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ABSTRACT

Bargaining failures and merger policy*

In this paper we study the optimal ex-ante merger policy in a model where merger proposals are the result of strategic bargaining among alternative candidates. We allow for firm asymmetries and, in particular, we emphasize the fact that potential synergies generated by a merger may vary substantially depending on the identity of the participating firms. The model demonstrates that, under some circumstances, relatively inefficient mergers may take place. That is, a particular merger may materialize despite the existence of an alternative merger capable of generating higher social surplus and even higher profits. Such bargaining failures have important implications for the ex-ante optimal merger policy. We show that a more stringent policy than the ex-post optimal reduces the scope of these bargaining failures and raises expected consumer surplus. We use a bargaining model that is flexible, in the sense that its strategic structure does not place any exogenous restriction on the dendogenous likelihood of feasible mergers.

JEL Classification: L13 and L41
Keywords: bargaining, endogenous mergers, merger policy and synergies

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1 Introduction

In this paper we study merger policy in a model where merger proposals arise endogenously as the outcome of strategic bargaining. We allow for firm asymmetries so that the identity of the merging partners affects the distribution of surplus. In particular, we emphasize the fact that potential synergies generated by a merger may vary substantially depending on the identity of the participating firms.

We make two points. First, we show that relatively inefficient mergers may arise endogenously. Merger policy is typically passive; that is, authorities restrict themselves to approve or reject the merger proposal that has been submitted. In this scenario, a particular merger may materialize despite the existence of an alternative merger capable of generating larger synergies and hence higher profits and consumer surplus. Second, we show that commitment to an ex-ante merger policy that is more stringent than the ex-post optimal policy can alleviate the negative consequences of such bargaining failures.

We use a specific non-cooperative bargaining protocol that treats all firms symmetrically and is very flexible. In particular, its strategic structure does not place any exogenous restriction on the endogenous likelihood of feasible mergers. Moreover, it generates a unique prediction.[1]

We study an abstract industry model that encompasses several standard models of oligopoly. As usual, merger policy focuses on the trade-off between market power and cost efficiencies. The benchmark model considers three firms. In isolation these firms are symmetric. Only two-firm mergers can generate synergies, hence a merger to monopoly will never be authorized. However, different mergers may generate different levels of synergy. In every period firms compete in the market place and also bargain about the possibility of submitting a merger proposal, if one has not been put forward to date. Once an agreement is reached, the market structure shifts from triopoly to duopoly for all future periods. Different pairs of firms can exploit different synergies by merging, and so different reductions in marginal costs. We focus on a scenario where any merger resulting in prices that are lower than in the status quo will be profitable and attractive for the firms involved. That is, the profits of the merged firm are higher than the sum of profits of the merging firms in the status quo, and all firms prefer to be part of a merger rather than be left out of the deal. In this case, it is not surprising that if a consumer surplus enhancing merger exists then firms always reach an agreement immediately. However, the issue is whether the bargaining process will select the most profitable and/or efficient merger. We show that

[1] Our protocol asymptotically implements a new solution concept for cooperative games that we have developed elsewhere (Burguet and Caminal, 2011)

whenever all three mergers would pass the merger review, then the most efficient merger will be proposed with probability one *only* if the cost efficiencies of alternative mergers are sufficiently lower. Otherwise, in the unique equilibrium of our model all feasible mergers occur with positive probability. All firms in such equilibrium are indifferent about their merging partners and as a result any pair may actually arise as a successful merger.

We assume that competition authorities are able to commit ex-ante to a certain approval rule. The authorities' goal is to maximize consumer surplus and they can perfectly monitor the level of synergies that the merger under review would generate. Hence, ex-post (at the moment of reviewing a specific merger proposal) the best merger policy is to accept a merger if and only if it reduces market prices. However, when bargaining failures are possible, making the approval rule more stringent than the optimal ex-ante policy causes two countervailing effects. On the one hand, it precludes some consumer-surplus enhancing mergers. On the other hand, it reduces the probability that the most efficient merger is destabilized by alternative, less efficient ones. If the deviation from the ex-post optimal rule is small, then the former negative effect is of second order, since foregone increases in consumer surplus are also small, but the latter positive effect is of first order. Therefore, the optimal ex-ante policy is always more stringent than the ex-post optimal policy.

The benchmark model describes the implications of alternative market structures using reduced-forms. Both the model and the main results are presented in Section 2. In Section 3 we provide foundations for the reduced forms, showing that some standard oligopoly models fit perfectly in the framework studied in the previous section.

In Section 4 we take a step in extending the analysis to n firms by considering the case where an industry is initially populated by four identical firms, although again only bilateral mergers can generate synergies. With four firms, two consequtive mergers may maximize consumer surplus, and hence the merger review process may become truly dynamic. The analysis is more complex and bargaining failures may take new forms. However, the insights of the three-firm case largely apply. The dynamic nature of policy becomes an issue, but a more stringent merger policy still alleviates the negative consequences of bargaining failures.

In Section 5 we relax the assumption that authorities can perfectly monitor the level of synergies. Instead, we assume that authorities can only observe a noisy signal, whose quality depends on the effort exerted by the firms applying for merger. In other words, firms are given the possibility of an *efficiency defense*, the chance to document their claims about the efficiencies that the merger may generate. However, they are required to sustain those claims with a minimum degree of accuracy. A merger rule then includes two instruments: (i) the minimum level of synergies that must be documented and (ii) the quality of the evidence backing the claim. A higher required quality makes monitoring mistakes less frequent and by imposing a higher cost on the merger, also discourages proposals with relatively small synergies. We show that, as in the benchmark case, the presence of bargaining failures induces a more stringent rule with respect to the threshold level of synergies required. The effect on the required quality level is less clear. Bargaining failures reduce the average efficiency gains of mergers, and hence the social return from a more accurate monitoring technology is lower. In this sense, bargaining failures diminish the authorities' incentive to demand more extensive information gathering efforts. However, given the existence of these bargaining failures, a higher quality requirement helps reducing competition from relatively inefficient mergers, just as in the benchmark model.

A final Section 6 contains conclusions and comments on several additional issues.

Our paper is closely related to three different strands of the literature: endogenous mergers, optimal merger approval rules, and non-cooperative bargaining.

Many studies have focused on how mergers are endogenously determined. This literature typically ignores merger control. Some authors (Barros, 1998; and Horn and Persson, 2001) have approached the problem using cooperative solution concepts for games in partition function form, since a merger creates externalities on non-merging firms. Other authors (Kamien and Zang, 1990; Gowrisankaran, 1999; Inderst and Wey, 2004; Fridolffson and Stennek, 2005a; and Nocke and Whinston, 2011) have set up non-cooperative games where both the market structure and the division of surplus are determined simultaneously. In some of these models there are restrictions on the subsets of firms that can participate in a merger. For example, Inderst and Wey (2004) assume that there is an exogenously designated target and in Nocke and Whinston (2011) nature selects one of the firms as the acquirer. In a similar spirit, Gowrisankaran (1999) assumes that the largest firm is the only one that can acquire a smaller firm. Hence, these studies fail to contemplate the possibility that each member of, say, a three-firm group considers merging with each of the other two, which is a crucial ingredient in the emergence of bargaining failures.[2] In other studies, these restrictions on the feasible merger combinations are not imposed, but all mergers are assumed symmetric and/or their non-cooperative games produce multiple

[2] In some real world situations a particular firm (perhaps, in financial distress) may appair as the natural target. This was the case, for instance, in the Nestlé-Perrier case following the 'benzene scandal', where Nestlé and the Agnelli's group competed to acquire the troubled French company. In other situations the industry may require an increase in concentration, but *a priori* all possible subsets could sensibly attempt a cost-reducing merger. A recent example is the US airlines industry. Before announcing their merger with Continental in 2010, United had been reported negotiating with US Airways. Moreover, the press speculated about almost all possible bilateral mergers involving these three firms plus American.

equilibrium outcomes (Kamien and Zang, 1990; and Fridolffson and Stennek, 2005a).[3]

The literature on optimal merger approval rules can be traced back to Besanko and Spulber (1993). In this research line, the paper closest to ours is Nocke and Whinston (2011). We borrow from them the specification of the merger review and, moreover, our results on the optimal merger rule are analogous to theirs, although emerging from different mechanisms.[4] They study a Cournot model with endogenous determination of merger proposals where one firm is exogenously selected to be the proposer. As in our paper, more stringent rules may serve to induce mergers which cause a higher increase in consumer surplus. They show that the larger the change in the (naively computed) Herfindhal index caused by the merger, the more stringent the optimal ex-ante rule is. In their paper, bargaining failures are ruled out by the merger selection mechanism. The ex-ante optimal merger rule differs from the ex-post optimal one in order to compensate for a misalignment between private and public incentives (profits versus consumer surplus). In contrast, in ours the preferred merger from the point of view of consumer surplus is also preferred from the point of view of industry profits, but no selection mechanism prevents the success of other, less attractive merger proposals. Nocke and Whinston allow for ex-ante asymmetric firms, and so the merger rule can be conditional on the ex-ante size of applicants, which renders a nice relationship between optimal stringency and the concentration index typically used in merger policy. Instead, we abstract from this asymmetry and focus on the differences in efficiency gains resulting from alternative mergers. As discussed in the last section, the insights from the two papers are complementary.

The merger problem we take up in this paper is similar (and equivalent, for some parameter values) to what has been termed the three-person/three-cake problem (see, for instance, Binmore, 1985), or in general a (restricted) game of coalition formation. Non cooperative analyses of this sort of problems abound, most using one version or another of a dynamic proponent-respondent game in the Rubinstein-Stahl tradition. (See Ray, 2007, for a general discussion including games with externalities, and Compte and Jehiel, 2010, for a recent example.) Our contribution in this strand is to propose a game designed so that the ordering of proposers is endogenous. That is, the agreed outcome is not a consequence of any arbitrary order of proposals: on the contrary, both order and outcome, are jointly determined by the primitives of the bargaining problem.[5]

[3] Kamein and Zang (1990) assume that each firm simoultaneously sets an asking price and bids for each of the other firms. Much closer to our modeling approach, Fridolffson and Stennek (2005a) set up a dynamic bargaining game, which will be discussed below.

[4] Our discussion of dynamic merger policy (Section 4) is also related to Motta and Vasconcelos (2005) and Nocke and Whinston (2010).

[5] If the ordering of proposers and movements, random or deterministic, is exogenous, we should agree with Ray

2 The benchmark model

We consider an industry where firms compete with each other but also bargain about the possibility of submitting a merger proposal. We embed bargaining and competition in a dynamic but stationary setting. First, merger opportunities, i.e., potential synergies, do not evolve with time. Second, we restrict ourselves to "stationary" equilibria, so that firms' strategies do not depend on history. This latter assumption means that firms, on the one hand, do not adhere to what could be considered as collusive behavior (and as a result they repeatedly play the equilibrium strategies of the static competition game) and on the other hand, when bargaining they ignore past moves. In this benchmark model we take a reduced form approach about the distributional implications of alternative market structures.

Time is a discrete variable indexed by $t = 0, 1, 2, ...$ (infinite horizon). At the beginning of the game there are three identical firms: $1, 2, 3$. The following sequence of moves takes place in period 0:

a) Competition authorities announce a rule for approving mergers that will remain fixed forever (full commitment).

b) Firms, but not authorities, learn about the synergies that can be exploited in case of a merger. Different pairs of firms may enjoy different reductions in marginal costs. All firms become perfectly informed about the marginal costs that would result from all possible mergers. Only two-firm mergers generate synergies and as a result authorities will never allow a merger to monopoly. Thus, there are only two possible market structures: the initial triopoly or a duopoly where the merged firm competes with the stand alone firm.

c) The three firms bargain about the possible submission of a merger proposal. They negotiate bilaterally within a protocol specified below. If two firms agree on submitting a merger proposal, authorities will learn the cost of the merged firm, and approve the merger if and only if it complies with the announced rule.

d) If no merger has been authorized then the triopoly game is played in the market; otherwise, the duopoly game is played.

In all subsequent periods, if a merger was successful in the past then the existing firms keep playing the duopoly game. If no merger proposal was ever submitted, firms again engage in bargaining, followed by competition.

Authorities are assumed to maximize the expected present value of consumer surplus, and

(2007, page 140) in that "a theory that purports to yield solutions that are independent of proposer ordering is suspect".

this appears to be what goes on in the real world. Both firms and competition authorities discount the future at the rate r. We will focus on the case that r is arbitrarily small, so that the friction built in the bargaining protocol is also arbitrarily small.

2.1 Static competition in a nutshell

The effect of synergies on the distribution of profits and consumer welfare will be represented by exogenous functions. In Section 3 we provide foundations for such reduced forms.

The three initial firms have access to the same constant returns to scale technology and hence face the same marginal cost c_0. The equilibrium level of profits of a single firm under triopoly is denoted by π_0, and the level of consumer surplus by CS_0.

