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ABSTRACT

A dynamic analysis of bank bailouts and constructive ambiguity

Bailout expectations have led banks to behave imprudently, holding too little capital and relying too much on short term funding to finance long term investments. This paper presents a model to rationalize a constructive ambiguity approach to liquidity assistance as a solution to forbearance. Faced with a bank that chooses capital and liquidity, the institution providing liquidity assistance can commit to a mixed strategy: never bailing out is too costly and therefore not credible, while always bailing out causes moral hazard. In equilibrium, the bank chooses above minimum capital and liquidity, unless either capital costs or the opportunity cost of liquidity are too high. We also find that the probability of a bailout is higher for a regulator more concerned about bank failure, and when the bailout penalty for the bank is higher; this suggests that forbearance is not entirely eliminated by adopting ambiguity.

JEL Classification: E58, G21, G28
Keywords: banking, commitment, lender of last resort, liquidity, regulation

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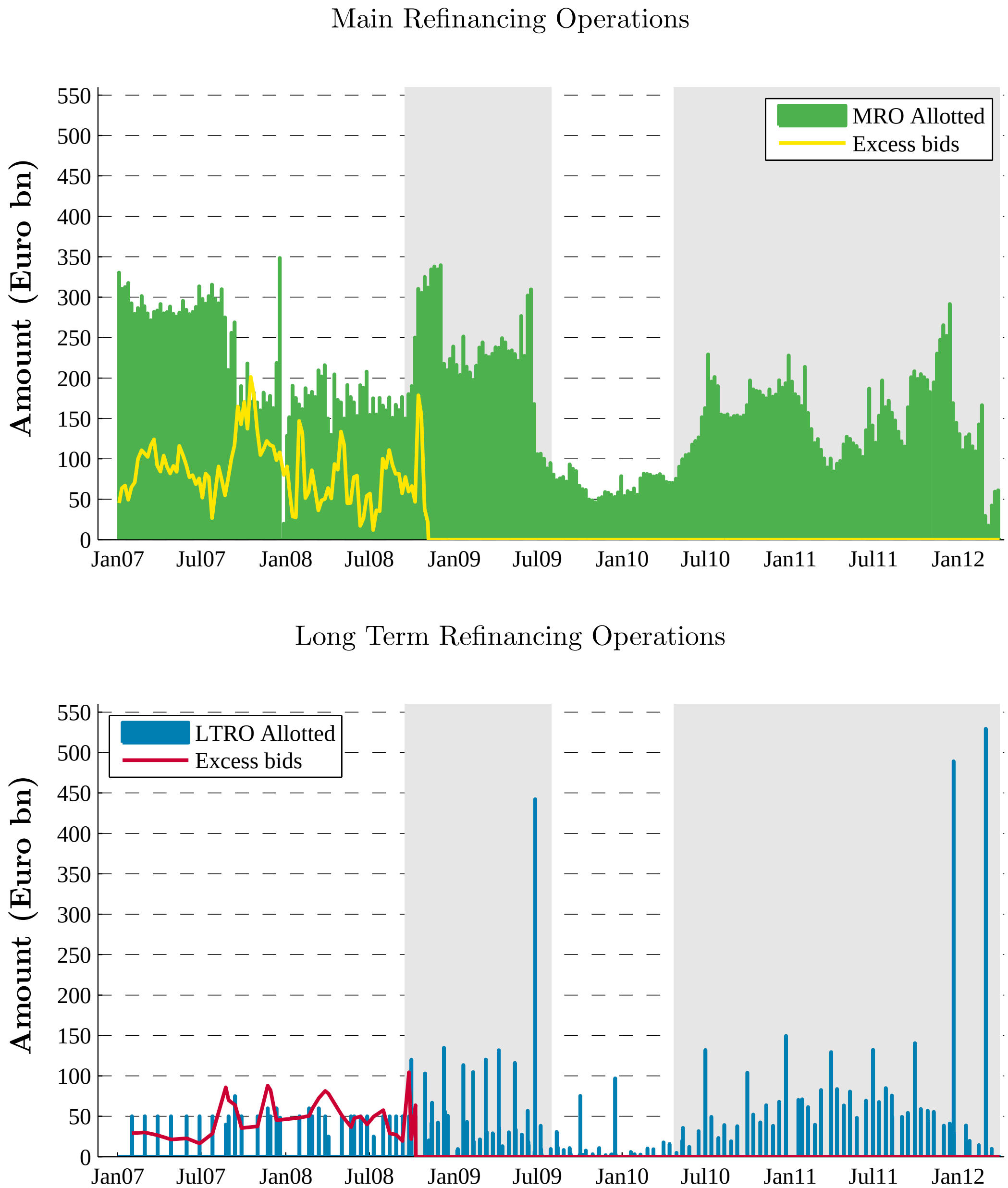
1 Introduction

After the recent financial crisis, calls for new regulation have dominated the academic debate. While this first centered on how to manage crises better, the debate has now moved towards reforming prudential regulation and setting up a sustainable financial system with safeguards. The current Basel capital requirements have not put much emphasis on banks' excessive maturity mismatches. Banks have relied increasingly on short term funding to invest in long-term assets (Brunnermeier and Pedersen, 2009). Apart from prudential regulation, the Lender of Last Resort (LLR) or bailout function of central banks has come under discussion. While central banks worldwide have intervened heavily in interbank markets to alleviate the crisis, they have also gathered much criticism. This has mainly focused on the forbearing behaviour of regulators, and the moral hazard their policies have generated: banks took excessive risks knowing that they would be provided with liquidity. They also held too little capital and were relying too much on short term funding to finance long term investments.

The large scale bailouts during the 08/09 crisis, not only by central banks but also by national governments (Levy and Schich, 2010), have proven them right. And although governments have slowly decreased their exposure to the banking system since 2010, the European Central Bank (ECB) has not ceased providing liquidity. To restore confidence in interbank markets as a response to the current Eurozone crisis, the ECB has even increased the intensity and maturity of its assistance. We can see this in figure 1: while its main refinancing operations (with 1 or 2 week maturity) have remained relatively stable since 2010, the ECB has increased its long term assistance (at least 3 month maturity). The recent outliers in the bottom figure represent the exceptionally large liquidity injections of December 2011 and February 2012, which also have a very long maturity of 3 years. Furthermore, the figure also shows that the ECB has honored all requests for liquidity since 2009 as there is so-called "full allotment" (no excess bids for liquidity). These two developments show a clear commitment by the central bank that it will provide banks with liquidity for a significant period of time; this resembles the Federal Reserve's promise to keep interest rates low until at least the end of 2014. Taken more broadly, this could even be interpreted as solvency instead of liquidity assistance.

However, these commitments have still not persuaded banks to provide funds to the real

Figure 1: ECB refinancing operations



This figure illustrates the short- and long term open market operations of the ECB since 2007. This is not a continuous process; especially the LTRO are performed relatively infrequently. To clarify: the bars depict how many funds have been provided to the system, while the red line indicates the amount of bids that exceeded the amount allotted. This means there has been full allotment from 2009 onwards. Furthermore, the grey areas indicate the global financial crisis and the current Euro crisis, respectively.
Source: `http://www.ecb.int/mopo/implement/omo/html/top_history.en.html`

sector or to reduce their holdings of (very) risky assets; if anything, these holdings have even increased. To alleviate this moral hazard problem facing central banks it has been argued that a central bank should adhere to an ambiguous bailout strategy (Freixas, 1999; Kocherlakota and Shim, 2007; Shim, 2011). This means that it will ex ante not state whether it will assist the bank or not; instead, the bank can expect to be bailed out only with some probability. This practice of so-called "constructive ambiguity" has been more common in the monetary policy context, where it also often linked to incomplete transparency (see Cukierman (2009), and Eijffinger and Hoeberichts (2002) and Demertzis and Hughes Hallett (2007) for evidence). However, as opposed to monetary policy, in the practice of assisting banks one often has to act very fast in deciding whether a bank will be assisted or not.

Furthermore, banking regulation is not a one-shot game: a bank raises funds and invests them continuously. More importantly, decisions that the bank makes now (i.e. regarding its capital structure) will have an impact on its future profitability and ability to withstand liquidity shocks. The regulator also takes this into account, as better capitalized banks and banks who have more liquid reserves are more likely to be assisted when they knock on the regulator's door for liquidity.

Recent investigations into the reform of the LLR function have focused on different aspects of the LLR, but not often in a dynamic context focusing on constructive ambiguity[1]. Kahn and Santos (2005), for instance, focus on the allocation of LLR responsibility between different agencies. More recently, Eijffinger and Nijskens (2011)[2] have analyzed the roles of the central bank and the fiscal authorities in providing liquidity and solvency assistance, respectively. However, both these analyses assume a static context without considering ambiguity. Regarding penalty rates and the LLR, Repullo (2005) and Castiglionesi and Wagner (2011) have both found that penalties increase risk taking by banks and regulatory forbearance. They focus, however, only on bank risk taking. The analyses most similar to ours are Goodhart and Huang (2005) and Shim (2011), who both allow for multiple time periods and ambiguity, but either do not incorporate bank incentives at all or do not allow for liquidity choice.

[1]A good overview of two decades of research on LLR and closure policy can be found in Freixas and Parigi (2008).

[2]The model presented in this paper borrows some features of theirs.

