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NETWORKS IN ECONOMICS

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ABSTRACT

Networks in Economics*

We provide an overview on networks in economics. We first look at the theoretical aspects of network economics using a game-theoretical approach. We derive some results on games on networks and network formation. We also study what happens when agents choose both links and actions. We then examine how these models can be used to address some applied and empirical-relevant questions by mainly focusing on labor-market networks and crime networks. We provide some empirical evidence on these two types of networks and address some policy implications of the models.

JEL Classification: A14, C72 and D85
Keywords: criminal networks, games on networks, labor networks, network formation and social networks

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1 Introduction

The study of social networks was initiated by sociologists more than a century ago and has grown to be a central field of sociology over the past fifty years (see e.g. Wasserman and Faust, 1994). Over that same period, a mathematical literature on the structure and properties of graphs has been developed and extensively studied (see, e.g. Bollobás, 1998). A recent awakening of interest in social networks has occurred in the computer science and statistical physics literatures, mainly over the past fifteen years (see Albert and Barabási, 2002; Newman, 2010, for an overview of these studies). While the importance of embeddedness of economic activity in social settings has been fundamental to sociologists for some time, it was largely ignored by economists until the last decade. This is surprising given that non-market interactions, i.e. interactions between agents that are not mediated by the market, are crucial to explain different economic phenomena such as stock market crashes, growth, education, religion, crime, etc. The studies of networks with economic perspectives and using game-theoretic modelling techniques have only emerged over the last decade (see Goyal, 2007; Jackson, 2008).

In the present article, we would like to survey the recent literature on networks in economics. As in all surveys and because of space constraint, we will not be able to cover all aspects of the literature and therefore we refer to other overviews such as Goyal (2007), Jackson (2008), de Martí and Zenou (2011) and Jackson and Zenou (2013a).[1] We will expose what we believe are the most important aspects of the economics of networks and will illustrate each aspect by a simple theoretical model.

This article is divided in two parts. In the first one, we will look at the theoretical aspects of network economics using a game-theoretical approach. In this part, we will look first at games on networks, which take networks as given and focus on the impact of their structure on individuals' outcomes. We will then analyze network formation and explain the way links between agents are formed. We will also study what happens when agents choose both links and actions. In the second part of this paper, we will see how the theoretical models can be used to address applied and empirical-relevant questions. We will mainly focus on labor-market networks and crime networks. We will also provide some empirical evidence on these two types of networks and address some policy implications of the models.

[1] See also Jackson and Zenou (2013b) for a collection of the most important published papers in the economics of networks.

2 Networks in economics: Theoretical aspects

2.1 Games on networks

We begin with a class of canonical and widely applicable games; specifically, games where there is a *fixed* and *given network* of interactions. Links indicate which players' strategies affect which others' payoffs. In particular, a given player's payoff depends only on the play of her neighbors. Of course, this still results in network effects since there may be chains of influence.

2.1.1 Network definitions

We provide some basic definitions on networks.

Players and Networks A set of players $N = \{1, \dots, n\}$ are connected by a network.

A *graph* or *network* is a pair $(N, \mathbf{g})$, where $\mathbf{g}$ is a network on the set of nodes N. To each network $\mathbf{g}$, we associate its adjacency matrix $\mathbf{G} = [g_{ij}]$. A graph is *undirected* if $\mathbf{G}$ is required to be symmetric so that $g_{ij} = g_{ji}$, and is *directed* otherwise. It is useful to use the notation $ij \in \mathbf{g}$ to indicate that $g_{ij} = 1$ and $ij \notin \mathbf{g}$ to indicate that $g_{ij} = 0$, and one can represent a graph by the set of links that are present (so one could alternatively represent $\mathbf{g}$ by its set of links). A weighted adjacency matrix, $\mathbf{G}$, is when g_{ij} can take other values than 0 and 1.

A relationship between two nodes i and j, represented by $ij \in \mathbf{g}$, is referred to as a link. Links are also referred to as edges or ties in various parts of the literature; and sometimes also directed links, directed edges, or arcs in the specific case of a directed network.

A *walk* in a network $(N, \mathbf{g})$ refers to a sequence of nodes, $i_1, i_2, i_3, \dots, i_{K-1}, i_K$ such that $i_k i_{k+1} \in \mathbf{g}$ for each k from 1 to K. The *length* of the walk is the number of links in it, or $K - 1$. A *path* in a network $(N, \mathbf{g})$ is a walk in $(N, \mathbf{g})$, $i_1, i_2, i_3, \dots, i_{K-1}, i_K$, such that all the nodes are distinct.

The *distance* $d(i, j)$ between two nodes i and j in the same component of a network is the length of a shortest path (also known as a *geodesic*) between them.

The *neighbors* of a node i in a network $(N, \mathbf{g})$ are denoted by $N_i(\mathbf{g})$, so that

$$N_i(\mathbf{g}) = \{j | ij \in \mathbf{g}\}$$

The *degree* of a node i in a network $(N, \mathbf{g})$ is the number of neighbors that i has in the network, so that $d_i(\mathbf{g}) = |N_i(\mathbf{g})|$. Unless otherwise stated, let us suppose that $g_{ii} = 0$, so that nodes are not linked to themselves.

The kth power $\mathbf{G}^k = \mathbf{G}^{(k \overset{times}{\cdots})}\mathbf{G}$ of the adjacency matrix $\mathbf{G}$ keeps track of indirect connections in $\mathbf{g}$. More precisely, the coefficient $g_{ij}^{[k]}$ in the (i, j) cell of $\mathbf{G}^k$ gives the number of paths of length k in g between i and j. In particular, $\mathbf{G}^0 = \mathbf{I}$. Note that, by definition, a path between i and j needs not to follow the shortest possible route between those agents. For instance, when $g_{ij} = 1$, the sequence $ij \rightarrow ji \rightarrow ij$ constitutes a path of length three in g between i and j.

2.1.2 Game on network definitions

When the network is fixed, there are two types of games: games with strategic complementarities and games with strategic substitutes. Let us define them formally. Players have effort spaces X_i, and payoff functions $u_i : X_i \rightarrow \mathbb{R}$. The action spaces are finite sets or subsets of a Euclidean space. A given player's payoff depends on other players' behaviors, but in particular only on those to whom the player is connected in the network. More formally, i's payoff depends only on x_i and $\{x_j\}_{j \in N_i(\mathbf{g})}$ and can be written as: $u_i(x_i, x_j)$, where $x_i \in X_i$ is the effort of player i. We have the following definitions:

A game exhibits *strategic complements* if it exhibits *increasing differences*; that is, for all i, j, with $i \neq j$, $x_i \geq x_i'$ and $x_j \geq x_j'$:

$$u_i(x_i, x_j) - u_i(x_i', x_j) \geq u_i(x_i, x_j') - u_i(x_i', x_j').$$

A game exhibits *strategic substitutes* if it exhibits *decreasing differences*; that is, for all i, j, with $i \neq j$, $x_i \geq x_i'$ and $x_j \geq x_j'$:

$$u_i(x_i, x_j) - u_i(x_i', x_j) \leq u_i(x_i, x_j') - u_i(x_i', x_j').$$

These notions are said to apply strictly if the inequalities above are strict whenever $x_i > x_i'$ and $x_j > x_j'$.

