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ABSTRACT

Switching Costs and Equilibrium Prices*

In a competitive environment, switching costs have two effects. First, they increase the market power of a seller with locked-in customers. Second, they increase competition for new customers. I provide conditions under which switching costs decrease or increase equilibrium prices. Taken together, the suggest that, if markets are very competitive to begin with, then switching costs make them even more competitive; whereas if markets are not very competitive to begin with, then switching costs make them even less competitive. In the above statements, by "competitive" I mean a market that is close to a symmetric duopoly or one where the sellers' discount factor is very high.

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* An earlier draft (April 2008) circulated under the title “Small Switching Costs Lead to Lower Prices." While that title remains valid, the discovery of new results makes it incomplete.

Submitted 03 May 2012

1. Introduction

Consumers frequently must pay a cost in order to switch from their current supplier to a different supplier (Klemperer, 1995; Farrell and Klemperer, 2007). These costs suggest some interesting questions: are markets more or less competitive in the presence of switching costs? Are prices higher or lower under switching costs? How do seller profits and consumer surplus vary as switching costs increase?

Empirical evidence regarding these questions is ambiguous: although most studies suggest that switching costs lead to higher prices, there is also evidence to the contrary. This ambiguity is mirrored by the theoretical literature: some papers provide sufficient conditions such that switching costs make markets less competitive, with others predict the opposite effect.

In this paper, I develop an analytical framework that is flexible enough to encompass several possibilities, including pro-and anti-competitive effects of switching costs. In developing and analyzing this framework, I hope to provide intuition for some of the main effects of switching costs of market competition.

Most of the previous economics literature has solved some variation of a simple two-period model (see Section 2.3.1. in Farrell and Klemperer, 2007) The equilibrium of this game typically involves a bargain-then-ripoff pattern: in the second period, the seller takes advantage of a locked-in consumer and sets a high price (rip-off). Anticipating this second-period profit, and having to compete against rival sellers, the first-period price is correspondingly lowered (bargain). One limitation of two-period models is that potentially they distort the relative importance of bargains and ripoffs. In particular, considering the nature of many practical applications, two-period models unrealistically create game-beginning and game-ending effects.

To address this problem, I consider an infinite-period model where the state variable indicates the seller to which a given consumer is currently attached. The dynamic counterpart of the bargain-then-ripoff pattern is given by two corresponding effects on a seller's dynamic pricing incentives: the harvesting effect (sellers with locked-in customers are able to price higher without losing demand) and the investment effect (sellers without locked-in customers are eager to cut prices in order to attract new customers).

The harvesting and investment effects work in opposite directions in terms of market average price. Which effect dominates? Conventional wisdom and the received economics literature suggest that the harvesting effect dominates (Farrell and Klemperer, 2007). However, recent research casts doubt on this assertion (e.g., Doganoglu, 2010; Dubé, Hitsch and Rossi, 2007; Fabra and Garcia (2012)).[1]

In this paper, I attempt to clarify the issue by providing conditions under which switching costs decrease or increase equilibrium prices. The bottom line is that, if markets are very competitive to begin with, then switching costs make them even more competitive; whereas if markets are not very competitive to begin with, then switching costs make them even less competitive. In the above statements, by "competitive" I mean a market that is close to a symmetric duopoly or one where the discount factor is very high, so that the competition for future customers is relatively more important than revenues from current

1. The idea that the effects of switching costs can be pro-competitive is not novel. See for example von Weizsacker (1984) and Klemperer (1987) for seminal contributions and Cabral and Villas-Boas (2005) for a reinterpretation of some of those results. Recent research has explored the competitive effects of switching costs in an infinite-period context.

customers. In other words, in very competitive markets the competition effect dominates, whereas in concentrated markets the harvesting effect dominates.

As mentioned earlier, the idea that switching costs may have both a pro-competitive and an anti-competitive effect is hardly novel. The value added by the present analysis results form (a) using an infinite-period model, so that artificial "beginning of the world" and "end of the world" effects are not present; (b) assuming buyers are rational, forward looking agents (and allowing for the possibility that buyers discount the future differently from sellers); (c) making very weak assumptions regarding functional forms (essentially, assuming that marginal revenue increases in price).

Although fairly general in several dimensions, I restrict my analysis to the case when sellers can discriminate between locked-in and not locked-in consumers. Examples of this scenario include consumer goods such as magazine subscriptions and bank accounts.[2] More important, the assumption of customer discrimination is present in customer markets such as ready-mixed concrete or wide-body aircraft, markets where sales contracts are typically tailored to each customer.

■ **Related literature.** As mentioned earlier, two-period models suffer from various limitations. To address these, a series of infinite-horizon models have been developed. In particular Beggs and Klemperer (1992) show that switching costs lead to higher equilibrium prices. Farrell and Shapiro (1988), To (1995), and Padilla (1995) also consider infinite-horizon models with overlapping generations of consumers. Broadly speaking, they obtain similar results. My approach differs from theirs in two important ways. First, I assume that switching costs are finite and that there is residual product differentiation, so that switching occurs along the equilibrium path. Second, I consider the case when the seller is able to discriminate between locked-in and not locked-in consumers.

Against this backdrop of dynamic models with predicted anti-competitive effects, a series of recent papers have shown that switching costs may lead to pro-competitive effects. Dubé, Hitsch and Rossi (2009) show, by means of numerical simulations, that if switching costs are small then the investment effect dominates, that is, switching costs increase market competitiveness (see also Shin et al, 2009). I analytically solve a version of their model. Analytical solution has three advantages. First, it leads to more general results, that is, results that are not dependent on specific assumptions regarding functional forms and parameter values. Second, the process of solving the model leads to a better understanding of the mechanics underlying the result that the average market price declines when switching costs increase. It also shows why there is an important difference between small and large switching costs. Finally, an analytical model allows me to derive additional results (Propositions 3–6) beyond the effect of small switching costs (Proposition 2).

Doganoglu (2010) considers the case of small switching costs and shows that, along the equilibrium path, locked-in customers switch to the rival seller with positive probability. Moreover, steady-state equilibrium prices are decreasing in switching costs. Doganoglu's (2010) approach differs from mine in various respects. He assumes uniformly distributed preferences and linear pricing strategies; by contrast, I make very mild assumptions regarding the distribution of buyer preferences and the shape of the seller's pricing strategies. Moreover, my analysis goes beyond the case of small switching costs. Similarly to Doganoglu (2010), Fabra and García (2012) develop a continuous-time dynamic model with switching

2. Klemperer (1995) suggests many other examples.

costs and conclude that switching costs should only raise concerns in concentrated markets.

Four other recent papers on dynamics with switching costs are Arie and Grieco (2012); Biglaiser, Crémer and Dobos (2010); Somaini and Einav (2012); and Pearcy (2011). Arie and Grieco (2012) find an additional force toward a pro-competitive effect of switching costs: marginal consumers with a small switching cost lead an incumbent seller to lower price, even if this implies a lower margin on locked-in, loyal buyers. Biglaiser, Crémer and Dobos (2010) consider the case when consumer switching cost varies across consumers. They show that even low switching cost customers have value for the incumbent: when there are more of them the incumbent's profits increase. Moreover, an increase in the switching costs of all consumers can lead to a decrease in the profits of the incumbent. Somaini and Einav (2012) develop a model of dynamic price competition with switching costs. While some of the features of their model are somewhat stylized, it has the important advantage of allowing for a generic number of competitors and being analytically solved in closed form. Like Somaini and Einav (2012), Pearcy (2011) considers the general n firm case and shows that the effect of switching costs is the more competitive the larger the number of firms. This result is consistent with the overall theme of the present paper, namely that switching costs "amplify" market competitiveness: making competitive markets more competitive and non-competitive markets less competitive. In fact, it adds an additional diminution to this result, namely variations in the number of firms.

The rest of the paper is organized as follows. In Sections 3–5, I consider the effect of switching costs on price for different levels of the switching cost and discount factor assuming that sellers are symmetric (aside from the switching cost). In Sections 6 and 7, I consider the case of asymmetric market structures, either because one of the seller's product is (on average) better than the rival or because one of the sellers unilaterally creates a switching cost. Section 8 discusses several possible extensions and Section 9 concludes the paper.

2. Model

Consider an industry where two sellers compete over an infinite number of periods for sales to n infinitely lived buyers. Each buyer purchases one unit each period from one of the sellers. A buyer's valuation for seller i's good is given by z_i and known to the buyer only. I assume that z_i is distributed according to cdf $\Phi(z_i)$, with a corresponding density $\phi(z_i)$. I assume that the valuations z_i are sufficiently high that the buyer always purchases from one of the sellers. Moreover, for now I assume that z_i is i.i.d. across sellers and periods. (Section 6 addresses the possibility that preferences are serially correlated.) Moreover — and this a crucial element in the model — if the buyer previously purchased from seller j, then his utility from buying from seller i in the current period is reduced by s, the cost of switching between sellers.

In each period, sellers set prices simultaneously and then each buyer chooses one of the sellers. I assume that sellers are able to discriminate between locked-in and not locked-in buyers (that is, buyers who are locked in to the rival seller). Without further loss of generality, I hereafter focus on the sellers' competition for a particular buyer. I focus on symmetric Markov equilibria where the state indicates which seller made the sale in the previous period. I denote the seller who made a sale in the previous period (the "insider" seller) with the subscript 1, and the other seller (the "ousider" seller) with the subscript 0.

