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Pierpaolo Benigno and Federica Romei

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Pierpaolo Benigno, LUISS, EIEF and CEPR
Federica Romei, LUISS

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Debt Deleveraging and the Exchange Rate*

Deleveraging from high debt can provoke deep recession with significant international side effects. The exchange rate of the deleveraging country will depreciate in the short run and appreciate in the long run. The real interest rate will fall by more than in the rest of the world. Bounds and policies that constrain the adjustment can prolong and deepen the recession. Early exit strategies from accommodating monetary policy can be quite harmful, as can such other policies as keeping interest rates too high during the deleveraging period. The analysis also applies to a monetary union facing internal adjustment of current account imbalances.

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Pierpaolo Benigno
LUISS Guido Carli
Dipartimento di Scienze Economiche e Aziendali
Viale Romania 32
00197 Rome
ITALY

Email: pbenigno@luiss.it

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Federica Romei
LUISS Guido Carli
Dipartimento di Scienze Economiche e Aziendali
Viale Romania 32
00197 Rome
ITALY

Email: fromei@luiss.it

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The decade leading up to the financial crisis was marked by divergences and disequilibria. Global imbalances have been at the center of the debate, with several economists warning against the unsustainability of the US external position. The euro area has experienced internal current account divergences, producing an enormous accumulation of debt. The crisis was most severe in the economies that had piled up too much private or public debt in one form or another. It is still being debated whether the divergences of the past actually caused the crisis or merely reflected other underlining problems.[1] In any case, the general tendency is for the crisis-ridden countries to reduce debt. Deleveraging raises interesting questions on macroeconomic adjustment and international spillovers. How do monetary policy and the exchange rate affect the adjustment? What happens in a monetary union in which some countries are forced to deleverage? What if mistaken policies are followed? These are some of the questions this work aims to answer.

Deleveraging is a costly process: it forces debtor countries to cut spending sharply and depresses demand. A healthy correction would involve an increase in spending in the rest of the world. But international relative prices are not immune to the adjustment, and the exchange rate can in fact accompany the process and attenuate its costs.[2] If the fall in demand is sharper for domestic goods, the excess supply of these goods globally lowers their prices relative to foreign prices and expands overall demand for them, thus easing the depressive impact of deleveraging. These changes in relative prices can be achieved by depreciation of the deleveraging country's currency, but if exchange rates are fixed, as in a monetary union, some deflation there should achieve adjustment but at the cost of a longer contraction. In the longer run, a country that has paid down part of its debt is richer than at first, since there is less debt to serve, so the demand for domestic goods is relatively higher. The exchange rate swings from short-term depreciation to appreciation in the long run.

Other relative price movements are also critical to a smooth adjustment. These are intertemporal relative prices such as real interest rates. The debt of some agents corresponds to assets of others, either domestic or foreign. In the course of the adjustment, to reduce their asset holdings creditors should increase consumption, which could be favored by a fall in the real interest rate. Given that the real exchange rate depreciates in the short run and appreciates in the long run, the real interest rate of the deleveraging country falls by more.

In shorts, a smooth adjustment to a deleveraging shock in some part of the world eco-

[1] An interesting discussion is in Obstfeld (2011), Obstfeld and Rogoff (2010).

[2] Krugman (2011a) discusses the importance of exchange-rate movements when there is debt deleveraging.

nomy requires short-run depreciation and long-run appreciation of the deleverager's real exchange rate and a sharper short-term fall in its real interest rate than in the rest of the world. Constraints or policies that impede these mechanisms can only prolong the contraction. For instance, the zero-lower-bound constraint on the nominal interest rate can keep the real rate from falling when prices are sticky, causing a longer contraction. And, fixed-exchange-rate or controlled-rate policy regimes can also lead to a more protracted stagnation. A monetary union falls into the latter category, since sluggish adjustment in relative prices makes deleveraging in some countries inherently costly. Other policies, such as keeping nominal interest rates too high or exiting too early from the zero lower bound, can be even more damaging.

We discuss these issues using a simple two-country world economy in which some agents are more impatient and borrowing-constrained, as in the model of Eggertsson and Krugman (2010). Our environment can also describe a two-country monetary union in which one is reducing debt. Deleveraging is modelled as a tightening debt constraint. There are multiple traded goods but consumers are biased towards those produced in their own country. We consider first an environment with flexible prices and wages, then one with nominal rigidities. We study deleveraging under alternative exchange-rate and monetary policy regimes. In the flexible price and wage model, debt deleveraging cuts the country's liabilities by 20% of GDP, which depreciates the currency by more than 10% in the short run and drives the real interest rate well below zero. In the model with nominal rigidities, a similar experiment can lead to a deep recession with important spillovers to the other country, in particular when the exchange rate is fixed. A floating-exchange-rate regime can attenuate the contraction, as would a monetary union with more symmetrical inflation-targeting policies. A prolonged stay at the zero lower bound helps the economic recovery, at the cost of excess inflation. A policy of high nominal interest rates can be very detrimental, pushing the world economy into deep recession and deflation.

This paper is closely related to Eggertsson and Krugman (2010), Guerrieri and Lorenzoni (2010) and Philippon and Midrigan (2011), who have studied debt deleveraging in closed economies. In the current debate on the unwinding of global imbalances, Feldstein (2011) and Krugman (2011a,b) have stressed the importance of exchange rate movements in correcting global imbalances. Earlier works by Obstfeld and Rogoff (2001, 2005, 2007) also studied the exchange-rate implications of a sudden improvement in one country's current account balance, conducting some comparative-static experiments. Our focus here is on dynamic adjustment and the role of monetary policy. Policies at the zero lower bound, in an open economy, have been explored by Svensson (2001, 2003), but in a different model without debt deleveraging. There is also substantial literature on open economies analyzing credit-constrained economies and the implications of relaxing or restricting credit access for the equilibrium economy: see among others Aghion et al. (2001), Aoki et al. (2010) and Mendoza (2010) and more recently Devereux and Yetman (2010).

This paper is organized as follows. Section 1 describes a deleveraging shock in a simple two-country open-economy endowment model. Section 2 extends the basic model to include nominal rigidities and endogenous output. Section 3 discusses the dynamic implications of deleveraging with alternative monetary policies and exchange-rate regimes, considering the zero lower bound on the nominal interest rates. Section 4 concludes. A separate appendix reports the main equations of the model and the solution method.

1 A simple model

We adopt a simple two-country endowment economy to study how debt deleveraging in one country spreads to the rest of the world economy. The two countries are Home, denoted by H, and Foreign, denoted by F. Each country has an endowment of a good. The two goods, H and F respectively, are traded frictionlessly. In country H, there are two groups of consumers with different discount factors. The first are savers, of mass θ, with discount factor β^*. The second are borrowers, of mass $(1-\theta)$, with discount factor β. Borrowers are more impatient than savers and have a lower discount factor, $0 < \beta < \beta^* < 1$. In what follows, the index "s" identifies the savers and "b" the borrowers in country H. All the consumers in country F have the same discount factor, β^*, coinciding with that of the savers in country H.

The inhabitants of country H maximize utility from consumption

$$\sum_{t=0}^{\infty}(\beta^j)^t u(C_t^j),$$

for $j = \{b, s\}$, where $\beta^s = \beta$ is the discount factor of savers, and $\beta^b = \beta^*$ that of borrowers. The consumption index C^j is a Cobb-Douglas aggregator of the consumption of the two goods, C_H^j (denoting Home goods) and C_F^j (denoting Foreign goods):

$$C^j = \left(\frac{C_H^j}{\alpha}\right)^{\alpha}\left(\frac{C_F^j}{1-\alpha}\right)^{1-\alpha}, \tag{1}$$

where $0 < \alpha < 1$ represents the share of consumption of goods H in the overall consumption basket, for a consumer of country H. Given the prices for the two goods, P_H and P_F, expressed in the currency of country H, the consumption-based price index of the Home country, P, is

$$P = P_H^{\alpha} P_F^{1-\alpha}.$$

Consumers in the Foreign country maximize their utility from consumption

$$\sum_{t=0}^{\infty} (\beta^*)^t u(C_t^*),$$

where the consumption basket C^* is:

$$C^* = \left(\frac{C_H^*}{1-\alpha^*}\right)^{1-\alpha^*} \left(\frac{C_F^*}{\alpha^*}\right)^{\alpha^*}, \tag{2}$$

and now α^*, with $0 < \alpha^* < 1$, is the weight given to goods F. The general price index in country F is:

$$P^* = P_H^{*(1-\alpha^*)} P_F^{*\alpha^*},$$

where P_H^* and P_F^* are the prices of goods H and F in the currency of country F.

The two goods are traded with no friction, and the law of one price holds

$$P_F = SP_F^*, \qquad P_H = SP_H^*,$$

where S is the nominal exchange rate, defined as units of Home currency per unit of Foreign currency. Preferences are biased towards domestic goods, since we assume that $\alpha = \alpha^* > 1/2$. For this reason, our model generates deviations from purchasing power parity (PPP), in which the real exchange rate (Q) is proportional to the terms of trade $T = P_F/P_H$

$$Q = \frac{SP^*}{P} = \left(\frac{P_H}{P_F}\right)^{1-2\alpha} = T^{2\alpha-1}. \tag{3}$$

Given preferences and prices, demands for the goods are:

$$C_H = \alpha \left(\frac{P_H}{P}\right)^{-1} C, \qquad C_F = (1-\alpha)\left(\frac{P_F}{P}\right)^{-1} C,$$

$$C_H^* = (1-\alpha^*)\left(\frac{P_H^*}{P^*}\right)^{-1} C^*, \qquad C_F^* = \alpha^* \left(\frac{P_F^*}{P^*}\right)^{-1} C^*.$$

Consumers in the Home country receive in every period t an endowment $Y_{H,t}$ of good H, which they can sell at the price $P_{H,t}$; they consume a bundle C_t^j of goods H and F at price P_t; borrow or lend resources $D_{t+1}^j/(1+i_t)$, in units of currency of country H, and pay back or receive the face value of the funds lent in the previous period D_t^j. A positive value for D^j denotes nominal debt. D^j is the only asset traded internationally and $1+i$ is the one-period risk-free gross nominal interest rate on domestic currency.[3] As a result, the flow budget constraint for consumers in the Home country is:

$$P_t C_t^j = P_{H,t} Y_{H,t} + \frac{D_{t+1}^j}{1+i_t} - D_t^j. \tag{4}$$

There is a limit on the amount of the debt that is proportional to nominal GDP

$$D_t^j \leq k(P_{H,t} Y_{H,t}), \tag{5}$$

where $k > 0$. Similar constraints have been used in other open-economy models, such as Aoki et al. (2010), Devereux and Yetman (2010) and Mendoza (2010). They are justified in terms of the guarantees that international creditors require when borrowers have limited commitment. As in Eggertsson and Krugman (2010), we do not model the source of this constraint but interpret it as the maximum size of the debt that can be considered safe at a given point in time. A change in this limit –in particular its reduction over time– constitutes the debt-deleveraging experiment analyzed here.[4]

Looking now at country F, the flow budget constraint is:

$$P_t^* C_t^* = P_{F,t}^* Y_{F,t}^* + \frac{D_{t+1}^*}{S_t(1+i_t)} - \frac{D_t^*}{S_t}, \tag{6}$$

where $Y_{F,t}^*$ represents the endowment of good F and D_t^* the holding of nominal debt in units of currency H. Consumers in country F face a borrowing limit in terms of their GDP:

$$\frac{D_t^*}{S_t} \leq k^* P_{F,t}^* Y_{F,t}^*, \tag{7}$$

for a positive k^*.

The optimal intertemporal allocation of consumption in country H is governed by the following Euler equations:

$$U_c(C_t^j) \geq \beta^j U_c(C_{t+1}^j) \frac{(1+i_t)P_t}{P_{t+1}}. \tag{8}$$

[3] Nominal bonds allow for meaningful asset trading even when consumption baskets are different across countries.

[4] We do not impose that k should be less than 1, since the pledgeable collateral can go beyond current income and up to the present discounted value of all future income. As will be clear in the next section, k should be strictly less than $1/(1-\beta)$.

Similarly, the Euler equation for consumers in country F is:

$$U_c(C_t^*) \geq \beta^* U_c(C_{t+1}^*) \frac{(1+i_t)S_t P_t^*}{S_{t+1} P_{t+1}^*}. \tag{9}$$

Both equations hold with equality when the borrowing limit is not binding.

