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No. 8997

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ISSN 0265-8003

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CEPR Discussion Paper No. 8997

ABSTRACT

Reference Dependence and Labor-Market Fluctuations*

We incorporate reference-dependent preferences into a search-and-matching model of the labor market, in which firms have all the bargaining power and productivity follows an AR(1) process. Motivated by Akerlof (1982) and Bewley (1999), we assume that existing workers are willing to exert unobserved, "intrinsically motivated" effort as long as their wage does not fall below a "reference wage", which (broadly following Kőszegi and Rabin (2006)) is equal to their lagged-expected wage. We formulate the model game-theoretically and show that it has a unique subgame perfect equilibrium that exhibits the following properties: existing workers experience downward wage rigidity as well as destruction of output following negative shocks due to layoffs or loss of morale; newly hired workers earn relatively flexible wages, but not as much as in the benchmark without reference dependence; market tightness is more volatile than under this benchmark. We relate these findings to the debate over the "Shimer puzzle" (Shimer (2005)).

JEL Classification: C72, D03, E24, E32 and J64
Keywords: negative-reciprocity, reference-dependence, search and matching, shimer puzzle, social preferences and wage rigidity

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*Spiegler acknowledges financial support from the European Research Council, Grant no. 230251. We thank Elhanan Helpman, Christopher Pissarides, Ariel Rubinstein, Ron Siegel, Eran Yashiv, Alwyn Young and seminar participants at Berlin, Brown, Haifa University, LSE, Oxford, Mannheim and Tel Aviv University, for helpful comments.

Submitted 24 May 2012

1 Introduction

Economists have long pondered over the observation that wages display downward rigidity and do not fall in recessions as much as one might expect on the basis of supply-and-demand analysis. An idea with a long pedigree, going back to Keynes (1936), Solow (1979), Akerlof (1982), Kahneman et al. (1986), Falk and Fehr (1999) and many others, is that reciprocal-fairness considerations deter employers from cutting wages during recessions. Specifically, the theory is that the labor contract's inherent incompleteness forces employers to rely to some extent on workers' intrinsic motivation. When workers feel that they have been treated unfairly, their intrinsic motivation is dampened and their output declines. According to this "morale hazard" theory, wage cuts relative to a "reference point" have such an effect, which is why employers avoid them.

Blinder and Choi (1990), and especially Bewley (1999), surveyed personnel managers and other labor-market actors, and found overwhelming support for the morale theory. As Bewley (1999) puts it:

> "My findings support none of the existing economic theories of wage rigidity, except those emphasizing the impact of pay cuts on morale. Other theories fail in part because they are based on the unrealistic psychological assumptions that people's abilities do not depend on their state of mind and that they are rational in the simplistic sense that they maximize a utility that depends only on their own consumption and working conditions..."

Fehr et al. (2009) review a large body of research on experimental labor markets that corroborates this view.

In this paper we incorporate a reciprocal-fairness account of the labor relation into a search-and-matching (S&M) model of the labor market in which "productivity" fluctuates according to an AR(1) process, and explore its theoretical implications for equilibrium wage and unemployment fluctuations. Following Akerlof (1982), our main departure from the standard S&M model in the Mortensen-Pissarides tradition (Pissarides (2000)) lies in the assumption that the labor contract is incomplete, such that part of the work effort is discretionary and relies on the worker's intrinsic motivation. The worker is willing to exert this unobserved effort as long as his wage does not fall too much below a "reference point". While the worker's decision whether to accept or reject a job offer is entirely standard (maximizing expected discounted wage and non-market benefits), his unobserved-effort decision follows a myopic reference-dependent rule.

How is the reference point determined? We assume that (negative) reciprocity considerations emerge only after the worker has experienced a relationship with his current employer. Therefore, a formerly unemployed worker enters his very first employment period only with the expectation of being paid the lowest admissible wage (normalized to zero). After his first period of employment, the worker has developed a relationship with his employer, and he cultivates an expectation to earn the equilibrium wage of existing workers, conditional on his current information. This expectation will constitute the worker's reference point at the next period.

Thus, the reference wage of an existing worker at period t is equal to his expected wage, calculated according to his "rational" expectations at period $t-1$. This "lagged expectations" approach to reference-point formation is due to an influential model of reference-dependent preferences due to Kőszegi and Rabin (2006). The justification for the expectation-based specification is that a given wage offer may be greeted as a pleasant surprise or as a demoralizing disappointment, depending on how it compares with the worker's former expectations: if he expected a big salary raise, failure to meet these expectations may hurt his morale, even if his current wage is higher than yesterday's wage. The justification for the "lagged" aspect is that it takes the reference point some time to adapt to changing circumstances, just as it takes people time to change a habit. (Another example of the "stickiness" of reference points is the reluctance of homeowners to lower their asking price when a boom in the real-estate market is followed by a downturn; see Genesove and Mayer (2001) for empirical evidence for this effect.)[1]

Note that since the worker's reference point changes over the course of his relationship with his employer, his effort decisions are dynamically inconsistent: if he could commit ex-ante to exerting discretionary effort throughout his employment, he would be willing to do so. However, commitment is infeasible, and the dynamic inconsistency ultimately generates the interesting effects of the model. In Appendix B, we present a slightly different formulation of the reference point, which *endogenizes* this distinction between newly hired and existing workers; our main results are robust to this variation.

Before giving an overview of our results, we wish to comment on our methodology, which is firmly in the microeconomic-theory tradition. We seek complete analytical characterizations of dynamic equilibria and high their qualitative features. This has several implications. First, we focus exclusively on the labor market. Second, while the standard Mortensen-Pissarides model mixes non-cooperative game-theoretic mod-

[1] See Crawford and Meng (2011) for an empirical implementation of the "lagged expectations" approach to reference dependence.

eling with the "cooperative" Nash bargaining solution, we formulate the model as an extensive-form game with moves of Nature and study its subgame perfect equilibria (SPE), as in Rubinstein and Wolinsky (1985). Third, we eliminate two degrees of freedom in the standard S&M model: workers have no bargaining power (firms make take-it-or-leave-it wage offers), and their non-market payoff is proportional to productivity. Doing so not only simplifies the analysis, but also ensures that all wage-rigidity effects are due to the novel behavioral element. Finally, for most of the paper, we impose a two-period exogenous separation process, which is innocuous in the reference-independent benchmark but facilitates analysis under reference dependence.[2]

As long as the magnitude of productivity shocks is not too large, our model generates a unique SPE, which displays the following features.

Wage rigidity. Equilibrium wage for existing workers displays downward rigidity with respect to current productivity shocks. Specifically, for intermediate noise realizations, the firm offers the reference wage, and therefore does not respond to productivity fluctuations. At high noise realizations, the firm pays the outside option. In certain special cases of the model, the wage is entirely rigid.

Destruction of output. At low noise realizations, the firm either fires existing workers or pays them their outside option (in which case, the workers reciprocate by shirking), depending on the importance of "morale" in the production function. Thus, existing workers experience layoffs or loss of morale in equilibrium.

History dependence. The outcome of the interaction with existing workers is sensitive to past realizations of productivity. Their reference point at period t is a function of productivity at $t-1$, and so are the critical noise realizations that determine the firms' wage and retention policies. In particular, the probability of layoffs or loss of morale at period t decreases with productivity at $t-1$. As the profit margin that characterizes the economy shrinks, layoffs become more frequent, but less sensitive to past productivity.

Entry-level wages. Newly matched workers are always hired in equilibrium and paid a wage below existing workers' wage. The entry-level wage is not rigid; it fluctuates with current productivity, although to a lesser extent than in the benchmark model without reference dependence. Unlike existing workers, the equilibrium wage of new hires is purely a function of current productivity.

[2]For a textbook treatment of the S&M model with a business cycle, see Shimer (2010). The only attempt to introduce reference dependence into an S&M model we are aware of is Kuang and Wang (2010), who follow a quantitative "macro" approach that complements our paper.

Increased volatility of market tightness. As in the standard S&M model, free entry implies that market tightness is determined by the firms' hiring incentive. We show that the ratio between tightness calculated at the extreme productivity levels is bigger than in the benchmark without reference dependence. This is a "global" measure of tightness volatility. We also show that under certain specifications of the model, the elasticity of tightness with respect to productivity is higher than in the reference-independent benchmark. This is a "local" measure of tightness volatility. The reason for this volatility effect is that as current productivity goes down, firms are more concerned about the destruction of value that the future combination of wage rigidity and negative productivity shocks may bring. This means that the incentive to hire is more sensitive to productivity fluctuations than in the benchmark.

In an influential paper, Shimer (2005) argued that the S&M model has shortcomings in accounting for real-life labor-market fluctuations, in the sense that the wage volatility it predicts is too large and the unemployment volatility it predicts is too small. A fast-growing literature ensued. One research direction, suggested by Shimer (2005) and Hall (2005), and challenged by Pissarides (2009), Kudlyak (2009) and Haefke et al. (2012), has centered around the hypothetical role of wage stickiness in addressing Shimer's puzzle.

Our results can be viewed in light of this debate. Since our paper follows a purely theoretical and qualitative approach, it cannot be viewed as an attempt to resolve Shimer's puzzle, which is quantitative in nature. However, we believe it helps understanding the questions that the puzzle has raised. First, the volatility effects our model generates are in the "right" direction. Second, as we show in Section 4, our model synthesizes the arguments raised by the two sides in the debate, showing they are not mutually contradictory after all. Finally, the model provides a behavioral foundation for the association between wage rigidity and enhanced tightness volatility.

2 A Model

Consider the following complete-information, infinite-horizon game. There is a continuum of players: a measure one of workers and an unbounded measure of firms (the latter assumption captures free entry among firms). We break the description into the following components: search and matching, separation, wage and output determination, the agents' information and their preferences.

Search and matching

Time is discrete. At each period t, firms and workers are matched according to the following process. An unemployed worker (including workers who lost their job at the beginning of period t, as described below) is automatically in the search pool. (That is, we abstract from questions of labor market participation.) An unmatched firm (including firms that dismissed workers at the beginning of the period, as described below) decides whether to be in the search pool, i.e., post a vacancy.[3]

If there are U_t unemployed workers and V_t open vacancies at this stage, then a measure $m(U_t, V_t) \leq \min\{U_t, V_t\}$ of unemployed workers are matched to vacancies at the beginning of period $t+1$. The matching function m satisfies the standard assumptions: it is continuous, strictly increasing in each of its arguments and exhibits constant returns to scale.

The matching probabilities for workers and firms at period t are thus $q_t = m(U_t, V_t)/U_t$ and $p_t = m(U_t, V_t)/V_t$, respectively. Note that $\lim_{V \to \infty} m(U, V)/V = 0$. We assume that if all firms post vacancies, then $p = 0$. Define *market tightness* at t as the ratio $\eta_t = U_t/V_t = p_t/q_t$. Since m exhibits constant returns to scale, it is easy to verify that q is a strictly decreasing function of p, given by the implicit function,

$$m(\frac{p}{q}, 1) = p \tag{1}$$

Thus, η_t is a strictly increasing function of p_t, and a strictly decreasing function of q_t. From now on we will be primarily interested in market tightness as an indicator of the state of unemployment, and we will suppress U and V.[4]

Separation and wage determination

Consider a worker who at time $t-1$ completes a tenure of $i-1$ consecutive periods of employment at the same firm, where $i = 1, 2, ...$ With probability $s(i)$, this pair will be separated at the beginning of period t for some unspecified exogenous reason. With probability $1 - s(i)$, the match will survive into the beginning of period t; during that period, we will refer to the worker as a worker of type i. For most of the paper, we will assume $s(1) = 0$ and $s(2) = 1$.