Any pair of firms incurs a sunk cost for merging, F, the same for all pairs, although the synergies that each pair can exploit are different. For most of the paper we will simplify the analysis by setting $F = 0$. If firms i and j merge the constant marginal cost of the new firm is denoted by c_{ij}. As a notational convention firms 1 and 2 are the partners to the most efficient and profitable merger. Of course, in making this convention, we must assume the identity of the firms is common knowledge among firms, but ignored by authorities. For simplicity, we also assume that the other two mergers are symmetric; i.e., $c_{13} = c_{23} \geq c_{12}$. We thus assume that from the authorities' point of view (c_{12}, c_{13}) are random variables distributed according to the density function $h(c_{12}, c_{13})$ that has no mass points and takes strictly positive values on $C \equiv \{(c_{12}, c_{13}) \mid 0 \leq c_{12} \leq c_0, c_{12} \leq c_{13} \leq c\}$, but $h(c_{12}, c_{13}) = 0$, if $(c_{12}, c_{13}) \notin C$. Also for simplicity, we restrict attention to the case that approval rules take the form of a threshold value, $\bar{c}$. Thus, at the beginning of the game authorities announce a cut-off, $\bar{c}$, and commit to approve a merger proposal if and only if the marginal cost of the merged firm is lower than $\bar{c}$.[6]

Let us now consider the duopoly game. Suppose a merger between firms i and j, with marginal cost c_{ij}, has been approved. Then the merged firm faces a stand alone firm, k, with marginal cost c_0. Obviously, i, j, k represent generic, different firms. Profits of the merged firm and stand alone firm are denoted by $\pi_{ij}(c_{ij})$ and $\pi_k(c_{ij})$, respectively. Consumer surplus is denoted by $CS(c_{ij})$. The following assumptions will be derived from first principles in the next section:

(A.1) There exists a value of the marginal cost of the merged firm, c_n, $0 < c_n < c_0$, such that $CS(c_n) = CS_0$. Moreover, $CS(c_{ij})$ is a continuously differentiable function with $\frac{dCS}{dc_{ij}} < 0$. Hence, a merger increases consumer surplus if and only if $c_{ij} \leq c_n$.

[6] Nocke and Whinston (2011) discuss in detail the optimality of cutoff policies in a related setup.

(A.2) $\pi_{ij}(c_{ij})$ is continuously differentiable with $\frac{d\pi_{ij}}{dc_{ij}} < 0$. Moreover, $\pi_{ij}(c_n) > 2\pi_0$. Hence, any merger that is socially desirable is also profitable for the merging firms.

(A.3) $\pi_k(c_{ij})$ is continuously differentiable with $\frac{d\pi_k}{dc_{ij}} > 0$. Moreover, $\pi_k(c_n) = \pi_0$. Hence, any merger that is socially desirable is detrimental to the stand alone firm.

In imposing (A.3) we implicitly assume that for all $c_{ij} > 0$ the stand alone firm, k, remains active. In the models reviewed in Section 3 there may exist a positive value of c_{ij}, denoted by c_M, such that $\pi_k(c_{ij}) = 0$ for all $c_{ij} \leq c_M$. In other words, c_M is the threshold below which the stand alone firm is ejected from the market and the merged firm becomes a monopolist. We will ignore this possibility, which does not affect the qualitative results but would only lengthen the presentation.

Since we focus for the moment on the case $\overline{c} \leq c_n$, these assumptions imply a feasible merger is not only profitable but also attractive. That is, all mergers that would be authorized promise a level of profits for the merged firm that exceeds the joint profits in the status quo. Moreover, all firms prefer to be part of a merger rather than standing alone. Under these conditions we can confidently expect that in equilibrium a merger occurs immediately with probability one. Beyond this unsurprising aggregate outcome, we emphasize the possibility that the merger proposal that arises in equilibrium is neither the most efficient nor the most profitable.

For a given realization of (c_{12}, c_{13}), we denote by π_{12}, π_3 the respective profits when the efficient merger takes place, and by $\pi_{13}\ (= \pi_{23})$, $\pi_1\ (= \pi_2)$ the profits when one of the less efficient merger materializes. Thus, these four numbers, $\pi_{12}, \pi_{13}, \pi_1, \pi_3$, are the crucial parameters of the bargaining game. If $c_{ij} \leq c_n$, from assumptions (A.2) and (A.3): $\frac{1}{2}\pi_{12} \geq \frac{1}{2}\pi_{13} > \pi_0 \geq \pi_1 \geq \pi_3$.

Finally we assume:

(A.4) Aggregate profits under the efficient merger are higher than under those less efficient; i.e., $\pi_{12} + \pi_3 \geq \pi_{13} + \pi_1$.

According to (A.4), private and social goals are aligned as far as the ranking of mergers is concerned. In fact, this assumption is not important for predicting the outcome of the bargaining process. Its significance is that it does clarify the nature of the bargaining failures and hence the main argument behind the characteristics of the ex-ante optimal policy.

2.2 The bargaining game

We model firms' bargaining as a game that is repeated in each period until an agreement is reached. There are two elements to any agreement, and so too for any terminal outcome of the protocol: the identity of the merging firms, and the division of the surplus resulting from the merger. Thus, in each period the protocol first allows firms to endogenously select the negotiating partners, and then allows those partners to discuss the realization of the merger and the division of the corresponding surplus. The first part of the protocol is as follows:

Selection of negotiating partners

(1) Nature selects one of the three firms, each with probability $\frac{1}{3}$. Let that firm be A.

(2) Firm A invites one of the other two firms to become its negotiation partner. Let us call it firm B.

(3) Firm B accepts or rejects. If it accepts then firms (A, B) enter into the negotiation stage. If firm B rejects then firms (B, C) enter into the negotiation stage.

The second part, negotiations between either (A, B) or (B, C) (let (F, E) represent in general that pair of firms), could be modeled in a variety of equivalent ways and we choose the simplest.

Actual negotiation between F and E.

(4) Nature selects one of the two firms, each with probability $\frac{1}{2}$. Let that firm be F.

(5) Firm F makes an offer to firm E: θ_F^E, understood as the per-period profits that E retains if merged with F.

(6) Firm E accepts or rejects F's offer. If E accepts then it gets θ_F^E per period ($\frac{1+r}{r}\theta_F^E$ discounted total payoff), firm F gets $\pi_{FE} - \theta_F^E$ ($\left(\pi_{FE} - \theta_F^E\right)\frac{1+r}{r}$ discounted total payoff) and the bargaining ends. If E rejects the offer then all firms obtain the equilibrium profits of the triopoly game in that period and bargaining is resumed in the next period.

Points (1) to (6) describe the timing of the perfect information stage-game played in each period until an agreement is reached. (See Figure 1.) The protocol is simple and flexible.[7] Perhaps its most novel feature is step (3). This minor complication with respect to standard protocols is what makes our protocol sufficiently flexible: crucially, by introducing it, nature's choice in step (1) does not impose upper or lower bounds on the probability that any given firm is part of a successful merger in any given period. For example, this would not be the case if we instead assumed that in node (3), if firm B rejects the offer then the game moves to the next period, or assumed that then firm A can still ask firm C. In that case, a negotiation between firms B and C would be impossible in that period. We elaborate on alternative specifications

[7] Note that we could have added a trivial possibility in (3): that firm C rejects being part of the negotaitions with B, and then the game moves to the next period without agreement. Without adding this possibility, firm C can always reject any offer in (6) if it becomes firm E, and make an offer that will be rejected for sure, if it is firm E. Thus any equilibrium outcome in the extended game where that option is played with positive probability would be also an equilibrium outcome of our game. We can also argue that nothing would change by adding yet another possibility, that firm C rejects firm B's invitation and instead invites firm A. The key is that: 1) any (equilibrium) probability distribution over the three possible pairs in the extended protocol can be obtained with (mixed) strategies in the protocol proposed here; and, 2) with the same continuation strategies for the negotiations stage, those strategies would also be equilibria in our protocol.

below.

We focus on subgame perfect Nash equilibria (SPE) in stationary strategies and are interested in situations with negligible bargaining friction. Thus, we will state results for the limit of equilibria as $r \to 0$. A stationary strategy for a firm in this game includes a probability distribution over the two potential invitees in step (2) in case the firm is chosen by nature in step 1, and a probability distribution over the two potential partners in step (3) in case the firm is invited in step (2). Also, a strategy includes an offer to be made to each potential partner in step (5) in case the firm is chosen by nature in step (4) and an answer to every possible offer received by either of the two possible partners in step (5). In principle, the "moves" in steps (3), (5), and (6) may depend on previous moves, in particular on previous moves in that period. Nevertheless, for simplicity and keeping with the stationarity assumption, we will only consider equilibria where B's choice of partner for steps (4) to (6) does not depend on who played the role of A, and offers and answers in (5) and (6) depend on the identity of E and F, but not on who played the role of A and B.

In the trivial case that $\overline{c} < c_{12} \leq c_{13}$ no merger would pass the requirements of the authorities, and hence there is no room for negotiation. Almost as trivial is the case $c_{12} \leq \overline{c} < c_{13}$, where only one merger is feasible. In the unique equilibrium for that case, the efficient merger is agreed in period 0 and firms 1 and 2 obtain an expected payoff per period $\frac{\pi_{12}(c_{12})}{2}$, whereas firm 3 obtains $\pi_3(c_{12})$.

The most interesting case is $c_{12} \leq c_{13} \leq \overline{c}$, where three mergers are feasible. Since all acceptable mergers are profitable and attractive, a merger is bound to occur immediately with probability one. However, there is still a question as to the identity of the merger. The following proposition characterizes the unique equilibrium outcome in this case.

Proposition 1 *Suppose all three mergers are feasible. Then, for r sufficiently small, there exists a unique SPE outcome, both in payoffs and probability distribution over mergers. A merger always occurs with probability 1 in the first period. However, the identity of the merger depends on parameter values: (i) If $\left(\frac{1}{2}+r\right)\pi_{12} - (1+r)\pi_{13} + \pi_3 \geq 0$ then the efficient merger between firms 1 and 2 occurs with probability one, (ii) if $\left(\frac{1}{2}+r\right)\pi_{12} - (1+r)\pi_{13} + \pi_3 < 0$ then all three potential mergers take place with positive probability. In particular, the probability of the efficient merger is:*

$$d = \frac{\pi_{12} - 2\pi_1 + 4r(\pi_{12} - \pi_{13})}{-\pi_{12} + 4\pi_{13} - 2\pi_1 - 4\pi_3}. \tag{1}$$

Proof. See Appendix. ■

The uniqueness result indicates that our bargaining protocol offers very sharp predictions. Even more important, these predictions include the possibility of inefficient equilibria. If mergers are sufficiently heterogeneous (case (i)) then the merger between firms 1 and 2 will occur with probability one. In the limit, as $r \rightarrow 0$, the per-period equilibrium payoffs are $u_1 = u_2 = \frac{1}{2}\pi_{12}$ and $u_3 = \pi_3$. In the negotiation stages firm 1 offers $\theta_1^2 = \pi_{12} - \frac{u_2}{1+r}$, and firm 2 accepts; similarly for firm 1. The existence of alternative feasible mergers is irrelevant (outside option principle): any offer that firm 3 may accept or any request that it would make, i.e., above π_3, would leave at most $\pi_{13} - \pi_3 < \frac{1}{2}\pi_{12}$ for firm 1 or 2, and hence these firms have no incentives to deviate, inviting to or accepting the invitation from firm 3. However, when mergers are not too heterogeneous and satisfy $\pi_{13} - \pi_3 > \frac{1}{2}\pi_{12}$, such equilibrium is impossible. Indeed, in the limit when $r \rightarrow 0$, if $d = 1$ we would still have $u_1 = u_2 = \frac{1}{2}\pi_{12}$ and $u_3 = \pi_3$. This time in the negotiation stages firm 1 (or, equivalently, firm 2) would like to negotiate with firm 3. In the actual negotiation, if firm 1 is the proposer in step (4) then it would offer $\theta_1^3 = \pi_3$, which would be accepted, rendering the deviation profitable. Likewise, firm 3 would gain when being the proposer to firm 1 in step (4). Thus, in stage (3) firm 1 would have to accept an invitation by firm 3 in a SPE, and firm 3 would have to invite firm 1 in stage (2), which contradicts $d = 1$.

Note that in this second case the core (of the cooperative game) is empty. When the core is empty a pure strategy equilibrium cannot exist in our (flexible) protocol. For such pure strategy equilibrium to exist, the bargaining protocol would have to impose exogenous "restrictions" on the merger proposals that firms can make in a given period. Indeed, the reasoning in the previous paragraph depends very little on the details of our particular protocol. All that is needed for that reasoning is that, if what they envision is that they will merge tomorrow, firms 1 and 2 would rather attempt today a mutually beneficial agreement with firm 3. This of course would be a contradiction, and so no such equilibrium exists. Therefore, any equilibrium will necessarily be characterized by bargaining failures.

The question is then, what could be an equilibrium when the core is empty? It is not difficult to realize that an equilibrium would have to put positive probability on all three potential mergers, and that firms 1 and 2 will have a symmetric treatment. If that is so, when firm 1 negotiates with 2, it obtains $\frac{1}{2}\pi_{12}$ per period. If instead firm 1 negotiates with 3, then its expected payoff per period as $r \rightarrow 0$ is $\frac{1}{2}(\pi_{13} + u_1 - u_3)$. Hence, firm 1 is indifferent if and only if:

$$u_1 - u_3 = \pi_{12} - \pi_{13}. \tag{2}$$

In equilibrium u_1 and u_3 are given by:

$$u_1 = \frac{1}{2}\frac{1+d}{2}\pi_{12} + \frac{1-d}{2}\pi_1, \tag{3}$$

$$u_3 = d\pi_3 + (1-d)\left(\pi_{13} - \frac{1}{2}\pi_{12}\right). \tag{4}$$

Indeed, if we solve equations (2),(3), and (4) for d we obtain (1) as $r \to 0$.