We define aggregate consumption of consumers in country H as $C_t = \theta C_t^s + (1-\theta)C_t^b$, where θ is the fraction of savers. Equilibrium in goods and asset markets implies

$$Y_{H,t} = T_t^{1-\alpha}[\alpha C_t + (1-\alpha)Q_t C_t^*], \tag{10}$$

$$Y_{F,t}^* = T_t^{-\alpha}[(1-\alpha)C_t + \alpha Q_t C_t^*], \tag{11}$$

$$D_t^* + \theta D_t^s + (1-\theta)D_t^b = 0. \tag{12}$$

Combining the equilibrium in the goods market, the terms of trade can be written as

$$T_t = \frac{Y_{H,t}}{Y_{F,t}^*}\left(\frac{(1-\alpha)C_t + \alpha Q_t C_t^*}{\alpha C_t + (1-\alpha)Q_t C_t^*}\right), \tag{13}$$

while the real exchange rate follows from $Q_t = T_t^{2\alpha-1}$.

Two results can be read directly from equation (13). First, a relative abundance of Home over Foreign goods lowers Home prices relative to the Foreign (expressed in the same currency), worsening the Home terms of trade and depreciating its real exchange rate. If prices of goods are rigid in the endowment currency or if the monetary authority strictly targets the domestic price level, this corresponds to a nominal depreciation. Under these assumptions, in what follows, we use terms of trade, real and nominal exchange rates interchangeably.[5]

Second, and more important, home bias in consumption is crucial in order for deleveraging to influence the exchange rate. In fact, if preferences are identical across countries ($\alpha = 1/2$), the terms of trade are independent of the distribution of wealth and just proportional to the ratio of the endowments of the two goods.[6] Instead, when there is home bias, the distribution of wealth and debt across countries can also affect relative prices through the demand channel. Imagine that deleveraging in the Home country reduces Home consumption. Since Home consumers demand more of their own goods, the fall

[5] In the model with nominal rigidities the decomposition of the terms of trade into prices and exchange rate movements will follow naturally from the interaction between price rigidities and monetary policy.

[6] This is a standard result that depends on the assumption of Cobb-Douglas preferences, as in Cole and Obstfled (1991).

in Home consumption depresses the demand for Home goods more than that for Foreign goods. The price of the Home goods relative to Foreign falls, worsening the Home terms of trade and depreciating the Home currency. In these cases, exchange rate management is a factor in the debt-deleveraging transmission mechanism. We will study these issues more extensively in the model with nominal rigidities.

1.1 Steady state

A deleveraging shock produced by a lowering of the debt limit k requires some time to be absorbed. In this section we abstract from the adjustment process and compare the initial and final steady-state equilibria. In the steady state, the debt of the borrowers in country H comes up against the borrowing limit because of their impatience to consume goods. By contrast, the savers in country H and all the consumers in country F are on their Euler equations, which link Home and Foreign real interest rates ($\bar{r}$ and $\bar{r}^*$) to the subjective discount factor β^*

$$(1+\bar{r}^*) = (1+\bar{r}) = \frac{1}{\beta^*}, \tag{14}$$

where an upper bar denotes variables at their steady-state levels. Debt of the borrowers is determined by the borrowing limit (5), and the steady-state level of consumption follows from their budget constraint

$$\bar{C}^b = \bar{T}^{\alpha-1}\bar{Y}_H[1-(1-\beta^*)k]. \tag{15}$$

From the savers' budget constraint (4), we derive their steady-state consumption

$$\bar{C}^s = \bar{T}^{\alpha-1}\bar{Y}_H[1-(1-\beta^*)\bar{d}^s], \tag{16}$$

where d_t^s are their outstanding debt obligations as a ratio to GDP (if positive) or assets (if negative), defined as $d_t^s = D_t^S/(P_{H,t}Y_{H,t})$. Note that $\bar{C}^s$ is not determined, since D_t^S is not determined. This indeterminacy, standard in open-economy models, is convenient, since it allows for a degree of freedom to initialize, consistent with the data, the starting point from which to study the transition path to a new equilibrium. In this new equilibrium, k is assumed low and $\bar{d}^s$ endogenously reaches a new level. Aggregate consumption in the Home country follows from a weighted average of (15) and (16)

$$\bar{C} = \bar{T}^{\alpha-1}\bar{Y}_H[1-(1-\beta^*)\bar{d}], \tag{17}$$

where $\bar{d}$ represents country H's external liabilities over GDP

$$\bar{d} = [\theta \bar{d}^s + (1-\theta)k].$$

Combining (3), (6) and (12) consumption in the Foreign country is given by

$$\bar{Q}\bar{C}^* = \bar{T}^{\alpha}\bar{Y}_F^* + \bar{T}^{\alpha-1}\bar{Y}_H(1-\beta^*)\bar{d}. \tag{18}$$

The steady-state terms of trade can be simply obtained by appropriately incorporating (17) and (18) into (13)

$$\bar{T} = \frac{\bar{Y}_H}{\bar{Y}_F^*}\left[1 + (1-\beta^*)\left(\frac{2\alpha-1}{1-\alpha}\right)\bar{d}\right]. \tag{19}$$

Interestingly, the terms of trade and the real exchange rate depend on the level of debt and the distribution of wealth, but only when there is home bias in consumption, i.e. when $\alpha > 1/2$. Keeping fixed $\bar{d}^s$ when we move from a high- to a low-debt equilibrium (k falls and $\bar{d}$ falls), equation (19) shows that the terms of trade improve in the long run. Indeed, consumption for the constrained borrowers is higher in the final than in the initial steady state, since they have less debt and can service it at less real cost. On the contrary, Foreign consumers have to lower consumption. Since there is home bias, the demand for Home goods increases relative to that of Foreign goods in the long run, the terms of trade of country H improve and the real exchange rate rises. There is one important caveat, namely that the level of debt or assets of the savers in country H does not vary during the exercise.[7] However, we will see in the next section that even considering dynamic adjustment, the response of savers in the Home country does not overturn the reduction of debt by the constrained agents, so that the total net external debt of the Home country is smaller in the new long-run equilibrium. These results characterize the comparison between the initial steady state and the long run qualitatively. The interesting part of the exercise, however, is the short-run adjustment, which is completely different in form, actually swinging from a short-run currency depreciation to a long-run appreciation.

1.2 Adjustment to a deleveraging shock in country H

We now study the dynamic adjustment to a deleveraging shock that hits the borrowers in country H. Let us say that for exogenous reasons borrowers face a fall in the maximum amount of debt that can be considered risk-free: the debt ceiling k drops from k_{high} to

[7] The discussion of this section clearly applies when there are no savers in country H, i.e., $\theta = 0$.

k_{low}. Given an initial condition on the asset position of the savers, d^s, the external debt position of country H with respect to GDP moves from $d = \theta d^s + (1-\theta)k_{high}$ to a new steady-state level $\bar{d} = \theta\bar{d}^s + (1-\theta)k_{low}$. The new long-run levels of $\bar{d}^s$ and $\bar{d}$ are determined along the transition path. The adjustment takes place in two periods, the short run and the long run.

In the long run, denoted by a bar, the results of section (1.1) apply. The real interest rate follows from (14) while $\bar{T}$, $\bar{C}$, $\bar{C}^*$ and $\bar{Q}$ solve equations (3), (13), (17) and (18), and depend on $\bar{d} = \theta\bar{d}^s + (1-\theta)k_{low}$, where $\bar{d}^s$ is determined in the adjustment from the short to the long run.

In the short run, the flow budget constraint of the borrowers in the Home country implies:

$$C^b = T^{\alpha-1}Y_H + \frac{k_{low}}{1+r}\overline{T}^{\alpha-1}\overline{Y}_H - k_{high}T^{\alpha-1}Y_H, \tag{20}$$

while that of the savers implies

$$C^s = T^{\alpha-1}Y_H + \frac{\overline{d}^s}{1+r}\overline{T}^{\alpha-1}\overline{Y}_H - d^s T^{\alpha-1}Y_H. \tag{21}$$

Note in particular that the consumption of savers depends on the initial asset position d^s and on the new equilibrium level $\overline{d}^s$. Combining (20) and (21), aggregate consumption in the Home country is

$$C = T^{\alpha-1}Y_H + \frac{\overline{d}}{1+r}\overline{T}^{\alpha-1}\overline{Y}_H - d\cdot T^{\alpha-1}Y_H,$$

and Foreign consumption follows specularly

$$QC^* = T^{\alpha}Y_F^* - \frac{\overline{d}}{1+r}\overline{T}^{\alpha-1}\overline{Y}_H + d\cdot T^{\alpha-1}Y_H.$$

Euler equations of the savers in the Home country and of the consumers in the Foreign country link short and long-run consumption through the real interest rate

$$\frac{1}{C^s} = \frac{1}{\bar{C}^s}\beta^*(1+r), \tag{22}$$

$$\frac{1}{C^*} = \frac{1}{\bar{C}^*}\beta^*(1+r^*), \tag{23}$$

where we have assumed log utility. In the short run, the Home and Foreign rates are related to the changes in the real exchange rate between the short and the long run

$$1 + r = (1 + r^*)\frac{\bar{Q}}{Q}. \tag{24}$$

Using short- and long-run consumption in the Euler equation (23) of country F, we obtain an expression for the short-run real interest rate

$$(1 + r) = \frac{1}{\beta^*}\left[\frac{\bar{T}^{\alpha}\bar{Y}_F^* + \bar{d}\cdot\overline{T}^{\alpha-1}\overline{Y}_H}{T^{\alpha}Y_F + d\cdot T^{\alpha-1}Y_H}\right] \tag{25}$$

and analogously using short and long-run consumption for the savers in country H, now in the Euler equation (22), we obtain another restriction on the real interest rate:

$$(1 + r) = \frac{1}{\beta^*}\frac{\bar{T}^{\alpha-1}\bar{Y}_H}{T^{\alpha-1}Y_H}\left[\frac{1-\bar{d}^s}{1-d^s}\right]. \tag{26}$$

The short-run real rate depends on movements in the terms of trade and debt positions between the short and the long run for a given path of output, which is exogenous and can be considered constant through the exercise. We have equations (25) and (26) to determine r, T and $\bar{d}^s$ given that $\overline{T}$ is also a function of $\bar{d}^s$ as discussed in the previous section. The additional equilibrium condition comes from combining (13), (20) and (21) into

$$T = \frac{Y_H}{Y_F^*}\left[1 + \frac{1-2\alpha}{1-\alpha}\left(\frac{\bar{d}}{1+r}\frac{\overline{T}^{\alpha-1}\overline{Y}_H}{T^{\alpha-1}Y_H} - d\right)\right],$$

which can be written using (26) as

$$T = \frac{Y_H}{Y_F^*}\left[1 + \frac{2\alpha-1}{1-\alpha}\left(\frac{d}{\bar{d}} - \frac{1-\bar{d}^s}{1-d^s}\beta^*\right)\bar{d}\right].$$

Some qualitative implications for the short-run terms of trade can be inferred already from this equation, again assuming home bias in consumption, which is necessary in order for the dynamic and the distribution of debt to affect the terms of trade. When the borrowers in country H are deleveraging, the savers there increase their debt and $\bar{d}^s > d^s$. If in the new steady state the external debt position of the Home country, $\bar{d}$, is lower than the initial position, so that overall the country is deleveraging with respect to the world, then it is easy to see that the terms of trade in the short run, T, will move to a higher level. Therefore, the exchange rate will come down in the short run but rise in the long run.

For a first assessment of the magnitude of the impact of deleveraging on the world economy, we calibrate the model assuming that the deleveraging process takes one year. In the next section, we consider a more general environment in which deleveraging is spread over several periods, but in a quarterly model. Here, in a yearly model, considering a steady-state real rate of 2.5% per year we can calibrate $\beta^* = 0.9756$. We set $\alpha = 0.76$, which is consistent with the share of US non-durable consumption spending that goes to US-made products, as shown in Hale and Hobijn (2011). The initial level of debt of the borrowers is calibrated to 50% of GDP, implying $(1-\theta)k_{high} = 0.5$. This value corresponds to the level of external liabilities in debt instruments as a ratio to GDP for the US economy in 2008, according to Gourinchas, Govillot and Rey (2010). The initial value of θd_s is chosen to match the US asset position in debt instruments in 2008, equal to 10% of GDP. The overall net liability position of country H implies, therefore, that d is set to 40% of GDP as the corresponding position of the US economy in 2008 as regards debt instruments. We set the share of borrowers in the country equal to 1/3 to imply $\theta = 2/3$. We imagine alternative scenarios in which the overall debt of the borrowers is reduced from 50% to 40%, 30% and 20%.

Figure 1 shows the adjustment of Home and Foreign consumption, Home and Foreign real interest rates, the terms of trade and Home net external liabilities after a deleveraging shock. The deleveraging of the borrowers in the Home country improves the long-run external position. External debt is reduced to 36%, 30% or less than 26% in the three alternative scenarios. As discussed in Section (1.1), the terms of trade improve in the long run because the Home country reduces its debt exposure and so has more resources available to buy goods. Since there is home bias in preferences, the demand for domestic goods rises together with their relative price. In quantitative terms, the Figure shows that in the long run the appreciation is negligible.