When the two parties are matched at the beginning of period t, the firm first chooses whether to employ the worker. We use $r_{i,t} \in \{0, 1\}$ to denote the firm's endogenous separation decision when facing a worker of type i, where $r_{1,t} = 1$ means that the

[3] For expositional simplicity, we assume that each firm can post at most one vacancy. This entails no loss of generality, as long as production is separable across vacancies.

[4] The requirement that m exhibits constant returns to scale is not only sufficient, but also *necessary* for the one-to-one correspondence between p and q.

firm chooses the employ the worker at t, and $r_{i,t} = 0$ means that the firm chooses to dismiss him. Conditional on employing a worker of type i at period t, the firm makes a take-it-or-leave-it, flat-wage offer $w_{i,t} \geq 0$. This is a "spot" contract that covers period t only (put differently, the firm can renegotiate the labor contract at every period).

The two parties are endogenously separated at period t if the firm fires the worker, or if the worker rejects the firm's wage offer. In this case (as well as following an exogenous separation), the worker joins the search pool of period t, while the firm chooses whether to be in the search pool of period t.

Output

Conditional on accepting an offer, an employed worker is committed to a minimal level of effort. On top of that, he chooses a level of discretionary effort $x_t \in \{0, 1\}$. We refer to $x = 1$ as "normal effort". The worker's output is $y_t = \theta_t[\gamma + (1-\gamma)x_t]$, where $\gamma \in [0, 1]$ is a parameter that captures the completeness of the labor contract (such that $1-\gamma$ captures the importance of discretionary effort in the production function), and θ_t represents "aggregate productivity" at period t. We will refer to θ_t as the "state" at t.

Productivity θ_t follows an AR(1) process:

$$\theta_t = (1-\rho)\mu + \rho\theta_{t-1} + \varepsilon_t$$

where $\rho \in (0, 1)$ and ε_t is *i.i.d.* across periods according to a density function f, which is continuously and symmetrically distributed around zero, with support $[-1, 1]$. We use F to denote the *cdf* induced by f. Slightly abusing notation, we often use $f(\theta_t \mid \theta_{t-1})$ to denote the density of θ_t conditional on θ_{t-1}. Let $\Psi(\theta_t) = (1-\rho)\mu + \rho\theta_t$ be the expected value of θ_{t+1} conditional on θ_t. Let $[\theta_*, \theta^*]$ denote the support of θ_t. Note that $\theta_* = \mu - 1/(1-\rho)$ and $\theta^* = \mu + 1/(1-\rho)$. Throughout this paper, we assume that $\mu(1-\rho) > 2$, such that $\theta^* - \theta_* < \mu$ - that is, the spread of θ_t is lower than its long-run mean.

Information

In each period $t \geq 1$, every agent observes the productivity realizations $\theta_0, ..., \theta_t$. The agent also observes his own private history. Finally, whenever a firm and a worker interact, they observe the history of wage offers since they were matched. They do not observe the negotiation history in other firm-worker matches. Finally, the firm does not observe its worker's effort decision x nor the output he generates (we discuss this assumption below).

Preferences

All agents in the model maximize their expected discounted sum of payoffs, using the same constant discount factor δ. The payoff flow for firms at each period is as follows. A firm outside the search pool earns zero. A firm in the search pool earns $-c$, where $c > 0$ is the cost of posting a vacancy. A firm in a relationship with a worker earns a payoff that equals output minus the wage paid.

An unemployed worker at period t receives a non-market payoff of $b\theta_t$, where $b \in (0,1)$. An employed type-i worker with reference wage $e_{i,t}$ gets a payoff of $w_{i,t}$ if $w_{i,t}/e_{i,t} \geq \lambda$, where $\lambda \in [0,1)$; otherwise, he receives a payoff of $w_{i,t} - x_{i,t}$. The interpretation is that when the worker's wage is sufficiently lower than his reference point, he perceives this as unfair treatment; his intrinsic motivation is damaged, and he strictly prefers not to exert his normal effort. Otherwise, the worker is indifferent between $x = 0$ and $x = 1$, and we assume that he chooses the latter.

Given the assumption that the worker's discretionary effort and output are unobserved, the worker's choice of x will be entirely myopic in any subgame perfect equilibrium: at any period t in which he accepts a wage offer w_t, he will play $x_t = 1$ if and only if $w_t/e_t \geq \lambda$. As a result, the worker will respond to wage offers as if he maximizes the discounted sum of expected wage and non-market earnings, independently of γ.[5]

These preferences thus capture what Fehr et al. (2009) call "negative reciprocity", but do not give room to "positive reciprocity" - namely, increased effort beyond the normal level following a wage offer sufficiently higher than the reference point. This asymmetry reflects findings in the literature: "Whereas the positive effects of fair treatment on behavior are usually small, the negative impact of unfair behavior is often large" (Fehr et al. (2009, p. 366)). It is also in the spirit of Prospect Theory (Kahneman and Tversky (1979)): losses relative to the reference point loom significantly larger than gains.

How is the reference point $e_{i,t}$ determined? We assume that when an unemployed worker is matched to a firm, he begins his first period of employment with "modest expectations" in the sense that his reference point, $e_{1,t}$, equals the lowest possible wage, which is zero. On the other hand, existing workers, who were employed by the same firm at period $t-1$ enter period t with a reference point equal to the wage they expected to earn at t conditional on being retained. That is, the expected wage is computed

[5] The worker's behavioral model is thus equivalent to a multiple-selves model, consisting of one long-run self who moves after wage offers, and an infinite collection of myopic selves, who make single discretionary effort choices. The long-run self maximizes discounted wage and non-market earnings, while the myopic self follows a simple rule: $x = 1$ if and only if $w_{i,t}/e_{i,t} \geq \lambda$.

according to the information the workers had at the end of period $t-1$. To sum up, we assume that at any period t, $e_{1,t}=0$ and $e_{i,t}=E(w_{i,t} \mid \theta_{t-1}, r_{i,t}=1)$ for $i>1$.

The equilibrium concept

Because workers' preferences in this model depend on their expectations (both of the moves of Nature and of the players' strategies), this is not strictly speaking a conventional game, but rather an example of an extensive-form psychological game (after Geanakoplos et al. (1989)). In general, extending standard game-theoretic solution concepts to this class of games may involve subtleties. However, in the present case, the standard concept of subgame perfect equilibrium (SPE) is defined and analyzed in a completely standard way, and we will follow this concept, which is appropriate for our setting.

A comment on contractual incompleteness

The assumption that firms do not observe their workers' output may appear strange. However, recall that although the model is presented in terms of one-to-one matching, this assumption is purely expositional and the entire analysis is valid for one-to-many matching where production is separable across vacancies. It is entirely realistic to assume that while the firm can only observe its aggregate output with some noise, it cannot monitor the contribution of any individual worker.

Even under this limited monitoring, one could argue that flat-wage contracts are too restrictive, and that firms could incentivize effort by conditioning the workers' compensation on the noisy signal, namely aggregate output. However, as the literature on moral hazard in teams has demonstrated (starting with Holmstrom (1982)), such incentives are limited in their ability to induce team effort. Furthermore, they are likely to exacerbate morale problems because they punish individual workers for a drop in output which is due to chance or to other workers' effort decisions. (Similar issues arise when the worker has multiple tasks and the firm can only monitor a subset of those.) Thus, morale considerations and limited monitoring of workers' effort complement each other in dissuading firms from elaborate incentive schemes toward flat-wage contracts (see Fehr et al. (2009) for an eloquent discussion of this point).

2.1 The Complete-Contract Benchmark

Let us first consider the benchmark model in which only monitored (or contractible) effort matters for the firm, i.e., $\gamma=1$. Here, reciprocal-fairness considerations - which impact the worker's discretionary effort - are irrelevant for the firm. In this case, our model reduces to a standard S&M model.

Proposition 1 *Let $\gamma = 1$. There is a unique SPE, in which firms choose $(r_t, w_t) = (1, b\theta_t)$ at every t and regardless of the worker's type, and workers accept any wage offer weakly above $b\theta_t$.*[6]

Equilibrium in the benchmark model exhibits several noteworthy features. First, equilibrium behavior is Markovian in a narrow sense: hiring/retention and wages at any period t are purely a function of θ_t. Second, wages are entirely flexible, in the sense that they are proportional to productivity. Third, there is no behavioral distinction between newly matched and existing workers. Finally, there are no layoffs.

Proposition 1 determines equilibrium market tightness via a free-entry property. A firm's expected discounted benefit from posting a vacancy at period t, conditional on finding a new match at the beginning of $t+1$, is equal to the expected discounted sum of the firm's payoffs over the duration of the employment relation. Formally, it is a function of the state at t, defined as follows:

$$\Pi^0(\theta_t) = (1-b)\sum_{i=1}^{\infty} \delta^i \left(\prod_{j=1}^{i} (1 - s(j)) \right) \Psi^i(\theta_t) \tag{2}$$

where $\Psi^i(\theta)$ is defined recursively: $\Psi^{i+1}(\theta) = \Psi(\Psi^i(\theta)))$. Note that Π^0 is an increasing function. If $c > \Pi^0(\theta_t)$, then in SPE no firm posts a vacancy at t, and market tightness is infinite. If $c \leq \Pi^0(\theta_t)$, then in equilibrium firms will be indifferent between searching and not searching. The probability p_t that a searching firm will find a match at the beginning of $t+1$ will be set such $c = p_t\Pi^0(\theta_t)$. Market tightness is derived from p_t according to (1). Hence, equilibrium tightness at t is purely a function of θ_t as well.

3 Equilibrium Analysis under Incomplete Contracts

We now analyze SPE in the case of $\gamma < 1$, in which non-contractible effort affects the worker's output. We impose a few parametric restrictions, which are relaxed in the sequel. First, we restrict attention to a simple exogenous job separation process, for which $s(1) = 0$ and $s(2) = 1$. That is, the employment relation lasts at most two periods. This could approximate industries in which firm-specific human capital depletes quickly as a result of rapid technological changes. However, we assume it mainly for tractability, and defer the discussion of other separation processes to Section 5. Second, we focus on the $\lambda \to 1$ limit (where existing workers play $x = 0$ whenever

[6] We omit the description of the workers' effort decision because it is irrelevant for the market outcome.

their wage falls below their reference point). The case of $\lambda < 1$ is addressed in Section 6. All proofs are relegated to an appendix.

It is useful to make two preliminary observations. First, in SPE, all newly matched workers at any given period are treated identically, and similarly, all existing workers at any given period are treated identically. The reason is that all agents on each side of the market are identical, and no firm-worker pair gets to observe the history of any pairwise interaction prior to their own match, thus preventing history-dependent asymmetries from emerging. In what follows we often refer to the way "the worker" or "the firm" behave at a given history, with the understanding that this pertains to all firms and all workers of the same type at the same period.