Because of the lack of space, we will only expose games with strategic complementarities since they have been more studied, have nice properties and have nicer and more natural applications in economics.[2]

[2]Prominent papers studying games with strategic substitutes are Bramoullé and Kranton (2007) and

2.1.3 A simple model with strategic complementarities[3]

The game Consider a game in which n agents, linked to each other in a network $\mathbf{g}$, must decide how much effort to exert in some activity. We denote by x_i the effort level of agent i and by $\mathbf{x} = (x_1, ..., x_n)^\top$ the vector of efforts of all agents ($^\top$ means transposition). Each agent i selects an effort $x_i \geq 0$, and obtains a payoff $u_i(\mathbf{x}, \mathbf{g})$ that depends on the effort profile $\mathbf{x}$ and on the underlying network $\mathbf{g}$, in the following way:

$$u_i(\mathbf{x}, \mathbf{g}) = a\, x_i - \frac{1}{2}x_i^2 + \phi \sum_{j=1}^{n} g_{ij} x_i x_j \tag{1}$$

where $a, \phi > 0$. In this linear-quadratic utility function, agents are ex ante homogeneous in terms of observable characteristics (i.e. they all have the same a) and their heterogeneity only stems from their position in the network. The first two terms of (1), $a\, x_i - \frac{1}{2}x_i^2$, give the individual benefits and costs of providing effort x_i. The last term of this utility function, $\phi \sum_{j=1}^{n} g_{ij} x_i x_j$, reflects the influence of direct links on own action. This peer effect component can be heterogeneous, and this *endogenous heterogeneity* reflects the different locations of individuals in the network $\mathbf{g}$ and the resulting effort levels. More precisely, bilateral influences are captured by the following cross derivatives, for $i \neq j$:

$$\frac{\partial^2 u_i(\mathbf{x}, \mathbf{g})}{\partial x_i \partial x_j} = \phi g_{ij} \geq 0 \tag{2}$$

When i and j are directly linked, i.e. $g_{ij} = 1$, the cross derivative is $\phi > 0$ and reflects *strategic complementarity* in efforts. When i and j are not direct friends, i.e. $g_{ij} = 0$, this cross derivative is zero.

The Bonacich network centrality Define

$$\mathbf{M}(\mathbf{g}, \phi) = (\mathbf{I} - \phi \mathbf{G})^{-1} = \sum_{k=0}^{+\infty} \phi^p \mathbf{G}^p$$

where $\mathbf{I}$ is the identity matrix. Denote by $\mathbf{1}$ the column vector of 1. We have the following definition:

Galeotti and Goyal (2010). For an overview of the literature on games on networks, see Jackson and Zenou (2012a).

[3] We assume perfect information, i.e. the players know everything about the network and other players' actions. For games on networks with incomplete information, see Calvó-Armengol and de Martí (2009), Galeotti et al. (2009), Hagenbach and Koessler (2011) and Acemoglu et al. (2011).

Definition 1 (Katz, 1953; Bonacich, 1987) *Given $\phi \geq 0$, a small enough scalar, the vector of Katz-Bonacich centralities of parameter ϕ in network* $\mathbf{g}$ *is defined as:*

$$\mathbf{b}(\mathbf{g}, \boldsymbol{\phi}) = (\mathbf{I} - \phi\mathbf{G})^{-1}\mathbf{1} = \mathbf{M}(\mathbf{g}, \boldsymbol{\phi})\mathbf{1} = \sum_{p=0}^{+\infty} \phi^p \mathbf{G}^p \mathbf{1}. \tag{3}$$

The Katz-Bonacich centrality of node i is thus $b_i(\mathbf{g}, \phi) = \sum_{j=1}^{n} m_{ij}(\mathbf{g}, \phi)$, and counts the *total* number of walks in $\mathbf{g}$ starting from i. It is the sum of all loops $m_{ii}(\mathbf{g}, \phi)$ from i to i itself, and of all the outer walks $\sum_{j\neq i} m_{ij}(\mathbf{g}, \phi)$ from i to every other player $j \neq i$, that is:

$$b_i(\mathbf{g}, \phi) = m_{ii}(\mathbf{g}, \phi) + \sum_{j\neq i} m_{ij}(\mathbf{g}, \phi).$$

By definition, $m_{ii}(\mathbf{g}, \phi) \geq 1$, and thus $b_i(\mathbf{g}, \phi) \geq 1$, with equality when $\phi = 0$.

Nash equilibrium[4]

Let us now characterize the Nash equilibrium of the game where agents choose their effort level $x_i \geq 0$ simultaneously. Denote by $\mu_1(\mathbf{G})$ the spectral radius of $\mathbf{G}$. Ballester et al. (2006) have shown the following result:

Proposition 1 *If $\phi\mu_1(\mathbf{G}) < 1$, the game with payoffs (1) has a unique interior Nash equilibrium in pure strategies given by:*

$$\mathbf{x}^* = a\mathbf{b}(\mathbf{g}, \phi) \tag{4}$$

This results shows that the Katz-Bonacich centrality is the right network index[5] to account for equilibrium behavior when the utility functions are linear-quadratic. In (1), the local payoff interdependence is restricted to direct network contacts. At equilibrium, though, this local payoff interdependence spreads all over the network through the overlap of direct link clusters. The Katz-Bonacich centrality precisely reflects how individual decisions feed into each other along any direct and indirect network path. The condition $\phi\mu_1(\mathbf{G}) < 1$ stipulates that local complementarities must be small enough compared to own concavity, which

[4]In game theory, the Nash equilibrium (named after John F. Nash) is a solution concept of a game involving two or more players, in which no player has anything to gain by changing only her own strategy (i.e., by changing *unilaterally*). If each player has chosen a strategy and no player can benefit by changing her strategy while the other players keep theirs unchanged, then the current set of strategy choices constitute a Nash equilibrium.

[5]There are many other centrality measures such as the betweeness or the closeness centrality. See Wasserman and Faust (1994) for an overview on the different possible centrality measures.

prevents multiple equilibria to emerge and, at the same time, rules out corner solutions. The condition $\phi\mu_1(\mathbf{G}) < 1$ also guarantees that $(\mathbf{I} - \phi\mathbf{G})$ is invertible and its series expansion well defined.

Proposition 1 has characterized the equilibrium effort x_i^* of each agent i as equals to her Bonacich centrality. We can now characterize the equilibrium utility of i as a function of her Bonacich centrality. It is easily verified that:

$$u_i(\mathbf{x}^*, \mathbf{g}) = \frac{1}{2}x_i^{*2} = \frac{1}{2}\left[a\, b_i\left(\mathbf{g}, \phi\right)\right]^2$$

In other words, the equilibrium utility of each agent i is positively related to her Katz-Bonacich centrality.