In the short run, the adjustment takes a different direction. Debt-constrained agents in the Home country must reduce their consumption drastically in order to repay the debt. Because of home bias, aggregate demand for goods H drops more sharply, so the terms of trade worsen, implying a sharp depreciation of the Home currency (in Figure 1, the exchange rate falls by as much as 20%). Since in the short run deleveraging borrowers reduce their demand for goods more than in the long run, the real interest rate falls, an offsetting factor that generates more consumption by the savers in country H and by all consumers in country F. The real interest rate falls more in H than in F, as is shown in equation (24), since the terms of trade (and the real exchange rate) rise in the short run before falling in the long run. Notice that starting from a real interest rate of 2.5% the deleveraging shock drives both Home and Foreign rates below zero; and when deleveraging is severe far below zero, to -10% or more. Overall the model shows that real interest rates and the terms of trade must move very significantly to accommodate

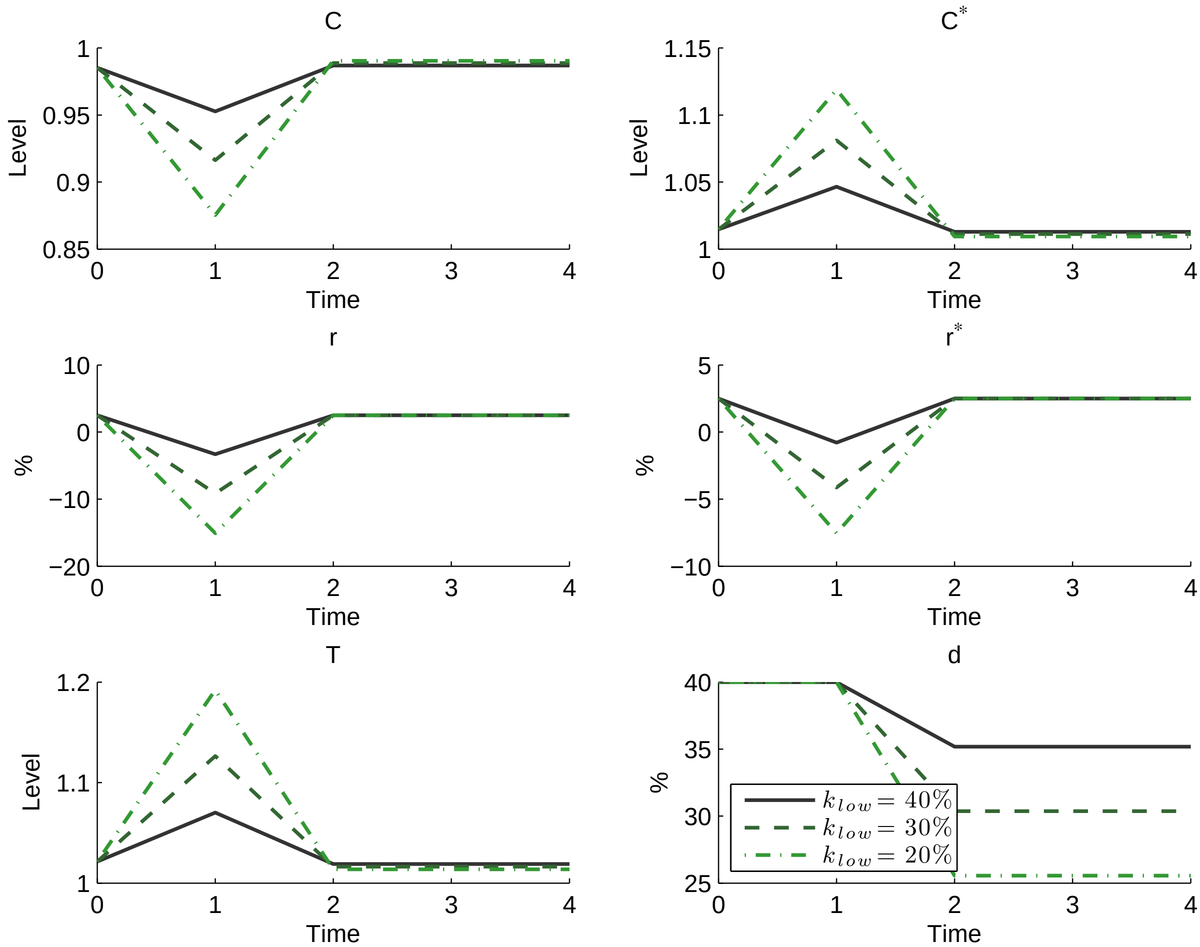

Figure 1: Responses of the level of Home and Foreign consumption (C, C^*), Home and Foreign real interest rates (r, r^*), the terms of trade (T) and the Home external debt position over GDP (d), to a deleveraging shock that brings the debt-to-GDP ratio down from 50% to 40%, 30%, 20%. The variables r, r^* and d are in percents the others are in levels.

the deleveraging shock. The adjustment is mitigated by the expansion of consumption in the foreign economy and by the expenditure-switching effect via the terms of trade. Any mechanism constraining these relative price movements can produce a larger fall in Home country consumption. If monetary policy sets too low an inflation target, the required drop in the real rate in country H will be constrained by the nominal zero lower bound, absolutely preventing a healthy adjustment. In the next section, we investigate the implications of the zero lower bound more thoroughly in a model with nominal rigidities and endogenous production. We will also study alternative exchange-rate regimes that can constrain or amplify the adjustment of international relative prices.

2 Nominal rigidities

The model described in the previous section shows that relative price movements are important shock absorbers in the event of deleveraging. Two relative prices in particular are crucial in the international context: one intratemporal relative price – the terms of trade – and one intertemporal – the real rate of interest. The terms of trade adjust through changes in the nominal exchange rate and/or in the prices of the goods produced in the two countries, while the real interest rate adjusts through changes in the nominal interest rate and/or the inflation rate. Monetary policy and the exchange-rate regime matter for the decomposition among these variables. When there are nominal rigidities, as posited here, they are also important for the dynamic adjustment of the real variables and in particular for output, which is now endogenous.

Three factors can delay the adjustment and create interesting dynamics. First, nominal rigidity slows the response of relative prices and can lead to a contraction in real output. Second, the zero lower bound on the nominal interest rate can prevent real rates from falling, depressing aggregate demand and output. Finally, the exchange-rate regime may either attenuate or amplify the response of real and nominal exchange rates.

The model used in this section closely follows those of the open-economy macro literature, such as Obstfeld and Rogoff (2001, 2005) and Benigno (2009), to which we add borrowing limits that are binding in equilibrium. The new elements here with respect to the simple model of the previous section are nominal rigidities, endogenous output and debt deleveraging on a longer horizon. Since there is an interesting dynamic here, the debt limit is written in real terms instead of the ratio debt-to-GDP. In this way debt-to-GDP becomes endogenous and deleveraging may be unsuccessful in the short run if gauged as reduction in the debt/GDP ratio. On the opposite side, the analysis in the previous section can be read as successful deleveraging that directly brings down the ratio of the borrowers' debt to GDP. Finally, to simplify exposition and analysis without losing the main mechanisms, we assume that in country H there are only borrowers. The deleveraging experiment in this section accordingly involves a reduction in the country's external debt position.

Households in country H, a continuum of measure one, have preferences over consumption and work hours as follows:

$$E_t\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\rho}}{1-\rho}-\int_0^1\frac{[L_t(j)]^{1+\eta}}{1+\eta}dj\right]\right\},$$

where $L_t(j)$ is hours worked of variety j and $\eta \geq 0$ the inverse of the labor-supply elasticity. Every household can supply all varieties of labor; C is a consumption bundle of goods H and F as in equation (1), with $\rho > 0$ the inverse of the intertemporal elasticity of substitution in consumption. However, differently from the previous section, we now assume that C_H is composed of a continuum of goods $c(h)$ of measure one all produced in country H, while C_F is a continuum of goods, $c(f)$, produced in country F:

$$C_H=\left[\int_0^1 c(h)^{\frac{\sigma-1}{\sigma}}dh\right]^{\frac{\sigma}{\sigma-1}} \qquad C_F=\left[\int_0^1 c(f)^{\frac{\sigma-1}{\sigma}}df\right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 0$. The price indices P_H and P_F are:

$$P_H=\left[\int_0^1 P(h)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}} \qquad P_F=\left[\int_0^1 P(f)^{1-\sigma}df\right]^{\frac{1}{1-\sigma}},$$

where $P(h)$ and $P(f)$ are the prices of the goods h and f denominated in the currency of country H. Households in country H face the following flow budget constraint:

$$P_tC_t=\int_0^1 W_t(j)L_t(j)dj+\Pi_t+\frac{D_t}{1+i_t}-D_{t-1} \tag{27}$$

where $W_t(j)$ is the nominal wage for variety j work and Π_t are firms' profits, which are distributed to the households in equal proportion. The borrowing limit is now written in real terms

$$\frac{D_t}{P_t}\leq z_t.$$

where z_t is the maximum amount of real debt that can be taken on in period t.

Similarly, preferences of households in country F are:

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{*1-\rho}}{1-\rho} - \int_0^1 \frac{[L_t^*(i)]^{1+\eta}}{1+\eta} di \right] \right\},$$

where C_t^* is the same as in equation (2) and $L_t^*(i)$ represents hours worked of variety i in foreign firms. The consumption bundles C_H^* and C_F^* and the appropriate consumption-based price indices P_H^* and P_F^* have the same structure as those of country H, whereas $P^*(h)$ and $P^*(f)$ are now the prices of the goods h and f expressed in the currency of country F. The law of one price holds for each traded good (i.e., $P(h) = SP^*(h)$ and $P(f) = SP^*(f)$) but, as explained in Section 1, there can be deviations from PPP because of Home bias. Households in country F also face a limit on their borrowing in real terms

$$\frac{D_t^*}{S_t P_t^*} \leq z_t^*$$

and their flow budget constraint is:

$$P_t^* C_t^* = \int_0^1 W_t^*(i) L_t(i) di + \Pi_t^* + \frac{D_t^*}{(1+i_t)S_t} - \frac{D_{t-1}^*}{S_t},$$

where $W_t^*(i)$ is nominal wages for the variety of work i and Π_t^* is Foreign profits. Turning to the consumer's optimality conditions, the stochastic versions of the Euler equations (8) and (9) still describe the intertemporal allocation of aggregate consumption and still hold with equality whenever the debt limit is not binding.

In both countries there is a continuum of firms, each producing one of the goods. Firms use all the varieties of labor offered in the country, combining them through the following technologies

$$y(h) = \left[\int_0^1 L^h(j)^{\frac{\tau-1}{\tau}} dj \right]^{\frac{\tau}{\tau-1}} \qquad y^*(f) = \left[\int_0^1 L^f(i)^{\frac{\tau-1}{\tau}} di \right]^{\frac{\tau}{\tau-1}},$$

where τ is the elasticity of substitution across varieties of labor, with $\tau > 1$. We assume that firms operate under monopolistic competition, setting their prices in a flexible way. It follows that $p_t(h) = P_{H,t} = \mu W_t$ for each h and $p_t^*(f) = P_{F,t}^* = \mu W_t^*$ for each f, where W_t and W_t^* are aggregate nominal wages in the respective currencies and the price markup is $\mu \equiv \sigma/(\sigma - 1)$. While prices are flexible, wages adjust in a staggered way following Calvo's model in which unions, grouping work of the same variety, have monopolistic power in setting wages. In each period, in country H (F), only a fraction $1 - \lambda$ ($1 - \lambda^*$) of the varieties of labor , with $0 < \lambda, \lambda^* < 1$, can have their wages reset according to the macroeconomic conditions and independently of the last adjustment. The remaining fraction of varieties of labor, of measure λ (λ^*), can only index their wages to the current inflation target, which does not necessarily coincide with actual inflation. It is clear that wage rigidity translates directly into price rigidity, since we do not have productivity shock. The resulting AS equations are standard for this kind of model. The set of equilibrium conditions is presented in detail in the Appendix.