Second, we can think about a worker's equilibrium behavior in terms of whether his choices satisfy "individual rationality" (IR) and "psychological incentive compatibility" (PIC) constraints, in analogy to IR/IC constraints in contract theory. Fix a history h following a wage offer. An SPE satisfies the *IR constraint* at h if he is weakly better off than if he rejects the firm's wage offer and sticks to his equilibrium strategy afterwards. An SPE satisfies the *PIC constraint* at h if the worker does not strictly prefer playing $x = 0$ at h. By assumption, newly matched workers' PIC constraint coincides with the constraint that wages are non-negative. Therefore, the PIC constraint is only relevant for existing workers. According to the one-deviation property of SPE, the IR constraint always holds in equilibrium, and the only question is at which histories it is binding. Note that in SPE, if the IR constraint holds with slack at h, the PIC constraint must be binding. The reason is simple: if the PIC constraint is violated or holds with slack, the firm can slightly lower its wage without changing the set of constraints it satisfies.

3.1 The Case of $\gamma < b$

Our analysis distinguishes between two ranges of values that γ can get. When $\gamma < b$, this means that discretionary effort plays an important role in the production function). SPE turns out to have a very simple structure in this case.

Proposition 2 *Let $\gamma < b$. In the $\lambda \to 1$ limit, the game has a unique SPE, which has the following properties.*

(i) An existing worker's period-t reference point $e_{2,t}$ is a function $e_2(\theta_{t-1})$, which is equal to the highest value that non-market benefits may attain at period t,

$$e_2(\theta_{t-1}) = b(\Psi(\theta_{t-1}) + 1). \tag{3}$$

(ii) An existing worker is retained at period t if and only if

$$\varepsilon_t \geq \varepsilon^*(\theta_{t-1}) \equiv b - (1-b)\Psi(\theta_{t-1}). \tag{4}$$

Conditional on being retained at t, his wage is

$$w_2(\theta_{t-1}, \theta_t) = e_2(\theta_{t-1}). \tag{5}$$

(iii) A newly matched worker at period t is always hired; his wage at period t is

$$w_1(\theta_t) = b\left(\theta_t - \delta \int_{\varepsilon^*(\theta_t)}^{1} (1-\varepsilon) f(\varepsilon) d\varepsilon\right). \tag{6}$$

(iv) Workers accept all wage offers on the equilibrium path and choose $x = 1$.

Existing workers' equilibrium wage in SPE is absolutely rigid, in the sense that it is independent of current productivity; it is purely a function of productivity in the previous period. Wage rigidity here has a flavor of "grade inflation": in the $\lambda \to 1$ limit, existing workers' reference wage is the expectation of the maximum between the outside option and the reference wage itself. This means that the reference wage must always be greater or equal to the expected outside option, which can only be true if the reference wage equals the highest possible value of the outside option. Because $\gamma < b$, a firm would rather dismiss a worker than paying him a wage below his reference point. Thus, existing workers always get their reference wage conditionally on being retained, and they exert discretionary effort in return. Finally, layoffs of existing workers occur with positive probability, when the reference wage is higher than the realized output. Layoffs are inefficient: firms are impelled to fire workers even though there are gains from retaining them.

The proof of Proposition 2 proceeds roughly as follows. First, we show that newly matched workers' IR constraint is always binding in SPE (or, equivalently, that they must earn strictly positive wages in any SPE). We do so by deriving an upper bound on the rent that existing workers can get in equilibrium, which translates into a lower bound on newly matched workers' wage. Here we make use of the assumption that $\mu(1-\rho) > 2$, namely that the magnitude of the business cycle is not too big relative to the long-run average productivity. Second, we observe that newly matched workers must always be indifferent between accepting an equilibrium wage offer (and sticking to their equilibrium strategy thereafter) and being permanently unemployed. This implies that an existing worker at period t would accept any wage above $b\theta_t$. As we observed above, his reference wage at t is the highest participation wage he could expect to get at t, given θ_{t-1}. Thus, we have derived existing workers' equilibrium wage, and the firm's retention policy immediately follows from that. To obtain newly matched workers' wage, we use their indifference to permanent unemployment, such that their equilibrium wage at t is equal to $b\theta_t$ minus the discounted rent they expect to receive as existing workers at $t+1$.

We proceed to describe several noteworthy properties of the SPE.

History dependence

Equilibrium behavior at any period t is Markovian with respect to an extended state (θ_{t-1}, θ_t). That is, the market outcome depends not only on the absolute productivity level (as in the $\gamma = 1$ benchmark), but also on changes in its level relative to the previous period. Also, unlike the $\gamma = 1$ benchmark, equilibrium wage exhibits a "seniority premium": existing (newly matched) workers earn wages above (below) the outside option.

Since the threshold $\varepsilon^*(\theta_{t-1})$ is decreasing in θ_{t-1}, layoffs become more frequent at period t as θ_{t-1} goes down. Note that the layoff frequency increases with b, but the sensitivity of this frequency to θ_{t-1} decreases with b. We will see that these features of the equilibrium have important implications for the volatility of market tightness.

IR and PIC constraints

An important property of the SPE (which is crucial for the proof) is that newly matched workers' IR constraint is always binding: their continuation payoff after every history is as if they earn $b\theta_t$ at every subsequent period t. In contrast, existing workers' IR constraint holds with slack except for the zero-probability event in which the highest possible noise value $\varepsilon_t = 1$ is realized. Their PIC constraint is always binding.

The structure of entry-level wages

The equilibrium wage paid to new hires is not necessarily increasing in θ_t, because it is affected by the rent the worker expects to get at $t+1$ due to reference dependence, and this rent increases with θ_t. However, if f is not too "jagged" (in the sense that there is an appropriate upper bound on the values f can get), $w_1(\theta_t)$ is increasing with θ_t. In this case, we may say that entry-level wage is flexible in the sense that it increases (strictly and continuously) with current productivity, albeit at a flatter rate than in the $\gamma = 1$ benchmark.

A quick glance at the formula (6) reveals that $w_{1,t}$ is bounded from below by $b(\theta_t - \delta)$. The assumption that $\mu > 2/(1-\rho)$ ensures that $\theta_t > 1$, hence $w_{1,t} > 0$. That is, the non-negativity constraint hold with slack. Note that if μ is sufficiently high (or b is sufficiently low), we have $\varepsilon^*(\theta_{t-1}) \leq -1$ for every θ_{t-1}, such that layoffs never occur in equilibrium, and entry-level wages hit the lower bound.

3.2 Volatility of Market Tightness

In order to study the equilibrium volatility of market tightness, we follow the S&M literature, and assume in this subsection that the matching function takes the following form

$$m(U_t, V_t) = kU_t^{\alpha} V_t^{1-\alpha} \tag{7}$$

$\alpha \in (0, 1)$, where k is sufficiently small so that match probabilities are always well-defined. This allows us to get an explicit, closed-form expression for market tightness. Let us first establish that in SPE, tightness at any period t is purely a function of θ_t. The expected discounted profit generated by a vacancy opened in period t conditional on getting a new match at the beginning of period $t+1$ is

$$\delta(1-b)\left[\Psi(\theta_t) + \delta \int_{-1}^{1} \int_{\varepsilon^*[\Psi(\theta_t)+\varepsilon_{t+1}]}^{1} [\Psi(\Psi(\theta_t) + \varepsilon_{t+1}) + \varepsilon_{t+2}] f(\varepsilon_{t+2}) f(\varepsilon_{t+1}) d\varepsilon_{t+2} d\varepsilon_{t+1}\right]$$

This expression is purely a function of θ_t, and we denote it by $J(\theta_t)$. Since the integrand is always positive, $\Psi(\cdot)$ is a strictly increasing function and $\varepsilon^*(\cdot)$ is a decreasing function, $J(\theta_t)$ is unambiguously strictly increasing in θ_t.

Lemma 1 *In the SPE characterized by Proposition 2, η_t is a function of θ_t given by the following equation:*

$$\eta_t(\theta_t) = \sqrt[\alpha]{\frac{c}{kJ(\theta_t)}}$$

as long as $c/J(\theta_t) < 1$. Otherwise, market tightness is infinite.

To understand why equilibrium market tightness is a well-defined function of current productivity, recall that η_t is a strictly increasing function of p_t, the probability that a searching firm finds a match at t. Because of free entry, p_t itself is a function of $J(\theta_t)$. Thus, although some aspects of equilibrium behavior at t - specifically, the outcome of the interaction with existing workers - depend on θ_{t-1}, tightness is only a function of θ_t.

Note that in the $\gamma = 1$ benchmark, $J(\theta_t)$ is reduced to $\Pi^0(\theta_t)$, as given by (2). Moreover, in SPE, if the probability of layoffs at $t+2$ conditional on filling a vacancy at the beginning of $t+1$ is zero, then $J(\theta_t) = \Pi^0(\theta_t)$. From (4) it follows that there exists a (unique) critical level of productivity θ, such that $\varepsilon^*(\Psi(\theta) - 1) \leq -1$ for every $\theta > \theta$, and $\varepsilon^*(\Psi(\theta) - 1) > -1$ for every $\theta < \theta$. That is, when a firm posts a vacancy at period t, it assigns positive probability to closing the vacancy at $t+2$, conditional on filling it at $t+1$, if and only if $\theta_t < \theta$. When $\theta_t < \theta$, $J(\theta_t) < \Pi^0(\theta_t)$. The critical productivity level θ will lie in the interior of the interval of possible values for θ whenever

$$\Psi(\theta_*) < \frac{1+b}{1-b} < \Psi(\theta^*)$$

Corollary 1 *In SPE, the ratio* $\eta(\theta')/\eta(\theta) > 1$ *is:*
(i) Strictly higher than in the $\gamma = 1$ *benchmark when* $\theta > \theta > \theta'$*.*
(i) Exactly as in the $\gamma = 1$ *benchmark when* $\theta, \theta' > \theta$*.*

Thus, wage rigidity affects tightness volatility only insofar as it leads to layoffs of existing workers. To see why, recall that market tightness at period t is determined by the incentive to hire during that same period, which in turn is a function of θ_t. As θ_t goes up, the distribution over the period-$(t+2)$ cutoff $\varepsilon^*(\theta_{t+1})$ undergoes a stochastic shift to the left, such that layoffs are less likely at $t+2$. If the rise in θ_t is sufficiently large, there are no layoffs at $t+2$. At that point, the incentive to hire at t is exactly as in the benchmark $\gamma = 1$, because firms can fully offset the wage-rigidity effect by setting a low wage for newly hired workers, such that their total benefit from the vacancy is the same as in the benchmark, and therefore tightness volatility is unaffected. Since the incentive to hire at t is more sensitive to fluctuations in θ_t than in the benchmark, tightness is more sensitive as well.

Corollary 1 deals with "global" elasticity. When the noise density f is uniform, we can also derive a sufficient condition for the elasticity of tightness to be pointwise higher than in the $\gamma = 1$ benchmark for every $\theta < \theta$.