Symmetry implies that the buyer's continuation values from being locked in to seller i

or seller j are the same. This greatly simplifies the analysis, for even forward looking buyers need not compute any value function. (Sections 6 and 7 deal with asymmetric competition, and I will then need to explicitly compute the buyer's value function.) In each period the buyer chooses the insider seller if and only if

$$z_1 - p_1 + \beta\, u_1 \geq z_0 - p_0 + \beta\, u_0 - s \tag{1}$$

where u_i is the buyer's expected discounted utility from being locked in to seller i and β is the buyer's discount factor.[3] By symmetry, the value, starting next period, of being locked in to the firm that the buyer is currently locked in is the same as the value of being locked in to the other firm (that is, assuming that the switching cost has already been paid). For this reason, $u_1 = u_0$.[4]

Define

$$\begin{aligned} z &\equiv z_1 - z_0 \qquad (2) \\ x &\equiv p_1 - p_0 - s \qquad (3) \end{aligned}$$

Then (1) may be re-written as $z \geq x$. In words, x is the critical level of the buyer's relative preference z such that the buyer chooses the insider. Define by q_1 and q_0 the probability that the buyer chooses the insider or the outsider, respectively. If z is distributed according to $F(z)$, then we have

$$\begin{aligned} q_1 &= 1 - F(x) \\ q_0 &= F(x) \end{aligned}$$

I make the following assumptions regarding the c.d.f. F and the corresponding density f:

Assumption 1. *(i) $F(z)$ is continuously differentiable; (ii) $f(z) = f(-z)$; (iii) $f(z) > 0$, $\forall z$; (iv) $f(z)$ is unimodal; (v) $F(z)/f(z)$ is strictly increasing.*

Many distribution functions, including the Normal and the t, satisfy Assumption 1. In many of the results that follow, I will use repeatedly the following lemma, which characterizes several properties of F that follow from Assumption 1:

Lemma 1. *Under Assumption 1, the following are strictly increasing in z:*

$$\frac{F(z)^2}{f(z)}\,, \qquad \frac{F(z)-1}{f(z)}\,, \qquad \frac{2\,F(z)-1}{f(z)}$$

Moreover, the following is increasing in z iff $z > 0$ (and constant in z at $z = 0$):

$$\frac{\big(1-F(z)\big)^2 + \big(F(z)\big)^2}{f(z)}$$

3. I will denote the sellers' discount factor by δ. In many applications, it may make sense to assume $\beta = \delta$. However I distinguish between the buyer and the seller discount factor throughout. Among other reasons, this has the advantage of better highlighting the role of forward looking by buyers vs sellers.

4. In Sections 6 and 7, where I explicitly consider an asymmetric duopoly, the equality $u_1 = u_0$ no longer holds and I will need to explicitly compute the buyer value functions.

Proof: See Appendix. ■

In this paper, I will focus on symmetric Markov equilibria. My first result shows that there exists only one such equilibrium.

Proposition 1. *There exists a unique symmetric Markov equilibrium.*

Proof: The seller value functions are given by

$$\begin{aligned} v_1 &= \big(1 - F(x)\big)\big(p_1 + \delta\, v_1\big) + F(x)\;\delta\, v_0 \\ v_0 &= F(x)\big(p_0 + \delta\, v_1\big) + \big(1 - F(x)\big)\;\delta\, v_0 \end{aligned} \tag{4}$$

where δ is the seller's discount factor. The corresponding first-order conditions are

$$\begin{aligned} -f(x)\big(p_1 + \delta\, v_1\big) + 1 - F(x) + f(x)\,\delta\, v_0 &= 0 \\ -f(x)\big(p_0 + \delta\, v_1\big) + F(x) + f(x)\,\delta\, v_0 &= 0 \end{aligned}$$

(Recall that, from (3), $\frac{d\,x}{d\,p_1} = 1$ and $\frac{d\,x}{d\,p_0} = -1$.) Solving for optimal prices, I get

$$\begin{aligned} p_1 &= \frac{1 - F(x)}{f(x)} - \delta\, V \\ p_0 &= \frac{F(x)}{f(x)} - \delta\, V \end{aligned} \tag{5}$$

where

$$V \equiv v_1 - v_0$$

Substituting (5) for p_1, p_0 in (4) and simplifying, I get

$$\begin{aligned} v_1 &= \frac{\big(1 - F(x)\big)^2}{f(x)} + \delta\, v_0 \\ v_0 &= \frac{F(x)^2}{f(x)} + \delta\, v_0 \end{aligned}$$

It follows that

$$x = \frac{1 - F(x)}{f(x)} - \frac{F(x)}{f(x)} - s = \frac{1 - 2\,F(x)}{f(x)} - s \tag{6}$$

$$V = \frac{\big(1 - F(x)\big)^2}{f(x)} - \frac{F(x)^2}{f(x)} = \frac{1 - 2\,F(x)}{f(x)} \tag{7}$$

Equation (6) may be rewritten as

$$x + \frac{2\,F(x) - 1}{f(x)} = -s \tag{8}$$

By Lemma 1, the left-hand side is strictly increasing in x, ranging from $-\infty$ to $+\infty$ as x itself ranges from $-\infty$ to $+\infty$. This implies there exists a unique solution x. From (7), there exists a unique V. Finally, from (5) there exist unique p_0, p_1. ■

In the next sections, I offer sets of conditions under which switching costs lead to an increase or a decrease in prices (and average price). In the next section, I show that average price decreases (resp. increases) in switching costs if the initial value of the switching cost is low (resp. high). In Section 4, I consider the case of a high discount factor. In Section 6 I consider the asymmetric case, that is, the case when one seller is known to have a better product (in the eyes of the consumer). Finally, Section 7 considers the case when switching costs are themselves asymmetric.

3. Switching cost and average price

My main goal is to characterize equilibrium pricing as a function of switching costs s. In this section, I consider the case of small switching costs and prove that average price is decreasing in s if s is small and increasing in s if s is large. Let $\bar{p}$ be the average price paid by the buyer, that is,

$$\bar{p} = q_1\, p_1 + q_0\, p_0$$

Proposition 2. *If s is sufficiently small, then average price $\bar{p}$ is decreasing in switching cost s. Conversely, if s is sufficiently large, then average price $\bar{p}$ is increasing in switching cost s.*

Proof: Lemma 1, (8), and the implicit function theorem imply that x is strictly decreasing in s, ranging from 0 to $-\infty$ as s ranges from 0 to $+\infty$. I will thus consider the derivative of average price with respect to x, from which I then derive the comparative statics with respect to s. Average price is given by

$$\bar{p} \equiv q_1\, p_1 + q_0\, p_0 = \big(1 - F(x)\big)\, p_1 + F(x)\, p_0$$

Substituting (5) for p_1, p_0 and (7) for V, and simplifying, I get

$$\begin{aligned}
\bar{p} &= \big(1 - F(x)\big)\left(\frac{1 - F(x)}{f(x)} - \delta\, V\right) + F(x)\left(\frac{F(x)}{f(x)} - \delta\, V\right) \\
&= \frac{\big(1 - F(x)\big)^2}{f(x)} + \frac{F(x)^2}{f(x)} - \delta\, V \\
&= \frac{\big(1 - F(x)\big)^2 + F(x)^2}{f(x)} + \delta\left(\frac{2\,F(x) - 1}{f(x)}\right) \qquad (9)
\end{aligned}$$

Lemma 1 implies that, at $x = 0$, the first term on the right-hand side of (9) is constant in x; and that the second term on the right-hand side of (9) is increasing in x. It follows that, if x is small, then $\frac{d\,\bar{p}}{d\,x} > 0$.

Consider now the case when $x \to -\infty$. Taking the derivative of (9) with respect to x; then taking the limit $x \to -\infty$; and noting that, as $x \to -\infty$, $F(x) \to 0$ and $1 - F(x) \to 1$; we get

$$\lim_{x \to -\infty} \frac{\partial}{\partial\, x}\left(\frac{\big(1 - F(x)\big)^2 + F(x)^2}{f(x)} + \delta\left(\frac{2\,F(x) - 1}{f(x)}\right)\right) = -\left(2 + \lim_{x \to -\infty}\frac{f'(x)}{f(x)}\right)(1 - \delta) < 0$$

where I use the fact that, as $x \to -\infty$, $f'(x) > 0$. It follows that, if x is sufficiently close to $-\infty$, then $\frac{d\,\bar{p}}{d\,x} < 0$. ■

Proposition 2 provides an analytical and function-form generalization of Figure 1 in Dubé, Hitsch and Rossi (2009), where the authors show that average price is a U-shaped function of s. To understand the intuition, it is useful to look at the sellers' first-order conditions. The insider seller's value function is given by

$$v_1 = \big(1 - F(x)\big)\big(p_1 + \delta\, v_1\big) + F(x)\,\delta\, v_0$$

where v_i is seller i's value. In words: with probability $1 - F(x)$, the insider seller makes a sale. This yields a short-run profit of p_1 and the continuation value of an insider, v_1. With probability $F(x)$, the insider loses the sale, makes zero short run profits, and earns a continuation value v_0.

Maximizing with respect to p_1, we get the insider seller's first-order condition:

$$p_1 = \frac{1 - F(x)}{f(x)} - \delta\, V \tag{10}$$

where $V \equiv v_1 - v_0$ is the difference, in terms of continuation value, between winning and losing the current sale. In other words, $-\delta\, V$ is the "cost," in terms of discounted continuation value, of winning the current sale.

Since $q_1 = 1 - F(x)$ and $P = p_1 - p_0 - s$, we have $\frac{d\,q_1}{d\,p_1} = f(x)$. It follows that (10) may be re-written as

$$\frac{p_1 - (-\delta\, V)}{p_1} = \frac{1}{\epsilon_1}$$

where $\epsilon_1 \equiv \frac{d\,q_1}{d\,p_1}\,\frac{p_1}{q_1}$. This is simply the "elasticity rule" of optimal pricing, with one difference: the future discounted value from winning the sale appears as a negative cost (or subsidy) on price.

We thus have two forces on optimal price, which might denoted by "harvesting" and "investing." If the seller is myopic ($\delta = 0$), then optimal price is given by the first term in the right-hand side of (10). The greater the value of s, the smaller the value of x (as shown in the proof of Proposition 2), and therefore the greater the value of p_1. We thus have harvesting, that is, a higher switching cost implies a higher price (by the insider seller, which is the more likely seller).