2.2 Network formation

In the previous section, we have seen how the position in the network of each player (i.e. her Katz-Bonacich centrality) affects her outcomes when the network is fixed. We now look at network formation but agents cannot decide how much effort to exert in some activity. They only choose with whom they want to form a link.

2.2.1 Static models of network formation

Myerson (1977, 1991) provides an early formulation of a network formation game. The structure of the game is simple: agents have to decide about their potential partners and their strategies consist in naming those with whom they want to form a link with. For a link to be formed, it has to be that two individuals name each other, i.e. there needs to be mutual consent in link creation. The Nash equilibrium concept defined above can then be used to find out which strategy profiles are stable and, hence, which networks are the possible outcomes of the game.

The main problem of using the Nash equilibrium concept is that it exacerbates the coordination problems that arise when all agents are simultaneously deciding about links. In particular, the empty network (i.e. nobody forms a link) is always a Nash equilibrium of this game since no deviation is profitable. Indeed, deviating means here to name someone as a friend but this will not generate a link because of mutual consent. In general, with the Myerson game, there are many equilibria, which reduces the attractiveness of this approach.

A solution to this problem has been proposed by Jackson and Wolinsky (1996). In their seminal paper, they introduce an alternative solution concept for network formation games, namely *pairwise stability*, which provides a network equilibrium notion. Let us define it.

Consider some payoff function $u(\mathbf{g}) = (u_1(\mathbf{g}), \ldots, u_n(\mathbf{g}))$ that assigns a payoff to every agent in N as a function of the underlying network $\mathbf{g}$ connecting them.

Definition 2 A network $\mathbf{g}$ is pairwise stable for the payoff function u if and only if:

$$\begin{array}{ll} (i) & \text{for all } ij \in \mathbf{g},\ u_i(\mathbf{g}) \geq u_i(\mathbf{g}{-}ij) \text{ and } u_j(\mathbf{g}) \geq u_j(\mathbf{g}{-}ij) \\ (ii) & \text{for all } ij \notin \mathbf{g}, \text{ if } u_i(\mathbf{g}) < u_i(\mathbf{g}{+}ij), \text{ then } u_i(\mathbf{g}) > u_j(\mathbf{g}{+}ij) \end{array}$$

In words, a network is pairwise-stable if (i) no player gains by cutting an existing link, and (ii) no two players not yet connected both gain by creating a direct link with each other. Pairwise-stability thus only checks for one-link deviations.[6] It requires that any mutually beneficial link be formed at equilibrium but does not allow for multi-link severance.[7] This notion takes into account the *individual incentives* to create and sever links and the necessary *mutual consent* between both sides for a link to be formed.[8]

Pairwise-stable networks can be interpreted as the limiting graphs of a dynamic procedure of network formation. Suppose, indeed, that players myopically add or sever links to improve their current status, and that only one link is added or removed at a time. When this process converges, the networks ultimately reached are pairwise-stable.[9]

Let us now define strong efficiency.

Definition 3 A network $\mathbf{g}^E \subset \mathbf{g}^N$ is strongly efficient if

$$W\left(\mathbf{g}^E\right) \equiv \sum_{i \in \mathbf{g}^E} u_i(\mathbf{g}^E) \geq W(\mathbf{g}) \equiv \sum_{i \in \mathbf{g}} u_i(\mathbf{g}) \text{ for all } \mathbf{g} \subset \mathbf{g}^N$$

[6] This weak equilibrium concept is often interpreted as a necessary conditions for stronger stability concepts.

[7] There are other network equilibrium concepts, which are often refinements from the pairwise-stability equilibrium notion. See Bloch and Jackson (2006) for a complete overview on this issue. In particular, they compare variations on three types of definitions: those based on a pairwise stability notion, those based on the Nash equilibria of a link formation game, and those based on equilibria of a link formation game where transfers are possible.

[8] Bala and Goyal (2000) extend the Myerson model described above by only considering directed networks, i.e. individuals form links unilaterally without requiring the consent of the other party to create a link.

[9] Note that network transitions only concern one link at at time. The dynamics may thus lock in (the set of limiting graphs need not be unique) or cycle. See Jackson and Watts (2002) for more details.

Observe that this concept of efficiency is *utilitarian* and not *Paretian* since this definition says that the socially efficient network is the one for which the sum of utilities of all agents is the highest among all possible networks with n agents.

2.2.2 A simple model: The connections model

Jackson and Wolinsky (1996) has proposed a simple model where the utility function of each individual i in a network $\mathbf{g}$ is given by:

$$u_i(\mathbf{g}) = \sum_{j \neq i} \delta^{d(i,j)} - \sum_{j \in N_i(g)} c = \sum_{j \neq i} \delta^{d(i,j)} - d_i(\mathbf{g})\, c \tag{5}$$

where $0 < \delta < 1$ gives the value of the link between two individuals i and j, $d(i, j)$ is the *geodesic distance* between i and j, i.e. the length of any shortest path between i and j,[10] and finally c is the cost of forming a link. In this formulation, when individual i has a direct link with j, she first benefits from this link by obtaining δ (since, for any direct link, $d(i, j) = 1$) but then she may benefit from the direct links of j by obtaining $\delta^{d(i,j)}$ (indirect connections, which are weighted by a *decay factor*).

Observe that if two agents create a link between them it is never detrimental to a third partie (*positive externalities*). This is because creating a new link does not affect the cost of a third player (she still has the same number of direct links) but can affect her gains since a new link can shorten the distance between a third player and another one. Here is an example of utilities for the following star network:

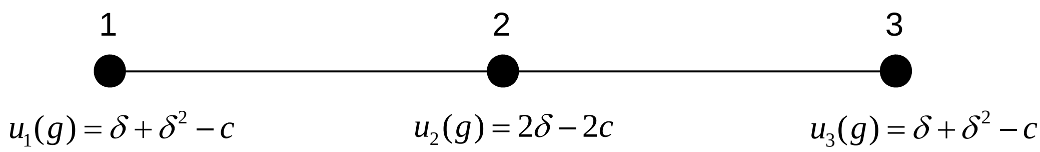

Figure 1: Utility values in a star-shaped network

We have the following result for the connections model (Jackson and Wolinsky, 1996):

Proposition 2 *In the connections model in which the utility of each player $i = 1, ..., n$ is given by (5), we have:*

[10] See Section 2.1.1.