Our approach differs from the abovementioned papers in that we allow for multiple time periods, but also explicitly take the banker's incentives into account. Furthermore, we focus explicitly on liquidity problems, leaving out solvency considerations. We set up a model of an economy consisting of one bank and one regulator with a Lender of Last Resort mandate from society. They operate in an environment without (functioning) interbank markets, i.e. a crisis episode. The bank can choose the structure of its balance sheet, while the regulator has to decide whether to assist the bank or not when it runs into trouble. In our analysis we want to focus on the incentives for the bank to hold too little capital and liquidity, and investigate the institutional details of capital and liquidity requirements in a dynamic context. Moreover, we assess the effects that failure costs and possible bailout penalties have on the choices of the bank and the regulator.

We find that it is optimal for the regulator to follow a mixed strategy: announcing that the bank will never be bailed out is too costly for society, and therefore not credible, while always bailing out the bank with certainty causes moral hazard by the banker. In response to this mixed strategy, the bank will choose capital and liquidity above the minimum requirements. However, when these requirements or the costs of capital and liquidity are too high, the bank will not keep more than the minimum capital or liquidity. For current LLR policy our results imply that the institution responsible for liquidity assistance should be ambiguous about whether it will assist a bank or not, avoiding regulatory capture.

This result is depending on the existence of a commitment technology for the regulator: it should be able to commit to this strategy of constructive ambiguity. A legally binding mandate from society, together with accountability and credibility of the regulator, can accomplish this. We will elaborate upon this in the next section, where we will also relate this commitment technology to the monetary policy literature.

Furthermore, our analysis also shows that charging a lump sum penalty for LLR assistance improves the bank's incentives to hold more capital and reserves. Finally, increasing the bankers' time horizon can have positive effects on bailout probability and capital, although the amount of liquid reserves decreases. In the next section we present our institutional environment in more detail.

2 Institutional setup

We consider an economy that consists of a single bank and a regulator, which we call the CBFS (Central Bank/Financial Supervisor), who both operate during two time periods. These periods consist of several stages. In the first stage, the decisions are made by both players. The bank chooses its liability structure by setting capital and liquidity and its asset structure by choosing between investing in risky assets and liquid reserves. The CBFS decides on its Lender of Last Resort policy by committing to a certain policy in the first stage of every period; however, this policy is not announced.

In the second stage of each period a liquidity shock occurs. This means that a fraction of deposits will be withdrawn randomly (as in i.e. Repullo (2005) and Eijffinger and Nijskens (2011)). The bank will have to use its own liquid reserves to cope with this shock; we assume that there is no access to an interbank market. This resembles a crisis situation, similar to that of the 2008 financial crisis and even the current situation in the interbank market.

Therefore, when the bank cannot cope with the liquidity shock itself, it can go to the CBFS for liquidity. This resembles the situation many European banks are in at the moment, with the ECB acting not only as a lender, but even as a full-fledged market maker of last resort. In our analysis, when the bank turns to the CBFS for liquidity, the latter has to decide whether to provide liquidity assistance to the bank or not[3]. In case of liquidity assistance, the bank will receive the amount of liquidity necessary to repay the withdrawing depositors, and it has to pay a lump sum penalty to society at the end of each period. This penalty explicitly does not accrue to the CBFS to not distort its incentives and those of the bank (Castiglionesi and Wagner, 2011).

This structure requires there exists a commitment technology for the CBFS, enabling it to commit to constructive ambiguity. We assume that this is so, since our CBFS is a credible authority with a sound reputation that has received a Lender of Last Resort mandate from society. As it is accountable to society but independent in its decision-making, it is plausible that the CBFS can commit to an ambiguity strategy in equilibrium; in the monetary policy literature this has been a standard assumption since the 1980's (Barro and Gordon, 1983;

[3]Note that we use the terms "liquidity assistance", "bailout" and "rescue" interchangeably. For our purposes, they have the same meaning.

Lohmann, 1992). Alternatively, we can think of the bank not having complete information about the CBFS's objective ex ante. Instead, it will form a belief about the CBFS's objective function that will prove to be correct in equilibrium. We will come back to this interpretation later.

In the third stage the return on the illiquid asset realizes. If this is positive, the bank will reap the rewards, pay back the regulator and continue into the next period. The bank keeps its capital, and profits are consumed or partly invested into new capital that can be put to productive use. If, however, the risky asset does not pay off, the bank fails, the CBFS loses its liquidity injection and a new bank owner will be put in place by the deposit insurance fund. The game between bank and CBFS starts again from scratch.

The choices of the bank have different effects on the equilibrium payoffs in our model. To begin with, when the banker finances the bank with his own capital (instead of deposits) this has several advantages. First of all, the size of the possible shock decreases as the ratio of deposits to total liabilities is lower. This also increases the probability of continuing into the next period. Furthermore, a higher capital ratio increases the probability that the CBFS assists the bank if necessary. Finally, profit in period 2 increases, since initial capital has positive value in period 2, but is already fully paid for in period 1. The disadvantage of funding the bank with capital is that it reduces profit in period 1, since the costs of capital are increasing more than proportionally with investment in capital (for a rationale, see i.e. Hellmann et al. (2000)). Liquid reserves have the benefit that they increase the capability of coping with liquidity shocks. This means that they also increase the probability of continuing into the next period. The disadvantage of liquidity, however, is its opportunity cost: it reduces the amount of assets available for risky investment, and thus the profits from this investment.

To summarize this institutional setup, we provide a small table of the players' choices in both time periods.

How does our approach differ from the existing literature? To begin with, there are not very many analyses of LLR assistance and ambiguity, and even less that take place in a dynamic context. A natural first example is the analysis by Freixas (1999), who analyzes

Table 1: Overview of players and their choices

Player	Choices
Bank	Capital, deposits, liquidity, risky assets
CBFS	Liquidity assistance policy

the optimal behaviour of the LLR in response to the choice of uninsured debt by banks. A crucial assumption is that the LLR finds rescuing banks costly. As never bailing out a bank is not credible (this would be even more costly, especially for large banks), the LLR engages in "constructive ambiguity": it follows a mixed strategy in rescuing the bank. A drawback of this analysis is that it only considers the liability side of the bank; no specific attention is paid to liquidity management.

Cordella and Levy-Yeyati (2003) also touch upon constructive ambiguity: they argue against it. Their analysis demonstrates that having a clear, unambiguous bailout policy creates a charter value effect that outweighs the moral hazard costs. Yet again, these authors do not take into account liquidity management and the effect this can have on the bank's demand for liquidity assistance.

The abovementioned analyses take a static perspective. To our knowledge there are only a few studies that employ a dynamic framework. A notable example is Goodhart and Huang (2005), who analyze the decision of whether a central bank should engage in open market operations to manage liquidity or whether it should provide direct LLR assistance. They conclude that a "too-big-to-fail" policy can be rationalized, but only when moral hazard is the sole concern. In case contagion is also a concern, this is the main reason for LLR assistance, leading to a "too-many-to-fail" policy. Although the authors provide a very thorough analysis of the central bank's incentives, they do not take into account the incentives of the bank manager; an issue that our analysis focuses on.

Another, more recent, example of LLR in a dynamic context is Shim (2011). He sets up a model containing hidden risk choice, private information on returns, limited commitment by the bank owner and costly liquidation. In his analysis, he finds that a combination of capital requirements and risk-based deposit insurance can implement an optimal allocation. This is coupled with a stochastic liquidation policy, i.e. constructive ambiguity. In contrast to our analysis, his focus lies more on capital regulation rather than on both liquidity and capital requirements.

Finally, we have to note that our model does not contain any uncertainty about the regulator's objectives (as in Cukierman and Meltzer (1986)). In this respect, our model differs from those by Caballero and Krishnamurthy (2008), Vinogradov (2010), Bosma (2011) or Cukierman and Izhakian (2011). We abstract from this uncertainty; in our analysis, the bank and the regulator know perfectly well what each other's objectives are, but each makes choices that are unobservable to the other ex ante. Nevertheless, this remains an important issue, and in section 4 we come back to this.

3 The Model

Our model takes the same basic assumptions about bank choices as in Eijffinger and Nijskens (2011), except for the choice of monitoring p. Instead, the bank chooses its capital ratio. To start with, let us consider economy consisting of one bank and one regulator. There are two time periods, indexed by $t = 1, 2$, where each time period consists of several stages that will be described below. Figure 2 on page 14 clarifies the description that will follow.

At $t = 1$ one unit of funds is required to set up a bank[4]. The bank owner faces only limited liability. He chooses how many of his own funds to invest in capital, denoted by i_t. The rest is raised by attracting deposits d_t, such that $i_t + d_t = 1$. The net deposit rate is normalized to zero (we assume deposits are insured, so they are risk-free), and the bank cannot influence this rate: there is a perfectly elastic supply of deposits at an exogenous rate of zero[5]. We also assume that the depositor base is sticky, so the amount of deposits chosen in period 1 is the same as that in period 2, so $d_1 = d_2 = d$.