In this region, d is always between $\frac{1}{3}$ and 1. When all mergers are identical, $\pi_{12} = \pi_{13}$, $d = \frac{1}{3}$. Since d is a continuous function of c_{12} and c_{13}, then as far as c_{12} and c_{13} are not too different the probability of the efficient merger is strictly lower than one.

Using different protocols, several papers (Inderst and Wey, 2004; Fridolffson and Stennek, 2005a; and Nocke and Whinston, 2011) also predicted that in situations where mergers are profitable and attractive then a merger occurs immediately with probability one. In Inderst and Wey (2004), nature chooses one of the firms as the target. The target firm sets a reserve price and the other firms bid to acquire the target. In Nocke and Whinston (2011), one of the firms is exogenously designated as the acquirer, which is entitled to make a take-it-or-leave-it offer to the target of her choice. Thus, in both cases there are exogenous restrictions on the set of feasible mergers. In contrast, Fridolffson and Stennek (2005) propose a bargaining game that treats all firms symmetrically. In their paper, the bargaining protocol imposes certain restrictions on the probability distribution over the set of feasible mergers, which tend to enhance the bargaining power of the weakest player. More specifically, in the spirit of Stähl and Rubinstein, they propose a protocol where in each period a proponent chosen by nature makes an offer to merge and share the surplus with one of the other two firms. If the respondent rejects the offer then the game moves to the next period.[8] That is, the bargaining protocol sets an upper bound of $\frac{2}{3}$ for d in any equilibrium without delay. This upper bound has nothing to do with players' decisions, but implies that player 3's payoff is higher than in our game. Moreover, also as a consequence of this rigidity, the game studied by Fridolffson and Stennek (2005) has multiple equilibria in the asymmetric case.[9] In particular, if firm 1 expects player 2 to accept firm $3'$s offer, then u_1 will be relatively low and firm 1 will also accept firm 3's offer (likewise for firm 2). However, if firm 1 expects firm 2 to reject firm $3'$s offer, then u_1 will be relatively higher, which will induce firm 1 to reject firm 3's offer and generate some delay. Clearly, u_1 and u_2 are higher in the second equilibrium.

[8] In fact, they frame their game in continuous time and bidding rounds occur at random points in time. However, they also focus on the limit case that the expected difference between two bidding rounds goes to zero. This is equivalent to the deterministic version we discuss in the text.

[9] Details are available upon request.

Our non-cooperative game asymptotically implements a solution concept for cooperative games, PSBN (Prediction for Simultaneous Bilateral Negotiations), that we have proposed in an earlier paper (Burguet and Caminal, 2011). We define PSBN as the generalization of a Nash Bargaining Solution to simultaneous bargaining negotiations. The underlying assumption is that all pairs simultaneously bargain à la Nash, and that in each of these negotiations fallback options are endogenous and determined by (consistent) beliefs about the consequences of failing to reach an agreement in that particular negotiation. Players are assumed to share the same beliefs about the probability distribution over the success of different coalitions. In our restricted model this is equivalent to sharing beliefs about the occurrence of each coalition: the probability of a coalition between 1 and 2 is equal to d, and the probability of a coalition between 1 and 3, or 2 and 3, is f. Of course, $d + 2f = 1$. Therefore, if negotiations between players 1 and 3 break down then they will expect that coalition $(1, 2)$ will succeed with (conditional) probability $\frac{d}{1-f}$, and coalition $(1, 3)$ with probability $\frac{f}{1-f}$. Other than that, the PSBN only imposes a weak restriction on the set of admissible beliefs: a coalition cannot have positive probability of success if both players prefer to reach an agreement with the third player, with one of the preferences being strict.

2.3 The ex-ante optimal merger policy

After characterizing the outcome of the bargaining game we are now ready to discuss the optimal merger approval rule. If we let $\Delta(c_{ij}) = CS(c_{ij}) - CS_0$ then the ex-ante expected change in consumer surplus obtained from a given policy rule, $\overline{c}$, can be written as:

$$\begin{aligned} W(\overline{c}) &= \int_0^{\overline{c}} \int_{c_{12}}^{\overline{c}} \{d(c_{12}, c_{13}) \Delta(c_{12}) + [1 - d(c_{12}, c_{13})] \Delta(c_{13})\} h(c_{12}, c_{13}) \, dc_{13} dc_{12} + \\ &+ \int_0^{\overline{c}} \int_{\overline{c}}^{c_0} \Delta(c_{12}) h(c_{12}, c_{13}) \, dc_{13} dc_{12}. \end{aligned}$$

We can now compute the effect of a change in $\overline{c}$ on W:

$$\frac{dW(\overline{c})}{d\overline{c}} = -\int_0^{\overline{c}} d(c_{12}, \overline{c}) [\Delta(c_{12}) - \Delta(\overline{c})] h(c_{12}, \overline{c}) \, dc_{12} + \int_{\overline{c}}^{c_0} \Delta(\overline{c}) h(\overline{c}, c_{13}) \, dc_{13}. \tag{5}$$

If $\overline{c} < c_n$, then an increase in $\overline{c}$ causes two effects on the expected consumer surplus. On the one hand, it expands the area where there is a bargaining failure; that is, the area where there is a positive probability, $1 - d$, that one of the relatively inefficient merger succeeds. Put differently, an increase in $\overline{c}$ intensifies the competition between the efficient and the less efficient mergers. This effect is reflected in the first term of (5). On the other hand, it expands the area where *some* merger that raises consumer surplus is accepted. This is captured by the second term of

(5). Figure 2 illustrates these two effects. With respect to the reference case where $\overline{c} = c_n$, the expected consumer surplus under a policy $\overline{c} < c_n$ is different in the two shaded areas: A and B. In area A the efficient merger no longer faces the competition from relatively inefficient mergers and hence the expected consumer surplus is higher: the probability of success of the efficient merger jumps from $d < 1$ to 1. In contrast, in area B the consumer surplus is lower: some consumer surplus enhancing mergers do not pass the tougher standard. The optimal policy must balance these two countervailing effects. However, for $\overline{c}$ close c_n, the loss in consumer surplus suffered when a consumer surplus-enhancing merger is rejected is close to zero, whereas the expected difference in consumer surplus between a merger with costs c_{12} and one with costs c_{13} does not approach zero. That is, the first term is of first order, while the second is of second order. Therefore, $\frac{dW}{d\overline{c}}\left(\overline{c} = c_n\right) < 0$ and hence the ex-ante optimal rule involves $\overline{c} < c_n$.

So far we have considered threshold values such that $\overline{c} \leq c_n$. It is quite straightforward to check that a value of $\overline{c}$ above c_n is dominated by $\overline{c} = c_n$. In this interval both effects have the same negative sign: an increase in $\overline{c}$ intensifies competition between the efficient and the inefficient merger. Moreover, such an increase expands the acceptance of mergers that imply a reduction in consumers surplus with respect to the status quo. Summarizing,

Proposition 2 *The optimal ex-ante merger policy is more stringent than the ex-post; i.e.,* $\overline{c} < c_n$.

The proposition demonstrates that a commitment to a merger rule which is more stringent than the optimal ex-post rule increases the expected consumer surplus by reducing the probability that the efficient merger is threatened by a relatively inefficient one. In other words, it reduces the negative consequences of failures in the bargaining process that lead to a submission of a merger proposal. A passive merger policy cannot implement the most efficient market structure, but a more stringent rule can help in alleviating some of its negative consequences.

The comparative statics of the optimal policy are rather straightforward. The gap between the ex-post and ex-ante optimal policies, $c_n - \overline{c}$, is higher as the density over area A increases and the density over area B decreases. In other words, $c_n - \overline{c}$ is high if: (i) it is likely that the efficient merger involves significant gains in consumer surplus but only small gains from the relatively inefficient ones, and (ii) it is unlikely that the efficient merger involves small gains in consumer surplus while the relatively inefficient ones involve losses. No doubt some market characteristics also have an influence on the relative size of these two areas. For instance, suppose that we are dealing with an n-firm industry, $n \geq 3$, but only three firms are genuine candidates to participate in a two-firm merger. Suppose also that potential synergies associated with these mergers are identical to those of our model, including the joint probability distribution on (c_{12}, c_{13}). Then, as n increases the distance between c_0 and c_n decreases, which implies that area B diminishes, and as a result $c_n - \bar{c}$ increases.

3 Foundations of the reduced forms

In this section we review some alternative models of competition that satisfy all conditions (A.1) through (A.4). Consider a differentiated goods model in the spirit of Dixit (1979). The representative consumer's utility function is quadratic in the varieties produced by each firm $i = 1, 2, 3$, and additively separable in the numeraire good, x. More specifically:

$$U(\mathbf{q}, x) = x + \alpha \sum_{i=1}^{3} q_i - \frac{\beta}{2} \sum_{i=1}^{3} q_i^2 - \frac{\gamma}{2} \sum_{i=1}^{3} \prod_{j \neq i} q_i q_j,$$

where q_i represents the quantities of variety i consumed, and $\mathbf{q} = (q_1, q_2, q_3)$. The inverse demand function for each good, $p_i = \frac{\partial U(q,x)}{\partial q_i}$, is then linear in all quantities, and the consumer surplus can be written as

$$\widehat{CS}(\mathbf{q}, x) = U(\mathbf{q}, x) - \sum_{i=1}^{3} p_i q_i = x + \frac{\beta - \gamma}{2} \sum_{i=1}^{3} q_i^2 + \frac{\gamma}{2} (\sum_{i=1}^{3} q_i)^2. \qquad (6)$$

Whether firms compete in quantities or in prices, the reaction function of a stand alone firm is unchanged by a merger. Thus, if for $c_{ij} = c_n$ a merger between firm i and j sets output equal to the pre-merger level q_0 for each of its varieties, then the stand alone firm k will also set output q_0, so that prices of all varieties will be unaffected. Thus, $\pi_{ij}(c_n) = (c_0 - c_n) 2q_0 + 2\pi_0 > 2\pi_0$, and $\pi_k(c_n) = \pi_0$. With quantity competition, a decrease in c_{ij} results in a higher quantity of each of the varieties produced by the merged firms, and a reduction in the output of firm k. Likewise, with price competition, a decrease in c_{ij} results in lower prices both for varieties produced by the merged firm and the variety produced by firm k, as long as firm k produces a positive output. However, the (direct) first effect dominates for each firm, so that the output of the varieties produced by the merged firm is larger and the output of firm k lower after the reduction. Thus, the profits of the merged firms are decreasing in c_{ij} whereas the profits of firm k are increasing in c_{ij}. Once again, the first effect is stronger, so that total profits of the industry (and total output) are decreasing in c_{ij}.

It follows that assumptions (A.2) through (A.4) are satisfied by this model. Finally, for both terms in the right hand side of (6) to be decreasing in c_{ij} it suffices that both the output and the effect of changes in c_{ij} on output are larger for the varieties of the merged firm than for the variety of firm k. As already mentioned, these conditions are satisfied when $c_{ij} < c_n$, both under quantity and price competition. Thus, condition (A.1) is also satisfied.

Lemma 3 *Whether firms compete in quantities or prices, assumptions (A.1) through (A.4) are satisfied under appropriate parameter restrictions.*

Proof. See the Appendix. ■

The parameter restrictions (on α, β, γ, and c_0) that the lemma refers to are presented in the proof and are only necessary to ensure that $c_n > 0$, that is, that there exists a possibility that a merger increases consumer surplus. If this was not the case, then some of the properties in (A.1) through (A.4) would not be satisfied but our results in the main section would be trivially satisfied.

When all firms produce homogeneous goods, $\beta = \gamma$, and firms compete in quantities, the lemma still holds and so the Cournot model with linear demand is a particular case. Moreover, with homogeneous good it is a simple exercise to show that assumptions (A.1) through (A.4) are satisfied under much more general conditions. For instance, assume homogeneous products and a strictly decreasing inverse demand function $p(Q)$ that satisfies $p'(Q)\,Q + p"(Q) < 0$. As in the differentiated goods case, and for the same purpose, we also impose conditions so that $c_n > 0$.

Lemma 4 *The Cournot model satisfies assumptions (A.1) through (A.4) under appropriate restrictions on the primitives.*

Proof. See the Appendix ■

Finally, we discuss to what extent a positive sunk cost of merging may alter the results of the benchmark model. Let $\overline{F}$ be the level of sunk costs that leaves firms i and j indifferent between merging and not merging if $c_{ij} = c_n$; i.e., $\overline{F} \equiv \pi_{ij}(c_n) - 2\pi_0$. The competition models that we have reviewed here satisfy $\overline{F} > 0$. If $F \leq \overline{F}$ then all acceptable mergers are still profitable (and attractive) and hence sunk costs are completely irrelevant. If $F > \overline{F}$ then some socially desirable mergers may not be proposed. Moreover, mergers may become non-profitable but attractive; that is, firms may prefer that no merger takes place, but if they expect that a merger will occur then they prefer to be part of it rather than being left out. As a result, there may be multiple equilibria: in one of them no merger takes place and in the others one of the mergers occurs with positive probability (preemptive mergers).[10] An explicit analysis of the policy implications in this scenario requires an equilibrium selection device and it will not be elaborated here.

[10] See Fridolfsson and Stenneck (2005a) for an analysis of the symmetric mergers case.