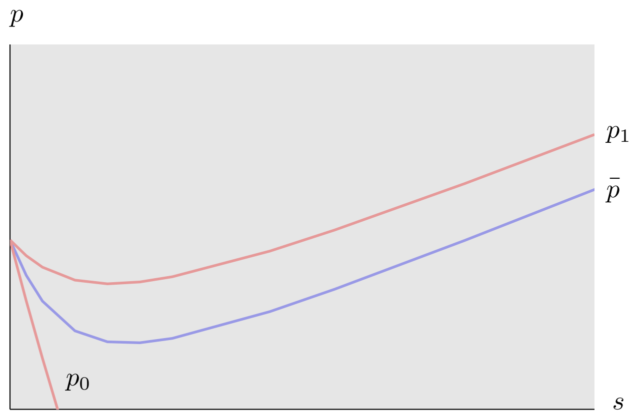
Suppose however that $\delta > 0$. Then we have a second effect, investing, which leads to lower prices. The greater the value of s, the greater the difference between being an insider and being an outsider, that is, the greater the value of V.

What is the relative magnitude of the harvesting and the investment effects on average price? First notice that harvesting leads to a higher price by the insider but a lower by the outsider. If fact, by symmetry, the effects are approximately of the same absolute value when s is close to zero. To see this, consider the outsider's first-order condition:

$$p_0 = \frac{F(x)}{f(x)} - \delta\, V \tag{11}$$

The "static" component of the first-order condition (corresponding to $\delta = 0$) moves in the opposite direction of p_1, as can be seen in (10). From a short-run point of view, the

Figure 1
Switching cost and equilibrium price. Red lines represent seller prices; blue lines represent average price.



switching cost s effectively introduces vertical product differentiation between the insider and the outsider, which in turn leads the insider to increase price and the outsider to reduce price by the same amount. This implies that, for s close to zero and in terms of average price, the harvesting effects approximately cancel out, since for $s = 0$ insider and ousider sell with equal probability.

Not so with the dynamic effect. In fact, as can be seen from (10) and (11), the "subsidy" resulting from the value of winning is the same for insider and outsider; and, to the extent that $v_1 > v_0$, the subsidy is positive for both players, that is, both want to price aggressively so as to secure a better position in the future. It follows that the effect on average price is unambiguously negative, and of first-order importance.

In other words, the harvesting effect is symmetric: the amount by which the insider increases its price is the same as the amount by which the outsider lowers its price. However, the investment effect is equal for both sellers — and negative.

Now consider the case when s is very high. A high value of s implies a low value of x, which in turn implies a low value of $F(x)$. In other words, a high s implies that the insider's probability of a sale, $1 - F(x)$, is close to 1. This implies that the average price paid by the buyer is essentially determined by the insider's price. Putting (5) and (8) together, we derive that the insider's price is given by

$$p_1 = \frac{1 - \delta - (1 - 2\,\delta)\,F(x)}{f(x)}$$

As $s \to \infty$, $x \to -\infty$ and $p_1 \to \infty$. This is the classic "ripoff" effect of switching costs: once the insider locks in the buyer, if valuations and the cost of switching are very high then the insider can set a very high price.

Figure 1 illustrates Proposition 2. (In this numerical illustration, I assume x is distributed according to a standardized normal.) On the horizontal axis, the value of switching cost varies from zero to positive values. On the vertical axis, three prices are plotted: the insider's price, the outsider's price, and average price. As the figure shows, average price is a U-shaped function of switching cost: for low values of s, higher switching costs have a pro-competitive effect, whereas for higher values of s, higher switching costs have an anti-competitive effect.

In sum, Proposition 2 suggests that the effect of switching costs on prices is largely an empirical question. Dubé, Hitsch and Rossi (2006) claim that, for various products, the value of switching cost lies in the region when the net effect of switching costs is to decrease average price.

In the next section, I add another dimension to the comparative statics of switching costs and market competition. Specifically, I show that, if the discount factor is sufficiently close to one, then switching costs lead to lower prices for *any* positive value of switching costs.

4. High discount factor

In the previous section, I showed that, for any preference distribution F satisfying Assumption 1 and any positive discount factor δ, average price is a U-shaped function of s. In this section, I provide an alternative sufficient condition for competitive switching costs. I show that, for any preference distribution F satisfying Assumption 1 and for any positive value of the switching cost s, if the discount factor is sufficiently close to 1 then average price is decreasing in s.[5]

Proposition 3. *If δ is close to 1, then average price $\bar{p}$ is decreasing in switching cost s.*

Proof: From (9),

$$\lim_{\delta \to 1} \bar{p} = 2\, \frac{F(x)^2}{f(x)}$$

Lemma 1 then implies that, if δ is sufficiently close to 1, then $\bar{p}$ is increasing in x. From the proof of Proposition 2, x is decreasing in s. (Notice that that statement does not depend on s being small.) The result then follows by the chain rule of differentiation. ■

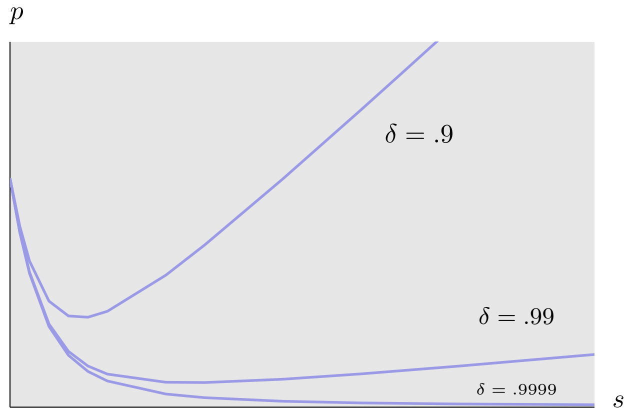
Figure 2 illustrates Proposition 3. It plots average price as a function of s for various values of δ. The curve corresponding to $\delta = .9$ corresponds to that in Figure 1. It is U shaped: for small values of s, average price is decreasing in s (Proposition 2). However, for high values of s, average price becomes increasing in s. As we consider higher values of δ, the U shape becomes more and more extended, so that, for a given range $[0, \bar{s}]$ of values of s, average price eventually becomes uniformly decreasing in s (Proposition 3).

I next attempt to provide an intuitive explanation for Proposition 3. In the proof of Proposition 1 and the discussion of Proposition 2, we saw that equilibrium prices are given by

$$\begin{aligned} p_1 &= \frac{1 - F(x)}{f(x)} - \delta\, V \\ p_0 &= \frac{F(x)}{f(x)} - \delta\, V \end{aligned} \tag{12}$$

5. Villas-Boas (2006) shows that prices are higher if sellers are more forward-looking. I will return to this paper in Section 8.

Figure 2
Switching cost and equilibrium average price as a function of the sellers' discount factor



(As I mentioned earlier, this is just the "elasticity rule" with the added element that sellers "subsidize" their cost by δV.) We also saw that the seller value functions are given by

$$\begin{aligned} v_1 &= \big(1 - F(x)\big)\big(p_1 + \delta\, v_1\big) + F(x)\ \delta\, v_0 \\ v_0 &= F(x)\big(p_0 + \delta\, v_1\big) + \big(1 - F(x)\big)\ \delta\, v_0 \end{aligned}$$

Substituting equilibrium prices into the value functions and simplifying we get

$$\begin{aligned} v_1 &= \frac{\big(1 - F(x)\big)^2}{f(x)} + \delta\, v_0 \\ v_0 &= \frac{F(x)^2}{f(x)} + \delta\, v_0 \end{aligned} \tag{13}$$

If $\delta = 0$, seller value is given by short-run profit, the first term on the right-hand side of the value functions. In a dynamic equilibrium, seller value is given by these short-term profits plus $\delta\, v_0$, *regardless of whether the seller wins or loses the current sale.*

This is an important point and one worth exploring in greater detail. To understand the intuition, it may be useful to think of an auction with two bidders with the same valuations. Specifically, each better gets w if he wins the auctions and l if he loses. The Nash equilibrium is for both bidders to bid $w - l$. If follows that equilibrium value is l for both bidders (winner or loser). In other words, the extra gain a bidder receives from being the winner, $w - l$, is bid away, so that a bidder can't expect more than l.

In the dynamic game at hand the analog of l is the continuation value if the seller loses the current sale, $\delta\, v_0$. So the idea is that all of the extra gain in terms of future value, $\delta\, V = \delta\,(v_1 - v_0)$, is bid away in terms of lower prices.

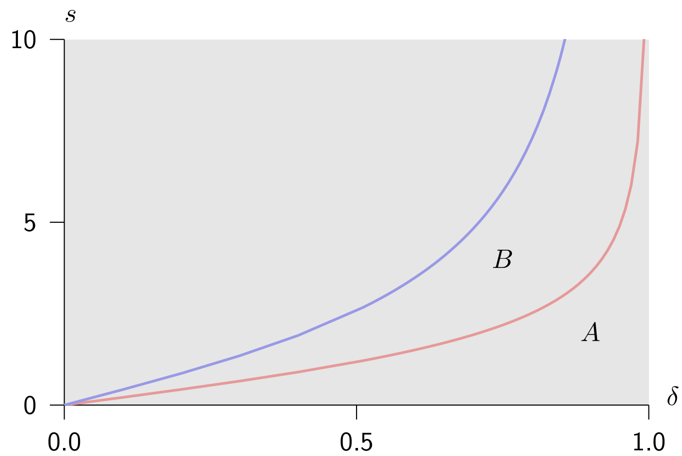
What does this imply in terms of equilibrium prices? From (19), we get

$$V = v_1 - v_0 = \frac{\big(1 - F(x)\big)^2}{f(x)} - \frac{F(x)^2}{f(x)} = \frac{1 - F(x)}{f(x)} - \frac{F(x)}{f(x)} \tag{14}$$

Substituting for V in (12) we get

$$\lim_{\delta \to 1} p_1 = \frac{1 - F(x)}{f(x)} - V = \frac{F(x)}{f(x)} \tag{15}$$

Figure 3
In region A, an increase in switching costs leads to a lower average equilibrium price. In regions A and B, average price is lower than it would be if switching costs were zero.