Capital investment entails a cost $\phi(i_t)$, which is a convex function. Capital investment

[4]This effectively normalizes period 1 bank size to one. This should not be a problem as we do not focus on too-big-to-fail issues. Alternatively, we can fix the size of liabilities by fixing the deposit rate or by assuming a decreasing deposit supply function

[5]This allows us to focus on the liquidity and capital choices of the bank, without having to consider competition issues. This assumption can be rationalized by considering, for instance, a large foreign market for deposits or by assuming that the outside option of depositors is equal to the offered deposit rate.

augments the capital stock k_t, according to the following law of motion:

$$k_t = k_{t-1} + i_t \tag{1}$$

with $k_0 = 0$. Since the total endowment is equal to 1, we can thus use this law of motion to determine that $d \equiv 1 - k_1$.

When he has set up the bank, the banker can choose to allocate funds towards two different assets. The long term asset a_t has a positive gross return $R > 1$[6] with probability p; with probability $1-p$ the return on a_t will be zero and the bank fails. The other asset l_t is a short term storage technology, which can be liquidated at any time during the period but generates a zero return for sure (risk-free). This can be summarized in the following balance sheet:

Assets	Liabilities
l_t	d_t
a_t	k_t
1	1

We can then write end-of-period bank value as follows:

$$V_t = Ra_t + l_t - d_t, \tag{2}$$

which, using $d \equiv 1 - k_1$, $a_t = d + k_t - l_t$ and the cost function $\phi(i_t)$, translates to expected end of period profit

$$\Pi_t = p[(R-1)(d + k_t - l_t) + i_t - \phi(i_t)]. \tag{3}$$

During each time period, a liquidity shock $\tilde{x}_t \sim U(0,1)$ occurs after the bank has made its decisions. It leads to a withdrawal of deposits amounting to $x_t d$, where x_t is the realization of $\tilde{x}_t$.

If the bank has enough liquid reserves relative to deposits, it can cope with the shock. This means that the withdrawn amount has to be smaller than the amount of liquid reserves, or $x_t d < l_t$. From this expression we can deduce a threshold $\underline{x}_t = l_t/d$, below which the bank

[6] For an interior solution, regularity requires that $R < 2$ as well. This seems reasonable, as $R > 2$ corresponds to a net return of more than 100% which is not very realistic.

can meet the liquidity demand. The probability that this happens is $Pr[x_t < \underline{x}_t] = \underline{x}_t$, since x_t is uniformly distributed.

However, when liquid reserves are not adequate to meet the liquidity demand after a shock ($x_t > \underline{x}_t$ with $Pr[x_t > \underline{x}_t] = (1 - \underline{x}_t)$), the bank will fail if it is not assisted by the CBFS. If it is assisted by the CBFS, the bank will have to pay a lump sum penalty T, that accrues to society via the deposit insurance fund. This penalty is smaller than the excess return on risky investment: $T < R - 1$. This gives the bank owner sufficient incentive to set up a bank. Additionally, the penalty is smaller than the costs of bankruptcy ($T < c$). If it is larger, the CBFS will always rescue the bank, which is not in the interest of the bank owner itself as this rescue will be expensive for the bank.

In the final step the return on the risky asset realizes. If this is equal to R, the remaining depositors are repaid, bank profits realize and the bank continues into next period. If it is equal to zero, the bank fails, depositors are reimbursed via the deposit guarantee fund and the current bank owner will get 0. A new bank owner, again with endowment 1, will recapitalize the bank in the next period.

Under the above assumptions, we can write expected per period profit as follows:

$$E[\Pi_t] = (\underline{x}_t + (1 - \underline{x}_t)q_t)\Pi_t - p(1 - \underline{x}_t)q_t T \tag{4}$$

From the perspective of the current bank owner period 2 profit only matters when the bank succeeds in period 1 and continues to period 2. We can write down a continuation probability that depends on the bank's own choices and that of the CBFS:

$$Pr[Continue]_t \equiv p(\underline{x}_t + (1 - \underline{x}_t)q_t)$$

Using this and denoting the discount factor by β we can connect the two periods:

$$E[\Pi] = E[\Pi_1] + p(\underline{x}_1 + (1 - \underline{x}_1)q_1)\beta E[\Pi_2], \tag{5}$$

which is the objective the bank wants to maximize by choosing l_1, i_1, l_2 and i_2. This equation tells us that the choices of liquidity and capital in period 1 do not only affect profit at $t = 1$, but also the probability that the bank will continue into period 2. This probability increases when x_1 increases due to liquidity or capital, but it is also dependent on q_1, which is determined by the CBFS.

Before we explain the regulator's objectives, one last remark about the choice of i_2 is in order. The bank owner can raise deposits only at $t = 1$. At $t = 2$, he can only use the profits from the previous period to increase capital and thus the size of the bank. We assume that the depositor base is fixed, and that no sale of capital is allowed. As will be explained below, the no sale constraint will never be met. Furthermore, capital investment in period 2 does not affect anything but the amount of available assets for investment. The capital investment i_2 is thus determined by a very simple cost benefit analysis (for more details see the appendix):

$$R = \phi'(i_2) \tag{6}$$

Additionally, capital and liquid reserves are subject to minimum requirements, which are denoted by $\underline{k}$ and $\underline{l}$ respectively. These will play a role in determining the equilibrium values of capital and liquidity, as we will see in the next section. In the end, the banker faces a trade-off between profits (by increasing leverage) on the one hand, and the risk of liquidity problems and facing the regulator on the other.

The CBFS is the only source of liquidity for the bank beyond its own liquid reserves. After observing a shock, the CBFS will decide whether it intervenes and provides the bank with liquidity, or whether it lets the bank fail. In the latter case, the remainder of the bank will be seized by the deposit insurance fund (a passive authority), which pays out the remaining depositors, and a new bank owner with endowment 1 will be put in place. Additionally, the CBFS will incur the costs of bank failure c, which can be thought of as disruptions in the payment system, misallocation of funds or the destruction of lending relationships; in general, c represents problems with financial intermediation that are related to decisions made by the CBFS.

As we have described in section 2, the CBFS can commit to a liquidity assistance strategy *ex ante*. This does not have to be a pure strategy in all periods; the CBFS can also announce a policy that specifies a certain probability q_t with which the bank will be rescued. In determining this probability, the CBFS will weigh the costs of intervening against the costs of letting the bank fail. The costs of letting the bank fail are the (social) costs of bank failure c. The costs of intervention will only realize when the bank fails at the end of the period, i.e. when the investment does not succeed with probability $1-p$. These costs consist of the amount of liquidity provided, and the social bank failure costs that arise since the bank has failed. The amount of liquidity provided is equal to $x_t d - l_t$. Denoting no bailout by f (for failure) and rescuing the bank by r, we can write the respective losses as follows:

$$L_t^f = c$$
$$E[L_t^r] = (1-p)(\underbrace{x_t d - l_t}_{liq.\ support} + c)$$

As we can see the bank's choices also determine the size of these costs: the more capital and liquidity the bank chooses, the lower the costs of assisting the bank are.

The expected value of x_t, conditional on it being larger than $\underline{x}_t$, is

$$\int_{\underline{x}_t}^{1} x_t df(x_t) = (1-\underline{x}_t)E[x_t|x_t > \underline{x}_t]. \tag{7}$$

Using equation (7) and the probability of bailout q_t we arrive at the following per period CBFS expected loss function:

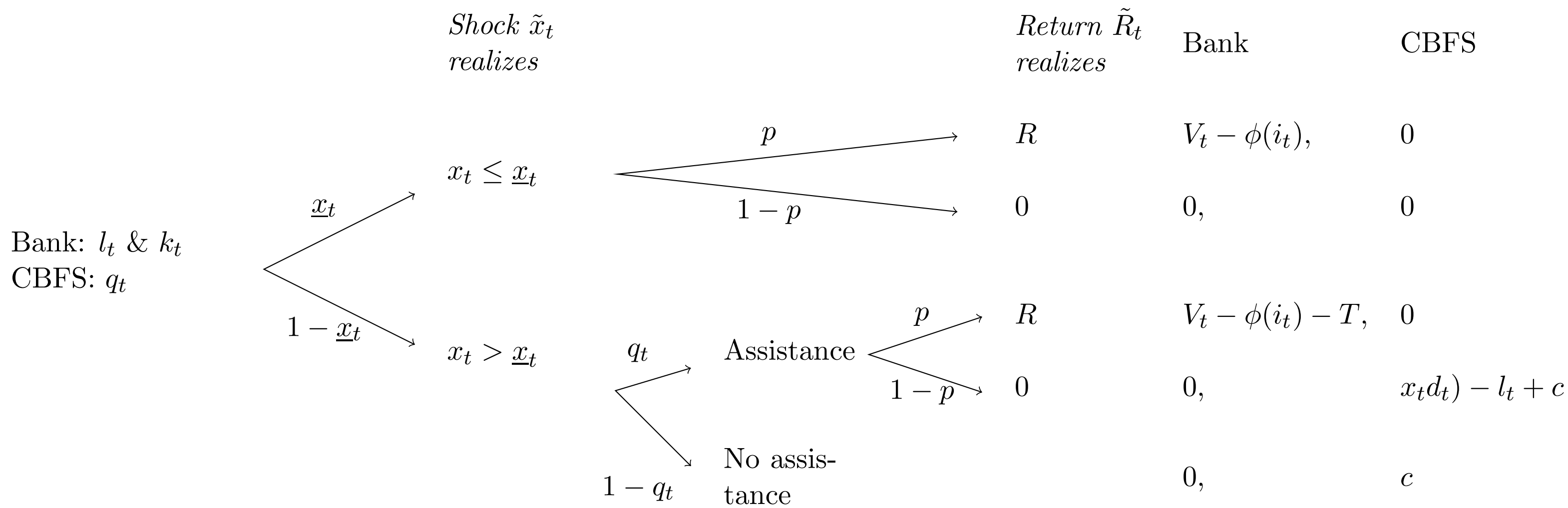
$$E[L_t] = (1-\underline{x}_t)[(1-q_t)c + q_t(1-p)(E[x_t|x_t > \underline{x}_t]d - l_t + c)]. \tag{8}$$

Aggregating across periods, we can write the CBFS loss function as follows (γ is the CBFS discount factor):

$$E[L] = E[L_1] + \gamma E[L_2]. \tag{9}$$

By choosing q_t, the CBFS will want to minimize its loss function. For regularity, we further assume that the CBFS will never intervene when the bank's capital and liquidity are at the bare minimum; if we would not assume this, the bank would clearly engage in moral hazard immediately. This will be formalized in the next section.