4 More than three firms

The main predictions of the benchmark model do not depend on the assumption that there are initially three firms in the market. However, the case of more than three firms opens the possibility of more than one merger and so adds a potentially dynamic occurrence of mergers. As a first step in generalizing our analysis in this direction, suppose that the industry is initially populated by four identical firms: $1, 2, 3, 4$. As in the benchmark model let us assume that only bilateral mergers can generate synergies, and also let us maintain a similar quasi-symmetric structure: $c_{12} \leq c_{13} = c_{14} = c_{23} = c_{24} \leq c_{34} = c_0$. Thus, there are two types of firms. Firms 1 and 2 are able to generate synergies and the most efficient merger is precisely the one that involves them both. A merger of either of them with one of the other two generates an intermediate level of synergies, while the merger between firms 3 and 4 generates no synergies at all.[11] As in the benchmark model, the identity of all firms is common knowledge among firms, but it is ignored by authorities.

The bargaining game described in Section 2.2 can easily be adapted in the same spirit to the presence of four players, so that no exogenous restrictions on the probability of each merger are imposed. In particular, only the selection of negotiation partners (stages (1) to (3) of the stage game) needs be modified. The new sequence should now read:

(1) Nature selects one of the four firms, each with probability $\frac{1}{4}$. Let that firm be A.

(2) Firm A invites one of the other three firms to become its negotiation partner. Let us call it firm B.

(3) Firm B either accepts the offer and then firms (A, B) enter into the negotiation stage, or it rejects it and chooses an alternative negotiation partner. Hence, either (B, C) or (B, D) enter into the negotiation stage.

Although only bilateral mergers generate synergies, authorities may still accept two merger proposals if they both generate sufficient synergies so that a duopoly is socially optimal. Thus, in general a policy rule may consist of two thresholds, $\overline{c}_1, \overline{c}_2$, where $\overline{c}_2$ could be a function of the marginal cost of the first merger, call it c^F, or equivalently, of the post-merger price.[12]

If two mergers are feasible, we can simply assume that after the first successful merger, the remaining two firms keep bargaining according to the protocol described in stages (4) to (6) and continue to do so in every period until an agreement is reached.

[11] We close this section with a comment on the case $c_{34} < c_0$; that is, that merger $(3, 4)$ can also generate synergies.

[12] In practice it makes little sense to think of rules that are conditional on the "number of mergers" previously realized. It would thus be reasonable to restrict to simpler rules that only depend on the effect of the merger on prices. Such restriction would be irrelevant for our purposes.

Denote by c_{n1} and $c_{n2}(c^F)$ the ex-post optimal myopic acceptance rule. That is, c_{ni} is the marginal cost of the ith merger, $i = 1, 2$, that leaves the market price unchanged. We will assume that $c_{13} \leq c_{n1}$ if and only if $c_{13} \leq c_{n2}(c_{n1})$. Nocke and Whinston (2010) have shown that this assumption is satisfied in the Cournot model. It can be shown that it also holds in the differentiated goods model of Dixit (1979), discussed in the previous section.[13] That is, if it is expected that a *mixed* merger between either firm 1 or 2 and one of the other firms will be approved, then the merger between the remaining firms would also be approved. Put differently, two consecutive mergers with the same level of efficiency are complements from a consumer surplus point of view.

Let us examine the merger equilibrium if the authority sets $\overline{c}_1 = c_{n1}$, $\overline{c}_2 = c_{n2}(c^F)$. We need to consider two alternative scenarios depending on whether one or two mergers are feasible.

First, consider the case that $c_{12} \leq c_{13} \leq c_{n1}$ and also assume that if a merger $(1, 3)$ is approved, then the merger $(2, 4)$ is not profitable.[14] In this case there are two types of equilibria analogous to those described by Proposition 1. Denote by π_3 the profits of each of the stand alone firms when firms 1 and 2 merge. If the following condition holds:

$$\frac{1}{2}\pi_{12} - \pi_{13} + \pi_3 \geq 0, \tag{7}$$

then the efficient merger occurs immediately with probability one, in the limiting case of $r \to 0$. That is, $u_1 = u_2 = \frac{1}{2}\pi_{12}$, and $u_3 = u_4 = \pi_3$. This equilibrium requires that firms 1 and 2 prefer to merge with each other, and firms 3 and 4 are unable to tempt 1 or 2. That is, the equilibrium payoff of firms 1 and 2, $\frac{1}{2}\pi_{12}$, should be higher than the maximum payoff any of them can obtain from deviating and inviting one of the other firms, $\pi_{13} - \pi_3$. This is exactly condition (7). This is one of the types of equilibrium. However, if condition (7) fails then the efficient merger cannot take place with probability one, and in the unique equilibrium all acceptable mergers will occur with positive probability, just as in case (ii) of Proposition 1.

Summarizing, when only one merger is feasible, the analysis of the four-firm case is identical to the three-firms case: if alternative mergers are not too different, then the merger that generates both lower profits and lower consumer surplus succeeds with positive probability (bargaining failure). Consequently, an approval rule slightly more stringent than c_{ni} would generate second order losses but first order gains.

Second, let us assume that $c_{12} \leq c_{13} \leq c_{n1}$ and the sunk cost of merging F is sufficiently low. To simplify the presentation set $F = 0$. Then, from assumption (A.2) all socially acceptable

[13] We still assume that conditions $(A1)$ to $(A4)$ are satisfied unless otherwise specified.

[14] In the Appendix we show that this is the case in the Cournot model with linear demand and fixed costs of merging for some range of parameter values.

mergers are also profitable. In this case, there are two competing market structures. On the one hand, if the merger $(1,2)$ is proposed first, then the equilibrium structure is a triopoly. On the other hand, if a mixed merger between either firm 1 or 2 and one of the other firms is proposed and approved, this will be followed by a merger by the remaining firms. The resulting market structure will be a duopoly. From a consumer surplus point of view, assume that the merger between 1 and 2 will be preferred if and only if $c_{13} \geq \widehat{c}(c_{12})$, where $\widehat{c}(c_{12})$ is an increasing function with $\widehat{c}(c_{n1}) = c_{n1}$. In the Cournot model with linear demand $\widehat{c}(c_{12}) = \frac{5c_{n1}+3c_{12}}{8}$. In contrast, the equilibrium of the bargaining game is determined by the total expected profits under the two market configurations. If 1 and 2 merge then they make π_{12}, and the standing alone firms make π_3 each. After the two symmetric mergers each of the merged firms makes duopoly profits that we can represent by $\widetilde{\pi}_{13}$. If the following condition holds:

$$\pi_{12} + 2\pi_3 \geq 2\widetilde{\pi}_{13}, \tag{8}$$

then firms 1 and 2 merge with probability one if r is close to 0. That is, in the limiting case of $r \to 0$ payoffs are $u_1 = u_2 = \frac{1}{2}\pi_{12}$ and $u_3 = u_4 = \pi_3$. Firms 1 and 2 are only willing to merge with each other and never accept an offer from firms 3 or 4. This will occur only if the maximum gains that firm 1 or 2 can make by deviating, $\widetilde{\pi}_{13} - \pi_3$, are lower than $\frac{1}{2}\pi_{12}$. Hence, an equilibrium of this type exists if and only if condition (8) holds. If condition (8) fails then as r goes to 0 two symmetric mergers take place with probability one. That is, the final market structure is a duopoly where each firm is the result of a mixed merger. [15]

It is important to note that in this second case, i.e., when two mergers are feasible, there are no bargaining failures in the private sense. Condition (8) indicates that the resulting market structure is the one that maximizes total profits. However, social and private objectives do not coincide. In particular, if c_{13} is close to c_{n1}, and c_{12} is not too far away from c_{n1}, then the merger between firms $(1,2)$ maximizes consumer surplus ($c_{13} > \widehat{c}(c_{12})$), but the alternative market structure will take place in equilibrium. More specifically, if $c_{12} = c_{13} = c_{n1}$ $(= c_{n2}(c_{n1}))$ then $\pi_{12} = \widetilde{\pi}_{13}$. The reason is that all merged firms face the same residual demand and have the same marginal cost. Moreover, from assumption (A.2) $\pi_{12} > 2\pi_0 = 2\pi_3$. Therefore, condition (8) does not hold. Thus, this new source of discrepancy between social and private objectives

[15] The intuition behind this latter equilibrium type requires considering positive discounting. For $r > 0$, firms 3 and 4 prefer to merge with either 1 or 2, but the latter are indifferent as to the merging partner. Thus, in equilibrium the merger between firms 1 and 2 occur with probability d, mergers $(1,3)$ and $(2,4)$ with probability $\frac{1-d}{2}$, and mergers $(1,4)$ and $(2,3)$ with probability $\frac{1-d}{2}$. For firms 1 and 2 to be indifferent it must the case that $\pi_{12} - u_1 = \widetilde{\pi}_{13} - u_3$. This equation, together with the determinants of u_1 and u_2, allows us to compute $d(r)$, whose limit is zero.Morevoer, firms 3 and 4 will not have incentives to deviate if $\widetilde{\pi}_{13} - u_1 \geq \pi_3$, which in the limit is equivalent to the failure of (8).

once again calls for a more stringent merger rule that generates second order losses but first order gains.

A crucial assumption of our model is that different mergers may generate different levels of synergies. However, we have explicitly assumed a quasi-symmetric structure: firms 1 and 2 are symmetric, and so are firms 3 and 4. Moreover, we have ruled out the possibility that firms $(3,4)$ may submit an eligible merger proposal. If we relax these two assumptions the model becomes far more complex, and the dynamic features of merger policy may take a more central role. New effects may appear. For example, if we let $c_{34} \leq c_{n1}$ and only one merger is possible then, under some parameter values, it can be shown that in equilibrium firms 1 and 2 prefer to merge with each other, and as a result firms 3 and 4 also prefer to meet each other. Under our bargaining protocol the successful merger will be the result of a kind of merger race. Formally, the result of the race will be determined by nature's choice in stage (1) of the protocol. Once again, a more stringent merger policy would also alleviate this type of bargaining failure. If on top of this we may have $c_{34} \leq c_{n2}(c_{12})$ and more than one merger is possible, then there may be two alternative duopoly configurations: $(1,2)$ and $(3,4)$ on the one hand, and $(1,3)$ and $(2,4)$ on the other (or the other equivalent permutation). Then, c_{n1} may eliminate mixed mergers and $c_{n2}(c^F)$ may eliminate merger $(3,4)$. Whether or not these two instruments are complements or substitutes may be an issue.

The analysis of the four-firm case suggests that there was hardly anything special about the three-firm case. Thus, bargaining failures and their implications for merger policy are likely to be part of the big picture under any initial market structure. However, it also indicates that the typology of cases grows with the number of firms, as does the complexity associated with the dynamic aspects. In a truly dynamic merger situation, where not all merger opportunities unfold simultaneously,[16] there are certainly new issues worth exploring but, once again, bargaining failures of the type described here are bound to be among them.

5 Imperfect cost monitoring

In this section we relax the assumption that authorities can perfectly observe the potential synergies generated by a merger proposal. Instead, we assume that they have access only to a noisy signal, whose quality depends on the effort exerted by the partners in the potential merger. That is, authorities allow for an *efficiency defense*, but they set the information standards that are required to substantiate such efficiency claims. Merger policy will then consist of two

[16] Like in Nocke and Whinston (2010).

instruments: the minimum quality of the noisy signal and the threshold of its realization. Also, assume that industries differ in how costly it is to generate any given quality of information, while the authorities are unable to distinguish between the different types. This second source of asymmetric information rules out the possibility that the information requirement can perfectly screen the cost c_{ij} and render the actual monitoring process redundant.

Thus, assume that a signal reveals the cost c_{ij} with probability $b(e)$. With probability $1-b(e)$ the signal takes the value c_0 so that it reveals nothing. The value of b is an increasing function of e, the verifiable effort spent in generating the signal's quality. In a particular industry, the cost of effort e is e with probability s and 0 with probability $1-s$. Let $b(e)$ be twice continuously differentiable, with $b'(e) > 0$, $b''(e) < 0$, $b(M) = 1$, for M very large, and $b(0) = 0$.[17] A merger policy is a pair $(\underline{e}, \overline{c})$: authorities require from firms a minimum level of effort, $\underline{e}$, and commit to accept the merger if and only if the realization of the signal is lower than $\overline{c}$.

Assume the industry is one where the effort is costly. If a merger results in cost $c_{ij} \leq \overline{c}$ and merging partners exert a costly effort $\underline{e}$, then their expected profits upon applying for approval are[18]

$$\Pi_{ij}(c_{ij}; \underline{e}) = b(\underline{e})\, \pi(c_{ij}) + [1 - b(\underline{e})]\, 2\pi_0 - \underline{e}.$$

$\Pi_{ij}(c_{ij}; \underline{e}) = 2\pi_0$ defines a threshold $\widehat{c}(\underline{e})$, with $\widehat{c}(\underline{e})$ decreasing in $\underline{e}$.[19] Firms will be willing to submit their proposal only if $c_{ij} \leq \min\{\overline{c}, \widehat{c}(\underline{e})\}$. Also $\frac{d\Pi_{ij}}{de} < 0$, so that there are no incentives to exert effort above the minimum requirement, $\underline{e}$.

If the industry is one where effort is costless, then if $c_{ij} \leq \overline{c}$ firms would be willing to submit their proposal and exert effort M so that c_{ij} is revealed. If $c_{ij} > \overline{c}$ then we assume that they will not bother.

