes our measurement approach conceptually. Section 3 shows how we estimate the risk price parameters and Section 4 describes the results from that estimation. Section 5 shows that our conclusions remain valid when there is priced unspanned consumption risk. Section 6 investigates what features of the model are responsible for which results and investigates an annual instead of a quarterly version of our model. Section 7 compares the properties of the wealth consumption ratio in the long-run risk and external habit models to the ones we estimate in the data. Finally, Section 8 concludes. An Online Appendix describes our data, presents proofs, details the robustness checks, and shows that our valuation approach remains valid in an incomplete markets model.

2 Measuring the Wealth-Consumption Ratio in the Data

Section 2.1 describes the framework for estimating the wealth-consumption ratio and the return on total wealth. Section 2.2 presents two methodologies to compute the wealth-consumption ratio. Section 2.3 links the wealth-consumption ratio to the cost of aggregate consumption risk and the propensity to consume out of wealth.

2.1 Model

State Evolution Equation We assume that the $N \times 1$ vector of state variables follows a Gaussian first-order VAR:

$$z_t = \Psi z_{t-1} + \Sigma^{\frac{1}{2}} \varepsilon_t, \tag{1}$$

with $\varepsilon_t \sim i.i.d. \mathcal{N}(0, I)$ and Ψ is a $N \times N$ matrix. The vector z is demeaned. The covariance matrix of the innovations is Σ; the model is homoscedastic. We use a Cholesky decomposition of the covariance matrix, $\Sigma = \Sigma^{\frac{1}{2}} \Sigma^{\frac{1}{2}\prime}$, which has non-zero elements only on and below the diagonal. We discuss the elements of the state vector in detail below. Among other elements, the

state z_t contains real aggregate consumption growth, the nominal short-term interest rate, and inflation. Denote consumption growth by $\Delta c_t = \mu_c + e_c' z_t$, where μ_c denotes the unconditional mean consumption growth rate and the $N \times 1$ vector e_c is the column of a $N \times N$ identity matrix that corresponds to the position of Δc in the state vector. Likewise, the nominal 1-quarter rate is $y_t^{\$}(1) = y_0^{\$}(1) + e_{yn}' z_t$, where $y_0^{\$}(1)$ is the unconditional average and e_{yn} the selector vector. Similarly, $\pi_t = \pi_0 + e_\pi' z_t$ is the (log) inflation rate between $t-1$ and t. All lowercase letters denote logs.

Stochastic Discount Factor We specify a stochastic discount factor (SDF) familiar from the no-arbitrage term structure literature, following Ang and Piazzesi (2003). The nominal pricing kernel $M_{t+1}^{\$} = \exp(m_{t+1}^{\$})$ is conditionally log-normal:

$$m_{t+1}^{\$} \quad = \quad -y_t^{\$}(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t' \varepsilon_{t+1}. \tag{2}$$

The real pricing kernel is $M_{t+1} = \exp(m_{t+1}) = \exp(m_{t+1}^{\$} + \pi_{t+1})$; it is also conditionally Gaussian.[2] The innovations in the vector ε_{t+1} are associated with a $N \times 1$ market price of risk vector Λ_t of the affine form:

$$\Lambda_t = \Lambda_0 + \Lambda_1 z_t,$$

The $N \times 1$ vector Λ_0 collects the average prices of risk while the $N \times N$ matrix Λ_1 governs the time variation in risk premia.

2.2 The Wealth-Consumption Ratio

We explore two methods to measure the wealth-consumption ratio. The first one uses consumption strips and avoids any approximation while the second approach builds on the Campbell (1991) approximation of log returns.

From Consumption Strips A consumption strip of maturity τ pays realized consumption at period τ, and nothing in the other periods. Under a no-bubble constraint on total wealth, the wealth-consumption ratio is the sum of the price-dividend ratios on consumption strips of all horizons (Wachter 2005):

$$\frac{W_t}{C_t} = e^{wc_t} = \sum_{\tau=0}^{\infty} P_t^c(\tau), \tag{3}$$

where $P_t^c(\tau)$ denotes the price of a τ period consumption strip divided by the current consumption. Appendix B proves that the log price-dividend ratio on consumption strips are affine in the state

[2]Note that the consumption-CAPM is a special case of this where $m_{t+1} = \log\beta - \alpha\mu_c - \alpha\eta_{t+1}$ and η_{t+1} denotes the innovation to real consumption growth and α the coefficient of relative risk aversion.

vector and shows how to compute them recursively.

If consumption growth were unpredictable and its innovations carried a zero risk price, then consumption strips would be priced like real zero-coupon bonds.[3] The consumption strips' dividend-price ratios would equal yields on real bonds (with coupon adjusted for growth μ_c). In this special case, all variation in the wealth-consumption ratio would be traced back to the real yield curve.

From Total Wealth Returns Consumption strips allow for an exact definition of the wealth-consumption ratio, but they call for the estimation of an infinite sum of bond prices. A second approximate method delivers both a more practical and elegant definition of the wealth-consumption ratio. In our empirical work, we check that both methods deliver similar results.

In our exponential-Gaussian setting, the log wealth-consumption ratio is an affine function of the state variables. To show this result, we start from the aggregate budget constraint:

$$W_{t+1} = R^c_{t+1}(W_t - C_t). \tag{4}$$

The beginning-of-period (or cum-dividend) total wealth W_t that is not spent on aggregate consumption C_t earns a gross return R^c_{t+1} and leads to beginning-of-next-period total wealth W_{t+1}. The return on a claim to aggregate consumption, the *total wealth return*, can be written as

$$R^c_{t+1} = \frac{W_{t+1}}{W_t - C_t} = \frac{C_{t+1}}{C_t}\frac{WC_{t+1}}{WC_t - 1}.$$

We use the Campbell (1991) approximation of the log total wealth return $r^c_t = \log(R^c_t)$ around the long-run average log wealth-consumption ratio $A^c_0 \equiv E[w_t - c_t]$.[4]

$$r^c_{t+1} \simeq \kappa^c_0 + \Delta c_{t+1} + wc_{t+1} - \kappa^c_1 wc_t. \tag{5}$$

The linearization constants κ^c_0 and κ^c_1 are non-linear functions of the unconditional mean wealth-consumption ratio A^c_0:

$$\kappa^c_1 = \frac{e^{A^c_0}}{e^{A^c_0} - 1} > 1 \quad \text{and} \quad \kappa^c_0 = -\log\left(e^{A^c_0} - 1\right) + \frac{e^{A^c_0}}{e^{A^c_0} - 1}A^c_0. \tag{6}$$

Proposition 1. *The log wealth-consumption ratio is approximately a linear function of the (demeaned) state vector* z_t

$$wc_t \simeq A^c_0 + A^{c\prime}_1 z_t,$$

[3]First, if aggregate consumption growth is unpredictable, i.e., $e'_c\Psi = \mathbf{0}$, then innovations to future consumption growth are not priced. Second, if prices of current consumption risk are zero, i.e., $e'_c\Sigma^{\frac{1}{2}}\Lambda_1 = 0$ and $e'_c\Sigma^{\frac{1}{2}}\Lambda_0 = 0$, then innovations to current consumption are not priced.

[4]Throughout, variables with a subscript zero denote unconditional averages.

where the mean log wealth-consumption ratio A_0^c is a scalar and A_1^c is the $N \times 1$ vector which jointly solve:

$$0 = \kappa_0^c + (1-\kappa_1^c)A_0^c + \mu_c - y_0(1) + \frac{1}{2}(e_c + A_1^c)'\Sigma(e_c + A_1^c) - (e_c + A_1^c)'\Sigma^{\frac{1}{2}}\left(\Lambda_0 - \Sigma^{\frac{1}{2}\prime}e_\pi\right) \quad (7)$$

$$0 = (e_c + e_\pi + A_1^c)'\Psi - \kappa_1^c A_1^{c\prime} - e'_{yn} - (e_c + e_\pi + A_1^c)'\Sigma^{\frac{1}{2}}\Lambda_1. \quad (8)$$

Appendix B proves the proposition. The proof conjectures an affine function for the log wealth-consumption ratio, imposes the Euler equation for the log total wealth return, and solves for the coefficients of the affine function as verification of the conjecture. The resulting expression for wc_t is an approximation only because it relies on the log-linear approximation of returns in equation (5). This log-linearization is the only approximation in our procedure. Once we estimate the market prices of risk Λ_0 and Λ_1 below, equations (7) and (8) allow us to solve for the mean log wealth-consumption ratio (A_0^c) and its dependence on the state (A_1^c).[5]

Consumption Risk Premium Proposition 1 and the total wealth return definition in (5) jointly imply the following log total wealth return:

$$r_{t+1}^c = r_0^c + \left[(e_c + A_1^c)'\Psi - \kappa_1^c A_1^{c\prime}\right] z_t + (e'_c + A_1^{c\prime})\Sigma^{\frac{1}{2}}\varepsilon_{t+1}, \quad (9)$$

$$r_0^c = \kappa_0^c + (1-\kappa_1^c)A_0^c + \mu_c, \quad (10)$$

where equation (10) defines the unconditional mean total wealth return r_0^c. The consumption risk premium, the expected log return on total wealth in excess of the log real risk-free rate $y_t(1)$ corrected for a Jensen term, follows from the Euler equation $E_t[M_{t+1}R_{t+1}^c] = 1$:

$$\begin{aligned} E_t\left[r_{t+1}^{c,e}\right] \equiv E_t\left[r_{t+1}^c - y_t(1)\right] + \frac{1}{2}V_t[r_{t+1}^c] &= -Cov_t\left[r_{t+1}^c, m_{t+1}\right] \quad (11) \\ &= (e_c + A_1^c)'\Sigma^{\frac{1}{2}}\left(\Lambda_0 - \Sigma^{\frac{1}{2}\prime}e_\pi\right) + (e_c + A_1^c)'\Sigma^{\frac{1}{2}}\Lambda_1 z_t, \end{aligned}$$

The first term on the last line is the average consumption risk premium. This is a key object of interest which measures how risky total wealth is. The second (mean-zero) term governs the time variation in the consumption risk premium.

Growth Conditions Given the no-bubble constraint, there is an approximate link between the coefficients in the affine expression of the wealth-consumption ratio and the coefficients of the

[5]Equations (7) and (8) form a system of $N+1$ non-linear equations in $N+1$ unknowns. It is a non-linear system because of equation (6), but is well-behaved and can easily be solved numerically.

strip price-dividend ratios $P_t^c(\tau) = \exp(A^c(\tau) + B^c(\tau)' z_t)$:

$$\exp(A_0^c) \simeq \sum_{\tau=0}^{\infty} \exp(A^c(\tau)) \text{ and } \exp(A_1^c) \simeq \sum_{\tau=0}^{\infty} \exp(B^c(\tau)). \tag{12}$$

A necessary condition for this first sum to converge and hence produce a finite average wealth-consumption ratio is that the consumption strip risk premia are positive and large enough in the limit (as $\tau \to \infty$):

$$(e_c + B^c(\infty))' \Sigma^{\frac{1}{2}} \left(\Lambda_0 - \Sigma^{\frac{1}{2}} e_\pi \right) > \mu_c - y_0(1) + \frac{1}{2} (e_c + B^c(\infty))' \Sigma (e_c + B^c(\infty)),$$

We refer to this inequality as the growth condition. Because average real consumption growth μ_c exceeds the average real short rate $y_0(1)$ in the data, the right-hand side of the inequality is positive. When all the risk prices in Λ_0 are zero, this condition is obviously violated. It implies a lower bound for the consumption risk premium.

Human Wealth The same way we priced a claim to aggregate consumption, we price a claim to aggregate labor income. Human wealth is the present value of the latter claim. We impose that the conditional Euler equation for human wealth returns is satisfied and obtain a log price-dividend ratio which is also approximately affine in the state: $pd_t^l = A_0^l + A_1^l z_t$. (See Proposition 2 in Online Appendix B.1.) By the same token, the conditional risk premium on the labor income claim is affine in the state vector (see equation A.5 in Online Appendix B.1).

2.3 Cost of Consumption Risk and Consumption-Wealth Effects

The computation of the wealth-consumption ratio implies an estimate of the marginal welfare cost of aggregate consumption growth risk, a central object of interest in this paper. Alvarez and Jermann (2004) define the marginal cost of consumption uncertainty by how much consumption the representative agent would be willing to give up at the margin in order to eliminate some consumption uncertainty.[6] Since our approach is preference-free, our marginal cost calculation applies to the entire class of representative agent dynamic asset pricing models.

Eliminating exposure to aggregate consumption growth risk is achieved by selling a claim to stochastically growing aggregate consumption and buying a claim to deterministically growing aggregate consumption. Denote trend consumption by C_t^{tr}. The marginal cost of consumption

[6]The literature on the costs of consumption fluctuations starts with Lucas (1987) who defines the *total* cost of aggregate consumption risk Ω as the fraction of consumption the consumer is willing to give up in order to get rid of consumption uncertainty: $U\left((1+\Omega(\alpha))C^{actual}\right) = U\left((1-\alpha)C^{trend} + \alpha C^{actual}\right)$, where α=0. Alvarez and Jermann (2004) define the marginal cost of business cycles as the derivative of this cost evaluated at zero, i.e., $\Omega'(0)$. While the total cost can only be computed by specifying preferences, the marginal cost can be backed out directly from traded assets prices.

uncertainty, ϖ_t, is defined as the ratio of the price of a claim to trend consumption (without cash-flow risk) to the price of a claim to consumption with cash-flow risk minus one:

$$\varpi_t = \frac{W_t^{tr}}{W_t} - 1 = \frac{WC_t^{tr}}{WC_t}\frac{C_t^{tr}}{C_t} - 1 = e^{wc_t^{tr} + c_t^{tr} + wc_t - c_t} - 1, \tag{13}$$

where wc^{tr} denotes the log price-dividend ratio on the claim to trend consumption, a perpetuity with cash-flows that grow deterministically at the average real consumption growth rate μ_c. The latter is approximately affine in the state variables: $wc_t^{tr} \simeq A_0^{tr} + A_1^{tr\prime} z_t$ (see Appendix B for a derivation). The risk premium on a claim to trend consumption is not zero but it approximately equals the risk premium on the real perpetuity:

$$E_t\left[r_{t+1}^{tr,e}\right] \equiv E_t\left[r_{t+1}^{tr} - y_t(1)\right] + \frac{1}{2}V_t[r_{t+1}^{tr}] \simeq A_1^{tr\prime}\Sigma^{\frac{1}{2}}\left(\Lambda_0 - \Sigma^{\frac{1}{2}\prime}e_\pi\right) + A_1^{tr\prime}\Sigma^{\frac{1}{2}}\Lambda_1 z_t \tag{14}$$

The marginal cost of business cycles is zero, on average, when innovations to current and future consumption growth jointly carry a zero price of risk so that $wc_t \approx wc_t^{tr}$. Even in the latter case, the marginal cost of consumption fluctuations will fluctuate because realized consumption is at times above and at times below trend.

Propensity to Consume out of Wealth A large literature studies households' average and marginal propensities to consume out of wealth. To the best of our knowledge, ours is the first estimator of these propensities that is consistent with both the budget constraint and no-arbitrage pricing of stock and bond prices. Specifically, the consumption-wealth ratio evaluated at the sample average state vector, $\exp\left(-A_0^c\right)$, is a no-arbitrage estimate of the *average* propensity to consume out of total wealth. We also obtain the *marginal* propensity to consume out of total wealth: $\left(1 + e_c' A_1^c\right)^{-1} \exp\left(-\left(A_0^c + A_1^{c\prime} z_t\right)\right)$.[7] The dynamics of the marginal cost of consumption fluctuations vary directly with the consumption-wealth ratio.

3 Estimating the Market Prices of Risk

In order to recover the dynamics of the wealth-consumption ratio and of the return on wealth, we need to estimate the market prices of risk Λ_0 and Λ_1. This section details our estimation procedure. Section 3.1 describes the state vector. Section 3.2 lists the additional restrictions we impose on our framework. Section 3.3 describes the estimation technique.

[7] In the literature, the marginal propensity to consume is the slope coefficient a_1 in the following regression: $\Delta c_{t+1} = a_0 + a_1 \Delta w_{t+1} + \epsilon_{t+1}$. From our estimates, we can back out the implied marginal propensity to consume as follows: $a_1^{-1} = \partial(\Delta w_{t+1})/\partial(\Delta c_{t+1}) = \partial(\Delta wc_{t+1} + \Delta c_{t+1})/\partial(\Delta c_{t+1}) = \partial(\Delta wc_{t+1})/\partial(\Delta c_{t+1}) + 1 = e_c' A_1^c + 1$. We multiply $a_1 = (e_c' A_1^c + 1)^{-1}$ by the consumption-wealth ratio to get an expression of the marginal propensity to consume out of wealth in levels (cents per dollars).

To implement the model, we need to take a stance on what observables describe the aggregate dynamics of the economy. The *de minimis* state vector contains the nominal short rate, realized inflation, and the cash flow growth dynamics of the two cash flows this paper sets out to price: consumption and labor income. In this section we lay out our benchmark model which contains substantially richer state dynamics than contained in these four variables. The richness stems from a desire to infer market prices of risk from a model that accurately prices bonds of various maturities, the equity market, and that takes into account some cross-sectional variation across stocks. Section 6 explores special cases of the benchmark model, with fewer state variables, in order to understand what elements are crucial for our main findings.

3.1 Benchmark State Vector

Our benchmark state vector is:

$$z_t = [CP_t, y_t^{\$}(1), \pi_t, y_t^{\$}(20) - y_t^{\$}(1), pd_t^m, r_t^m, r_t^{fmpc}, r_t^{fmpy}, \Delta c_t, \Delta l_t]'.$$

The first four elements represent the bond market variables in the state, the next four represent the stock market variables, the last two variables represent the cash flows. The state contains in order of appearance: the Cochrane and Piazzesi (2005) factor (CP), the nominal short rate (yield on a 3-month Treasury bill), realized inflation, the spread between the yield on a 5-year Treasury note and a 3-month Treasury bill, the price-dividend ratio on the CRSP stock market, the real return on the CRSP stock market, the real return on a factor mimicking portfolio for consumption growth, the real return on a factor mimicking portfolio for labor income growth, real per capita consumption growth, and real per capita labor income growth. We recall that lower-case letters denote natural logarithms. This state vector is observed at quarterly frequency from 1952.I until 2011.IV (240 observations). In a robustness check, we also consider annual data from 1952 to 2011. Appendix A describes data sources and definitions in detail. The Cholesky decomposition of the residual covariance matrix, $\Sigma = \Sigma^{\frac{1}{2}}\Sigma^{\frac{1}{2}\prime}$, allows us to interpret the shock to each state variable as the sum of the shocks to all the preceding state variables and an own shock that is orthogonal to all previous shocks. Consumption and labor income growth are ordered after the bond and stock variables because we use the prices of risk associated with the first eight innovations to value the consumption and labor income claims.

The goal of our exercise is to price claims to aggregate consumption and labor income using as much information as possible from traded assets. Thus, the choice of state variables is motivated by a desire to capture all important dynamics of bond and stock prices. Many of the state variables have a long tradition in finance as predictors of stock and bond returns.[8]

[8]For example, Ferson and Harvey (1991) study the yield spread, the short rate and consumption growth as predictors of stocks while Cochrane and Piazzesi (2005) emphasize the importance of the CP factor to predict

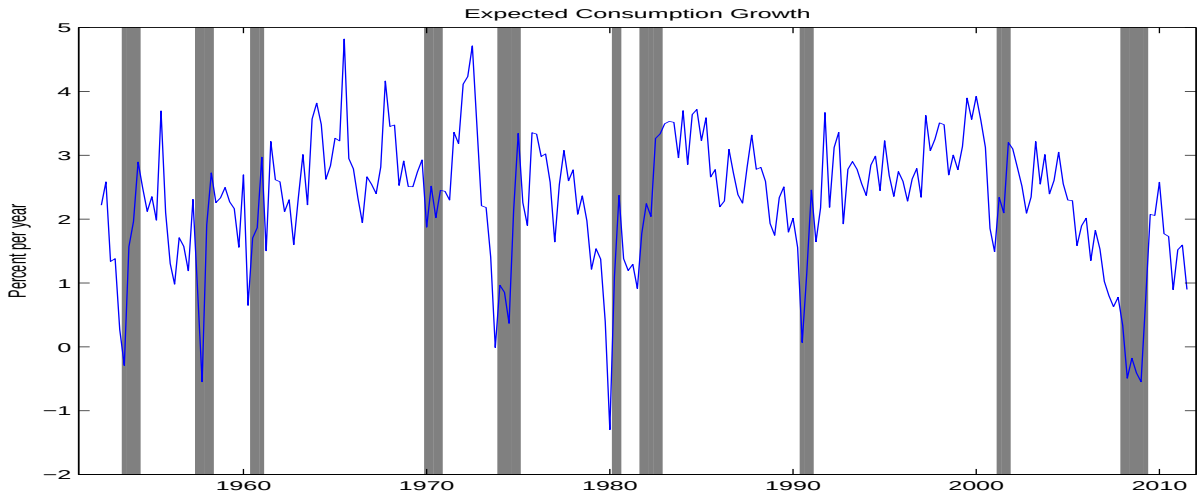
Expected Consumption Growth Equally important is a rich specification of the cash flows we want to price, consumption and labor income growth. First, our state vector includes variables like interest rates (Harvey (1988)), the price-dividend ratio, and the slope of the yield curve (Ang, Piazzesi, and Wei (2006)) that have been shown to forecast future consumption growth. The predictability of future consumption growth by stock and bond prices whose own shocks carry non-zero prices of risk results in a risk premium to future consumption growth innovations and thus to create a wedge between the risky and the trend consumption claims. Having richly specified expected consumption growth dynamics alleviates the concern that the model misses important (priced) shocks to expected consumption growth. Second, the modest correlation (29%) of the aggregate stock market portfolio with consumption growth motivates us to use additional information from the cross-section of stocks to learn more about contemporaneous shocks to consumption and labor income claims. We use the 25 size- and value-portfolio returns to form a consumption growth factor mimicking portfolio (fmp) and a labor income growth fmp. The consumption (labor income) growth fmp has a 36% (36%) correlation with actual consumption. Pricing these fmp well alleviates the concern that our model misses important shocks to current consumption innovations.

Our state variables z_t explain 29% of variation in Δc_{t+1}. The volatility of annualized expected consumption growth is 0.49%, more than one-third of the volatility of realized consumption growth, while the first-order autocorrelation of expected consumption growth is 0.70 in quarterly data. This shows non-trivial consumption growth predictability, in line with the literature. Figure 1 plots the (annualized) one-quarter-ahead expected consumption growth series implied by our VAR. The shaded areas are NBER recessions. Expected consumption growth experiences the largest declines during the Great Recession of 2007.IV-2009.II, the 1953.II-1954.II recession, the 1957.III-1958.II recession, the 1973.IV-1975.I recession, the double-dip NBER recession from 1980.I to 1982.IV, and somewhat smaller declines during the less severe 1960.II-1961.I, 1990.III-1991.I and 2001.I-2001.IV recessions. Hence, the innovations to expected consumption growth are highly cyclical. That cyclical risk, alongside the long-run risk in expected consumption growth implied by the VAR, should be priced in asset markets. Finally, most of the cyclical variation in consumption growth is captured by traded asset returns. The correlation of unspanned (orthogonal) consumption growth with the NBER dummy is only -.01 and not statistically different from zero. Moreover, these unspanned innovations are essentially uncorrelated over time; the first-order autocorrelation is -0.05.

bond returns.

Figure 1: Consumption Growth Predictability

The figure plots (annualized) expected consumption growth at quarterly frequency, as implied by the VAR model: $E_t[\Delta c_{t+1}] = \mu_c + I_c'\Psi z_t$, where z_t is the N-dimensional state vector.



3.2 Restrictions

With ten state variables and time-varying prices of risk our model has many parameters. On the one hand, the richness offers the possibility to accurately describe bond and equity prices without having to resort to latent state variables. On the other hand, there is the risk of over-fitting the data. To guard against this risk and to obtain stable estimates, we impose restrictions on our benchmark estimation.

We start by imposing restrictions on the dynamics of the state variable, that is, in the companion matrix Ψ. Only the bond market variables -first block of four- govern the dynamics of the nominal term structure; Ψ_{11} below is a 4×4 matrix of non-zero elements. For example, this structure allows for the CP factor to predict future bond yields, or for the short-term yield and inflation to move together. It also imposes that stock returns, the price-dividend ratio on stocks, or the factor-mimicking portfolio returns do not predict future yields or bond returns; Ψ_{12} is a 4×4 matrix of zeroes. The second block of Ψ describes the dynamics of the log price-dividend ratio and log return on the aggregate stock market, which we assume depends not only on their own lags but also on the lagged bond market variables. The elements Ψ_{21} and Ψ_{22} are 2×4 and 2×2 matrices of non-zero elements. This allows for aggregate stock return predictability by the short rate, the yield spread, inflation, the CP factor, the price dividend-ratio, and its own lag, all of which have been shown in the empirical asset pricing literature. The fmp returns in the third block have the same predictability structure as the aggregate stock return, so that Ψ_{31} and Ψ_{32} are 2×4 and 2×2 matrices of non-zero elements. In our benchmark model, consumption and labor income growth do not predict future bond and stock market variables; Ψ_{14}, Ψ_{24}, and Ψ_{34}

are all matrices of zeroes. Finally, the VAR structure allows for rich cash flow dynamics: expected consumption growth depends on the first nine state variables and expected labor income growth depends on all lagged state variables; Ψ_{41}, Ψ_{42}, and Ψ_{43} are 2×4, 2×2, and 2×2 matrices of non-zero elements, and Ψ_{44} is a 2×2 matrix with one zero in the upper-right corner. In sum, our benchmark Ψ matrix has the following block-diagonal structure:

$$\Psi = \begin{pmatrix} \Psi_{11} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \Psi_{21} & \Psi_{22} & \mathbf{0} & \mathbf{0} \\ \Psi_{31} & \Psi_{32} & \mathbf{0} & \mathbf{0} \\ \Psi_{41} & \Psi_{42} & \Psi_{43} & \Psi_{44} \end{pmatrix}.$$

Section 6 also explores various alternative restrictions on Ψ. These do not materially alter the dynamics of the estimated wealth-consumption ratio. We estimate Ψ by OLS, equation-by-equation, and we form each innovation as follows $z_{t+1}(\cdot) - \Psi(\cdot, :)z_t$. We compute their (full rank) covariance matrix Σ.