But, as we can see from (12), the right-hand side of (15) is simply the equilibrium value of p_0 when $\delta = 0$. In words, as the discount factor tends to 1, the insider's price level converges to the the outsider's static price level (i.e., when $\delta = 0$). But we know that, in a static model, increasing vertical product differentiation (in particular, increasing the switching cost) leads to a lower price by the "outsider" seller. That is, as we increase s from zero to a positive value, keeping $\delta = 0$, then the insider's price increases and the outsider's price decreases. If $s = 0$, then equilibrium price is the same regardless of the value of δ. Finally, putting it all together, we conclude that, as $\delta \to 1$ and $s > 0$, the high price is at the level of the lower price when $s = 0$; and so switching costs lead to lower average price.

■ **Numerical simulations.** Figure 3 summarizes the main results in the paper so far. The red curve represents the points at which the derivative of average price with respect to switching cost is zero. At points to the SE of this curve, an increase in switching cost implies a lower average price. Propositions 2 and 3 state two important properties of this curve: points with s sufficiently small (Proposition 2) or δ sufficiently high (Proposition 3) belong to region A. The figure suggests that this characterization is "tight," that is, Propositions 2 and 3 describe the essential properties of the boundary of region A.

The results in Pearcy (2011), setting $N = 2$, imply a qualitatively similar figure. In fact, as mentioned earlier, the idea that a low s or a high δ lead to a competitive effect of switching costs is present in various papers. One important element added by my analysis so far is to derive results that are largely independent of functional form assumptions. The specific values of Figure 3 assume a specific functional form for z (standardized normal); but Propositions 1–3 only require Assumption 1.

So far, I have examined how average price changes when switching costs increase. An alternative interesting comparison is between average price with $s > 0$ and average price when $s = 0$. The blue curve in Figure 3 depicts points such that average price is the same as when $s = 0$. For points to the SE of this curve, average price is lower with switching costs than without switching costs.

5. Profits and welfare

So far I have been dealing with the the impact of switching costs on average price, one of the central questions in the academic and public policy debate. What can be said about seller profits and consumer welfare? Tentatively, Propositions 2 and 3 suggest that under some conditions buyers are better off, and sellers worse off, with switching costs than without. However, paying a lower price is only half of the story for a buyer. To the extent that there is private information about preferences, switching actually occurs along the equilibrium path. We must therefore subtract the costs from switching when considering expected buyers surplus. As to the sellers, the fact that average price declines does not imply that sellers are uniformly worse off with switching costs. In fact, as shown earlier, the insider may be able to increase its price as a result of a higher switching cost. In sum, it is not obvious whether switching costs benefit buyers and sellers. My next result provides some answers to this question.

Proposition 4. *If s is small, then there exist $\delta'(s)$ and $\delta''(s)$, where $0 < \delta'(s) < \delta''(s) < 1$, such that an increase in switching cost s leads to*

1. *An increase in the insider's value if and only if $\delta < \delta'(s)$.*
2. *A decrease in the outsider's value for all δ.*
3. *A decrease in industry value (that is, the joint value of insider and outsider) for all δ.*
4. *An increase in consumer surplus if and only if $\delta > \delta''(s)$.*
5. *A decrease in welfare for all δ.*

Proof: See Appendix. ■

Note that one of the implications of Proposition 4 is that, if $\delta'(s) < \delta < \delta''(s)$, then all agents (insider, outsider, buyer) are worse off with switching costs than without.[6] More generally, Proposition 4 raises an interesting question: if switching costs are frequently created by sellers; and if largely sellers lose as a result of switching costs; then why do sellers create switching costs? Part of the answer to the question is given by point 1 in Proposition 4: if δ is sufficiently small, then an increase in switching costs increases the value of the incumbent seller. In Section 7 I take this issue of step further by considering the possibility that one of the sellers unilaterally increases the cost of switching away from its product.

6. Customer recognition

In the preceding sections I have assumed that, other than the switching cost, sellers believe that the consumer is on average indifferent between the two sellers. In real-world customer markets, however, sellers typically know something about buyer preferences. I now explicitly consider the possibility of *customer recognition*, that is, the possibility that sellers have

6. Biglaiser, Crémer and Dobos (2010) also show that an increase in the switching costs of all consumers can lead to a decrease in the profits of the incumbent. However, this takes place in a different context and for different reasons. See also Section 8.

some information about buyer preferences.[7] I model this by assuming that the buyer has a preference for seller i that is given by z_i, distributed according to cdf $\Phi_i(z_i)$. In the previous sections, I assumed that $z_i - z_j$ is distributed according to a cdf $F(z)$ which is symmetric about zero. Now I assume that $z_A - z_B = d + z$, where d is a constant common knowledge to sellers and z is distributed according to a cdf $F(z)$ which is symmetric about zero.

We now have buyer preferences that are serially correlated. This changes the problem substantially. In particular, a buyer is no longer indifferent between being attached to one seller or the other, for a buyer anticipates that, on average, one of the sellers will provide higher utility. Specifically, a buyer currently locked in to seller A now prefers to stay with that seller if and only if

$$d + z_A - p_{1A} + \beta\, u_A \geq -s + z_B - p_{0B} + \beta\, u_B$$

By contrast, a buyer currently locked-in to seller B prefers to stay with that seller if and only if

$$z_B - p_{0B} + \beta\, u_B \geq -s + d + z_A - p_{1A} + \beta\, u_A$$

The critical values x_i leading the buyer to purchase from seller i when seller i is the insider are now given by

$$\begin{aligned} x_A &\equiv p_{1A} - p_{0B} - \beta\,(u_A - u_B) - s - d \\ x_B &\equiv p_{1B} - p_{0A} + \beta\,(u_A - u_B) - s + d \end{aligned} \tag{16}$$

As before, seller i (the insider) sells with probability $q_{1i} = 1 - F(x_i)$.

We now have two "forces" toward seller asymmetry: d and s. The first one is permanent and favors one seller against the other (seller A, if $d > 0$). The second is transitory and favors the insider against the outsider. As before, the question at hand is: what impact does an increase in switching costs s have on the average price paid by the buyer? Contrary to Propositions 2 and 3, my next result provides sufficient conditions such that switching costs lead to higher prices:

Proposition 5. *If d is sufficiently large, then an increase in switching cost s leads to an increase in average price $\bar{p}$ if and only if $\delta < \delta'(d)$, where $0 < \delta'(d) < 1$.*

Proof: See Appendix. ■

Although Proposition 5 predicts the opposite effect of an increase in s than Proposition 2, the underlying mechanism is the same. The "harvesting" effect of an increase in switching cost is to increase the insider's price and decrease the outsider's. If $d = 0$ (as Proposition 2 assumes) then the two effects cancel out at $s = 0$ and are of second order for s close to zero. The reason is that, at $s = 0$, insider and outsider sell with equal probability; that is, in terms of average price, the insider and the outsider prices weigh equally. If $d > 0$, however, then the insider's price is the main determinant of average price. In the limit as $d \to \infty$, it is the only determinant of average price. It follows that the net harvesting effect is to increase average price. Moreover, this effect is of first order. It follows that, unless δ is

7. Chen and Pearcy (2010) also allow for customer recognition. Specifically, they model the joint distribution of consumer preferences over time with a copula to parametrize the degree of temporal dependence in preferences. For additional models with customer recognition, see Chen (1997), Villas-Boas (1999, 2006), Fudenberg and Tirole (2000), Taylor (2003), Doganoglu (2005).

high enough, then the harvesting effect dominates and average price increases as the result of an increase in s.

Proposition 5 is in line with the finding in Fabra and Garcia (2012) that "switching costs should only raise concerns in concentrated markets." In fact, a market with high d is a "concentrated" market, if we focus on the particular customer with high d. But Proposition 5 is about the value of d, not the distribution of market shares. It is conceivable that overall market shares are 50–50 and still the effect of switching costs is very anticompetitive.

■ **Closed-form solution.** For most of the paper I have made minimal assumptions regarding the distribution of $F(z)$. The disadvantage of that approach is that no analytical closed form solution is feasible. If we make additional assumptions, then a closed-form solution may be possible. Specifically, suppose that z is uniformly distributed. Without further loss of generality, suppose that $F(z) = \frac{1}{2} + z$.[8] Suppose also that $\beta = 0$. Then a closed-form solution can be derived for all values of d (seller A's advantage) and s (switching cost). The solution is given by (see Appendix for details)

$$\begin{aligned}
x_A &= -\frac{1}{3}\,s - \left(3 - \frac{8}{3}\,\delta\,s\right)^{-1} d \\
x_B &= -\frac{1}{3}\,s + \left(3 - \frac{8}{3}\,\delta\,s\right)^{-1} d \\
p_{1A} &= \frac{1}{2} + \frac{1-2\,\delta}{3}\,s + \frac{3-4\,\delta\,s}{9-8\,\delta\,s}\,d \\
p_{0A} &= \frac{1}{2} - \frac{1+2\,\delta}{3}\,s + \frac{3-4\,\delta\,s}{9-8\,\delta\,s}\,d \\
p_{1B} &= \frac{1}{2} + \frac{1-2\,\delta}{3}\,s - \frac{3-4\,\delta\,s}{9-8\,\delta\,s}\,d \\
p_{0B} &= \frac{1}{2} - \frac{1+2\,\delta}{3}\,s - \frac{3-4\,\delta\,s}{9-8\,\delta\,s}\,d
\end{aligned}$$

Computing the steady state average price is a little more difficult. For most of the paper I considered variations around $s = 0$ (and $d = 0$), so that all prices have equal weight. Not so when $d, s > 0$. Now I need to derive the steady-state probability that each price is actually charged. Specifically, the steady state average price is given by