If effort is costly, in the region where bargaining failures may occur $(\Pi_{12} - 2\Pi_{13} + 2\Pi_3 < 0)$ the probability that the efficient merger is proposed, d_c, can be written by simply reinterpreting equation (1) in the limiting case of $r \to 0$:

$$d_c(c_{12}, c_{13}; \underline{e}) = \frac{b(\underline{e})(\pi_{12} - 2\pi_1) - \underline{e}}{b(\underline{e})(-\pi_{12} + 4\pi_{13} - 2\pi_1 - 4\pi_3) - 3\underline{e}},$$

and $\frac{\partial d_c}{\partial \underline{e}} > 0$. That is, a higher effort requirement affects the efficient merger relatively less. As a result it reduces the probability of a bargaining failure even in those cases where the least

[17] The specification of the noisy signal allows for type I errors, but type II errors are avoided This simplifies the analysis significantly.

[18] The profits of a standing alone firm, Π_3 are similarly affected: with probability b they will be π_3 and with probability $1-b$ it will be π_0.

[19] We are assuming that in equilibrium b will be sufficiently high so all socially desirable mergers are profitable and attractive. If b is too low in equilibrium, we may have some non profitable mergers that are attractive, just as in the case discussed at the end of Section 3.

efficient mergers are still profitable. If firms' effort is costless then the probability that the efficient merger is submitted, d_{nc}, does not depend on $\underline{e}$, and is given by (1) as $r \to 0$.

Suppose that $\widehat{c}(\underline{e}) > \overline{c}$. In that case, whether effort is costly or not, firms will choose to submit a proposal if and only if $c_{ij} \leq \overline{c}$, and then $\widehat{c}(\underline{e})$ will play no role in such a decision. However, higher $\underline{e}$ implies higher $b(\underline{e})$, which has a positive effect on consumer surplus because it raises both d_c and the fraction of approved mergers. Thus, the optimal policy cannot be such that $\widehat{c}(\underline{e}) > \overline{c}$. Hence, we can restrict the analysis to the case $\widehat{c}(\underline{e}) \leq \overline{c}$. Then, $\widehat{c}(\underline{e})$ is the threshold relevant for mergers with costly effort, and $\overline{c}$ is only relevant for mergers with costless effort.

This last observation means that, absent bargaining failures, the optimal value of $\overline{c}$ is c_n. However, when $d_c, d_{nc} < 1$ the derivative of the expected consumer surplus with respect to $\overline{c}$ ($\leq c_n$) is simply (5). Hence, the presence of bargaining failures reduces the optimal threshold value of $\overline{c}$, just like in the benchmark model.

The effect of $\underline{e}$ on consumer surplus, and how this effect is affected by the presence of bargaining failures is more involved. Let $g(c_{12})$ and $G(c_{12})$ be the (marginal) density and distribution function of c_{12},[20] respectively, and assume away bargaining failures. That is, assume that the merger between 1 and 2 is the only one proposed. The expected consumer surplus would then be

$$W = (1-s)\, b(\underline{e}) \int_0^{\widehat{c}} \Delta(c_{12})\, dG(c_{12}) + s \int_0^{\overline{c}} \Delta(c_{12})\, dG(c_{12}), \tag{9}$$

and so the derivative of W with respect to $\underline{e}$ is $(1-s)$ times

$$b(\underline{e})\, \widehat{c}'(\underline{e})\, \Delta(\widehat{c})\, g(\widehat{c}) + b'(\underline{e}) \int_0^{\widehat{c}} \Delta(c_{12})\, dG(c_{12}).$$

The second term is positive, since $b' > 0$. It represents the effect that $\underline{e}$ has on the informativeness of the signal and then on the proportion of consumer surplus-enhancing mergers that are approved. The first term is negative, since $\widehat{c}' < 0$. It represents the loss of marginal, consumer surplus-enhancing mergers that do not apply when the cost of gathering the required information is higher.

When we take into account the possibility that mergers other than $(1,2)$ apply, then the expression of W is more involved. It will be similar to (9), where the second term is still independent of $\underline{e}$. The first term, corresponding to the expected consumer surplus when effort is costly, will be

$$(1-s) b(\underline{e}) \int_0^{\widehat{c}} \Phi(c_{12}, \widehat{c}) dG(c_{12})$$

[20] That is, $g(c_{12}) = \int_{c_{12}}^{c_0} h(c_{12}, c_{13})\, dc_{13}$.

where

$$\Phi(c_{12},\widehat{c})=\int_{c_{12}}^{c_0}\left[d_c\Delta\left(c_{12}\right)+\left(1-d_c\right)\Delta\left(c_{13}\right)\right]dH\left(c_{13}\left|c_{12}\right.\right),$$

and $dH\left(c_{13}\left|c_{12}\right.\right)$ is the density of c_{13} conditional on c_{12}. Note that $\Phi(c_{12},\widehat{c})$ depends on $\widehat{c}$ since d_c does (for instance, $d_c=1$ whenever $c_{13}>\widehat{c}$). In general, lower $\widehat{c}$ implies higher d_c and since $\Delta\left(c_{12}\right)>\Delta\left(c_{13}\right)$, then higher $\Phi(c_{12},\widehat{c})$. The derivative of W with respect to $\underline{e}$ is $(1-s)$ times

$$b\left(\underline{e}\right)\widehat{c}'\left(\underline{e}\right)\Phi(\widehat{c},\widehat{c})g\left(\widehat{c}\right)+b'\left(\underline{e}\right)\int_0^{\widehat{c}}\Phi(c_{12},\widehat{c})dG\left(c_{12}\right)+b(\underline{e})\int_0^{\widehat{c}}\widehat{c}'\left(\underline{e}\right)\frac{\partial\Phi(c_{12},\widehat{c})}{\partial\widehat{c}}dG\left(c_{12}\right). \qquad (10)$$

The comparison with the optimal value of $\widehat{c}$ in the absence of bargaining failures is complicated since $\Phi(c_{12},\widehat{c})<\Delta\left(c_{12}\right)$. In general, the quality of the proposals is diminished and hence there is less incentive to monitor them carefully, which makes a higher level of $\widehat{c}$ (lower $\underline{e}$) more attractive. However, with the new effect directly linked to the effect of $\underline{e}$ on the probability of these bargaining failures, the third term in (10) is positive since both $\widehat{c}'$ and $\frac{\partial\Phi}{\partial\widehat{c}}$ are negative. Thus, this new effect calls for tougher merger policies with respect to $\underline{e}$.

Summarizing, under imperfect cost monitoring, bargaining failures still induce authorities to commit to a lower threshold level of the noisy signal than the ex-post optimal one (a more stringent policy in this sense), but they may or may not require the signal to be of higher quality.

6 Discussion

In this paper we have made two main points: (a) passive merger policy opens the door to relatively inefficient mergers because of potential bargaining failures, and (b) a commitment to a more stringent policy rule may alleviate this inefficiency. These two ideas have first been illustrated in a stylized model with three ex-ante identical firms, but we have also shown that some of the reduced forms are compatible with several standard oligopoly models. We have also suggested that results are likely to hold in the general case where an arbitrary number of firms are involved in the merging process. Finally, we have examined how to specify point (b) in the event merger policy includes more than one instrument. However, anticipating how results could or would change if we modify the model in other directions is more difficult to assert.

Three important issues remain to be briefly discussed. First, if firms are ex-ante asymmetric, then the policy rule in principle may well depend on the relative position of firms in the status quo. Second, bargaining failures could be eliminated if firms were allowed to reach binding agreements that include monetary transfers between the merged and non-merged firms. Third, competition authorities may care not only about the effect of a merger on consumer surplus, but may also put some weight on industry profits.

Let us consider the following change in the previous model. Suppose that firms have different marginal costs in the triopoly game. Then, the merger rule could be conditioned on the relative position of firms in the status quo. In a Cournot (homogeneous product) model where an exogenously designated buyer can make a take-it-or-leave-it offer to one of the potential target firms, Nocke and Whinston (2011) have shown that private and social incentives are misaligned: mergers that cause a higher increase in the (naively computed) Herfindhal index tend to be more profitable for similar levels of consumer surplus. As a result, the optimal ex-ante policy rule includes multiple requirements for mergers that cause varied increases in the Herfindhal index. For the merger that causes the lowest increase in the Herfindhal index, the optimal ex-ante policy requires that the merger does not decrease consumer surplus with respect to the status quo. For the rest of the mergers the optimal policy is more stringent: it requires a minimum increase in consumer surplus, the size of which is larger as the Herfindhal index increases. In our model with ex-ante symmetric firms, private and social incentives are perfectly aligned as far as the choice of the merger is concerned: the merger that maximizes consumer surplus is also the one that maximizes aggregate profits. Thus, in our setting a commitment to a more stringent merger policy is desirable, exclusively due to failures in the bargaining process. The insights from Nocke and Whinston (2011) and ours are potentially complementary. However, it is not obvious how ex-ante asymmetries and bargaining failures may interact, and hence what would be the consequences of both factors combined in the implementation of the optimal merger policy. This issue is left for future research.

One of the crucial characteristics of our bargaining protocol is that only bilateral agreements are feasible. In our framework whenever a merger involves lower consumer surplus then it also implies lower industry profits. Thus, bargaining failures are mainly associated with the inability of the merging partners to compensate the outsider. Alternatively, if firms could sign binding contracts involving potential transfers from the merged to non-merged firms then we would expect that the probability of a relatively inefficient merger would be substantially diminished.

Obviously, allowing for transfer payments among firms is a delicate issue, as it could be used to implement collusive arrangements. Allowing transfer payments may also be counterproductive if there exists the possibility of preemptive mergers, which were discussed at the end of Section 3. In this case, since a merger lowers aggregate profits but raises consumer surplus, if and when transfer payments are allowed then firms may be able to reach an overall agreement that destroys the merger equilibrium.[21]

[21] Fridolfsson and Steneck (2005b) have noticed that divesture clauses in merger proposals can actually be equivalent to transfer payments between merged and non-merged firms. They have shown that when mergers

Let us now consider the case where competition authorities maximize a weighted average of consumer surplus and profits. Extending the analysis in this direction involves considering mergers that are profitable but unattractive, in the sense that firms prefer not to participate and be left out of the merger. In this case, Fridolfsson and Stennek (2005a and 2005b) show that the bargaining game is a war of attrition, and if all mergers are symmetric then there are multiple equilibria. In particular, there is an equilibrium where a merger takes place later in the game (delay). In our model with asymmetric mergers it can be shown that for some parameter values there is a probability that the relatively inefficient merger succeeds. Hence the identity problem is also an issue in this scenario. Consequently, the main insight of our paper seems to extend quite easily to the case where authorities have a more general objective function.

7 References

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are profitable but unattractive firms play a war of attrition. Whether or not divesture clauses may help in implementing the aggregate profit maximizing outcome in alternative scenarios remains an open question.

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8 Appendix

8.1 Proof of Lemma 3

We are restricting attention to parameter values that imply $c_n > 0$. Thus, we let $(3\beta+2\gamma)c_0 > \gamma\alpha$ if competition is in quantities, and $\gamma(\beta-\gamma)\alpha < (\beta+\gamma)(2\beta-\gamma)c_0$ if competition is in prices. Also, in order to simplify the discussion we will only consider cases where $q_k > 0$. This requires that $\beta\alpha > (\beta+\gamma)c_0$ if competition is in quantities and $(\beta-\gamma)\alpha > (\beta - \frac{\gamma^2}{\beta+\gamma})c_0$,[22] if competition is in prices.

The inverse demand function for each good is

$$p_i = \frac{\partial U(q,x)}{\partial q_i} = \alpha - \beta q_i - \gamma \sum_{j\neq i} q_j, \tag{11}$$

where p_i is the price of variety i in terms of the numeraire. Thus, the consumer surplus as a function of $(\mathbf{q}, x)$ can be written as

$$\widehat{CS}(\mathbf{q}, x) = U(q,x) - \sum_{i=1}^{3} p_i q_i = x + \frac{\beta-\gamma}{2}\sum_{i=1}^{3} q_i^2 + \frac{\gamma}{2}(\sum_{i=1}^{3} q_i)^2.$$

Quantity competition: In the triopoly game each firm solves

$$\max_{q_i} \left(\alpha - c_0 - \beta q_i - \gamma \sum_{j\neq i} q_j \right) q_i,$$

with reaction function

$$q_i = R_0(q_{-i}) = \frac{\alpha - c_0 - \gamma \sum_{j\neq i} q_j}{2\beta}. \tag{12}$$

[22] In price competition, if the intercept of the demand for firm k is below c_0, then the equilibrium price for firm k is c_0.

Then the symmetric equilibrium output of each firm, q_0, is

$$q_0 = \frac{\alpha - c_0}{2(\beta+\gamma)}. \tag{13}$$

Assume firms i and j merge with cost of each variety c_{ij}. Firm k's reaction function is still (12). For the merged firm the first order condition that determines q_i is now

$$(\alpha - c_{ij} - 2\beta q_i - 2\gamma q_j - \gamma q_k = 0, \tag{14}$$

and similarly for q_j. In equilibrium, $q_i = q_j$. Thus, let $q_{ij}(c_{ij})$ denote this common quantity of varieties i and j, and $q_k(c_{ij})$ denote the equilibrium quantity for firm k. These values are

$$\begin{aligned} q_{ij}(c_{ij}) &= \frac{2\beta\,(\alpha - c_{ij}) - \gamma(\alpha - c_0)}{4\beta(\beta+\gamma) - 2\gamma^2}, \\ q_k(c_{ij}) &= \frac{2(\gamma+\beta)(\alpha - c_0) - 2\gamma\,(\alpha - c_{ij})}{4\beta(\beta+\gamma) - 2\gamma^2}. \end{aligned} \tag{15}$$

We assume that indeed the condition for interior solution is satisfied. We let c_n be the value of c_{ij} that satisfies $q_{ij}(c_{ij}) = q_0$, where q_0 is the solution under triopoly defined in (13). We will see below that this value is well defined and below c_0. Since the reaction function of firm k has not changed, then also $q_k(c_n) = q_0$. Finally, $CS(c_n) = CS_0$.