The zero restrictions on Ψ imply zero restrictions on the corresponding elements of the market price of risk dynamics in Λ_1. For example, the assumption that the stock return and the price-dividend ratio on the stock market do not predict the bond variables implies that the market prices of risk for the bond market shocks cannot fluctuate with the stock market return or the price-dividend ratio. The entries of Λ_1 in the first four rows and the fifth and sixth column must be zero. Likewise, because the last four variables in the VAR do not affect expected stock and fmp returns, the prices of stock market risk cannot depend on the last four state variables. Finally, under our assumption that all sources of aggregate uncertainty are spanned by the innovations to the traded assets (the first eight shocks), the part of the shocks to consumption growth and labor income growth that is orthogonal to the bond and stock innovations is not priced. We relax this assumption in section 5. Thus, $\Lambda_{1,41}$, $\Lambda_{1,42}$, $\Lambda_{1,43}$, and $\Lambda_{1,44}$ are zero matrices. This leads to the following structure for Λ_1:

$$\Lambda_1 = \begin{pmatrix} \Lambda_{1,11} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \Lambda_{1,21} & \Lambda_{1,22} & \mathbf{0} & \mathbf{0} \\ \Lambda_{1,31} & \Lambda_{1,32} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix},$$

We impose corresponding zero restrictions on the mean risk premia in the vector Λ_0: $\Lambda_0 = [\Lambda_{0,1},\ \Lambda_{0,2},\ \Lambda_{0,3}\ \mathbf{0}]'$, where $\Lambda_{0,1}$ is 4×1, and $\Lambda_{0,2}$ and $\Lambda_{0,3}$ are 2×1 vectors.

The matrix $\Lambda_{1,11}$ contains the bond risk prices, $\Lambda_{1,21}$ and $\Lambda_{1,22}$ contain the aggregate stock risk prices, and $\Lambda_{1,31}$ and $\Lambda_{1,32}$ the risk prices on the factor mimicking portfolios (fmp) of aggregate consumption and labor income growth. While all zeroes in Ψ lead to zeroes in Λ_1 in the corresponding entries, the converse is not true. That is, not all entries of the matrices $\Lambda_{1,11}$,

$\Lambda_{1,21}$, $\Lambda_{1,22}$, $\Lambda_{1,31}$, and $\Lambda_{1,32}$ must be non-zero even though the corresponding elements of Ψ all are non-zero. Whenever we have a choice of which market price of risk parameters to estimate, we follow a simple rule: we associate non-zero risk prices with traded assets instead of non-traded variables. In particular, we set the rows corresponding to the prices of CP risk, inflation risk, and pd^m risk equal to zero because these are not traded assets, while the rows corresponding to the short rate, the yield spread, the stock market return and the fmp returns are non-zero. Our final specification has five non-zero elements in Λ_0 and twenty-six in Λ_1 (two rows of four and three rows of six). This specification is rich enough for the model to match the time-series of the traded asset prices that are part of the state vector.

The structure we impose on Ψ and on the market prices of risk is not overly restrictive. A Campbell-Shiller decomposition of the wealth-consumption ratio into an expected future consumption growth component (Δc_t^H) and an expected future total wealth returns component (r_t^H), detailed in Appendix B, delivers the following expressions:

$$\Delta c_t^H = e_c' \Psi (\kappa_1^c I - \Psi)^{-1} z_t \quad \text{and} \quad r_t^H = \left[(e_c + A_1^c)' \Psi - \kappa_1^c A_1^{c\prime} \right] (\kappa_1^c I - \Psi)^{-1} z_t.$$

Despite the restrictions on Ψ and Λ_t, both the cash flow component and the discount rate component depend on all state variables. In the case of Δc_t^H, this is because expected consumption growth depends on all lagged stock and bond variables in the state. In the case of r_t^H, there is additional dependence through A_1^c, which itself is a function of the first nine state variables. The cash flow component does not directly depend on the risk prices (other than through κ_1^c) while the discount rate component depends on all risk prices of stocks and bonds through A_1^c. This flexibility implies that our model can theoretically accommodate a large consumption risk premium. This happens when the covariances between consumption growth and the other aggregate shocks are large and/or when the unconditional risk prices in Λ_0 are sufficiently large. In fact, in our estimation, we choose Λ_0 large enough to match the equity premium. A low estimate of the consumption risk premium and hence a high wealth-consumption ratio are not a foregone conclusion.

3.3 Estimation

We estimate Λ_0 and Λ_1 from the moments of bond yields and stock returns. We relegate a detailed discussion of the estimation strategy to Appendix B. While all moments pin down all market price of risk estimates jointly, it is useful to organize the discussion as if the estimation proceeded in four steps. These steps can be interpreted as delivering good initial guesses from which to start the final estimation.

The model delivers a nominal (and real) term structure where bond yields are affine functions

of the state variables. In a first step, we estimate the risk prices in the *bond market block* $\Lambda_{0,1}$ and $\Lambda_{1,11}$ by matching the time series for the short rate, the slope of the yield curve, and the CP risk factor. Because of the block diagonal structure, we can estimate these risk prices separately. In a second step, we estimate the risk prices in the *stock market block* $\Lambda_{0,2}$, $\Lambda_{1,21}$, and $\Lambda_{1,22}$ jointly with the bond risk prices, taking the estimates from the first step as starting values. Here, we impose that the model delivers expected excess stock returns similar to the VAR. In a third step, we estimate the fmp risk prices in the *factor mimicking portfolio block* $\Lambda_{0,3}$, $\Lambda_{1,31}$, and $\Lambda_{1,32}$ taking as given the bond and stock risk prices. Again, we impose that the risk premia on the fmp coincide between the VAR and the SDF model. The stock and bond moments used in the first three steps exactly identify the 5 elements of Λ_0 and the 26 elements of Λ_1. In other words, given the structure of Ψ, they are all strictly necessary to match the levels and dynamics of bond yields and stock returns.

For theoretical as well as for reasons of fit, we impose several additional constraints. We obtain these constraints from matching additional nominal yields, imposing the present-value relationship for stocks, imposing a human wealth share between zero and one, and imposing the growth condition on the consumption claim. To avoid over-parametrization, we choose not to let these constraints identify additional market price of risk parameters. We re-estimate all 5 parameters in Λ_0 and all 26 parameters in Λ_1 in a final fourth step where we impose the constraints, starting from the third-step estimates. Our final estimates for the market prices of risk from the last-stage estimation are listed at the end of Appendix B alongside the VAR parameter estimates. Online Appendix B provides more detail on the over-identifying restrictions.

4 Estimation Results

We first verify that the model does an adequate job describing the quarterly dynamics of the bond yields and of stock returns. We then study the variations in the total wealth and human wealth. In the interest of space, we present auxiliary figures in the Online Appendix; they are denoted by the letter "A."

4.1 Model Fit for Bonds and Stocks

Our model fits the nominal term structure of interest rates reasonably well (see Figure A.1). We match the 3-month yield exactly. For the 5-year yield, which is part of the state vector through the yield spread, the average pricing error is -1 basis points (bp) per year. The annualized standard deviation of the pricing error is only 33bp, and the root mean squared error (RMSE) is 33bp. For the other four maturities, the mean annual pricing errors range from -7bp to +62bp, the volatility

of the pricing errors range from 33 to 58bp, and the RMSE from 33 to 65bp.[9] While these pricing errors are somewhat higher than the ones produced by term-structure models, our model has no latent state variables and only two term structure factors (two priced sources of risk which we associate with the second and fourth shocks). It also captures the level and dynamics of long-term bond yields well, a part of the term structure rarely investigated, but important for our purposes of evaluation a long-duration consumption claim. On the dynamics, the annual volatility of the nominal yield on the 5-year bond is 1.40% in the data and 1.35% in the model.

The model also does a good job capturing the bond risk premium dynamics. The model produces a nice fit between the Cochrane-Piazzesi factor, a measure of the 1-year nominal bond risk premium, in model and data (see right panel of Figure A.2). The annual mean pricing error is -15bp and standard deviation of the pricing error is 70bp. The 5-year nominal bond risk premium, defined as the difference between the 5-year yield and the average expected future short term yield averaged over the next 5 years, is also matched closely by the model (left panel of Figure A.2). The long-term and short-term bond risk premia have a correlation of 74%. Thus, our model is able to capture the substantial variation in bond risk premia in the data. This is important because the bond risk premium turns out to constitute a major part of the consumption risk premium and of the marginal cost of consumption fluctuations.

The model also manages to capture the dynamics of stock returns quite well. The model matches the equity risk premium that arises from the VAR structure (bottom panel of Figure A.3). The average equity risk premium (including Jensen term) is 6.41% per annum in the data, and 6.41% in the model. Its annual volatility is 3.31% in the data and 3.25% the model. The model, in which the price-dividend ratio reflects the present discounted value of future dividends, replicates the price-dividend ratio in the data quarter by quarter (top panel).

As in Ang, Bekaert, and Wei (2008), the long-term nominal risk premium on a 5-year bond is the sum of a real rate risk premium (defined the same way for real bonds as for nominal bonds) and the inflation risk premium. The right panel of Figure A.4 decomposes this long-term bond risk premium (solid line) into a real rate risk premium (dashed line) and an inflation risk premium (dotted line). The real rate risk premium becomes gradually more important at longer horizons. The left panel of Figure A.4 decomposes the 5-year yield into the real 5-year yield (which itself consists of the expected real short rate plus the real rate risk premium), expected inflation over the next 5-years, and the 5-year inflation risk premium. The inflationary period in the late 1970s-early 1980s was accompanied by high inflation expectations and an increase in the inflation risk premium, but also by a substantial increase in the 5-year real yield.[10] Separately identifying real

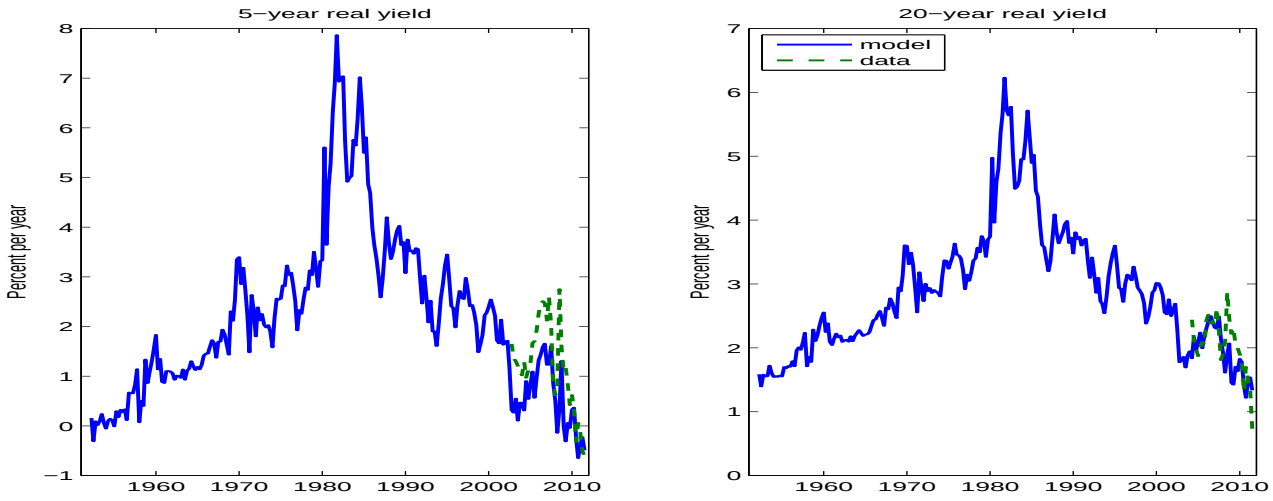
[9]Note that the largest errors occur on the 20-year yield, which is unavailable between 1986.IV and 1993.II. The standard deviation and RMSE on the 10-year yield are only half as big as on the 20-year yield.

[10]Inflation expectations in our VAR model have a correlation of 76% with inflation expectations from the Survey of Professional Forecasters (SPF) over the common sample 1981-2011. The 1-quarter ahead inflation forecast error series for the SPF and the VAR have a correlation of 79%. Realized inflation fell sharply in the first quarter of

rate risk and inflation risk based on nominal term structure data alone is challenging.[11] We do not have long enough data for real bond yields, but stocks are real assets that contain information about the term structure of real rates. They can help with the identification. For example, high long real yields in the late 1970s-early 1980s lower the price-dividend ratio on the stock market stock, which indeed was low in the late 1970s-early 1980s (top panel of Figure A.3). High nominal yields combined with high price-dividend ratios would have suggested low real yields instead.

Figure 2: Dynamics of the Real Term Structure of Interest Rates

The figure plots the observed and model-implied 5-, 7-, 10-, 20-, and 30-year real bond yields. Real yield data are constant maturity yields on Treasury Inflation Indexed Securities from the Federal Reserve Bankk of St.-Louis (FRED II). We use the longest available sample for each maturity.



Average real yields range from 1.49% per year for 1-quarter real bonds to 2.87% per year for 20-year real bonds. Despite the short history of Treasury Inflation Indexed Bonds, potential liquidity issues early in the sample, and the dislocation in the TIPS market/rich pricing of nominal Treasuries (Longstaff, Fleckenstein, and Lustig 2010), it is nevertheless informative to compare model-implied real bond yields to observed real yields. Despite the fact that these real yields were not used in estimation, Figure 2 shows that the fit over the common sample is reasonably good both in terms of levels and dynamics.

Finally, the model matches the expected returns on the consumption and labor income growth factor mimicking portfolios (fmp) very well (See Figure A.6). The annual risk premium on the consumption growth fmp is 1.08% in the data and model, with volatilities of 1.59 and 1.54%. Likewise, the risk premium on the labor income growth fmp is 3.48% in data and model, with

1981. Neither the professional forecasters nor the VAR anticipated this decline, leading to a high realized real yield. The VAR expectations caught up more quickly than the SPF expectations, but by the end of 1981, both inflation expectations were identical.

[11]Many standard term structure models have a likelihood function with two local maxima with respect to the persistence parameters of expected inflation and the real rate.

volatilities of 2.41 and 2.51%.

To summarize, Table 1 provides a detailed overview of the pricing errors on the assets used in estimation. Panel A shows the pricing errors on the equity portfolios, Panels B and C the pricing errors on nominal bonds. Panel A shows that the volatility and RMSE of the pricing errors on the equity risk premium are about 15 bp per year; those on the factor mimicking portfolio returns are 6 and 37 bp. Panel B shows the pricing errors on nominal bonds that were used in estimation. The three month rate is matched perfectly since it is in the state vector and carries no risk price. The pricing error on the 5-year bond is only 1 bp on average, with a standard deviation and RMSE of about 33 bp. One- through four-year yields have RMSEs between 39bp and 46bp per year. The seven-year bond has a RMSE of 35 bp, the ten-year bond one of 37 bp. The largest pricing errors occur on bonds of 20- and 30-year maturity, around 65bp. One mitigating factor is that these bonds have some missing data over our sample period, which makes the comparison of yields in model and data somewhat harder to interpret. Another is that there may be liquidity effects at the long end of the yield curve that are not captured by our model. Finally, the RMSE on the CP factor is comparable to that on the 5-year yield once its annual frequency is taken into account.[12]

We conclude that our pricing errors are low given that we jointly price bonds and stocks, use no latent state variables, and include much longer maturity bonds than what is typically done in the literature.

4.2 The Wealth-Consumption Ratio

With the estimates for Λ_0 and Λ_1 in hand, it is straightforward to use Proposition 1 and solve for A_0^c and A_1^c from equations (7)-(8). Table 2 summarizes the key moments of the log wealth-consumption ratio obtained in quarterly data in column 3. The numbers in parentheses are small sample bootstrap standard errors, computed using the procedure described in Appendix B.9.

Comparison to Stocks We can directly compare the moments of the wealth-consumption ratio with those of the price-dividend ratio on equity. The wc ratio has an annualized volatility of 19% in the data, considerably lower than the 29% volatility of the pd^m ratio. The wc ratio in the data is a persistent process; its 1-quarter (4-quarter) serial correlation is .97 (.87). This is similar to the .94 (.77) serial correlation of pd^m. The annual volatility of changes in the wealth consumption ratio is 4.51%, and because of the low volatility of aggregate consumption growth changes, this translates into a volatility of the total wealth return on the same order of magnitude (4.59%). The corresponding annual volatility of 9.2% is about half the 17.2% volatility of stock returns.

[12]The CP factor is constructed from annual returns while the yields are quarterly. To annualize the volatility of yield pricing errors, we multiply the quarterly pricing errors by $2 = \sqrt{4}$. To compare the two, the volatility and RMSE of CP should be divided by a factor of two.

Table 1: Pricing Errors

This table reports the pricing errors on the asset pricing moments used in the estimation, as well as some over-identifying restrictions. The pricing error time series are computed as the difference between the predicted asset pricing moment by the model and the observed asset pricing moment in the data. The table reports time-series averages (Mean), standard deviations (Stdev), and root-mean squared errors (RMSE). Panel A reports pricing errors on the equity market portfolio, the consumption growth factor-mimicking portfolio ($fmpc$), and the labor income growth factor-mimicking portfolio ($fmpl$). It also reports how well the model matches the price-dividend ratio on the aggregate stock market. Panel B shows nominal bond yield pricing errors for the bond maturities that were used in estimation. Panel C shows bond yield errors for bond maturities that were not used in estimation, as well as the Cochrane-Piazzesi (CP) ratio. All moments are annualized and are multiplied by 100, except for the price-dividend ratio which is annualized in levels.

Panel A: Equity Portfolio Returns and PD						
	Equity Mkt.	$fmpc$	$fmpl$			pd ratio
Mean	0.0014	-0.0003	0.0004			-0.1134
Stdev	0.1517	0.0579	0.3662			0.1932
RMSE	0.1514	0.0578	0.3655			0.2237
Panel B: Nominal Bond Yields Used in Estimation						
	$y^{\$}(1)$	$y^{\$}(4)$	$y^{\$}(12)$	$y^{\$}(20)$	$y^{\$}(40)$	$y^{\$}(80)$
Mean	-0.0000	-0.0698	-0.0446	-0.0094	0.2026	0.6212
Stdev	0.0000	0.4649	0.3859	0.3325	0.3586	0.5761
RMSE	0.0000	0.4652	0.3857	0.3318	0.3719	0.6532
Panel C: CP and Nominal Bond Yields Not Used in Estimation						
	$y^{\$}(8)$	$y^{\$}(16)$	$y^{\$}(28)$	$y^{\$}(120)$		CP
Mean	-0.0399	-0.0391	-0.0484	0.0316		-0.1531
Stdev	0.4258	0.3587	0.3537	0.6638		0.7006
RMSE	0.4254	0.3585	0.3535	0.6612		0.7157

The change in the wc ratio and the total wealth return have weak autocorrelation, suggesting that total wealth returns are hard to forecast by their own lags. The correlation between the (quarterly) total wealth return and consumption growth is mildly positive (.21).

How risky is total wealth compared to equity? According to our estimation, the consumption risk premium (calculated from equation 11) is 60 bp per quarter or 2.38% per year. This results in a mean wealth-consumption ratio of 5.81 in logs (A_0^c), or 83 in annual levels ($\exp\{A_0^c - \log(4)\}$). The consumption risk premium is only one-third as big as the equity risk premium of 6.41%. Correspondingly, the wealth-consumption ratio is much higher than the price-dividend ratio on equity: 83 versus 26. A simple back-of-the-envelope Gordon growth model calculation sheds light on the mean of the wealth-consumption ratio. The discount rate on the consumption claim is 3.51% per year (a consumption risk premium of 2.38% plus a risk-free rate of 1.49% minus a Jensen term of 0.37%) and its cash-flow growth rate is 2.31%: $83 = 1/(.0351 - .0231)$. The standard errors on the moments of the wealth-consumption ratio and total wealth return are sufficiently small so that the corresponding moments of the price-dividend ratio or stock returns are outside the 95% confidence interval of the former. The main conclusion of our measurement exercise is that total wealth is (economically and statistically) significantly less risky than equity.

Table 2: Moments of the Wealth-Consumption Ratio

This table displays unconditional moments of the log wealth-consumption ratio wc, its first difference Δwc, and the log total wealth return r^c. The table also reports the time-series average of the conditional consumption risk premium, $E[E_t[r_t^{c,e}]]$, where $r^{c,e}$ denotes the expected log return on total wealth in excess of the risk-free rate and corrected for a Jensen term. The last row denotes the share of human wealth in total wealth. The first column reports moments from the long-run risk model (LRR model), simulated at quarterly frequency. All reported moments are averages and standard deviations (in parentheses) across the 5,000 simulations of 220 quarters of data. The second column reports the same moments for the external habit model (EH model). The last two columns report the data at quarterly and annual frequencies respectively. The standard errors are obtained by bootstrap, as described in Appendix B.9.

Moments	LRR Model quarterly	EH model quarterly	data quarterly	data annual
$Std[wc]$	2.35%	29.33%	18.57%	24.68%
(s.e.)	(.43)	(12.75)	(4.30)	(7.81)
$AC(1)[wc]$	0.91	0.93	0.97	
(s.e.)	(.03)	(.03)	(.03)	
$AC(4)[wc]$	0.70	0.74	0.87	0.86
(s.e.)	(.10)	(.11)	(.08)	(.21)
$Std[\Delta wc]$	0.90%	9.46%	4.51%	12.13%
(s.e.)	(.05)	(2.17)	(1.16)	(3.33)
$Std[\Delta c]$	1.43%	0.75%	0.46%	1.24 %
(s.e.)	(.08)	(.04)	(.03)	(.14)
$Corr[\Delta c, \Delta wc]$	-0.06	0.90	0.12	0.04
(s.e.)	(.06)	(.03)	(.06)	(.16)
$Std[r^c]$	1.64%	10.26%	4.59%	12.34 %
(s.e.)	(.09)	(2.21)	(1.16)	(3.42)
$Corr[r^c, \Delta c]$	0.84	0.91	0.21	0.15
(s.e.)	(.02)	(.03)	(.07)	(.15)
$E[E_t[r_t^{c,e}]]$	0.40%	2.67%	0.60%	2.34%
(s.e.)	(.01)	(1.16)	(.16)	(.88)
$E[wc]$	5.85	3.86	5.81	4.63
(s.e.)	(.01)	(.17)	(.49)	(.53)
2011 Wealth (in millions)			3.49	3.57
$(s.e.)$			(0.27)	(.52)
Human wealth share			0.92	0.92
$(s.e.)$			(0.03)	(.02)

Comparison to Claim to Trend Consumption The claim to trend consumption is the second benchmark for the risky consumption claim. Table 3 reports the same moments as Table 2 but for a claim to deterministically growing consumption. We estimate a risk premium on the trend claim of 64 bp per quarter or 2.58% per annum. The difference with the consumption risk premium is 4bp per quarter and not statistically different from zero. Because the claims to risky and to trend consumption differ only in terms of their consumption cash flow risk, the small difference in risk premia shows that the market assigns essentially zero compensation to current consumption innovations. This is the result of two offsetting forces. One the one hand, quarterly consumption innovations are positively correlated to market equity and consumption-fmp portfolio shocks, both of which carry a positive price of risk. This equity exposure adds to the

consumption risk premium. On the other hand, quarterly consumption innovations *hedge* both shocks to the level (second orthogonalized shock) and the slope (fourth orthogonalized shock) of the term structure. Consumption innovations are positively correlated with level innovations, which carry a negative risk price, and they are negatively correlated with slope shocks, which carry a positive risk price. Both of these term structure exposures lower the consumption risk premium. Put differently, the claim to trend consumption has a higher exposure to interest rate shocks than the claim to risky consumption because of the interest rate hedging benefits of the latter. Exposure to stock market risk (almost) offsets the lower bond market risk exposure so that the two claims end up with nearly the same risk premium.

Table 3: Moments of a Claim to Trend Consumption

This table displays unconditional moments for the consumption perpetuity, the claim to deterministically growing aggregate consumption. We report its log wealth-consumption ratio wc^{tr}, its first difference Δwc^{tr}, and the log total wealth return $r^{c,tr}$. The last panel reports the time-series average of the conditional consumption risk premium, $E[E_t[r_t^{c,tr,e}]]$, where $r^{c,tr,e}$ denotes the expected log return on total wealth in excess of the risk-free rate and corrected for a Jensen term. We report the estimated moments in the data at quarterly and annual frequencies respectively.