Figure 2: Sequence of events at period t



Note: this sequence is followed in time period 1 and repeated in period 2. Furthermore, the CBFS incurs no loss when the bank fails in case the liquidity shock is small: as the CBFS has not been required to make a decision, it will not be held responsible for any bank failures.

4 A dynamic equilibrium

To establish the equilibrium of our dynamic game, we first solve the CBFS's problem, as this is the most straightforward one. The CBFS will want to minimize $E[L]$ w.r.t q_t. Closer scrutiny of this objective shows that this problem is not truly dynamic in q_t; the CBFS's problem is actually two separate problems. Therefore, the conditions for an interior solution for both q_1 and q_2 follow from the CBFS's FOC in both periods:

$$\frac{\partial E[L]}{\partial q_1} = (1 - \underline{x}_1)(-c + (1-p)(\frac{1}{2}(\underline{x}_1 + 1)d_1 - l_1 + c)) = 0 \tag{10}$$

$$\frac{\partial E[L]}{\partial q_2} = \gamma(1 - \underline{x}_2)(-c + (1-p)(\frac{1}{2}(\underline{x}_2 + 1)d_2 - l_2 + c)) = 0. \tag{11}$$

Taking into account that $(1 - \underline{x}_t)$ is a probability and γ a discount factor we know that these are always nonnegative, so the above conditions translate to

$$1 - \frac{2pc}{1-p} = l_1 + k_1 \quad \text{and} \quad 1 - \frac{2pc}{1-p} = l_2 + k_1, \tag{12}$$

which again translates to

$$l_1 = l_2 = 1 - \frac{2pc}{1-p} - k_1. \tag{13}$$

For this condition to hold as an interior equilibrium, in which the CBFS plays a mixed strategy and the bank chooses liquidity and capital above the minimum, we have to assume that $1 - \frac{2pc}{1-p} > \underline{l} + \underline{k}$. As is mentioned above, this means that the CBFS will never provide liquidity when both liquidity and capital are at their minimum values.

The bank will maximize its expected profit $E[\Pi]$ w.r.t. l_1, i_1, l_2 and i_2. The First Order Conditions (FOC) for this problem we have put in the appendix because of space considerations. As we now have all conditions to establish the reaction functions of bank and CBFS, we can solve them to obtain an equilibrium. As follows from the proposition below, this equilibrium does not involve situations in which there will be either always or never a bailout.

Proposition 1: *in equilibrium, the CBFS will not play a pure "always bailout" strategy in*

any period. The CBFS is also not able to credibly commit to a "never bailout" strategy.
Proof: see appendix. ■

We can intuitively explain the proof for a mixed strategy as follows; it goes by contradiction and its intuition resembles that in Freixas (1999). To start with, an unconditional "always rescue" policy ($q_t = 1$ for $t = 1, 2$) will generate clear moral hazard problems: the bank will choose its capital and liquidity buffers to be as low as possible, which is too costly for the CBFS. It is thus never optimal to provide assistance with probability 1. A "never rescue" policy ($q_t = 0$ for $t = 1, 2$) is also not sustainable, albeit for more subtle reasons: in this case the bank will self-insure against liquidity shocks. It will choose less leverage at $t = 1$ and more liquidity in both periods, even above the capital and reserve requirements. Technically, this leads to capital and liquidity being too high for the CBFS to be able to sustain a strategy of never rescuing the bank. More intuitively, this policy is not credible for the CBFS to commit to, as always letting the bank fail will be excessively costly.

For "never bailout" and "always bailout" equilibria to be ruled out, only a few additional parameter assumptions have to be made. One is that the penalty that the bank faces in case of rescue is smaller than the profit it can make on its risky investment ($T < R - 1$). If this is not the case, the bank owner will not want to start up the bank as his expected profit will always be negative. Furthermore, the probability of success and return should not be too large, lest the CBFS will choose to always bail out the bank as a high probability of success reduces the cost of liquidity assistance: $\frac{2p}{1-p}c < 1$. This means that the condition on T and R can be specified even stricter (as we show in the appendix): $T < \frac{2p}{1-p}c(R - 1)$, which is a necessary condition for a "pure bailout" to be ruled out. The last assumption is that the penalty should also be smaller than the social cost of bankruptcy ($T < c$) to prevent distortion of the CBFS's incentives.

The only sustainable equilibrium is thus a mixed one: the probability of rescue in any period lies between 0 and 1. A mixed strategy Nash equilibrium always exists in a finite game such as ours, which means that there is an equilibrium with $\{q_1, q_2\} \in (0, 1)$, $\{l_1, l_2\} \in (\underline{l}, 1)$, $i_1 = k_1 \in (\underline{k}, 1)$ and $i_2 \in (0, 1)$. In this completely mixed equilibrium, the bank chooses capital and liquidity above the minimum required, while the CBFS plays a mixed bailout strategy.

However, as proposition 2 below states, the equilibrium can also be only partly mixed; the bank will choose either minimum capital or minimum liquidity in equilibrium.

Proposition 2: *there exists a unique equilibrium consisting of a mixed strategy for the CBFS and, depending on minimum capital and liquidity requirements, the convexity of the cost of capital and the return on risky investment, different strategies for the bank. In this mixed strategy equilibrium, the level of liquidity is the same in both periods, while there is a trade-off between capital and liquidity in period 1.*

More specifically:

1. *If capital costs are high enough ($\phi(\cdot)$ is sufficiently convex), the bank will choose capital in period 1 to be at the minimum required: $i_1 = k_1 = \underline{k}$. Liquidity in both periods will be higher than when $k_1 > \underline{k}$, to fulfill condition* (13).
2. *If R is high enough, and $\phi(\cdot)$ not too convex, the bank will keep liquidity at the minimum required: $l_1 = l_2 = \underline{l}$. Capital will be higher than when $l_1 = l_2 > \underline{l}$, to fulfill condition* (13).

Proof: see appendix. ■

This proposition explains that, when capital costs are too high (i.e. quite convex), the bank will choose to satisfy the CBFS's indifference constraint by choosing more liquidity and minimum capital. On the other hand, when the return on the risky asset is too high, the bank will keep less liquid reserves and choose a higher capital ratio at $t = 1$[7]. The following corollary elaborates upon this.

Corollary 1: *minimum capital and liquidity requirements increase the likelihood of partial corner solutions; an equilibrium with either l_t or i_t at the minimum requirement is more likely when $\underline{k}$ and $\underline{l}$ increase.*

Proof: following from Proposition 2, when the cost of capital or R is high, the bank will

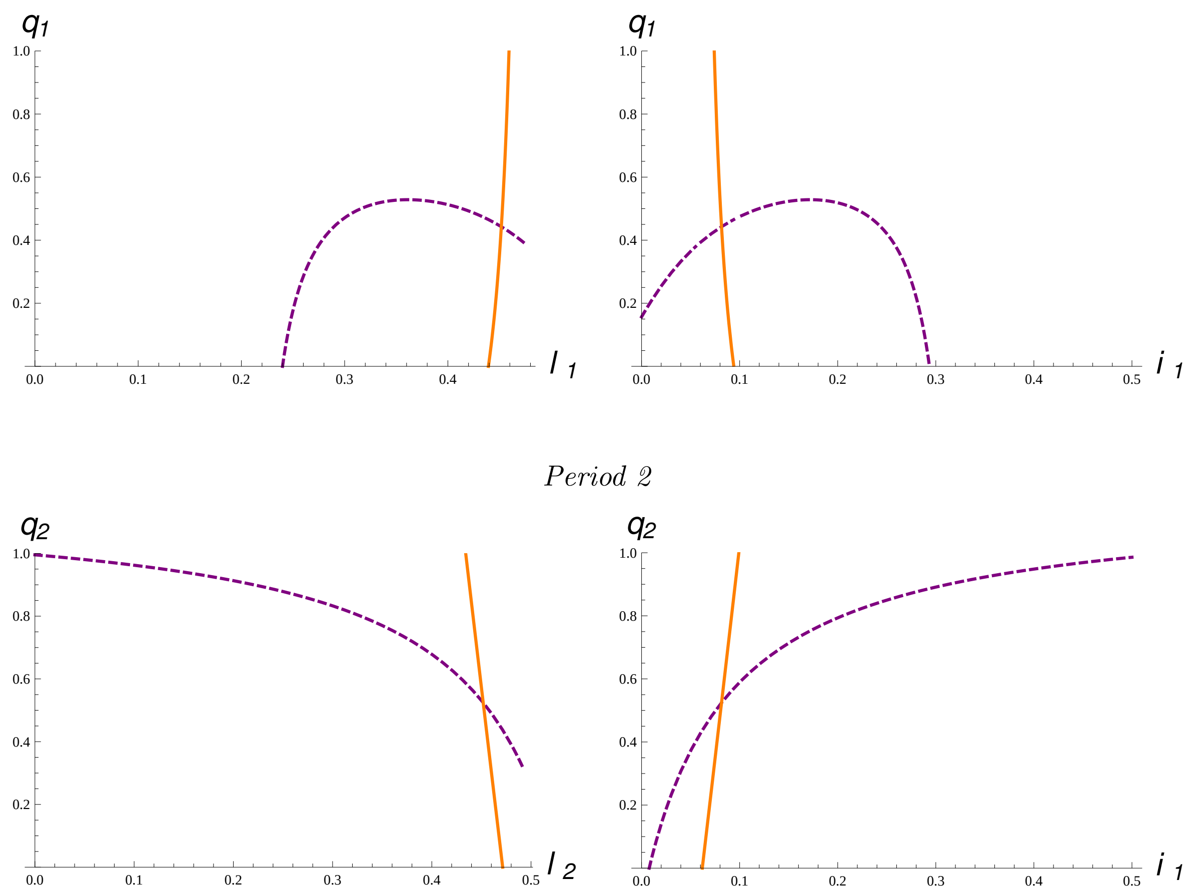
[7]Note that there may also be parameterizations of $\phi(\cdot)$ and R for which $i_1 = k_1 = \underline{k}$ and $l_1 = l_2 = \underline{l}$. As we have assumed that $\underline{l} + \underline{k} < 1 - \frac{2pc}{1-p}$, this will result in an equilibrium with $q_1 = q_2 = 0$, which we have ruled out.