Proposition 3 *Assume* F *is uniform. If*

$$\frac{1}{4\rho b(1-b)} < \Psi(\theta_*) < \frac{1+b}{1-b}$$

then for every $\theta < \theta$*, the elasticity of* η_t *with respect to* θ_t *is higher than in the* $\gamma = 1$ *benchmark.*

Note that if b is too close to zero, there are no layoffs, and therefore the mechanism that generates the volatility effect disappears. At the other extreme, if b is too close to one, or if ρ is too close to zero, then the cutoff $\varepsilon^*(\theta_{t-1})$ is relatively insensitive to θ_{t-1}, which means that the value of a vacancy is relatively insensitive to the value of θ at the time it was posted. It follows that the enhanced elasticity of tightness due to wage rigidity is likely to be more marked when b takes intermediate values and ρ is high.

3.3 The Case of $b \leq \gamma < 1$

We now consider the case in which non-discretionary effort is relatively unimportant for production. Specifically, the output of a demoralized worker is still above his equilibrium participation wage. The important departure from the $\gamma < b$ case is thus that existing workers are not fired in SPE after low realizations of θ_t; instead, they are retained and paid their outside option $b\theta_t$. The firm's effective dilemma at low realizations of θ_t is between two alternatives: paying the worker his reference wage and getting the normal output θ_t in return, and paying him his outside option $b\theta_t$ and getting the lower output $\gamma\theta_t$ in return. The firm will opt for the latter whenever $\theta_t(\gamma - b) > \theta_t - e_{2,t}$.

Proposition 4 *Let $b \leq \gamma < 1$. In the $\lambda \to 1$ limit, the game has a unique SPE, which has the following properties.*

(i) An existing worker's period-t reference point $e_{2,t}$ is a function $e(\theta_{t-1})$ which is the unique solution of the following equation:

$$e(\theta_{t-1}) \equiv \int_{\theta_*}^{\frac{e(\theta_{t-1})}{1-\gamma+b}} [b\theta_t \cdot f(\theta_t \mid \theta_{t-1})]d\theta_t + \int_{\frac{e(\theta_{t-1})}{1-\gamma+b}}^{\theta^*} [\max(e(\theta_{t-1}), b\theta_t) \cdot f(\theta_t \mid \theta_{t-1})]d\theta_t \quad (8)$$

(ii) An existing worker is always retained; his wage at period t is

$$w_2(\theta_{t-1}, \theta_t) = \begin{cases} e(\theta_{t-1}) & if \quad \theta_t \in (\frac{e(\theta_{t-1})}{b+1-\gamma}, \frac{e(\theta_{t-1})}{b}) \\ b\theta_t & otherwise \end{cases} \quad (9)$$

(iii) A newly matched worker at period t is always hired; his wage at period t is a function $w_1(\theta_t)$ given by

$$w_1(\theta_t) \equiv b\theta_t - \delta \int_{\frac{e(\theta_{t+1})}{b+1-\gamma}}^{\frac{1}{b}e(\theta_{t+1})} (e(\theta_{t+1}) - b\theta_{t+1}) \cdot f(\theta_{t+1} \mid \theta_t)d\theta_{t+1} \quad (10)$$

(iv) Workers accept all wage offers on the equilibrium path; newly matched workers always play $x = 1$*, whereas existing workers play* $x = 1$ *if and only if* $\theta_t \geq \frac{e(\theta_{t-1})}{b+1-\gamma}$.

Note that in contrast to the $\gamma < b$ case, existing workers' wages are rigid only in intermediate levels of productivity. When θ is sufficiently low, workers are paid their participation wage and do not exert normal effort. When productivity is high, existing workers' participation wage exceeds their reference point and therefore induces normal effort. In the middle range of productivity, non-discretionary effort is important for profits, but participation wages are too low relative to the workers' reference point. Firms are impelled to give rents to workers in this range.

Observe that the interval of productivity realizations for which existing workers are paid their rigid reference wage shrinks as γ goes up. In the $\gamma \to 1$ limit, the interval vanishes and the equilibrium converges to the $\gamma = 1$ benchmark. Also, the enhanced tightness volatility effect captured by Corollary 1 continues to hold, because as in the case of $\gamma < b$, low noise realizations result in loss of output - except that now it is due to loss of intrinsic motivation rather than layoffs.

3.4 General Finite-Horizon Separation

Our analysis in this section was based on the assumption that $s(1) = 0$ an $s(2) = 1$. Let us consider a generalization of this exogenous separation process, in which $s(i) = 0$ for every $i = 1, ..., T-1$, and $s(T) = 1$, where $T \geq 2$. Assume $\gamma < b$. Characterization of SPE would proceed along the same lines as in Proposition 2, with two differences. First, in order to ensure that the non-negativity constraint holds with slack, a stronger condition on the magnitude of the business cycle relative to long-run average productivity would be required. The inequality $\theta_* > T - 1$ is a sufficient condition. Second, the expressions for the worker's wage as a function of his tenure could be somewhat cumbersome.

Subject to these qualifications, the qualitative features of SPE constitute a straightforward extension of the $T = 2$ case. The equilibrium period-t wage of a worker of any type $i = 2, ..., T$ is exclusively a function of θ_{t-1}. current productivity, hence it is rigid w.r.t current productivity. In particular, the equilibrium treatment of a worker of type T is the same as in the $T = 2$ case. Newly matched workers' wage is exclusively a function of θ_t. Finally, the "seniority premium" effect is generalized as well: fixing the state (θ_{t-1}, θ_t), a worker's wage is strictly increasing in his type i. As a result, the layoff rate rises with the worker's tenure.

4 Discussion

In this section we discuss our results in comparison with alternative S&M models of the labor market. We will demonstrate that the combination of effects that our model generates - wage rigidity for existing workers, flexible entry-level wages, a seniority premium, endogenous job destruction that is sensitive to changes in productivity, and enhanced volatility of market tightness - cannot be reproduced by these alternative models.

4.1 Wage Rigidity and the Shimer Puzzle

The enhanced tightness volatility captured by Corollary 1 relates our equilibrium characterization to Shimer's puzzle. Shimer himself suggested that incorporating wage rigidity into S&M models may be an appropriate response to his finding. Hall (2005) proposed an example of such a model, replacing the assumption that wages are determined by a Nash-Bargaining formula with the assumption that the wage is constant across all states of the economy, as long as it is in the bargaining set in each state. The latter is an IR requirement: wage-rigidity effects should not cause parties to turn down individually rational offers.

Two features of Hall's model are noteworthy in comparison to our model. First, Hall imposes wage rigidity a priori, without deriving it from explicit behavioral or institutional considerations. In contrast, our model generates wage rigidity from workers' reference-dependent preferences. Second, Hall's analysis does not distinguish between newly hired and existing workers; what he refers to as "the wage" applies to all workers, regardless of their tenure.

The latter feature was criticized by Pissarides (2009), Kudlyak (2009) and Haefke et al. (2012), who argue - echoing Bewley (1999) and Fehr et al. (2009) - that this distinction does seem to exist in reality. They claim that if one observes wage rigidity in aggregate data, one cannot infer anything about the wages of newly hired workers, since these form a tiny minority of the stock of employed workers at any given point in time. In particular, Haefke et al. (2012) construct a time series for wages of new hires using micro-data on earnings and hours worked from the Current Population Survey (CPS) outgoing rotation groups. They find that the wage for newly hired workers is much more volatile than the aggregate wage and responds one-to-one to productivity.

If one wanted to reconcile Hall's model of wage determination with this critique, one would have to impose wage rigidity only on existing workers. However, Pissarides (2009) and Haefke et al. (2012) show that by doing so, one loses the modified S&M

model's ability to generate increased tightness volatility. The reason is that in Hall's model, wage rigidity never causes the firm-worker relationship to break down. A newly matched pair fully incorporates all future rigidities into their negotiation, such that the agreed-upon wage offsets all future departures from the "normal" surplus-division rule. As a result, the firms' hiring incentives are unaffected by the anticipated rigidity of existing workers' wage.

How do our results fit into this interesting exchange? On one hand, our model respects the distinction between newly hired and existing workers, and derives rigid wages for the latter only. On the other hand, seemingly in contradiction to Pissarides (2009) and Haefke et al. (2012), it generates increased tightness volatility relative to the benchmark model.

The key to resolving this apparent inconsistency is the incompleteness of the labor contract and the workers' changing reference point. The standard S&M model assumes complete contracts, and Hall (2005) shares this feature. When complete contracts are feasible, the rule for dividing the surplus does not affect the size of the surplus. This independence breaks down in our model. When a firm violates an existing worker's PIC constraint by paying him a wage below his reference point, the bargaining set effectively shrinks due to the worker's loss of morale, potentially to the point where all gains from mutual agreement are dissipated. It follows that the value of a new firm-worker match is not neutral to anticipated wage rigidity.

Our model does respect Hall's desideratum that wage rigidity should not cause workers to turn down individually rational offers. Indeed, existing workers' equilibrium acceptance decisions are the same as in the benchmark model. However, the labor relation in our model involves decisions outside the scope of the labor contract, which are determined by the workers' changing reference point. The adverse effects of wage rigidity on the value of vacancies are traced to these incontractible decisions.

4.2 Idiosyncratic Shocks and Endogenous Job Destruction

Since endogenous destruction of output plays a major part in our tightness volatility result, it is natural to ask whether other mechanisms of endogenous job destruction would generate similar patterns. The most well-known S&M model that exhibits endogenous job destruction, due to Mortensen and Pissarides (1994) - referred to as the MP model henceforth - generates this effect through idiosyncratic productivity shocks. To create an MP-like model that is comparable to ours, modify the benchmark model as follows. In each period, each vacancy is subjected to a random productivity shock, such that an employed worker produces an output of $\theta_t + \nu_t$, where v_t is *iid* with zero mean across firms and periods.

SPE wage offers in this model are exactly as in the benchmark: workers are always offered $b\theta_t$ when they are employed. Hiring and retention decisions are as follows: $r_t = 1$ if and only if $\theta_t + \nu_t - b\theta_t \geq 0$. Thus, in each period, depending on the state of the economy and the distribution of idiosyncratic shocks, some fraction of firms will choose not to hire a newly matched worker, or to fire an existing one. Clearly, there is no distinction between newly hired and existing workers.

This MP-like variation on the benchmark model implies *lower* volatility of market tightness - the exact opposite of our effect. To see why, note that we could reinterpret our benchmark model as an MP model in which firms make their hiring/retention decisions *before* the realization of their idiosyncratic shock. By the zero-mean property of the shocks, firms will always choose $r = 1$. When the noise realization v_t for a given firm turns out to lie below $v_t < (1 - b)\theta_t$, the firm's pre-commitment to play $r = 1$ is inefficient ex-post; if the firm could delay its decision until *after* it has learned its idiosyncratic shock, it would efficiently close the vacancy.

This is a simple value-of-information argument: enabling firms to move after learning their idiosyncratic shock raises the expected value of a vacancy. The magnitude of this effect is decreasing in the value of θ at the time the firm contemplates posting a vacancy, because knowledge of v_t is less likely change the firm's hiring/retention decision at t. But this means that the MP-like variation on the benchmark model *narrows* the gap between the firm's hiring incentive at different states of the economy, and consequently it shrinks tightness volatility.

This comparison highlights the feature that endogenous separations in our model destroy value. The worker's changing reference point and the firm's inability to offer a complete labor contract imply that vacancies will be closed even though the two parties would have agreed ex-ante that it would be efficient to keep them. In contrast, vacancies in the MP model are closed if and only if it is efficient to do so. This difference translates to tightness volatility effects in opposite directions.