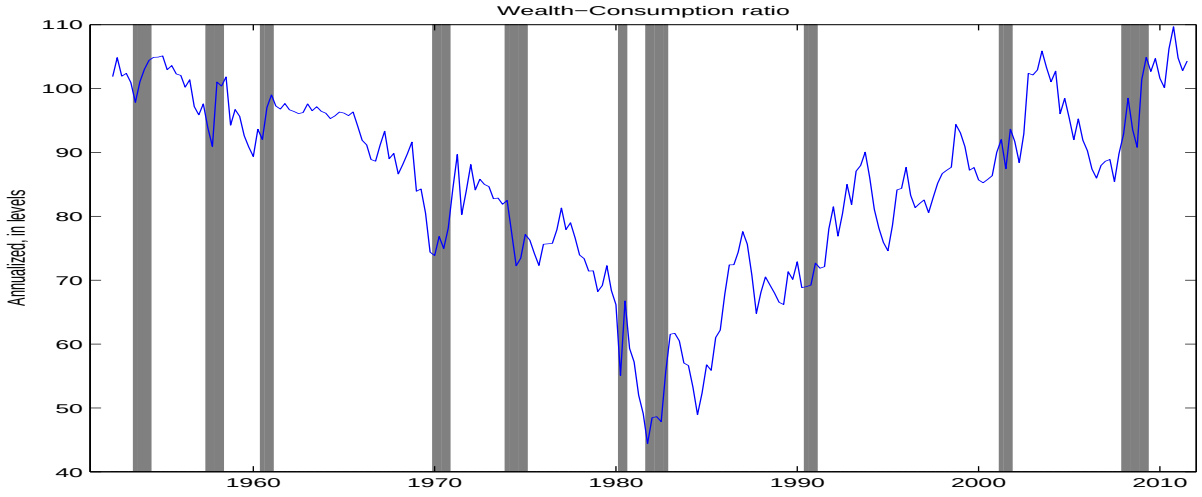
Moments	data quarterly	data annual
$Std[wc^{tr}]$	21.32	25.79
$AC(1)[wc^{tr}]$	0.96	×
$AC(4)[wc^{tr}]$	0.86	0.87
$Std[\Delta wc^{tr}]$	5.30	11.60
$Corr[\Delta c, \Delta wc^{tr}]$	0.06	0.01
$Std[r^{c,tr}]$	5.31	11.67
$Corr[r^{c,tr}, \Delta c]$	0.06	0.02
$E[E_t[r_t^{c,tr,e}]]$	0.64	2.05
$E[wc^{tr}]$	5.78	4.92
2011 Wealth (in millions)	3.97	6.05

Wealth Creation and Destruction Figure 3 plots the wealth-consumption ratio in levels, alongside NBER recessions (shaded bars). Its dynamics are to a large extent inversely related to the long real yield dynamics in Figure 2. For example, the 5-year real yield increases from 3.5% per annum in 1979.I to 6.9% in 1981.III while the wealth-consumption ratio falls from 68 to 49. This corresponds to a loss of $318,000 in real per capita wealth in 2005 dollars, where real per capita wealth is the product of the wealth-consumption ratio and observed real per capita consumption. Similarly, the low-frequency decline of the real yield in the twenty-five years after 1981 corresponds to a gradual rise in the wealth-consumption ratio. One striking way to see that total wealth behaves differently from equity is to study it during periods of large stock market declines. During the bear markets of 1973.III-1974.IV, 2000.I-2002.IV, and 2007.II-2009.I, the change in US households' real per capita stock market wealth (including mutual fund holdings)

was -46%, -61%, and -65%, respectively. In contrast, real per capita total wealth changed by -12%, +23%, and +11%, respectively.[13] Over the full sample, the total wealth return has a correlation of only 27% with the value-weighted real CRSP stock return, while it has a correlation of 94% with realized one-quarter holding period returns on the 5-year nominal government bond.[14] Likewise, the quarterly consumption risk premium has a correlation of 55% with the quarterly equity risk premium, lower than the 62% correlation with the quarterly nominal bond risk premium on a 5-year bond.

Figure 3: The Log Wealth-Consumption Ratio in the Data

The figure plots $\exp\{wc_t - log(4)\}$, where wc_t is the quarterly log total wealth to total consumption ratio. The log wealth consumption ratio is given by $wc_t = A_0^c + (A_1^c)' z_t$. The coefficients A_0^c and A_1^c satisfy equations (7)-(8).



To show more formally that the consumption claim behaves like a real bond, we compute the discount rate that makes the current wealth-consumption ratio equal to the expected present discounted value of future consumption growth. This is the solid line measured against the left axis of Figure 4. Similarly, we calculate a time series for the discount rate on the dividend claim, the dotted line measured against the right axis. For comparison, we plot the yield on a long-term real bond (50-year) as the dashed line against the right axis. The correlation between the consumption discount rate and the real yield is 99.95%, whereas the correlation of the dividend discount rate and the real yield is only 46%. In addition, the consumption and dividend discount

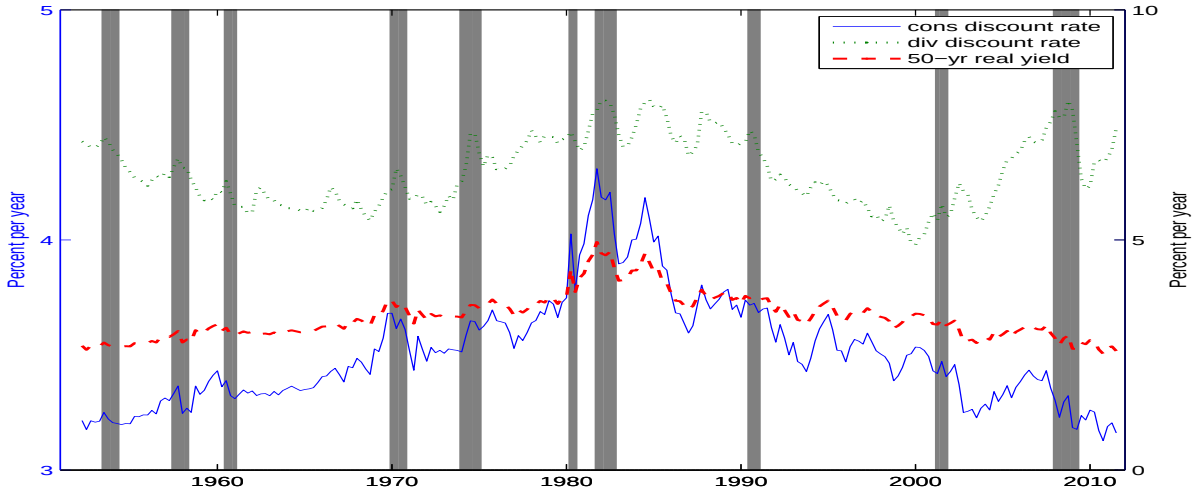
[13] During the Great Recession, total per capita wealth is estimated to fall between 2008.II and 2008.IV. In addition to this absolute decline, we argue below that total wealth fell substantially relative to trend wealth over a multi-year period surrounding the Great Recession. Finally, we note that our model might understate the total wealth destruction during the Great Recession if flight-to-safety effects made nominal Treasuries yields artificially low.

[14] A similarly low correlation of 18% is found between total wealth returns and the Flow of Fund's measure of the growth rate in real per capita household net worth, a broad measure of financial wealth. The correlation of the total wealth return with the Flow of Fund's growth rate of real per capita housing wealth is 4%.

rates only have a correlation of 48%, reinforcing our conclusion that the data suggest a large divergence between the perceived riskiness of a claim to consumption and a claim to dividends in securities markets.

Figure 4: Discount Rates on Consumption and Dividend Claim

The figure plots the discount rate on a claim to consumption (solid line, measured against the left axis, in percent per year), the discount rate on a claim to dividend growth (dashed line, measured against the right axis, in percent per year), and the yield on a real 50-year bond (dotted line, measured against the right axis, in percent per year). The discount rates are the rates that make the price-dividend ratio equal to the expected present-discounted value of future cash flows, for either the consumption claim or the dividend claim.



A second way of showing that the consumption claim is bond-like is to study yields on consumption strips. We decompose the yield on the period-τ strip in two components. The first component is the yield on a security that pays a certain cash flow $(1+\mu_c)^\tau$. The underlying security is a real perpetuity with a cash flow which grows at a deterministic consumption growth rate μ_c. The second component is the yield on a security that pays off $C_\tau/C_0 - (1+\mu_c)^\tau$; it captures pure consumption cash flow risk. Appendix B.4 shows that the log price-dividend ratios on the consumption strips are approximately affine in the state, and details how to compute the yield on its two components. In our model, consumption strip yields are mostly comprised of a compensation for variation in real rates (labeled "real bond yield -μ_c" in Figure A.5), not consumption cash flow risk (labeled "yccr"). Other than at short horizons, the consumption cash flow risk security has a yield that is approximately zero.

Predictability Properties Our analysis so far has focused on unconditional moments of the total wealth return. The conditional moments of total wealth returns are also very different from those of equity returns. The familiar Campbell and Shiller (1988) decomposition for the wealth-consumption ratio shows that the wealth-consumption ratio fluctuates either because it predicts

future consumption growth rates (Δc_t^H) or because it predicts future total wealth returns (r_t^H):

$$V\left[wc_t\right] = Cov\left[wc_t, \Delta c_t^H\right] + Cov\left[wc_t, -r_t^H\right] = V\left[\Delta c_t^H\right] + V\left[r_t^H\right] - 2Cov\left[r_t^H, \Delta c_t^H\right].$$

The second equality suggests an alternative decomposition into the variance of expected future consumption growth, expected future returns, and their covariance. Finally, it is straightforward to break up $Cov\left[wc_t, r_t^H\right]$ into a piece that measures the predictability of future excess returns, and a piece that measures the covariance of wc_t with future risk-free rates. Our no-arbitrage methodology delivers analytical expressions for all variance and covariance terms (See Appendix B).

We draw three main empirical conclusions. First, the mild variability of the wc ratio implies only mild total wealth return predictability. This is in contrast with the high variability (and predictability) of pd^m. Second, 104.9% of the variability in wc is due to covariation with future total wealth returns while the remaining -4.9% is due to covariation with future consumption growth. Hence, the wealth-consumption ratio predicts future returns (discount rates), not future consumption growth rates (cash flows). Using the second variance decomposition, the variability of future returns is 111.5%, the variability of future consumption growth is 1.7% and their covariance is -13.2% of the total variance of wc. This variance decomposition is similar to the one for equity. Third, 74% of the 104.9% covariance with returns is due to covariance with future *risk-free rates*, and the remaining 30.9% is due to covariance with future *excess* returns. The wealth-consumption ratio therefore mostly predicts future variation in interest rates, not in risk premia. The exact opposite holds for equity: the bulk of the predictability of the pd^m ratio for future stock returns is predictability of excess returns (51% out of 66.5%). In sum, the conditional asset pricing moments also reveal interesting differences between equity and total wealth. Again, they point to the link between the consumption claim return and interest rates.

4.3 Cost of Consumption Fluctuations and Wealth Effects

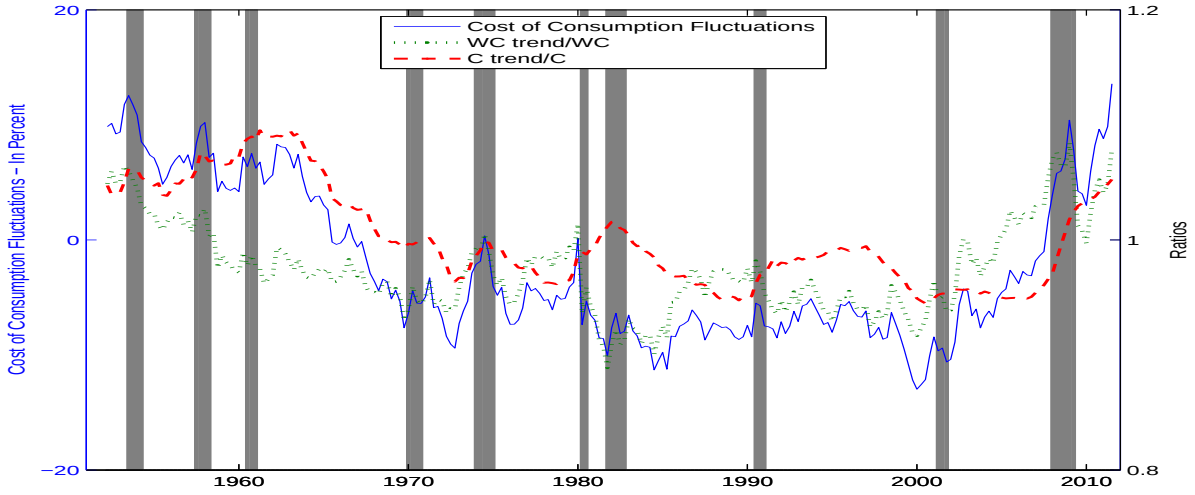
Figure 5 shows the cost of consumption fluctuations (ϖ) in our benchmark model. It breaks down this cost into the ratio of the wealth-consumption ratios of the trend claim to that of the risky consumption claim and the ratio of trend consumption to consumption (see equation 13). The average cost of consumption fluctuations is slightly negative, consistent with the slightly lower consumption risk premium arising from the hedging properties of consumption discussed above.

More interesting than the mean is the substantial amount of variation in the marginal cost. At the end of the sample, the cost of consumption fluctuations skyrockets. This happens because consumption is far below trend and because the ultra-low interest rates result in a much higher trend wc ratio than risky wc ratio (recall the former's greater interest rate sensitivity). In other

words, total wealth falls far below trend during the Great Recession. Thus, relative to trend, our model implies a large wealth destruction during the Great Recession.

Figure 5: Cost of Consumption Fluctuations

The figure plots the marginal cost of consumption fluctuations $\varpi_t = \frac{W_t^{tr}}{W_t} - 1 = \frac{WC_t^{tr}}{WC_t}\frac{C_t^{tr}}{C_t} - 1$ against the left axis (solid line). It also plots the two ratio terms $\frac{WC_t^{tr}}{WC_t}$ (dotted line) and $\frac{C_t^{tr}}{C_t}$ (dashed line) that constitute ϖ_t.



The average propensity to consume out of total wealth is 1.2 cents for every dollar of wealth (= 1/83). The marginal propensity to consume out of (the last dollar increase in) total wealth is 0.75 cents. Our estimates for the marginal propensity are at the low end of the range of numbers in the literature – see (Poterba 2000).[15] In contrast, if the consumption claim was priced like equity, the average propensity to consume would be much higher: 3.9 cents (1/26) out of every dollar. Such a number is in the ballpark of the 5 cent estimate that is suggested by Modigliani (1971) and a large literature that follows it.

There is considerable variation in the marginal propensity to consume. It peaks at 1.40 cents per dollar in 1981.IV, when real interest rates peak, and it bottoms out at 0.57 cents in 2010.IV, when real interest rates bottom out. The 50% decline in the propensity to consume occurs despite the massive wealth creation over the 1981 to 2010 period. The logic of the budget constraint imposes that the propensity to consume must drop when expected total wealth returns drop. The latter are highly correlated with real interest rates which fall substantially over this period. Our estimated decline is consistent with Ludvigson and Steindel (1999), who report a large drop in the marginal propensity to consume out of stock market wealth after 1986.

Time-variation in the wealth-consumption ratio implies that the wealth effect decreases during

[15] In part this is because we consider total wealth, the infinite present discounted value of all future labor and dividend income. If we were to limit ourselves to a 35-year "career" for labor income, the marginal propensity to consume would be three times higher at 2.3 cents per last dollar of career human wealth.

periods of abnormal total wealth creation, while it increases during periods of abnormal total wealth destruction. Previously, Poterba (2000) and others had speculated that consumers may respond more strongly to wealth destruction than creation. Moreover, macro-economists have long been puzzled by the dramatic destruction of capital in 1973 and 1974, inferred from the stock market's steep decline (e.g., Hall (2001)). Our findings suggest these events in the stock market only had a minor impact on total U.S. wealth, and are consistent with Hobijn and Jovanovic (2001)'s account of this episode.

4.4 Human Wealth Returns

Our estimates indicate that the bulk of total wealth is human wealth. The human wealth share fluctuates between 86 and 99%, with an average of 92% (see last row of Table 2). Interestingly, Jorgenson and Fraumeni (1989) calculates a similar 90% human wealth share. The average price-dividend ratios on human wealth is slightly above the one on total wealth (93 versus 83 in annual levels). The risk premium on human wealth is very similar to the one for total wealth (2.31 versus 2.38% per year). The price-dividend ratios and risk premia on human wealth and total wealth have a 99.87% (99.95%) correlation.

Existing approaches to measuring total wealth make ad hoc assumptions about expected human wealth returns. The model of Campbell (1996) assumes that expected human wealth returns are equal to expected returns on financial assets. This is a natural benchmark when financial wealth is a claim to a constant fraction of aggregate consumption. Shiller (1995) models a constant discount rate on human wealth. Jagannathan and Wang (1996) assume that expected returns on human wealth equal the expected labor income growth rate; the resulting price-dividend ratio on human wealth is constant. The construction of *cay* in Lettau and Ludvigson (2001a) makes that same assumption. Our approach avoids having to make arbitrary assumptions on unobserved human wealth returns.[16]

Our estimation results indicate that expected excess human wealth returns have an annual volatility of 2.9%. This is substantially higher than the volatility of expected labor income growth (0.6%), but lower than that of the expected excess returns on equity (3.3%). Lastly, average (real) human wealth returns (3.8%) are much lower than (real) equity returns (7.9%), but higher than (real) labor income growth (2.3%) and the (real) short rate (1.5%).

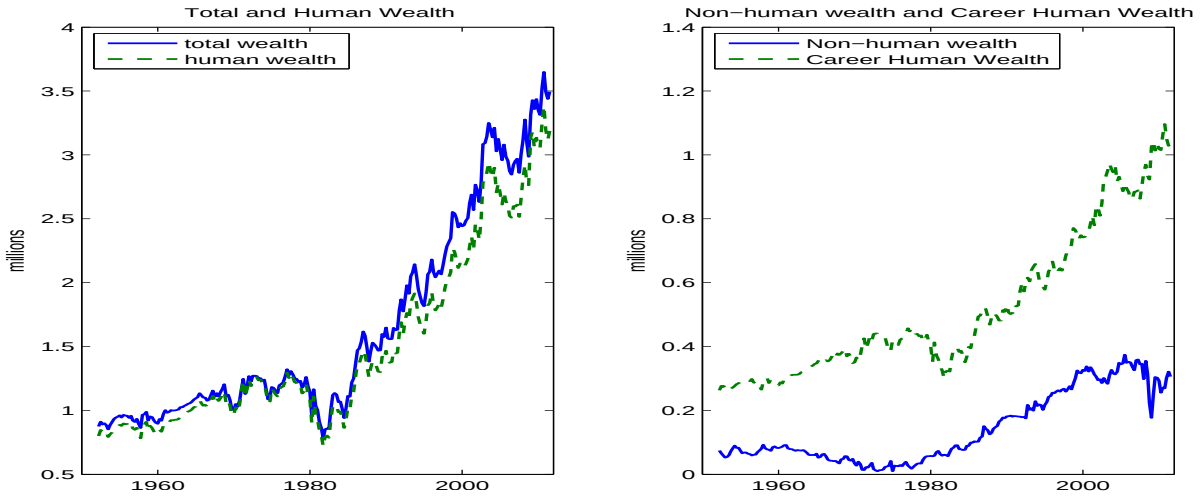
How much human wealth do our estimates imply? In real 2005 dollars, total per capita wealth increased from \$0.87 million to \$3.49 million between 1952 and 2011. The thick solid line in the left panel of Figure 6 shows the time series. Of this, \$3.2 million was human wealth in 2011 (dashed line in left panel), while the remainder is non-human wealth (solid line in right panel).

[16]These models can be thought of as special case of ours, imposing additional restrictions on the market prices of risk Λ_0 and Λ_1. Our work rejects these additional assumptions.

To judge whether this is a reasonable number, we compute the fraction of human wealth that accrues in the first 35 years.[17] In 2011, this implies a human wealth value of $1.04 milllion per capita (dashed line in the right panel). This amount is the price of a 35-year annuity with a cash flow of $38,268 which grows at the average labor income growth rate of 2.31% and is discounted at the average real rate of return on human wealth of 3.81%. This model-implied annual income of $38,268 compares to U.S. per capital labor income of $24,337 at the end of 2011. Another reference point for the "first 35 years" human wealth number is per capita residential home equity from the Flow of Funds. In 2011, home equity is a factor 51 smaller than human wealth. Unlike the massive destruction of home equity, human wealth has grown substantially over the last five years and is the main driver behind the overall wealth accumulation.[18]

Figure 6: Real Per Capita Wealth Estimates

The left panel of the figure plots total wealth and human wealth as estimated from the data. The right panel plots their difference, which we label non-human wealth. It also plots the present discounted value of the first 35 years of labor income.



Finally, we compare *non-human* wealth, the difference between our estimates for total and for human wealth, with the Flow of Funds series for household net worth. The latter is the sum of equity, bonds, housing wealth, durable wealth, private business wealth, and pension and life insurance wealth minus mortgage and credit card debt. Our non-human wealth series is on average 3.3 times the Flow of Funds series. This ratio varies over time: it is 10.1 at the beginning and 1.7 at the end of the sample, and it reaches a low of 0.46 in 1975. We chose not to use the Flow of Funds net worth data in our estimation because many of the wealth categories are hard

[17]This fraction is the price of the first 140 quarterly labor income strips divided by the price of all labor income strips. The labor income strip prices are computed just like the consumption strip prices. On average, 35% of human wealth pertains to the first 35 years.

[18]The destruction of housing wealth has been linked to a reversal of a financial market liberalization in credit markets by Favilukis, Ludvigson, and Van Nieuwerburgh (2011). This increased risk premia, more than offsetting the large fall in real rates during the Great Recession.

to measure accurately or are valued at book value (e.g., private business wealth). Arguably, only the equity component for publicly traded companies is measured precisely, and this may explain why the dynamics of the household net worth series are to a large extent driven by variation in stock prices.[19] It is reassuring that our non-human wealth measure exceeds the net worth series. After all, our series measures the present discounted value of all future non-labor income. This includes the value of growth options that will accrue to firms that have not been born yet, the same way human wealth includes labor income from future generations.

Total stock market wealth of $32,900 per capita in 2011.III represents 0.94% (1.03%) of our per capita total (human) wealth estimate of $3.5 ($3.2) million. To gauge the plausibility of these numbers, consider that stock market wealth is 18.6% of total household net worth according to the Flow of Funds. With a standard capital income share of 30% and no risk adjustment, this would translate in a 5.3% share of equity in total wealth. Our numbers are lower because we find that human wealth is substantially less risky than stock market wealth, requiring labor income cash flows to be discounted substantially less than equity dividends.

5 Non-Traded Consumption Risk

So far we have assumed that all aggregate shocks are spanned by stock and bond prices. This assumption is satisfied in all structural dynamic asset pricing models that we are aware of. Even in incomplete markets models, asset prices will reflect changes in the income or wealth distribution (e.g., Constantinides and Duffie (1996)).

In the absence of spanning, it is impossible to conclusively bound the wealth-consumption ratio, except by writing down a fully specified general equilibrium model. However, it is possible to put reasonable bounds on the non-traded consumption risk premium in our model. In particular, we relax our assumption that traded assets span all aggregate shocks by freeing up the 9^{th} element of Λ_0, the risk price of the non-traded consumption growth shock that is orthogonal to the eight traded asset shocks. Table 4 reports the consumption risk premium (Column 2), the average wealth-consumption ratio (Column 3), the maximum conditional Sharpe ratio (Column 4) and the Sharpe ratio on a one-period ahead consumption strip (Column 5) for different values of the price of non-traded consumption risk, governed by the 9th-element of Λ_0 (Column 1). This parameter does not affect the prices of any traded assets, so this exercise does not change any of the model's implications for observables.[20]

[19] Lettau and Ludvigson (2001a, 2001b) also use Flow of Funds data to measure household financial wealth. Lettau and Ludvigson (2001a)'s measure $-cay$ falls during the stock market crashes of 1974 and 2000-02. It has a correlation of only 0.24 with our wealth-consumption measure.

[20] Freeing up $\Lambda_0(9)$ also affects the risk premium and price-dividend ratio on human wealth, in quantitatively similar ways. We also experimented with freeing up the price of risk on the shock to labor income growth that is orthogonal to all previous shocks, including the aggregate consumption growth shock. Increasing this $\Lambda_0(10)$ has

The first line reports our benchmark case in which the non-traded consumption risk is not priced. The consumption risk premium is 2.38% per annum, the maximum Sharpe ratio is 0.6 and the conditional Sharpe ratio on the one-period ahead consumption strip is .06. Increasing $\Lambda_0(9)$ increases the consumption risk premium, lowers the wealth-consumption ratio, and increases the Sharpe ratio on the consumption strip. How far should we increase $\Lambda_0(9)$? A first answer is to bound the maximal Sharpe ratio ($std_t[m_{t+1}]$). Cochrane and Saa-Requejo (2000) and Alvarez and Jermann (2004) choose a "good deal" bound of one, which they argue is high because it is twice the 0.5 Sharpe ratio on equities in the data.[21] Since we work with quarterly log returns, the Sharpe ratio on equities is only 0.19, and that same good deal bound of one is more than twice as conservative. This bound is reached for $\Lambda_0(9)$ around 0.8, and implies a consumption risk premium of 3.91% per annum and an average wealth-consumption ratio of 36. Even then, the consumption risk premium is still 2.5% short of the equity premium, so that our conclusion that total wealth has different risk-return characteristics than equity remains valid. In order to match the equity premium by increasing the price of non-traded consumption risk, we would need an increase in the maximum Sharpe ratio to twice the good-deal bound or ten times the Sharpe ratio on equity. This does not seem reasonable for two reasons. First, such non-traded risk would certainly differ across households and would beg the question of why no market exists to share this risk. Second, a high risk premium on total consumption would imply a low wealth-consumption ratio, which in turn would suggest an extremely high marginal cost of business cycles.

A second answer would be to evaluate the Sharpe ratios on the consumption strip return in Column 5. When we set $\Lambda_0(9)$ to 0.1, this Sharpe ratio doubles compared to $\Lambda_0(9) = 0$. I.e., the implied price of non-traded consumption cash flow risk is much higher than that of traded consumption cash flow risk on a *per unit of risk* basis. Allowing the consumption strip to have the same Sharpe ratio as equity (0.19), would imply a value for $\Lambda_0(9)$ around 0.10. At this value the consumption risk premium is only about 0.2% per year higher than in our benchmark case. At $\Lambda_0(9) = 0.8$, the conditional Sharpe ratio on the consumption strip is 0.8, four times higher than the Sharpe ratio on equity and 13 times higher than the Sharpe ratio on the traded consumption strip.

Finally, Appendix C shows that our methodology for pricing aggregate consumption and labor income claims remains valid if the data are generated from an economy inhabited by heterogeneous agents who face idiosyncratic labor income risk which they cannot perfectly insure away and who may face binding borrowing or asset market participation constraints.

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no effect on the consumption risk premium and the wealth-consumption ratio. It only affects the risk premium on human wealth. Quantitatively, the effects are similar to those presented in Table 4. The same is true when we simultaneously increase $\Lambda_0(9)$ and $\Lambda_0(10)$.

[21] In related work, Bernardo and Ledoit (2000) bound the gain-loss ratio which summarizes the attractiveness of a zero-price portfolio. It is equivalent to a restriction on admissible pricing kernels, precluding the existence of arbitrage and approximate arbitrage opportunities.

Table 4: Non-traded Consumption Risk

The first column reports the market price of risk $\Lambda_0(9)$ that is associated with the innovation to consumption growth that is orthogonal to all innovations to the preceding stock and bond innovations. The second column reports the consumption risk premium. The third column reports the average wealth/consumption ratio. The fourth column is the maximum Sharpe ratio computed as $\sqrt{\Lambda_0'\Lambda_0}$. The last column shows the conditional Sharpe ratio on a one-period ahead consumption strip: $\left(e_c'\Sigma^{1/2}\Lambda_0\right)/\sqrt{e_c'\Sigma^{1/2}\Sigma^{1/2\prime}e_c}$.