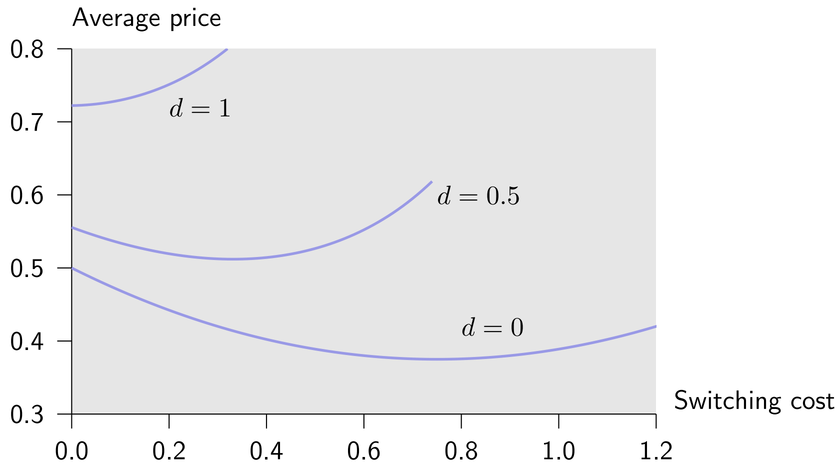
$$\bar{p} = \sum_{i=A,B} \mu_i (q_{1i}\,p_{1i} + q_{0j}\,p_{0j})$$

($j \neq i$) where μ_i is the steady-state probability that seller i is the insider. In the linear case, $q_{1i} = \frac{1}{2} - x_i$. Moreover, the steady state probabilities μ_i solve the system $[\mu_A\ \mu_B]\ M = [\mu_A\ \mu_B]$, where M, the Markov transition matrix, has q_{1i} in the main diagonal and q_{0j} in the off-diagonal terms, $i = A, B$, $j \neq i$. This implies that

$$\mu_i = \frac{\frac{1}{2} + x_j}{1 + x_i + x_j}$$

8. Strictly speaking, this functional form violates pars (i) and (iii) of Assumption 1. However, these do not play a crucial role in Proposition 5 or the analytical derivation that follows. Moreover, one can think of a smooth approximation to the uniform distribution that satisfies all of the parts of Assumption 1.

Figure 4
Switching cost and average price: linear case ($F(z) = \frac{1}{2} + z$, $\delta = .5$, $\beta = 0$)



$i = A, B$, $j \neq i$. We thus conclude that average price in the steady state is given by

$$\bar{p} = \sum_{i=A,B} \frac{\frac{1}{2} + x_j}{1 + x_i + x_j} \left(\left(\frac{1}{2} - x_i \right) p_{1i} + \left(\frac{1}{2} + x_i \right) p_{0j} \right)$$

Figure 4 plots equilibrium price as a function of switching cost s for various values of firm A's product advantage, d.[9] The case when $d = 0$ corresponds to the case considered in Section 3. Consistent with Proposition 2, we see that average price is a U-shaped function of switching cost: switching costs are pro-competitive for low values of s but anti-competitive for high values of s. As the value of d increases, we observe to things. First, average price is uniformly higher. This is not surprising and corresponds to the standard oligopoly intuition that greater asymmetry between firms leads to higher market power and consequently higher prices. Second, we observe that a higher d also increases the slope of average price with respect to s. In fact, if d is high enough (e.g., $d = 1$) then average price is increasing in s even for low values of s.

One of the advantages of a closed-form solution is that we can make more complete statements regarding comparative statics. Whereas in previous sections I was able to characterize broadly the shape of comparative statics, I can now explicitly derive the second cross partial derivative of average price $\bar{p}$ with respect to s and d. It is given by

$$\frac{\partial^2 \bar{p}}{\partial s \, \partial d} = \frac{24 \, d \left(9 + 6 \, \delta + 8 \, \delta \, s \, (1 - 2 \, \delta)\right)}{(9 - 8 \, \delta \, s)^3} \tag{17}$$

It can be shown that $3 - 4 \, \delta \, s > 0$, a necessary condition for the solution to be interior, implies that (17) is positive. In words, as the value of d increases, the impact of an increase in s becomes less and less competitive (or more and more anti-competitive). Or, as Figure 4 confirms: for a given value of s, as we increase d, the slope of the average price curve increases — eventually becoming positive.

9. I only plot points corresponding to an interior solution, that is, one where $0 < F(x) < 1$.

7. Endogenous asymmetric switching cost

Suppose that only seller A creates a switching cost. In other words, it costs s_A for a consumer to switch from seller A to seller B, but it costs zero for the consumer to switch from seller B to seller A. As in the previous section, I need to keep track of the seller's identity. Moreover, I need to explicitly compute the buyer's value functions. A buyer who is currently locked-in to seller A, chooses seller A again if and only if

$$z_A - p_{1A} + \beta\, u_A \geq -s_A + z_B - p_{0B} + \beta\, u_B$$

If the buyer is locked-in to seller B, however, then he chooses seller B if and only if

$$z_B - z_A \geq p_{1B} - p_{0A} - \beta\, u_B + \beta\, u_A$$

This implies that the critical values of the buyer's relative preference leading to a switch away from the insider seller are now given by

$$\begin{aligned} x_A &\equiv p_{1A} - p_{0B} - \beta\, u_A + \beta\, u_B - s_A \\ x_B &\equiv p_{1B} - p_{0A} - \beta\, u_B + \beta\, u_A \end{aligned} \tag{18}$$

As before, the insider seller's demand is given by $q_{1i} = 1 - F(x_i) = F(-x_i)$, $i = A, B$. As before, I attempt to evaluate the impact of increasing switching costs (unilaterally, in the present context) on prices and profits.

Proposition 6. *If s_A is sufficiently small, then an increase in seller A's switching cost s_A leads to*

1. *An decrease in average price.*
2. *An increase in seller A's steady state value.*
3. *A decrease in seller B's steady state value.*

Proof: See Appendix. ■

Notice that point 2 in Proposition 6 is different, and stronger, than point 1 in Proposition 4. The latter refers to an increase in value by the insider seller, whereas the former refers to an increase in value along the steady-state. Consider a seller with an equal number of buyers who are locked in and buyers who are not locked in. In this case, Proposition 4 would predict a decline in seller value following a uniform increase in switching costs, whereas Proposition 6 states that, in the steady state, seller value would increase following a unilateral increase in switching cost.

Proposition 6 also suggests that the metagame where sellers choose levels of switching cost has the nature of a prisoner's dilemma. Specifically, consider the following metagame. In a first stage, sellers simultaneously choose $s \in \{s_L, s_H\}$, where $s_H > s_L$ and both s_L and s_H are small. In a second stage, sellers play the infinite period game we have been considering up to now. Then Propositions 2 and 6 imply that the metagame has the structure of a prisoner's dilemma: choosing s_H is a dominant strategy for each seller (by Proposition 6), but the payoff from (s_H, s_H) is lower than the payoff from (s_L, s_L) (by Proposition 2).

This result bears some resemblance to Caminal and Matutes (1990) (see also Banerjee and Summers, 1987), with the important difference that I consider an infinite period price competition model rather than a two-period model.

8. Discussion

In this section, I discuss the extent to which my results depend on consumer discounting, product differentiation, and price discrimination. I also look at the issue of heterogeneous switching costs as well as the role of market shares.

■ **Consumer discounting.** The effects of the seller's discount factor has already been discussed in previous sections. Basically, the greater δ, the greater the competition effect is, which leads to more aggressive pricing. Consider now the effect of buyer discounting. In Sections 6 and 7, I explicitly modeled the buyer's dynamic problem. Specifically, buyers discount the future according to discount factor β and compute the expected future value of being attached to a given seller. What is the effect of buyer's discount? In other words, how does the effect of switching costs on prices depend on the extent to which buyers are forward looking? In the proof of Proposition 5, I show that, as $d \to \infty$ (that is, as one of the sellers becomes arbitrarily large), then the derivative of average price with respect to switching cost at $s = 0$ is given by

$$\frac{\partial \bar{p}}{\partial s}\bigg|_{s=0} = \frac{\partial p_{1A}}{\partial s}\bigg|_{s=0} = \frac{1}{3}\,(1 - 2\,\delta)\,(1 + \beta)$$

The above equation suggest that the effect of buyer discounting is to "magnify" the effect of switching costs. In fact, as β changes from 0 to 1, the effect of switching costs is multiplied by 2. Consider specifically the case when $\delta < \frac{1}{2}$, such that that the harvesting effect dominates and average price goes up as switching costs increase. Intuitively, a forward looking buyer is more "locked in" to a dominant seller. The forward looking buyer knows that it is being ripped off, but it also knows that switching is not going to be of much help, since most likely the buyer will switch back to the dominant seller again, at a discounted loss of $\beta\, s$. The higher β, the higher this loss is, and the less tempted to switch the buyer is. This effect is similar to the role of consumer discounting discussed in Farrell and Klemperer (2007) and Villas-Boas (2006).

■ **Product differentiation.** What is the effect of product differentiation? One natural way of modeling product differentiation is to assume that the distribution of z is given by $F(z/\sigma)$. Whereas up to now we have implicitly assumed that $\sigma = 1$ (fixed degree of product differentiation), we can now consider comparative statics with respect to the value of σ. Notice that my limit results (small s, large s, large δ, large d) do not depend on the shape of $F(z)$ beyond what's assumed in Assumption 1. In this sense, the results do not depend on the degree of product differentiation. However, the value of d considered in Section 6 is likely to be higher (on average) the higher the value of σ. In fact, the value of d is best thought of as a random draw from $F(z/\sigma)$. In this sense, we would predict that, in markets with a greater degree of product differentiation, the effect of switching costs is more anticompetitive.