3 International spillovers from debt deleveraging with nominal rigidities

We can now inquire into the international transmission of debt deleveraging and its dynamic adjustment in the model with nominal rigidities. We analyze the effects of a deleveraging shock that takes three years to be completed. The model is calibrated quarterly, so the time of deleveraging, T, is set at 12 quarters. We set $\beta^* = 0.9938$ to imply 2.5% real annual return on a yearly basis and the discount factor of country H is set to $\beta = 0.99$. We set the parameter $\alpha = 0.76$ as in previous section and calibrate the parameters σ and τ to 7.66, implying steady-state mark-ups in goods and labor market equal to 15%. The inverse of the elasticity of substitution in consumption, ρ, is set to 2, consistent with a number of studies, and the inverse of the labor supply elasticity, η, is set to 1.5, which is in the range of the estimates of De Walque et al (2005) in a two-country model of the euro area and the US. The degree of wage rigidities is also taken from De Walque et al. (2005); λ and λ^* are set equal to 0.8, which is consistent with their estimates and implies a duration of wages of 5 quarters in both countries (this too in line with other micro studies). The upper bound on real debt, $z_{\max}$, is chosen such that in the initial steady state it corresponds to a level of net foreign liabilities in debt instruments equal to 40% of GDP, on an annualized basis, approximately the peak reached in the US economy in 2008. Given that $z_{\max}$ represents real debt, it should be set equal to 1.5988.[8] We consider a deleveraging that moves $z_{\max}$ to $z_{\min}$ such that in the final steady state the ratio of external debt liabilities to GDP in the domestic economy reaches 30%. Deleveraging in our experiment lasts 3 years and involves a proportional reduction of real debt in each quarter. Real debt is exogenously reduced in each period, but the path of the debt/GDP ratio is endogenously determined along the adjustment. When deleveraging begins, agents in the economy know its length and size. The adjustment depends critically on the specification

[8] Since GDP is quarterly, 1.5988 corresponds to a value of 0.3977 when GDP is annualized.

of monetary policy.[9]

3.1 Fixed versus floating exchange rate

In this section we compare the equilibrium responses to deleveraging in fixed and floating exchange-rate regimes. With some qualifications, the fixed-rate regime can also describe a monetary union. There are obviously many possible equilibrium outcomes within these two broad categories, depending on the specification of policy. To pin down comparisons, we first specify policies under the fixed-rate regime and under monetary union. Afterward, we identify the floating-rate regime for appropriate comparisons. Two monetary policy rules close the model described in the previous section. Under fixed exchange rates or monetary union, one should be of the form $S_t = S_{t-1}$, which indeed fixes the exchange rate.[10] The second policy rule identifies an inflation-targeting regime. Here it comes a subtle difference, distinguishing monetary union from fixed-exchange-rate regime. In the former it would seem more natural to have a symmetrical monetary policy and perhaps to assume, like the European Central Bank, that inflation targeting involves a weighted average of national inflation rates.

Among the possible choices, the relevant inflation rates can be related to the GDP deflators, P_H and P_F^*. An inflation target for the whole area can be of the form $1/2\pi_{H,t} + 1/2\pi_{F,t}^* = \bar{\pi}_W$ where $\pi_{H,t}$ is the inflation rate associated with the index P_H and $\pi_{F,t}^*$ with P_F^*; we set the target $\bar{\pi}_W$ at 2% on a yearly basis. A fixed-exchange-rate system can be more asymmetrical: one country may be a follower by pegging its exchange rate, while the other serves as a leader by targeting its own domestic inflation. For the sake of brevity, in this section we only consider a regime in which home GDP inflation, $\pi_{H,t}$, is targeted to $\bar{\pi}_H$ and set to 2%.

A first interesting finding is that the equilibrium is not possible, in that nominal interest rates fall below zero. Therefore, to comply with the zero lower bound, we characterize the benchmark fixed-exchange-rate policy as one in which nominal interest rates hold at zero until time $T_1 - 1$, the shortest time needed for them to stay at that level. This means that at T_1 following regular policies the implied equilibrium nominal interest rate is positive. Given T_1, the benchmark fixed-exchange-rate regime is thus described by

[9]The model solution is non-trivial, considering that deleveraging lasts longer than a quarter and the zero lower bound may or may not bind. Details are discussed in the Appendix.

[10]See Benigno et al. (2007) for a discussion of interest-rate rules that can determine a fixed exchange rate.

the following policies

$$\begin{cases} i_t = 0 & 1 \leq t < T_1 \\ S_t = S_{t-1} & 1 \leq t < T_1 \\ & \\ \pi_{H,t} = \bar{\pi}_H & t \geq T_1 \\ S_t = S_{t-1} & t \geq T_1 \end{cases}, \tag{28}$$

while, given the same T_1, the benchmark monetary-union regime is characterized by[11]

$$\begin{cases} i_t = 0 & 1 \leq t < T_1 \\ S_t = S_{t-1} & 1 \leq t < T_1 \\ & \\ \frac{1}{2}\pi_{H,t} + \frac{1}{2}\pi^*_{F,t} = \bar{\pi}_W & t \geq T_1 \\ S_t = S_{t-1} & t \geq T_1 \end{cases}. \tag{29}$$

The floating-exchange-rate regime is designed for ready comparison with the above two regimes, to determine whether significant gains from exchange-rate flexibility are built into the model.[12] To this end, we should specify policies so that nominal interest rates are at the zero lower bound for the same period as in the benchmark regime. However, there are two caveats. First, to be consistent with a perfect-foresight equilibrium, uncovered interest-rate parity requires that if interest rates are zero in both countries in a given quarter, the equilibrium exchange rate should be fixed in the next quarter. Second, if nominal interest rates are set to zero until $T_1 - 1$, when standard inflation targeting is adopted (including a quarter T_1 of fixed exchange rate), real and nominal equilibrium indeterminacy arises. This depends mainly on the fact that zero-interest-rate polices do not define down the initial level of the exchange rate (see Benigno et al., 2007). Considering

[11] In our numerical example, the shortest exit time from zero-lower-bound policies is the same under the fixed exchange rates and monetary union.

[12] Obviously, since we can specify monetary policies freely, there can be floating exchange-rate regimes which can be definitely worse than the above-defined fixed exchange-rate regime.

these two arguments, we specify policies under a flexible exchange rate as follows:

$$\begin{cases} i_t = 0 & t = 1 \\ S_t = S_{t-1}(1+\gamma_t) & t = 1 \\ & \\ i_t = 0 & 1 < t < T_1 \\ S_t = S_{t-1} & 1 < t < T_1 \\ & \\ \pi_{H,t} = \bar{\pi}_H & t = T_1 \\ S_t = S_{t-1} & t = T_1 \\ & \\ \pi_{H,t} = \bar{\pi}_H & t > T_1 \\ \pi^*_{F,t} = \bar{\pi}_F & t > T_1 \end{cases}, \tag{30}$$

where after period T_1 countries revert to standard inflation-targeting policies with targets $\bar{\pi}_H = \bar{\pi}_F = 2\%$ on an annual basis. Notice that this policy specification is consistent with both countries being at the zero lower bound over the horizon $1 \leq t < T_1$. Moreover, it allows for an initial depreciation or appreciation of the currency, of magnitude γ_t, before entering the zero lower bound.

Figures 2 and 3 compare the benchmark fixed-exchange-rate policy described by (28), the symmetrical monetary-union regime (29) and floating-exchange-rate policies of the form (30) where the first-quarter depreciation γ is set to 0% or 5 %. Several results emerge. First, the shortest period at the zero lower bound under the benchmark fixed-exchange-rate policy is 13 quarters, one more than the deleveraging period. As Figure 2 shows, under fixed exchange rates the contraction in domestic consumption, as in domestic and foreign output, is quite deep and prolonged. In the first quarter, Home consumption falls by 9% with respect to the steady state and recovers only when deleveraging ends. Home output falls by 6% and Foreign output by an average of around 2% during the deleveraging period. Foreign consumption, however, expands slightly at the beginning to remain flat afterward.

First, compare this equilibrium with that implied by the monetary union, where the only difference is the latter's symmetrical inflation-targeting policy as opposed to the more asymmetrical policies of the fixed-exchange-rate system, upon exiting the zero lower bound. Even with this small difference, the policy under monetary union improves the allocation along many dimensions. The fall in Home consumption and output is more than cut in half, while Foreign consumption expands significantly. Intuitively this result depends on the symmetrical inflation-targeting policy, which leaves much more flexibility

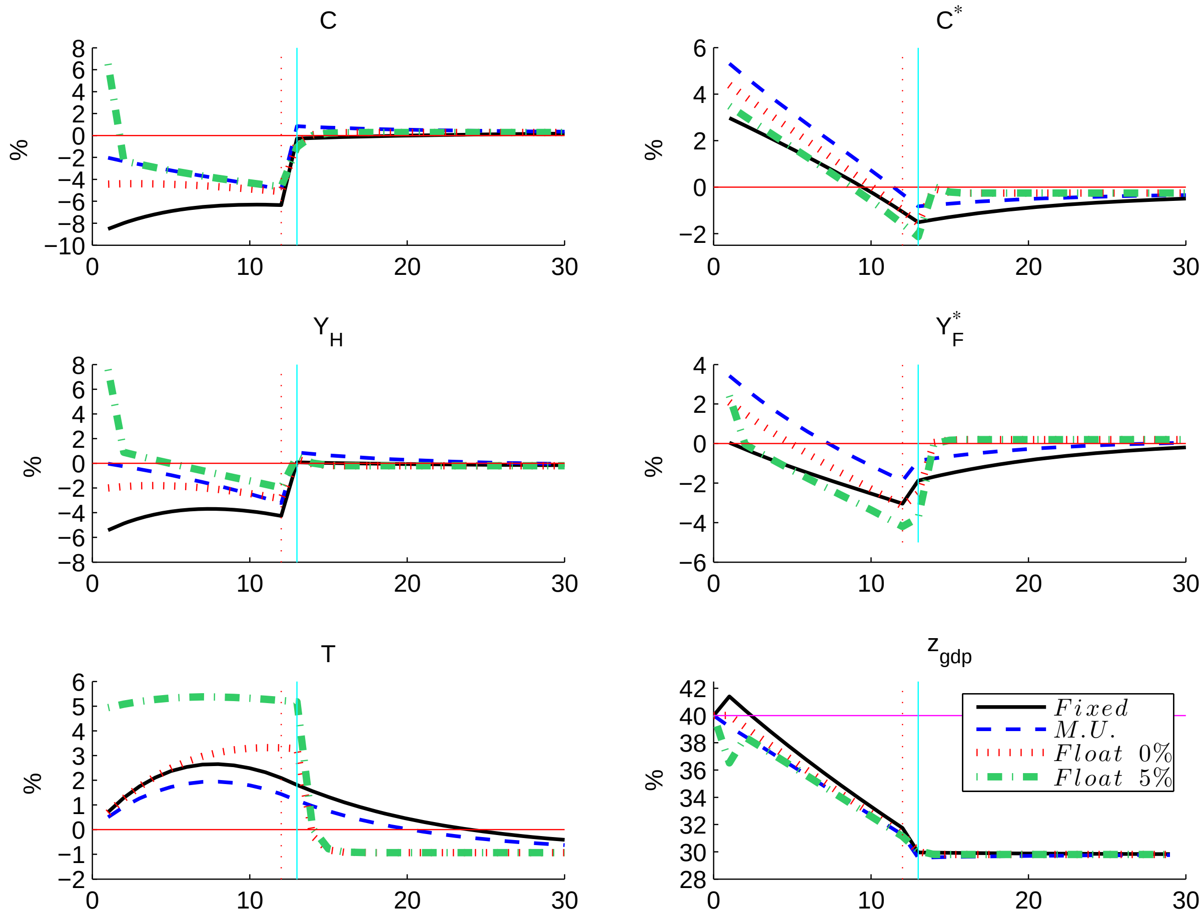

Figure 2: Comparison between impulse responses under the fixed-exchange-rate regime "Fixed" (28), the monetary-union regime "M.U." (29), and the floating-exchange-rate regime "Float" (30), where the initial depreciation rate is chosen to be 0% and 5%, respectively. Deleveraging lasts 12 quarters. Exiting from zero lower bound is after 13 quarters. Variables are Home and Foreign consumption (C , C^*), Home and Foreign output (Y_H, Y_F), terms of trade (T), the level of external liabilities of country H with respect to its GDP (z_{gdp}). All variables except for z_{gdp} are in percentage deviations from the steady state.