choose minimum capital or minimum liquidity. If these minimum levels are higher, they will be reached more easily. In other words, $\phi(\cdot)$ or R have to increase less for corner solutions to hold. ■

The above corollary states that when minimum capital or liquidity requirements increase, the bank will be more likely to choose a capital or liquidity ratio at the minimum when the opportunity costs for both variables increase.

Figure 3: Reaction functions of the bank and the CBFS

Period 1



Note: the solid line represents the bank's reaction function, while the dashed line represents the CBFS's.

To clarify the reasoning behind Propositions 1 and 2 and Corollary 1, figure 3 displays the reaction functions of the bank and the CBFS[8]. We immediately see that the CBFS will

[8]The parameter values used for this figure are $\underline{l} = 5\%$, $\underline{k} = 5\%$, $R = 1.2$, $p = 0.7$, $\beta = 0.95$, $T = 9\%$ and $c = 10\%$. The cost function is $\phi(i_t) = i_t + 4i_t^2$.

never set q_1 to 1, so an "always bailout" strategy is not feasible. The equilibrium is clearly an interior one. Note that in equilibrium liquidity is indeed equal in both periods, and that both right hand figures feature i_1 on the horizontal axis: investment in period 2 does not play a role in determining bailout probability in our model. Finally, we can see the reasoning behind Proposition 2 and Corollary 1: if the opportunity cost of either liquidity or capital increases, the bank's reaction functions shift to the left. If this shift is strong enough, or the minimum requirement on liquidity or capital is high enough, the intersection point of the reaction functions may lie at the minimum requirement.

As a final note on the solution, the existence of this equilibrium is of course under the implicit assumption that the regulator can commit to a mixed strategy over multiple time periods. In section 2 we have already noted that this assumption derives from the monetary policy literature, where the central bank is a credible, transparent and independent authority that can commit ex ante to a specific strategy.

Several mechanisms can serve as the basis for this commitment technology. Of course, in a repeated the game the most straightforward commitment device is reputation as in Barro and Gordon (1983). We can also think of the objective or technology of the CBFS as being ambiguous in itself. Cukierman and Meltzer (1986) already suggested this: they provide a theory of a monetary policymaker whose preferences are stochastically determined, but who has more information about their realization than the public. This ambiguity can also mean, for instance, that the bank does not know the exact magnitude of the bank failure costs c that are imposed on the CBFS if the bank fails (Bosma, 2011); the CBFS does know this. In equilibrium, the bank then has to rely on signals about these failure costs to determine its belief. As has been suggested recently, the policymaker can additionally be explicitly ambiguous about the announcement of its policy (Vinogradov, 2010; Cukierman and Izhakian, 2011). However, the effects of this strategy are not always beneficial.

Our model can be thought of as building on any of these commitment technologies; we do not specify the technology explicitly since we want to focus on the interaction between the bank and the CBFS. Exploring these different commitment mechanisms any further is, therefore, beyond the scope of this paper.

5 Comparative Statics

The institutional structure of the CBFS will determine equilibrium values. Specifically, the bailout penalty T that the bank has to pay when assisted and the social costs of bankruptcy c will play a role. Note that these are defined as fractions of the bank's size in period 1, which means they lie between zero and one.

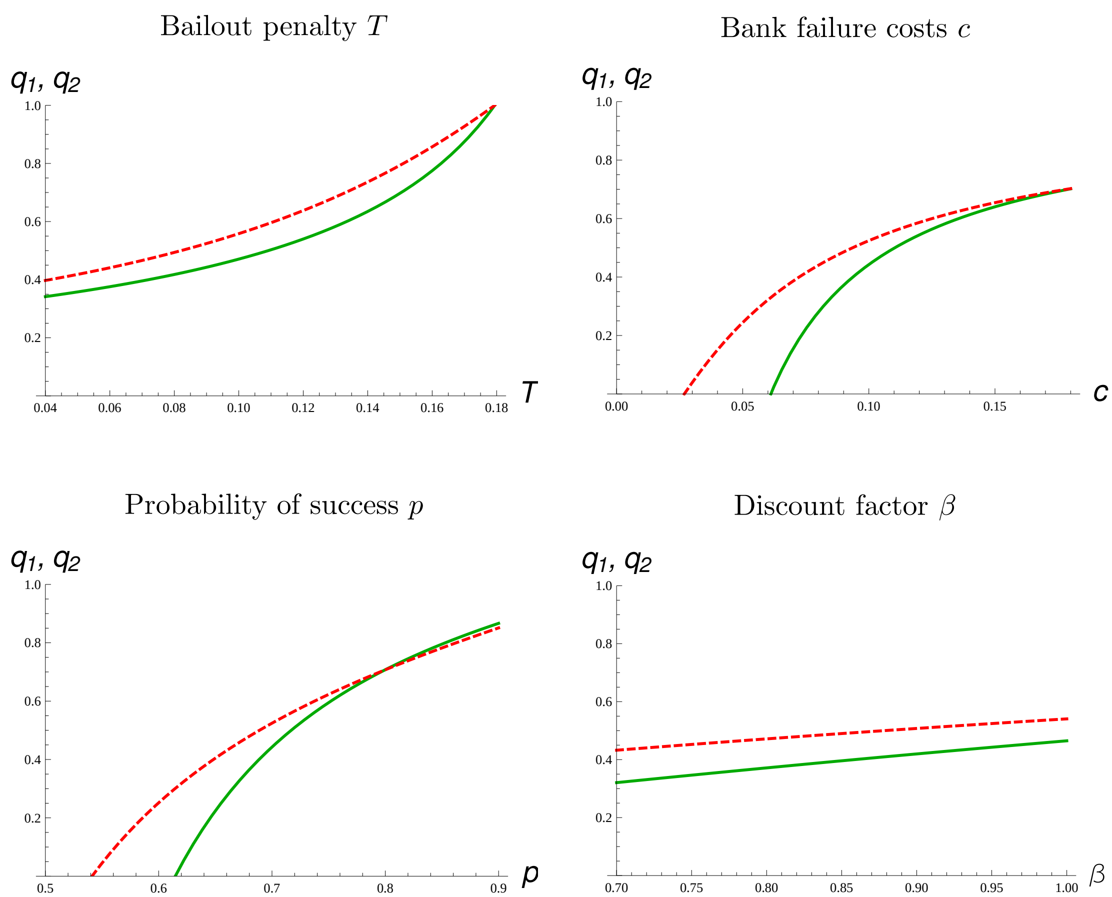
Proposition 3: *the probability of bailout in both periods increases with the bailout penalty T and the social costs of bankruptcy c.*
Proof: see appendix. ■

An increase in the bailout penalty T means that the bank owner has to pay more to society in the event of a bailout. We can think of this as an increase in public indignation leading to a demand for bankers to pay more to society if they need bailout assistance. The bailout penalty then increases the probability of liquidity assistance, as it rewards prudent behaviour. This means that the bank will want to invest more in capital to avoid having to pay the penalty. Although this investment increases the probability of bailout by the CBFS, it also increases the probability that the bank can survive without any assistance and thus does not have to pay T.

A higher cost of bank failure c means that the CBFS will incur a higher loss in case the bank fails. A higher c can reflect a change in the CBFS's mandate, increasing the responsibility the CBFS bears for the banking system. It can also reflect a stronger connection between the banking sector and the real economy (i.e. through the payment system), which means that the failure of a bank has more severe implications for the rest of the economy. Therefore, higher social costs of bankruptcy decrease the liquidity and capital levels needed to sustain a mixed strategy; a more concerned CBFS will require less investment by the banker to be able to provide liquidity assistance in equilibrium.