$\Lambda_0(9)$	cons. risk premium	$E[WC]$	$std_t(m_{t+1})$	SR on strips
0	2.38%	85	0.58	0.06
0.05	2.48%	78	0.58	0.10
0.10	2.57%	73	0.59	0.15
0.50	3.33%	46	0.77	0.52
0.80	3.91%	36	**0.99**	0.80
1.00	4.30%	31	1.16	0.98
1.50	5.29%	24	1.61	1.45
2.00	6.30%	19	2.08	1.91
3.00	8.35%	14	3.06	2.84

6 Robustness

6.1 Smaller Models

The results of our estimation exercise are robust to different specifications of the law of motion for the state z. Appendix D considers five alternative models that have fewer state variables than our benchmark exercise and lists the goodness of fit for each of these. The variations are selected to give insight into what drives our main result. For brevity, we discuss only the main findings here.

The simplest model (labeled Model 2 in the appendix) has a simplified term structure and deliberately ignores any equity moments. Despite its simplicity, it generates a reasonably good fit to the nominal bond yields, and even to the nominal bond risk premium. It implies a lower consumption risk premium of 2.11% per year (compared to 2.38% in the benchmark) and thus a higher mean wealth-consumption ratio of 113 (versus 83). The wealth-consumption ratios have a correlation of 99% because the term structure fit of the simple model is comparable to that of the benchmark model. The marginal cost of consumption fluctuations, which averages 36.2%, is substantially higher than in the benchmark model (average of -2.4%), and the two have a time series correlation of only 63%. The reason for the higher cost of consumption fluctuations is that there is more consumption cash flow risk in this simple model. Instead of hedging it as in the benchmark model, consumption cash flow risk adds interest rate exposure which increases the consumption risk premium, *ceteris paribus*. The lower bond risk premium more than offsets the higher consumption risk so that the overall consumption risk premium ends up lower than in the benchmark. This model illustrates that the inverse relationship between real rates and the wealth

consumption ratio as well as the low consumption risk premium and high wealth-consumption ratio are generic features that arise even in a very simple model. But, because it does not price stock-based moments (by construction), the question of whether the low consumption risk premium is the result of ignoring important sources of risk when pricing the consumption claim. To dispel this possibility, we consider both bond- and stock-based moments in our benchmark model.

We consider another simple model (labeled Model 3) that prices aggregate equity moments well but that has only a one-factor term structure. This model yields in a much worse fit for bond yields, but a similar mean consumption risk premium of 2.24% and mean wealth-consumption ratio of 96. The volatility of the WC ratio is lower at 14% and the time series correlation with the WC ratio of the benchmark is 93.0%, the lowest among all alternative models we consider. The consumption risk premium only has a correlation of 67% with that in the benchmark (68% with Model 2), again the lowest among the alternatives. Clearly consumption risk premium dynamics are substantially affected by giving up on a reasonably fitting term structure model.

Combining the two simple models results in a better fit (Model 4) but otherwise similar results. When we add the CP factor to the state and the estimation (Model 5), results change meaningfully. In particular, the hedging benefits of consumption cash flows can be traced back to adding the CP factor. The addition of CP results in a large drop of the consumption risk premium from 53% in Model 4 to -2% in Model 5, on average. The addition of CP also forces the model to match the one-year bond risk premium more closely, at the expense of the long-term bond risk premia and yields. The 20-year nominal bond yield in Model 5, for example, is 27bp higher than that in Model 4 without CP, translating in a lower wealth-consumption ratio. The final model is one without CP but with the factor mimicking portfolios. That model looks similar to Model 4, except that there is more consumption cash flow risk priced and the cost of consumption fluctuations is higher.

In sum, the wealth-consumption dynamics are very similar across models and are largely driven by the similar dynamics of real yields. Insisting on matching the one-year bond risk premium (the CP factor), leads to consumption that carries a much lower price of risk and results in higher long-term mean yield estimates. These two effects result in a lower mean cost of consumption fluctuations, and have offsetting effects on the mean wealth-consumption ratio. Without the CP factor, our results would indicate a consumption risk premium that were lower still and a mean wealth-consumption ratio that was higher still, further reinforcing our conclusions.

6.2 Simpler VAR Dynamics

A second set of robustness exercises explores changes to our benchmark results when we simplify the VAR dynamics. Appendix D explores four different sets of additional zero restrictions on

the matrix Ψ. In particular, we zero out either all non-significant elements of Ψ, only the non-significant elements in the stock market block, only those in the fmp block, or only those in the consumption and labor income block. The dynamics of the resulting wealth-consumption ratios and consumption risk premia are extremely highly correlated (above 99%) across our benchmark model and these four variations, in large part because expected consumption growth dynamics and real yield dynamics are so highly correlated. The cost of consumption fluctuations also have correlations across models above 90% with the benchmark model. The main difference is in the average wealth-consumption ratio and risk premia across models rather than in the dynamics. In the last model with the restricted consumption and labor growth dynamics, we have the lowest mean wealth-consumption ratio among all models, at 62, and the highest consumption risk premium at 2.94% per year. The latter remains well below the observed equity risk premium of 6.41% so that our main conclusions are unaffected.

6.3 Annual Estimation

We repeat our analysis at annual frequency. The annual exercise is useful because annual VAR dynamics may be able to capture lower-frequency correlations between consumption growth and traded asset prices than the quarterly results.[22]

The model structure and estimation procedure are identical, except that the short rate is now the one-year constant maturity bond yield. We find that annual consumption growth has a significantly positive covariance with stock returns (t-stat is 3.15), which contributes to a better spanning of annual consumption growth risk by the traded assets than in the quarterly model. Indeed, our state variables in z_t explain 50% of variation in annual Δc_{t+1}, compared to 29% in our benchmark quarterly exercise.

The main results from the annual estimation, which are listed in the last column of Table 2 are similar to those of the quarterly model. The consumption risk premium is nearly identical at 2.34% (versus 2.38%). The mean wealth-consumption ratio is 103 compared to an annualized number of 83 in the benchmark results. The dynamics of the wealth-consumption ratio still mirror those of long-term real bond yields. Our main message that the consumption claim is much less risky than equity remains unaffected. The human wealth share is 92%, just as in the quarterly benchmark.

The main difference with the quarterly results is a much higher marginal cost of consumption fluctuations. The latter is 34% on average in the annual model compared to the quarterly model where we found a small negative cost. The high cost arises because the risk premium of the trend consumption claim is substantially lower than that of the risky claim (2.05% versus 2.34%),

[22]While it would be interesting to go back to the Great Depression, the necessary bond yield data are not available prior to 1953, so that the annual sample spans the same 1952-2011 period as our quarterly sample.

leading to a much higher mean trend wealth-consumption ratio (see Table 3). The difference with the quarterly results can be traced back to differences in annual versus quarterly consumption dynamics. In annual data, consumption innovations have strong negative correlation with the level (second orthogonal) shock and a less negative correlation with the slope (fourth orthogonal) shock. Both exposures increase the consumption risk premium above that in the quarterly model. Basically, consumption cash flow shocks no longer hedge interest rate risk but rather contribute to the interest rate risk that is already present through the discount rate (the same risk the consumption trend is also exposed to). Despite the mean differences, the dynamics of the cost of consumption fluctuations are similar between annual and quarterly models. The annual series also shows a large destruction of wealth relative to trend during the Great Recession. These dynamics results from similar interest rate behavior and from similar deviations of consumption from its trend.

7 Structural Models

We end the paper with a short comparison of our results to those implied by leading asset pricing models.[23] We focus on the long-run risk (LRR) model of Bansal and Yaron (2004) and the external habit (EH) model of Campbell and Cochrane (1999), each of which has received an enormous amount of attention in the modern asset pricing literature. Just like in the affine model we estimated, the log wealth-consumption ratio is linear in the state variables in each of these two models. We do *not* attempt to formally test the two models, only to point out their implications for the wealth-consumption ratio.[24] Interestingly, they have quite different implications for the wealth-consumption ratio. We refer the reader to the NBER working paper version of our work for a detailed derivation of the wealth-consumption ratios in these two models and a description of our simulations. We present our main findings in Columns 1 and 2 of Table 2.

The LRR model produces a *wc* ratio that matches many features of the data. It is high on average and not very volatile. For the standard calibration of the LRR model, the mean annual wealth-consumption ratio is 87, very close to our estimate in the data ($e^{A_0^{c,LRR}-\log(4)}$). The high *wc* ratio corresponds to a low consumption risk premium of 1.6% per year. The volatility of the *wc* ratio is low at 2.35% and so is the volatility of the change in the wealth-consumption ratio. Both are somewhat lower than our estimates. The persistence of the model's state variables induces substantial persistence in the *wc* ratio: its auto-correlation coefficient is 0.91 (0.70) at the 1-quarter (4-quarter) horizon. The log total wealth return has a volatility of 1.64% per quarter

[23] A comprehensive discussion appears in the NBER working paper version of this paper.

[24] The LRR and EH models are not nested by our model. Their state displays heteroscedasticity, which translates into market prices of risk Λ_t that are affine in the square root of the state. Our model has conditionally homoscedastic state dynamics and linear market prices of risk, but more shocks and therefore richer market price of risk dynamics.

in the LRR model. Low autocorrelation in Δwc and Δc generates low autocorrelation in total wealth returns. Our main conclusion is that, just as in the data, total wealth is much less risky than equity in the LRR model.

The benchmark EH model has almost the opposite implications for the wealth-consumption ratio. First, the wc ratio is volatile in the EH model: it has a standard deviation of 29.3%, which is 27 (11) percentage points higher than in the LRR model (in the data). The wealth-consumption ratio inherits a high volatility and persistence from the surplus consumption ratio. The change in the wc ratio has a volatility of 9.46%, much higher than that of consumption growth. The high volatility of Δwc ratio translates into a highly volatile total wealth return (10.26% per quarter). As in the LRR model, the total wealth return is strongly positively correlated with consumption growth. In the EH model this happens because most of the action in the total wealth return comes from changes in the wc ratio. The latter are highly positively correlated with consumption growth, in contrast with the LRR model. Most importantly, the consumption risk premium is high because total wealth is risky; the quarterly consumption risk premium is 267 bp, which translates into 10.7% per year. The high consumption risk premium implies a low annual mean wealth-consumption ratio of 12. In the EH model, the properties of total wealth returns are similar to those of equity returns. The equity risk premium is only 1.2 times higher than the consumption risk premium and the volatility of the pd^m ratio is only 1.2 times higher than the volatility of the wc ratio. For comparison, in the LRR model, these ratios are 3.5 and 6 and in the data they are 2.7 and 1.5, respectively. The EH model essentially equates the riskiness of total wealth and equity, and as a result, it overstates the representative agent's aversion to consumption risk.

In contrast to the LRR model, the EH model asserts that all variability in returns arises from variability in risk premia. Since there is no consumption growth predictability, 100% of the variability of wc is variability of the discount rate component. The same is true for stocks. A key strength of the EH model is its ability to generate a lot of variability in expected equity returns, all of which comes from the discount rate channel. The flip side is that the same mechanism generates too much variability in expected excess total wealth returns. Finally, the EH model implies that almost all the covariance with future returns comes from covariance with future excess returns, not future risk-free rates. In the total wealth data, there is evidence for substantial risk-free rate predictability.

In sum, the two leading asset pricing models have very different implications for the wealth consumption ratio, despite the fact that they both match unconditional equity return moments. In the LRR model, as in the data, the consumption claim looks more like a bond, whereas in the EH model it looks more like a stock. The properties of the wealth-consumption ratio could serve as useful, indeed primitive, asset pricing moments that structural asset pricing models should aim to match.

8 Conclusion

We develop a new methodology for estimating the wealth-consumption ratio in the data, based on no-arbitrage conditions that are familiar from the term structure literature. Our method combines restrictions on stocks and bonds in a novel way, because we are pricing a claim that a priori has bond-like and stock-like features. We find that a claim to aggregate consumption is much less risky than a claim to aggregate dividends: the consumption risk premium is only one-third of the equity risk premium. This suggests that the stand-in households' portfolio is much less risky than what one would conclude from studying the equity component of that portfolio. The consumption claim looks much more like a real bond than like a stock.

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Online Appendix for "The Wealth-Consumption Ratio"

NOT FOR PUBLICATION

A Appendix: Data

Our data are quarterly and span the period 1952.I-2011.IV. They are compiled from the most recent data available. In robustness analysis, we also consider data sampled at annual frequency for 1952-2011.

A.1 Macroeconomic Series

Labor income Labor income is computed from NIPA Table 2.1 as wage and salary disbursements (line 3) + employer contributions for employee pension and insurance funds (line 7) + government social benefits to persons (line 17) - contributions for government social insurance (line 24) + employer contributions for government social insurance (line 8) - labor taxes. As in Lettau and Ludvigson (2001a), labor taxes are defined by imputing a share of personal current taxes (line 25) to labor income, with the share calculated as the ratio of wage and salary disbursements to the sum of wage and salary disbursements, proprietors' income (line 9), and rental income of persons with capital consumption adjustment (line 12), personal interest income (line 14) and personal dividend income (line 15). The series is seasonally-adjusted at annual rates (SAAR), and we divide it by 4. Because net worth of non-corporate business and owners' equity in farm business is part of financial wealth, it cannot also be part of human wealth. Consequently, labor income excludes proprietors' income.

Consumption Non-housing consumption consists of non-housing, non-durable consumption and non-housing durable consumption. Consumption data are taken from Table 2.3.5. from the Bureau of Economic Analysis' National Income and Product Accounts (BEA, NIPA). Non-housing, non-durable consumption is measured as the sum of non-durable goods (line 6) + services (line 13) - housing services (line14).

Non-housing durable consumption is unobserved and must be constructed. From the BEA, we observe durable *expenditures*. The value of the durables (Flow of Funds, see below) at the end of two consecutive quarters and the durable expenditures allows us to measure the implicit depreciation rate that entered in the Flow of Fund's calculation. We average that depreciation rate over the sample; it is δ=5.19% per quarter. We apply that depreciation rate to the value of the durable stock at the beginning of the current period (= measured as the end of the previous quarter) to get a time-series of this period's durable *consumption*.

We use housing services consumption (BEA, NIPA, Table 2.3.5, line 14) as the dividend stream from housing wealth. The BEA measures rent for renters and imputes a rent for owners. These series are SAAR, so we divide them by 4 to get quarterly values.

Total consumption is the sum of non-housing non-durable, non-housing durable, and housing consumption.

Population and deflation Throughout, we use the disposable personal income deflator from the BEA (Table 2.1, implied by lines 36 and 37) as well as the BEA's population series (line 38).

A.2 Financial Series

Stock market return We use value-weighted quarterly returns (NYSE, AMEX, and NASDAQ) from CRSP as our measure of the stock market return. In constructing the dividend-price ratio, we use the repurchase-yield

adjustment advocated by Boudoukh, Michaely, Richardson, and Roberts (2007). We add the dividends over the current and past three quarters in order to avoid seasonality in dividend data.

Additional cross-sectional stock returns In the formation of the factor mimicking portfolios, we use the 25 size and value equity portfolio returns from Kenneth French. We form log real quarterly returns.

Bond yields We use the nominal yield on a 3-month Treasury bill from Fama (CRSP file) as our measure of the risk-free rate. We also use the yield spread between a 5-year Treasury note and a 3-month Treasury bill as a return predictor. The 5-year yield is obtained from the Fama-Bliss data (CRSP file). The same Fama-Bliss yields of maturities 1-, 2-, 3-, 4-, and 5-years are used to form annual forward rates and to form 1-year excess returns in the Cochrane-Piazzesi excess bond return regression.

In addition to the -month and 5-year bond yields which enter through the state variables, we use nominal bond yields at 1-, 3-, 10-, and 20-year maturities as additional moments to match. For the 1- and 3-year maturities, we use Fama-Bliss data. For the 10- and 20-year maturities, we use yield data from the Federal Reserve Bank of Saint Louis (FRED II). For the latter, we construct the spread with the 5-year yield from FRED. The 10- and 20-year yields we use in estimation are the sum of the 5-year Fama-Bliss yield and the 10-5 and 20-5 yield spread from FRED. This is to adjust for any level differences in the 5-year yield between the two data sources. The 20-year yield data are missing from 1987.I until 1993.III. The estimation can handle these missing observations because it minimizes the sum of squared differences between model-implied and observed yields, where the sum is only taken over available dates.

In order to plot the average yield curve in Figures A.8 and A.11, and only for this purpose, we also use the 7-5 year and the 30-5 year spread from FRED II. We add them to the 5-year yield from Fama-Bliss to form the 7-year and 30-year yield series. Since the 7-year yield data are missing from 1953.4-1969.6, we use spline interpolation (using the 1-, 2-, 5-, 10-, and 20-year yields) to fill in the missing data. The 30-year bond yield data are missing from 1953.4-1977.1 and from 2002.3-2006.1. We use the 20-year yield in those periods as a proxy. In the period where the 20-year yield is absent, we use the 30-year yield data in that period as a proxy. The resulting average 5-year yield is 5.83% per annum (straight from Fama-Bliss), the average 7-year yield is 6.00%, 10-year yield is 6.15%, 20-year is 6.36%, and the average 30-year yield is 6.32%.

Cochrane and Piazzesi's (2005) factor Cochrane and Piazzesi (2005) show that a linear combination of forward rates is a powerful predictor of one-year excess bond returns. Following their procedure, we construct 1- through 5-year forward rates from our quarterly nominal yield data, as well as one-year excess returns on 2- through 5-year nominal bonds. We regress the average of the 2- through 5-year excess returns on a constant, the one-year yield, and the 2- through 5-year forward rates. The regression coefficients display a tent-shaped function, very similar to the one reported in Cochrane and Piazzesi (2005). The state variable CP_t is the fitted value of this regression.

Factor mimicking portfolios We regress real per capita consumption growth on a constant and the returns on the 25 size and value portfolios (Fama and French 1992). We then form the fmp return series as the product of the 25 estimated loadings and the 25 portfolio return time series. In the estimation, we impose that the fmp weights sum to one and that none of the weights are greater than one in absolute value. We follow the same procedure for the labor income growth fmp. The consumption (labor income) growth fmp has a 35.84% (36.01%) correlation with consumption (labor income) growth. These two fmp returns have a mutual correlation of 71.35%. The fmp returns are lower on average than the stock return (2.34% and 3.94% versus 6.47% per annum) and are less volatile (7.07% and 14.53% versus 17.20% volatility per annum).

B Appendix: No-Arbitrage Model

B.1 Proof of Proposition 1

Proof. To find A_0^c and A_1^c, we need to solve the Euler equation for a claim to aggregate consumption. This Euler equation can either be thought of as the Euler equation that uses the nominal log SDF $m_{t+1}^{\$}$ to price the nominal total wealth return $\pi_{t+1} + r_{t+1}^c$ or the real log SDF $m_{t+1}^{\$} + \pi_{t+1}$ to price the real return r_{t+1}^c:

$$\begin{aligned}
1 &= E_t[\exp\{m_{t+1}^{\$} + \pi_{t+1} + r_{t+1}^c\}] \\
&= E_t[\exp\{-y_t^{\$}(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1} + \pi_0 + e_\pi' z_{t+1} + \mu_c + e_c' z_{t+1} + A_0^c + A_1^{c\prime} z_{t+1} + \kappa_0^c - \kappa_1^c\,(A_0^c + A_1^{c\prime} z_t)\}] \\
&= \exp\{-y_0^{\$}(1) + \pi_0 - e_{yn}' z_t - \frac{1}{2}\Lambda_t'\Lambda_t + e_\pi'\Psi z_t + \kappa_0^c + (1-\kappa_1^c)A_0^c + \mu_c - \kappa_1^c A_1^{c\prime} z_t + (e_c' + A_1^{c\prime})\,\Psi z_t\} \times \\
&\quad E_t\left[\exp\{-\Lambda_t'\varepsilon_{t+1} + (e_c + e_\pi + A_1^c)'\Sigma^{\frac{1}{2}}\varepsilon_{t+1}\}\right]
\end{aligned}$$

First, note that because of log-normality of ε_{t+1}, the last line equals:

$$\exp\left\{\frac{1}{2}\left(\Lambda_t'\Lambda_t + (e_c + e_\pi + A_1^c)'\Sigma(e_c + e_\pi + A_1^c)' - 2(e_c + e_\pi + A_1^c)'\Sigma^{\frac{1}{2}}\Lambda_t\right)\right\}$$

Substituting in for the expectation, as well as for the affine expression for Λ_t, we get

$$\begin{aligned}
1 &= \exp\{-y_0^{\$}(1) + \pi_0 - e_{yn}' z_t + \kappa_0^c + (1-\kappa_1^c)A_0^c + \mu_c - \kappa_1^c A_1^{c\prime} z_t + (e_c + e_\pi + A_1^c)'\Psi z_t\} \times \\
&\quad \exp\{\frac{1}{2}(e_c + e_\pi + A_1^c)'\Sigma(e_c + e_\pi + A_1^c) - (e_c + e_\pi + A_1^c)'\Sigma^{\frac{1}{2}}(\Lambda_0 + \Lambda_1 z_t)\}
\end{aligned}$$

Taking logs on both sides, an collecting the constant terms and the terms in z, we obtain the following:

$$\begin{aligned}
0 &= \{-y_0^{\$}(1) + \pi_0 + \kappa_0^c + (1-\kappa_1^c)A_0^c + \mu_c + \frac{1}{2}(e_c + e_\pi + A_1^c)'\Sigma(e_c + e_\pi + A_1^c) - (e_c + e_\pi + A_1^c)'\Sigma^{\frac{1}{2}}\Lambda_0\} + \\
&\quad \{-e_{yn}' - \kappa_1^c A_1^{c\prime} + (e_c + e_\pi + A_1^c)'\Psi - (e_c + e_\pi + A_1^c)'\Sigma^{\frac{1}{2}}\Lambda_1\} z_t
\end{aligned}$$

This equality needs to hold for all z_t. This is a system of $N+1$ equations in $N+1$ unknowns:

$$0 = -y_0^{\$}(1) + \pi_0 + \kappa_0^c + (1-\kappa_1^c)A_0^c + \mu_c + \frac{1}{2}(e_c + e_\pi + A_1^c)'\Sigma(e_c + e_\pi + A_1^c) - (e_c + e_\pi + A_1^c)'\Sigma^{\frac{1}{2}}\Lambda_0, \quad \text{(A.1)}$$

$$0 = (e_c + e_\pi + A_1^c)'\Psi - \kappa_1^c A_1^{c\prime} - e_{yn}' - (e_c + e_\pi + A_1^c)'\Sigma^{\frac{1}{2}}\Lambda_1. \quad \text{(A.2)}$$

The real short yield $y_t(1)$, or *risk-free rate*, satisfies $E_t[\exp\{m_{t+1} + y_t(1)\}] = 1$. Solving out this Euler equation, we get:

$$\begin{aligned}
y_t(1) &= y_t^{\$}(1) - E_t[\pi_{t+1}] - \frac{1}{2}e_\pi'\Sigma e_\pi + e_\pi'\Sigma^{\frac{1}{2}}\Lambda_t \\
&= y_0(1) + \left[e_{yn}' - e_\pi'\Psi + e_\pi'\Sigma^{\frac{1}{2}}\Lambda_1\right] z_t \quad \text{(A.3)} \\
y_0(1) &\equiv y_0^{\$}(1) - \pi_0 - \frac{1}{2}e_\pi'\Sigma e_\pi + e_\pi'\Sigma^{\frac{1}{2}}\Lambda_0 \quad \text{(A.4)}
\end{aligned}$$

The real short yield is the nominal short yield minus expected inflation minus a Jensen adjustment minus the inflation risk premium. Using the expression (A.4) for $y_0(1)$ in equation (A.1) delivers equation (7) in the main

text. □

Proposition 2. *The log price-dividend ratio on human wealth is a linear function of the (demeaned) state vector* z_t

$$pd_t^l = A_0^l + A_1^l z_t$$

where the following recursions pin down A_0^l *and* A_1^l*:*

$$\begin{aligned} 0 &= \kappa_0^l + (1-\kappa_1^l)A_0^l + \mu_l - y_0(1) + \frac{1}{2}(e'_{\Delta l} + A_1^{l\prime})\Sigma(e_{\Delta l} + A_1^l) - (e'_{\Delta l} + A_1^{l\prime})\Sigma^{\frac{1}{2}}\left(\Lambda_0 - \Sigma^{\frac{1}{2}\prime}e_\pi\right), \\ 0 &= \left(e_{\Delta l} + e_\pi + A_1^l\right)'\Psi - \kappa_1^l A_1^{l\prime} - e'_{yn} - (e_{\Delta l} + e_\pi + A_1^l)'\Sigma^{\frac{1}{2}}\Lambda_1. \end{aligned}$$

The proof is identical to the proof of Proposition 1, and obtains by replacing μ_c by μ_l and the selector vector e_c by $e_{\Delta l}$. The linearization constants κ_0^l and κ_1^l relate to A_0^l through the analog of equation (6).

The conditional risk premium on the labor income claim is affine in the state vector and given by:

$$E_t\left[r_{t+1}^{l,e}\right] = (e_{\Delta l} + A_1^l)'\Sigma^{\frac{1}{2}}\left(\Lambda_0 - \Sigma^{\frac{1}{2}\prime}e_\pi\right) + (e_{\Delta l} + A_1^l)'\Sigma^{\frac{1}{2}}\Lambda_1 z_t.$$

We use μ_l to denote unconditional labor income growth and $e_{\Delta l}$ selects labor income growth in the VAR.