Price competition: Assume now that firms set prices. We still have that the inverse demand system is given by (11). Inverting that system, we obtain the demand system,

$$q_i = \frac{1}{D}\left[\alpha(\beta-\gamma) - (\beta+\gamma)p_i + \gamma(p_j + p_k)\right],$$

where $D = (\beta-\gamma)(\beta+2\gamma)$. We write this as

$$q_i = \frac{1}{D}\left[A - Bp_i + G(p_j + p_k)\right],$$

where $A = \alpha(\beta-\gamma)$, $B = (\beta+\gamma)$ and $G = \gamma$. The first order condition for profit maximization by firm i is

$$A + Bc_0 - 2Bp_i + G(p_j + p_k) = 0. \tag{16}$$

Solving this system we obtain the pre-merger equilibrium prices as

$$p_0 = \frac{A + Bc_0}{2(B-G)}.$$

Assume firms i and j merge. Firm k's first order condition is still (16), whereas for the merged firm in its variety i the first order condition is

$$A + (B-G)c_{ij} - (2B-G)p_i + G(p_j + p_k) = 0.$$

Solving this system for $p_i = p_j$, we obtain

$$\begin{aligned} p_{ij} &= \frac{A(G+2B) + 2B(B-G)c_{ij} + BGc_0}{4B(B-G) - 2G^2}, \\ p_k &= \frac{AB + B(B-G)c_0 + G(B-G)c_{ij}}{2B(B-G) - G^2}. \end{aligned}$$

The quantities are

$$\begin{aligned} q_{ij} &= \frac{1}{D}\left[A - (B-G)p_{ij} + Gp_k\right], \\ q_k &= \frac{1}{D}\left[A - 2Bp_k + 2Gp_{ij}\right]. \end{aligned}$$

Note that,

$$\frac{dq_{ij}}{dc_{ij}} = \frac{2(B-G)}{D\left(4B(B-G)-2G^2\right)}\left(-(B-G)B+G^2\right) < 0,$$

since $(B-G)=\beta>\gamma=G$ and $B=\beta+\gamma>G$. Likewise,

$$\frac{dq_k}{dc_{ij}} = \frac{2G(B-G)B}{D\left(4B(B-G)-2G^2\right)} > 0.$$

Moreover,

$$2\frac{dq_{ij}}{dc_{ij}} + \frac{dq_k}{dc_{ij}} < 0,$$

since $2\left(-(B-G)B+G^2\right)+GB = -2B^2+3GB+2G^2 = -2(\beta+\gamma)^2+3(\beta+\gamma)\gamma+2\gamma^2 = -2\beta^2-\beta\gamma+3\gamma^2$, which is negative. Thus an increase in c_{ij} reduces the total quantity sold in the market, adding up all varieties.

Let c_n be such that $p_k(c_n)=p_0$. Note again that $p_{ij}(c_n)=p_0$. Indeed, the reaction function of firm k has not changed, so that any other prices by the merged firms would imply a different price by firm k.

We now show that all assumptions (A.1) through (A.4) are satisfied, both with price and quantity competition.

(A.4): $\pi_{ij}+\pi_k$ is decreasing in c_{ij}:

Assume quantity competition. Then, using the first order conditions of the firms' maximization problems,

$$\begin{aligned}\frac{d(\pi_{ij}+\pi_k)}{dc_{ij}} &= -2q_{ij}+2\frac{\partial q_{ij}}{\partial c_{ij}}\frac{\partial p_k}{\partial q_{ij}}q_k+\frac{\partial q_k}{\partial c_{ij}}\frac{\partial p_{ij}}{\partial q_k}q_{ij}\\ &= -2q_{ij}+2\frac{2\beta}{D}\gamma q_k-\frac{2\gamma}{D}\gamma q_{ij},\end{aligned}$$

where $D=4\beta(\beta+\gamma)-2\gamma^2$. Thus, the sign is the same as the sign of

$$-4\beta(\beta+\gamma)q_{ij}+\beta\gamma q_k<0.$$

Alternatively, assume price competition. Then, taking into account the first order conditions of firms' maximization problems for price competition,

$$\begin{aligned}\frac{d(\pi_{ij}+\pi_k)}{dc_{ij}} &= -2q_{ij}+2\frac{\partial q_{ij}}{\partial p_k}\frac{\partial p_k}{\partial c_{ij}}(p_{ij}-c_{ij})+\frac{\partial q_k}{\partial p_{ij}}\frac{\partial p_{ij}}{\partial c_{ij}}(p_k-c_0)\\ &= -2q_{ij}+2\frac{G}{D}\left[\frac{\partial p_k}{\partial c_{ij}}(p_{ij}-c_{ij})+\frac{\partial p_{ij}}{\partial c_{ij}}(p_k-c_0)\right]=\\ &\quad -2\frac{1}{D}\left[A-(B-G)p_{ij}+Gp_k\right]+2\frac{G}{D}\left[\frac{\partial p_k}{\partial c_{ij}}(p_{ij}-c_{ij})+\frac{\partial p_{ij}}{\partial c_{ij}}(p_k-c_0)\right].\end{aligned}$$

The second square bracket is positive. For interior solutions $q_k>0$, so that $A-2Bp_k+2Gp_{ij}>0$, and then the first square bracket is larger than $(2B+G)p_k-(G+B)p_{ij}$. Note that $\frac{\partial p_k}{\partial c_{ij}}, \frac{\partial p_{ij}}{\partial c_{ij}}<1$, $(p_k-c_0)<p_k$ and $(p_{ij}-c_{ij})<p_{ij}$. Thus, the expression is smaller than $2\frac{1}{D}$ times

$$\begin{aligned}&-\left[(2B+G)p_k-(G+B)p_{ij}\right]+G\left[p_{ij}+p_k\right]\\ =\;&-2Bp_k+(2G+B)p_{ij}\\ =\;&-2(\beta+\gamma)p_k+(3\gamma+\beta)p_{ij}<0,\end{aligned}$$

where the last inequality follows from $\beta>\gamma$ and the fact that, since $\frac{\partial p_k}{\partial c_{ij}}<\frac{\partial p_{ij}}{\partial c_{ij}}$, then for $c_{ij}<c_n$, $p_k>p_{ij}$. Q.E.D.

(A.2) and (A.3): $\frac{d\pi_{ij}}{dc_{ij}} < 0$ and $\frac{d\pi_k}{dc_{ij}} > 0$:

Under quantity competition the proof is trivial, since q_k is increasing in c_{ij} and q_{ij} is decreasing in c_{ij}. Also, for price competition

$$\frac{d\pi_k\left(c_{ij}\right)}{dc_{ij}} = \frac{\partial q_k}{\partial p_{ij}}\frac{\partial p_k}{\partial c_{ij}}\left(p_k - c_k\right) > 0,$$

and that, together with the previous lemma, proves that $\frac{d\pi_{ij}}{dc_{ij}} < 0$. Q.E.D.

Note that, if c_n is well defined, then $\pi_{ij}\left(c_n\right) = \left(p_0 - c_n\right)2q_0 > 2\left(p_0 - c_0\right)q_0 = 2\pi_0$, and $\pi_k\left(c_n\right) = \left(p_0 - c_0\right)q_0 = \pi_0$. Thus, (A.2) indeed follows from the lemma when c_n is well defined. We now turn to (A.1).

(A.1): $CS(c_{ij})$ is a differentiable, decreasing function of c_{ij} for $c_{ij} < c_0$, and so c_n is well defined, where $CS(c_n) = CS_0$:

Assume quantity competition. Note that by (15) $\frac{dq_{ij}(c_{ij})}{dc_{ij}} < 0$. Thus an increase in c_{ij} will induce a reduction in $q_{ij}(c_{ij})$. Then it suffices to show that $\widehat{CS}\left(\left(q_{ij}, q_{ij}, R_0(q_{ij})\right), x\right)$ is differentiable and increasing in q_{ij} for the relevant domain, $c_{ij} \leq c_0$. And indeed,

$$\begin{gathered}\frac{dCS\left(\left(q_{ij}, q_{ij}, R_0(q_{ij})\right), x\right)}{dq_{ij}} = \\ (\beta-\gamma)(2q_{ij} + R_0' q_k) + \gamma(2 + R_0')(2q_{ij} + q_k),\end{gathered} \tag{17}$$

where R_0' is the slope of the reaction function of firm k. The second term is positive, since $R_0' > -1$. Also, for $c_{ij} = c_0$

$$2q_{ij} + R_0' q_k = (\alpha - c_0)\frac{2(2\beta-\gamma) + 2(\beta-\gamma)R_0'}{4\beta(\beta+\gamma) - 2\gamma^2},$$

and since $R_0' > -1$, this is positive. Finally, $2q_{ij} + R_0' q_k$ is decreasing in c_{ij} since $\frac{dq_{ij}(c_{ij})}{dc_{ij}} < 0$ and $\frac{dq_k(c_{ij})}{dc_{ij}} > 0$. Thus, for $c_{ij} < c_0$ the expression is also positive, and we conclude that (17) is positive for all $c_{ij} \leq c_0$, which proves the result.

Consider now price competition. Recall that $\widehat{CS}(\mathbf{q}(c_{ij}), x) = x + \frac{\beta-\gamma}{2}\sum_{i=1}^{3} q_i^2 + \frac{\gamma}{2}(\sum_{i=1}^{3} q_i)^2$. We have already shown that $\sum_{i=1}^{3} q_i$ is decreasing in c_{ij}. Now,

$$\frac{d\sum_{i=1}^{3} q_i^2}{dc_{ij}} = (4q_{ij}\frac{\partial q_{ij}}{\partial c_{ij}} + 2q_k\frac{\partial q_k}{\partial c_{ij}}).$$

For $c_{ij} \leq c_n$, $q_{ij} > q_k$ so that since $2\frac{dq_{ij}}{dc_{ij}} + \frac{dq_k}{dc_{ij}} < 0$, this term is negative. Now, for all $c_{ij} > c_n$, we have $q_k > q_{ij}$. Also, we can write $\widehat{CS}(q(c_{ij}), x) = \frac{\beta}{2}(\sum_{i=1}^{3} q_i)^2 - \frac{\beta-\gamma}{2}(2q_{ij}^2 + 4q_{ij}q_k)$. Note that

$$\begin{aligned}\frac{d(2q_{ij}^2 + 4q_{ij}q_k)}{dc_{ij}} &= 4(q_{ij}+q_k)\frac{\partial q_{ij}}{\partial c_{ij}} + 4q_{ij}\frac{\partial q_k}{\partial c_{ij}} \\ &> 4(q_{ij}+q_k)\frac{\partial q_{ij}}{\partial c_{ij}} + 4\frac{q_{ij}+q_k}{2}\frac{\partial q_k}{\partial c_{ij}} \\ &= 2(q_{ij}+q_k)\left(2\frac{\partial q_{ij}}{\partial c_{ij}} + \frac{\partial q_k}{\partial c_{ij}}\right),\end{aligned}$$

where we have used the fact that $q_k > q_{ij}$ in the inequality. Moreover, note that

$$\frac{d}{dc_{ij}}(\sum_{i=1}^{3} q_i)^2 = 2(2q_{ij} + q_k)\left(2\frac{\partial q_{ij}}{\partial c_{ij}} + \frac{\partial q_k}{\partial c_{ij}}\right).$$

Therefore,

$$\begin{aligned}\frac{dCS}{dc_{ij}} &= \frac{\beta}{2}\frac{d}{dc_{ij}}(\sum_{i=1}^{3}q_i)^2 - \frac{\beta-\gamma}{2}\frac{d(2q_{ij}^2+4q_{ij}q_k)}{dc_{ij}} \\ &< \frac{\gamma}{2}\frac{d}{dc_{ij}}(\sum_{i=1}^{3}q_i)^2 + \frac{\beta-\gamma}{2}2q_{ij}\left(2\frac{\partial q_{ij}}{\partial c_{ij}}+\frac{\partial q_k}{\partial c_{ij}}\right) < 0,\end{aligned}$$

since $2\frac{\partial q_{ij}}{\partial c_{ij}} + \frac{\partial q_k}{\partial c_{ij}} < 0$. Q.E.D.

8.2 Proof of Lemma 4

Let $p(Q)$ be twice continuously differentiable, and for all Q such that $p(Q) > 0$, satisfy (i) $p'(Q) < 0$, (ii) $p'(Q) + p''(Q)\,Q < 0$, and (iii) $\lim_{Q\to\infty} p(Q) = 0$. Condition *(ii)* implies that individual profit functions are strictly concave. The first order condition for firm i's profit maximization problem, whether in duopoly or triopoly, is

$$p'(Q)\,q_i + p(Q) - c_i = 0. \tag{18}$$

Also, condition (ii) plus constant marginal costs imply that the reaction function of a firm is decreasing in the output of rival firms with slope greater than -1. Finally, in the duopoly game, a higher value of c_{ij} results in lower q_{ij}, and hence higher q_k and lower aggregate output. (See, for instance, Proposition 2.4 in Corchón, 1996.) Thus, CS is decreasing in c_{ij}.