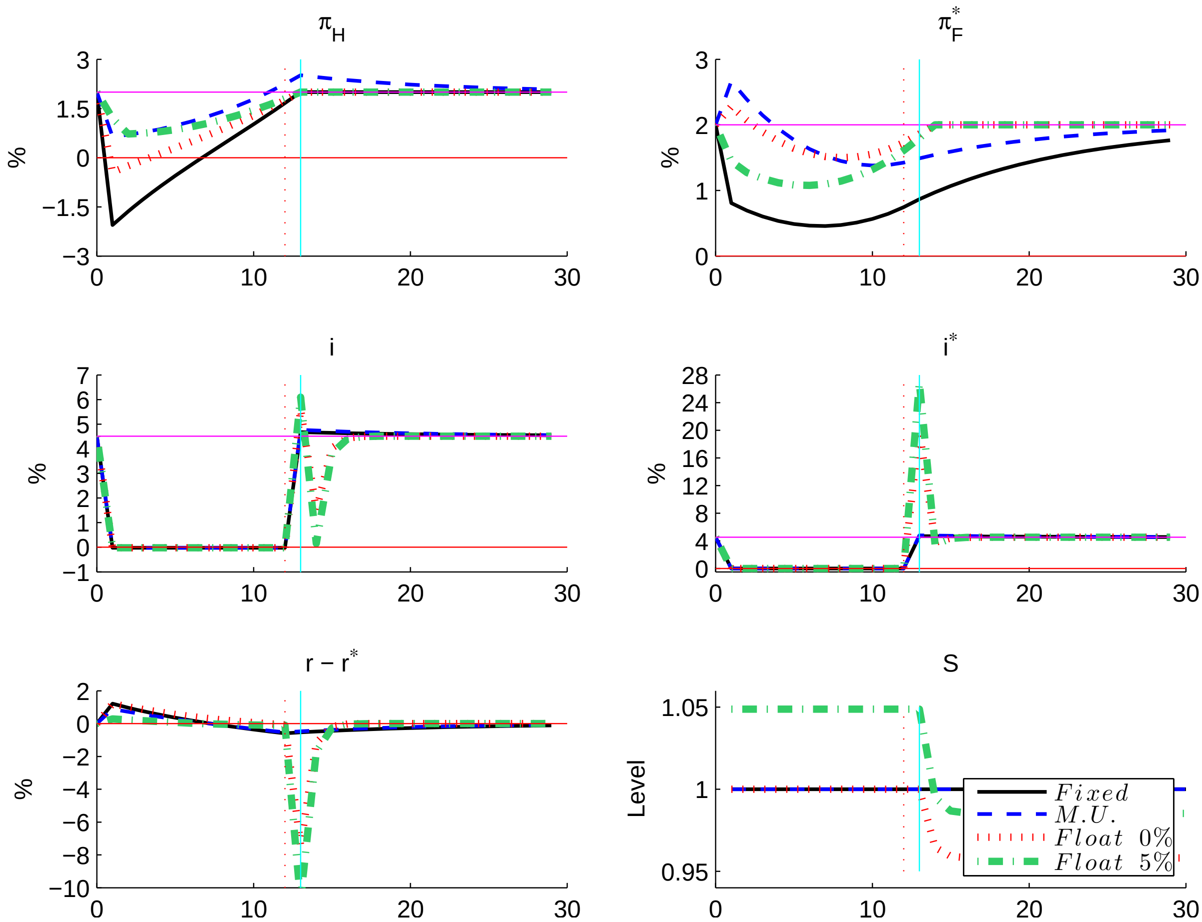

Figure 3: Comparison between impulse responses under the fixed-exchange-rate regime "Fixed" (28), the monetary-union regime "M.U." (29), and the floating-exchange-rate regime "Float" (30), where the initial depreciation rate is chosen to be 0% and 5%, respectively. Deleveraging lasts 12 quarters. Exiting from zero lower bound is after 13 quarters. Variables are Home and Foreign producer inflation rates (π_H, π_F^*), Home and Foreign nominal interest rates (i, i^*), the real interest rate differential $(r - r^*)$, the level of the nominal exchange rate (S); inflation and interest rates are in percents and annual rates.

to the inflation rates of each country after exiting the zero lower bound. In particular, as is shown in Figure 3, the inflation rate of country H rises above the target level of 2% for several quarters. This movement can push up prices also in the short run and significantly in country F, helping to avoid deflation in both countries during deleveraging. The consequent fall in the real rate substantially stimulates Foreign consumption.

Notice that under fixed exchange rates the period of subdued inflation is longer in country F because, asymmetrically, it bears most of the adjustment burden. Indeed the inflation rate in country H sticks to the 2% target, after exiting the zero lower bound. Recall that the adjustment requires the real exchange rate to depreciate in the short run and appreciate in the long run. Since the nominal exchange rate is fixed, the adjustment should occur in the short run through low inflation or deflation in country H and higher inflation in country F. But in the long run inflation should be higher in country H. Since under fixed exchange rates inflation in country H is targeted to 2%, the long-run adjustment comes through a much lower inflation in country F which creates a longer economic contraction. This low medium-run inflation is initiated by relatively low inflation at the start of deleveraging, which deepens the recession in the world economy. A higher inflation rate in country H after exiting the zero lower bound allows for higher inflation in country F and therefore for a smooth adjustment.

In the floating-exchange-rate regime the contraction is smaller, and there is also an improvement in foreign output and consumption. The adjustment of the exchange rate allows for a large appreciation after exiting the zero lower bound, which substantially helps the recovery in both countries. In the short run conditions are relatively worse than in monetary union, but entering with a depreciated Home currency can also help to alleviate the initial costs of the recession, in particular in country H. Notice that when policies revert to floating rates the significant appreciation of the currency creates large movements in short-term interest rates.[13] The Figures suggest that monetary union could be more flexible in the short term than the floating-rate regime analyzed here, as the latter constrains the inflation rate in each country to be strictly targeted to 2% upon exiting the zero lower bound. As noted, policies that can push inflation in the deleveraging country above the target level are quite helpful in mitigating the recession.[14] Interestingly, the ratio of external debt to GDP under fixed exchange rates worsens substantially in the short run before falling to the new long-run value.

[13] With a larger initial depreciation, Home interest rates fall back to the zero lower bound in some periods.

[14] Clearly, we can find floating-rate regimes that are better than monetary union when they allow for more flexiblity of the inflation rates after exiting the zero lower bound period. We discuss these issues in the next section.

3.2 Alternative monetary policies

Now, we discuss alternative monetary policies seeking to determine the features that mitigate or aggravate the recession that necessarily follows deleveraging. To save space, we base the analysis here on the fixed-exchange-rate regime defined in (28).[15] Through this analysis, we can also describe a world economy in which exchange rates are free to float but for exogenous reasons remain stable both during and after deleveraging.

3.2.1 A prolonged stay at the zero lower bound

Our benchmark fixed-exchange-rate regime assumed that the exit time was the earliest at which regular policies would imply non-negative interest rates. But interest rates can stay at the zero lower bound for longer than this. In this section, we study how the equilibrium changes when exit is delayed. In particular we consider delays from one to three quarters with respect to the benchmark exit time at $T_1 = 13$. The results are shown in Figures 4 and 5. The equilibrium is very sensitive to any lengthening, even by just one quarter, the stay at the zero lower bound. On the positive side, a longer stay can mitigate the costs of deleveraging and actually produce a substantial expansion in country F. The fall in the real interest rate is faster and more protracted, alleviating the cost of deleveraging in the country H and stimulating more consumption in F. On the negative side, too long stay can cause rapid inflation in both countries, with significant and prolonged overshooting of the 2% target. Since deleveraging ends in quarter 12, the analysis suggests that exiting from the zero lower bound two quarters later can lower the costs of deleveraging without missing the inflation target by much. Interestingly, in this case the inflation rate in country H goes above the target about when deleveraging ends. But a commitment to exit too late can generate excessive inflation.

Figure 4 shows that deleveraging – the reduction of debt in proportion to GDP – becomes successful even in the short run because nominal output increases, sharply reducing the value of debt in terms of the size of the economy. Finally, most of the adjustment is effected by the real interest rate, while the terms of trade worsen by less in the short run because of the increased demand of domestic goods. The analysis also suggests that the equilibrium is highly sensitive to even small variations in the policies that act on demand. This is because this model, like Eggertsson and Krugman (2010), has a sort of upward sloping demand equation for the world economy, so that changes in demand may produce larger shifts in the equilibrium.[16]

[15] Similar analysis performed for the monetary union and floating rates produce qualitatively similar conclusions.

[16] Indeed, if we increase the flexibility of wages, the reaction is even larger as in the "paradox of

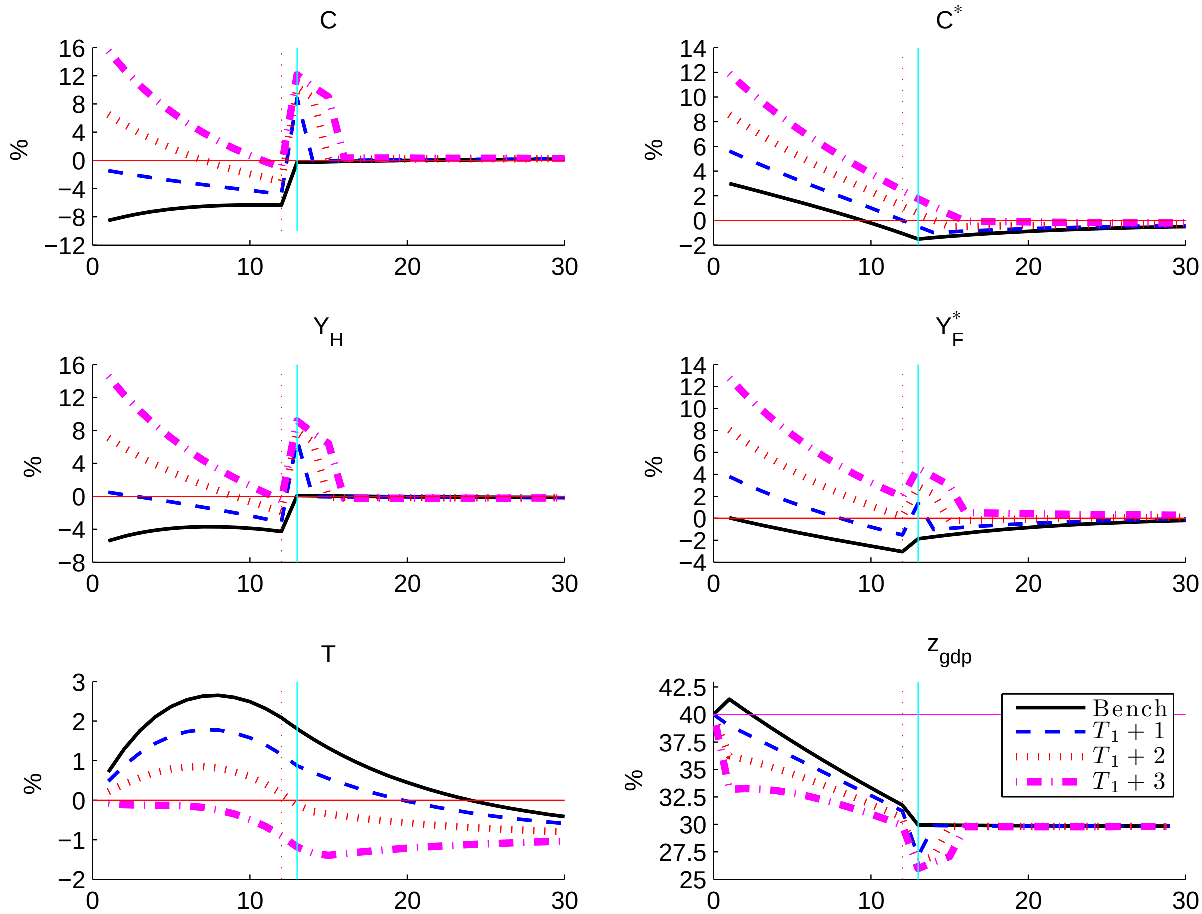

Figure 4: Comparison between impulse responses under fixed-exchange-rate regime "Fixed" (28) for different lengths of the period at the zero lower bound. Deleveraging lasts 12 quarters. Exiting from zero lower bound under "Fixed" is at $T_1 = 13$ quarters. Variables are: Home and Foreign consumption (C, C^*), Home and Foreign output (Y_H, Y_F), terms of trade (T), the level of external liabilities of country H with respect to its GDP (z_{gdp}). All variables except for z_{gdp} are in percentage deviations from the steady state.