B.2 Nominal and Real Term Structure

Proposition 3. *Nominal bond yields are affine in the state vector:*

$$y_t^\$(\tau) = -\frac{A^\$(\tau)}{\tau} - \frac{B^\$(\tau)'}{\tau} z_t,$$

where the coefficients $A^\$(\tau)$ *and* $B^\$(\tau)$ *satisfy the following recursions*

$$A^\$(\tau+1) = -y_0^\$(1) + A^\$(\tau) + \frac{1}{2}\left(B^\$(\tau)\right)'\Sigma\left(B^\$(\tau)\right) - \left(B^\$(\tau)\right)'\Sigma^{\frac{1}{2}}\Lambda_0, \tag{A.5}$$

$$\left(B^\$(\tau+1)\right)' = \left(B^\$(\tau)\right)'\Psi - e'_{yn} - \left(B^\$(\tau)\right)'\Sigma^{\frac{1}{2}}\Lambda_1, \tag{A.6}$$

initialized at $A^\$(0) = 0$ *and* $B^\$(0) = \boldsymbol{0}$*.*

Proof. We conjecture that the $t+1$-price of a τ-period bond is exponentially affine in the state

$$\log(P_{t+1}^\$(\tau)) = A^\$(\tau) + \left(B^\$(\tau)\right)' z_{t+1}$$

and solve for the coefficients $A^\$(\tau+1)$ and $B^\$(\tau+1)$ in the process of verifying this conjecture using the Euler equation:

$$\begin{aligned} P_t^\$(\tau+1) &= E_t[\exp\{m_{t+1}^\$ + \log\left(P_{t+1}^\$(\tau)\right)\}] \\ &= E_t[\exp\{-y_t^\$(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1} + A^\$(\tau) + \left(B^\$(\tau)\right)' z_{t+1}\}] \\ &= \exp\{-y_0^\$(1) - e'_{yn}z_t - \frac{1}{2}\Lambda_t'\Lambda_t + A^\$(\tau) + \left(B^\$(\tau)\right)'\Psi z_t\} \times \\ &\quad E_t\left[\exp\{-\Lambda_t'\varepsilon_{t+1} + \left(B^\$(\tau)\right)'\Sigma^{\frac{1}{2}}\varepsilon_{t+1}\}\right] \end{aligned}$$

We use the log-normality of ε_{t+1} and substitute for the affine expression for Λ_t to get:

$$P_t^{\$}(\tau+1) = \exp\{-y_0^{\$}(1) - e'_{yn} z_t + A^{\$}(\tau) + \left(B^{\$}(\tau)\right)' \Psi z_t + \frac{1}{2}\left(B^{\$}(\tau)\right)' \Sigma \left(B^{\$}(\tau)\right) - \left(B^{\$}(\tau)\right)' \Sigma^{\frac{1}{2}} (\Lambda_0 + \Lambda_1 z_t)\}$$

Taking logs and collecting terms, we obtain a linear equation for $\log(p_t(\tau+1))$:

$$\log\left(P_t^{\$}(\tau+1)\right) = A^{\$}(\tau+1) + \left(B^{\$}(\tau+1)\right)' z_t,$$

where $A^{\$}(\tau+1)$ satisfies (A.5) and $B^{\$}(\tau+1)$ satisfies (A.6). The relationship between log bond prices and bond yields is given by $-\log\left(P_t^{\$}(\tau)\right)/\tau = y_t^{\$}(\tau)$ □

Real bond yields, $y_t(\tau)$, denoted without the \$ superscript, are affine as well with coefficients that follow similar recursions:

$$A(\tau+1) = -y_0(1) + A(\tau) + \frac{1}{2}(B(\tau))' \Sigma (B(\tau)) - (B(\tau))' \Sigma^{\frac{1}{2}} \left(\Lambda_0 - \Sigma^{\frac{1}{2}\prime} e_\pi\right), \tag{A.7}$$

$$(B(\tau+1))' = (e_\pi + B(\tau))' \Psi - e'_{yn} - (e_\pi + B(\tau))' \Sigma^{\frac{1}{2}} \Lambda_1. \tag{A.8}$$

For $\tau = 1$, we recover the expression for the risk-free rate in (A.3)-(A.4).

B.3 Dividend Strips

We define the return on equity conform the literature as $R_{t+1}^m = \frac{P_{t+1}^m + D_{t+1}^m}{P_t^m}$, where P_t^m is the end-of-period price on the equity market. A log-linearization delivers:

$$r_{t+1}^m = \kappa_0^m + \Delta d_{t+1}^m + \kappa_1^m pd_{t+1}^m - pd_t^m. \tag{A.9}$$

The unconditional mean stock return is $r_0^m = \kappa_0^m + (\kappa_1^m - 1)A_0^m + \mu_m$, where $A_0^m = E[pd_t^m]$ is the unconditional average log price-dividend ratio on equity and $\mu_m = E[\Delta d_t^m]$ is the unconditional mean dividend growth rate. The linearization constants κ_0^m and κ_1^m are different from the other wealth concepts because the timing of the return is different:

$$\kappa_1^m = \frac{e^{A_0^m}}{e^{A_0^m}+1} < 1 \text{ and } \kappa_0^m = \log\left(e^{A_0^m}+1\right) - \frac{e^{A_0^m}}{e^{A_0^m}+1} A_0^m. \tag{A.10}$$

Even though these constants arise from a linearization, we *define* log dividend growth so that the return equation holds exactly, given the CRSP series for $\{r_t^m, pd_t^m\}$. Our state vector z contains the (demeaned) return on the stock market, $r_{t+1}^m - r_0^m$, and the (demeaned) log price-dividend ratio $pd^m - A_0^m$. The definition of log equity returns allows us to back out dividend growth:

$$\begin{aligned} \Delta d_{t+1}^m &= \mu^m + \left[(e_{rm} - \kappa_1^m e_{pd})'\Psi + e'_{pd}\right] z_t + (e_{rm} - \kappa_1^m e_{pd})' \Sigma^{\frac{1}{2}} \varepsilon_{t+1} \\ \mu^m &= r_0^m - kappa_0^m + A_0^m(1-\kappa_1^m) \end{aligned}$$

Proposition 4. *Log price-dividend ratios on dividend strips are affine in the state vector:*

$$p_t^d(\tau) = A^m(\tau) + B^{m\prime}(\tau) z_t,$$

where the coefficients $A^m(\tau)$ and $B^m(\tau)$ follow recursions

$$\begin{aligned} A^m(\tau+1) &= A^m(\tau) + \mu_m - y_0(1) + \frac{1}{2}\left(e_{rm} - \kappa_1^m e_{pdm} + B^c(\tau)\right)' \Sigma \left(e_{rm} - \kappa_1^m e_{pdm} + B^m(\tau)\right) \\ &\quad - \left(e_{rm} - \kappa_1^m e_{pdm} + B^m(\tau)\right)' \Sigma^{\frac{1}{2}} \left(\Lambda_0 - \Sigma^{\frac{1}{2}} e_\pi\right), \qquad \text{(A.11)} \\ B^m(\tau+1)' &= \left(e_{rm} - \kappa_1^m e_{pdm} + e_\pi + B^m(\tau)\right)' \Psi + e_{pdm}' - e_{yn}' - \left(e_{rm} - \kappa_1^m e_{pdm} + e_\pi + B^m(\tau)\right)' \Sigma^{\frac{1}{2}} \Lambda_1 \qquad \text{(A.12)} \end{aligned}$$

initialized at $A^m(0) = 0$ and $B^m(0) = 0$.

Proof. We conjecture that the log $t+1$-price of a τ-period strip, scaled by the dividend in period $t+1$, is affine in the state

$$p_{t+1}^d(\tau) = \log\left(P_{t+1}^d(\tau)\right) = A^m(\tau) + B^m(\tau)' z_{t+1}$$

and solve for the coefficients $A^m(\tau+1)$ and $B^m(\tau+1)$ in the process of verifying this conjecture using the Euler equation:

$$\begin{aligned} P_t^d(\tau+1) &= E_t[\exp\{m_{t+1}^\$ + \pi_{t+1} + \Delta d_{t+1}^m + \log\left(p_{t+1}^m(\tau)\right)\}] \\ &= E_t[\exp\{-y_t^\$(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1} + \pi_0 + e_\pi' z_{t+1} + \Delta d_{t+1}^m + A^m(\tau) + B^m(\tau)' z_{t+1}\}] \\ &= \exp\{-y_0^\$(1) - e_{yn}' z_t - \frac{1}{2}\Lambda_t'\Lambda_t + \pi_0 + e_\pi'\Psi z_t + \mu_m + \left[(e_{rm} - \kappa_1^m e_{pd})'\Psi + e_{pd}'\right] z_t + A^m(\tau) + B^m(\tau)'\Psi z_t\} \times \\ &\quad E_t\left[\exp\{-\Lambda_t'\varepsilon_{t+1} + (e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^m(\tau))' \Sigma^{\frac{1}{2}} \varepsilon_{t+1}\right] \end{aligned}$$

We use the log-normality of ε_{t+1} and substitute for the affine expression for Λ_t to get:

$$\begin{aligned} P_t^d(\tau+1) &= \exp\{-y_0^\$(1) - e_{yn}' z_t + \pi_0 + \mu_m + A^m(\tau) + \left[(e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^m(\tau))'\Psi + e_{pd}'\right] z_t + \\ &\quad \frac{1}{2}\left(e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^m(\tau)\right)' \Sigma \left(e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^m(\tau)\right) - \left(e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^m(\tau)\right)' \Sigma^{\frac{1}{2}} (\Lambda_0 + \Lambda_1 z_t) \end{aligned}$$

Taking logs and collecting terms, we obtain a log-linear expression for $p_t^d(\tau+1)$:

$$p_t^d(\tau+1) = A^m(\tau+1) + B^m(\tau+1)' z_t,$$

where

$$\begin{aligned} A^m(\tau+1) &= A^m(\tau) + \mu_m - y_0^\$(1) + \pi_0 + \frac{1}{2}\left(e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^c(\tau)\right)' \Sigma \left(e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^m(\tau)\right) \\ &\quad - \left(e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^m(\tau)\right)' \Sigma^{\frac{1}{2}} \Lambda_0, \\ B^m(\tau+1)' &= \left(e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^m(\tau)\right)' \Psi + e_{pd}' - e_{yn}' - \left(e_{rm} - \kappa_1^m e_{pd} + e_\pi + B^m(\tau)\right)' \Sigma^{\frac{1}{2}} \Lambda_1 \end{aligned}$$

We recover the recursions in (A.11) and (A.12) after using equation (A.4). □

B.4 Consumption Strips

Proposition 5. *Log price-dividend ratios on consumption strips are affine in the state vector:*

$$p_t^c(\tau) = A^c(\tau) + B^{c\prime}(\tau) z_t,$$

where the coefficients $A^c(\tau)$ and $B^c(\tau)$ follow recursions:

$$\begin{aligned} A^c(\tau+1) &= A^c(\tau) + \mu_c - y_0(1) + \frac{1}{2}\left(e_c + B^c(\tau)\right)' \Sigma \left(e_c + B^c(\tau)\right) - \left(e_c + B^c(\tau)\right)' \Sigma^{\frac{1}{2}} \left(\Lambda_0 - \Sigma^{\frac{1}{2}} e_\pi\right), \\ B^c(\tau+1)' &= \left(e_c + e_\pi + B^c(\tau)\right)' \Psi - e_{yn}' - \left(e_c + e_\pi + B^c(\tau)\right)' \Sigma^{\frac{1}{2}} \Lambda_1. \end{aligned}$$

initialized at $A^c(0) = 0$ and $B^c(0) = 0$.

The proof is analogous to that of Proposition 4. The proposition implies that $B^c(\infty)' = \left(e_c + e_\pi + B^c(\infty)\right)' \Psi - e_{yn}' - \left(e_c + e_\pi + B^c(\infty)\right)' \Sigma^{\frac{1}{2}} \Lambda_1$.
If we set μ_c to zero and eliminate e_c from the above recursions, then they collapse to those that govern the coefficients for the log price of real zero coupon bonds in equations (A.7) and (A.8).
We can decompose the yield on a τ-period consumption strip from Proposition 5, $y_t^c(\tau) = -p_t^c(\tau)/\tau$, into the yield on a τ-period real coupon bond, with coupon adjusted for deterministic consumption growth, plus the yield on the consumption cash-flow risk security $y_t^{ccr}(\tau)$:

$$y_t^c(\tau) = \left(y_t(\tau) - \mu_c\right) + y_t^{ccr}(\tau).$$

The former can be thought of as the period-τ coupon yield on a real perpetuity with cash-flows that grow at a deterministic rate μ_c, while the latter captures the cash-flow risk in the consumption claim. We have that $y_t^{ccr}(\tau) = -p_t^{ccr}(\tau)/\tau$. Since the log price-dividend ratio of the consumption strips and the log real bond prices are both affine, so is the log price-dividend ratio of the consumption cash-flow risk security: $\log p_t^{ccr}(\tau) = A^{ccr}(\tau) + B^{ccr}(\tau) z_t$. It is easy to show that its coefficients follow the recursions:

$$\begin{aligned} A^{ccr}(\tau+1) &= A^{ccr}(\tau) + \frac{1}{2}\left(e_c + B^{ccr}(\tau)\right)' \Sigma \left(e_c + B^{ccr}(\tau)\right) + \left(e_c + B^{ccr}(\tau)\right)' \Sigma B(\tau) \\ &- \left(e_c + B^{ccr}(\tau)\right)' \Sigma^{\frac{1}{2}} \left(\Lambda_0 - \Sigma^{\frac{1}{2}} e_\pi\right), \\ B^{ccr}(\tau+1)' &= \left(e_c + B^{ccr}(\tau)\right)' \Psi - \left(e_c + B^{ccr}(\tau)\right)' \Sigma^{\frac{1}{2}} \Lambda_1. \end{aligned}$$

B.5 Trend Consumption

Proposition 6. *The log price-dividend ratio on a claim to trend consumption is approximately a linear function of the (demeaned) state vector z_t*

$$wc_t^{tr} \simeq A_0^{tr} + A_1^{tr\prime} z_t,$$

where the mean A_0^{tr} is a scalar and A_1^{tr} is the $N \times 1$ vector which jointly solve:

$$0 = \kappa_0^{tr} + (1 - \kappa_1^{tr}) A_0^{tr} + \mu_c - y_0(1) + \frac{1}{2}(A_1^{tr})' \Sigma (A_1^{tr}) - (A_1^{tr})' \Sigma^{\frac{1}{2}} \left(\Lambda_0 - \Sigma^{\frac{1}{2}\prime} e_\pi\right) \tag{A.13}$$

$$0 = (e_\pi + A_1^c)' \Psi - \kappa_1^c A_1^{tr\prime} - e_{yn}' - (e_\pi + A_1^{tr})' \Sigma^{\frac{1}{2}} \Lambda_1. \tag{A.14}$$

The linearization constants κ_1^{tr} and κ_2^{tr} are defined analogously to equation (6). The derivation is analogous to that of the wealth-consumption ratio and results from setting $e_c = 0$ in Proposition 1.

B.6 Campbell-Shiller Variance Decomposition

By iterating forward on the total wealth return equation (5), we can link the log wealth-consumption ratio at time t to expected future total wealth returns and consumption growth rates:

$$wc_t = \kappa_0^c \sum_{j=1}^{H} (\kappa_1^c)^{-j} + \sum_{j=1}^{H} (\kappa_1^c)^{-j} \Delta c_{t+j} - \sum_{j=1}^{H} (\kappa_1^c)^{-j} r_{t+j}^c + (\kappa_1^c)^{-H} wc_{t+H}. \tag{A.15}$$

Because this expression holds both ex-ante and ex-post, one is allowed to add the expectation sign on the right-hand side. Imposing the transversality condition as $H \to \infty$ kills the last term, and delivers the familiar Campbell and Shiller (1988) decomposition for the price-dividend ratio of the consumption claim:

$$wc_t = \frac{\kappa_0^c}{\kappa_1^c - 1} + E_t \left[\sum_{j=1}^{\infty} (\kappa_1^c)^{-j} \Delta c_{t+j} \right] - E_t \left[\sum_{j=1}^{\infty} (\kappa_1^c)^{-j} r_{t+j} \right] = \frac{\kappa_0^c}{\kappa_1^c - 1} + \Delta c_t^H - r_t^H. \tag{A.16}$$

where the second equality follows from the definitions

$$\Delta c_t^H \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^c)^{-j} \Delta c_{t+j} \right] = e_c' \Psi (\kappa_1^c I - \Psi)^{-1} z_t, \tag{A.17}$$

$$r_t^H \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^c)^{-j} r_{t+j} \right] = \left[(e_c + A_1^c)' \Psi - \kappa_1^c A_1^{c\prime} \right] (\kappa_1^c I - \Psi)^{-1} z_t, \tag{A.18}$$

where I is the $N \times N$ identity matrix. The first equation for the cash-flow component Δc_t^H follows from the VAR dynamics, while the second equation for the discount rate component r_t^H follows from Proposition 1 and the definition of the total wealth return equation (5).
Using expressions (A.18) and (A.17) and the log-linearity of the wealth-consumption ratio, we obtain analytical expressions for the following variance and covariance terms:

$$V[wc_t] = A_1^{c\prime} \Omega A_1^c \tag{A.19}$$

$$Cov\left[wc_t, \Delta c_t^H\right] = A_1^{c\prime} \Omega (\kappa_1^c I - \Psi')^{-1} \Psi' e_c \tag{A.20}$$

$$Cov\left[wc_t, -r_t^H\right] = A_1^{c\prime} \Omega \left[A_1^c - (\kappa_1^c I - \Psi)^{-1} \Psi' e_c' \right] \tag{A.21}$$

$$V\left[\Delta c_t^H\right] = e_c' \Psi (\kappa_1^c I - \Psi)^{-1} \Omega (\kappa_1^c I - \Psi')^{-1} \Psi' e_c \tag{A.22}$$

$$V\left[r_t^H\right] = \left[(e_c' + A_1^{c\prime}) \Psi - \kappa_1^c A_1^{c\prime} \right] (\kappa_1^c I - \Psi)^{-1} \Omega (\kappa_1^c I - \Psi')^{-1} \left[\Psi' (e_c + A_1^c) - \kappa_1^c A_1^c \right] \tag{A.23}$$

$$Cov\left[r_t^H, \Delta c_t^H\right] = \left[(e_c' + A_1^{c\prime}) \Psi - \kappa_1^c A_1^{c\prime} \right] (\kappa_1^c I - \Psi)^{-1} \Omega (\kappa_1^c I - \Psi')^{-1} \Psi' e_c \tag{A.24}$$

where $\Omega = E[z_t' z_t]$ is the second moment matrix of the state z_t.

B.7 Estimation

B.7.1 Block 1: Bonds

The first four elements in the state, the Cochrane-Piazzesi factor, the nominal 3-month T-bill yield, the inflation rate, and the yield spread (5-year T-bond minus the 3-month T-bill yield), govern the term structure of interest rates. In contrast to most of the term structure literature, all factors are observable. The price of a τ-period

nominal zero-coupon bond satisfies:

$$P_t^{\$}(\tau) = E_t\left[e^{m_{t+1}^{\$} + \log P_{t+1}^{\$}(\tau-1)}\right].$$

This defines a recursion with $P_t^{\$}(0) = 1$. The corresponding bond yield is $y_t^{\$}(\tau) = -\log(P_t^{\$}(\tau))/\tau$. Bond yields in this class of models are an affine function of the state: $y_t^{\$}(\tau) = -\frac{A^{\$}(\tau)}{\tau} - \frac{B^{\$}(\tau)'}{\tau} z_t$. Appendix B.2 formally states and proves this result and provides the recursions for $A^{\$}(\tau)$ and $B^{\$}(\tau)$ in equations (A.5) and (A.6). Given the block-diagonal structure of Λ_1 and Ψ, only the risk prices in $\Lambda_{0,1}$ and $\Lambda_{1,11}$ affect the yield loadings. That is why, in a first step, we can estimate the bond block separately from the stock block. We do so by matching the time series for the short rate, the slope of the yield curve and the CP risk factor.

First, we impose that the model prices the 1-quarter and the 20-quarter nominal bond correctly. The condition $A^{\$}(1) = -y_0^{\$}(1)$ guarantees that the one-quarter nominal yield is priced correctly on average, and the condition $B^{\$}(1) = -e_{yn}$ guarantees that the nominal short rate dynamics are identical to those in the data. The short rate and the yield spread are in the state, which implies the following expression for the 20-quarter bond yields:

$$y_t^{\$}(20) = y_0^{\$}(20) + (e_{yn}' + e_{spr}')z_t.$$

Matching the 20-quarter yield implies two sets of parameter restrictions:

$$\frac{-1}{20} A^{\$}(20) = y_0^{\$}(20), \tag{A.25}$$

$$\frac{-1}{20}\left(B^{\$}(20)\right)' = (e_{yn} + e_{spr})'. \tag{A.26}$$

Equation (A.25) imposes that the model matches the unconditional expectation of the 5-year nominal yield $y_0^{\$}(20)$. This provides one restriction on Λ_0; it identifies its second element. To match the dynamics of the 5-year yield, we need to free up one row in the bond block of the risk price matrix $\Lambda_{1,11}$; we choose to identify the second row in $\Lambda_{1,11}$. We impose the restrictions (A.25) and (A.26) by minimizing the summed square distance between model-implied and actual yields.

Second, we match the time-series of the CP risk factor ($CP_0 + e_{cp}' z_t$) in order to replicate the dynamics of bond risk premia in the data. We follow the exact same procedure to construct the CP factor in the model as in the data, using the model-implied yields to construct forward rates. By matching the mean of the factor in model and data, we can identify one additional element of Λ_0; we choose the fourth element. By matching the dynamics of the CP factor, we can identify four more elements in $\Lambda_{1,11}$, one in each of the first four columns; we identify the fourth row in $\Lambda_{1,11}$. We impose the restriction that the CP factor is equal in model and data by minimizing their summed squared distance. We now have identified two elements (rows) in $\Lambda_{0,1}$ (in $\Lambda_{1,11}$). The first and third elements (rows) in $\Lambda_{0,1}$ (in $\Lambda_{1,11}$) are zero.

B.7.2 Block 2: Stocks

In the second step, we turn to the estimation of the risk price parameters in $\Lambda_{1,21}$ and $\Lambda_{1,22}$. We do so by imposing that the model prices excess stock returns correctly; we minimize the summed squared distance between VAR- and SDF-implied excess returns:

$$\begin{aligned} E_t^{VAR}[r_{t+1}^{m,e}] &= r_0^m - y_0(1) + \frac{1}{2} e_{rm}' \Sigma e_{rm} + \left((e_{rm} + e_\pi)' \Psi - e_{yn}'\right) z_t, \\ E_t^{SDF}[r_{t+1}^{m,e}] &= e_{rm}' \Sigma^{\frac{1}{2}} \left(\Lambda_0 - \Sigma^{\frac{1}{2}\prime} e_\pi\right) + (e_{rm} + e_\pi)' \Sigma^{\frac{1}{2}} \Lambda_1 z_t, \end{aligned}$$

where r_0^m is the unconditional mean stock return and e_{rm} selects the stock return in the VAR. Matching the unconditional equity risk premium in model and data identifies one additional element in Λ_0; we choose the sixth element (the second element of $\Lambda_{0,2}$). Matching the risk premium dynamics allows us to identify the second row in $\Lambda_{1,21}$ (4 elements) and the second row in $\Lambda_{1,22}$ (2 more elements). Choosing to identify the sixth element (row) of Λ_0 (Λ_1) instead of the fifth row is an innocuous choice. But it is more natural to associate the prices of risk with the traded stock return rather than with the non-traded price-dividend ratio. These six elements in $\Lambda_{1,21}$ and $\Lambda_{1,22}$ must all be non-zero because expected returns in the VAR depend on the first six state variables. The first element of $\Lambda_{0,2}$ and the first rows of $\Lambda_{1,21}$ and $\Lambda_{1,22}$ are zero.

B.7.3 Block 3: Factor Mimicking Portfolios

In addition, we impose that the risk premia on the fmp coincide between the VAR and the SDF model. As is the case for the aggregate stock return, this implies one additional restriction on Λ_0 and N additional restrictions on Λ_1:

$$\begin{aligned} E_t^{VAR}[r_{t+1}^{fmp,e}] &= r_0^{fmp} - y_0(1) + \frac{1}{2} e'_{fmp}\Sigma e_{fmp} + \left((e_{fmp}+e_\pi)'\Psi - e'_{yn}\right) z_t, \\ E_t^{SDF}[r_{t+1}^{fmp,e}] &= e'_{fmp}\Sigma^{\frac{1}{2}}\left(\Lambda_0 - \Sigma^{\frac{1}{2}\prime} e_\pi\right) + (e_{fmp}+e_\pi)'\Sigma^{\frac{1}{2}}\Lambda_1 z_t, \end{aligned}$$

where r_0^{fmp} is the unconditional average fmp return. There are two sets of such restrictions, one set for the consumption growth and one set for the labor income growth fmp. Matching average expected fmp returns and their dynamics identifies both elements of $\Lambda_{0,3}$. Matching the risk premium dynamics allows us to identify both rows of in $\Lambda_{1,31}$ (4 elements) and $\Lambda_{1,32}$ (4 more elements).

B.7.4 Over-identifying Restrictions in Detail

Additional Nominal Yields We minimize the squared distance between the observed and model-implied yields on nominal bonds of maturities 1, 3, 10, and 20 years. These additional restrictions help improve the model's ability to price distant cash-flows. This is important given that the dynamics of the wealth-consumption ratio will turn out to be largely driven by the behavior of long yields. We impose several other restrictions that force the term structure to be well-behaved at long horizons. None of these additional term structure constraints, however, are binding at the optimum.[25]

Consumption and Dividend Strips While we imposed that expected excess equity returns coincide between the VAR and the SDF model, we have not yet imposed that the return on stocks reflects cash flow risk in the equity market. To do so, we require that the price-dividend ratio in the model, which is the expected present discounted value of all future dividends, matches the price-dividend ratio in the data, period by period.[26] Given

[25] We impose that the average nominal and real yields at maturities 200, 500, 1000, and 2500 quarters are positive, that the average nominal yield is above the average real yield at these same maturities, and that the nominal and real yield curves flatten out. The last constraint is imposed by penalizing the algorithm for choosing a 500-200 quarter yield spread that is above 3% per year and a 2500-500 quarter yield spread that is above 2% per year. Together, they guarantee that the infinite sums we have to compute are well-behaved.