(*i*) *A pairwise stable network has at most one (non-empty) component.*

(*ii*) *For* $c < \delta - \delta^2$*, the unique pairwise stable network is the complete graph* g^N*.*

(*iii*) *For* $\delta - \delta^2 < c < \delta$ *a star encompassing all players is pairwise stable, but not necessarily the unique pairwise stable graph.*

(*iv*) *For* $\delta < c$*, any pairwise stable network that is non-empty is such that each player has at least two links.*

This proposition characterizes the pairwise-stable equilibria. In particular, it says that if $\delta - \delta^2 < c < \delta$, then the star-shaped is a pairwise stable equilibrium. Let us check if this is true for the network described in Figure 1. If the star agent, player 2, wants to delete a link (she cannot create any new link), her net utility is: $c - \delta$. If $c < \delta$, this will never happen. If a peripheral agent (say player 1) deletes a link, her net utility is: $c - \delta$ while if she creates a link (with player 3), her net utility is: $\delta - c - \delta^2$. Any of these two actions will never take place if $\delta - \delta^2 < c < \delta$. It is easily verified that this same condition holds for a star-shaped network with n agents to be pairwise stable. As can be seen in this proposition, one of the main problems of pairwise stable networks is that it is very difficult *to provide a full characterization of the set of equilibrium networks.*[11]

Let us now give a result in terms of efficiency:

Proposition 3 *The unique strongly efficient network in the symmetric connections model, where the utility of each player* $i = 1, ..., n$ *is given by (5), is:*

(1) *the complete graph* g^N *if* $c < \delta - \delta^2$*.*

(2) *a star encompassing all players if* $\delta - \delta^2 < c < \delta + \frac{(n-2)}{2}\delta^2$

(3) *no links (the empty network) if* $\delta + \frac{(n-2)}{2}\delta^2 < c$*.*

There is therefore a tension between stability and efficiency. Take part (*iv*) of Propositions 2. In the high cost range ($c > \delta$) the only (non-degenerated) paiwise stable networks are those who are overconnected from an efficiency perspective. For example, when $c > \delta$, the star cannot be pairwise stable but it is an efficient network.

[11] It is indeed well-known that non-cooperative games of network formation with nominal lists of intended links are plagued by coordination problems (Myerson, 1991; Jackson, 2008). Cooperative-like stability concepts solve them partially, but heavy combinatorial costs still jeopardize a full characterization.

2.2.3 A second model: The co-author model

In this case, for $n_i > 0$,[12] the utility function of a player $i = 1, ..., n$ in network $\mathbf{g}$ is given by:

$$u_i(\mathbf{g}) = \sum_{j \in N_i(\mathbf{g})} \left[\frac{1}{d_i} + \frac{1}{d_j} + \frac{1}{d_i d_j} \right]$$

where $d_i \equiv d_i(\mathbf{g})$ is the number of direct links player i has (or equivalently the number of projects player i is involved in). Here each player is a researcher and a link ij is a joint project (or joint paper or collaboration) between players i and j.

Each researcher has a fixed amount of time to spend on research and so the time each researcher i spends on a given project is inversely related to the number of project d_i he is involved in. The synergy between two researchers i and j are captured by the term $1/(d_i d_j)$. Here the more projects each researcher is involved with, the lower the synergy that is obtained per project. There is no direct cost of forming a link but an indirect cost because of congestion.

Contrary to the connections model, when two agents create a link, it is *never* beneficial to a third partie (*negative externalities*). This is because creating a new link does affect the time the researcher will spend on other projects with the researchers she is already connected to.

Proposition 4 *In the co-author model,*

- (i) *if n is even, then the strongly efficient network is a graph consisting of $n/2$ separate pairs (dyads), and*
- (ii) *a pairwise stable network can be partitioned into fully intraconnected components, each of which has a different number of members. If k is the number of members of one such component and k' is the next largest in size, then $k > (k')^2$.*

2.3 Choosing both actions and links

There are different models that have combined actions' and links' choices. For example, Bloch and Dutta (2009) proposed a model where both link intensities and network formation are modeled. For games with strategic substitutes, Galeotti and Goyal (2010) has developed

[12] It is assumed that for $n_i = 0$, $u_i(\mathbf{g}) = 0$.

a static model where players choose both actions and links while López-Pintado (2008) investigates a dynamic one. Because we have focused on games with strategic complements, we would like now to expose the static model of Cabrales et al. (2011).[13]

Consider a simultaneous move game of network formation (or social interactions) and investment. $N = \{1, ..., n\}$ is a finite set of players, and $T = \{1, \ldots, t\}$ is a finite set of types for these players. We let n be a multiple of t, that is, $n = mt$ for some integer $m \geq 1$, so that there is the same number of players of each type. The case $n = t$ is referred to as *the baseline game* and the general case $n = mt$ as *the* $m-$*replica* of this baseline game. For each player $i \in N$, we denote by $\tau(i) \in T$ her type. In an $m-$replica game, there are exactly m players of each type $\tau \in T$. This replica game allows us to take limits as the population becomes large without having to specify the types of the new individuals that are added.

Let $c > 0$. Player i's utility is equal to:

$$u_i(\mathbf{x}, \mathbf{s}) = a_{\tau(i)}\, x_i + \phi \sum_{j=1, j\neq i}^{n} g_{ij}(\mathbf{s})\, x_i x_j - \frac{1}{2} c x_i^2 - \frac{1}{2} s_i^2 \tag{6}$$

where $x_i \geq 0$ is the *productive* effort taken by player i, with $\mathbf{x} = (s_1, ..., s_n)$ being a profile of productive efforts while $s_i \geq 0$ is the *socialization* effort of player i, with $\mathbf{s} = (s_1, ..., s_n)$ being a profile of socialization efforts. This utility is very similar to the one defined in (1) with the difference that the socialization effort s_i is also included. In (6), the returns to the investment are the sum of a private component and a synergistic component. The private returns are heterogeneous across players and depend on their type. We denote by $\mathbf{a} = (a_1, ..., a_t)$ the profile of these private returns, where $0 < a_1 \leq a_2 \leq ... \leq a_t$. Payoffs have non-negative cross effects, i.e.

$$\frac{\partial^2 u_i(\mathbf{x}, \mathbf{s})}{\partial x_i \partial x_j} = \phi g_{ij}(\mathbf{s}), \text{ for all } i \neq j, \tag{7}$$

reflecting strategic complementarities in productive investments ($\phi \geq 0$ corresponds to the level of synergistic returns). As in (1), the size $\phi g_{ij}(\mathbf{s}) \geq 0$ of these complementarities depends on the profile of socialization efforts, and varies across different pairs of players.

The key innovation is $g_{ij}(\mathbf{s})$, which determines link formation. Players i and j interact with a link intensity given by:

[13] See König et al. (2012) for a dynamic model of network formation with strategic complements in efforts.

$$g_{ij}(\mathbf{s}) = \frac{s_i s_j}{\sum_{j=1}^{n} s_j} \tag{8}$$

By definition, links are symmetric, that is, $g_{ij} = g_{ji}$. We also allow for self-loops (when $i = j$). From (8), we can determine the adjacency matrix $\mathbf{G}$ of this network where each cell ij is $0 < g_{ij} < 1$.