4.3 Moral Hazard and Efficiency Wages

Our model is essentially an efficiency-wage model: in equilibrium, firms pay (existing) workers a wage above their reservation value, in order to induce unobserved effort. The mechanism that generates this effect is based on reciprocal fairness considerations, but there could be others. Shapiro and Stiglitz (1984) assume that when a worker shirks, he is caught and fired with some probability. In order for the worker to have an incentive to exert effort, the firm must offer him a wage above his outside option.

Costain and Jansen (2010) and Malcolmson and Mavroeidis (2010) incorporated the Shapiro-Stiglitz efficiency wage model into an S&M model. To illustrate the similarities and differences between such a model and ours, we briefly analyze the following modification of the benchmark model. Leave the output function unaltered, and assume $\gamma < b$, but suppose that the firm can observe the worker's discretionary effort decision with probability α. The workers' preferences are modified as follows: an employed worker's payoff is $w - dx$, where d is his cost of discretionary effort.

Since $\gamma < b$, the incentive constraint that induces workers to exert effort must hold in order for firms to earn positive profits. In SPE, both this constraint and the IR constraint will be binding. As a result, equilibrium wage at period t will be $b\theta_t + d/(1-\alpha)$. Firms will therefore choose $r_t = 1$ if and only if $\theta_t \geq d/(1-\alpha)(1-b)$. This means that separation will be more frequent when productivity is low, hence the effect on tightness volatility will be in the same direction as in our model. However, the equilibrium wage is linear in θ, as in the benchmark model, which means that the model does not generate wage rigidity. In addition, it makes no distinction between newly hired and existing workers.

5 Stationary Exogenous Separation

Our focus in previous sections on a two-period process of exogenous separation enabled us to obtain a complete analytical characterization of SPE. In this section we provide partial equilibrium characterizations under the stationary exogenous separation process most often assumed in the literature: $s(1) = 0$ and $s(i) = s \in (0, 1)$ for every $i > 1$. We assume throughout that $\gamma < b$. Complete characterization of SPE under this process is an open problem. In this section we will present two examples of simple, tractable SPE that are Markovian w.r.t (θ_{t-1}, θ_t). We begin with an equilibrium that involves no destruction of output. Such an equilibrium exists when μ is sufficiently high.

Proposition 5 *Assume that $\Psi(\theta_*) > \frac{1+b}{1-b}$. Then, the following behavior constitutes an SPE in the $\lambda \to 1$ limit:*

(i) $r_{i,t} = x_{i,t} = 1$ at any period t and for any worker type i.

(ii) At any period t, a newly matched worker ($i = 1$) earns $b[\theta_t - \delta(1-s)]$.

(iii) At any period t, an existing worker ($i > 1$) earns $b[\Psi(\theta_{t-1}) + 1 - \delta(1-s)]$.

In this equilibrium, the stationarity of the separation process enables identical treatment of all existing workers. Their wage at any period t is absolutely rigid w.r.t current productivity; it is equal to the highest possible wage that newly matched workers could earn at the same period, given θ_{t-1}. Newly matched workers' wage is linear in current productivity, and lies below the benchmark level. Since there is no destruction of output in equilibrium, tightness volatility is the same as in the $\gamma = 1$ benchmark.

Note that in the $s \to 0$ limit, newly matched workers earn the same wage as in the SPE obtained under the two-period separation process (see Proposition 2), when $\Psi(\theta_*) > (1+b)/(1-b)$. In both cases, this restriction enables us to sustain $r_{2,t} = 1$ at any t in SPE. And in both cases, existing workers earn a total discounted rent of b relative to the outside option; the difference is that under the two-period separation process, the rent is earned in a single period, whereas in the stationary model it is smoothed over the entire infinite horizon.

Let us now turn to an equilibrium that does exhibit destruction of output, in the relatively simple case in which productivity shocks are $i.i.d$ - i.e., $\rho = 0$. We make certain parametric restrictions for expositional simplicity.

Proposition 6 *Assume $\mu > 2$. If b is sufficiently close to one, there exists a stationary SPE in the $(\lambda, \delta(1-s)) \to (1,1)$ limit, in which:*

(i) $r_{1,t} = 1$ for every period t, and $r_{2,t} = 1$ if and only if $\varepsilon_t \geq \varepsilon^*$, where $\varepsilon^* \in (-1,1)$ is the unique solution of the equation

$$\mu(1-b) = \int_{-1}^{\varepsilon^*} [\varepsilon^* - b - \varepsilon(1-b)] f(\varepsilon) d\varepsilon$$

(ii) At any period t, a newly matched worker ($i = 1$) earns

$$w_{1,t}(\varepsilon_t) = b\left(\mu + \varepsilon_t - \int_{\varepsilon^*}^{1} (1-\varepsilon) f(\varepsilon) d\varepsilon\right)$$

(iii) At any period t, an existing worker ($i > 1$) earns a constant wage of

$$w^* = b\left(\mu + 1 - \int_{\varepsilon^*}^{1} (1-\varepsilon) f(\varepsilon) d\varepsilon\right)$$

(iv) All employed workers choose $x = 1$.

This SPE is Markovian w.r.t ε_t. That is, only current productivity shocks are relevant for equilibrium behavior. Existing workers' wage is constant in equilibrium; it is equal to the highest possible wage for a newly matched worker. Existing workers are dismissed whenever the current productivity shock falls below the cutoff ε^*. Newly matched workers' equilibrium wage is equal to their $\gamma = 1$ benchmark wage minus a constant. The restriction on b ensures that layoffs will occur with positive probability in equilibrium. When $\varepsilon_t = \varepsilon^*$, the short-run loss from retaining an existing worker and paying him his reference wage is exactly offset by the long-run gains that his continual employment generates. Note that since $\rho = 0$, the elasticity of market tightness with respect to current productivity is zero, because the value of a vacancy filled at the beginning of period $t+1$ is a function of θ_{t+1}, which is independent of θ_t, the level productivity at the time the vacancy was most recently posted.

6 Weakening the Propensity for Negative Reciprocity

Recall that the parameter $\lambda \in (0,1)$ captures the worker's propensity for negative reciprocity. As λ gets lower, the employer can get away with larger wage cuts relative to the worker's reference point, without triggering a reduction in the worker's effort. So far, we analyzed SPE in the $\lambda \to 1$ limit. In this sub-section, we discuss SPE when λ is bounded away from one.

For simplicity, we assume that $f \equiv U[-1,1]$ and consider the case of $\gamma < b$. Under these restrictions, the game has a unique SPE, which is qualitatively very similar to the SPE derived under the $\lambda \to 1$ limit when $\gamma \in (b,1)$ (see Section 3.2). For every θ_{t-1}, existing workers' reference wage at period t is the unique solution below $\lambda b(\Psi(\theta_{t-1})+1)$ to the equation

$$e(\theta_{t-1}) = \frac{\int_{\varepsilon^*(\theta_{t-1})}^{1} \max[\lambda e(\theta_{t-1}), b(\Psi(\theta_{t-1}) + \varepsilon)]d\varepsilon}{1 - \varepsilon^*(\theta_{t-1})}$$

where

$$\varepsilon^*(\theta_{t-1}) = \max[\lambda e(\theta_{t-1}) - \Psi(\theta_{t-1}), -1]$$

Treatment of existing workers follows the three-piece partition of the set of productivity shocks, which we already observed in Section 3.2. At any period t, firms retain an existing worker if and only if $\varepsilon_t \geq \varepsilon^*(\theta_{t-1})$. Moreover, the reference wage increases with θ_{t-1} at a rate below $1/\lambda$, such that the cutoff $\varepsilon^*(\theta_{t-1})$ decreases with θ_{t-1}. Conditional on retaining an existing worker, firms pay him

$$w_2(\theta_{t-1}, \theta_t) \equiv \max[\lambda e(\theta_{t-1}), b\theta_t]$$

Thus, existing workers' wage exhibits downward rigidity - it coincides with the flexible $\gamma = 1$ benchmark following high productivity shocks, and it is rigid w.r.t current productivity following intermediate shocks.

As in the basic model, a newly matched worker at period t is always hired in equilibrium; his wage at period t is a function $w_1(\theta_t)$ which is positive-valued and uniquely determined by

$$w_1(\theta_t) \equiv b\theta_t - \frac{\delta}{2} \int_{\varepsilon^*}^{\frac{\lambda}{b} e(\theta_t) - \Psi(\theta_{t-1})} (\lambda e(\theta_t) - b\theta_{t+1}) d\varepsilon$$

Workers accept all wage offers on the equilibrium path and choose $x = 1$. Finally, as far as market tightness is concerned, the same observations made in Section 3.2 apply here.

7 Concluding Remarks

Our objective in this paper was to formalize the idea that reciprocal fairness and morale considerations affect the labor market's response to macroeconomic fluctuations, in the context of an S&M model. In our model, as in Akerlof (1982), workers' morale (and consequently their willingness to exert unobserved effort) is damaged when their wage falls below a reference point. Following Kőszegi and Rabin (2006), we assumed that existing workers' reference point is a function of their lagged wage expectations. The equilibrium predictions of the model are that existing workers' wages display downward rigidity with respect to macroeconomic shocks, while entry-level wages are lower and more flexible. The main open problem is to provide a complete characterization of SPE under general exogenous separation processes. Extending the model to other bargaining protocols is an additional interesting avenue for future research.

We believe that the model is capable of producing additional insights, some of which were made informally by Bewley (1999) on the basis of his survey. Here we make do with a brief description.

Part-time jobs. Suppose that a firm's hiring/retention decision is not binary, but any real number $r \in [0, 1]$, such that an interior r corresponds to a part-time job.

Suppose further that wages are stated for full-time positions, such that an employed worker's total wage earnings are rw. It makes sense to assume that an existing worker's reciprocal-fairness considerations will rely on w (expected versus actual), rather than on rw. This means that if a firm moves its worker from full- to part-time employment without cutting w below its lagged-expected value, this will not be construed as unfair behavior, and the worker will exert normal effort. It follows that following a bad productivity shock, a firm may prefer this option to the alternative of keeping the worker at full-time employment while lowering his wage, even in circumstances where this would have been sub-optimal in a reference-independent model. Exploring this role of part-time jobs in curbing the effects of wage rigidity is left for future research.

The role of inflation. Discussions of wage rigidity often involve a distinction between real and nominal wages and the mitigating role of inflation. In a model with reference dependence, this distinction is traced to an assumption as to whether the reference point is formed in nominal or real terms. If the reference point is stated in terms of (lagged-expected) nominal wages, then it is not surprising that unexpected inflation can have real (yet temporary) effects on the labor market, by lowering the reference point in real terms, and therefore making the PIC constraint less likely to be binding.

We would like to conclude the paper with a discussion of alternative reference-point formation rules. In Appendix B, we examine a close variation on our model, in which the reference point of workers of *any* type is equal to their lagged-expected monetary earnings, thus endogenizing the distinction between newly matched and existing workers. The main qualitative results of our model are reproduced under this alternative specification. Another variant would abandon the lagged-expectation component, and assume that the worker's reference point at period t is equal to his *actual* wage earnings at period $t-1$. The technical difficulty with this specification is that equilibrium behavior depends on the entire history of realizations of θ. The reason is that the market outcome at $t-1$ is a function of the worker's reference point at $t-1$, which is a function of the market outcome at $t-2$, and so forth. In contrast, in our model the worker's reference point at t is determined by the worker's expectations at $t-1$, and hence it is a function of θ_{t-1} rather than the equilibrium outcome at $t-1$. This ensures the simple Markovian structure of SPE in our model.