[26] This constraint is not automatically satisfied from the definition of the stock return: $r_{t+1}^m = \kappa_0^m + \Delta d_{t+1}^m + \kappa_1^m pd_{t+1}^m - pd_t^m$. The VAR implies a model for expected return and the expected log price-dividend ratio dynamics, which implies expected dividend growth dynamics through the definition of a return. These dynamics are different from the ones that would arise if the VAR contained dividend growth and the price-dividend ratio instead. The reason is that the state vector in the first case contains r_t and pd_t^m, while in the second case it contains Δd_t^m

a no-bubble-constraint for equities, the sum of the price-dividend ratios on dividend strips of all horizons equals the price-dividend ratio (Wachter 2005):

$$\frac{P_t^m}{D_t^m} = e^{pd_t^m} = \sum_{\tau=0}^{\infty} P_t^d(\tau), \tag{A.27}$$

where $P_t^d(\tau)$ denotes the price of a τ period dividend strip divided by the current dividend. Appendix B.3 formally states and proves that the log price-dividend ratios on dividend strips is approximately affine in the state vector: $\log\left(P_t^d(\tau)\right) = A^m(\tau) + B^{m\prime}(\tau) z_t$. It also provides the recursions for $A^m(\tau)$ and $B^m(\tau)$. See Bekaert, Engstrom, and Grenadier (2010) for a similar result. Using (A.27) and the affine structure, we impose the restriction that the price-dividend ratio in the model equals the one in the data by minimizing their summed squared distance. Imposing this constraint not only affects the price of equity risk (the sixth row of Λ_t) but also the real term structure of interest rates (the second and fourth rows of Λ_t). Real yields turn out to play a key role in the valuation of real claims such as the claim to real dividends (equity) or the claim to real consumption (total wealth). As such, the price-dividend ratio restriction turns out to be useful in sorting out the decomposition of the nominal term structure into an inflation component and the real term structure.

We also impose the no-bubble constraint in equation (12) that the wealth-consumption ratio equals the sum of the consumption strip price-dividend ratios.

Human Wealth Share We define the labor income share, lis_t, as the ratio of aggregate labor income to aggregate consumption. It is 0.826 on average in our sample. The human wealth share is the ratio of human wealth to total wealth; it is a function of the labor income share and the price-dividend ratios on human and total wealth: $hws_t = lis_t \frac{e^{pd_t^l}-1}{e^{wc_t}-1}$. We impose on the estimation that hws_t lies between 0 and 1 at each time t. At the optimum, this constraint is satisfied.[27]

B.8 Point Estimates

Below, we report the point estimates for the VAR companion matrix Ψ, the Cholesky decomposition of the covariance matrix $\Sigma^{.5}$ (multiplied by 100), and the market price of risk parameters Λ_0 and Λ_1 for our benchmark specification. We recall that the market price of risk parameter matrix Λ_1 pre-multiplies the state z_t, which has a (non-standardized) covariance matrix Ω.

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and pd_t^m. For the two models to have identical implications for expected returns and expected dividend growth, one would need to include pd_{t-1}^m as an additional state variable. We choose to include returns instead of dividend growth rates because the resulting properties for expected returns and expected dividend growth rates are more desirable. For example, the two series have a positive correlation of 20%, a number similar to what Lettau and Ludvigson (2005) estimate. See Lettau and Van Nieuwerburgh (2008), Ang and Liu (2007), and Binsbergen and Koijen (2010) for an extensive discussion of the present-value constraint.

[27] We impose that aggregate labor income grows at the same rate as aggregate consumption ($\mu_l = \mu_c$). We rescale the level of consumption to end up with the same average labor income share (after imposing $\mu_l = \mu_c$) as in the data (before rescaling).

$$\Psi = \begin{bmatrix}
\mathbf{0.5429} & \mathbf{0.7650} & -0.3922 & \mathbf{0.9475} & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0605 & \mathbf{0.9893} & \mathbf{0.0751} & \mathbf{0.4266} & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0519 & \mathbf{0.1669} & \mathbf{0.7020} & 0.1496 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{0.0734} & -0.0752 & 0.0094 & \mathbf{0.4123} & 0 & 0 & 0 & 0 & 0 & 0 \\
0.4109 & -0.2856 & -2.3795 & -3.0151 & \mathbf{0.9345} & -0.0003 & 0 & 0 & 0 & 0 \\
0.1036 & 0.3773 & -2.0304 & 0.9121 & -0.0434 & 0.0992 & 0 & 0 & 0 & 0 \\
0.0006 & 0.4722 & -1.1454 & -0.1641 & -0.0086 & \mathbf{0.0728} & 0 & 0 & 0 & 0 \\
0.2124 & 0.2100 & -0.5441 & -0.5424 & 0.0030 & \mathbf{0.1258} & 0 & 0 & 0 & 0 \\
-0.0048 & 0.1020 & -0.0414 & \mathbf{0.3583} & \mathbf{0.0029} & -0.0006 & -0.0044 & 0.0071 & \mathbf{0.3708} & 0 \\
0.0456 & -0.0441 & -0.0940 & -0.1228 & 0.0032 & -0.0063 & 0.0322 & -0.0162 & \mathbf{0.5856} & -0.1223
\end{bmatrix}$$

$$\Sigma^{\cdot 5} \times 100 = \begin{bmatrix}
1.2067 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0452 & 0.2198 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0430 & 0.0468 & 0.3375 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0368 & -0.0947 & 0.0046 & 0.0989 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.2434 & -0.4005 & -0.5932 & -0.5909 & 8.7028 & 0 & 0 & 0 & 0 & 0 \\
0.8102 & -0.2304 & -0.6966 & -0.6537 & 7.4960 & 3.6770 & 0 & 0 & 0 & 0 \\
0.5170 & -0.1007 & 0.1218 & -0.2464 & 1.9769 & 0.7068 & 2.6517 & 0 & 0 & 0 \\
1.0995 & 0.2947 & 0.5349 & -0.4281 & 2.8884 & 1.2009 & 3.8412 & 5.0047 & 0 & 0 \\
0.0401 & 0.0159 & 0.0038 & -0.0269 & 0.0753 & 0.0735 & 0.0858 & -0.0015 & 0.3568 & 0 \\
0.1141 & 0.0122 & -0.1086 & -0.1010 & 0.0550 & -0.0412 & 0.1328 & 0.2263 & 0.3423 & 0.7007
\end{bmatrix}$$

$$\Lambda_0' = \begin{bmatrix} 0 & -0.3176 & 0 & 0.1663 & 0 & 0.4447 & -0.0132 & 0.1104 & 0 & 0 \end{bmatrix}$$

$$\Lambda_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
33.5522 & -152.4822 & 80.6774 & -294.3807 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-30.2473 & 118.4352 & -16.4773 & 167.8749 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2.2141 & 1.1240 & -34.9664 & 44.0152 & -1.1080 & 2.7185 & 0 & 0 & 0 & 0 \\
-3.4354 & -6.0395 & -7.3666 & -2.7468 & -0.0322 & 1.8921 & 0 & 0 & 0 & 0 \\
1.4903 & 12.4732 & 10.2902 & 18.1533 & 0.4822 & 0.3419 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

We compute OLS standard errors for the elements of Ψ and report the coefficients with a t-stat greater than 1.98 in bold. Bootstrap standard errors on the market price of risk parameters are available upon request. They are derived as part of the method explained in Appendix B.9.

The implied point estimates for A_1^c are given by:

$$A_1^c = \begin{bmatrix} \text{-0.5191} \\ \text{-25.8220} \\ \text{1.8370} \\ \text{-26.0851} \\ \text{0.0854} \\ \text{-0.0067} \\ \text{-0.0070} \\ \text{0.0113} \\ \text{0.5866} \\ \text{0} \end{bmatrix}$$

while the implied point estimate for the constant is $A_0^c = 5.8082$.

B.9 Bootstrap Standard Errors

We obtain standard errors on the moments of the estimated wealth-consumption ratio by bootstrap. More precisely, we conduct two bootstrap exercises leading to two sets of standard errors. In each exercise, we draw with replacement from the VAR innovations ε_t. We draw row-by-row in order to preserve the cross-correlation structure between the state innovations (Step 1). Given the point estimates for Ψ and Σ as well as the mean vector μ, we recursively reconstruct the state vector (Step 2). We then re-estimate the mean vector, companion matrix, and innovation covariance matrix (Step 3). With the new state vector and the new VAR parameters in hand, we re-estimate the market price of risk parameters in Λ_0 and Λ_1 (Step 4). Just as in the main estimation, we use 2500 quarters to approximate the infinite-horizon sums in the strip price-dividend ratio calculations. We limit the estimation in Step 4 to 500 function evaluations for computational reasons. In some of the bootstrap iterations, the optimization in Step 4 does not find a feasible solution. This happens, for example when no parameter choices keep the human wealth share less than hundred percent or the consumption or labor income claim finite. We discard these bootstrap iterations. These new market price of risk parameters deliver a new wealth-consumption ratio time series (Step 5). With the bootstrap time series for consumption growth and the wealth-consumption ratio, we can form all the moments in Table 2. We repeat this procedure 1,000 times and report the standard deviation across the bootstrap iterations. We conduct two variations on the above algorithm. Each bootstrap exercise takes about 12 hours to compute on an 8-processor computer. The more conservative standard errors from the second bootstrap exercise are the ones reported in Table 2.

In the first exercise, we only consider sampling uncertainty in the last four elements of the state: the two factor mimicking portfolios, consumption growth, and labor income growth. We assume that all the other variables are observed without error. The idea is that national account aggregates are measured much less precisely than traded stocks and bonds. This procedure takes into account sampling uncertainty in consumption growth and its correlations with yields and with the aggregate stock market. Given our goal of obtaining standard errors around the moments of the wealth-consumption ratio, this seems like a natural first exercise. The second column of Table A.1 reports the standard errors from this bootstrap exercise in parentheses. For completeness, it also reports the mean across bootstrap iterations.

In a second estimation exercise, we also consider sampling uncertainty in the first six state variables (yields and stock prices). Redrawing the yields that enter in the state space (the 1-quarter yield and the 20-1 quarter yield spread) requires also redrawing the additional yields that are used in estimation (the 4-, 12-, 40-, and 80-quarter

Figure A.1: Dynamics of the Nominal Term Structure of Interest Rates

The figure plots the observed and model-implied 1-, 4-, 12-, 20-, 40-, and 80-quarter nominal bond yields. Note that the 20-year yield is unavailable between 1986.IV and 1993.II.

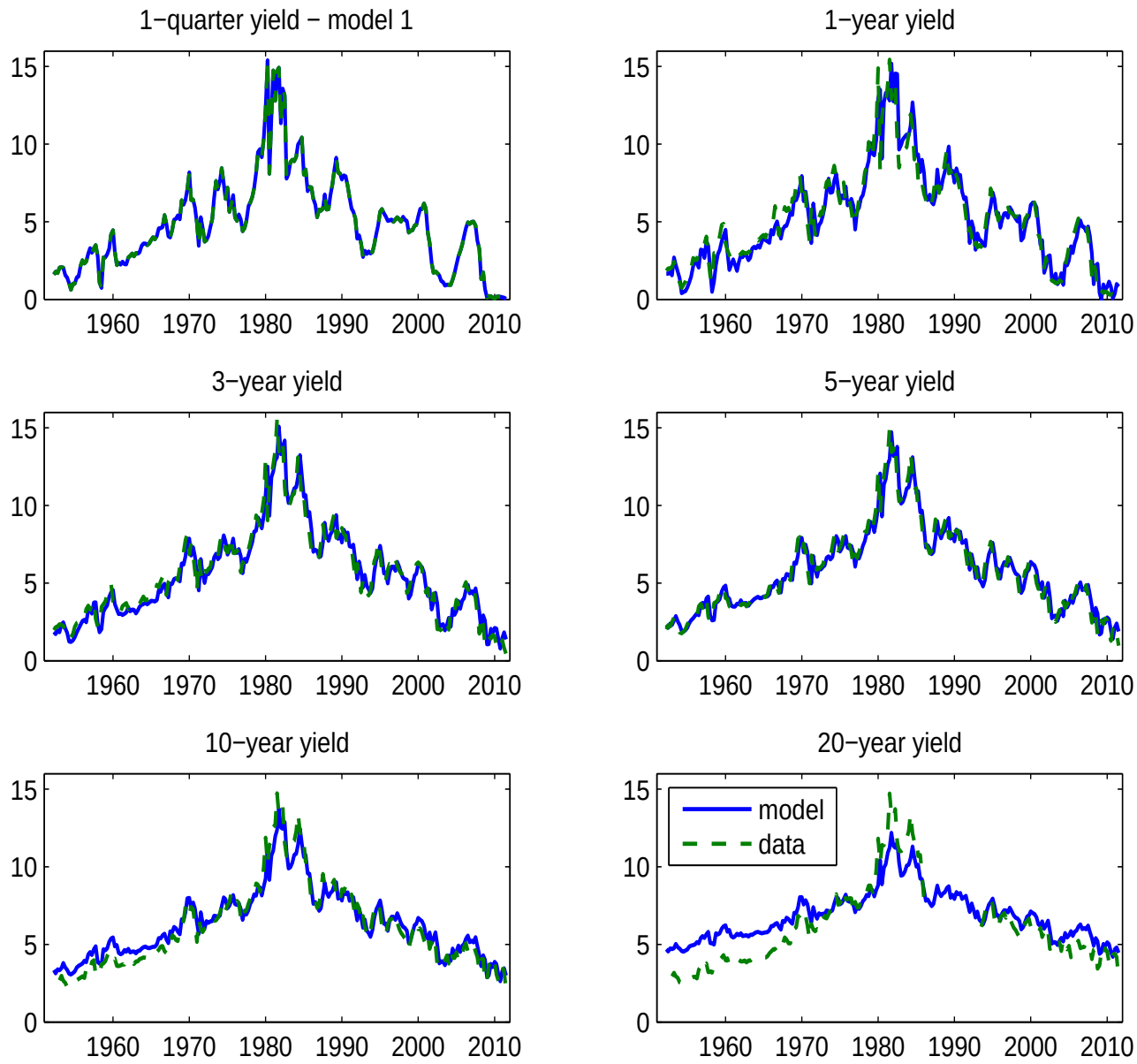


Figure A.2: Nominal Bond Risk Premia

The left panel plots the 5-year nominal bond risk premium on a 5-year nominal bond in model and data. It is defined as the difference between the nominal 5-year yield and the expected future 1-quarter yield averaged over the next 5 years. It represents the return on a strategy that buys and holds a 5-year bond until maturity and finances this purchase by rolling over a 1-quarter bond for 5 years. The right panel plots the Cochrane-Piazzesi factor in model and data. It is a linear combination of the one-year nominal yield and 2- through 5-year forward rates. This linear combination is a good predictor of the one-quarter bond risk premium.

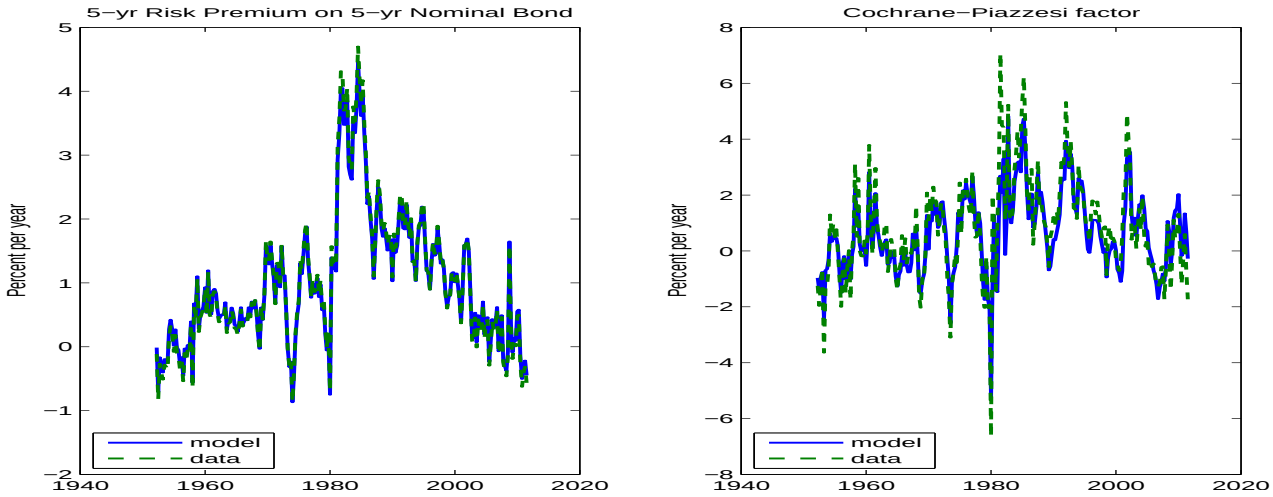


Figure A.3: The Stock Market

The figure plots the observed and model-implied price-dividend ratio and expected excess return on the overall stock market.

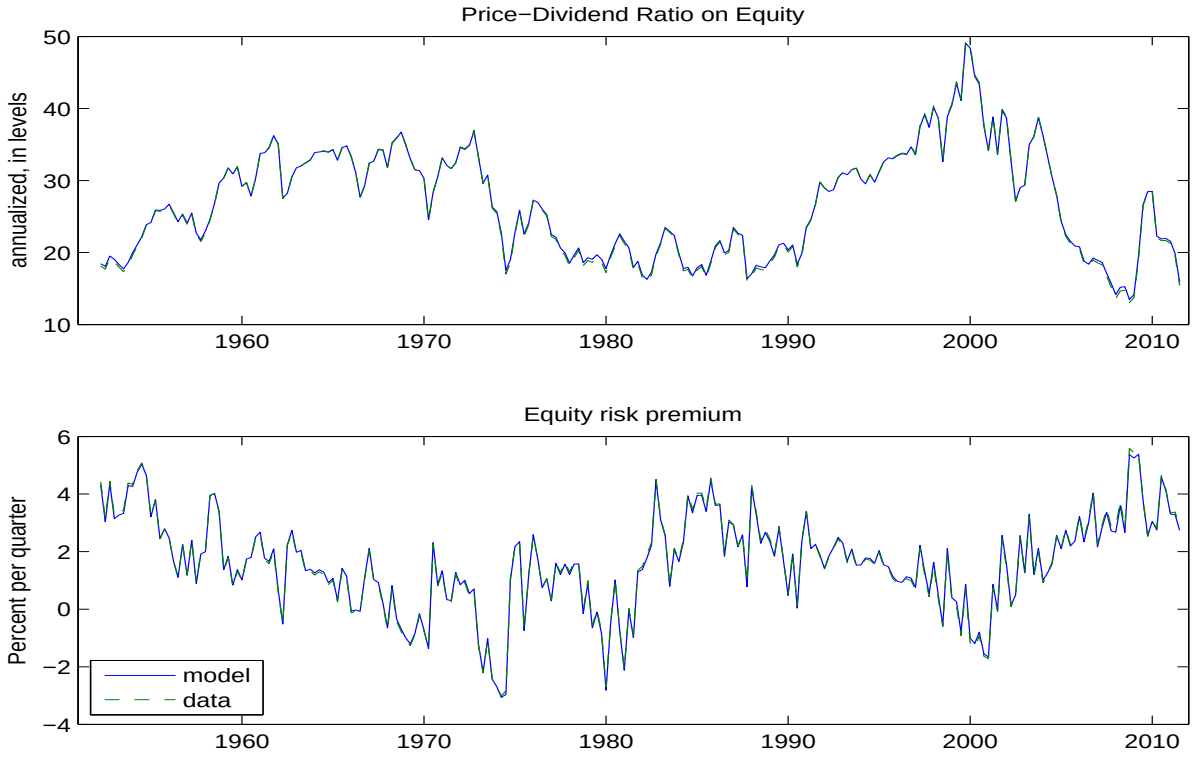


Figure A.4: Decomposing the 5-Year Nominal Yield

The left panel decomposes the 5-year yield into the real 5-year yield, expected inflation over the next 5-years, and the inflation risk premium. The right panel decomposes the average nominal bond risk premium into the average real rate risk premium and inflation risk premium for maturities ranging from 1 to 120 quarters. The nominal (real) bond risk premium at maturity τ is defined as the nominal (real) τ-quarter yield minus the average expected future nominal (real) 1-quarter yield over the next τ quarters. The τ-quarter inflation risk premium, labeled as IRP, is the difference between the τ-quarter nominal and real risk premia.

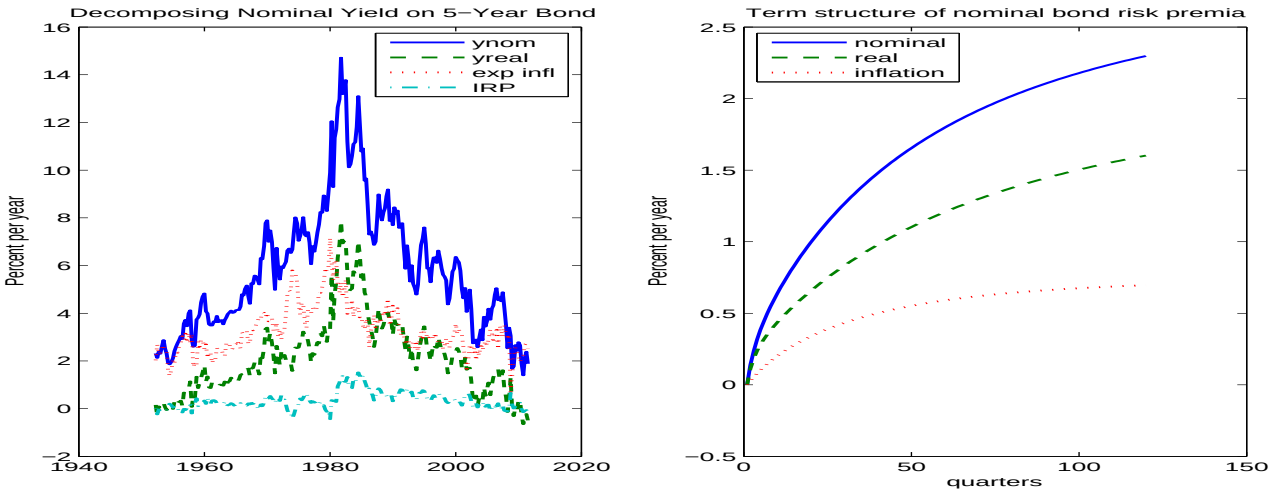


Figure A.5: Decomposing the Yield on a Consumption Strip

The figure decomposes the yield on a consumption strip of maturity τ, which goes from 1 to 1000 quarters, into a real bond yield minus deterministic consumption growth on the one hand and the yield on a security that only carries the consumption cash flow risk on the other hand. See B.4 for a detailed discussion of this decomposition.

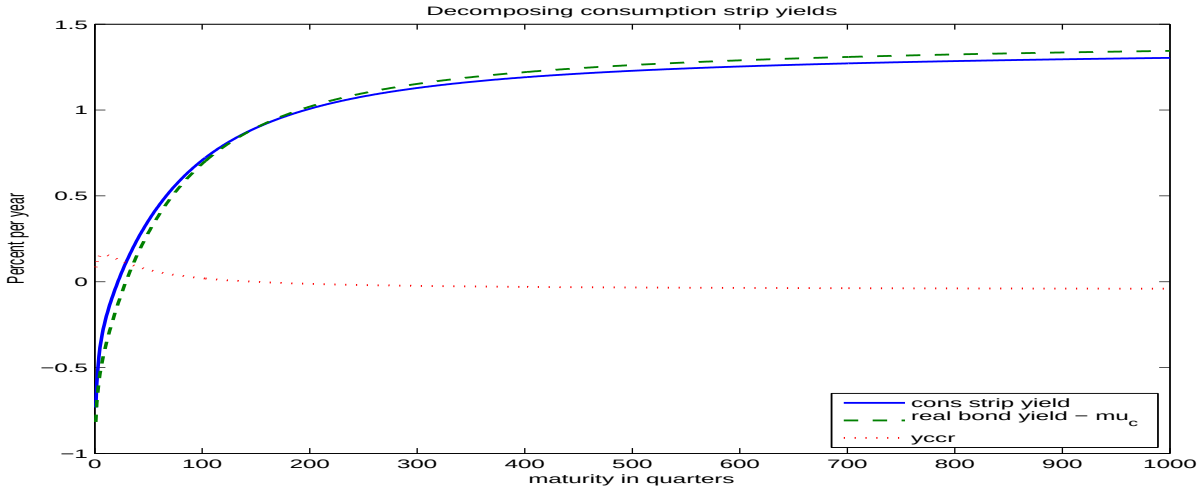
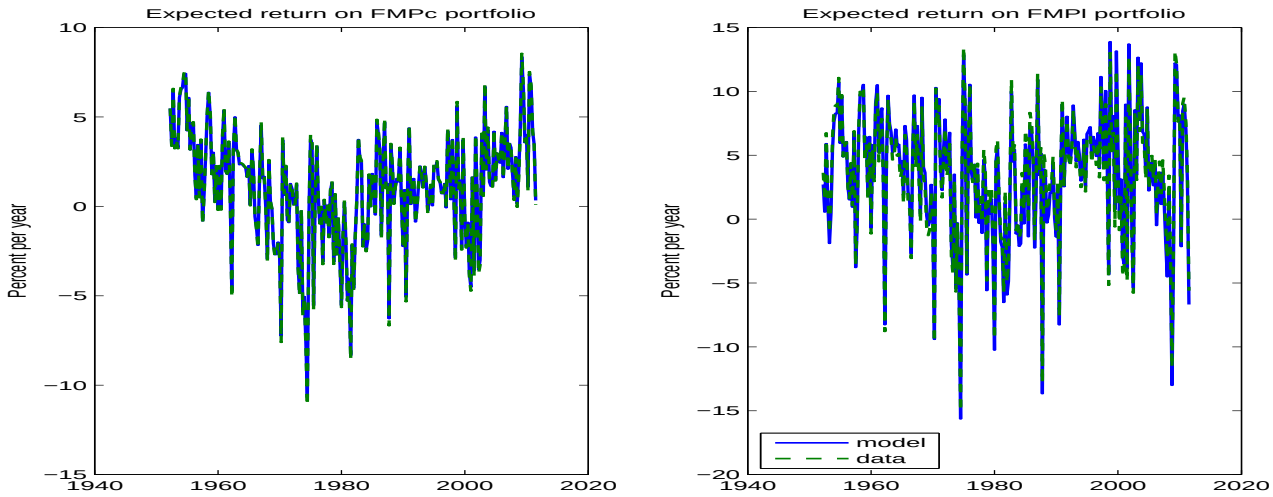


Figure A.6: Factor Mimicking Portfolios

The left panel plots the expected excess return on the consumption growth factor mimicking portfolio (FMPc). The right panel plots the expected excess return on the labor income growth factor mimicking portfolio (FMPl).



yields) and in the formation of the Cochrane-Piazzesi factor (the 4-, 8-, 12-, and 16- quarter yields). Otherwise, the bootstrapped time-series for the yields in the state space would be disconnected from the other yields. For this second exercise, we augment the VAR with the following yield spreads: 4-20, 8-20, 12-20, 16-20, 40-20, and 80-20 quarter yield spreads. We let these spreads depend on their own lag and on the lagged 1-quarter yield. Additional dependence on the lagged 20-1 quarter yield makes little difference. In Step 1, we draw from the yield spreads-augmented VAR innovations. This allows us to take into account the cross-dependencies between all the yields in the yield curve. In addition to recursively rebuilding the state variables in Step 2, we also rebuild the six yield spreads. With the bootstrapped yields, we reconstruct the forward rates, one-year excess bond returns, re-estimate the excess bond return regression, and re-construct the Cochrane-Piazzesi factor. Steps 3 through 5 are the same as in the first exercise. One additional complication arises because the bootstrapped yields often turn negative for one or more periods. Since negative nominal yields never happen in the data and make no economic sense, we discard these bootstrap iterations. We redraw from the VAR innovations until we have 1,000 bootstrap samples with strictly positive yields at all maturities. This is akin to a rejection-sampling procedure. One drawback is that there is an upward bias in the yield curve. The average one-quarter yield is 0.8% per annum higher in the bootstrap sample than in the data. This translates in a small downward bias in the average wealth consumption ratio: the average log wealth-consumption ratio is 0.17 lower in the bootstrap than in the data. The third column of Table A.1 reports the standard errors from this bootstrap exercise in parentheses. As expected, the standard errors from the second bootstrap exercise are somewhat bigger. However, their difference is small.