A key innovation of this paper is that the synergistic effort $\mathbf{s}$ is *generic* within a community −a scalar decision. Socializing is not equivalent to elaborating a nominal list of intended relationships, as in the literature on network formation surveyed by Jackson (2008) and exposed in Section 2.2. We have seen (see Proposition 2) that this leads to too many equilibria and that it is very difficult to provide a full characterization of the set of equilibrium networks. To avoid this problem, here, network formation is not the result of an earmarked socialization process so that agents put efforts in socialization that *may* result in links between agents. This choice of a model without earmarked socialization greatly improves the tractability of the analysis. Unlike with richer models of link formation a la Jackson and Wolinsky (1996) exposed in Section 2.2, we can resort to off-the-shelf Nash equilibrium analysis without being burdened by the extreme (combinatorial) multiplicity problems of the other models. As a result, the authors can perform a standard type of equilibrium analysis that equates marginal costs and benefits of both production and socialization. Define:

$$\Phi(\mathbf{a}) = \phi \frac{\sum_{\tau=1}^{t} a_\tau^2}{\sum_{\tau=1}^{t} a_\tau}. \tag{9}$$

Cabrales et al. (2011) demonstrate the following result:

Proposition 5 *Suppose that* $2\,(c/3)^{3/2} > \Phi(\mathbf{a}) > 0$. *Then, there exists an* m^* *such that for all* $m-$*replica games with* $m \geq m^*$, *there are exactly two stable interior pure strategy Nash equilibria. These pure strategy Nash equilibria are such that, for all players* i *of type* τ, *the strategies* (s_i, x_i) *converge to* $(s^*_{\tau(i)}, x^*_{\tau(i)})$ *as* m *goes to infinity, where* $s^*_{\tau(i)} = a_{\tau(i)} s$, $x^*_{\tau(i)} = a_{\tau(i)} x$, *and* (s, x) *are positive solutions to:*

$$\begin{cases} s = \Phi(\mathbf{a}) x^2 \\ x\,[c - \Phi(\mathbf{a}) s] = 1 \end{cases} \tag{10}$$

In words, when $\Phi(\mathbf{a})$ is small enough compared to the infra-marginal cost for a productive investment, the system of two equations (10) with two unknowns has exactly two positive solutions. As m gets large, each such solution gets arbitrarily close to a pure strategy Nash equilibrium of the corresponding $m-$replica game. We get two approximate Nash equilibria. Besides, as m gets large, every pure strategy Nash equilibrium gets arbitrarily close to a solution of (10). The equilibrium multiplicity identified in Proposition 5 reflects an inter-twinned coordination problem in the socialization process and in the production technology.

The authors are then able to compare the actions and payoffs of players across the two approximate equilibria characterized in Proposition 5. They show that the equilibrium actions can be ranked component-wisely and the equilibrium payoffs can be Pareto-ranked accordingly. There is a Pareto-superior approximate equilibrium $(\mathbf{s}^*, \mathbf{x}^*)$ and a Pareto-inferior approximate equilibrium $(\mathbf{s}^{**}, \mathbf{x}^{**})$ while the socially efficient outcome lies in between the two equilibria. Formally, $(\mathbf{s}^*, \mathbf{x}^*) \geq \left(\mathbf{s}^E, \mathbf{x}^E\right) \geq (\mathbf{s}^{**}, \mathbf{x}^{**})$ and $\mathbf{u}\left(\mathbf{s}^E, \mathbf{x}^E\right) \geq \mathbf{u}(\mathbf{s}^*, \mathbf{x}^*) \geq \mathbf{u}(\mathbf{s}^{**}, \mathbf{x}^{**})$, where $\geq$ is the component-wise ordering. This simple model has shown how the set of equilibria can be reduced and equilibria can be characterized when agents do not direct their links but instead choose some effort in socialization.

3 Networks in economics: Applications and empirical aspects

We would like now to study two important applications of economic networks: labor market and crime networks.

3.1 Labor-market networks

There is a host of evidence showing that social networks are pervasive in the labor market. For instance, Holzer (1987, 1988) documents that, in the US, among 16-23 year old workers who reported job acceptance, 66 percent use informal search channels while only 11 percent use state agencies and 10 percent newspapers.[14] Topa (2001) argues that the observed spatial distribution of unemployment in Chicago is consistent with a model of local interactions and information spillovers, and may thus be generated by an agent's reliance in informal methods of job search such as networks of personal contacts. Similarly, Bayer et al. (2008) document that people who live close to each other, defined as being in the same census block, tend to

[14] See also Granovetter (1995) for additional evidence.

work together, that is, in the same census block.[15] Let us now provide a simple model that captures these facts.

3.1.1 The model

Let us describe the model of Calvó-Armengol and Jackson (2004). Time evolves in discrete periods indexed by t. The vector σ_t describes the employment status of the workers at time t. If individual i is employed at the end of period t, then $\sigma_{it} = 1$ and if i is unemployed then $\sigma_{it} = 0$.

A period t begins with some agents being employed and others not, as described by the vector $\boldsymbol{\sigma}_{t-1} = (\sigma_{1t-1}, ..., \sigma_{nt-1})$ that gives the employment status of all workers from the last period. Next, information about job openings arrives. In particular, any given individual hears about a job opening with probability γ that is between 0 and 1. This job arrival process is independent across individuals. If the individual is unemployed, then she will take the job. However, if the individual is already employed then she will pass the information along to a friend, picked at random among her unemployed friends. As stated above, the graph or network $\mathbf{g}$ summarizes the links of all agents, where $g_{ij} = 1$ indicates that i and j know each other (strong tie), and share their knowledge about job information, while $g_{ij} = 0$ indicates that they do not know each other. Finally, the last thing that happens in a period is that some agents lose their jobs. This happens randomly according to an exogenous breakup rate, δ, which is between 0 and 1. We are able to write the probability $\mathbb{P}_{ij}$ of the joint event that individual i learns about a job and this job ends up in individual j's hands. It is equal to:

$$\mathbb{P}_{ij}(\boldsymbol{\sigma}) = \begin{cases} \gamma & \text{if} \quad \sigma_i = 0 \text{ and } i = j \\ \gamma / \sum_{k:\sigma_k=0} g_{ik} & \text{if} \quad \sigma_i = 1,\ \sigma_j = 0, \text{ and } g_{ij} = 1 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where the vector $\boldsymbol{\sigma}$ describes the employment status of all the individuals at the beginning of the period. In (11), γ is the probability of obtaining a job information without using friends and relatives. Three cases may then arise. If individuals i and j are unemployed ($\sigma_i = \sigma_j = 0$), then the probability that j will obtain a job is just γ since individual i will never transmit any information to j. If individual i is already employed and her friend j is not ($\sigma_i = 1$, $\sigma_j = 0$), then individual i transmits this job information to all her direct

[15] For a survey of the literature on social interactions and the labor market, see Ioannides and Loury (2004).

unemployed neighbors, who total number is $\sum_{k:\sigma_k=0} g_{ik}$. We assume that all unemployed neighbors are treated on equal footing, meaning that the employed worker who has the job information does not favor any of her direct neighbors. As a result, the probability that an unemployed worker j is selected among the $\sum_{k:\sigma_k=0} g_{ik}$ unemployed direct neighbors of an employed worker j is given by: $\gamma / \sum_{k:\sigma_k=0} g_{ik}$. Finally, if individual j is employed, then she does not need any job information, at least in the current period.