The reference point that conditions the worker's effort decision could be a function of variables other than the worker's own (expected) wage. For instance, it could be the wage earned by his peers. Alternatively, the reference point could represent a fair share of his output. In fact, we have analyzed such a model, under the assumption that the worker considers receiving a fraction $\beta < b$ of lagged-expected output to be fair. The main results are qualitatively the same as the ones presented here. Exploring richer models of reference dependence and reciprocal fairness in the context of S&M models is an important challenge for future research.

8 Appendix A: Proofs

Let us first introduce some notation that will serve us in several proofs. Fix an SPE.

Unemployed workers' payoff. Recall that for a given firm-worker pair, the only observable aspect of the history prior to their match is the sequence of realizations of θ. In particular, it does not matter whether the worker's unemployment at t is due to a matching failure, a firm's decision not to hire him, or his own decision to reject a wage offer. Therefore, we can denote an unemployed worker's equilibrium continuation payoff at t by $W_0(\theta_0, ..., \theta_t)$, without loss of generality.

Employed workers' payoff. Let h_t be the information set of a given firm-worker matched pair at period t, where the worker is of type i at t. Let (h_t, w_t) denote the immediate concatenation in which the firm hires/retains the worker and makes the offer w_t. Let $W_i(h_t, w_t)$ denote the worker's equilibrium continuation payoff at (h_t, w_t), where the subscript i clarifies the worker's type at t. Let $W_0(h_t)$ denote his continuation payoff if he rejects the wage offer that the firm makes and thus becomes unemployed at t. By definition, $W_i(h_t, w_t) \geq W_0(h_t)$.

Employed workers' rent. Let $B(\theta)$ denote a worker's continuation payoff from the strategy of rejecting all wage offers when the current state is θ. Define $R(h_t, w_t) = W(h_t, w_t) - B(\theta_t)$. Note that $R(h_t, w_t) \geq 0$, since workers can always implement the strategy of rejecting all offers. In addition, $R(\cdot)$ is bounded from above because firms will never make offers that generate negative profits.

8.1 Proof of Proposition 1

Define R^* as the maximum of $R(h, w)$ over all histories (h, w). In general, the maximum need not be well-defined, and complete rigor demands it to be replaced with the sup. However, this would complicate our analysis in a way we find superfluous. Thus, to simplify exposition, we deal with the case in which R^* is well-defined and attained in some finite history (h_t, w_t) by a worker of some type i.

Suppose that $w_t = 0$ - i.e., the non-negativity constraint is binding at (h_t, w_t). Since $w_t - b\theta_t < 0$, it cannot be the case that $R(h_t, w_t) = R^*$. Thus, the non-negativity constraint must hold with slack at (h_t, w_t). It follows that the IR constraint is binding at (h_t, w_t) - otherwise, the firm can slightly lower the worker's wage without changing his subsequent behavior.

By the definition of R^*, $W_i(h_t, w_t) \geq W_1(h_t, w_t)$ and

$$W_1(h_t, \theta_{t+1}, w_{t+1}) - B(\theta_{t+1}) \leq W_i(h_t, w_t) - B(\theta_t) \tag{11}$$

for any realization of θ_{t+1} and a wage offer w_{t+1} made to a newly matched worker at $t+1$. Observe that

$$W_0(h_t) = b\theta_t + \delta[q_t \cdot EW_1((h_t, \theta_{t+1}, w_{t+1}) \mid \theta_t) + (1 - q_t) \cdot EW_0((h_t, \theta_{t+1}) \mid \theta_t)] \tag{12}$$

where q_t is the probability that an unemployed worker at t finds a match. The determinants of q_t are immaterial for our purposes. Since $W_0(h_t, \theta_{t+1}) \leq W_1(h_t, \theta_{t+1}, w_{t+1})$, we obtain from (12) that

$$W_0(h_t) \leq b\theta_t + \delta E(W_1(h_t, \theta_{t+1}, w_{t+1}) \mid \theta_t)$$

Since the IR constraint is binding at (h_t, w_t), $W_i(h_t, w_t) = W_0(h_t)$. Using (11) we may therefore conclude that

$$W_i(h_t, w_t) = W_0(h_t) \leq b\theta_t + \delta W_i(h_t, w_t) + \delta EB(\theta_{t+1} \mid \theta_t) - \delta B(\theta_t)$$

Since $b\theta_t + \delta EB(\theta_{t+1} \mid \theta_t) = B(\theta_t)$, we have $W(h_t, w_t) \leq B(\theta_t)$, hence $R^* = 0$.

By the definition of R^*, it follows that for any worker type i and any (h_t, w_t) along the equilibrium path, $W_i(h_t, w_t) = B(\theta_t)$. Thus, if the worker accepts the wage offer, we have

$$W_i(h_t) = w_t + \delta E(W_{i+1}(h_t, \theta_{t+1}, w_{t+1}) \mid \theta_t) = b\theta_t + \delta EB(\theta_{t+1} \mid \theta_t)$$

and this implies $w_t = b\theta_t$. Finally, there cannot be a SPE in which a worker rejects an offer of $b\theta_t$ at some period t because the firm could profitably deviate by slightly raising the wage.

8.2 Proof of Proposition 2

We first prove a pair of lemmas that will serve us in several proofs. In particular, they hold for any γ. Define R^{**} as the maximum of $R(h,w)$ over all histories (h,w) in which a *newly matched worker* responds to a wage offer.

Lemma 2 *Let (h_t, w_t) be a history in which a newly matched worker responds to a wage offer, for which $R(h_t, w_t) = R^{**}$. If the IR constraint is binding at (h_t, w_t), then $R^{**} = 0$.*

Proof. By the definition of R^*, $W_1(h_t, \theta_{t+1}, w_{t+1}) - B(\theta_{t+1}) \leq W_1(h_t, w_t) - B(\theta_t)$. The proof that $R^{**} = 0$ reproduces exactly the same steps that led us to conclude that $R^* = 0$ in the proof of Proposition 1. (Note that here we simply assume that IR is binding at (h_t, w_t), rather than deriving this property.) ■

Lemma 3 *In SPE, $w_{1,t} > 0$ at any period t.*

Proof. If $\gamma = 1$, this follows from Proposition 1. Assume $\gamma < 1$. Let $\bar{w}_i^t$ denote the participation wage of a worker of tenure $i = 1, 2$ at period t (implicitly, given the history). Let W_0^t be the reservation payoff of workers at period t (again, the history is implicit) - that is, their payoff if they reject their wage offer at t and join the pool of unemployed workers. Recall that this payoff is independent of the worker's type at t. Let R_i^t denote the rent (i.e., excess payoff above the reservation payoff) that a worker of type i gets at period t. If the worker is unemployed at t, we write $R_i^t = 0$. The following equations hold, by the definition of these objects:

$$\bar{w}_2^t + \delta E(W_0^{t+1} \mid \theta_t) = W_0^t$$
$$\bar{w}_1^t + \delta E(W_0^{t+1} \mid \theta_t) + \delta E(R_2^{t+1} \mid \theta_t) = W_0^t$$

Therefore,

$$\bar{w}_1^t = \bar{w}_2^t - \delta E(R_2^{t+1} \mid \theta_t) \tag{13}$$

Moreover, since

$$W_0^t = b\theta_t + \delta E(W_0^{t+1} \mid \theta_t) + \delta q_t E(R_1^{t+1} \mid \theta_t)$$

we obtain

$$\bar{w}_2^t = b\theta_t + \delta q_t E(R_1^{t+1} \mid \theta_t) \tag{14}$$
$$\bar{w}_1^t = b\theta_t + \delta q_t E(R_1^{t+1} \mid \theta_t) - \delta E(R_2^{t+1} \mid \theta_t) \tag{15}$$

If the IR constraint of a newly matched worker is binding at t, then his period-t wage is equal to his period t reservation wage and $R_1^t = 0$. If his PIC constraint is binding at t, then the actual wage at t is zero, and $R_1^t = -\bar{w}_1^t$. If $\bar{w}_1^t < 0$ ($\bar{w}_1^t > 0$), then the PIC (IR) constraint is binding. Therefore, $R_1^t = \max\{0, -\bar{w}_1^t\}$.

Let R^* and R_* denote the maximum and minimum values that R_1^t can attain at any t. By definition, $R_* \geq 0$. Assume that $R^* > 0$. Let w_* denote the minimum value that $\bar{w}_1^t$ may obtain at any t. Then $R^* = -w_*$, where $w_* < 0$. From (15) it follows that

$$\bar{w}_1^t = b\theta_t + \delta q_t E(R_1^{t+1} \mid \theta_t) - \delta E(R_2^{t+1} \mid \theta_t)$$

where $\delta E(R_2^{t+1} \mid \theta_t)$ is smaller or equal to the sum

$$\delta[\max_{\theta_{t+1}|\theta_t} (b\theta_{t+1} + \delta q_{t+1} E(R_1^{t+2} \mid \theta_{t+1})) + E(b\theta_{t+1} + \delta q_{t+1} E(R_1^{t+2} \mid \theta_{t+1}) \mid \theta_t)]$$

which in turn is lower or equal to

$$\delta b + \delta^2 \max_{\theta_{t+1}|\theta_t} [q_{t+1} E(R_1^{t+2} \mid \theta_{t+1})] - \delta^2 E[q_{t+1} E(R_1^{t+2} \mid \theta_{t+1}) \mid \theta_t]$$

Note that

$$\begin{aligned} \max_{\theta_{t+1}|\theta_t} [q_{t+1} E(R_1^{t+2} &\mid \theta_{t+1})] \leq R^* \\ E[q_{t+1} E(R_1^{t+2} &\mid \theta_{t+1}) \mid \theta_t] \geq 0 \\ q_t E(R_1^{t+1} &\mid \theta_t) \geq 0 \end{aligned}$$

Hence, for any t,

$$\bar{w}_1^t \geq b(\min \theta_t - \delta) - \delta^2 R^* \tag{16}$$

Since $R^* = -w^*$, inequality (16) holds for every t only if it holds at the lowest possible value of $\bar{w}_1^t$, i.e., only if

$$w_* \geq b(\min \theta_t - \delta) - \delta^2(-w_*)$$

which implies

$$w_* \geq \frac{b(\min \theta_t - \delta)}{1 - \delta^2} > 0$$

where the last inequality follows from out assumption that $\min \theta_t < 1$. But this contradicts our assumption that $w_* < 0$. It follows that $R^* = 0$, and this establishes the result. ■

The rest of the proof proceeds in two steps. First, we use the above lemmas to derive the retention decision, reference point and equilibrium wages for existing workers. Second, we compute the hiring decision and equilibrium wages for newly matched workers. Since by assumption $e_{1,t} = 0$, Lemma 3 implies that the PIC constraint of newly matched workers holds with slack after every history. Therefore, their IR constraint must be binding after every history.