Although the qualitative nature of my results does not depend on the value of σ, I should note that there is an important discontinuity at $\sigma = 0$. So, a more precise statement would be that the qualitative results do not depend on the value of σ *so long as* σ *is positive.* If $\sigma = 0$, so that there is no residual product differentiation, then the insider seller sells with probability 1 regardless of the level of switching cost. By contrast, if the degree of product differentiation is strictly greater than zero, then, as the level of switching costs goes to zero the insider seller sells with probability 50%. This is one of the important differences of my paper (as well as other papers that feature product differentiation) with respect to much of the previous literature: because of residual product differentiation, switching does take place along the equilibrium path.

■ **Price discrimination.** Although I make only very weak assumptions regarding the nature of product differentiation, I do make some important assumptions regarding the nature of pricing and the dynamics of buyer preferences. First, as mentioned in the introduction, I assume that sellers can discriminate between buyers who are locked in and buyers who are not. If sellers cannot discriminate, then we must simultaneously consider all buyers (not just one) and the seller's optimal price will strike a balance between harvesting locked-in buyers and investing on new buyers. Beggs and Klemperer (1992) argue that the balance tends to favor higher prices than without switching costs. The contrast between my result and that of Beggs and Klemperer (1992) bears some relationship to the literature of oligopoly price discrimination (Corts, 1998). When the technology of price discrimination allows firms to target each others most loyal or locked-in consumers, oligopolists typically would like to commit not to price discriminate as this would soften overall price competition.

■ **Uncertain and heterogeneous switching cost.** I have assumed throughout that the value of s is known to buyer and sellers. In many real-world situations, the buyer has better information about switching cost than sellers. Suppose that sellers believe that s is distributed according to $G(s)$, whereas the buyer knows the precise value of s. Assume also that s is i.i.d. across periods and independent of buyer valuations z_i. Then all of the results in the paper go through if we redefine variables appropriately. First, we let s denote the average of s (according to cdf G). Second, we denote by $F(z)$ the convolution of the original $F(z)$ and the cdf $G(s)$ shifted by the average of s. In other words, we can simply include uncertainty about s in the overall uncertainty about valuations z_i.[10]

A related issue is that of heterogeneity in switching costs, that is, the case when different buyers have different values of the switching cost. To the extent that sellers can discriminate across buyers, the results in the paper still apply. If however discrimination is not possible, then heterogeneity can matter a lot, especially in a dynamic context. This problem lies beyond the scope of the present paper and is discussed in Biglaiser, Crémer and Dobos (2010).

■ **Market share and firm value.** My assumption of price discrimination implies that I can look at competition for each individual buyer. In aggregate terms, this leads to market share effects similar to those in previous literature. As shown in Proposition 2, a small increase in switching cost implies that the incumbent seller increases its price, whereas the outsider seller decreases its price. Now consider a seller with a market share ν, that is, for a fraction ν this seller is the inside seller. Then we can show that (a) seller average price is linear and increasing in market share; (b) seller value is linear and increasing in market share. Biglaiser, Crémer and Dobos (2010) also provide conditions under which firm value is linear and increasing in market share.

10. This is related to the classical problem of estimating switching costs empirically, namely separating switching costs from consumer heterogeneity.

9. Conclusion

In a competitive environment, switching costs have two effects. First, they increase the market power of a seller with locked-in customers. Second, they increase competition for new customers. In this paper, I derived conditions under which switching costs decrease or increase equilibrium prices. Overall, the paper's message is that, if markets are very competitive to begin with, then switching costs make them even more competitive; whereas if markets are not very competitive to begin with, then switching costs make them even less competitive. In the above statements, by "competitive" I mean a market that is close to a symmetric duopoly or one where the sellers' discount factor is very high.

Appendix

Proof of Lemma 1: First notice that

$$\frac{F(z)^2}{f(z)} = F(z)\,\frac{F(z)}{f(z)}$$

Since $F(z)$ is increasing and $\frac{F(z)}{f(z)}$ is strictly increasing (by Assumption 1), it follows that the product is strictly increasing.

Next notice that, by part (ii) Assumption 1,

$$\frac{F(z)-1}{f(z)} = \frac{-F(-z)}{f(z)} = \frac{-F(-z)}{f(-z)}$$

Since $\frac{F(z)}{f(z)}$ is strictly increasing, $\frac{-F(-z)}{f(-z)}$ is strictly increasing too.

Next notice that

$$\frac{2\,F(z)-1}{f(z)} = \frac{F(z)-1}{f(z)} + \frac{F(z)}{f(z)}$$

I have just proved that $\frac{F(z)-1}{f(z)}$ is strictly increasing. We thus has the sum of two strictly increasing functions, the result being a strictly increasing function.

Finally, taking the derivative of the fourth expression I get

$$\begin{aligned}
&\frac{d}{d\,z}\left(\frac{\big(1-F(z)\big)^2+\big(F(z)\big)^2}{f(z)}\right) = \\
&\qquad = \frac{\Big(-2\big(1-F(z)\big)\,f(z)+2\,F(z)\,f(z)\Big)\,f(z)}{\big(f(z)\big)^2} - \frac{f'(z)\,\Big(\big(1-F(z)\big)^2+\big(F(z)\big)^2\Big)}{\big(f(z)\big)^2} \\
&\qquad = 4\left(F(z)-\frac{1}{2}\right) - f'(z)\,\xi
\end{aligned}$$

where $\xi = \Big(\big(1-F(z)\big)^2+\big(F(z)\big)^2\Big)/\big(f(z)\big)^2$ is positive. The result then follows from Assumption 1. ■

Proof of Proposition 4: From the proof of Proposition 1, we know that

$$\begin{aligned}
v_1 &= \frac{\big(1-F(x)\big)^2}{f(x)} + \delta\,v_0 \\
v_0 &= \frac{F(x)^2}{f(x)} + \delta\,v_0
\end{aligned} \tag{19}$$

where the value of x is given by

$$x + \frac{2\,F(x)-1}{f(x)} = -s \tag{20}$$

By Assumption 1 and Lemma 1, the left-hand side of (20) is increasing in x. It follows that, by the implicit function theorem, x is decreasing in s. In particular $\partial\,x\,/\,\partial\,s\,|_{\,s\,=\,0} \;\; = -\frac{1}{3}$.

Lemma 1 also implies that $F(x)^2/f(x)$ is increasing in x and $(1-F(x))^2/f(x)$ decreasing in x. From (19), this implies that v_0 is decreasing in x, whereas v_1 is increasing in x if and only if δ is sufficiently low. Specifically, at $s = 0$ we have

$$\begin{aligned}\frac{\partial v_0}{\partial s} &= \frac{1}{1-\delta}\,\frac{\partial x}{\partial s}\\ \frac{\partial v_1}{\partial s} &= \left(1+\delta\,\frac{-1}{1-\delta}\right)\,\frac{\partial x}{\partial s} = -\frac{1-2\,\delta}{1-\delta}\,\frac{\partial x}{\partial s}\\ \frac{\partial\,(v_0+v_1)}{\partial s} &= \frac{2\,\delta}{1-\delta}\,\frac{\partial x}{\partial s}\end{aligned}$$

Since $\partial x \,/\, \partial s < 0$, we conclude that, at $s = 0$, both v_0 and $v_0 + v_1$ are decreasing in s, whereas v_1 is increasing in s if and only if $\delta < \frac{1}{2}$.

Next consider consumer welfare. Recall that the distribution of z_i $(i = A, B)$ is given by $\phi(z_i)$ and define

$$E(z) \equiv \int_{z_i - z_j \geq z} z_i \; d\Phi(z_i)\, d\Phi(z_j)$$

In words, $E(z)$ is the buyer's expected valuation given that he chooses a particular seller by using the threshold z of differences in valuations (times the probability of choosing that particular seller). Per period expected consumer surplus is given by

$$u = E(x) + E(-x) - \big(1 - F(x)\big)\; p_1 - F(x)\,(p_0 + s) \tag{21}$$

Notice that if $E(x)$ is the expected value of z_i given that $z_i - z_j \geq x$, then $E(-x)$ is the expected value of z_j given that $z_j - z_i \geq -x$, where the latter is the same condition as $z_i - z_j \leq x$, that is, the negation of $z_i - z_j \geq x$. Define

$$e(z) \equiv \frac{d\,E(z)}{d\,z}$$

Differentiating (21) with respect to s, we get

$$\frac{\partial u}{\partial s} = \Big(e(x) - e(-x) + f(x)\,(p_1 - p_0 - s)\Big)\;\frac{\partial x}{\partial s} - \big(1 - F(x)\big)\;\frac{\partial p_1}{\partial s} - F(x)\,\frac{\partial p_0}{\partial s} - F(x)$$

Evaluating at $s = 0$, we get

$$\left.\frac{\partial u}{\partial s}\right|_{s\,=\,0} = -\frac{\partial \bar{p}}{\partial s} - \frac{1}{2}$$

Differentiating (8) we get $\partial x \,/\, \partial s = -\frac{1}{3}$. Differentiating (9), we get $\frac{\partial \bar{p}}{\partial x} = 2\,\delta$. It follows that

$$\left.\frac{\partial u}{\partial s}\right|_{s\,=\,0} = \frac{2}{3}\,\delta - \frac{1}{2}$$

It follows that consumer surplus increases if and only if $\delta > \frac{3}{4}$. Finally, the result regarding total welfare is trivial: since the market is covered, all price effects are simply a transfer between buyers and sellers. The net effects on consumer welfare come from "transportation cost" (an effect of second order at $s = 0$) and switching costs (a first-order effect). This implies that an increase in s has a first-order negative effect on total welfare. ■

Proof of Proposition 5: Seller i's value functions ($i = A, B;\; j \neq i$) are given by

$$\begin{aligned} v_{1i} &= \Big(1 - F(x_i)\Big)\big(p_{1i} + \delta\, v_{1i}\big) + F(x_i)\;\delta\, v_{0i}\\ v_{0i} &= F(x_j)\Big(p_{0i} + \delta\, v_{1i}\Big) + \big(1 - F(x_j)\big)\;\delta\, v_{0i}\end{aligned} \tag{22}$$