The first result obtained by Calvó-Armengol and Jackson (2004) is the following.

Proposition 6 *The higher $d_i(\mathbf{g})$, the number of strong ties (direct friends) individual i has, the higher is her individual probability of finding a job.*

Indeed, if an individual has more strong ties (direct friends), then she is more likely to hear on average about more jobs through them but her chance of finding a job directly does not increase since γ is not affected by the size of the network. This result is quite intuitive since, when the number of direct connections increases, the source of information about jobs is larger and people find it easier to obtain a job through their friends and relatives. This is the first prediction of this model, which implies that workers have a greater chance of finding a job, the higher is the number of their strong ties. Observe that the individual probability of finding a job through strong ties for individual j is obviously not given by (11) since $\mathbb{P}_{ij}(\mathbf{s})$ is the probability that only one individual, i, who hold a strong tie with j, and who is aware of some job, will transmit this information to individual j. To determine the individual probability of obtaining a job for j, one has to do the calculation for all the direct friends of i.

We would now like to study the impact of weak ties (indirect friends or path-connected friends) on the individual probability of finding a job. Calvó-Armengol and Jackson (2004) show that, in steady-state, there is a *positive correlation* in employment status between two path-connected workers. This result is not at all easy to obtain since, in the short run, the correlation is negative. Indeed, in a static model, if an employed worker is directed linked to two unemployed workers, then if she is aware of a job, she will share this job information with her two unemployed friends (see (11)). These two persons, who are path-connected (path of length two) are thus in competition and one (randomly chosen) will obtain the job and be employed while the other will stayed unemployed. So their employment statuses will be negatively correlated (see Calvó-Armengol, 2004).

Let us now give the intuition why this negative correlation result does *not* hold in a dynamic labor-market model. Consider the star-shaped network described in Figure 2 with three individuals, i.e. $n = 3$ and $g_{12} = g_{23} = 1$. Suppose the employment status of these three workers from the end of the last period is $\boldsymbol{\sigma}_{t-1} = (0, 1, 0)$. In the figure, a black node represents an employed worker (individual 2), while unemployed workers (1 and 3) are represented by white nodes. Conditional on this state $\boldsymbol{\sigma}_{t-1}$, the employment states σ_{1t} and σ_{3t} are negatively correlated. As stated above, this is due to the fact that individuals 1 and 3 are "competitors" for any job information that is first heard by individual 2.

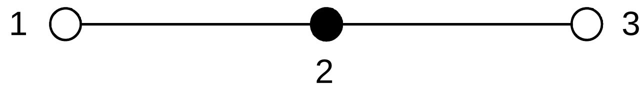

Figure 2: Employment correlations in a star-shaped network

Despite this negative (conditional) correlation in the short run, individual 1 can benefit from individual 3's presence in the longer run. Indeed, individual 3's presence helps improve individual 2's employment status. Also, when individual 3 is employed, individual 1 is more likely to hear about any job that individual 2 hears about. These two aspects counter the local (conditional) negative correlation, and help induce a positive correlation between the employment status of individuals 1 and 3.

Proposition 7 *Under fine enough subdivisions of periods, the unique steady-state long-run distribution on employment is such that the employment statuses of any path-connected agents are positively correlated.*

The proposition shows that, despite the short-run conditional negative correlation between the employment of competitors for jobs and information, in the longer run any interconnected workers' employment is positively correlated. This implies that there is a clustering of agents by employment status, and employed workers tend to be connected with employed workers, and vice versa. The intuition is clear: conditional on knowing that some set of agents are employed, it is more likely that their neighbors will end up receiving information about jobs, and so on. The benefits from having other agents in the network outweigh the local negative correlation effects, if we take a long-run perspective.

Proposition 8 *The longer the length of two-path connected individuals (i.e weak ties), the lower is the correlation in employment statuses between these two individuals.*

Indeed, the correlation between two agents' employment is (weakly) decreasing in the number of links that each an agent has, and the correlation between agents' employment is higher for direct compared to indirect connections. The decrease as a function of the number of links is due to the decreased importance of any single link if an agent has many links. The difference between direct and indirect connections in terms of correlation is due to the fact that direct connections provide information, while indirect connections only help by indirect provision of information that keeps friends, friends of friends, etc., employed. In other words, the longer the path in the social network between two individuals, the weaker is the effect of job transmission.

3.1.2 Empirical test

Using individual-level data from the UK Labour Force Survey, Patacchini and Zenou (2012a) test this model by looking at the employment prospects of ethnic minorities. To the traditional determinants of employment rates (sex, age, education, years since arrival in UK, percentage of high-skilled leaving nearby, etc...), they add the local ethnic employment density bands based on travel-time between areas.

Because they do not have direct data on social networks, Patacchini and Zenou (2012a) conjecture that the social space is highly correlated to the physical space for ethnic minorities in relatively small areas. This approximation is based on the fact that, in relatively small areas, ethnic minorities of a given group (say Indians or Pakistanis) are likely to interact with other ethnic minorities from the same group and thus exchange information about jobs. In other words, ethnic employment density is interpreted as a proxy for the strength of social contacts in delivering information about jobs.

Patacchini and Zenou (2012a) test Propositions 6 and 8 described above, which show that the individual probability of finding a job increases with the number of strong ties and weak ties, and the longer the length of weak ties, the lower is this probability.

They find that the higher is the percentage of employed workers from a given ethnic group living nearby, the higher is the probability of finding a job through social networks (Proposition 6). This effect decays, however, very rapidly with distance, losing significance beyond approximately an hour travel time (Proposition 8). They argue that local social interactions between people of the same ethnicity can explain this positive relationship and its spatial trend.

Let us now investigate our second application: criminal networks.

3.2 Criminal networks

It is well-established that delinquency is, to some extent, a group phenomenon, and the source of crime and delinquency is located in the intimate social networks of individuals (see e.g. Sutherland, 1947, Sarnecki, 2001 and Warr, 2002). Indeed, delinquents often have friends who have themselves committed several offenses, and social ties among delinquents are seen as a means whereby individuals exert an influence over one another to commit crimes.

3.2.1 A simple model

Glaeser et al. (1996) where among the first to model criminal social interactions. In their model, criminal interconnections act as a social multiplier on aggregate crime. They impose, however, a specific network structure, the circle, for the location of criminals. Following Calvó-Armengol and Zenou (2004) and Ballester et al. (2010), we would like to propose a more general model that can encompass any social network. For that, we will use the model of Section 2.1.3 by reinterpreting it in terms of criminal activities.