If the condition

$$p'(Q_0)\,2q_0 + p(Q_0) > 0 \tag{19}$$

holds then in the duopoly game a merged firm with marginal costs $c_{ij} = 0$ will choose a level of output higher than $2q_0$, which implies that aggregate output will be higher than $Q_0 \equiv 3q_0$. Therefore, by the monotonicity of $CS(c_{ij})$, $c_n > 0$. It can be shown that condition (19) is equivalent to the elasticity of demand at Q_0 being higher than $\frac{2}{3}$. A sufficient assumption involving only the primitives that implies (19) is that $p(0) - c_0$ is positive but sufficiently small.

Equilibrium output, consumer surplus and profits are continuously differentiable, since demand is continuously differentiable. At a cost $c_{ij} = c_0$, the left hand side of (18) evaluated at $Q = Q_0$ and $q_{ij} = \frac{2}{3}Q_0$ $(> \frac{1}{3}Q_0)$ is negative, since $p' < 0$. Thus, at that cost $q_{ij} < \frac{2}{3}Q_0$, and so $Q_1 < Q_0$. Therefore, $CS(c_0) < CS_0$ and $c_n < c_0$. Assumption (A.1) holds.

Since q_{ij} is decreasing in c_{ij}, and q_k is increasing in q_{ij}, then $\pi_{ij}(c_{ij})$ is decreasing in c_{ij}. At cost c_n, $q_{ij} = 2q_k = \frac{2}{3}Q_0$, and since $c_n < c_0$, $\pi_{ij}(c_n) > 2\pi_0$. Assumption (A.2) holds.

Similarly, $\pi_k(c_{ij})$ is increasing in c_{ij}, and $\pi_k(c_n) = \pi_0$. Assumption (A.3) holds.

Now, using the envelope theorem, we can write

$$\frac{d\pi_{ij}+\pi_k}{dc_{ij}} = -q_{ij} + p'\frac{dq_{ij}}{dc_{ij}}\left[q_{ij}\frac{dq_k}{dq_{ij}} + q_k\right] = q_{ij}(-1 + p'\frac{dq_{ij}}{dc_{ij}}\left[\frac{dq_k}{dq_{ij}} + \frac{q_k}{q_{ij}}\right],$$

where, as mentioned, $\frac{dq_k}{dq_{ij}} < 0$, and $\frac{dq_{ij}}{dc_{ij}} < 0$. Thus, $\frac{d\pi_{ij}+\pi_k}{dc_{ij}} < q_{ij}(-1 + p'\frac{dq_{ij}}{dc_{ij}}\frac{q_k}{q_{ij}})$. Note that $\frac{q_k}{q_{ij}} < 1$ and, totally differentiating in (18) for the merged firm,

$$\frac{dq_{ij}}{dc_{ij}} = \frac{1}{2p' + p''q_{ij}}.$$

Thus, if $p'' < 0$, then $p'\frac{dq_{ij}}{dc_{ij}} < \frac{1}{2}$ and $\frac{d\pi_{ij}+\pi_k}{dc_{ij}} < 0$. If $p'' > 0$, then $2p' + p''q_{ij} < 2p' + p''Q_1 < p'$, since we are assuming that $p' + p''Q_1 < 0$. Therefore, $p'\frac{dq_{ij}}{dc_{ij}} < 1$, and we conclude that $\frac{d\pi_{ij}+\pi_k}{dc_{ij}} < 0$. Assumption (A.4) holds.

8.3 Proof of Proposition 1

A strategy for firm i consists of $\left(\mu_i^j, \lambda_i^j, \lambda_i^k\right)$ for the selection of negotiating partners and $\left(\theta_i^j, \rho_i^j, \theta_i^k, \rho_i^k\right)$ for the actual negotiation phase. μ_i^j is the probability that firm i invites firm j to be its negotiation partner in node (2), if i is chosen by nature in node (1). Given the definition of the game, the probability that i proposes k is $\mu_i^k = 1- \mu_i^j$. λ_i^j is the probability that firm i accepts firm j's invitation to become a negotiation partner in node (3), and λ_i^k is the probability that i accepts firm k's invitation. In line with the restriction to stationary strategies, we will assume that $\lambda_i^j = 1 - \lambda_i^k$. That is, if invited in step (2), firm i chooses its partner for the negotiation phase independently of who gave it that possibility, firm k or firm j. Therefore, in case nature chooses firm i, the probability that firms (i, j) negotiate in nodes (5) and (6) is $\mu_i^j \lambda_j^i$, the probability that (i,k) negotiate is $\mu_i^k \lambda_k^i = \left(1 - \mu_i^j\right) \lambda_k^i$, and the probability that (j,k) negotiate is $\mu_i^j \lambda_j^k + \mu_i^k \lambda_k^j = \mu_i^j \left(1 - \lambda_j^i\right) + \left(1 - \mu_i^j\right)\left(1 - \lambda_k^i\right)$. Also, θ_i^j is the (per period) offer that firm i makes to firm j with probability ρ_i^j in node (5) if the former is chosen by nature in node (4) as the proponent. θ_i^k and ρ_i^k are the corresponding values in a negotiation with k. In order to avoid open-set technical problems, and also to save in notation, we assume that in node (6) the respondent accepts with probability one any offer above or equal to the value of continuation. That is why we do not include these decisions in the definition of a strategy. As we will see in the analysis below, this is innocuous and in particular does not rule out the possibility of delay in case of indifference.[23] Then, in any equilibrium $\theta_i^j = \frac{r\pi_0 + u_j}{1+r}$ whenever $\pi_{ij} > \frac{u_i + u_j + 2r\pi_0}{1+r}$ (and we can also restrict to such offer when $\pi_{ij} = \frac{u_i+u_j+2r\pi_0}{1+r}$ and $\rho_i^j > 0$).

Again, note that in line with the restriction to stationary strategies, we are implicitly assuming that the answer to invitations to negotiate in node (3) and the offer in node (5) do not depend on who made the invitation to negotiate or who answered to that invitation, but only on the identity of the partner.

Let us denote u_i^{ij} the equilibrium per-period payoff that firm i expects in step (4) before nature chooses who will make an offer, and given that firms $(i.j)$ will be negotiating. That is, in any node of the extensive form game reached immediately after i has chosen j as the negotiation partner, or j has chosen i. Finally, let us d_{ij} denote the probability that merger (i, j) succeeds. We derive several properties of any equilibrium outcome.

8.3.1 Property 1: At least in one negotiation there is a strictly positive surplus; i.e., there exist a pair (i, j) such that $\pi_{ij} > \frac{u_i+u_j+2r\pi_0}{1+r}$.

Suppose not; i.e., for all (i, j)

$$\pi_{ij} \leq \frac{u_i + u_j + 2r\pi_0}{1+r}, \tag{20}$$

which implies that whenever firm i is one of the negotiation partners it gets $\frac{u_i+r\pi_0}{1+r}$, whether the merger materializes or not. If $d_{jk} = 0$ then $u_i = \frac{u_i+r\pi_0}{1+r}$, so that $u_i = \pi_0$. Thus, (20) implies that $u_j > \pi_{ij}$ and $u_k > \pi_{ik}$. This, together with $d_{jk} = 0$, is a contradiction. If $d_{ij} > 0$ then $u_i = d_{jk}\pi_i + (1 - d_{jk}) \frac{u_i+r\pi_0}{1+r}$. Hence, $u_i \leq \pi_0$. Similarly, $u_j \leq \pi_0$. Therefore, $\frac{u_i+u_{j+2r\pi_0}}{1+r} \leq 2\pi_0 < \pi_{ij}$. We have reached a contradiction.□

[23]Indeed, apart from open-set issues, in a SPE there could be indifference between accepting and rejecting a partner's offer only if the sum of the continuation values for both partners is equal to what they have to share. In ths case, the fact that the proponent can choose any value ρ in $[0, 1]$ already allows for any probability of delay.

8.3.2 Property 2: It cannot be the case that there is a strictly positive surplus in exactly two negotiations .

Suppose that in two and only two negotiations there is a strictly positive surplus, i.e.,

$$\begin{aligned} \frac{u_i+u_j+2r\pi_0}{1+r} &\geq \pi_{ij}, \\ \frac{u_i+u_k+2r\pi_0}{1+r} &< \pi_{ik}, \\ \frac{u_j+u_k+2r\pi_0}{1+r} &< \pi_{jk}. \end{aligned}$$

These inequalities imply that:

$$u_k + r\pi_0 < \frac{1+r}{2}\left(\pi_{ik}+\pi_{jk}-\pi_{ij}\right). \tag{21}$$

Since $u_i^{ik} > \frac{u_i+r\pi_0}{1+r} = u_i^{ij}$ then $\lambda_i^k = (\rho_i^k =)1$. Similarly, $\lambda_j^k = 1$. As a result, $d_{ij} = 0$ and $d_{ik} + d_{jk} = 1$. Hence, we can write:

$$\begin{aligned} u_i &= d_{ik}\frac{1}{2}\left(\pi_{ik}+\frac{u_i-u_k}{1+r}\right)+(1-d_{ik})\,\pi_i, \\ u_j &= d_{ik}\pi_j+(1-d_{ik})\,\frac{1}{2}\left(\pi_{jk}+\frac{u_j-u_k}{1+r}\right), \\ u_k &= d_{ik}\frac{1}{2}\left(\pi_{ik}+\frac{u_k-u_i}{1+r}\right)+(1-d_{ik})\,\frac{1}{2}\left(\pi_{jk}+\frac{u_k-u_j}{1+r}\right). \end{aligned}$$

If we solve the system for u_k then it turns out that for any $d_{ik} \in [0,1]$, the solution violates inequality (21). We have reached a contradiction.$\square$

8.3.3 Property 3: If firm i strictly prefers to negotiate with firm j and viceversa, then $i = 1$ and $j = 2$.

Consider first the case where there is only one negotiation with a strictly positive surplus. Then we show that it has to be the negotiation between firms i and j. Indeed, if $\pi_{ij} > \frac{u_i+u_j+2r\pi_0}{1+r}$, then

$$u_i^{ij} = \frac{1}{2}\left(\pi_{ij}+\frac{u_i+r\pi_0}{1+r}-\frac{u_j+r\pi_0}{1+r}\right) > \frac{u_i+r\pi_0}{1+r},$$

whereas $u_i^{ik} = \frac{u_i+r\pi_0}{1+r}$. The same applies to j, so that in equilibrium $\lambda_i^j = \lambda_j^i = 1$. Also, this implies that $\mu_i^j = \mu_j^i = 1$, and then $u_i = u_j = \frac{1}{2}\pi_{ij}$. Thus, $\pi_{ik} \leq \frac{u_i+u_k+2r\pi_0}{1+r} = \frac{\pi_{ij}+2r\pi_0}{1+r}$ only if $(i,j) = (1,2)$.

Alternatively, if all three negotiations involve a strictly positive surplus, then suppose that $(i,j) = (1,3)$. That is,

$$\begin{aligned} u_1^{13} &> u_1^{12}, \\ u_3^{13} &> u_3^{23}. \end{aligned}$$

These inequalities imply that $\lambda_3^1 = \lambda_1^3 = (\rho_1^3 = \rho_3^1 =)1$, and also $\mu_3^1 = \mu_1^3 = 1$. Thus, $u_1 = u_3 = \frac{1}{2}\pi_{13}$, and $u_2 = \pi_2$. Then

$$\begin{aligned} u_1^{12} &= \frac{1}{2}\left(\pi_{12}+\frac{\frac{1}{2}\pi_{13}+r\pi_0}{1+r}-\frac{\pi_2+r\pi_0}{1+r}\right) \\ &> \frac{1}{2}\left(\pi_{13}+\frac{\frac{1}{2}\pi_{13}+r\pi_0}{1+r}-\frac{\frac{1}{2}\pi_{13}+r\pi_0}{1+r}\right) = u_1^{13}, \end{aligned}$$

which is a contradiction. A similar contradiction would obtain if we assume that $(i, j) = (2, 3)$.$\square$

8.3.4 Property 4: Preference cycles cannot occur: If i weakly prefers to negotiate with j, j weakly prefers to negotiate with k, and k weakly prefers to negotiate with i, then they all must be indifferent.