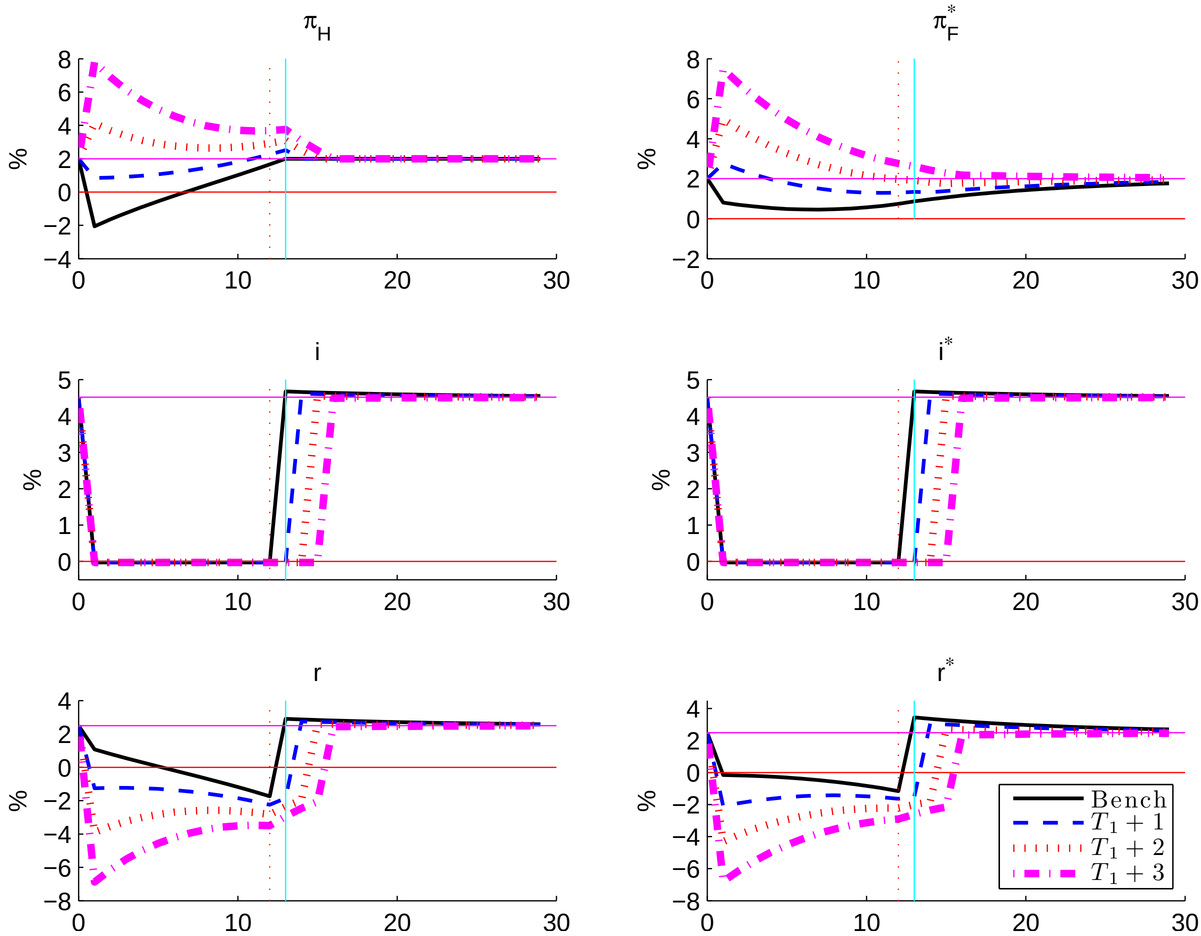

Figure 5: Comparison between impulse responses under fixed-exchange-rate regime "Fixed" (28) for different lengths of the period at the zero lower bound. Deleveraging lasts 12 quarters. Exiting from zero lower bound under "Fixed" is at $T_1 = 13$ quarters. Variables are: Home and Foreign producer inflation rates (π_H, π_F^*), Home and Foreign nominal interest rates (i, i^*), Home and Foreign real interest rates (r, r^*); all variables are in percents and annual rates.

3.2.2 Mistaken policies

Going to the zero lower bound is a policy option, not a requirement. If the natural rate of interest falls, policymakers may counteract the deleveraging-induced recession by expansionary monetary policy, that drives nominal interest rates to zero. In the previous section, we studied the benefits of these policies even over long time horizons.

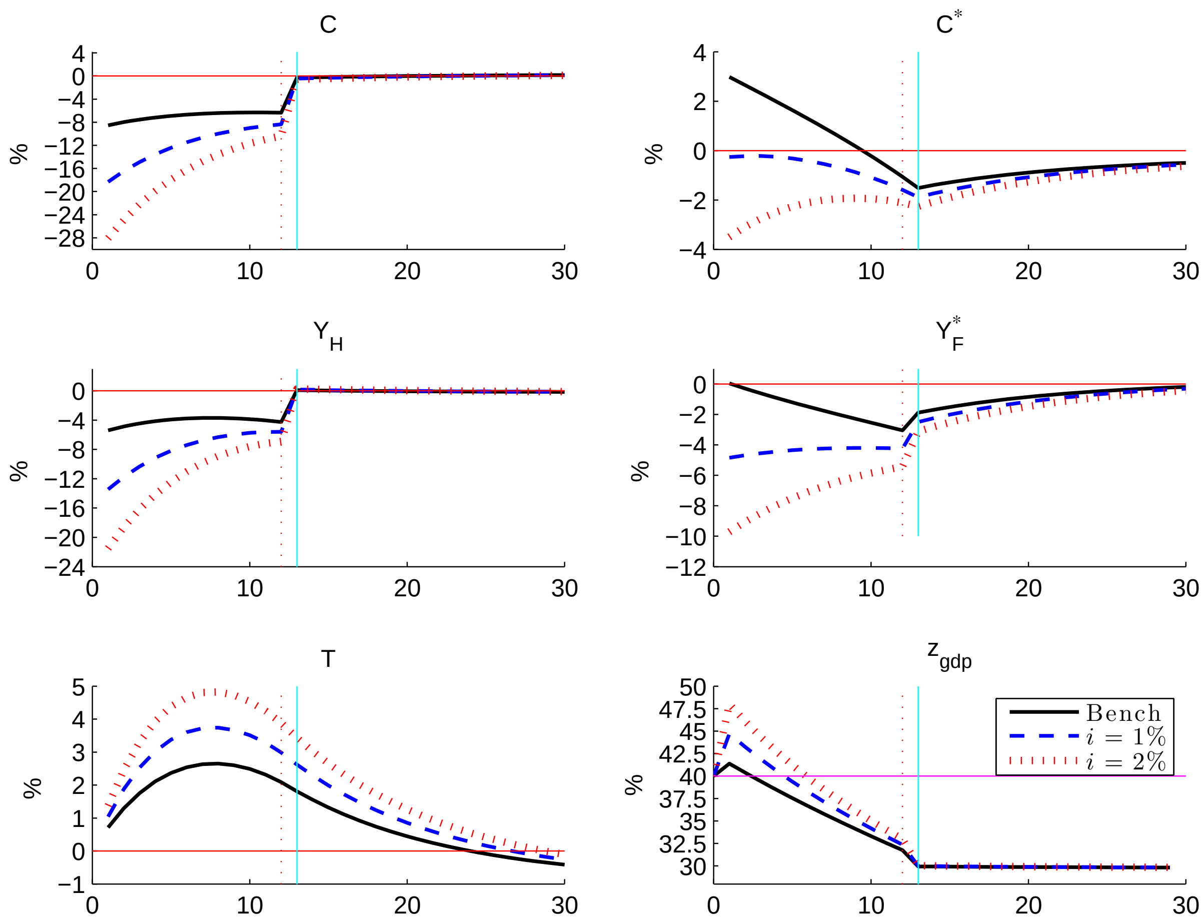

Figure 6: Comparison between impulse responses under fixed-exchange-rate regime "Fixed" (28) for different nominal-interest-rate targets until quarter $T_1 = 13$. Deleveraging lasts 12 quarters. Exiting from zero lower bound under "Fixed" is at $T_1 = 13$ quarters. Nominal interest rates under "Fixed" are at the zero-lower bound until T_1, under other polices at 1%, 2%. Variables are: Home and Foreign consumption (C, C^*), Home and Foreign output (Y_H, Y_F), terms of trade (T), the level of external liabilities of country H with respect to its GDP (z_{gdp}). All variables except for z_{gdp} are in percentage deviations from the steady state.

However, monetary policymakers may be reluctant to push zero interest rates to this limit. Let us accordingly look at what happens when policymakers are blocked by worrying about lowering interest rates too far. This experiment sheds light on the adjustment of economies in which even if the policy rate is low borrowing and lending rates are high owing

flexibility" described in Eggertsson and Krugman (2010).

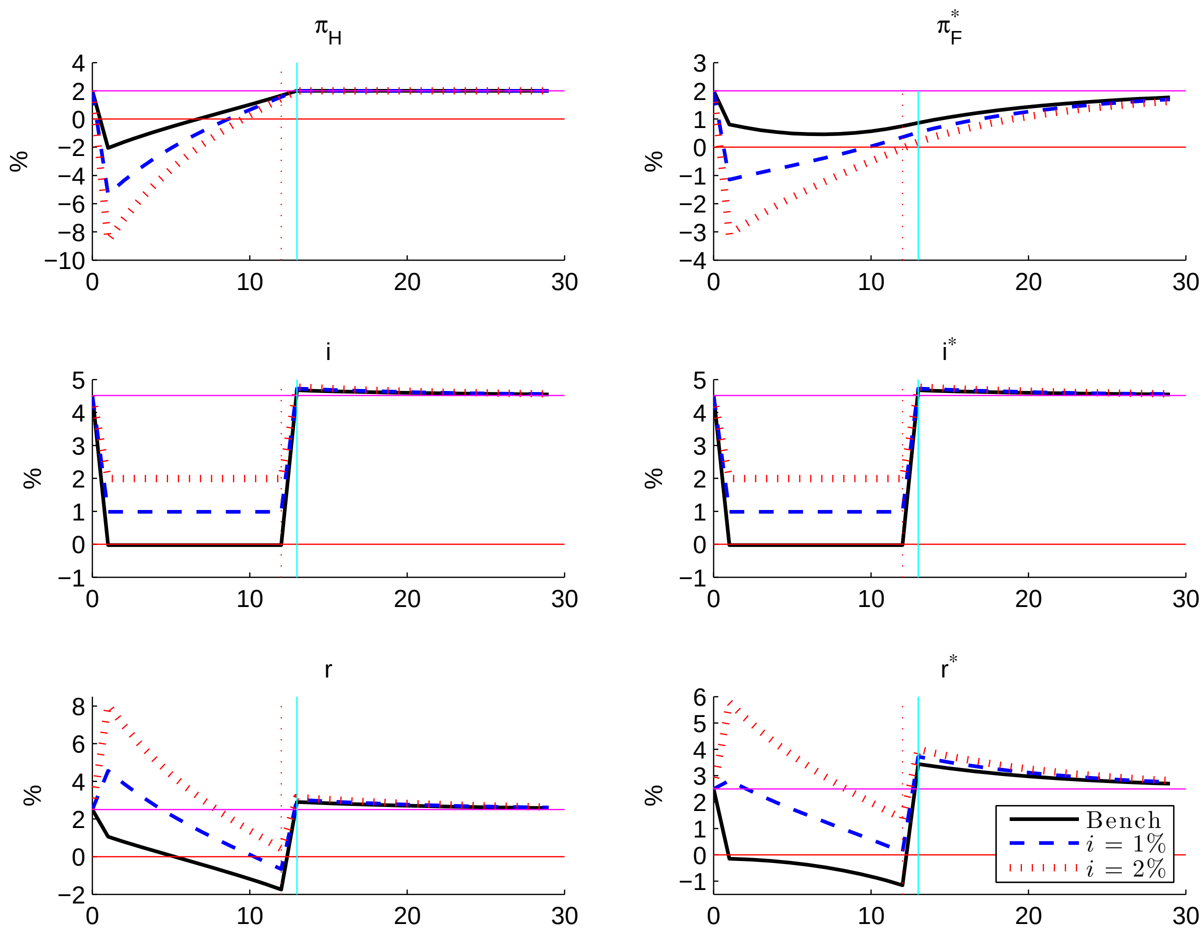

Figure 7: Comparison between impulse responses under fixed-exchange-rate regime "Fixed" (28) for different nominal-interest-rate targets until quarter $T_1 = 13$. Deleveraging lasts 12 quarters. Exiting from zero lower bound under "Fixed" is at $T_1 = 13$ quarters. Nominal interest rates under "Fixed" are at the zero-lower bound until T_1, under other polices at 1%, 2%. Variables are: Home and Foreign producer inflation rates (π_H, π^*_F), Home and Foreign nominal interest rates $(i$,$i^*)$, Home and Foreign real interest rates (r, r^*); all variables are in percents and annual rates.

to credit market malfunctioning. In this section, starting from our benchmark of fixed exchange rates we study the equilibrium in which nominal interest rates are kept constant until time T_1 but at a higher level, 1% or 2%. Figures 6 and 7 show how costly these mistaken polices can be in terms of the recession's depth and duration. The contraction of consumption in country H can be deep indeed, like the contraction of output in both economies. For example, when nominal interest rates are kept at 2% the real rate rises well above normal levels in both countries because deflation is sharp. Consumption falls by two or even three times as much as in the benchmark case. Interestingly, when the interest rate is kept high consumption falls substantially in country F as well, worsening the recession in both economies. In fact, they can slip into a deep, prolonged deflation-recession mode with excessively high real interest rates. In the short run, because of the contraction in nominal spending, deleveraging results in overshooting the desirable level of the country's external debt, reaching as high as 47% of GDP. Slowly debt comes to its new steady-state level.