Step 1: Existing workers

Let us first show that an existing worker at period t will accept a wage offer $w_{2,t}$ if and only if $w_{2,t} \geq b\theta_t$. This is his last period of employment. If he rejects the firm's offer, he will be unemployed and earn a payoff of $b\theta_t$ at t. We have seen that newly matched workers' IR is binding after every history. By Lemma 2, it follows that the worker's equilibrium continuation payoff from period $t+1$ onwards is the same as if he were to receive $b\theta_s$ in every period $s \geq t+1$. Therefore, the existing worker's participation constraint at t will be binding if he receives a payoff of $b\theta_t$. Note that this result holds for all γ.

Next, let us show that since $\gamma < b$, $r_{2,t} = 1$ if and only if $\theta_t \geq e_{2,t}$, and $w_{2,t} \geq \max(e_{2,t}, b\theta_t)$ conditional on $r_{2,t} = 1$. If $w_{2,t} < \lambda e_{2,t}$, an existing worker at t will choose $x = 0$, and the firm's output will be $\gamma\theta_t$. Since $w_{2,t} \geq b\theta_t$, the firm's profit is below $\theta_t(\gamma - b) < 0$. Since this is the last period of the firm's interaction with the worker, it would rather fire the worker. Thus, conditional on $r_{2,t} = 1$, $w_{2,t} = \max\{\lambda e_{2,t}, b\theta_t\}$. Note that when $\theta_t = e_{2,t}$, this means that the firm pays $w_{2,t} = e_{2,t}$ and it is indifferent between retaining and firing the worker. At any $\theta_t < e_{2,t}$, the firm fires the worker.

It follows that $e_{2,t}$ is determined by the equation

$$e_{2,t} = E[\max\{\lambda e_{2,t}, b(\theta_t)\} \mid \theta_{t-1}] \tag{17}$$

We first claim that for any $\lambda < 1$, $w_{2,t} \leq b(\Psi(\theta_{t-1}) + 1)$ in every state θ_t (hence $e_{2,t} \leq b(\Psi(\theta_{t-1}) + 1)$). Assume the contrary, i.e., that $w_{2,t} > b(\Psi(\theta_{t-1}) + 1) \geq b\theta_t$ at some period-t history. Let w_2^* be the highest wage paid to existing workers at any history. Then, $e_{2,t} \leq w_2^*$. Therefore, if the employer at t lowered the wage to $\max(b\theta_t, \lambda w_2^*)$, the worker would still accept the offer and exert normal effort, hence the deviation would be profitable. Therefore, w_2^* cannot be ever offered in equilibrium, a contradiction. It follows that $w_{2,t} \leq b(\Psi(\theta_{t-1}) + 1)$ at any t in equilibrium. When $\lambda \to 1$ the unique solution to (17) is $e_{2,t} = b(\Psi(\theta_{t-1})+1)$.Therefore, $w_{2,t} = b(\Psi(\theta_{t-1})+1)$ conditional on $r_{2,t} = 1$. This completes the treatment of existing workers at any period t in the $\lambda \to 1$ limit. From now on, all quantities will refer to this limit case.

Step 2: Newly matched workers

A newly matched worker at period t expects to earn the discounted sum of payoffs in periods t and $t+1$:

$$w_{1,t} + \delta E[r_{2,t+1} w_{2,t+1} + (1 - r_{2,t+1}) b\theta_{t+1} \mid \theta_t] \tag{18}$$

We have already noted that a new worker's SPE continuation payoff is as if he receives $b\theta_t$ in every period t. Hence, in any SPE, the expected, discounted sum in (18) must equal $b\theta_t + \delta E(b\theta_{t+1} \mid \theta_t)$. By Step 1, $r_{2,t+1} = 1$ and $w_{2,t+1} = e_{2,t+1}$ if and only if $\theta_{t+1} \geq e_{2,t+1}$, where $e_{2,t+1} = b(\Psi(\theta_t) + 1$. It follows that $w_{1,t}$ is given by (**??**). To see why $r_{1,t} = 1$ regardless of the history, note that in the second period of the interaction between the firm and the worker, the firm necessarily earns non-negative profits. The newly matched worker at t plays $x = 1$ because we saw that his PIC constraint is satisfied. Therefore, he generates an output of θ_t. Since he is paid at most $b\theta_t$, the firm earns strictly positive profits, and therefore would always prefer to hire the worker.

8.3 Proof of Proposition 3

Let θ be the realized productivity at the time of opening a vacancy. To simplify the exposition, denote $\Psi(\theta) = z$ and let $a \in [-1, 1]$. For a given z, define

$$l(z) = \frac{\delta^2 \int_{-1}^{1} \left(\int_{-1}^{\varepsilon^*(z+a)} [\Psi(z+a) + \varepsilon] f(\varepsilon) d\varepsilon \right) f(a) da}{z + \delta\Psi(z)}$$

Assume $\theta < \theta$, so that $\varepsilon^*(z-1) > -1$. Define $\bar{a}$ by $\varepsilon^*(z + \bar{a}) = -1$, and let $a^* = \min(1, \bar{a})$. By Lemma 1, the elasticity of $\eta(\theta)$ with respect to θ is higher than in the $\gamma = 1$ benchmark if and only if $\partial l(z)/\partial z < 0$. Straightforward differentiation and algebraic manipulation confirms that this is indeed the case, under the parametric restrictions assumed.

8.4 Proof of Proposition 4

From Lemma 3 it follows that in any SPE, the PIC constraint of new hires will have slack after every history. Therefore, the IR constraint of new hires must be binding after every history. We have observed in the proof of Proposition 2 that existing workers' participation constraint at period t is binding if he receives a payoff of $b\theta_t$, independently of γ. Therefore, at period t, the firm must offer the worker at least $b\theta_t$, conditional on retaining him. Because $\gamma > b$, firing a worker at time t is strictly inferior to retaining him and paying him the participation wage $b\theta_t$, even if the worker plays $x = 0$ in return. Hence, $r_{2,t} = 1$ at any t. It follows that if $\theta_t - e_{2,t} \geq \theta_t(\gamma - b)$, the firm will pay the worker $w_{2,t} = \max(b\theta_t, e_{2,t})$ and the worker will play $x = 1$ in return, and if $\theta_t - e_{2,t} < \theta_t(\gamma - b)$, the firm will pay the worker $w_{2,t} = b\theta_t < e_{2,t}$ and the worker will play $x = 0$ in return.

It follows that $e_{2,t}$ is determined by equation (8). To establish existence of a solution to this equation, note first that the R.H.S is a continuous function of $e_{2,t}$. Second, from the proof of Proposition 2 it follows that for any $\lambda < 1$ the maximal value that the R.H.S can take is $\bar{e} = b(\Psi(\theta_{t-1}) + 1)$. (This proof is independent of γ.) Hence, we can view the R.H.S as a continuous mapping from $[0, \bar{e}]$ to itself. By Brouwer's fixed-point theorem, this mapping has a fixed point. To see that this fixed point is unique, differentiate both sides of the equation w.r.t $e_{2,t}$. For brevity, denote $e_{2,t} = e$ and $f(\theta_t \mid \theta_{t-1}) = f(\theta_t)$. Let F denote the *cdf* induced by f. The derivative of the L.H.S w.r.t e is 1, while the derivative of the R.H.S w.r.t e is

$$-\frac{(1-b)e}{1-\gamma+b} \cdot f(\frac{be}{1-\gamma+b}) + F(\frac{e}{b}) - F(\frac{e}{1-\gamma+b}) < 1$$

Since the slope of the R.H.S is always strictly lower than the slope of the L.H.S, there can be at most one point in which the functions on the two sides of the equation intersect, hence precisely one fixed point. We have thus proved parts $(i) - (ii)$ of the result.

From the previous paragraphs it follows that new worker at time t expects to earn the following discounted sum of payoffs at times t and $t+1$,

$$w_{1,t} + \delta E[b\theta_{t+1} \mid \theta_{t+1} < \frac{e(\theta_t)}{1-\gamma+b} \quad or \quad \theta_{t+1} > \frac{e(\theta_t)}{b}] + \delta E[e(\theta_t) \mid \frac{e(\theta_t)}{1-\gamma+b} < \theta_{t+1} < \frac{e(\theta_t)}{b}] \tag{19}$$

We have already noted that a new worker's SPE continuation payoff is as if he receives $b\theta_t$ in every period t. Hence, by equating (19) to $b\theta_t + \delta E(b\theta_{t+1} \mid \theta_t)$ and using the expression for $e(\theta_t)$ given by (8), we obtain that $w_{1,t}$ is given by (10). Finally, $r_{1,t} = 1$, by the same argument as in the proof of Proposition 2.

8.5 Proof of Proposition 5

Since $\theta_* > 1$, the wage that newly matched workers are assumed to receive is strictly above zero, hence their PIC constraint holds with slack. Let us assume that their IR constraint is binding at every history. Therefore, at period t, a worker of any type i will accept any wage offer that gives him a continuation payoff weakly above the payoff from permanent unemployment, namely $B(\theta_t)$. By assumption, if the worker accepts a wage offer w_t, then at any future period $t' > t$, he will be regarded as an existing worker, and the firm will choose to retain the worker (whenever the two are not exogenously separated) and pay him $b[\Psi(\theta_{t'-1}) + 1 - \delta(1-s)]$. In other words, as long as the two parties are not exogenously separated, the worker will get his outside option plus a fixed rent of $b(1-\delta(1-s))$. The total expected discounted rent is b conditional on being re-matched with the firm at the beginning of period $t+1$. Therefore, the worker will accept the offer w_t if and only if

$$w_t + \delta EB(\theta_{t+1} \mid \theta_t) + \delta(1-s)b \geq B(\theta_t) = b\theta_t + \delta EB(\theta_{t+1} \mid \theta_t)$$

or, equivalently

$$w_t \geq b(\theta_t - \delta(1-s)) \tag{20}$$

By the same reasoning as in the proof of Proposition 2, in the $\lambda \to 1$ limit, the reference point for a worker of type $i > 1$ at t is equal to his highest possible participation wage at t, given θ_{t-1}. Therefore, $e_{i,t} = b[\Psi(\theta_{t-1}) + 1 - \delta(1-s)]$. If $\Psi(\theta_*) > (1+b)/(1-b)$, a firm paying an existing worker his reference wage at t will not incur a loss, and therefore in equilibrium $r_{i,t} = 1$ for any t and all $i > 1$. Let us turn to newly matched workers. As noted above, the R.H.S of inequality (20) is strictly positive, such that if the firm offered it, the worker's PIC constraint would hold with slack. We have confirmed that firms make positive profits at every period when retaining existing workers. Thus, $r_{1,t} = x_{1,t} = 1$ at every period t, and $w_{1,t}$ is as guessed.