The effect on q_1 and q_2 of T and c is illustrated in the top row of figure 4 below. This figure shows that for low values of T and c, the probability of bailout in period 1 is always lower than that in period 2. The intuition behind this is that at period 1, the CBFS will want to signal that it is not very concerned about bank failure, to sustain the belief of the bank that it indeed is little concerned about failure and that the bank should keep high capital and liquidity also in the next period. In period 2 (the last period) this motive for the CBFS is no longer present, so the bailout probability can be higher. This discrepancy between periods 1 and 2 disappears as soon as c is high enough: it is no longer possible for the CBFS to sustain the bank's belief that it is not concerned about bankruptcy. A high T (above 18% of bank size under our parametrization) leads the bank to invest so much in capital that the CBFS will choose $q_t = 1$ in both periods. However, since it violates our parameter assumptions on T this will not occur in equilibrium.

Figure 4: The effect of different parameters on bailout probability q_t



Note: the solid line represents q_1, while the dashed line represents q_2.

Note also that there is a minimum level of c for the CBFS to be concerned; if c is too low, the CBFS will not be willing to assist the bank for any level of liquidity and capital. This minimum level is lower for q_2 than for q_1, again demonstrating that the CBFS has no concerns about the future in period 2. The CBFS will thus require less liquidity and capital investment efforts from the bank in period 2 to warrant a certain bailout probability.

Besides the institutional details of the CBFS, the probability that the bank's investment succeeds and the discount factor (the inverse of the rate of time preference) are important in determining the equilibrium outcome.

Proposition 4: *the probability of success p increases the probability of liquidity assistance, but decreases the bank's investment in capital and liquid reserves. The discount factor β increases the probability of assistance in both periods via an increase in period 1 capital.*
Proof: see appendix. ■

An increase in p increases the probability that the bank succeeds at the end of each period, which means that the CBFS has to worry less about the repayment of its liquidity injection. In other words, the default or solvency risk of the bank is lower. Therefore, the probability of bailout is positively affected by an increase in the probability of success. However, since the CBFS will be more lenient, the bank also has to invest less in capital and liquidity to satisfy the condition for a mixed strategy equilibrium.

The effect of β is more subtle. Increasing the discount factor β increases the importance of period 2 for the banker; it decreases banker myopia. He will thus want to increase both expected period 2 profit $E[\Pi_2]$ and the probability of arriving at period 2. Increasing liquid reserves decreases the amount of assets available for investment and thus the investment return in period 2. Investment in capital in period 1 increases $E[\Pi]$ and the probability of continuation after period 1 by decreasing the size of the liquidity shock and increasing the bailout probability q_1. Therefore, in period 1 the bank will want to invest more in capital and less in liquidity as the importance of period 2 increases.

The bottom row of figure 4 shows these effects. A first observation tells us that the discount factor has limited effect; even with a discount rate of more than 40%, the probability of bailout is still far above zero. Furthermore, we can see that there exists a minimum value of the success probability p for the CBFS to be willing to assist the bank. If the probability of success is too low, the probability that the CBFS loses the liquidity it lent to the bank is too high. This is again, and analogous to the minimum value of the bank failure cost c, lower for q_2 than for q_1: when setting q_1, period 2 still matters, while there is no concern about the future anymore when setting q_2. The probability of success is therefore more important in period 1 than in period 2.

6 Conclusion

Calls for new banking regulation have been numerous during the aftermath of the financial crisis. One of the main questions has been how to design proper system of financial regulation, consisting of both prudential regulation and a financial safety net. This system should provide protection to depositors, other debtors and the economy as a whole, while also preventing moral hazard by banks and other financial institutions.

In our model, we analyze the game between a bank and a regulator in a dynamic context, taking into account that the regulator can implement a mixed strategy in providing liquidity assistance. We find that unconditional liquidity assistance leads to too much moral hazard, while a policy without any assistance is not credible. Therefore, a mixed strategy, conditional on the choices of liquidity and capital by the bank, is the equilibrium solution. The bank chooses above minimum capital and liquidity, unless capital costs or the opportunity cost of liquidity are too high. In case one of either type of costs is too high, the equilibrium can still be sustained. When both are high, however, the bank will have to choose capital and liquidity to be at the minimum. In this case, liquidity assistance costs will be too high for any size of the liquidity shock, so the regulator will never bail out the bank and there will be no equilibrium. We also find that the probability of a bailout is higher for a regulator more concerned about bank failure, a bank more concerned about the future, a higher success probability and a higher the bailout penalty for the bank. This last finding suggests that forbearance arising from penalty rates is not entirely eliminated.

As a starting point, our model takes the same basic assumptions as in Eijffinger and Nijskens (2011); the only difference is that monitoring choice is replaced by the choice of capital. We add to the existing literature by analyzing LLR policy over multiple periods, while taking into account explicitly both the regulator's and the bank's incentives. A novel result is that the only possible strategy for the regulator is a mixed one: constructive ambiguity is the only solution to our game. Furthermore, we provide the bank with two different variables to fulfill the requirements for liquidity assistance: both capital and liquidity choice can be altered to maximize the expected profit over all periods. Our final major addition to the literature is that we find an indirect forbearance effect of bailout penalties: even though these penalties are not paid to the regulator directly, they increase the probability of bailout.

Our results can have important policy implications for reforming LLR policy. The institution responsible for liquidity assistance (preferably an independent institution like the central bank) should not state explicitly what its line of action will be. Instead, it should be ambiguous about whether it will assist a bank or not, and retain some discretion up until the point that a bank will ask for assistance. Furthermore, our analysis also shows that it is useful to let the bank pay a (lump sum) penalty when it receives assistance, as this indeed improves the incentives to hold more capital and reserves. Finally, we find that decreasing the myopia of bankers can have positive effects on bailout probability and capital, but a negative effect on liquidity holdings.

We have presented a model to analyze the possibility of constructive ambiguity in bailing out illiquid banks, under the assumption that the central bank can commit to this strategy ex ante. An important prerequisite for our results to hold is, therefore, the existence of a commitment technology for the regulator. As we have argued, this commitment may be provided by a mandate coupled with accountability. This should be provided to a credible authority with a good reputation, as is often the case in monetary policy. A regulator with these characteristics can internalize bank behaviour without being subject to regulatory capture, as it will be able to commit to a strategy of constructive ambiguity. This implies that a more general setup should also encompass an analysis of this commitment technology. Therefore, a further investigation into (political) commitment mechanisms is warranted to grasp better the dynamic effects of bailout policies. This, however, remains for future research.

References

Barro, R. and Gordon, D. (1983). Rules, discretion and reputation in a model of monetary policy. *Journal of monetary economics*, 12(1):101–121.

Bosma, J. (2011). Communicating bailout policy and risk taking in the banking industry. DNB Working Papers 277, Netherlands Central Bank, Research Department.

Brunnermeier, M. and Pedersen, L. (2009). Market liquidity and funding liquidity. *Review of Financial Studies*, 22(6):2201–2238.

Caballero, R. J. and Krishnamurthy, A. (2008). Collective risk management in a flight to quality episode. *Journal of Finance*, 63(5):2195–2230.

Castiglionesi, F. and Wagner, W. (2011). Turning Bagehot on his head: lending at penalty rates when banks can become insolvent. *Journal of Money, Credit and Banking*, forthcoming.

Cordella, T. and Levy-Yeyati, E. (2003). Bank bailouts: Moral hazard vs. value effect. *Journal of Financial Intermediation*, 12(4):300–330.

Cukierman, A. (2009). The limits of transparency. *Economic Notes*, 38(1-2):1–37.

Cukierman, A. and Izhakian, Y. (2011). Bailout uncertainty in a microfounded general equilibrium model of the financial system. (8453).

Cukierman, A. and Meltzer, A. (1986). A theory of ambiguity, credibility, and inflation under discretion and asymmetric information. *Econometrica: Journal of the Econometric Society*, pages 1099–1128.

Demertzis, M. and Hughes Hallett, A. (2007). Central bank transparency in theory and practice. *Journal of Macroeconomics*, 29(4):760–789.

Eijffinger, S. and Hoeberichts, M. (2002). Central bank accountability and transparency: Theory and some evidence. *International Finance*, 5(1):73–96.

Eijffinger, S. and Nijskens, R. (2011). Complementing Bagehot: illiquidity and insolvency resolution. CEPR Discussion Papers 8603, Centre for Economic Policy Research.

Freixas, X. (1999). Optimal bail-out policy, conditionality and creative ambiguity. CEPR Discussion Paper 2238, Centre for Economic Policy Research.

Freixas, X. and Parigi, B. M. (2008). Lender of last resort and bank closure policy. CESifo Working Paper Series 2286, CESifo GmbH.

Goodhart, C. and Huang, H. (2005). The lender of last resort. *Journal of Banking & Finance*, 29(5):1059–1082.

Hellmann, T. F., Murdock, K. C., and Stiglitz, J. E. (2000). Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough? *American Economic Review*, 90(1):147–165.

Kahn, C. and Santos, J. (2005). Allocating bank regulatory powers: lender of last resort, deposit insurance and supervision. *European Economic Review*, 49(8):2107–2136.

Kocherlakota, N. and Shim, I. (2007). Forbearance and prompt corrective action. *Journal of Money, Credit and Banking*, 39(5):1107–1129.