The corresponding first-order conditions are

$$\begin{aligned} -f(x_i)\big(p_{1i} + \delta\, v_{1i}\big) + 1 - F(x_i) + f(x_i)\,\delta\, v_{0i} &= 0 \\ -f(x_j)\big(p_{0i} + \delta\, v_{1i}\big) + F(x_j) + f(x_j)\,\delta\, v_{0i} &= 0 \end{aligned}$$

Solving for p_{1i}, p_{0i}, we get

$$\begin{aligned} p_{1i} &= \frac{1 - F(x_i)}{f(x_i)} - \delta\, V_i \\ p_{0i} &= \frac{F(x_j)}{f(x_j)} - \delta\, V_i \end{aligned} \tag{23}$$

where $V_i \equiv v_{1i} - v_{0i}$. Substituting (23) for p_{ki} in (22), I get

$$\begin{aligned} v_{1i} &= \frac{\big(1 - F(x_i)\big)^2}{f(x_i)} + \delta\, v_{0i} \\ v_{0i} &= \frac{F(x_j)^2}{f(x_j)} + \delta\, v_{0i} \end{aligned} \tag{24}$$

and so

$$V_i = v_{1i} - v_{0i} = \frac{\big(1 - F(x_i)\big)^2}{f(x_i)} - \frac{F(x_j)^2}{f(x_j)} \tag{25}$$

Substituting (25) for V_i in (23), we get

$$\begin{aligned} p_{1i} &= \frac{1 - F(x_i)}{f(x_i)} - \delta\, \frac{\big(1 - F(x_i)\big)^2}{f(x_i)} + \delta\, \frac{F(x_j)^2}{f(x_j)} \\ p_{0i} &= \frac{F(x_j)}{f(x_j)} - \delta\, \frac{\big(1 - F(x_i)\big)^2}{f(x_i)} + \delta\, \frac{F(x_j)^2}{f(x_j)} \end{aligned} \tag{26}$$

This parallels the derivation starting in (4), only that now value functions and prices are indexed by seller identity.

Next consider the buyer value functions, u_i, which I measure *before* the buyer learns his valuations z_A, z_B. The values of u_i are recursively given by

$$u_i = E(x_i) + E(-x_i) + \big(1 - F(x_i)\big)\big(d_i - p_{1i} + \beta\, u_i\big) + F(x_i)\big(d_j - s - p_{0j} + \beta\, u_j\big) \tag{27}$$

$i = A, B,\ j \neq i$.

Recall that the preference thresholds are given by

$$\begin{aligned} x_A &\equiv p_{1A} - p_{0B} - \beta\,(u_A - u_B) - s - d \\ x_B &\equiv p_{1B} - p_{0A} + \beta\,(u_A - u_B) - s + d \end{aligned} \tag{28}$$

Notice that, as $d \to \infty$, $x_A \to -\infty$ and $x_B \to +\infty$.[11] This implies that, in the limit as $d \to \infty$, $F(x_A) \to 0$ and $F(x_B) \to 1$.

In what follows, I use the notation, for a generic variable x,

$$\dot{x} \equiv \lim_{d\to\infty} \frac{d\,x}{d\,s}$$

11. To see why, suppose that x_A and x_B remain bounded while $d \to \infty$. Then from (26) and (27) all prices and value functions are bounded, which by (28) contradicts the hypothesis that x_i are bounded.

Taking derivatives of (26) with respect to s and then limits as $d \to \infty$ I get

$$\begin{aligned} \dot{p}_{1A} &= -\dot{x}_A - 2\,\delta\,\dot{x}_A + 2\,\delta\,\dot{x}_B \\ \dot{p}_{0A} &= +\dot{x}_B - 2\,\delta\,\dot{x}_A + 2\,\delta\,\dot{x}_B \\ \dot{p}_{1B} &= -\dot{x}_B \\ \dot{p}_{0B} &= +\dot{x}_A \end{aligned} \tag{29}$$

Taking derivatives of (27) with respect to s and then limits as $d \to \infty$ I get

$$\begin{aligned} \dot{u}_A &= -\dot{p}_{1A} + \beta\,\dot{u}_A \\ \dot{u}_B &= -1 - \dot{p}_{0A} + \beta\,\dot{u}_A \end{aligned} \tag{30}$$

where I note that $\lim_{x_i \to \infty}\ e(x_i) = \lim_{x_i \to \infty}\ e(-x_i) = 0$. Taking derivatives of (28) with respect to s and then limits as $d \to \infty$ I get

$$\begin{aligned} \dot{x}_A &\equiv \dot{p}_{1A} - \dot{p}_{0B} - \beta\,(\dot{u}_A - \dot{u}_B) - 1 \\ \dot{x}_B &\equiv \dot{p}_{1B} - \dot{p}_{0A} + \beta\,(\dot{u}_A - \dot{u}_B) - 1 \end{aligned} \tag{31}$$

The system formed by (29), (30) and (31) includes 8 equations and 8 unknowns. Solving form $\dot{p}_{1A}$, I get

$$\dot{p}_{1A} = \frac{1}{3}\,(1 - 2\,\delta)\,(1 + \beta)$$

Since $q_{1A} \to 1$ and $q_{0B} \to 1$ as $d \to 1$, average price is determined by p_{1A}, whereas changes in average price are determined by $\dot{p}_{1A}$. The result follows. ■

Proof of Proposition 6: The buyer's value functions, measured *before* the buyer learns his valuations z_A, z_B, are recursively given by

$$\begin{aligned} u_A &= E(x_A) + E(-x_A) + \big(1 - F(x_A)\big)\big(-p_{1A} + \delta\,u_A\big) + F(x_A)\big(-s_A - p_{0B} + \beta\,u_B\big) \\ u_B &= E(x_B) + E(-x_B) + \big(1 - F(x_B)\big)\big(-p_{1B} + \delta\,u_B\big) + F(x_B)\big(-p_{0A} + \beta\,u_A\big) \end{aligned}$$

In what follows, I use the notation, for a generic variable x,

$$\hat{x} \equiv \left.\frac{d\,x}{d\,s_A}\right|_{s_A = 0}$$

Note that, at $s_A = 0$, we have a symmetric outcome where $x_A = x_B = 0$, $u_A = u_B = u$, and $p_{1A} = p_{0B} = p_{0A} = p_{1B} = p$. Differentiating the buyer value functions with respect to s_A at $s_A = 0$ and defining $e(x) \equiv \frac{d\,E(x)}{d\,x}$, I then get

$$\begin{aligned} \hat{u}_A &= e(0)\,\hat{x}_A - e(0)\,\hat{x}_A + \frac{1}{2}\big(-\hat{p}_{1A} + \beta\,\hat{u}_A\big) - f(0)\,\hat{x}_A\big(-p + \beta\,u\big) + \\ &\quad + \frac{1}{2}\big(-1 - \hat{p}_{0B} + \beta\,\hat{u}_B\big) + f(0)\,\hat{x}_A\big(-p + \beta\,u\big) \\ &= \frac{1}{2}\big(\beta\,\hat{u}_A + \beta\,\hat{u}_B - \hat{p}_{1A} - \hat{p}_{0B} - 1\big) \\ \hat{u}_B &= \frac{1}{2}\big(\beta\,\hat{u}_A + \beta\,\hat{u}_B - \hat{p}_{1B} - \hat{p}_{0A}\big) \end{aligned} \tag{32}$$

This is intuitive: a buyer's expected valuation increases by the increase in future expected valuation, $\delta\,\frac{1}{2}\,(\hat{u}_A + \hat{u}_B)$, minus the increase in expected price paid this period, which is given by $\frac{1}{2}\,(\hat{p}_{1A} + \hat{p}_{0B})$ if the buyer is attached to seller A and $\frac{1}{2}(\hat{p}_{1B} + \hat{p}_{0A})$ if the buyer is attached to seller B. Moreover, if the buyer is attached to seller A, buyer welfare further decreases by an additional $\frac{1}{2}\,s$, the probability that an immediate switch to seller B will take place.

Differentiating (18) with respect to s_A at $s_A = 0$, I get

$$\begin{aligned} \hat{x}_A &= \hat{p}_{1A} - \hat{p}_{0B} - \beta\,(\hat{u}_A - \hat{u}_B) - 1 \\ \hat{x}_B &= \hat{p}_{1B} - \hat{p}_{0A} - \beta\,(\hat{u}_B - \hat{u}_A) \end{aligned} \tag{33}$$

The derivation of value functions and first-order conditions is identical to those in the proof of Proposition 5, leading to (26), with the difference that the values of x_i are now different. Differentiating with respect to s_A at $s_A = 0$, and noting that $f'(0) = 0$, I get

$$\begin{aligned} \hat{p}_{1i} &= -(1-\delta)\,\hat{x}_i + \delta\,\hat{x}_j \\ \hat{p}_{0i} &= \delta\,\hat{x}_i + (1+\delta)\,\hat{x}_j \end{aligned} \tag{34}$$

The system formed by (32), (33) and (34) includes 8 equations and 8 unknowns. Its solution is given by

$$\begin{aligned} \hat{p}_{1A} &= \frac{1}{3}\Big(1 - \delta\,(1-\beta) - \beta/2\Big) \\ \hat{p}_{0A} &= -\frac{1}{3}\Big(\delta\,(1-\beta) - \beta/2\Big) \\ \hat{p}_{1B} &= -\frac{1}{3}\Big(\delta\,(1-\beta) + \beta/2\Big) \\ \hat{p}_{0B} &= -\frac{1}{3}\Big(1 + \delta\,(1-\beta) - \beta/2\Big) \end{aligned} \tag{35}$$

Recall that these are variations with respect to the equilibrium values at $s_A = 0$. The above values indicate that seller A, by creating a switching cost s_A, is able to increase its price when the buyer is locked-in, specifically by $\frac{1}{2}\big(1-\delta\big)\,ds_A$. If the buyer is locked-in to seller B, however, then seller A must decrease its price by $\frac{\delta}{2}\,ds_A$.