8.6 Proof of Proposition 6

Consider the acceptance decision of a worker at any period t. The analysis proceeds along the same lines as in the proof of Proposition 5, with two differences. First, the stationary separation probability at any $t' > t$ is not s, but the endogenous separation rate $F(\varepsilon^*)$. In addition, conditional on being employed at t', the worker receives the outside option plus a rent of

$$b[1 - \varepsilon_t - \int_{\varepsilon^*}^{1} (1-\varepsilon) f(\varepsilon) d\varepsilon]$$

The total expected discounted rent is therefore

$$\frac{b\int_{\varepsilon^*}^1 [1-\varepsilon-\int_{\varepsilon^*}^1 (1-\varepsilon)f(\varepsilon)d\varepsilon]f(\varepsilon)d\varepsilon}{F(\varepsilon^*)}$$

which is equal to

$$b\int_{\varepsilon^*}^1 (1-\varepsilon)f(\varepsilon)d\varepsilon$$

Therefore, the worker will accept the offer w_t if and only if

$$w_t + \delta E B_{t+1} + b\int_{\varepsilon^*}^1 (1-\varepsilon)f(\varepsilon)d\varepsilon \geq V_t$$

Since $\rho = 0$, $B_t = b(\theta_t + \delta\mu/(1-\delta))$. Therefore, the worker's participation wage is

$$b\left(\mu + \varepsilon_t - \int_{\varepsilon^*}^1 (1-\varepsilon)f(\varepsilon)d\varepsilon\right)$$

as we guessed. As before, existing workers' reference point is equal to the highest possible participation wage:

$$b\left(\mu + 1 - \int_{\varepsilon^*}^1 (1-\varepsilon)f(\varepsilon)d\varepsilon\right)$$

as guessed.

It remains to verify our expression for ε^*. When a firm interacts with an existing worker at period t, it pays him a wage of $b(\mu + 1 - \int_{\varepsilon^*}^1 (1-\varepsilon)f(\varepsilon)d\varepsilon)$ and generates an output of $\mu + \varepsilon_t$. If b is sufficiently close to one, the firm will incur a loss at period t for low realizations of ε_t. In each future period t' in which the worker is employed, the firm earns a profit of

$$(\mu + \varepsilon_{t'}) - b(\mu + 1 - \int_{\varepsilon^*}^1 (1-\varepsilon)f(\varepsilon)d\varepsilon)$$

The cutoff ε^* is the noise realization ε_t for which the loss at period t is exactly offset by the expected discounted profits at all periods $t' > t$.

8.7 Appendix B: Endogenizing the distinction between new and existing workers

In the paper, we assumed a reference-point formation rule that imposed an exogenous distinction between newly matched and existing workers. One could argue that there are endogenous reasons for such a distinction. In particular, they have different employment prospects: the probability that a newly hired worker at t is employed at $t+1$ is a function of his employer's equilibrium retention policy, while the probability that an unemployed worker at t is employed at $t+1$ is a function of market tightness at t and firms' hiring policy.

In this appendix, we modify the reference-point formation rule in order to capture this consideration and endogenize the distinction between the reference points of workers of different types. Before we do so, we introduce an element which has been neutralized in the basic model, namely the workers' value of leisure (equivalently, the cost of monitored, contractible effort). Assume it is A, such that if a worker is unemployed at t, his total payoff is $A + b\theta_t$, where $b\theta_t$ is his *monetary* non-market benefit (and A is his non-pecuniary non-market benefit). In the basic model, we set A to zero, as a simplifying assumption we could afford to make without compromising essential features of the model. We are no longer able to do so.[7]

Assume that at any period t and for any worker type $i = 1, 2$, the worker's reference point is equal to his *expected monetary earnings* conditional on his information at the end of period $t-1$. This reference point formation rule puts newly matched and existing workers in the same footing a priori. However, the difference in their employment prospects translates into different reference points. Specifically, the period-t reference points for newly matched and existing workers are

$$
\begin{aligned}
e_{1,t} &= (1 - q_{t-1}) \cdot b\Psi(\theta_{t-1}) + q_t \cdot E[r_{1,t}w_{1,t} + (1 - r_{1,t})b\theta_t] \\
e_{2,t} &= E[r_{2,t}w_{2,t} + (1 - r_{2,t})b\theta_t]
\end{aligned}
$$

where the expectation over$(\theta_t, (r_{i,t}, w_{i,t})_{i=1,2})$ is conditional on the worker's information set at the end of $t-1$. In the $\lambda \to 1$ limit, the worker exerts normal effort unless his wage falls below his lagged-expected monetary earnings.

SPE characterization under this alternative reference-point formation rule differs

[7]When $A > \theta^*(\gamma - b)$, Proposition 2 and Corollary 1 hold unaltered, except that we need to add the constant A to workers' wage. When the condition is violated, equilibrium characterization is more complex, as both layoffs and wage cuts may occur with positive probability in equilibrium, depending on the realization of ε_t.

from the analysis of the model of Section 2, because it is no longer true that newly matched workers' IR constraint is always binding in equilibrium. However, when A is sufficiently large and $m(1,1)$ is sufficiently small, there exists a SPE that exhibits this feature, and consequently replicates the main qualitative features of SPE in the basic model. For simplicity, we focus on the case of $\gamma < b$.

Proposition 7 *Let $\gamma < b$. If*

$$m(1,1) < \min\left[\frac{c}{\Pi(\theta^*)}, 1 - \frac{2b}{A}\right] \tag{21}$$

the game has a SPE in the $\lambda \to 1$ limit with the following properties.

(i) An existing worker's period-t reference point $e_{2,t}$ is a function $e(\theta_{t-1})$ which is the unique solution of the following equation:

$$e(\theta_{t-1}) \equiv \int_{\Psi(\theta_{t-1})-1}^{e(\theta_{t-1})} b\theta_t f(\theta_t \mid \theta_{t-1}) d\theta_t + \int_{e(\theta_{t-1})}^{\Psi(\theta_{t-1})+1} \max[e(\theta_{t-1}), A + b\theta_t] f(\theta_t \mid \theta_{t-1}) d\theta_t \tag{22}$$

(ii) The firm retains an existing worker if and only if $\theta_t \geq e(\theta_{t-1})$; his wage at period t is a function $w_2(\theta_{t-1}, \theta_t)$ which is uniquely determined by

$$w_2(\theta_{t-1}, \theta_t) \equiv \max(e(\theta_{t-1}), A + b\theta_t) \tag{23}$$

(iii) A newly matched worker at period t is always hired; his wage at period t is a function $w_1(\theta_t)$ which is positive-valued and uniquely determined by

$$w_1(\theta_t) \equiv A + b\theta_t - \delta \int_{e(\theta_t)}^{\frac{e(\theta_t)-A}{b}} [e(\theta_t) - (A + b\theta_{t+1})] f(\theta_{t+1} \mid \theta_t) d\theta_{t+1} \tag{24}$$

(iv) Workers accept all wage offers on the equilibrium path and choose $x = 1$.

Proof. Our method of proof is as follows. First, we construct a unique SPE under the assumption that the PIC constraint of newly matched workers holds with slack. Then, we show that this assumption holds under (21).

Step 1: Existing workers

By the same reasoning as in the basic model, an existing worker at period t will accept a wage offer $w_{2,t}$ if and only if $w_{2,t} \geq A + b\theta_t$. Furthermore, $r_{2,t} = 1$ if and only if $\theta_t \geq e_{2,t}$, and $w_{2,t} = \max\{e_{2,t}, A + b\theta_t\}$ conditional on $r_{2,t} = 1$. It follows that $e_{2,t}$ is determined by equation (22). To establish existence of a solution to this equation, note first that the R.H.S is a continuous function of $e_{2,t}$. Second, the maximal value that the R.H.S can take is $\bar{e} = A + b(\Psi(\theta_{t-1}) + 1)$, hence we can view the R.H.S as a continuous mapping from $[0, \bar{e}]$ to itself. By Brouwer's fixed-point theorem, this mapping has a fixed point. To see that this fixed point is unique, differentiate both sides of the equation w.r.t $e_{2,t}$. For brevity, denote $e_{2,t} = e$ and $f(\theta_t \mid \theta_{t-1}) = f(\theta_t)$. Let F denote the *cdf* induced by f. The derivative of the L.H.S w.r.t e is 1, while the derivative of the R.H.S w.r.t e is

$$-(1-b)ef(e) + F(\frac{e-A}{b}) - F(e) < 1$$

Since the slope of the R.H.S is always strictly lower than the slope of the L.H.S, there can be at most one point in which the functions on the two sides of the equation intersect, hence precisely one fixed point. We have thus proved parts $(i) - (ii)$ of the result.

Step 2. Newly matched workers

A new worker at time t expects to earn the following discounted sum of payoffs at times t and $t+1$,

$$w_{1,t} + \delta E[r_{2,t+1}w_{2,t+1} + (1 - r_{2,t+1})b\theta_{t+1} \mid \theta_t] \tag{25}$$

By Lemma 2, a new worker's SPE continuation payoff is as if he receives $b\theta_t$ in every period t. Hence, in any SPE, the expected, discounted sum in (25) must equal

$$A + b\theta_t + \delta E(A + b\theta_{t+1} \mid \theta_t)$$

By Step 1, $r_{2,t+1} = 1$ if and only if $\theta_{t+1} \geq e(\theta_t)$. In addition, conditional on $r_{2,t} = 1$, $w_{2,t} = e(\theta_t)$ if and only if $e(\theta_t) \geq A + b\theta_{t+1}$. Therefore, $w_{1,t}$ is given by (24). To see why $r_{1,t} = 1$ regardless of the history, note that in the second period of the interaction between the firm and the worker, the firm necessarily earns non-negative profits. The newly matched worker at t plays $x = 1$ because by assumption, his PIC constraint is satisfied. Therefore, he generates an output of θ_t. Since he is paid at most $A + b\theta_t$, the firm earns strictly positive profits, and therefore would always prefer to hire the worker.

Step 3: Verifying that newly matched workers' PIC holds with slack

By the expression for newly matched workers' wage (24),

$$w_{1,t} > A + b(\Psi(\theta_{t-1}) - 2)$$

On the other hand, by the same expression and the definition of newly matched workers' reference point,

$$\begin{aligned} e_{1,t} &\leq (1 - q_{t-1}) \cdot b\Psi(\theta_{t-1}) + q_{t-1} \cdot (A + b\Psi(\theta_{t-1})) \\ &\leq q_{t-1}A + b\Psi(\theta_{t-1}) \end{aligned}$$

In order to prove the result, it suffices to show that the lower bound on $w_{1,t}$ is always higher than the upper bound on $e_{1,t}$. A bit of algebra gives us the following sufficient condition:

$$q_{t-1} < 1 - \frac{2b}{A}$$

Recall that by the free entry assumption, the following inequality holds in any equilibrium:

$$p_{t-1} \geq \frac{c}{\Pi(\theta^*)}$$

By the assumption that m satisfies constant returns to scale, $p_{t-1} > m(1,1)$ if and only if $q < m(1,1)$. Therefore, if $m(1,1)$ satisfies condition (21), the sufficient condition for newly matched workers' PIC constraint to hold with slack is satisfied. ■

Define the cutoff $\varepsilon^*(\theta_{t-1})$ as in the case of $\gamma < b$ in the basic model: this is the noise realization ε_t for which firms are indifferent between retaining and dismissing existing workers, under the SPE characterized by Proposition 7.

Remark 1 *$\varepsilon^*(\theta_{t-1})$ decreases with θ_{t-1}.*

The proof involves straightforward differentiation of the R.H.S of (22) and therefore omitted. Thanks to this property, Corollary 1 continues to hold. Since $\varepsilon^*(\theta_{t-1})$ lacks a simple closed-form solution, it is hard to obtain sharper results regarding the elasticity of tightness w.r.t current productivity.

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