Levy, A. and Schich, S. (2010). The design of government guarantees for bank bonds: Lessons from the recent financial crisis. *OECD Journal: Financial Market Trends*, 2010(1):3.

Lohmann, S. (1992). Optimal commitment in monetary policy: credibility versus flexibility. *The American Economic Review*, 82(1):273–286.

Repullo, R. (2005). Liquidity, Risk Taking, and the Lender of Last Resort. *International Journal of Central Banking*, 1(2):47–80.

Shim, I. (2011). Dynamic prudential regulation: Is prompt corrective action optimal? *Journal of Money, Credit and Banking*, 43(8):1625–1661.

Vinogradov, D. (2010). Destructive effects of constructive ambiguity in risky times. Technical report, Essex Business School.

Appendix

First Order Conditions (FOC):

$$\frac{\partial E[\Pi]}{\partial l_1} = p\left(\frac{\partial \underline{x}_1}{\partial l_1}((1-q_1)(V(l_1,i_1)-\phi(i_1))+q_1T)-(\underline{x}_1+(1-\underline{x}_1)q_1)(R-1) + \frac{\partial \underline{x}_1}{\partial l_1}(1-q_1)\beta E[\Pi_2]\right)=0 \tag{14}$$

$$\frac{\partial E[\Pi]}{\partial i_1} = p\left(\frac{\partial \underline{x}_1}{\partial i_1}((1-q_1)(V(l_1,i_1)-\phi(i_1))+q_1T)+(\underline{x}_1+(1-\underline{x}_1)q_1)(1-\phi'(i_1)) + \frac{\partial \underline{x}_1}{\partial i_1}(1-q_1)\beta E[\Pi_2] + (\underline{x}_1+(1-\underline{x}_1)q_1)\beta p\left[\frac{\partial \underline{x}_2}{\partial i_1}((1-q_2)(V(l_2,i_2,i_1)-\phi(i_2))+q_2T)+(\underline{x}_2+(1-\underline{x}_2)q_2)\right]\right) = 0 \tag{15}$$

$$\frac{\partial E[\Pi]}{\partial l_2} = \beta p(\frac{\partial \underline{x}_2}{\partial l_2}((1-q_2)(V(l_2,i_2,i_1)-\phi(i_2))+q_2T)-(\underline{x}_2+(1-\underline{x}_2)q_2)(R-1))=0 \tag{16}$$

$$\frac{\partial E[\Pi]}{\partial i_2} = \beta p(\underline{x}_2+(1-\underline{x}_2)q_2)(R-\phi'(i_2))=0 \tag{17}$$

From the first order conditions in equations (14) and (16) we can derive the expressions for equilibrium q_1^* and q_2^*:

$$q_1^* = \frac{V(l_1,i_1)+\beta E[\Pi_2]-\phi(i_1)-l_1(R-1)}{V(l_1,i_1)+\beta E[\Pi_2]-\phi(i_1)+(R-1)(1-i_1-l_1)-T} \tag{18}$$

$$q_2^* = \frac{V(l_2,i_2,i_1)-\phi(i_2)-l_2(R-1)}{V(l_2,i_2,i_1)-\phi(i_2)+(R-1)(1-i_1-l_2)-T} \tag{19}$$

Proof of Proposition 1:

Our goal is to show that there does not exist a subgame perfect equilibrium in which the CBFS plays a pure strategy in any period. In other words, no equilibrium with $q_1 = \{0,1\}$ and/or $q_2 = \{0,1\}$ can be sustained. The proof makes use of backward induction and proceeds in steps.

Step 1:

Let us first consider period 2, in which the CBFS chooses q_2 and the bank chooses l_2 and i_2. Assuming that $q_2 = 1$, i.e. a full bailout at $t = 2$, we first observe that from equation

(16) it follows that $i_1 = 1 - \frac{T}{R-1}$. Furthermore, as stated in the text we make an auxiliary (technical) assumption that $\frac{T}{R-1} < \frac{2pc}{1-p}$. This means that the penalty cannot be too large relative to the investment return and the cutoff point for the CBFS. Then, we have several different situations at $t = 1$:

1. $q_1 = 1$, from which follows that $l_2 > 1 - \frac{2pc}{1-p} - i_1 = \frac{T}{R-1} - \frac{2pc}{1-p}$. Equation (15) requires that $l_1 + \beta p l_2 = \frac{T}{(R-1)^2}(\phi'(1 - \frac{T}{R-1}) - (1+\beta p))$. However, a balance sheet constraint also has to be fulfilled: $l_1 + \beta p l_2 \leq 1 + \beta p(1+i_2)$. From (17) we can deduce that $\phi'(i_2) = R$. We can transform this to $i_2 = \psi(R)$ by taking the function $\psi(\cdot)$ as the inverse of $\phi'(\cdot)$, or $\psi(\cdot) = \phi'^{-1}(\cdot)$ which is increasing. This leaves us with the condition

$$\frac{T}{(R-1)^2}(\phi'(1 - \frac{T}{R-1}) - (1+\beta p)) \leq 1 + \beta p(1 + \psi(R)). \tag{20}$$

   As $\phi(\cdot)$ is convex, $\phi'(\cdot) > \psi(\cdot)$ and $\frac{\partial(1-\frac{T}{R-1})}{\partial R} > 1$, this cannot hold for reasonably large R.

2. $q_1 = 0$, from which follows that $l_1 < 1 - \frac{2pc}{1-p} - i_1 = \frac{T}{R-1} - \frac{2pc}{1-p}$. We have assumed that $T < R - 1$ and $\frac{2pc}{1-p} < 1$. If we additionally restrict the parameter space such that $\frac{T}{R-1} - \frac{2pc}{1-p} < 0$, the above condition on l_1 cannot hold.

3. $q_1 \in (0, 1)$, requiring that $l_1 = 1 - \frac{2pc}{1-p} - i_1 = \frac{T}{R-1} - \frac{2pc}{1-p}$. As $\frac{T}{R-1} - \frac{2pc}{1-p} < 0$, this is also no equilibrium.

Step 2:

Having established that a full bailout at $t = 2$ is not sustainable in equilibrium, we now consider the situation where $q_2 = 0$. This means the CBFS never assists the bank at $t = 2$. We know that this means that $l_2 < 1 - \frac{2pc}{1-p} - i_1$. The following situations can occur at $t = 1$:

1. $q_1 = 0$, which requires that $l_1 < 1 - \frac{2pc}{1-p} - i_1$. From condition (16) we can establish that

$$p(\frac{\partial \underline{x}_2}{\partial l_2}(V(l_2, i_2) - \phi(i_2)) - (\underline{x}_2)(R-1)) = 0, \text{or} \tag{21}$$

$$V(l_2, i_2) - \phi(i_2) = l_2(R-1) \tag{22}$$

Using this and our earlier condition $i_2 = \psi(R)$, we can write i_1 as

$$i_1 = \phi(\psi(R)) - \psi(R)R + (R-1)(2l_2 - 1) \tag{23}$$

which can only be positive if

$$l_2 > \frac{R(1+\psi(R)) - (1+\phi(\psi(R))}{2(R-1)}. \tag{24}$$

Furthermore, $l_2 < 1 - \frac{2pc}{1-p} - i_1$ must hold. Using (23) this translates to

$$l_2 < \frac{R(1+\psi(R)) - (\frac{2pc}{1-p} + \phi(\psi(R))}{2R-1} \tag{25}$$

Some algebra shows that the derivative w.r.t R of the RHS of condition (24) is larger than that of condition (25) when $\psi(R) > \frac{4pc}{1-p} - 1$. This means that, for large enough R and reasonable p and c, the two conditions cannot hold simultaneously and $q_1 = 0$ cannot be an equilibrium.

2. $q_1 = 1$, leading to $l_1 > 1 - \frac{2pc}{1-p} - i_1$ and $i_1 = 1 - \frac{T}{R-1}$, which cannot be an equilibrium as we have shown above in step 1.

3. $q_1 \in (0,1)$, which requires that $l_1 = 1 - \frac{2pc}{1-p} - i_1$. Applying the same reasoning as in the $q_1 = 0$ case, this cannot be an equilibrium for reasonably large R.

Step 3:

We now move to period 1, noting that in period 2 the CBFS will always play a mixed strategy in the form of $q_2 \in (0,1)$; this establishes the relation $l_2 = 1 - \frac{2pc}{1-p} - i_1$. We now only have to show that $q_1 = 0$ and $q_1 = 1$ are not possible:

1. $q_1 = 0$, which requires that $l_1 < 1 - \frac{2pc}{1-p} - i_1$. Using condition (14) we can set up a necessary condition for l_1:

$$l_1 = \frac{\beta E[\Pi_2] + R - 1 - (\phi(i_1) - i_1)}{2(R-1)} < l_2 \tag{26}$$

We claim that this condition cannot hold if $\phi(\cdot)$ convex enough, since i_1 will be too low to sustain an l_1 below l_2. This requires that $\frac{dl_1}{di_1} < 0$, for which we apply the Implicit

Function Theorem to condition (14):

$$\frac{dl_1}{di_1} = -\frac{\frac{\partial E[\Pi]}{\partial l_1 i_1}}{\frac{\partial E[\Pi]}{\partial l_1^2}} = \frac{\beta p \frac{l_2}{(1-i_1)^2}((1-q_2)(V_2 - \phi(i_2)) + q_2 T) + 1 - \phi'(i_1)}{R-1} \tag{27}$$

This equation is negative for sufficiently convex $\phi(\cdot)$. Also, i_1 will decrease towards $\underline{k}$ when $\phi(\cdot)$ is very convex, which means that l_2 is fixed by $\underline{k}$. Thus, if i_1 decreases towards $\underline{k}$, l_1 increases and will be larger than l_2 for plausible parameter values.