Where higher interest rates are a symptom of credit market malfunctioning due to widespread illiquidity problems and risk of bank defaults, this analysis suggests that the sort of credit-easing policies followed by many central banks during the recent financial crisis can reduce the costs of deleveraging by narrowing spreads and restoring normal flow in credit markets.

3.2.3 Raising the inflation target in the future

There has been a considerable debate over possible unconventional monetary policies when policymakers cannot lower nominal interest rates any further. We have shown that keeping nominal interest rates at zero for a protracted period can help recovery in deleveraging economy. The work of Krugman (1998) and Eggertsson and Woodford (2003) sparked interest in the idea of temporarily setting a higher inflation target, once the liquidity trap is past. By increasing expectations of future inflation, this should drive prices already up even in the short run because of their forward-looking behavior, thus attenuating the costs of deleveraging by lowering the current real interest rate. This was clear in section (3) where we compared the fixed-exchange-rate regime with the monetary union and showed that the benefits of a milder recession were due to the overshooting of the inflation rate in country H after exiting the zero lower bound. In the present section we assume, under fixed exchange rates, that the inflation target in the country H is raised to $\bar{\pi}_H^{high} = 2.5\%$ for 2 quarters after the end of the zero lower-bound period. Figures 8 and 9 compare the adjustment to the deleveraging shock under the benchmark fixed-exchange-rate policy and the new policy.

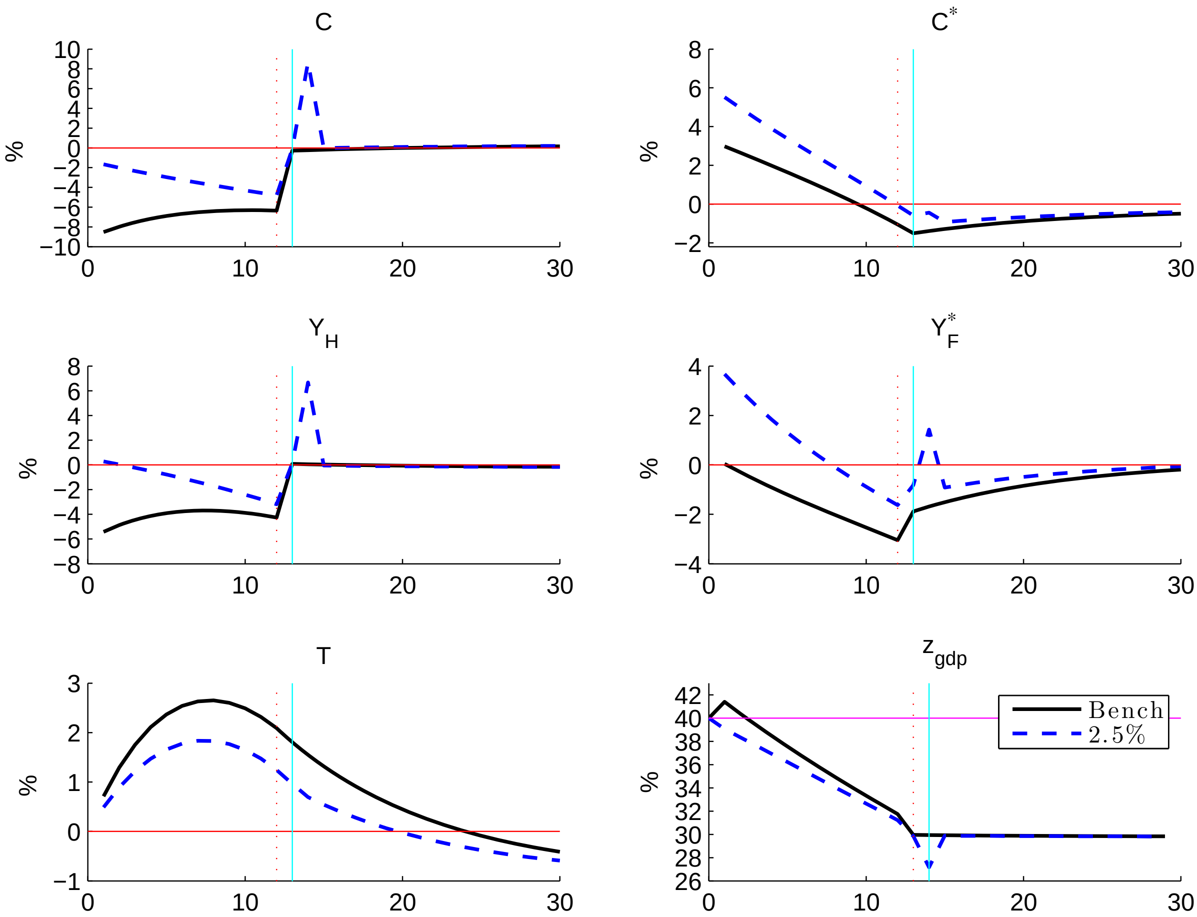

Figure 8: Comparison between impulse responses under the fixed-exchange-rate regime "Fixed" (28), and under this regime with the inflation target raised to 2.5% from period T_1 to $T_1 + 2$. Deleveraging lasts 12 quarters. $T_1 = 13$. Variables are: Home and Foreign consumption $(C\,, C^*)$, Home and Foreign output (Y_H, Y_F), terms of trade (T), the level of external liabilities of country H with respect to its GDP (z_{gdp}). All variables except for z_{gdp} are in percentage deviations from the steady state.

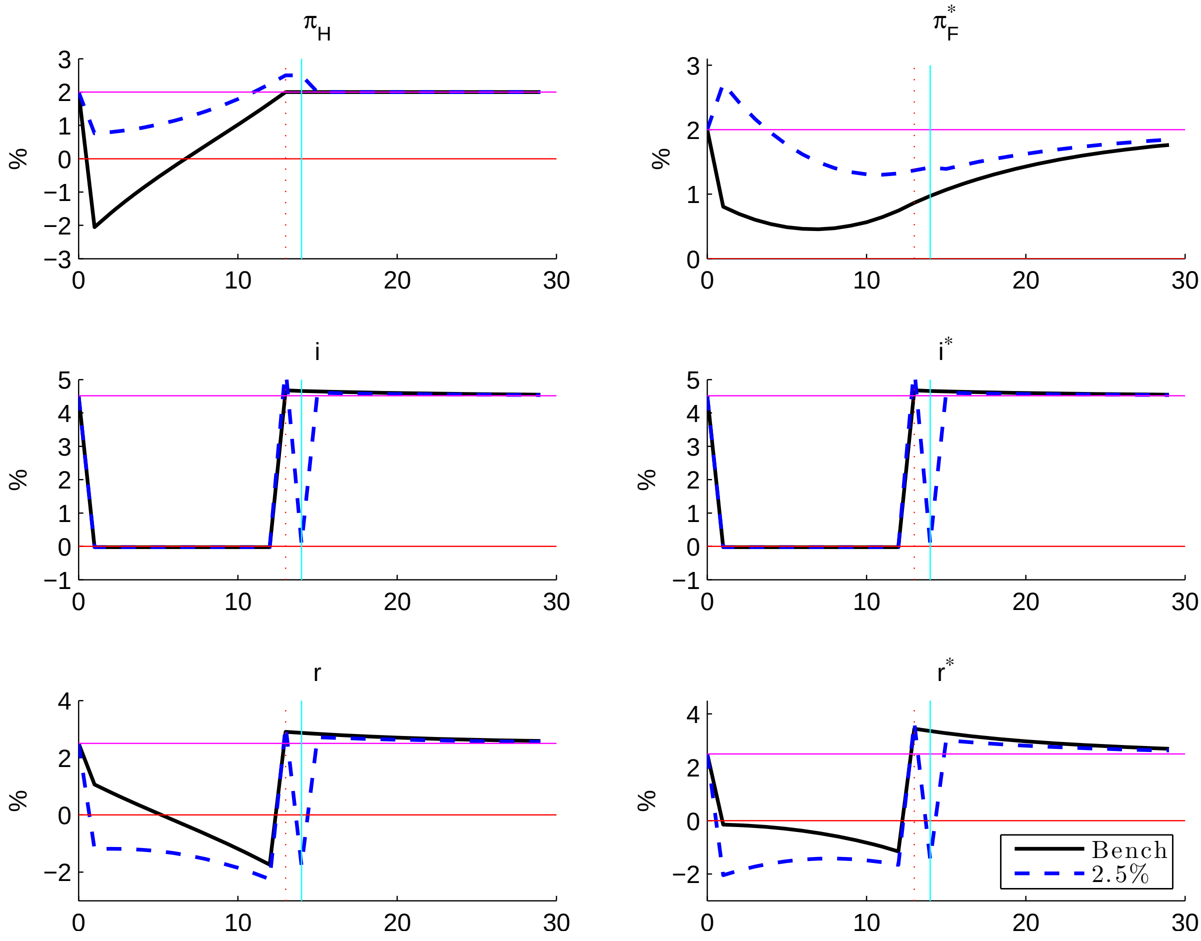

Figure 9: Comparison between impulse responses under the fixed-exchange-rate regime "Fixed" (28), and under this regime with the inflation target raised to 2.5% from period T_1 to $T_1 + 2$. Deleveraging lasts 12 quarters. $T_1 = 13$. Variables are: Home and Foreign producer inflation rates (π_H, π_F^*), Home and Foreign nominal interest rates (i, i^*), Home and Foreign real interest rates (r, r^*); all variables are in percents and annual rates.

The figures show that raising the inflation target can reduce the real cost of deleveraging, producing a milder recession in country H and possibly even a strong expansion in country F. The real rates fall more sharply in both countries, deflation is less pronounced in country H and avoided entirely in country F. But when the new higher target is implemented it causes swings in consumption and in the real rate in contrast with the smooth adjustment when the stay at the zero lower bound is longer. A higher inflation target pushes consumption up especially in the last period when it is in place, as is evident in the standard New-Keynesian forward-looking AS equation. And consumption falls significantly when policies goes back to normal, the real rate will also fall, pushing the nominal interest rate close to zero.[17]

4 Conclusion

We have examined the international implications of debt deleveraging in one country within the world economy or a monetary union. Deleveraging reduces aggregate demand and may lead to recession, as economic agents save to repay the debt. There are interesting international spillovers through trade and the exchange rate. A smooth adjustment requires movements in two relative prices; namely the exchange rate and the real interest rate. The exchange rate, which is an international relative price, should move in such a way as to rebalance resources across countries. The deleveraging country's currency will depreciate in the short run and appreciate in the long-run. This depends critically on home bias in consumers' preferences. Since in the short run agents who are paying down their debt have less resources for consumption, the price of home goods should fall relative to the foreign, and a fall in the exchange rate will assist this adjustment. Once the debt has been repaid, however, agents have more resources to spend and in particular on domestic goods. The other important relative price in the adjustment, the real interest rate, will come down and fall more sharply in the deleveraging country.

In this study, we have concentrated on the role of monetary policy and alternative exchange-rate regimes in mitigating or amplifying the costs of debt deleveraging. The zero lower bound on nominal interest rates is a significant constraint in our analysis, because the natural rate of interest falls substantially. Floating exchange rates help ease the recession, whereas under fixed exchange rates international relative prices move sluggishly. We have also shown that alternative times of exit from the zero lower bound can have a major impact on real economy, as can mistaken policies of keeping interest rates too high

[17] Temporary targets higher than 2.5% would bring nominal interest rates back to the zero lower bound for some time after deleveraging ends.

for too long.

We have analyzed a very simple two-country open-economy model; the consequent limitations are essentially the price paid for the simplifications used. First of all, the debt constraint in this model is exogenous and deleveraging is interpreted as a progressive lowering of this limit. It would clearly be interesting to have more endogeneity along these dimensions, but this limitation is shared with other recent works in the field. Second, in the real world debt deleveraging affects a variety of agents in the economy: households, banks, firms and governments. Distinguishing them in the model would enhance realism and possibly enable us to differentiate the effects of deleveraging on the economy according on which agents are paying down their debt. It is likely that, however, the qualitative results implied by our simple framework would hold also in a more complex context. Finally, the asset market structure has been kept very simple – only one asset traded internationally. This is a significant limitation, since the portfolio position of a country is much more complex and diversified involving assets and liabilities, in different currencies and instruments ranging from equity to debt. This is an interesting avenue for future research.