Denote by x_i the delinquency effort level (i.e., how often they commit crime) of delinquent i, and by $\mathbf{x} = (x_1, ..., x_n)^\top$ the population delinquency profile. Delinquents in network $\mathbf{g}$ decide how much effort to exert. Each agent i selects an effort $x_i \geq 0$, and obtains a payoff $u_i(\mathbf{x}, \mathbf{g})$ that depends on the effort profile $\mathbf{x}$ and on the underlying network $\mathbf{g}$, in the following way:

$$u_i(\mathbf{x}, \mathbf{g}) = \underbrace{\alpha\, x_i}_{\text{Proceeds}} - \underbrace{\frac{1}{2} x_i^2}_{\text{moral cost of crime}} - \underbrace{p \cdot f \cdot x_i}_{\text{cost of being caught}} + \underbrace{\phi \sum_{j=1}^{n} g_{ij} x_i x_j}_{\text{positive peer effects}} \tag{12}$$

where $\phi > 0$. This utility has a standard cost/benefit structure (as in Becker, 1968). The proceeds from crime are given by $\alpha\, x_i$ and are increasing in own effort x_i. The costs of committing crime are captured by the probability of being caught $0 < p < 1$ times the fine $f\, x_i$, which increases with own effort x_i, as the severity of the punishment increases with one's involvement in crime. Individuals have a *moral* cost of committing crime equals to $\frac{1}{2}x_i^2$, which is also increasing in own crime effort x_i. Finally, the new element in this utility function is the last term $\phi \sum_{j=1}^{n} g_{ij}x_i x_j$, which reflects the influence of friends' behavior on own action. Denote $a \equiv \alpha - pf > 0$. Then (12) can be written as:

$$u_i(\mathbf{x}, \mathbf{g}) = a\, x_i - \frac{1}{2}x_i^2 + \phi \sum_{j=1}^{n} g_{ij}x_i x_j \tag{13}$$

which is exactly the same as (1). We can thus apply Proposition 1 and show that, if $\phi\mu_1(\mathbf{g}) < 1$, there is exists a unique Nash equilibrium in crime effort given by:

$$\mathbf{x}^* = a\,(\mathbf{I}{-}\phi\mathbf{G})^{-1}\,\mathbf{1} = a\,\mathbf{b}\,(\mathbf{g}, \phi)$$

3.2.2 Policy implications

One interesting aspect of *criminal networks* is that we can propose alternative policies than those advocated by Becker (1968) and others. In particular, because of the network aspect, we can implement a *key player* policy, which consists in finding and getting rid of the key player, i.e., the delinquent who, once removed, leads to the highest aggregate delinquency reduction.

Given that delinquent removal has both a direct and an indirect effect on the group outcome, the choice of the key player results from a compromise between both effects. In particular, the key player need not necessarily be the one exerting the highest delinquency effort or, equivalently, the one with the highest Katz-Bonacich centrality measure. The planner's objective is thus to generate the highest possible reduction in aggregate delinquency level by picking the appropriate delinquent. Formally, the planner's problem is the following:

$$\max\{x^*(\mathbf{g}) - x^*(\mathbf{g}^{[-i]}) \mid i = 1, ..., n\},$$

where $x^*(\mathbf{g}) = \sum_i x_i^*(\mathbf{g})$ is the total equilibrium level of crime in network $\mathbf{g}$ and $\mathbf{g}^{[-i]}$ is the network when individual i has been removed. In terms of adjacency matrix, network $\mathbf{g}^{[-i]}$ corresponds to the adjacency matrix $\mathbf{G}^{[-i]}$, which is equal to $\mathbf{G}$ for which the row and column of individual i has been removed. The program above is equivalent to:

$$\min\{x^*(\mathbf{g}^{[-i]}) \mid i = 1, ..., n\} \tag{14}$$

From Ballester et al. (2006, 2010), we now define a new network centrality measure of player i, the *intercentrality index*, that solves this program.

$$c_i(\mathbf{g}, \phi) = \frac{b_i(\mathbf{g}, \phi)^2}{m_{ii}(\mathbf{g}, \phi)} \tag{15}$$

where $m_{ii}(\mathbf{g}, \phi)$ is the i^{th} element on the diagonal of $\mathbf{M} = (\mathbf{I} - \phi \mathbf{G})^{-1}$. The Katz-Bonacich centrality of player i counts the number of paths in $\mathbf{g}$ stemming from i; the inter-centrality counts the total number of such paths that hit i. It is the sum of i's Katz-Bonacich centrality and i's contribution to every other player's Katz-Bonacich centrality. Holding $b_i(\mathbf{g}, \phi)$ fixed, $c_i(\mathbf{g}, \phi)$ decreases with the proportion of i's Katz-Bonacich centrality due to self-loops, $m_{ii}(\mathbf{g}, \phi)/b_i(\mathbf{g}, \phi)$.

Let us now provide an example showing that the key player is not always the most active delinquent in a network.

Example Consider the following network $\mathbf{g}$:

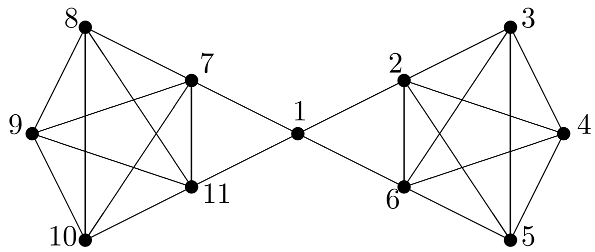

Figure 3: Network with eleven delinquents.

We distinguish three different types of equivalent actors in this network, which are the following:

Type	Players
1	1
2	2, 6, 7 and 11
3	3, 4, 5, 8, 9 and 10

For $a = 1$, the following table computes, for delinquents of types 1, 2 and 3, the value of delinquency efforts x_i, centrality measures $b_i(\mathbf{g}, \phi)$ and intercentrality measures $c_i(\mathbf{g}, \phi)$ for two different values of ϕ, 0.1 and 0.2. In each column, a variable with a star identifies the highest value.

ϕ	0.1			0.2		
Player Type	x_i	$b_i(\mathbf{g}, \phi)$	$c_i(\mathbf{g}, \phi)$	x_i	$b_i(\mathbf{g}, \phi)$	$c_i(\mathbf{g}, \phi)$
1	0.077	1.75	2.92	0.072	8.33	41.67*
2	0.082*	1.88*	3.28*	0.079*	9.17*	40.33
3	0.075	1.72	2.79	0.067	7.78	32.67

First note that type-2 delinquents always display the highest Bonacich centrality measure. These delinquents have the highest number of direct connections. Besides, they are directly connected to the bridge delinquent 1, which gives them access to a very wide and diversified span of indirect connections. For low values of ϕ, the direct effect on delinquency reduction prevails, and type-2 delinquents are the key players $-$those with highest intercentrality measure d_i. When ϕ is higher, though, the most active delinquents are not anymore the key players. Now, indirect effects matter a lot, and eliminating delinquent 1 has the highest joint direct and indirect effect on aggregate delinquency reduction. Note that the network $\mathbf{g}^{[-1]}$ has twenty different links, while $\mathbf{g}^{[-2]}$ has nineteen links. In fact, when ϕ is small enough, the key player problem minimizes the number of remaining links in a network, which explains why type-2 delinquents are the key player when $\phi = 0.1$ in this example.