2. $q_1 = 1$, leading to $l_1 > 1 - \frac{2pc}{1-p} - i_1$ and $i_1 = 1 - \frac{T}{R-1}$, which cannot be an equilibrium as we have shown above where $q_2 = 1$.

This establishes that no pure strategies are possible for the CBFS: $\{q_1, q_2\} \in (0,1)$ is the only strategy sustainable in equilibrium.

Proof of Proposition 2:

As stated in the text, an interior mixed strategy equilibrium always exists. As we have shown in proposition 1, $q_1, q_2 \in (0,1)$, which means that $l_1 = 1 - \frac{2pc}{1-p} - i_1$ and $l_2 = 1 - \frac{2pc}{1-p} - i_1$, establishing that $l_1 = l_2$.

If $\phi(\cdot)$ is sufficiently convex, the FOC on i_1 in equation (15) will always be negative ($\frac{\partial E[\Pi_2]}{\partial \phi} < 0$):

$$\begin{aligned}\frac{\partial E[\Pi]}{\partial i_1 \partial \phi} = {} & p(\frac{\partial \underline{x}_1}{\partial i_1}(-\phi'(i_1) + (1-q_1)\beta \frac{\partial E[\Pi_2]}{\partial \phi}) - (\underline{x}_1 + (1-\underline{x}_1)q_1)\phi''(i_1) \\ & - (\underline{x}_1 + (1-\underline{x}_1)q_1)\beta p \frac{\partial \underline{x}_2}{\partial i_1}\phi'(i_2)) < 0\end{aligned} \tag{28}$$

This means that the bank will choose capital to be at the minimum required, establishing part 1 of proposition 2.

When $\phi(\cdot)$ is less convex and R is high enough, the FOCs on l_1 and l_2 will be negative since i_1 is relatively high and thus l_1 and l_2 relatively low (see condition (13) in the main text):

$$\frac{\partial E[\Pi]}{\partial l_1 \partial R} = p\left(\frac{1}{1-i_1}(1-q_1)((1-l_1)+p\beta(\underline{x}_2+(1-\underline{x}_2)q_2)(1+i_2-l_2)) \right.$$
$$\left. -(\underline{x}_1+(1-\underline{x}_1)q_1)\right) \tag{29}$$

$$\frac{\partial E[\Pi]}{\partial l_2 \partial R} = \beta p\left(\frac{\partial \underline{x}_1}{\partial l_2}(1-q_2)(1+i_2-l_2)-(\underline{x}_2+(1-\underline{x}_2)q_2)\right). \tag{30}$$

These expressions are negative when i_1 is relatively high, establishing that the bank chooses liquidity at $\underline{l}$ in both periods, establishing part 2 of proposition 2.

Proof of Proposition 3:

To gauge the effect of T on q_1 and q_2, we can employ the Implicit Function Theorem to determine the sign of the derivatives of q_1 and q_2 w.r.t T. Using equations (14) and (16) these can be written as follows:

$$\frac{dq_1}{dT} = -\frac{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial T}}{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial q_1}} \tag{31}$$

$$\frac{dq_2}{dT} = -\frac{\frac{\partial^2 E[\Pi]}{\partial l_2 \partial T}}{\frac{\partial^2 E[\Pi]}{\partial l_2 \partial q_2}} \tag{32}$$

where $\frac{\partial^2 E[\Pi]}{\partial l_2 \partial T} = \beta p q_2 \frac{\partial \underline{x}_2}{\partial l_2} > 0$. Since, in equilibrium, $l_1 = l_2$, we also know that $\frac{\partial^2 E[\Pi]}{\partial l_1 \partial T} = p\frac{\partial \underline{x}_1}{\partial l_1}(q_1-(1-q_1)\beta\frac{\partial \Pi_2}{\partial T}) > 0$. Therefore, the numerators of $\frac{dq_1}{dT}$ and $\frac{dq_2}{dT}$ are negative. The denominators are, respectively:

$$\frac{\partial^2 E[\Pi]}{\partial l_1 \partial q_1} = p(\frac{\partial \underline{x}_1}{\partial l_1}(V(l_1,i_1)+\beta E[\Pi_2]-\phi(i_1)-T)+(1-\underline{x}_1)(R-1)) < 0 \tag{33}$$

$$\frac{\partial^2 E[\Pi]}{\partial l_2 \partial q_2} = \beta p(\frac{\partial \underline{x}_2}{\partial l_2}(V(l_2,i_2 i_1)-\phi(i_2)-T)+(1-\underline{x}_2)(R-1)) < 0 \tag{34}$$

As both the numerator and denominator are negative, $\frac{dq_1}{dT}$ and $\frac{dq_2}{dT}$ are positive.

The effect of c is more straightforward: as c only appears in the indifference condition of the CBFS, $l_1 = l_2 = 1 - \frac{2pc}{1-p} - i_1$, we can substitute this condition for l_1 and l_2 in the FOCs.

Again applying the implicit function theorem leads us to

$$\frac{dq_1}{dc} = -\frac{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial c}}{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial q_1}} \tag{35}$$

$$\frac{dq_2}{dc} = -\frac{\frac{\partial^2 E[\Pi]}{\partial l_2 \partial c}}{\frac{\partial^2 E[\Pi]}{\partial l_2 \partial q_2}}, \tag{36}$$

of which we already know that the denominators are negative. The respective numerators are

$$\frac{\partial^2 E[\Pi]}{\partial l_1 \partial c} = p2((1-q_1)\frac{1}{1-i_1}(R-1)\frac{2p}{1-p} < 0 \tag{37}$$

$$\frac{\partial^2 E[\Pi]}{\partial l_2 \partial c} = \beta p2(1-q_2)\frac{\partial \underline{x}_2}{\partial l_2}((R-1)\frac{2p}{1-p} < 0 \tag{38}$$

As again both the numerator and denominator are negative, $\frac{dq_1}{dc}$ and $\frac{dq_2}{dc}$ are positive.

Proof of Proposition 4:

An increase in p works through the same channel as an increase in c: we substitute $1 - \frac{2pc}{1-p} - i_1$ for $l1$ and l_2 in their respective FOCs, and then calculate the total derivative of q_t w.r.t. p:

$$\frac{dq_1}{dp} = -\frac{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial p}}{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial q_1}} \tag{39}$$

$$\frac{dq_2}{dp} = -\frac{\frac{\partial^2 E[\Pi]}{\partial l_2 \partial p}}{\frac{\partial^2 E[\Pi]}{\partial l_2 \partial q_2}}. \tag{40}$$

We already know that the denominators are negative. The respective numerators are

$$\frac{\partial^2 E[\Pi]}{\partial l_1 \partial p}|_{l_1 = 1 - \frac{2p}{1-p}c - i_1} = p2((1-q_1)\frac{1}{1-i_1}(R-1)\frac{2}{(1-p)^2}c > 0 \tag{41}$$

$$\frac{\partial^2 E[\Pi]}{\partial l_2 \partial p}|_{l_2 = 1 - \frac{2p}{1-p}c - i_1} = \beta p2(1-q_2)\frac{1}{1-i_1}(R-1)\frac{2}{(1-p)^2}c > 0 \tag{42}$$

An increase in β increases the importance of period 2 for the banker, so he will want to increase $E[\Pi_2]$. As β affects the marginal benefit of i_1 positively ($\frac{dMB(i_1)}{d\beta} > 0$, see equation (15)), without affecting its costs, the bank will want to increase i_1 to accomplish this. An increase in i_1 increases $E[\Pi_2]$, q_1 and q_2 as $\frac{dq_1}{di_1}$ is positive. The effect of β on q_1 and q_2 is reflected in the derivations below:

$$\frac{dq_1}{d\beta} = -\frac{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial \beta}}{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial q_1}} \tag{43}$$

$$\frac{dq_2}{d\beta} = -\frac{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial \beta}}{\frac{\partial^2 E[\Pi]}{\partial l_1 \partial q_2}} \tag{44}$$

We already know that the denominators of these expressions are negative, since $\frac{\partial^2 E[\Pi]}{\partial l_1 \partial q_1} < 0$, $\frac{\partial^2 E[\Pi]}{\partial l_2 \partial q_2} < 0$ and $l_1 = l_2$ in equilibrium. The (shared) numerator is:

$$\frac{\partial^2 E[\Pi]}{\partial l_1 \partial \beta} = p\frac{\partial \underline{x}_1}{\partial l_1}(1 - q_1)E[\Pi_2] > 0, \tag{45}$$

so $\frac{dq_1}{d\beta} > 0$ and $\frac{dq_2}{d\beta} > 0$.

Through the CBFS indifference condition $l_1 = l_2 = 1 - \frac{2pc}{1-p} - i_1$ we can also see that l_1 and l_2 decline as i_1 increases. Therefore, an increase in β increases i_1, q_1 and q_2 and decreases l_1 and l_2, as required.