The paper focuses on the positive implications of deleveraging under alternative monetary policies. An open issue, which we plan to address in future work, is the optimal design of monetary policy given the objective of maximizing welfare in the world economy.

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A Model equilibrium conditions

The model of Section 3 is represented by the following 15 equilibrium conditions

$$(C_t^*)^{-\rho} = \beta^* E_t \left\{ (C_{t+1}^*)^{-\rho} \frac{(1+i_t)Q_t}{Q_{t+1}\Pi_{t+1}} \right\},$$

$$(C_t^*)^{-\rho} = \beta^* E_t \left\{ (C_{t+1}^*)^{-\rho} \frac{(1+i_t^*)}{\Pi_{t+1}^*} \right\},$$

$$C_t = T_t^{\alpha-1} Y_{H,t} + \frac{D_t}{(1+i_t)P_t} - \frac{D_{t-1}}{P_{t-1}} \frac{1}{\Pi_t}$$

$$Y_{F,t}^* = T_t^{-\alpha} \left[(1-\alpha) C_t + \alpha Q_t C_t^* \right]$$

$$Y_{H,t} = T_t^{1-\alpha} \left[\alpha C_t + (1-\alpha) Q_t C_t^* \right].$$

$$\left(\frac{1 - \lambda^* \left(\frac{\Pi_{F,t}^*}{\bar{\Pi}_t^*} \right)^{\tau-1}}{1-\lambda^*} \right)^{\frac{1+\eta\tau}{\tau-1}} = \frac{F_t^*}{K_t^*}$$

$$F_t^* = (C_t^*)^{-\rho} T_t^{1-\alpha} Y_{F,t}^* + \beta^* \lambda^* E_t \left[F_{t+1}^* \left(\frac{\Pi_{F,t+1}^*}{\bar{\Pi}_{t+1}^*} \right)^{\tau-1} \right]$$

$$K_t^* = \tilde{\mu} (Y_{F,t}^*)^{1+\eta} + \beta^* \lambda^* E_t \left[K_{t+1}^* \left(\frac{\Pi_{F,t+1}^*}{\bar{\Pi}_{t+1}^*} \right)^{\tau(1+\eta)} \right]$$

$$\left(\frac{1 - \lambda \left(\frac{\Pi_{H,t}}{\bar{\Pi}_t} \right)^{\tau-1}}{1-\lambda} \right)^{\frac{1+\eta\tau}{\tau-1}} = \frac{F_t}{K_t}$$

$$F_t = (C_t)^{-\rho} T_t^{\alpha-1} Y_{H,t} + \beta^* \lambda E_t \left[F_{t+1} \left(\frac{\Pi_{H,t+1}}{\bar{\Pi}_{t+1}} \right)^{\tau-1} \right]$$

$$K_t = \tilde{\mu} Y_{H,t}^{1+\eta} + \beta^* \lambda E_t \left[K_{t+1} \left(\frac{\Pi_{H,t+1}}{\bar{\Pi}_{t+1}} \right)^{\tau(1+\eta)} \right]$$

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_{F,t}^*}{\Pi_{H,t}} \frac{S_t}{S_{t-1}}$$

$$\Pi_t = \Pi_{H,t}^{\alpha} \Pi_{F,t}^{*1-\alpha} \left(\frac{S_t}{S_{t-1}} \right)^{1-\alpha}$$

$$\Pi_t^* = \Pi_{H,t}^{1-\alpha}\Pi_{F,t}^{*\alpha}\left(\frac{S_t}{S_{t-1}}\right)^{\alpha-1}$$

$$Q_t = T_t^{2\alpha-1}$$

which need to be solved for the following 18 unknowns C_t, C_t^*, i_t, Q_t, Π_t, i_t^*, Π_t^*, T_t, $Y_{H,t}$,.$Y_{F,t}$, $\frac{D_t}{P_t}$, $\Pi_{F,t}^*$, F_t^*, K_t^*, $\Pi_{H,t}$, F_t, K_t, $\frac{S_t}{S_{t-1}}$ given the inflation targets $\bar{\Pi}_t^*$ and $\bar{\Pi}_t$ where two further restrictions come from the policy rules, specified in the text. Notice that $\tilde{\mu}$ is a composite mark-up including the mark-ups in the goods and labor markets. Finally, the last restriction is the binding borrowing constraint in the Home country

$$\frac{D_t}{P_t} = z_t$$

Moreover, the zero-lower-bound constraint requires that i_t and $i_t^* \geq 0$.

B Model Solution

In the exercises of Section 3, the model is solved given the monetary policies specified in the text and the processes of the exogenous disturbances. One of these exogenous variable is z_t, the debt limit, which is assumed to follow the process

$$z_t = z_{t-1} - v_t$$

where $v_t = v$ for $1 \leq t \leq T$, and $v_t = 0$ for $t > T$. In particular v measures the amount of deleveraging per quarter during the deleveraging period and T is the length of the period in quarters. To study the policy experiments discussed in the main text, it is also convenient to add three more exogenous state variables. In particular, when the nominal interest rate is targeted either to zero or to some other value, we add the exogenous state variable $\bar{\imath}_t$, such that $1+i_t = 1+\bar{\imath}_t$ for $1 \leq t < T_1$, where T_1 denotes the time at which the economy exits from interest-rate targeting or from the zero lower bound. In particular, we assume a process for the new exogenous state variable of the form

$$1 + \bar{\imath}_t = (\beta^*)^{-1}\bar{\Pi} - \varepsilon_t$$

where $\varepsilon_t = \varepsilon$ for $1 \leq t < T_1$ and ε measures the reduction needed to bring the nominal interest rate down from the steady-state to the desired level.

We add also the exogenous state $\bar{\pi}_{H,t}$ which is used in Section 3.2 in studying the effects of inflation-targeting policy with a high target starting from time T_1 to T_2. In particular, in this case, inflation (in logs) in country H is set to $\pi_{H,t} = \bar{\pi}_{H,t}$ where $\bar{\pi}_{H,t} = \bar{\pi}_{H,high}$ for $T_1 \leq t < T_2$ and $\bar{\pi}_{H,t} = \bar{\pi}$ for $t \geq T_2$. Therefore we write a process for $\bar{\pi}_{H,t}$ of the form

$$\bar{\pi}_{H,t} = \bar{\pi} + u_t$$

where $u_t = \bar{\pi}_{H,high} - \bar{\pi}_H$ for $T_1 \leq t < T_2$ and $u_t = 0$ for $t \geq T_2$. Finally, in Section 3.1, under the flexible-exchange-rate policy, the exchange rate falls in the first period by the factor γ_t, so that $\frac{S_t}{S_{t-1}} = 1 + \gamma_t$. We add γ_t as a state variable with the following process

$$\gamma_t = \bar{\gamma}_t$$

where $\bar{\gamma}_t = \gamma$ for $t = 1$, conditional on the floating-exchange-rate regime, and $\bar{\gamma}_t = 0$ for $t > 1$.

Notice that for $t \geq T_2$ and for any of the policies considered in the text, all the "shocks" are zero, i.e. $v_t = \varepsilon_t = u_t = \bar{\gamma}_t = 0$. Therefore the model described in the previous section together with the processes for $(\bar{\imath}_t\ \bar{\pi}_{H,t}, \gamma_t, z_t)$ outlined above (considering $v_t = \varepsilon_t = u_t = \bar{\gamma}_t = 0$), given the policy rules specified in the text, can be written in the compact form

$$E_t F(y_t, y_{t+1}; x_t, x_{t+1}) = 0$$

where y_t collects the endogenous non-predetermined variables while x_t collects the state variables including the endogenous state variables $x_{1,t}$ and the exogenous state variables $x_{2,t} = (\bar{\imath}_t\ \bar{\pi}_{H,t}, \gamma_t, z_t)$. Note that the function F depends on the policy rules posited and is a vector of functions of dimension equal to the sum of the dimension of y_t and x_t. Therefore, for $t \geq T_2$, in a log-linear approximation, the solution will be of the form

$$\tilde{y}_t = F_1 \tilde{x}_{1,t-1} + F_2 \tilde{x}_{2,t-1} \tag{B.1}$$

$$\tilde{x}_{1,t} = V_1 \tilde{x}_{1,t-1} + V_2 \tilde{x}_{2,t-1} \tag{B.2}$$

$$\tilde{x}_{2,t} = M_2 \tilde{x}_{2,t-1}$$

for well-defined matrices F_1, F_2, V_1, V_2 where M_2 is a square matrix of zeros except for a one in the last element of the last row.

Instead, for periods $1 \leq t < T_2$ the model described above can be written compactly as

$$E_t \tilde{F}_t(y_t, y_{t+1}; x_t, x_{t+1}) = 0$$

where the vector of functions might now depend on time and be different across subperiods depending on the policy rules posited: for example, whether the system is or is not at the zero lower bound. Moreover, the dimension of $\tilde{F}_t$ is now the sum of the dimension of y_t and $x_{1,t}$ since we are not adding the processes for the exogenous state variables. It can be shown that in a log-linear approximation the above system implies the following restriction

$$A_1 E_t y_{t+1} + A_2 \tilde{x}_{1,t} = B_{1,t} y_t + B_{2,t} \tilde{x}_{1,t-1} + B_{3,t} \tilde{x}_{2,t} \tag{B.3}$$

for well-defined matrices $A_1, A_2, B_{1,t},\ B_{2,t}\ B_{3,t}$ where only $B_{1,t},\ B_{2,t}\ B_{3,t}$ depend on the policy regime posited. For the period $1 \leq t < T_2$ we can write the processes for the exogenous state variables generically as

$$\tilde{x}_{2,t} = M_2 \tilde{x}_{2,t-1} + m_{\varepsilon,t}\varepsilon_t + m_{v,t} v_t + m_{u,t} u_t + m_{\gamma,t}\bar{\gamma}_t \tag{B.4}$$

where the shocks $v_t, \varepsilon_t, u_t, \bar{\gamma}_t$ are constant or zero depending on the specification of the policy considered and the vectors $m_{\varepsilon,t}, m_{v,t}, m_{u,t}, m_{\gamma,t}$ are constant or zero depending as well on the specification of the policy considered.

For period $1 \leq t < T_2$, the solution has the form

$$\tilde{y}_t = F_{1,t}\tilde{x}_{1,t-1} + F_{2,t}\tilde{x}_{2,t-1} + h_{\varepsilon,t}\varepsilon + h_{v,t} v + h_{u,t} u_t + h_{\gamma,t}\bar{\gamma}$$

$$\tilde{x}_{1,t} = V_{1,t}\tilde{x}_{1,t-1} + V_{2,t}\tilde{x}_{2,t-1} + g_{\varepsilon,t}\varepsilon + g_{v,t} v + g_{u,t} u_t + g_{\gamma,t}\bar{\gamma}$$

where the matrices $F_{1,t}, F_{2,t}, V_{1,t}, V_{2,t}$ and the vectors $g_{\varepsilon,t}, g_{v,t}, g_{u,t}, g_{\gamma,t}$ and $h_{\varepsilon,t}, h_{v,t}, h_{u,t}, h_{\gamma,t}$ are uniquely identified by the equilibrium conditions given in (B.3), for the process (B.4) and given the terminal conditions implied by the solution at $t = T_2$ shown in (B.1), (B.2). In particular, it can be shown that $F_{1,t},\ V_{1,t}$ and $F_{2,t}, V_{2,t}$ can be obtained by solving the following systems of equations

$$\begin{bmatrix} A_1 F_{1,t+1} + A_2 & -B_{1,t} \end{bmatrix} \begin{bmatrix} V_{1,t} \\ F_{1,t} \end{bmatrix} = B_{2,t}$$

$$\begin{bmatrix} A_1 F_{1,t+1} + A_2 & -B_{1,t} \end{bmatrix} \begin{bmatrix} V_{2,t} \\ F_{2,t} \end{bmatrix} = B_{3,t} M_2 - A_1 F_{2,t+1} M_2$$

given that $F_{2,T_2} = F_2$ and $F_{1,T_2} = F_1$. Moreover, $g_{\varepsilon,t}, g_{v,t}, g_{u,t}, g_{\gamma,t}$ and $h_{\varepsilon,t}, h_{v,t}, h_{u,t}, h_{\gamma,t}$

can be obtained by solving the following systems of equations

$$\begin{bmatrix} A_1 F_{1,t+1} + A_2 & -B_{1,t} \end{bmatrix} \begin{bmatrix} g_{j,t} \\ h_{j,t} \end{bmatrix} = -A_1 h_{j,t+1} + B_{3,t} m_{j,t} - A_1 F_{2,t+1} m_{j,t}$$

for each $j = \varepsilon, v, u, \gamma$ given that $F_{2,T_2} = F_2$, $F_{1,T_2} = F_1$ and $h_{j,T_2} = 0$.