In a steady state and with $s_A = 0$, all prices are paid by the consumer with equal probability. It follows that

$$\frac{\partial\,\bar{p}}{\partial\,s} = -\frac{1}{3}\,\delta\,(1-\beta) \tag{36}$$

Differentiating (24), we get

$$\begin{aligned} \hat{v}_{1i} &= -\hat{x}_i + \delta\,\hat{v}_{0i} \\ \hat{v}_{0i} &= \hat{x}_j + \delta\,\hat{v}_{0i} \end{aligned}$$

Substituting (35) for $\hat{x}_i, \hat{x}_j$ and solving, we get

$$\begin{aligned} \hat{v}_{1A} &= \gamma\,\Big(1 + (1-\beta)\,(1-2\,\delta)\Big) \\ \hat{v}_{0A} &= \gamma\,\beta \\ \hat{v}_{1B} &= -\gamma\,\Big(1 - (1-\beta)\,(1-2\,\delta)\Big) \\ \hat{v}_{0B} &= -\gamma\,(2-\beta) \end{aligned}$$

where $\gamma \equiv \frac{1}{6\,(1-\delta)}$.

At $s_A = 0$, both states are visited with equal probability. It follows that steady state values are given by

$$\begin{aligned} \hat{\bar{v}}_A &= \frac{1}{2}\big(\hat{v}_{0A} + \hat{v}_{1A}\big) = \frac{1}{6}\left(1 + \beta\,\frac{\delta}{1-\delta}\right) \\ \hat{\bar{v}}_B &= \frac{1}{2}\big(\hat{v}_{0B} + \hat{v}_{1B}\big) = -\frac{1 + \delta\,(1-\beta)}{6\,(1-\delta)} \\ \hat{\bar{v}} &= \frac{1}{4}\big(\hat{v}_{0A} + \hat{v}_{1A} + \hat{v}_{0B} + \hat{v}_{1B}\big) = -\frac{\delta\,(1-\beta)}{6\,(1-\delta)} \end{aligned} \tag{37}$$

Equations (36) and (37) imply the result. ■

■ **Closed form solution in linear case (cf Section 6).** Substituting $F(x) = \frac{1}{2} + x$ in (25), we get

$$v_{1i} - v_{0i} = \left(\frac{1}{2} - x_i\right)^2 - \left(\frac{1}{2} + x_j\right)^2 = (x_i + x_j)\,(x_i - x_j - 1) \tag{38}$$

Substituting (38) for $v_{1i} - v_{0i}$ and $F(x) = \frac{1}{2} + x$ in (23), we get

$$\begin{aligned} p_{1i} &= \frac{1}{2} - x_i - \delta\,(x_i + x_j)\,(x_i - x_j - 1) \\ p_{0i} &= \frac{1}{2} + x_j - \delta\,(x_i + x_j)\,(x_i - x_j - 1) \end{aligned}$$

which implies

$$\begin{aligned} p_{1A} - p_{0B} &= \left(\frac{1}{2} - x_A - \delta\,(x_A + x_B)\,(x_A - x_B - 1)\right) - \left(\frac{1}{2} + x_A - \delta\,(x_A + x_B)\,(x_B - x_A - 1)\right) \\ &= -2\,x_A - 2\,\delta\,(x_A + x_B)\,(x_A - x_B) \\ p_{1B} - p_{0A} &= \left(\frac{1}{2} - x_B - \delta\,(x_A + x_B)\,(x_B - x_A - 1)\right) - \left(\frac{1}{2} + x_A - \delta\,(x_A + x_B)\,(x_A - x_B - 1)\right) \\ &= -2\,x_B - 2\,\delta\,(x_A + x_B)\,(x_B - x_A) \end{aligned}$$

Plugging this back into (16), we get

$$\begin{aligned} x_A &= -2\,x_A - 2\,\delta\,(x_A + x_B)\,(x_A - x_B) - s - d \\ x_B &= -2\,x_B - 2\,\delta\,(x_A + x_B)\,(x_B - x_A) - s + d \end{aligned} \tag{39}$$

From this we get

$$\begin{aligned} x_A - x_B &= -2\,(x_A - x_B) - 4\,\delta\,(x_A + x_B)\,(x_A - x_B) - 2\,d \\ x_A + x_B &= -2\,(x_A + x_B) - 2\,s \end{aligned}$$

Solving the system with respect to $x_A + x_B$ and $x_A - x_B$, we get

$$\begin{aligned} x_A + x_B &= -\frac{2}{3}\,s \\ x_A - x_B &= -2\left(3 - \frac{8}{3}\,\delta\,s\right)^{-1} d \\ x_A &= -\frac{1}{3}\,s - \left(3 - \frac{8}{3}\,\delta\,s\right)^{-1} d \\ x_B &= -\frac{1}{3}\,s + \left(3 - \frac{8}{3}\,\delta\,s\right)^{-1} d \end{aligned}$$

Substituting these equations for x_i in the above price equations and simplifying we get the expressions in the text.

References

Arie, Guy, and Paul L E Grieco (2012), "Do Firms Compensate Switching Consumers?," Kellogg and Penn State.

Banerjee, A, and L. Summers (1987), "On Frequent Flyer Programs and Other Loyalty-Inducing Arrangements," HIER DP # 1337.

Beggs, Alan W, and Paul Klemperer (1992), "Multi-period Competition with Switching Costs," *Econometrica* **60**, 651–666.

Biglaiser, G., J. Cremer, and G. Dobos (2010), "The Value of Switching Costs," UNC-Chapel Hill, Toulouse School of Economics and Gazdasagi Versenyhivatal.

Cabral, Luís, and J Miguel Villas-Boas (2005), "Bertrand Supertraps," *Management Science* **51**, 599–613.

Caminal, Ramon, and Carmen Matutes (1990), "Endogenous Switching Costs in a Duopoly Model," *International Journal of Industrial Organization* **8**, 353–373.

Chen, Yongmin (1997), "Paying Customers to Switch," *Journal of Economics and Management Strategy* **6** (4), 877–897.

Chen, Yongmin, and Jason Pearcy (2010), "Dynamic Pricing: When to Entice Brand Switching and When to Reward Consumer Loyalty," *Rand Journal of Economics* **41** (4), 674–685.

Corts, Kenneth S (1998), "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment," *Rand Journal of Economics* **29**, 306–323.

Doganoglu, Toker (2010), "Switching Costs, Experience Goods and Dynamic Price Competition," *Quantitative Marketing and Economics* **8** (2), 167–205.

Dubé, Jean-Piere, Günter J Hitsch, and Peter E Rossi (2009), "Do Switching Costs Make Markets Less Competitive?," *Journal of Marketing Research* **46**, 435–445.

Fabra, Natalia, and Alfredo García (2012), "Dynamic Price Competition With Switching Costs," CEPR Discussion Paper 8849.

Farrell, Joseph, and Paul Klemperer (2007), "Coordination and Lock-In: Competition with Switching costs and Network Effects," in M. Armstrong and R. Porter (Eds.), *Handbook of Industrial Organization*, Vol. 3, Amsterdam: North Holland Publishing.

Farrell, Joseph, and Carl Shapiro (1988), "Dynamic Competition with Switching Costs," *Rand Journal of Economics* **19**, 123-137.

Fudenberg, Drew, and Jean Tirole (2000), "Customer Poaching and Brand Switching," *Rand Journal of Economics* **31**, 634–657.

Garcia, A. (2011), "Dynamic Price Competition, Switching Costs, and Network Effects," University of Virginia.

Klemperer, Paul (1987a), "Markets with Consumer Switching Costs," *Quarterly Journal of Economics* **102** (2), 375–394.

Klemperer, Paul (1987b), "The Competitiveness of Markets with Switching Costs," *RAND Journal of Economics* **18** (1), 138–150.

Nilssen, T. (1992), "Two Kinds of Consumer Switching Costs," *RAND Journal of Economics* **23** (4), 579–589.

Padilla, A Jorge (1992), "Revisiting Dynamic Duopoly with Consumer Switching Costs," *Journal of Economic Theory* **67**, 520–530.

Pearcy, Jason (2011), "Bargains Followed by Bargains: When Switching Costs Make Markets More Competitive," Montana State University.

Rhodes, A. (2011), "Re-examining the Effects of switching costs," Oxford University.

Somaini, Paulo, and Liran Einav (2012), "A Model of Market Power in Customer Markets," Stanford University.

Shin, J., K. Sudhir, L. M. B. Cabral, J.-P. Dubé, G. J. Hitsch, and P. E. Rossi (2009), "Commentaries and Rejoinder to 'Do Switching Costs Make Markets Less Competitive'," *Journal of Marketing Research* **46** (4), 446–552.

Taylor, Curtis (2003), "Supplier Surfing: Competition and Consumer Behavior in Subscription Markets," *Rand Journal of Economics* **34**, 223–46.

To, Theodore (1995), "Multiperiod Competition with Switching costs: An Overlapping Generations Formulation," *Journal of Industrial Economics* **44**, 81–87.

Viard, B. (2007), "Do Switching Costs Make Markets More or Less Competitive? The Case of 800-Number Portability," *RAND Journal of Economics* **38** (1), 146–163.

Villas-Boas, J. Miguel (1999), "Price Competition with Customer Recognition," *Rand Journal of Economics* **30**, 604–631.

Villas-Boas, J. Miguel (2006), "Dynamic Competition with Experience Goods," *Journal of Economics and Management Strategy* **15**, 37–66.

Villas-Boas, J. Miguel (2011), "Notes on Switching Costs and Dynamic Competition," U C Berkeley.

von Weizsacker, C.C. (1984), "The Costs of Substitution," *Econometrica* **52**, 1085–1116.