Table A.1: Bootstrap Standard Errors

This table displays bootstrap standard errors on the unconditional moments of the log wealth-consumption ratio wc, its first difference Δwc, and the log total wealth return r^c. The last but one row reports the time-series average of the conditional consumption risk premium, $E[E_t[r_t^{c,e}]]$, where $r^{c,e}$ denotes the expected log return on total wealth in excess of the risk-free rate and corrected for a Jensen term. The second and third columns report the results from two bootstrap exercises, described above. The table reports the mean and standard deviation (in parentheses) across 1,000 bootstrap iterations.

Moments	Bootstrap 1	Bootstrap 2
$Std[wc]$	16.26%	15.24%
(s.e.)	(3.39)	(4.30)
$AC(1)[wc]$	.95	.93
(s.e.)	(.00)	(.03)
$AC(4)[wc]$	.83	.74
(s.e.)	(.01)	(.08)
$Std[\Delta wc]$	4.86%	5.07%
(s.e.)	(0.98)	(1.16)
$Std[\Delta c]$	.44%	.44%
(s.e.)	(.03)	(.03)
$Corr[\Delta c, \Delta wc]$	.02	.12
(s.e.)	(.06)	(.06)
$Std[r^c]$	4.90%	5.16%
(s.e.)	(2.21)	(1.16)
$Corr[r^c, \Delta c]$	.12	.21
(s.e.)	(.07)	(.07)
$E[E_t[r_t^{c,e}]]$	.46%	0.53%
(s.e.)	(.11)	(.16)
$E[wc]$	6.29	5.69
(s.e.)	(.48)	(.49)
2006 Wealth (in millions)	2.65	2.52
(s.e.)	(0.32)	(0.27)
hws	0.86	0.86
boot std	(0.03)	(0.03)

C Appendix: Consumption and Labor Income Claims in Incomplete Markets Models

An important question is whether our methodology for pricing aggregate consumption and labor income claims remains valid if the data are generated from a world with heterogeneous agents who face idiosyncratic labor income risk which they cannot perfectly insure away and who may face binding borrowing or asset market participation constraints. We argue that our methodology remains valid in such environments, as long as households have access to at least a savings account. We show here how to compute total and human wealth in such models. In order to make this point as clearly as possible, we first consider an economy without aggregate risk, in which a risk-free bond is the sole financial asset. We then generalize our result by adding aggregate uncertainty and more assets.

C.1 Model Without Aggregate Risk

The first economy we consider is a standard Bewley model. Agents are ex-ante identical, but ex-post heterogeneous because they are hit by idiosyncratic labor income shocks. Incomplete markets prevents sharing this risk. We consider an economy with a unit measure of households. Each household lives forever and maximizes its expected utility:

$$E\{\sum_{t=0}^{\infty}\beta^t u(c_t)\},$$

where c and β denote the household's consumption and subjective time discount factor. Households receive stochastic labor income $\eta_t \in \Gamma$. We assume that the endowment space is finite and Markovian. We denote by $\pi(\eta'|\eta)$ the transition matrix from state η to state η'. $\Pi(\eta)$ denotes the stationary distribution of η. Since there are many households, the law of large numbers applies and $\Pi(\eta)$ corresponds to the fraction of households with endowment y.

We abstract from aggregate uncertainty: the aggregate endowment $\bar{\eta}$ is constant over time. If markets were complete, households would be able to fully insure away their labor income risk, and their consumption would be constant. Here, we assume that markets are incomplete. Households have only access to a savings account, with interest rate r. They can also borrow, up to a limited amount b. The budget and borrowing constraints are:

$$a' = \eta + (1+r)a - c, \tag{A.28}$$

$$a' \geq -b, \tag{A.29}$$

where a and a' correspond to the household's wealth today and next period.

We focus on a stationary equilibrium where aggregate quantity and prices are constant over time. The agent is characterized by a state vector (a, η). To solve for the steady-state of this economy, we first look at each household's Bellman equation given the interest rate r. We then solve for the equilibrium interest rate.

The Bellman equation is:

$$v(a,\eta) = Max\{u(c) + \beta\sum_{\eta'}\pi(\eta'|\eta)v(a',\eta')\},$$

subject to the budget and borrowing constraints in equations A.28 and A.29. The borrowing constraint may bind. Let μ denote the Lagrange multiplier on the borrowing constraint. The Bellman equation becomes:

$$v(a,\eta) = Max\{u(\eta + (1+r)a - a') + \beta\sum_{\eta'}\pi(\eta'|\eta)v(a',\eta') + \mu(a'+b)\},$$

The first-order and envelope conditions are thus:

$$\begin{aligned} u'(c) &= \beta \sum_{\eta'} \pi(\eta'|\eta) \frac{\partial v(a', \eta')}{\partial a'} + \mu, \\ \frac{\partial v(a)}{\partial a} &= u'(c)(1+r), \\ \mu(a'+b) &= 0 \end{aligned}$$

The Euler equation is:

$$u'(c) \geq E\{\beta(1+r)u'(c')\},$$

with equality if $a' > -b$. This means that, for a given η, if the household holds enough "cash in hand", e.g if $\eta + (1+r)a$ is high enough, then the household's standard Euler equation is satisfied. On the other hand, if the "cash in hand" is too low, $a' = -b$, the Euler equation is not satisfied and the borrowing constraint depresses current consumption.

Denote by Φ the joint cross-sectional distribution of assets and endowments: $\Phi(a, \eta)$. The interest rate r is determined using the household's equilibrium conditions derived above, along with market clearing conditions on the bond and goods markets:

$$\begin{aligned} \int \int c(a, \eta) \Phi(da, d\eta) &= \int \eta \Pi(d\eta), \\ \int \int a'(a, \eta) \Phi(da, d\eta) &= 0. \end{aligned}$$

Household's consumption and savings choices determine next period's distribution of assets and endowments Φ': the policy function $a'(a, \eta)$ implies a law of motion for Φ. Let the transition function $Q((a, \eta), (A, \Gamma))$ describe the probability and mass of households in state (a, η) now that will end up in $(a', \eta') \in (A, \Gamma)$ next period. The law of motion of $\Phi(a, \eta)$ is thus:

$$\Phi'(A, \Gamma) = \int \int Q((a, \eta), (A, \Gamma) \Phi(da, d\eta),$$

We focus on a stationary equilibrium so that $\Phi'(A, \Gamma) = \Phi(A, \Gamma)$ for all (A, Γ). As a result, the equilibrium interest rate r depends on the entire cross-sectional distribution of assets and endowments: $r(\Phi(A, \Gamma))$. As we have seen, for some agents, the Euler equation is not satisfied and they are borrowing-constrained. This information is encoded in the aggregate state Φ and thus impacts the level of the risk-free rate. Borrowing constraints and heterogeneity matter.

Nevertheless, we can still easily compute total and human wealth in this economy. The budget constraint holds the key. Starting from $a_{t+1} = \eta_t + (1+r)a_t - c_t$, we iterate in order to obtain:

$$a_t = \sum_{n=1}^{\infty} (\frac{1}{1+r})^n [c_{t+n} - \eta_{t+n}].$$

To derive this result, we assume that the usual transversality condition (or no-Ponzi condition) holds: $\lim_{n \to \infty} (\frac{1}{1+r})^n a_{t+n} = 0$. Note that in our stationary example, the interest rate is constant. Below, we can generalize our results to economies where the aggregate state Φ and thus r evolve over time.

Taking expectations on both side of the inter-temporal budget constraint leads to the definition of total and

human wealth:

$$\begin{aligned}\text{Total Wealth} &= \sum_{n=1}^{\infty} E_t[(\frac{1}{1+r})^n c_{t+n}],\\ \text{Human Wealth} &= \sum_{n=1}^{\infty} E_t[(\frac{1}{1+r})^n \eta_{t+n}].\end{aligned}$$

Aggregate total wealth and aggregate human wealth are the sums of these objects across individuals.

Two important points should be noted. First, our definition of total wealth and human wealth derives from each agent's budget constraint so that wealth estimates are consistent with future consumption. Second, even if some agents are borrowing constrained in this highly incomplete economy, the risk-free rate is the right way to discount future labor income and consumption streams. Incompleteness gets reflected in the risk-free rate itself (Krueger and Lustig 2009).

C.2 Models With Aggregate Risk

We now generalize this result to an economy with aggregate uncertainty and many assets. We continue to assume that all agents have access to a savings account. Some agents can also participate in financial markets and have access to more financial assets. As in our main estimation, we assume that financial markets span the aggregate sources of risk: there is a full set of contingent claims whose payoffs span all aggregate states of the world. Agents continue to face idiosyncratic labor income risk and incomplete markets, and potentially face both borrowing and participation constraints. We show below that we can define total and human wealth, using the same methodology as in the Bewley economy above. As before, we can easily value aggregate total and human wealth, even if agents are heterogenous and face different constraints. The discount rate is no longer the risk-free interest rate but rather the economy's stochastic discount factor. This stochastic discount factor is the same one that prices tradeable securities, such as stocks and bonds. Hence, market incompleteness and binding borrowing or participation constraints do not invalidate our approach, they merely change that stochastic discount factor.

Environment Let $z_t \in Z$ be the aggregate state vector. We use z^t to denote the history of aggregate state realizations. Section 2.1 describes the dynamics of the aggregate state z_t of this economy, including the dynamics of aggregate consumption $C_t(z^t)$ and aggregate labor income $L_t(z^t)$.

We consider an economy that is populated by a continuum of heterogeneous agents, whose labor income is subject to idiosyncratic shocks. The idiosyncratic shocks are denoted by $\ell_t \in \mathcal{L}$, and we use ℓ^t to denote the history of these shocks. The household labor income process is given by:

$$\eta_t(\ell^t, z^t) = \widehat{\eta}_t(\ell^t, z_t) L_t(z^t).$$

Let $\Phi_t(z^t)$ denote the distribution of household histories ℓ^t conditional on being in aggregate node z^t. The labor income shares $\hat{\eta}$ aggregate to one:

$$\int \widehat{\eta}_t(\ell^t, z_t) d\Phi_t(z^t) = 1.$$

Trading in securities markets A non-zero measure of these households can trade bonds and stocks in securities markets that open every period. These households are in partition 1. We assume that the returns of these securities span Z. In other words, the payoff space is $R^{Z\times t}$ in each period t. Households in partition 2 can only trade one-period riskless discount bonds (a cash account). We use A^j to denote the menu of traded assets

for households in segment $j \in \{1, 2\}$. However, none of these households can insure directly against idiosyncratic shocks ℓ_t to their labor income by selling a claim to their labor income or by trading contingent claims on these idiosyncratic shocks.

Law of One Price We assume free portfolio formation, at least for some households, and the law of one price. There exists a unique pricing kernel Π_t in the payoff space. Since there is a non-zero measure of households that trade assets that span z_t, it only depends on the aggregate shocks z_t. Formally, this pricing kernel is the projection of any candidate pricing kernel on the space of traded payoffs $\underline{X}_t = \mathcal{R}^{Z \times t}$:

$$\frac{\Pi_t}{\Pi_{t-1}} = proj(M_t | \mathcal{R}^{Z \times t}).$$

We let P_t be the arbitrage-free price of an asset with payoffs $\{D_t^i\}$:

$$P_t^i = E_t \sum_{\tau=t}^{\infty} \frac{\Pi_\tau}{\Pi_t} D_\tau^i. \tag{A.30}$$

for any non-negative stochastic dividend process D_t^i that is measurable w.r.t z^t.

Household Problem We adopted the approach of Cuoco and He (2001): We let agents trade a full set of Arrow securities (contingent on both aggregate and idiosyncratic shock histories), but impose *measurability* restrictions on the positions in these securities.
After collecting their labor income and their payoffs from the Arrow securities, households buy consumption in spot markets and take Arrow positions $a_{t+1}(\ell^{t+1}, z^{t+1})$ in the securities markets subject to a standard budget constraint:

$$c_t + E_t \left[\frac{\Pi_{t+1}}{\Pi_t} a_t(\ell^{t+1}, z^{t+1}) \right] + \sum_{j \in A^j} P_t^j s_{t+1}^j \leq \theta_t,$$

where s denotes the shares in a security j that is in the trading set of that agent. In the second term on the left-hand side, the expectations operator arises because we sum across all states of nature tomorrow and weight the price of each of the corresponding Arrow securities by the probability of that state arising. Wealth evolves according to:

$$\theta_{t+1} = a_t(\ell^{t+1}, z^{t+1}) + \eta_{t+1} + \sum_{j \in A^j} \left[P_{t+1}^j + D_{t+1}^j \right] s_t^j,$$

subject to a measurability constraint:

$$a_t(\ell^{t+1}, z^{t+1}) \textit{ is measurable w.r.t. } A_t^j(\ell^{t+1}, z^{t+1}),\ j \in \{1, 2\},$$

and subject to a generic borrowing or solvency constraint:

$$a_t(\ell^{t+1}, z^{t+1}) \geq B_t(\ell^t, z^t).$$

These measurability constraints limit the dependence of total household financial wealth on (z^{t+1}, ℓ^{t+1}). For example, for those households in partition 2 that only trade a risk-free bond, $A_t^2(\ell^{t+1}, z^{t+1}) = (\ell^t, z^t)$, because their net wealth can only depend on the history of aggregate and idiosyncratic states up until t. The households in partition 1, who do trade in stock and bond markets, can have net wealth that additionally depends on the aggregate state at time $t+1$: $A_t^1(\ell^{t+1}, z^{t+1}) = (\ell^t, z^{t+1})$.

Pricing of Household Human wealth In the absence of arbitrage opportunities, we can eliminate trade in actual securities, and the budget constraint reduces to:

$$c_t + E_t\left[\frac{\Pi_{t+1}}{\Pi_t} a_t(\ell^{t+1}, z^{t+1})\right] \leq a_{t-1}(\ell^t, z^t) + \eta_t.$$

By forward substitution of $a_t(\ell^{t+1}, z^{t+1})$ in the budget constraint, and by imposing the transversality condition on household net wealth:

$$\lim_{t\to\infty} \Pi_t a_t(\ell^t, z^t) = 0,$$

it becomes apparent that the expression for financial wealth is :

$$\begin{aligned} a_{t-1}(\ell^t, z^t) &= E_t\left[\sum_{\tau=t}^{\infty} \frac{\Pi_\tau}{\Pi_t}\left(c_\tau(\ell^\tau, z^\tau) - \eta_\tau(\ell^\tau, z^\tau)\right)\right] \\ &= E_t\left[\sum_{\tau=t}^{\infty} \frac{\Pi_\tau}{\Pi_t} c_\tau(\ell^\tau, z^\tau)\right] - E_t\left[\sum_{\tau=t}^{\infty} \frac{\Pi_\tau}{\Pi_t} \eta_\tau(\ell^\tau, z^\tau)\right] \end{aligned}$$

The equation states that non-human wealth (on the left) equals the present discounted value of consumption (total wealth) minus the present discounted value of labor income (human wealth). The value of a claim to $c - y$ is *uniquely* pinned down, because the object on the left hand side is traded financial wealth.

Pricing of Aggregate Human Wealth Let Φ_0 denote the measure at time 0 over the history of idiosyncratic shocks. The (shadow) price of a claim to aggregate labor income at time 0 is given by the aggregation of the valuation of the household labor income streams:

$$\begin{aligned} &\int E_0\left[\sum_{t=0}^{\infty} \frac{\Pi_t}{\Pi_0}\left(\widehat{c}_t(\ell^t, z_t) C_t(z^t) - \widehat{\eta}_t(\ell^t, z_t) L_t(z^t)\right)\right] d\Phi_0 \\ = \; & E_0\left[\sum_{t=0}^{\infty} \frac{\Pi_t}{\Pi_0} \int \left(\widehat{c}_t(\ell^t, z_t) d\Phi_t(z^t) C_t(z^t) - \widehat{\eta}_t(\ell^t, z_t) d\Phi_t(z^t) L_t(z^t)\right)\right], \\ = \; & E_0\left[\sum_{t=0}^{\infty} \frac{\Pi_t}{\Pi_0} [C_t(z^t) - L_t(z^t)]\right], \end{aligned}$$

where we have used the fact that the pricing kernel Π_t does not depend on the idiosyncratic shocks, the labor income shares integrate to one $\int \widehat{\eta}_t(\ell^t, z_t) d\Phi_t(z^t) = 1$, and the consumption shares integrate to one $\int \widehat{c}_t(\ell^t, z_t) d\Phi_t(z^t) = 1$, in which $\Phi_t(z^t)$ is the distribution of household histories ℓ^t conditional on being in aggregate node z^t.

Under the maintained assumption that the traded assets span aggregate uncertainty, this implies that aggregate human wealth is the present discounted value of aggregate labor income and that total wealth is the present discounted value of aggregate consumption, and that the discounting is done with the projection of the SDF on the space of traded payoff space. Put differently, the discount factor is the same one that prices tradeable securities, such as stocks and bonds. This result follows directly from aggregating households' budget constraints. The result obtains despite the fact that human wealth is non-tradeable in this model, and therefore, markets are incomplete. Since the above argument only relied on iterating forward on the budget constraint, we did not need to know the exact form of the equilibrium SDF. Chien, Cole, and Lustig (2011) show that the SDF in this environment depends on the evolution of the wealth distribution over time. More precisely, for each agent, one needs to keep track of a cumulative Lagrange multiplier which changes whenever the measurability constraints or the borrowing constraints bind. One cross-sectional moment of these cumulative multipliers suffices to keep track of ho the

wealth distribution affects asset prices. Because of the law of large numbers, that moments only depends on the aggregate history z^t. Similar aggregation results are derived in Constantinides and Duffie (1996) and in the limited commitment models of Lustig (2007) and Lustig and Van Nieuwerburgh (2007). To sum up, in the presence of heterogeneous agents who cannot trade idiosyncratic labor income risk, there is an additional source of aggregate risk which captures the evolution of the wealth distribution. But asset prices will fully reflect that source of aggregate risk so that our procedure remains valid in such a world.

No Spanning If the traded payoffs do not span the aggregate shocks then the preceding argument still goes through for the projection of the candidate SDF on the space of traded payoffs:

$$\frac{\Pi_t^*}{\Pi_{t-1}^*} = proj\left(M_t|\underline{X}_t\right).$$

We can still price the aggregate consumption and labor income claims using Π^*. In this case, the part of non-traded payoffs that is orthogonal to the traded payoffs, may be priced:

$$\begin{aligned} E_t\left[\left(C_{t+1} - proj\left(C_{t+1}|\underline{X}_{t+1}\right)\right)\Pi_{t+1}^*\right] &\neq 0, \\ E_t\left[\left(Y_{t+1} - proj\left(Y_{t+1}|\underline{X}_{t+1}\right)\right)\Pi_{t+1}^*\right] &\neq 0, \end{aligned}$$

where we assume that $\underline{X}$ includes a constant so that the residuals are mean zero. In the main text, we compute good-deal bounds.

D Appendix: Robustness

The results of our estimation exercise are robust to different specifications of the state vector z and restrictions on the benchmark law of motion for the state z. We compare ten models to gain insight in what part of the model structure drives which result.

Model 1 is the benchmark model from the paper. It has the ten elements in the VAR and the companion matrix Ψ, as specified in equation (1) of the paper. Models 2 through 6 simplify the benchmark model, starting from two minimal models, and gradually building up to the benchmark model. Models 7 through 10 have the same state space as the benchmark Model 1, but simplify the dynamics of the state space, i.e., Ψ. These variations are selected to give insight into what drives our main result, as well as to verify the robustness of our results.

Table A.2 summarizes the goodness of fit of all ten models; each column refers to a model. The first ten rows report the root mean-squared pricing errors (RMSE) on ten key asset pricing moments the model is trying to fit. We estimate the market price of risk by minimizing a function of these RMSEs. The last four rows express the goodness fit the models as well as a log difference with the benchmark model. The first comparison is based on the simple average RMSE (labeled SM). The second comparison is based on a weighted average RMSE (labeled WM), where the weights reflect the importance of each moment in the optimization routine. It shows that the benchmark Model 1 has the best fit of all models we consider, justifying its label.

We now turn to a detailed discussion of these models and what they teach us about the wealth-consumption ratio.

D.1 Simpler Models

Model 2 The simplest model we consider, Model 2, only contains the short rate (nominal 1-quarter bond yield), inflation, the yield spread (20-quarter minus 1-quarter yield), consumption growth, and labor income growth (in that order). It strips out from the state the CP factor, the PD ratio on the stock market, the excess return on the stock market, and the factor mimicking portfolios for consumption growth and labor income growth (computed from the cross-section of stocks). Model 2 is the most basic model that still allows us to price the claims to aggregate consumption and labor income. Inflation is necessary to go from nominal to real pricing kernels and yields. The term structure with two maturities in the state vector is quite basic. The model purposely ignores all equity moments.

Consistent with the logic explained in the estimation section of the paper, we estimate two elements in the constant market price of risk vector Λ_0 and the two corresponding rows (of three elements each) in the matrix Λ_1, which governs the time variation in prices of risk. These are the first and third elements corresponding to the short rate and the yield spread. This is the minimal structure needed to provide a reasonably good fit to the term structure of interest rates. The model also does a surprisingly good job at matching the dynamics of the 5-year nominal bond risk premium and a decent job at matching the dynamics of the CP factor. Because the CP factor, a linear combination of 1- through 5-year yields, is not in the state, the model implicitly puts less weight on matching that part of the term structure and instead pays more attention to matching the long end of the yield curve. This results in a lower estimate for the 20-year nominal yields than in Model 1 (6.74% in Model 2 versus 6.85% in Model 1), bringing it closer to the data (6.23%). The model-implied 20-year real yield is correspondingly lower (2.49% in Model 2 versus 2.87% in Model 1). Similarly, the annualized risk premium (average excess return over a 1-period bond yield) on a (hypothetical) 50-year real bond is 1.29% in Model 2 versus 1.84% in Model 1.

Model 2 clearly illustrates our main conclusion: that the real term structure of interest rates is the key determinant of the wealth-consumption ratio. The lower long-term real yields in Model 2 translate into a higher mean wealth-consumption ratio : 113 versus 83 in Model 1. The average consumption risk premium, the excess return on the claim to aggregate consumption, is 2.11% in Model 2 versus 2.38% in Model 1. While the mean levels differ, the dynamics of the WC ratio in Model 2 are nearly identical to that in Model 1: their correlation is 99.03% and the volatilities are nearly identical (18.9% versus 18.6%). The correlation between the consumption risk premia in the two model is 97.6%. The main reason for the similar dynamics in wealth-consumption ratios and consumption risk premia is that long-term real yields comove strongly: the 20-year real yields have a time series correlation of 99.86%.

Offsetting the strong real yield correlation is the fact that the cash flow risk in the consumption claim (as opposed to the real rate risk) is priced somewhat differently in Model 2 than in Model 1. Because the state vectors differ, expected consumption growth dynamics differ for Models 1 to 6. And innovations to future consumption growth are priced differently across models. For example, the yield on a claim to the risky part of aggregate consumption 50 years from now is about zero in Model 1. This implies that the entire yield on the 50-year consumption strip (trend growth plus risky fluctuations around that trend) equals the yield on a 50-year coupon bond (with cash flow adjusted for trend consumption growth). In contrast, the risky 50-year consumption strip has a yield that is 20 basis points above that on the corresponding real bond in Model 2, implying substantially more cash-flow risk. The reason for this difference is that real bond yields and yields on the risky part of the consumption strip are strongly negatively correlated in Model 1 and strongly positively correlated in Model 2. Hence, the cash flow risk of the consumption claim actually *hedges* the real rate risk in Model 1 while the exposures go in the same direction in Model 2. The extra consumption cash flow risk in Model 2 translates in a higher consumption risk premium *ceteris paribus*. These differences in consumption cash flow risk show up clearly in the form of higher costs of consumption fluctuations in Model 2. However, the much lower bond risk premium more than offsets the

higher consumption cash flow risk so that the consumption risk premium still ends up lower than in the benchmark Model 1.

In summary, Model 2 is simple and generates results that are qualitatively and quantitatively similar to our benchmark results. But, by construction, Model 2 does an awful job at pricing the stock-based moments. This immediately raises the question of whether the low consumption risk premium and high wealth-consumption ratio are the result of ignoring important sources of risk when pricing the consumption claim. To dispel this possibility, we consider both bond- and stock-based moments in our benchmark model.

Model 3 To understand the role of equity-based moments better, we propose Model 3 which focusses on fitting the equity return and the price-dividend ratio on the stock market while ignoring the bond-based moments. The state vector contains the short rate, inflation, the PD ratio on the stock market, the excess return on the stock market, consumption growth, and labor income growth (in that order). It strips out from the state the CP factor, the yield spread, and the factor mimicking portfolios for consumption growth and labor income growth. This is the minimal model that enables us to fit the aggregate stock market facts.