Suppose not. If there is a strictly positive surplus in all three negotiations, so that all end up in agreement, then $u_i + u_j + 2r\pi_0 \leq (1+r)\,\pi_{ij}$, for all i, j, and then

$$\pi_{ij} - \frac{u_j + r\pi_0}{1+r} \geq \pi_{ik} - \frac{u_k + r\pi_0}{1+r}, \tag{22}$$

$$\pi_{jk} - \frac{u_k + r\pi_0}{1+r} \geq \pi_{ij} - \frac{u_i + r\pi_0}{1+r}, \tag{23}$$

$$\pi_{ik} - \frac{u_i + r\pi_0}{1+r} \geq \pi_{jk} - \frac{u_j + r\pi_0}{1+r}. \tag{24}$$

If we add up these three inequalities then this can only be satisfied if the three hold with equality. Suppose instead that only a negotiation between firms 1 and 2 generate a strictly positive surplus, $u_1 + u_2 + 2r\pi_0 < (1+r)\,\pi_{12}$, but for the rest of the pairs the inequality is (weakly) reversed. Let $i = 1$ and $j = 2$. Then $u_2^{23} = \frac{u_2 + r\pi_0}{1+r}$ and for 2 to prefer negotiations with $k = 3$, but $\frac{u_2+r\pi_0}{1+r} \geq \pi_{12} - \frac{u_1+r\pi_0}{1+r}$, which contradicts $u_1 + u_2 + 2r\pi_0 < (1+r)\,\pi_{12}$. Similarly if $i = 2$ and $j = 1$.$\square$

8.3.5 Property 5: If firm 1 strictly prefers to negotiate with firm 2, and viceversa, then $d_{12} = 1$.

If $u_1^{12} > u_1^{13}$ then $\lambda_1^2 = 1$. Similarly, if $u_2^{12} > u_2^{23}$ then $\lambda_2^1 = 1$. Since $u_1^{13} \geq \frac{u_1+r\pi_0}{1+r}$ and $u_2^{23} \geq \frac{u_2+r\pi_0}{1+r}$, then $\pi_{12} = u_1^{12} + u_2^{12} > \frac{u_1+u_2+2r\pi_0}{1+r}$. That is, $\rho_1^2 = \rho_2^1 = 1$. Finally, if $\mu_i^3 > 0$, $i = 1, 2$, then with positive probability there will be a meeting between firm 3 and either 1 or 2. Conditional on this, i's payoff is either u_i^{i3}, $\frac{u_i+r\pi_0}{1+r}$, or π_2. Since $u_i^{12} > u_i^{i3} \geq \frac{u_i+r\pi_0}{1+r}$, this cannot be part of an equilibrium unless $u_i^{12} = \pi_2 < \pi_0$. Thus, $u_i^{12} = \pi_2 > \frac{u_i+r\pi_0}{1+r}$ implies that $u_i < \pi_2$, which is a contradictions, since π_2 can be guaranteed by firms 1 and 2 by simply refusing any deal. Thus, $\mu_i^{12} = 1$ for $i = 1, 2$, and so $d_{12} = 1$.$\square$

8.3.6 Property 6: Firms 1 and 2 obtain the same expected payoff

Without loss of generality suppose $u_1 > u_2$. First, suppose that the negotiation between 1 and 2 is the only one that generates a strictly positive surplus. In this case, $u_1^{12} > \frac{u_1+r\pi_0}{1+r} = u_1^{13}$ and $u_2^{12} > \frac{u_2+r\pi_0}{1+r} = u_2^{23}$, and from Property 5, $d_{12} = 1$, which implies $u_1 = u_2 = \frac{1}{2}\pi_{12}$. Contradiction.

Suppose now that all three negotiations generate a strictly positive surplus. In this case firm 3 strictly prefers to negotiate with firm 2 rather than firm 1: $u_3^{23} > u_3^{13}$. Then from Property 4 there are two possibilities; (a) $u_2^{23} > u_2^{12}$, and (b) $u_1^{12} > u_1^{13}$ and $u_2^{23} \leq u_2^{12}$. Case (a) is ruled out by Property 3. In case (b), note that $u_1^{12} > u_1^{13}$ implies that $\lambda_1^2 = 1$, and $u_3^{23} > u_3^{13}$ implies that $\lambda_3^2 = 1$. Hence, $d_{13} = 0$ and $d_{12} + d_{23} = 1$. Thus,

$$u_1 = d_{12}u_1^{12} + d_{23}\pi_1 \leq u_1^{12}, \tag{25}$$

$$u_2 = d_{12}u_2^{12} + d_{23}u_3^{23}. \tag{26}$$

If $u_2^{23} < u_2^{12}$ then from Property 5, $d_{12} = 1$, and equations (25) and (26) imply that $u_1 = u_2$. If $u_2^{23} = u_2^{12}$ then $u_2 = u_2^{12}$, which together with inequality (25) contradicts that $u_1 > u_2$.$\square$

8.3.7 Property 7: There are two possible types of equilibria: (I) $u_1^{12} > u_1^{13}$ and $u_2^{12} > u_2^{23}$, (II) $u_i^{ij} = u_i^{ik}$ for all i, j, k.

Since $u_1 = u_2$ (Property 6) then $u_3^{23} = u_3^{13}$. Thus, both $u_2^{12} \leq u_2^{23}$, $u_1^{12} \geq u_1^{13}$, and $u_2^{12} \geq u_2^{23}$, $u_1^{12} \leq u_1^{13}$ would violate Property 4, unless both inequalities hold with equality. Thus, besides the case where all firms are indifferent, there are two other cases to consider: (a) $u_2^{12} < u_2^{23}$, $u_1^{12} < u_1^{13}$ and (b) $u_2^{12} > u_2^{23}$, $u_1^{12} > u_1^{13}$. Case (a) cannot be part of an equilibrium, since in this case $\lambda_1^2 = \lambda_2^1 = 0$ and hence $d_{12} = 0$. Moreover, $\pi_{13} > \frac{u_1+u_3+2r\pi_0}{1+r}$, and $\pi_{23} > \frac{u_2+u_3+2r\pi_0}{1+r}$. Therefore, $d_{13} + d_{23} = 1$. In this case:

$$\begin{aligned} u_1 &= d_{13}\frac{1}{2}\left(\pi_{13} - \frac{u_3 - u_1}{1+r}\right) + d_{23}\pi_1, \\ u_2 &= d_{13}\pi_1 + d_{23}\frac{1}{2}\left(\pi_{13} - \frac{u_3 - u_2}{1+r}\right), \\ u_3 &= \frac{1}{2}\left(\pi_{13} - \frac{u_1 - u_3}{1+r}\right). \end{aligned}$$

Since $u_1 = u_2$ then $d_{13} = d_{23}$. If we solve the above system we find that $u_3 = \frac{(1+2r)\pi_{13}-\pi_1}{1+4r} > \frac{\pi_{13}}{2}$. As a result $u_1^{13} < \frac{\pi_{13}}{2} < \frac{\pi_{12}}{2} = u_1^{12}$. We have reached a contradiction.□

We can now proceed to characterize the two types of equilibria.

8.3.8 Equilibrium type I

Consider an equilibrium with $u_1^{12} > u_1^{13}$ and $u_2^{12} > u_2^{23}$. From Property 4, $d_{12} = 1$. For instance, $\lambda_1^2 = \lambda_2^1 = \mu_1^2 = \mu_2^1 = \rho_1^2 = \rho_2^1 = 1$. Hence

$$u_1 = u_2 = \frac{1}{2}\pi_{12},$$

$$u_3 = \pi_3.$$

Thus, a profitable deviation for either firm 1 or firm 2 exists if and only if $\frac{1}{2}\pi_{12} < \frac{1}{2}\left(\pi_{13} - \frac{\pi_3}{1+r} + \frac{1}{2(1+r)}\pi_{12}\right)$. Therefore, $d_{12} = 1$ is an equilibrium if and only if:

$$\left(\frac{1}{2} + r\right)\pi_{12} - (1+r)\pi_{13} + \pi_3 \geq 0.$$

8.3.9 Equilibrium type II

Consider an equilibrium with $u_i^{ij} = u_i^{ik}$ for all i, j, k. If $u_i^{ij} = u_i^{ik} = \frac{u_i+r\pi_0}{1+r}$, that implies $u_j^{ij} = \frac{u_j+r\pi_0}{1+r}$ and $u_k^{ik} = \frac{u_k+r\pi_0}{1+r}$. Then, since $u_k^{ik} = u_k^{jk}$ and $u_j^{ij} = u_j^{jk}$, we reach a contradiction with Property 1. Thus, $u_i^{ij} = u_i^{ik} > \frac{u_i+r\pi_0}{1+r}$, so that by Property 3 there is positive surplus in all negotiations. Thus, all three negotiations would end in agreement. Thus, $d_{12} + d_{13} + d_{23} = 1$ and

$$\begin{aligned} u_1 &= (d_{12} + d_{13})\frac{1}{2}\pi_{12} + d_{23}\pi_1, \\ u_2 &= (d_{12} + d_{23})\frac{1}{2}\pi_{12} + d_{13}\pi_1, \\ u_3 &= d_{12}\pi_3 + (d_{13} + d_{23})\left(\pi_{13} - \frac{1}{2}\pi_{12}\right). \end{aligned}$$

Since $u_1 = u_2$ then $d_{13} = d_{23}$. If we let $d_{12} = d$, and hence $d_{13} = d_{23} = \frac{1-d}{2}$, and since

$$\pi_{12} - \frac{u_3 + r\pi_0}{1+r} = \pi_{13} - \frac{u_1 + r\pi_0}{1+r}, \tag{27}$$

we can solve this system to obtain

$$d = \frac{\pi_{12} - 2\pi_1 + 4r(\pi_{12} - \pi_{13})}{-\pi_{12} + 4\pi_{13} - 2\pi_1 - 4\pi_3}.$$

Note that $(\frac{1}{3} \leq)\ d < 1$ if and only if $\left(\frac{1}{2} + r\right)\pi_{12} - (1+r)\pi_{13} + \pi_3 < 0$. One such equilibrium would be $\lambda_i^j = \frac{1}{2}$,for all i, j, and $\mu_1^2 = \mu_2^1 = \frac{3d-1}{2}$ and $\mu_3^1 = \frac{1}{2}$. Finally, if $\left(\frac{1}{2} + r\right)\pi_{12} - (1+r)\pi_{13} + \pi_3 \geq 0$ then for r sufficiently small $\frac{u_1 + u_3 + 2\pi_0}{1+r} > \frac{1}{2}\pi_{13}$, and firms 1 and 2 cannot be indifferent between negotiating with each other or with firm 3.

Summarizing, for any r sufficiently close to 0 the equilibrium exists and the equilibrium outcome is unique. Q.E.D.

8.4 More than three firms

Consider the Cournot model with linear demand. In particular, the inverse demand function is $p = 1 - Q$, where Q is aggregate output. Thus, if we let n be the number of competing firms, and provided all firms are active, the equilibrium price can be written as:

$$p(n) = \frac{1 + \sum_{i=1}^{n} c_i}{n+1},$$

where c_i is the marginal cost of firm i, $i = 1, ..., n$.

Let us first consider the ex-post optimal myopic acceptance rule. That is, the rule that maximizes consumer surplus taking into account only the effect of the current merger proposal. Compared to the status quo, the marginal cost of the first approved merger proposal, c^F, must be lower than c_{n1}, which is given by:

$$\frac{1 + 4c_0}{5} = \frac{1 + 2c_0 + c_{n1}}{4},$$

i.e.,

$$c_{n1} = \frac{6c_0 - 1}{5}.$$

Thus, we assume that $c_0 \in \left[\frac{1}{6}, \frac{1}{2}\right]$ in order to guarantee that there is room for socially efficient mergers and all firms are active for any realization of synergies. Let us characterize the set of parameter values such that there is only one merger, but the identity of the merger remains an issue: $c_{13} \leq c_{n1}$.

A first mixed merger is profitable if and only if:

$$\pi_{13} - 2\pi_0 = \frac{(1 + 2c_0 - 3c_{13})^2}{16} - \frac{(1 - 2c_0 + c_{13})^2}{8} \geq F. \tag{28}$$

Similarly, a second mixed merger is not profitable if and only if:

$$\widetilde{\pi}_{13} - 2\pi_2 = \frac{(1 - c_{13})^2}{9} - \frac{(1 - 2c_0 + c_{13})^2}{4} \leq F. \tag{29}$$

It is easy to check that for all $c_0 \in [\frac{1}{6}, \frac{1}{2}]$, and all $c_{13} \in [0, c_{n1}]$, there exists a non-empty interval of values of F that satisfy both (28) and (29). Thus, in equilibrium there is a single merger, but the identity of the merger depends on independent conditions on (c_{12}, c_{13}), as discussed in the main text.

Figure 1

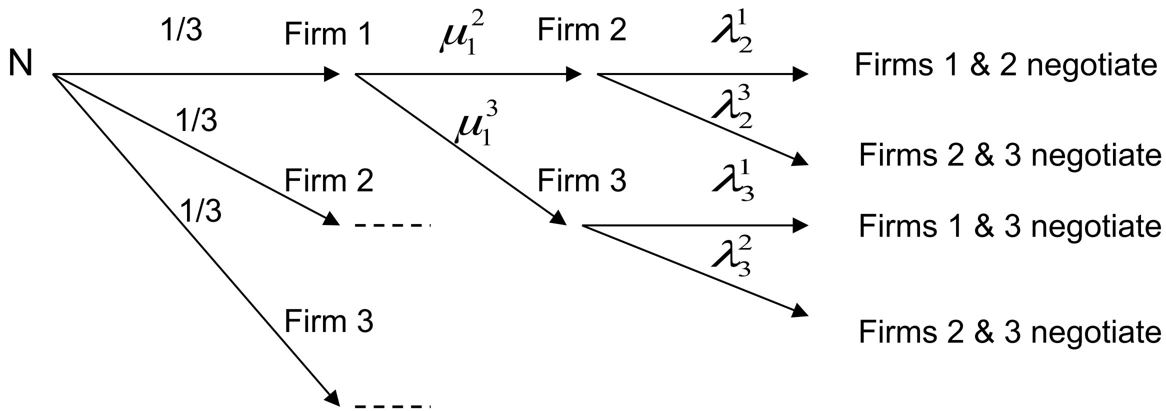
N
1/3
1/3
1/3
Firm 1
Firm 2
Firm 3
μ_1^2
μ_1^3
Firm 2
Firm 3
λ_2^1
λ_2^3
λ_3^1
λ_3^2
Firms 1 & 2 negotiate
Firms 2 & 3 negotiate
Firms 1 & 3 negotiate
Firms 2 & 3 negotiate

Figure 2