3.2.3 Empirical test

Testing the idea of key players in crime is quite complicated since we need very detailed information about networks and crime outcomes. Fortunately, there is a unique database on friendship networks from the National Longitudinal Survey of Adolescent Health (AddHealth).[16] It collects data on students in grades 7-12 from a nationally representative sample of roughly 130 private and public schools in years 1994-95. In terms of networks, the most interesting aspect of the AddHealth data is the information on friendships. Indeed, the friendship information is based upon actual friends nominations. Pupils were asked to identify their best friends from a school roster (up to five males and five females). As a result, $g_{ij} = 1$ if either i or j or both have nominated each other. Otherwise, $g_{ij} = 0$.

Using this dataset, Liu et al. (2011) try to identify key players for juvenile crime in the United States. First, as in the example above, they find that a little bit more than 20 percent

[16]The AddHealth website *http://www.cpc.unc.edu/projects/addhealth* describes survey design and data in details.

key players are not the most active deliquents. This is because they have a crucial position in the network in terms of betweenness centrality. They also find that, compared to other criminals, "key" criminals are less likely to be a female, are less religious, belong to families whose parents are less educated and have the perception of being socially more excluded. They also feel that their parents care less about them, are less likely to come from families where both parents are married and have more trouble getting along with teachers. An interesting feature is that key players are more intelligent (i.e. higher mathematics scores) than the average criminal and are more likely to have friends who are older (i.e. in higher grades), more religious and whose parents are more educated. Also, even though key players themselves do not have a better self-esteem, are not more physically developed nor are they more likely to be urbanites than other criminals, their friends are.

The authors also try to determine how efficient is a key-player policy as opposed to a random-player policy, i.e. a policy that removes a criminal at random from the network. Figure 4 displays the results.

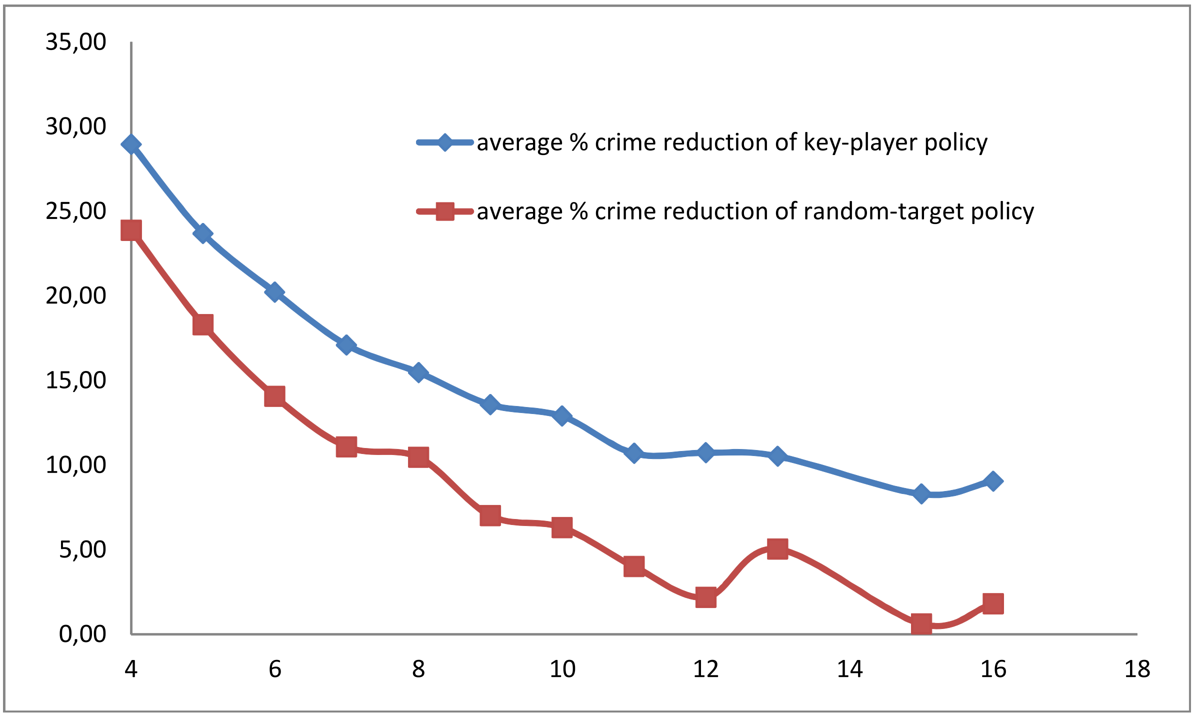

Figure 4: Difference between a key-player and a random-target policy

To plot this figure, Liu et al. (2011) put together networks of the same size and calculate the average crime reduction for this size of networks under the two policies. For example, for all networks of size 4 (horizontal axis), the average crime reduction is 29.94 percent on average when the key-player policy (vertical axis blue curve) is implemented while it is 23.86 percent when a random-target policy is implemented (vertical axis red curve). The difference in crime reduction between these two policies can be large, especially for big networks. This can justify why a key-player policy, though expensive, could be implemented.

4 Concluding Remarks

The study of networks in economics is still at an early stage and much more research needs to be done. In this article, we have provided an overview on this literature by exposing its recent developments. We have seen that the models of network formation are often plagued by too many equilibria and that the full characterization of all possible equilibria is a very difficult task. On the contrary, the study of games on networks, where the network is fixed and the impact of the network structure on individuals' outcomes is analyzed, is much more developed since the existence, uniqueness and characterization of equilibrium has been established. We have also seen how these models can easily been used to analyze issues related to labor and crime networks (and also education, R&D, cities, etc.; see Jackson and Zenou, 2012a).

We believe that, in the future, more effort should be devoted to the formation of networks in a dynamic framework using some microfoundations based on economic choices. Indeed, following the physics literature, some economic models have been used to model dynamic network formation but links are usually formed in a random or probabilistic way.[17] Some recent papers have, however, tried to provide more microfounded models where links between agents are formed based on a careful cost and benefit analysis so that agents maximize their utility (see, in particular, Christakis et al., 2010; Mele, 2011; König et al., 2012).

We also believe that more work should be done to estimate empirically economic networks. In Section 3, we have shown how labor and crime networks could be estimated and what kind of policy implications they imply. Recent papers (Bramoullé et al., 2009; Calvó-Armnegol et al., 2009) have shown how econometric issues such as the reflection problem (Manski,

[17]In these models, usually, one agent at a time is selected to form or create links. This greatly simplifies the analysis and avoids the coordination problems highlighted in Section 2.2 where all agents could delete or create links simultaneously.

1993) and correlated effects can be solved using a network approach. This is a growing field (see, in particular, Patacchini and Zenou, 2012b, Banerjee et al., 2012) and we hope that many more tests on different economic aspects where networks are prevalent such as crime, microfinance, religion, smoking, teenage pregnancy, etc. will be performed in the future.

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