We estimate two elements in Λ_0 and six elements in Λ_1, associated with the short rate and the excess return on equity (2 in row 1 and 4 in the equity row, consistent with the structure of full model). Because it implies a one-factor structure for the nominal yield curve, it does substantially worse than Model 1 and 2 in fitting the term structure of yields of all maturities as well as the CP factor; See Table A.2. The model implies an annual 20-year real yield of 2.66%, a 50-year real bond risk premium of 1.50%, a consumption risk premium of 2.24%, and a mean wealth-consumption ratio of 96. These numbers are in between those of Model 1 and Model 2. Hence, a model that is substantially less rich on the term structure side but fits the equity excess return and price-dividend series very well generates qualitatively similar conclusions, at least with respect to the mean wealth-consumption ratio and the associated consumption risk premium.

The dynamics of the wealth-consumption ratio do differ somewhat from those of the benchmark model. The volatility of the WC ratio is lower at 14% and the time series correlation with the WC ratio of Model 1 is 93.0%, which is the lowest among all alternative models we consider. There are episodes in the sample where this results in the WC ratio of Model 3 going up when the WC ratio of all other models goes down. The reason for the gap is that, while the long-term real yield is still highly correlated with that in Model 1 (91.7%), the consumption risk premium is much less so (67.1%). The low correlation between the consumption risk premium in Models 2 and 3 (68.8%) shows that emphasizing the pricing of risk in the stock market at the expense of the risk pricing in the bond market makes a tangible difference. In particular, Model 3 fails completely at capturing the dynamics of the nominal bond risk premium as described by the CP factor. This has implications for the cost of consumption fluctuations.

Model 4 Model 4 combines Models 2 and 3. The state vector contains the short rate, inflation, the yield spread, the PD ratio, the excess return on the stock market, consumption growth, and labor income growth (in that order). It leaves out the CP factor and the factor mimicking portfolios for consumption growth and labor income growth. Model 4 adds a second term structure factor to Model 3. Alternatively, it adds the aggregate stock market moments to Model 2. Model 4 generates small bond pricing and stock pricing errors, but to sizeable pricing errors on CP and the factor mimicking portfolio returns. Naturally, model 4 fits better than either Models 2 or 3, but the log difference in simple (weighted) average pricing errors is still 62% (54%) with the benchmark Model 1. See Table A.2. In terms of the long-term real yield and bond risk premia, Model 4 is very close to Model 2. In terms of the consumption risk premium, it is close to Model 3. The mean wealth-consumption ratio is 96 in Model 4, similar to the 83 average in Model 1. The wealth-consumption ratios in Models 1 and 4 have a

correlation of 99.85%.

Like Model 3, Model 4 implies a positive correlation between real bond prices and the prices on the consumption strip with only cash flow risk; in Model 4 that correlation is higher still. This results in a substantial risk premium for the consumption strip of about 30bps at 50-year horizon. The mean cost of consumption fluctuations is 52.9% for Model 4 (compared to 36.2% for Model 2 and 16.2% for Model 3).

Model 5 In Model 5, we add back the CP factor to the state vector of Model 4. This is the full model, except without the two factor mimicking portfolio returns in the state. This model results in a substantially better fit for the CP factor. In order to fit the CP factor better, the model focuses more on the 1- to 5-year bond yields and less on the long end of the term structure. The result is higher model-implied nominal and real long-term bond yields. The 20-year nominal (real) yield is 7.01% (2.99%), the highest among all models. The 50-year real bond risk premium is 2.05% per year. The mean wealth-consumption ratio is 69 compared to 96 in Model 4 (and 83 in Model 1). The wealth-consumption ratios in model 1 and 5 have a correlation of 99.97%.

Going from Model 4 to Model 5, there is a dramatic change in consumption cash flow risk. In Model 5, the prices of strips that pay the risky part of aggregate consumption and the prices of real bonds are strongly negatively correlated, so that the cash flow part of the consumption strip hedges the real rate risk in these strips. This lowers the risk premium and the overall yield on the consumption claim. It is the low consumption cash flow risk that is responsible for the lower cost of consumption fluctuations in Model 5 (-1.8%), relative to Model 4 (52.9%). The higher consumption risk premium of 2.67% is therefore solely attributable to the higher real yields and bond risk premia, not to higher consumption cash flow risk.

Model 6 In Model 6, we add back the two factor mimicking portfolios in the state but leave out the CP factor. Naturally, this results in a substantially better fit for the factor mimicking portfolios at the expense of the fit for the CP factor. The bond pricing is similar to that in Models 2 and 4 with relatively low long-term rates and real bond risk premia, a low consumption risk premium, and a high mean wealth-consumption ratio (115). The factor mimicking portfolios add priced sources of equity risk which add to the riskiness of the consumption claim. Adding the factor mimicking portfolios (going from Model 4 to Model 6) substantially increases the mean cost of consumption fluctuations from 52.9% in Model 4 to 68.1% in Model 6. Despite the additional consumption cash flow risk, the overall consumption risk premium is lower in Model 6 than in Model 4 because the real bond risk premium is lower.

Summary Figure A.7 shows the wealth-consumption ratio for the benchmark model 1 and the 5 simpler (and sequentially more complex) models. They show that the wealth-consumption ratios are highly correlated across models, with all numbers lying between 91.23% and 99.98%. If we exclude Model 3, the correlation is never below 98%. The high comovement is largely driven by the high comovement in the real yield curve across models. For example, the real 20-year yield across all 6 models varies between 91.68% and 99.97%. Again, the lowest correlation comes from Model 3; all other have real yield correlations in excess of 99%. Since all models price the short-term bond perfectly, and because one factor accounts for a lot of the comovement across bonds of various maturities, that conclusion is not surprising.

The main difference between models, therefore, is in the level rather than the dynamics of the wealth-consumption ratio. The average varies from 69 in Model 5 to 115 in Model 6. As discussed above, this difference is largely accounted for by differences in the mean (real) yield curve, which is plotted in Figure A.8. The long-term real yield is highest in Models 1 and 5, the only two that contain the CP factor. Including the CP factor forces the estimation to focus more on the 1- through 5-year part of the yield curve in order to better match the one-year bond

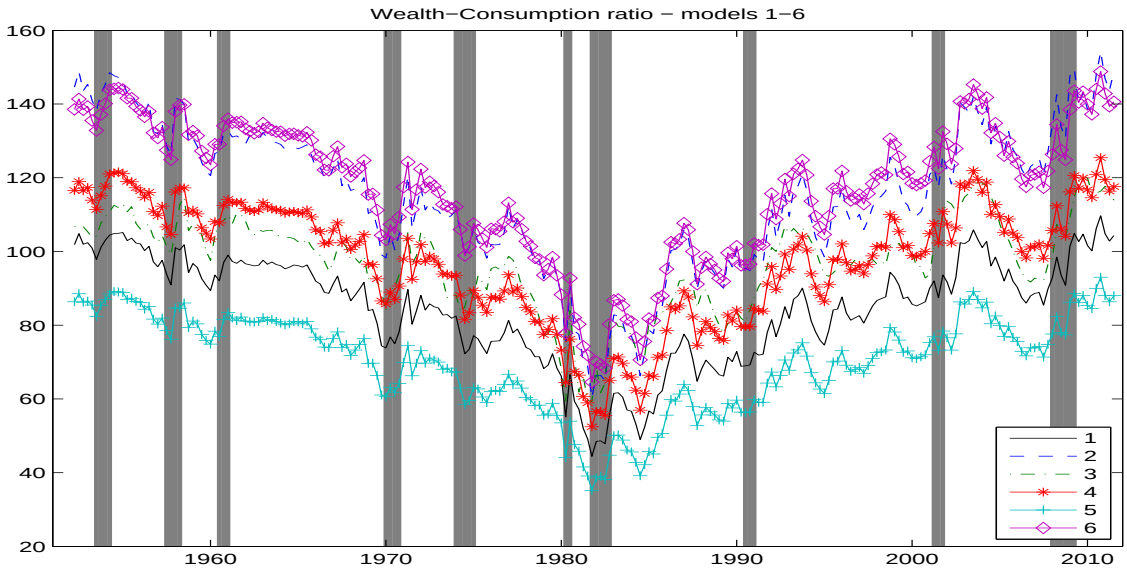


Figure A.7: Wealth-Consumption Ratio, Models 1-6

risk premium. This leads the estimation to choose a higher long-term bond yield, and a lower wealth-consumption ratio.

Finally, the mean cost of consumption fluctuations (CCF), and to a lesser degree their dynamics, differ substantially across models. Figure A.9 plots the CCF for Models 1 through 6. Consumption cash flow risk is priced quite differently in Models 1 and 5 with the CP factor in the state than in the other models, resulting in much lower CCF than in Models 2, 3, 4, or 6. This difference underscores the importance of including the CP factor, a measure of the one-year bond risk premium, when it comes to measuring the cost of consumption fluctuations.

D.2 Changes to State Dynamics

A different dimension of the "what drives what" question is the specification of the VAR which governs the dynamics of the state variables. We study four models, Models 7-10, which have the same state variables as the benchmark Model 1, but simplified VAR dynamics. Table A.2 lists the asset pricing errors of these models in its four last columns.

Model 7 In Model 7, we zero out all non-significant elements of the Ψ matrix. This leads to only two zeros in the 4×4 bond block. However, the *pd* ratio only loads on its own lag, and expected stock returns are only significantly predicted by the lagged *pd* ratio. The factor mimicking portfolio returns depend on the lagged excess stock return, and the fmp for consumption growth additionally on lagged inflation. The consumption growth rate is predicted significantly by the yield spread, the pd ratio, and its own lag. Labor income growth is predicted only by lagged consumption growth. After imposing these zero restrictions, we re-estimate the constrained companion matrix.

Despite the substantial changes to the state vector's dynamics, we find similar results. The wealth-consumption ratio in Model 7 has a correlation of 99.98% with that in Model 1. The model-implied 20-year real yields are essentially perfectly correlated because the dynamics of the bond block did not change much. The average long-term real yield and real bond risk premium are higher, though, which results in a lower mean wealth-consumption

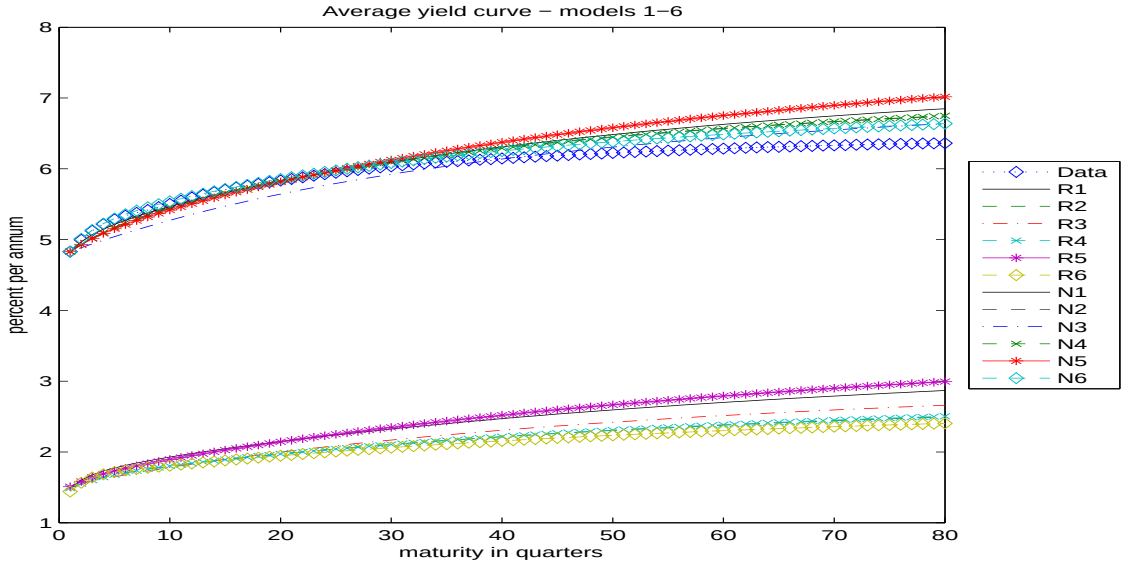


Figure A.8: Average Yield Curve, Models 1-6

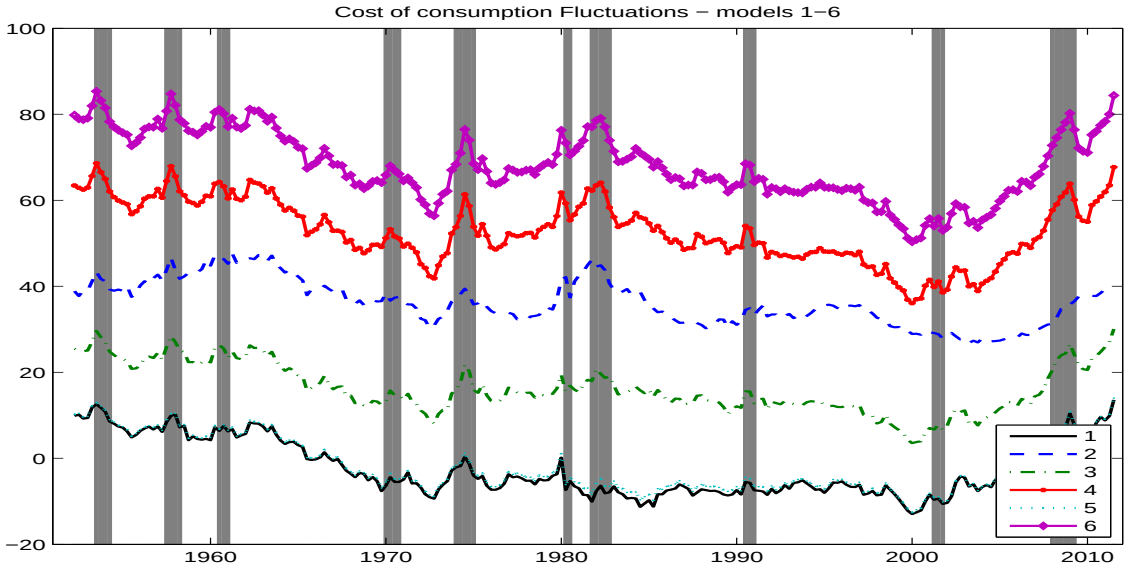


Figure A.9: Cost of Consumption Fluctuations, Models 1-6

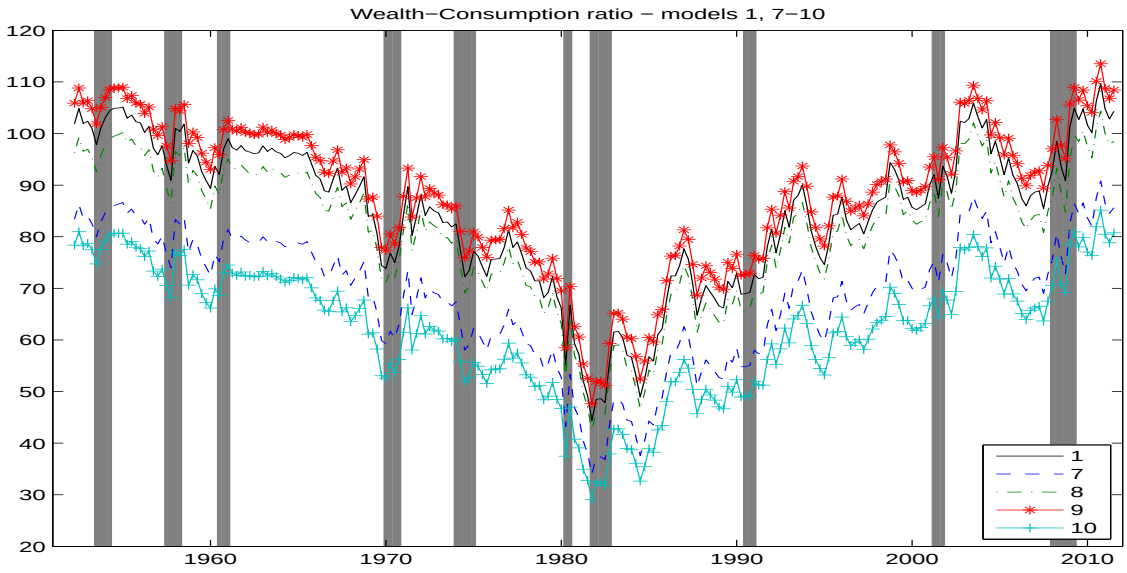


Figure A.10: Wealth-Consumption Ratio Models 1 and 7-10

ratio (67 in Model 7 versus 83 in Model 1) and a higher consumption risk premium (2.74% versus 2.38%). Some of the increase in the consumption risk premium is due to a higher reward for consumption cash flow risk. The reason is that the prices of risky consumption cash flow strips are less negatively correlated with real bond prices than in the benchmark model. The higher consumption cash flow risk leads to a cost of consumption fluctuations which is slightly higher on average (4.0% compared to -2.4% in Model 1). While expected consumption growth in Model 7 still has a high correlation of 96.3% with that in Model 1, that small difference is sufficient to lead to noticeable differences in the consumption cash flow risk premium and the cost of consumption fluctuations.

Models 8 and 9 In Model 8, we only zero out only the elements in the pd and stock return equation that are not significant. In Model 9, we zero out only the elements in the factor mimicking portfolio return equations that are not significant. In both cases, this leads to very minor changes to the benchmark results because the term structure implications of Models 1, 8, and 9 are nearly identical and the expected consumption growth dynamics are identical.

Model 10 In Model 10, we only zero out the elements in the consumption and labor income growth equations that are not significant. This model is similar to Model 7, which clarifies that the change in results between Model 1 and 7 is largely due to the changed consumption and labor income growth dynamics. In Model 10, we have the lowest mean wealth-consumption ratio among all models, at 62, and the highest consumption risk premium at 2.94% per year. The cost of consumption fluctuations is 9.9% on average.

Summary Figure A.10 shows the wealth-consumption ratio for the benchmark model 1 and the four simpler models in terms of VAR dynamics, Models 7-10. They show that the wealth-consumption ratios are highly correlated across models, with all numbers lying between 99.62% and 99.99%. The high comovement is largely driven by the high comovement in the real yield curve across models and the high correlation in expected consumption growth dynamics. For example, the real 20-year yield across the five models is nearly perfect.

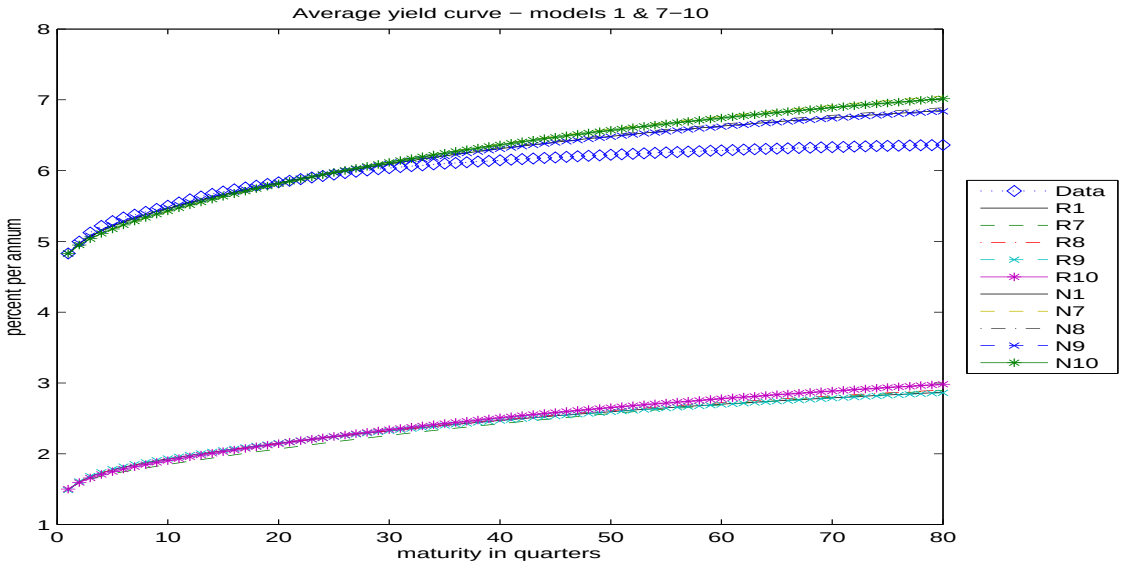


Figure A.11: Average Yield Curve, Models 1 and 7-10

The main difference between models, therefore, is in the level rather than the dynamics of the wealth-consumption ratio. The average varies from 62 in Model 10 to 87 in Model 9. As discussed above, this small difference is largely accounted for by small differences in the mean (real) yield curve, which is plotted in Figure A.11. Despite these differences, the mean wealth-consumption ratios of all 10 models (ranging from 62 to 115) are all more than double the mean price-dividend ratio on stocks, which is 26 in our sample.

The range across all 10 models for the consumption risk premium is between 52bp and 74bp per quarter or between 2.07% for Model 6 and 2.94% for Model 10 per year. It deserves emphasis that even the highest value, implies a consumption risk premium less than half as big as the mean equity risk premium of 6.4%. Also, the narrow range of estimates (87 basis points per year) is testimony to the robustness of our results.

Finally, Figure A.12 shows that the cost of consumption fluctuations is much less affected by zeroing out elements of the VAR dynamics, compared to changing the elements in the state vector itself.

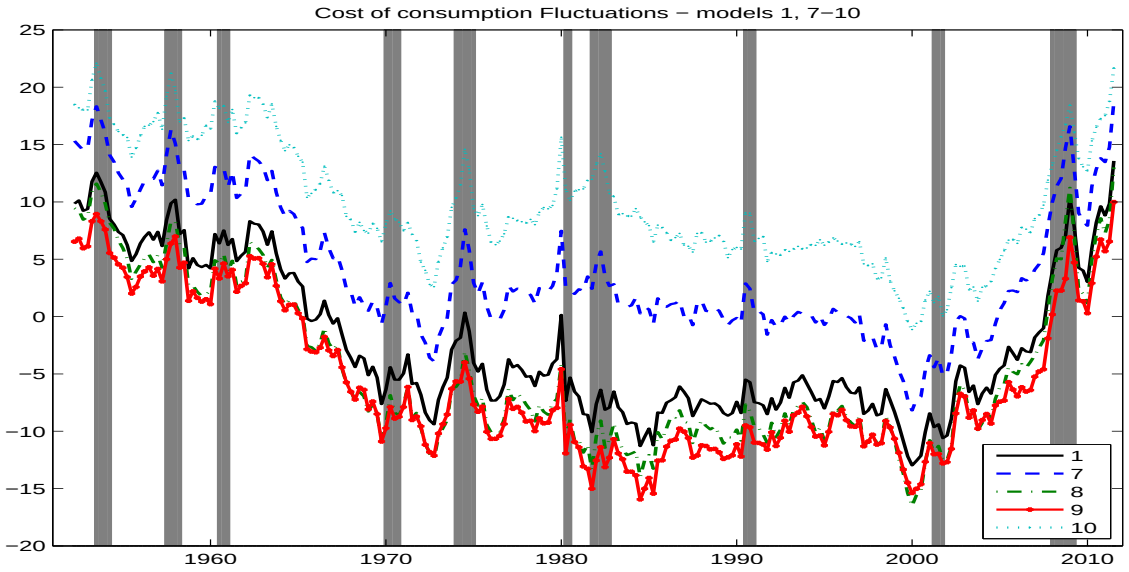


Figure A.12: Cost of Consumption Fluctuations, Models 1 and 7-10

Table A.2: Model Comparison: Root Mean Squared Errors

Model:	1	2	3	4	5	6	7	8	9	10
1q yield	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1y yield	0.465	0.500	0.495	0.495	0.500	0.575	0.464	0.473	0.467	0.462
3y yield	0.386	0.452	0.519	0.417	0.407	0.448	0.385	0.389	0.386	0.384
5y yield	0.332	0.382	0.553	0.346	0.338	0.353	0.334	0.332	0.332	0.333
10y yield	0.372	0.331	0.641	0.345	0.355	0.344	0.388	0.368	0.374	0.383
20y yield	0.653	0.589	0.847	0.628	0.658	0.620	0.700	0.651	0.654	0.687
CP	0.716	1.474	1.745	1.389	0.771	1.424	0.842	0.812	0.764	0.755
PD	0.224	39.141	0.392	0.381	0.235	0.411	0.262	0.244	0.229	0.278
Equity RP	0.151	3.163	0.390	0.222	0.154	0.194	0.146	0.137	0.157	0.177
FMPc RP	0.058	1.154	0.952	0.933	0.611	0.049	0.587	0.006	0.406	0.061
FMPl RP	0.365	1.612	1.738	1.765	1.332	0.486	0.234	0.490	0.552	1.019
SM	0.338	4.436	0.752	0.629	0.487	0.446	0.395	0.355	0.393	0.413
SM (%)	–	-257.4%	-79.9%	-62.0%	-36.5%	-27.6%	-15.4%	-4.7%	-14.9%	-19.9%
WM	0.528	5.522	1.125	0.907	0.628	0.806	0.631	0.564	0.581	0.582
WM (%)	–	-234.8%	-75.6%	-54.1%	-17.4%	-42.4%	-17.9%	-6.6%	-9.6%	-9.7%

The table reports root mean squared errors (expressed in percent) for six yields, ranging in maturity from 3-months to 20-years, the Cochrane-Piazzesi factor (a measure of the nominal bond risk premium which is in the same units as a yield), the log price-dividend ratio (same units as yield), and the equity risk premium on the market portfolio, the consumption growth factor mimicking portfolio, and the labor income growth factor mimicking portfolio. The 10 different models are in the columns, with the first column being the benchmark model. The row SM reports the simple mean across the 11 RMSEs. The row $SM(\%)$ reports the log difference in SM with Model 1. The row WM reports the weighted mean across the 11 RMSEs. The row $WM(\%)$ reports the log difference in WM with Model 1. The weights in the weighted mean reflect the weight each of these moments receives in the minimization problem we are solving to find the market prices of risk, with one exception. For the calculation of WM we cap the weight on the 20-year yield at 30% while the estimation weights it more heavily. The weights are held constant across columns.